NONSIMILAR SOLUTION OF THE LAMINAR BOUNDARY LAYER IN AN OSCILLATORY FLOW BY AN INTEGRAL MATRIX METHOD

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THESIS

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ABSTRACT

The development of a numerical procedure for the treatment of nonsimilar, unsteady, laminar boundary layers is presented. The solution is obtained from the laminar, isothermal, incompressible boundary layer equations employing a modification of the integral matrix procedure of Bartlett and Kendall. Solutions of example problems are presented for steady Blasius and Howarth flows, and for oscillating Blasius flow. Agreement with the known classical results is satisfactory and establishes the general feasibility of the method. Core storage requirements of 130,000 bytes allow consideration of as many as 25 nodal points across the boundary layer, 50 time increments per oscillation cycle and 50 streamwise stations. Solution of oscillating Blasius flow considering 8 nodal points and 16 time increments requires 13.49 seconds for one streamwise station utilizing IBM 360/67 time sharing capabilities.

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TABLE OF SYMBOLS

b_1	Rate of change of the mean free stream velocity with distance from the leading edge
d_0, d_1, d_2	Coefficients defined in the finite difference representation of streamwise derivatives (defined in equations (33) and (34) for two and three-point difference relations)
f	Stream function (defined by equation (14))
P	Pressure
r_o	Local radius of the body in a meridian plane for an axisymmetric shape
s	Distance along the body from a stagnation point or the leading edge
T_0, T_1, T_2	Coefficients defined in the finite difference representation of time derivatives (defined in equations (37) and (38) for two and three-point difference relations)
t	Time
U_o	Mean free stream velocity at the leading edge
U	s velocity component outside the boundary layer
u	Velocity component parallel to the body surface
v	Velocity component normal to the body surface
$XM_1, XM_3, ...$	Truncated series obtained in Taylor series expansion of $\int_{i-1}^{i} f'^2 d\eta$ (defined by equation (A2))
y	Distance from the surface into the boundary layer measured normal to the surface
$ZM_1, ZM_2, ...$	Truncated series obtained in Taylor series expansion of integrals involving nonsimilar terms (defined by equation (A7))

α^*	Normalizing parameter (defined by equation (16))
β	Streamwise pressure gradient parameter (defined by equation (22))
${}_k\Delta_{k-1}$	Time difference between two time grid points denoted by the subscripts k and k-1 (defined by equation (39))
${}_\ell\Delta_{\ell-1}$	Logarithmic distance between two streamwise positions denoted by the subscripts ℓ and ℓ-1 (defined by equation (35))
$\Delta f_i, \Delta f_i', \ldots$	Corrections for $f_i, f_i', \ldots$ during Newton-Raphson iterations
$\delta\eta$	Distance between two boundary layer nodal points (defined by equation (30))
δt	Time difference between two time grid points (defined by equation (42))
ε	Ratio of the amplitude of the free stream oscillation to the local mean free stream velocity
η	Transformed coordinate in a direction normal to the surface (defined by equation (10))
μ	Shear viscosity
ξ	Transformed streamwise coordinate (defined by equation (9))
ρ	Density
ω	Frequency of the free stream oscillation

SUBSCRIPTS

e	Pertains to the boundary layer edge
i	Pertains to the i^{th} nodal point in the boundary layer, starting with i=1 at the surface
k	Pertains to the k^{th} time grid in the oscillation cycle
ℓ	Pertains to the ℓ^{th} streamwise position

m	Pertains to the m^{th} iteration during the Newton-Raphson iteration
w	Pertains to the wall or surface of the body

SUPERSCRIPTS

n	Equal to unity for axisymmetric bodies and zero for two-dimensional bodies
' (prime)	Represents partial differentiation with respect to η

MATRIX NOTATION

$[A]$	Coefficients of corrections matrix in Newton-Raphson recurrence formulas prior to any matrix reduction (defined by equation (60))
$[AL]$	Coefficients of "linear" corrections in Newton-Raphson recurrence formulas corresponding to linear equations prior to any matrix reduction (defined by equation (61))
$[ANL]$	Coefficients of "linear" corrections in Newton-Raphson recurrence formulas corresponding to nonlinear equations prior to any matrix reduction (defined by equation (61))
$[BA1]$	Product of $[AL]^{-1}[BL]$ (defined in equation (66))
$[BL]$	Coefficients of "nonlinear" corrections in Newton-Raphson recurrence formulas corresponding to linear equations prior to any matrix reduction (defined by equation (61))
$[BNL]$	Coefficients of "nonlinear" corrections in Newton-Raphson recurrence formulas corresponding to nonlinear equations prior to any matrix reduction (defined by equation (61))
$[\overline{BNL}]$	Coefficients of "nonlinear" corrections in Newton-Raphson recurrence formulas corresponding to nonlinear equations after matrix reduction (defined in equation (69))
$[E]$	Matrix of errors in Newton-Raphson recurrence formulas prior to any matrix reduction (defined in equation (60))
$[EL]$	Matrix of errors in Newton-Raphson recurrence formulas corresponding to linear equations prior to any matrix reduction (defined by equation (61))

$[ELA]$ Product of $[AL]^{-1}[-EL]$ (defined in equation (67))

$[ENL]$ Matrix of errors in Newton-Raphson recurrence formulas corresponding to nonlinear equations prior to any matrix reduction (defined by equation (61))

$[\overline{ENL}]$ Matrix of errors in Newton-Raphson recurrence formulas corresponding to nonlinear equations after matrix reduction (defined by equation (70))

$[\Delta V]$ Matrix of corrections to primary variables in Newton-Raphson recurrence formulas (defined by equation (60))

$[\Delta VL]$ Matrix of corrections to "linear" primary variables in Newton-Raphson recurrence formulas (defined by equation (61))

$[\Delta VNL]$ Matrix of currections to "nonlinear" primary variables in Newton-Raphson recurrence formulas (defined by equation (61))

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I. INTRODUCTION

A. GENERAL

All aerodynamic flows are to some degree influenced by unsteadiness. Rotating stall in turbomachinery, aerodynamic stall flutter and hydrofoil flow fields are but a few practical problems in which the nonsteady fluid flow phenomena are of prime importance. Additionally, the recent emphasis on the development of rotary wing aircraft has created a new demand for knowledge of the behavior of boundary layers in unsteady flow.

B. ANALYTICAL HISTORY

Early analytical treatments by Stokes, Rayleigh and Schlichting (Ref. 1) considered only a special case of the general problem: that of unsteady viscous fluid flow with no mean flow or pressure gradient. Lighthill (Ref. 2) made the first significant investigation of small sinusoidal oscillations superimposed on a mean flow. Lighthill's analysis was based on a small perturbation treatment of the boundary layer equations for sinusoidal flow, with only the first order terms retained. The determination of the existance of the "quasi-steady" regime at low frequencies, and the "shear wave" regime at higher frequencies was the principal result of this analysis. Glauert (Ref. 3) extended Lighthill's work by considering the boundary layer in the vicinity of a stagnation point. Carrier and DiPrima (Ref. 4), using a system of equations derived using a modified Oseen linearization, confirmed the existance of the "shear wave" solution and the phase advance of the wall shear determined by Lighthill. Nickerson (Ref. 5) initially retained perturbation terms beyond the first order, thus widening the scope of Lighthill's work.

In addition to his analytical development, Nickerson made hot-wire measurements of the laminar boundary layer on a flat plate oscillating in a Blasius mean flow. His work partially confirmed both his own and Lighthill's analyses, but mechanical difficulties precluded the completion of the experimental program. Hill and Stenning (Ref. 6) imposed sinusoidal oscillations directly on mean Blasius and Howarth flows and were able to confirm the analytical results of both Lighthill and Nickerson. Recently an integral analysis of the unsteady boundary layer was introduced by Koob and Abbott (Ref. 7), who succeeded in demonstrating the method for the case of an accelerating flat plate.

C. APPLICATION OF NUMERICAL METHODS

With the advent of the high-speed digital computer, it has become possible to apply numerical methods to the analytical studies of viscous fluid flows. Fromm (Ref. 8) and Chorin (Ref. 9) introduced modified finite difference methods for the treatment of viscous flows which could feasibly be applied to the boundary layer problem. Smith and Clutter (Ref. 10) and Kleinstein and Nabi (Refs. 11 and

12) applied finite difference methods to the steady boundary layer equations. Successive approximation, linearization and integral methods have also been applied to the boundary layer equations (Refs. 13, 14 and 15). A major drawback of most of the methods mentioned is that in order to reach a solution to a given problem large computing times are necessary.

The introduction of the Boundary Layer Integral Matrix Procedure by Bartlett and Kendall (Ref. 16) provided a major breakthrough in the field of fast computer solution of the steady flow boundary layer problem. The method was developed as a means of obtaining rapid computations in the laminar regime for steady, multicomponent, chemically reacting boundary layers; and the logical extension of the technique was to encompass the unsteady flow condition.

The purpose of this investigation is to develop a computer program capable of solving the unsteady, laminar boundary layer problem by adapting the rapid solution techniques of the Boundary Layer Integral Matrix Procedure.

II. ANALYSIS

A. GOVERNING EQUATIONS

In this development, utilizing the integral matrix method, it is convenient to make the assumption of incompressible isothermal flow. The resulting laminar boundary layer equations for two-dimensional (n=0) or axisymmetric (n=1) flow are as follows:

Continuity:

$$\frac{1}{r_o^{\,n}} \frac{\partial (r_o^{\,n} u)}{\partial s} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial s} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial s} + \frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial t} = -\frac{1}{\rho}\frac{\partial P}{\partial s} \tag{3}$$

where s and y are respectively the streamwise and normal components, u and v are the corresponding velocity components, U is the streamwise velocity component outside the boundary layer, r_o is the radius of the body perpendicular to the axis of revolution for an axisymmetric body, n is zero for a two-dimensional body and unity for an axisymmetric body, t is time, ρ is density, μ is viscosity and P is pressure.

B. BOUNDARY CONDITIONS

The boundary conditions considered for the governing equations as stated are:

$$u_w = 0 \tag{4}$$

$$v_w = 0 \tag{5}$$

$$u_e = U(s,t) \tag{6}$$

$$\frac{\partial u_e}{\partial y} = 0 \tag{7}$$

where the subscripts w and e refer respectively to wall and boundary layer edge conditions. Equation (4) is the requirement of zero slip at the wall, equation (5) implies no mass transfer at the wall, and equations (6) and (7) are representative of the assumption that the boundary layer edge velocity is equal to the free stream velocity.

The free stream velocity may be written:

$$U = (U_o - b_1 s)(1 + \varepsilon \cos \omega t) \tag{8}$$

where U_o is the mean free stream velocity at the leading edge, b_1 is the rate of change of the mean free stream velocity with distance from the leading edge, ε is the ratio of the amplitude of the free stream oscillation to the local mean free stream velocity and ω is the frequency of the oscillation.

C. TRANSFORMATION OF EQUATIONS

Paralleling the development of Bartlett and Kendall (Ref. 16) a transformation of variables is performed utilizing a combination of the Levy and Mangler and the Howarth-Dorodnitsyn transformations which is known by a variety of names including Levy-Lees, Lees-Dorodnitsyn and Mangler-Dorodnitsyn. This transformation is as follows:

$$\xi = \int_o^s u_e \rho_e \mu_e r_o^{2n} ds \tag{9}$$

$$\eta = \frac{r_o^n u_e}{\sqrt{2\xi}} \int_o^y \rho \, dy \tag{10}$$

Simplifying equations (9) and (10) to encompass the assumptions of incompressibility and constant temperature the transformations become:

$$\xi = \rho\mu \int_o^s u_e r_o^{2n} \, ds \tag{11}$$

$$\eta = \frac{r_o^n \rho u_e}{\sqrt{2\xi}} y \tag{12}$$

The transformation to Levy-Lees variables is greatly simplified by use of the operator:

$$u\frac{\partial(\ )}{\partial s} + v\frac{\partial(\ )}{\partial s} = \alpha^* u_e r_o^n \sqrt{2\xi} \left\{ f'\frac{\partial(\ )}{\partial \xi} - \left(\frac{f}{2\xi} + \frac{\partial f}{\partial \xi}\right)\frac{\partial(\ )}{\partial \eta} \right\} \tag{13}$$

where f is the stream function defined as:

$$f - f_w = \int_o^{\eta} \frac{u}{u_e} d\eta \qquad (14)$$

the prime represents partial differentiation with respect to η so that:

$$f' = \frac{u}{u_e} \qquad (15)$$

and α^* is a normalizing parameter:

$$\alpha^* = \frac{\rho \mu u_e r_o^{\,n}}{\sqrt{2\xi}} \qquad (16)$$

In addition, the partial derivatives are:

$$\frac{\partial(\ )}{\partial s} = \alpha^* r_o^{\,n} \sqrt{2\xi} \left\{ \frac{\partial(\ )}{\partial \xi} + \frac{\partial \eta}{\partial \xi} \frac{\partial(\ )}{\partial \eta} \right\} \qquad (17)$$

$$\frac{\partial(\ )}{\partial y} = \frac{\alpha^*}{\mu} (\ )' \qquad (18)$$

$$\frac{\partial^2(\ )}{\partial y^2} = \left(\frac{\alpha^*}{\mu}\right)^2 (\ )'' \qquad (19)$$

Applying equation (15) to the velocity in the partial derivative with respect to time yields:

$$\frac{\partial u}{\partial t} = u_e \frac{\partial f'}{\partial t} + f' \frac{\partial u_e}{\partial t} \qquad (20)$$

Using equations (13) through (20) and applying equation (6) to equation (3) results in the following transformed momentum equation:

$$f''' + ff'' + \beta(1 - f'^2)$$

$$-2\,\{f'\,\frac{\partial f'}{\partial \ell n\xi} - f''\frac{\partial f}{\partial \ell n\xi}\} - \frac{\rho\mu}{\alpha^{*2}}\,\{\frac{\partial f'}{\partial t} + (f' - 1)\frac{\partial \ell n u_e}{\partial t}\} = 0 \quad (21)$$

where

$$\beta = 2\,\frac{\partial \ell n u_e}{\partial \ell n \xi} \quad (22)$$

Transforming the boundary conditions, equations (4) through (7) become:

$$f_w' = 0 \quad (23)$$

$$f_w = 0 \quad (24)$$

$$f_e' = 1 \quad (25)$$

$$f_e'' = 0 \quad (26)$$

D. TAYLOR SERIES EXPANSIONS

At any streamwise station s, the boundary layer may be divided into N-1 strips which are joined by N nodal points η_i, where i is 1 at the wall and N at the boundary layer edge. The primary dependent variables f_i and their normal derivatives are related by Taylor series expansions such that the f_i' are represented by connected cubics with continuous first and second derivatives at the nodal points. This is commonly called a spline fit.

Expanding the primary variables in Taylor series and truncating at f_i''' the following linear equations result:

$$-f_{i+1} + f_i + f_i' \delta\eta + f_i'' \frac{\delta\eta^2}{2} + f_i''' \frac{\delta\eta^3}{8} + f_{i+1}''' \frac{\delta\eta^3}{24} = 0 \quad (27)$$

$$-f_{i+1}' + f_i' + f_i'' \delta\eta + f_i''' \frac{\delta\eta^3}{3} + f_{i+1}''' \frac{\delta\eta^2}{6} = 0 \quad (28)$$

$$-f_{i+1}'' + f_i'' + f_i''' \frac{\delta\eta}{2} + f_{i+1}''' \frac{\delta\eta}{2} = 0 \quad (29)$$

$$\delta\eta = \eta_{i+1} - \eta_i \quad (30)$$

$$_i f_{i+1}'''' = \frac{f_{i+1}''' - f_i'''}{\delta\eta} \quad (31)$$

Equations (27) through (29) represent a set of 3(N-1) equations where f is represented as a quartic, f' as a cubic, f'' as a quadratic and f''' is considered to vary linearly between η_i and η_{i+1}.

E. AXIAL AND TIME DERIVATIVES

The axial derivatives of equation (21) are treated as logarithmic two or three-point differences such that:

$$2 \left[\frac{\partial(\ )}{\partial \ell n \xi}\right]_{\ell} = d_o(\ )_{\ell} + d_1(\ )_{\ell-1} + d_2(\ )_{\ell-2} \quad (32)$$

where for two-point differences:

$$d_o = \frac{2}{_{\ell}\Delta_{\ell-1}} \qquad d_1 = -d_o \qquad d_2 = 0 \quad (33)$$

and for three-point differences:

$$d_o = 2\frac{{}_{\ell}\Delta_{\ell-1} + {}_{\ell}\Delta_{\ell-2}}{{}_{\ell}\Delta_{\ell-1}\ {}_{\ell}\Delta_{\ell-2}} \qquad d_1 = -2\frac{{}_{\ell}\Delta_{\ell-2}}{{}_{\ell}\Delta_{\ell-1}\ {}_{\ell-1}\Delta_{\ell-2}} \qquad d_2 = 2\frac{{}_{\ell}\Delta_{\ell-1}}{{}_{\ell}\Delta_{\ell-2}\ {}_{\ell-1}\Delta_{\ell-2}} \tag{34}$$

In equations (32) through (34), $(\ )_{\ell-1}$ implies the previous streamwise station.

$$_{\ell}\Delta_{\ell-1} = \ell n\xi_{\ell} - \ell n\xi_{\ell-1} = \ell n(\xi_{\ell}/\xi_{\ell-1}) \tag{35}$$

Differencing the time derivatives in the same way:

$$\left[\frac{\partial(\ )}{\partial t}\right]_k = T_o(\ )_k + T_1(\ )_{k-1} + T_2(\ )_{k-2} \tag{36}$$

where for two-point differences:

$$T_o = \frac{1}{_k\Delta_{k-1}} \qquad T_1 = -T_o \qquad T_2 = 0 \tag{37}$$

and for three-point differences:

$$T_o = \frac{_k\Delta_{k-1} + {}_k\Delta_{k-2}}{_k\Delta_{k-1}\ {}_k\Delta_{k-2}} \qquad T_1 = -\frac{_k\Delta_{k-2}}{_k\Delta_{k-1}\ {}_{k-1}\Delta_{k-2}} \qquad T_2 = \frac{_k\Delta_{k-1}}{_k\Delta_{k-2}\ {}_{k-1}\Delta_{k-2}} \tag{38}$$

In equations (36) through (38), $(\ )_{k-1}$ implies the previous time.

$$_k\Delta_{k-1} = t_k - t_{k-1} \tag{39}$$

Equations (37) through (39) may be greatly simplified by considering the utilization of equal time spacings. The resulting equations for the treatment of time derivatives over equal time spacings are as follows:

Two point differences:

$$T_o = \frac{1}{\delta t} \qquad T_1 = -T_o \qquad T_2 = 0 \tag{40}$$

Three-point differences:

$$T_o = \frac{3}{2\delta t} \qquad T_1 = -\frac{2}{\delta t} \qquad T_2 = \frac{1}{2\delta t} \tag{41}$$

where:

$$\delta t = t_k - t_{k-1} = t_{k-1} - t_{k-2} \tag{42}$$

F. INTEGRATION OF THE BOUNDARY LAYER EQUATION

The transformed streamwise momentum equation, equation (21), is integrated at constant ξ from η_{i-1} to η_i, yielding:

$$\Big[f'' \Big]_{i-1}^{i} + \int_{i-1}^{i} ff'' d\eta + \beta \int_{i-1}^{i} (1 - f'^2)\, d\eta$$

$$-2 \int_{i-1}^{i} \left(f' \frac{\partial f'}{\partial \ell n \xi} - f'' \frac{\partial f}{\partial \ell n \xi}\right) d\eta - \frac{\rho\mu}{\alpha^{*2}} \int_{i-1}^{i} \frac{\partial f'}{\partial t}\, d\eta$$

$$- \frac{\rho\mu}{\alpha^{*2}} \int_{i=1}^{i} (f' - 1) \frac{\partial \ell n u_e}{\partial t}\, d\eta = 0 \tag{43}$$

Expanding equation (43) in Taylor series and applying equation (36) to the time derivative terms:

$$\int_{i-1}^{i} \frac{\partial f'}{\partial t} d\eta = \left[\frac{\partial f}{\partial t} \right]_{i-1}^{i}$$

$$= [T_o f_k + T_1 f_{k-1} + T_2 f_{k-2}] \quad (44)$$

$$\int_{i-1}^{i} (f' - 1) \frac{\partial \ell n u_e}{\partial t} d\eta = \frac{\partial \ell n u_e}{\partial t} \left\{ \left[f \right]_{i-1}^{i} - \delta\eta \right\}$$

$$= \left\{ \left[f \right]_{i-1}^{i} - \delta\eta \right\} [T_o \ell n u_{e_k} + T_1 \ell n u_{e_{k-1}} + T_2 \ell n u_{e_{k-2}}] \quad (45)$$

the integrated boundary layer equation becomes:

$$\Big[f'' + f'(1 + d_o) f + (d_1 f_{\ell-1} + d_2 f_{\ell-2}) f' - \frac{\rho\mu}{\alpha^{*2}} \{ [T_o f_k + T_1 f_{k-1} + T_2 f_{k-2}]$$

$$- f [T_o \ell n u_{e_k} + T_1 \ell n u_{e_{k-1}} + T_2 \ell n u_{e_{k-2}}] \Big]_{i-1}^{i}$$

$$- (1 + \beta + 2d_o) [f_i' XM_1 + f_i'' XM_2 + f_i''' XM_3 + f_{i-1}''' XM_4]$$

$$-2 [f_i' ZM_1 + f_i'' ZM_2 + f_i''' ZM_3 + f_{i-1}''' ZM_4]$$

$$+ \delta\eta \{ \beta + \frac{\rho\mu}{\alpha^{*2}} [T_o \ell n u_{e_k} + T_1 \ell n u_{e_{k-1}} + T_2 \ell n u_{e_{k-2}}] \} = 0 \quad (46)$$

The Taylor series expansions, XM and ZM, of the integrals in equation (43) are listed in Appendix A.

G. RECURRENCE FORMULAS

The boundary layer equations, equation (46), and the Taylor series expansions of the primary variables, equations (27) through (29), together with the boundary conditions, equations (23) through (26), are solved by Newton-Raphson iteration of 4N simultaneous equations in 4N primary variable unknowns. These are summarized as follows:

Primary Variables	No. of Equations
f	N
f'	N
f''	N
f'''	N
No. of Eqns. Req:	4N

	Equation Nos.	No. of Equations
Taylor Series Expansions	(27)-(29)	3(N-1)
Boundary Layer Equations	(46)	N-1
Boundary Conditions	(34)-(26)	4
No. of Eqns. Available:		4N

The Newton-Raphson solution technique is illustrated by considering two simultaneous nonlinear equations:

$$F(x,y) = 0 \qquad G(x,y) = 0 \tag{47}$$

which have the solution x=X and y=Y. Defining x_m and y_m as the values of x and y for the m^{th} iteration, the desired solution f(X,Y) is expressed in the Taylor series expansions:

$$0 = F(X,Y) = F(x_m,y_m) + (X - x_m)\frac{\partial F(x_m,y_m)}{\partial x} + (Y - y_m)\frac{\partial F(x_m,y_m)}{\partial y} + \ldots$$

$$0 = G(X,Y) = G(x_m,y_m) + (X - x_m)\frac{\partial G(x_m,y_m)}{\partial x} + (Y - y_m)\frac{\partial G(x_m,y_m)}{\partial y} + \ldots \tag{48}$$

In the Newton-Raphson method (X,Y) is replaced by (x_{m+1}, y_{m+1}) and the nonlinear terms in x and y appearing in equations (48) are ignored yielding the set of recurrence formulas:

$$\Delta x_m \frac{\partial F(x_m, y_m)}{\partial x} + \Delta y_m \frac{\partial F(x_m, y_m)}{\partial y} = -F(x_m, y_m)$$

$$\Delta x_m \frac{\partial G(x_m, y_m)}{\partial x} + \Delta y_m \frac{\partial G(x_m, y_m)}{\partial y} = -G(x_m, y_m) \qquad (49)$$

where:

$$\Delta x_m = x_{m+1} - x_m \qquad \Delta y_m = y_{m+1} - y_m \qquad (50)$$

The Δx_m and Δy_m are corrections to be added to x_m and y_m respectively yielding x_{m+1} and y_{m+1}. $F(x_m, y_m)$ and $G(x_m, y_m)$ are the values of the original functions F(x,y) and G(x,y) evaluated for the value of the variables during the m^{th} iteration. Noting that $F(x_m, y_m)$ and $G(x_m, y_m)$ approach zero as the corrections approach zero, it is appropriate to consider them as errors associated with the original equations (47).

Differentiating the Taylor series expansions, equations (27) through (29), with respect to the primary variables in accordance with the derivation of equation (49) yields the following recurrence formulas for the m^{th} iteration:

$$(-1)\Delta f_{i+1} + (1)\Delta f_i + (\delta\eta)\Delta f_i' + \left(\frac{\delta\eta^2}{2}\right)\Delta f_i'' + \left(\frac{\delta\eta^3}{8}\right)\Delta f_i''' + \left(\frac{\delta\eta^3}{24}\right)\Delta f_{i+1}''' = -\,\text{ERROR} \qquad (51)$$

$$(-1)\Delta f'_{i+1}+(1)\Delta f'_i+(\delta\eta)\Delta f''_i+(\frac{\delta\eta^2}{3})\Delta f'''_i+(\frac{\delta\eta^2}{6})\Delta f'''_{i+1}=-\text{ERROR} \quad (52)$$

$$(-1)\Delta f''_{i+1}+(1)\Delta f''_i + (\frac{\delta\eta}{2})\Delta f'''_i + (\frac{\delta\eta}{2})\Delta f'''_{i+1} = -\text{ERROR} \quad (53)$$

where Δf_i represents the correction for f_i, the coefficients of the corrections represent the partial derivatives of the Taylor series expansions with respect to the primary variables, and the ERRORs are equations (29) through (31) respectively, evaluated for the values of the variables during the iteration.

The recurrence formulas for the boundary layer equations are:

$$\Big[\Delta f'' + \Delta f'(1+d_o)f + f'(1+d_o)\Delta f + \Delta f'(d_1 f_{\ell-1} + d_2 f_{\ell-2})$$

$$-\frac{\rho\mu}{\alpha^{*2}}\{T_o\Delta f - \Delta f(T_o \ell n u_{e_k} + T_1 \ell n u_{e_{k-1}} + T_2 \ell n u_{e_{k-2}})\}\Big]_{i-1}^{i}$$

$$-2(1+\beta+2d_o)\{\Delta f'_i XM_1+\Delta f''_i XM_2 + \Delta f'''_i XM_3 + \Delta f'''_{i-1} XM_4\}$$

$$-2\{\Delta f'_i ZM_1 + \Delta f''_i ZM_2 + \Delta f'''_i ZM_3 + \Delta f_{i-1} ZM_4\} = -\text{ERROR} \quad (54)$$

where ERROR is the left-hand-side of equation (46) evaluated for the values of the variables during the iteration.

Finally, the recurrence formulas for the boundary conditions, equations (23) through (26), are:

$$\Delta f'_w = -\text{ ERROR } = -(f'_w)_m \quad (55)$$

$$\Delta f_w = -\text{ ERROR } = -(f_w)_m \quad (56)$$

$$\Delta f_e' = -\text{ ERROR} = 1 - (f_e')_m \quad (57)$$

$$\Delta f_e'' = -\text{ ERROR} = -(f_e'')_m \quad (58)$$

Equation (54) may be further refined by collecting terms and writing the equation in terms of correction coefficients as follows:

$$C1\Delta f_i + C2\Delta f_{i-1} + C3\Delta f_i' + C4\Delta f_{i-1}'$$
$$+ C5\Delta f_i'' + C6\Delta f_{i-1}'' + C7\Delta f_i''' + C8\Delta f_{i-1}''' = -\text{ ERROR} \quad (59)$$

The correction coefficients C1 through C8 are given in Appendix B.

H. MATRIX FORMULATION

The coefficients for the recurrence formulas, equations (51) through (53) and (55) through (59), evaluated for the m^{th} iteration form a square matrix $\left[A\right]$ with 4N rows and columns. The matrix equation is:

$$\left[A\right]\left[\Delta V\right] = -\left[E\right] \quad (60)$$

where the column matrix $\left[\Delta V\right]$ is the matrix of corrections to the primary variables, and the column matrix $\left[E\right]$ is the matrix of errors. The matrix $\left[A\right]$ contains many zeros, and great savings in computer storage space and computation time may be realized through proper ordering of the equations and variables.

The equations are first divided into linear and nonlinear sets, and at the same time, the variables are classified as linear or nonlinear, resulting in the partitioned matrix equation:

$$\begin{bmatrix} AL & BL \\ \hline ANL & BNL \end{bmatrix} \begin{bmatrix} \Delta VL \\ \hline \Delta VNL \end{bmatrix} = - \begin{bmatrix} EL \\ \hline ENL \end{bmatrix} \tag{61}$$

The division of variables into linear and nonlinear sets is generally arbitrary except that the division must be made to insure that the square matrix AL is not singular. Adopting the variable groupings of Bartlett and Kendall (Ref. 16) the linear and nonlinear corrections are arranged as follows:

$$\Delta VL(\Delta f_2, \Delta f_3, \ldots, \Delta f_e, \Delta f'_w, \Delta f'_2, \ldots, \Delta f'_e, \Delta f''_w, \Delta f''_2, \ldots, \Delta f''_{e-1})$$

$$\Delta VNL(\Delta f_w, \Delta f''_e, \Delta f'''_w, \Delta f'''_2, \ldots, \Delta f'''_e)$$

The linear and nonlinear equations are sequenced within the partitioned square matrix $\begin{bmatrix} A \end{bmatrix}$ as shown in Figures 1 and 2, which demonstrate the case of 4 nodes across the boundary layer.

Expanding equation (61), the resulting linear and nonlinear equations are, respectively:

$$\begin{bmatrix} AL \end{bmatrix} \begin{bmatrix} \Delta VL \end{bmatrix} + \begin{bmatrix} BL \end{bmatrix} \begin{bmatrix} \Delta VNL \end{bmatrix} = \begin{bmatrix} -EL \end{bmatrix} \tag{62}$$

$$\begin{bmatrix} ANL \end{bmatrix} \begin{bmatrix} \Delta VL \end{bmatrix} + \begin{bmatrix} BNL \end{bmatrix} \begin{bmatrix} \Delta VNL \end{bmatrix} = \begin{bmatrix} -ENL \end{bmatrix} \tag{63}$$

Solving equation (62) for the linear corrections:

AL / BL	Δf_2	Δf_3	Δf_e	$\Delta f'_w$	$\Delta f'_2$	$\Delta f'_3$	$\Delta f'_e$	$\Delta f''_w$	$\Delta f''_2$	$\Delta f''_3$	Δf_w	$\Delta f''_e$	$\Delta f'''_w$	$\Delta f'''_2$	$\Delta f'''_3$	$\Delta f'''_e$
TAYLOR f_2	-1			$\delta\eta_{12}$				$\frac{\delta\eta_{12}^2}{2}$			1		$\frac{\delta\eta_{12}^3}{8}$	$\frac{\delta\eta_{12}^3}{24}$		
TAYLOR f_3	1	-1			$\delta\eta_{23}$				$\frac{\delta\eta_{23}^2}{2}$					$\frac{\delta\eta_{23}^3}{8}$	$\frac{\delta\eta_{23}^3}{24}$	
TAYLOR f_e		1	-1			$\delta\eta_{3e}$				$\frac{\delta\eta_{3e}^2}{2}$					$\frac{\delta\eta_{3e}^3}{8}$	$\frac{\delta\eta_{3e}^3}{24}$
B.C. f'_w				1												
f'_2				1	-1			$\delta\eta_{12}$					$\frac{\delta\eta_{12}^2}{3}$	$\frac{\delta\eta_{12}^2}{6}$		
f'_3					1	-1			$\delta\eta_{23}$					$\frac{\delta\eta_{23}^2}{3}$	$\frac{\delta\eta_{23}^2}{6}$	
f'_e						1	-1			$\delta\eta_{3e}$					$\frac{\delta\eta_{3e}^2}{3}$	$\frac{\delta\eta_{3e}^2}{6}$
f''_2								1	-1				$\frac{\delta\eta_{12}}{2}$	$\frac{\delta\eta_{12}}{2}$		
f''_3									1	-1				$\frac{\delta\eta_{23}}{2}$	$\frac{\delta\eta_{23}}{2}$	
f''_e										1		-1			$\frac{\delta\eta_{3e}}{2}$	$\frac{\delta\eta_{3e}}{2}$

FIGURE 1. LINEAR PORTION OF [A] MATRIX

ANL | BNL	Δf_2	Δf_3	Δf_e	$\Delta f'_w$	$\Delta f'_2$	$\Delta f'_3$	$\Delta f'_e$	$\Delta f''_w$	$\Delta f''_2$	$\Delta f''_3$	Δf_w	$\Delta f''_e$	$\Delta f'''_w$	$\Delta f'''_2$	$\Delta f'''_3$	$\Delta f'''_e$
B.C. f_w											1					
MOMENTUM WALL — 2	C1			C4	C3			C6	C5		C2		C8	C7		
MOMENTUM 2 — 3	C2	C1			C4	C3			C6	C5				C8	C7	
MOMENTUM 3 — e		C2	C1			C4	C3			C6		C5			C8	C8
B.C. f'_e							-1									
B.C. f''_e												-1				

FIGURE 2. NONLINEAR PORTION OF [A] MATRIX

$$\left[\Delta VL\right] = -\left[AL\right]^{-1}\left[BL\right]\left[\Delta VNL\right] + \left[AL\right]^{-1}\left[-EL\right] \tag{64}$$

Examination of the $\left[AL\right]$ and $\left[BL\right]$ matrices reveals that the product $\left[AL\right]^{-1}\left[BL\right]$ may be performed given only the nodal spacing and must be determined only once for a given problem, thus:

$$\left[\Delta VL\right] = -\left[BA1\right]\left[\Delta VNL\right] + \left[ELA\right] \tag{65}$$

where:

$$\left[BA1\right] = \left[AL\right]^{-1}\left[BL\right] \tag{66}$$

$$\left[ELA\right] = \left[AL\right]^{-1}\left[-EL\right] \tag{67}$$

Applying equation (65) to equation (63), and solving for the nonlinear corrections:

$$\left[\Delta VNL\right] = \left[\overline{BNL}\right]^{-1}\left[\overline{ENL}\right] \tag{68}$$

where:

$$\left[\overline{BNL}\right] = \left[BNL\right] - \left[ANL\right]\left[BA1\right] \tag{69}$$

$$\left[\overline{ENL}\right] = \left[-ENL\right] - \left[ANL\right]\left[ELA\right] \tag{70}$$

I. SOLUTION PROCEDURE

The matrix solution proceeds according to the following algorithm:

1. Given the nodal spacing determine $\left[AL\right]^{-1}$ and form the product $\left[BA1\right]$.

2. Given initial values of the primary variables determine $\left[-EL\right]$ and $\left[-ENL\right]$, and form the product $\left[ELA\right]$.

3. Form $\left[\overline{BNL}\right]$ and $\left[\overline{ENL}\right]$ from the coefficients of $\left[ANL\right]$ and $\left[BNL\right]$.

4. Invert $\left[\overline{BNL}\right]$ and form the product $\left[\overline{BNL}\right]^{-1}\left[\overline{ENL}\right]$, which is the solution for $\left[\Delta VNL\right]$.

5. Solve for $\left[\Delta VL\right]$ from equation (64) using $\left[\Delta VNL\right]$ as determined in step 4.

6. Add the linear $\left[\Delta V\right]$ to the corresponding primary variables to complete the iteration.

7. Compute the new errors $\left[-EL\right]$ and $\left[-ENL\right]$, and test for a maximum allowable error. If this criterion has not been reached, return to step 2 using these errors to commence the new iteration.

III. RESULTS AND DISCUSSION

The computer program has been used to investigate three basic categories of boundary layer problems using a flat plate model; steady Blasius flow (zero pressure gradient), steady Howarth flow (uniform adverse pressure gradient), and oscillating Blasius flow. The velocity profiles which resulted from these solutions are compared to previously reported results and presented in Figures 3, 4, and 5.

A. STEADY BLASIUS FLOW

In Figure 3, computed values corresponding to three nodal spacing choices are superimposed on the Blasius profile. Agreement between the numerical solution and the classical solution is better than 0.08 percent for as few as 10 nodal points.

B. STEADY HOWARTH FLOW

Results for a steady flow in a uniform adverse pressure gradient are compared with the solution of Howarth (Ref. 11) in Figure 4. The comparison of the computer solution with that of Howarth is accomplished by correlating the dimensionless parameter x^* $(=b_1 s/U_o)$, which characterizes Howarth's velocity profiles. Agreement between the two solutions is generally good with differences no greater than 4.35 percent for 22 nodal points. The differences encountered may be reduced somewhat by increasing the free stream velocity of the test problem. This results in larger

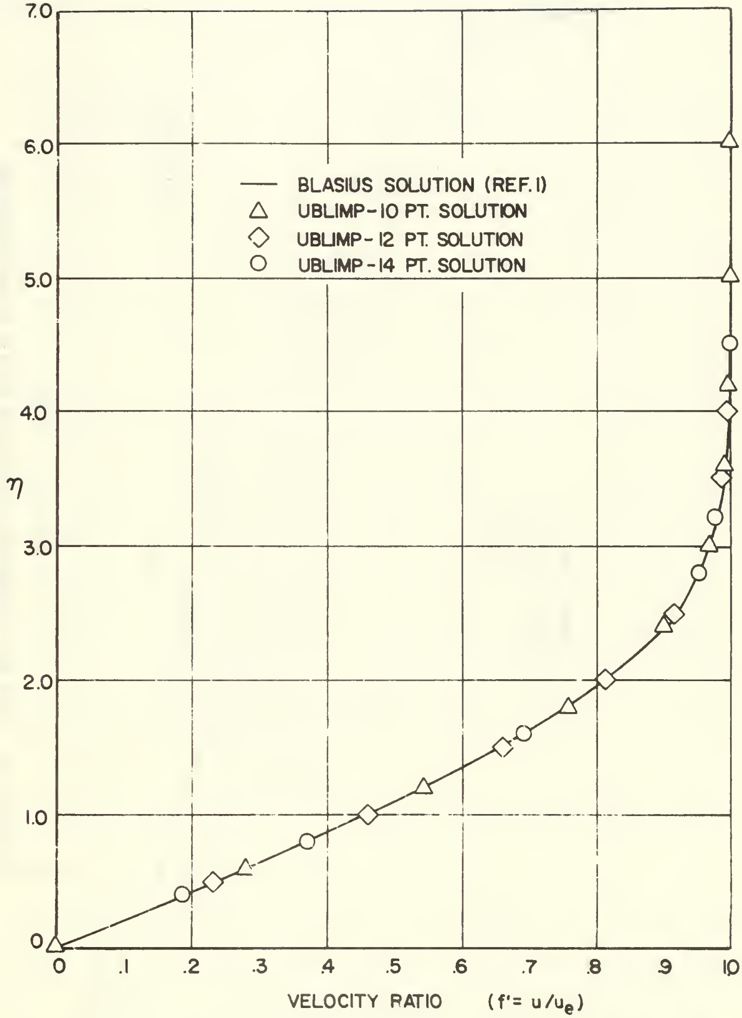

FIGURE 3. STEADY BLASIUS FLOW DATA COMPARISON

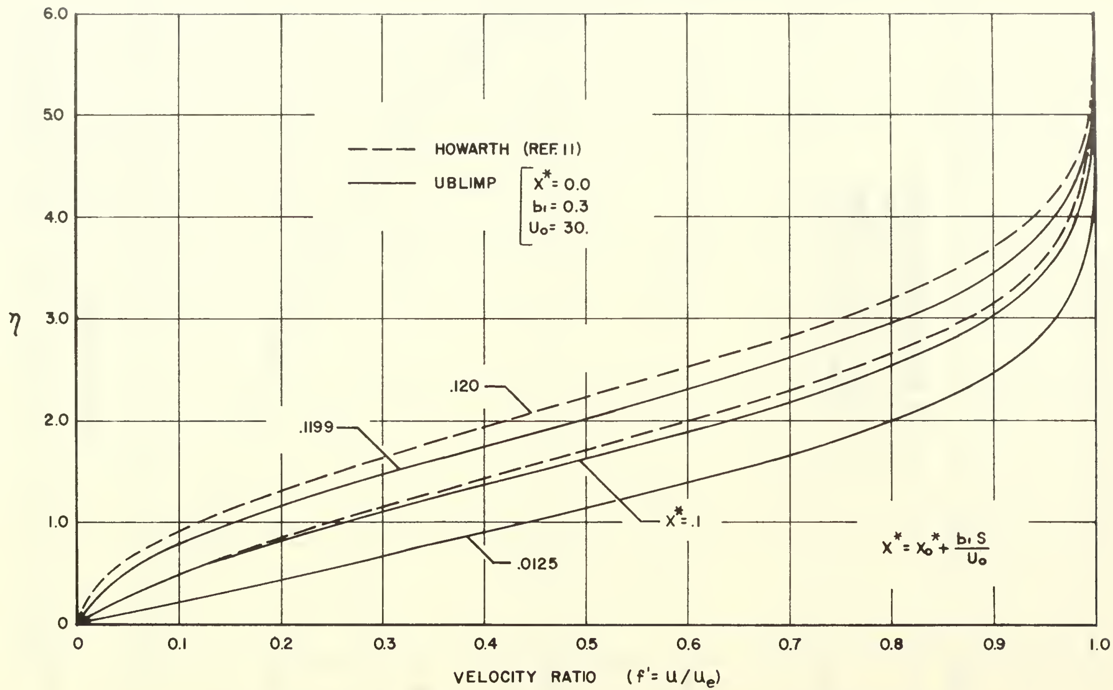

FIGURE 4. STEADY HOWARTH FLOW DATA COMPARISON

stream-wise pressure differences in the equations of motion and consequently lesser roundoff errors.

C. OSCILLATING BLASIUS FLOW

The unsteady capabilities of the numerical procedure were investigated in the solution of Blasius flow with sinusoidal oscillations superimposed on the mean flow. Utilizing the critical oscillation frequency ω_o (=.6U_o/s, for a flat plate) of Lighthill's analysis (Ref. 2) to determine the solution approximation with which to compare the numerical results, the profile of velocity fluctuations which resulted was compared with the "quasi-steady" analysis as shown in Figure 5. Agreement between the calculated velocities and the Lighthill profile is within 15 percent deep in the boundary layer and within 5.5 percent in the outer regions. Noting that Lighthill's analysis is based on linearized small perturbation theory and that the amplitude of oscillation in the test problem of 0.1 U_o cannot be considered a small perturbation, better agreement could scarcely be expected.

D. NUMERICAL PROCEDURE

The computer program has been operated on both IBM 360/67 and CDC 6600 computer installations. Core storage requirements are 130,000 bytes for a maximum array size encompassing 25 nodal points across the boundary layer, 50 streamwise stations and 50 time increments per oscillation cycle. A

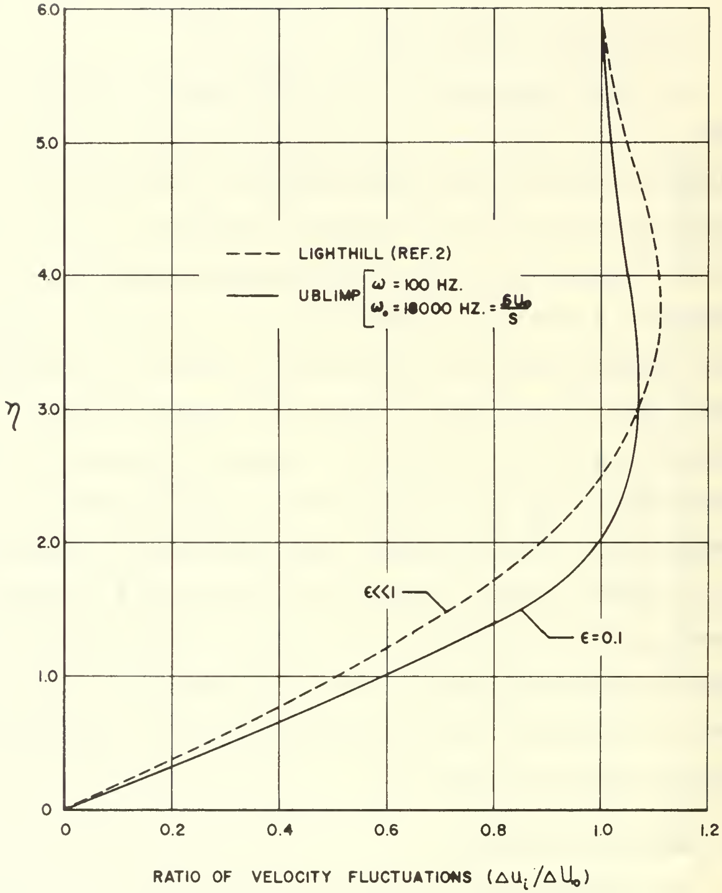

FIGURE 5. OSCILLATING BLASIUS FLOW DATA COMPARISON

typical 25 nodal point Blasius solution was performed in 7 seconds on the CDC 6600.

Limitations in the present numerical capabilities of the program precluded a more extensive investigation of parametric effects, however the results qualitatively demonstrated the feasibility of the computational procedure. Further refinements to the computer program should enable a more extensive investigation to include quantitative comparisons with experimental results.

IV. CONCLUSIONS AND RECOMMENDATIONS

The feasibility of the Unsteady Boundary Layer Integral Matrix Procedure in the treatment of simple steady and unsteady flow problems has been demonstrated. Favorable comparison with classical results provides a firm base for continued investigation in the unsteady regime.

Continued research utilizing the program should include the following revisions:

1. Alteration of the present time derivative component arrays (FTM1) and FTM2) to insure that all values of the components are stored for each nodal point, instead of retaining only the values at the wall.

2. Incorporation of Blasius profiles as first estimates for the primary variables in order to decrease the time required to complete the first nodal iteration procedure.

3. A study of array usage aimed at reducing the required storage and subsequent conversion to double precision computation in order to minimize round-off interference.

4. Addition of the capability of handling compressible flows in order that higher velocities may be introduced.

5. Inclusion of turbulent boundary layer capabilities perhaps through the use of eddy transport properties.

APPENDIX A

TAYLOR SERIES EXPANSIONS OF INTEGRALS (Ref. 16)

$$\int_{i-1}^{i} ff'' \, d\eta = \Big[ff' \Big]_{i-1}^{i} - \int_{i-1}^{i} f'^2 \, d\eta \qquad (A1)$$

$$\int_{i-1}^{i} f'^2 \, d\eta = f_i' XM_1 + f_i'' XM_2 + f_i''' XM_3 + f_{i-1}''' XM_4 \qquad (A2)$$

$$XM_1 = \delta\eta \left(f_i' - f_i'' \frac{\delta\eta}{2} + f_i''' \frac{\delta\eta^2}{8} + f_{i-1}''' \frac{\delta\eta^2}{24} \right) \qquad (A3)$$

$$XM_2 = -\delta\eta^2 \left(\frac{1}{2} f_i' - f_i'' \frac{\delta\eta}{3} + f_i''' \frac{11\delta\eta^2}{120} + f_{i-1}''' \frac{\delta\eta^2}{30} \right) \qquad (A4)$$

$$XM_3 = \delta\eta^3 \left(\frac{1}{8} f_i' - f_i'' \frac{11\delta\eta}{120} + f_i''' \frac{11\delta\eta^2}{420} + f_{i-1}''' \frac{5\delta\eta^2}{504} \right) \qquad (A5)$$

$$XM_4 = \delta\eta^3 \left(\frac{1}{24} f_i' - f_i'' \frac{\delta\eta}{30} + f_i''' \frac{5\delta\eta^2}{504} + f_{i-1}''' \frac{\delta\eta^2}{252} \right)$$

$$2\int_{i-1}^{i} \left(f' \frac{\partial f'}{\partial \ell n \xi} - f'' \frac{\partial f}{\partial \ell n \xi} \right) d\eta = -\left[d_o ff' + d_1 f_{\ell-1} f' + d_2 f_{\ell-2} f' \right]_{i-1}^{1}$$

$$+2d_o \left[f_i' XM_1 + f_i'' XM_2 + f_i''' XM_3 + f_{i-1}''' XM_4 \right]$$

$$+2 \left[f_i' ZM_1 + f_i'' ZM_2 + f_i''' ZM_3 + f_{i-1}''' \right] \qquad (A7)$$

$$ZM_1 = \delta\eta \left(YM_1 - YM_2 \frac{\delta\eta}{2} + YM_3 \frac{\delta\eta^2}{8} + YM_4 \frac{\delta\eta^2}{24} \right) \qquad (A8)$$

$$ZM_2 = -\delta\eta^2 \left(\frac{1}{2} YM_1 - YM_2 \frac{\delta\eta}{3} + YM_3 \frac{11\delta\eta^2}{120} + YM_4 \frac{\delta\eta^2}{30} \right) \qquad (A9)$$

$$ZM_3 = \delta\eta^3 \left(\frac{1}{8} YM_1 - YM_2 \frac{11\delta\eta}{120} + YM_3 \frac{11\delta\eta^2}{420} + YM_4 \frac{5\delta\eta^2}{504} \right) \qquad (A10)$$

$$ZM_3 = \delta\eta^3 (\frac{1}{24} YM_1 - YM_2 \frac{\delta\eta}{3} + YM_3 \frac{5\delta\eta^2}{504} + YM_4 \frac{\delta\eta^2}{252} \qquad \text{(A11)}$$

$$YM_1 = d_1 f'_{\ell-1,i} + d_2 f'_{\ell-2,i} \qquad \text{(A12)}$$

$$YM_2 = d_1 f''_{\ell-1,i} + d_2 f''_{\ell-2,i} \qquad \text{(13)}$$

$$YM_3 = d_1 f'''_{\ell-1,i} + d_2 f'''_{\ell-2,i}$$

$$YM_4 = d_1 f'''_{\ell-1,i-1} + d_2 f'''_{\ell-2,i-1} \qquad \text{(A15)}$$

APPENDIX B

BOUNDARY LAYER EQUATION CORRECTION COEFFICIENTS

$$C1 = (1+d_o)f'_i - \frac{\rho\mu}{\alpha^{*2}}\{T_o(1-\ell nu_e)-(T_1\ell nu_{e_{k-1}} + T_2\ell nu_{e_{k-2}})\} \quad (B1)$$

$$C2 = -(1+d_o)f'_{i-1} + \frac{\rho\mu}{\alpha^{*2}}\{T_o(1-\ell nu_e)-(T_1\ell nu_{e_{k-1}} + T_2\ell nu_{e_{k-2}})\} \quad (B2)$$

$$C3 = (1+d_o)f_i+(d_1 f_{\ell-1,i}+d_2 f_{\ell-2,i})-2(1+\beta+2d_o)XM_1-2ZM_1 \quad (B3)$$

$$C4 = -(1+d_o)f_{i-1} - (d_1 f_{\ell-1,i-1} + d_2 f_{\ell-2,i-1}) \quad (B4)$$

$$C5 = 1 - 2(1+\beta+2d_o)XM_2 - 2ZM_2 \quad (B5)$$

$$C6 = -1 \quad (B6)$$

$$C7 = -2(1+\beta+2d_o)XM_3 - 2ZM_3 \quad (B7)$$

$$C8 = -2(1+\beta+2d_o)XM_4 - 2ZM_4 \quad (B8)$$

APPENDIX C

PROGRAM DESCRIPTION

A. GENERAL

The computer program referred to as UBLIMP, is written in FORTRAN IV source language and has been operated on both IBM 360/67 and CDC 6600 computer installations. Core storage requirements are 130,000 bytes for a maximum array size encompassing 25 nodal points across the boundary layer, 50 streamwise stations and 50 time increments per oscillation cycle.

The program consists of a main program (UBLIMP) and 21 subroutines which are divided into two sections as shown in Figure 6. LINK 0, common to both sections, contains the main program and certain service routines which are common to both sections in the fully expanded program. LINK 1 sets up the boundary conditions and LINK 2 iterates to solve the boundary layer equations.

The program may be applied to steady or unsteady flow, two-dimensional cartesian or axisymmetric bodies with similar or nonsimilar profiles. It allows quadratic or cubic curve fits of the primary variables. Program options are controlled by the control variable KR and are detailed in the input instructions for the UBLIMP program in Appendix D. A program linkage schematic is shown in Figure 7 and a functional flow chart is shown in Figure 8. Correlation

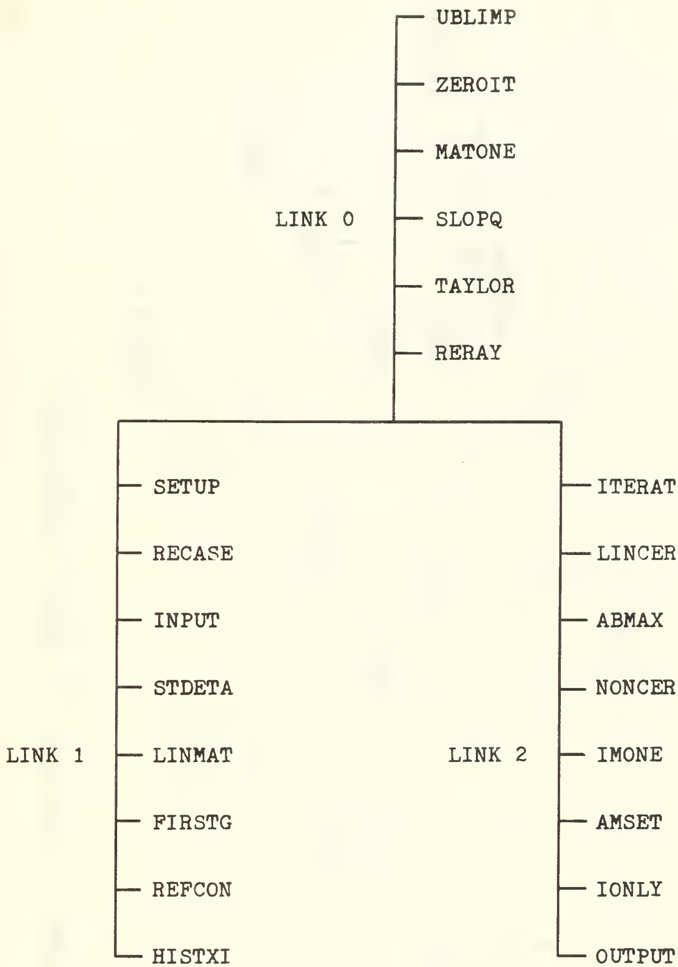

FIGURE 6. COMPUTER PROGRAM OVERLAY STRUCTURE

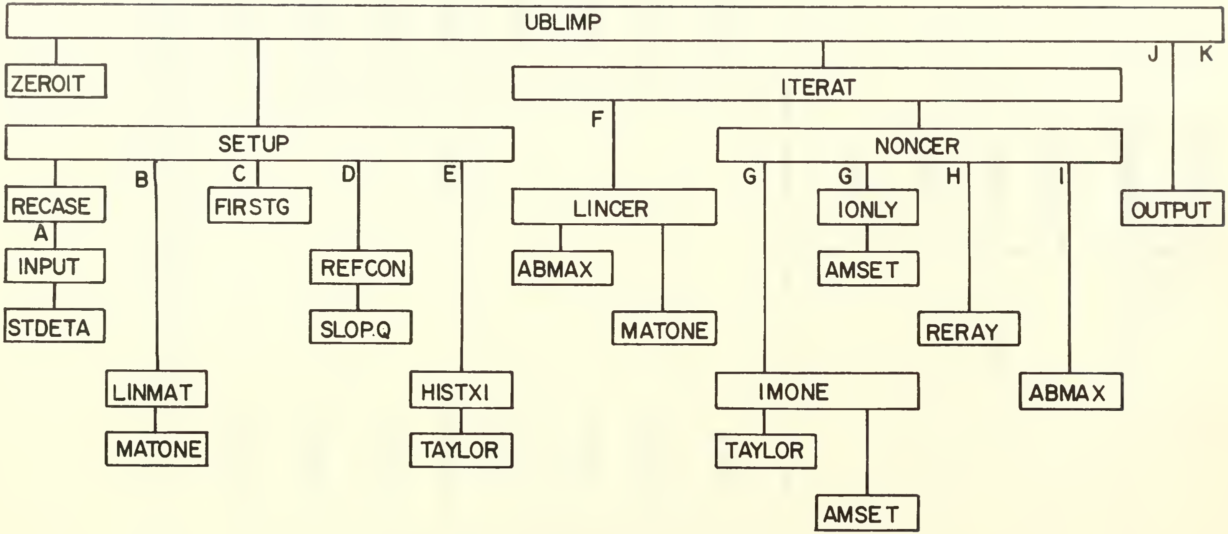

FIGURE 7. SCHEMATIC OF UBLIMP SUBROUTINES

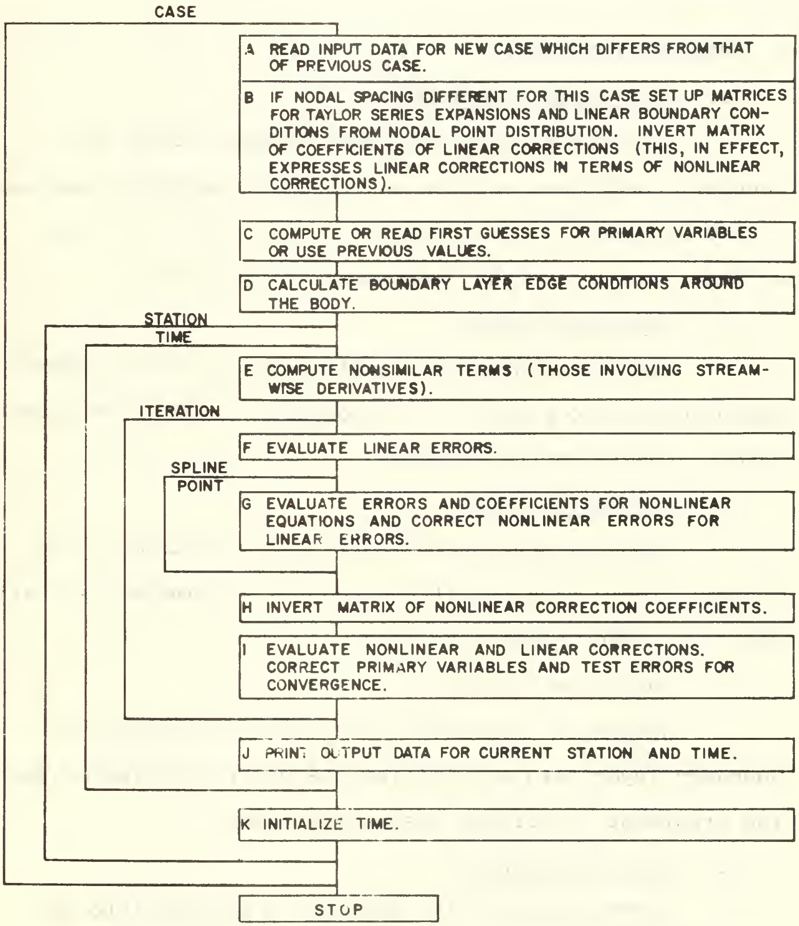

FIGURE 8. FUNCTIONAL FLOW CHART OF UBLIMP PROGRAM

between the two is indicated by letters running from A through K.

B. PROGRAM FUNCTIONS

1. Main Program (UBLIMP)

The main program provides linkage between the boundary conditions section and iterative solution section of the program. Initial core zeroing and the call for solution output are also initiated by the routine.

2. Subroutine ZEROIT

ZEROIT performs the initial zeroing of all named common blocks to eliminate the possibility of introducing random numbers in the solution.

3. Subroutine SETUP

SETUP is the control routine for setting up the boundary layer edge conditions and the streamwise derivatives for a new time, station or case.

4. Subroutine RECASE

RECASE is the control routine for the input of boundary layer data and initializes control variables for the treatment of surface discontinuities.

5. Subroutine INPUT

INPUT reads all the data for a problem into the computer.

6. Subroutine STDETA

STDETA permits simplification of the input data deck by providing standard nodal point distributions should the user not care to provide one.

7. Subroutine LINMAT

LINMAT sets up the matrices for the Taylor series expansions and linear boundary conditions from the eta spacing and solves to express the linear corrections in terms of the nonlinear corrections.

8. Subroutine MATONE

MATONE performs operations on a column of a matrix of coefficients $\left[BL \right]$ or on a column of a matrix of errors $\left[-EL \right]$ (designated E in the subroutine call list) such as to form $\left[AL \right]^{-1} \left[E \right]$.

9. Subroutine FIRSTG

FIRSTG computes first estimates of the primary variables based on the nodal point distribution should the user not care to provide them.

10. Subroutine REFCON

REFCON calculates the boundary layer edge conditions and sets up the wall boundary conditions.

11. Subroutine SLOPQ

SLOPQ defines a cubic equation for a set of points, calculates the average slope at each point and integrates the cubic equation between each pair of points.

12. Subroutine HISTXI

HISTXI computes terms involving the axial derivatives and time derivatives and stores those upstream quantities needed for these difference relations.

13. Subroutine TAYLOR

TAYLOR calculates the coefficients in the Taylor series expansions of the integrals which appear in the integral form of the boundary layer equations.

14. Subroutine ITERAT

ITERAT is the control program for performing the boundary layer iteration.

15. Subroutine LINCER

LINCER evaluates errors for the linear equations and determines the maximum linear error.

16. Subroutine ABMAX

ABMAX searches a given array for the entry with the maximum absolute value.

17. Subroutine NONCER

NONCER performs that portion of a boundary layer iteration having to do with the solution of the nonlinear corrections of the nonlinear equations, computes the damping factor and applies it to the corrections and then applies the damped corrections to the primary variables.

18. Subroutine IMONE

IMONE evaluates the coefficients of (i-1) corrections for the i^{th} nonlinear equations, where i is the i^{th} nodal point in the boundary layer.

19. Subroutine AMSET

AMSET calculates the contributions of the $\left[ANL\right]$ and applies them to $\left[\overline{BNL}\right]$ and $\left[\overline{ENL}\right]$.

20. <u>Subroutine IONLY</u>

IONLY evaluates the coefficients of the i^{th} corrections for the i^{th} nonlinear equations, where i is the i^{th} nodal point in the boundary layer.

21. <u>Subroutine RERAY</u>

RERAY replaces a matrix $[C]$ with its inverse and forms the product of the inverse with another matrix $[D]$.

22. <u>Subroutine OUTPUT</u>

OUTPUT prints a standard boundary layer output block for a converged solution, or at the end of each iteration if desired.

APPENDIX D

INPUT INSTRUCTIONS FOR THE UBLIMP PROGRAM

GROUP 1 CONTROL CARD, TITLE

CARD 1 FORMAT(20I1,15A4)

FIELD 1 (Columns 1-20) This is the variable KR which is used to control the various program options.

COLUMN 1 Determines whether a new set of ETA values is to be input for the present case.

0 Uses resident values from previous case.

1 Values input by user (mandatory for first case).

COLUMN 2 Designates the type of first guesses to be utilized for primary variables.

0 Uses built-in relations to calculate first guesses.

1 First guesses input by user.

2 Uses resident values from previous case (cannot be used for first case).

COLUMN 3 Determines the treatment of streamwise derivatives.

0 Performs similar solution at each streamwise station.

1 Considers two-point difference relations at all stations except that a similar solution is performed at the first station for non-blunt bodies and at the first two stations for blunt bodies.

2 Considers three-point difference relations at all stations except that a similar solution is performed at the first station and a two-point solution is performed at the second station for non-blunt bodies; similar solutions are performed at the first and second stations and a two-point solution is performed at the third station for blunt bodies; and a two-point solution is performed for the first station after a discontinuity.

COLUMN 4 Determines when output block is to be printed.

0 Output block printed for converged solution or for nonconverged solution after 50 iterations.

1 Output block printed after each iteration.

COLUMN 5 Determines treatment of Entropy Layer. (Not used in present program. KR(5)=4 is input to maintain input data sequencing.)

COLUMN 6 Designates body shape.

0 Axisymmetric blunt body.

1 Planar blunt body

2 Axisymmetric sharp body.

3 Planar sharp body.

4 Axisymmetric or planar shape which has no sharp tip or blunt stagnation point, such as a nozzle.

COLUMN 7 Steady, zero pressure-gradient noise filter.

0 Normal computations of DUDS.

1 DUDS is forced to zero.

COLUMN 8 Designates form in which wall mass fluxes are input. KR(8) is not utilized if wall mass fluxes are not input.

0 Wall fluxes input in LBS/SEC FT**2

1 Wall fluxes input in normalized form (divided by -ALPHASTAR).

COLUMN 9 Together with KR(11) this designates the type of wall boundary conditions.

0 Assigned stream function at the wall.

1 Assigned mass flux at the wall. KR(9)=1 is used if zero mass flux is used.

COLUMN 10 Determines the type of curve fits employed to represent the primary variables. (KR(10)=0 is recommended for accuracy for most problems, however for severe problems KR(10)=1 is better since the cubics can become poorly behaved.)

0 Utilizes connected cubics.

1 Utilizes connected quadratics except for the outermost segment where connected cubics are used.

COLUMN 11 Together with KR(9) this designates the type of wall boundary conditions. (Presently KR(11)=0 is the only active value and requires an assignment of wall temperature.)

COLUMN 12 Provides access to standard nodal-point distributions built into the program.

0 User inputs data for nodal-points.

1 8 points with more concentration at the wall.

2 8 points equally spaced.

3 10 points equally spaced.

4 12 points equally spaced.

5 14 points equally spaced.

6 15 points with more concentration at the wall.

7 18 points equally spaced.

8 22 points equally spaced.

9 25 points equally spaced.

COLUMN 13 Permits the assignment of a convergence damping factor. (This factor is overridden if a smaller damping factor is computed internally by some constraint.)

0 No damping factor is assigned.

J If J is greater than zero, corrections are damped uniformly by J/10.

COLUMN 14 Non-zero entry causes a complete set of primary variables to be output for future use as first guess inputs.

COLUMN 15 Non-zero entry provides debug output for first guesses and linear matrices.

0 No debug.

1 First guesses are dumped.

2 Linear matrices before and after inversion are also dumped.

COLUMN 16 Non-zero input indicates the pressure profile has been input in the form of pressure coefficients.

COLUMN 17 Non-zero entry provides debug output for coefficients.

0 No debug.

Y For (Y+1-ITS) greater than zero, where ITS is the number of the current boundary layer iteration, the coefficients which combine to make up the nonlinear equations are dumped and the derivatives of the nonlinear equations with respect to the nonlinear variables are dumped before and after inversion.

COLUMN 18 Not used.

COLUMN 19 Non-zero entry provides debug output for linear and nonlinear errors.

COLUMN 20 Determines which of the two internal sets of first guesses are to be used if the user does not provide them.

0 Straight line Blasius inputs are calculated for the stream function and velocity ratio, and zero is input for the first and second derivatives of the velocity ratio.

1 Straight line Blasius inputs are calculated for all the primary variables.

FIELD 2 (Columns 21-80) CASE. Title of the case (ALPHA-NUMERIC). Used for identification of printed output.

GROUP 2 TIMES AND STATIONS

CARD 1 FORMAT(I3)

FIELD 1 (Columns 1-3, right-justified) NITEM Number of time grid points per cycle when considering an unsteady solution, otherwise input 1. (max=50)

CARD 2 FORMAT(E10.4)

FIELD 1 (Columns 1-10) TIME(1) Value to be used in the output block to identify a time of solution.

CARD 3 FORMAT(I3)

FIELD 1 (Columns 1-3, right-justified) NS. Number of streamwise stations. (max=50)

CARD 4 FORMAT(8E10.4)

FIELD 1,2,...8 per card(10 columns per field) S(IS) Streamwise distance upon which the boundary layer solution is based in feet. Blunt body problems should start with S=0.0. Sharp body or nozzle problems must start with some finite value of S. The boundary layer is assumed similar up to and including this first station. A negative entry for S indicates a discontinuity at that station, and alters the differencing scheme for axial derivatives.

GROUP 3 NODAL DATA (Skip this GROUP for KR(12)≠ 0)

CARD 1 FORMAT(I3)

FIELD 1 (Columns 1-3, right-justified) NETA Number of nodal points across the boundary layer. (maximum of 25)

CARD 2 FORMAT(8E10.4)

FIELD 1,2,...8 per card(10 columns per field) ETA(I) ETA stations across the boundary layer starting at the wall (ETA=0.0). It is recommended that a value between 5.0 and 6.0 be assigned at the boundary layer edge. UBLIMP values of ETA are equivalent to Blasius values divided by the square root of two.

GROUP 4 BODY SHAPE DATA

CARD 1 FORMAT(2E10.4) USED ONLY IF KR(6)=0 or 1.

FIELD 1 (Columns 1-10) CONE. Cone half angle in sphere-cone shape bodies. Leave blank for other body shapes.

FIELD 2 (Columns 11-20) RNOSE. Effective nose radius in feet. Used to calculate stagnation point velocity gradient from Newtonian relations.

CARD 2 FORMAT(8E10.4) USED ONLY IF KR(6)=0, 2 or 4

FIELD 1,2,...8 per card(10 columns per field) ROKAP(IS) This is the local body radius in feet normal to the body centerline raised to the n power, where n is unity for axisymmetric bodies and zero for planar bodies. Therefore, ROKAP is unity for planar bodies and the local body radius for axisymmetric bodies. For planar bodies, this card set is used only if KR(6)=4. (A special input format can be used for spherical-nosed bodies as follows. Set ROKAP(1) equal to minus the nose radius. The nose radius is then set to -ROKAP(1) and ROKAP(1) is set to zero. If subsequent ROKAP are input as zeros, the program computes ROKAP from S for a spherical nose. The first non-zero entry is then ROKAP at the tangent point. If this again followed by zeros, linear interpolation is used to the next non-zero entry to yield ROKAP along a conical afterbody.)

GROUP 5 STAGNATION DATA

CARD 1 FORMAT(E10.4)

FIELD 1 (Columns 1-10) PTET. Local stagnation pressure in atmospheres.

GROUP 6 FIRST GUESS INFORMATION

CARD 1 FORMAT(5(2XE14.7))

FIELD 1,2,...5 per card(16 columns per field) F(I,J). First guesses for stream function (F(1,J)), velocity ratio (F(2,J)), shear function (F(3,J)), and derivative of shear function (F(4,J)) across the boundary layer.

GROUP 7 STREAMWISE DISTRIBUTIONS FOR EDGE CONDITIONS

CARD 1 FORMAT(5(2XE14.7))

FIELD 1,2,...5 per card(16 columns per field) PRE(IS). Ratio of local static to stagnation pressure for the mean flow condition. When KR(16) is non-zero PRE is input in the form of negative pressure coefficients referred to PTET.

CARD 2 FORMAT(Blank) This card is inserted to maintain data continuity. In more complex formulations this card corresponds to a control flag for updating the pressure profile during the computation.

GROUP 8 STREAMWISE DISTRIBUTIONS FOR WALL INPUT CONDITIONS

CARD 1 FORMAT(E10.4)

FIELD 1 (columns 1-10) T. Wall temperature in degrees R (for the present this is the free stream static temperature).

CARD 2 FORMAT(8E10.4) USED ONLY FOR KR(9)=0

FIELD 1,2,...8 per card(10 columns per field) FW(IS). Wall stream function (negative for mass addition).

CARD 3 FORMAT(Blank) USED ONLY IF CARD 2 IS USED. This card is inserted to maintain data continuity. In more complex formulations this card corresponds to a control flag for updating the wall stream function profile during the computation.

CARD 4 FORMAT(8E10.4) USED ONLY FOR KR(9)=1

FIELD 1,2,...8 per card(10 columns per field) RHOVW(IS). Total mass flux at the wall (LB/SEC FT**2 or dimensionless for KR(8)=0 or 1 respectively). These values are positive for mass injection.

CARD 5 FORMAT(Blank) USED ONLY IF CARD 4 IS USED. This card is inserted to maintain data continuity. In more complex formulations this card corresponds to a control flag for updating the total mass flux profile during the computation.

GROUP 9 FREE STREAM DATA

CARD 1 FORMAT(3E10.4)

FIELD 1 (Columns 1-10) UMFS. Mean free stream velocity in FT/SEC.

FIELD 2 (Columns 11-20) FREQ. Frequency of mean free stream oscillations in HZ.

FIELD 3 (Columns 21-30) EPS. Ratio of the amplitude of the free stream oscillation to the local mean free stream velocity.

LAST CARD FORMAT(A1) The purpose of this entry is to permit a test on whether or not a new case is to follow. In the event a case does not converge in the alloted number of iterations, any remaining cards for that case are read and then ignored until either a period or a comma is encountered in column 1. A comma in column 1 indicates that another case is to follow, while a period in column 1 indicates that this is the last card in the input deck.

The preceding input instructions should permit the user to fully exercise the UBLIMP program with a minimum of difficulty. For more detailed information on various routines the reader is referred to reference 18, which is the user manual prepared for the BLIMP program.

SAMPLE COMPUTER PROBLEM

The UBLIMP computer program was exercised for steady Blasius flow, steady Howarth flow and oscillating Blasius flow. The solution of steady Blasius flow for 8 nodal points across the boundary layer and 6 streamwise stations is presented as a sample problem. Solution time was 3.93 seconds utilizing the time sharing capabilities of the Naval Postgraduate School IBM 360/67 computer system.

INPUT DATA DECK

```
102043001002000000001                        STEADY BLASIUS FLOW.
  1
  6
  .001     .2       .4       .6       .8      1.0
 1.0
    .9994944     .9994944     .9994944     .9994944     .9994944
    .9994944
(blank)
520.
(blank)
(blank)
30.
.
```

```
                         PRINTED OUTPUT

      UNSTEADY BOUNDARY LAYER INTEGRAL MATRIX PROCEDURE

                  DEPARTMENT OF AERONAUTICS
                  NAVAL POSTGRADUATE SCHOOL
                     MONTEREY, CALIFORNIA

                       INVESTIGATION OF

                      STEADY BLASIUS FLOW.

                       CONTROL NUMBERS
   1  2  3  4  5  6  7  8  9 10 11 12 13 14 15 16 17 18 19 20
   1  0  2  0  4  3  0  0  1  0  0  2  0  0  0  0  0  0  0  1

                          ETA VALUES
0.0        8.000E-01 1.600E 00 2.400E 00 3.200E 00 4.000E 00 5.000E 00
6.000E 00

TEMPERATURE     TOTAL        MEAN        OSCIL.       OSCIL.
               PRESSURE    VELOCITY     AMPLITUDE    FREQUENCY
  (DEG R)       (ATM)      (FT/SEC)     PARAMETER     (HERTZ)
 5.200E 02    1.000E 00    3.000E 01     0.0          0.0

          DENSITY         =0.07650995 LB/FT CU
          VISCOSITY       = 3.719E-07 LB SEC/FT SQ
          TIME GRID       = 0.0       SEC
          CYCLE TIME      = 0.0       SEC
```

STATION INPUT DATA

DISTANCE,FT	1.000E-03	2.000E-01	4.000E-01	6.000E-01	8.000E-01
	1.000E 00				
ROKAP	1.000E 00	1.000E 00	1.000E 00	1.000E 00	1.000E 00
	1.000E 00				
PRESSURE RATIO, MEAN	9.995E-01	9.995E-01	9.995E-01	9.995E-01	9.995E-01
	9.995E-01				
ENTROPY DROP,BTU/LB R	0.0	0.0	0.0	0.0	0.0
	0.0				
MASS FLUX,LB/SEC FT**2	0.0	0.0	0.0	0.0	0.0
	0.0				

STATION COMPUTED DATA
TIME= 0.0 SEC

STREAMWISE DIMENSION (FEET)	XI (LB/ SEC)**2	BETA	EDGE VELOCITY (FT/SEC)	STATIC PRESSURE (ATM)
1.000E-03	8.536E-10	-3.052E-09	3.000E 01	9.995E-01
2.000E-01	1.707E-07	2.035E-07	3.000E 01	9.995E-01
4.000E-01	3.414E-07	-1.017E-07	3.000E 01	9.995E-01
6.000E-01	5.122E-07	-2.747E-06	3.000E 01	9.995E-01
8.000E-01	6.829E-07	4.069E-07	3.000E 01	9.995E-01
1.000E 00	8.536E-07	1.628E-05	3.000E 01	9.995E-01

TIME 0.0 SECONDS * * * STREAMWISE DIMENSION 0.10000E-02 FEET * * * *

ITS	ITERATED VALUES TIME	FPPW	DAMP		MAX LINEAR ERROR		MAX MOMENTUM ERROR
1	0.0	0.6177	0.0852	7	1.000E 00	4	-3.976E-01
2	0.0	0.6970	0.1597	7	9.148E-01	4	-3.300E-01
3	0.0	0.7187	0.2736	7	7.687E-01	4	-2.231E-01
4	0.0	0.6564	0.4665	7	5.584E-01	5	-1.025E-01
5	0.0	0.5008	0.9494	7	2.979E-01	3	9.781E-02
6	0.0	0.4699	1.0000	7	1.507E-02	4	4.533E-02
7	0.0	0.4695	1.0000	4	9.537E-07	5	1.790E-03
8	0.0	0.4695	1.0000	6	1.907E-06	8	-3.551E-06

EDGE VELOCITY (FT/SEC)	STATIC PRESSURE (ATM)	XI (LB/ SEC)**2	BETA	FLUX NORM PARAMETER	ROKAP (FT)
3.000E 01	9.995E-01	8.536E-10	0.0	2.066E-02	1.000E 00

WALL FR. COEFF, CF	MOM TRANS COEFF, RHO*UE*CF/2	WALL SHEAR (LB/SQ FT)	MASS FLUXES MECH REM (LB/SQ FT SEC)	TOTAL GAS
8.451E-03	9.698E-03	9.043E-03	-0.0	0.0

REYNOLDS NUMBER	STROUHAL NUMBER	MOMENTUM THICKNESS, THETA (FT)	DISPLACE. THICKNESS, DELSTAR (FT)	SHAPE FACTOR, DELSTAR/ THETA
6.172E 03	0.0	8.451E-06	2.190E-05	2.592E 00

NODAL INFORMATION

DISTANCE FROM WALL (FT)	ETA	STREAM FUNCTION (F)	VELOCITY RATIO (FP=U/UE)	SHEAR FUNCTION (FPP)	FPPP	VELOCITY (FT/SEC)
0.0	0.0	0.0	0.0	4.695E-01	6.403E-03	0.0
1.440E-05	8.000E-01	1.495E-01	3.713E-01	4.511E-01	-5.229E-02	1.114E 01
2.880E-05	1.600E 00	5.829E-01	6.976E-01	3.420E-01	-2.205E-01	2.093E 01
4.320E-05	2.400E 00	1.232E 00	9.011E-01	1.676E-01	-2.154E-01	2.703E 01
5.761E-05	3.200E 00	1.991E 00	9.801E-01	4.695E-02	-8.628E-02	2.940E 01
7.201E-05	4.000E 00	2.784E 00	9.977E-01	6.894E-03	-1.386E-02	2.993E 01
9.001E-05	5.000E 00	3.783E 00	1.000E 00	-1.719E-05	3.861E-05	3.000E 01
1.080E-04	6.000E 00	4.783E 00	1.000E 00	1.029E-09	-4.238E-06	3.000E 01

TIME 0.0 SECONDS * * * STREAMWISE DIMENSION 0.20000E 00 FEET * * * *

ITS	ITERATED VALUES TIME	FPPW	DAMP		MAX LINEAR ERROR		MAX MOMENTUM ERROR
1	0.0	0.4695	1.0000	3	1.907E-06	4	-4.897R-06

EDGE VELOCITY (FT/SEC)	STATIC PRESSURE (ATM)	XI (LB/ SEC)**2	BETA	FLUX NORM PARAMETER	ROKAP (FT)
3.000E 01	9.995E-01	1.707E-07	0.0	1.461E-03	1.000E 00

WALL FR. COEFF, CF	MOM TRANS COEFF, RHO*UE*CF/2	WALL SHEAR (LB/SQ FT)	MASS FLUXES MECH REM (LB/SQ FT SEC)	TOTAL GAS
5.976E-04	6.858E-04	6.394E-04	-0.0	0.0

REYNOLDS NUMBER	STROUHAL NUMBER	MOMENTUM THICKNESS, THETA (FT)	DISPLACE. THICKNESS, DELSTAR (FT)	SHAPE FACTOR, DELSTAR/ THETA
1.234E 06	0.0	1.195E-04	3.098E-04	2.592E 00

NODAL INFORMATION

DISTANCE FROM WALL (FT)	ETA	STREAM FUNCTION (F)	VELOCITY RATIO (FP=U/UE)	SHEAR FUNCTION (FPP)	FPPP	VELOCITY (FT/SEC
0.0	0.0	0.0	0.0	4.695E-01	6.398E-03	0.0
2.037E-04	8.000E-01	1.495E-01	3.714E-01	4.511E-01	-5.229E-02	1.114E 01
4.073E-04	1.600E 00	5.829E-01	6.976E-01	3.420E-01	-2.205E-01	2.093E 01
6.110E-04	2.400E 00	1.232E 00	9.011E-01	1.676E-01	-2.154E-01	2.703E 01
8.147E-04	3.200E 00	1.991E 00	9.801E-01	4.695E-02	-8.627E-02	2.940E 01
1.018E-03	4.000E 00	2.784E 00	9.977E-01	6.894E-03	-1.386E-02	2.993E 01
1.273E-03	5.000E 00	3.783E 00	1.000E 00	-1.782E-05	3.560E-05	3.000E 01
1.528E-03	6.000E 00	4.783E 00	1.000E 00	2.059E-09	5.272E-08	3.000E 01

TIME 0.0 SECONDS * * * STREAMWISE DIMENSION 0.40000E 00 FEET * * * *

ITS	ITERATED VALUES TIME	FPPW	DAMP		MAX LINEAR ERROR		MAX MOMENTUM ERROR
1	0.0	0.4695	1.0000	3	9.537E-07	8	9.759E-06

EDGE VELOCITY (FT/SEC)	STATIC PRESSURE (ATM)	XI (LB/ SEC)**2	BETA	FLUX NORM PARAMETER	ROKAP (FT)
3.000E 01	9.995E-01	3.414E-07	0.0	1.033E-03	1.000E 00

WALL FR. COEFF, CF	MOM TRANS COEFF, RHO*UE*CF/2	WALL SHEAR (LB/SQ FT)	MASS FLUXES MECH REM (LB/SQ FT SEC)	TOTAL GAS
4.226E-04	4.849E-04	4.521E-04	-0.0	0.0

REYNOLDS NUMBER	STROUHAL NUMBER	MOMENTUM THICKNESS, THETA (FT)	DISPLACE. THICKNESS, DELSTAR (FT)	SHAPE FACTOR, DELSTAR/ THETA
2.469E 06	0.0	1.690E-04	4.381E-04	2.592E 00

NODAL INFORMATION

DISTANCE FROM WALL (FT)	ETA	STREAM FUNCTION (F)	VELOCITY RATIO (FP=U/UE)	SHEAR FUNCTION (FPP)	FPPP	VELOCITY (FT/SEC)
0.0	0.0	0.0	0.0	4.695E-01	6.420E-03	0.0
2.880E-04	8.000E-01	1.495E-01	3.714E-01	4.511E-01	-5.230E-02	1.114E 01
5.761E-04	1.600E 00	5.829E-01	6.976E-01	3.420E-01	-2.205E-01	2.093E 01
8.641E-04	2.400E 00	1.232E 00	9.011E-01	1.676E-01	-2.154E-01	2.703E 01
1.152E-03	3.200E 00	1.991E 00	9.801E-01	4.695E-02	-8.627E-02	2.940E 01
1.440E-03	4.000E 00	2.784E 00	9.977E-01	6.894E-03	-1.387E-02	2.993E 01
1.800E-03	5.000E 00	3.783E 00	1.000E 00	-1.739E-05	4.356E-05	3.000E 01
2.160E-03	6.000E 00	4.783E 00	1.000E 00	4.119E-09	-8.768E-06	3.000E 01

TIME 0.0 SECONDS * * * STREAMWISE DIMENSION 0.60000E 00 FEET * * * *

ITS	ITERATED VALUES TIME	FPPW	DAMP		MAX LINEAR ERROR		MAX MOMENTUM ERROR
1	0.0	0.4695	1.0000	3	9.537E-07	8	-2.644E-05

EDGE VELOCITY (FT/SEC)	STATIC PRESSURE (ATM)	XI (LB/ SEC)**2	BETA	FLUX NORM PARAMETER	ROKAP (FT)
3.000E 01	9.995E-01	5.122E-07	0.0	8.434E-04	1.000E 00

WALL FR. COEFF, CF	MOM TRANS COEFF, RHO*UE*CF/2	WALL SHEAR (LB/SQ FT)	MASS FLUXES MECH REM (LB/SQ FT SEC)	MASS FLUXES TOTAL GAS
3.450E-04	3.959E-04	3.692E-04	-0.0	0.0

REYNOLDS NUMBER	STROUHAL NUMBER	MOMENTUM THICKNESS, THETA (FT)	DISPLACE. THICKNESS, DELSTAR (FT)	SHAPE FACTOR, DELSTAR/ THETA
3.703E 06	0.0	2.070E-04	5.365E-04	2.592E 00

NODAL INFORMATION

DISTANCE FROM WALL (FT)	ETA	STREAM FUNCTION (F)	VELOCITY RATIO (FP=U/UE)	SHEAR FUNCTION (FPP)	FPPP	VELOCITY (FT/SEC)
0.0	0.0	0.0	0.0	4.695E-01	6.350E-03	0.0
3.528E-04	8.000E-01	1.495E-01	3.713E-01	4.511E-01	-5.224E-02	1.114E 01
7.055E-04	1.600E 00	5.829E-01	6.976E-01	3.420E-01	-2.206E-01	2.093E 01
1.058E-03	2.400E 00	1.232E 00	9.011E-01	1.676E-01	-2.153E-01	2.703E 01
1.411E-03	3.200E 00	1.991E 00	9.801E-01	4.695E-02	-8.632E-02	2.940E 01
1.764E-03	4.000E 00	2.784E 00	9.977E-01	6.891E-03	-1.383E-02	2.993E 01
2.205E-03	5.000E 00	3.783E 00	1.000E 00	-1.543E-05	1.958E-05	3.000E 01
2.646E-03	6.000E 00	4.783E 00	1.000E 00	8.235E-09	1.131E-05	3.000E 01

TIME 0.0 SECONDS * * * STREAMWISE DIMENSION 0.80000E 00 FEET * * * *

ITERATED VALUES

ITS	TIME	FPPW	DAMP		MAX LINEAR ERROR		MAX MOMENTUM ERROR
1	0.0	0.4695	1.000	4	9.537E-07	8	-6.526E-05

EDGE VELOCITY (FT/SEC)	STATIC PRESSURE (ATM)	XI (LB/ SEC)**2	BETA	FLUX NORM PARAMETER	ROKAP (FT)
3.000E 01	9.995E-01	6.829E-07	0.0	7.304E-04	1.000E 00

WALL FR. COEFF, CF	MOM TRANS COEFF, RHO*UE*CF/2	WALL SHEAR (LB/SQ FT)	MASS FLUXES MECH REM (LB/SQ FT SEC)	MASS FLUXES TOTAL GAS (LB/SQ FT SEC)
2.988E-04	3.429E-04	3.197E-04	-0.0	0.0

REYNOLDS NUMBER	STROUHAL NUMBER	MOMENTUM THICKNESS, THETA (FT)	DISPLACE. THICKNESS, DELSTAR (FT)	SHAPE FACTOR, DELSTAR/ THETA
4.937E 06	0.0	2.390E-04	6.195E-04	2.592E 00

NODAL INFORMATION

DISTANCE FROM WALL (FT)	ETA	STREAM FUNCTION (F)	VELOCITY RATIO (FP=U/UE)	SHEAR FUNCTION (FPP)	FPPP	VELOCITY (FT/SEC)
0.0	0.0	0.0	0.0	4.695E-01	6.411E-03	0.0
4.073E-04	8.000E-01	1.495E-01	3.714E-01	4.511E-01	-5.229E-02	1.114E 01
8.147E-04	1.600E 00	5.829E-01	6.976E-01	3.420E-01	-2.205E-01	2.093E 01
1.222E-03	2.400E 00	1.232E 00	9.011E-01	1.676E-01	-2.154E-01	2.703E 01
1.629E-03	3.200E 00	1.991E 00	9.801E-01	4.694E-02	-8.632E-02	2.940E 01
2.037E-03	4.000E 00	2.784E 00	9.977E-01	6.894E-03	-1.380E-02	2.993E 01
2.546E-03	5.000E 00	3.783E 00	1.000E 00	-1.645E-05	-1.945E-05	3.000E 01
3.055E-03	6.000E 00	4.783E 00	1.000E 00	1.647E-08	5.239E-05	3.000E 00

TIME 0.0 SECONDS * * * STREAMWISE DIMENSION 0.10000E 01 FEET * * * *

ITS	ITERATED VALUES TIME	FPPW	DAMP	MAX LINEAR ERROR		MAX MOMENTUM ERROR	
1	0.0	0.4695	1.0000	3	9.537E-07	7	-4.801E-05

EDGE VELOCITY (FT/SEC)	STATIC PRESSURE (ATM)	XI (LB/ SEC)**2	BETA	FLUX NORM PARAMETER	ROKAP (FT)
3.000E 01	9.995E-01	8.536E-07	0.0	6.533E-04	1.000E 00

WALL FR. COEFF, CF	MOM TRANS COEFF, RHO*UE*CF/2	WALL SHEAR (LB/SQ FT)	MASS FLUXES MECH REM (LB/SQ FT SEC)	MASS FLUXES TOTAL GAS (LB/SQ FT SEC)
2.673E-04	3.067E-04	2.860E-04	-0.0	0.0

REYNOLDS NUMBER	STROUHAL NUMBER	MOMENTUM THICKNESS, THETA (FT)	DISPLACE. THICKNESS, DELSTAR (FT)	SHAPE FACTOR, DELSTAR/ THETA
6.172E 06	0.0	2.672E-04	6.926E-04	2.592E 00

NODAL INFORMATION

DISTANCE FROM WALL (FT)	ETA	STREAM FUNCTION (F)	VELOCITY RATIO (FP=U/UE)	SHEAR FUNCTION (FPP)	FPPP	VELOCITY (FT/SEC)
0.0	0.0	0.0	0.0	4.695E-01	6.396E-03	0.0
4.554E-04	8.000E-01	1.495E-01	3.714E-01	4.511E-01	-5.231E-02	1.114E 01
9.108E-04	1.600E 00	5.829E-01	6.976E-01	3.420E-01	-2.205E-01	2.093E 01
1.366E-03	2.400E 00	1.232E 00	9.011E-01	1.676E-01	-2.154E-01	2.703E 01
1.822E-03	3.200E 00	1.991E 00	9.801E-01	4.694E-02	-8.629E-02	2.940E 01
2.277E-03	4.000E 00	2.784E 00	9.977E-01	6.897E-03	-1.381E-02	2.993E 01
2.846E-03	5.000E 00	3.783E 00	1.000E 00	-1.935E-05	-2.210E-05	3.000E 01
3.416E-03	6.000E 00	4.783E 00	1.000E 00	3.293E-08	6.087E-05	3.000E 01

COMPUTER PROGRAM

```
C       UNSTEADY BOUNDARY LAYER INTEGRAL MATRIX PROCEDURE
C                              (UBLIMP)

      REAL MU
      COMMON/BUMCOM/BUMP,CORMA,EASE,ICORM,WDOT,I777,ISP,IX
      COMMON/EDGCOM/PEM(50),PTE(50),DUES,UE(50,50),PE(50,50)
     1,RHO,MU,T,DSIP(50),IDSIP
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KP17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
      COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSF,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/PRPCOM/PR,SC,XM(5)
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
      CALL ZEROIT
    1 FORMAT(A1)
    2 FORMAT(F10.4)
    3 FORMAT(1H1,///////////////5X10HRUN TIME = F8.2,8H SECO
     1NDS )
      DATA IAST/1H,/
      DATA LAST/1H./
      KIN=4
      KOUT=8
      B(1)=.5
      B(2)=.333333333
      B(3)=.166666666
      B(4)=.125
      B(5)=.041666666
      B(6)=.033333333
      B(7)=.013888888
      B(8)=.003968254
      RHO=.07651
      MU=3.719E-07
      IT=1
   46 MWE=-1
      EASE=1.
      I777=0
      IS=1
   41 ITEM=1
   42 CALL SETUP
   43 CALL ITERAT
      CALL OUTPUT
      IF(NON)43,44,40
   44 ITEM=ITEM+1
      DO 49 I=1,NETA
      DO 49 J=1,5
      FTM2(J,I)=FTM1(J,I)
      FTM1(J,I)=F(J,I)
   49 CONTINUE
      IF(ITEM-NITEM) 42,42,45
   45 IS=IS+1
      IF(IS-NS) 41,41,40
   40 READ(KIN,1) JAST
      IF(IAST-JAST) 47,46,47
   47 IF(LAST-JAST) 40,48,40
   48 STOP
      END


      SUBROUTINE ZEROIT
      COMMON/BUMCOM/BUMCO(8,1)
      COMMON/EDGCOM/EDGCO(5,1),EDGC(5,10),EDG(1,100),ED(5,10
     100)
      COMMON/ERRCOM/ERRCO(5,1),ERRC(2,10),ERR(3,100)
      COMMON/ETACOM/ETACO(9,1),ETAC(8,10),ETA(2,100),ET(2,10
```

```
  100)
      COMMON/HISCOM/HISCO(6,1),HISC(3,10),HIS(4,100)
      COMMON/INTCOM/INTCO(5,1),INTC(7,10)
      COMMON/NONCOM/NONC(2,10),NON(8,100)
      COMMON/PRMCOM/PRMCO(8,1),PRMC(7,10),PRM(2,100)
      COMMON/PRPCOM/PRPCO(7,1)
      COMMON/TEMCOM/TEMCO(5,1),TEMC(3,10),TEM(4,100)
      COMMON/VARCOM/VAPCO(5,1),VARC(7,10),VAR(3,100)
      COMMON/WALCOM/WALCO(3,1),WAL(1,100)
      Z=0.0
      DO 3 I=1,1000
      DO 1 J=1,2
      ET(J,I)=Z
    1 CONTINUE
      DO 2 J=1,5
      ED(J,I)=Z
    2 CONTINUE
    3 CONTINUE
      DO 8 I=1,100
      EDG(1,I)=Z
      WAL(1,I)=Z
      DO 4 J=1,2
      ETA(J,I)=Z
      PRM(J,I)=Z
    4 CONTINUE
      DO 5 J=1,3
      ERR(J,I)=Z
      VAR(J,I)=Z
    5 CONTINUE
      DO 6 J=1,4
      HIS(J,I)=Z
      TEM(J,I)=Z
    6 CONTINUE
      DO 7 J=1,8
      NON(J,I)=Z
    7 CONTINUE
    8 CONTINUE
      DO 14 I=1,10
      DO 9 J=1,2
      ERRC(J,I)=Z
      NONC(J,I)=Z
    9 CONTINUE
      DO 10 J=1,3
      HISC(J,I)=Z
      TEMC(J,I)=Z
   10 CONTINUE
      DO 11 J=1,5
      EDGC(J,I)=Z
   11 CONTINUE
      DO 12 J=1,7
      INTC(J,I)=Z
      PRMC(J,I)=Z
      VARC(J,I)=Z
   12 CONTINUE
      DO 13 J=1,8
      ETAC(J,I)=Z
   13 CONTINUE
   14 CONTINUE
      DO 15 J=1,3
      WALCO(J,1)=Z
   15 CONTINUE
      DO 20 J=1,6
      HISCO(J,1)=Z
   20 CONTINUE
      DO 16 J=1,5
      EDGCO(J,1)=Z
      ERRCO(J,1)=Z
      INTCO(J,1)=Z
      TEMCO(J,I)=Z
      VAPCO(J,1)=Z
   16 CONTINUE
      DO 21 J=1,7
```

```
      PRPCO(J,1)=Z
   21 CONTINUE
      DO 18 J=1,8
      BUMCO(J,1)=Z
      PRMCO(J,1)=Z
   18 CONTINUE
      DO 19 J=1,9
      ETACO(J,1)=Z
   19 CONTINUE
      RETURN
      END


      SUBROUTINE SETUP
      REAL MU
      DIMENSION HIST1(278),HIST2(51),HIST3(375),HIST4(100),V
     1MAT(158)
      COMMON/EDGCOM/PEM(50),PTE(50),DUES,UE(50,50),PE(50,50)
     1,RHO,MU,T,DSIP(50),IDSIP
      COMMON/HISCOM/TZ,T1,T2,TF(25),C1,C2,C3,C4,BETA,ZM(5,25
     1),XI(50),HF(25,5),HUE,HHUE,DLX2,C3M(50),BETAM(50)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,FPS
      COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
      COMMON/WALCOM/FW(50),RHOVW(50),IFW,IRHOVW,JRHOVW
      EQUIVALENCE (HIST1,XI),(HIST2,PTE),(HIST3,F),(HIST4,FW
     1),(VMAT,TZ)
    2 FORMAT(1H18X4HTIMEE12.5,56H SECONDS - - - - - - - - -
     1- - - - - - - - - - - - - - - - - -,6X,A5,3X,A2,1H.,A2)
    3 FORMAT(1H1,3X4HTIME,E12.5,34HSECONDS * * * STREAMWISE
     1DIMENSION , E12.5,13H FEET * * * * /)
    4 FORMAT(1H18X4HCASEE12.5,56H - - - - - - - - - - - - - -
     1- - - - - - - - - - - - - - - - - -,6X,A5,3X,A2,1H.,A2)
    5 FORMAT(1H1,2X12H* * * * CASE ,E12.5,27H * * * STREAMWI
     1SE DIMENSION ,E12.5,13H FEET * * * * /)
 7771 FORMAT(1P8E10.3)
 9901 FORMAT(I3,7E10.3)
      J=MOD(ITEM,2)+1
      IF(MWE)101,154,154
  101 CALL RECASE
      NBT=3
      NBT2=9
      IS=1
      IT=1
      IF(KR(1)) 104,104,103
  103 CALL LINMAT
  104 KR17=KR(17)
  154 IF(IS+ITEM-2) 153,153,1572
 1572 IF(NITEM-1)1574,1262,1574
 1574 WRITE(NBT)HIST1,HIST2,HIST3,HIST4,VMAT
      IF(ITEM-1) 157,156,157
  156 IDUM=NBT
      NBT=NBT2
      NBT2=IDUM
      REWIND NBT
      REWIND NBT2
      GO TO 155
  157 IF(IS-1) 153,153,155
  155 READ(NBT2)HIST1,HIST2,HIST3,HIST4,VMAT
      GO TO 1262
  153 CONTINUE
      IF(IT-1) 105,105,106
C     INITIAL GUESSES FOR PRINCIPAL DEPENDENT VARIABLES.
C     CALCULATE (KR(2)=0), INPUT (KR(2)=1), OR USE VALUES
C     FROM PREVIOUS CASE (KR(2)=2). NOTE: LATTER REQUIRES
C     SAME ETA VALUE. ITS UTILITY IS FOR REPEATED SIMILARITY
C     SOLUTIONS. IT OBVIOUSLY CANNOT BE USED FOR FIRST CASE.
  105 CALL FIRSTG
```

```
  106 CALL REFCON
      KR(2)=2
      IS=1
 1262 IF(TIME(1))1053,1054,1054
 1053 TIME(1)=-TIME(1)
      WRITE(KOUT,5)TIME(ITEM),S(IS)
      TIME(1)=-TIME(1)
      GO TO 126
 1054 WRITE(KOUT,3)TIME(ITEM),S(IS)
      WRITE(6,3)TIME(ITEM),S(IS)
C     COMPUTE HISTORIC INFORMATION
  126 CALL HISTXI
      MWF=0
C     START OF ITERATION LOOP
  158 ITS=0
      KR(17)=KR17
  159 RETURN
      END


      SUBROUTINE RECASE
      REAL MU
      COMMON/EDGCOM/REM(50),PTE(50),DUES,UE(50,50),PE(50,50)
     1,RHO,MU,T,DSIP(50),IDSIP
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
      COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/PRPCOM/PR,SC,XM(5)
      COMMON/WALCOM/FW(50),RHOVW(50),IFW,IRHOVW,JRHOVW
    1 FORMAT(1H1///////15X49HUNSTEADY BOUNDARY LAYER INTEGRA
     1L MATRIX PROCEDURE //
     127X25HDEPARTMENT OF AERONAUTICS /27X25HNAVAL POSTGRADU
     2ATE SCHOOL /30X20HMONTEREY, CALIFORNIA //32X16HINVESTI
     3GATION OF //10X15A4,//32X15
     4HCONTROL NUMBERS /11X58H1  2  3  4  5  6  7  8  9 10 1
     51 12 13 14 15 16 17 18 19 20 /9X20I3//35X10HETA VALUES
     6 /4(5X1P7E10.3/)/)
    2 FORMAT(20X17HNOSE RADIUS      = F10.5,13H FT          )
    3 FORMAT(20X17HCONE HALF ANGLE  = F10.5,13H DEG         )
    4 FORMAT(20X17HDENSITY          = F10.8,13H LB/FT CU    )
    5 FORMAT(20X17HVISCOSITY        = 1PE10.3,13H LB SEC/FT S
     1Q )
    6 FORMAT(20X17HTIME GRID        = 1PE10.3,13H SEC
     1   )
    7 FORMAT(20X17HCYCLE TIME       = 1PE10.3,13H SEC
     1   )
    8 FORMAT(1H1//////////31X18HSTATION INPUT DATA //1X11HDI
     1STANCE,FT   ,12X5(1X1PE10.3)/9(24X5(1X1PE10.3)/))
    9 FORMAT(1X5HROKAP ,18X5(1X1PE10.3)/9(24X5(1X1PE10.3)/))
   10 FORMAT(1X20HPRESSURE RATIO, MEAN ,3X/9(24X5(1X1PF10.3)
     15(1X1PE10.3)/))
   11 FORMAT(/5X68HTEMPERATURE          TOTAL             MEAN
     1         OSCIL.          OSCIL. /21X53HPRESSURE        VELO
     2CITY          AMPLITUDE FREQUENCY /7X66H(DEG R)          (A
     3TM)          (FT/SEC)          PARAMETER          (HERTZ) /5(5X
     41PE10.3)///)
   12 FORMAT(1X21HENTROPY DROP,BTU/LB R ,2)/9(24X5(1X1PE10.3
     1X5(1X1PE10.3)/))
   13 FORMAT(1X22HMASS FLUX,LB/SEC FT**2 ,1X5(1X1PE10.3)/9(2
     14X5(1X1PE10.3)/))
      IF (ITEM-1)302,302,194
  302 CALL INPUT
      WRITE(KOUT,1)CASE,KR,(ETA(JN),JN=1,NETA)
      WRITE(KOUT,11)T,PTET,UMFS,EPS,FREQ
      NITEM=NTIME
      NTIME=1
C     COMPUTE INFORMATION NEEDED TO CONSIDER DISCONTINUITIES
```

```
 303 J=1
 102 MSD(1)=1
     IDISC(1)=0
     S(1)=ABS(S(1))
     IF (NS-1) 105,105,1021
1021 DO 101 IS=2,NS
     IDISC(IS)=0
     IF(S(IS)) 103,101,101
 103 NSD(J)=IS-MSD(J)+1
     S(IS)=-S(IS)
     IDISC(IS)=1
     J=J+1
     MSD(J)=IS
 101 CONTINUE
 105 NSD(J)=NS-MSD(J)+1
     NDISC=J-1
     IF(IABS(KR(6)-2)-1)207,208,207
 207 IF(NS-1)234,234,2071
2071 IIS=2
     LNZ=1
     IF(ROKAP(1)) 223,226,226
 223 RADNO=-ROKAP(1)
     ROKAP(1)=0.
     DO 229 IS=2,NS
     IF(ROKAP(IS)) 224,224,225
 224 IF(KR(6)) 221,221,222
 222 ROKAP(IS)=S(IS)*SIN(RADNO/57.29578)
     GO TO 229
 221 ROKAP(IS)=RADNO*SIN(S(IS)/RADNO)
 229 CONTINUE
     GO TO 234
 225 IF (IS-NS) 2251,234,234
2251 IIS=IS+1
     LNZ=IS
 226 DO233 IS=IIS,NS
     IF(ROKAP(IS)) 233,233,227
 227 IF(IS-1-LNZ) 232,232,228
 228 LNZ=LNZ+1
     ROKAP(LNZ)=ROKAP(LNZ-1)+(S(LNZ)-S(LNZ-1))/(S(IS)-S(LNZ
    1-1))*(ROKAP(IS)-ROKAP(LNZ-1))
     GO TO 227
 232 LNZ=IS
 233 CONTINUE
 234 VKAP=1.
     GO TO 210
 208 DO 209 IS=1,NS
 209 ROKAP(IS)=1.
     VKAP=0.
 210 CONTINUE
 181 STEF = .481E-12
     IF(KR(6)-1)182,182,183
 182 WRITE(KOUT,2)RADNO
     WRITE(KOUT,3)CONE
 183 WRITE(KOUT,4)RHO
     WRITE(KOUT,5)MU
     WRITE(KOUT,6)DELT
     WRITE(KOUT,7)CYCLE
     WRITE(KOUT,8)(S(I),I=1,NS)
     WRITE(KOUT,9)(ROKAP(I),I=1,NS)
     WRITE(KOUT,10)(PPF(I),I=1,NS)
     WRITE(KOUT,12)(DSIP(I),I=1,NS)
     WRITE(KOUT,13)(RHOVW(I),I=1,NS)
 184 RETURN
     END


     SUBROUTINE INPUT
     REAL MU
     COMMON/EDGCOM/PEM(50),PTE(50),DUES,UE(50,50),PE(50,50)
    1,RHO,MU,T,DSIP(50),IDSIP
     COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
    1B2(24),LAR(100),BA1(73,28)
```

```
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMES,FREQ,EPS
      COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
      COMMON/WALCOM/FW(50),RHOVW(50),IFW,IRHOVW,JRHOVW
    1 FORMAT(20I1,15A4)
    2 FORMAT(2I3,E10.4)
    3 FORMAT(8E10.4)
    4 FORMAT(5(2XE14.7))
      READ (KIN,1)KR,CASE
      READ (KIN,2)NTIME
      READ (KIN,2)NS
      READ (KIN,3)(S(IS),IS=1,NS)
      IF (KR(1))103,103,100
  100 IF(KR(12))101,101,102
  101 READ (KIN,2)NETA
      READ (KIN,3)(ETA(I),I=1,NETA)
      GO TO 103
  102 CALL STDETA
  103 IF (KR(6)-1)104,104,105
  104 READ (KIN,3)CONE,RNOSE
  105 IF(IABS(KR(6)-2)-1)106,107,106
  106 READ (KIN,3)(ROKAP(IS),IS=1,NS)
  107 READ (KIN,3)PTET
 1071 IF (KR(2)-1)112,110,112
  110 READ (KIN,4)((F(I,J),J=1,NETA),I=1,4)
  112 IF(ITEM-1)115,113,115
  113 IDSIP=1
      IPRE=1
      DO 114 I=1,NS
  114 DSIP(I)=0.
  115 IF(KR(5)-5)118,116,118
  116 IF(IDSIP-ITEM)118,117,118
  117 READ (KIN,3)(DSIP(I),I=1,NS)
      READ (KIN,2)IDSIP
  118 IF (IPRE-ITEM)120,119,120
  119 READ (KIN,4)(PRE(I),I=1,NS)
      READ (KIN,2)IPRE
  120 JRHOVW=0
      IF(ITEM-1)121,121,122
  121 IFW=1
      IRHOVW=1
  122 READ (KIN,3)T
  129 IF(ITEM-1)130,130,131
  130 IF(KR(9)-1)131,135,138
  131 IF(IFW-ITEM)132,133,132
  132 IF(IFW-1)134,135,134
  133 READ (KIN,3) (FW(I),I=1,NS)
      READ (KIN,2)IFW
  134 GO TO 138
  135 IF(IRHOVW-ITEM)136,137,136
  136 GO TO 138
  137 READ (KIN,3) (RHOVW(I),I=1,NS)
      READ (KIN,2)IRHOVW
  138 READ(KIN,3)UMES,FREQ,EPS
      JRHOVW=0
      IF (FREQ)141,141,139
  139 CYCLE=1./FREQ
      BAG=FLOAT(NTIME)
      DELT=CYCLE/BAG
      TIME(1)=0.
      DO 140 I=2,NTIME
  140 TIME(I)=TIME(I-1)+DELT
      GO TO 142
  141 TIME(1)=0.
      DELT=0.
      CYCLE=0.
  142 CONTINUE
      IF(KR(16))143,145,143
```

```
143 DO 144 I=2,NS
144 PRE(I)=PRE(I)*RHO*UMFS*UMFS/136175.57+1.
145 RETURN
    END

    SUBROUTINE STDETA
    COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
   1B2(24),LAR(100),BA1(73,28)
    COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
   1,IT,NTIME,NAW,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
   2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
    IF(KR(12)-2)4,5,1
  1 IF(KR(12)-4)6,7,2
  2 IF(KR(12)-6)8,9,3
  3 IF(KR(12)-8)10,11,12
  4 NETA=8
    ETA(1)=0.0
    ETA(2)=0.5
    ETA(3)=1.0
    ETA(4)=1.5
    ETA(5)=2.0
    ETA(6)=3.0
    ETA(7)=5.0
    ETA(8)=6.0
    GO TO 13
  5 NETA=8
    ETA( 1)=0.00
    ETA( 2)=0.80
    ETA( 3)=1.60
    ETA( 4)=2.40
    ETA( 5)=3.20
    ETA( 6)=4.00
    ETA( 7)=5.00
    ETA( 8)=6.0
    GO TO 13
  6 NETA=10
    ETA( 1)=0.00
    ETA( 2)=0.60
    ETA( 3)=1.20
    ETA( 4)=1.80
    ETA( 5)=2.40
    ETA( 6)=3.00
    ETA( 7)=3.60
    ETA( 8)=4.20
    ETA( 9)=5.00
    ETA(10)=6.00
    GO TO 13
  7 NETA=12
    ETA( 1)=0.00
    ETA( 2)=0.50
    ETA( 3)=1.00
    ETA( 4)=1.50
    ETA( 5)=2.00
    ETA( 6)=2.50
    ETA( 7)=3.00
    ETA( 8)=3.50
    ETA( 9)=4.00
    ETA(10)=4.50
    ETA(11)=5.00
    ETA(12)=6.00
    GO TO 13
  8 NETA=14
    ETA( 1)=0.00
    ETA( 2)=0.40
    ETA( 3)=0.80
    ETA( 4)=1.20
    ETA( 5)=1.60
    ETA( 6)=2.00
    ETA( 7)=2.40
    ETA( 8)=2.80
    ETA( 9)=3.20
```

```
      ETA(10)=3.60
      ETA(11)=4.00
      ETA(12)=4.50
      ETA(13)=5.00
      ETA(14)=6.00
      GO TO 13
    9 NETA=15
      ETA(1)=0.00
      ETA(2)=0.20
      ETA(3)=0.40
      ETA(4)=0.60
      ETA(5)=0.80
      ETA(6)=1.00
      ETA(7)=1.20
      ETA(8)=1.40
      ETA(9)=1.60
      ETA(10)=2.00
      ETA(11)=2.50
      ETA(12)=3.25
      ETA(13)=4.00
      ETA(14)=5.00
      ETA(15)=6.00
      GO TO 13
   10 NETA=18
      ETA( 1)=0.00
      ETA( 2)=0.30
      ETA( 3)=0.60
      ETA( 4)=0.90
      ETA( 5)=1.20
      ETA( 6)=1.50
      ETA( 7)=1.80
      ETA( 8)=2.10
      ETA( 9)=2.40
      ETA(10)=2.70
      ETA(11)=3.00
      ETA(12)=3.30
      ETA(13)=3.60
      ETA(14)=3.90
      ETA(15)=4.20
      ETA(16)=4.60
      ETA(17)=5.00
      ETA(18)=6.00
      GO TO 13
   11 NETA=22
      ETA(1)=0.000
      ETA(2)=0.125
      ETA(3)=0.250
      ETA(4)=0.375
      ETA(5)=0.500
      ETA(6)=0.625
      ETA(7)=0.750
      ETA(8)=0.875
      ETA(9)=1.000
      ETA(10)=1.125
      ETA(11)=1.250
      ETA(12)=1.375
      ETA(13)=1.500
      ETA(14)=1.750
      ETA(15)=2.000
      ETA(16)=2.250
      ETA(17)=2.500
      ETA(18)=3.000
      ETA(19)=3.500
      ETA(20)=4.000
      ETA(21)=5.000
      ETA(22)=6.000
      GO TO 13
   12 NETA=25
      ETA( 1)=0.00
      ETA( 2)=0.20
      ETA( 3)=0.40
      ETA( 4)=0.60
```

```
      ETA( 5)=0.80
      ETA( 6)=1.00
      ETA( 7)=1.20
      ETA( 8)=1.40
      ETA( 9)=1.60
      ETA(10)=1.80
      ETA(11)=2.00
      ETA(12)=2.20
      ETA(13)=2.40
      ETA(14)=2.60
      ETA(15)=2.80
      ETA(16)=3.00
      ETA(17)=3.20
      ETA(18)=3.40
      ETA(19)=3.60
      ETA(20)=3.80
      ETA(21)=4.00
      ETA(22)=4.25
      ETA(23)=4.50
      ETA(24)=5.00
      ETA(25)=6.00
   13 RETURN
      END


      SUBROUTINE LINMAT
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
      COMMON/TEMCOM/SPDUM2(10),DER(50),DUMM1(25),SLOPE(25),R
     1EDUM(25),SDUM1 (50),SDUM2(50),FWDUM(50),XICON(50),FWCO
     2N(50),DUDS(50)
      DO 104 I=2,NETA
      DETA(I-1)=ETA(I)-ETA(I-1)
      DSQ(I-1)=DETA(I-1)*DETA(I-1)
      B1(I-1)=B(3)*DSQ(I-1)
      B2(I-1)=2.*B1(I-1)
  104 DCU(I-1)=DETA(I-1)*DSQ(I-1)
      MAT1I=3*NETA-2
      MAT1J=NETA+2
      DO 105 I=1,MAT1I
      DO 105 J=1,MAT1J
  105 BA1(I,J)=0.
      BA1(1,1)=1.
      BA1(MAT1I,2)=-1.
      DO 106 I=2,NETA
      IF (KR(10)) 1041,1041,1042
 1042 IF (I-NETA) 1044,1041,1044
 1041 DUM1=B(5)
      DUM2=B(4)
      DUM3=B(3)
      DUM4=B(2)
      DUM5=B(1)
      DUM6=B(1)
      GO TO 1043
 1044 DUM1=0.
      DUM2=B(3)
      DUM3=0.
      DUM4=B(1)
      DUM5=0.
      DUM6=1.
 1043 BA1 (I-1,I+1)=DUM2*DCU(I-1)
      BA1 (I-1,I+2)=DUM1*DCU(I-1)
      J=I+NETA-1
      BA1 (J,I+1)=DUM4*DSQ(I-1)
      BA1 (J,I+2)=DUM3*DSQ(I-1)
      J=J+NETA-1
      BA1 (J,I+1)=DUM6*DETA(I-1)
  106 BA1 (J,I+2)=DUM5*DETA(I-1)
 9060 FORMAT(2X1P12E10.3)
```

```
9064 FORMAT(2X37HLINEAR MATRIX FOR MOMENTUM EQUATIONS,I3,2H
    1 XI3,27H, BEFORE AND AFTER SOLUTION)
     IF(KR(15)-1) 9062,9062,9061
9061 WRITE(KOUT,9064) MAT1I,MAT1J
     DO 9063 I=1,MAT1I
9063 WRITE(KOUT,9060)(BA1(I,J),J=1,MAT1J)
9062 DO 107 LI=1,MAT1J
 107 CALL MATONE(BA1(1,LI))
     IF (KR(15)-1)9072,9072,9066
9066 DO 9067 I=1,MAT1I
9067 WRITE(KOUT,9060)(BA1(I,J),J=1,MAT1J)
9072 NLEQ=MAT1I
     NNLEQ=MAT1J
     NAM=NNLEQ-1
     NRNL=1
9902 CONTINUE
     RETURN
     END


     SUBROUTINE MATONE(E)
     DIMENSION E(1)
     COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
    1B2(24),LAR(100),BA1(73,28)
     COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
    1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
    2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
     I=MAT1I
     DO 201 J=3,NETA
     I=I-1
 201 E(I)=E(I)+E(I+1)
     I=NETA
     M=NETA+NETA-1
     K=0
     DO 202 J=2,NETA
     I=I+1
     M=M+1
     K=K+1
 202 E(I)=E(I-1)-E(I)+DETA(K)*E(M)
 203 E(1)=-E(1)+DETA(1)*E(K+1)+DSQ(1)/2.*E(I+1)
     I=I+1
     K=K+1
     M=1
     DO 204 J=3,NETA
     I=I+1
     K=K+1
     M=M+1
 204 E(M)=E(M-1)-E(M)+DETA(M)*E(K)+DSQ(M)/2.*E(I)
     RETURN
     END


     SUBROUTINE RERAY(N,C,D,NNN,IS,ND)
     DIMENSION D(ND,1),SD(28),C(ND,1),L(28),S(28),LL(28),LL
    1L(28)
     COMMON/INTCOM/KR(20),KIN,KOUT
C    DIRECT INVERSION PROCEDURE  --  C IS REPLACED BY C**-1
     N1=N+1
     DO 11 I=1,N
     LLL(I)=I
 113 L(I)=I
  11 CONTINUE
     IX=-1
     IF(IS+2) 111,109,111
 106 FORMAT(11H  L(I),I=1,I3,5X (30I3))
 107 FORMAT(15H  ((C(I,J),J=1,I3,12H),(D(J),J=1,I3, 6H),I=1
    1,I3,15H)  BEFORE RERAY)
 108 FORMAT(2X1P11E10.3/(12X1P10E10.3))
 109 WRITE(KOUT,107) N,NNN,N
     WRITE(KOUT,106)N,(L(I),I=1,N)
     IX=0
     DO 110 I=1,N
```

```
  110 WRITE(KOUT,108)(C(I,J),J=1,N),(D(I,J),J=1,NNN)
  111 IS=-1
C     TRIANGULATE MATRIX
      DO 15 I=1,N
      DO 160 M=1,N
  160 S(M)=ABS(C(I,M))
      IF(IS) 18,16,16
   18 IS=0
      GO TO 12
C     REDUCE ROW I BY PRECEEDING ROWS
   16 DO 17 J=2,I
      K=L(J-1)
      DIV=-C(I,K)
      IF(DIV) 161,17,161
  161 C(I,K)=0.
      DO162 M=1,N
      DIVC=DIV*C(J-1,M)
      S(M)=AMAX1(S(M),ABS(DIVC))
  162 C(I,M)=C(I,M)+DIVC
      IF(NNN) 17,17,163
  163 DO 164 M=1,NNN
  164 D(I,M)=D(I,M)+DIV*D(J-1,M)
   17 CONTINUE
C     SEEK MAXIMUM PIVOT
   12 DIV=0.
      DO 13 JJ=I,N
      M=L(JJ)
      IF(ABS (C(I,M))-DIV)13,13,121
  121 DIV=ABS (C(I,M))
      K=M
      J=JJ
   13 CONTINUE
      SD(I)=DIV/S(K)
      L(J)=L(I)
      L(I)=K
      IF(SD(I)-1.E-8) 130,130,14
  130 C(I,K)=0.
      IF(SD(I)) 131,131,12
C     SINGULAR MATRIX RETURN
  131 IS=-1
      WRITE(KOUT,151) (II,L(II),SD(II),II=1,I)
      RETURN
   14 DIV=C(I,K)
      C(I,K)=1.0
      K=LLL(J)
      LLL(J)=LLL(I)
      LLL(I)=K
      LL(K)=I
C     NORMALIZE ROW
      IF(NNN) 143,143,141
  141 DO 142 J=1,NNN
  142 D(I,J)=D(I,J)/DIV
  143 DO 15 J=1,N
   15 C(I,J)=C(I,J)/DIV
      IF(IX) 152,150,152
  151 FORMAT(24H PIVOT ROW/COL/RES.RATIO 5(I4,1H/I3,1H/E9.2,
     11H,))
  150 WRITE(KOUT,151) (I,L(I),SD(I),I=1,N)
C     DIAGONALIZE MATRIX
  152 NM=N-1
      DO 20 I=1,NM
      K=L(I+1)
      DO 20 J=1,I
      DIV=-C(J,K)
      IF(DIV)19,20,19
   19 C(J,K)=0.
      IF(NNN) 191,191,192
  192 DO 193 M=1,NNN
  193 D(J,M)=D(J,M)+DIV*D(I+1,M)
  191 DO 201 M=1,N
  201 C(J,M)=C(J,M)+DIV*C(I+1,M)
   20 CONTINUE
```

```
C      INTERCHANGE COLUMNS
       DO 30 II=1,N
       I=II
   21  J=L(I)
       L(I)=I
       IF(J-I)22,30,22
   22  IF(IS)25,23,25
   23  DO 24 M=1,N
       S(M)=C(M,I)
   24  C(M,I)=C(M,J)
       IS=I
       I=J
       GO TO 21
   25  IF(IS-J)26,28,26
   26  DO 27 M=1,N
   27  C(M,I)=C(M,J)
       I=J
       GO TO 21
   28  DO 29 M=1,N
   29  C(M,I)=S(M)
       IS=0
   30  CONTINUE
C      INTERCHANGE ROWS
       DO 40 II=1,N
       I=II
   31  J=LL(I)
       LL(I)=I
       IF(J-I)32,40,32
   32  IF(IS)35,33,35
   33  DO 34 M=1,N
       S(M)=C(I,M)
   34  C(I,M)=C(J,M)
       IF(NNN) 343,343,341
  341  DO 342 M=1,NNN
       SD(M)=D(I,M)
  342  D(I,M)=D(J,M)
  343  IS=I
       I=J
       GO TO 31
   35  IF(IS-J)36,38,36
   36  DO 37 M=1,N
   37  C(I,M)=C(J,M)
       IF(NNN) 373,373,371
  371  DO 372 M=1,NNN
  372  D(I,M)=D(J,M)
  373  I=J
       GO TO 31
   38  DO 39 M=1,N
   39  C(I,M)=S(M)
       IF(NNN) 393,393,391
  391  DO 392 M=1,NNN
  392  D(I,M)=SD(M)
  393  IS=0
   40  CONTINUE
       IF(IX) 411,409,411
  407  FORMAT(15H  ((C(I,J),J=1,I3,12H),(D(J),J=1,I3, 6H),I=1
     1,I3,15H)   AFTER REPAY   )
  409  WRITE(KOUT,407) N,NNN,N
       DO 410 I=1,N
  410  WRITE(KOUT,108)(C(I,J),J=1,N),(D(I,J),J=1,NNN)
  411  RETURN
       END

       SUBROUTINE FIRSTG
       COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
       COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
       COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
```

```
     2NE,DELT,CYCLE
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
      IF(KR(2)-1) 112,152,152
  112 DO 128 I=1,NETA
      F(1,I)=0.
      DUM1=ETA(I)-4.
      IF(DUM1) 115,116,116
  115 DUM1=-0.1*DUM1
      DUM2=0.25*ETA(I)
      F(2,I)=DUM2
      F(3,I)=DUM1
      GO TO 114
  116 F(2,I)=1.0
      F(3,I)=0.
  114 F(4,I)=0.
  128 CONTINUE
      IF(KR(20))152,152,200
  200 DO 208 I=1,NETA
      IF(ETA(I)-5.)201,201,202
  201 F(1,I)=ETA(I)
      F(2,I)=ETA(I)/5.
      F(3,I)=.1*(5.-ETA(I))
      GO TO 203
  202 F(1,I)=5.
      F(2,I)=1.
      F(3,I)=0.
  203 IF(ETA(I)-2.)204,204,205
  204 F(4,I)=-.1*ETA(I)
      GO TO 208
  205 IF(ETA(I)-5.)206,206,207
  206 F(4,I)=-.067*(5.-ETA(I))
      GO TO 208
  207 F(4,I)=0.
  208 CONTINUE
      F(1,1)=0.
      F(2,1)=0.
      F(2,NETA)=1.
      F(3,NETA)=0.
      F(4,NETA)=0.
  152 CONTINUE
 9901 FORMAT(4(2X7E10.3/))
 9904 FORMAT(2X19HDEBUG FIRST GUESSES)
      IF(KR(15)) 9902,9903,9902
 9902 CONTINUE
      WRITE(KOUT,9904)
      WRITE (KOUT,9901)((F(I,J),J=1,NETA),I=1,4)
 9903 CONTINUE
      RETURN
      END


      SUBROUTINE REFCON
      REAL MU
      COMMON/EDGCOM/PEM(50),PTE(50),DUES,UE(50,50),PE(50,50)
     1,RHO,MU,T,DSIP(50),IDSIP
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/HISCOM/TZ,T1,T2,TF(25),C1,C2,C3,C4,BETA,ZM(5,25
     1),XI(50),HF(25,5),HUE,HHUE,DLX2,C3M(50),BETAM(50)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
      COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/TEMCOM/SPDUM2(10),DER(50),DUMM1(25),SLOPE(25),R
     1EDUM(25),SDUM1 (50),SDUM2(50),FWDUM(50),XICON(50),FWCO
     2N(50),DUDS(50)
      COMMON/WALCOM/FW(50),RHOVW(50),IFW,IRHOVW,JRHOVW
    1 FORMAT(1H1//////////29X21HSTATION COMPUTED DATA /30X5H
     1TIME= 1PE10.3,4H SEC /)
    2 FORMAT(/5X68HSTREAMWISE          XI                BETA
```

```
     1        EDGE              STATIC /5X69HDIMENSION          (LB/
     2                                VELOCITY        PRESSURE /7X6
     35H(FEET)            SEC)**2                              (FT/SEC)
     4             (ATM)  /)
   3 FORMAT(5(5X1PE10.3))
9901 FORMAT(5(5X1PE10.3))
1532 IF(ITEM-1)605,1538,605
1538 IDSIP=1
     IPRE=1
     DO 1540 I=1,NS
1540 DSIP(I)=0.
1534 IF (KR(6)-3)1535,1541,1535
1535 IF (PRE(1))1541,1542,1541
1542 DO 1543 I=2,NS
     IF(PRE(I)) 1544,1543,1544
1544 L=I
     GO TO 1545
1543 CONTINUE
1545 RNOSE=S(L)/SQRT(1.-PRE(L))
     DO 1546 I=2,L
1546 PRE(I-1)=1.-S(I-1)/RNOSE*S(I-1)/RNOSE
1541 DO 1531 I=1,NS
     PTE(I)=PTET
1531 PEM(I)=PTET*PRE(I)
8002 PI=3.14159
     DO 8 I=1,NS
     DO 8 II=1,NITEM
     DPDS=(PTET-PEM(I))/RHO
     DPDS=136175.57*DPDS
     UE(I,II)=SQRT(DPDS)
     UE(I,II)=UE(I,II)*(1.+EPS*COS((FREQ*S(I)/UE(I,II))+
    1FLOAT(II)*2.*PI/FLOAT(NITEM)))
     PE(I,II)=PTET-(RHO*UE(I,II)**2./136175.57)
   8 CONTINUE
 205 IF(KP(15)) 9903,9904,9903
9903 CONTINUE
     WRITE(KOUT,9901)UE(IS,ITEM),PTE(IS),T,RHO,MU
9904 CONTINUE
C    FORCES DUDS TO ZERO FOR FLAT PLATE TEST.
     IF (KR(7))81,83,81
  81 DO 82 JS=2,NS
  82 UE(JS,ITEM)=UE(JS-1,ITEM)
  83 CONTINUE
C    END OF EDGE PROPERTY LOOP, START OF BETA AND XI
C    CALCULATION.
 605 XI(1)=0.
     J=NDISC+1
     DO 111 II=1,J
     K=NSD(II)
     M=MSD(II)
     MM=M+1
     LL=K+M-1
     IF (II-1) 6052,6052, 402
6052 IF (KP(6)-1) 400,401,402
C    AXISYMETRIC BLUNT
 400 DO 403 I=M,LL
     SDUM2(I)=S(I)*S(I)
     IF(S(I)) 403,403,4031
4031 DUM1=ROKAP(I)/S(I)
     XICON(I)=UE(I,ITEM)/S(I)*RHO/4.*MU*DUM1*DUM1
 403 SDUM1(I)=SDUM2(I)*SDUM2(I)
     QI=4.
     BETAM(1)=0.5
     GO TO 406
C    PLANAR BLUNT
 401 DO 404 I=M,LL
     SDUM2(I)=S(I)
     IF(S(I)) 404,404,4041
4041 XICON(I)=UE(I,ITEM)/S(I)*RHO/2.*MU
 404 SDUM1(I)=S(I)*S(I)
     QI=2.
     BETAM(1)=1.
```

```
      GO TO 406
  402 IF (KR(6)-2) 408,407,408
C     AXISYMETRIC  SHARP
  407 DO 409 I=M,LL
      SDUM2(I)=S(I)*S(I)
      DUM1=ROKAP(I)/S(I)
      XICON(I)=RHO*UE(I,ITEM)*MU*DUM1/3.*DUM1
  409 SDUM1(I)=S(I)*SDUM2(I)
      XI(1)      =XICON(1)*S(1)*S(1)*S(1)
      QI=3.
      IF (II-1) 4051,4051,406
C     PLANAR  SHARP
  408 DO 405 I=M,LL
      SDUM2(I)=S(I)
      XICON(I)=RHO*UE(I,ITEM)*MU*ROKAP(I)*ROKAP(I)
  405 SDUM1(I)=S(I)
      QI=1.
      IF (II-1) 4052,4052,406
 4052 XI(1)      =XICON(1)*S(1)
 4051 MM=M
      FW(1)=0.
  406 CONTINUE
      CALL SLOPQ(K,S(M),UE(M,ITEM),DUDS(M),DER(M))
C     FORCES DUDS TO ZERO FOR FLAT PLATE TEST.
      IF (KR(7))84,86,84
   84 DO 85 JS=1,NS
   85 DUDS(JS)=0.0
   86 CONTINUE
      IF (KR(6)-2) 4066,4062,4062
 4066 IF (M-1) 4061,4061,4062
 4061 IF (RNOSE) 4063,4063,4064
C     DUES COMPUTED BY SLOPQ
 4063 DUES=DUDS(1)
      GO TO 4065
C     DUES FROM EFFECTIVE NOSE RADIUS USING NEWTONIAN FLOW
 4064 DUES=SQRT(2./RHO*PE(1,ITEM)*32.17405*2116.2)/RNOSE
 4065 XICON(1)=RHO*MU/(2.*VKAP+2.)*DUES
 4062 CALL SLOPQ(K,SDUM1(M),XICON(M),DER(M),XI(M))
      IF (LL-MM) 111,4101,4101
 4101 DO 410 L=MM,LL
  410 BETAM(L)=2./QI*XI(L)/UE(L,ITEM)*S(L)/SDUM1(L)*DUDS(L)/
     1XICON(L)
  111 CONTINUE
 9803 FORMAT(8E10.4)
C     CALCULATION OF C3 MATRIX
      DO 138 I=1,NS
      IF (KR(6)-1) 137,137,158
  137 IF (I-1) 139,139,158
  139 C3M(I)=-SQRT(BETAM(I)/(DUES*RHO*MU))
      GO TO 138
  158 C3M(I)=-SQRT(2.*XI(I))/(RHO*ROKAP(I)*UE(I,ITEM)*MU)
  138 CONTINUE
      JRHOVW=1
 7043 IF(JRHOVW)7047,7047,7045
C     CALCULATE FW IF RHOVW GIVEN
 7045 J=NDISC+1
      DO 7046 II=1,J
      K=NSD(II)
      M=MSD(II)
      LL=K+M-1
      DO 209 I=M,LL
      IF (KR(8)) 7049,7049,2291
 7049 RHOVW(I)=RHOVW(I)*C3M(I)
 2291 IF (II-1) 7048,7048, 230
 7048 IF (KR(6)) 229,229,230
C     VALID AT AXISYMETRIC STAGNATION POINT ONLY
  229 FWCON(I)=-RHOVW(I)/(2.*C3M(I))
      IF( I-1) 209,209,232
C     MODIFICATION FOR AXISYMETRIC BLUNT AWAY FROM
C     STAGNATION POINT.
  232 FWCON( I)=FWCON( I)/S( I)*ROKAP( I)
      GO TO 209
```

```
C       VALID FOR ALL PLANAR
  230 FWCON(I)=-RHOVW(I)/C3M(I)
      IF(KR(6)-2) 209,236,209
C       MODIFICATION FOR AXISYMETRIC SHARP
  236 FWCON( I)=FWCON( I)/S( I)*ROKAP( I)/2.
  209 CONTINUE
      FWDUM(1)=FWCON(1)*S(1)
      IF(KR(6)-2) 241,237,241
C       MODIFICATION FOR AXISYMETRIC SHARP
  237 FWDUM(1)=FWDUM(1)*S(1)
  241 CONTINUE
 7046 CALL SLOPQ(K,SDUM2(M),FWCON(M),DER(M),FWDUM(M))
      DO 126 I =1,NS
      IF(I -1) 124,124,123
  124 IF (KR(6)-1) 133,133,123
  133 IF (S(I)) 113,113,123
  113 FW(1)=RHOVW(1)
      GO TO 126
  123 FW(I)=FWDUM(I)/SQRT(2.*XI(I))
  126 CONTINUE
 7047 WRITE(KOUT,1)TIME(ITEM)
      WRITE(KOUT,2)
      DO 7050 I=1,NS
 7050 WRITE(KOUT,3)S(I),XI(I),BETAM(I),UE(I,ITEM),PE(I,ITEM)
      RETURN
      END

      SUBROUTINE SLOPQ(N,X,Y,S,Z)
      DIMENSION X(1),Y(1),S(1),Z(1)
      IF(N-1) 9,9,8
    8 S(2)=(Y(2)-Y(1))/(X(2)-X(1))
      S(1)=S(2)
      QC=S(2)
      DO 7 I=1,N
      IF(I+1-N)2,1,6
    1 QB=QC
      IF (I-2)7,6,5
    2 XOT=X(I)-X(I+1)
      XTT=X(I+1)-X(I+2)
      XTO=X(I+2)-X(I)
      AA=Y(I)/(XOT*XTO)
      XOTT=XOT*XTT
      AB=Y(I+1)/XOTT
      AC=Y(I+2)/(XTT*XTO)
      AAA=AA*XTT
      ABB=AB*XTO
      ACC=AC*XOT
      QA=QC
      QB=S(I)
      QC=S(I+1)
      S(I)=AA*(XTO-XOT)+ABB-ACC
      S(I+1)=AB*(XOT-XTT)+ACC-AAA
      S(I+2)=AC*(XTT-XTO)+AAA-ABB
    3 IF(I-2)7,5,4
    4 S(I)=(S(I)+QA)/2.
    5 S(I)=(S(I)+QB)/2.
    6 XD=X(I)-X(I-1)
      YS=Y(I)+Y(I-1)
      SD=S(I)-S(I-1)
      SS=S(I)
      Z(I)=Z(I-1)+XD/2.*(YS-XD/6.*SD)
      S(I)=SS
    7 CONTINUE
    9 RETURN
      END

      SUBROUTINE HISTXI
      REAL MU
      DIMENSION FD(4)
      COMMON/EDGCOM/PEM(50),PTE(50),DUES,UE(50,50),PE(50,50)
```

```
      1,RHO,MU,T,DSIP(50),IDSIP
       COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
      1B2(24),LAR(100),BA1(73,28)
       COMMON/HISCOM/TZ,T1,T2,TF(25),C1,C2,C3,C4,BETA,ZM(5,25
      1),XI(50),HF(25,5),HUF,HHUE,DLX2,C3M(50),BETAM(50)
       COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
      1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
      2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
       COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
      1OSF,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
      2NF,DELT,CYCLE
       COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
C      INITIALIZE AXIAL VARIATION TERMS
       NUL=0
       DLX1=0.
       DO 120 I=1,4
  120  FD(I)=0.
       IF (KR(6)-1) 400,400,401
  400  IF (IS-3) 154,153,153
  401  IF(IS-2)154,153,153
  153  IF(KR(3)-1) 154,143,143
  143  DLX1=ALOG(XI(IS)/XI(IS-1))
       IF (KR(6)-1) 402,402,403
  402  IF (IS-3) 154,155,157
  403  IF(IS-2) 154,155,157
  157  IF(KR(3)-1) 154,155,144
  154  DZ=0.
       D1=0.
       D2=0.
       M=NETA-1
       DO 140 I=1,4
       DO 140 J=1,M
       ZM(I,J)=0.0
  140  CONTINUE
       DO 141 I=1,NETA
       DO 142 J=1,3
       HF(I,J)=0.
  142  CONTINUE
       HF(I,4)=0.
  141  HF(I,5)=0.
       DLX2=0.
       GO TO 130
C      COMPUTE TWO- OR THREE-POINT DIFFERENCE RELATIONS
  155  DZ=2./DLX1
       D1=-DZ
       D2=0.
       GO TO 145
  144  J=IDISC(IS-1)
       IF(J)121,121,155
  121  DZ=DLX1+DLX2
       D1=-DZ/(DLX1*DLX2)*2.
       D2=DLX1/(DZ*DLX2)*2.
       DZ=-D1-D2
  145  DLX2=DLX1
  122  FD(3)=D1*F(4,1)+D2*HF(1,4)
       DO 147 I=2,NETA
       FD(1)=D1*F(2,I)+D2*HF(I,2)
       FD(2)=D1*F(3,I)+D2*HF(I,3)
       FD(4)=FD(3)
       FD(3)=D1*F(4,I)+D2*HF(I,4)
  147  CALL TAYLOR(DETA(I-1),FD(2),FD(1),ZM(1,I-1))
C      SAVE HISTORIC VALUES
  162  DO 164 I=1,NETA
       HF(I,4)=F(4,I)
       HF(I,5)=D1*F(1,I)+D2*HF(I,1)
       DO 164 J=1,3
       HF(I,J)=F(J,I)
  164  CONTINUE
C      COMPUTE GROUPINGS WHICH DEPEND ON DZ
  130  C1=1.+DZ
       BETA=BETAM(IS)
       C2=-2.*(C1+BETA+DZ)
```

```
      C3=C3M(IS)
      C4=RHO*MU*C3*C3
C     COMPUTE TWO OR THREE POINT DIFFERENCE TIME DERIVATIVES
      IF(KR(6)-1)200,200,201
  200 IF(ITEM-2)202,203,204
  201 IF(ITEM-2)202,203,203
  202 TZ=0.
      T1=0.
      T2=0.
      GO TO 205
  203 TZ=1./DELT
      T1=-TZ
      T2=0.
      GO TO 205
  204 TZ=1.5/DELT
      T1=-2./DELT
      T2=.5/DELT
C     SAVE HISTORIC VALUES.
  205 DO 206 I=1,NETA
      TF(I)=T1*FTM1(1,I)+T2*FTM2(1,I)
  206 CONTINUE
  207 CONTINUE
 9904 FORMAT(6XI2,6XI2/(8X1P10E10.3))
      IF(KR(17)) 9905,9906,9905
 9905 CONTINUE
      WRITE(KOUT,9907)
 9907 FORMAT(2X26HDEBUG IS,DLX1 ... ZM,HF,TF)
      NETAM1=NETA-1
      WRITE(KOUT,9904) IS,ITEM,DLX1,DLX2,DZ,D1,D2,TZ,T1,T2,
     1C1,C2,C3,C4,FD,((ZM(I,J),J=1,NETAM1),I=1,4),((HF(I,J),
     2J=1,5),I=1,NETA)
 9906 CONTINUE
      RETURN
      END


      SUBROUTINE TAYLOR (D,FM,F,P)
      DIMENSION FM(1),F(1),P(1)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,,UMES,FREQ,EPS
      D2=D*D
      IF (KR(10)) 1,1,2
    2 IF (I-NETA) 4,1,4
    4 FD=0.
      P(1)=(((FM(3)/6.-FD/24.)*D-F(2)/2.)*D+F(1))*D
      P(2)=(((FD/30.-FM(3)/8.)*D+F(2)/3.)*D-F(1)/2.)*D2
      P(3)=0.
      P(4)=(((FM/20.-FD/72.)*D-F(2)/8.)*D+F(1)/6.)*D2*D-P(3)
     1(3)
      GO TO 3
    1 FD=F(3)-FM(3)
      P(1)=(((F(3)/6.-FD/24.)*D-F(2)/2.)*D+F(1))*D
      P(2)=(((FD/30.-F(3)/8.)*D+F(2)/3.)*D-F(1)/2.)*D2
      P(4)=-(((FD/252.-F(3)/72.)*D+F(2)/30.)*D-F(1)/24.)*D2*
     1D
      P(3)=(((F(3)/20.-FD/72.)*D-F(2)/8.)*D+F(1)/6.)*D2*D-P(
     14)
    3 CONTINUE
      RETURN
      END


      SUBROUTINE ITERAT
      COMMON/BUMCOM/BUMP,CORMA,EASE,ICORM,WDOT,I777,ISP,IX
      COMMON/ERRCOM/ELE(73),ELA(73),ELEM,ELM(3),ELMM,IELM,NE
     1LM,ILMM,DEL(73),ENLE(28),ENL(28),ENLEM,ENLM(3),ENLMM,I
     2ENLM,NENLM,INLMM,DENL(28),DRNL(6)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMES,FREQ,EPS
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
```

```
    1 FORMAT(22H NON-CONVERGENT OUTPUT /)
    5 FORMAT(6XI3,1XF9.3,2(1XF9.4),2(2XI3,2X1PF10.3))
    7 FORMAT(/10X62HITERATED VALUES           DAMP           MAX LI
     1NEAR        MAX MOMENTUM /6X62HITS      TIME          FPPW
     2                   ERROR                   ERROR)
      CLOCKB=0.
  181 ITS=ITS+1
      CALL LINCER
      NON=0
      CALL NONCER
      FPPW=F(3,1)
      IF(KR(4)+KR(17)+KR(19)+NON)189,189,192
  189 IF(ITS-1)192,192,194
  192 WRITE(KOUT,7)
  194 CLOCK1=0.
      BTIME=CLOCK1-CLOCKB
      BTIME=.01*BTIME
      CLOCKB=CLOCK1
      WRITE(KOUT,5)ITS,BTIME,FPPW,EASE,IFLM,ELMM,IFNLM,FNLEM
 1921 IF(ELMM+FNLMM+.001/(EASE*BUMP)-.002)162,162,159
  162 NON=0
      RETURN
  159 IF(ITS-50) 161,160,160
  160 WRITE(KOUT,1)
      WRITE(6,1)
      IF (ELMM+FNLMM-.02) 162,162,1601
 1601 NON=1
      RETURN
C     ITERATE OR OUTPUT
  161 IF(KR(4)) 181,181,193
  193 NON=-1
      RETURN
      END


      SUBROUTINE LINCER
      COMMON/ERRCOM/FLF(73),ELA(73),FLEM,ELM(3),ELMM,IFLM,NE
     1LM,ILMM,DFL(73),FNLE(28),ENL(28),FNLEM,ENLM(3),ENLMM,I
     2FNLM,NFNLM,INLMM,DFNL(28),DRNL(6)
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
      COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/TEMCOM/SPDUM2(10),DER(50),DUMM1(25),SLOPE(25),R
     1EDUM(25),SDUM1 (50),SDUM2(50),FWDUM(50),XICON(50),FWCO
     2N(50),DUDS(50)
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
C     EVALUATE LINEAR ERRORS FOR MOMENTUM AND ENERGY
      NELM=2
      NFNLM=NELM
      DO 401 I=2,NETA
      IF(KR(10)) 4000,4000,4001
 4001 IF (I-NETA) 4003,4000,4003
 4000 DUM1=B(4)
      DUM2=B(5)
      DUM3=B(2)
      DUM4=B(3)
      DUM5=B(1)
      DUM6=1.0
      GO TO 4002
 4003 DUM1=B(3)
      DUM2=0.
      DUM3=B(1)
      DUM4=0.
      DUM5=1.0
      DUM6=0.
 4002 FLF(I-1)=-(F(1,I-1)+DETA(I-1)*F(2,I-1)+DSQ(I-1)*B(1)*F
     1(3,I-1)+DCU(I-1)*(DUM1*F(4,I-1)+DUM2*F(4,I))-F(1,I))
```

```
      M=I+NETA-1
      FLE(M)=-(F(2,I-1)+DETA(I-1)*F(3,I-1)+DSQ(I-1)*(DUM3*F(
     14,I-1)+DUM4*F(J,I))-F(2,I))
      M=I+2*NETA-2
  401 FLE(M)=-(F(3,I-1)+DETA(I-1)*DUM5*(F(4,I-1)+DUM6*F(4,I)
     1)-F(3,I))
      FLE(NETA)=-F(2,1)
C     DETERMINE MAXIMUM LINEAR ERRORS
  404 CALL ABMAX(MAT1I,FLE,ELMM,IFLM)
C     FORM PRODUCT OF A**-1 AND LINEAR ERRORS
  469 CALL MATONE(FLE)
  473 CONTINUE
      RETURN
      END


      SUBROUTINE ABMAX(N,X,XM,I)
      DIMENSION X(1)
      I=1
      XM=ABS (X(1))
      IF (N-1) 4,4,5
    5 DO 3 J=2,N
      XT=ABS (X(J))
      IF(XM-XT) 2,3,3
    2 XM=XT
      I=J
    3 CONTINUE
    4 XM=X(I)
      RETURN
      END


      SUBROUTINE NONCER
      REAL MU
      DIMENSION CORAR(25)
      COMMON/BUMCOM/BUMP,CORMA,EASE,ICORM,WDOT,I777,ISP,IX
      COMMON/EDGCOM/PEM(50),ETE(50),DUES,UE(50,50),PE(50,50)
     1,RHO,MU,T,DSIP(50),IDSIP
      COMMON/ERRCOM/FLE(73),FLA(73),FLEM,ELM(3),ELMM,IFLM,NE
     1LM,ILMM,DEL(73),FNLE(28),ENL(28),FNLEM,ENLM(3),ENLMM,I
     2ENLM,NENLM,INLMM,DENL(28),DRNL(6)
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/HISCOM/T7,T1,T2,TE(25),C1,C2,C3,C4,BETA,ZM(5,25
     1),XI(50),HE(25,5),HUE,HHUE,DLX2,C3M(50),BETAM(50)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMES,FREQ,EPS
      COMMON/NONCOM/AM(28,28),DVNL(28),C5,C6,C7,C8,C9,C10,C1
     11,C12
      COMMON/PRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/PPPCOM/PP,SC,XM(5)
      COMMON/TEMCOM/SPDUM2(10),DER(50),DUMM1(25),SLOPE(25),R
     1EDUM(25),SDUM1 (50),SDUM2(50),FWDUM(50),XICON(50),FWCO
     2N(50),DUDS(50)
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
      COMMON/WALCOM/FW(50),RHOVW(50),IFW,IRHOVW,JRHOVW
      EQUIVALENCE (CORAR(1),AM(1))
      EASE=AMIN1(EASE*2.,1.)
      IF (ITS - 1) 10,5,10
    5 EASE=1.
      BUMP = 1.0
    4 ICORM=1
      CORMA = 1.E + 10
   10 CONTINUE
      IX=0
C     EVALUATE COEFFICIENTS AND ERRORS FOR NONLINEAR
C     EQUATIONS.
C     INITIALIZE AM MATRIX
      DO 15 I=1,NNLEQ
```

```
      FNLE(I)=0.
      DO 15 J=1,NNLEQ
   15 AM(I,J) = 0.
C     EVAL. GROUPINGS WHICH CHANGE DURING ITERATION BUT ARE
C     NOT F(ETA).
      IF(ITEM-2)30,31,32
   30 C5=0.
      GO TO 33
   31 C5=TZ*ALOG(UE(IS,ITEM))+T1*ALOG(UE(IS,ITEM-1))
      GO TO 33
   32 C5=TZ*ALOG(UE(IS,ITEM))+T1*ALOG1))+T2*ALOG(UE(IS,ITEM-
     1(UE(IS,ITEM-2))
   33 CONTINUE
C---- EVAL COEFS, AM, AND ERRORS FOR EDGE BOUN CONDS
C     NEXT,  EVALUATE NONLINEAR ERRORS
   45 FNLE(NETA+1)=-(1.-F(2,NETA))
      FNLE(NETA+2)=-F(3,NETA)
      FNLE(1)=FW(IS)-F(1,1)
C     NEXT, EVALUATE ORIGINAL AM COEFFICIENTS
      AM(1,1)=1.
      AM(NETA+2,2)=-1.
      FNLE(NETA+1)=FNLE(NETA+1)+FLE(2*NETA-1)
      DO 50 J=1,MAT1J
   50 AM(NETA+1,J)=AM(NETA+1,J)+BA1(2*NETA-1,J)
C     START OF MAJOR DO LOOP FOR EVAL OF COEFFS AND ERRORS
C     AT EACH STATION.
      DO 120 I=1,NETA
C     TEST TO BYPASS COMMANDS THAT CANNOT BE PERFORMED AT
C     ETA(1).
      IF (I-1)75,75,55
   55 CALL IMONE
C---- EVAL GROUPINGS WHICH ARE USED AT I-1 AS WELL AS AT I
   75 C6=HF(I,5)
      C7=F(3,I)+F(2,I)*(C1*F(1,I)+C6)
      C8=C4*(TZ-C5)
      C9=C1*F(2,I)-C8
      C10=C6+C1*F(1,I)
      C11=-C4*TF(I)
      C12=C9*F(1,I)
      IF (I-1) 100,105,100
C     BACK TO CONSERVATION EQUATIONS
  100 CALL IONLY
  105 IF (KR(17)) 120,120,115
  110 FORMAT(21H ALL THE COEFFICIENTS/(1X1P12E10.3))
  115 WRITE(KOUT,110)C1,C2,C3,C4,C5,C6,C7,C8,C9,C10,C11,C12
      IX=-2
  120 CONTINUE
  145 IF(KR(19)) 170,190,170
  170 CONTINUE
      WRITE(KOUT,175)
  175 FORMAT(2X10HDEBUG FNLE )
  180 FORMAT(/(12X1P10E10.3))
      WRITE(KOUT,180)(FNLE(J),J=1,MAT1I)
  190 CONTINUE
      DO 189 III=1,MAT1J
  189 ENL(III)=FNLE(III)
      CALL RERAY(MAT1J,AM,ENL,1,IX,28)
      IF (KR(17)) 245,265,245
  245 CONTINUE
      WRITE(KOUT,255)
  255 FORMAT(2X9HDEBUG FLE )
      WRITE(KOUT,180)(FLE(J),J=1,MAT1I)
  265 CONTINUE
C     DETERMINE MAXIMUM NONLINEAR ERRORS
      CALL ABMAX(MAT1J,FNLE,FNLEM,IFNLM)
      ELMM = ABS(ELMM)
      ENLMM=ABS(FNLEM)
C     EVALUATE NONLINEAR CORRECTIONS
      DO 615 I=1,MAT1J
  615 DVNL(I)=ENL(I)
  625 CONTINUE
C     EVALUATE LINEAR CORRECTIONS
```

```
      DO 630 I=1,MAT1I
      DO 630 J=1,MAT1J
  630 FLE(I) = FLE(I) - DVNL(J) * BA1(I,J)
      FLE(NETA)=0.
      DO 620 I=1,NETA
      L=I+NETA-1
  620 CORAR(I)=FLE(L)
  665 CONTINUE
      IF (.33 + CORAR(ICORM) / CORMA) 670,675,675
  670 IF (EASE-.2) 675,671,671
  671 BUMP = 2.*BUMP
  675 CALL ABMAX(NETA,CORAR,CORMA,ICORM)
      IF (KR(17)) 680,680,685
  680 IF (KR(19)) 690,705,690
  685 CONTINUE
      KR(17) = KR(17) - 1
  690 CONTINUE
  695 FORMAT(2X23HDEBUG CORRECTIONS FL,NL )
      WRITE(KOUT,695)
      WRITE(KOUT,180)(FLE(J),J=1,MAT1I)
      WRITE(KOUT,180)(DVNL(J),J=1,MAT1J)
  705 CONTINUE
C     CORRECT PRIMARY VARIABLES
      DUM=.1/BUMP
      EASE=AMIN1(3.*EASE,1.,DUM/ABS(CORMA))
  710 IF (KR(13)) 720,720,715
  715 DUM = KR(13)
      EASE = AMIN1(DUM / 10.,EASE)
  720 IF (EASE - 1.0) 725,740,740
  725 DO 730 I=1,MAT1I
  730 FLE(I) = FLE(I) * EASE
      DO 735 I=1,MAT1J
  735 DVNL(I) = DVNL(I) * EASE
  740 CONTINUE
  750 DO 765 J=2,NETA
      F(1,J) = F(1,J) + FLE(J - 1)
  765 CONTINUE
      M = NETA
      DO 780 J=1,NETA
      M = M + 1
      F(2,J) = F(2,J) + FLE(M - 1)
  780 CONTINUE
      DO 785 J=2,NETA
      M = M + 1
  785 F(3,J - 1) = F(3,J - 1) + FLE(M - 1)
  800 F(1,1)=F(1,1)+DVNL(1)
      F(3,NETA)=F(3,NETA)+DVNL(2)
      DO 835 J=1,NETA
      F(4,J)=F(4,J)+DVNL(J+2)
  835 CONTINUE
      IF (ITS - 49) 850,840,850
  840 IF (I777 - 777) 845,850,845
  845 I777 = 777
      ITS = 30
  850 CONTINUE
      RETURN
      END

      SUBROUTINE IMONE
      DIMENSION RRR(5)
      COMMON/ERRCOM/FLE(73),ELA(73),FLEM,ELM(3),ELMM,IFLM,NE
     1LM,ILMM,DFL(73),FNLE(28),ENL(28),FNLEM,ENLM(3),ENLMM,I
     2FNLM,NENLM,INLMM,DFNL(28),DRNL(6)
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAP(100),BA1(73,28)
      COMMON/HISCOM/TZ,T1,T2,TF(25),C1,C2,C3,C4,BETA,ZM(5,25
     1),XI(50),HF(25,5),HUE,HHUE,DLX2,C3M(50),BETAM(50)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
      COMMON/NONCOM/AM(28,28),DVNL(28),C5,C6,C7,C8,C9,C10,C1
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```
     11,C12
      COMMON/PRPCOM/PR,SC,XM(5)
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
C     EVALUATE GROUPINGS WHICH CONTRIBUTE TO (I-1) PORTION
C     OF COEFFS VARIABLES WITH DIMENSION (NETA-1)
      IF(I-2)401,4002,401
 4002 XM(5)=0.
C     EVALUATE XM, WHICH CONTRIBUTES TO ERRORS AND TO COEFFS
C     AT (I) AND (I-1)
  401 CALL TAYLOR (DETA(I-1),F(2,I-1),F(2,I),XM)
C     EVAL PORTION OF NLE DEPENDENT ON XM,... AND GROUPINGS
C     EVAL AT (I-1).
  403 C72=F(2,I)*XM(1)+F(3,I)*XM(2)+F(4,I)*XM(3)+F(4,I-1)*XM
     1(4)
      XM(5)=XM(5)+C72
      C73=F(2,I)*ZM(1,I-1)+F(3,I)*ZM(2,I-1)+F(4,I)*ZM(3,I-1)
     1+F(4,I-1)*ZM(4,I-1)
      FNLE(I)=-(-C7-C11+DETA(I-1)/2.*(BETA+C4*C5)+.5*C2*C72-
     12.*C73)
      RRR(1)=FNLE(I)
C     EVAL PORTION OF ORIG COEFFS OF AM DEPENDENT UPON PARAM
C     EVAL AT (I-1).
  405 AM(I,I+1)=C2*XM(4)-2.*ZM(4,I-1)
  407 IF(I-2) 418,417,418
  417 AM(2,1)=-C9
C     EVAL I-1 PORTION OF COEFS, AND FORM CONTRIBUTIONS TO
C     AM AND ERRORS.
  418 LPI=NETA-2+I
      DUM1=-C10
  421 CALL AMSET (DUM1,LPI,2,RRR)
  411 LPI=LPI+NETA
      DUM1=-1.
  423 CALL AMSET (DUM1,LPI,3,RRR)
      IF(I-2)424,425,424
  424 CALL AMSET (-C9,I-2,4,RRR)
  425 CONTINUE
      RRR(5)=FNLE(I)
 9999 FORMAT(2X,I5,2X1P5E12.4)
      IF(KR(17)) 9901,9902,9901
 9901 CONTINUE
      WRITE(KOUT,9001)
 9001 FORMAT(34H  DEBUG FNLE FROM IMONE PLUS COEFS  )
      WRITE(KOUT,9999)I,RRR
 9902 CONTINUE
      RETURN
      END


      SUBROUTINE AMSET (CAM,LPI,NN,RRR)
      DIMENSION RRR(5)
      COMMON/ERRCOM/FLE(73),ELA(73),FLEM,ELM(3),ELMM,IFLM,NE
     1LM,ILMM,DFL(73),FNLE(28),ENL(28),FNLEM,ENLM(3),ENLMM,I
     2FNLM,NENLM,INLMM,DFNL(28),DRNL(6)
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMFS,FREQ,EPS
      COMMON/NONCOM/AM(28,28),DVNL(28),C5,C6,C7,C8,C9,C10,
     1C11,C12
      FNLE(I)=FNLE(I)-CAM*FLE(LPI)
      IF(NN-1)2,1,1
    1 RRR(NN)=FNLE(I)
    2 DO 3 J=1,MAT1J
    3 AM(I,J)=AM(I,J)-CAM*BA1(LPI,J)
      RETURN
      END


      SUBROUTINE IONLY
      DIMENSION RRR(4)
      COMMON/ERRCOM/FLE(73),ELA(73),FLEM,ELM(3),ELMM,IFLM,NE
```

```
     1LM,ILMM,DEL(73),FNLE(28),ENL(28),FNLEM,ENLM(3),ENLMM,I
     2FNLEM,NENLM,INLMM,DFNL(28),DRNL(6)
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/HISCOM/T7,T1,T2,TE(25),C1,C2,C3,C4,BETA,ZM(5,25
     1),XI(50),HE(25,5),HUE,HHUE,DLX2,C3M(50),BETAM(50)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,,UMES,FREQ,EPS
      COMMON/NONCOM/AM(28,28),DVNL(28),C5,C6,C7,C8,C9,C10,
     1C11,C12
      COMMON/PRPCOM/PR,SC,XM(5)
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
C     ADD CONTRIBUTIONS OF I TO NONLINEAR ERRORS
C     EVALUATE GROUPINGS WHICH ARE USED ONLY AT I (NOT AT
C     I-1).
      FNLE(I)=FNLE(I)-(C7+DETA(I-1)/2.*(BETA+C4*C5)+C11)
      RRR(1)=FNLE(I)
C     EVAL PORTION OF ORIG COEFFS OF AM DEPENDENT UPON PARAM
C     EVAL AT I.
  403 AM(I,I+2)=AM(I,I+2)+C2*XM(3)-2.*ZM(3,I-1)
C     ADD CONTRIBUTIONS OF I DEPENDENT COEFFS TO AM AND
C     NONLINEAR ERRORS.
      IF(I-NETA)439,438,439
  438 AM(I,2)=AM(I,2)+1.+C2*XM(2)-2.*ZM(2,I-1)
  439 CALL AMSET (C9,I-1,2,RRR)
      LPI=NETA-1+I
      DUM1=C10+C2*XM(1)-2.*ZM(1,I-1)
  444 CALL AMSET (DUM1,LPI,3,RRR)
  418 IF(I-NETA)459,405,459
  459 LPI=2*NETA-1+I
      DUM1=1.+C2*XM(2)-2.*ZM(2,I-1)
  461 CALL AMSET (DUM1,LPI,4,RRR)
  405 CONTINUE
 9999 FORMAT(2X,I5,2X1P4E12.4)
      IF(KR(17)) 9901,9902,9901
 9901 CONTINUE
      WRITE(KOUT,9001)
 9001 FORMAT(34H  DEBUG FNLE FROM IONLY PLUS COEFS )
      WRITE (KOUT,9999) I,RRR
 9902 CONTINUE
      RETURN
      END


      SUBROUTINE OUTPUT
      REAL MU
      DIMENSION CM(5),U(25),Y(25)
      COMMON/EDGCOM/FEM(50),PTE(50),DUES,UE(50,50),PE(50,50)
     1,RHO,MU,T,DSIP(50),IDSIP
      COMMON/ETACOM/ETA(25),DETA(24),DSQ(24),DCU(24),B1(24),
     1B2(24),LAR(100),BA1(73,28)
      COMMON/HISCOM/T7,T1,T2,TE(25),C1,C2,C3,C4,BETA,ZM(5,25
     1),XI(50),HE(25,5),HUE,HHUE,DLX2,C3M(50),BETAM(50)
      COMMON/INTCOM/KR(20),KIN,KOUT,MAT1I,MAT1J,NETA,I,IS,NS
     1,IT,NTIME,NAM,NLEQ,NNLEQ,NRNL,ITS,CASE(15),B(8),MWE,NO
     2N,ITEM,NITEM,KR17,NBT,NBT2,IDENT,UMES,FREQ,EPS
      COMMON/NONCOM/AM(28,28),DVNL(28),C5,C6,C7,C8,C9,C10,
     1C11,C12
      COMMON/FRMCOM/TIME(50),PRE(50),PTET,S(50),ROKAP(50),RN
     1OSE,VKAP,NDISC,IDISC(50),NSD(10),MSD(10),IPRE,RADNO,CO
     2NE,DELT,CYCLE
      COMMON/PRPCOM/PR,SC,XM(5)
      COMMON/TEMCOM/SPDUM2(10),DER(50),DUMM1(25),SLOPE(25),R
     1EDUM(25),SDUM1 (50),SDUM2(50),FWDUM(50),XICON(50),FWCO
     2N(50),DUDS(50)
      COMMON/VARCOM/F(5,25),FTM1(5,25),FTM2(5,25)
      COMMON/WALCOM/FW(50),RHOVW(50),IFW,IRHOVW,JRHOVW
    1 FORMAT(/8X65HEDGE            STATIC           XI
     1        FLUX NORM       ROKAP /6X57HVELOCITY      PRESSURE
     2(LB/               BETA         PARAMETER /6X57H(FT/SEC)
     3(ATM)           SEC)**2                                 (FT)
```

```
    4 /3X6(2X1PE10.3)/)
  2 FORMAT(/10X54HWALL FR.     MOM TRANS          WALL
   1    MASS FLUXES /11X56HCOEFF,          COEFF,           SHEAR
   2          MECH          TOTAL /13X53HCF           RHO*UE*CF/2
   3                   REM          GAS /35X31H(LB/SQ FT)
   4   (LB/SQ FT SEC) /6X5(3X1PE10.3)/)
  3 FORMAT(/10X58HREYNOLDS       STROUHAL        MOMENTUM       D
   1ISPLACE.      SHAPE /11X58HNUMBER          NUMBER        THIC
   2KNESS,    THICKNESS,      FACTOR, /10X60H
   3              THETA          DELSTAR          DELSTAR/ /38X30H(F
   4T)            (FT)           THETA /6X5(3X1PE10.3)/)
  4 FORMAT(/31X17HNODAL INFORMATION //3X50HDISTANCE
   1           STREAM      VELOCITY      SHEAR /2X75HFROM WALL
   2   ETA          FUNCTION      RATIO        FUNCTION        FPPP
   3 VELOCITY /5X72H(FT)                             (F)        (FP=U/UE
   4)        (FPP)                  (FT/SEC) /)
  5 FORMAT(1X7(1X1PE10.3))
  6 FORMAT(1P8E10.3)
    DATA IBLANK/2H   /
    IF (KR(11)-1) 300,300,301
300 IF (KR(9))302,302,301
302 RHOVW(IS)=C1*F(1,1)+HF(1,5)
301 C89=-C3M(IS)*MU
    DUM1=-1./C3M(IS)
    DUM2=RHOVW(IS)/C3M(IS)
    IF (UE(IS,ITEM)-1.)3012,3011,3012
3011 UE(IS,ITEM)=0.
3012 CONTINUE
3051 CONTINUE
    DEP(1)=0.0
    DEP(2)=0.0
    VMECH=-DUM2
    DUM4=VMECH*100.
    IF (DUM4-DUM2) 1901,190,190
1901 VMECH=0.
190 IF(ABS(BETA)-.0001) 303,304,304
303 BETA=0.
304 Y(1)=0.
    DO 182 I=2,NETA
182 Y(I)=Y(I-1)+C89*DETA(I-1)
    SHEAR=MU*UE(IS,ITEM)/C89*F(3,1)/32.17405
    WRITE(KOUT,1)UE(IS,ITEM),PE(IS,ITEM),XI(IS),BETA,DUM1,
   1ROKAP(IS)
212 DUM1=RHO*UE(IS,ITEM)/MU
    CF=MU/C89*F(3,1)
    DUM4=1.
214 RES=DUM1*S(IS)
    WFC=CF*2./RHO/UE(IS,ITEM)
    DUM3=C89*(F(1,NETA)-F(1,1))
    DELST=Y(NETA)-DUM3
    REDELS=DUM1*DELST
    THMOM=DUM3-C89*XM(5)
    RETHMO=DUM1*THMOM
    SHAPE=DELST/THMOM
    STR=FREQ/UE(IS,ITEM)
    DUM1=DUM1*S(IS)
    STR=STR*S(IS)
    WRITE(KOUT,2)WFC,CF,SHEAR,VMECH,DUM2
    WRITE(KOUT,3)DUM1,STR,THMOM,DELST,SHAPE
    WRITE(KOUT,4)
    DO 183 I=1,NETA
C     COMPUTE TRUE VALUES OF F(I,J) AND ETA
    REF1=F(1,I)
    REF2=F(2,I)
    REF3=F(3,I)
    REF4=F(4,I)
    RETA=ETA(I)
    U(I)=REF2*UE(IS,ITEM)
    ETAN=RETA/(ETA(NETA))
183 WRITE(KOUT,5)Y(I),RETA,REF1,REF2,REF3,REF4,U(I)
1930 IF (IDENT-IBLANK) 194,309,194
194 IF(NON) 309,308,309
```

```
308 DUM3=RHO*UE(IS,ITEM)
    DUM4=-1./C3
309 CONTINUE
    IF(KR(14))310,311,310
310 WRITE(KOUT,6)((F(I,J),J=1,NETA),I=1,4)
311 RETURN
    END
```

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13. ABSTRACT

The development of a numerical procedure for the treatment of nonsimilar, unsteady, laminar boundary layers is presented. The solution is obtained from the laminar, isothermal, incompressible boundary layer equations employing a modification of the integral matrix procedure of Bartlett and Kendall. Solutions of example problems are presented for steady Blasius and Howarth flows, and for oscillating Blasius flow. Agreement with the known classical results is satisfactory and establishes the general feasibility of the method. Core storage requirements of 130,000 bytes allow consideration of as many as 25 nodal points across the boundary layer, 50 time increments per oscillation cycle and 50 streamwise stations. Solution of oscillating Blasius flow considering 8 nodal points and 16 time increments requires 13.49 seconds for one streamwise station utilizing IBM 360/67 time sharing capabilities.

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