PREDICTION OF OPTIMAL FLIGHT PROFILES FOR JET AIRCRAFT UNDER SHORT RANGE AND LOW FUEL CONDITIONS

by

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UNITED STATES NAVAL POSTGRADUATE SCHOOL

THESIS

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ABSTRACT

There are many factors, such as aircraft configuration and weight, winds aloft, airspeeds flown, altitude, distance, etc., which affect fuel consumption in turbojet aircraft. For any given combination of these factors a flight path can be determined that will result in the least fuel consumed for a ground distance covered. Under divert conditions from aircraft carriers at sea to fields ashore the choice of the optimal flight path is critical. The many possible combinations of factors lead to the adoption of computer flight planning. Pilots can avail themselves of computer solutions during flight planning and briefing sessions, and after take-off can receive further information via UHF radio. Typical flight handbooks display fuel flow data, etc. in such a manner that the pilot must "guesstimate" entry parameters such as average horizontal weight, or weight prior to descent. Several iterative procedures are developed that provide exact solutions to these important figures. Thus the computer flight planning system will provide more accurate solutions, and free the pilot from this chore so that he may better spend his time briefing tactics.

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CHAPTER I

PURPOSE OF THE STUDY

Naval Aviation has been described as hours and hours of boredom interrupted by moments of sheer terror. The simple word " BINGO " provides for some of these anxious moments. Bingo is well known to everyone as a friendly game of chance, but the word carries a special meaning to a U.S. Navy carrier pilot. It causes concern both ashore and at sea, and especially in the cockpit of at least one airplane that is flying circles around some aircraft carrier somewhere at sea.

"Fortress 501, this is Atlas tower. Bingo 100°/125 miles. Change to departure control, 316.6, for radar vector."

"Atlas tower, this is Fortress 501. Roger, switching 316.6."

This conversation might have occurred west of San Clemente Island off the coast of Southern California. To the pilot of Fortress 501 the message is unmistakable. He is to turn to 100° magnetic and land at the field 125 miles away, rather than attempt to land aboard Atlas, U.S.S. INTREPID, CVA-11.

In most cases an aircraft will receive a BINGO because of a flight deck accident, or because the pilot is having difficulty landing aboard the carrier due to bad weather and/or a badly pitching flight deck. The high fuel consumption of jet aircraft at low altitudes precludes having the pilot hold until the deck is clear, or in the latter case attempting more landings aboard.

When the pilot of Fortress 501 switches to 316.6, he will receive a radar vector to the nearest suitable landing field, the weather at the field, and the approach control and tower frequencies. But he will have to determine his own flight profile; that is, how high to climb, what airspeeds to fly, and at what distance from his destination to commence an idle descent. Since the aircraft is usually low on fuel when the pilot commences the BINGO, the pilot attempts to fly the flight profile that minimizes fuel burned. There are no other constraints on the pilot, for he is only interested in running out of ocean before he runs out of fuel.

The factors that affect fuel consumption in a jet aircraft are altitude, aircraft weight, and of course, the air distance travelled. Jet aircraft burn enormous amounts of fuel at low altitudes and during the climb to higher altitudes. During an idle descent from altitude the fuel flow is only one-sixth that of the climb portion. A heavier aircraft burns more than a lighter one, and a headwind increases air distance and thus increases fuel burned. With these factors in mind, the pilot must decide what combination of climb, cruise, and descent flight paths will take him to his destination with the least fuel consumed.

A wrong choice could result in the loss of an aircraft and perhaps a pilot. A lesser error might result in a safe landing, but several hundred pounds of jet fuel will have been wasted. This latter error might also result in some premature gray hairs on an intrepid naval aviator.

Since a determination had to be made at what fuel weight to BINGO the pilot, the knowledge that all pilots would fly an optimal (minimum fuel) flight path would allow the BINGO to be delayed for perhaps one more pass at the flight deck. Also, any "gravy" could be reduced from the bingo weight. This lower weight would increase operational readiness by keeping all aircraft aboard the carrier where they belong, and where all commanders want them. If an aircraft is sent to the beach, the carrier must remain in the area to send messages concerning ship's position and overhead times so that the stray bird can come home to roost. Since carrier skippers like to hide their ships out at sea, there are obvious tactical advantages in keeping all aircraft aboard.

An ideal way to ensure that all pilots fly an optimal flight path for all distances, aircraft weight, and wind conditions would be by computer flight planning. Thus when Fortress 501 contacted departure control on 316.6 he might hear,

"Roger, Fortress 501. Climb at 285 knots to 19,000 feet. Cruise at 260 knots indicated. Commence idle descent when 44 miles from destination fix." Departure control could then follow with the standard weather and radio aids information.

This thesis considers only the short BINGO problem in its development of a computer flight planning program. This applies to a small, but important segment of aviation. The obvious extension is to use this type of flight planning for all long distance cross-country flights. The only modification necessary is to incorporate several horizontal legs at altitude sandwiched between a climb and a descent leg, rather than just one as does the BINGO program. The cross-country program would also require a change of optimal flight altitude to a higher one when the aircraft weight decreases enough to warrant a climb in order to remain optimal.

CHAPTER II

REPORT OF THE STUDY

The BINGO problem may be stated mathematically as a classic minimization problem:

$$\min\ (W_0 - W_3)$$

$$\text{s.t.}\quad h \leq 40 \times 10^3, \quad \text{in even thousands}$$

$$X_c + X_h \leq X_t - X_d$$

$$W_3 > B \quad \text{where } B = \text{aircraft empty weight}$$

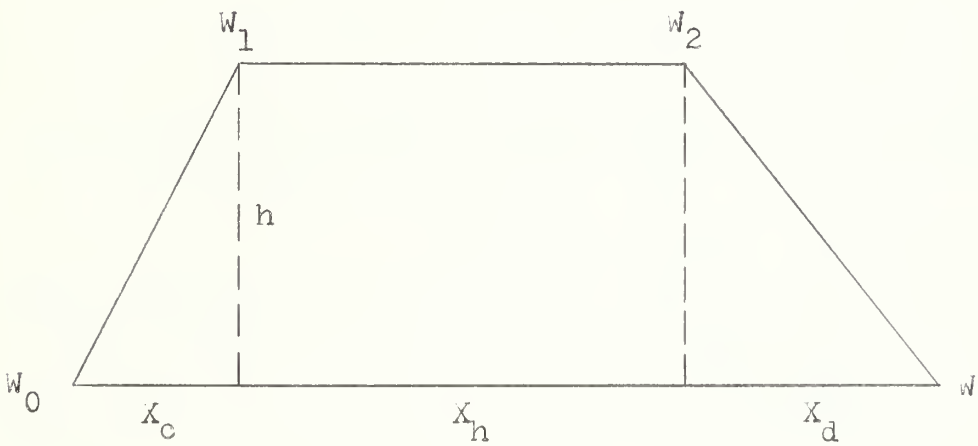

Figure 1

Definitions and abbreviations:

W_0 = aircraft weight at start of flight

W_1 = weight at end of climb portion

W_2 = weight at end of horizontal portion

W_3 = weight at destination

h = altitude in thousands of feet

X_c = ground covered in climb portion

X_h = ground covered in horizontal portion

X_d = ground covered during idle descent

X_t = total distance to destination

t = time

CAS = calibrated airspeed, velocity of the air over the pitot tube

TAS = true airspeed, CAS corrected to sea level

$$TAS = \frac{1}{d} \quad , \text{ where } \quad d = \frac{P_h}{P_0}$$

P_h = density of the air at altitude h

P_0 = density of air at sea level

GSPD = ground speed, aircraft speed relative to the ground (TAS corrected for wind)

F = fuel burned in pounds

1/f = rate of fuel flow in pounds per nautical mile

DI = drag index for the particular aircraft configuration

A flight profile depicted by Figure (1) is used because of standard operating procedure, and also because the aircraft instrumentation is such that it is the only profile that a pilot can fly accurately. An arcing path such as a semi-circle or a cycloid is impossible.

In order to fly a flight profile in the form of a cycloid or a semi-circle a pilot would have to rely on his VSI (vertical speed indicator) to accurately control his rate of climb throughout the entire flight. This instrument is inaccurate at best since it is a pressure instrument and fluctuates with slight changes in pressure. The VSI also tends to lag the true pressure changes. A climb, level, descent flight profile can be accurately flown because the pilot need only control his airspeed and altitude. Both these cockpit indications are excellent in all aircraft, and a pilot gets acquainted with them from his first step into a cockpit.

The objective function, $W_0 - W_3$, is the fuel consumed in travelling the distance X_t, since the only weight loss will be the fuel burned. The constraint $W_3 > B$ is obvious. If the destination cannot be reached, even by flying an optimal flight profile, then the aircraft should rendezvous with a tanker aircraft if one is available, or else continue landing attempts aboard the carrier. It is not obvious that $X_c + X_h \leq X_t - X_d$ is a necessary constraint. A possible flight path could be to continue horizontally until the destination is reached, and then commence a circling descent such that $X_d = 0$. Navy flight tests show that an idle descent at the proper airspeed for X_d will use less fuel than any other possible flight path covering the distance X_d. This type of idle descent is standard operating procedure for jet aircraft.

The altitude, h, must necessarily be less than or equal to the service ceiling of the aircraft being flown. For this study $h \leq 40{,}000$ ft., the service ceiling for the A4C, is used. Even thousands of feet are used, resulting in 40 possible flight profiles. Using every 500 feet will result in 80 possible profiles, and using every 250 feet will result in 160. This study uses even thousands merely for ease of presentation. Only a minor change is required to solve for any desired number of possible profiles.

The following assumptions were made concerning the flight path of Figure 1.

1. the aircraft makes an instantaneous transition from the climb to the level flight attitude, and from the level to the descent.
2. all airspeed changes are instantaneous.
3. the aircraft can remain on track despite any cross-wind.
4. wind information is known, and there are no up-drafts or downdrafts.

Performance data such as fuel flow and airspeed were obtained from NATOPS Flight Manual [1] for the A4C aircraft. Although only the A4C is considered here, similar data can be used for other aircraft in the inventory. All performance data was gathered by U.S. Navy flight tests. The peculiar form of the graphs (Figure 6 - 13) makes it difficult to form mathematical functions for such values as maximum range airspeed. This precludes formulating the problem as a standard Lagrange minimization problem.

The computer program developed is one that calculates the fuel burned for all altitudes up to 40,000 feet, the service ceiling of the A4C, and chooses that flight profile that results in the least fuel burned. The first altitude tried is level at 1000 feet for the entire distance, X_t. The remaining thirty-nine are flown as follows.

(1) a 100% power climb to h-thousand feet

(2) level at h-thousand feet at the max-range airspeed. (that airspeed that gives the most miles per pound of fuel)

(3) an idle descent at that airspeed that covers X_d with the least fuel burned

Aircraft performance data is presented as follows.

CLIMB PORTION:

$$F = F\,(W_0, h, DI)$$

$$CAS = C\,(W_0, h, DI)$$

$$time = t\,(W_0, h, DI)$$

HORIZONTAL PORTION:

$$f = f\,\left(\frac{W_1 + W_2}{2}, h, DI\right)$$

$$F = \frac{X_h}{f} = W_1 - W_2 \qquad (1)$$

$$F = F\left(X_h, \frac{W_1 + W_2}{2}, h, DI\right) \qquad (2)$$

$$CAS = C\left(\frac{W_1 + W_2}{2}, h, DI\right)$$

DESCENT PORTION:

$$F = F(W_2, h, DI)$$

$$CAS = C(W_2, DI) \qquad (3)$$

$$X_d = X(W_2, h, DI) \qquad (4)$$

$$time = t(h, DI) \qquad (5)$$

Future functional notation will omit the drag index, DI. It will be held constant at DI = 50, the value for an aircraft with no external stores. This is almost always the case in a BINGO situation.

The first flight profile tried is level at h = 1000 feet. Since this differs from the others only insofar as there are no climb or descent portions, a typical climb, level, descent profile will adequately explain both possibilities. For the climb to any altitude, W_0 and h are known. From Figures 6, 7, and 8, F, CAS, and the time to climb are easily determined. CAS is converted to TAS, and the predicted wind information (averaged over h-thousand feet) converts TAS to GSPD. $W_0 - F = W_1$, and GSPD x time = X_c. Each 5000 feet the CAS is changed to the optimal climb speed for the next 5000 foot portion. If h > 5 the climb portion of the flight results in $([h/5] + 1)$ iterations.

At the start of the horizontal portion, W_1, h, and $(X_t - X_c)$ are known. Figures 9 and 10 however, require the average weight during the horizontal leg, $(W_1 + W_2)/2$, as an entry parameter. Recall that the fuel burned during the horizontal leg is an implicit function of W_1 and W_2. From equation 2,

$$F = F(X_h, \frac{W_1 + W_2}{2}, h)$$

The procedure for using the graph in Figure 10 is to enter with the average weight, proceed horizontally to the altitude h, read vertically downward to the DI = 50 line, and horizontally to the left to read f, the fuel flow.

Three different methods were tried to solve the implicit function for the average weight.

METHOD I

Approximate the altitude lines in Figure 10 with straight lines as shown in Figure 2.

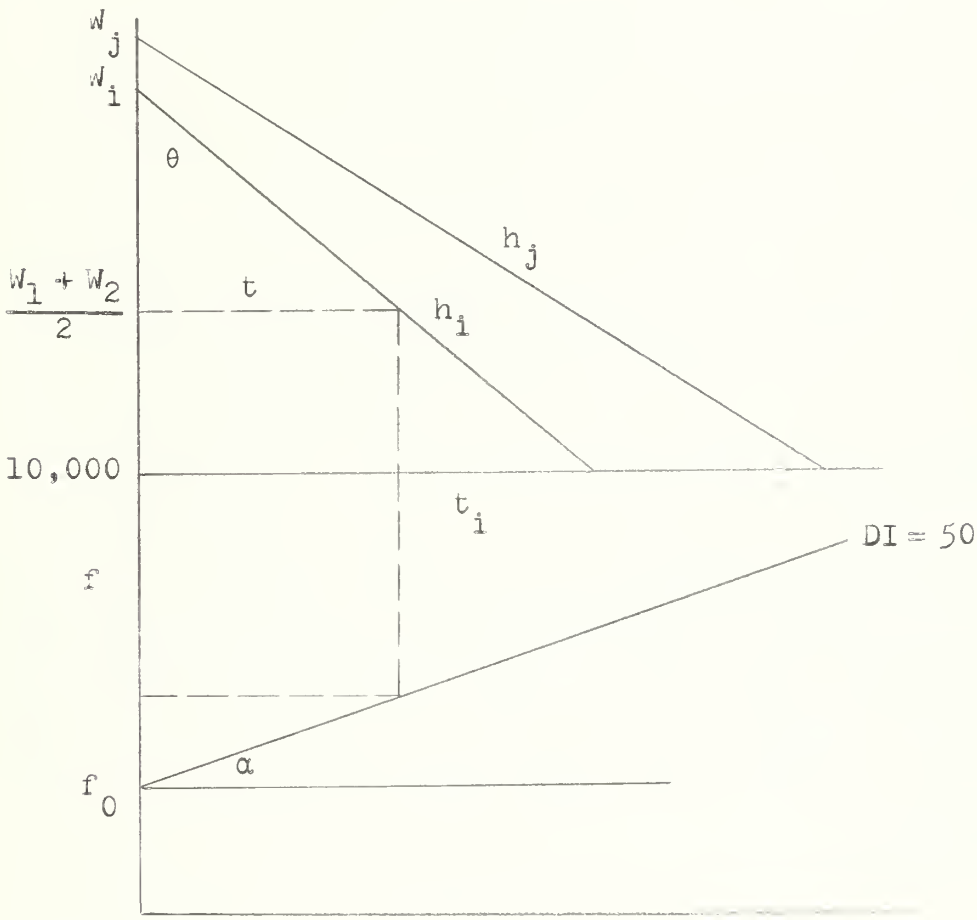

Figure 2

From geometry we obtain:

$$\tan \theta = \frac{t_i}{(W_i - 10{,}000)} = \frac{t}{(W_i - \frac{W_1 + W_2}{2})}$$

$$t = (W_i - \frac{W_1 + W_2}{2}) \cdot \tan \theta$$

Knowing t, we solve for fuel flow, f, using

$$f = f_o + t \cdot \tan \alpha$$

$$f = f_o + (W_i - \frac{W_1 + W_2}{2}) \tan \theta \tan \alpha$$

Since $F = \frac{X_h}{f} = W_1 - W_2$ from equation 1

$$W_1 - W_2 = \frac{X_h}{f_o + (W_i - \frac{W_1 + W_2}{2}) \tan \theta \cdot \tan \alpha} \qquad (6)$$

where W_i is defined in Figure 2.

Solving this quadratic for W_2 will allow a good approximation to the average weight, $(\frac{W_1 + W_2}{2})$. This approximation is believed to be quite accurate, since the altitude lines are fairly straight. An exact approach is given in method two.

METHOD II:

Allowing for the fact that the altitude lines are non-linear, a more precise but lengthy method for solving for the average weight is the method of successive approximations. Referring again to the graph of Figure 10, we see that the average weight is a number such that, if it is used to enter the table, it produces a fuel used, F, such that $W_1 - F = W_2$. When W_2 is combined with W_1 to get the average $(\frac{W_1 + W_2}{2})$, this average will exactly equal the average we used to enter the table. Because of the construction of the table, the true average is the only entry number that will give the average back again.

The first approximation to the average is W_1. The aircraft continuously loses weight, thus we know that this approximation is too high. Entering the table with W_1 yields an F_1. This yields an average $\frac{W_1 + (W_1 - F_1)}{2} \neq W_1$. Decrease W_1 by some $\lambda > 0$. Entering the table with $W_1 - \lambda$ gives F_2. Form another average:

$$\frac{W_1 + (W_1 - F_2)}{2} \overset{?}{=} W_1 - \lambda$$

Continue in this manner until

$$\frac{W_1 + (W_1 - F_{n+1})}{2} = W_1 - n\lambda$$

This method is easily suited for computations on a digital computer, but it proved too lengthy. The greater the accuracy desired, the smaller λ must be. But a small λ requires a large number of iterations. This method was abandoned for the third and final method.

METHOD III:

Recall from Figure 10, $F = F(\frac{W_1 + W_2}{2}, h)$. The graph shows that the fuel burned is directly proportional to the aircraft weight. As a first approximation to the true average weight, use W_1. This is too large, therefore it results in a fuel burned, F_1, that is too large.

$$AV_1 = W_1$$

$$F_1 = F(AV_1, h)$$

From equation 6

$$F_1 = \frac{X_h}{f_o + (W_i - AV_1)\tan\theta\tan\alpha}$$

Let

$X_h = D$ a constant

$f_o + W_i \tan\theta\tan\alpha = B$ a constant

$\tan\theta\tan\alpha = C$ a constant

The first approximation to the fuel used on the horizontal leg is:

$$F_1 = \frac{D}{B + CX_1}, \text{ where } X_1 = AV_1 \qquad (7)$$

Since F_1 is too large, as a next approximation to the average weight decrease AV_1 by $\frac{1}{2}F_1$. Thus in equation 7, multiply by $-\frac{1}{2}$, and add W_1 to both sides. We get

$$AV_2 = W_1 - \frac{1}{2}F_1 = \frac{-D}{2(B + CX_1)} + W_1$$

$$AV_2 = X_2 = \frac{A}{B + CX_1} + W_1$$

$$(\text{where } A = -\frac{D}{2})$$

Entering Figure 10 with X_2 yields

$$F_2 = \frac{D}{B + CX_2}$$

and

$$AV_3 = X_3 = \frac{A}{B + CX_2} + W_1$$

This iterative process yields a sequence $\{X_n\}$:

$$X_1 = W_1$$

$$X_2 = \frac{A}{B + CX_1} + W_1$$

$$X_3 = \frac{A}{B + CX_2} + W_1$$

$$\vdots$$

$$X_n = \frac{A}{B + CX_{n-1}} + W_1$$

where

$A = -150$

$B = .34$

$C = -6.4 \times 10^{-6}$

$W_1 = 1.5 \times 10^4$

Solving for the first few points yields a nest of closed intervals as shown in Figure 3.

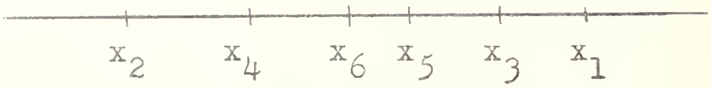

Figure 3

At this point it can be seen why $(W_1 - \frac{1}{2}F_1)$ was used as the second approximation to the average weight. Since we know that X_1 is well above the average, it is desired to decrease it by a quantity large enough to drive the second approximation below the average. A number like .8 or .9 could have been used to multiply F_1, but the use of $\frac{1}{2}$ is sufficient and causes the sequence to converge faster than numbers like .8 or .9.

The manner in which the successive points alternate led to applying the theory of continued fractions in an attempt to show that the sequence $\{X_n\}$ converges. Hall and Knight[2] show that each successive convergent of a continued fraction is alternately less than and greater than the true value of the continued fraction.

Rewriting equation 8 yields

$$X_n = W_1 + \frac{A}{B + CX_{n-1}}$$

Back-substituting for X_{n-1} by X_{n-2}; for X_{n-2} by X_{n-3}; etc. yields:

$$X_n = W_1 + \cfrac{A}{B + C\left(W_1 + \cfrac{A}{B + C\left(W_1 + \cfrac{A}{B + C\left(W_1 + \cdots + \cfrac{A}{B + CX_o}\right.}\right.}\right.}$$

The form of the classic continued fraction is obvious. Factoring out the A in the numerator, and multiplying in each convergent by C yields:

$$X^* = \lim_{n \to \infty} X_n = W_1 + A\left(\cfrac{1}{(B + CW_1) + \cfrac{CA}{(B + CW_1) + \cfrac{CA}{(B + CW_1) + \cdots}}}\right.$$

Dividing numerator and denominator of each successive fraction by (CA)

$$X^* = W_1 + A\left(\cfrac{1}{(B + CW_1) + \cfrac{1}{\left(\frac{B + CW_1}{CA}\right) + \cfrac{1}{(B + CW_1) + \cfrac{1}{\left(\frac{B + CW_1}{CA}\right) + \cdots}}}}\right.$$

In the continued fraction in the brackets, let

$$a = B + CW_1 > 0$$

$$b = \frac{B + CW_1}{CA} > 0$$

Then

$$\theta^* = \cfrac{1}{a + \cfrac{1}{b + \cfrac{1}{a + \cfrac{1}{b + \cdots}}}} \tag{9}$$

As we let $n \to \infty$, replace the 3^{rd}, 4^{th}, etc. convergents by θ

$$\theta^* = \cfrac{1}{a + \cfrac{1}{b + \theta^*}}$$

$$\theta^* = \frac{1}{\frac{a(b+\theta^*)+1}{b+\theta^*}} = \frac{b+\theta^*}{a(b+\theta^*)+1}$$

Solving the resulting quadratic yields:

$$\theta^* = \frac{-ab \pm (a^2b^2 + 4ab)^{\frac{1}{2}}}{2a}$$

Thus we have that

$$X^* = W_1 + A\left(\frac{-ab - (a^2b^2 + 4ab)^{\frac{1}{2}}}{2a}\right)$$

The minus sign is selected because $X^* < W_1$.

This third method gives the average weight for any given value of W_1, X_h, and h. For most values, the sequenc converges in 6 to 8 iterations, whereas the second method required more than 50.

It is interesting to note that in the continued fraction, equation 9, for real but unequal odd and even convergents, Van Vleck's theorem can be applied. For a or b imaginary, Stieltjes theorem applies. Proofs of these theorems are given in Wall[3]. Having solved the implicit function for $\frac{W_1 + W_2}{2}$, Figure 9 yields CAS. It is converted to TAS, and the wind at altitude h changes TAS to GAPD

$$\text{time} = X_h \,/\, \text{GSPD}$$

$$\text{air distance} = X_a = \text{time} \times \text{TAS}$$

Figure 10 gives F as a function of X_a and $\frac{W_1 + W_2}{2}$.

Since optimal CAS is a function of the aircraft weight, as the weight decreases during the horizontal leg the CAS should be changed to remain optimal. In this program, when the aircraft weight decreases by 500 pounds, CAS is recomputed. This results in breaking up X_h into segments of length $\underline{X_h} = 500 \times f$. This results in $([X_h/\underline{X_h}] + 1)$ iterations of the horizontal leg computations.

At this point it should be noted that in order to solve for the fuel burned in the horizontal leg by either of the three methods, we require X_h. At the start of the leg we know only X_t and X_c. Since $X_h = X_t - (X_c + X_d)$ the value of X_d must be determined prior to the start of the horizontal leg. From equation 4 recall that $X_d = X\,(W_2, h)$. The problem resolves to this: in order to solve for W2, we need X_d. But we need W_2 before we can solve for X_d. Another method of successive approximations is developed that results in a recurrent sequence that is shown to converge. Once it is shown that we can find X_d (and consequently X_h) at the start of the horizontal leg, then all previous procedures are justified.

The air distance covered during the descent is $X_{ad} = X\ (h,\ DI,\ wind)$. From equation 3, $CAS = C\ (W_2,\ DI)$. From equation 5, $t = t\ (h,\ DI)$.

From Figure 11

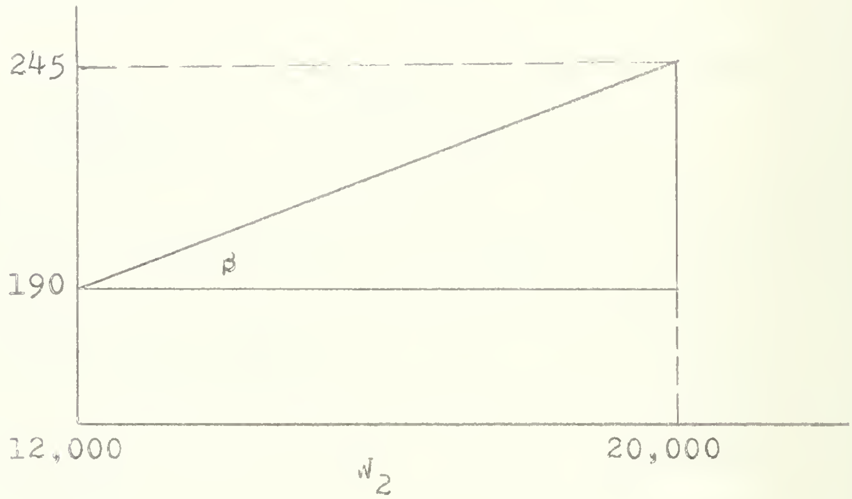

Figure 4

$$CAS = 190 + \tan \beta\ (W_2 - 12000)$$

Let $(190 - 12000 \tan \beta) = a$

$\tan \beta = b$

$$CAS = a + bW_2$$

and

$$TAS = \frac{1}{d} \cdot CAS = k_o \cdot CAS \quad \text{where } k_o = \frac{1}{d}$$

$$\text{time in descent} = t_d = \frac{X_{ad}}{TAS} = \frac{X_d}{GSPD}$$

$$X_d = t_d \times GSPD$$

$$GSPD = TAS \pm \text{wind} = k_1 \times TAS$$

where k_1 is a constant to correct TAS for wind.

$$GSPD = k_1 \; (k_o \times CAS)$$

$$GSPD = k_1 \; (k_o[a + bW_2])$$

Since

$$X_d = t_d \times GSPD$$

$$X_d = t_d \; (ak_1k_o + bk_1k_oW_2)$$

Let

$$ak_1k_ot_d = \alpha$$

$$bk_1k_ot_d = \beta$$

$$X_d = \alpha + \beta W_2 \tag{10}$$

Begin the iterative process by letting $(W_2)_0 = W_1$. This large value will yield an X_d that is too large. Since $X_h = X_t - (X_c + X_d)$, the resulting X_h will be too small. Fuel burned on the horizontal leg is directly proportional to the distance, therefore F_h will be smaller than the true value. $(W_2)_1 = W_1 - F_h$. This value is still too large, bu is closer to the true value of W_2 than was the first approx mation, W_1.

From equation 6 we have

$$W_1 - W_2 = \frac{X_h}{f_o + W_i \tan\theta \tan\alpha - \tan\theta \tan\alpha \cdot \left(\frac{W_1 + W_2}{2}\right)}$$

Let

$$f_o + W_i \tan\theta \tan\alpha = B$$

and

$$-\frac{\tan\theta \tan\alpha}{2} = C$$

$$W_i - W_2 = \frac{X_h}{B + C\ (W_1 + W_2)}$$

$$BW_1 - BW_2 + C\ (W_1 - W_2)(W_1 + W_2) = X_h$$

Solving the quadratic for W_2 yields

$$W_2 = \frac{-B \pm (B^2 - 4C\ [-CW_1^2 - BW_1 + X_h])^{\frac{1}{2}}}{2C}$$

But from equation (10)

$$X_d = \alpha + \beta(W_2)_o$$

$$X_h = (X_t - X_c) - X_d = (X_t - X_c) - \alpha - \beta(W_2)_o$$

Let

$$(X_t - X_c - \alpha) = d$$

$$X_h = d - \beta(W_2)_o$$

and

$$(W_2)_1 = \frac{-B \pm (B^2 - 4C\,[-CW_1^2 - BW_1 + [d - \beta(W_2)_o]])^{\frac{1}{2}}}{2C}$$

Grouping constants,

$$(W_2)_1 = A - (B - C(W_2)_o)^{\frac{1}{2}}$$

where

$$A = 5.65 \times 10^4$$

$$B = 1.81 \times 10^9$$

$$C = 2.35 \times 10^2$$

$$(W_2)_o = W_1 = 1.6 \times 10^4$$

The iterative process thus described results in the sequence $\{X_n\}$:

$$X_o = W_1$$

$$X_1 = A - (B - CX_o)^{\frac{1}{2}}$$

$$X_2 = A - (B - CX_1)^{\frac{1}{2}}$$

.

.

.

$$X_n = A - (B - CX_{n-1})^{\frac{1}{2}} \qquad (10.1)$$

Calculating the first few values results in a monotonic non-increasing sequence of points as in Figure 5. In order to show that such a sequence converges, we refer to the theory of contraction mappings as illustrated by Lyusternik and Yanpolskii[4].

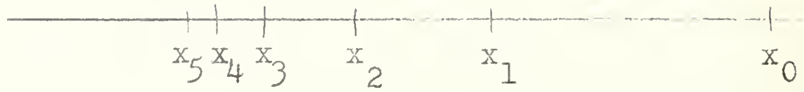

Figure 5

Let f be a continuous operator from E_n into E_n, and $Y = f(X)$. In order to solve for the equation $X = f(X)$ we set up the iterative sequence of elements: $x_0, x_1, x_2, \ldots,$ where x_0 is an arbitrary element of E_n. Here we have $X_{m+1} = f(X_m)$. If the sequence $\{X_m\}$ is convergent to some X^*, then X^* is a solution of the equation $X = f(X)$, and X^* is called a fixed point of the transformation $Y = f(X)$.

The principle of contraction mappings provides a condition for the existence of a fixed point of the transformation $Y = f(X)$.

DEFINITIONS:

(1) An operator f from E_n into E_n is said to be a contraction mapping if there exists a constant q, $(0 < q < 1)$, such that for any $x, x_1 \in E_n$

$$| f(x_1) - f(x) | \leq q \, | x_1 - x | .$$

(2) A sequence $\{X_n\}$ having the property that, given any $\varepsilon > 0$, there exists a subscript N such that $| X_m - X_n | < \varepsilon$ for all $m > N$, $n > N$, is said to be fundamental.

BOLZANO-CAUCHY CRITERION:

A necessary and sufficient condition for the sequence $\{X_n\}$ to be convergent is that it be fundamental.

The following shows that the principle of contraction mappings provides for a fixed point of the transformation $y = f(X)$.

Let f be a continuous operator from E into E_n, such that f is a contraction mapping and in C^2, i.e. twice continuously differentiable in all components. Then there exists a solution X^* to the equation $X = f(X)$, and this solution is unique. The iterative sequence formed by successive approximations is convergent to X^* whatever the initial approximation X_o.

PROOF:

Form the iterative sequence of elements

$$X_o = w$$

$$X_1 = f(X_o)$$

$$X_2 = f(X_1)$$

$$\vdots$$

$$X_n = f(X_{n-1}) \qquad (11)$$

$$X_{n+1} = f(X_n) \qquad (12)$$

Subtract equation (11) from (12) and take absolute values

$$|X_{n+1} - X_n| = |f(X_n) - f(X_{n-1})|$$

But f is a contraction mapping by hypothesis.

$$|X_{n+1} - X_n| = |f(X_n) - f(X_{n-1})| \leq q\,|X_n - X_{n-1}|$$

$$|X_{n+1} - X_n| \leq q\,|X_n - X_{n-1}| \qquad (13)$$

$$|X_2 - X_1| \leq q\,|X_1 - X_o|$$

$$|X_3 - X_2| \leq q\,|X_2 - X_1| = q^2\,|X_1 - X_o|$$

$$\vdots$$

$$|X_{n+1} - X_n| \leq q^n\,|X_1 - X_o| \qquad (14)$$

If the sequence, equation (14), can be shown to be fundamental, then by the Bolzano-Cauchy criterion it is convergent. Choose $\varepsilon > 0$, and N large. To show that $|X_m - X_n| < \varepsilon$ for all $m > N$, $n > N$, we add and subtract all values of X_k, $m < k < n$.

$$|X_m - X_{m-1} + X_{m-1} - X_{m-2} + X_{m-2} - \cdots - X_{n+1} + X_{n+1} - X_n|$$

$$< |X_m - X_{m-1}| + |X_{m-1} - X_{m-2}| + \cdots + |X_{n+1} - X_n|$$

From equation (14),

$$|X_m - X_{m-1}| \leq q^{m-1}\,|X_1 - X_o|$$

$$|X_{m-1} - X_{m-2}| \leq q^{m-2}\,|X_1 - X_o|$$

$$\vdots$$

$$|X_{n+1} - X_n| \leq q^n\,|X_1 - X_o|$$

$$\left|X_m - X_{m-1} + X_{m-1} - \cdots - X_{n-1} + X_{n-1} - X_n\right| \leq (q^n + q^{n+1} + \cdots + q^{m-2} + q^{m-1}) \cdot \left|X_1 - X_0\right|$$

$$\leq q^n (1 + q + q^2 + \cdots + q^{m-n-1}) \cdot \left| X_1 - X_0\right|$$

Since $q < 1$, the partial geometric series is less than $\frac{1}{1-q}$

$$\left|X_m - X_{m-1} + X_{m-1} - \cdots - X_{n-1} + X_{n-1} - X_n\right| < q^n \left(\frac{1}{1-q}\right) \cdot \left| X_1 - X_0 \right|$$

We can choose n large enough so that the sequence is fundamental and therefore convergent. Since $q < 1$, we can make the right hand side arbitrarily small. In fact, $\left| X_n - X_{n-1}\right| \to 0$ as $n \to \infty$.

From equation (11)

$$X_n = f(X_{n-1})$$

$$\left| X_n - f(X_n) \right| = \left| f(X_{n-1}) - f(X_n) \right| \leq q \left|X_n - X_{n-1}\right|$$

$$\left| X_n - f(X_n) \right| \leq q \left| X_n - X_{n-1}\right| \to 0$$

Since f is in C^2, as $X_n \to X^*$, $f(X_n) \to f(X^*)$.

Thus

$$\left| X^* - f(X^*) \right| = 0$$

and

$$X^* = f(X^*)$$

Q.E.D.

If $|f'(X)| < q < 1$, then f is a contraction mapping. From the mean value theorem of differential calculus

$$f(b) - f(a) = (b - a) \cdot f'(X) \quad \text{for some } X,\ a \leq X \leq b.$$

$$f'(X) = \frac{f(b) - f(a)}{b - a}$$

If $|f'(x)| < q < 1$, then $|f(b) - f(a)| < |b - a|$

and $|f(b) - f(a)| \leq q\,|b - a|$

From equation 10.1 the function under study is:

$$(f) = A - (B - CX)^{\frac{1}{2}}$$

$$|f'| = \left| \frac{C}{2(B - CX)^{\frac{1}{2}}} \right|$$

To show that the function is a contraction mapping it is necessary to show that $|f'| < q < 1$ for all values of X that will be encountered. It is easily seen that when $X = \frac{B}{C}$ the derivative is not defined, and that when $X > \frac{B}{C}$ it is imaginary.

The values of the constants in the equation are:

$A = 5.65 \times 10^4$

$B = 1.81 \times 10^9$

$C = 2.35 \times 10^2$

$X_0 = W_1 = 1.6 \times 10^4$

We desire to show that $X < \frac{B}{C} = 7.7 \times 10^6$ for all X that are encountered in the iterative process. The first approximation to X is:

$$X_0 = W_1 = 1.6 \times 10^4$$
$$X_1 = 5.65 \times 10^4 - (1.81 \times 10^9 - 2.35 \times 10^2 \times 1.6 \times 10^4)^{\frac{1}{2}}$$
$$X_1 = 5.65 \times 10^4 - (18.3 \times 10^8)^{\frac{1}{2}}$$
$$X_1 = 1.39 \times 10^4 < X_0$$
$$X_2 = 5.65 \times 10^4 - (1.81 \times 10^9 - 2.35 \times 10^2 \times X_1)^{\frac{1}{2}}$$
$$X_2 = 1.35 \times 10^4 < X_1 < X_0$$

It can easily be shown that each value of X will be smaller than the one that precedes it, since at each iteration a larger value will be subtracted from the constant A. Thus the sequence X_n is monotonic non-increasing, and its upper bound is W_1. Since $W_1 < \frac{B}{C}$, all values of X that will be encountered are also strictly less than $\frac{B}{C}$. Solving for the first derivative when $X = W_1$ yields:

$$|f'| = \frac{2.35 \times 10^2}{2(4.243 \times 10^4)} < 1$$

Thus $|f'| < 1$ for all values of X in the iterative sequence. This is so because $\{X_n\}$ is monotonic non-increasing, and any value of $X < W_1$ increases the denominator of the derivative, thereby decreasing its value. Thus f is a contraction mapping, and the sequence $\{X_n\}$ converges. Lyusternik and Yanpol'skii[4] show that if f is a contraction mapping, then the sequence is convergent to X^* as fast as a geometric progression with ratio q. Thus the computer iteration procedure is not lengthy since in this case q = .0027.

The knowledge that we can solve for X_d, and thus X_h, at the start of the horizontal leg of the flight assures us that we can correctly find the average weight on the leg using the procedures described earlier. We can solve $W_1 - F = W_2$. Knowing W_2 we solve for the fuel used in the descent using $F = f(W_2, h)$, and $W_3 = W_2 - F$. The fuel burned during this profile $(W_0 - W_3)$ is stored, along with the altitude and proper airspeeds for the profile.

Once fuel figures for all profiles are calculated, a searching procedure selects the global minimum and prints it as the optimal fuel along with corresponding altitude and airspeed figures. The possibility of a tie is virtuall eliminated by reading fuel figures to four decimal places. Should several local minima arise, the search is designed so that the global minimum is always selected.

A listing of the Fortran IV program and a sample output is given in Appendix B. All variable names are written such that they are easily recognizable. TAS is true airspe DIST is distance, etc. As inputs, the program only require the course and distance to the field, the type aircraft (A4 = 1, A7 = 2, etc. This allows for the difference in aircraft), the weight of fuel aboard the aircraft at time o BINGO, and the wind information. The wind can be up-dated at intervals as set by meterological readings.

CHAPTER III

RESULTS AND CONCLUSIONS

For a fixed distance, X_t nautical miles, $(100 \leq X_t \leq 250)$, the winds aloft and the aircraft weight combine to determine the optimal flight profile. Under a zero wind condition and normal BINGO fuel weight, the optimal flight profile is to climb to the service ceiling and commence an idle descent at X_d, or to climb until X_d is reached, and then to start the idle descent such that X_h is zero. For fuel weights higher than the normal BINGO weights, the optimal altitude is lower than the service ceiling because of the excess fuel used in climbing to high altitudes when the aircraft is heavy.

Since zero wind conditions are rarely if ever encountered, the wind velocity aloft is a most important factor in determining the optimal flight profile. In order to test the effect of winds on a standard BINGO problem, a typical wind pattern for winter months along the coastal region of Southern California was obtained from the weather facility at the Naval Auxiliary Landing Field in Monterey.

The following winds are typical of a northern hemisphere cyclonic low pressure area.

ALTITUDE	WIND
0 - 5000 ft	270°/ 10 kts
6 -10000 ft	240°/ 15 kts
11 -15000 ft	210°/ 20 kts
16 -20000 ft	180°/ 30 kts
21 -25000 ft	150°/ 40 kts
26 -30000 ft	120°/ 60 kts
31 -35000 ft	095°/ 80 kts
36 -40000 ft	080°/ 100 kts

The BINGO situation considered is such that a pilot leaves a carrier off the coast of Southern California and flies 090° / 160 miles to NAS North Island near San Diego. The fuel weight at start of BINGO is 2200#.

For this example a flight profile with the service ceiling as the optimum altitude would require 1499.4#. A guess by a "seat of the pants" acquaintance of mine would use 25000 ft as the optimum altitude and would require 1435.3#. The computer flight plan predicts the optimal flight profile as follows.

Optimum altitude is	20,000	ft
Climb speed is	314.4	kts
Cruise speed is	250.0	kts
Start descent when	32.1	miles out
Descent speed is	195.3	kts
Fuel required is	1135.6	lbs

The savings in jet fuel of 300# for the optimal over the guess, and 364# for the optimal over the climb to servi ceiling is substantial when it is considered that this savings would allow two more landing attempts either at the carrier or at the field ashore. Multiply this savings by thousands of flights per month if the system were incorporated in the fleet, and it is easily seen that the jet fuel saved would soon become significant. The two extra landing attempts would probably result in fewer aircraft ramaining on the beach over night.

In order to demonstrate the usefulness of extending the present BINGO computer program to long distance flights, a flight of 600 miles was flown with a fuel weight of 6000 pounds at the start. The entire distance is flown with a course of 115° magnetic. This is not realistic since even West - East cross-country flights sometimes require course changes of more than 20°. Also, the present program does not change altitudes to higher ones when the aircraft weight decreases to a value that suggests a climb in order to remain optimal. Nevertheless, some fuel values for different profiles indicate the savings in fuel that would be realized if the optimal profile is flown. For example:

Fuel (lbs.)	Altitude (ft.)	
5596.6	5,000	
4646.1	10,000	
4333.9	15,000	
→ 3842.8	20,000	
4517.1	25,000	
4348.8	30,000	
3976.6	35,000	
→ 3733.5	40,000	Service ceiling

In this case the service ceiling is the optimal altitude. But if this altitude can not be reached because of poor engine performance in a particular airplane, then 20,000 ft. would be the best altitude.

An experienced pilot may often fly a near-optimal flight profile merely by an educated guess or by carefully managing the fuel flow. But he can never do better than the computer prediction, and will probably do worse most of the time since his wind information is sketchy if he has any at all.

The professional doubter may complain that even mete ology doesn't know the accurate wind information at altitu so why even bother with a computer solution? We can only reply that some good estimate is better than none at all, and that modern meteorological equipment can measure and predict the winds quite accurately.

The incorporation of a computer system to predict optimal flight profiles may at first meet with some inerti from fleet pilots, especially the more experienced ones. The "seat of the pants" pilot may reject the computer decision as hocus-pocus or just plain incorrect. But even the most experienced aviator may encounter vertigo some dark, rainy night and he may discover that flying the airplane is about the only job he can handle. Figuring ou an optimal flight profile to the beach may take a back seat to survival. It is in this situation that informatio from departure control would be most welcome.

The cost involved in establishing and operating a system such as described above may appear to be too large when compared to the small savings in relatively inexpensive jet fuel. But the possible loss of a Phantom II and/or a pilot may shift the balance in the system's favor.

One possible reaction against the use of computer flight planning for cross-country flights is that often the air traffic control center will not allow a climb to the desired altitude because they must wait until they can safely fit the aircraft into the West-East (or vice-versa) traffic flow, which may be considerable in this jet age. This will result in remaining at a non-optimal altitude for many minutes. But this drawback exists for non-computer flights as well. And it is still better to climb from a non-optimal altitude to an optimal one when cleared to climb, than to go from one non-optimal altitude to another.

Another great advantage in long range computer flight planning occurs when several alternate routes are available. Then the program can be written to choose the optimal altitude as well as the best route of flight. The many possible routes and altitudes would require too much computation for a pilot during his flight briefing, but a high speed computer can solve such a problem in minutes. Thus the pilot can spend his pre-flight time briefing the mission and tactics, and let the machine do the arithmetic.

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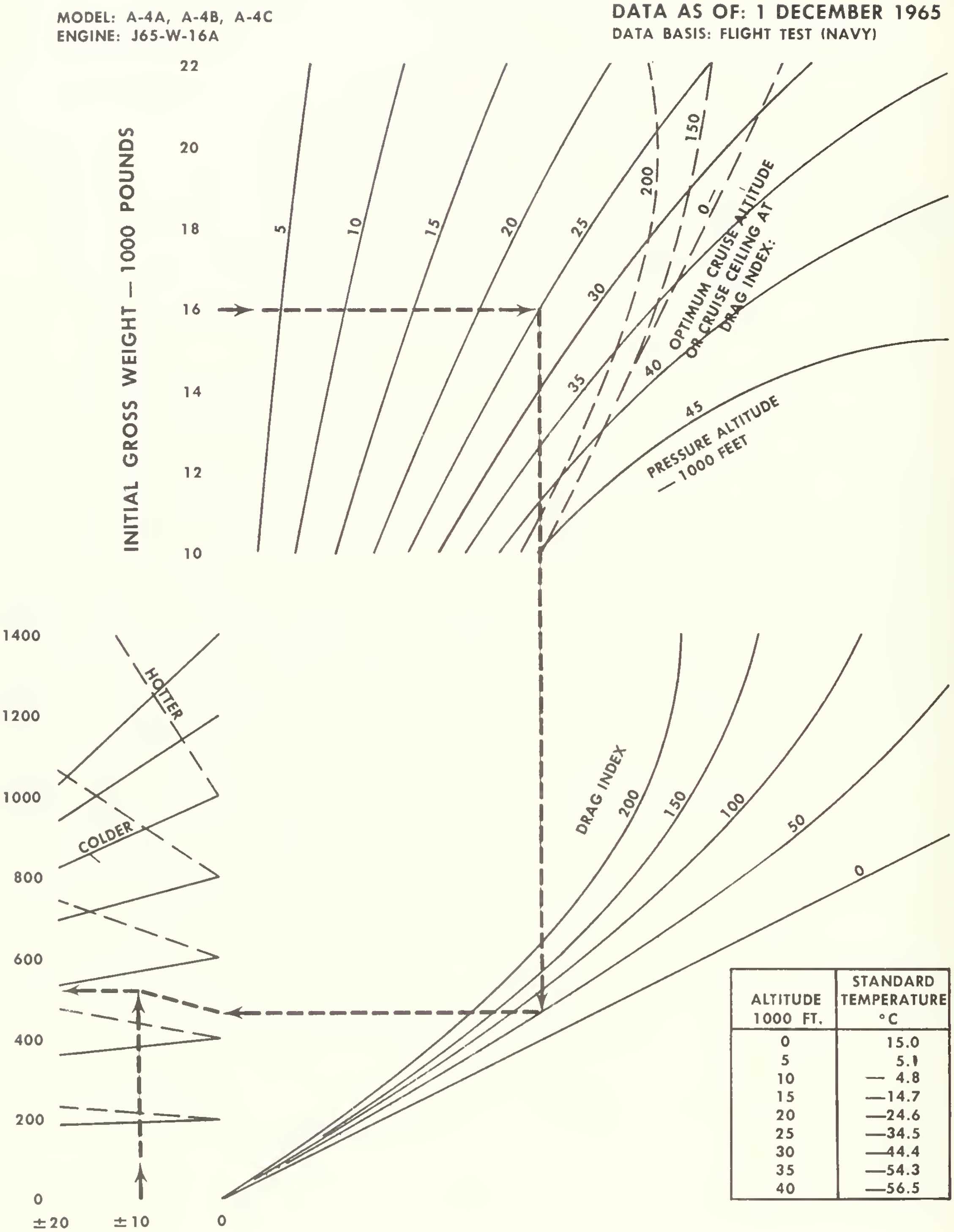
2. Hall, H. S. and Knight, S. R. Higher Algebra. London: Macmillan and Company, Ltd., Reprinted 1964.

3. Wall, H. S. Analytic Theory of Continued Fractions. New York: D. Van Nostrand Company, Inc., 1948.

4. Lyusternik, L. A. and Yanpol'skii, A. R. Mathematical Analysis. Translated by D. E. Brown. London: Pergamon Press, Ltd., 1965.

APPENDIX A

Aircraft Performance Charts for the Navy A4C Aircraft



ALTITUDE 1000 FT.	STANDARD TEMPERATURE °C
0	15.0
5	5.1
10	— 4.8
15	—14.7
20	—24.6
25	—34.5
30	—44.4
35	—54.3
40	—56.5

Figure 6. Climb Fuel

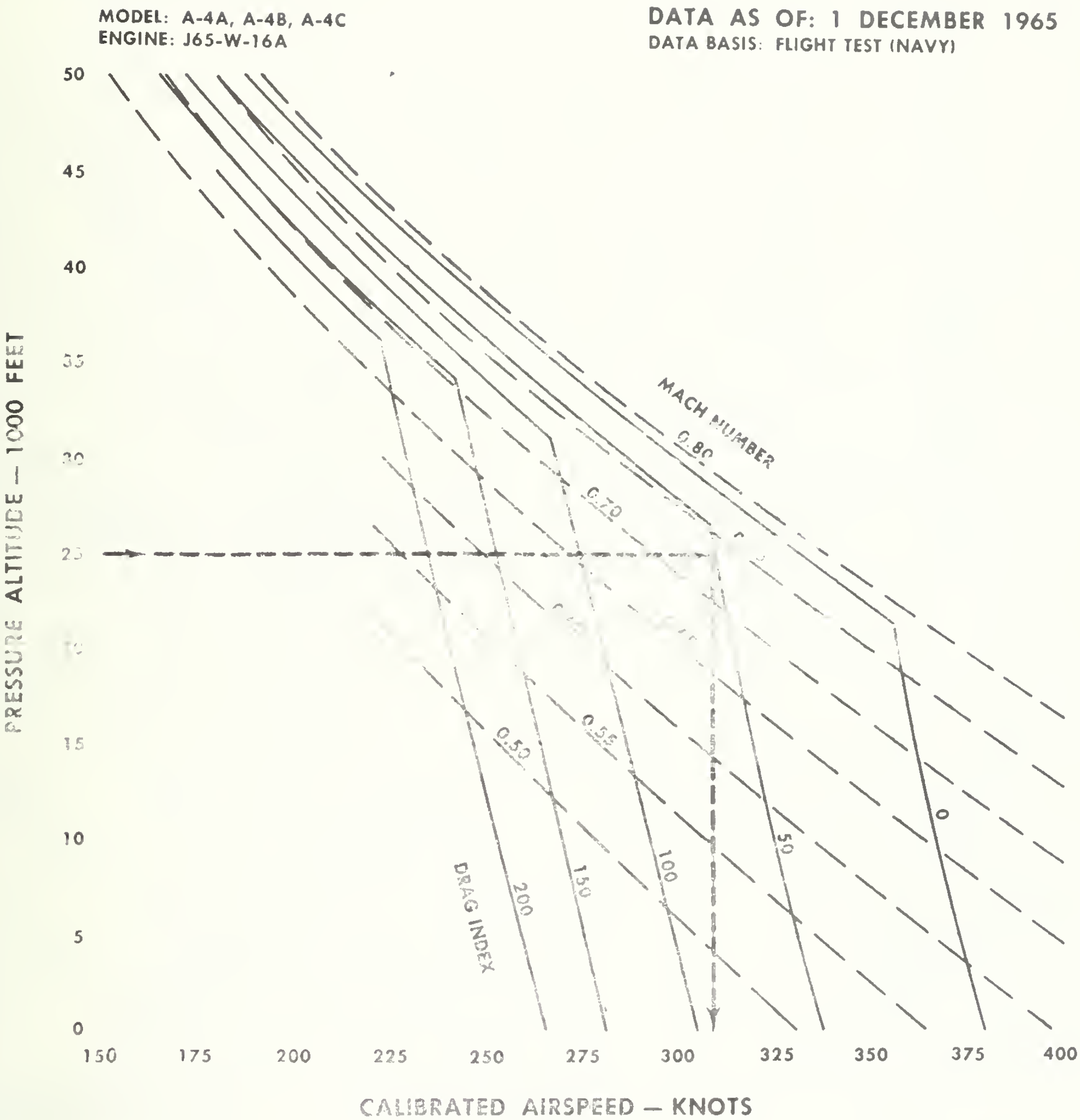
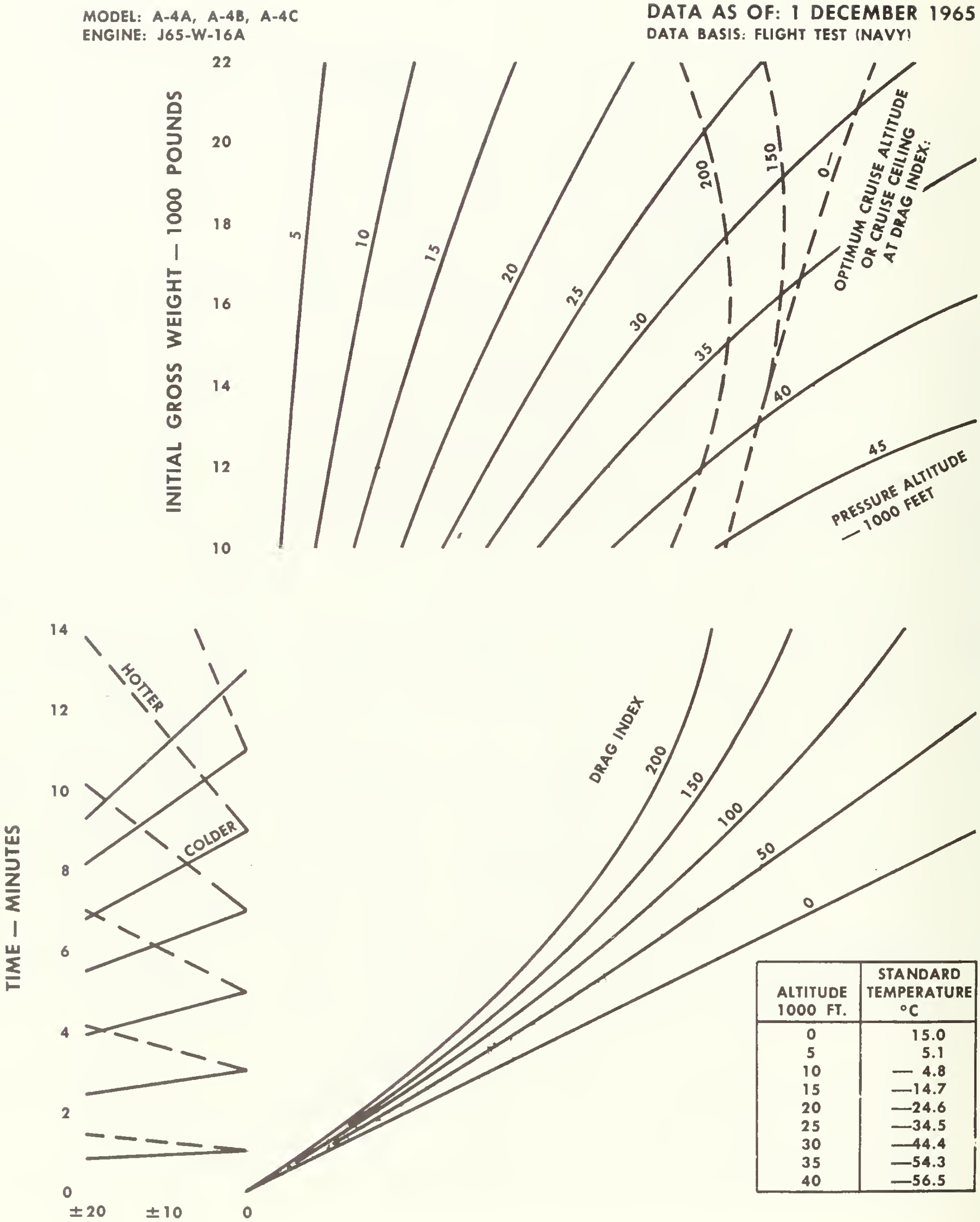


Figure 7. Climb Speed Schedule



ALTITUDE 1000 FT.	STANDARD TEMPERATURE °C
0	15.0
5	5.1
10	— 4.8
15	—14.7
20	—24.6
25	—34.5
30	—44.4
35	—54.3
40	—56.5

Figure 8. Climb Time

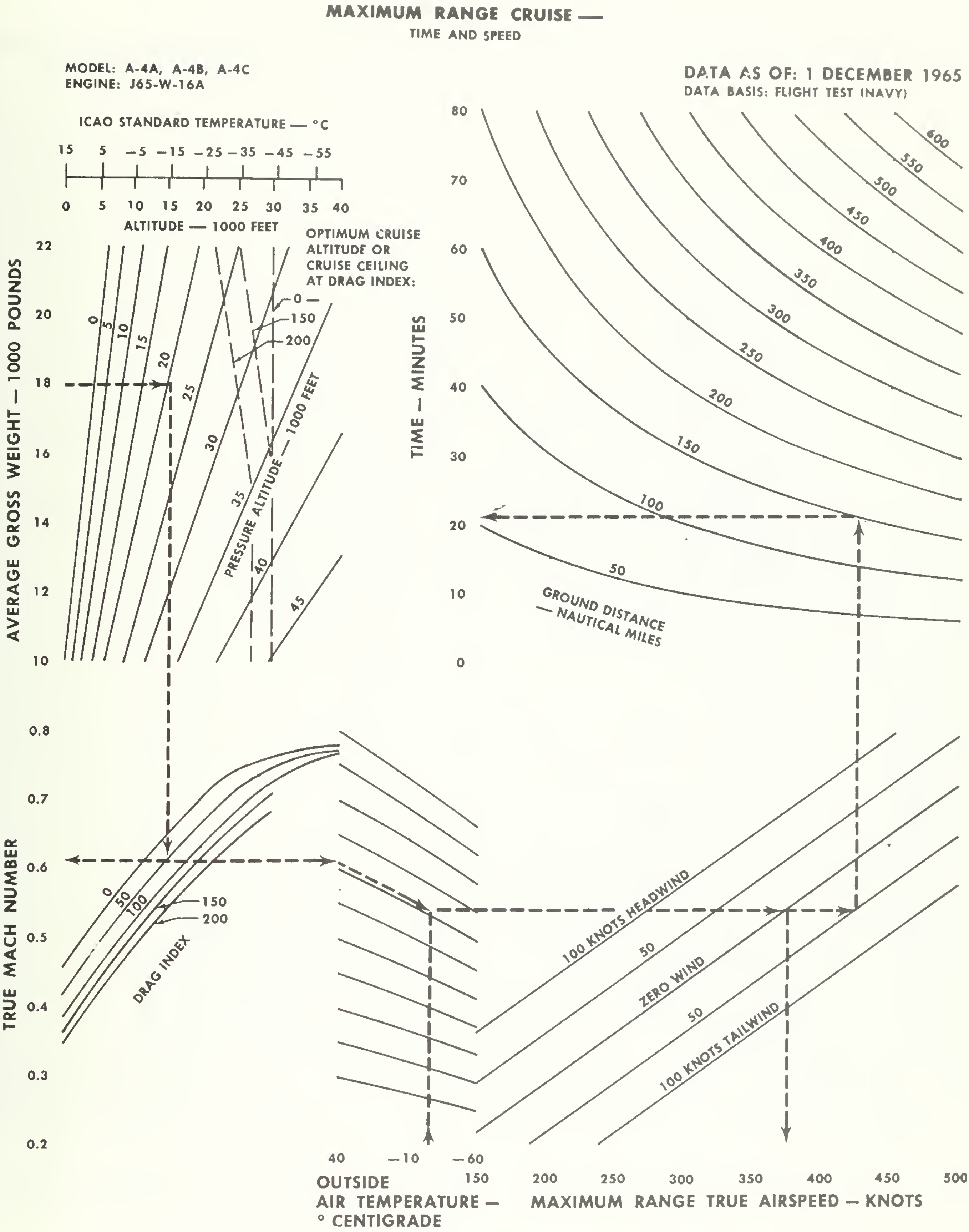


Figure 9. Maximum Range Cruise - Time and Speed

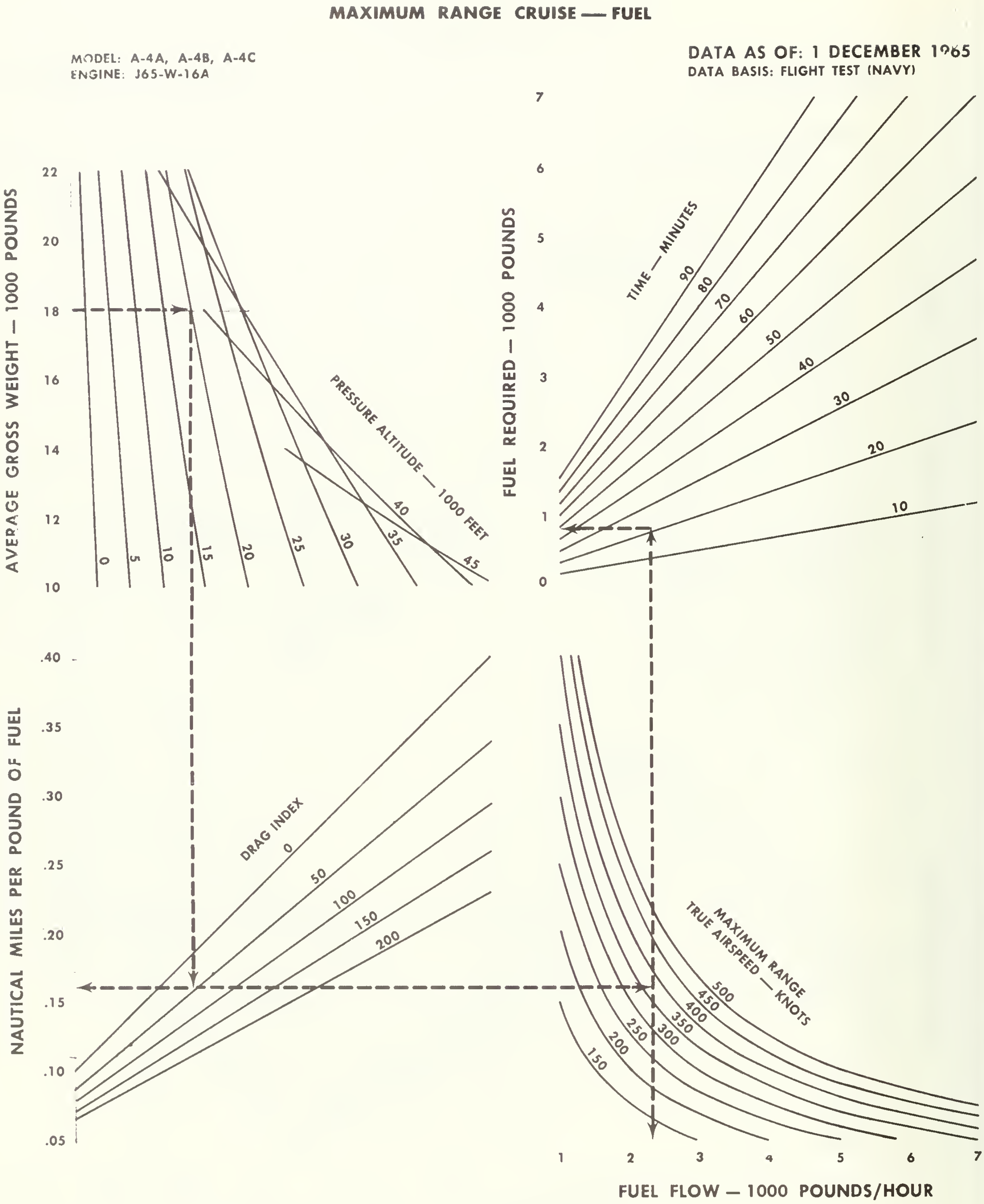
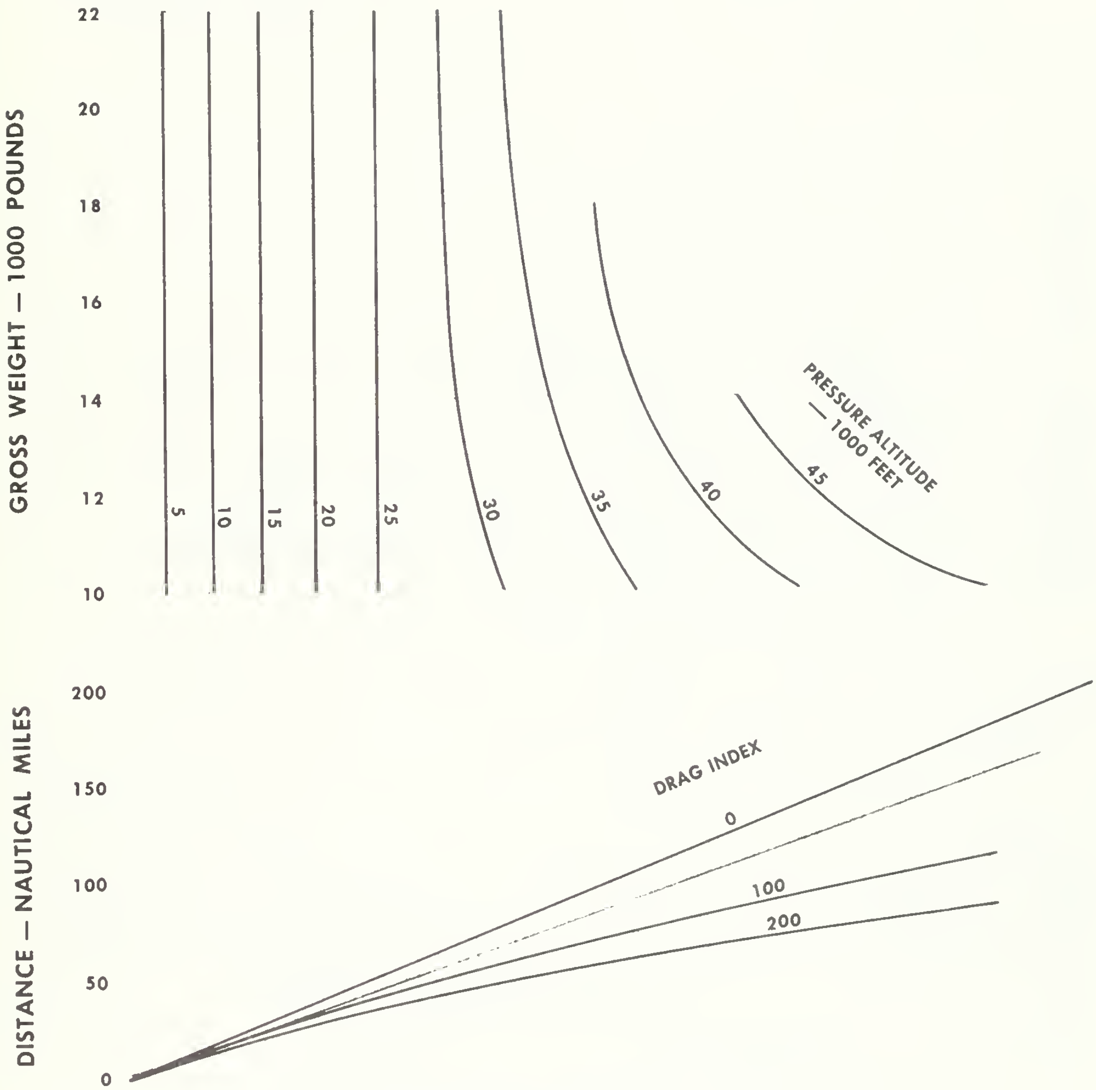


Figure 10. Maximum Range Cruise - Fuel

MODEL: A-4A, A-4B, A-4C
ENGINE: J65-W-16A

DATA AS OF: 1 DECEMBER 1965
DATA BASIS: FLIGHT TEST (NAVY)



DESCENT SPEED SCHEDULE — KCAS

	GROSS WEIGHT — 1000 POUNDS						
DRAG INDEX	10	12	14	16	18	20	22
0	180	200	215	230	240	255	270
100	165	180	195	210	220	235	245
200	155	170	180	195	205	220	230

Figure 11. Descent Distance

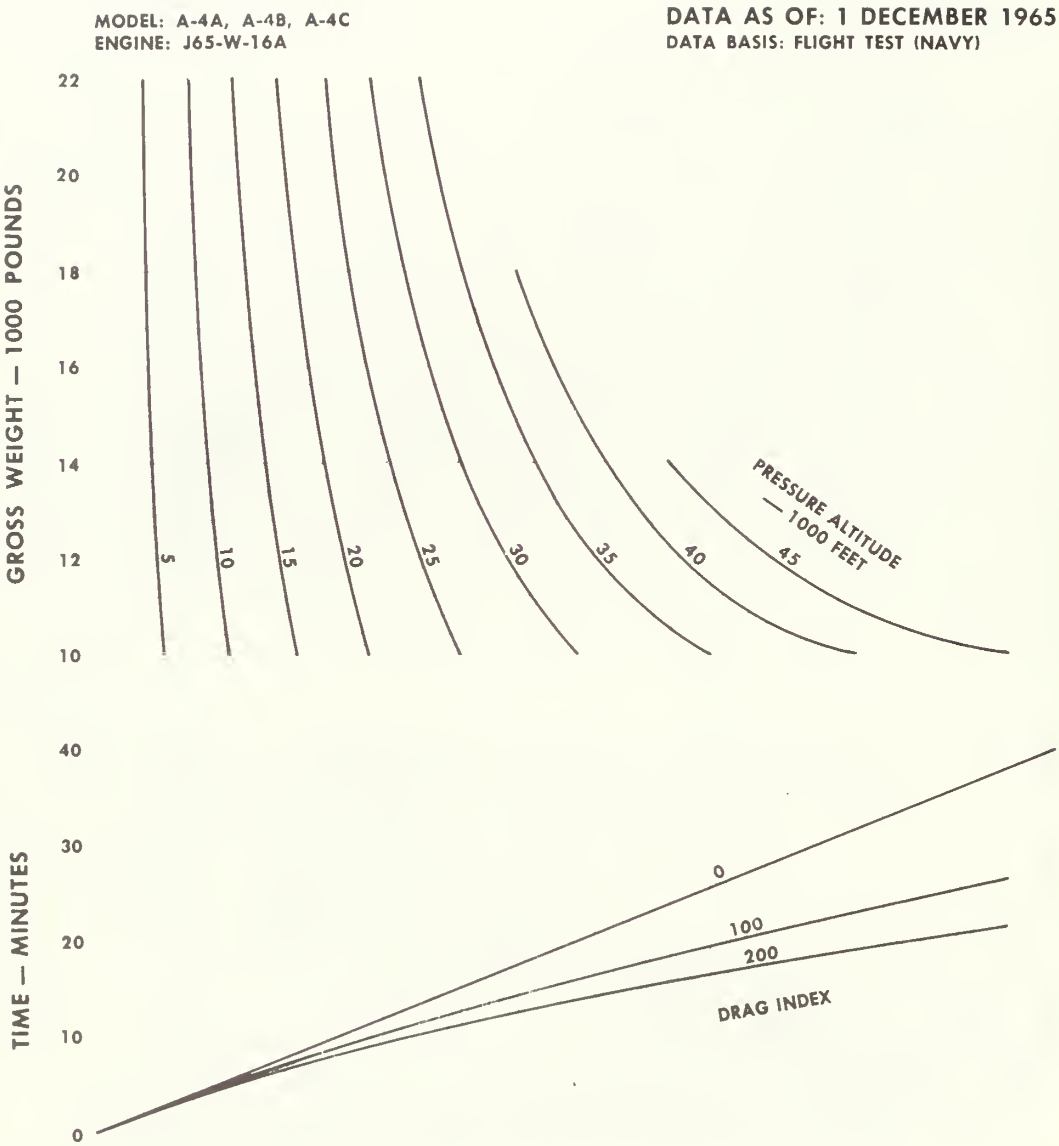


Figure 12. Descent Time

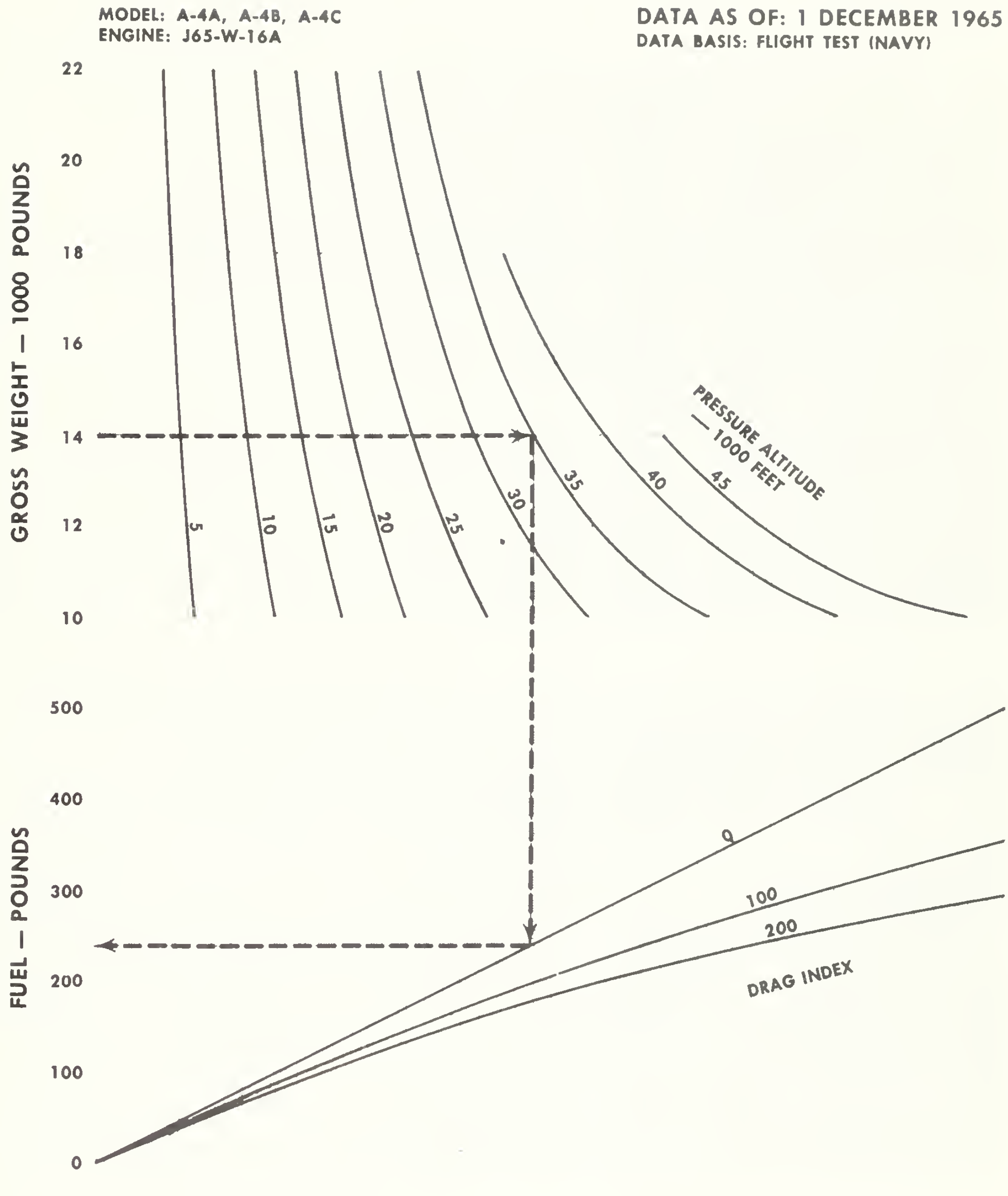


Figure 13. Descent Fuel

APPENDIX B

Program BINGO and Sample Output

```
C       PROGRAM BINGO

        DIMENSION WNDCRS(40), WNDSPD(40), BURN(40), CLIMB(40)
        DIMENSION HORIZ(40), DESC(43), OPALT(40),OUT(40)
        COMMON WNDCRS,WNDSPD,CRS
C
C       FIRST DATA CARD: BEARING AND DISTANCE TO FIELD
C       FUEL ON BOARD, AND AIRCRAFT TYPE
C
        READ(5,90) CRS,DIST,FUEL,ACTYPE
        WRITE(6,90) CRS,DIST,FUEL,ACTYPE
   90   FORMAT(4F10.2)
        DO 92 I=1,40,5
        J=I+1
        K=I+2
        L=I+3
        M=I+4
        READ(5,91) WNDCRS(I),WNDSPD(I),WNDCRS(J),WNDSPD(J),
       *WNDCRS(K),WNDSPD(K),WNDCRS(L),WNDSPD(L),
       *WNDCRS(M),WNDSPD(M)
   91   FORMAT(10F5.1)
   92   CONTINUE
C
C       EACH CARD CONTAINS 5000 FEET OF WIND INFO
C
        IF(ACTYPE.EQ.1.) GO TO 40
        GO TO 41
   40   BASIC=11000.
C
C       LEVEL AT 1000 FT
C
   41   ALT=1000
        WT=FUEL+BASIC
        AFUEL=FUEL
        ADIST=DIST
        BDIST=DIST
        I=1
        BURN(I)=0
        REDIST=BDIST
        BWT=WT
        JI=1
        AWT=WT
   42   CONTINUE
        START=AVHRWT(WT,I,DIST,JI)
        WT=START
        CAS=HRSPD(WT,I)
        TAS=CTAS(CAS,I)
        GSPD=WIND(I,TAS)
        FUEL=HRFUEL(WT,I)
        WDIST=REDIST
        TIME=WDIST/GSPD
        WDIST=TIME*TAS
        IF(WNDCRS(I) GE.180.) GO TO 731
        CHECK=WNDCRS(I)+180.
        GO TO 732
  731   CHECK=WNDCRS(I)-180.
  732   CONTINUE
        DIFF=CRS-CHECK
        CORR=ABS(DIFF)
        IF(CORR.LT 90.) GO TO 733
        IF(CORR.GT.270.) GO TO 733
        GO TO 734
  733   TEST=WDIST-DIST
        TEST=ABS(TEST)
        IF(TEST.LE.2.) GO TO 797
        WT=BWT
        DIST=DIST-2.
        GO TO 42
  734   TEST=WDIST-DIST
        TEST=ABS(TEST)
        IF(TEST.LE 2.) GO TO 797
        WT=BWT
```

```
      DIST=DIST+2.
      GO TO 42
  797 CONTINUE
      IF(JI.GT.1) GO TO 63
      HORIZ(I)=CAS
   63 CONTINUE
      WT=BWT
      SHDIST=FUEL*500.
      REDIST=REDIST-SHDIST
      ARG=ABS(REDIST)
      JI=2
      IF(REDIST.LT.SHDIST) GO TO 61
      BURN(I)=BURN(I)+500.
      BWT=BWT-500.
      WT=BWT
      DIST=REDIST
      WDIST=REDIST
      GO TO 42
   61 BURN(I)=BURN(I)+500.+(ARG/FUEL)
      CLIMB(I)=0.
      DESC(I)=0.
      WRITE(6,17)
   17 FORMAT(1H1,35X,'    BURN          CLIMB       HORIZ ',
     *3X,' DESC         ALT   ')
      WRITE(6,703)
  703 FORMAT(     35X,'     ----          -----       ----- ',
     *3X,' ----         ---   ')
C
      WRITE(6,72) BURN(1),CLIMB(1),HORIZ(1),DESC(1),I
   72 FORMAT(////,35X,4F10.4,I7)
C
C
C     START CLIMBING IN 1000 FT INCREMENTS
C
      WT=AFUEL+BASIC
      N=1
      MN=1
      MM=10
      DO 1000 I=2,40
      BURN(I)=0.
      JI=1
      CLDIST=0.
      MI=I/MM
      IF(MI.EQ.1) GO TO 56
      GO TO 57
   56 MM=MM+5
C
C     COMPUTE THE CLIMB FUEL AND DISTANCE
C
      DO 820 J=5,I,5
      ALT=I*1000
      OPALT(I)=ALT
      CAS=CLSPD(J)
      TAS=CTAS(CAS,J)
      TIME=CLTIME(WT,J)
      TIME=TIME/(J/5)
      TIME=TIME/60.
      VERT=5000./TIME
      VERT=VERT/6080.
      TAS=SQRT(TAS**2-VERT**2)
      GSPD=AVWIND(J,MN)
      MN=MN+5
      GSPD=GSPD+TAS
      FUEL=CLFUEL(WT,I)
      BURN(I)=FUEL
      CLDIST=CLDIST+GSPD*TIME
  820 CONTINUE
   57 CONTINUE
      DIST=BDIST
      WT=AFUEL+BASIC
      ALT=I*1000.
      OPALT(I)=ALT
```

```
      CAS=CLSPD(I)
      CLIMB(I)=CAS
      DELTA=ALT-1000.
      TAS=CTAS(CAS,I)
      TIME=CLTIME(WT,I)
      TIME=TIME/60.
      VERT=DELTA/TIME
      VERT=VERT/6080.
      TAS=SQRT(TAS**2-VERT**2)
      GSPD=AVWIND(I,1)
      GSPD=GSPD+TAS
      FUEL=CLFUEL(WT,I)
      BURN(I)=FUEL
      CLDIST=GSPD*TIME
C
C     COMPUTE THE DESCENT FUEL AND DISTANCE
C
      TODIST=DIST
      AWT=WT-BURN(I)
      BEFORE=BURN(I)
      BWT=AWT
      WAIT=AWT
      NN=1
      NK=5
      DO 555 J=1,NK
      WT=AWT
      CAS =DSSPD(WAIT)
      DESC(I)=CAS
      DSDIST=DDIST(I)
      TAS=CTAS(CAS,I)
      TIME=DSDIST/TAS
      MN=1
      GSPD=AVWIND(I,MN)
      GSPD=GSPD+TAS
      DSDIST=GSPD*TIME
      OUT(I)=DSDIST
C
C     COMPUTE THE HORIZONTAL FUEL AND DISTANCE
C
      DIST=TODIST-(CLDIST+DSDIST)
      ADIST=DIST
      REDIST=DIST
      IF(DIST.GE.0.) GO TO 95
   93 WRITE(6,94) I
   94 FORMAT(/,12X,' FOR THIS DIST/FUEL COMBINATION, ',
     *' STAY BELOW ',I7,' THOUSAND FEET ')
      GO TO 1000
   95 CONTINUE
      START=AVHRWT(WT,I,DIST,JI)
      WT=START
      CAS=HRSPD(WT,I)
      TAS=CTAS(CAS,I)
      GSPD=WIND(I,TAS)
      FUEL=HRFUEL(WT,I)
      WDIST=ADIST
      TIME=WDIST/GSPD
      WDIST=TIME*TAS
      IF(WNDCRS(I).GE.180.) GO TO 426
      CHECK=WNDCRS(I)+180.
      GO TO 427
  426 CHECK=WNDCRS(I)-180.
  427 CONTINUE
      DIFF=CRS-CHECK
      CORR=ABS(DIFF)
      IF(CORR.LT.90.) GO TO 428
      IF(CORR.GT.270.) GO TO 428
      GO TO 429
  428 TEST=WDIST-DIST
      TEST=ABS(TEST)
      IF(TEST.LE.8.) GO TO 431
      WT=AWT
      NN=NN+1
```

```
      DIST=DIST-8.
      GO TO 95
  429 TEST=WDIST-DIST
      TEST=ABS(TEST)
      IF(TEST.LE.8.) GO TO 431
      WT=AWT
      NN=NN+1
      DIST=DIST+8.
      GO TO 95
  431 CONTINUE
      IF(JI.GT.1) GO TO 727
      HORIZ(I)=CAS
  727 CONTINUE
      JI=2
      FUEL=WDIST/FUEL
      WAIT=AWT-FUEL
  555 CONTINUE
      BURN(I)=BURN(I)+FUEL
      WT=WAIT
      FUEL=DSFUEL(WT,I)
      BURN(I)=BURN(I)+FUEL
      IF(BURN(I).LE.BURN(N)) GO TO 51
      GO TO 52
   51 N=I
   52 CONTINUE
      WRITE(6,71) BURN(I),CLIMB(I),HORIZ(I),DESC(I),I
   71 FORMAT(35X,4F10.4,I7)
 1000 CONTINUE
      WRITE(6,73) OPALT(N)
   73 FORMAT(/////////,41X,' OPTIMUM ALTITUDE IS ',F9.1,
     *' FEET ')
      WRITE(6,74) CLIMB(N)
   74 FORMAT(/,41X,' CLIMB SPEED IS ',F6.1,' KNOTS ')
      WRITE(6,75) HORIZ(N)
   75 FORMAT(/,41X,' CRUISE SPEED IS ',F6.1,' KNOTS ')
      WRITE(6,76) OUT(N)
   76 FORMAT(/,41X,' START DESCENT WHEN ',F6.1,' OUT ')
      WRITE(6,77) DESC(N)
   77 FORMAT(/,41X,' DESCENT SPEED IS ',F6.1,' KNOTS ')
      WRITE(6,78) BURN(N)
   78 FORMAT(/,41X,' FUEL USED WILL BE ',F9.1,' POUNDS ')
      END



      FUNCTION AVHRWT(WT,I,DIST,JI)
C
C     COMPUTE THE AHERAGE HORIZONTAL WEIGHT
C
      HOLD=WT
      ALT=I*1000.
      DO 3333 JJ=1,5
      FUEL=HRFUEL(WT,I)
      FUEL=DIST/FUEL
      WT=HOLD-(FUEL/2.)
      IF(I.GT.1) GO TO 3333
      WRITE(6,3332) WT,I
 3332 FORMAT(F10.3,5X,I7)
 3333 CONTINUE
      AVHRWT=WT
      WT=HOLD
      RETURN
      END
```

```
      FUNCTION CLSPD(I)
C
C     THIS IS THE CLIMB SPEED SCHEDULE
C
      ALT=I*1000.
      IF(ALT.LE.26500.) GO TO 10
      CLSPD=307.-((ALT-26500.)/13500.)*70.
      GO TO 11
   10 CLSPD=337.-(ALT/26500.)*30
   11 CONTINUE
      RETURN
      END


      FUNCTION CLTIME(WT,I)
C
C     TIME NEEDED TO CLIMB TO ALTITUDE
C
      ALT=I
      IF(WT.LE.12000.) GO TO 761
      IF(WT.LE.13000.) GO TO 762
      IF(WT.LE.14000.) GO TO 763
      IF(WT.LE.15000.) GO TO 764
      IF(WT.LE.16000.) GO TO 765
      CLTIME=12.
      GO TO 766
  761 IF(ALT.LE.20.) GO TO 7611
      IF(ALT.LE.30.) GO TO 7612
      CLTIME=7.1-((40.-ALT)/10.)*2.7
      GO TO 766
 7611 CLTIME=2.7*(ALT/20.)
      GO TO 766
 7612 CLTIME=4.4-((30.-ALT)/10.)*1.7
      GO TO 766
  762 IF(ALT.LE.10.) GO TO 7621
      IF(ALT.LE.30.) GO TO 7622
      CLTIME=8.-((40.-ALT)/10.)*3.2
      GO TO 766
 7621 CLTIME=(ALT/10.)*1.2
      GO TO 766
 7622 CLTIME=4.8-((30.-ALT)/20.)*3.6
      GO TO 766
  763 IF(ALT.LE.10.) GO TO 7631
      IF(ALT.LE.30.) GO TO 7632
      CLTIME=9.-((40.-ALT)/10.)*3.8
      GO TO 766
 7632 CLTIME=5.2-((30.-ALT)/20.)*3.8
      GO TO 766
 7631 CLTIME=1.4*(ALT/10.)
      GO TO 766
  764 IF(ALT.LE.15.) GO TO 7641
      IF(ALT.LE.30.) GO TO 7642
      CLTIME=10.-((40.-ALT)/10.)*4.1
      GO TO 766
 7641 CLTIME=2.4*(ALT/15.)
      GO TO 766
 7642 CLTIME=5.9-((30.-ALT)/15.)*3.5
      GO TO 766
  765 IF(ALT.LE.15.) GO TO 7651
      IF(ALT.LE.35.) GO TO 7652
      CLTIME=11.4-((40.-ALT)/5.)*3.2
      GO TO 766
 7651 CLTIME=2.6*(ALT/15.)
      GO TO 766
 7652 CLTIME=8.2-((35-ALT)/20.)*5.6
  766 CONTINUE
      RETURN
      END
```

```
      FUNCTION HRFUEL(WT,I)
C
C     COMPUTE THE FUEL USED IN LEVEL FLIGHT
C
      ALT=I
      IF(WT.LE.13000.) GO TO 655
      IF(WT.LE.14000.) GO TO 651
      IF(WT.LE.15000.) GO TO 654
      IF(WT.LE.16000.) GO TO 652
      GO TO 652
  651 IF(ALT.LE.20.) GO TO 6511
      HRFUEL=.29-((40.-ALT)/15.)*.09
      HRFUEL=HRFUEL-.025*(WT/14000.)
      GO TO 657
 6511 HRFUEL=.20-((25.-ALT)/25.)*.1
      HRFUEL=HRFUEL-.02 *(WT/14000.)
      GO TO 657
  655 IF(ALT.LE.20.) GO TO 6551
      HRFUEL=.30-((40.-ALT)/15.)*.09
      HRFUEL=HRFUEL-.025*(WT/13000.)
      GO TO 657
 6551 HRFUEL=.22-((25.-ALT)/25.)*.1
      HRFUEL=HRFUEL-.02*(WT/13000.)
      GO TO 657
  654 IF(ALT.LE.20.) GO TO 6541
      HRFUEL=.27-((40.-ALT)/15.)*.09
      HRFUEL=HRFUEL-.025*(WT/15000.)
      GO TO 657
 6541 HRFUEL=.19-((25.-ALT)/25.)*.1
      HRFUEL=HRFUEL-.02*(WT/15000.)
      GO TO 657
  652 IF(ALT.LE.10.) GO TO 6521
      IF(ALT.LE.15.) GO TO 6522
      IF(ALT.LE.30.) GO TO 6523
      HRFUEL=.22
      HRFUEL=HRFUEL-.03 *(WT/16000.)
      GO TO 657
 6521 HRFUEL=.1+(ALT/10.)*.04
      HRFUEL=HRFUEL-.02*(WT/16000.)
      GO TO 657
 6522 HRFUEL=.15-((15.-ALT)/5.)*.01
      HRFUEL=HRFUEL-.023*(WT/16000.)
      GO TO 657
 6523 HRFUEL=.21-((30.-ALT)/15.)*.06
      HRFUEL=HRFUEL-.029*(WT/16000.)
  657 CONTINUE
      RETURN
      END

      FUNCTION DSSPD(WT)
C
C     THIS IS THE DESCENT SPEED SCHEDULE
C
      IF(WT.LE.12000.) GO TO 21
      DSSPD=190.+((WT-12000.)/8000.)*55.
      GO TO 22
   21 DSSPD=190.
   22 CONTINUE
      RETURN
      END
```

```
      FUNCTION WIND(I,TAS)
      DIMENSION WNDCRS(40),WNDSPD(40)
      COMMON WNDCRS,WNDSPD,CRS
C
C     COMPUTE WIND COMPONENTS TO CONVERT TAS INTO GSPD
C
      IF(WNDCRS(I).GE.180.) GO TO 300
      CHECK=WNDCRS(I)+180.
      GO TO 301
  300 CHECK=WNDCRS(I)-180.
  301 CONTINUE
      DIFF=CRS-CHECK
      CORR=ABS(DIFF)
      IF(CORR.LT. 90.) GO TO 330
      IF(CORR.GT.270.) GO TO 340
      IF(CORR.GE 180.) GO TO 3101
      GO TO 311
 3101 CORR=CORR-180.
      RAD=(CORR/180.)*3.1415
       WIND=TAS-COS(RAD)*WNDSPD(I)
      GO TO 313
  311 CORR=CORR-90.
      RAD=(CORR/180.)*3.1416
       WIND=TAS-SIN(RAD)*WNDSPD(I)
      GO TO 313
  330 RAD=(CORR/180.)*3.1416
       WIND=TAS+COS(RAD)*WNDSPD(I)
      GO TO 313
  340 CORR=CORR-270.
      RAD=(CORR/180.)*3.1416
       WIND=TAS+SIN(RAD)*WNDSPD(I)
  313 CONTINUE
      RETURN
      END


      FUNCTION CTAS(CAS,I)
C
C     THIS CONVERTS CAS INTO TAS
C
      ALT=I
      IF(ALT.LE. 5.) GO TO 410
      IF(ALT.LE.10.) GO TO 411
      IF(ALT.LE.15.) GO TO 412
      IF(ALT.LE.20.) GO TO 413
      IF(ALT.LE.25.) GO TO 414
      IF(ALT.LE.30.) GO TO 415
      IF(ALT.LE.35.) GO TO 416
      IF(ALT.LE.40.) GO TO 417
      GO TO 417
  410 SIGMA=1.+(ALT/5.)*.0773
      GO TO 499
  411 SIGMA=1.0773+((ALT-5 )/5.)*.0864
      GO TO 499
  412 SIGMA=1.1637+((ALT-10.)/5.)*.0962
      GO TO 499
  413 SIGMA=1.2602+((ALT-15.)/5.)*.1094
      GO TO 499
  414 SIGMA=1.37+((ALT-20.)/5.)*.1238
      GO TO 499
  415 SIGMA=1.4938+((ALT-25 )/5.)*.1411
      GO TO 499
  416 SIGMA=1.6349+((ALT-30.)/5.)*.1615
      GO TO 499
  417 SIGMA=1.7964+((ALT-35.)/5.)*.2191
  499 CTAS=CAS*SIGMA
      RETURN
      END
```

```
      FUNCTION CLFUEL(WT,I)
C
C     COMPUTE THE FUEL USED IN THE CLIMB
C
      ALT=I
      IF(WT.LE.12500.) GO TO 61
      IF(WT.LE.13500.) GO TO 62
      IF(WT.LE.14500.) GO TO 63
      IF(WT.LE.15500.) GO TO 64
      CLFUEL=760.-((40.-ALT)/40.)*760.
      GO TO 69
   61 CLFUEL=500.-((40.-ALT)/40.)*500.
      GO TO 69
   62 CLFUEL=560.-((40 -ALT)/40.)*560.
      GO TO 69
   63 CLFUEL=620.-((40 -ALT)/40.)*620.
      GO TO 69
   64 CLFUEL=700.-((40.-ALT)/40.)*700.
   69 IF(I.LT.10) GO TO 691
      IF(I.LE.20) GO TO 693
      IF(I.LT.30) GO TO 692
      CLFUEL=CLFUEL+(ALT/40.)*600.
      GO TO 691
  692 CLFUEL=CLFUEL+(ALT/40.)*400.
      GO TO 691
  693 CLFUEL=CLFUEL+(ALT/40.)*200.
  691 CONTINUE
      RETURN
      END
```

```
      FUNCTION DDIST(I)
C
C     DISTANCE COVERED DURING IDLE DESCENT
C
      ALT=I
      IF(ALT.LE. 5.) GO TO 41
      IF(ALT.LE.10.) GO TO 42
      IF(ALT.LE.15.) GO TO 43
      IF(ALT.LE.20.) GO TO 44
      IF(ALT.LE.25.) GO TO 45
      IF(ALT.LE.30.) GO TO 46
      IF(ALT.LE.35.) GO TO 47
      DDIST=80.+((ALT-35.)/5.)*20.
      GO TO 48
   41 DDIST=((ALT-1.)/4.)*6
      GO TO 48
   42 DDIST=6.+((ALT-5.)/5 )*8.
      GO TO 48
   43 DDIST=14.+((ALT-10.)/5.)*9.
      GO TO 48
   44 DDIST=23.+((ALT-15.)/5.)*10.
      GO TO 48
   45 DDIST=33.+((ALT-20.)/5.)*11.
      GO TO 48
   46 DDIST=44.+((ALT-25.)/5.)*18.
      GO TO 48
   47 DDIST=62.+((ALT-30.)/5.)*18.
   48 CONTINUE
      RETURN
      END
```

```
      FUNCTION DSFUEL(WT,I)
C
C     COMPUTE THE FUEL USED IN THE DESCENT
C
      ALT=I
      IF(WT.LE.12000.) GO TO 771
      IF(WT.LE.13000.) GO TO 772
      IF(WT.LE.14000.) GO TO 773
      IF(WT.LE.15000.) GO TO 774
      IF(WT.LE.16000.) GO TO 775
      DSFUEL=210.*(ALT/40.)
      GO TO 777
  771 DSFUEL=280.*(ALT/40.)
      GO TO 777
  772 DSFUEL=260.*(ALT/40.)
      GO TO 777
  773 DSFUEL=240.*(ALT/40.)
      GO TO 777
  774 DSFUEL=225.*(ALT/40.)
      GO TO 777
  775 DSFUEL=215.*(ALT/40.)
  777 CONTINUE
      RETURN
      END
```

```
      FUNCTION HRSPD(WT,I)
C
C     HORIZONTAL SPEED SCHEDULE
C
      ALT=I
      IF(WT.LE.14000.) GO TO 551
      IF(WT.LE.15000.) GO TO 552
      IF(WT.LE.16000.) GO TO 553
      GO TO 557
  551 IF(ALT.LE.30.) GO TO 5511
      HRSPD=220.+((40.-ALT)/10.)*10.
      GO TO 557
 5511 HRSPD=230.+((30.-ALT)/30.)*60.
      GO TO 557
  552 IF(ALT.LE.25.) GO TO 5521
      HRSPD=240.+((40.-ALT)/15.)*15.
      GO TO 557
 5521 HRSPD=255.+((25.-ALT)/25.)*50.
      GO TO 557
  553 IF(ALT.LE.25.) GO TO 5531
      HRSPD=245.+((40.-ALT)/15.)*15.
      GO TO 557
 5531 HRSPD=260.+((25.-ALT)/25.)*50.
  557 CONTINUE
      RETURN
      END
```

```
      FUNCTION AVWIND(I,MN)
      DIMENSION WNDCRS(40),WNDSPD(40)
      COMMON WNDCRS,WNDSPD,CRS
C
C     COMPUTE THE AVERAGE WIND IN CLIMB AND DESCENT
C
      AVWIND=0.
      DO 444 N=MN,I
      IF(WNDCRS(N).GE.180.) GO TO 404
      CHECK=WNDCRS(N)+180.
      GO TO 405
  404 CHECK=WNDCRS(N)-180.
  405 CONTINUE
      DIFF=CRS-CHECK
      CORR=ABS(DIFF)
      IF(CORR.LT.90.) GO TO 407
      IF(CORR.GT.270.) GO TO 408
      IF(CORR.GE.180.) GO TO 409
      GO TO 410
  409 CORR=CORR-180.
      RAD=(CORR/180.)*3.1416
      ADD=-COS(RAD)*WNDSPD(N)
      GO TO 444
  410 CORR=CORR-90.
      RAD=(CORR/180.)*3.1416
      ADD=-SIN(RAD)*WNDSPD(N)
      GO TO 444
  407 RAD=(CORR/180.)*3.1416
      ADD=COS(RAD)*WNDSPD(N)
      GO TO 444
  408 CORR=CORR-270.
      RAD=(CORR/180.)*3.1416
      ADD=SIN(RAD)*WNDSPD(N)
  444 AVWIND=AVWIND+ADD
      AI=I
      AVWIND=AVWIND/AI
      RETURN
      END
```

BURN	CLIMB	HORIZ	DESC	ALT
1520.3477	0.0	287.9998	0.0	1
1436.8254	334.7358	285.9998	198.0575	2
1388.4229	333.6038	283.9998	197.9612	3
1344.5466	332.4717	281.9998	197.8650	4
1304.7539	331.3396	279.9998	197.7687	5
1228.1814	330.2075	277.9998	197.6725	6
1195.5281	329.0754	275.9998	197.5762	7
1165.8674	327.9434	273.9998	197.4800	8
1138.9519	326.8113	271.9998	197.3837	9
1164.0527	325.6792	269.9998	196.9437	10
1259.1255	324.5471	267.9998	196.8131	11
1236.3667	323.4150	265.9998	196.6825	12
1216.0037	322.2830	263.9998	196.5519	13
1198.0518	321.1509	261.9998	196.4212	14
1181.8977	320.0188	260.0000	196.2906	15
1048.8237	318.8867	257.9998	196.1600	16
1039.0364	317.7546	256.0000	196.0294	17
1030.6504	316.6226	254.0000	195.8987	18
1023.5820	315.4905	252.0000	195.7681	19
1017.7534	314.3584	250.0000	195.6375	20
1356.7603	313.2263	248.0000	194.7850	21
1428.3113	312.0942	246.0000	194.6200	22
1447.1089	310.9622	244.0000	194.4550	23
1427.0532	309.8301	242.0000	194.2900	24
1410.2793	308.6980	240.0000	194.1250	25
1208.3538	307.5659	238.0000	193.9600	26
1199.5144	304.0740	236.0000	193.7950	27
1194.4165	298.2222	234.0000	193.6300	28
1191.4863	292.3704	232.0000	193.4650	29
1340.2283	286.5183	230.0000	192.2687	30
1349.7646	280.6665	229.0000	192.0694	31
1354.3699	274.8147	228.0000	191.8700	32
1361.1338	268.9629	227.0000	191.6706	33
1369.8821	263.1111	226.0000	191.4712	34
1380.4607	257.2590	225.0000	191.2719	35
1388.2974	251.4074	224.0000	191.0725	36
1398.0271	245.5556	223.0000	190.8731	37
1409.5244	239.7037	222.0000	190.6737	38
1422.6638	233.8519	221.0000	190.4744	39
1437.3345	228.0000	220.0000	190.2750	40

OPTIMUM ALTITUDE IS 20000.0 FEET

CLIMB SPEED IS 314.4 KNOTS

CRUISE SPEED IS 250.0 KNOTS

START DESCENT WHEN 34.4 OUT

DESCENT SPEED IS 195.6 KNOTS

FUEL USED WILL BE 1017.8 POUNDS

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ABSTRACT

There are many factors, such as aircraft configuration and weight, winds aloft, airspeeds flown, altitude, distance, etc., which affect fuel consumption in turbo-jet aircraft. For any given combination of these factors a flight path can be determined that will result in the least fuel consumed for a ground distance covered. Under divert conditions from aircraft carriers at sea to fields ashore the choice of the optimal flight path is critical. The many possible combinations of factors lead to the adoption of computer flight planning. Pilots can avail themselves of computer solutions during flight planning and briefing sessions, and after take-off can receive further information via UHF radio. Typical flight handbooks display fuel flow data, etc. in such a manner that the pilot must "guesstimate" entry parameters such as average horizontal weight, or weight prior to descent. Several iterative procedures are developed that provide exact solutions to these important figures. Thus the computer flight planning system will provide more accurate solutions, and free the pilot from this chore so that he may better spend his time briefing tactics.

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
COMPUTER FLIGHT PLANNING						
COMPUTER SIMULATION						
OPTIMAL FLIGHT PROFILES						