1965-05

A formulation of the allocation of attack aircraft to fixed location targets

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Monterey, California: U.S. Naval Postgraduate School

http://hdl.handle.net/10945/12235

A FORMULATION OF THE ALLOCATION OF ATTACK AIRCRAFT TO FIXED LOCATION TARGETS

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Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
OPERATIONS RESEARCH

United States Naval Postgraduate School
Monterey, California

MAY 1965

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OPERATIONS RESEARCH
from the
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ABSTRACT

Allocation of resources has become a classical problem in optimization by mathematical programming. In the field of military applications attack aircraft assignment has been treated widely by deterministic and/or linear models. However, destruction of a target is no certainty nor is damage inflicted on a target linear with respect to the number of weapons delivered on it. Recent extensions in the field of nonlinear programming in conjunction with the widespread use of electronic digital computers permit a more realistic approach to this problem. This paper formulates a stochastic nonlinear model for assigning a force of attack aircraft on a single sortie against fixed location targets. The number of aircraft alive at weapon release on any pass of a series against a given target is treated as a random variable. The total value of damage to all targets is taken as the measure of effectiveness and a particular form of the objective function derived. The parameters of the model and the form of the constraint equations are also discussed.

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CHAPTER I

THE PROBLEM

I. BACKGROUND

Allocation of resources has become a classical problem in optimization by mathematical programming. Linear programming has been used extensively to provide a solution. It is valid when the relationship between variables is linear, however, linearity is not always the case. For example, an individual may find the usefulness derived from owning five automobiles not necessarily five times that derived from owning just one. In the past, for cases such as this, it was customary for one to either be satisfied with a linear approximation or to search by other means for a solution. The extension of nonlinear programming over the past five years to its present state provides the analyst with a mathematical programming technique for treating the allocation problem with certain types of nonlinearity in a direct rather than approximate fashion. Dorn [18], Graves and Wolfe [17], Hadley [25], and Wolfe [13] have published comprehensive works on the status of nonlinear programming.

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A particular allocation problem in the field of military applications that has received a great deal of attention is that of attack aircraft assignment. Numerous studies and papers have been published on the subject, and several which have been found to be of particular interest are: _Limited War Operations_(U) [10], _Tactical Air Warfare Force Allocation_(U) [21], Dresher [4], and Haering [29].

II. ASSUMPTIONS

It was through participation in the Center for Naval Analyses Study: _Tactical Air Warfare, 1964_, that the necessity for a formulation such as developed in this paper became apparent. The allocation of attack aircraft to fixed location targets is treated under the following broad assumptions:

(A1.1)[1] An optimal allocation is one in which the total value of damage to all targets is a maximum, subject to the constraints imposed by the availability of aircraft, fuel, and weapons.

(A1.2) The damage inflicted on a target is nonlinear with respect to the number of aircraft passes made against it.

[1](A _._) identifies assumptions,
(D _._) identifies definitions.

(A1.3) There is a probabilistic interpretation of the number of aircraft alive on a particular pass against a given target. That is, given ten aircraft assigned to attack a target, there is a probability that all ten aircraft will not be alive on any particular pass against that target.

III. THE FORMULATION

Under these assumptions the allocation of attack aircraft was formulated as a stochastic nonlinear mathematical program with its attendant objective function and constraint equations. The objective function in this case is a nonlinear function of the number of attack aircraft alive on each pass against each target. Its particular form is derived and a discussion of the constraint equations given in Chapter II.

The number of passes against a target is treated as a random variable in the following way: the number of aircraft alive at release on a particular pass against a target is a random variable, therefore the number of passes against a target is a random variable which is the sum of the number of aircraft alive at release on each pass. Their probability distributions are derived in Chapter III and Appendix A.

Chapter IV is devoted to a discussion of the parameters of the model. These include target value, fuel required,

racks available for ordnance, the probabilities of detection, engagement, kill, acquisition, and pass survival.

Conclusions and recommendations comprise the final chapter of this paper.

CHAPTER II

THE OBJECTIVE FUNCTION AND CONSTRAINT EQUATIONS

I. THE OBJECTIVE FUNCTION

The measure of effectiveness chosen for the allocation of attack aircraft was the total value of damage inflicted upon enemy targets. The problem is to maximize the total expected value of damage subject to certain constraints.

(A2.1) Assume that each aircraft assigned the i^{th} target delivers the same number of preferred weapons on each pass against that target. A preferred weapon implies that for a specific target there exists a weapon which is most effective in destroying that target.

(A2.2) On any pass against a target assume that the target is either killed or not killed. This implies that a target can be destroyed on one pass.

(A2.3) Aircraft make passes until all weapons are expended or until the aircraft is killed.

(D2.1) The base-aircraft index specifies an aircraft type located at a specific base. For example,

$j = 1$ denotes the set of A4 aircraft from base number one,

$j = 2$ denotes the set of F4 aircraft from base

[illegible]

number one,

$\vdots$

$j = J - 1$ denotes the set of A4 aircraft from base number S,

$j = J$ denotes the set of F4 aircraft from base number S.

(D2.2) N_{ij} denotes the number of j^{th} base-aircraft type assigned the i^{th} target, $i = 1,2,\ldots, I$ and $j = 1,2,\ldots,J$.

(D2.3) $\underline{N}_i$ denotes the vector $(N_{i1}, N_{i2}, \ldots , N_{iJ})$.

(D2.4) N_j denotes the total number of the j^{th} base-aircraft type available.

(D2.5) R_{ij} denotes the number of passes per aircraft planned by the j^{th} base-aircraft type assigned the i^{th} target.

(D2.6) PT_{ib}, a constant, denotes the probability the i^{th} target is killed by exactly one pass given that b preferred weapons are delivered on that pass.

(D2.7) $W_i(\underline{N}_i)$ denotes the random variable which represents the total number of passes against the i^{th} target. $W_i(\underline{N}_i)$ will be written as W_i when convenient. The range of possible values is:

$$W_i = 0,1,2,\ldots \sum_{j=1}^{J} N_{ij}R_{ij}.$$

(D2.8) Let Y_i be a random variable such that

$$Y_i = \begin{cases} 0 \text{ if the } i^{th} \text{ target is not killed} \\ 1 \text{ if the } i^{th} \text{ target is killed at least once.} \end{cases}$$

Then

$$P\left[Y_i = 0\right] = \sum_{\text{all } w} P\left[Y_i = 0 \mid W_i = w\right] P\left[W_i = w\right]$$

$$= \sum_{\text{all } w} (1 - PT_{ib})^w P\left[W_i = w\right],$$

and

$$P\left[Y_i = 1\right] = 1 - P\left[Y_i = 0\right]$$

$$= 1 - \sum_{\text{all } w} (1 - PT_{ib})^w \ P\left[W_i = w\right].$$

(D2.9) Let v_i denote the pre-assigned value of the i^{th} target, and let $V = \sum_{i=1}^{I} v_i$ be the total value of the target complex.

(D2.10) Let D_i denote the random variable which represents the value of damage to the i^{th} target such that:

$$D_i = \begin{cases} 0 & \text{if } Y_i = 0 \\ v_i & \text{if } Y_i = 1. \end{cases}$$

Then,

$$E\left[D_i\right] = \sum_{\text{all } d} d \cdot P\left[D_i = d\right]$$

$$= v_i P\left[Y_i = 1\right]$$

$$= v_i \left\{ 1 - \sum_{\text{all } w} (1 - PT_{ib})^w P\left[W_i = w\right] \right\}.$$

The total value of expected damage (TD) to the target

complex is:

$$\text{(D2.11)} \quad TD = \sum_{i=1}^{I} E\left[D_i\right]$$

$$= \sum_{i=1}^{I} v_i - \sum_{i=1}^{I} v_i \sum_{\text{all } w} (1 - PT_{ib})^{w} \; P\left[W_i = w\right]$$

$$= V - \sum_{i=1}^{I} v_i \sum_{\text{all } w} (1 - PT_{ib})^{w} \; P\left[W_i = w\right],$$

$$w = 0, 1, \ldots, \sum_{j=1}^{J} N_{ij} R_{ij}.$$

To maximize TD one must minimize the expression

$$\sum_{i} v_i \sum_{w} (1 - PT_{ib})^{w} \; P\left[W_i = w\right], \text{ which}$$

is simply the sum of the product of the value of the i^{th} target and the probability the i^{th} target is not killed.

Therefore the objective function is

$$\text{(2.1)} \quad \min \sum_{i=1}^{I} v_i \sum_{\text{all } w} (1 - PT_{ib})^{w} \; P\left[W_i = w\right].$$

II. THE CONSTRAINT EQUATIONS

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The minimization of the objective function is subject to certain constraints on the resources available. The four constraints given below are a non-exhaustive subset of those which could be imposed, but are representative of the most important ones:

1. The number of aircraft allocated to targets must be less than or equal to the total number of aircraft

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available, i.e.,

(2.2) $$\sum_{i=1}^{I} N_{ij} \leq N_j \quad , \quad N_{ij} \geq 0 \; ,$$

where N_j = the total number of j^{th} base-aircraft type available, and N_{ij} = the number of j^{th} base-aircraft type assigned the i^{th} target.

2. The total fuel required must be less than or equal to the total fuel available, i.e.,

(2.3) $$\sum_{i=1}^{I} f_{ij}N_{ij} \leq F_j \quad ,$$

where F_j = the total fuel available for the j^{th} base-aircraft type, and f_{ij} = the fuel required for the j^{th} base-aircraft type to strike the i^{th} target.

3. The total ordnance loading must be less than or equal to the ordnance available, i.e.,

(2.4) $$\sum_{i=1}^{I} b_{ijn}N_{ij} \leq B_{jn} \quad ,$$

where B_{jn} = the number of weapons of type n available for the j^{th} element, and b_{ijn} = the number of preferred weapons of type n the j^{th} element carries to the i^{th} target.

4. There exists an upper limit on the number of aircraft which one is willing to lose on a given mission, i.e.,

(2.5) $$\sum_{i=1}^{I} L_{ij}N_{ij} \leq L_j \quad ,$$

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where L_j = the maximum acceptable number of j^{th} base-aircraft losses, and L_{ij} = the expected percentage attrition of the j^{th} base-aircraft type assigned the i^{th} target.

There are numerous ramifications to these constraints which will not be covered in detail here. For example there are additional restrictions to (2.3), (2.4), and to (2.5) in that, respectively, f_{ij} is constrained by the amount of fuel an aircraft can carry, b_{ijn} is constrained by the maximum ordnance load and rack restrictions of the aircraft type, and L_{ij} is a function of the aircraft type, speed, penetration altitude, the enemy defenses, and the number of passes made against the target.

The allocation problem has now been formulated as:

$$(2.1) \quad \min \sum_{i=1}^{I} v_i \sum_{\text{all } w} (1 - PT_{ib})^w P \left[W_i = w \right] ,$$

subject to :

$$(2.2) \quad \sum_{i=1}^{I} N_{ij} \leq N_j \quad , \quad N_{ij} \geq 0 \, ,$$

$$(2.3) \quad \sum_{i=1}^{I} f_{ij} N_{ij} \leq F_j \, ,$$

$$(2.4) \quad \sum_{i=1}^{I} b_{ijn} N_{ij} \leq B_{jn} \, ,$$

$$(2.5) \quad \sum_{i=1}^{I} L_{ij} N_{ij} \leq L_j \, .$$

III. NONLINEAR PROGRAMS

Rosen [7], [11] and Fiacco and McCormick [20], [22], [30] have developed algorithms which have been successful in solving nonlinear programming problems subject to linear or nonlinear constraints.

The Sequential Unconstrained Minimization Technique (SUMT) of Fiacco and McCormick and the Gradient Projection (GP) technique of Rosen are both available through the IBM Share General Program Library as Share Distribution 3189, RAC SUMT, and Share Distribution 1399, SD GP 90 respectively.

The use of SUMT is precluded in this allocation problem due to the non-differentiable nature of the objective function. That is, W_i is a discrete integer-valued random variable. The concluding remarks of the Share write up of RAC SUMT indicate that special subroutines are being developed to handle non-differentiable functions.

The program GP 90 should handle the non-differentiable objective function since it uses two-sided differences in place of the gradients. Thus it is unnecessary to explicitly evaluate the gradients of the objective function.

CHAPTER III

THE PROBABILITY DISTRIBUTIONS

I. ASSUMPTIONS

The probability distributions of two of the random variables, the number of aircraft alive at release on a particular pass and the total number of passes against a target, are presented in this chapter.

The following assumptions were made:

(A3.1) An individual aircraft is assigned only one target per sortie.

(A3.2) A raid is composed of one base-aircraft type, but more than one raid can be assigned to a target.

(A3.3) Enemy fighters, if scrambled against a raid, are sent in numbers sufficient to engage each aircraft in that raid.

II. DEFINITIONS

The following definitions were useful:

(D3.1) $PD_{ij}(n)$ denotes the probability that a raid of size n is detected enroute from the j^{th} base-aircraft location to the i^{th} target.

(D3.2) $PE_{ij}(N,n)$ denotes the probability that a raid of size n is engaged enroute from the j^{th} base-aircraft location to the i^{th} target given that the raid is detected

and given that a total of N aircraft are employed in strike operations.

(D3.3) PK denotes the probability that any aircraft in a raid is killed enroute given that the raid is engaged.

(D3.4) $PA_{ij}(n)$ denotes the probability that a raid from the j^{th} base-aircraft location finds the i^{th} target given that n aircraft survive enroute.

(D3.5) $PR_{ijk}(n)$ denotes the probability that any aircraft in a raid from the j^{th} base-aircraft location survives until the k^{th} release against the i^{th} target given that n aircraft are alive commencing the first pass, $k = 1$, or given that n aircraft are alive at the $(K - 1)^{st}$ release for the second and subsequent passes, $K \geqq 2$; $k = 1,2,3,\ldots,R_{ij}$.

(D3.6) $X_{ijk}(N_{ij})$ denotes the random variable which represents the number of the j^{th} base-aircraft type alive at release on the k^{th} pass against the i^{th} target given that N_{ij} are assigned. The range of possible values is: $X_{ijk}(N_{ij}) = 0,1,2,\ldots,N_{ij}$. For simplicity $X_{ijk}(N_{ij})$ is sometimes written as X_{ijk}.

III. DISTRIBUTION OF THE NUMBER OF AIRCRAFT ALIVE AT RELEASE ON THE FIRST PASS

Under the above assumptions the event that x aircraft, $x > 0$, are alive at release on the first pass against a

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target can occur in three mutually exclusive and exhaustive ways:

A raid of size N is undetected enroute, finds the target and x aircraft among N survive target defenses on the first pass,

OR a raid of size N is detected enroute, but unengaged, finds the target and x aircraft among N survive target defenses on the first pass,

OR a raid of size N is detected enroute, engaged, some n aircraft survive enroute, $n \geqq x$, those n find the target and then x among n survive target defenses on the first pass.

The extension of this type of reasoning used in conjunction with the laws of elementary probability theory produced the probability distribution of the number of aircraft alive at release on the first pass, X_{ij1}, presented in Table I.

IV. DISTRIBUTION OF THE NUMBER OF AIRCRAFT ALIVE AT RELEASE ON SUBSEQUENT PASSES

The probability distribution of X_{ijk}, $k \geqq 2$, was obtained from the distribution of X_{ij1} by making the following observations:

The stochastic process, $\left\{X_{ijk}, k = 1,2,3,\ldots,R_{ij}\right\}$, is a discrete parameter finite Markov chain. That is,

$$P\left[X_{ijk} = x_k \mid X_{ij1} = x_1, X_{ij2} = x_2, \ldots, X_{ijk-1} = x_{k-1}\right]$$
$$= P\left[X_{ijk} = x_k \mid X_{ijk-1} = x_{k-1}\right]$$

[illegible]

The number of the j^{th} base-aircraft type alive at release on the k^{th} pass against the i^{th} target is less than or at most equal to the number alive on the $(k-1)^{st}$ pass. That is,

$$X_{ijk} \leq X_{ijk-1} \qquad k = 2,3,4,\ldots,R_{ij}.$$

Therefore the conditional distribution of X_{ijk}, $k \geq 2$, given that $X_{ijk-1}=m$, is binomial with parameters $PR_{ijk}(m)$ and m as presented in Table II.

By the theorem of total probabilities:

$$P\left[X_{ijk} = x\right] = \sum_{m=x}^{N_{ij}} P\left[X_{ijk} = x \mid X_{ijk-1} = m\right] P\left[X_{ijk-1} = m\right]$$

The synthesis of these facts yielded the probability distribution of X_{ijk}, $k \geq 2$, presented in Table III.

V. DISTRIBUTION OF THE TOTAL NUMBER OF PASSES AGAINST A TARGET

The derivation of the probability distribution of W_i necessitated one further assumption:

(A3.5) The number of a given base-aircraft type alive at release on any pass over a given target is statistically independent of any other base-aircraft types alive over the same target on any pass. That is, X_{ijk} is assumed independent of $X_{ij'k'}$ provided $j \neq j'$.

Under all foregoing assumptions the probability distribution of W_i as presented in Table IV is derived in Appendix A.

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TABLE I

PROBABILITY DISTRIBUTION OF $X_{1j1}(N_{1j})$

$$P\left[X_{1j1}(0) = x\right] = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$P\left[X_{1j1}(N_{1j}) = x\right] = \begin{cases} \left[1 - PD_{1j}(N_{1j})PE_{1j}(\Sigma N_j, N_{1j})\right]\left\{1 - PA_{1j}(N_{1j})\left[1 - (1 - PR_{1j1}(N_{1j}))^{N_{1j}}\right]\right\} \\ + PD_{1j}(N_{1j})PE_{1j}(\Sigma N_j, N_{1j})\left\{(PK)^{N_{1j}} + \sum_{n=1}^{N_{1j}} \binom{N_{1j}}{n} (1 - PK)^{n} (PK)^{N_{1j} - n} \right. \\ \left. \cdot PA_{1j}(n)\left[1 - PR_{1j1}(n)\right]^{n}\right\} & \text{if } x = 0 \text{ and } N_{1j} \geqq 1 \\ \left[1 - PD_{1j}(N_{1j})PE_{1j}(\Sigma N_j, N_{1j})\right] PA_{1j}(N_{1j}) \\ \cdot \binom{N_{1j}}{x} \left[PR_{1j1}(N_{1j})\right]^{x} \left[1 - PR_{1j1}(N_{1j})\right]^{N_{1j} - x} \\ + PD_{1j}(N_{1j})PE_{1j}(\Sigma N_j, N_{1j})\left\{\sum_{n=x}^{N_{1j}} \binom{N_{1j}}{n} (1 - PK)^{n} (PK)^{N_{1j} - n} \right. \\ \left. \cdot PA_{1j}(n)\binom{n}{x}\left[PR_{1j1}(n)\right]^{x} \left[1 - PR_{1j1}(n)\right]^{n - x}\right\} & \text{if } x = 1,2,\ldots,N_{1j} \text{ and } N_{1j} \geqq 1 \\ 0 & \text{elsewhere} \end{cases}$$

TABLE II

PROBABILITY DISTRIBUTION OF $X_{ijk}(N_{ij})$ GIVEN $X_{ijk-1}(N_{ij}) = m$

<u>For $k = 0$, 1 or $k > R_{ij}$</u>

$$P\left[X_{ijk}(N_{ij}) = x \;\middle|\; X_{ijk-1}(N_{ij}) = m\right] = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{elsewhere} \end{cases}$$

<u>For $k = 2,3,4,\ldots,R_{ij}$</u>

$$P\left[X_{ijk}(0) = x \;\middle|\; X_{ijk-1}(0) = m\right] = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$P\left[X_{ijk}(N_{ij}) = x \;\middle|\; X_{ijk-1}(N_{ij}) = m\right] = \begin{cases} \binom{m}{x}\left[PR_{ijk}(m)\right]^{x}\left[1 - PR_{ijk}(m)\right]^{m-x} & \text{if } x = 0,1,2,\ldots,m \leqq N_{ij} \text{ and } N_{ij} \geqq 1 \\ 0 & \text{elsewhere} \end{cases}$$

TABLE III

PROBABILITY DISTRIBUTION OF $X_{ijk}(N_{ij})$, $k \neq 1$

<u>For $k = 0$ or $k > R_{ij}$</u>

$$P\left[X_{ijk}(N_{ij}) = x\right] = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{elsewhere} \end{cases}$$

<u>For $k = 2,3,4,\ldots,R_{ij}$</u>

$$P\left[X_{ijk}(0) = x\right] = \begin{cases} 1 & \text{if } x = 0 \\ 0 & \text{elsewhere} \end{cases}$$

$$P\left[X_{ijk}(N_{ij}) = x\right] = \begin{cases} \sum_{m=x}^{N_{ij}} \binom{m}{x} \left[PR_{ijk}(m)\right]^{x} \left[1 - PR_{ijk}(m)\right]^{m-x} P\left[X_{ijk-1}(N_{ij}) = m\right] & \text{if } x = 0,1,2,\ldots,N_{ij} \text{ and } N_{ij} \geq 1 \\ 0 & \text{elsewhere} \end{cases}$$

TABLE IV

PROBABILITY DISTRIBUTION OF $W_1(\underline{N}_1)$

$$P\left[W_1(\underline{N}_1) = w\right] = \begin{cases} \prod_j P\left[X_{1j1} = 0\right] & \text{if } w = 0 \\ \sum_{j=1}^{J} A_j P_{1j} & \text{if } w = 1,2,3,\ldots,\sum_j N_{1j}B_{1j} \\ 0 & \text{elsewhere} \end{cases}$$

Where A_n and P_{1n}, $n = 1,2,3,\ldots,J$, are defined in Appendix A.

CHAPTER IV

THE PARAMETERS

I. TARGET VALUE

The value of a target may be characterized by a time dependent index, i.e., target value will change as the tactical situation changes over time. Furthermore, the relative values of targets in given tactical situations will depend strongly upon the doctrine adopted by the local military commander. If an index of target value is to provide a basis for decision, it must be formulated with consideration given to the whole social, political, economic, and ethical processes within which military action takes place. The intent of this section is not to formulate a scheme for target value indexing but to present some aspects of the problem which should be considered in any indexing plan.

Kaplan [2] delimits areas of empirical study by which each element in the concept of military worth can be measured or appraised. These areas include values, objectives, welfare, achievement, and worth.

Kaysen [3] outlines criteria for selecting target systems as:

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[illegible]

1. Military importance -- a rough classification of the value to enemy military operations of all types of equipment and supplies used by enemy forces;

2. Percent direct (or indirect) military use -- the share of total output of a product or service which goes into military use;

3. Depth -- an indication of the time available to the enemy for the organization of substitute consumption, alternate production, etc., before suffering military damage;

4. Economic vulnerability -- includes

   a. Ratio of capacity to output,

   b. Substitutability for processes and equipment,

   c. Substitutability for product,

   d. Process and plant layout vulnerability,

   e. Recuperability;

5. Physical vulnerability;

6. Location and size of the target system.

In addition to the above criteria for indexing target value the effect on enemy morale should be considered.

Hesse and Mitchell in ''Limited War Campaigns: Dacca Method'' [10] have derived an index of target value which incorporates several of the criteria mentioned above. They consider military targets from a balanced ton standpoint using weighting factors for consumable and reuseable equipment, supplies, and manpower. This index

is quite involved, but it provides a method of assigning target values for most conceivable military targets. It should serve well as a useful working tool for a first look at target value in the allocation problem.

II. FUEL AND ORDNANCE LOADINGS

To maximize damage to the enemy requires that aircraft be put over their targets with optimal ordnance loadings. A basic prerequisite to attainment of this objective is determination of fuel required to carry out the given mission.

The following basic mission profile has been postulated to determine fuel requirements:

1. Warm-up and take-off,
2. Climb to cruise altitude and cruise to descent point for run-in to the target,
3. Descent to target run-in altitude,
4. Run-in to the target,
5. Ordnance delivery,
6. Run-out from the target,
7. Climb to cruise altitude and return to base,
8. Descent to the base for landing,
9. Landing and reserve.

For ease of computation the following assumptions have been made:

(A4.1) Aircraft carry full internal fuel and use

fully loaded 300 gallon external fuel tanks as required;

(A4.2) A fuel weight of six and one-half pounds per gallon was used;

(A4.3) No fuel is burned and no distance over the ground is covered in descents;

(A4.4) All external store racks for ordnance are equivalent on any particular aircraft type;

(A4-5) There are no asymmetric load restrictions for launching aircraft;

(A4-6) Five minutes fuel at normal rated power was allotted for warm-up and take-off;

(A4-7) Thirty minutes fuel at sea level maximum endurance was allotted for landing and reserve.

Utilizing the mission profile and the assumptions stated above one may formulate the basic fuel required equation as

(4.1) Total fuel required = Take-off fuel + (2 x cruise fuel rate x (Range - Climb distance - Run-in distance)) + (2 x Run-in distance x Run-in fuel rate) + (Time over target x Target fuel rate) + Landing and reserve fuel.

After determining the total fuel required the next step is to ascertain if the fuel required is less than

the fuel available. Successful mission performance is constrained by

(4.2) Fuel available - Fuel required ≥ 0.

Substituting for Fuel available in (4.2) one obtains

(4.3) (Internal fuel + External fuel) - Fuel required ≥ 0.

Solving for External fuel results in

(4.4) External fuel $\geq$ Fuel required - Internal fuel.

Once (4.4) has been solved it is possible to determine the number and size of external fuel tanks required and subsequently to determine the number of external store stations available for ordnance loading.

The maximum ordnance load may be obtained by solving

(4.5) Ordnance load = Maximum take-off weight - (Basic aircraft weight + Internal fuel + External fuel).

Appendix B contains a program (PROGRAM ORDLOAD) written in FORTRAN IV for the CDC-1604 digital computer which computes for a given aircraft mission the fuel required, the number of store stations available for ordnance, and the maximum ordnance loading. This program uses the assumptions and the algorithms outlined above for computations. Inputs required to the program, definitions, a program listing, and a sample output are given in Appendix B.

III. OTHER PARAMETERS

The probability that an aircraft survives to make passes against a target is a function of the probability of detection by the enemy, the probability of engagement by the enemy, the probability of being killed enroute to the target by the engaging force, the probability of acquiring the target, and the probability of being alive at the weapons release point on the k^{th} pass against the target. Values for all of the above are required to obtain the distribution of W_i. To minimize the objective function one also needs values for the probability that the i^{th} target is killed by one pass given that b weapons are released on that pass. These probabilities will be considered as input parameters.

Derivation of each of these probability distributions is beyond the scope of this paper; however, the remainder of this section will be devoted to discussion and/or references to derivations of these distributions. Throughout the investigation of these parameters, past data will in some instances be adequate and reliable, and the parameters could be estimated by Bayesian techniques. World War II, Korea, and Viet Nam experiences should result in extensive information which may be used to increase confidence in the estimates used.

Koopman [1] derives an expression for detection

probability both for visual and for radar detection. Electronic and other intelligence data coupled with characteristics of friendly radar should allow an estimate to be obtained for detection probability. Crout, Fay, and Harvey [5] have formulated a model describing a bomber's penetration into hostile territory in the face of a given area defense strategy. Specifically an expression is developed to compute the probability that an attacking aircraft is killed before penetrating to a given depth. This model then accounts for the probability of detection, engagement, and kill.

Simultaneously with passing through enemy area defenses friendly aircraft must attempt to acquire their assigned targets. Acquisition includes the processes of search, detection, identification, and flying the aircraft into a position to make a weapons pass against the target. Visual acquisition of the target is, of course, the most reliable and informative method of locating a given target. Due to circumstances such as weather, terrain, camouflage, aircraft speed, and aircraft altitude a pilot will sometimes have to rely on radar, infrared, microwave, or vectoring from personnel not in the aircraft to acquire his target. A combination of these methods should increase the probability an aircraft acquires its target. Erickson [9] presents a study on the visual capabilities of a pilot searching for ground

targets. Kamrass and Heckroth [15] include a section on target acquisition which discusses detection probabilities and an analysis of types of sensors and methods of operation which will be most effective for reconnaissance and strike missions.

Once an aircraft has acquired a target and is positioned for a weapons release pass it will usually be in the area of effectiveness of the point defenses of the target. These point defenses can run the gamut from no defense to a completely automatic system of surface-to-air missiles with a ground level to outer space capability. The probability an aircraft can successfully release weapons over the target is highly dependent upon the type of defensive weapon encountered. Selection of Aircraft for Tactical Air Missions [24] discusses and gives values for probabilities of kill and the effect of various penetration aids associated with several types of point defense weapons.

If an aircraft survives the point defenses, it will release weapons against the target. The probability of killing the target on one pass will be a function of the type and number of weapons released on that pass, the skill of the individual pilot, the weather, the type of terrain, and the type of target. The Naval Ordnance Test Station (NOTS) has worked on the development of weapons and the formulation of models which allow the probabilities

of kill of those weapons to be computed. NOTS technical publications include those of a probabilistic nature -- Kusterer [8] , W.B. Simecka [16] , Verry [19] , K.D. Simecka [26] , and Strang [28] ; and those of a computational nature -- Weldon and Young [14] , W. B. Simecka, et. al. [23] , and ''Conventional Air-Delivered Strike Weapons'' [27] .

An alternative to derivation of the various probability distributions for all the parameters would be to use three estimates of each parameter, i.e., a best, an optimistic, and a pessimistic estimate. These will be referred to as BOP estimates. After obtaining solutions to the allocation problem using these BOP estimates, a sensitivity analysis should be performed to discover which parameters significantly affect the results. Following this analysis high confidence estimates for the most sensitive ''parameters'' could be obtained using the distribution theory.

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

I. CONCLUSIONS

This analysis has demonstrated that the allocation of aircraft to fixed location targets is a complex problem. If it had been reasonable to assume that attack aircraft make a single pass, as in strategic nuclear warfare, then the nonlinear model reported by Fiacco and McCormick [22] could have been used. However in limited warfare, for which this model was developed, it is not reasonable to assume a single pass. Furthermore, we believe that a deterministic treatment of repeated passes would not suitably reflect the expected outcome in even an approximate fashion. The substantiation of this assertion is dependent upon the generation of the probability distribution of W_i.

The objective function for this model is non-differentiable; this property restricts the class of usable and currently available nonlinear programming algorithms to that of Rosen [7].

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[illegible]

[illegible]

[illegible]

II. RECOMMENDATIONS

It is recommended that:

1. This model be coded for use on an electronic digital computer utilizing the algorithm of Rosen [7].

2. The list of targets be grouped into type or character classes and an investigation made of the feasibility of using common values within each group for the parameters $PA_{ij}(n)$, $PR_{ijk}(n)$ and v_i respectively.

3. The list of targets be grouped into geographical classes and an investigation made of the feasibility of using common values within each group for the parameters $PD_{ij}(n)$ and $PE_{ij}(N,n)$ respectively.

4. A sensitivity analysis using BOP (best, optimistic, pessimistic) estimates of the parameters be made to determine those requiring further analysis.

5. The feasibility of invoking the Central Limit Theorem on the expression $W_i = \sum_j \sum_k X_{ijk}$ be determined.

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APPENDIXES

APPENDIX A

DERIVATION OF THE PROBABILITY

DISTRIBUTION OF $W_i(\underline{N}_i)$

The event $W_i = 0$ occurs if and only if $\sum_j \sum_k X_{ijk} = 0$, or if and only if $X_{ijk} = 0$ for all j and all k. It was noted that $X_{ijk} \leq X_{ijk-1}$, $k = 2,3,4,\ldots,R_{ij}$, so that $X_{ijk} = 0$ for all j and all k if and only if $X_{ij1} = 0$ for all j. It was assumed that X_{ijk} is independent of $X_{ij'k'}$ provided that $j \neq j'$, so that:

$$P\left[W_i = 0\right] = P\left[\sum_j \sum_k X_{ijk}=0\right]=P\left[X_{ijk}= 0 \text{ for all j and all k}\right]$$

$$= P\left[X_{ij1} = 0 \text{ for all j}\right] = \prod_j P\left[X_{ij1} = 0\right]$$

The distribution of $W_i \geq 1$ was obtained in a slightly different manner. The event $W_i = 1$ occurs if and only if for exactly one j, $X_{ij1} = 1$ and $X_{ij2} = 0$, but the event $W_i = 2$ occurs if for exactly one j, $X_{ij1} = 2$ and $X_{ij2} = 0$, or if for exactly one j, $X_{ij1} = 1, X_{ij2} = 1$ and $X_{ij3} = 0$, or if for exactly two j's, $X_{ij1} = 1$ and $X_{ij2} = 0$. This enumeration can be continued and was up to $W_i = 4$, it produced a lengthy table from which a recursion relation was developed. The use of this relation in conjunction with the probability distribution of X_{ijk} produced the probability distribution of W_i in the following manner.

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

(D6.1) Let faco denote: for all combinations of. Where ''combinations'' is used in the sense of combinatorial analysis, e.g., Parzen [6], then

$$P\left[W_i = 1\right] = \sum_j \left[P\, X_{ij1} = 1\right] P\left[X_{ij2} = 0 \mid X_{ij1} = 1\right] \prod_{n \neq j} P\left[X_{in1} = 0\right]$$

$$P\left[W_i = 2\right] = \sum_j \sum_{a_1=0}^{1} P\left[X_{ij1} = 2-a_1\right] P\left[X_{ij2} = a_1 \mid X_{ij1} = 2 - a_1\right]$$

$$\cdot P\left[X_{ij3} = 0 \mid X_{ij2} = a_1\right] \prod_{n \neq j} P\left[X_{in1} = 0\right]$$

$$+ \sum_{\substack{\text{faco} \\ j_1, j_2}} P\left[X_{ij_11} = 1\right] P\left[X_{ij_12} = 0 \mid X_{ij_11} = 1\right]$$

$$\cdot P\left[X_{ij_21} = 1\right] P\left[X_{ij_22} = 0 \mid X_{ij_21} = 1\right] \prod_{n \neq j_1, j_2} P\left[X_{in1} = 0\right]$$

NOTE: $P\left[X_{ijk} = x \mid X_{ijk-1} = m\right] = 0$ for all $x > m$ and

$P\left[X_{ijk} = 0 \mid X_{ijk-1} = 0\right] = 1$, Table II.

$$P\left[W_i = 3\right] = \sum_j \sum_{a_1=0}^{2} \sum_{a_2=0}^{a_1-1} P\left[X_{ij1} = 3 - a_1\right]$$

$$\cdot P\left[X_{ij2} = a_1 - a_2 \mid X_{ij1} = 3 - a_1\right] P\left[X_{ij3} = a_2 \mid X_{ij2} = a_1 - a_2\right]$$

$$\cdot P\left[X_{ij4} = 0 \mid X_{ij3} = a_2\right] \prod_{n \neq j} P\left[X_{in1} = 0\right]$$

(continued on the following page)

$$+ \sum_{\substack{\text{faco}\\ j_1, j_2}} \sum_{a_1=1}^{2} \sum_{b_{11}=0}^{3-a_1-1} \sum_{b_{21}=0}^{a_1-1} P\left[X_{ij_11} = 3 - a_1 - b_{11}\right]$$

$$\cdot \; P\left[X_{ij_12} = b_{11} \mid X_{ij_11} = 3 - a_1 - b_{11}\right] P\left[X_{ij_13} = 0 \mid X_{ij_12} = b_{11}\right]$$

$$\cdot \; P\left[X_{ij_21} = a_1 - b_{21}\right] P\left[X_{ij_22} = b_{21} \mid X_{ij_21} = a_1 - b_{21}\right]$$

$$\cdot \; P\left[X_{ij_23} = 0 \mid X_{ij_22} = b_{21}\right] \prod_{n \neq j_1, j_2} P\left[X_{in1} = 0\right]$$

$$+ \sum_{\substack{\text{faco}\\ j_1, j_2, j_3}} P\left[X_{ij_11} = 1\right] P\left[X_{ij_12} = 0 \mid X_{ij_11} = 1\right]$$

$$\cdot \; P\left[X_{ij_21} = 1\right] P\left[X_{ij_22} = 0 \mid X_{ij_21} = 1\right]$$

$$\cdot \; P\left[X_{ij_31} = 1\right] P\left[X_{ij_32} = 0 \mid X_{ij_31} = 1\right] \prod_{n \neq j_1, j_2, j_3} P\left[X_{in1} = 0\right]$$

By induction the general term of the probability distribution of W_i was obtained and is presented in Table IV, where all sums are defined in the positive sense, i.e., the sum $\sum_{y=d}^{e} f(y)$ is defined if and only if $e \geq d$ and the following definitions were used:

(D6.2) R denotes the $\max_{i,j} (R_{ij})$

(D6.3) r denotes the $\max (R, J)$

Summation Operators:

(D6.4) $$B_i = \prod_{k_i=2}^{r-1} \sum_{b_{ik_i}=0}^{b_{ik_i-1}-1}$$

(D6.5) $$C_j = \prod_{k=2}^{j-1} \sum_{a_k=j-k}^{a_{k-1}-1}$$

(D6.6) $$A_1 = \sum_{a_1=0}^{w-1} \prod_{k=2}^{r-1} \sum_{a_k=0}^{a_{k-1}-1}$$

(D6.7) $$A_2 = \sum_{a_1=1}^{w-1} \sum_{b_{11}=0}^{w-a_1-1} B_1 \sum_{b_{21}=0}^{a_1-1} B_2$$

(D6.8) $$A_j = \sum_{a_1=j-1}^{w-1} C_j \left[\prod_{t=1}^{j-1} \sum_{b_{t1}=0}^{a_{t-1}-a_t-1} B_t \right] \sum_{b_{j1}=0}^{a_{j-1}-1} B_j$$

$j = 3, 4, \ldots, J$ and $a_0 = w$

Summation Operands:

$$\text{(D6.9)} \quad G_{tm}(i) = P\left[X_{ij_t 1} = a_{t-1} - a_t(1-d_{tm}) - b_{t1}\right]$$

$$\cdot \quad P\left[X_{ij_t 2} = b_{t1} - b_{t2} \;\middle|\; X_{ij_t 1} = a_{t-1} - a_t(1-d_{tm}) - b_{t1}\right]$$

$$\cdot \prod_{k_t=2}^{r-2} P\left[X_{ij_t k_t+1} = b_{tk_t} - b_{tk_t+1} \;\middle|\; X_{ij_t k_t} = b_{tk_t-1} - b_{tk_t}\right]$$

$$\cdot \quad P\left[X_{ij_t r} = b_{tr-1} \;\middle|\; X_{ij_t r-1} = b_{tr-2} - b_{tr-1}\right]$$

$$\cdot \quad P\left[X_{ij_t r+1} = 0 \;\middle|\; X_{ij_t r} = b_{tr-1}\right]$$

$$\text{where } d_{ij} = \begin{cases} 0 & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}$$

$$\text{(D6.10)} \quad P_{i1} = \sum_{j_1} \prod_{k=1}^{r-2} P\left[X_{ij_1 k} = a_{k-1} - a_k\right]$$

$$\cdot \quad P\left[X_{ij_1 k+1} = a_k - a_{k+1} \;\middle|\; X_{ij_1 k} = a_{k-1} - a_k\right]$$

$$\cdot \quad P\left[X_{ij_1 r} = a_{r-1} \;\middle|\; X_{ij_1 r-1} = a_{r-2} - a_{r-1}\right]$$

$$\cdot \quad P\left[X_{ij_1 r+1} = 0 \;\middle|\; X_{ij_1 r} = a_{r-1}\right] \prod_{n \neq j_1} P\left[X_{in1} = 0\right]$$

$$\text{(D6.11)} \quad P_{im} = \sum_{\substack{\text{faco} \\ j_1, \ldots, j_m}} \prod_{t=1}^{m} G_{tm}(i)$$

$$\cdot \prod_{n \neq j_1, \ldots, j_m} P\left[X_{in1} = 0\right]$$

$$m = 2, 3, \ldots, J-1 \text{ and } J \geqq 3:$$

$$\text{(D6.12)} \quad P_{iJ} = \prod_{t=1}^{J} G_{tJ}(i) \qquad J \geqq 2$$

Therefore under the above definitions, given values for R and J, the probability distribution of W_i presented in Table IV can be expressed in closed form for computational purposes.

APPENDIX B

PROGRAM ORDLOAD

The FORTRAN IV program to compute the fuel and ordnance loadings is presented in this Appendix. A brief description of the imput data, the definition of variables, a program listing, and a sample output will be given. Data for the sample problem was selected at random, and the resultant output bears little semblance to any realistic situation that might occur. The program has been dimensioned to handle six aircraft type, six bases, and fifty targets. These dimensions may be expanded as necessary to suit a particular user's need. Common blocks have also been established to facilitate incorporation of this program as a sub-routine in a larger allocation problem.

The data input is presented below. Variables with subscript (I) require one data card for each aircraft type.

Card 1: FORMAT (3I10,F10.0):

Fields 1-10: NTYPE = the number of different aircraft type, $1 \leq$ NTYPE ≤ 6;

Fields 11-20: JBASE = the number of friendly bases, $1 \leq$ JBASE ≤ 6;

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

Fields 21-30: KTGT = the number of targets, $1 \leq KTGT \leq 50$;

Fields 31-40: TIME = the time (tenths of hours) over the target for delivery of ordnance.

Card 2,...,NTYPE+1: FORMAT (5F10.0,I10):

Fields 1-10: GAXTOW(I) = maximum gross aircraft take-off weight (pounds);

Fields 11-20: BACFTW(I) = basic aircraft weight (pounds);

Fields 21-30: ENTFW(I) = maximum internal fuel capacity (pounds);

Fields 31-40: EXTFW(I) = maximum external fuel capacity (pounds);

Fields 41-50: GAXBMW(I) = maximum gross ordnance load (pounds);

Fields 51-60: IRACK(I) = maximum number of external store racks on an aircraft;

Card NTYPE+2,...,(2xNTYPE)+1: FORMAT (5F10.0):

Fields 1-10: FTAKE(I) = fuel (pounds) required for warm-up and take-off;

Fields 11-20: FCLIMB(I) = fuel (pounds) required for climb to cruise altitude;

Fields 21-30: FLOIT(I) = loiter fuel rate (pounds per minute)-- not required in the current formulation of the program;

Fields 31-40: DASHSP(I) = speed (knots) over the target at military power;

Fields 41-50: FLAND(I) = landing and reserve fuel (pounds).

Card (2xNTYPE)+2,...,(3xNTYPE)+1: FORMAT (2F10.0):

Fields 1-10: DISTML(I) = distance (nautical miles) covered in the climb to cruising altitude:

Fields 11-20: DISTRI(I) = run-in distance (nautical miles) to the target.

Card (3NTYPE)+2: FORMAT (2F10.0):

Fields 1-10: ALT1 = cruise altitude (thousands of feet);

Fields 11-20: ALT2 = run-in altitude (thousands of feet).

Card (3xNTYPE)+3,...,(4xNTYPE)+2: FORMAT (4F10.0):

Fields 1-10: SFC(I) = sea level cruise specific fuel consumption (pounds per nautical mile = ppnm);

Fields 11-20: SFCMIL(I) = sea level military power specific fuel consumption (ppnm);

Fields 21-30: SFCK(I) = cruise specific fuel consumption altitude correction factor (ppnm per thousand feet altitude);

Fields 31-40: SFCMILK(I) = military power specific fuel consumption altitude correction factor (ppnm per thousand feet altitude).

Card (4xNTYPE)+3,...,(4xNTYPE)+2+(JBASExKTGT): FORMAT (F10.0):

Field 1-10: RANGE(J,K) = range (nautical miles) from Jth base to the Kth target.

Definitions of other variables used in the program are:

FCRUSA(I) = cruise altitude fuel consumption rate (ppnm);

FCRUSL(I) = run-in altitude fuel consumption rate (ppnm);

FDASHM(I) = run-in altitude military power fuel consumption rate (ppnm);

FAVAIL(I) = maximum total fuel available for aircraft type I;

FRQRD(I,J,K) = fuel required for aircraft type I to complete mission from base J to target K and return;

EXTFRQ(I,J,K) = external fuel required for aircraft type to complete mission from base J to target K and return;

KRACK(I,J,K) = the number of external store racks available on aircraft type I to target K from base J;

ORDLD(I,J,K) = ordnance load (pounds) aircraft I can carry to target K from base J.

A complete program listing and a sample output, which uses the data at the end of the program listing, are given on the last four pages of this Appendix. The sample

output tables the aircraft number, the base number, the target number, the range from base J to target K, the total fuel required (lbs.), the internal and external fuel required (lbs.), the number of ordnance racks available, and the ordnance load capability (pounds).

```
-COOP,,RUSSELL BOX R,S/1S/2S,15,15000,4.                                ORDL0000
-FTN,L,A,E.                                                             ORDL0001
      PROGRAM ORDLOAD                                                   ORDL0002
      DIMENSION GAXTOW(6),BACFTW(6),ORDFW(6),ENTFW(6),EXTFW(6),GAXBMW   ORDL0010
     1(6),FTAKE(6),FCLIMB(6),FCRUSA(6),FCRUSL(6),FDASHM(6),             ORDL0020
     2FLOIT(6),FLAND(6),DISTML(6),DISTRI(6),FRQRD(6,6,50)               ORDL0030
     3,FAVAIL(6),DASHSP(6),SFC(6),SFCMIL(6),                            ORDL0040
     4SFCK(6),SFCMILK(6),RANGE(6,50),EXTFRQ(6,6,50),ORDLD(6,6,          ORDL0050
     550),KRACK(6,6,50),IRACK(6)                                        ORDL0060
      COMMON/WEIGHT/GAXTOW,BACFTW,ORDFW,ENTFW,EXTFW,GAXBMW,FRQRD,FAVAIL,ORDL0100
     1ORDLD/FUEL/FTAKE,FCLIMB,FCRUSA,FCRUSL,FDASHM,FLOIT,FLA            ORDL0110
     2ND/DIST/DISTML,DISTRI/ALT/ALT1,ALT2                               ORDL0120
      READ 1000,NTYPE,JBASE,KTGT,TIME                                   ORDL0180
 1000 FORMAT(3I10,F10.0)                                                ORDL0190
C     NTYPE = NO. OF DUFFERENT TYPE AIRCRAFT = 1,2,....,6               ORDL0200
C     JBASE = NO. OF BASES = 1,2,....,6                                 ORDL0202
C     KTGT = NO. OF TARGETS = 1,2,....,50                               ORDL0204
    1 DO 10 I=1,NTYPE                                                   ORDL0210
   10 READ 1010,GAXTOW(I),BACFTW(I),ENTFW(I),EXTFW(I),GAXBMW(I),IRACK(I)ORDL0230
 1010 FORMAT (5F10.0,I10)                                               ORDL0240
    2 DO 11 I=1,NTYPE                                                   ORDL0245
   11 READ 1020, FTAKE(I),FCLIMB(I),FLOIT(I),DASHSP(I),FLAND(I)         ORDL0250
 1020 FORMAT(5F10.0)                                                    ORDL0260
    3 DO 12 I=1,NTYPE                                                   ORDL0265
   12 READ 1030,DISTML(I),DISTRI(I)                                     ORDL0270
 1030 FORMAT (2F10.0)                                                   ORDL0280
      READ 1040,ALT1,ALT2                                               ORDL0300
 1040 FORMAT (2F10.0)                                                   ORDL0310
      DO 20 I=1,NTYPE                                                   ORDL0320
      READ 1050,SFC(I),SFCMIL(I),SFCK(I),SFCMILK(I)                     ORDL0330
 1050 FORMAT(4F10.0)                                                    ORDL0340
      FCRUSA(I)=SFC(I)-SFCK(I)*ALT1                                     ORDL0350
      FCRUSL(I)=SFC(I)-SFCK(I)*ALT2                                     ORDL0360
      FDASHM(I)=SFCMIL(I)-SFCMILK(I)*ALT2                               ORDL0370
   15 FAVAIL(I)=ENTFW(I)+EXTFW(I)                                       ORDL0380
   20 CONTINUE                                                          ORDL0390
```

```
C      RANGE(J,K)=RANGE OF JTH BASE TO KTH TARGET                        ORDL0400
       DO 30 J=1,JBASE                                                   ORDL0410
       DO 30 K=1,KTGT                                                    ORDL0420
       READ 1060, RANGE(J,K)                                             ORDL0430
 1060 FORMAT( F10.0)                                                     ORDL0440
   30 CONTINUE                                                           ORDL0450
      PRINT 2000                                                         ORDL0454
 2000 FORMAT(1H1,////////,52X,26HFUEL    INTERNAL  EXTERNAL,/,11X,8HAIRCORDL0455
     1RAFT,3X,4HBASE,5X,6HTARGET,4X,5HRANGE,4X,8HREQUIRED,4X,4HFUEL,6X,4ORDL0456
     2HFUEL,4X,8HORDRACKS,3X,7HORDLOAD//)                                ORDL0457
   31 DO 40 I=1,NTYPE                                                    ORDL0460
      DO 40 J=1,JBASE                                                    ORDL0470
      DO 40 K=1,KTGT                                                     ORDL0480
   32 FRQRD(I,J,K)=FTAKE(I)+2.*FCLIMB(I)+2.*FCRUSA(I)*(RANGE(J,K)-       ORDL0490
     1DISTML(I)-DISTRI(I))+2.*FCRUSL(I)*DISTRI(I)+TIME*DASHSP(I)*FDASHM ORDL0500
     2(I)+FLAND(I)                                                       ORDL0510
   33 IF(FRQRD(I,J,K).GE. FAVAIL(I)) 35,36                               ORDL0520
   35 PRINT 2005,I,J,K,RANGE(J,K),FRQRD(I,J,K)                           ORDL0530
 2005 FORMAT (14X,I2,7X,I2,8X,I2,6X,F5.0,5X,F5.0,31H FUEL AVAILABLE IS IORDL0540
     1NSUFFICIENT,/)                                                     ORDL0550
      GO TO 40                                                           ORDL0560
   36 EXTFRQ(I,J,K)=FRQRD(I,J,K)-ENTFW(I) $ IF(EXTFRQ(I,J,K).GE.EXTFW(   ORDL0570
     1I))35,360                                                          ORDL0580
  360 IF(EXTFRQ(I,J,K).GT.1950.)361,362                                  ORDL0590
  362 IF(EXTFRQ(I,J,K).LE.0.)3621,3622                                   ORDL0600
 3622 KRACK(I,J,K)=IRACK(I)-1  $ EXTFRQ(I,J,K)=1950. $ GO TO 367         ORDL0610
 3621 EXTFRQ(I,J,K)=0. $ KRACK(I,J,K)=IRACK(I) $ GO TO 367               ORDL0610
  361 IF(EXTFRQ(I,J,K).LE.3900.)364,363                                  ORDL0620
  364 KRACK(I,J,K)=IRACK(I)-2   $ EXTFRQ(I,J,K)=3900.  $ GO TO 367       ORDL0630
  363 IF(EXTFRQ(I,J,K).LE.5850.)366,365                                  ORDL0640
  366 KRACK(I,J,K)=IRACK(I)-3   $ EXTFRQ(I,J,K)=5850. $ GO TO 367        ORDL0650
  365 KRACK(I,J,K)=IRACK(I)-4   $   EXTFRQ(I,J,K)=7800.                  ORDL0660
  367 ORDLD(I,J,K)=GAXTOW(I)-(BACFTW(I)+ENTFW(I)+EXTFRQ(I,J,K))          ORDL0670
 2010 FORMAT (14X,I2,7X,I2,8X,I2,6X,F5.0,5X,F5.0,5X,F5.0,5X,F5.0,7X,I1,8ORDL0680
     1X,F5.0,/)                                                          ORDL0690
   37 PRINT 2010,I,J,K,RANGE(J,K),FRQRD(I,J,K),ENTFW(I),EXTFRQ(I,J,K),KRORDL0700
```

```
     1ACK(I,J,K),ORDLD(I,J,K)                                                       ORDL0710
   40 CONTINUE                                                                      ORDL0720
      END                                                                           ORDL0730
         END                                                                        ORDL0740
         FINIS                                                                      ORDL0750
-EXECUTE.                                                                           ORDL0760
         2         2         4        .2                                            DATA0010
    48000.    20000.    12000.     7800.    16000.          8                       DATA0020
    25000.    15000.     6000.     3900.    10000.          6                       DATA0030
      900.     1500.       65.      500.     3000.                                  DATA0040
      600.     1200.       35.      470.     1300.                                  DATA0050
       90.       50.                                                                DATA0060
      100.       50.                                                                DATA0070
       35.       1.0                                                                DATA0080
      20.0      25.0       .20       .40                                            DATA0090
       12.      20.0       .20       .35                                            DATA0100
      150.                                                                          DATA0110
      200.                                                                          DATA0120
      250.                                                                          DATA0130
      300.                                                                          DATA0140
      350.                                                                          DATA0150
      400.                                                                          DATA0160
      450.                                                                          DATA0170
      500.                                                                          DATA0180
```

AIRCRAFT	BASE	TARGET	RANGE	FUEL REQUIRED	INTERNAL FUEL	EXTERNAL FUEL	CRDRACKS	ORDLOAD
1	1	1	150	11600	12000	0	8	16000
1	1	2	200	12900	12000	1950	7	14050
1	1	3	250	14200	12000	3900	6	12100
1	1	4	300	15500	12000	3900	6	12100
1	2	1	350	16800	12000	5850	5	10150
1	2	2	400	18100	12000	7800	4	8200
1	2	3	450	19400	12000	7800	4	8200
1	2	4	500	20700 FUEL AVAILABLE IS INSUFFICIENT.				
2	1	1	150	7327	6000	1950	5	2050
2	1	2	200	7827	6000	1950	5	2050
2	1	3	250	8327	6000	3900	4	100
2	1	4	300	8827	6000	3900	4	100
2	2	1	350	9327	6000	3900	4	100
2	2	2	400	9827	6000	3900	4	100
2	2	3	450	10327 FUEL AVAILABLE IS INSUFFICIENT				
2	2	4	500	10827 FUEL AVAILABLE IS INSUFFICIENT				