AN ANALYSIS OF CONNECTED REPLENISHMENT OPERATIONAL DATA: THE DISTRIBUTION OF CONREP SERVICE TIME

by

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UNITED STATES
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THESIS

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ABSTRACT

Most analytic and computer simulation models of the Navy's CONREP phase of underway replenishment operations have assumed exponentially distributed service times for two reasons: (1) lack of better estimate of the true distribution, and (2) such an assumption leads to more tractable mathematical solutions.

By analyzing several different combinations of replenishment vessels and combatant ships, it is demonstrated through goodness-of-fit tests that gamma distributions whose parameters can be estimated from actual operational data are more precise estimates of the actual underlying CONREP service time distributions. Furthermore, it is shown that the distribution can be made Erlang by minor adjustments to the parameter estimates of the gamma distribution. Such a procedure might be desirable for analytical models employing Laplace transforms.

The data is subjected to linear regression techniques in an effort to develop meaningful and accurate functional relationships between service time and customer needs. The results indicate that the standard error of the estimating relationships would be too large to be of any practical use in planning an underway replenishment operation.

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I. INTRODUCTION

The United States Navy recognizes that one of the most practical ways to increase the effectiveness of our naval forces in maintaining superiority at sea is through the application of Underway Replenishment (UNREP). The purpose of these operations is to improve unit endurance by supplying needed logistic items to fleet units with minimum increase in vulnerability and with minimum diversion from performance of combat missions. The resulting ability of ships to remain on or near station is equivalent to adding additional combatants.

The importance of UNREP has been reemphasized within the Department of the Navy in that UNREP responsibility is now a NAVSHIPSYSCOM designated Project Office (PMS-90) [1]. NAVSHIPS Instruction 5430.73 of 25 August 1967 is the implementing document. Prior to this the responsibility for UNREP was solely a technical one and located in various BuShips branches.

There are basically three ways in which an UNREP may be accomplished. The primary method used is called Connected Replenishment (CONREP). This operation consists of intership horizontal transfers via rigs connecting the supply ships and the supported units. An increasingly important method, which involves the use of UH-34 or UH-46 helicopters to deliver certain items, is that of Vertical Replenishment (VERTREP). It has been envisioned that VERTREP will someday enable all but non-nuclear ships to avoid the inherent vulnerability imposed during CONREP. The third method entails a combination of CONREP and VERTREP.

Considerable effort has and is being expended to reduce the total time to conduct an UNREP. These efforts include extensive training, improved delivery techniques and equipment, and newer designs in supply vessels. Several attempts have been made to model the underway replenishment operations, both from the analytic and computer simulation approach.

McCullough [2] made an analytic approximation of the replenishment process by using a multi-stage cyclic-queueing model. The model considered M supply ships, called stages, placed in series. These ships serviced N combatant units, each of which passed by the supply ships in succession. In order to achieve a tractable solution, the operation was permitted to repeat itself continuously in order to study the steady state behavior of the system. A strong assumption used in this study was that of exponential services times. The most basic objection to this approach is that it is not really representative of what actually happens during CONREP. First, the cyclic nature of this model is a great simplification of the more complex sequences possible in these operations. Secondly, the operation is finite, and therefore no steady state solution ever exists. The assumption of exponential service times in order to make the model more amenable to analysis is not valid, a fact that this paper will attempt to prove. Included in McCullough's work is an incremental time step computer simulation model to test various sequences and to make comparisons with the results obtained in the analytic model. Here again, a basic assumption was the use of specified exponential service times for the supply ships, which did not vary between different types of customers.

Gordon and Copes [3] developed a deterministic model to investigate the optimal scheduling of underway replenishment operations by considering the process as a job scheduling problem. The service times of each combination of supply ship and customer were assumed to be deterministic and known, and general expressions were obtained for the total time to complete the CONREP and the total waiting time of the ships involved. The most serious deficiency of this approach is the assumption of fixed service times. It is more reasonable to assume that such service times are in fact random variables, and that questions relating to the determination of the average time needed to complete an underway replenishment operations must take this into account.

Waggoner [4] has developed an analytic model for the case of two supply ships and L combatants using queueing theory concepts and a random walk in the plane. The distribution of the total time to complete the finite operation is obtained in terms of its Laplace transform, which although frequently uninvertable, can provide information regarding the mean and variance of CONREP completion times. Here again, however, exponential service times were assumed. Waggoner concedes the point that although the exponential distribution frequently fits many realistic queueing models, some doubt exists as to its applicability in the case of CONREP operations.

II. OBJECTIVES

The objectives of this paper are twofold:

1. To determine from operational data more accurate estimates of the distribution of service times between various participants in the CONREP situation. If this can be accomplished, then more accurate results from future analytical and computer simulation models on CONREP operations can be expected.

2. Through the use of scatter diagrams and statistical regression techniques, determine if functional relationships exist between the amount of a specified type of material received by the combatant and the service time necessary to provide this material.

IV. GENERAL APPROACH TO CONREP DATA ANALYSIS

A. DETERMINATION OF CONREP SERVICE TIME DISTRIBUTION AND PARAMETER ESTIMATES

From previous experience, it was felt that service times in a CONREP operation would vary in at least two ways. First, the total requirement and replenishment configuration would differ among combatants serviced by the same replenishment ship. Secondly, variation would occur among servers replenishing the same type of customers, since delivery rates vary from server to server.

One of the first steps taken to determine the distribution of service times was to look carefully at the data from several specific situations, and from the results find a pattern which might make it possible to make a statement regarding the underlying distribution of CONREP service times in a general sense.

The three pilot cases chosen were those consisting of replenishment ship types AOE, AFS, and AE servicing DD (Destroyer) type customers. These cases were selected primarily due to the large number of sample points available, but also because it would permit an evaluation of the hypothesis held a priori.

Frequency histograms were constructed from these sets of data, one of which is shown in figure 1. Because of the distinctive skewed shapes of these graphs, it was decided to investigate the possibility of fitting a "gamma" distribution to each of these situations. The gamma distribution is discussed in detail in Appendix A.

Since the two parameters of the gamma distribution can be estimated from the sample mean and variance, these statistics were computed (utilizing appropriate computer subroutines). With these

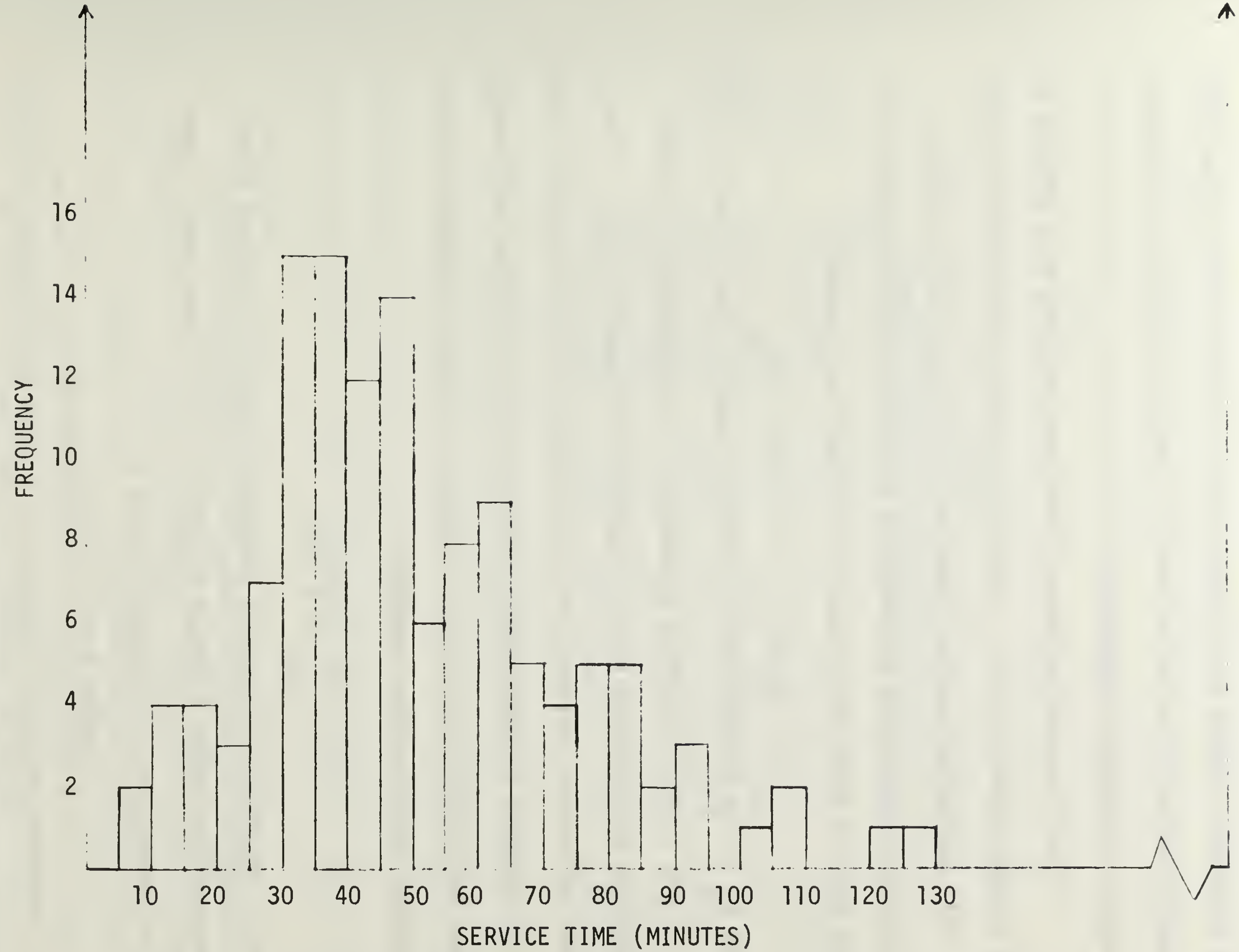

Figure 1. FREQUENCY HISTOGRAM OF DATA FOR DD (CUSTOMER), AE (SERVER)

parameters estimated, another subroutine developed for this analysis called GAMDIS was utilized to provide the necessary computations necessary to conduct goodness-of-fit tests with the observed data. Since the subroutine GAMDIS is a rather unique program involving numerical integration and is not to be found in the usual reference literature, a copy is provided in Appendix B.

The chi-square goodness-of-fit test was used as a measure of the appropriateness of fit. Having the expected and observed numbers of units possessing service times which fall into "cells" which represent an exhaustive and mutually exclusive set of intervals for which data is held, the problem is to test the hypothesis

$$H: \quad p_i = p_{io} \quad (i = 1,2,3,\ldots,k)$$

where p_i is the probability of finding a service time in the i_{th} cell, p_{io} is the observed relative frequency of the actual data falling within the same intervals, and k denotes the number of cells. Ten minute cells were utilized, with the exception of the last cell, which had an interval of time ranging from the end of the final cell having a data point within it to plus infinity. The test procedure is to calculate the chi-square statistic

$$\chi^2 = \sum_{i=1}^{k} (O_i - E_i)^2 / E_i$$

where O_i represents the number observed in the i_{th} cell and $E_i = np_{io}$ represents the number expected in cell i if H is true, with n equal to the sample size. The decision rule is

$$\text{reject H if } \chi^2 \geq \chi^2_{(1-\alpha)(k-3)}.$$

It is necessary to use k-3 degrees of freedom since the two population parameter estimates are obtained from the data.

The results obtained employing the above procedures on the three pilot cases are shown in Table I. In the table, MEAN T_s denotes average service time, VAR. is the sample variance, STD. DEV. is the sample standard deviation, $\hat{r}$ and $\hat{\lambda}$ are estimates of the parameters of the gamma distribution as computed from the data, χ^2 is the chi-square statistic, and $\chi^2_{k,.95}$ is the tabled chi-square value with k degrees of freedom, and significance level of $\alpha = .05$.

Table I.
Statistics on Determination of Estimates of Parameters for Gamma Distribution for Pilot Cases

SERVER	MEAN T_s	VAR.	STD. DEV.	$\hat{r}$	$\hat{\lambda}$	χ^2	$\chi^2_{k,.95}$
COMBINED	50.2	462.7	21.5	5.4	.11	24.5	35.2
AOE	48.0	308.0	17.5	7.5	.16	23.3	26.7
AE	50.6	562.2	23.7	4.5	.09	21.7	35.2
AFS	51.7	233.8	15.0	11.9	.23	8.4	25.0

These results indicated that, for the particular cases chosen, suitable parameter estimates could be obtained from the data which enabled gamma distributions to be fitted to each situation. As indicated, the calculated value of the chi-square is somewhat less than the tabled value in each case.

As noted in Appendix A, the gamma distribution becomes Erlang if the parameter r is a positive integer. Since the Erlang is somewhat easier to use in analytic work involving the use of Laplace transforms, it was decided to adjust the value of $\hat{r}$ in each case to the nearest integer, and then find the value of $\hat{\lambda}$ which minimized χ^2. The results of this procedure as applied to the three pilot cases are contained in Table II.

Table II.
Statistics on Determination of Estimates of Parameters for Gamma Distribution for Pilot Cases, $\hat{r}$ Converted to Nearest Positive Integer

SERVER	$\hat{r}$	$\hat{\lambda}$	X^2	$X^2_{.95}$
COMBINED	5.0	.11	26.2	35.2
AOE	7.0	.14	22.3	26.7
AE	4.0	.08	19.4	35.2
AFS	12.0	.23	8.3	25.0

It is interesting to note that an improvement in X^2 was obtained in all but one case. For this reason, and also due to the advantage of working with integer values of $\hat{r}$ whenever possible, the practice was followed throughout the remainder of the effort associated with service time distribution computations. Since it was always necessary to compute the actual value of the $\hat{r}$ estimate and its associated X^2, it was possible to note that the practice improved (lowered) the X^2 value approximately 60% of the time. It never raised the X^2 high enough to cause it to fail a goodness-of-fit solely because of this adjustment.

It should be mentioned at this point that no consideration had yet been given to the possibility that the distribution of service time might also vary depending upon whether the CONREP was conducted during daylight or at night. Eventually surmising that this might be the case, the data was separated and the analysis rerun. As expected, the average service time for night time operations was generally higher, and adjustments were necessary in the estimated parameters $\hat{r}$ and $\hat{\lambda}$ in each case. Therefore all remaining calculations were conducted on data which had been carefully separated between day and night operations. Fortunately, all data sources clearly indicate this important factor.

B. LINEAR REGRESSION MODEL TO DETERMINE THE RELATIONSHIP BETWEEN CONREP SERVICE TIME AND AMOUNT OF MATERIAL TRANSFERRED

Prior to undertaking this part of the data analysis, it was felt that nearly linear relationships existed between customer needs and the times necessary to provide them. If these relationships could be found, they could be a valuable aid to those responsible for planning an UNREP. For example, if this technique could be utilized to determine the total time of an UNREP, then this knowledge would aid planners and participants alike since more effective use of combatants and Underway Replenishment Groups (URGs) would result.

In order to provide a visual aid in determining what relationships actually existed between service times and requirements, scatter diagrams were produced using BIMED 02D [10], a statistical subroutine contained in the function library of the IBM 360 computer facility at the Naval Postgraduate School. The cases chosen as representative of the many possible combinations were those involving AOs and AEs as replenishment vessels and combatant types CVA, CLG, and DD. This selection was chosen since AOs provide an assortment of liquid fuels, while AEs supply only dry material, namely, ammunition and explosives.

Before discussing the scatter diagrams further, it is necessary to digress for the moment to describe the unique problems presented by the AO server situation. These supply vessels have the capability of supplying three different fuels, namely, Navy Standard Fuel Oil (NSFO), Diesel Fuel (JP-5), and Aviation Gasoline (AVGAS). One customer, the conventionally powered aircraft carrier, frequently takes all three types of fuel simultaneously through separate hoses. Other customers, say helicopter equipped, conventionally powered cruisers and destroyers, will take NSFO or JP-5 and perhaps AVGAS. As can be seen on the scatter diagrams, there is a wide range of fuel type and requirements combinations. Thus the problem of finding a single, controlling independent variable for the AO server situation would appear to have no solution. The alternative is to attempt to find a method of predicting the response based on the magnitudes of a number of variables.

The scatter diagrams for 16 situations involving the aforementioned participants are shown in figures 3 through 18 in Appendix C.

In the special case of the AO serving CVA/CVS ships, each of the independent variables, barrels of NSFO, JP-5, and AVGAS, were plotted individually against total CONREP service time. In all other cases, only the dominant independent variable was plotted with time. All daylight CONREP situations are plotted separately from night operations.

A discussion of the scatter diagrams is contained in the next section.

The technique utilized to discover the functional relationships between needs and service times was that of linear regression. In the AO-CVA/CVS cases, usually involving three independent variables, multiple linear regression was applied. The aim was to determine if a regression model of the form

$$T_S = \alpha + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + E$$

could be calculated and applied in this case, where T_S represents the response dependent variable, total service time, and X_1, X_2, and X_3

represent the independent variables, barrels of NSFO, JP-5, and AVGAS respectively. E denotes the error term in the model.

Since the remainder of the cases involve only one independent variable, the attempt was to find a simple linear regression model of the form

$$T_S = \alpha + \beta X + E$$

where X represents barrels of NSFO for the other AO serving situations, and short tons of explosive material for all AE serving situations.

BIMED 03R, a multiple regression subroutine contained in the function library of the IBM 360 computer facility at the Naval Post-graduate School, was utilized for all regression work.

No attempt was made to fit higher order polynomials to the CONREP data.

V. RESULTS OF THE DATA ANALYSIS

A. DETERMINATION OF CONREP SERVICE TIME DISTRIBUTIONS AND PARAMETER ESTIMATES

The techniques described in Section IV for the determination of the parameters of the gamma distribution from the data were applied to a total of 29 out of a possible 36 separate cases, consisting of the combinations of AO, AE, AOE, AFS, AF, and AKS serving CVA/CVS, CLG/DLG/DDG, and DD type customers, for day and night operations separately.

The results of this determination are contained in Tables III through VIII. The following abbreviations are used in the tables.

N	=	total number of sample points considered or available. A maximum of 30 was utilized to reduce size of problem.
$\bar{T}_s$	=	average CONREP service time of sample.
$\hat{\sigma}_{ts}$	=	sample standard deviation of CONREP service time.
$\hat{r}$	=	estimate of parameter r for gamma distribution to nearest integer.
$\hat{\lambda}$	=	estimate of parameter λ for gamma distribution.
X^2	=	chi-square statistic for goodness of fit.
k	=	degrees of freedom.
$X^2_{k,.95}$	=	tabled value of chi-square statistic with k degrees of freedom and significant level $\alpha = .05$.

Statistics were not computed for situations in which eight or less data points were held.

Only two cases, AO-DD (Night) and AFS-CVA/CVS (Day) failed the chi-square goodness-of-fit tests at the $\alpha = .05$ level. Both would have passed at the .03 level.

Eight out of 29 cases had estimates of the r parameter of 9 or greater, indicating that these cases could be estimated by a normal population with parameters of mean and standard deviation indicated in the tables.

Table III.
Statistics on AO Serving Situation

	CVA/CVS		CLG/DLG/DDG		DD	
	DAY	NIGHT	DAY	NIGHT	DAY	NIGHT
N	30	22	30	30	30	30
$\bar{T}_s$	114.3	123.4	58.0	72.1	81.2	78.3
$\hat{\sigma}_{ts}$	34.1	30.4	21.8	22.1	41.1	29.6
$\hat{r}$	11.0	16.0	7.0	11.0	4.0	7.0
$\hat{\lambda}$	.09	.13	.11	.15	.05	.09
X^2	20.5	15.9	20.2	9.9	27.3	28.4*
k	19	17	13	12	17	17
$X^2_{k,.95}$	30.1	27.6	22.4	21.0	27.6	27.6

Table IV.
Statistics on AE Serving Situation

	CVA/CVS		CLG/DLG/DDG		DD	
	DAY	NIGHT	DAY	NIGHT	DAY	NIGHT
N	30	30	30	30	30	30
$\bar{T}_s$	98.6	161.2	71.8	181.3	41.2	83.3
$\hat{\sigma}_{ts}$	61.2	52.8	51.5	123.0	23.7	36.1
$\hat{r}$	3.0	9.0	2.0	2.0	3.0	5.0
$\hat{\lambda}$	.03	.06	.02	.01	.06	.07
X^2	25.8	27.6	34.7	62.4	12.0	25.3
k	26	27	26	51	10	16
$X^2_{k,.95}$	38.9	40.1	38.9	68.7	18.3	26.3

Table V.
Statistics on AOE Serving Situation

	CVA/CVS		CLG/DLG/DDG		DD	
	DAY	NIGHT	DAY	NIGHT	DAY	NIGHT
N	19	5	23	3	30	11
$\bar{T}_s$	141.7	INSUFFICIENT DATA	81.2	INSUFFICIENT DATA	51.6	70.9
$\hat{\sigma}_{ts}$	58.9		41.1		18.3	35.7
$\hat{r}$	6.0		4.0		8.0	4.0
$\hat{\lambda}$	.03		.05		.17	.05
χ^2	28.0		27.1		12.7	18.8
k	31		17		8	13
$\chi^2_{k,.95}$	45.0		27.6		15.5	22.4

Table VI.
Statistics on AFS Serving Situation

	CVA/CVS		CLG/DLG/DDG		DD	
	DAY	NIGHT	DAY	NIGHT	DAY	NIGHT
N	13	3	14	13	30	24
$\bar{T}_s$	145.8	INSUFFICIENT DATA	108.3	71.5	52.4	58.5
$\hat{\sigma}_{ts}$	61.6		60.0	30.3	12.2	21.3
$\hat{r}$	5.0		3.0	5.0	18.0	7.0
$\hat{\lambda}$	.03		.02	.07	.35	.12
χ^2	46.5*		27.3	22.0	6.8	4.3
k	30		26	12	6	9
$\chi^2_{k,.95}$	43.8		38.9	21.0	12.6	16.9

Table VII.
Statistics on AF Serving Situation

	CVA/CVS		CLG/DLG/DDG		DD	
	DAY	NIGHT	DAY	NIGHT	DAY	NIGHT
N	9	6	16	9	30	20
$\bar{T}_s$	113.4	INSUFFICIENT DATA	61.9	69.2	55.2	54.3
$\hat{\sigma}_{ts}$	45.3		29.2	27.7	17.4	21.1
$\hat{r}$	6.0		4.0	6.0	10.0	6.0
$\hat{\lambda}$	.05		.06	.08	.18	.11
χ^2	25.9		16.7	9.1	8.2	11.1
k	17		13	11	7	9
$\chi^2_{k,.95}$	27.6		22.4	19.7	14.1	16.9

Table VIII.
Statistics on AKS Serving Situation

	CVA/CVS		CLG/DLG/DDG		DD	
	DAY	NIGHT	DAY	NIGHT	DAY	NIGHT
N	3	6	20	5	30	19
$\bar{T}_s$	INSUFFICIENT DATA	INSUFFICIENT DATA	41.3	INSUFFICIENT DATA	36.2	45.9
$\hat{\sigma}_{ts}$			13.5		11.9	20.4
$\hat{r}$			9.0		9.0	5.0
$\hat{\lambda}$			.21		.24	.10
χ^2			2.7		6.3	8.0
k			6		6	9
$\chi^2_{k,.95}$			12.6		12.6	16.9

B. LINEAR REGRESSION MODEL TO DETERMINE THE RELATIONSHIP BETWEEN CONREP SERVICE TIME AND AMOUNT OF MATERIAL TRANSFERRED

With few exceptions the scatter diagrams in Appendix C tend to discourage the notion that meaningful, accurate linear relationships exist between combatant ship requirements and service time. This is especially true in the case of the AO servicing various units. There is a definite improvement in those situations involving the AE as the server. On several of these diagrams, it is possible to fair in a line which would serve as a linear model. It is questionable from the results of the scatter diagrams that the variability around a single regression line could be low enough to be acceptable. In order to determine this variability more precisely, the multiple regression techniques mentioned in Section IV were employed. The results of these efforts are shown in Table IX, for the AO and AE server situation only.

As an example, this table indicates that a linear regression model for predicting the amount of time necessary for an AO to supply quantities of NSFO, JP-5, and AVGAS could be computed using the following formula:

$$T_S = 68.1 + .0050\,X_1 + .0014\,X_2 + .0062\,X_3 \text{ (minutes)}$$

where X_1 denotes barrels of NSFO, X_2 barrels of JP-5, and X_3 barrels of AVGAS required. The standard error (i.e., the standard deviation) of the estimate is 27.0 minutes. Three cases were selected from the dependent sample to illustrate the accuracy of this procedure. The results were as follows:

NSFO (BBLS)	JP-5 (BBLS)	AVGAS (BBLS)	T_S CALCULATED	T_S ACTUAL	DIFFERENCE OBSERVED
5681	3535	1212	108	96	+12
4139	12452	0	106	99	+ 7
4595	0	0	90	158	-68

If this sample can be considered representative, it illustrates quite clearly that magnitudes of the errors that might be expected.

As expected and in agreement with the scatter diagram, the standard estimate errors for the cases involving the AE as server were less than those involving AOs in four out of six cases.

Table IX.

Results of Linear Regression Analysis on AO and AE Server Situations

AO-CVA/CVS (Day)

Sample size	45
Percent variance explained	36.4
Multiple correlation coefficient	.604
Standard error of the estimate	27.0 min.
α - intercept value	68.1 min.

Dependent variables	Mean (BBLS)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficients
NSFO	6577.9	3460.5	0.0050	0.00125	0.53
JP-5	6267.8	4638.2	0.0014	0.00093	0.22
AVGAS	273.2	655.3	0.0062	0.00639	0.15

AO-CVA/CVS (Night)

Sample size	20
Percent variance explained	22.6
Multiple correlation coefficient	.4757
Standard error of the estimate	32.8 min.
α - intercept value	54.9 min.

Dependent variables	Mean (BBLS)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficients
NSFO	6939.5	4579.7	0.0027	0.00173	0.36
JP-5	5454.8	3136.9	-0.0013	0.00251	-0.13
AVGAS	258.3	514.4	0.0212	0.01489	0.34

AO-CLG/DLG/DDG (Day)

Sample size	51
Percent variance explained	43.2
Standard error of the estimate	14.4 min.
α - intercept value	24.8 min.

Dependent variable	Mean (BBLS)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
NSFO	2025.1	879.0	0.0141	0.00230	0.66

AO-CLG/DLG/DDG (Night)

Sample size	49
Percent variance explained	10.0
Standard error of the estimate	24.0 min.
α - intercept value	56.5 min.

Dependent variable	Mean (BBLS)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
NSFO	2111.0	1470.9	0.0054	0.00236	0.32

Table IX. (cont.)

AO-DD (Day)

Sample size	99
Percent variance explained	22.0
Standard error of the estimate	30.8 min.
α - intercept value	25.3 min.

Dependent variable	Mean (BBLS)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
NSFO	1320.9	533.2	0.0305	0.00583	0.47

AO-DD (Night)

Sample size	40
Percent variance explained	0.02
Standard error of the estimate	60.8 min.
α - intercept value	80.9 min.

Dependent variable	Mean (BBLS)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
NSFO	1490.7	449.0	0.0020	0.02164	0.0151

AE-CVA/CVS (Day)

Sample size	30
Percent variance explained	85.5
Standard error of the estimate	23.4 min.
α - intercept value	-30.4 min.

Dependent variable	Mean (S.T.)*	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
AE Material	207.2	126.9	0.4332	0.03419	0.923

AE-CVA/CVS (Night)

Sample size	30
Percent variance explained	68.2
Standard error of the estimate	31.9 min.
α - intercept value	48.6 min.

Dependent variable	Mean (S.T.)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
AE Material	285.6	122.0	0.376	0.04861	0.826

* S.T. denotes Short Tons.

Table IX. (cont.)

AE-CLG/DLG/DDG (Day)

Sample size	30
Percent variance explained	86.7
Standard error of the estimate	20.7 min.
α - intercept value	80.3 min.

Dependent variable	Mean (S.T.)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
AE Material	50.7	53.8	0.965	0.07144	0.93

AE-CLG/DLG/DDG (Night)

Sample size	25
Percent variance explained	77.8
Standard error of the estimate	62.5 min.
α - intercept value	67.4 min.

Dependent variable	Mean (S.T.)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
AE Material	90.8	85.7	1.337	0.1489	.882

AE-DD (Day)

Sample size	30
Percent variance explained	79.5
Standard error of the estimate	7.3 min.
α - intercept value	23.7 min.

Dependent variable	Mean (S.T.)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
AE Material	18.0	16.0	0.568	0.0543	0.892

AE-DD (Night)

Sample size	30
Percent variance explained	46.3
Standard error of the estimate	39.1 min.
α - intercept value	36.4 min.

Dependent variable	Mean (S.T.)	Std. Dev.	Reg. Coe.	Std. Err. Reg. Coe.	Partial Corr. Coefficient
AE Material	27.9	21.4	1.67	0.34	0.68

VI. CONCLUSIONS

It has been shown that the gamma distribution, with parameters determined from operational data, can be used to describe the probability distribution of CONREP service times for a large number of practical situations. With minor adjustments in the gamma parameter estimates, the Erlang distribution, with its increased flexibility in analytical models employing Laplace transforms, can be used effectively for the same purpose.

It is conceivable that previous work in analytical and computer simulation studies which assumed exponentially distributed CONREP service times can be extended to incorporate this new information, with a resultant increase in the accuracy of these models in depicting actual underway replenishment operations.

The second part of the analysis indicates that, with few exceptions, estimates of service time utilizing linear regression techniques would be too inaccurate, as indicated by the standard error of the estimates derived from the data, to serve the purposes for which such a model might be used. Although certain cases appeared to be quite accurate, the overall results discourage enthusiasm for further effort along these lines. It is noted that some cases appear to lend themselves to non-linear estimates, such as might be obtained through polynomial regression, since some curvature can be detected in several of the scatter diagrams. A larger data base than that which was available for this study would be necessary to make meaningful conclusions along these lines.

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APPENDIX A

The Gamma Distribution

If a process consists of r successive events and if the total elapsed time of this process can be regarded as the sum of r independent exponential variates each with parameter λ, the probability distribution of this sum will be a gamma distribution with parameters λ and r. A form of the gamma distribution may be fitted to many positively skewed distributions of statistical data.

The gamma distribution is described by the following density function

$$g(x;r,\lambda) = \frac{\lambda^r x^{(r-1)} e^{-\lambda x}}{(r-1)!}$$

where $\lambda > 0$, $r > 0$, and x are non-negative.

The sum of r (where r is a positive integer) exponential variables each having the same parameter λ is also called an Erlang distribution. Mathematically, the Erlang distribution is a convolution of r exponential variates, i.e., the distribution of the sum of r exponential variables. Because of the restriction on r, a process which can be represented by an Erlang distribution is easier to work with in an analytic sense, especially if Laplace transform techniques are utilized.

The expected value and variance of the gamma distribution are given by

$$EX = \frac{r}{\lambda}$$

$$VX = \frac{r}{\lambda^2} .$$

From these relationships it follows that

$$r = \frac{(EX)^2}{VX}$$

$$\lambda = \frac{EX}{VX} .$$

Therefore, a gamma distribution, like a normal distribution, can be specified by its mean and its variance.

If $r = 1$, the gamma distribution is identical to the exponential distribution

$$f(x) = \lambda e^{-\lambda x} \qquad x \geq 0.$$

As r increases, the gamma distribution approaches a normal distribution asymptotically. r is often referred to as the "shape" parameter, while λ is known as the "scale" parameter. Figure 2 illustrates the graphs of $g(x;r,\lambda)$ for several values of r and $\lambda = 1$. In the gamma distribution the ratio of the standard deviation (the square root of the variance) to the mean (expected value) is the reciprocal of the square root of r. When this ratio is 1/3 or smaller (i.e., when r is 9 or greater), the gamma distribution closely approximates a normal distribution with the same mean and standard deviation.

The cumulative distribution function does not exist in explicit form for the gamma distribution, although the values of the so-called incomplete gamma function have been tabled by Pearson [7]. The computer subroutine entitled GAMDIS was constructed for and used in this analysis. The subroutine employs numerical integration techniques involving the trapezoidal rule [8]. GAMDIS has the function of computing probabilities and expected numbers necessary in making tests of hypotheses in chi-square goodness-of-fit tests. Because of the potential usefulness of this subroutine to others engaged in future efforts of this nature, a copy of the subroutine, written in FORTRAN with explanatory comment cards, is included as Appendix B.

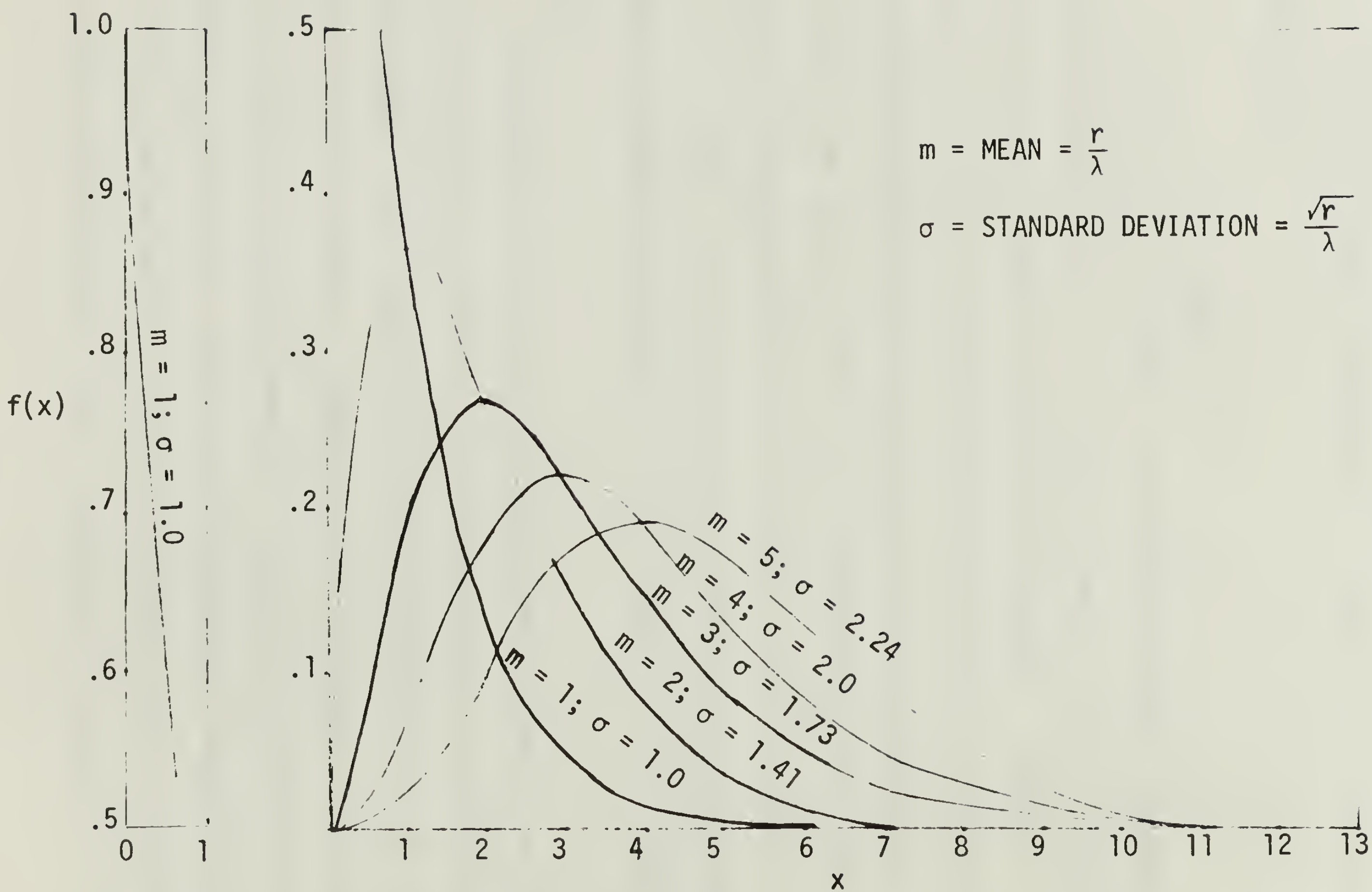

Figure 2. GAMMA PROBABILITY DISTRIBUTION IN THE SPECIAL CASE WHERE $\lambda = 1$.

APPENDIX B

Subroutine GAMDIS

Subroutine GAMDIS is called by specifying the estimates of the gamma distribution, R and XLAMDA. These estimates are computed from the sample mean and sample variance of the service times under consideration in accordance with the following relationships:

$$R = \hat{r} = \frac{(\text{sample mean})^2}{\text{sample variance}}$$

$$XLAMDA = \hat{\lambda} = \frac{\text{sample mean}}{\text{sample variance}}$$

Other calling parameters for the subroutine include NBU, the number of units in the sample, and NBCL, the number of cells for which expected numbers of units having appropriate service times are to be calculated and labeled by vector EXPECT.

The subroutine performs the following computations:

1. Defines the limit of the total area for which numerical integration will be utilized (LIMIT = NBCL*10).

2. Defines constants based on given values of R and XLAMDA which will be used in specifying the gamma density function (F) used in numerical integration.

3. Sets up subintervals necessary to obtain an accurate measurement of the probabilities associated with each interval.

4. Compute the probabilities mentioned in 3., (SUM2), plus the remainder of probability associated with the last interval (SUM4).

5. Computes the expected number of units for each cell using NBU, SUM2, and SUM4.

6. Returns the computations of 5. to the main program via vector EXPECT.

```
C     * * * * * * * * * * * * * * * * * * * * * * * *
      SUBROUTINE GAMDIS (R,XLAMDA,NBU,NBCL,EXPECT)
C     * * * * * * * * * * * * * * * * * * * * * * * *
      DIMENSION A(500),FBAR(500),AJ(1001),EXPECT(50)
C     R=SHAPE PARAMETER OF GAMMA DISTRIBUTION
C     XLAMDA = SCALE PARAMETER
C     NBU = TOTAL NUMBER OF SAMPLE UNITS
C     NBCL = TOTAL NUMBER OF 'CELLS' TO BE CONSIDERED
C     EXPECT = VECTOR OF EXPECTED NUMBER OF UNITS IN EACH 'CELL' BASE
C     ON GIVEN R AND XLAMDA
      LIMIT=NBCL*10
      PLAMBA=XLAMDA**R
      N=10
      N1=10
      AN=N
      AN1=N1
      GX=GAMMA(R)
      CONS=PLAMBA/GX
      L=0
      SUM3=0
      DO 15 JJ=10,LIMIT,10
      RJ=JJ
      BLIM=RJ
      ALIM=RJ-10.0
      XX=(BLIM-ALIM)/AN
      A(1)=ALIM
      NK=N+1
      DO 20 K=2,NK
      AK=K-1
   20 A(K)=ALIM+XX*AK
      SUM2=0
      DO 50 K=1,N
      SUM1=0
      NK=N1+1
      DO 40 I=1,NK
      AI=I-1
      XXX=XX/AN1
      AJ(I)=A(K)+XXX*AI
      IF (AJ(I).LE.0.0) GOTO 40
   70 YY=AJ(I)
      E=EXP(-XLAMDA*YY)
C     F = VALUE OF GAMMA DENSITY FUNCTION AT X=YY
      F=CONS*YY**(R-1.)*E
      SUM1=SUM1+F
   40 CONTINUE
C     FBAR(K) = AVERAGE VALUE OF F IN SUBINTERVAL
      FBAR(K)=SUM1/(AN1+1.)
C     T1 = AREA OF SUBINTERVAL
      T1=(A(K+1)-A(K))*FBAR(K)
C     SUM2 = PROBABILITY ASSOCIATED WITH THIS 'CELL'
      SUM2=SUM2+T1
   50 CONTINUE
      L=L+1
      EXPECT(L)=SUM2*NBU
C     SUM3 = CUMULATIVE PROBABILITY THROUGH LAST COMPUTATION
      SUM3=SUM3+SUM2
   15 CONTINUE
C     SUM4 = PROBABILITY ASSOCIATED WITH LAST CELL
      SUM4=1.0-SUM3
      EXPECT(L+1)=SUM4*NBU
      RETURN
      END
```

APPENDIX C

Scatter Diagrams of Operational CONREP Data for Situations Involving AO, AE Servers, CVA/CVS, CLG, and DD Customers

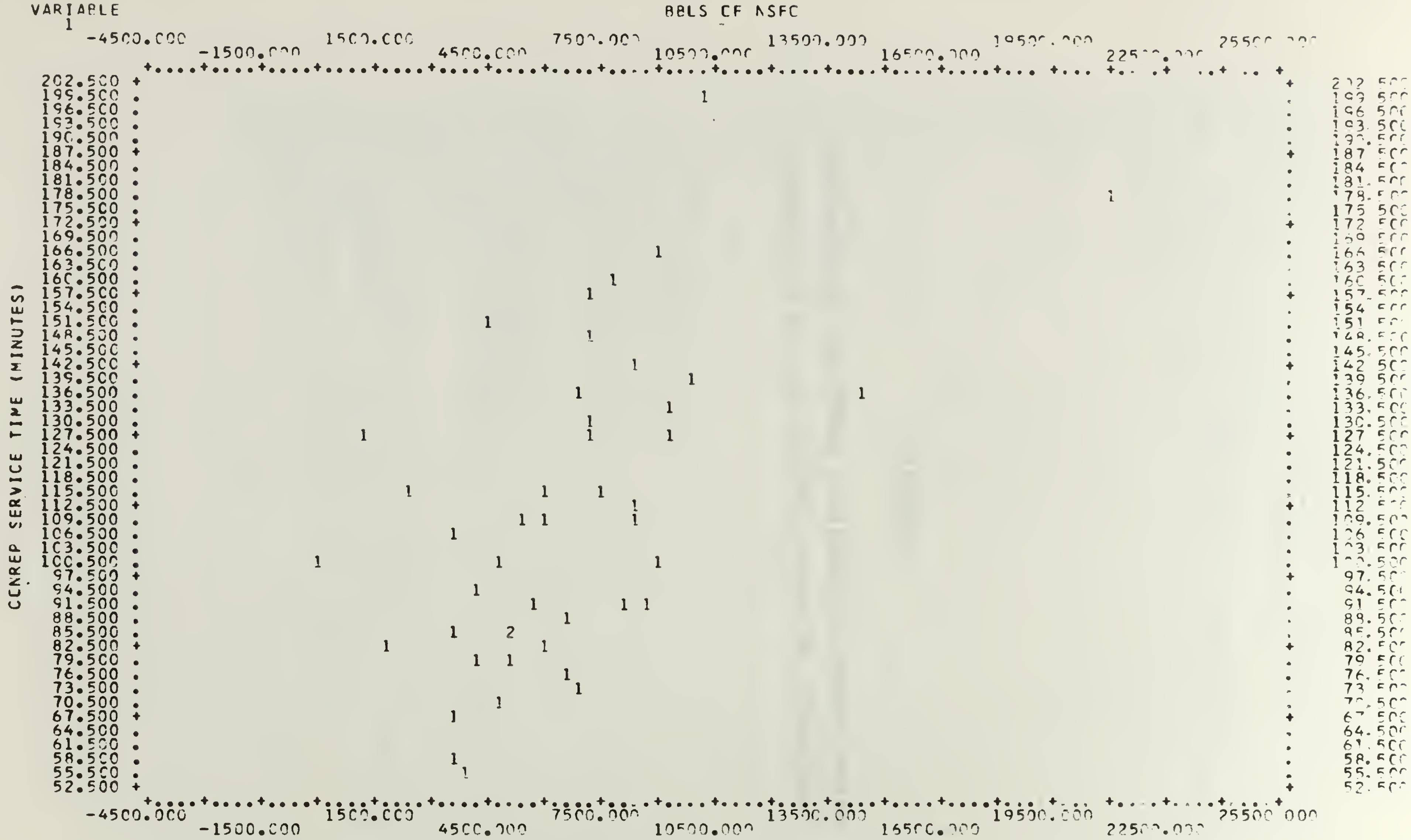

FIGURE 3. BIMED C2D SCATTER DIAGRAM FOR AC-CVA/CVS (DAY) SITUATION, NFSO VS SERVICE TIME

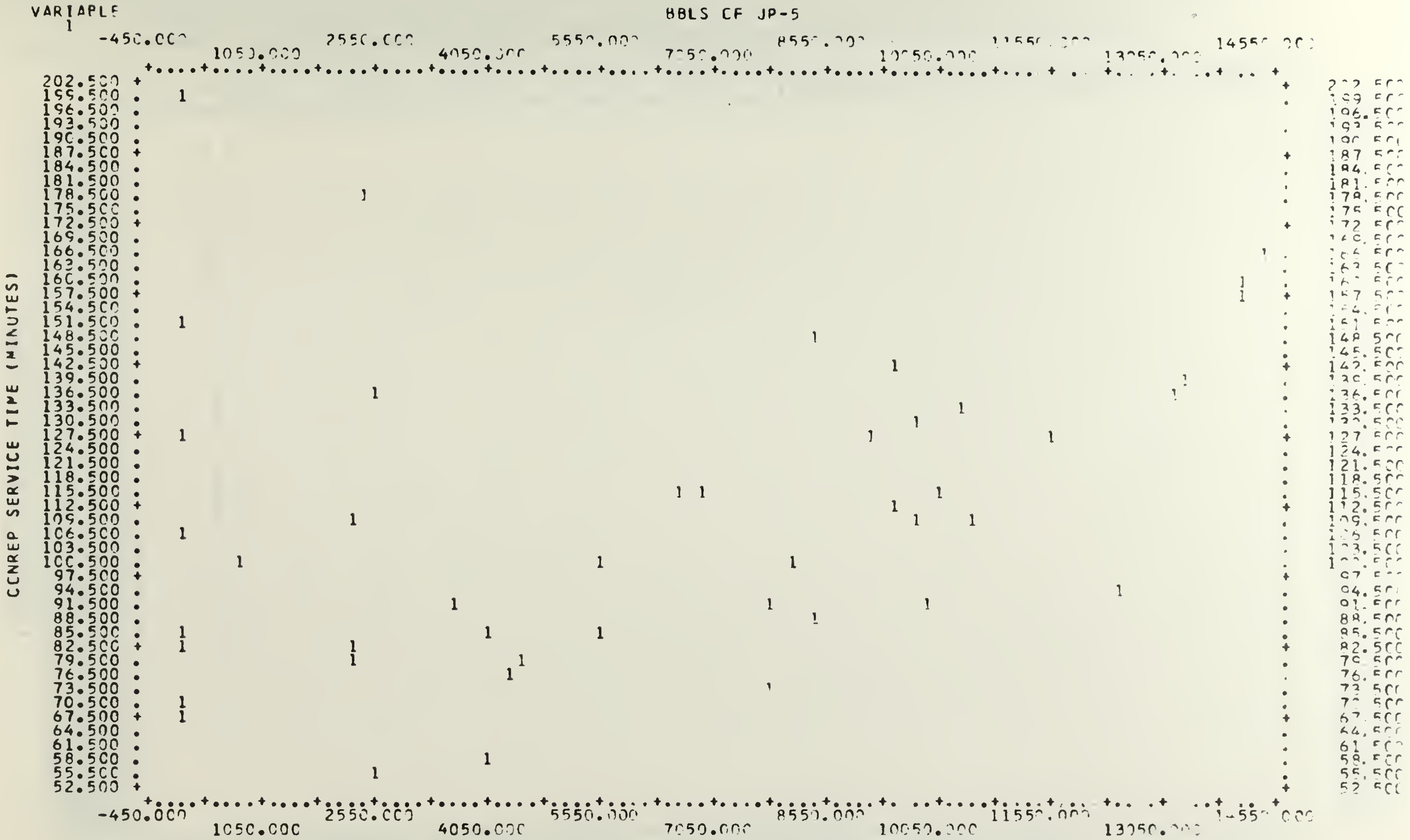

FIGURE 4. BIMED C2D SCATTER DIAGRAM FOR AC-CVA/CVS (DAY) SITUATION, JP-5 VS SERVICE TIME

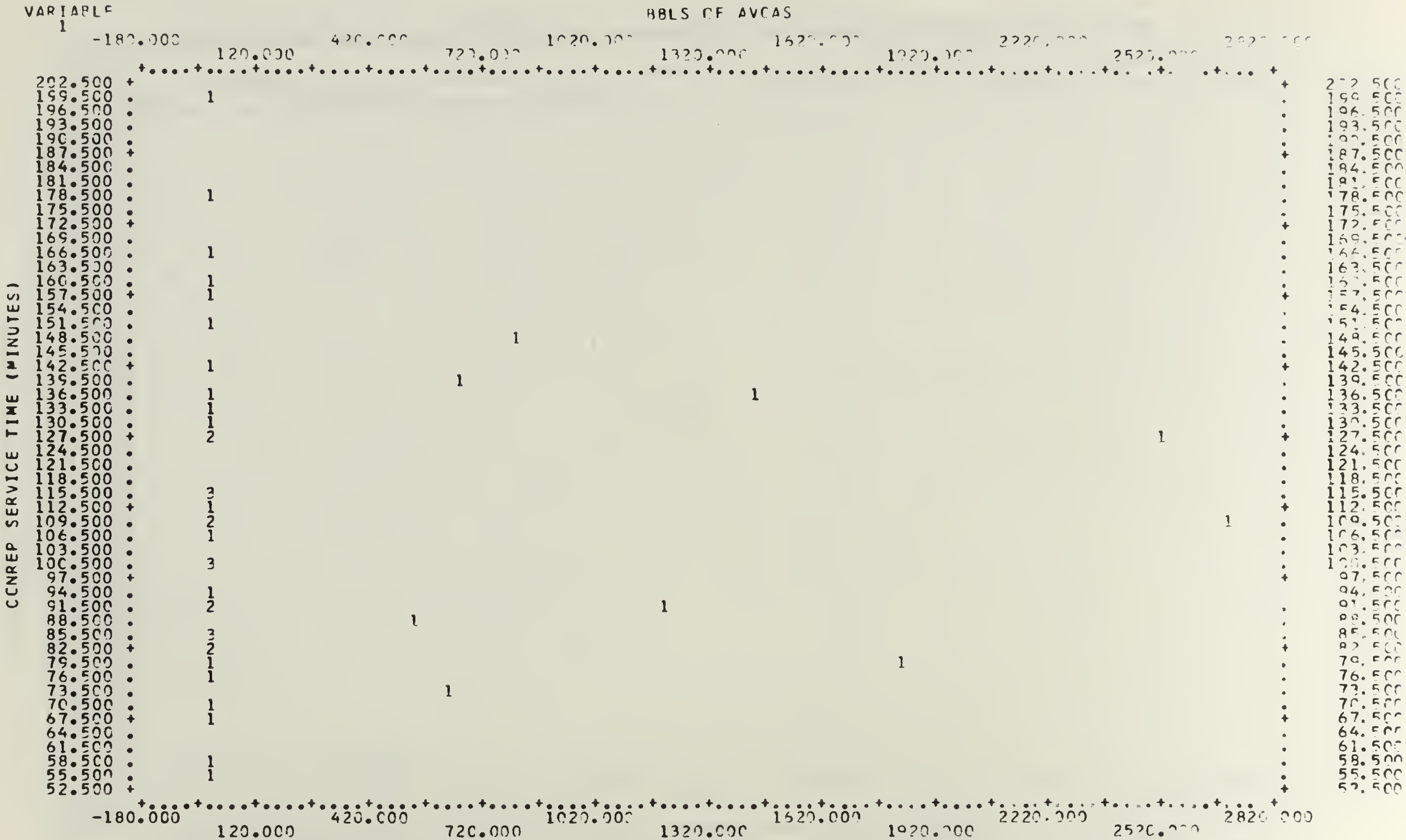

FIGURE 5. BIMED 02D SCATTER DIAGRAM FOR AC-CVA/CVS (DAY) SITUATION, AVGAS VS SERVICE TIME

FIGURE 6. BIMED O2D SCATTER DIAGRAM FOR AC-CVA/CVS (NIGHT) SITUATION, NSFO VS SERVICE TIME

FIGURE 7. BIMED O2D SCATTER DIAGRAM FOR AC-CVA/CVS (NIGHT) SITUATION, JP-5 VS SERVICE TIME

FIGURE 8. BIMEC 02C SCATTER DIAGRAM FOR AC-CVA/CVS (NIGHT) SITUATICN, AVGAS VS SERVICE TIME

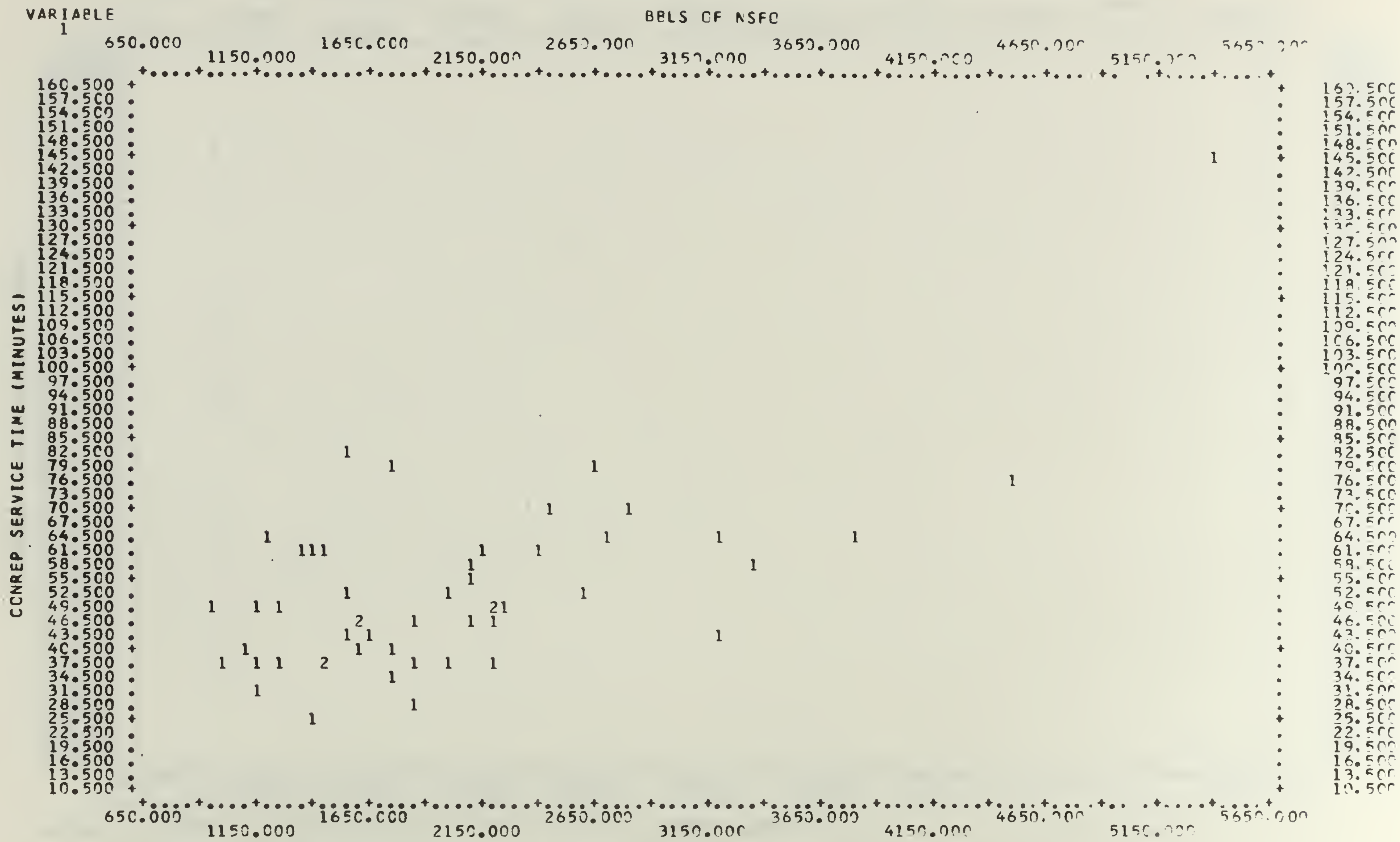

FIGURE 9. BIMED O2D SCATTER DIAGRAM FOR AO-CLG/DLG (DAY) SITUATION, NSFO VS SERVICE TIME

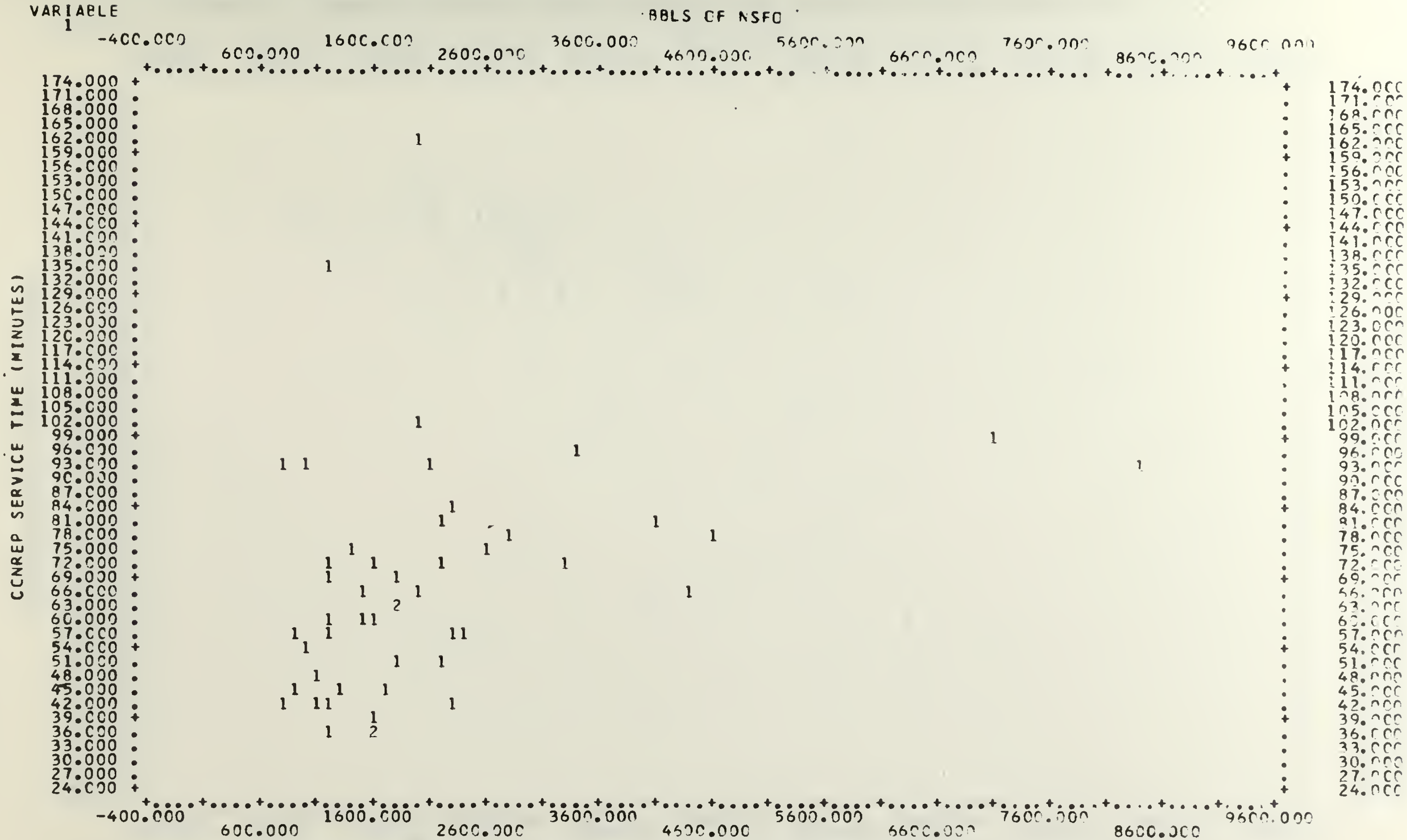

FIGURE 10. BIMED 02D SCATTER DIAGRAM FOR AC-CLG/DLG (NIGHT) SITUATION, NSFO VS SERVICE TIME

FIGURE 11. BIMED O2D SCATTER DIAGRAM FOR AC-CC (DAY) SITUATION, NFSC VS SERVICE TIME

FIGURE 12. BIMED O2C SCATTER DIAGRAM FOR AC-CD (NIGHT) SITUATION, NSFC VS SERVICE TIME

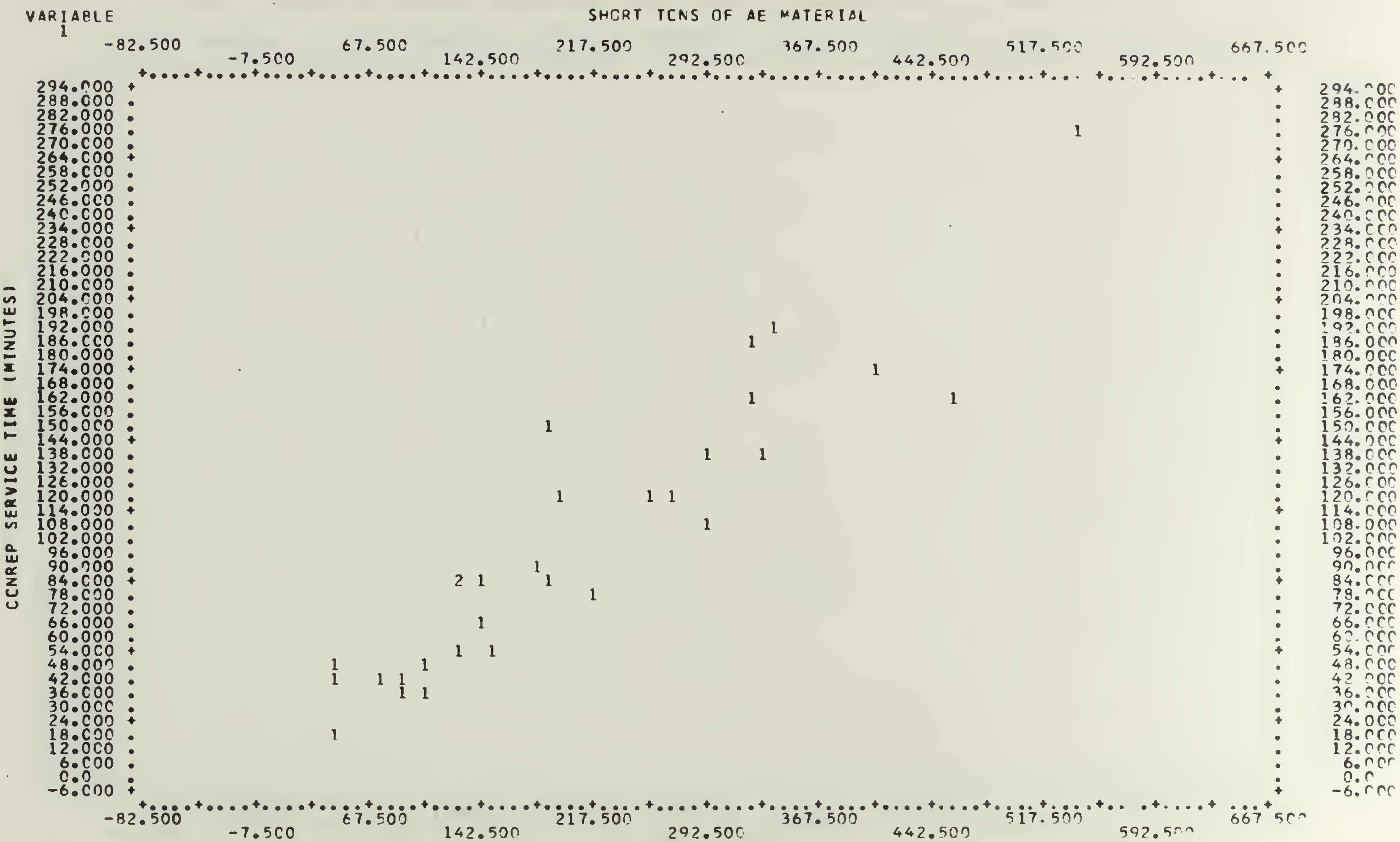

FIGURE 13. BIMED O2D SCATTER DIAGRAM FOR AE-CVA/CVS (DAY) SITUATION, AE MATERIAL VS SERVICE TIME

FIGURE 14. BIMED 02D SCATTER DIAGRAM FOR AE-CVA/CVS (NIGHT) SITUATION, AE MATERIAL VS SERVICE TIME

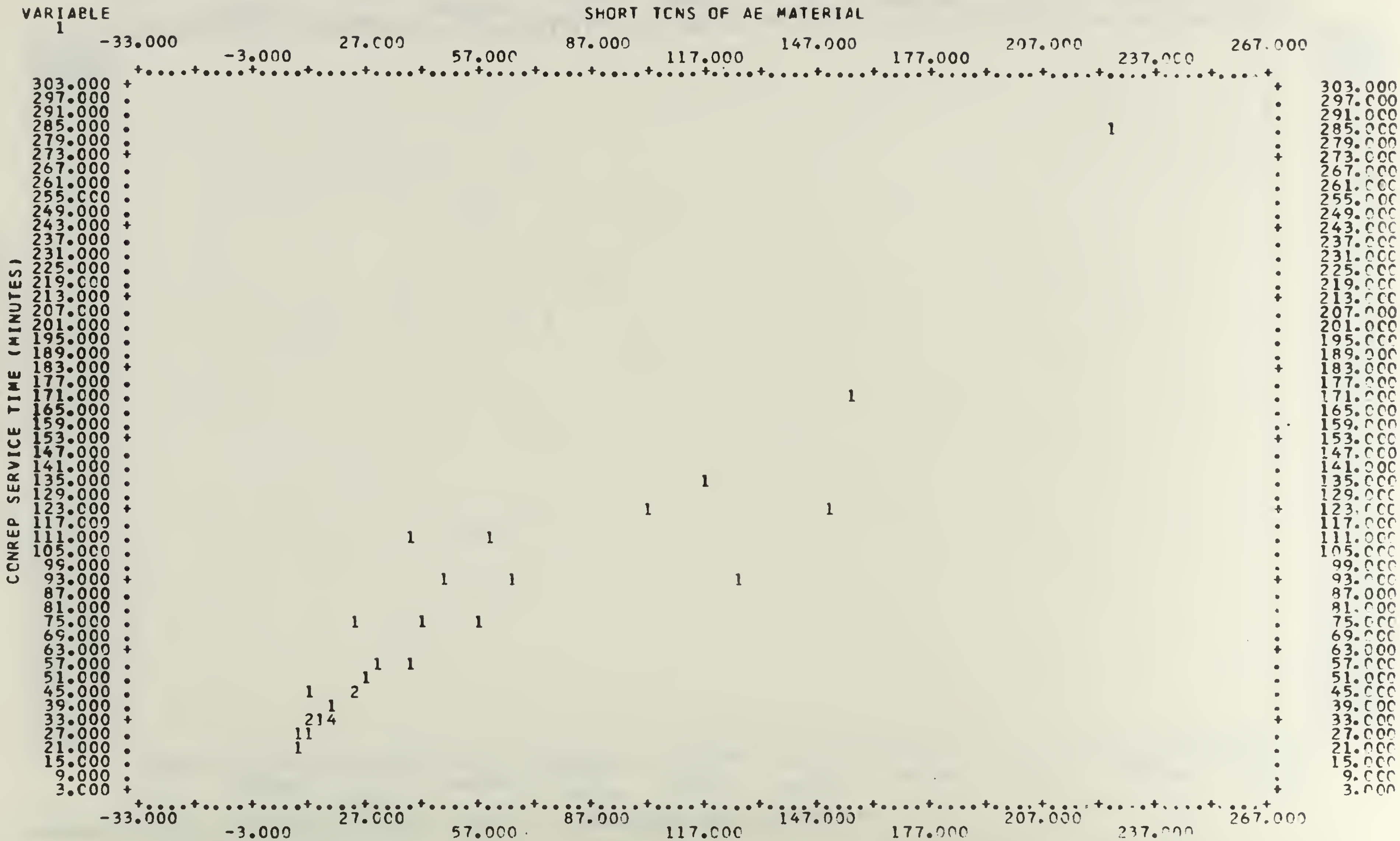

FIGURE 15. BIMED O2C SCATTER DIAGRAM FOR AE-CLG/DLG (DAY) SITUATION, AE MATERIAL VS SERVICE TIME

FIGURE 16. BIMED C2D SCATTER DIAGRAM FOR AE-CLG/DLG (NIGHT) SITUATION, AE MATERIAL VS SERVICE TIME

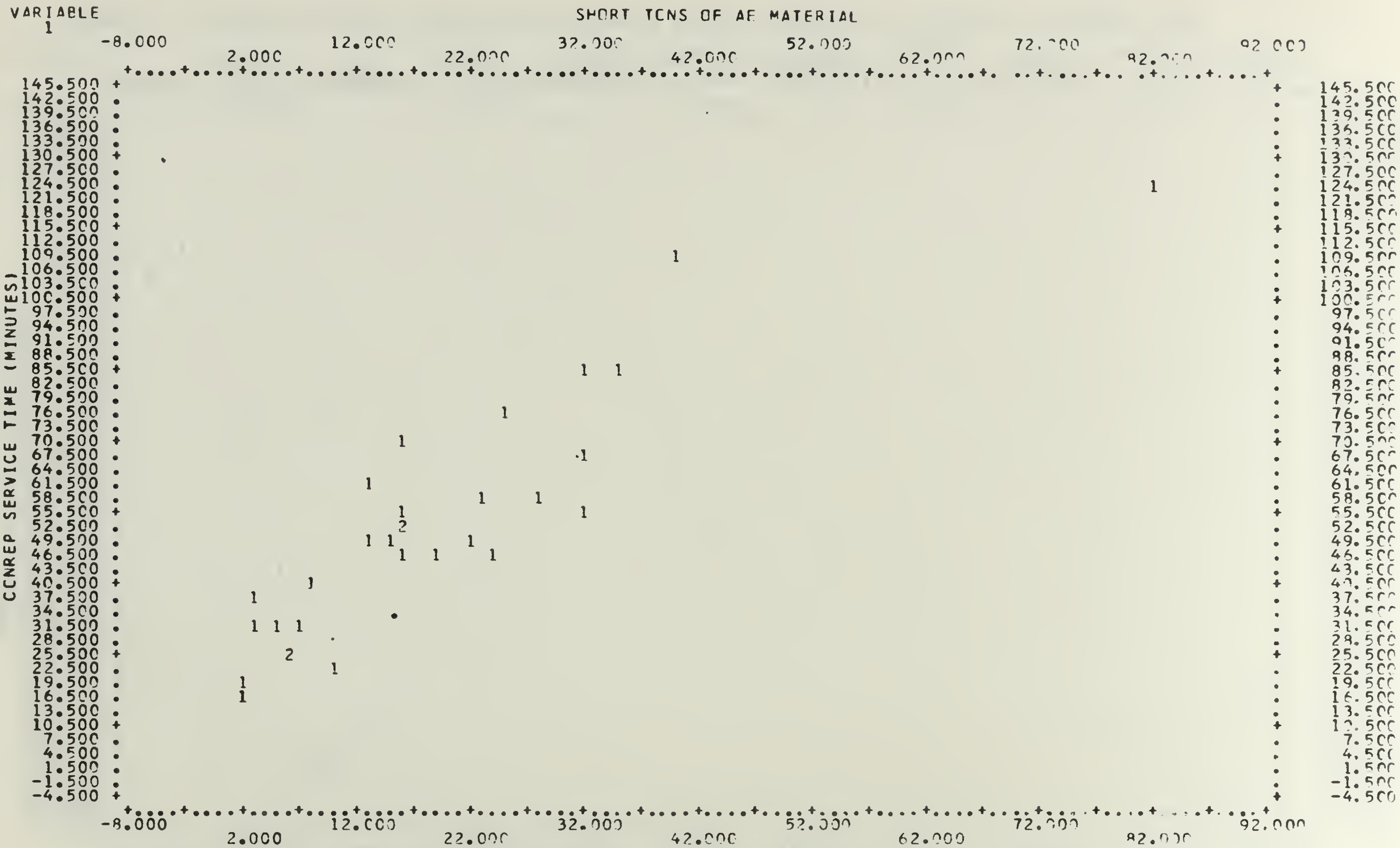

FIGURE 17. BIMEC O2D SCATTER DIAGRAM FOR AE-CD (DAY) SITUATION, AE MATERIAL VS SERVICE TIME

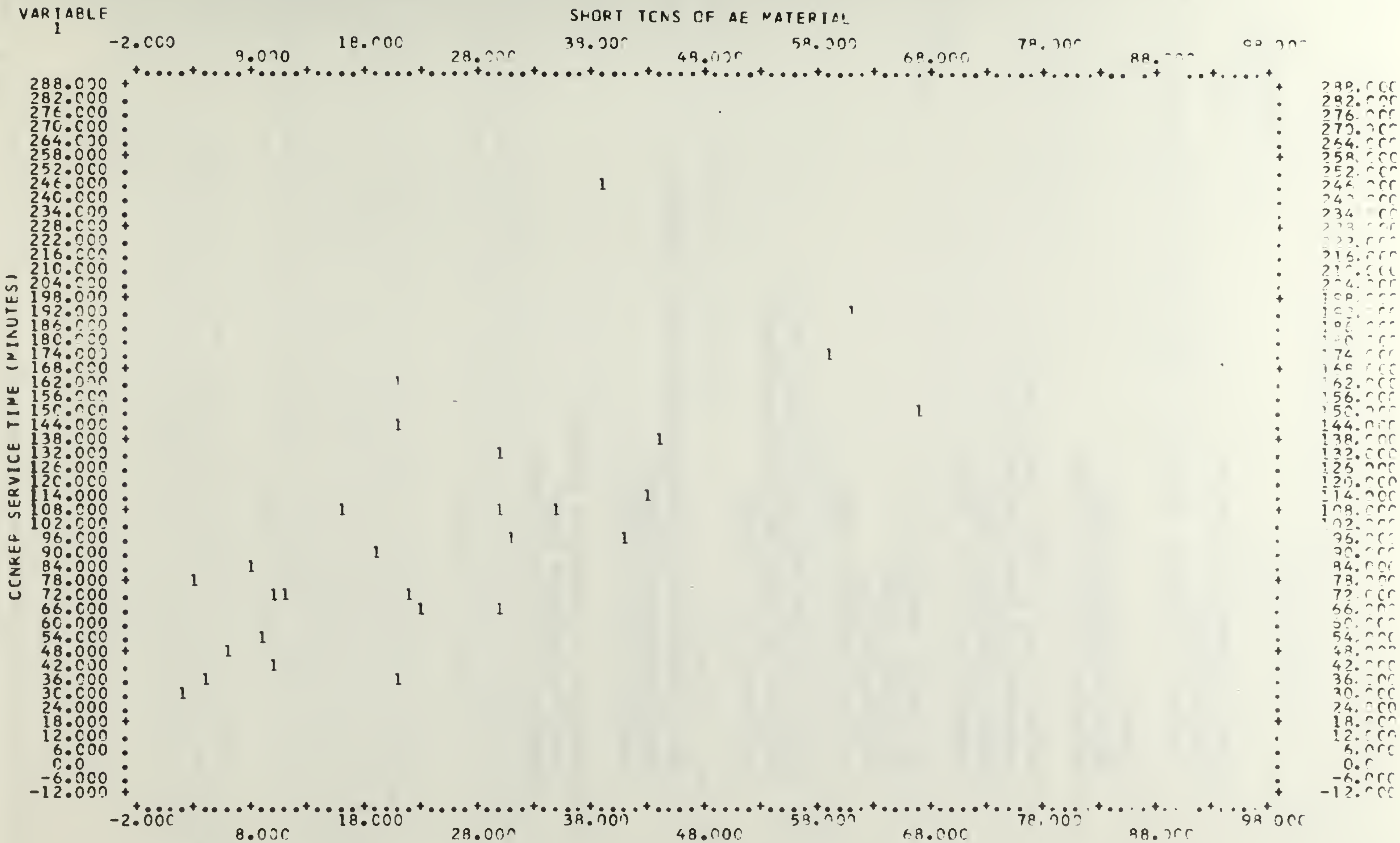

FIGURE 18. BIMED C2D SCATTER DIAGRAM FOR AE-CD (NIGHT) SITUATION, AE MATERIAL VS SERVICE TIME

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13. ABSTRACT

Most analytic and computer simulation models of the Navy's CONREP phase of underway replenishment operations have assumed exponentially distributed service times for two reasons: (1) lack of better estimate of the true distribution, and (2) such an assumption leads to more tractable mathematical solutions.

By analyzing several different combinations of replenishment vessels and combatant ships, it is demonstrated through goodness-of-fit tests that gamma distributions whose parameters can be estimated from actual operational data are more precise estimates of the actual underlying CONREP service time distributions. Furthermore, it is shown that the distribution can be made Erlang by minor adjustments to the parameter estimates of the gamma distribution. Such a procedure might be desirable for analytical models employing Laplace transforms.

The data is subjected to linear regression techniques in an effort to develop meaningful and accurate functional relationships between service time and customer needs. The results indicate that the standard error of the estimating relationships would be too large to be of any practical use in planning an underway replenishment operation.

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