NPS ARCHIVE
1963
CANON, R.

An Investigation Of The Theoretical Effect Of Various Parameters On Bulk Petroleum Inventory Costs At Military Installations

Library
U. S. Naval Postgraduate School
Monterey, California

AN INVESTIGATION OF THE THEORETICAL EFFECT OF VARIOUS PARAMETERS ON BULK PETROLEUM INVENTORY COSTS AT MILITARY INSTALLATIONS

by

Roscoe H. Canon, Jr.
B.S., University of Texas, 1953

Submitted to the School of Business, Department of Chemical and Petroleum Engineering, and the Faculty of the Graduate School of the University of Kansas in partial fulfillment of the requirements for the degree of Master of Science.

Library
U. S. Naval Postgraduate School
Monterey, California

ACKNOWLEDGMENTS

The author is greatly indebted to Dr. Floyd W. Preston, under whose guidance this thesis was brought to completion; and to the Bureau of Naval Personnel and Bureau of Supplies and Accounts, whose sponsorship made it possible.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

Purpose

The purpose of this thesis is to examine the theoretical manner in which product, storage, and tanker characteristics affect bulk petroleum inventory costs at military installations. This is being done in order to develop a more scientific rather than intuitive understanding of the interactions and relative importance of the various variables which are involved. The problem will be approached from a mathematical standpoint by constructing a model with inventory cost, the variable of interest, expressed as a function of certain independent variables, some of which may be capable of management manipulation or control.

This model will be constructed in such a manner as to allow differentiation in order to determine the economic order quantity. The nature and general magnitude of the effect of each variable on the economic order quantity will be examined, over a reasonable range, and compared with the effects of all other variables to determine if any pertinent conclusions are warranted.

The economic order quantity has been selected as the variable of interest because, unlike other variables, its contribution to the total variable cost of inventory is not

easily recognized. In other words, an increase in safety stock will obviously increase total variable costs, but it may either increase or decrease the economic order quantity. Since the order quantity also affects total variable costs, the net effect us not readily apparent.

Other Work

Studies carried out in this general area by Creole Petroleum Corporation indicate that models of this type fall generally into two classifications: the planning model and the operating model. Where the model is of a planning nature it is used to study an operation in view of anticipated changes. The results of these studies are interpreted and presented to management in the form of recommendations. Experience has indicated that one of the most difficult planning problems involving the transportation and storage of petroleum products is the specification of optimum pier facilities and tankage required at marine terminals. This type of model is usually extremely complex as its evaluation requires an understanding of the interaction of numerous cost variables and probability distributions, a Monte Carlo type of analysis, and the services of a high speed computor.

The most extensive use of operational models has been in short range scheduling. The overall objectives of such a model might be to (1) minimize all terminal and transportation costs concerned with the tanker movement of refined products from source of procurement to terminal, (2) meet

planned inventory targets, and (3) assure that the demands of customers are serviced without excessive or unnecessary delay. Studies carried out in this area indicate that models of this type vary a great deal in complexity depending upon the circumstances. Costs which were found to be pertinent included the terminal cost of avoiding a runout or an excess, the terminal inventory carrying costs, the cost penalty for not meeting planned inventory targets at terminals, and the additional costs for multiple-port discharges. Initial analysis also indicated that safety stocks had to be provided at each terminal to take into account uncertainties due to variations in demand.

The first step in the development of either type of model, as mentioned above, is an exploratory study to determine whether or not an incentive is present for the development of more refined analytical procedures. This incentive may manifest itself in either or both of the following forms: (1) decreased operating or investment costs, or (2) improved methods of control over the operation.[1] The purpose of this thesis is to conduct such an exploratory study but in a very generalized manner.

---

[1]D. S. McArthur and others, "Operations Research Applied to Marine Transportation and Tankage Problems," Proceedings, Fifth World Petroleum Congress, Section VIII, Paper 2, Fifth World Petroleum Congress, Inc., New York, 1959, pp. 15-19.

U. S. Navy Distribution System for Bulk Petroleum Products

In order to provide a frame of reference for the problem to be considered, a brief and very general description of the U. S. Navy distribution system for bulk petroleum products will be described.

In general, the system consists of a series of product sources, a series of marine terminals, a series of customers, and a centralized control point. Product sources consist of commercial terminals located near refineries or major marketing areas. These sources have been predetermined by purchase policies and may be located anywhere in the world. These product sources provide product to the military terminals.

The military marine terminals serve as distribution points for the product received from commercial sources. They may supply products to other terminals or to the military customer. They are located on a world-wide basis with their size and location governed primarily by military factors.

Military customers consist of aircraft, ships, automotive equipment, and mobile or stationary power equipment. Control over inventories at the marine terminals is exercised by the centralized control point through a system of periodic and situation reports submitted by each terminal. These reports can be generally categorized as dealing with the subjects of inventory, demand, tanker transportation, or storage capacity. Reports on inventories and historical

demand are generally combined and are submitted in the form of stock reports or financial reports on a monthly and quarterly basis. Tanker transportation reports are of the situation type only. These reports cover the details of tanker loadings and discharges as they occur. Storage reports indicate available storage at each terminal and are made on an annual basis. Interim storage reports are made as changes caused by casualties or new storage occur.

The problem of the centralized control point is to receive this information and utilize it in such a manner as to maintain adequate product inventories at military terminals. Many factors such as storage capacity, tanker size, and variation in demand combine to make a problem which is extremely complex. The author feels that this complexity has lead most people to consider the problem as insoluble; hence, the treatment of distribution and inventory control problems has been on a symptomatic basis only and has stopped short of determining and controlling, as much as possible, the basic causes underlying these problems. It is the feeling of the author that a scientific approach, as proposed herein, will reveal certain underlying mechanics which are fundamental to all problems and that the correlations developed in this thesis will eventually lead to more detailed studies of the problem.

Scope of Study

The development and evaluation of the economic order quantity will be carried out in three separate parts with a single chapter being devoted to each part. Before proceeding with a brief outline of the proposed study, some overall assumptions and limitations will be described in order to establish a general frame of reference for the discussion.

Among the problems that will not be considered is the one of war reserve determinations. The author feels that this problem is based on economic and military considerations which are essentially independent of the economic problem that is to be evaluated in this paper. For the same reason, consideration will not be given to the military factors governing the relative location and size of fuel terminals nor to local management practices dealing with operations and maintenance at fuel terminals. Throughout the study, continuous functions will be used, in lieu of discrete functions, due to the ease with which they can be manipulated. In addition, unless otherwise indicated, all variables will be assumed to be independent of time. The assumptions and limitations, set forth above, are not expected to detract from the usefulness of the generalized relationships which are to be developed.

The first part of the study, to which Chapter II is devoted, addresses itself to the problem of developing cost

functions which are pertinent to either the construction or evaluation of the cost model. A brief description will be given of the criteria for determining applicable costs. Following this, consideration will be given to product costs and quantity discounts, unit tanker transportation costs, carrying costs associated with inventory holdings, and storage space costs. Cost penalties for early or late deliveries of product to the terminal of interest involve military considerations and will therefore not be considered.

Chapter III is devoted to the second part of the study in which consideration will be given to the effects of lead time and uncertainty on the inventory problem. In particular, the following topics will be covered: variation in storage capacity, variation in demand, surge capacity and safety stock, and inventory in transit. Reliable data on variations in transportation lead time is not available, so the effect of this variable will not be considered. The final chapter will be devoted to the development and analysis of the economic model. Pertinent variables developed in the previous two chapters will be combined to form a function descriptive of the total variable cost of inventory. This function will be differentiated to obtain mathematical description of the economic order quantity. The effects of various independent variables on the economic order quantity will then be evaluated and compared to determine whether or not any significant conclusions are warranted.

CHAPTER II

VARIABLE COSTS OF INVENTORY

Description of Costs Considered

The economic purchase or order quantity for a given inventory operation is determined by minimizing the total of all variable costs connected with that operation.[2] One might well ask how variable costs are identified. In this connection, the criterion proposed by J. F. Magee appears to be reasonable and hence shall be used. This criterion specifies that variable costs shall represent only those out-of-pocket expenditures or foregone opportunities whose magnitudes are directly affected by inventory policies.[3] Thus, in a government operation, material, direct-labor and overhead costs are out-of-pocket; however, they are so stable that they can be considered to be independent of order size and should therefore not be considered as variables. On the other hand, tanker transportation costs are out-of-pocket and vary with order size as well, and therefore should be considered. Storage space which is available but which cannot be used for

---

[2]Thomson M. Whitin, The Theory of Inventory Management, (Second Edition), Princeton University Press, Princeton, New Jersey, 1957, p. 57.

[3]John F. McGee, Production Planning and Inventory Control, McGraw-Hill Book Company, Inc., 1958, p. 27.

other productive purposes may not be considered as an out-of-pocket expense; however, storage space which is rented (out-of-pocket) or which could be used for other productive purposes (foregone opportunity) should be considered as a justifiable cost.[4]

In view of the foregoing, this paper will consider the relative magnitude and effect on inventory costs of each of the following: product purchase price, price discounts for quantity purchases, tanker transportation rates, risk and interest factors, and storage space. In order to simplify mathematical treatment of costs in a later chapter, all cost data will be expressed either on a per-barrel basis or as a percentage of annual per-barrel costs.

Product Costs and Quantity Discounts

Table I is a listing of representative purchase prices for refined petroleum products in cargo lot quantities. These prices will be used as a frame of reference in later numerical examples. Quantity discount practices on cargo lot quantities of refined petroleum products will be assumed to be typified by the following data.[5] During fiscal year 1961, 4.5 per cent of the contracts for cargo liftings of

---

[4] Ibid.

[5] Personal communication, U. S. Navy Fuel Supply Office, Washington, D. C., August 8, 1962.

petroleum products, for use by the U. S. Navy, had discount provisions. In terms of money value, 11.0 per cent of the products under contract for cargo size liftings were subject to quantity discounts. These discounts ranged from 1.5 to 2.0 per cent of the base price of each contract. The above data suggests that although discounting does exist, the practice is by no means widespread. Because of this and the small magnitude of the discounts, the problem of quantity discounts will not be further considered in this paper.

TABLE I

GULF COAST SPOT PRICES FOR CARGOES ON AUGUST 1, 1962

Product	$/gal.	$/bbl.	A.P.I. Gravity
Gasoline (98 octane)	0.11625[a]	4.88[b]	62[c]
Kerosine (41-43)	0.0925[a]	3.90[b]	42[a]
Bunker C	-	2.00[a]	12[c]

[a] "Price Statistics," Oil and Gas Journal, 60:216, August 6, 1962.

[b] Computed to nearest cent.

[c] Petroleum Conversion Factors and Capacity Tables for Logistics Planning and Reference, Office of the Assistant Secretary for Defense (Supply and Logistics), Petroleum Office, Washington 25, D. C.

Tanker Transportation Costs

Generalized formulas for tanker freight rates are exceedingly difficult to obtain for a variety of reasons. First of all tanker rates are to some extent a product of the market place and are thus subject to the laws of supply and demand. As a result, single voyage tanker freight rates may vary by a factor as great as 5, while time charter freight rates may vary by a factor as great as 0.5.[6] Other important factors which affect tanker freight rates are construction costs, crew costs, tanker size, and employment patterns. For example, tankers built in the United States cost as much as 40 or 50 per cent more than tankers built in foreign countries,[7] while total annual crew costs on tankers registered in the United States are 4 to 5 times as great as crew costs on tankers registered in foreign countries.[8] In general, the larger the size of a tanker, the smaller will be the unit cost of transportation. For example, the unit cost of transporting one long ton of petroleum in a tanker of 16,600 tons deadweight may be as much as 2.5 times greater

---

[6]P. H. Frankel, "Short-Term and Long-Term Tanker Freight Rates and the Significance of Their Fluctuations," Proceedings, Third World Petroleum Congress, Section IX, E. J. Brill, Leiden, Netherlands, 1951, pp.189-90.

[7]H. N. Emerson, "Oil -- No. 1 Transportation Job," The Oil and Gas Journal, 55:230, November 18, 1957.

[8]Harry Benford, "Engineering Economy in Tanker Design," Transactions, The Society of Naval Architects and Marine Engineers, 65:814-15, 1957.

than transporting the same quantity of petroleum on a tanker of 100,000 tons deadweight.[9] Tanker utilization policies such as multiple-port loadings and discharges,[10] and light-loadings[11] also affect unit transportation costs. There are also many other factors which affect tanker freight rates: those cited have been used merely to illustrate the complexity of the rate structure. The complex nature of tanker rates probably accounts for the non-availability in the published literature of generalized formulas for tanker freight rates.

In spite of the above difficulties, an attempt will be made to develop a limited approximation to a generalized formula for tanker freight rates. This formula will ignore variations in tanker freight rates due to market conditions, and will, insofar as possible, attempt to reflect approximate long-term freight rates for tankers built and registered in the United States.

---

[9] Loren F. Kahle, and A. J. Kelly, Jr., "The Role of Sea Transportation in the Petroleum Industry," Proceedings, Fifth World Petroleum Congress, Section VIII, Paper 1, Fifth World Petroleum Congress, Inc., New York, 1959, p. 7.

[10] D. S. McArthur, and others, "Operations Research Applied to Marine Transportation and Tankage Problems," Proceedings, Fifth World Petroleum Congress, Section VIII, Paper 2, Fifth World Petroleum Congress, Inc., New York, 1959, p. 19.

[11] J. Bes, Tanker Chartering and Management, Uitgeverij v/h C. DE BOER JR., le Weteringplantsoen 8, Amsterdam, 1956, pp. 92-7.

A freight rate formula can be synthesized from the following data given by Emerson[12]:

one way distance: 1840 nautical miles (2120 statue miles)
tariff: \$2.85/long ton (\$2.54/short ton)

tanker costs: 20% fixed
80% variable

If the freight rate is assumed to be a straight-line function of distance, the above data may be used to compute a freight rate formula as follows:

$$R_1 = (2.85)(0.20) + \frac{(2.85)(0.80)(d)}{1840}$$

$$R_1 = 0.57 + 0.00124d \qquad (1)$$

where R_1 is the freight rate in dollars per long ton, and d is the standardized sea distance between loading and discharge ports, in nautical miles. Another source[13] represents the average tanker freight rate with the following equation:

$$r_1 = 0.0945 + 0.000177d \qquad (2)$$

where r_1 is the freight rate in dollars per barrel. The average U. S. crude oil has a 35° A.P.I. gravity and a

---

[12]H. N. Emerson, "Oil -- No. 1 Transportation Job," The Oil and Gas Journal, 55:231, November 18, 1957.

[13]Military Petroleum Indoctrination Course sponsored by the Military Petroleum Supply Agency in conjunction with the Union Oil Co. of California, Tidewater Oil Co., and Richfield Oil Co., Los Angeles, September 26-October 7, 1960.

specific volume of 7.537 barrels per long ton.[14] Multiplying equation (2) by 7.537 yields

$$R_2 = 0.712 + 0.00134d \qquad (3)$$

where R_2 is the freight rate in dollars per long ton. A third source[15] provides the following freight rate formula applicable to T-2 (16,600 tons deadweight) tankers:

$$R_3 = A(0.89 + 0.00128d + K) \qquad (4)$$

where R_3 is the freight rate in dollars per long ton, K represents an additional charge for canal transits in dollars per long ton, and A represents a cost adjustment factor whose value is 1.00 for black cargoes and 1.10 for clean cargoes. Black cargoes are defined as residual type products such as crude, asphalt, Navy Special Fuel Oil, Bunker C, and No. 6 Fuel Oil; while clean cargoes are defined as distillate type products such as kerosene, gasoline, jet fuel, diesel oil, and solvent napthas.[16] Unit freight rates on tankers are higher for clean cargoes than for dirty cargoes due to the higher degree of tank corrosion and higher cost of tank

---

[14] Petroleum Conversion Factors and Capacity Tables for Logistics Planning and Reference, Office of the Assistant Secretary for Defense (Supply & Logistics), Petroleum Office, Washington 25, D. C., September 1953, pp. 2-3.

[15] COMSTS INSTRUCTION 7600.3, Commander, Military Sea Transportation Service, Washington 25, D. C.

[16] Ibid.

cleaning associated with the carriage of distillate type products.[17] The corresponding coefficients in equations (1), (3), and (4), which are in general agreement with each other, have been averaged to produce a single representative equation for tanker freight rates, as follows:

$$R_a = A(0.724 + 0.00129d) \qquad (5)$$

where R_a is the freight rate in dollars per long ton. Canal tolls will not be considered in this paper, hence the K term was dropped from equation (5).

The variation of tanker freight rates as a function of tanker size will now be considered. Relative unit transportation costs versus tanker size, from various sources, have been tabulated in Table II. The data in Column E in Table II indicates that the variation of unit freight rates with size is essentially independent of the voyage length. Note also that Column B represents relative costs for ships of American registry, operated with American crews, and presumably built in the United States; while Column D represents relative costs for ships of foreign registry, operated with foreign crews, and presumably built in foreign yards. A straight line relationship was found to exist between the

---

[17] P. H. Frankel, and W. L. Newton, "Current Economic Trends in Location and Size of Refineries in Europe," Proceedings, Fifth World Petroleum Congress, Section IX, Paper 10, Fifth World Petroleum Congress, Inc., New York, 1959, pp. 91-2.

TABLE II

RELATIVE UNIT TRANSPORTATION COSTS VERSUS TANKER SIZE

Tanker Dead-weight, long tons	Relative Unit Cost of Oil Transportation						
	A[a]	B[b]	C[c]	D[d]	E[e]		G[g]
16,000	....	....	1.00	....	....	....	....
16,600	1.00	....	....	1.00	....	....	....
18,000	....	1.00	....	....	....	....	....
19,000	0.90	....	....	....	....	....	....
20,000	....	....	0.95	....	....	....	1.00
22,000	....	0.87	....	....	....	....	....
25,000	....	....	....	....	1.00	1.00	....
26,700	....	0.75	....	....	....	....	....
30,000	0.63	....	....	....	....	....	....
31,000	....	0.70	....	....	....	....	....
32,000	....	....	0.73	....	....	....	....
32,800	....	....	....	....	0.91	0.90	....
35,000	....	0.65	....	....	....	....	....
38,000	....	....	0.67	....	....	....	....
45,000	0.51	....	0.61	0.66	....	....	0.74
60,000	....	....	0.56	....	....	....	....
70,000	0.43	....	....	....	....	....	....
80,000	....	....	0.52	....	....	....	....
85,000	0.40	....	....	....	....	....	....
90,000	....	....	0.50	....	....	....	....
100,000	0.38	....	....	....	....	....	....
Round-Trip Distance, naut. mi.		4,000		25,000	17,100	3,800	4,000
Notes		Am. Reg. and Crew		For. Reg. and Crew	Same ships used for both comparisons		
Date	59	Oct. 55	59	Sep. 59	Mar. 61		Mar. 61

[a]Loren F. Kahle and A. J. Kelly, Jr., "The Role of Sea Transportation in the Petroleum Industry," Proceedings, Fifth World Petroleum Congress, Section VIII, Paper 1, Fifth World Petroleum Congress, Inc., New York, 1959.

[b]Address by Mr. J. D. Rogers, Executive Vice-President of the Esso Shipping Company at the American Merchant Marine Conference, New Orleans, October, 1955, cited by J. Bes, Tanker Chartering and Management, Uitgeverij v/h, C. DE BOER JR., le Weteringplantsoen 8,

Amsterdam, 1956, p. 25.

[c]P. H. Frankel and W. L. Newton, "Current Economic Trends in Location and Size of Refineries in Europe," Proceedings, Fifth World Petroleum Congress, Section IX, Paper 10, Fifth World Petroleum Congress, Inc., New York, 1959.

[d]"How Big Tankers Cut Cost," Petroleum Week, McGraw-Hill Book Company, New York, 9:59, September 18, 1959.

[e]Ben F. Boyd, "A Study of Some of the Effects of Supertankers on Military Petroleum Logistics," Unpublished Master's Thesis, The University of Kansas, Lawrence, Kansas, 1961, p. 26, citing responses to questionaires sent to major oil companies, independent tanker operators and shipbuilding firms.

[g]Ibid., p. 28.

logarithm of the tanker deadweight and the reciprocal of the relative unit transportation cost. This relationship is illustrated in Figure 2. Each of the curves in Figure 2 is labeled with a letter in order to identify it with the corresponding column of data in Table II. Note in Figure 2, that data appear to fall into two different groupings, and that data from Column B (American tankers) falls in one grouping while the data from Column D (foreign tankers) falls in the other grouping. This suggests that the relationship between tanker size and relative unit transportation cost is slightly different for American and foreign tankers, and that Group I relationships in Figure 2 represent American ships and costs while Group II relationships represent foreign ships and costs. Assuming this inference to be correct, the data of column (or curve) A was chosen as representative of

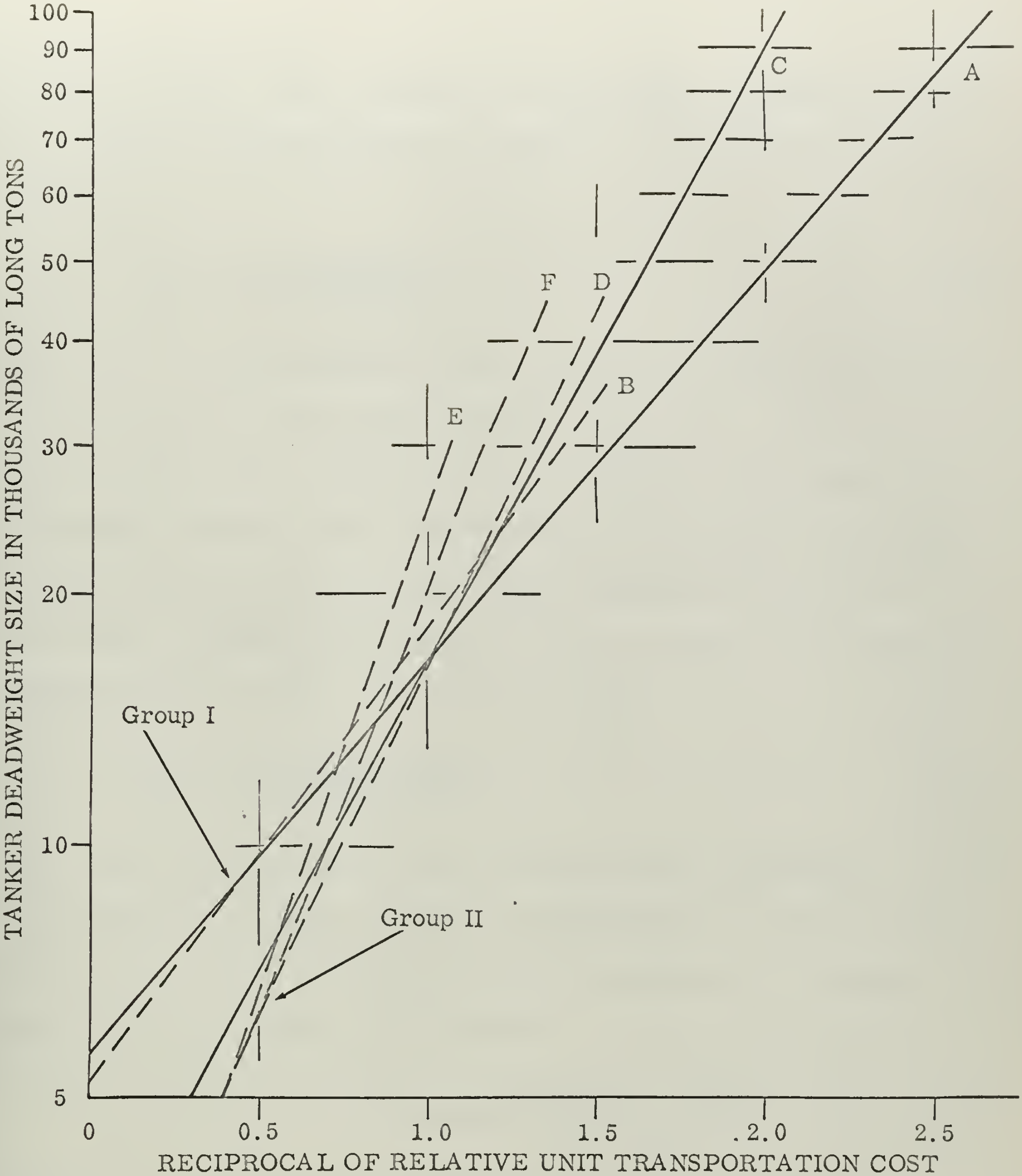

FIGURE 1: Tanker Size Versus Relative Unit Transportation Cost (see Table II for particulars).

American tanker costs because of its greater range and more recent date. The equation of curve A, in Figure 2, is as follows:

$$C_1 = \frac{0.4715}{\log (D/5600)} \tag{6}$$

where D is the deadweight of the tanker in long tons and C_1 is the relative unit cost of tanker transport. This equation is cumbersome, however, so an alternate form will be used which utilizes the first term of the series expansion for the logarithm of a number. The resulting equation, which has a maximum deviation of 1.25 per cent from the original data in Table II, is as follows:

$$C_2 = \frac{0.2846D + 8114}{D - 3762} \tag{7}$$

where C_2 is the relative unit cost of tanker transport. Multiplication of equation (5) by equation (7) will yield a formula for tanker freight rates as a function of voyage length and tanker size, as follows:

$$R_b = A(0.724 + 0.00129d)\frac{(0.2846D + 8114)}{(D - 3762)} \tag{8}$$

where R_b is the adjusted freight rate in dollars per long ton.

Now consider the effect of light loading and multiple-port loadings and discharges on tanker freight rates. Rate Order No. 438 issued by the U.S.A. War Shipping Administration on February 27, 1946, and now administered by the U. S.

Maritime Administration, specifies that differential charges for extra loading and discharge ports shall be based on the entire cargo handled by the vessel, and that where part cargoes are carried, dead freight is collectable for the vessel's unused capacity.[18] Thus, equation (8) is applicable only when considering a fully loaded ship with a single port loading and a single port discharge.

No data are available on differential freight charges for extra loading and discharge ports, so a mileage penalty has been used for this type of contingency. The mileage penalty is based on two assumptions: (1) that, on the average, a ship will lose one steaming day for each additional loading port over one and each additional discharge port over one; and (2) the average sea speed of a tanker is 17.0 knots. The mileage penalty is then (17.0)(24) or 408 nautical miles. This figure has been rounded off to 400 nautical miles and introduced into equation (8) as follows:

$$R_c = A(0.00128[d + 400(x - 2)] + 0.724)\frac{(0.2846D + 8114)}{(D - 3762)}$$

$$R_c = A(0.00128d + 0.512x - 0.30)\frac{(0.2846D + 8114)}{(D - 3762)} \qquad (9)$$

where d is now the total voyage length in nautical miles, x is the total number of loading and discharge ports, and R_c is the new unit freight rate in dollars per long ton.

---

[18]J. Bes, Tanker Chartering and Management, Uitgeverij v/h C. DE BOER JR., le Weteringplantsoen 8, Amsterdam, 1956, pp. 92-7.

As noted previously, if less than a full cargo is carried, dead freight is collectable on the unused cargo carrying capacity of the ship. In other words, the total bill for a cargo is the same regardless of how much cargo is transported. Thus, the unit freight rate as expressed by equation (9) must be multiplied by the following factor in order to give the proper unit freight rate for light loading.

$$\frac{1}{L} = \frac{O + B + U}{O + B} \tag{10}$$

where L is the voyage load-factor, O is the order size in long tons for the product and terminal under consideration, B is the quantity in long tons of other cargo carried on the voyage, and U is the unused cargo carrying capacity of the tanker in long tons. Multiplying equation (9) by equation (10) yields

$$R = 0.1457A(0.0025d + x - 0.586)\frac{(D + 28,500)(O + B + U)}{(D - 3672)(O + B)} \tag{11}$$

where R is the adjusted unit freight rate in dollars per long ton. Although the above equation is descriptive of unit tanker transportation rates, it is not in a useful form. In order to facilitate mathematical manipulation, later on in the discussion, the tanker deadweight must be expressed as a function of the order size, and the quantities R and

O must be expressed in terms of barrels instead of long tons. These relations are developed in the following paragraphs.

The cargo capacity of a ship is equivalent to the total deadweight of the ship on the applicable load line at the sailing port minus the bunkers, provisions, stores, and spare parts required for the voyage, including the usual safety margines.[19] Bunkers are by far the most important item affecting the cargo carrying capacity of the ship.[20] With this in mind the data in Table III will be used to develop an approximate correlation between tanker carrying capacity, total deadweight, and length of voyage. By using the averaged data from Table III, the following approximate relationship for cargo capacity can be derived:

$$f(D) = O + B + U = D - \frac{d(0.390D)(0.531)(1.15)}{(16.58)(2240)}$$

$$D = \frac{O + B + U}{1 - 6.41d/(10^6)} \qquad (12)$$

where f(D) is the cargo carrying capacity in long tons, and all other terms are as previously defined. The adjustment factor of 1.15 is used in the above equation to account for the additional weight taken up by the reserve fuel oil,

---

19 *Ibid.*, 143.

20 *Ibid.*, 151.

TABLE III

TYPICAL TANKER CHARACTERISTICS

Deadweight, long tons	Draft, ft. at summer free-board	Normal Shaft-Horse-power	Fuel Rate, lb./SHP-HR	Sea Speed, knots	Norm. SHP/Dead-weight
16,620[a]	30.17	6,000			0.361
25,400[b]	33.48	12,500	0.535	16.75	0.492
27,000[b]	33.33	12,500	0.535	16.50	0.463
28,250[b]	33.14	12,500	0.535	16.50	0.443
29,350[b]	33.00	13,600	0.535	16.75	0.463
33,150[b]	34.13	13,600	0.535	16.50	0.410
46,100[b]	37.83	13,600	0.535	16.00	0.295
67,450[b]	43.61	20,900	0.522	16.25	0.310
71,300[b]	44.05	20,900	0.522	16.50	0.293
106,600[b]	49.15	39,000	0.522	17.50	0.366
Average	-	-	0.531	16.58	0.390

[a]M. Mack Earle, "The Conversion of T2 Tankers for Great Lakes and Seaway Service," Transactions, The Society of Naval Architects and Marine Engineers, 68:980,94, 1960.

[b]W. O. Nichols, M. L. Rubin, and R. V. Danielson, "Some Aspects of Large Tanker Design," Transactions, The Society of Naval Architects and Marine Engineers, 68:804-6, 1960.

provisions, stores and spare parts.[21] Substituting this expression into equation (11) and simplifying, yields:

[21]I. Jung, and G. Ohlsson, "Technical and Economic Data for Turbine Powered Tankers," International Shipbuilding Progress, 4:541, October 1957.

$$R = A(0.1457)(0.0025d + x - 0.586)\frac{(0 + B + U)}{(0 + B)}\frac{(0 + B + U + 28,500 - 0.1828d)}{(0 + B + U - 3672 + 0.0235d)} \qquad (13)$$

An expression for converting long tons to barrels may be developed with the aid of the following formula:[22]

$$S.G. = \frac{\bar{v}_w}{\bar{v}_o} = \frac{141.5}{131.5 + g} \qquad (14)$$

where S.G. is the specific gravity of the oil, $\bar{v}_w$ is the specific volume of water in barrels per long ton, $\bar{v}_o$ is the specific volume of the oil in barrels per long ton, and g is the A.P.I. gravity of the oil all at 60°F. By substituting the specific volume of water, 6.404 barrels/long ton,[23] into equation (14) and rearranging, the following expression for the specific volume of oil at 60°F, in barrels per long ton, will result

$$\bar{v}_o = \frac{(6.404)(131.5 + g)}{141.5} \qquad (15)$$

where all units are as previously defined. Equation (15) may be used to obtain the following equalities:

---

[22] Petroleum Conversion Factors and Capacity Tables for Logistics Planning and Reference, Office of the Assistant Secretary for Defense (Supply & Logistics), Petroleum Office, Washington 25, D. C., September 1953, p. 1.

[23] Ibid., p. 8.

$$R = r\bar{v}_o = \frac{r(6.404)(131.5 + g)}{141.5} \qquad (16)$$

$$O = \frac{Q}{\bar{v}_o} = \frac{Q(141.5)}{(6.404)(131.5 + g)} \qquad (17)$$

where r is the unit transportation rate in dollars per barrel, Q is the order quantity in barrels, and all other terms are as previously defined. Substituting equations (16) and (17) into equation (13) will yield a unit transportation rate formula in terms of dollars per barrel. The resulting equation, which has been simplified, is as follows:

$$r = \frac{3.22A[0.0025d + x - 0.586][22.1Q + (131.5 + g)(B + U)]}{[131.5 + g][22.1Q + (131.5 + g)(B)]}$$

$$\frac{[22.1Q + (131.5 + g)(B + U + 28{,}500 - 0.1828d)]}{[22.1Q + (131.5 + g)(B + U - 3672 + 0.0235d)]} \qquad (18)$$

where

r = unit tanker transportation rate ; \$/bbl

A = 1.00 for black cargoes

= 1.10 for clean cargoes

Q = order quantity, bbl

B = other liquid tanker cargo, long tons

U = unused tanker cargo carrying capacity, long tons

x = total number of loading and discharge ports

d = standardized voyage distance, nautical miles

g = A.P.I. gravity of product at 60°F.

Equation (18) is the final form of the unit tanker transportation rate formula. To be sure, it is only an approximation; however, it is believed to be sufficiently accurate to evaluate the general nature of bulk petroleum economics.

Carrying Costs

Inventory carrying costs generally consist of risk costs, attributable to loss, obsolescence, and depreciation; and interest charges. Product obsolescence is not considered to be a problem in the area of bulk petroleum supply, due to the rapid turnover experienced in this commodity area. Data for fiscal year 1960[24] indicates that Navy peace-time operating stocks of bulk petroleum products were turned over on the average of once every 42 days. This indicates that inventories of bulk products could easily be eliminated from the system if specification changes were contemplated.

Product loss may occur as a result of leaks, evaporation, fire, and overissues. Product depreciation may be attributed to deterioriation or contamination. The most common forms of deterioration are weathering, gum formation, and loss of additives. These changes occur while the product lies in storage and become more marked as the product ages. The changes may be initiated or hastened by the conditions

[24]BUSANDA Notices 7330 for 4th Qtr. of FY 59 and for 1st, 2nd, 3rd, and 4th Qtrs. of FY 60, Bureau of Supplies and Accounts, Navy Department, Washington 25, D. C.

of storage, and are not normally observable by personnel handling the product; therefore, discovery before issue is dependent upon adequate sampling and testing programs. Product contamination is brought about by the addition of some material not normally present such as dirt, rust, water, or another petroleum product. Such an admixture may modify the usual qualities of the product permanently or may add new and undesirable characteristics. In either case, the contaminated product may be unsuitable for its intended use.[25] Product loss may occur as the result of leaks, evaporation, fire, and overissues. Both loss and contamination may result from accident, inability or neglect to follow prescribed procedures, gross carelessness, or sabotage.

Interest charges, in a military inventory problem such as this, are used as a means for allocating funds. In a sense, this interest rate is similar to the internal interest rate that a business might use; however, in the case of the military such factors as military necessity, vulnerability to enemy attack, and efficient capital utilization are the fundamental considerations rather than the profit motive.

Because of the rather complex and subjective nature of all carrying costs, they are generally grouped together

---

[25]Bureau of Naval Personnel, Fundamentals of Petroleum, NAVPERS 10883, Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 1953, pp. 88-9.

in one lump sum and expressed as a percentage of the average annual money value of the inventory.[26] The letter, I, will be used herein to designate carrying charges. For purposes of example, annual carrying charges varying from 10 to 30 per cent will be considered.

Storage Space Costs

Storage space costs may be considered as a variable cost of inventory only when the storage space in question is rented (out-of-pocket expense) or can be used for other productive purposes.[27] There is room for argument on whether or not the use of storage space costs is appropriate in an economic model for military installations; therefore, both positions will be explored.

Construction costs for various types of storage tanks are set forth in Table IV. The data in columns A, B, C, for cone roof, pontoon roof, and double deck floating roof tanks are the most recent, but do not include costs for underground storage tanks. Since underground tanks are more expensive than other types of tanks, recent cost data on underground tanks were sought in order to establish an upper limit for

---

[26] Thomson M. Whitin, The Theory of Inventory Management, (Second Edition), Princeton University Press, Princeton, New Jersey, 1957, p. 32.

[27] John F. McGee, Production Planning and Inventory Control, McGraw-Hill Book Company, Inc., 1958, p. 27.

TABLE IV

STORAGE TANK COSTS IN DOLLARS PER TANK

Tank Capacity, bbl.	Type of Tank				
	A[a]	B[b]	C[c]	D[d]	E[e]
	Cone Roof	Pontoon Roof	Double Deck Floating Roof	Cone Roof	Navy Under-ground
14,000	20,000	28,000	29,500	......	......
20,000	......	......	......	21,000	59,000
30,000	......	......	......	27,750	71,250
36,000	39,000	50,000	55,000	......	......
40,000	......	......	......	34,000	84,000
50,000	......	......	......	40,000	98,750
60,000	......	......	......	46,500	114,000
70,000	......	......	......	52,500	129,500
80,000	77,000	89,000	105,000	60,000	146,000
90,000	......	......	....	62,250	....
100,000	......	......	....	72,000	....
Cost Basis: Date F.O.B.	March 20, 1960 Florida			March, 1956 Factory	

NOTE: All tanks except Navy underground tank conform to A.P.I. Standards. Navy underground tanks are cylindrical with flat roofs and flat bottoms. All tank costs include errection and normal accessories but exclude foundation and painting.

[a]N. H. Prater and John Mylo, "Equipment Cost Data File," Hydrocarbon Processing and Petroleum Refiner, 40:132, August 1961.

[b]Ibid.

[c]N. H. Prater and John Mylo, "Equipment Cost Data File," Hydrocarbon Processing and Petroleum Refiner, 40:174, June 1961.

[d]O. T. Zimmerman and Irvin Lavine, "Want Equipment Costs for Estimates?" Petroleum Refiner, 35:116,22-3, August 1956.

[e]Ibid.

storage space costs. Older data, from a single source, cited construction cost data for cone roof and Navy type underground tanks. These data are shown in columns D and E of Table IV. The data on cone roof tanks, in column D, was included in order to provide a reasonable basis for adjusting the cost data on underground tanks to a more recent date. A straight line relationship was found to exist between tank capacity and tank cost. This is illustrated in Figure 3, where all curves are identified to correspond to the columns in Table IV. The cost equations for the cone roof and underground tanks are as follows:

$$c_1 = 7{,}900 + 0.864h \tag{19}$$

$$c_2 = 7{,}900 + 0.641h \tag{20}$$

$$c_3 = 28{,}560 + 1.443h \tag{21}$$

where c_1 is the construction cost of a cone roof tank in 1960 dollars, c_2 is the cost of a cone roof tank in 1956 dollars, c_3 is the cost of underground tanks in 1956 dollars, and h is the tank capacity in barrels. Note that the only difference in costs between cone roof tanks constructed in 1956 and 1960 is reflected in the slopes of equations (19) and (20). The same relationship was assumed to exist for underground storage tanks. An estimate of 1960 construction costs for underground storage tanks was obtained by multiplying the slope of equation (21) by the ratio of the slopes

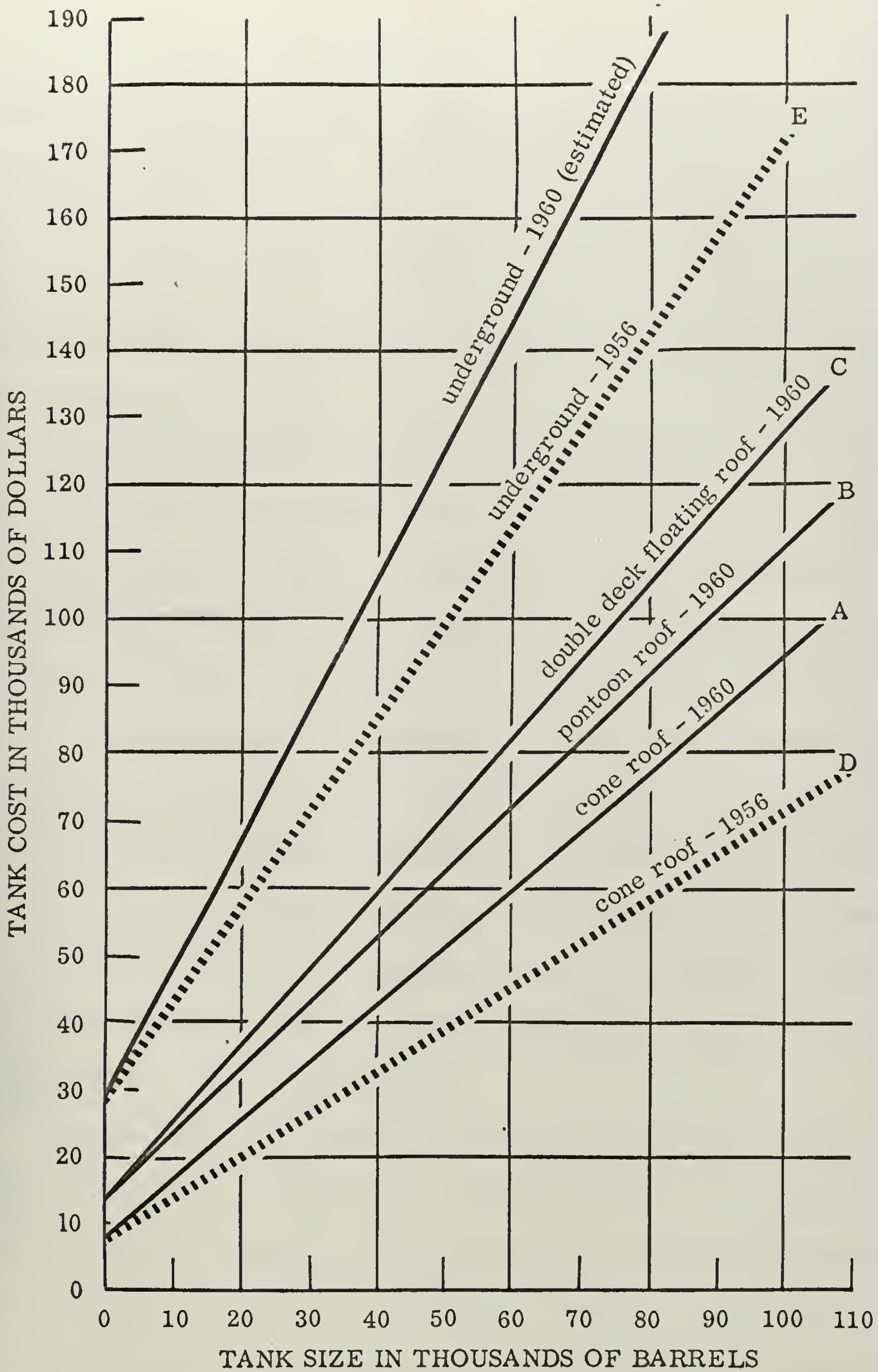

FIGURE 2: Storage Tank Costs in Dollars (see Table IV for particulars).

of equations (19) to (20). The resulting equation, which is plotted in Figure 3, is as follows:

$$c_4 = 28,560 + 1.945h \qquad (22)$$

where c_4 is the construction cost of an underground tank in 1960 dollars.

Equation (22) represents the estimated, 1960, construction costs for underground tanks built in Florida. This cost includes all normal accessories such as manholes, ladders, vents, flange connections, etc., but does not include foundation costs. Foundation costs may vary anywhere between 20 and 100 per cent of the cost of the tank, depending upon the nature of the subsoil. Tank costs will also vary from one locality to another because of freight differentials from the place of manufacture. Thus, equation (22) can only be considered as a broad approximation to storage space costs. With this in mind, the value for the unit annual cost of storage will now be developed.

Note that the slope of equation (22) has units of dollars per barrel. This slope, multiplied by two adjusting factors, will be used as the basis for computing storage costs. One factor of 1.50 will be used to compensate for foundation costs as well as the $28,560 cost in equation (22) which is not included in the slope. The other factor of 0.05 will be used to convert the total cost of a barrel of

storage to an annual cost, based on an estimated useful tank life of 20 years. The resulting unit annual cost of storage, then, is (1.50) (0.05) (1.945) - 0.146 dollars per barrel per year.

CHAPTER III

EFFECTS OF LEAD TIME AND UNCERTAINTY

Description of Variables Considered

If the storage capacity and demand for each product at each terminal was accurately known in advance, the matter of how much product to order and when to order it would be a relatively straightforward task made difficult only by the problems of defining and measuring costs. The principal limitations inhibiting the exclusive employment of cost data in this type of inventory problem are the timeliness of inventory and storage information; delivery delays; and variations or uncertainties in storage capacity and demand.[28]

Inventory and storage information is normally submitted by each terminal to the inventory control point on a periodic basis. The timeliness of information, the amount of time elapsing between the submission and use of this information shall be called administrative lead time and shall be measured in days. Delivery delays, the amount of time elapsing between the effective date of an order and the delivery date of the product, shall be called transportation lead time. Transportation lead time shall be measured in days and expressed as a function of the standardized sea distance

---

[28] John F. McGee, Production Planning and Inventory Control, McGraw-Hill Book Company, Inc., 1958, p. 65.

between loading and discharge ports, assuming an average tanker speed of 400 nautical miles per day. The sum of the administrative lead time and the transportation lead time will simply be called lead time and will be expressed by the following equation

$$T = t + 0.0025d_p \tag{1}$$

where T is the lead time in days, t is the administrative lead time in days, and d_p is the standard sea distance between loading and discharge ports.

Lead time, by itself, causes no problem in scheduling inventory replenishments when there are no uncertainties. If lead time, demand, and storage capacity are known orders for new product may be placed sufficiently far in advance to prevent stock depletion. In a similar manner, variations in demand and storage capacity cause no problem when lead time is zero, i.e., when instantaneous stock replenishment is available. The combination of lead time and uncertainty however does cause problems. The uncertainties connected with demand and storage capacity, during the lead time period create a possibility for either one or both of two undesirable occurrences: (1) depletion of the inventory a number of days before replenishment is effected or, (2) inventory replenishment when storage capacity is inadequate to handle the entire replenishment. The solution to the first problem

is to maintain some additional inventory or safety stock on hand which can be drawn upon in case of emergency, but not to count on this inventory in determining when to place a replenishment order. The second problem may be solved by maintaining storage space or surge capacity, in addition to that required for operating and safety stocks, which can be used in case of emergency, but not to count on this additional space in determining when to place a replenishment order. The objective in both cases is to arrive at a reasonable balance between the cost of extra inventory and storage capacity and the protection obtained against stock exhaustion or run-over. In general, the greater the safety stock and surge capacity, the smaller the risk of stock exhaustion or run-over; however, the amount of protection which each additional unit of safety stock or surge capacity buys diminishes rapidly. Thus, the question is: how much safety stock and surge capacity can be economically justified?[29] Some of the basic variables required to answer this question will be developed in the following sections.

Variation in Storage Capacity

Variations in storage capacity for a given product at a given terminal are caused by the intermittent removal of storage tanks from service for purposes of repair, alteration,

---

[29] Ibid., pp. 68-9.

or cleaning. Since no quantitative data are available which might indicate the frequency and duration of such occurrences, an estimate will be used. This estimate, which is probably on the high side, will assume that a tank is out of service approximately 10 days during each year, or approximately 0.272 per cent of the time.

Although storage tank down time is very small, the impact of an out-of-service tank on the inventory problem is tremendous. Most military terminals will store a single product in from two to ten tanks of approximately the same size. Thus, the loss of one tank will reduce storage capacity for that product between 10 and 50 per cent. Such an occurrence, in all probability, would invalidate any normal replenishment plan. For this reason, then, the problem of storage capacity variation will not be considered further in this paper.

Variation in Demand

Demand data for two different products at three different terminals over a 24-month period is shown in Table V. Due to suboptimization practices at each terminal, the data should be regarded as representing approximate rather than exact demand for each monthly period. For example, a terminal, for one reason or another, may choose July 2, July 30, and August 31, as successive cut-off periods for purposes of determining monthly demand. Both July and August are 31 day

TABLE V

MONTHLY DEMAND DATA FOR BULK PETROLEUM PRODUCTS AT VARIOUS TERMINALS[a]

Monthly Sequence	Product				
	Navy Special Fuel Oil			JP-5	
	Terminal A	Terminal B	Terminal C	Terminal A	Terminal B
1	294.9	89.0	16.4	32.2	20.3
2	192.3	27.7	18.4	31.5	5.7
3	105.0	62.8	9.9	0.3	6.3
4	183.2	45.0	16.3	19.6	1.5
5	96.3	39.7	24.2	0.0	8.1
6	124.7	57.4	21.3	0.3	0.8
7	122.7	66.5	11.6	0.0	25.4
8	267.6	82.7	63.6	27.8	14.7
9	161.9	19.7	54.8	0.0	0.3
10	124.5	69.3	54.7	17.1	1.6
11	178.6	79.2	59.4	15.1	11.0
12	164.9	20.2	37.4	1.8	26.8
13	217.5	127.9	16.9	0.0	22.0
14	200.0	117.7	54.9	29.1	22.4
15	112.9	186.2	25.6	0.3	18.8
16	257.0	107.6	69.2	33.3	32.3
17	146.0	180.3	53.7	2.3	41.4
18	239.5	98.7	19.4	3.0	11.4
19	204.5	189.8	24.0	29.5	41.3
20	83.3	127.5	28.6	10.2	27.2
21	182.1	71.5	16.3	0.3	19.4
22	191.0	68.5	34.2	27.2	20.0
23	186.6	235.8	75.4	25.5	47.5
24	257.0	129.2	53.3	19.5	28.0

[a]Personal communication, U. S. Navy Fuel Supply Office, Washington 25, D. C., August 8, 1962.

months; however, reported demand for July would cover a 28 day period while demand for August would cover a 32 day period. Since cut-off dates are not reported, a good correlation of monthly demand data is not possible.

In spite of this difficulty, an attempt has been made to illustrate that demand data may reasonably be assumed to

have a standardized normal probability distribution. This has been done by plotting the unadjusted data in Table V on normal probability paper as illustrated in Figure 3. Five straight lines, each representing an estimate of the normal curve applicable to each set of data points, have also been plotted. The fit is considered to be good considering the general lack of precision in the data which has been previously mentioned; therefore, a standardized normal distribution will be considered as being descriptive of demand characteristics.

The normal distribution has two parameters: the mean, μ, and the standard deviation, σ.[30] By assuming, for a particular product at a given terminal, that the probability distribution of demand remains the same for each instant of time considered,[31] the mean demand and standard variation of demand over the lead time period may be expressed as follows:

$$\mu_t = T\mu \qquad (2)$$

$$\sigma_t = T\sigma \qquad (3)$$

where μ_t is the mean demand over the lead time period in barrels, σ_t is the standard deviation of demand over the lead

---

[30] H. D. Brunk, An Introduction to Mathematical Statistics, Ginn and Company, New York, 1960, p. 142.

[31] Thomson M. Whitin, The Theory of Inventory Management, (Second Edition), Princeton University Press, Princeton, New Jersey, 1957, p. 57.

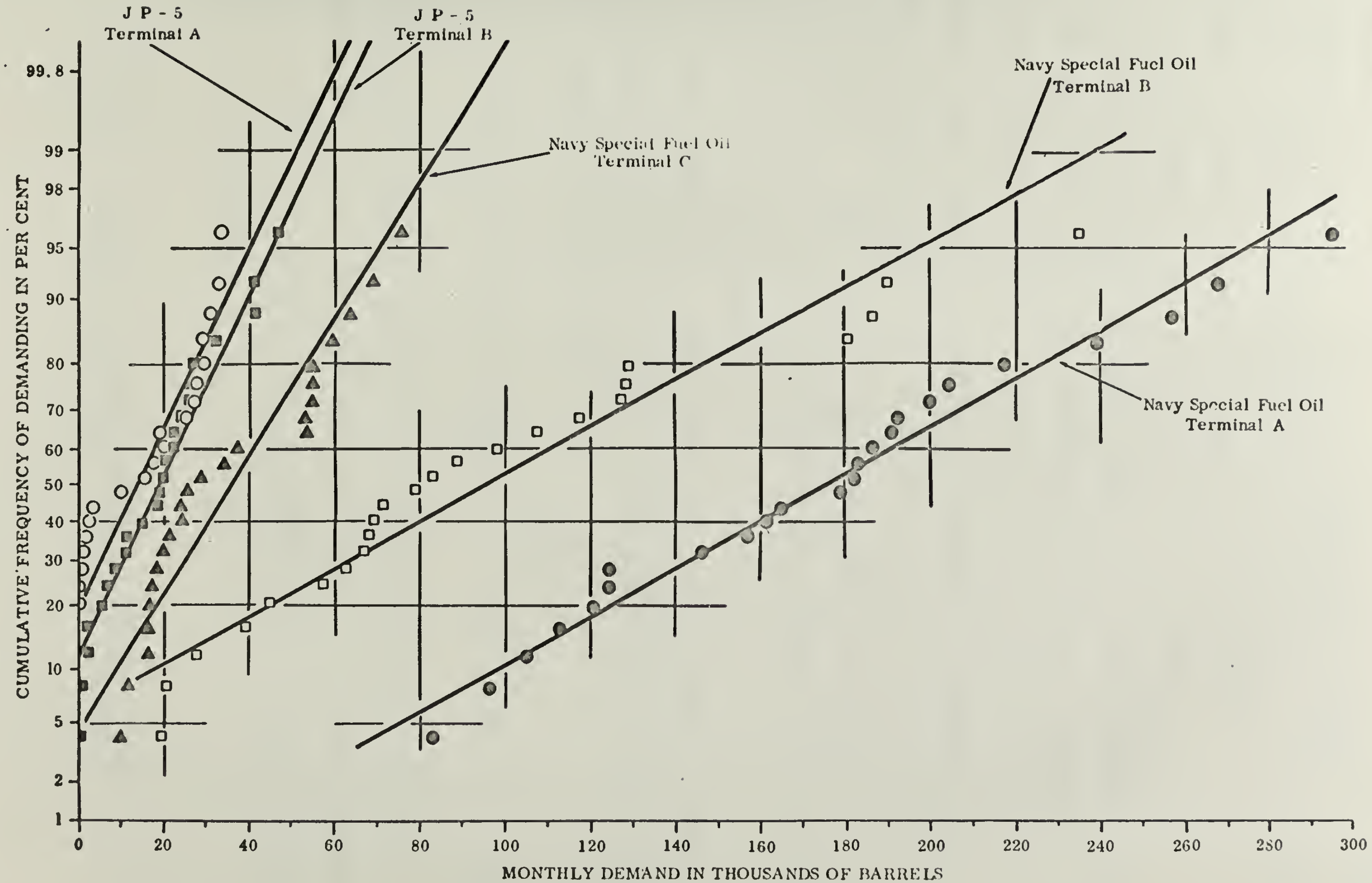

FIGURE 3: Monthly Demand for Bulk Petroleum Products at Various Terminals Versus Cumulative Frequency of Demand Plotted on Normal Probability Paper (see Table V for particulars).

time period in barrels, T is the lead time in days, μ is mean daily demand in barrels per day, and σ is the standard deviation of daily demand in barrels per day.[32]

The normal curve for Terminal A, in Figure 3, indicates that the average monthly demand for that terminal is 174,500 barrels, while the standard deviation of monthly demand is 60,500 barrels. Dividing these figures by an average of 30.4 days per month will yield an average daily demand of 5,740 barrels and a daily standard deviation of 1990 barrels. These figures will be used as a point of reference in the next chapter where the general nature of the overall problem is examined.

The probability that demand will not exceed a specified maximum value is given by the standardized normal distribution function as follows:

$$P(X \leq x) = F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{x - \mu}{\sigma}} e^{-t^2/2}\, dt \qquad (4)$$

where X is the random variable of demand in barrels, x is a selected value of demand in barrels, and F(x) is the distribution function of demand. By expressing x in terms of $\mu+\alpha\sigma$, equation (4) becomes:

---

[32]H. D. Brunk, An Introduction to Mathematical Statistics, Ginn and Company, New York, 1960, pp. 86, 88.

$$P(X \leq \mu + \alpha\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{-t^2/2}\, dt \qquad (5)$$

where α is a unitless multiplier whose purpose will be explained later. Equation (5) may be evaluated by successive integrations by parts or by reference to an appropriate table.[33] Since a mathematical function is desired and evaluation of this equation is tedious, the following empirical approximation has been developed:

$$P_A = P(X \leq \mu + \alpha\sigma) = 1 - (0.5)(10^{-0.3102\alpha - 0.18096\alpha^2}) \qquad (6)$$

The term P_A will be used hereafter as the abbreviation for the more complex probability expression. The probability values obtained from the above expression, for $0 \leq \alpha \leq 4.417$, have a maximum deviation of approximately one per cent when compared against tabulated values[34] of equation (5).

In a similar manner, the probability that demand will be equal to or greater than a specified minimum value is given by the following equation:

$$P(X \geq x) = F(x) = \frac{1}{\sqrt{2\pi}} \int_{\frac{x - \mu}{\sigma}}^{\infty} e^{-t^2/2}\, dt \qquad (7)$$

---

[33] Ibid., p. 379.

[34] Ibid.

By expressing x in terms of μ-$\alpha\sigma$, equation (7) becomes:

$$P(X \geq \mu - \alpha\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\infty} e^{-t^2/2} \, dt \qquad (8)$$

Since the standardized normal probability distribution is symmetrical, equation (8) may be expressed as:

$$P(X \geq \mu - \alpha\sigma) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\alpha} e^{-t^2/2} \, dt$$

$$P(X \geq \mu - \alpha\sigma) = P(X \leq \mu + \alpha\sigma)$$

$$P(X \geq \mu - \alpha\sigma) = 1 - (0.5)(10^{-0.3102\alpha - 0.18096\alpha^2}) \qquad (9)$$

Substituting P_B for the term on the left-hand side, yields:

$$P_B = 1 - (0.5)(10^{-0.3102\alpha - 0.18096\alpha^2}) \qquad (10)$$

an expression of the same form and with the same limitations as equation (6).

Surge Capacity and Safety Stock

The interactions of some of the previously defined concepts are schematically illustrated in Figure 4. This illustration will now be used to develop the method for specifying surge capacity and safety stock. Consider the problem of scheduling replenishments of a single product at a given terminal. A periodic report of inventory and demand has been received from the terminal. Assume that the inventory cut-off date has also been reported. Estimates of mean

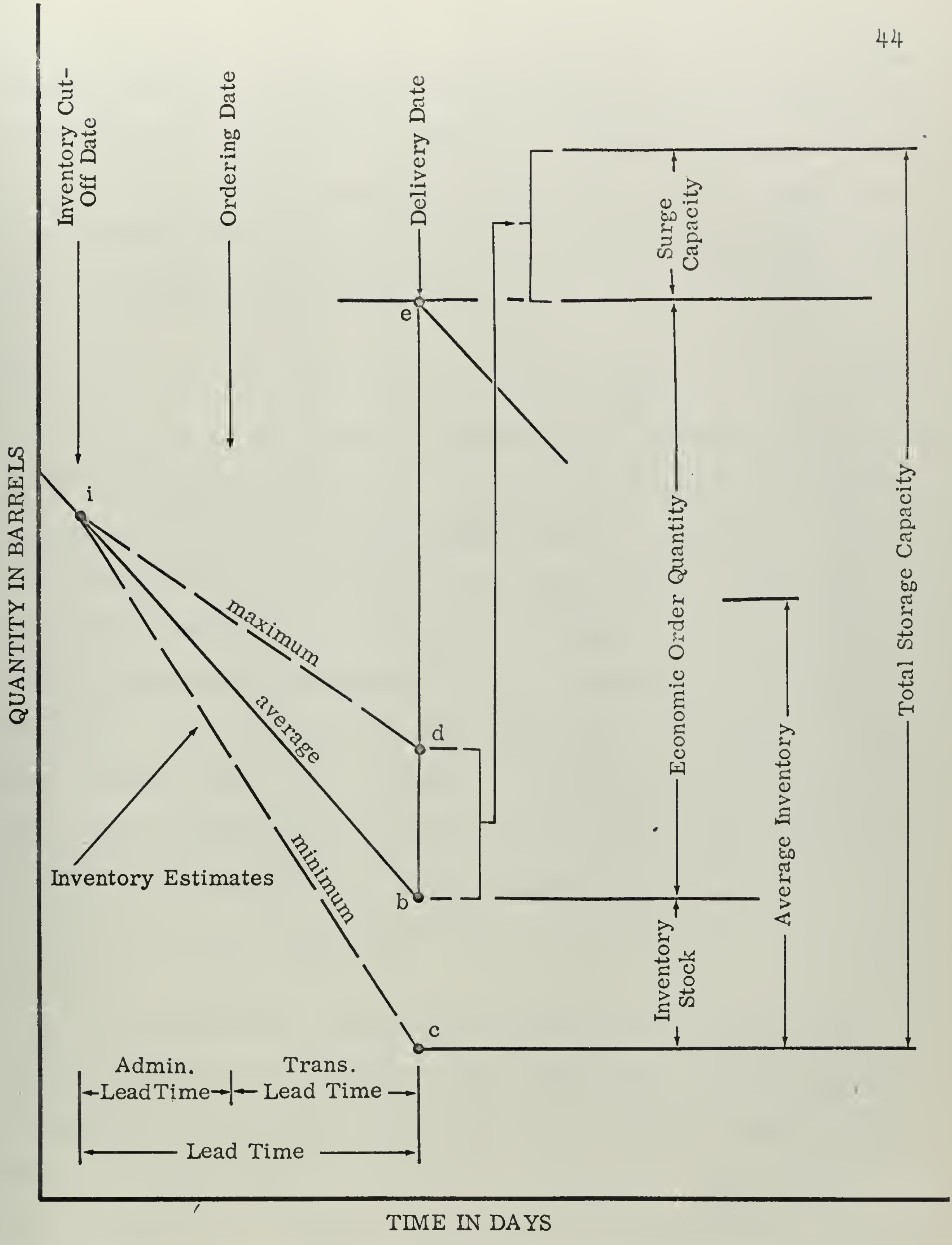

FIGURE 4: Schematic Diagram of Inventory Problem

daily demand and the standard deviation of daily demand are available as a result of analyzing historical demand data.

The expected quantity of operating stock, total stock less safety stock, which will be on hand at some future date may be computed as follows:

$$E(i - s) = i - s - \mu(t_2 - t_1) = i - s - \mu(\Delta t) \tag{11}$$

where E(i-s) is the expected quantity of operating stock on hand in barrels at time t_2, i is the total stock on hand in barrels at time t, s is the safety stock in barrels, t_1 is the inventory cut-off date, t_2 is some date following t_1, and Δt is the elapsed time in days between t_1 and t_2. The above equation is represented by the straight line $\overline{ib}$ in Figure 4. For the special case when E(i-s) is equal to zero, the elapsed time, Δt, is equal to the lead time. The lead time may then be computed by setting equation (11) equal to zero and then solving for Δt as follows:

$$T = \Delta t = \frac{i - s}{\mu} \tag{12}$$

The expected inventory on hand at some future time is, in a manner of speaking, very misleading because actual inventories hardly ever coincide with predicted inventories. They are either above or below the forecast. This problem may be handled by making an upper and lower estimate of inventory on hand at some future date. The upper estimate, represented by line $\overline{id}$, in Figure 4, has the following equation:

$$C = i - s - (\mu - \alpha\sigma)(\Delta t) \qquad (13)$$

where C is the estimate of the maximum quantity of operating stock on hand in barrels at some future time. The above equation however, represents any one of a family of curves, depending upon which value is assigned to the multiplier term, α. This value of α, as seen from equations (9) and (10), establishes the probability for future demand being greater than a given minimum value. Since future stock positions are computed by subtracting future demand from current inventory, the same probability applies to future operating stocks being less than a given value. An equation for estimating maximum future stock position, as a function of the probability or reliability of this estimate, may be obtained by solving equation (10) for the value of α, as follows:

$$\alpha = -0.857 + 2.35\sqrt{-\log(1 - P_B) - 0.168} \qquad (10)$$

and substituting this value of α into equation (13) as follows

$$C = i - s - [\mu - \sigma(-0.857 + 2.35\sqrt{-\log(1 - P_B) - 0.168})](\Delta t) \qquad (14)$$

In this type of inventory problem being considered, estimates of stock position on the delivery date are the only estimates of any practical value. These estimates may be obtained by introducing lead time, as a variable, into the estimating

equation. Thus the terms (i-s) and Δt in equation (14) may be replaced by substitution from equation (12) to produce

$$C = T\mu - [\mu - \sigma(-0.857 + 2.35\sqrt{-\log(1 - P_B) - 0.168})]T$$

$$C = T\sigma(-0.857 + 2.35\sqrt{-\log(1 - P_B) - 0.168}) \qquad (15)$$

Replacing T by its equivalent as given in equation (1) and replacing P_B by $(1-P_D)$, equation (15) becomes

$$C = (t + 0.0025d_p)\sigma(-0.857 + 2.35\sqrt{-\log(P_D) - 0.168}) \qquad (16)$$

where P_D is the risk associated with the estimate, C, of maximum stock position on the delivery date. Now examine Figure 5 again. If on the delivery date an economic order quantity of stock, represented by the line $\overline{be}$, were to arrive at the terminal, and the stock position was at point d, an additional amount of storage space, equivalent to the line $\overline{bd}$, would be required to accommodate the delivered product. This additional amount of storage space is the surge capacity which is equivalent to the C term in equation (16). The risk or probability that the stock position will be greater than C or that a full delivery would be more than sufficient to fill the available storage tanks, on the delivery date, is given by the term P_D. Thus, an inventory control manager may utilize equation (16) to establish the magnitude of the surge capacity by specifying the maximum risk of run-over he is willing to tolerate on the delivery date.

Equation (16) does not take into account the frequency of exposure to risk. Exposure to risk occurs each time the operating stock approaches zero. The annual frequency of exposure to risk is equivalent to the annual demand divided by the economic order quantity. Thus, the annual risk may be expressed as:

$$P_c = (P_D)\frac{365\mu}{Q} \qquad (17)$$

where μ is the average daily demand in barrels, Q is the economic order quantity in barrels, P_D is the risk or probability of run-over on single date, and P_c is the annual risk of run-over.[35]

Solving equation (17) for P_D and substituting the resulting expression into equation (16) yields

$$C = (t + 0.0025d_p)\sigma \left(-0.857 + 2.35\sqrt{\log(365\mu/QP_c) - 0.168}\right) \qquad (18)$$

where: C = surge capacity in barrels

t = administrative lead time in days

d_p = standard sea distance between loading and discharge ports in nautical miles

σ = standard deviation of daily demand in barrels

---

[35] John F. McGee, Production Planning and Inventory Control, McGraw-Hill Book Company, Inc., 1958, pp. 77-8.

P_c = maximum tolerable annual risk of run-over expressed as a decimal fraction

Q = economic order size in barrels

μ = mean or average daily demand in barrels

By exactly the same process of reasoning a similar equation may be derived which will indicate the amount of terminal safety stock required for a specified annual risk of run-out. The final form of the equation would be as follows:

$$S = (t + 0.0025d_p)\sigma \left(-0.857 + 2.35\sqrt{\log(365\mu/QP_s) - 0.168}\right) \quad (19)$$

where S is the safety stock in barrels, P_s is the maximum tolerable annual risk of run-out expressed as a decimal fraction, and all other terms are as previously defined. A minus sign preceding the entire expression on the right-hand side has been dropped since it only signifies that the safety stock is algebraically below the operating stock.

Inventory in Transit

Transportation lead time, in addition to its contribution to surge capacity and safety stock determinations, also contributes to another inventory function, inventory in transit. Inventory in transit may be considered as the amount of stock needed to "buy" the time required to ship product

from the supply source to the terminal.[36] Assuming that the average speed of a tanker is 400 nautical miles per day, the average inventory in transit may be stated as

$$q = 0.0025\mu d_p \tag{20}$$

where q is the average inventory in transit in barrels, and all other terms are as previously defined.[37] The average inventory in transit is a part of total average inventory which must be considered when computing inventory carrying charges.

---

[36] *Ibid.*, p. 67.

[37] *Ibid.*, p. 83.

CHAPTER IV

THE ECONOMIC MODEL

General Method of Approach

In the previous two chapters, various cost, lead time, and demand functions of inventory have been developed. In this chapter these functions will be combined to produce a single function which is descriptive of total variable inventory costs. This function will be used to determine the economic order quantity; i.e., that order quantity which minimizes the total variable cost of inventory.[37]

The resulting economic order quantity will be a function of certain independent variables. Base values will be selected for each of the independent variables in order to establish a standard value for economic order quantity function against which the individual effects of variations in the independent variables over reasonable ranges can be measured. The relative effects of each of the independent variables will then be compared to determine whether or not any obvious conclusions are warranted.

Total Variable Cost of Inventory

The total variable cost of inventory will be considered as the sum of the annual carrying cost, the annual

---

[37]Thomson M. Whitin, The Theory of Inventory Management, (Second Edition), Princeton University Press, Princeton, New Jersey, 1957, p. 32.

storage cost, and the annual procurement cost. These will be considered separately and then summed. As indicated previously, the cost to place and supervise an order is not considered a variable; hence, it will not be considered. Although the costs of running out of stock or over-filling storage capacity will, to a certain extent, be implicitly considered when finite values for safety stock and surge capacity are used, small probabilities favorable to these undesirable events will still exist. The incremental costs associated with these small probabilities are dependent upon military and political factors, which are considered beyond the scope of this paper. These costs will therefore not be included in the following analysis.

The annual carrying cost of inventory may be computed by taking the average dollar value of the inventory and multiplying it by the carrying cost rate. The average dollar value of inventory is the average inventory multiplied by the delivered unit cost of the product. If Q is the order quantity and S is the safety stock; and each new order quantity is assumed to arrive, on the average, when the old order quantity is just exhausted; the average terminal inventory will be $\frac{1}{2}Q + S$.[38] The average inventory in transit plus the average terminal inventory is the total average inventory. The delivered unit cost of the product is the

---

[38] _Ibid._

sum of the unit purchas cost and the unit transportation cost. Thus, the annual carrying cost of inventory is

$$ACC = (S + Q/2 + q)(z + r)(I) \qquad (1)$$

where ACC is the annual carrying cost of inventory in dollars, S is the safety stock in barrels, Q is the order quantity in barrels, q is the average inventory in transit in barrels, z is the unit purchase price of product in dollars per barrel, r is the unit transportation rate in dollars per barrel, and I is the annual carrying cost rate expressed as a decimal fraction.

The annual storage cost may be computed by multiplying the total required storage by the annual storage cost rate. The total required storage is merely the sum of the safety stock, ordering quantity, and surge capacity. In algebraic terms, the annual storage cost is as follows:

$$ASC = (S + Q + C)(w) \qquad (2)$$

where ASC is the annual storage cost in dollars, C is the surge capacity in barrels, and w is the annual storage cost rate.

The annual procurement cost is the product of the annual demand and the unit purchase price of the product. This cost may be expressed as:

$$APC = 365\mu(z + r) \qquad (3)$$

where APC is the annual procurement cost in dollars, and is the average daily demand in barrels.

As stated previously, the total variable cost of inventory is the sum of equations (1) through (3). Thus

$$TVC = ACC + ASC + APC$$

$$TVC = (S + Q/2 + q)(z + r)(I) + (S + Q + C)(w) + 365\mu(z + r)$$

$$TVC = (z + r)[(S + Q/2 + q)I + 365\mu] + (S + Q + C)(w) \qquad (4)$$

where TVC is the total variable cost of inventory in dollars. The terms r, C, S, and q; however, represent functions which have been developed in the previous two chapters. These functions are:

$$r = \frac{3.22A[0.0025d + x - 0.586][22.1Q + (131.5 + g)(B + U)]}{[131.5 + g][22.1Q + (131.5 + g)(B)]}$$

$$\frac{[22.1Q + (131.5 + g)(B + U + 28{,}500 - 0.1828d)]}{[22.1Q + (131.5 + g)(B + U - 3672 + 0.0235d)]} \qquad (5)$$

$$C = (t + 0.0025d_p)\sigma$$

$$(-0.857 + 2.35\sqrt{\log(365\mu/QP_c) - 0.168}) \qquad (6)$$

$$S = (t + 0.0025d_p)\sigma$$

$$(-0.857 + 2.35\sqrt{\log(365\mu/QP_s) - 0.168}) \qquad (7)$$

$$q = 0.0025\mu d_p \qquad (8)$$

Substituting these equations into equation (4) yields

$$TVC = \left[z + \frac{3.22A[0.0025d + x - 0.586][22.1Q + (131.5 + g)(B + U)][22.1Q + (131.5 + g)(B + U + 28,500 - 0.1828d)]}{[131.5 + g][22.1Q + (131.5 + g)(B)][22.1Q + (131.5 + g)(B + U - 3672 + 0.0235d]} \right]$$

$$\left[365\mu + I[Q/2 + 0.0025\mu d_p + \sigma(t + 0.0025d_p)(-0.857 + 2.35 \sqrt{\log(365\mu/QP_s) - 0.168})] \right] + w\left[Q + \sigma(t + 0.0025d_p)(-1.714 + 2.35\sqrt{\log(365\mu/QP_s) - 0.168} + 2.35\sqrt{\log(365\mu/QP_c) - 0.168}) \right] \tag{9}$$

where:

A = 1.00 for black cargoes

= 1.10 for clean cargoes

B = other liquid tanker cargo carried on the voyage, long tons

d = standardized voyage distance (i.e., from first loading port to last discharge port), in nautical miles

d_p = standardized sea distance, for the cargo of interest, between applicable loading and discharge ports, in nautical miles

g = A.P.I. gravity of product @ 60°F

I = inventory carrying cost rate, as a decimal fraction

P_c = maximum tolerable annual risk of run-over, as a decimal fraction

P_s = maximum tolerable annual risk of run-out, as a decimal fraction

Q = order quantity, in barrels

t = administrative lead time, in days

TVC = total variable cost of inventory, in dollars per year

U = minimum unused tanker cargo-carrying capacity on voyage, long tons

w = storage cost rate, in dollars per barrel-year

x = total number of tanker loading and discharge ports

z = purchase price of product, in dollars per barrel

μ = mean or average demand, in barrels per day

σ = standard deviation of demand, in barrels per day

Equation (9) will be used in the following section to determine the economic order quantity.

Economic Order Quantity

As mentioned previously, the economic order quantity is that order quantity which will minimize the total variable cost of inventory. The magnitude of the economic order quantity may be determined by taking the derivative of the total variable cost of inventory with respect to the order quantity, setting the derivative equal to zero, and solving for the (economic) order quantity. The net result of these operations on equation (9) is an extremely complex expression in which the economic order quantity is most easily determined by a reiterative trial and error procedure. This expression may be simplified to some extent by assuming that the safety stock and surge capacity are equal ($P_s = P_c$), and the tanker

is fully loaded at some point in its voyage (U = 0). The resulting expression, which must still be evaluated by a trial and error procedure, is as follows:

$$\Big[365\mu/I + Q/2 + 0.0025\mu d_p + \sigma(t + 0.0025d_p)(-0.857 + 2.35 \sqrt{\log(365\mu/QP) - 0.168})\Big]\Big[44.1(32{,}172 - 0.2064d)\Big] = \Big[[22.1Q + (131.5 + g)(B - 3672 + 0.0235d)][(22.1Q)/(131.5 + g) + B + 28{,}500 - 0.1828d] + [22.1Q + (131.5 + g)(B - 3672 + 0.0235d)]^2 [(zI + 2w)/(3.22AI)(0.0025d + x - 0.586)]\Big]\Big[1 - (1.021\sigma)(t + 0.0025d_p)/(Q)(\sqrt{\log(365\mu/QP) - 0.168})\Big] \quad (10)$$

where Q is now the economic order quantity in barrels, P is the maximum tolerable risk of run-out or runover, and all other terms are as previously defined. The above equation will be used in the following section to determine the effect of the indicated variables on the size of the economic order quantity.

Evaluation and Analysis of Economic Order Quantity

Prior to evaluating equation (10) for actual values of the economic order quantity, recall that certain assumptions have been made in deriving this equation. Some of these assumptions are the cost factors used, no variation in transportation lead time, no penalties for run-out or run-over, no

limitations with regard to tanker draft, and no order costs. In the aggregate, these assumptions may not be valid; thus, firm conclusions concerning the actual size of the economic order quantity should not be made. In other words, actual values for the economic order quantity should be used only for the purpose of making relative comparisons.

Base values and ranges for the independent variables in equation (10) have been listed in Table VI. Wherever possible, data developed in Chapters II and III has been incorporated into the table. Other values have merely been assumed. The base values were used to calculate standard values for the economic order quantity and the total cargo deadweight. These standard values are 278,000 barrels and 43,900 long tons respectively. The independent variables in each group were then allowed to assume their middle and maximum values as indicated, while leaving all other variables at their base values. The economic order quantity and total cargo deadweight were calculated for the middle and maximum values of each group of variables, and listed in the table for later comparison with the standard values. In each case, the total cargo deadweight has been determined by calculating the weight of the economic order quantity (O_e) and adding the weight of any other cargo carried by the tanker (B). Total cargo deadweight gives a rough approximation of tanker deadweight; i.e., see equation (12) in Chapter II, set $O = O_e$,

TABLE VI

VARIATION IN ECONOMIC ORDER QUANTITY AND TOTAL CARGO DEADWEIGHT AS FUNCTIONS OF VARIOUS GROUPS OF INDEPENDENT VARIABLES

Independent Variables					Economic Order Quantity (Q_e) in thousands of bbl.		Total Cargo Deadweight (D) in thousands of long tons	
Group	Symbol[a]	Values						
		Base	Middle	Maximum	Middle	Maximum	Middle	Maximum
I	d d_p	1,000 1,000	4,000 4,000	8,000 8,000	439	565	69.5	89.4
II	B	0	10,000	20,000	213	148	43.7	43.4
III	B d x	0 1,000 2	10,000 2,000 3	10,000 4,000 3	299	384	57.2	70.5
IV	B d d_p x	0 1,000 1,000 2	10,000 4,000 4,000 3	10,000 8,000 8,000 3	388	500	71.3	89.0
V	I	0.10	0.20	0.30	191	145	30.2	22.8
VI	P	0.10	0.05	0.01	277	274	43.8	43.3
VII	t	5	10	20	280	283	44.2	44.8
VIII	z A g	2.00 1.00 12	3.90 1.10 42	4.88 1.10 62	222	205	28.3	22.8
IX	w	0.000	0.080	0.146	240	219	38.0	34.6
X	μ	1,000	3,000	5,740	468	627	74.0	100.0
XI	σ	500	1,000	1,990	280	284	44.5	45.2

[a]See text following equation (9) for definitions and units.

and assume $U = 0.0$, so that $D = O_e + B$. Thus comparisons of tanker requirements for cargo delivery may also be made.

The standard values for economic order size and cargo deadweight are based, among other things, on a given voyage distance ($d = 1000$ nautical miles), a single port loading, and a single port discharge of a single product ($d_p = d$, $x = 2$, and $B = 0.00$). The effect of changing the voyage distance may be observed by examining the figures in Group I of Table VI. Analysis indicates that the economic order quantity and total cargo deadweight are roughly proportional to the cube root of the voyage distance (d). The effect that other tanker cargo (B) has on the economic order quantity is indicated in Group II of Table VI. Examination indicates that variation of B from its base value 0.00 to 10,000 and 20,000 long tons will result in the variation of total cargo deadweight from the base value of 43,900 to 43,700 and 43,400 long tons respectively. Total cargo deadweight is thus essentially independent of B, the quantity of other cargo carried by the tanker. In other words, the weight of the economic order quantity is decreased by an amount equivalent to the weight of other cargo carried. Prior to jumping to conclusions, examine Groups III and IV. Group III represents a single port lifting of cargo, with the discharge of the cargo of interest at the first port of discharge, and the discharge of the other cargo at the second port of discharge ($d_p = 1000 \leq d$, $x = 3$, $B \geq 0$). Group IV represents a single port lifting of

cargo at one port, with a two-port discharge where the cargo of interest is discharged last ($d_p = d \geq 1000, \quad = 3, B \geq 0$). Note that in each group, the previously mentioned cube root relationship exists between the total cargo deadweight (D) and the total voyage distance (d). Furthermore, for equal values of total voyage distance, in each of the first four groups, the total cargo deadweight has approximately the same value. These relationships have been summarized below from the data in Table VI.

Total Voyage Distance (d) in naut. miles	Other Cargo Carried (B) in long tons	Total Cargo Carried (D) in long tons	$\beta = D(d)^{-\frac{1}{3}}$	Group
1,000	0 10,000 20,000	43,900. 43,700 43,400	4,390 4,370 4,340	I,II,III,IV II II
2,000	10,000	57,200	4,540	III
4,000	0 10,000 10,000	69,500 70,500 71,300	4,375 4,440 4,490	I III IV
8,000	0 10,000	89,400 89,000	4,470 4,450	I IV

The foregoing analysis suggests that the economic order size is dependent, to a great extent, upon unit transportation rates as determined by total voyage distance (d) and total cargo (or tanker) deadweight (D). In addition, the maximum economic order size as determined by the total cargo deadweight, may be reduced by loading other cargo on the same tanker. Increasing the voyage distance (d) and total number of ports (x) in order to discharge the other part of

the cargo will, of course, cancel some or all of the reduction in the economic order quantity obtained by loading this other cargo.

Now, examine the variables, I, P, and t, which are susceptible to immediate management control. The effect of the annual holding cost rate, I, on the economic order quantity may be determined by examining the data in Group V. This data indicates that, in a very rough manner, the economic order quantity is directly proportional to the reciprocal of the holding cost rate. The data in Group VI and VII indicate the size of the economic order quantity is practically independent of the risk factor, P, or the administrative lead time, t. Thus, the annual carrying cost rate, I, is the only effective variable which management may use to control the economic lot size.

The effect that choice of product has on the economic lot size is indicated by the data in Group VIII of Table VI. As product cost (z) increases from its base value of 2.00 to 3.90 and 4.88 dollars per barrel, the economic order quantity decreases from its base value of 278,000 to 222,000 and 205,000 barrels respectively, and the total cargo deadweight decreases from its base value of 43,900 to 28,300 and 22,800 long tons respectively. Note that the economic order quantity decreases to 80 and 74 per cent of its base value while the total cargo deadweight decreases to 62 and 52 per cent of its base value.

This apparent discontinuity is attributable to the fact that A.P.I. gravity (g) increases as product cost increases. Since specific gravity decreases as A.P.I. gravity increases, a unit volume of product with a high A.P.I. gravity will weigh less than a unit volume of product with a low A.P.I. gravity (see equation (15) in Chapter II).

Now examine the remaining three variables, w, μ, and σ. The data in Group IX indicates that the economic lot size will decrease as unit storage cost rates (w) are increased, while the data in Group X indicates that changes in average daily demand (μ) have the opposite effect on the economic order quantity. Variations in the standard deviation of daily demand (σ), however, have negligible effect on the economic order quantity.

Summary and Conclusions

The analysis in the preceding section indicated that of the thirteen variables tested for their effect on the economic order size, only nine appeared to have any significant influence on the order size. Although the four remaining variables, d_p, P, t, and σ , do not contribute to the determination of the economic order quantity, equations (6) and (7) indicate that they exert a large influence on the size of the safety stock and the surge capacity. Thus, the total variable cost of inventory, as it is affected by surge capacity and safety stock, may be reduced by decreasing the

distance (d_p) between the terminal of interest and its source of supply, the administrative lead time (t), and the standard deviation of daily demand (σ); or by increasing the risk (P) of run-out or run-over. For a given level of risk, the easiest way to reduce the safety stock or surge capacity would be to reduce the administrative lead time.

Of the nine variables which affect economic order quantity, only four, B, x, d, and I, can be controlled to any extent by management. As mentioned previously, the economic order quantity will be lowered in almost direct proportion to the reciprocal of the holding cost rate. This suggests that a rational interpretation of holding cost rate should be developed for a military inventory system, rather than the traditional "interest rate" used in industry. Perhaps some function of inventory velocity would be appropriate. The economic order size may also be reduced by decreasing the total voyage distance (d), the number of loading and discharge ports (x); and by increasing the quantity of other cargo carried (B). This suggests that economies can be achieved by utilizing single port loadings and single port discharges of multiple cargoes. Such a policy would require the utilization of tankers capable of carrying two, three, or four types of cargo as fleet oilers do. In addition to the economic benefits, such a policy would have obvious military advantages. Tankers used for terminal replenishment

could also be used as substitutes for fleet oilers. In terms of military operations, the risk of losing a tanker would be spread over several types of products rather than a single and perhaps critical product.

Since this study has indicated that economic as well as military advantages may accrue as a result of a more scientific approach to the problems involved in handling military petroleum inventories, further work in this general area is recommended. As a first step, more accurate estimates of cost and demand variables should be developed. Particular emphasis should be placed on the unit tanker transportation rate as this variable has a great influence on the economic problem. In this connection, the work of Benford[39] is recommended as an excellent reference. The effect of other variables such as light loading, variations in transportation lead time, and cost penalties for stock run-out and run-over should also be investigated. Should these investigations confirm the generalization that split (multiple product) cargoes are economically advisable, other investigations utilizing linear programming techniques should be conducted to determine optimizing parameters for terminal and system management. These studies would yield such information as

---

Harry Benford, "Engineering Economy in Tanker Design," Transactions, The Society of Naval Architects and Marine Engineers, 65:775-838, 1957.

optimum tanker and terminal size, optimum storage and inventory distributions between products both aboard tankers and at terminals, and optimum routes between sources of supply and terminals.

BIBLIOGRAPHY

BIBLIOGRAPHY

Books

Bes, J., Tanker Chartering and Management, Uitgeverij v/h C. DE BOER JR., le Weteringplantsoen 8, Amsterdam, 19.

Brunk, H. D., An Introduction to Mathematical Statistics, Ginn and Company, New York, 1960.

McGee, John F., Production Planning and Inventory Control, McGraw-Hill Book Company, Inc., 1958.

Whitin, Thomson M., The Theory of Inventory Management, (Second Edition), Princeton University Press, Princeton, New Jersey, 1957.

Articles and Periodicals

Anonymous, "How Big Tankers Cut Cost," Petroleum Week, McGraw-Hill Book Company, New York, Volume 9, Number 12, September 18, 1959, p. 59, cited by Boyd, Ben F., "A Study of Some of the Effects of Supertankers on Military Petroleum Logistics," Unpublished Master's Thesis, The University of Kansas, Lawrence, Kansas, 1961, p. 24.

Anonymous, "Price Statistics," Oil and Gas Journal, Petroleum Publishing Company, Tulsa, Volume 60, Number 32, August 6, 1962, p. 216.

Benford, Harry, "Engineering Economy in Tanker Design," Transactions, The Society of Naval Architects and Marine Engineers, Volume 65, 1957, pp. 775-838.

Earle, M. Mack, "The Conversion of T2 Tankers for Great Lakes and Seaway Service," Transactions, The Society of Naval Architects and Marine Engineers, Volume 68, 1960,

Emerson, H. N., "Oil -- No. 1 Transaction Job," The Oil and Gas Journal, Petroleum Publishing Company, Tulsa, November 18, 1957, pp. 226-32.

Frankel, P. H., "Short-Term and Long-Term Tanker Freight Rates and the Significance of Their Fluctuations," Proceedings Third World Petroleum Congress, Section IX, E. J. Brill, Leiden, Netherlands, 1951, pp. 185-195.

Frankel, P. H. and W. L. Newton, "Current Economic Trends in Location and Size of Refineries in Europe," Proceedings, Fifth World Petroleum Congress, Section IX, Paper 10, Fifth World Petroleum Congress, Inc., New York, 1959.

Jung, I., and G. Ohlsson, "Technical and Economic Data for Turbine Powered Tankers," International Shipbuilding Progress, Volume 4, October 1957.

Kahle, Loren F., and A. J. Kelly, Jr., "The Role of Sea Transportation in the Petroleum Industry," Proceedings, Fifth World Petroleum Congress, Section VIII, Paper 1, Fifth World Petroleum Congress, Inc., New York, 1959.

McArthur, D. S., and others, "Operations Research Applied to Marine Transportation and Tankage Problems," Proceedings Fifth World Petroleum Congress, Section VIII, Paper 2, Fifth World Petroleum Congress, Inc., New York, 1959.

Nichols, W. O., M. L. Rubin, and R. V. Danielson, "Some Aspects of Large Tanker Design," Transactions, The Society of Naval Architects and Marine Engineers, Volume 68, 1960, pp. 743-835.

Prater, N. H., and John Mylo, "Equipment Cost Data File," Hydrocarbon Processing & Petroleum Refiner, Gulf Publishing Company, Houston, Volume 40, Number 6, June 1961, p. 174.

Prater, N. H., and John Mylo, "Equipment Cost Data File," Hydrocarbon Processing & Petroleum Refiner, Gulf Publishing Company, Houston, Volume 40, Number 8, August 1961, p. 132.

Zimmerman, O. T., and Irvin Lavine, "Want Equipment Costs for Estimates?" Petroleum Refiner, Gulf Publishing Company, Houston, Volume 35, Number 8, August 1956, pp. 116-127.

U. S. Government Publications

Bureau of Naval Personnel, Fundamentals of Petroleum, NAVPERS 10883, Superintendent of Documents, U. S. Government Printing Office, Washington 25, D. C., 1953.

Unpublished Material

Boyd, Ben, F., "A Study of Some of the Effects of Supertankers on Military Petroleum Logistics," Unpublished Master's Thesis, The University of Kansas, Lawrence, Kansas, 1961, pp. 26-28, citing responses to questionnaires sent to major oil companies, independent tanker operators and shipbuilding firms.

BUSANDA Notices 7330 for 4th Qtr. of FY 59 and for 1st, 2nd, 3rd, and 4th Qtrs. of FY 60, Bureau of Supplies and Accounts, Navy Department, Washington 25, D. C.

COMSTS INSTRUCTION 7600.3, Commander, Military Sea Transportation Service, Washington 25, D. C.

Military Petroleum Indoctrination Course sponsored by the Military Petroleum Supply Agency in conjunction with the Union Oil Co. of California, Tidewater Oil Co., and Richfield Oil Co., Los Angeles, September 26 - October 7, 1960.

Personal communication, U. S. Navy Fuel Supply Office, Washington 25, D. C., August 8, 1962.

Petroleum Conversion Factors and Capacity Tables for Logistics Planning and Reference, Office of the Assistant Secretary for Defense (Supply & Logistics), Petroleum Office, Washington 25, D. C., September 1953.

Miscellaneous

Address by Mr. J. D. Rogers, Executive Vice-President of the Esso Shipping Company at the American Merchant Marine Conference, New Orleans, October 1955, cited by Bes, J., _Tanker Chartering and Management_, Uitgeverij v/h, C. DE BOER JR., le Weteringplantsoen 8, Amsterdam, 1959, p. 25.

Library
U. S. Naval Postgraduate School
Monterey, California

thesC1944
An investigation of the theoretical effe
3 2768 001 02008 4
DUDLEY KNOX LIBRARY