CONTINENTAL SHELF WAVES OVER A CONTINENTAL SLOPE

by

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THESIS

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October 1969

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LIST OF SYMBOLS

a	dimensionless variable
A	constant of integration
B	constant of integration
D	depth of deep water
e	2.7183
f	Coriolis force (0.729×10^{-4}/sec)
Fa (z)	Laguerre function of the first kind
g	gravitational acceleration (9.80 m/sec^2)
Ga (z)	Laguerre function of the second kind
h	depth
i	square root of (-1)
k	subscript
m	wave number
p	dimensionless variable
s	slope
t	time
u	velocity in x direction
U	portion of u that varies with x
v	velocity in y direction
V	portion of v that varies with x
W_1	width of continental shelf
W_2	width of continental slope
x	horizontal coordinate perpendicular to coastline
y	horizontal coordinate parallel to coastline
z	dimensionless variable

ACKNOWLEDGEMENT

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ζ	wave amplitude
η	instantaneous wave height
ξ	dimensionless variable
σ	wave angular frequency
ω	dimensionless variable (σ/f)

I. INTRODUCTION

Considerable interest has recently been shown in trapped waves travelling along the boundaries of continents. A "waveguide" effect exists over the continental shelf. That is, wave energy is confined (essentially by refraction) to the continental shelf. Two general types of these waves exist:

A. Edgewaves which are characterized by wavelengths of hundreds of kilometers (km) and periods of hours (almost always less than a pendulum day).

B. Shelf waves, which are generally even longer, have periods greater than one pendulum day, and travel southward along the west coast of an ocean in the northern hemisphere (as do Kelvin waves).

STOKES (1846) showed that such a wave guide effect exists over a uniformly sloping beach or continental shelf, with the amplitude of the gravity waves decaying exponentially to seaward. URSELL (1952) showed that Stoke's edgewaves were the fundamental mode of a family of waves ordered by the number of modes parallel to the coast. REID(1958) studied long waves on uniformly sloping shelves of infinite width, including the effect of the Coriolis force. Reid showed that the sea surface may react as an "inverse barometer" and that atmospheric pressure systems may be a driving force for edgewaves. He found that the Coriolis force could cause the wave period to vary from 46% less than to

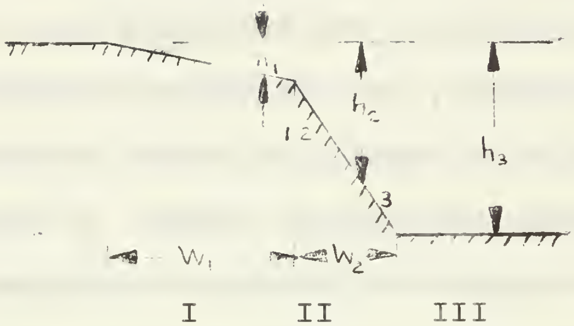
86% greater than that for the non-rotating case, depending on the direction of travel. A new quasigeostrophic wave is now possible, analogous to a Kelvin wave, having no small scale counterpart.

ROBINSON (1964) initiated a study of the continental shelf wave and studied the data of HAMON (1962, 1963) relating tidal and barometric conditions at several stations on the eastern and western coasts of Australia. In this model the continental shelf ends abruptly, at which point the depth becomes infinite. He found that an inverse barometer effect was exhibited but that the propagated shelf waves had a celerity double that of his calculations for the western boundary. MOOERS and SMITH (1968) studied the relation of sea level and barometric conditions along the Oregon coast for a period of nearly one year. Their statistical results show a barometric factor of -1.2 cm/mb and predominant sea level oscillations of 0.1 and 0.35 cycles per day in the summer. They conclude that a shelf wave of period three days is travelling north. MYSAK and HAMON (1969) found shelf waves off the coast of North Carolina, but found no coupling between the sea surface and air pressure in the frequency range 0-0.5 cpd. ADAMS and BUCHWALD (1969) show that an equally suitable driving force for shelf waves is the longshore component of the geostrophic wind. This may account for the exaggerated frequency response of the sea level on the east coast observed by Hamon.

MYSAK (1967, 1968) extended Robinson's work, and discussed the effect of a continental shelf of finite width on the frequency of Hamon's Australian waves. His theoretical solutions correspond more closely to the observations, although he still cannot account for the extremely low readings along the eastern boundary. He attributes the discrepancy mainly to the presence of stratified water and currents in the deep water beyond the continental shelf. A significant discrepancy exists between the dispersion relation and that for waves over an infinitely wide continental shelf.

This paper is a study of the effects of a continental slope and finite ocean depth upon the present one-slope models of MYSAK (1968). A sharp discontinuity in the depth of water beyond the continental shelf is not a common occurrence in the world ocean. It is interesting to study the two-slope situation where a gently sloping continental shelf (slope, $s < 0.002$) and steeper continental slope ($s \approx 0.05$) form a transition zone between the coast-line and deep water. Three parameters: the slope of the continental shelf, the slope of the continental slope, and the depth of the deep water should have possible effects on shelf waves. These are investigated below.

II. ANALYSIS



$h_1 = s_1 x \, (x \le W_1)$

$h_2 = s_2 x + W_1(s_1 - s_2)$

$(W_1 \le x \le W_2 + W_1)$

$h_3 = \text{constant}$

$= s_1 W_1 + s_2 W_2$

Fig. 1. Profile of the two-slope model.

The model is characterized by a gradually sloping continental shelf of finite width (Region I) adjoining a steeper continental slope (Region II) which terminates in water of uniform depth (Region III). A representative slope and width of the continental shelf of .002 and 100 km (Mysak, 1968a) are used below. A representative depth of the deep ocean is 5000m and is the greatest depth of Region III. Three slopes will be used for the continental slope: .03, .05 and .08, with .05 used as the standard for comparison with Mysak's model.

The shallow water equations are used:

$$\partial u/\partial t - fv + g\,\partial\zeta/\partial x = \partial v/\partial t + fu + g\,\partial\zeta/\partial y = 0 \qquad (1)$$

$$\partial/\partial x(hu) + \partial/\partial y(hv) + \partial\zeta/\partial t = 0 \qquad (2)$$

where (u, v) are the (x, y) velocity components and ζ is the free surface height.

Consider a wave, moving in the y direction, specified by

$$u_k = U_k(x)e^{i(\sigma t-my)}$$
$$v_k = V_k(x)e^{i(\sigma t-my)} \qquad (3)$$
$$\zeta_k = \eta_k(x)e^{i(\sigma t-my)}$$

where k denotes the region.

Using (1) and (3), u and v are

$$u_k = \frac{ig}{f^2-\sigma^2} \left(fm\eta - \sigma \frac{\partial \eta}{\partial x}\right)_k e^{i(\sigma t-my)} \qquad (4)$$

$$v_k = \frac{-g}{f^2-\sigma^2} \left(\sigma m\eta - f \frac{\partial \eta}{\partial y}\right)_k e^{i(\sigma t-my)} \qquad (5)$$

Eliminating u, v from (2) in Region I gives the equation

$$h_1\eta_1'' + s_1\eta_1' + \left(\frac{\sigma^2-f^2}{g} - \frac{s_1 fm}{\sigma} - h_1 m^2\right)\eta_1 = 0 \qquad (6)$$

After making the substitutions

$$h_1 = s_1 x$$
$$p_1 = \frac{\sigma^2-f^2}{gs_1} - \frac{fm}{\sigma}$$

(6) can be written

$$x\eta_1'' + \eta_1' + (p_1-m_2 x)\eta_1 = 0 \qquad (7)$$

This equation in η_1 has as its solution (REID, 1958)

$$\eta_1 = \{A_1 Fa_1(z_1) + B_1 Ga_1(z_1)\} e^{-z_1/2} \tag{8}$$

where $Fa_1(z_1)$ is Kummer's function, and $Ga_1(z_1)$ is a second (independent) solution (SLATER, 1960):

$$Fa(z) \equiv 1 + az + \frac{a(a+1)z^2}{4} + \frac{a(a+1)(a+2)z^3}{36} + \cdots$$

$$+ \frac{a(a+1)\cdots(a+n-1)z^n}{(n!)^2} + \cdots \tag{9}$$

and

$$Ga(z) \equiv Fa(z)\ln z + az(\frac{1}{a} - 2) + a(a+1)z^2(\frac{1-a-3a^2}{a(a+1)})\cdots$$

$$+ a(a+1)\cdots(a+n-1)z^n(\frac{1}{a} + \frac{1}{a+1} + \cdots + \frac{1}{a+n-1} - 2 - 1 - \cdots \frac{2}{n}) + \cdots \tag{10}$$

Here, $a_1 = \frac{m-p_1}{2m}$, $z_1 = 2mx$, and A_1, B_1 are constants of integration. Since $Ga_1(z_1)$ approaches infinity as z approaches zero, B_1 must equal zero:

$$\eta_1 = A_1 Fa_1(z_1) e^{-z_1/2} \tag{11}$$

Substituting (11) into (4) gives

$$U_1 = -\frac{iqme^{-z_1/2}}{\sigma^2 - f^2} A_1\left\{(f+\sigma)Fa_1(z_1) - 2\sigma \frac{dFa_1(z_1)}{dz_1}\right\} = 0 \tag{12}$$

Similarly, in Region II,

$$\eta_2 = e^{-z_2/2}\{A_2 Fa_2(z_2) + B_2 Ga_2(z_2)\} \tag{13}$$

and

$$U_2 = -\frac{igme^{-z_2/2}}{\sigma^2-f^2}\left\{A_2\left[(\sigma-f)Fa_2(z_2) - 2\sigma\frac{dGa_2(z_2)}{dz_2}\right]\right.$$

$$\left. + B_2\left[(\sigma-f)Ga_2(z_2) - 2\sigma\frac{dGa_2(z_2)}{dz_2}\right]\right\} \qquad (14)$$

where

$$z_2 = 2m\xi_2\ , \quad a_2 = \frac{m-p_2}{2m}\ ,$$

$$\xi_2 = \frac{h_2}{s_2} \text{ and } p_2 = \frac{\sigma^2-f^2}{gs_2} - \frac{fm}{\sigma}\ .$$

In Region III the counterpart of (6) is

$$h_3\eta_3'' + \left(\frac{\sigma^2-f^2}{g} - h_3m^2\right)\eta_3 = 0\ . \qquad (15)$$

Since η must be bounded for large x,

$$\eta_3 = A_3e^{-\ell x} \text{ where} \qquad (16)$$

$$\ell = \left[m^2 - \frac{\sigma^2-f^2}{gh_3}\right]^{\frac{1}{2}} \text{ and}$$

$$U_3 = -\frac{igA_3}{\sigma^2-f^2}(fm+\sigma\ell)e^{-\ell(x-W_1-W_2)} \qquad (17)$$

There are now equations defining η and U in each region. The next step is to patch together the solutions for η and U at the points $x = W_1$, and $x = (W_1+W_2)$ thus eliminating the constants A_k, B_k . The patching conditions are

$$[\zeta]_1^2 = [\zeta]_2^3 = 0 \qquad \text{(surface height continuity)} \qquad (18)$$

$$[hu]_1^2 = [hu]_2^3 = 0 \qquad \text{(normal flux continuity)} \qquad (19)$$

where

$$[\ ]_k^j \equiv [\ ]_j - [\ ]_k \ .$$

The following abbreviations will be used:

$$F_j \equiv Fa_j(z_j) \qquad G_j \equiv Ga_j(z_j)$$

The subscript, 1, refers to the solution for Region I (continental shelf) where it joins Region II (continental slope). The subscript, 2, refers to the Region II where it joins Region I. The subscript, 3, refers to the continental slope where it joins Region III, the flat bottom. The functions subscripted 3 have the same form as those subscripted 2 with the exception of the variable, z_3, which is determined by the distance from the origin.

Using (18) and (19) between Regions I and II and setting $A_1 = 1$, the constants of integration A_2 and B_2 can be solved for:

$$A_2 = \frac{G_2F_1' - F_1G_2'}{G_2F_2' - F_2G_2'} \qquad (20)$$

$$B_2 = \frac{F_1F_2' - F_2F_1'}{G_2F_2' - F_2G_2'} \qquad (21)$$

Using (18) and (19) between Regions II and III gives the final equation in terms of m and σ only:

$$\frac{G_2F_1' - F_1G_2'}{G_2F_2' - F_2G_2'} \left\{(\ell-m)F_3 + 2mF_3'\right\}$$

$$+ \frac{F_1F_2' - F_2F_1'}{G_2F_2' - F_2G_2'} \left\{(\ell-m)G_3 + 2mG_3'\right\} = 0 \qquad (22)$$

Because there is no way to solve (22) analytically, it is necessary to find the roots numerically using the IBM 360/67 computer system at the Naval Postgraduate School. For a fixed $\omega = \sigma/f$ a search routine was used to find the several m's satisfying the equation. The computer work is described in Appendix A.

III. RESULTS AND CONCLUSIONS

A. REVIEW

There are a number of questions to be answered. MYSAK (1968a) in his finite-width model found:

1. Shelf-wave numbers are inversely related to the shelf width, for a fixed frequency.

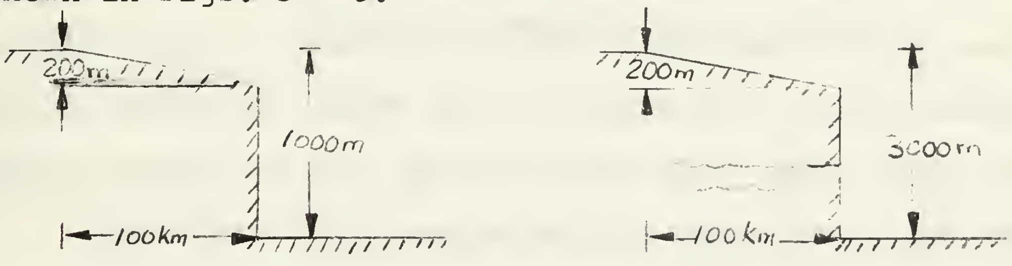
2. There is a low wave number cut-off for edgewaves which is a function of the shelf width. That is, as the shelf width increases, the smallest possible wave number decreases (the largest possible wavelength increases).

3. The fundamental mode edgewaves of REID (1958) with periods greater than one pendulum day do not exist over a continental shelf of finite width.

It should be noted that Mysak's solution is only approximate in that the maximum shelf depth h is assumed much less than the ocean depth, D, leading to an approximation of the equation expressing continuity at the edge of the shelf (Mysak's equation (10)). His results (labeled MA below) depend on this approximation. Exact solutions of Mysak's equation (6) (corresponding to equation (22) in this work), were also generated so that comparisons could be made among MA, an exact solution for Mysak's model (ME) and results of the two-slope model studied in Section II(TS).

B. COMPARISON OF MYSAK'S APPROXIMATE SOLUTION WITH AN EXACT SOLUTION FOR MYSAK'S MODEL

Nine cases were studied in order to compare ME and MA. Two of the cases are illustrated in Fig. 2. In all cases, the continental shelf is 100 km wide with a slope of .002, duplicating Mysak's sample calculation. The bottom depth varied from 500m to 5000m. The shelf wave results are shown in Figs. 5 - 9.



(Vertical exaggeration = 100:1)

Fig. 2. Profile of the finite-width model.

Note that:

1. The approximate solution consistently gives a smaller wave number for a particular ω and mode. It appears to be the limiting condition for the exact solution.

2. Except for the fundamental mode, an error of less than 1% exists between corresponding modes of MA and ME in cases where the depth ratio h/D is smaller than .067.

3. For the fundamental mode there is still a significant error introduced in m when using MA for large depth ratios and frequencies (>10% for $\omega > 0.8$ when D=5000m).

4. The edgewave results are shown in Figs. 18 - 20. Neither MA nor ME gives a fundamental edgewave mode similar to that of Reid. This can be seen directly from Mysak's equation (10) in the approximate case. Because the argument is positive by definition, a must be negative in order for the Laguerre function to have roots (zeroes). This in turn requires that either $f > \sigma$ or $\sigma >> f$.

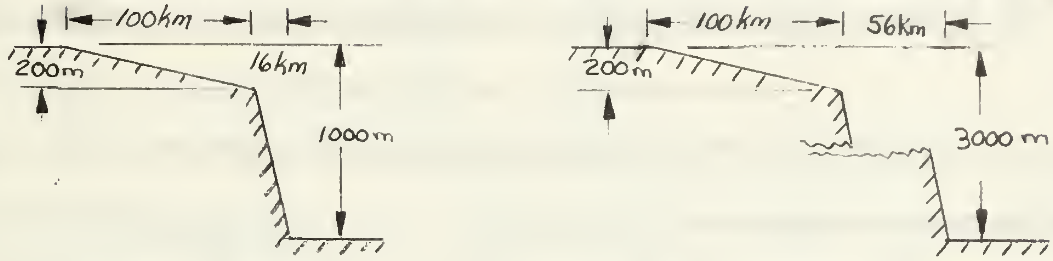
5. A low wave number cut-off does exist for ME edgewaves, which diminishes with an increase in depth and increases with an increase of $|\omega|$ (Table I). The cut-offs are always lower than those for MA, and are quite symmetric with respect to direction of travel.

Table I. ME edgewave wave number cut-off (mW)

Deep-water depth (m)	First mode	Second mode
1000	0.58	1.32
3000	0.32	0.76
5000	0.26	0.58

As pointed out by Mysak and Reid, the edgewave dispersion relation is not symmetrical with respect to direction of travel, due to the influence of the Coriolis parameter.

C. COMPARISON OF THE MYSAK MODEL WITH THE TWO-SLOPE MODEL



(Vertical exaggeration = 100:1)

Fig. 3. Two-slope model.

The Mysak model was compared with a two-slope model whose continental slope was .05 (Fig. 3). The results are shown in Figs. 5 - 9. Note that:

1. Except for the fundamental mode for small ω, there is little similarity between the dispersion relations for the two models, especially for the large deep-water depths (i.e., wide continental slopes). For a deep-water depth of 5000 m, both mode 2 and 3 waves of TS have smaller wave numbers than mode 2 of ME for any given frequency.

2. The fundamental TS shelf wave does not asymptotically approach $\omega = 1.0$ for large wave numbers as does the corresponding ME wave. In fact, the fundamental wave now behaves like the fundamental edgewave of Reid.

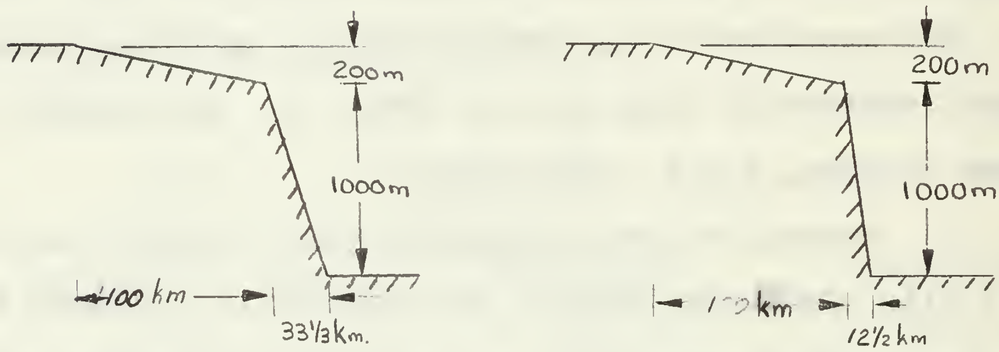
3. A low wave number cutoff is still present for edgewaves, but at a significantly lower wave number (for

mode 1, nearly half that of ME). The cutoffs are no longer symmetric with respect to direction of travel (Table II).

Table II Two-slope edgewave wave number cutoff (mW_1)

Deep-water depth	$\omega < 0$		$\omega > 0$	
m	Mode 1	Mode 2	Mode 1	Mode 2
1000	0.42	1.32	0.28	1.28
3000	0.26	0.76	0.16	0.76
5000	0.22	0.58	0.12	0.58

D. COMPARISON OF CONTINENTAL SLOPES



Slope = 0.03 Slope = 0.08

(Vertical exaggeration = 100:1)

Fig. 4. Two-slope models with different continental slopes.

Three values were used for the value of the continental slope: .03, .05, and .08 (Fig. 4). The results are shown in Figs. 10 - 18, for deep-water depths from 500 m to 5000 m.

1. Wave numbers for a particular mode of trapped waves over a fixed deep-water depth decrease with an increase of continental slope width.

2. For the fundamental mode over a constant gradient continental slope, wave numbers increase with an increase in slope width (i.e., an increase in deep-water depth). All other modes decrease with an increase in width.

3. A curve is not available for mode 3 for a continental slope of 0.03 and deep-water depth of 5000 m. This is attributed to unknown problems of the computer routine. This problem does not occur elsewhere.

4. Discontinuities appear in the dispersion relations for modes 2 and 3 with a continental slope 0.05 (in the deep-water depth range 2200-3400 m) and with a continental slope 0.08 (for depths greater than 2800 m), suggesting the presence of complex values of m. A similar phenomenon is not observed for a slope 0.03. Equation (22) was investigated for complex values of m and real ω (Appendix B) and complex roots were found for a slope of 0.05 and depth 2800 m (Fig. 22). Since the surface height is the real part of ζ (x, y, t) complex values of m imply a spatial growth rate exp $\{\mathcal{Im}(my)\}$ in the positive y direction. The roots m are complex conjugates, so that one wave grows and one decays at this rate. Then the most unstable wave (i.e., the one with the maximum growth rate) would be expected to dominate the shelf-wave spectrum.

5. No complex wave numbers were found for the edge-waves studied. Continental slope width is inversely proportional to wave number as in Mysak's results.

E. RECOMMENDED FURTHER STUDY

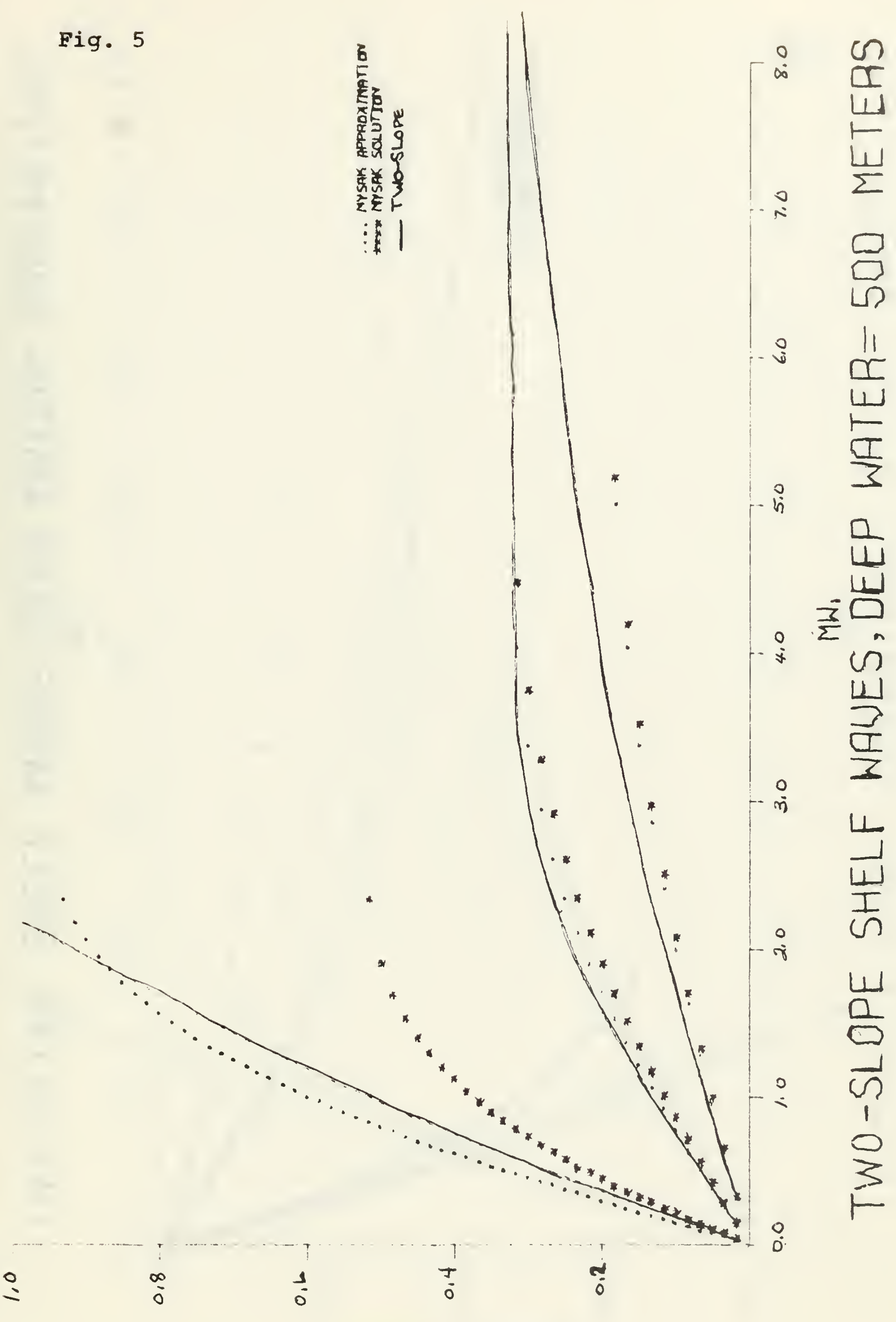
The next step would be to study further the effect of different continental shelf slopes using the TS model. More study is mandatory in D 4 above, both to:

1. find its limits and

2. determine if this is a mathematical curiosity or a physical reality.

Future investigators should seek to avoid approximations to their models. As this study has shown, the results for the exact equations can be significantly different than the approximations.

Fig. 5



TWO-SLOPE SHELF WAVES, DEEP WATER= 500 METERS
MW,
OMEGA
MYSAK APPROXIMATION
MYSAK SOLUTION
TWO-SLOPE
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.8
1.0

Fig. 6

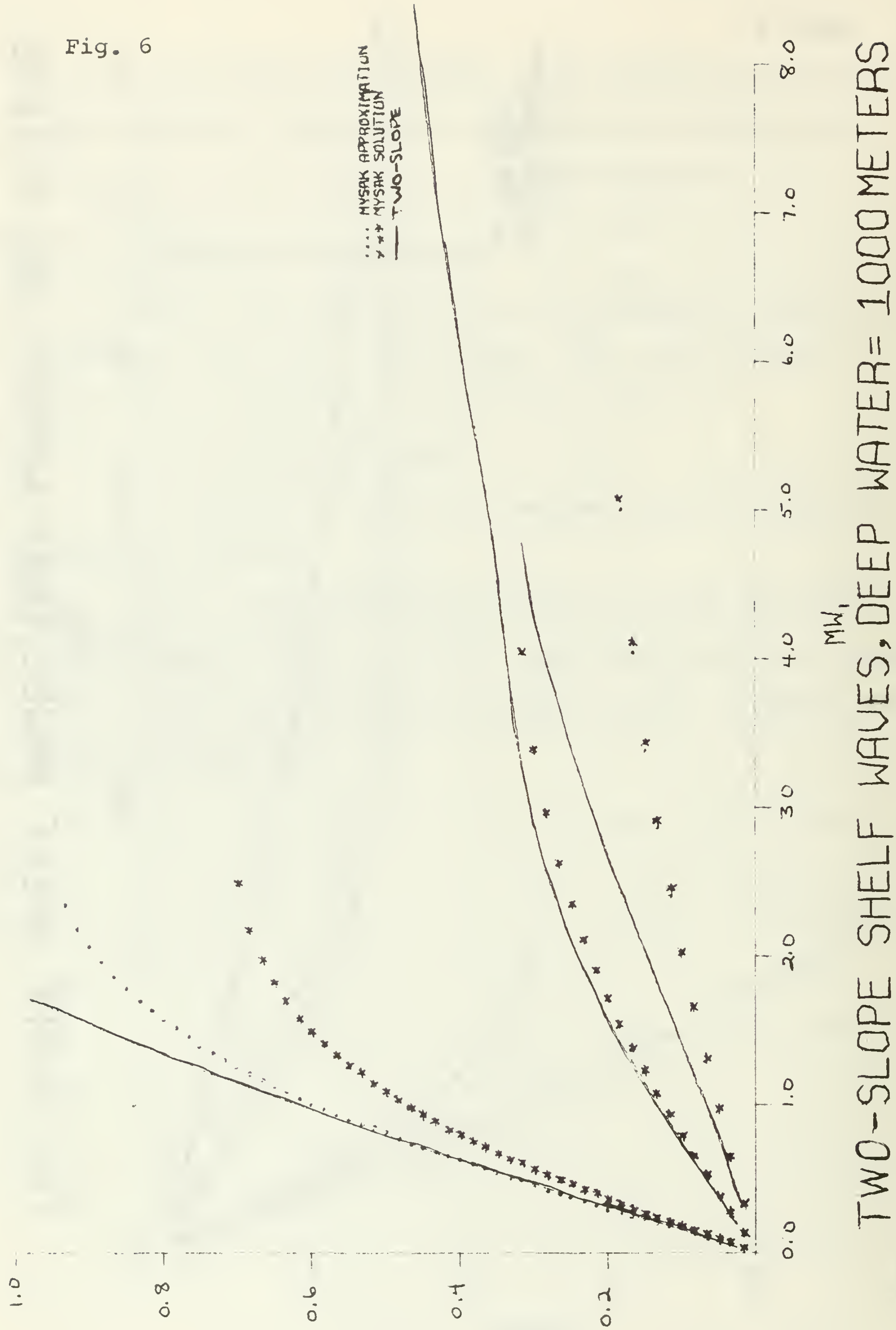
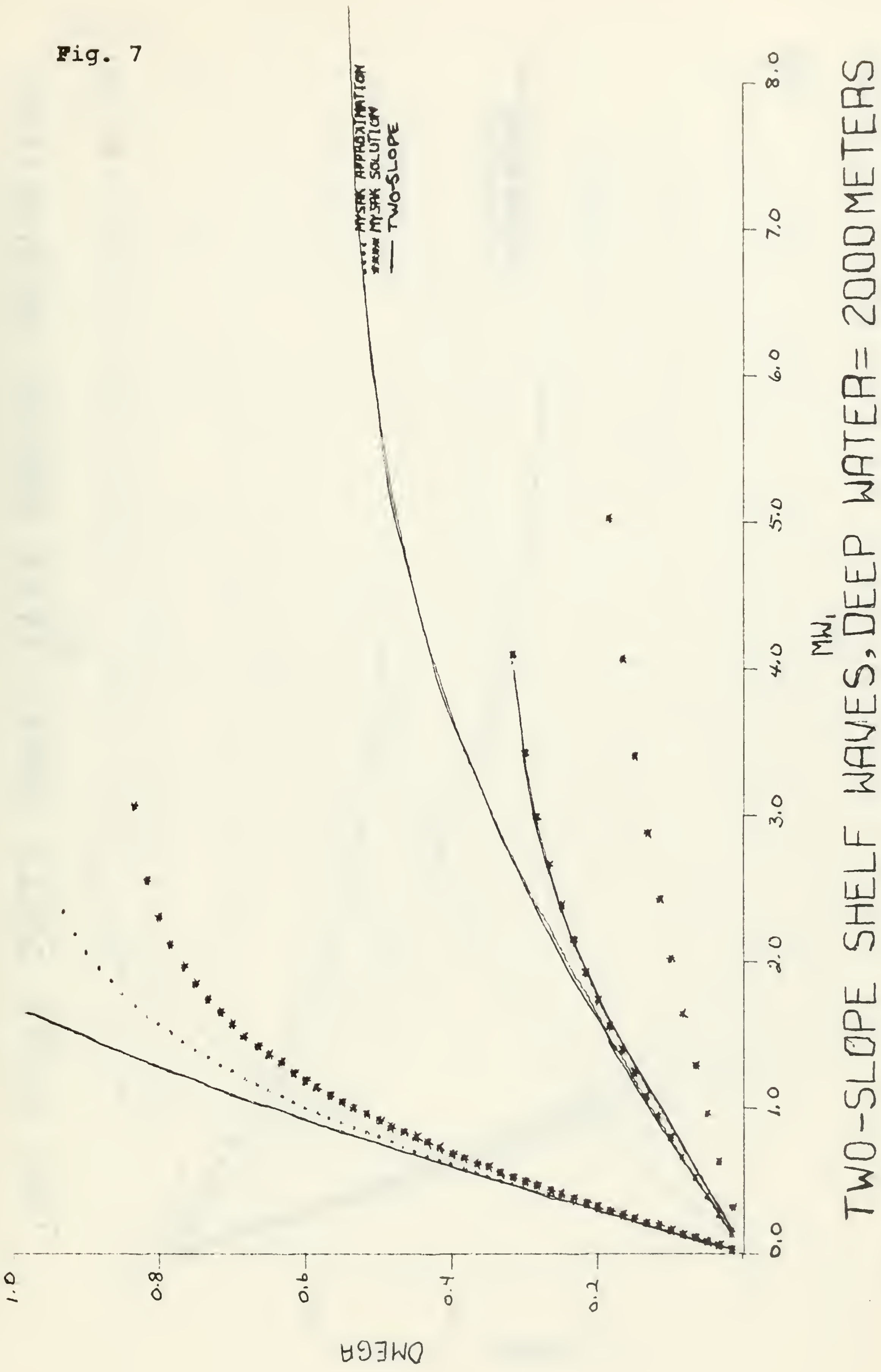
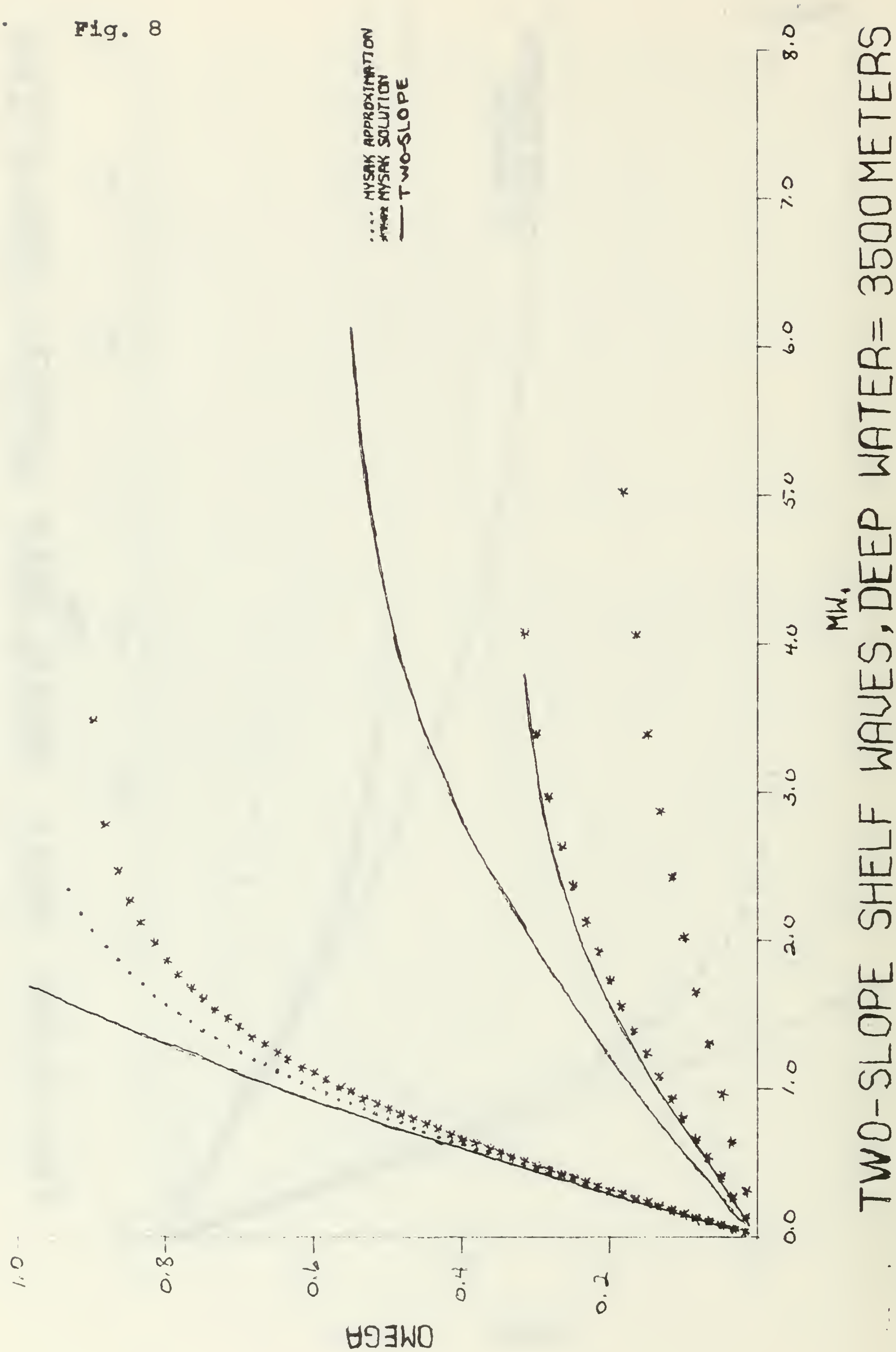

MYSAK APPROXIMATION
MYSAK SOLUTION
TWO-SLOPE
OMEGA
MW_1
TWO-SLOPE SHELF WAVES, DEEP WATER= 1000 METERS

Fig. 7



TWO-SLOPE SHELF WAVES, DEEP WATER= 2000METERS
MW1
OMEGA
MYSAK APPROXIMATION
MYSAK SOLUTION
TWO-SLOPE
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.6
0.8
1.0

Fig. 8



TWO-SLOPE SHELF WAVES, DEEP WATER= 3500 METERS
OMEGA
MW
MYSAK APPROXIMATION
MYSAK SOLUTION
TWO-SLOPE
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.6
0.8
1.0

Fig. 9

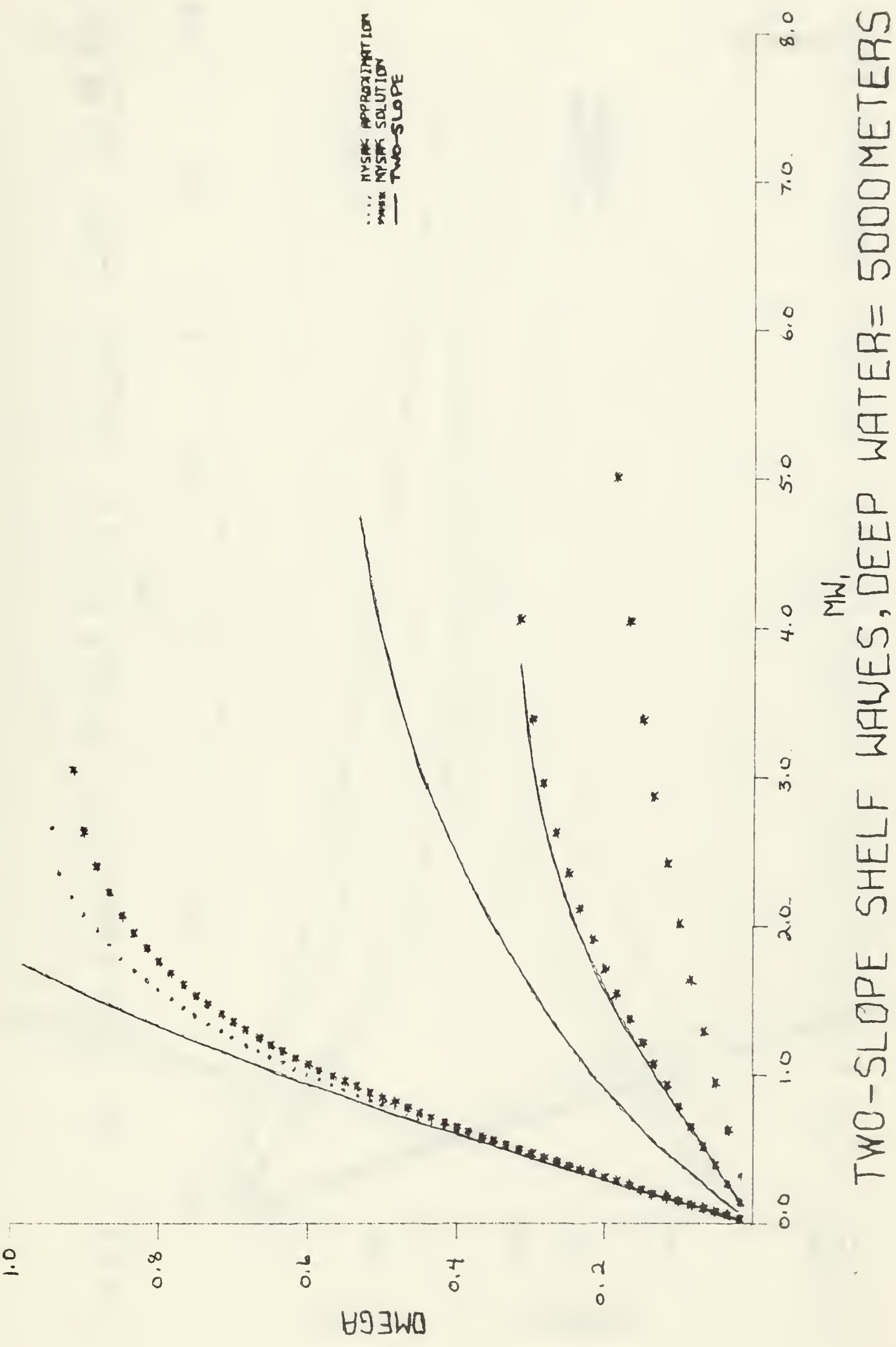
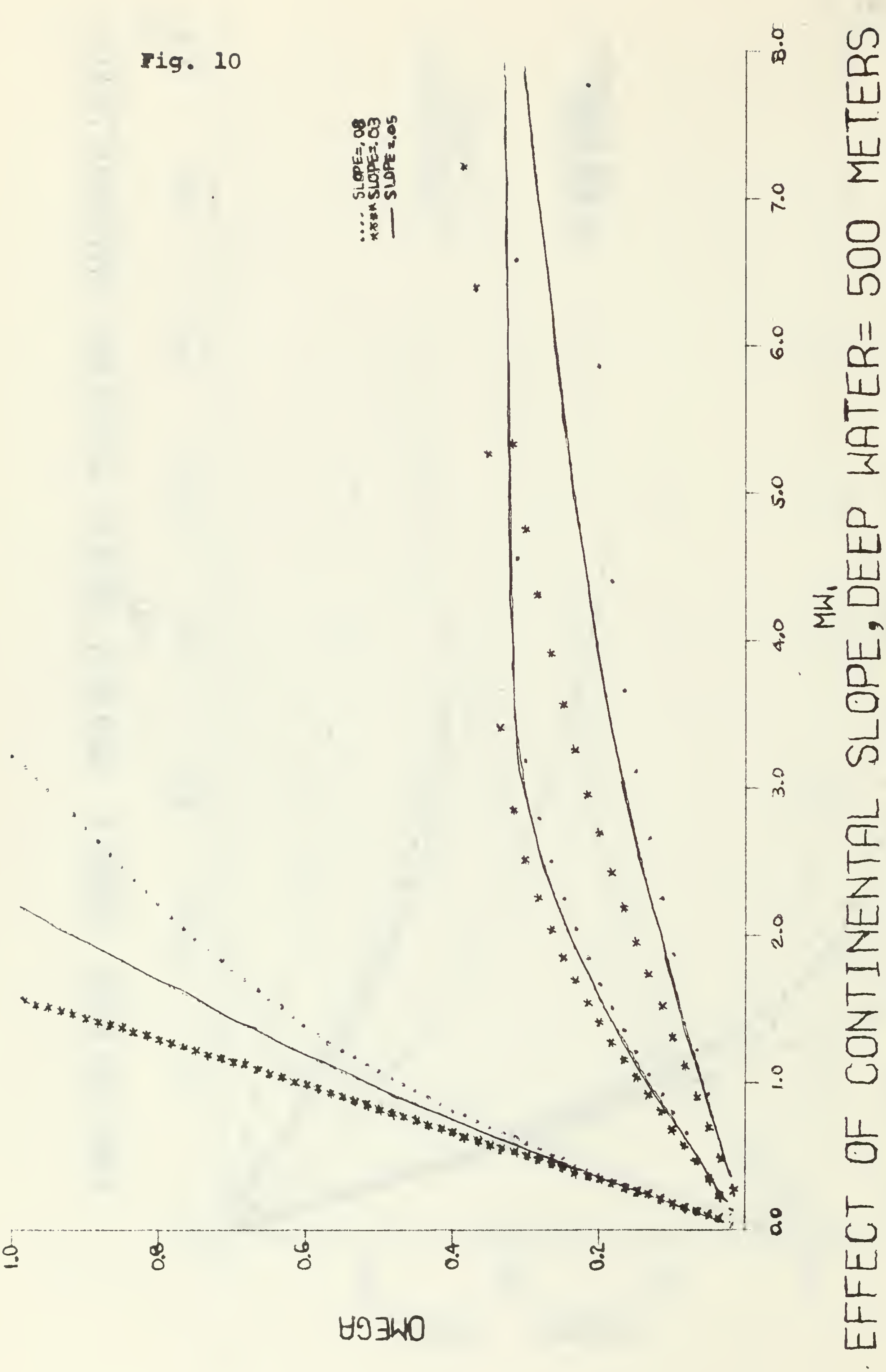

TWO-SLOPE SHELF WAVES, DEEP WATER= 5000METERS
MW,
OMEGA
MYSAK APPROXIMATION
MYSAK SOLUTION
TWO-SLOPE
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.6
0.8
1.0

Fig. 10



SLOPE=.08
SLOPE=.03
SLOPE=.05
OMEGA
0.0
0.2
0.4
0.6
0.8
1.0
MW
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 500 METERS

Fig. 11

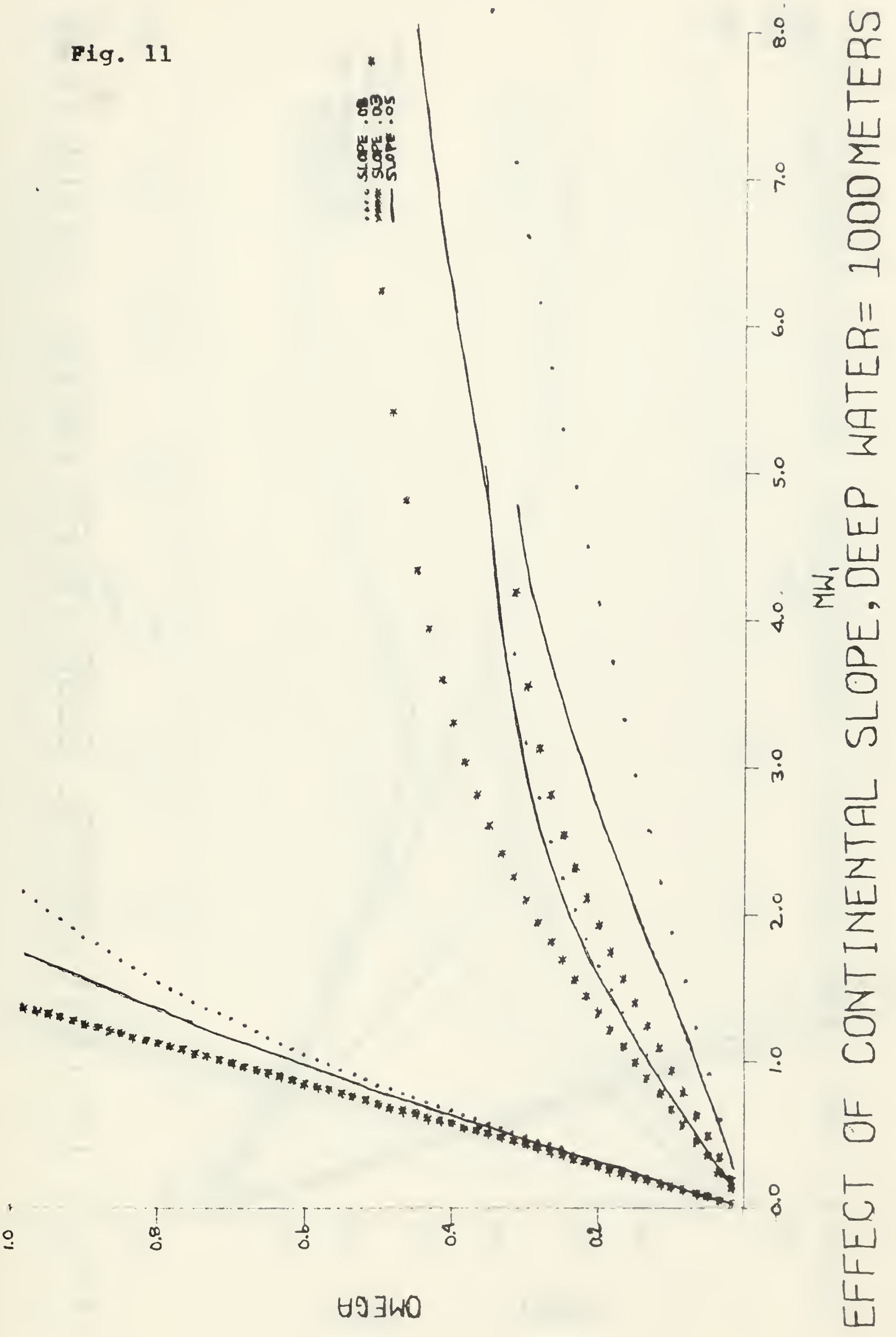
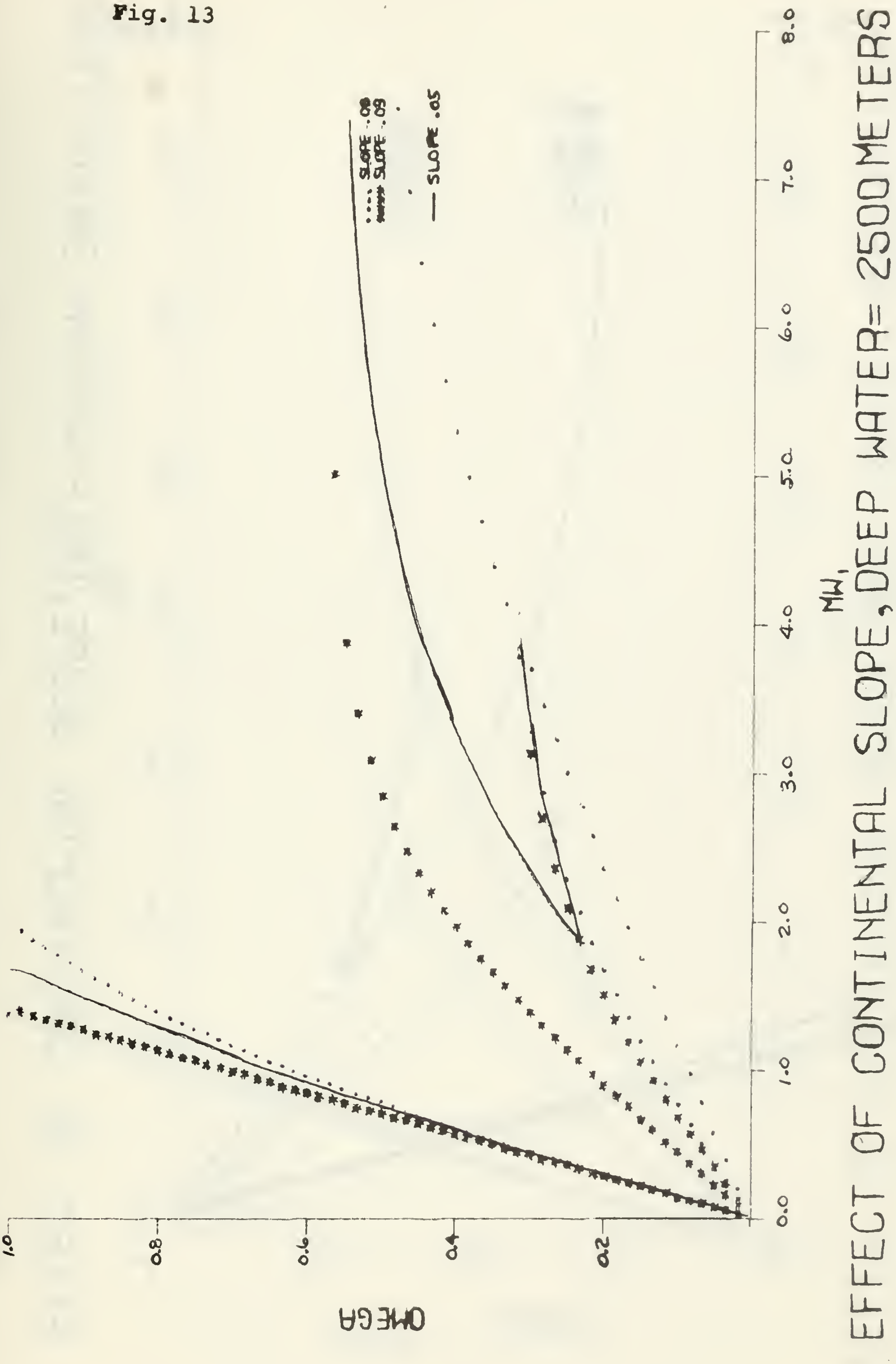

EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 1000METERS
OMEGA
MW1
SLOPE
SLOPE .03
SLOPE .05
0.0
0.2
0.4
0.6
0.8
1.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0

Fig. 12

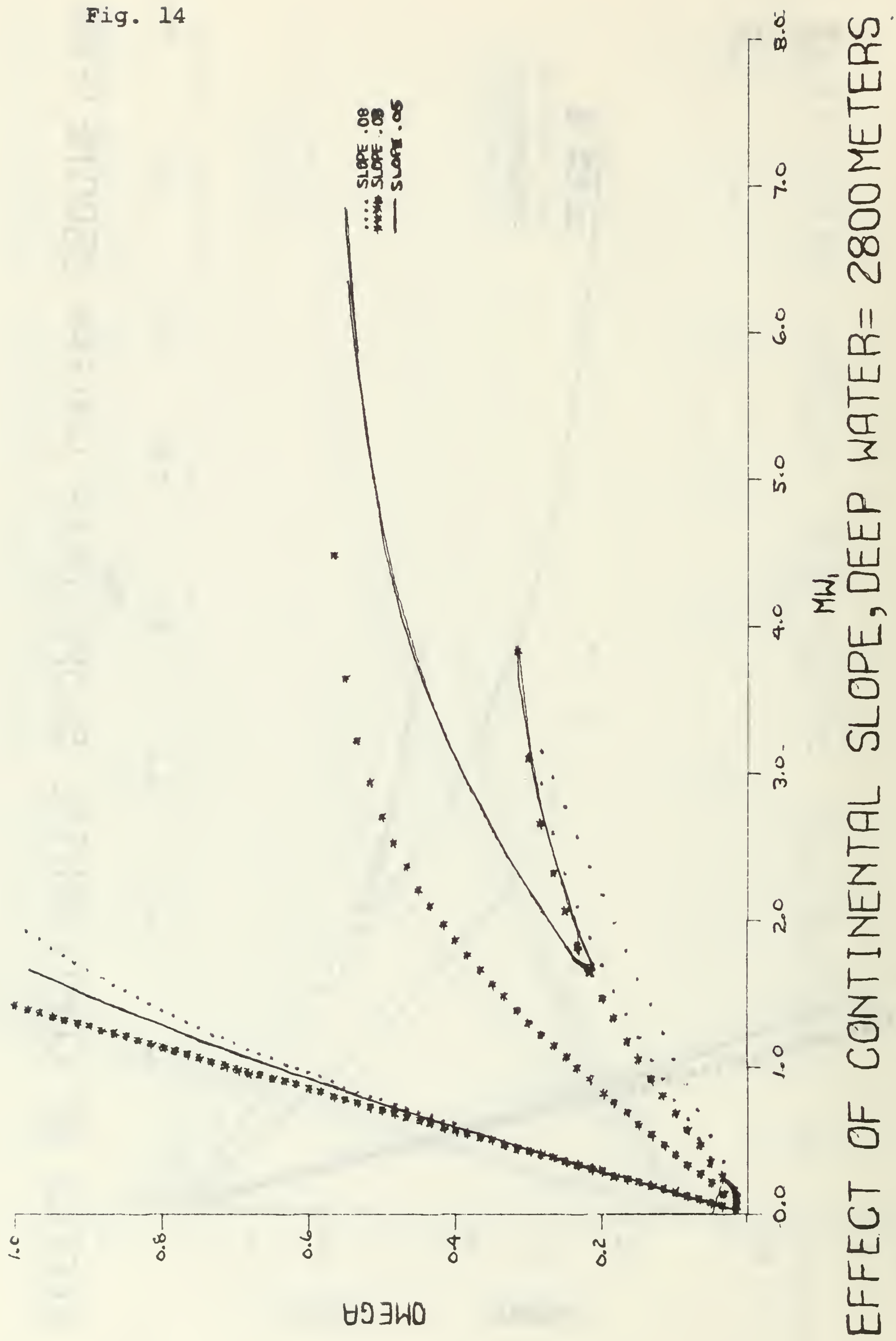
EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 2000METERS

Fig. 13



EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 2500 METERS
OMEGA
MW,
SLOPE .05
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.6
0.8
1.0

Fig. 14



EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 2800 METERS
OMEGA
MW,
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.6
0.8
1.0
SLOPE .08
SLOPE .08
SLOPE .05

Fig. 15

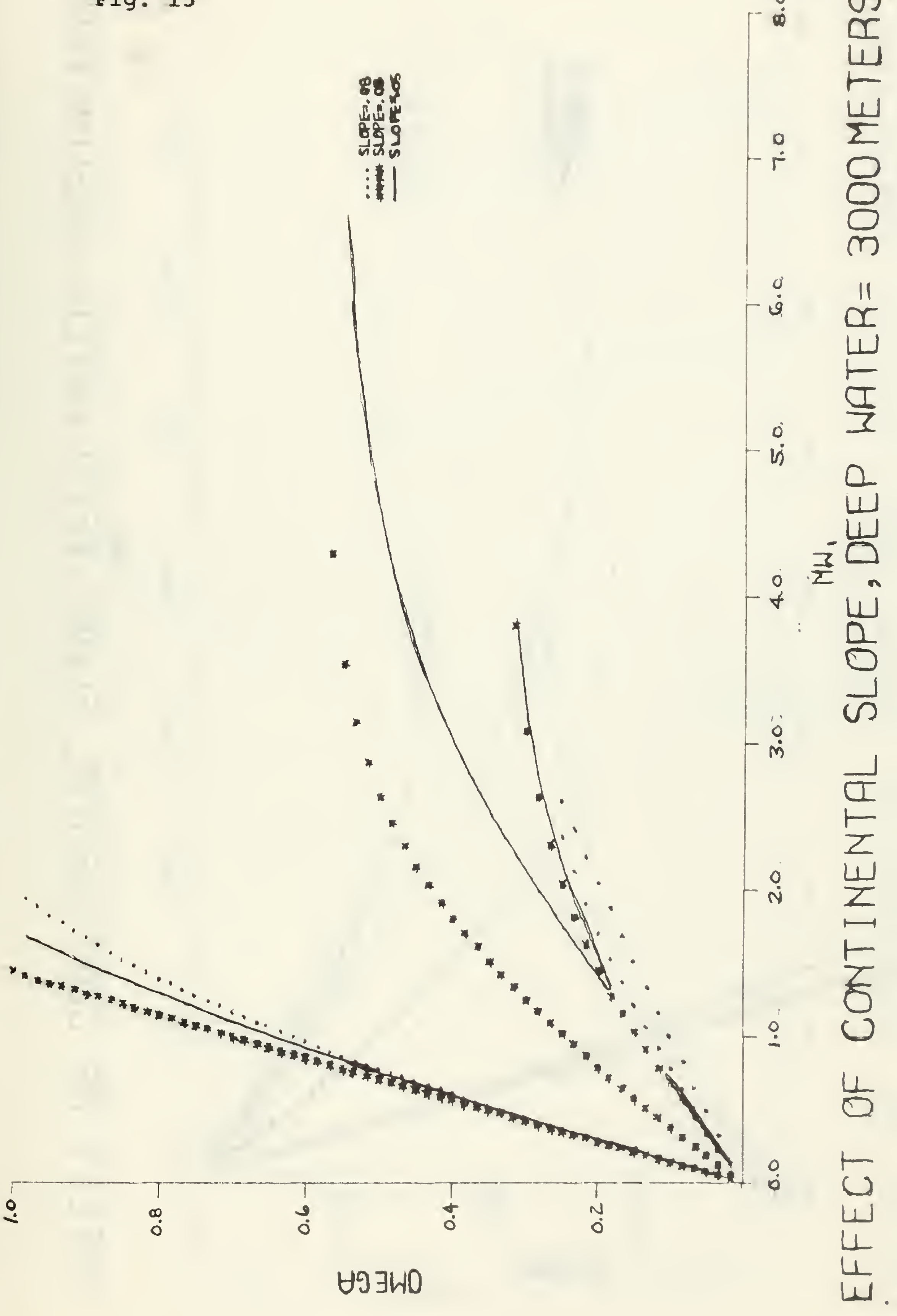
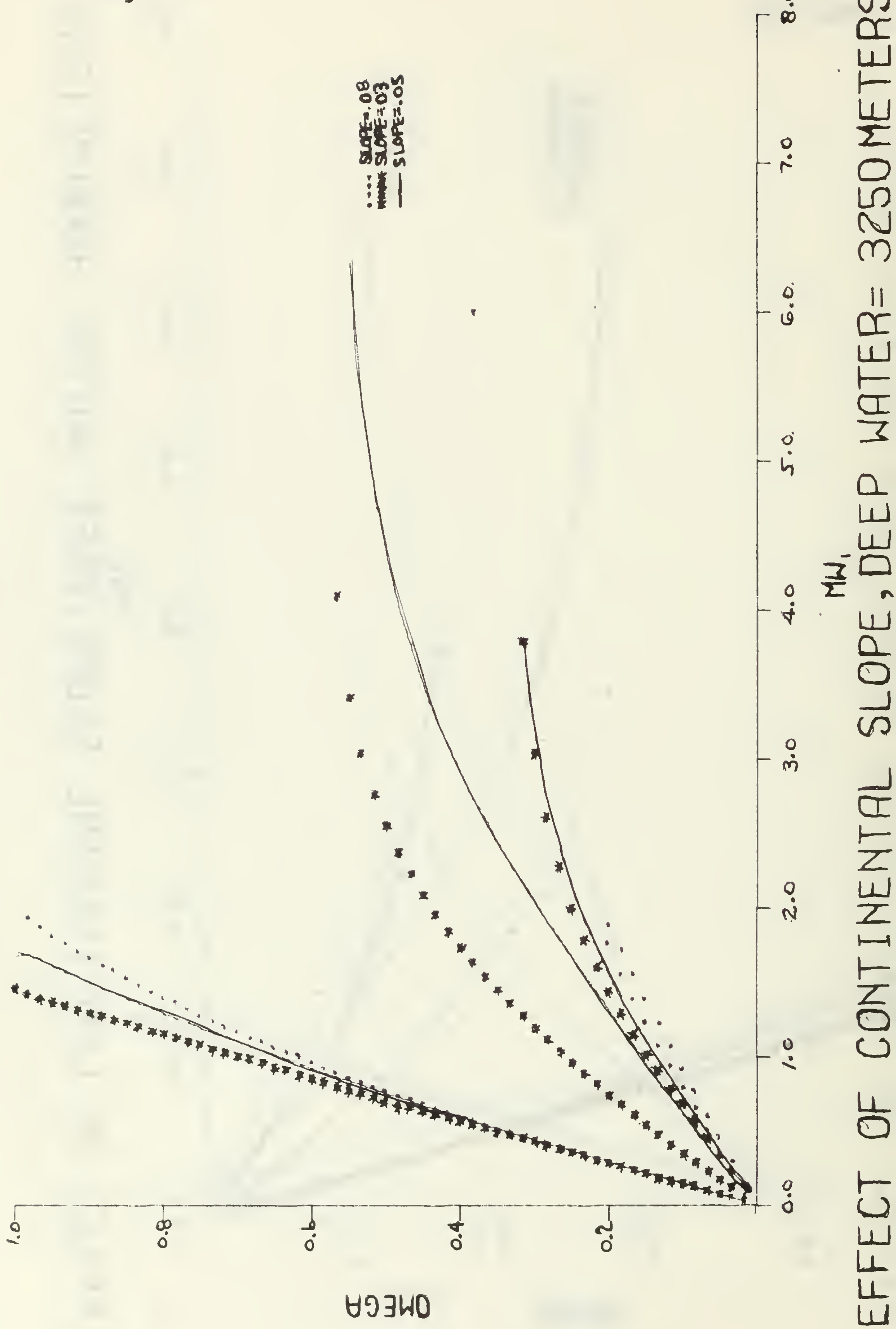

OMEGA
MW
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.6
0.8
1.0
EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 3000METERS

Fig. 16



SLOPE=.08
SLOPE=.03
SLOPE=.05
OMEGA
1.0
0.8
0.6
0.4
0.2
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
MW1
EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 3250 METERS

Fig. 17

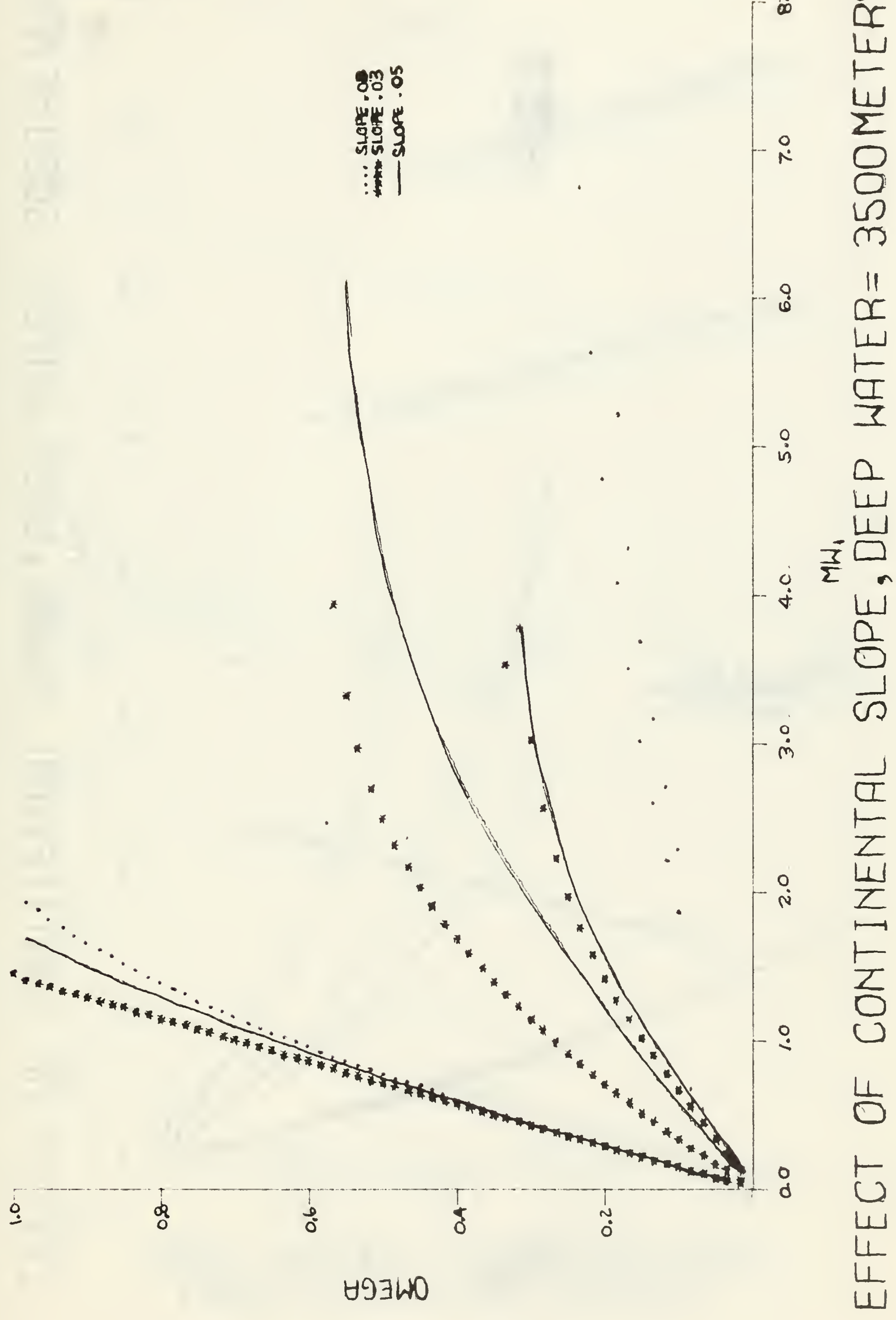

EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 3500METERS
OMEGA
MW,
SLOPE .03
SLOPE .05
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
0.2
0.4
0.6
0.8
1.0

Fig. 18

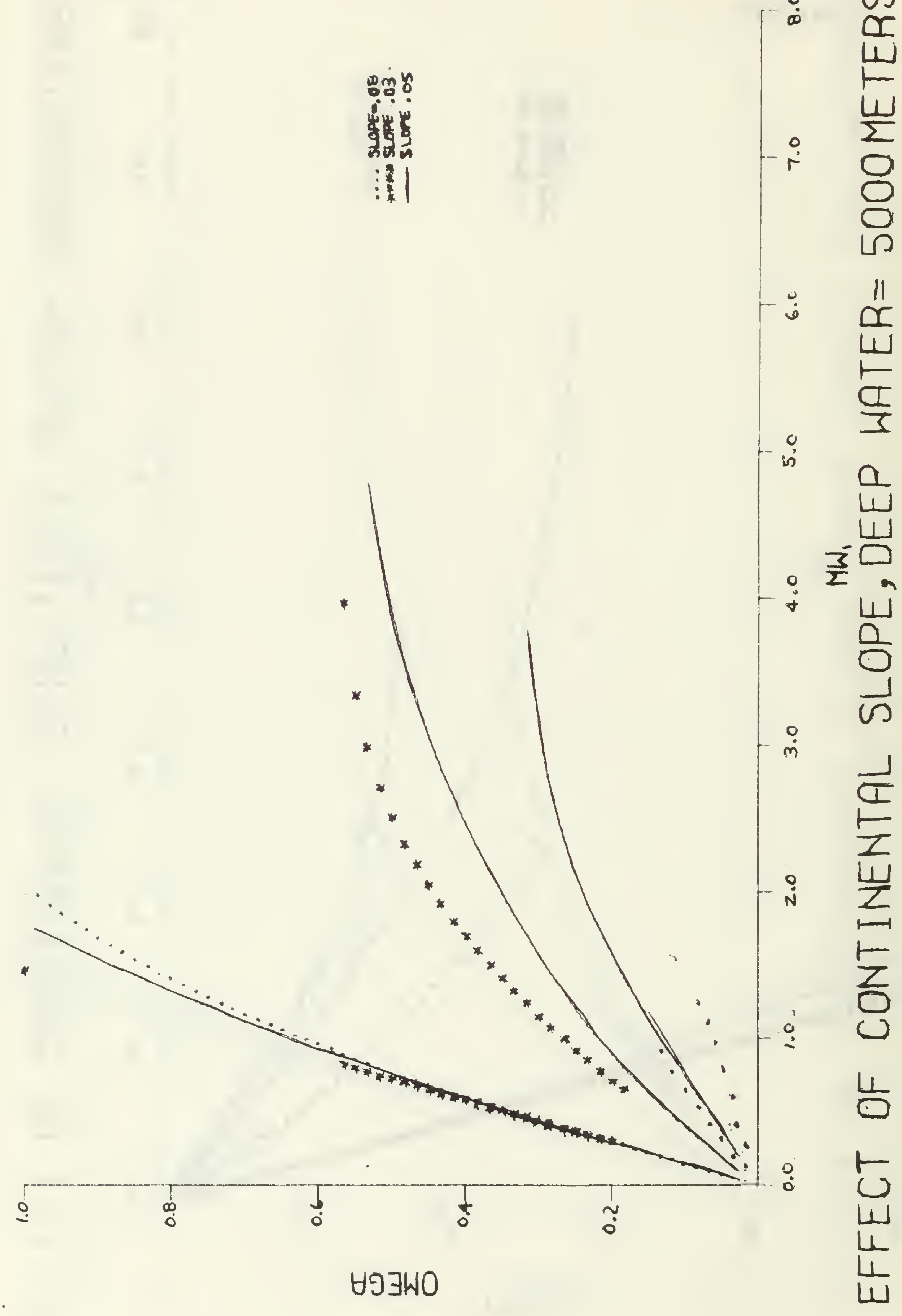
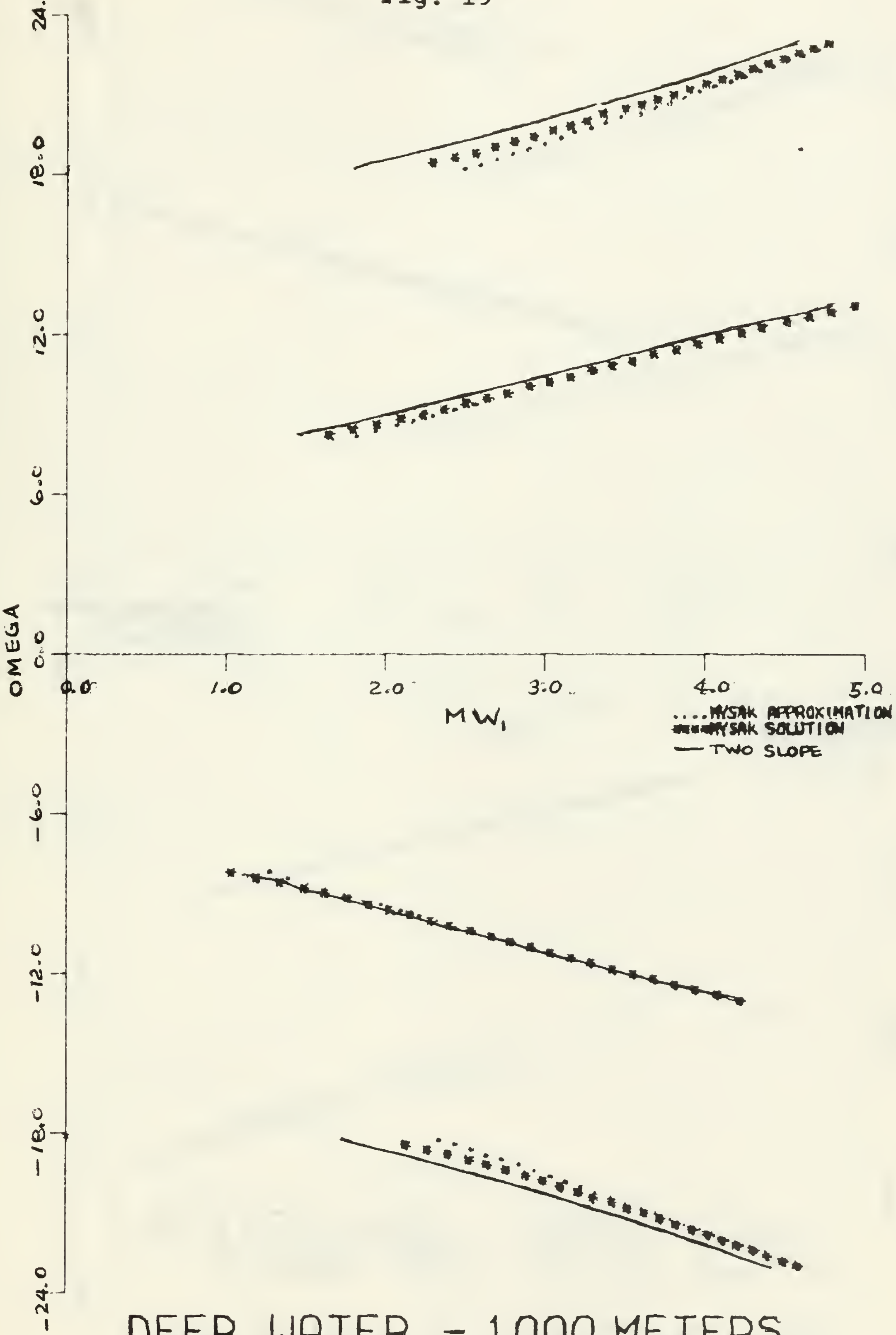
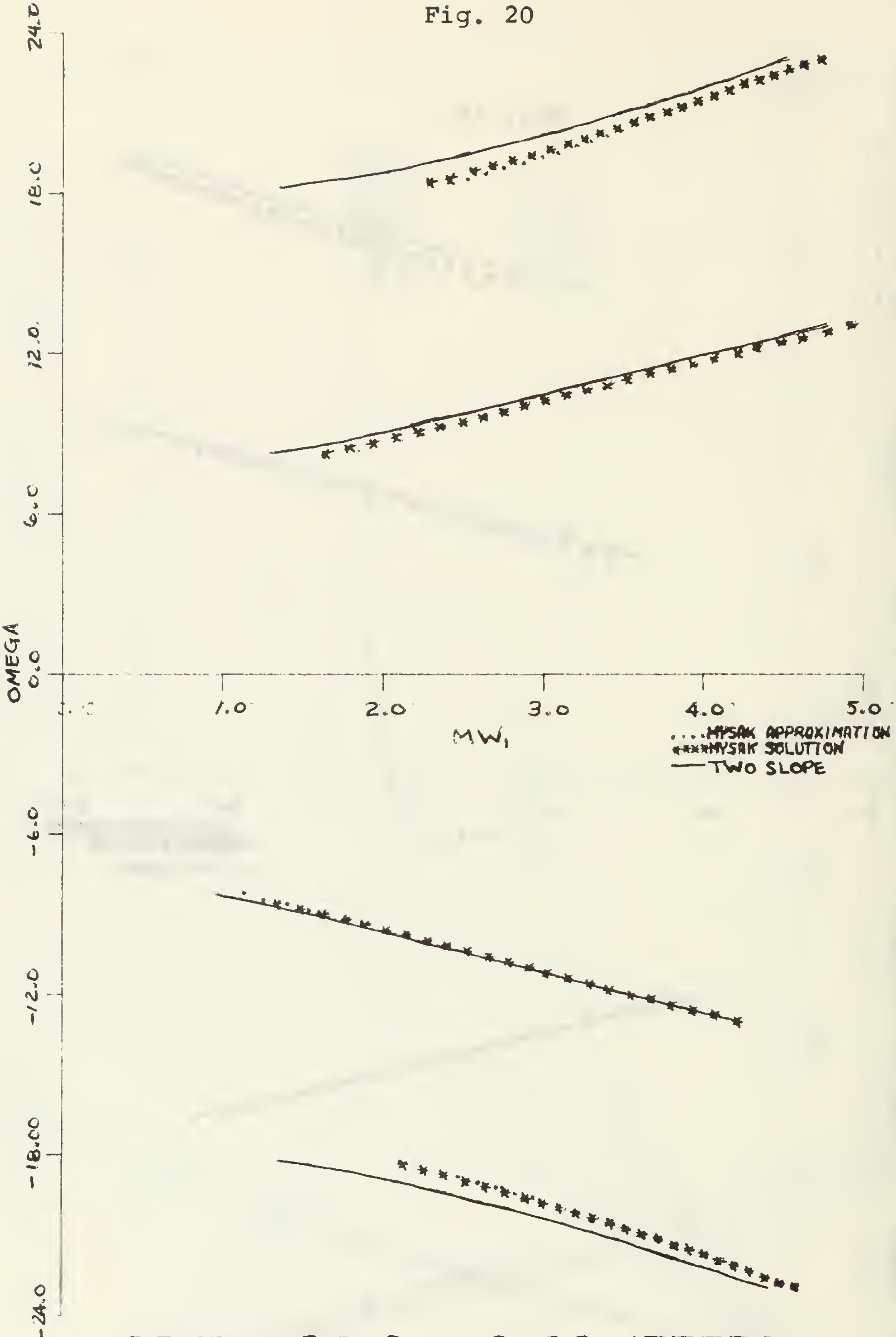

.... SLOPE=.08
**** SLOPE .03
—— SLOPE .05
OMEGA
0.0
0.2
0.4
0.6
0.8
1.0
MW
0.0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
EFFECT OF CONTINENTAL SLOPE, DEEP WATER= 5000METERS

Fig. 19



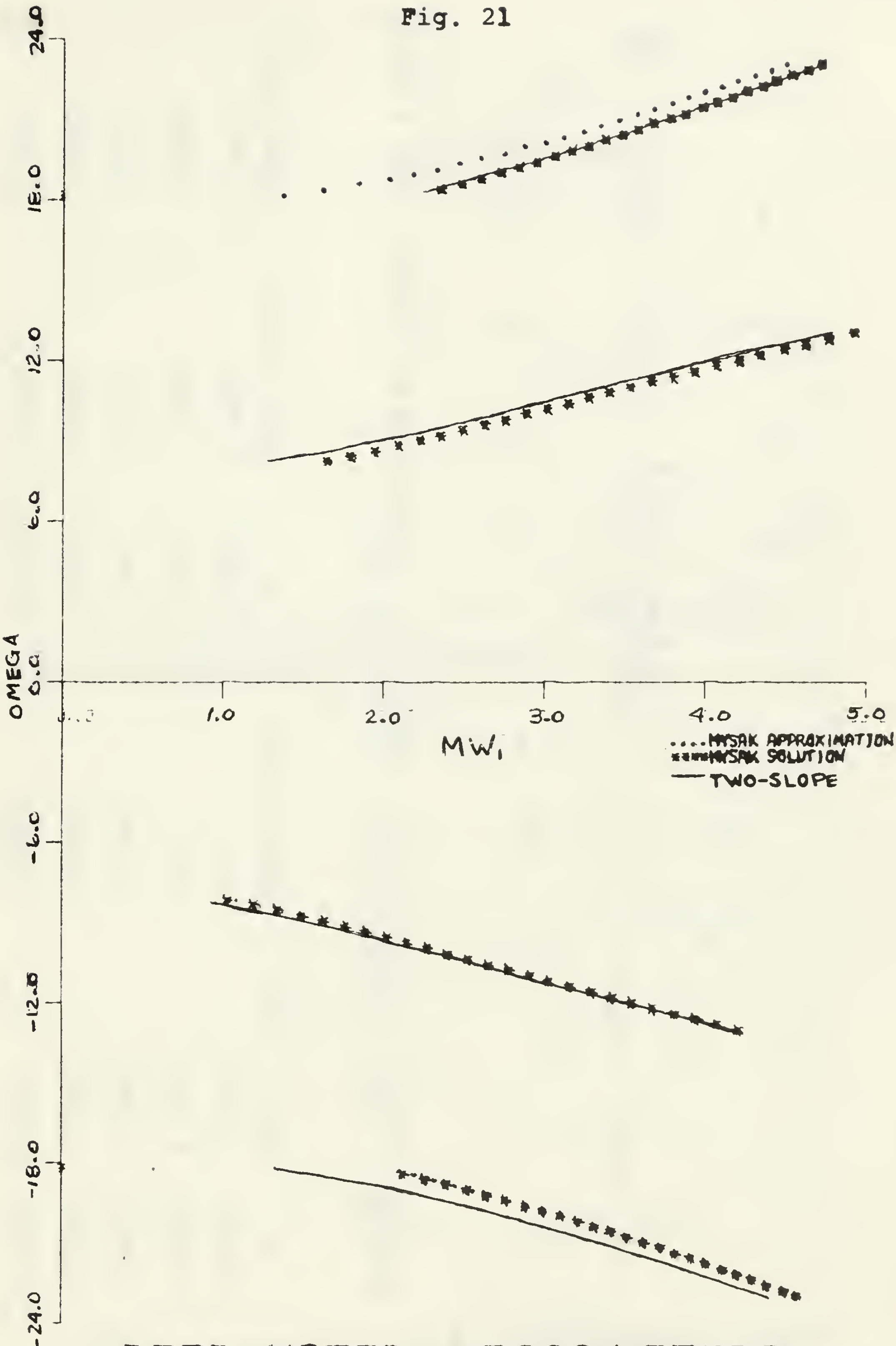
DEEP WATER = 1000 METERS
TWO-SLOPE EDGE WAVES

Fig. 20



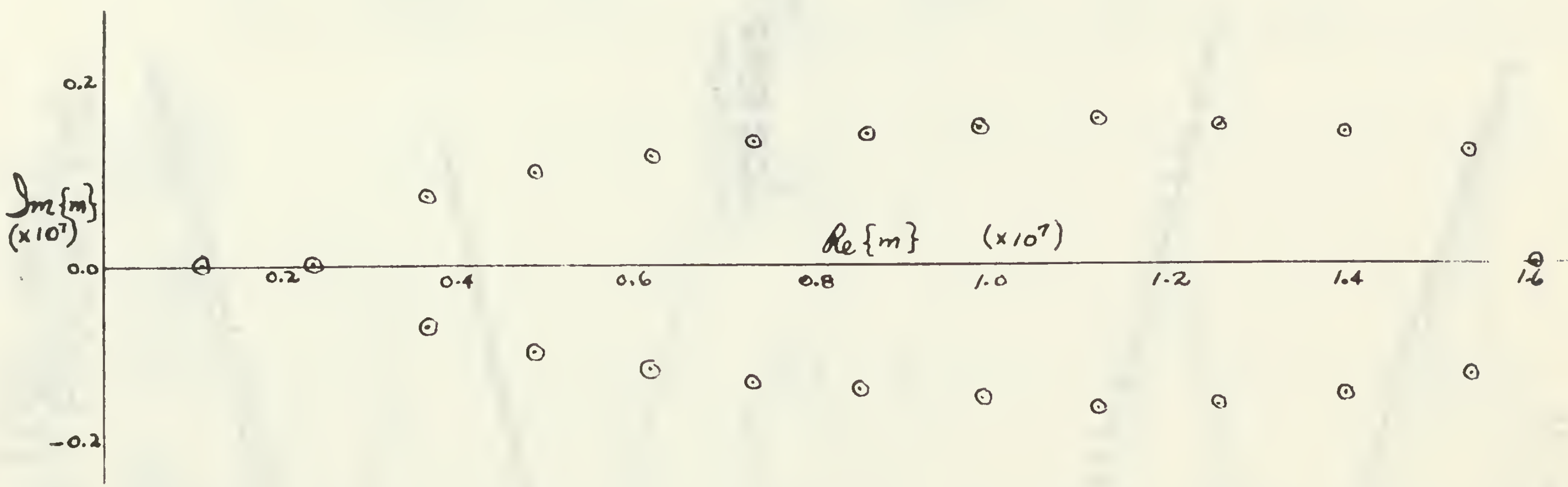
DEEP WATER = 3000 METERS
TWO-SLOPE EDGE WAVES

Fig. 21



DEEP WATER = 5000 METERS
TWO-SLOPE EDGE WAVES

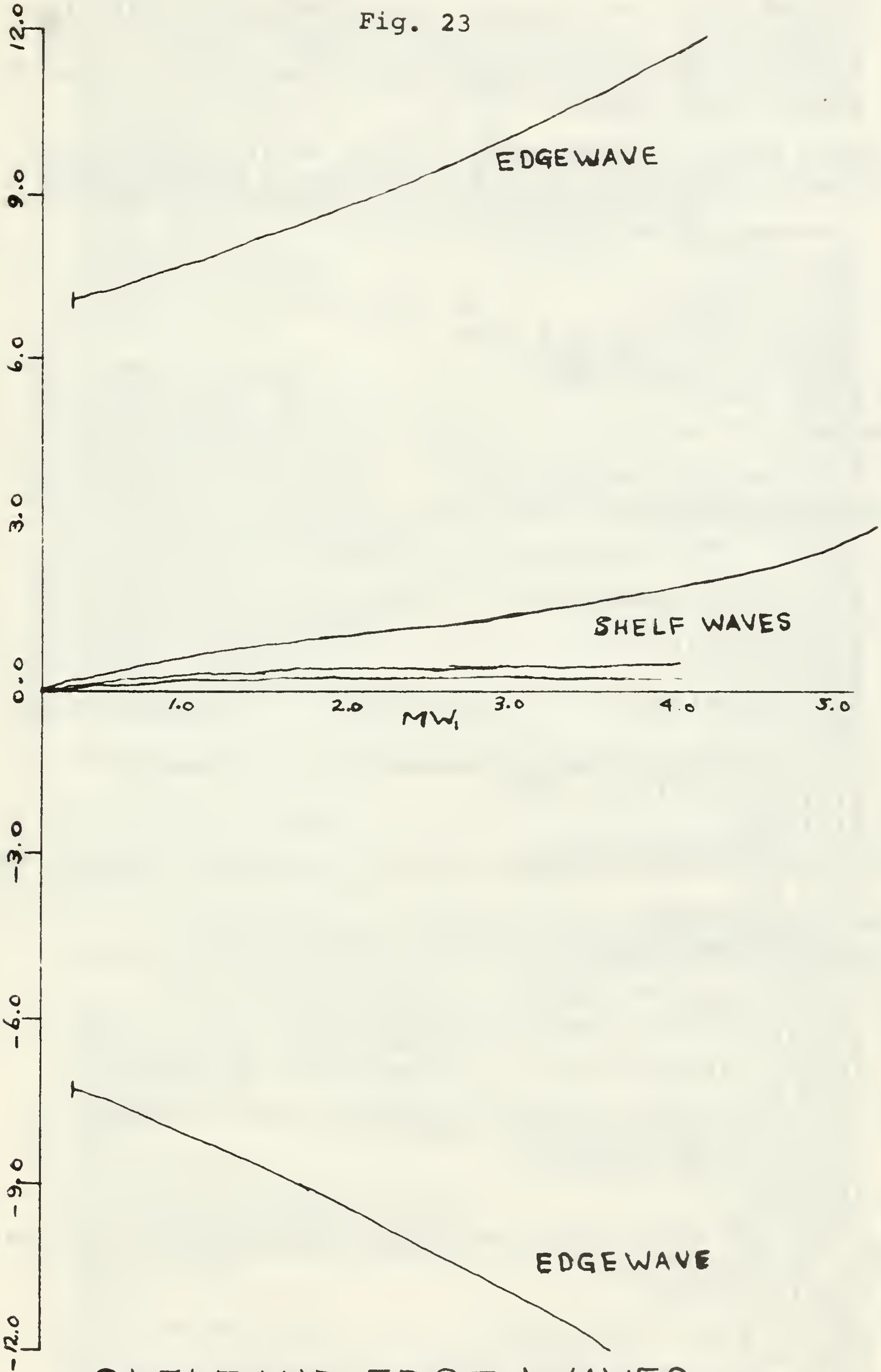
Fig. 22



MODE TWO COMPLEX ROOTS, DEPTH=2800 METERS

ω	$\Re\{m\}$ ($x10^7$)	$\Im\{m\}$ ($x10^7$)	ω	$\Re\{m\}$ ($x10^7$)	$\Im\{m\}$ ($x10^7$)
0.0167	0.1185	0.0	0.1333	0.9927	0.1562
0.0333	0.2396	0.0	0.1500	1.1242	0.1605
0.0500	0.3652	0.0783	0.1667	1.2597	0.1591
0.0667	0.4900	0.1021	0.1833	1.4002	0.1495
0.0833	0.6117	0.1205	0.2000	1.5462	0.1248
0.1000	0.7372	0.1367	0.2167	1.6112	0.0
0.1167	0.8637	0.1477	0.2333	1.7081	0.0

Fig. 23



SHELF AND EDGE WAVES
DEPTH = 5000 METERS, SLOPE = .05

APPENDIX A

NUMERICAL SOLUTION OF ONE-AND TWO-SLOPE MODELS

```
// EXEC FORTCLGP,PARM.FORT='LIST,MAP',REGION.GO=95K,TIME.GO=
//FORT.SYSIN DD *
      REAL*8 L,M,H,COR,SIG,SLO(2),W(2),ANS,X(3),Z(3),P(2),
     1 GRAV,DELTA
      COMMON SLO,M,L,H,COR,SIG,W,X,Z,GRAV, A,KCOL
      DIMENSION AE(2),A(3),ANSI(3000),QM(60),PT(60,8),KLUE(8
      DATA PT,KLUE/480*0.0,8*0/
      READ(5,102) KOMAND

 IF KOMAND EQUALS 0(BLANK CARD IN DATA DECK) THE MYSAK
SOLUTION AND APPROXIMATION ARE COMPUTED. IF KOMAND EQUALS
ANY INTEGER(THROUGH 999) THE 2-SLOPE SOLUTION IS COMPUTED..

      IF(KOMAND.NE.0) GOTO 4
      CALL MYSAK
      GOTO 500
    4 GRAV=9.8D02
      DELTA=0.200D-08
      TWOPI=DARCOS(-1.D0)* 2.D0
      READ(5,52) W,COR,SLO
      WRITE(6,56)W,COR,SLO
   89 WW=W(1)+W(2)
      KOUNT=0
      KTRL=1
      IM=8
      H=W(1)*SLO(1) +W(2)*SLO(2)
      CRIT=TWOPI/H*0.400D-01
      NUM=CRIT/DELTA
       WRITE(6,62)
      KEY=0
    3 DO 2 K=1,60
      M=DFLOAT(KTRL)*DELTA
      IK=1
      CS=FLOAT(K)/6.E01
      SIG=-CS*COR
      QM(K)=CS
      DO 5 J=KTRL,NUM
      LOOK=0
      DO 1 I=1,2
      P(I)=(SIG*SIG-COR*COR)/(GRAV*SLO(I))-COR*M/SIG
      AE(I)=(M-P(I))/(2.D0*M)
    1 A(I)=AE(I)
      A(3)=A(2)
      CALL FINSIG(ANS,KEY)
      IF(KOOL.NE.0) GOTO 100
      GOTO 41
    9 DO 8 MZ=J,500
      MZ1=MZ-1
    8 ANSI(MZ)=ANSI(MZ1)
      GOTO 5
   41 IF(ANS.LE.0.D0) LOOK=1
      ANSI(J)=ANS
   21 IF(L.LE.0.99D-19)ANSI(J)=0.D0
      JJ=J-1
      IF(ANSI(J)*ANSI(JJ).LE.0.D0.AND.J.GT.1) GOTO 30
      GOTO 5
   30 PT(K,IK)=(ANSI(J)/(ANSI(JJ)-ANSI(J))*DELTA+M)*WW
      IF(IK.EQ.1.AND.J.GE.2) KTRL=J-1
       WRITE (6,61) PT(K,IK)
      IF(IK.EQ.IM) GOTO 31
      IK=IK+1
    5 M=M+DELTA
   60 FORMAT(1H ,2HM=,D10.3,2HTO,D10.3,40X,15HCORIOLIS/SIGMA
     *=,D10.3)
  100 DO 32 IMP=IK,IM
   32 KLUE(IMP)=K-1
      IM=IK-1
   31 WRITE (6,60) DELTA,M,CS
    2 CONTINUE
      PT(60,1)=7.5D00
      DO 44 LEAK=1,IM
   44 KLUE(LEAK)=60
```

```
      DO 90 KK=1,8
      MJ=KLUE(KK)
      WRITE(7,71) H,KK,MJ
      WRITE(7,72)(PT(II,KK) ,II=1,MJ)
   90 WRITE(6,92) MJ,(PT(II,KK),II=1,MJ)
      WRITE(6,62)
      CALL PLOTP(PT(1,1),QM,KLUE(1),1)
      DO 33 ISK=2,8
      IF(KLUE(ISK).LE.4) GOTO 34
   33 CONTINUE
   34 ISK1=ISK-1
      DO 35 KRIC=2,ISK1
   35 CALL PLOTP (PT(1,KRIC),QM,KLUE(KRIC),2)
      CALL PLOTP (PT(1,ISK ),QM,KLUE(ISK),3)
      WRITE (6,65)
      READ(5,88) W(2)
      WRITE(6,56) W,COR,SLO
      IF(W(2).NE.0.D0) GOTO  89
  500 STOP

   52 FORMAT(5E10.3)
   53 FORMAT(5E10.3)
   54 FORMAT (1H ,5D20.7)
   55 FORMAT(1H0,12X,1HM,19X,1HL,17X,5HSIGMA,17X,6HANSWER,
     215X,1HH,/,5D20.7)
   56 FORMAT(1H0,5D20.7)
   57 FORMAT(1H ,3D25.12)
   58 FORMAT(1H0,8X,4HP(1),16X,4HP(2),16X,4HA(1),16X,4HA(2),
     416X,4HA(3),/,5D20.9)
   61 FORMAT(17H MIN OCCURS AT M=,E12.4)
   62 FORMAT(1H1)
   63 FORMAT (18H ZERO OCCURS AT M=,E12.4)
   65 FORMAT(1H0,39X,2HMW)
   71 FORMAT(E20.7,2I10)
   72 FORMAT (5E16.7)
   88 FORMAT (D10.3)
   92 FORMAT(1H0,I5,(/,5E20.7))
  102 FORMAT(I3)

      END

      SUBROUTINE MYSAK

THIS PROGRAM SOLVES FOR MYSAK'S SOLUTION AND APPROXIMATION.
FOR 2-SLOPE RETURN TO THE MAIN PROGRAM.

      REAL*8 D,DD,COR,W,GRAV,DEL,OMEGA,LAMDA,DDELTA,ARGU,ANS
     1      ,EPRI,SQT,ANSK,ANSWER,QUANT,M,SIG,DELTA
      DIMENSION QM(60),ANSI(600),ANSJ(600),PT(60,8),KLUE(8),
     1      PTT(60,8),KLU(8)
      DATA PT,PTT,KLUE,KLU/960*0.0,16*0/
      READ (5,52) D,DD,COR, W
      WRITE(6,56) D,DD,COR, W
      WRITE(6,62)
       GRAV=9.8D2
      KOUNT=0
   89 IM=4
      KTRL=1
    3 DO 2 K=1,59
      CS=FLOAT(K)/6.E1
      M=DFLOAT(KTRL)*0.200D-08
      DELTA=0.200D-08
      WRITE(6,60) DELTA,M,CS
      SIG=CS*COR
      IK=1
      IKK=1
      DO 5 J=KTRL,500
      JJ=J
      IF(JJ.EQ.500.AND.IK .LE.IM) GOTO 30
      IF(JJ.EQ.500.AND.IKK.LE.IM) GOTO 31
      LOOK=0
```

```
      DELL=DELTA
      QM(K)=CS
      KEY=0
      A=0.5D0-(COR/SIG+(SIG*SIG-COR*COR)/(GRAV*D/W*M))/2.D0
      DEL=D/DD
      DDELTA= COR*COR*W*W/(GRAV*D)
      LAMDA=M*W
      OMEGA=SIG/COR
      ARGU=2.D0*LAMDA
      CALL DLAP(ANS,ARGU,A,NN)
      CALL DRDLAP(A,ARGU,FPRI)
      QUANT=1.0D0+(DDELTA*DEL*(1.0D0-(OMEGA*OMEGA))/(LAMDA*
     1   LAMDA)
      SQT=DSQRT(QUANT)
      ANSWER=(SQT-(1.0D0/OMEGA)-DEL*(1.0D0-1.0D0/OMEGA))*ANS
     1   +2.0D0*DEL*FPRI
   16 ANSI(J)=ANS
   18 ANSJ(J)=ANSWER
      IF(IK.GT. IM.AND.IKK.GT.IM) GOTO 2
      JJJ=JJ-1
      IF(J.GE.2.AND.ANSI(JJ)*ANSI(JJJ).LE.0.D0) GOTO 11
      IF(J.GE.2.AND.ANSJ(JJ)*ANSJ(JJJ).LE.0.D0) GOTO 12
      GOTO 5
   11 DIF=M+(ANSI(JJ)/(ANSI(JJJ)-ANSI(JJ))*DELTA)
      IF(IK.EQ.1.AND.JJ.GT.2) KTRL=JJ-2
      PT(K,IK)= DIF
      IK=IK+1
      DIF1=M+(ANSJ(JJ)/(ANSJ(JJJ)-ANSJ(JJ))*DELTA)
      IF(ANSJ(JJ)*ANSJ(JJJ).GT.0.D0)GOTO 13
      PTT(K,IKK)= DIF1
      IKK=IKK+1
      WRITE(6,65) DIF,DIF1
      GOTO 1
   13 WRITE(6,64) DIF
      GOTO 1
   12 DIF1=M+(ANSJ(JJ)/(ANSJ(JJJ)-ANSJ(JJ))*DELTA)
      PTT(K,IKK)= DIF1
      IKK=IKK+1
      WRITE(6,66) DIF1
    1 IF(IK.GT.IM.AND.IKK.GT.IM) GOTO 2
    5 M=M+DELTA
      GOTO 2
   30 DO 32 IMP=IK, IM
   32 KLUE(IMP)=K-1
      IF(JJ.EQ.500.AND.IKK.LE.IM) GOTO 31
      GOTO 34
   31 DO 33 IMK=IKK, IM
   33 KLU(IMK) =K-1
   34 IF(IK.LT.IKK) IKK=IK
      IM=IKK-1
    2 CONTINUE
      DO 14 KIN=1,IM
      KLU(KIN)=59
   14 KLUE(KIN)=59
      DO 105  MZZ=1,4
      MJ=KLUE(MZZ)
      WRITE(7,700)DD,MZZ,MJ
      WRITE(7,701) (PT (II,MZZ),II=1,MJ)
      MJ=KLU(MZZ)
      WRITE(7,700) DD,MZZ,MJ
      WRITE(7,701) (PTT(II,MZZ),II=1,MJ)
      DO 300 KPR=1,60
      PT(KPR,MZZ)=PT(KPR,MZZ)*W
  300 PTT(KPR,MZZ)=PTT(KPR,MZZ)*W
      MJ=KLUE(MZZ)
      WRITE(7,700) DD,MZZ,MJ
      WRITE(7,701) (PT (II,MZZ),II=1,MJ)
      MJ=KLU (MZZ)
      WRITE(7,700) DD,MZZ,MJ
      WRITE(7,701) (PTT(II,MZZ),II=1,MJ)
  105 CONTINUE
    6 DO 90  KK=1,8
```

```
      MJ=KLUE(KK)
      MJJ=KLU(KK)
      WRITE(6,92) MJJ,(PTT(II,KK),II=1,MJJ)
   90 WRITE(6,92) MJ,(PT(II,KK),II=1,MJ)
      WRITE(6,71)
      CALL PLOTP(PTT(1,1),QM,KLU(1),1)
      DO 35 MLK=2,7
      CALL PLOTP(PTT(1,MLK),QM,KLU(MLK),2)
   35 CONTINUE
      CALL PLOTP(PTT(1,8),QM,KLU(8),3)
      WRITE(6,72)
      WRITE(6,73)
      CALL PLOTP(PT(1,1),QM,KLUE(1),1)
      DO 36 MOP=2,7
      CALL PLOTP(PT(1,MOP),QM,KLUE(MOP),2)
   36 CONTINUE
      CALL PLOTP(PT(1,8),QM,KLUE(8),3)
      WRITE(6,72)
      READ(5,88) DD
      WRITE(6,56) D,DD,COR,W
      IF(DD.NE.0.D0) GOTO 89
      RETURN

   52 FORMAT(4E10.3)
   53 FORMAT(5E10.3)
   54 FORMAT (1H ,5D20.7)
   55 FORMAT(1H0,12X,1HM,19X,1HL,17X,5HSIGMA,17X,6HANSWER,15
     *5D20.7)
   56 FORMAT(1H0,4D20.7)
   57 FORMAT(1H ,3D25.12)
   58 FORMAT(1H0,8X,4HP(1),16X,4HP(2),16X,4HA(1),16X,4HA(2),
     4/,5D20.9)
   60 FORMAT(3H M=,D10.3,2HTO,D10.3,40X,15HSIGMA/CORIOLIS=,
             D10.3)
   62 FORMAT(1H1)
   64 FORMAT(35H THE APPROXIMATION HAS A ROOT AT M=,E15.6,40
             H.THE EQUATION DOES NOT HAVE A ROOT HERE.)
   65 FORMAT(35H THE APPROXIMATION HAS A ROOT AT M=,E15.6,27
             H.THE EQUATION HAS A ROOT AT,E15.6)
   66 FORMAT(27H THE EQUATION HAS A ROOT AT,E15.6,49H.THE AP
             PROXIMATION DOES NOT HAVE A SOLUTION HERE.)
   71 FORMAT(1H1,27X,26HPLOT OF MYSAK'S ENTIRE EQN)
   72 FORMAT(1H0,40X,2HMW)
   73 FORMAT(1H1,25X,29HPLOT OF MYSAK'S APPROXIMATION)
   88 FORMAT(D10.3)
   92 FORMAT(1H0,I5,(/,5E20.7))
  700 FORMAT(E20.7,2I10)
  701 FORMAT(5F12.4)

      END


      SUBROUTINE FINSIG(ANS,KEY)
      REAL*8 L,M,X(3),F(3),FPRI(3),G(3),GPRI(3),C,H,GRAV,COR
     ,SIG,W(2), SLO(2),ANS,Z(3)
      COMMON SLO,M,L,H,COR,SIG,W,X,Z,GRAV, A,KOOL
      DIMENSION A(3)
       KOOL=0
      C=(M*M*GRAV*H+COR*COR-SIG*SIG)/(GRAV*H)
      IF (C.LE.0.D0)GOTO 11
      L=DSQRT(C)
      X(1)=W(1)
      X(2)=SLO(1)*W(1)/SLO(2)
      X(3)=X(2)+W(2)
      DO 2 I=1,3
      Z(I)=X(I)*2.D0*M
      CALL DLAP(F(I),Z(I),A(I),NN)
      CALL DLAPGG(F(I),Z(I),G(I),A(I))
      CALL DRDLAP(A(I),Z(I),FPRI(I))
      CALL DGDLAP(A(I),Z(I),FPRI(I),G(I),GPRI(I))
      IF(F(I).GE.0.1D30.OR.G(I).GE.0.1D30.OR.FPRI(I).GE.
     50.1D30.OR.GPRI(I).GE.0.1D30) GOTO 4
```

```
    2 CONTINUE
      ANS=(L-M)*(F(3)*(F(1)*GPRI(2)-G(2)*FPRI(1))+G(3)*(F(2)
     2*FPRI(1)-F(1)*FPRI(2)))+2*M*(FPRI(3)*(F(1)*GPRI(2)-
     3G(2)*FPRI(1))+GPRI(3)*(F(I)*FPRI(1)-F(1)*FPRI(2)))
      IF(KEY.EQ.1) WRITE(6,51) ANS,M,L
      IF(ANS.GE.0.1D04) GOTO 4
    1 RETURN
   11 L=9.99D-20
      GOTO 1
    4 KOOL=1
      WRITE(6,61) M
      GOTO 1

   51 FORMAT(1H ,3D20.7)
   61 FORMAT(34H0SOLUTION DOES NOT EXIST BEYOND M=,E12.4)

      END


      SUBROUTINE DLAP(Y,X,A,NN)

DLAP SOLVES FOR LAGUERRE FUNCTIONS OF THE FIRST KIND.GENER-
ALLY THE ARGUMENT,A,WOULD BE EXPECTED TO BE NEGATIVE.WHEN A
IS POSITIVE,NN IS SET TO 1.

      REAL*8 Y,X,YY,YX,YZ,YK,YN
      YN=A
      YY=1.D0
      YK=1.D0
      YX=1.D0
      YZ=1.D0
      IF(A) 3,1,2
    2 NN=1
    3 YX=X/YZ*YN/YZ*YX
      YY=YY+YX
      YN=YN+1.D0
      YZ=YZ+1.D0
      IF(DABS(YX).GT.0.50E-07) GOTO 3
    1 Y=YY
      RETURN

   50 FORMAT(1H ,3(5X,E14.7))
   51 FORMAT (1H ,5D20.7)

      END


      SUBROUTINE DLAPGG(Y,X,DL,A)

DLAPGG SOLVES FOR LAGUERRE FUNCTIONS OF THE SECOND KIND.WHEN
A APPROACHES A NEGATIVE INTEGER,CONTROL IS TRANSFERRED TO
DLAPG.WHEN CONVERGENCE DOES NOT TAKE PLACE BEFORE AN ANSWER
IS OBTAINED,CONTROL IS RETURNED TO THE CALLING ROUTINE AND A
MESSAGE PRINTED:WHEN X=0, THIS FUNCTION IS INDETERMINATE.

      REAL*8 DN,DQ,DX,D2,X,Y,DL,DA
      KOUNT=0
      IF(X.EQ.0) GOTO 6
      DN=1.D0
      DQ=1.D0
      DX=1.D0
      DL=Y*DLOG(X)
      DA=A
      D2=0.D0
    4 DX=DX*DA/DN*X/DN
      D2=D2+(1.D0/DA)-(2.D0/DN)
      DQ=DX*D2
      DL=DL+DQ
      DA=DA+1.D0

1/DA APPROACHING INFINITY?
```

```
      IF(DABS(DA).LE.0.1D-02) GOTO 1
C
C ANSWER TOO LARGE WITHOUT CONVERGENCE?
C
      IF (DABS(DX).GE.1.D65.OR.DABS(DX).LE.1.D-65) GOTO 10
C
C CONVERGENCE?
      IF(DABS(DQ).LE.0.5D-08.AND.KOUNT.GT.2) GOTO 3
      DN=DN+1.D0
      KOUNT=KOUNT+1
      GOTO 4
    1 CALL DLAPG(Y,X,DL,A)
      GOTO 3
    2 WRITE(6,54)KOUNT
    3 RETURN
   10 WRITE(6,57) KOUNT,DX,DL
      GOTO 2
    6 WRITE(6,53)
      GOTO 3

   53 FORMAT(59H X IS 0.LAGUERRE FUNCTION OF THE SECOND KIND
     *          DOES NOT EXIST)
   54 FORMAT(1H ,100X,I3)
   55 FORMAT(1H ,6D20.7,/,D20.7)
   57 FORMAT(29H  OVERFLOW APPROACHING.AFTER,I3,15H ITER-
     *          ATIONS,DX=,E12.5,9H AND DL=,E12.5)

      END


      SUBROUTINE DLAPG(Y,X,DL,A)

C DLAPG SOLVES FOR LAGUERRE FUNCTIONS OF THE SECOND KIND WHEN
C A IS A NEGATIVE INTEGER.TO AVOID DIVIDING BY ZERO,THE NTH
C TERM IS THE SUM OF N-1 MULTIPLICATIONS.

      REAL*8 Y,X,DA,ZZ,DZ,D1,D2,DN,DL,D11,DX,DQ
      KOUNT=0
      DN=1.D0
      DQ=1.D0
      DX=1.D0
      DL=Y*DLOG(X)
      DA=A
      DZ=1.D0
      D2=0.D0
   22 DX=DX*X
       D11=0.D0
      DO 1 I=0,KOUNT
       D1=1.D0
      KOOK=I+1
      DO2 K=0,I
      IF(K.EQ.I) GOTO 4
      IF(DABS(D1).LE.0.5D-10) GOTO 1
    2 D1=D1*(DA+DFLOAT(K))
    4 IF(KOOK.GT.KOUNT) GOTO 1
      DO 3 L=KOOK,KOUNT
      IF(DABS(D1).LE.0.5D-10) GOTO 1
    3 D1=D1*(DA+DFLOAT(L))
    1 D11=D11+D1
       D2=D2-(2.D0/DN)
      DQ=DQ*DN*DN
      IF(DQ. GE.0.1D40) GOTO 56
      ZZ=(D11 +D1*(DA+DFLOAT(KOUNT))*D2)*DX/DQ
      KOUNT=KOUNT+1
      IF(KOUNT.GT.25) GOTO 56
      DL=DL+ZZ
      IF(DABS(ZZ).LE.0.5D-08) GOTO 21
      DN=DN+ 1.D0
      GOTO 22
   21 RETURN
   56 WRITE(6,53) ZZ
      GOTO 21
```

```
   51 FORMAT(1H ,4(5X,E15.8))
   53 FORMAT(36H G(X) DID NOT CONVERGE. LAST TERM WAS,E15.8)

      END

       SUBROUTINE DRDLAP(A,X,RESULT)

DRDLAP SOLVES THE FIRST DERIVATIVE OF THE LAGUERRE FUNCTION
OF THE FIRST KIND.

      REAL*8 YDA,YDD,RESULT,YDB,YDN,X
      YDA=A+1.D0
      RESULT=A
      YDN=1.D0
      YDD=A
   22 YDB=YDN+1.D0
      YDD=YDD*YDA*X/ (YDN*YDB)
      RESULT=RESULT+YDD
      IF(DABS(YDD).LT.0.5D-8) GOTO 21
      YDA=YDA+1.D0
      YDN=YDN+1.D0
      GOTO 22
   21 RETURN

   23 FORMAT(1H ,2(5X,E15.8))
   54 FORMAT (1H ,6D20.7)

      END

      SUBROUTINE DGDLAP(A,X,RESULT,PLG,RESUL)

DGDLAP SOLVES THE FIRST DERIVATIVE OF THE SECOND KIND OF
LAGUERRE FUNCTION.WHEN X=0, THIS FUNCTION DOES NOT EXIST.

      REAL*8 X,RESUL,RESULT,XA,PLG,XN,X3,X4,X2,XX,XZ,XO
      IF(X.EQ.0) GOTO 10
      XN=2.D0
      XA=A
      RESUL=RESULT* DLOG(X)+PLG/X +1.D0-(2.D0*XA)
       X2=1.D0/XA-2.D0
      XX=XA
   21 XA=XA+1.D0
      IF(DABS(XA).LE.0.5D-4) GOTO 22
      XX=XX*X/(XN*XN-XN)*XA
      X2=X2 -2.D0/XN +1.D0/XA
      XZ=XX*X2
      RESUL=RESUL +XZ
      IF(DABS(XZ).LE.0.5D-8) GOTO 10
      XN=XN +1.D0
      IF (DABS(XZ).GE.1.D65) GOTO 23
      GO TO 21
   22 KISS=1
       XN=2.D0
      XA=A
      RESUL=RESULT*DLOG(X) + PLG/X +1.D0-(2.D0*XA)
      XO=1.D0
      XX=1.D0
      X2=-2.D0
    5 XX=XX*X
      X3=0.D0
      DO 6 II=0,KISS
      X4=1.D0
      KIN=KISS+1
      DO 7 JJ=0,II
      IF(JJ.EQ.II) GOTO 9
      IF(DABS(X4).LE.0.5D-10) GOTO 6
    7 X4=X4*(XA+DFLOAT(JJ))
    9 IF(KIN.GT.KISS) GOTO 6
      DO 8 LL=KIN,KISS
```

```
      IF(DABS(X4).LE.0.5D-10) GOTO 6
    8 X4=X4*(XA+DFLOAT(LL))
    6 X3=X3+X4
      X2=X2-(2.D0/XN)
      XQ=XQ*XN*DFLOAT(KISS)
      XZ=(X3+X4*(XA+DFLOAT(KISS))*X2)*XX/XQ
      KISS=KISS+1
      RESUL=RESUL+XZ
      IF(DABS(XZ).LE.0.5D-8) GOTO 10
      XN=XN+1.D0
      IF(KISS.LT.25) GOTO 5
      WRITE(6,54) XZ
   23 N=XN
      WRITE(6,24) N
   10 RETURN

   24 FORMAT(40H OVERFLOW ABOUT TO OCCUR IN DGDLAP AFTER,I3,
     2 7H TERMS.)
   51 FORMAT(1H ,4(5X,E15.8))
   54 FORMAT(1H ,36HG'(X) DID NOT CONVERGE.LAST TERM WAS,
     1E15.8)

      END
//GO.FT06F001 DD DCB=(RECFM=FA,BLKSIZE=133),SPACE=(CYL,(15,1
//GO.SYSIN DD *
```

APPENDIX B

NUMERICAL SOLUTION FOR TWO-SLOPE COMPLEX ROOTS

```
// EXEC FORTCLGP,PARM.FORT='LIST,MAP',REGION.GO=100K,TIME.GO
//FORT.SYSIN DD *

      COMPLEX*16 M,L,ANS,X(3),Z(3),P(2),A(3),AE(2),ANSI(1620
      REAL*8 H,COR,SIG,SLO(2),W(2),GRAV,PAR1,PAR2,ANSWER(162
      COMMON M,L,X,Z,A,H,COR,SIG,SLO,W,GRAV,KOOL
      DATA ANSI/1620*(0.E0,0.E0)/
      THIS PROGRAM SOLVES FOR THE COMPLEX ROOTS OF THE TWO
      SLOPE MODEL. IN THIS CASE,THE ROOT WAS  FOUND FOR OM-
      EGA=12/60 AT J= 10 AND K= 30.THIS GIVES AN ANSWER OF
      (1.5459E-07,0.1249E-07) FOR THE ROOT.
       GRAV=9.8D2
      READ(5,52) W,COR,SLO
      WRITE(6,56)W,COR,SLO
   89 WW=W(1)+W(2)
      H=W(1)*SLO(1) +W(2)*SLO(2)
      WRITE(6,100)
      PAR1=1.541234E-07
      SIG=-12.00*COR/6.00E01
      DO 2 J=1,20
      PAR1=PAR1 + 0.500D-10
      M=PAR1 *(1.D0,0.D0)
      DO 1 I=1,2
      P(I)=(SIG*SIG-COR*COR)/(GRAV*SLO(I))-COR*M/SIG
      AE(I)=(M-P(I))/(2.D0*M)
    1 A(I)=AE(I)
      A(3)=A(2)
      CALL FINSIG(ANS,KEY)
      ANSI(J)=ANS
      ANSWER(J)=CDABS(ANSI(J))
      DO 2 K=221,260
      PAR2= DFLOAT(K)* 0.500D-10
      M=PAR1*(1.D0,0.D0) + PAR2* (0.D0,1.D0)
      DO 8 I=1,2
      P(I)=(SIG*SIG-COR*COR)/(GRAV*SLO(I))-COR*M/SIG
      AE(I)=(M-P(I))/(2.D0*M)
    8 A(I)=AE(I)
      A(3)=A(2)
      CALL FINSIG(ANS,KEY)
      KOFE=(K-220)*20 + J

      PRINTS ANSI(21)- ANSI(820)

      ANSI(KOFE)=ANS
      ANSWER(KOFE)=CDABS(ANSI(KOFE))
      M=M-(2.D0*PAR2*(0.D0,1.D0))
      DO 9 I=1,2
      P(I)=(SIG*SIG-COR*COR)/(GRAV*SLO(I))-COR*M/SIG
      AE(I)=(M-P(I))/(2.D0*M)
    9 A(I)=AE(I)
      A(3)=A(2)
      CALL FINSIG(ANS,KEY)
      KOFF=(K-180)*20 + J

      PRINTS ANSI(821)- ANSI(1620)

      ANSI(KOFF)=ANS
      ANSWER(KOFF)=CDABS(ANSI(KOFF))
    2 CONTINUE
      DO 4 LPA=1,81
      K1=(LPA-1)*20+1
      K2=LPA*20
      WRITE(6,101)(ANSI(LP),ANSWER(LP) ,LP=K1,K2)
    4 CONTINUE

   52 FORMAT(5E10.3)
   56 FORMAT(1H0,5D20.7)
  100 FORMAT(1H1)
  101 FORMAT(1H0,(9D14.5))

      STOP
```

```
      END

      SUBROUTINE FINSIG(ANS,KEY)
      COMPLEX*16 M,L,X(3),Z(3),A(3),F(3),FPRI(3),G(3),GPRI
     * (3),C,ANS
      REAL*8 H,COR,SIG,SLO(2),W(2),GRAV,CRAPS
      COMMON M,L,X,Z,A,H,COR,SIG,SLO,W,GRAV,KOOL
       KOOL=0
      CRAPS=0.1D30
      C=(M*M*GRAV*H+COR*COR-SIG*SIG)/(GRAV*H)
      L=CDSQRT(C)
      X(1)=W(1)
      X(2)=SLO(1)*W(1)/SLO(2)
      X(3)=X(2)+W(2)
      DO 2 I=1,3
      Z(I)=X(I)*2.DO*M
      CALL DLAP(F(I),Z(I),A(I),NN)
      CALL DLAPGG(F(I),Z(I),G(I),A(I))
      CALL DRDLAP(A(I),Z(I),FPRI(I))
      CALL DGDLAP(A(I),Z(I),FPRI(I),G(I),GPRI(I))
      IF(CDABS(F(I)).GE.CRAPS.OR.CDABS(G(I)).GE.CRAPS.OR CD-
      ABS(FPRI(I)).GE.CRAPS.OR.CDABS(GPRI(I)).GE.CRAPS) GOTO
     ? J
    2 CONTINUE
      ANS=(L-M)*(F(3)*(F(1)*GPRI(2)-G(2)*FPRI(1))+G(3)*(F(2)
     2*FPRI(1)-F(1)*FRRI(2)))+2*M*(FPRI(3)*(F(1)*GPRI(2)-
     3G(2)*FPRI(1))+GPRI(3)*(F(I)*FPRI(1)-F(1)*FPRI(2)))
      IF(KEY.EQ.1) WRITE(6,51) ANS,M,L
      IF(CDABS(ANS).GE.0.1D4) GOTO 4
    1 RETURN
    4 KOOL=1
      WRITE(6,61) M
      GOTO 1

   51 FORMAT(1H ,3D20.7)
   61 FORMAT(34HOSOLUTION DOES NOT EXIST BEYOND M=,E12.4)

      END


      SUBROUTINE DLAP(Y,X,A,NN)

C DLAP SOLVES FOR LAGUERRE FUNCTIONS OF THE FIRST KIND.GENER-
C ALLY THE ARGUMENT,A,WOULD BE EXPECTED TO BE NEGATIVE.WHEN A
C IS POSITIVE,NN IS SET TO 1.

      COMPLEX*16 Y,X,A,YN,YX
      REAL*8 YY,YZ
      YN=A
      YY=1.DO
      YK=1.DO
      YX=1.DO
      YZ=1.CO
    3 YX=X/YZ*YN/YZ*YX
      YY=YY+YX
      YN=YN+1.DO
      YZ=YZ+1.DO
      IF(CDABS(YX).GT.0.5D-07) GOTO 3
    1 Y=YY
      RETURN

   50 FORMAT(1H ,3(5X,E14.7))
   51 FORMAT (1H ,5D20.7)

      END


      SUBROUTINE DLAPGG(Y,X,DL,A)
      COMPLEX*16 Y,X,DL,A,DA,DQ,DX,D2
      REAL*8 DN
```

```
CLAPGG SOLVES FOR LAGUERRE FUNCTIONS OF THE SECOND KIND.WHEN
A APPROACHES A NEGATIVE INTEGER,CONTROL IS TRANSFERRED TO
DLAPG.WHEN CONVERGENCE DOES NOT TAKE PLACE BEFORE AN ANSWER
IS OBTAINED,CONTROL IS RETURNED TO THE CALLING ROUTINE AND A
MESSAGE PRINTED;WHEN X=0, THIS FUNCTION IS INDETERMINATE.

      KOUNT=0
      IF(CDABS(X).EQ.0.D0) GOTO 6
      DN=1.D0
      DQ=1.D0
      DX=1.D0
      DL=Y*CDLOG(X)
      DA=A
      D2=0.D0
    4 DX=DX*DA/DN*X/DN
      D2=D2+(1.D0/DA)-(2.D0/DN)
      DQ=DX*D2
      DL=DL+DQ
      DA=DA+1.D0

1/DA APPROACHING INFINITY?

      IF(CDABS(DX).GE.0.1D65.OR.CDABS(DX).LE.0.1D-60) GOTO 1

ANSWER TOO LARGE WITHOUT CONVERGENCE?

      IF(CDABS(DA).LE.0.1D-2) GOTO 1

CONVERGENCE?

      IF(CDABS(DQ).LE.0.5D-8.AND.KOUNT.GT.2) GOTO 3
      DN=DN+1.D0
      KOUNT=KOUNT+1
      GOTO 4
    1 CALL DLAPG(Y,X,DL,A)
      GOTO 3
    2 WRITE(6,54)KOUNT
    3 RETURN
   10 WRITE(6,57) KOUNT,DX,DL
      GOTO 2
    6 WRITE(6,53)
      GOTO 3

   53 FORMAT(59H X IS 0.LAGUERRE FUNCTION OF THE SECOND KIND
     # DOES NOT EXIST)
   54 FORMAT(1H ,100X,I3)
   55 FORMAT(1H ,6D20.7,/,D20.7)
   57 FORMAT(29H  OVERFLOW APPROACHING.AFTER ,I3,15H ITERAT-
     / IONS,DX=,E12.5,9H AND DL=,E12.5)

      END


      SUBROUTINE DLAPG(Y,X,DL,A)
      COMPLEX*16 Y,X,A,DL,DA,ZZ,D1,D11,DX
      REAL*8 D2,DN,DQ

DLAPG SOLVES FOR LAGUERRE FUNCTIONS OF THE SECOND KIND WHEN
A IS A NEGATIVE INTEGER.TO AVOID DIVIDING BY ZERO,THE NTH
TERM IS THE SUM OF N-1 MULTIPLICATIONS.

      KOUNT=0
      DN=1.D0
      DQ=1.D0
      DX=1.D0
      DL=Y*CDLOG(X)
      DA=A
      DZ=1.D0
      D2=0.D0
   22 DX=DX*X
       D11=0.D0
      DO 1 I=0,KOUNT
```

```
      D1=1.D0
     KOOK=I+1
     DO2 K=0,I
     IF(K.EQ.I) GOTO 4
     IF(CDABS(D1).LE.0.5D-10) GOTO 1
   2 D1=D1*(DA+DFLOAT(K))
   4 IF(KOOK.GT.KOUNT) GOTO 1
     DO 3 L=KOCK,KOUNT
     IF(CDABS(D1).LE.0.5D-10) GOTO 1
   3 D1=D1*(DA+DFLOAT(L))
   1 D11=D11+D1
      D2=D2-(2.D0/DN)
     DQ=DQ*DN*DN
     IF(DQ. GE.0.1D40) GOTO 56
     ZZ=(D11 +D1*(DA+DFLOAT(KOUNT))*D2)*DX/DQ
     KCUNT=KOUNT+1
     IF(KOUNT.GT.25) GOTO 56
     DL=DL+ZZ
     IF(CDABS(ZZ).LE.0.5D-8) GOTO 21
     DN=DN+ 1.D0
     GOTO 22
  21 RETURN
  56 WRITE(6,53) ZZ
     GOTO 21

  51 FORMAT(1H ,4(5X,E15.8))
  53 FORMAT(1H ,35HG(X) DID NOT CONVERGE.LAST TERM WAS,E15.

     END


      SUBROUTINE DRDLAP(A,X,RESULT)
     COMPLEX*16 A,X,RESULT,YDA,YDD
     REAL*8 YDB,YDN

DRDLAP SOLVES THE FIRST DERIVATIVE OF THE LAGUERRE FUNCTION
OF THE FIRST KIND.

     YDA=A+1.DC
     RESULT=A
     YDN=1.D0
     YDD=A
  22 YDB=YDN+1.D0
     YDD=YDD*YDA*X/ (YDN*YDB)
     RESULT=RESULT+YDD
     IF(CDABS(YDD).LT.0.5D-8) GOTO 21
     YDA=YDA+1.D0
     YDN=YDN+1.D0
     GOTO 22
  21 RETURN

  23 FORMAT(1H ,2(5X,E15.8))
  54 FORMAT (1H ,6D20.7)

     END


     SUBROUTINE DGDLAP(A,X,RESULT,PLG,RESUL)
     COMPLEX*16 A,X,RESULT,PLG,RESUL,XA,X2,XX,XZ,X4,X3
     REAL*8 XN

DGDLAP SOLVES THE FIRST DERIVATIVE OF THE SECOND KIND OF
LAGUERRE FUNCTION.WHEN X=0, THIS FUNCTION DOES NOT EXIST.

     IF(CDABS(X).EQ.0.C0) GOTO 10
     XN=2.D0
     XA=A
     RESUL=RESULT*CDLOG(X)+PLG/X+1.D0-(2.D0*XA)
      X2=1.D0/XA-2.D0
     XX=XA
  21 XA=XA+1.D0
     IF(CDABS(XA).LE.0.5D-4) GOTO 22
```

```
      XX=XX*X/(XN*XN-XN)*XA
      X2=X2 -2.DO/XN +1.DO/XA
      XZ=XX*X2
      RESUL=RESUL +XZ
      IF(CDABS(XZ).LE.0.5D-8) GOTO 10
      XN=XN +1.DO
      IF(CDABS(XZ).GE.0.1D60) GOTO 23
      GO TO 21
   22 KISS=1
       XN=2.DO
      XA=A
      RESUL=RESULT*CDLOG(X)+PLG/X+1.DO-(2.DO*XA)
      XQ=1.DO
      XX=1.DO
      X2=-2.DO
    5 XX=XX*X
      X3=0.DO
      DO 6 II=0,KISS
      X4=1.DO
      KIN=KISS+1
      DO 7 JJ=0,II
      IF(JJ.EQ.II) GOTO 9
      IF(CDABS(X4).LE.0.5D-10) GOTO 6
    7 X4=X4*(XA+DFLOAT(JJ))
    9 IF(KIN.GT.KISS) GOTO 6
      DO 8 LL=KIN,KISS
      IF(CDABS(X4).LE.0.5D-10) GOTO 6
    8 X4=X4*(XA+DFLOAT(LL))
    6 X3=X3+X4
      X2=X2-(2.DO/XN)
      XQ=XQ*XN*DFLOAT(KISS)
      XZ=(X3+X4*(XA+DFLOAT(KISS))*X2)*XX/XQ
      KISS=KISS+1
      RESUL=RESUL+XZ
      IF(CDABS(XZ).LE.0.5D-8) GOTO 10
      XN=XN+1.DO
      IF(KISS.LT.25) GOTO 5
      WRITE(6,54) XZ
   23 N=XN
      WRITE(6,24) N
   10 RETURN

   24 FORMAT(40H OVERFLOW ABOUT TO OCCUR IN DGDLAP AFTER,I 3
     + 7H TERMS.)
   51 FORMAT(1H ,4(5X,E15.8))
   54 FORMAT(1H ,36HG'(X) DID NOT CONVERGE.LAST TERM WAS,
     , E15.8)

      END
//GO.FT06F001 DD DCB=(RECFM=FA,BLKSIZE=133),SPACE=(CYL,(15,1
//GO.SYSUDUMP DD SYSOUT=A
//GO.SYSIN DD *
 0.100D 08 0.520D 07 0.729D-04 0.200D-02 0.500D-01
```

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13. ABSTRACT

A numerical study is made of the effect of a continental slope and shelf of finite width on trapped shelf and edgewaves. A comparison is made between a numerical solution for a continental shelf of finite width and its simplified analytic solution. It is shown that certain modes of these quasigeostrophic waves can undergo exponential growth or decay under special conditions.

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