1967

A Game Theoretic Analysis of the Convoy-ASW Problem

Kiland, Ingolf Norman

Monterey, California. U.S. Naval Postgraduate School

A GAME THEORETIC ANALYSIS OF THE CONVOY-ASW PROBLEM

INGOLF NORMAN KILAND, JR.
and
JERRY ALLEN KOTCHKA

A GAME THEORETIC ANALYSIS OF THE CONVOY-ASW PROBLEM

by

Ingolf Norman Kiland, Jr.
Lieutenant, United States Navy
B.S., Naval Academy, 1959

and

Jerry Allen Kotchka
Lieutenant, United States Navy
B.S., Naval Academy, 1962

[redacted]

[redacted]

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
December 1967

ABSTRACT

The problem of allocation of ASW forces assigned to an oceanic convoy in a submarine warfare environment is postulated as a two-person game with the payoff function being based on the "formula of random search". The opponents in the game are a convoy system and a submarine system. A submarine is given the option of attacking the convoy system either from afar with surface-launched missiles or near with torpedoes. The convoy system is defended by units capable of destroying submarines exercising either of their options. The optimal allocation of forces for both sides is shown to be a set of pure strategies which are dependent on the parameters of the model.

TABLE OF CONTENTS

Section		Page
I.	INTRODUCTION	5
	General	
	Definition of Problem	
II.	FORMULATION OF THE MODEL	9
	Payoff Function	
	Expected Blue Losses	
	Expected Red Losses	
	Determination of Probabilities	
	Explicit Form of the Payoff Function	
III.	CRITERION OF THE MODEL	17
IV.	BLUE'S OPTIMAL STRATEGY	20
	Development	
	Example	
V.	DISCUSSION OF THE MODEL	33
	Assumptions	
	Payoff Function and Alternatives	
	Use of Game Theory	
VI.	EXTENSIONS AND FURTHER STUDIES	42
VII.	SUMMARY AND CONCLUSION	45
BIBLIOGRAPHY		46

ACKNOWLEDGEMENT

We wish to express our gratitude to Professor Alan W. McMasters for the direction he provided to our efforts and the ideas he contributed to this study. Further, his precise review of our many drafts resulted in many improvements in the organization and final wording of this thesis.

I. INTRODUCTION

GENERAL. The oceanic crossing of a convoy system in a submarine warfare environment is a problem of major concern to the U. S. Navy. As in all warfare the lines of communication must be maintained open to insure the support of the front lines of defense. In present day warfare a primary means of accomplishing this goal is by large oceanic convoys. An enemy would desire to make this task impossible or at least severely limit its success. In a non-nuclear conflict an enemy would probably send submarines against the convoy, since they are less susceptible to detection and attack than either aircraft or surface ships. The convoy would have to protect itself with anti-submarine warfare vehicles if it expects to succeed in its mission.

Since the results of a given convoy crossing will depend on the course of action taken by each force, modeling the defense of a convoy as a two-person game is intuitively appealing. In previous studies the convoy problem has been analytically treated by various deterministic or probabilistic mathematical techniques by Boice [1], Cooper [2], and others. However, these approaches were founded on the assumption that the opposition has some well defined tactic. The game theoretic approach does not require this assumption but more realistically considers that the submarine system also has an allocation problem.

DEFINITION OF THE PROBLEM. Throughout this paper we will refer to the convoy system and the submarine system which oppose each other as Blue and Red forces respectively. The convoy system is assumed to consist of Blue logistic units and attacking units. The role of the Blue logistic units is to transport men and equipment that are necessary for conducting a foreign campaign across an ocean. The Blue attacking units are to defend the logistic units. The submarines' primary role is the destruction of the convoy logistic units before they reach their destination.

Once contact with a convoy has been made, there are two methods of attack open to the Red forces. The most accurate means of delivery of a weapon is for a submarine to penetrate the convoy's screen and make a close-in torpedo attack. By standing off outside torpedo range a submarine may be able to use surface-launched missiles which are more destructive weapons than torpedoes. In our analysis a submarine will be allowed the option of attacking the convoy system either from afar with surfaced-launched missiles or near with torpedoes.

Since a submarine cannot attack until he is at least within missile range we will assume that all Red forces will be located somewhere within missile range of the convoy's path.

During both torpedo and missile attacks, a submarine's susceptibility to detection is increased considerably over normal cruising conditions. A missile attack requires that a submarine surface before launching the missile. For a submarine to realize an optimum attack position and torpedo fire-control solution, during a torpedo attack, he must operate at various speeds on various courses and occasionally broach his periscope.

Because Red may be detected before he can attack, it seems reasonable to assume that the Blue ASW forces will have an opportunity of attacking the Red submarine before the convoy absorbs the Red attack.

As the convoy proceeds on its oceanic crossing two separate areas of the total ocean will be of primary concern to both sides. The first, called the area of interest, is the area surrounding the path of the convoy in which any submarine present can conduct a torpedo attack as the convoy passes. Thus the area of interest is a function of Red's effective torpedo attacking radius. The second area, called the area of concern, consists of the total oceanic area adjacent to the area of interest from which a submarine can attack with a surface-launched missile. Thus the area of concern is a function of Red's effective missile radius. It is assumed that Blue has knowledge of, or is capable of estimating the size of these areas.

Since the enemy submarines can be expected to have a reasonable knowledge of the originating and terminal points of the convoy, they will be able to estimate the route utilized by the convoy. Thus it is seen that both sides can be expected to know both the size and the location of both areas throughout the convoy crossing.

Within this setting, the commanders of both forces are faced with the problem of how "best" to allocate their respective attacking units between the areas of interest and concern. In this paper we will concentrate our attention, during formulation, to <u>the Blue commander's problem</u>. It will be evident, however, that we will resolve both commanders' problems as a consequence of the game theoretic approach to the solution of the Blue commander's problem.

II. FORMULATION OF THE MODEL

PAYOFF FUNCTION. Since both sides are confronted with an allocation problem between the same two areas of the ocean, the measure of the payoff should be a function of those forces of both sides that are deployed in each area. Because each player is trying to destroy his opponent's ships a logical measure of the outcome of each player's actions is the expected losses incurred by both sides in each of the two areas.

Each player would use the payoff function to guide his decision making. The logical reaction of each player would be to try to maximize his opponent's losses while minimizing his own. However, it is important to realize that the nature of the ASW problem precludes the equivalence between maximizing an opponent's losses with minimizing one's own losses. For example, the minimizing of Blue losses is not equivalent to maximizing Red losses because a Blue attacking unit need not kill, but only prevent the submarine from attacking, to minimize Blue losses. However, to maximize Red losses, Blue must kill the Red units.

By minimizing the convoy system's losses the Blue commander realizes an immediate benefit because the final value of the logistic units completing the convoy crossing is maximized. By maximizing Red losses the Blue force also receives a long-run return because the Red units destroyed represent no threat to future convoys. Similarly, if the Red commander is seeking to maximize Blue convoy losses he is fulfilling his mission. At the same time, if he is minimizing his own losses then he, too, is receiving both a long-and short-run return.

The payoff for the Blue commander's problem will be taken as the difference between Blue's expected losses and a weighted function of Red's expected losses. Blue selects the weighting factor in a manner such that the latter losses are commensurable to the former from his viewpoint. The Blue commander's objective will be to minimize this linear combination.

Thus the payoff function, D, being the difference between Blue's expected losses and weighted Red's expected losses, may be expressed as:

$$D = DB - U\,(DR)\ , \qquad (1)$$

where DB = the total expected number of Blue logistic losses (Blue ships);

DR = the total expected number of Red submarine losses (Red ships);

U = the weighting factor equating a unit Red loss to a unit Blue loss (Blue ships/Red ships).

In practice there is a difference in worth of a submarine as compared to a logistic vessel. For this reason a weighting factor or utility index, U, is used for each Red submarine loss which equates the value of one Blue logistic loss to one Red submarine loss.

The structure of the payoff function, D, implies that a positive value of D corresponds to a gain by Red; a negative value of D corresponds to a gain for Blue.

EXPECTED BLUE LOSSES. In the development of an expression for the expected Blue losses we will assume that the Red forces consist of submarines which have identical capabilities and effectiveness. In addition, each Red submarine is assumed to be able to render either torpedo or surface-launches missile attacks, but not both concurrently. Finally, since a large convoy provides a "noisy" target, the probability of detection of the convoy by a Red unit can be considered to be unity.

To prevent multiple detection of Red by Blue, and at the same time to ensure a reasonable degree of survival, the submarines are assumed to act independently of one another and to be deployed uniformly over each of the areas.

We assume that each submarine has a limited supply of torpedoes and missiles; consequently, to conserve his weapons for the logistic units he does not expend his weapons on the ASW vehicles. It is obvious that Red has no choice in the case of ASW aircraft, but he may have a choice if Blue is using ships as the ASW vehicles.

The Blue attacking units are assumed also to act independently of each other. Further, we will assume that all units will be assigned equal areas in which to conduct a random search for Red submarines.

The independent, random search by both Red and Blue, combined with the limitations of Blue's detection equipment characteristics, sea and weather conditions, and operator performance allows the use of the "formula of random search" for determining the probability of detection of a Red unit by a Blue unit. [2].

The total expected number of Blue losses, DB, may therefore be expressed as:

$$DB = mP_iC_i + (M - m)P_cC_c \qquad (2)$$

where M = the total number of Red submarines;

m = the number of Red submarines in the area of interest;

M - m = the number of Red submarines in the area of concern;

P_i = the probability of a Red submarine in the area of interest, A_i, survives an attack by a Blue unit;

P_c = the probability a Red submarine in the area of concern, A_c, survives an attack by a Blue unit;

C_i = the expected number of Blue logistic units destroyed (i.e., effectiveness constant) by a Red submarine deployed in the area of interest;

C_c = the expected number of Blue logistic units destroyed by a Red submarine deployed in the area of concern.

The terms C_i and C_c represent effectiveness constants and are functions of the performance of a submarine. They are dependent on the number of Blue logistic units a Red submarine is able to take under attack, the accuracy of its attacks, and the effectiveness of its weapons.

<u>EXPECTED RED LOSSES</u>. Based on the assumptions associated with equation (2), we can express the total number of expected Red losses, DR, as:

$$DR = mPS_i + (M - m)PS_c \qquad (3)$$

where PS_i = the probability a Red submarine in A_i is destroyed by a Blue attacking unit, and

PS_c = the probability a Red submarine in A_c is destroyed by a Blue attacking unit.

Since P_i is the probability a Red unit survives, $PS_i = 1 - P_i$. Similarly, $PS_c = 1 - P_c$.

DETERMINATION OF PROBABILITIES. To determine the probabilities of kill it is necessary to define another area. The Blue attacking units are deployed in an area which is called the area of search, AS. The area of search has the convoy as its center at all times and is made up of two parts. The first is an area denoted AS_i which is partially congruent with the area of interest. The second, AS_c, is partially congruent with the area of concern.

The formation or composition of the logistic convoy may be thought of geometrically as a square, and the total area of search, $AS = AS_i + AS_c$, may then be considered to be a square with dimensions J by J, where J/2 is the maximum range of the Red missiles. For symmetry, the area AS_c has dimensions L_c by J and the area AS_i has dimensions L_i by J. Thus $J = L_i + L_c$ where L_i and L_c are measured perpendicular to the convoy's track (see figure 1).

It should be noted that as the convoy transits, AS_i will sweep out the area of interest, A_i, and AS_c will sweep out the area of concern, A_c.

The time, T, for the convoy to travel the distance J at a speed of S is given by

$$T = \frac{J}{S} \quad .$$

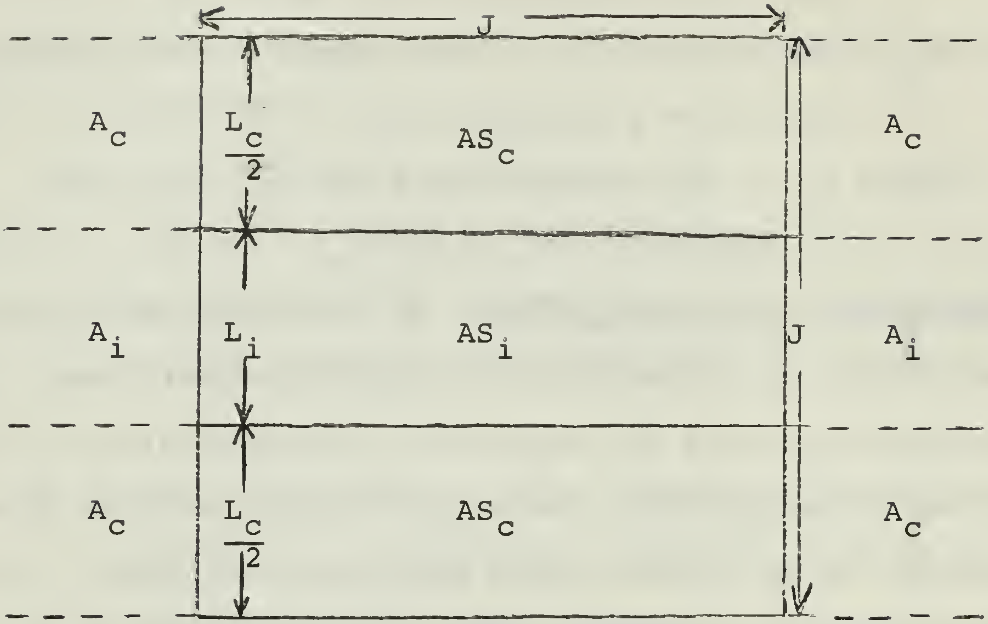

Figure 1

Because all attacking units are assigned equal areas for individual search,

$$\widehat{AS}_i = \frac{AS_i}{n} \quad ,$$

where $\widehat{AS}_i$ = the area assigned to each Blue attacking unit in AS_i;

n = the number of Blue attacking units in AS_i.

Similarly in the area of concern:

$$\widehat{AS}_c = \frac{AS_c}{N - n}$$

where $\widehat{AS}_c$ = the area assigned to each Blue attacking unit in AS_i;

N = the total number of Blue attacking units available;

N - n = the number of Blue attacking units in AS_c.

The probability, PS, that a Red submarine is killed in any area, given it is present, is expressed by

$$PS = P_w P_l P_d \ ,$$

where P_w = the probability a weapon launched by Blue is effective, given a weapon is launched;

P_l = the probability that a Blue attacking unit gets into an attack position and launches a weapon, given that a Red submarine is detected by a Blue attacking unit;

P_d = the probability a Red submarine is detected by a Blue attacking unit, given the submarine is in the area being searched.

Using the "formula for random search" as the probability of detection, P_d, the probability a Red submarine is killed in the appropriate area is:

$$PS = P_w P_l \left(1 - \exp - \frac{wWT}{\widehat{AS}}\right) \qquad (4)$$

where w = the relative speed of the Blue attacking unit and Red submarine;

W = the effective sweep width of the Blue attacking unit in either A_i or A_c.

Both w and W are assumed to be constant over the duration of the convoy's oceanic crossing.

Upon substitution for T and AS in equation (4), the probability a Red submarine in the area of interest is killed, PS_i, is

$$PS_i = (P_w)_i (P_l)_i \left[1 - \exp - \frac{wW_i Jn}{S(AS_i)}\right] . \qquad (5)$$

Since w, W_i, J, S, and AS_i are constant parameters of the model, we set $k_i = \frac{wW_iJ}{S(AS_i)}$, where k_i represents Blue's detection constant in the area of interest. In a like manner, since $(P_w)_i$ and $(P_1)_i$ are constant parameters we set $K_i = (P_w)_i(P_1)_i$, where K_i is a measure of Blue's attacking performance in the area of interest.

Similarly, we will denote k_c as Blue's detection constant in the area of concern and K_c as a measure of Blue's attacking performance in the area of concern.

Equations (4) and (5) can now be rewritten as

$$PS_i = K_i[1-\exp-k_in] \; ; \qquad (6)$$

$$PS_c = K_c[1-\exp-k_c(N-n)] \; . \qquad (7)$$

EXPLICIT FORM OF THE PAYOFF FUNCTION. Substitution of equations (6) and (7) in equations (2) and (3) gives the explicit forms of the expected Blue and Red losses. These forms, upon substitution into equation (1), result in the following expression for the Blue commander's payoff function:

$$D(m,n)=mC_i[1-K_i(1-\exp-k_in)] + (M-m)C_c\{1-K_c[1-\exp-k_c(N-n)]\}$$
$$- U\{mK_i(1-\exp-k_in) + (M-m)K_c[1-\exp-k_c(N-n)]\}. \qquad (8)$$

III. CRITERION OF THE MODEL

In analyzing this military conflict situation as a game of strategy, a player's skill and intelligence should be used to determine the payoff. The formulation of the model was intended to structure Blue's allocation problem as a two-person, finite, zero-sum, non-cooperative game.

It is two-person since there are only two opponents, Blue and Red. Since each player's resources are discrete units and have an upper limit, both have a finite number of possible alternative allocations. The game is therefore finite. The game is non-cooperative because neither side communicates with the other.

Consideration of Blue's allocation problem in the zero-sum sense means that whatever Blue does not win (i.e., Blue losses and Red survivals) will be considered to be a gain for Red and Red's gain is measured by Blue on the same scale as he measures his own payoff. It follows then that the sum of Blue's and Red's payoff determined in this manner will be zero.

A finite two-person, zero-sum game in which both opponents play simultaneously without information about the other's action is called a rectangular or matrix game. For such games, a payoff matrix or array of the payoffs to either player resulting from all combinations of the players' strategies can be constructed. If M and N are the total number of available Red and Blue attacking units, respectively, then a payoff matrix, D, can be constructed such that the number of the rows and the number of the columns is equal to the number of possible units that Red and Blue could respectively deploy into the area of interest. This completely describes all possible outcomes since units not allocated in the area of interest are allocated to the area of concern. An element of D, denoted d_{ij}, represents the expected outcome for a crossing in which Red uses i attacking units and Blue uses j units in the area of interest. Thus D is a M by N matrix with elements d_{ij} such that $i = 1,2,\ldots,M$ and $j = 1,2,\ldots,N$.

In solving for his optimal strategy in a matrix game, Blue will apply the minimax criterion. Under this criterion Blue makes use of the following three presuppositions.

First, Blue feels that Red's motives are diametrically opposed to his own. Blue is trying to get the convoy across the ocean and Red is trying to prevent this deed.

Second, Blue realizes Red could very closely approximate Blue's payoff matrix and determine Blue's optimal strategy.

Third, Blue feels that if Red knew Blue's allocation then Red would allocate his forces to reduce Blue's payoff as much as possible. These three presuppositions indicate that Blue considers Red a rational and intelligent opponent.

With these factors as his decision basis, Blue begins his selection of his optimal strategy for allocation of forces between the areas of interest and concern by investigating the worst that could happen (i.e., the largest value of D) for each of his possible alternatives. He then takes the alternative corresponding to the minimum of these as his optimal strategy. This is the well known minimax strategy of game theory. Although it is pessimistic in nature, the use of this criterion provides an upper bound on the worst that could happen to Blue.

For any finite two-person, zero-sum game each player's optimal strategy is either a pure strategy or a mixed strategy. A pure strategy for Blue in our problem implies that Blue always uses the same allocation between the areas for the same given set of parameters of the model. A mixed strategy under these same conditions implies choosing an allocation prior to each crossing in accordance with some particular probability distribution. We will show that only strategies which are pure strategies will be optimal for this allocation problem.

IV. BLUE'S OPTIMAL STRATEGY

DEVELOPMENT. In the determination of Blue's optimal strategy we will initially ignore the integer requirement on the number of Blue and Red forces allocated to any area.

If Blue plays some strategy n then the worst that can happen to him is that D(m,n) will take on a value of $\max_{0 \le m \le M} D(m,n)$. Therefore, under the minimax criterion Blue selects n yielding $\min_{0 \le n \le N} \max_{0 \le m \le M} D(m,n) = v_1$. In a like manner, Red would select m in order to $\max_{0 \le m \le M} \min_{0 \le n \le N} D(m,n) = v_2$.

The payoff function is now examined in the light of the following saddle point theorem [6]:

> Theorem: "Let f be a real-valued function such that f(x,y) is defined whenever $x \in A$ and $y \in B$ (A and B are sets); then a point, (x_o, y_o), such that $x_o \in A$ and $y_o \in B$ is called a saddle point of f if the following conditions are satisfied:
>
> (i) $f(x,y_o) \le f(x_o,y_o)$ for all $x \in A$
>
> (ii) $f(x_o,y_o) \le f(x_o,y)$ for all $y \in B$.
>
> Then a necessary and sufficient condition that
>
> $$\max_{x \in A} \min_{y \in B} f(x,y) = \min_{y \in B} \max_{x \in A} f(x,y) = f(x_o,y_o)$$
>
> is that f possesses a saddle point."

Therefore, for our problem, if v_1 and v_2 exist and are equal then the optimal solution to the game is the set of pure strategies (m_0,n_0). To prove that they exist and are equal it must be shown that a saddle point exists at (m_0,n_0) such that the following relation holds:

$$D(m,n_0) \le D(m_0,n_0) \le D(m_0,n) .$$

Inspection of equation (8) shows that D is continuous in both m and n for $-\infty \le m \le \infty$ and $-\infty \le n \le \infty$. Closer inspection shows that D is convex in n for any given m and linear in m for any given n. These properties suggest that a saddle point (m_0, n_0) may be obtained by taking the partial derivatives of D with respect to both m and n, setting both partials equal to zero, and solving for the values of m and n which satisfy the resulting system of equations.

The partial derivatives are

$$\frac{\partial D}{\partial m} = C_i - K_i(C_i+U)(1-e^{-k_i n}) - C_c + K_c(C_c+U)(1-e^{-k_c(N-n)})$$

$$\frac{\partial D}{\partial n} = -mK_i k_i(C_i+U)e^{-k_i n} + (M-m)K_c k_c(C_c+U)e^{-k_c(N-n)} .$$

When these derivatives are set equal to zero, we get

$$C_i - K_i(C_i+U)(1-e^{-k_i n}) = C_c - K_c(C_c+U)(1-e^{-k_c(N-n)}) , \quad (9)$$

$$m = M\left[\frac{K_c k_c(C_c+U)e^{-k_c(N-n)}}{K_i k_i(C_i+U)e^{-k_i n} + K_c k_c(C_c+U)e^{-k_c(N-n)}}\right] . \quad (10)$$

Equations (9) and (10) form a system of two equations with two unknowns. Since (9) is a function of n only, it can be solved for the value of n_0. This value of n_0 can then be substituted in (10) to determine m_0.

Since D is strictly convex in n for any given m, it follows that $D(m_0,n_0) \le D(m_0,n)$ for all $-\infty \le n \le \infty$. Further, since D is linear in m for any given n, it follows that $D(m,n_0) = D(m_0,n_0)$ for all $-\infty \le m \le \infty$ because n_0 was selected to give $\frac{\partial D}{\partial m} = 0$. Thus (m_0, n_0) obtained from equations (9) and (10) is a saddle point.

We will designate m* and n* as the optimal strategies for Red and Blue; as such they must satisfy the requirements of $0 \leq m^* \leq M$ and $0 \leq n^* \leq N$. If m_0 and n_0 fall within the feasibility region of m* and n* then it follows from the saddle point theorem that $m_0 = m^*$ and $n_0 = n^*$. We will refer to any pair of optimal strategies corresponding to this situation as an Internal-Saddle-Point (ISP) solution.

A special property of ISP solutions is that $0 < m^* < M$. This property is a consequence of equation (10). We can rewrite (10) to get the following form for m_0 as a function of n_0:

$$m_0 = M\left[1 + \frac{K_i k_i (C_i + U) e^{-k_i n_0}}{K_c k_c (C_c + U) e^{-k_c (N - n_0)}}\right]^{-1} .$$

The term in brackets of this expression is positive and greater than unity for $-\infty < n_0 < \infty$; therefore $0 < m_0 < M$. It follows that $0 < m^* < M$ whenever $0 \leq n_0 \leq N$.

A further consequence of the special property is that the following cases will <u>never</u> <u>occur</u>:

$$(m_0, n_0) = (M, 0) ,$$

$$(m_0, n_0) = (0, N) .$$

It is clear that the saddle-point solution will not necessarily provide integer values for m* and n* due to our relaxing of the integer requirement. To obtain the "best" integer solution we will evaluate the payoff function for the four integer solutions closest to m* and n* and choose that integer solution having its D value closest to $D(m^*, n^*)$. We will refer to this integer solution as the pseudo-saddle-point. We recognize the theoretical difficulties in rounding off non-integer solutions to obtain integer values, but we feel that the approach is reasonable for a practical problem, particularly one whose parameters are somewhat inexact. This will be discussed in more detail later in the paper.

Suppose now that n_0 lies outside the interval $[0,N]$. What will the optimal strategies be? In answering this question we will consider two cases; the first (Case I) corresponds to $n_0 < 0$ and the second (Case II) to $n_0 > N$.

To facilitate the study of these cases, the expressions for D when $m = 0$ and $m = M$ are useful. They are

$$D(0,n) = M\left[C_c - K_c(C_c+U)(1-e^{-k_c(N-n)})\right], \qquad (11)$$

$$D(M,n) = M\left[C_i - K_i(C_i+U)(1-e^{-k_i n})\right]. \qquad (12)$$

Inspection of equation (8) shows that $D(m,n)$ can be written as the following convex combination of $D(0,n)$ and $D(M,n)$:

$$D(m,n) = \lambda D(M,n) + (1-\lambda)\, D(0,n)$$

where $\lambda = \frac{m}{M}$. Obviously $0 \leq \lambda \leq 1$ if m is required to lie in the region $0 \leq m \leq M$. Figure 2 is a sketch of equations (11) and (12) as a function of n. This figure shows an ISP solution (i.e., $m^* = m_0$ and $n^* = n_0$). The value of n where the curves for $D(0,n)$ and $D(M,n)$ intersect is n_0 because the bracketed terms of (11) and (12) are, in fact, the right and left sides respectively of equation (9).

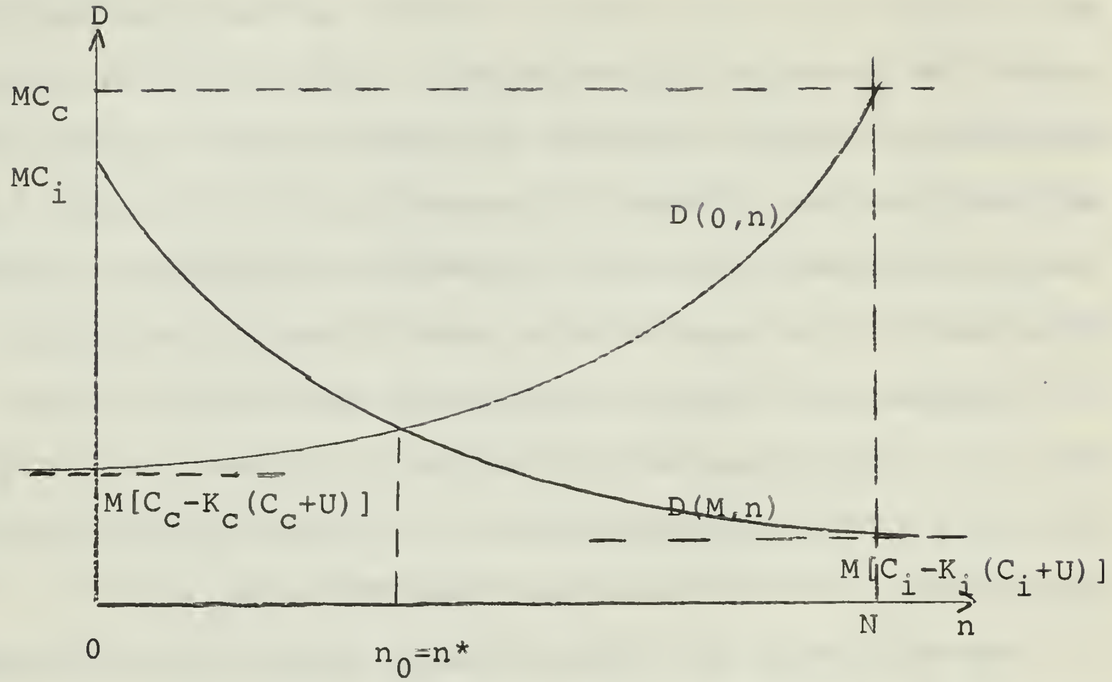

Figure 2

From figure 2 it is easy to see that Case I ($n_0 < 0$) will occur when $D(0,n) > D(M,N)$ for $0 \leq n \leq N$. The optimal strategies in this case must then be $n^* = 0$ and $m^* = 0$ since $D(m,0) < D(0,0) < D(0,n)$ for any $0 \leq n \leq N$ and any $0 \leq m \leq M$.

Case II ($n_0 > N$) will occur when $D(0,n) < D(M,n)$ for $0 \leq n \leq N$. The optimal strategy in this case is then $m^* = M$ and $n^* = N$ since $D(m,N) < D(M,N) < D(M,n)$ for any $0 \leq n \leq N$ and any $0 \leq m \leq M$.

From these observations and the derivation of equation (9) it follows that Case I will occur when

$$\left.\frac{\partial D}{\partial m}\right|_{n=0} < 0$$

and Case II will occur when

$$\left.\frac{\partial D}{\partial m}\right|_{n=N} > 0.$$

In addition, it is worthwhile to observe that

$$\frac{\partial D}{\partial m} < 0 \text{ when } n < n_0;$$

$$\frac{\partial D}{\partial m} > 0 \text{ when } n > n_0.$$

Further, $\frac{\partial D}{\partial m}$ is a strictly monotonic decreasing function of n; that is,

$$\left.\frac{\partial D}{\partial m}\right|_{n=n_2} < \left.\frac{\partial D}{\partial m}\right|_{n=n_1} \quad \text{when } n_1 < n_2.$$

These results suggest a procedure for determining the optimal strategy cases (Case I, Case II, or ISP) on the basis of the values of the parameters of a particular problem. The development of this procedure is based on the following lemmas and theorems.

Lemma 1: The necessary and sufficient condition for Case I ($n_0 < 0$) to occur is $\left.\frac{\partial D}{\partial m}\right|_{n=0} < 0$.

Proof: Assume $n_0 < 0$. Since n_0 is defined as the value of n giving $\frac{\partial D}{\partial m} = 0$ and $\frac{\partial D}{\partial m}$ is a strictly monotonic decreasing function of n, it follows that $\frac{\partial D}{\partial m} < 0$ for $n = 0$.

Next, assume $\left.\frac{\partial D}{\partial m}\right|_{n=0} < 0$. From the definition of n_0 we know $\left.\frac{\partial D}{\partial m}\right|_{n=n_0} = 0$. From the monotonicity of $\frac{\partial D}{\partial m}$ it then follows that $n_0 < 0$.

The expression for $\frac{\partial D}{\partial m}$ when $n = 0$ is

$$\left.\frac{\partial D}{\partial m}\right|_{n=0} = C_i - C_c + K_c(C_c+U)(1-e^{-k_cN}),$$

and $\left.\frac{\partial D}{\partial m}\right|_{n=0} < 0$ is equivalent to

$$C_c > \frac{C_i + K_c U(1-e^{-k_c N})}{1-K_c(1-e^{-k_c N})} \triangleq \hat{\hat{C}}_c . \tag{13}$$

Therefore, as a consequence of Lemma 1, we can state the following theorem.

<u>Theorem 1</u>: The optimal strategies are $(m^*, n^*) = (0,0)$ if and only if $C_c > \hat{\hat{C}}_c$.

We can make similar statements about Case II ($n_0 > N$).

<u>Lemma 2</u>: The necessary and sufficient condition for Case II ($n_0 > N$) to occur is $\left.\frac{\partial D}{\partial m}\right|_{n=N} > 0$.

The proof is similar to that of Lemma 1. In this case, $\left.\frac{\partial D}{\partial m}\right|_{n=N} > 0$ is equivalent to

$$C_c < C_i - K_i(C_i+U)(1-e^{-k_i N}) \triangleq \hat{C}_c . \tag{14}$$

And, as a consequence of Lemma 2, we can state the following theorem.

<u>Theorem 2</u>: The optimal strategies are $(m^*,n^*) = (M,N)$ if and only if $C_c < \hat{C}_c$

Finally, as a consequence of theorems 1 and 2 and the definitions of m_0 and n_0, we have the following corollary.

<u>Corollary</u>: The optimal strategies are $(m^*,n^*) = (m_0,n_0)$ if and only if $\hat{C}_c \leq C_c \leq \hat{\hat{C}}_c$.

From the theorems and the corollary we realize that the necessary and sufficient conditions for each case can be determined by an investigation of the relationship of C_c to

$\hat{C}_c$ and $\hat{\hat{C}}_c$. To solve a given problem for the optimal strategies, we would first calculate the values of $\hat{C}_c$ and $\hat{\hat{C}}_c$ and compare the value of C_c with these calculated values. If either Case I or II results then the optimal strategies are easily specified. If the ISP Case arises then the optimal strategies must be determined using equations (9) and (10). The following numerical example illustrates this procedure.

EXAMPLE. Suppose the following parameters are given:

$C_c = C_i = 1.0 = U = K_c$, $K_i = k_i = 0.5$, $k_c = 0.0405$, and $M = N = 10$.

From equations (13) and (14) we get $\hat{\hat{C}}_c = 2.0$ and $\hat{C}_c = 0.67$. Because $C_c = 1.0$ we have $\hat{C}_c \leq C_c \leq \hat{\hat{C}}_c$ and, from the corollary, an ISP solution is optimal. Using equations (9) and (10) we calculate the saddle-point solutions to be $m_0 = 3.54$ and $n_0 = 1.78$. Since both m_0 and n_0 are well within their respective feasible ranges, the optimal non-integer strategies are $m^* = 3.54$, $n^* = 1.78$ and $D(3.54, 1.78) = 4.37$. However, only integer values of m and n are permissible, thus the payoff function, $D(m,n)$, must be evaluated for the four integer solutions closest to m^* and n^* to determine the pseudo-saddle-point solution. These four values are: $D(3,1) = 4.50$, $D(3,2) = 4.22$, $D(4,1) = 4.76$, $D(4,2) = 4.16$. Since we should choose that integer solution closest to the non-integer saddle-point solution, we pick D equal to 4.50. This pseudo-saddle-point solution yields integer strategies of $m = 3$ and $n = 1$.

The relationship between m^* and n^* for ISP solutions to problems in general is suggested by figure 3. This figure

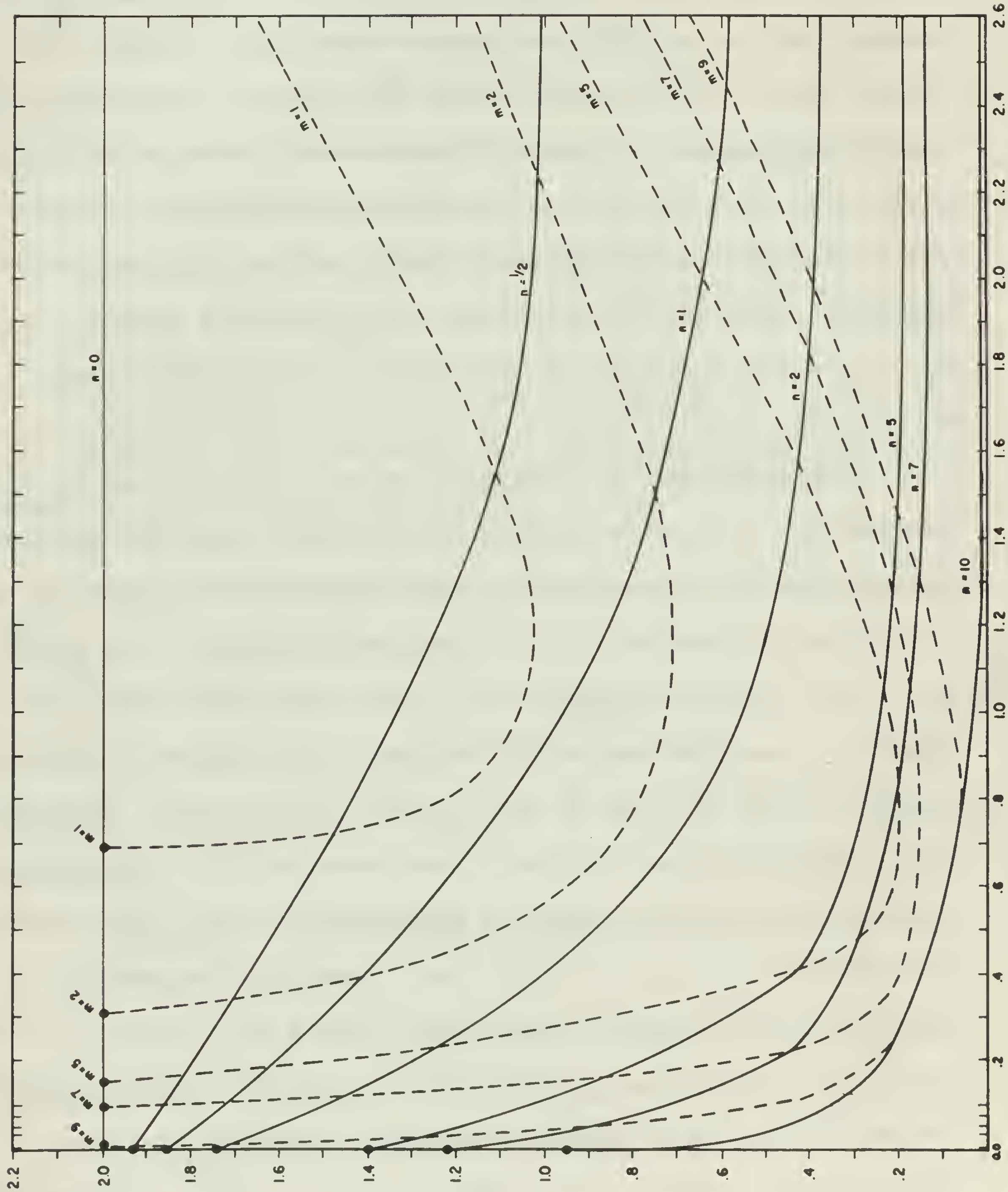
n=0
n=1/2
n=1
n=2
n=5
n=7
n=10
m=1
m=2
m=5
m=7
m=9
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
0.0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6

Figure 3

shows the various optimal strategy regions and constant value curves of m* and n* plotted on the C_c - k_i plane for values of the other parameters of our sample problem. Although it may not be evident from this figure, it should be noted that as k_i increases the constant m lines asymptotically approach the value of $\hat{C}_c$. The $\hat{\hat{C}}_c$ line corresponds also to the n = 0 curve and thus for very large values of k_i all solutions will be Case I (m* = 0, n* = 0).

From figure 3 it may be seen that an increase of k_i, with all other parameters fixed results in a decrease of n* and an initial decrease then increase of m*. This seems reasonable since as Blue's detection constant in the area of interest increases, it forces Red to maintain more of his effort in the area of concern, where he is not as vulnerable to detection, and thus Blue is forced to direct his attention to the area of concern. However, as Blue increases his effort in the area of concern Red will desire to shift more of his units to the area of interest.

An increase of C_c, with all other parameters fixed, results in a decrease of m* and n*. This implies the submarines' effectiveness in the area of concern has increased and as such Red would want to allocate more to this area. Supposedly Blue's estimate of C_c would increase also and thus he would allocate more ASW units to the area of concern. Hence both m* and n* decrease since both sides are re-allocating their units to the area of concern.

Associated with any particular set of m*, n* within the region of ISP solutions will be either one or two different sets of C_c, k_i values. Each case having only a single set of C_c, k_i values occurs where a constant m line is tangent to a constant $n > 0$ line or where a constant m line intersects with the $n = 0$ line.

As we have shown, the optimal strategies can be calculated if the values of all the parameters are known. In reality, these values will probably be the result of the Blue commander's judgment because it would be difficult, for example, for him to know precisely the submarines' effectiveness with surfaced-launched missiles. The value of U, in particular, is completely subjective. Consequently, an awareness of the influences of the various parameters is important.

An understanding of the parametric influences may be facilitated by consideration of the optimal strategy in the $C_c - k_i$ plane. Figure 4 illustrates the shape of these regions for parameter values of our example. A large amount of information about the influences of the various parameters can be obtained by a study of the behavior of $\hat{C}_c$ and $\hat{\hat{C}}_c$ in this plane. For example, if U were to be increased from 1.0 to 2.0, then $\hat{\hat{C}}_c$ would increase and $\hat{C}_c$ would decrease as indicated by the dashed curves in figure 4. Thus, the ISP solution area of the $C_c - k_i$ plane increases. Conversely, as U decreases this area decreases.

The changes in $\hat{C}_c$, $\hat{\hat{C}}_c$, and D per unit change of any of the parameters of the model can also be obtained by taking partial derivatives with respect to the particular parameter or by direct calculation if the other parameters are known.

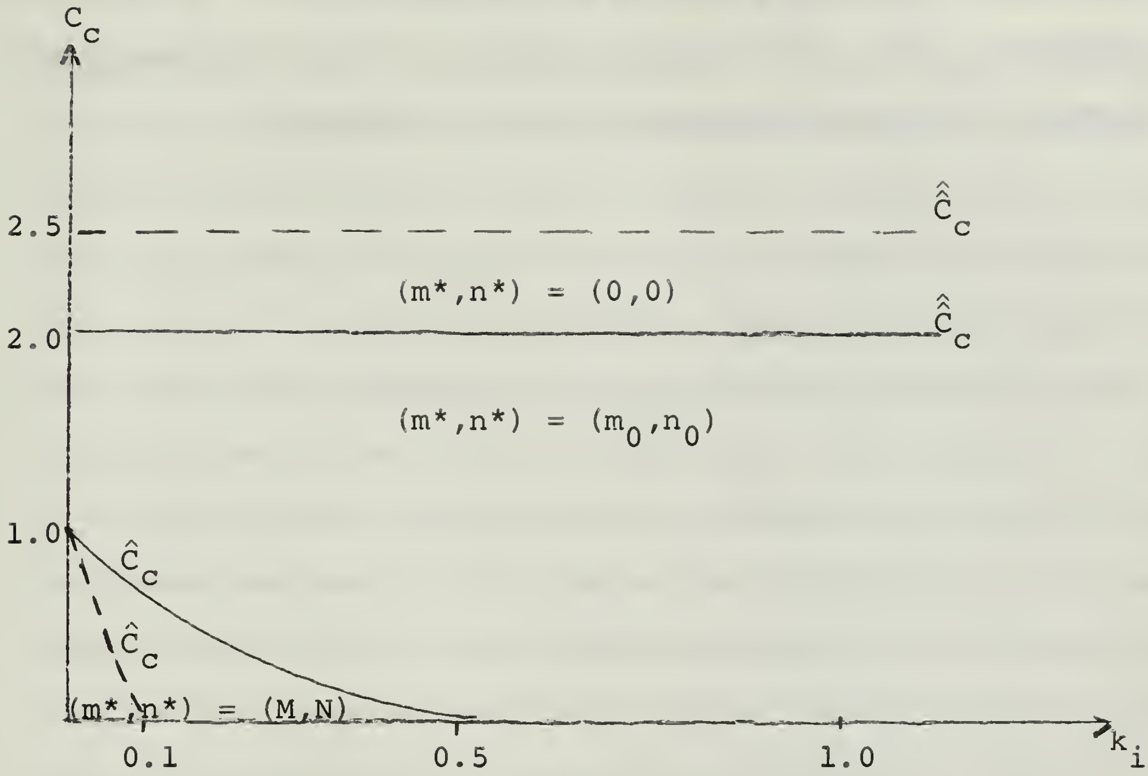

Figure 4

An analysis of the effects of changing the various parameters would indicate how best a planner might change his optimal strategy if the opportunity arises. In a sense it gives a planner a limited option of regulating the outcome of the payoff function if he has knowledge of and control of some of the input parameters. For example, a planner may have the ability to direct more effort or funds into one or more parameters which he is able to adjust. Through an examination of the model he could determine how best to change his controllable parameters in order to realize the most benefit. As another example, a planner may be able to obtain sufficiently reasonable estimates of the input parameters so that a "ball-park" optimal solution can be determined. He could then determine from a sensitivity analysis which parameters merit further study to obtain more precise estimates.

V. DISCUSSION OF THE MODEL

ASSUMPTIONS. We have greatly simplified the model by assuming both that the area of interest is like a "road" across the ocean with the area of concern laying on either side and that the areas assigned to the ASW units to search coincide with the areas of interest and concern. What has been ignored here is, first, the kinematics of search theory imposed by the capabilities of the units of both sides, and second, the possibility that the convoy could be attacked with missiles from the front and rear. This limits the application of the model but not the use of a game theoretic approach in the analysis.

We have assumed that both Blue and Red know the location and size of the areas of interest and concern. Red knows the size of the areas since both areas are a function of the capabilities of Red's weapons. Blue knows the location of the convoy route since he chooses it. Blue can usually approximate the effective range of Red's weapons and will use these estimates to assign his ASW units. Thus, we can say that Blue fairly well knows the size of both the area of interest and the area of concern. We have assumed that Red knows the location of the convoy route since this would probably lead to the worst possible outcome of a crossing as far as Blue is concerned, and this assumption is thus consistent with the pessimistic attitude of a player using the minimax criterion.

Use of the "formula for random search" for the probability of a Blue unit detecting a Red unit that is present is valid when both Blue and Red are moving independently and randomly. Even though the "formula" is by nature pessimistic, its use is consistent with the minimax criterion. The assumption of Blue attacking units moving independently and randomly is justified when a Blue attacking unit is individually assigned an area to search. This usually occurs when they are either limited in number available, detached from the immediate area of the convoy to search in some large remote area, or dispersed because of the threat of a nuclear attack. The assumption may not be justified if the ASW mission is conducted under coordinated, systematic, multi-vehicle search plans. This type of search may occur when the available number of both ships and aircraft are limited. Such search plans negate both the assumption of each Blue unit searching equal areas, and the assumption of homogenity of the Blue attacking force.

The assumption of an independent, random, uniform deployment of Red submarines is quite reasonable because any mutual interference that one submarine may have on another is avoided.

The assumption that a Blue attacking unit always has an opportunity to detect and attack a Red unit before the Red unit attacks is a matter of conjecture. In many regards this assumption is reasonable because the submarine must usually commit some act that will increase his likelihood of detection (i.e., expose his periscope when making a torpedo attack or surfacing while conducting a missile attack).

Although the primary mission of the Red submarines is the destruction of the convoy logistic units, it is quite possible that they may attack the escort vehicles. For example, if an ASW aircraft carrier is involved with the convoy most submarine commanding officers would prefer to sink the carrier before they begin to sink the logistic units. The destruction of a carrier obviously removes a major threat to the Red forces.

Some of the parameters which we have assumed to be constant in the model may in fact be quite variable. For example, when we formulated the detection constant in an area, we treated the relative speed between the Red and Blue units to be a constant given value determined exogenous to the model. The assumption of fixed relative speed between Blue and Red units is reasonable when there is a large speed differential between Red and Blue units such as when Blue uses ASW aircraft units against Red submarines. The assumption, however, is generally questionable. When ships are used as Blue attacking units they will generally operate at low speeds in order to enhance their sonar detection capabilities. The behavior of these ships and the Red submarines, operating at various speeds either to avoid detection or to establish an accurate firing position, would result in a highly variable relative speed. Also, if nuclear submarines are employed by Red, the relative speed is probably not constant regardless of the type vehicle used by Blue since the speeds of the nuclear submarines can vary over a wide range.

In summary, although the validity of some of the assumptions may be debatable, the assumptions are realistic enough to permit the model to be useful as a first approximation to the study of a convoy crossing in a submarine environment.

PAYOFF FUNCTION AND ALTERNATIVES. In this study the goal of the Blue commander has been to minimize the payoff function, D = DB - U (DR). It has been pointed out that this payoff function implies that Blue receives both a short-run and a long-run return. It should be noted that if long-run returns are considered, then this implies that the Blue commander would be willing to risk an increase in logistic losses to gain a larger increase in submarine losses. The amount of risk the Blue commander is willing to take is represented by the value of U. If the value of U is small this implies he is willing to take only a small risk in the increase of logistic losses. Conversely if U is large he is willing to take a high risk.

If the Blue commander had other objectives then an alternative payoff function might be appropriate. For instance, if circumstances prevail which dictate a one-convoy-only situation then the influence of the long-run benefits received by sinking a submarine would be ignored because the Blue commander's primary interest would be to prevent the submarines from attacking the logistic units. The objective of the Blue commander in this case would be to minimize only Blue losses. This implies that Blue does not have to destroy the submarines but only prevent them from attacking to assure the successful crossing of the logistic units. A contingency such as this will exist when the value of the cargo of the logistic units is of exceptionally high value (such as that of a country's total amphibious assault force). It should be emphasized that using this payoff function in a multiple convoy situation could very well lead to suboptimization.

If the size of the enemy's entire submarine force and other pertinent information, such as resupply rate, are known then a Lanchester approach might prove very interesting [8]. In this case the payoff function would be a relationship using the exchange ratio, $\frac{DB}{DR}$. The user, however, should be forewarned of the usual criticism of a ratio type of objective function; it is easy to lose sight of the magnitude of the losses.

USE OF GAME THEORY. According to game theory, instead of using the pseudo-saddle-point solution when a non-integer ISP occurs, we should use a mixed strategy. From the nature of the payoff function Blue's mixed strategy will assign positive probability to some set of the min (M,N) alternatives. The use of the pseudo-saddle-point solution, however, offers a more realistic approach to this convoy allocation problem. This can be seen for several reasons.

Since the Blue commander is faced with a single decision that determines the allocation strategy for the complete crossing, it seems reasonable that when the non-integer ISP case occurs he would choose a strategy near the saddle-point.

Clearly, the pseudo-saddle-point solution is appealing when there are a large number of units to allocate since the round-off procedure would imply only a small percentage change from the non-integer outcome.

Further, the uncertainty of the Blue commander's estimates of the values of the parameters of the model suggests a variation in the location of the true saddle-point. One might be able, for example, to make some confidence statements about a calculated value of the saddle-point based on the distribution associated with some parameter. As such, the round-off procedure will possibly keep the allocation strategy within the location of the true saddle-point. The variability of the parameter estimates also allows the pseudo-saddle-point to be used when a small number of units is to be allocated.

Because game theory has been used in this study to analyze the convoy system allocation, it is necessary to realize that there are limitations or restrictions that are inherent in a game theoretic approach to actual conflict situations. As Quade points out [9]:

> "Game theory does not cover all the diverse factors which enter into behavior in the face of a conflict of interest. There are certain important limitations. First, the theory assumes that all the possible outcomes can be specified and that each participant is able to assign to each a measure of preference, or utility, so that the one with a larger numerical utility is preferred to one with a smaller utility. Second, all the variables which determine the payoff and the values of the payoff can be specified; that is, a detailed description of all possible actions is required."

To what extent can we satisfy these limitations in our problem? First, all possible outcomes can be determined in our problem if the upper bound on the number of forces on each side is known or can be estimated with a high degree of confidence. The assumption that this parameter will be specified is very reasonable since a credible estimate of force size is usually available. Second, the payoff matrix in the convoy allocation problem is a function of losses and survivals and, hence, is readily adaptable to some measure of preference. Finally, even though knowledge of all actual parameters or variables in a complex problem is quite impossible, a reasonable estimate of the major or more significant parameters in such a problem is conceivable.

Whether a game theoretic approach can be used for determining an actual strategy in war or only for planning purposes appears to be dependent on the accurate description of all the necessary parameters and the degree of confidence in the estimate of their values. The convoy allocation model attempts to include those parameters which represent all the major aspects of the situation. However, both a more detailed model and a more precise investigation into the assumptions would undoubtedly be required in the determination of actual wartime strategies. Nonetheless, the model formulated in this study appears useful for planning or policy analysis. A planner can not only use the model to understand the general nature of the problem but also to investigate the influence of changes in parameter values. Both can be valuable when future models of the convoy allocation problem are considered.

Because of the structure of the payoff matrix and the nature of the payoff function required by the first and second limitation respectively, the game theoretic approach provides a systematic analysis of both the alternative courses of action and the effects of changes in parameters.

Even though the game is one in which Blue and Red are assumed to be diametrically opposed, it is possible that they will not have precisely opposite objectives. This may be taken into account by permitting each player to assign a different value to the weighting factor, U, which compares the value of a Blue logistic unit to a Red submarine. Thus, even though a matrix of outcomes in terms of absolute losses is the same for both sides, each player would generate his own payoff matrix and use it to determine his optimal allocation strategy. The zero-sum problem would occur only if both players use the same value for U. The general problem is undoubtedly a nonzero-sum game.

From a philosophical point of view, we have only determined Blue's optimal strategy. The Red strategy derived corresponding to Blue's optimal strategy is the strategy that Blue contends is optimal for Red to use. This is the strategy that Blue will assume that Red will actually employ when Blue plans his courses of action. However, this is clearly not Red's optimal allocation strategy if two different payoff matrices exist.

What guarantee is there that analyzing Blue's payoff matrix in the context of a zero-sum game and using the minimax criterion will give acceptable results to a decision maker? Suppose that internal saddle point solutions are obtained from both players' matrices and further that the two solutions are not identical. Clearly, if both sides use their optimal strategies with respect to their own payoff matrix they may be playing non-optimal strategies with respect to their opponent's payoff matrix. However, if each side plays his own minimax strategy then neither can receive a worse payoff as far as they are each concerned. Thus, the minimax solution of a player's own payoff matrix in our problem provides Blue and Red with upper and lower estimates, respectively, of the payoff they will receive in the combined problem. Each side would play these strategies if they had no information about their opponent's value of U.

If a player can accurately determine his opponent's payoff matrix then he may want to use a different strategy than the one based on his own payoff matrix. Suppose Blue knows not only Red's payoff matrix but also that Red uses the maximin criterion to determine his optimal strategy. The best course of action for Blue to take after evaluating Red's payoff matrix is to play that strategy which minimizes his own payoff when Red uses his maximin optimal strategy. In this case Blue's payoff would be at least as large as that for the minimax solution. Thus, a purpose or need for a continuous and persistent effort to obtain reliable intelligence of an enemy's intentions or knowledge of his actions is quite apparent.

VI. EXTENSIONS OF THE MODEL AND FURTHER STUDIES

The use of the pseudo-saddle-point solution needs further justification because the theoretical optimal solution for integer-valued strategies to the saddle-point payoff matrix indicates mixed strategies are best. The probability density function associated with the mixed strategies should be investigated to determine if it is unimodal in the vicinity of the saddle-point and if it has a small variance. It seems that this might be the case when the payoff function is reasonably flat in the region of the saddle-point as occurred in our sample problem. Such a study would indicate the validity of the round-off procedure.

A worthwhile study would be the investigation of the case where the Blue attacking units and Red units are not assumed to be homogeneous in effectiveness. For example, Blue could be allowed to use destroyers, aircraft, and submarines simultaneously as attacking units and Red could have several different types of submarines with different capabilities. If submarines are used as Blue attacking units, they would probably be deployed independently of one another in an area beyond the area of search of the aircraft and destroyers. Their purpose would be to provide a loose barrier patrol oriented towards the general direction of the expected Red threat. Whereas our model presupposes the deployment of Red prior to the convoy transit, the use of a Blue submarine barrier would require a change in the model to allow for attrition of the Red threat as it approaches the region of the convoy's anticipated track.

The effect of allowing Blue attacking units to be vulnerable to the Red submarines should be studied. This is particularly important because ASW carriers are oftem employed by Blue. By the very nature of the target a submarine commanding officer would take delight in the sinking of a carrier!

An interesting extension would be the study of the effect of relaxing the assumption that the deployment of the Blue attacking units as independent units in equal, non-overlapping areas. One approach might be to require coordinated, systematic search and attack plans which correspond more to actual naval operations. The use of systematic plans, which are usually based on acceptable assumptions, generally increases the probability of detection since they utilize current available data from the environment and other sources. However, it is important to note that systematic search plans will rule out the use of the "formula of random search". This "formula" gave the mathematical property of convexity to the payoff function and hence greatly facilitated the optimization of this model.

It might be beneficial to point out that time is present in the model in a limited manner since detection is usually a function of time. Yet, the model is still static in nature since the optimal strategies are derived for the complete crossing. A better model would be one that permits several changes in optimal strategies as the convoy crosses. After a certain time, possibly measured in number of engagements with Red units, the model would be updated to conform with a task-force commander's actions. A dynamic programming formulation might be appropriate for such a model. This model could also be applied to the problem of several sequential convoys.

VII. SUMMARY AND CONCLUSION

A game theoretic approach has been applied to an oceanic convoy situation in an enemy submarine environment. Providing the capabilities and limitations of both opponents can be specified, a procedure for determining the optimum allocation of both forces has been presented. The method is dependent upon the planner's ability to estimate the detection and kill effectiveness parameters of both opponents. The use of the minimax criterion, while providing a pessimistic outlook, does assure an upper bound on the worst that could happen to either side.

To the authors' knowledge this is the first study of an oceanic convoy crossing which utilizes game theory as an analytical technique. The results of the study have shown that a game theoretic approach provides both opponents with a flexible model from which a systematic solution to the allocation problem can be obtained. More significantly, it requires each player to consider his opponent's possible courses of action.

BIBLIOGRAPHY

1. Boice, F. B., Probability Model for a Convoy Threatened by a Submarine Launched Missile, (Naval Postgraduate School, 1965).

2. Cooper, R. E. and W. P. Hughes, Jr., Convoy Screening Against a Mixed Submarine Threat, (Naval Postgraduate School, 1964).

3. Dorfman, R., P. A. Samuelson, and R. M. Solow, Linear Programming and Economic Analysis, (New York: McGraw-Hill, 1958).

4. Hitch, C. J., and R. N. McKean, The Economics of Defense in the Nuclear Age, (Cambridge: Harvard University Press, 1961).

5. Matheson, J. D., An Application of Game Theory to an Offense-Defense Resource Allocation Problem, (Analytic Services Inc., AR-65-7, October 1965).

6. McKinsey, J. C. C., Introduction to the Theory of Games, (New York: McGraw-Hill, 1952).

7. Morse, P. M., B. O. Koopman, and others, OEG Report Number 56, (Operations Evaluation Group, 1946).

8. Morse, P. M., and G. E. Kimball, Methods of Operations Research, (New York: The Technology Press of Massachusetts Institute of Technology and John Wiley and Sons, Inc., 1950).

9. Quade, E. S., Analysis for Military Decisions, (The RAND Corporation, R-387-PR, November 1964).

10. Straugh, R., A Preliminary Treatment of Mobile SLBM Defense: A Game Theoretic Analysis, (The RAND Corporation, RM-4439, ARPA, June 1965).

INITIAL DISTRIBUTION LIST

		No. Copies
1.	Defense Documentation Center Cameron Station Alexandria, Virginia 22314	20
2.	Library Naval Postgraduate School Monterey, California 93940	2
3.	Chief of Naval Operations (OP 96) Navy Department Washington, D. C. 20350	1
4.	Asst. Prof. Alan W. McMasters Department of Operations Analysis Naval Postgraduate School Monterey, California 93940	2
5.	Lt. Ingolf N. Kiland, Jr USN Staff COMSUBLANT Norfolk, Virginia 23511	1
6.	Lt. Jerry A. Kotchka USN 405 Grandview Avenue Tiltonsville, Ohio 43963	1
7.	Mr. Lawrence J. Low Director, Naval Warfare Research Center Stanford Research Institute Menlo Park, California 94025	1

Unclassified
Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author)
Naval Postgraduate School
Monterey, California 93940

2a. REPORT SECURITY CLASSIFICATION
Unclassified

2b. GROUP

3. REPORT TITLE
A Game Theoretic Analysis of the Convoy-ASW Problem

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)
Thesis

5. AUTHOR(S) (Last name, first name, initial)
Kiland, Ingolf N., Jr., Lieutenant, U. S. Navy
Kotchka, Jerry A., Lieutenant, U. S. Navy

6. REPORT DATE
December 1967

7a. TOTAL NO. OF PAGES
49

7b. NO. OF REFS
10

8a. CONTRACT OR GRANT NO.

b. PROJECT NO.

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES
[redacted]

11. SUPPLEMENTARY NOTES

12. SPONSORING MILITARY ACTIVITY
Naval Postgraduate School

13. ABSTRACT

The problem of allocation of ASW forces assigned to an oceanic convoy in a submarine warfare environment is postulated as a two-person game with the payoff function being based on the "formula of random search". The opponents in the game are a convoy system and a submarine system. A submarine is given the option of attacking the convoy system either from afar with surface-launched missiles or near with torpedoes. The convoy system is defended by units capable of destroying submarines exercising either of their options. The optimal allocation of forces for both sides is shown to be a set of pure strategies which are dependent on the parameters of the model.

DD FORM 1 JAN 64 1473

Unclassified
Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Game Theory						
Convoy-ASW						
Allocation of Forces						
Random Search						

UNCLASSIFIED

[redacted]

[redacted]

UNCLASSIFIED