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VORTEX FLOW IN AN ACTUAL GAS by<br>IT. COMDR. W. C. VICKREY; JR., U.S.N.

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$\square$

## SUMMARY

Applications of the vortex have beer used in engineering for many years. Since the recent publication of Rudolph Hilsch's work in Germany, it is widely known that there is a transfer of energy from the center to the outside of a vortex in an actual gas. However, there is no theory which satisfactorily explains this transfer of energy in a vortex.

In this investigation, an effort was made to determine the characteristics of a vertex in an actual gas, the vortex being formed inside a pipe by the introduction of air tangent to the inside wall of the pipe.

It was found that the equilibrium condition for a vortex in an actual gas is a state in which there is no heat transfer and in which there are no viscous forces acting. These conditions call for the static temperature and $\frac{v e l o c i t y}{r a d i u s}$ being constant and for the total temperature, static pressure, and tctal pressure increasing with increasing radius.

The investigation was conducted by the author in the Mechanical Engineering Department, Rensselaer Polytechnic Institute, Troy, New York

Subscript (R) refers to quantities at the outer radius of the vortex.

Subscript (c) refers to quantities at the center of the vortex.
$F=$ force.
$g$ = acceleration of gravity in ft. per sec. per sec.
$k$ = thermal conductivity in Btu per hr. per square ft. per degree per ft.
$\mathrm{M}=$ Nach number.
$n^{\prime}=$ Nach number as computed using the uncorrected static pressure reading.
$P_{0}=$ total pressure.
$P^{\prime}=$ static pressure as obtained from probe before correcting for instrument error.
$P=$ static pressure.
$P_{1}=$ pressure before metering nozzle.
$P_{2}=$ pressure after metering nozzle $=$ ouppiy pressure to vortex.
$P_{3}=$ static pressure in center of vortex at closed end of pipe.
$P_{4}=$ static pressure at a radius of $\frac{1}{2}$ inch at closed end of pipe.
$P_{5}=$ static pressure at a radius of $5 / 8$ inch at closed end of pipe.

SYMBOLS (cont'd)
$P_{6}=$ static pressure on pipe wall at a distance of $7 / 16$ inch from closed end of pipe.
$q=$ heat transferred in Btu per hr.
$=$ = radius in inches.
$R=$ maximum radius in inches.
R = specific gas constant for air.
$\mathrm{m}_{0}=$ total temperature as obtained from thermocouple before correcting for instrument error.
$\mathrm{T}_{0}=$ total temperature.
$T=$ static temperature.
$\mathrm{v}=$ velocity in ft. per sec.
$W=$ weight flow in lbs. air per sec.
$\mathrm{X}=$ distance from closed end of pipe in inches.
$d p=\frac{v_{R}{ }^{2}}{2 g R T}$ as computed using the pressure ratio in the vortex.
$A T=v_{K}^{2}$ as computed using the total temperature ratio in the vortex.
$\gamma=$ adabatic gas constant. In this report, $(\gamma=1.395)$
*: was used.
z = density, slugs per cu. ft.
$\mu$ ecofficient of viscosity in sluss per sec. per sq. ft. per ft,
$\omega=$ angular velocity in radians per sec.

## VORTEX FLOW IN AN ACTUAL GAS

## INTRODUCTION

The classical free vortex derived from potential flow theory is based upon reversibility and constant total energy, neglecting the heat transfer, viscosity, and diffusion which are present in an actual gas. Writing an equation for the summation of radial forces in a vortex, and solving this equation in accordance with the above assumptions leads to the result that vr = constant. This requires that the velocity become infinite and the absolute pressure become zero at the center of the vortex where the radius is zero. These results are, of course, physically impossible. Also, it is known that the assumption of constant tctal energy is not true, since large total temperature gradients have been observed from the center to the periphery of a vortex.

The purpose of this investigation was to determine the characteristics of vortex flow in an actual gas. Rudolph Hilsch, a German low-temperature physicist, used a vortex to obtain low temperatures, drawing off cold air from the center of the vortex. ${ }^{1}$ High total temperatures were observed at the outer radius. Hilsch gave credit to an unidentified Frenchman for the discovery of
this phenomenon. Also in Germany, the vortex has been successfully used as a dust separator. ${ }^{2}$ There are other applications of the vortex in engineering, for example, centrifugai n purifiers, ana centrifugal compressor design. So ior as is known, however, there has been no $\therefore$ icory evcived, surforted by experimental data, which. :..I explain the pronomena occuring in a vortex.

The U.S. Navy and the U.S. Air Forces have conducter flights through tropical cyclones for the purpose of collecting data. The author has been unable to obtain copies of this data or any information concerning any theory which may have been developed from the data so obtained.
C.J. Ricketts and J.L. Genta ${ }^{3}$ investigated the vortex about a year ago. Unfortunately, due to instrumentation difficulties, Ricketts and Genta were unable to obtain sufficient data to build up a theory.

Textbooks on fluid mechanics and aerodynamics present the vortex as being a free vortex with a thin, solid, cylindrical core rotating with constant angular velocity. In this way, the physically impossible situation of the velocity becoming infinite in the center is avoided. However, no account is taken of the decrease in total energy which actually occurs from the periphery to the center of such a vortex.

Professor Neil P. Bailey, head of the Department of Mechanical Engineering, Rensselaer Polytechnic Institute, Troy, New York, and members of his staff, have proposed a theory that a vortex in an actual gas comes to a state of cquilibrium in which there is no heat transfer and in which there are no viscous forces acting. Such a zeory accounts for the total temperature gradient in a vortex, however this theory has not yet been checked with actual date.

This report is concerned with the case of a vortex in an actual gas in a cylindrical container, the vortex being induced by the injection of air tangent to the wall of the container. The investigation was conducted in the Mechanical Engineering Department, Rensselaer Polytechnic Institute, Troy, New York, during the period February-Hay 1948.

The author wishes to acknowledge the invaluable guidance and advice of Professor Neil P. Bailey. Thanks also are due to members of Professor Bailey's staff for helpful advice and the services of the machine shop.

## EOUIPIENT AND PROCEDURE

To obtain a vortex, a vortex valve designed by Professor Neil P. Bailey was used. (See Fig. 2). On one and of this valve is a series on vanes spaced around the periphery, these vares being adjustable so that the angle : which the flow enters the pipe may be varied. For this nnvestigation, the vanes were set at an angle of ten degrees with the tangent to the inside pipe wall. The end of the valve which has the vanes is inserted into the plenum chamber, thus the flow enters through the vanes and is discharged out the two inch diameter copper pipe.

To obtain the static pressures in the center of the vortex alone the pipe axis, a probe was traversed along the pipe axis; the probe being entered from the open end of the pipe. This probe was a stainless steel tube, 0.101 inch outside diameter, with a hole in the side $7 / 16$ inch from the end.

The center traverse for total temperature was made with an iron-constantan thermocouple. The thermocouple wires were led through insulators, which, in turn, were led through a length of $\frac{-1}{4}$ inch outside diameter copper tubing; thus making a stiff probe which could be traversed a.long the pipe axis.

To obtain measurements at any radius away from the center of the vortex, it was not practicable to use probes
traversed along the pipe as "as done for the center traverses. The forces which existed in the vortex away from the center were sufficient to break these long slender probes. If the probes were made strong enough to resist Dreakage, hey would still be bent or deflected so timat it was imposs:ble to posj.tion the probe at any desired. jont. The vibration was so violent as to make accurate readings impossible. Also, as soon as the probe was movec away from the center of the vortex, the flow was altered as indicated by a distinct change in sound.

These conditions made it necessary to obtain data by traversing probes along the radius. In this manner, a much shorter and much smaller probe could be used. Such a probe would be rigid, could be accurately positioned, and being very small would not disturb the flow as much as the probes which were inserted axially.

Before designing the probes, it was necessary to know a little about the character of the flow. It was known that there was a velocity component parallel to the pipe axis, but it was not known whether or not there was a radial component of velocity. To try to determine this, an open end impact tube with a right angle bend at the end was used. The probe was carefully positioned so that the open end was at the pipe axis, then the probe was traversed horizontally along the radius. At various points along
the radius, the probe was rotated and its position noted when the resing "as a mayimum. As nearly as could be determined by eye, the bert end o. the probe was vertical at the ooirt $\sim$ maximum rearing, thus indicating that the flow was in concentric circles, there being no rarial component. This was done for various supply pressures $1_{\sim_{1}}$ ) to twenty inches of mercury gage. Above this supply pressure, no check could be mane bec:use the probe vas bent by the forces in the vortex.
$\therefore$ Ithough the above check was admittedly approximate, the probes to be used vere desisned assuming that there was no racial component of flow. The axial component of flow would not afect the sccuracy of the resdins from the type of probes hich were in mind. Investigation wes limiter to a supply pressure of arout twerty inches of mercury gage, inasmuch as the possibility of radial flow "as not investigater at higher supply pressures.

Total nressure, static pressure, and total temperature were desired along the radius; and probes were made to obtain these quantities. All three probes were made of stainless steel tubing. The outside diameter was 0.048 inches, and the inside diameter was 0.029 inches. The probes ere 11 approximately three inches long. For the total pressure tube, one end of the steel
tube was closed and a 0.0135 inch diameter hole was drilled in the tube wall 0.10 inch from the closed end. A $3 / 8$ inch diameter brass hose fitting was soldered on the other en. This probe was tested against an open chd impact thbe and gave the same reading. The total riessure was obtaired by rotating the probe for the razimum reading.

For static pressure, a Ser's disk ${ }^{4}$ was used. A small brass circular disk was soldered on the end of the stainless steel probe, the plane of the disk being perpendicular to the axis of the probe. This disk was 0.2 inch in diameter and 0.02 inch thick. The hole in the disk was the same as the outside diameter of the steel tube, so the end of the tube was brought out flush ith the bottom of the disk. The bottom of the disk was flat, and the top side was beveled to produce a sharp edge. A $3 / 8$ inch diameter brass hose fitting was solrered on the other end. This probe was tested in straight flow, using an ordinary static pressure traverse tube with a hole in the side as the standard. The Ser's disk was found to give low readings, therefore it was compared with the static pressure traverse tube at various Mach numbers and a correction curve was made up. (See Curve No. l). Mach number was determined from total and static pressure. Although the Ser's disk was tested in straight flow,
it was used in circular flow. This gives the possibility of some component of velocity pressure being measured due to the fact that, the flow is curved and is not exactly parallel to the Ter's disk. This might cause the static pressure readirg to be slightly high and tend to cancel out the low reading which it gave in straight flow. Therefore, the value of Curve No. 1 is questionable. However, all data presente. in this report were reduced using the corrections from Curve No. l: Some of the data were reduced using the corrections of Curve No. 1 and again without this correction, and the conclusions to be drawn from the data were not affected. Obtaining static pressures presented the greatest problem, and no doubt the static pressures obtained can be improved upon. It is believed, though, that the effect of the flow curvature is not sufficient to affect the validity of the conclusions regarding the character of the flow.

It may be noted that the smaller the disk, the smaller would be the effect of the velocity component in a curved flow. However, smaller disks were tried, and they gave such large errors in straight flow that they were discarded.

The thermocouple was made of 30 gage iron and constantan wire. The wires were led through the 0.048 inch diameter steel tube and brought out through a hole in the
side of the tube approximately 0.1 inch Irom the end, this end of the tube being closed. The thermocouple junction was placed about even with the outside of the steel tube, thereby forming a good impact area. Then tested, this thermocouple gave readings slightly lower than total temDerature, so a curve of error versus Mach number was made up. (See Curve No. 2). The thermocouple was tested by placing it in a nozzle attached to the plenum chamber. It was assumed that there was constant total energy from the plenum chamber through the nozzle, and the reading of a testing thermometer in the plenum chamber was taken as standard. Mach number was determined from total and static pressure.

All three of the probes discussed above, which were used for measurements along the radius, were led through brass plugs. These plugs had a packing gland and packing nut on one end and were threaded on the other end. At points along the pipe where it was desired to obtain measurements, a small square brass pad was soldered to the pipe to increase the thickness. The pad and pipe were then drilled and tapped to receive the plug with the probe. Dividers and a scale were used to position the probes along the radius.

For temperature measurement, a Leeds and Northrup potentiometer indicator No. $8657-C$ was used, with the cold
junction at room temperature.
All pressures were measured with mercury manometers.
Air was supplied by a Schramm air compressor, Model 210, rated at 206 cu . ft. of free air per min. at 1175 r.p.m.

The air compressor was driven by a Westinghouse three phase, 60 cycle, 220-440 volt, 50 horsepower induction motor with a rated full load speed of 1175 r.p.m.

A schmatic drawing of the set-up is shown in Figure . 1. The plenum chamber was supported on a lathe beत.

The vortex valve was originally six inches from the closed end to the open end. This six inch length was used to obtain the center traverses for static pressure and total temperature shown on Curves No. 3 and 4. A back flow of air from the room down the center of the pipe existed due to the low pressure in the center of the vortex. On the basis of Curves No. 3 and 4, it was thought that a radial traverse could be made at $X=2.5$ inches and be free from this back flow. Accordingly, the pipe was instrumented and traverses were made at $X=2.5$ inches. However, a back flow down the center was still present, as indicated by the direction in which the total pressure tube pointed for maximum reading. This direction was indicated by a scribe mark on the hose fittirg in the
same plane as the impact hole. To try to get away from this back flow, the vortex valve was increased in length from six inches to two feet, and holes for traversing were provided at: $X=1.4$ inches; $X=2.5$ inches; $X=3.8$ inches; $X=8.1$, nches; $X=14.1$ inches; and $X=20.1$ inches. Plugs were provided to seal off the holes not in use. When traverses were again made, a small back flow was still indicated from $X=8.1$ inches to $X=24$ inches. Next, a $13 / 4$ inch diameter disk was used to plug out the back flow. This disk was moved axially until it appeared that all back flow had ceased. This position for the disk was about $1 / 8$ inch outside the pipe. If placed farther outside, room air leaked around the edge of the disk and entered the pipe. If placed inside the pipe, the arnular passage between the disk and pipe wall was not large enough to let all the flow pass, resulting in some of the flow going to the center of the pipe at the disk and flowing back up the pipe. The data thus obtained with the back flow cut out was of the same character as the data obtained with a small back flow present.

All tables and curves presented in this report are from data taken in the two foot long pipe without cutting out back flow, except that Curves No. 3 and 4 and Table No. $X$ are from data taken in the six inch long pipe, also without cutting out the back flow.

Preliminary investigation showed considerable agreement with the theory proposed by Professor Neil P. Bailey. (This theory was discussed briefly in the introduction to this report). Therefore, the general procedure in this investigation vas as follows:
(a) To obtain reliable data in a vortex.
(b) To compare this data with Professor Bailey's theory in an effort to prove or disprove this theory.
(c) If the theory in (b) is disproved, develop a theory based on the data obtained.

Since the procedure in this investigation was to prove or disprove the theory proposed by Professor Ifeil P. Bailey, and if the theory was disproved, to develop a theory based $\because=$ the deta: the theory will be presented at this point so that it may be freely referred to in the discussion of the results.

As mentioned in the introduction to this report, the theory is based upon the assumption that at ecuilibrium:
(a) there is no heat transfer.
(b) there are no viscous forces acting.

From assumption (a), we may write:

$$
\begin{equation*}
q=-k 2 \pi r \frac{d T}{d r}=0 \tag{1}
\end{equation*}
$$

This recuires that:

$$
\begin{equation*}
\frac{d T}{d r}=0 \tag{2}
\end{equation*}
$$

And integrating eq. (2) gives:

$$
\begin{equation*}
T=\text { const. } \tag{3}
\end{equation*}
$$

Since the flow is curvilinear, it is the angular velocity gradient, $\left(\frac{d w}{d r}\right)$, of which adjacent layers are conscious and not the linear velocity gradient, ( $\frac{d V}{d r}$ ). From assumption (b) we may write:

$$
\begin{equation*}
F_{\text {viscous }}=-\mu 2 \pi r \frac{d w}{d r}=0 \tag{4}
\end{equation*}
$$

This requires that:

$$
\begin{equation*}
\frac{d \omega}{d r}=0 \tag{5}
\end{equation*}
$$

And integrating ec. (5) gives:

$$
\begin{equation*}
\Leftrightarrow=\text { const. } \tag{6}
\end{equation*}
$$

Hence, since a $=\frac{\mathrm{V}}{\mathrm{r}}$ :

$$
\begin{equation*}
\frac{V}{r}=\text { const. } \tag{7}
\end{equation*}
$$

or $\mathrm{v}=$ constr. rr
At equilibrium, the pressure gradient must balance the centrifugal forces, or :

$$
\begin{equation*}
d P=\frac{v^{2}}{r} d r \tag{8}
\end{equation*}
$$

Substituting $P=\frac{p}{g R^{1} T}$ from the gas equation:

$$
\begin{equation*}
d P=\frac{\mathrm{v}^{2}}{\mathrm{r}} \frac{\mathrm{gR}^{\prime} \mathrm{T}}{\mathrm{gR}^{\prime} \mathrm{T}} \mathrm{dr} \tag{9}
\end{equation*}
$$

Rearranging:

$$
\begin{equation*}
\frac{d P}{P}=\frac{v^{2}}{r} \frac{1}{g R^{\prime} T} d r \tag{10}
\end{equation*}
$$

multiplying and dividing by (r), eq. (10) may be written:

$$
\begin{equation*}
\frac{d P}{P}=\frac{v^{2}}{r^{2}} \quad \frac{1}{g R^{1} T} \quad r d r . \tag{11}
\end{equation*}
$$

From eq. (7), $\frac{v}{r}=$ constr. ; and from eq. (3), $T=$ const.; therefore eq. (11) may be written:

$$
\begin{equation*}
\int \frac{d P}{P}=\left(\frac{v}{r}\right)^{2} \frac{I}{g R^{1} T} \int r d r \tag{12}
\end{equation*}
$$

Since $\frac{v}{r}=$ const. $=\frac{v_{R}}{R}$ :

$$
\begin{equation*}
\int \frac{d P}{P}=\left(\frac{v_{R}}{R}\right)^{2} \quad \frac{1}{g R^{\top} T} \int r d r \tag{13}
\end{equation*}
$$

Integrating ec. (13) from the center of the vortex to any radius:

$$
\begin{equation*}
\int_{P c}^{P} \frac{d P}{P}=\left(\frac{v_{R}}{R}\right)^{2} \frac{I}{g R^{r} T} \int_{0}^{r} r d r \tag{14}
\end{equation*}
$$

Or: $\ln \left(\frac{p_{0}}{P_{C}}\right)=\left(\frac{v_{R}}{R}\right)^{2} \frac{l}{g^{R} T T} \quad \frac{r^{2}}{2}$
Rearranging:

$$
\begin{equation*}
\left.\ln \left(\frac{P_{p}}{p_{C}}\right)=\frac{v_{R}^{2}}{2 V^{2} T} r^{2}\right\}^{2} \tag{16}
\end{equation*}
$$

Letting:

$$
\begin{equation*}
\frac{v_{R^{2}}}{2 g R^{\prime} T} \tag{17}
\end{equation*}
$$

And substituting this into eq. (16) and taking the anti-log:

$$
\begin{equation*}
\frac{p}{P_{c}}=e^{d P} \tag{18}
\end{equation*}
$$

Equation (18) describes the mamer in which the static pressure increases from the center of the vortex to the maximum radius of the vortex. Integrating eq. (13) from the center of the vortex to the maximum radius of the
 Integrating and rearranging gives:

$$
\begin{equation*}
\ln \left(\frac{P_{R}}{P_{c}}\right)=\frac{v_{R}{ }^{2}}{2 R^{\top} T} \tag{20}
\end{equation*}
$$

Using eq. (17) in eq. (20) gives:

$$
d=\ln \left(\frac{P_{R}}{P_{C}}\right) \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots(21)
$$

Equation (21) provides a means for evaluating the constant (d) if the pressures at the center and outer radius of the vortex are known, Equation (21) may also be written:

$$
\begin{equation*}
\frac{P_{R}}{P_{c}}=e^{d} \tag{22}
\end{equation*}
$$

Integrating eq. (12) from the center of the vortex to the outer radius, we obtain:

$$
\begin{equation*}
\ln \left(\frac{P_{R}}{P_{c}}\right)=\left(\frac{v}{r}\right)^{2} \quad \frac{R^{2}}{2 R^{T} T} \tag{23}
\end{equation*}
$$

Solving for $\left.{ }^{2}\right)^{2}$ :

$$
\begin{equation*}
\left(\frac{v}{r}\right)^{2}=\frac{2 g R!T}{R^{2}} \ln \left|\frac{P_{R}}{P_{c}}\right\rangle \tag{24}
\end{equation*}
$$

Combining equations (21) and (24):

$$
\begin{equation*}
\left(\frac{v}{r}\right)^{2}=d \frac{2 \pi R \cdot T}{R^{2}} \tag{25}
\end{equation*}
$$

From "The Thermodynamics of Air at High Velocities" by Neil P. Bailey ${ }^{5}$ (hereafter referred to simply as H.V.T.), the following relationship between static temperature, total temperature, and Mach number is obtained:

$$
\begin{equation*}
\frac{T_{0}}{T}=1+\frac{r-1}{2} \mathrm{~m}^{2} \tag{26}
\end{equation*}
$$

Also from H.V.T.:

$$
\begin{equation*}
M^{2}=\frac{v^{2}}{3 g R^{T} T} \tag{27}
\end{equation*}
$$

Combining equations (26) and (27):

$$
\begin{equation*}
\frac{\mathrm{T}_{0}}{\mathrm{~T}}=1+\frac{x-1}{2} \frac{\mathrm{v}^{2}}{\ddot{d} \mathrm{~g}^{1} \mathrm{~T}} \tag{28}
\end{equation*}
$$

Multiplying and dividing by $\left(r^{2}\right)$ in the last term:

$$
\begin{equation*}
\frac{T_{0}}{T}=1+\frac{i-1}{\varepsilon} \frac{r^{2}\left(\frac{V}{r}\right)^{2}}{r g R^{1} T} \tag{29}
\end{equation*}
$$

Su+たむituting ec. (25) for $\left(\frac{\mathrm{v}}{\mathrm{r}}\right)^{2}$ and simplifying:

$$
\begin{equation*}
\frac{T_{0}}{T}=1+\frac{\gamma-I}{\partial} \quad\left(\frac{r}{R}\right)^{2} a \tag{30}
\end{equation*}
$$

From eq. (7a), the velocity is zero at the center of the vortex, therefore static temperature and total temperature are the same at the center of the vortex. From eq. (3), the static temnerature is a constant, therefore the total temperature ai che center of the vortex may be substituted for the static tenperature at any radius, and eq. (30) may be written:

$$
\begin{equation*}
\frac{T_{0}}{T_{O_{C}}}=1+\frac{r-1}{r}\left(\frac{r}{R}\right)^{2} \gamma \tag{31}
\end{equation*}
$$

Eqnation (31) describes the manner in which the total temperature increases from the center of the vortex to the outer radius.

Taking conditions at the maximum radius in eq. (28):

$$
\begin{equation*}
\frac{{ }^{1} o_{R}}{T}=1+\frac{1-1}{3} \frac{V_{R^{2}}}{2 \mathrm{R}^{1} \mathrm{~T}} \tag{32}
\end{equation*}
$$

Combining equations (17) and (32); and substituting $T=T_{O_{C}}$ :

$$
\begin{equation*}
\frac{T_{a_{R}}}{T_{O_{C}}}=1+\frac{\gamma-1}{\gamma} \tag{33}
\end{equation*}
$$

Solving for ( ) :

$$
\begin{equation*}
\Leftrightarrow=\frac{x}{x-1}\left[\frac{T_{0}}{T_{O_{C}}}-1\right] \tag{34}
\end{equation*}
$$

Equation (34) provides a means of evaluating the cons tant ( $($ ) if the total temperature is known at the center and the maximum radius of the vortex.
F.: In H.V.T., the reiationship between static pressure, total pressure, end rach number is:

$$
\begin{equation*}
\left.\frac{P_{0}}{P}=\left[1+\frac{x-1}{2}\right]^{2}\right]^{x-1} \tag{35}
\end{equation*}
$$

Combining equations (26) and (35):

$$
\begin{equation*}
\frac{\mathrm{P}_{0}}{\mathrm{P}}=\left(\frac{\mathrm{T}_{0}}{\mathrm{~T}}\right) \frac{\sigma}{\gamma-1} \tag{36}
\end{equation*}
$$

Combining equations (30) and (36):

$$
\begin{equation*}
\frac{P_{0}}{P}=\left[1+\frac{r-1}{r}\left(\frac{r}{R}\right)^{2} a\right] \frac{r}{x-1} \tag{37}
\end{equation*}
$$

From equation (18):

$$
\begin{equation*}
P=P_{c} e^{d\left(\frac{r}{R}\right)^{2}} \tag{38}
\end{equation*}
$$

Combining ecuatigns (37) and (38):

$$
\begin{equation*}
\frac{P_{0}}{P_{c}}=e^{d\left(\frac{r}{R}\right)^{2}}\left[1+\frac{r-1}{r}\left(\frac{r}{R}\right)^{2} d\right] \frac{r}{r-1} \tag{39}
\end{equation*}
$$

It has already been shown that the velocity at the center of the vortex is zero. Therefore, the static pressure and total pressure are the same at the center of the vortex, and ( $P_{O_{C}}$ ) may be substituted for $P_{c}$ in ecuation (39):

$$
\frac{P_{0}}{\mathrm{P}_{O_{c}}}=e^{x\left(\frac{r}{R}\right)^{2}}\left[1+\frac{\frac{r}{3}}{3}\left(\frac{r}{R}\right)^{2}\right]^{\frac{\gamma}{r-1}} \ldots \ldots \ldots(40)
$$

Equation (40) describes the manner in which the total pressure increases from the center of the vortex to the outer radius. From the equations developed above, it can be seen what the characteristics of the vortex are according to this new theory. From equation (3), the static temperature is constant. From ecuations (7) and (7a), (Y) is a constant, ar the velocity is zero at the cente: and increases with increasing radius. From equation (18),
the static pressure is a minimum in the center and increases with increasing radius. From equation (31), the total energy is nnt constant, the total temperature being a minimum in the center and increasing with increasing radius. And tirally, from esuation (40), the total pressure is a minimum in the center and increases vith increasing raciars,

## ITSULTS AND DISCUSSION

The data taken in this investigation consists of static pressure, total pressure, and total temperature taken along the radius at various points along the pipe as the vortex develops and starts to dissipate. From these data, ach number, velocity, static temperature, and $\left(\frac{V}{r}\right)$ were computed. $\left(\frac{P_{C}}{F_{C}}\right)$ and $\left(\frac{T}{T_{0}}\right)$ were also computed from the data for various radil for comparison with the theoretical values as computed from equations (18) and (31) in the section on theory.

Data taken at $X=2.5$ inches in the two foot long pipe using approyimately ten inches of mercury (gage) supply pressure, and quantities derived from these data, are presented in Tables $I$ and $I I$ and in Curves 5 and 5-A. A comparison of static pressure ratio and total temperature ratio with thoon is presented in Table III and Cucies $5-B$ and 5-C.

Data taken at $X=8.1$ inches in the two foot long pipe
using approximately ten inches of mercury (gage) supply pressure, and cuantities derived from these data are presented in Tarles IV and $V$ and Curves 6 and 6-A. A comparison o? at 子tic pressure ratio and total temperature ratio witn th: Y is presented in Table VI and Curves $6-B$ and $6 \cdot C$.

Data tokn: $a t \mathrm{X}=14.1$ inches in the two foot long pipe using approyimately ten inches of mercury (gage) supply pressure, and çuantities derived fyom these data are presented in Tables VII and VIII and Curves 7 and 7-A. A comparison of static pressure ratio and total temperature ratio with theory is presented in Table IX and Curves 7-B and 7-C.

Data taken at $X=2.5$ inches in the six inch long pipe using approxtmately ten inches of mercury (gage) supply pressire ara presented in Table X.

Data taken a.t $\therefore=1.4$ inches in the two foot long pipe using 21 inches of mercury (gage) supply pressure are presented in Table XI.

In this investigation, $r: 0.9$ inch was taken as the maximum radius of the vortex due to the difficulty of ovtaining reliable raadings on the pipe wall.

In comparins tar enve of (i) versus ir) on Curyse 5-A, 6-A, and 7-f. $\because i t w i l l$ be noted that $\left(\frac{V}{T}\right)$ is greater at the outer radii than at the inner radii at $X=2.5$
inches; while at. $X=14.1$ inches, $\left(\frac{V}{r}\right)$ is greater at the inner radia than at the outer radii. The theory calls for $\left(\frac{y}{r}\right)$ bein, $\because$ stant at equilibrium. Before the vortex is fuユu expect the.t $\because \dot{\sim}$; suld be greater at the outer radii than at the innon adii As a limiting case, one could take the moment ac wan the air supply is turned on. The air in the pipe is at rest and $\left(\frac{V}{r}\right)$ equals zero. As the air enters at high velocity at the outer periphery of the pipe, the $\left(\frac{V}{r}\right)$ will be high at these outer radii; while at the inner radii, $\left(\frac{V}{r}\right)$ will still be zero. By viscous action, the inner mass of air will be pulled around until equilibrium is reached, at which time there will be no viscous forces acting, and ( $\frac{V}{r}$ ) will be constant, as was shown in the thecretical development. As the effect of friction at tire floe wail begins to be felt, the flow at the outer radii will commence to decrease in velocity, thus causing $\left(\frac{v}{-}\right)$ to be less at the outer radii than at the inner radii. Yiscous forces viil agein start to act due to relative morion between adjacent layers, the velocity beirg reduced tomars the center until the angular velocio; is zero, and the ronter: rill dissipate.

At $X=8 . j$ iciones Curve 6..A) it is seen that (V) in constant from the oonter out to $r=0.6$ inches, and that from this point on out, $\left(\frac{V}{r}\right)$ falls off gradually, apparently
due to friction effects. By the time the flow reaches $X=14.1$ inches (Curve 7-A), the effect of friction is very noticeable and ( $\frac{V}{r}$ ) decrease sharply from the center to the outsia the vortex quite definitely dissipating. At $X=2.5$ incies (Curve $5-A$ ), ( $\frac{v}{r}$ ) increases sharply from the centor to the outside, the vortex being not fully developed at this point. The slight drop in $\left(\frac{V}{r}\right)$ at the maximum radius is probably due to friction. It would not be expected that the vortex would be found in equilibrium exactly in accordance with the theory, due to friction at the wall, which the development of the theory did not take into account. The curves of $\left(\frac{V}{r}\right)$ versus (r) are, then, quite in accord with the theory.

It will be noted that, in drawing curves through the experimental points, the point $r=0.1$ has $b \in e n$ neglected. The worst case is at $X=8.1$ inches (Curve 6-A). To bring this point on to the curves requires a ratio of total pressure to static pressure of 1.004 instead of the ratio 1.001 which was obtained, ?his error is well within the experimental error in reading which might be expected. Also, a small error in velocity becones greatly magnified when तivided by $O_{\mathrm{I}}-\mathrm{t}$, o obtain $\left(\frac{\mathrm{V}}{\mathrm{r}}\right)$. It was; therefore, consirered necess: $\because=$ neglect this point; since with the instrumentation used, it was not possible to obtain the required degree of accuracy at such low Mach numbers.

The comparison of the static pressure ratio in the vortex with the static pressure ratio as determined by theory is shovir on Curves 5-3, 6-B, and 7-B. If one again takes noment at which the air is turned on, the ratio ( $\frac{P_{C}}{C}$ ) à unity across the vortex, or less than is called for by the theory. As the vortex develops, this ratio will blu...i 'lp until the vortex is fully developed, and the pressure ratio as determined by test will agree with the pressure ratio as determined by theory. As friction affects are felt and the static pressure at the outside increnses as the Mach number decreases, this pressure ratio will become greater than that called for by theory. At $X=2.5$ inches (Clirve $5-B$ ), it is seen that the the ratio $\left(\bar{P}_{C}\right)$ as determined by test is less thon the theoretical value; at $X=8.1$ inches (Curve $6-B$ ), the agreement between test and theory is quite good; and at $X=14.1$ inches (Curve $7-B$ ), the ratio by test is greater than theoretical. As with the curve of $\left(\frac{V}{r}\right)$, the prescure ratio ( ${\underset{\sim}{c}}_{C}$ ) as determined by test is in very good agreement with the theory.

An examination of Curves $5-\mathrm{C}, 5-\mathrm{C}$, and 7-C comparing the total temperatine ratio ( $\frac{T_{0}}{T_{0}}$ ) in the vortex with the thsoretical curven : onas that ${ }^{C}$ the test is in close agreement with the theory at $\mathrm{X}=2.5$ inches. Beyond thí point, heat is being transferred back to the center of
the vortex. A comparison of these results with the results for the pressure ratio would indicate that equilibrium in heat transfer is attained before equilibrium in balance of forces is reached.

It will be remem'ered that the theory calls for the static temperature being constan. $\quad$ ir once more, the limiting case is taken just at the moment at which tive air supply is turned on; the total temperature is constant, and the rotic ( $\frac{T_{O}}{T_{O C}}$ ) is unity, or less than theoretically called for. As air $a t$ high velocity enters at the outer radii, the static temperature at these outer radii will be less than at the center, and a transfer of heat from the center to the outside will occur. This process will continue until equilibrium is reached, at which time the total temperature gradient will be a maximum and the static temperature gradient will be zero. As friction effects are felt and the Mach number decreases at the outer radii, the static temperature at these outer radii will rise above the static temperature at the inner radii and heat will be transferred back toward the center, causing the ratio ( $\frac{T_{0}}{T_{O_{C}}}$ ) to become greater than the theoretical value.

The curves of static temperature versus radius for the three values of (X) investigated are shown on Curves 5, 6, and 7. It can be seen that the static temperature is approximately constant, as called for by theory. The peculiar shape of the curve at $X=2.5$ inches (Curve 5) is not readily explained. As has been mentioned previoushy.
there was a slig't recirculation of room air down the center of the fipe. Since this room air is about 30 degrees Fahren it cooler than the air supplied to the vortex from tif inenum chamber, it was thought possible that the center had been cooled and that the static temperature wive should have a negative slope at this point, indicetinc that the transfer of heat to the outer radii was rot Jet complete. To check this point, data was taken at $X=2.5$ inches with the back flow cut out, as described in the section on "Equipment and Procedure". All other conditions were the same ${ }_{\wedge}^{25}$ ror Curve No. 5. The resulting data was almost an exact duplication of the data taken originally, therefore it was concluded that the back flow was not sufficient to have any noticeable effect on the vortex. An examination of the total te:nperatire curves and total temperature ratio curves reveals the fact that actually the transfer of heat is completed, or nearly complo ted, by the time the flow reaches $X=3.5$ inches. The maximum toter. temperature gradiont observed was at $X=2,5$ inches (Curve 5), indicating that the transfer of heat had been cumpleted befons in, next point of investigation was reached at $X=8$, jrinose The comparison of $\left(\frac{T_{0}}{T_{0}}\right)$ by test and by theora at $\therefore=2.5$ inches (Curve 5-C), indicates that the thonsfer of heat back to the center has
already started at the outer radii, since the test poirts fall above the theoretical points at the outer radii. It will be rememberod that the effect of friction was noted at the outer rain at $X=2.5$ inches on the velocity and $\left(\frac{V}{r}\right)$ curves al.s.. All of this would lead one to expect the static tomperature curve at $X=2.5$ inches to rise toward the cuior radii. The curve, iol fact, doos this, but at the oxtrme outer radii it again falls off. This may well be due to heat transfor through the copper pipe. A temperature differential exists across the pipe, and since the flow is circular, it has boen in contact with the pipe a much greater distance than the axial distance of 2.5 inches. For the small weight flow involved, (0.066 lbs. per sec.), a small heat transfer would have a notice. able effect on the temperature.

The results of the investigation, then, lend very strong support to tine theory that a vortex in an actual gas comes to a state of equilibrium in which there is no heat transfer and in wind then are no viscous forces acting.

In Tablo $X$ are presented data taken at $X=2.5$ jnches in the si-: ion long pipe with approximately ten jrchas of merowir: iedqe; supply pressure。 Upon roducires this data, it was andicated that making the pipe shortor had the effect of specding up the development of the
vortex and of productng slightly larger temperature and pressure gradients. At $X=2.5$ inches, in the short pipe, the vortex was dofinitely dissipating.

In Table $\mathrm{X}^{\top}$ re presented data taken at $\mathrm{X}=2.4$ inches in the foot long pipe "ith 21 inches or mercury (gage) sppiy pressure. Reducing these data incicated tha inoreasing the supply pressure increased the temperature ant pressure gradients and resulted in higher flow Nach numbers.

It is of interest to note, on Curve 4, that a temperature of minus 16.6 degrees Fahrenheit was observed in the center of a vortex in the six inch long pipe, with a supply pressure of 60.4 inches of mercury (gage) and an air supply total temperature of 107 degrees Fahrenheit.

From the results of this investigation, it may be concluded that $\overline{\text { a rex }}$ in an actual gas reaches a state of equilibriun $:$ mich there is no heat transfer and in which there 9 m n viscous forces acting. As was shown in the sectia': temperature anc. $\because \because$ are constant, and that the total temperature, stati: pressure, and total pressure increase with increasing radius. The above statements apply to a vortex which is induced by the injection of air at the outer radius.

It is recommended that an investigation be made into the case where the air is introduced at the inner radius, as in a vaneless diffuser. The same vortex valve used for this inveatigation sould be modified to serve for the recommended investiryticn. Air could be introduced through the chpper pipe and discharged out through the vanes, just the reverse of tine process in this investiga. tion. If a oortainer werf planeA zround the vanes, the equiralent of a vaneless diffusfr would be obtained.

## BIBLIOGRA PFY

1. Popular Scicr 2 Monthly, May 1947, pp. 144-146
2. Schulz, F.'. "Contributions to the Theory of Cyclone Dust Collectors", The Engineers Digest, $\because r 1.5$, No. 2, February, 1948; pp. 49$\because$.
3. Ricketts, C. . and Genta, J. L. - "Energy Transformation in a Free Vortex;" a graduate thesis dnne in the Department of Itechanical Encineering, Rensselaer Polytechnic Institute, Troy, N.Y., June, 1947.
4. Dodge, R. A. and Thompson, I. J. - Fluid Rechanics: Firsi Edition, McGraw Hill, 1937, pp. ar 3:
 Voiocieites": Journal of Aeronautical Sejpres: $\quad$ On, No, 3, July, 1944.

## CALCULATIONS

In order to illustrate the manner in which the data
 will be worker , ! here.

$$
\frac{F_{0}}{F^{\prime}}=\frac{29.90}{28.77}=1.04
$$

From H.V.T, ${ }^{5}$ :

$$
\left(\frac{P_{0}}{F}\right)^{\frac{\gamma-1}{\gamma}}=\left[1+\frac{\gamma-1}{8} M^{2 i}\right]
$$

Curve No. 2 in H. $\therefore$. T. is a graphical solution of the above equation, Using tins curve and $\frac{\mathrm{Y}_{0}}{\mathrm{p}^{1}}=1.04$, wo obtain:

$$
1:=0.238
$$

Using Curve No. I of this report, the error of the static pressure reading for $1: 0.238$ is 0.20 per cent. Then the true static mossura jas:

$$
\begin{gathered}
P=\frac{P I}{P 7}: \frac{P_{0}}{P}: \frac{29}{28.82}=1.087
\end{gathered}
$$

From Curve in. 2 is foe the tum mot mien is. 290

From Tune No. 2 the nerorthor $\mathrm{N}: 0.220$, the error of the themanounis goading is D. cos per sent. Then the tone total monomature 15

$$
r_{0}=\frac{T_{0}^{\prime}}{.90 \% 3}=\frac{0597}{.5973}=55 x^{\circ} \mathrm{R}
$$

From H.V.T:

$$
\frac{T_{0}}{T}=1+\frac{\pi-1}{2} M^{2}
$$

From a tabular s-intion of this equation, for $M=0.229$ :

$$
\frac{T_{0}}{T}=1.011
$$

Then the stat cmperature is:

$$
T=\frac{T_{0}}{1.011}=\frac{554}{1.011}=548^{\circ} \mathrm{R}
$$

Curve No. I in H . is a curve of Mach number versus the function . This curve was used to determine the velocity:

$$
\begin{aligned}
& \because=263.3 \mathrm{ft} . / \mathrm{sec} \\
& \frac{v}{\mathrm{r}}=\frac{263.3}{.5}=526 \mathrm{ft} . / \mathrm{in} . \mathrm{sec} . \\
& { }=\frac{0.5}{0.9}=0.5555 }
\end{aligned}
$$

From the test $\approx i i^{+} \therefore$

$$
\frac{30}{r_{c}}=\frac{30.2}{28.04}=1.0849
$$

From equation icily) of tine report;

$$
\alpha_{p}=\ln \left(\frac{P_{R}}{P_{c}}\right)=\ln 1.0859=0.0815
$$

From equation (18) of this report:2

$$
\frac{P}{P_{c}}(\text { theory })=e^{d\left(\frac{R}{R}\right)^{2}}=1.0855
$$

From the test data:

$$
\frac{p}{p}(t \circ s t)=\frac{28.82}{28.04}=1.0278
$$

From the teat Fatal

$$
\frac{T_{0}}{T_{O_{C}}}=\frac{5.32 .5}{542.5}=1.0387
$$

From equation (34) of this report:

$$
\alpha_{T}=\frac{\gamma}{-1}\left[\frac{T_{O_{R}}}{T_{O_{C}}}-I\right]=.1366
$$

From equation (31) of this report:

$$
\frac{T_{0}}{\mathrm{~T}_{O_{C}}} \text { (theory): } I+\frac{r-1}{3} \text { ( }{ }_{\mathrm{F}}^{2} r^{2}=2.0 .12
$$

From the test, data:

$$
\frac{T_{0}}{T_{O_{C}}}(\text { test })=\frac{554.0}{512.5}=1.021
$$

## TABLE I

DATA OBTAINED AT $X=2.5$ INCHES

| $\begin{gathered} T \\ \text { Inches } \end{gathered}$ | Pollig. <br> Gage | $\begin{aligned} & \mathrm{P}_{\mathrm{O} \prime \mathrm{Hg}} . \\ & \text { Abs. } \\ & \hline \end{aligned}$ | pl ${ }^{\prime \prime} \mathrm{Hg}$. Gage. | $\begin{aligned} & \text { pilHg. } \\ & \text { Abs. } \\ & \hline \end{aligned}$ | $\begin{aligned} & T_{0}^{\prime} \\ & \text { Iivo } \end{aligned}$ | $\begin{gathered} \mathrm{T}_{0}^{\prime} \\ \mathrm{og}_{\mathrm{g}} \\ \hline \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | 5.48 | 35.23 | 0.23 | 29.98 | 3.03 | 565 |
| 0.8 | 3.79 | 33.54 | -0.38 | 29.37 | 2.95 | 562.3 |
| 0.7 | 1.86 | 31.61 | -0.80 | 28.95 | 2.82 | 557.9 |
| 0.6 | 0.41 | 30.16 | -1.01 | 28.74 | 2.70 | 553.7 |
| 0.5 | -0.47 | 29.28 | -1. 16 | 28.59 | 2.54 | 548.2 |
| 0.4 | -0.88 | 28.87 | -1.25 | 28.50 | 2.45 | 545 |
| 0.3 | $-1.17$ | 28.58 | -1.36 | 28.39 | 2.37 | 542.3 |
| 0.2 | -1.32 | 28.43 | -1. 42 | 28.33 | 2.325 | 540.8 |
| 0.1 | $-1.44$ | 28.31 | -1.46 | 28.29 | 2.31 | 540.3 |
| 0 | -1.49 | 28.26 | -1.49 | 28.26 | 2.31 | 540.3 |
| $P_{1}-P_{2}$ $P_{2}$ | $=1.16$ $=10.4$ | in. Hg $42 \mathrm{in}$. | . Gage | $\begin{aligned} \text { Barometer } & =29.754 \\ { }^{\mathrm{T}} \mathrm{O}_{2} & =108^{\circ} \mathrm{F} \end{aligned}$ |  |  |
| $P_{3}$ | $=-1$. | 57 in. | . Gage | Room Temp. $=76^{\circ} \mathrm{F}$. |  |  |
| $\mathrm{P}_{4}$ | $=-1$. | 07 in. | Ig. Gage | W |  | 665 |
| $\mathrm{P}_{5}$. | $=-0.45$ | 45 in. H | Hg. Gage | Iron-Constantan Thermocouple, |  |  |
|  | $=2.79$ | in. Hg | . Gage | Reference $0^{\circ} \mathrm{F}$. |  |  |

TABLE II

$X=2.5$ INCHES

| $\begin{gathered} r \\ \text { Inches } \end{gathered}$ | M' | $\begin{gathered} \mathrm{P} \\ \mathrm{Hg}, \mathrm{Abs} . \end{gathered}$ | M | $\begin{gathered} \mathrm{T}_{0} \\ \text { Deg. } \mathrm{R}_{.} \end{gathered}$ | $\begin{gathered} \mathrm{T} \\ \mathrm{Deg}_{.} \mathrm{R} . \end{gathered}$ | $\begin{gathered} \mathrm{V} \\ \mathrm{ft} . / \mathrm{sec} \end{gathered}$ | $\begin{aligned} & \mathrm{v} / \mathrm{r} \\ & \mathrm{ft} . / \mathrm{in} . \sec \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | . 489 | 30.38 | . 468 | 568.5 | 544 | 535 | 594 |
| 0.8 | . 441 | 29.70 | . 422 | 565.3 | 546 | 486 | 608.5 |
| 0.7 | . 358 | 29.17 | . 342 | 560 | 546 | 393.5 | 561.5 |
| 0.6 | . 262 | 28.80 | . 254 | 555 | 546.5 | 292 | 486 |
| 0.5 | . 185 | 28.60 | . 177 | 549 | 545.5 | 203.2 | 406 |
| 0.4 | . 128 | 28.50 | . 128 | 546 | 544 | 146.4 | 366 |
| 0.3 | . 076 | 28.39 | . 076 | 543 | 541.5 | 86.9 | 239.5 |
| 0.2 | . 04 | 28.33 | . 04 | 541 | 541 | 45.6 | 228.2 |
| 0.1 | . 01 | 28.29 | . 01 | 540.3 | 540.3 | 11.4 | 114 |
| 0 | 0 | 28.26 | 0 | 540.3 | 540.3 | 0 | ind. |

TABLE III

$$
x=2.5 \text { INChes }
$$



## TABLE IV

DATA OBTAINED AT $X=8.1$ INCHES

| Inches | $\begin{gathered} \mathrm{P}_{0}{ }^{\text {GHg }} \\ \text { Gage } \\ \hline \end{gathered}$ | Po"Hg. <br> Abs. | P ${ }^{\prime \prime} \mathrm{Hg}$. <br> Gage | P'"Hg. <br> Abs. | $\begin{aligned} & T_{0}^{\prime} \\ & \text { Milli volts } \end{aligned}$ | $\begin{aligned} & \text { To' } \\ & \text { Deg. R. } \\ & \hline \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | 2.93 | 32.66 | 0.50 | 30.23 | 2.925 | 561.5 |
| 0.8 | 2.49 | 32.22 | 0.12 | 29.85 | 2.86 | 559.3 |
| 0.7 | 1.74 | 31.47 | -0.30 | 29.43 | 2.79 | 556.8 |
| 0.6 | 0.89 | 30.62 | -0.68 | 29.05 | 2.73 | 554.7 |
| 0.5 | 0.17 | 29.90 | -0.96 | 28.77 | 2.67 | 552.7 |
| 0.4 | -0.45 | 29.28 | -1.21 | 28.52 | 2.59 | 550 |
| 0.3 | -0.97 | 28.76 | -1. 45 | 28.28 | 2.52 | 547.5 |
| 0.2 | -1.35 | 28.38 | -1.58 | 28.15 | 2.44 | 544.7 |
| 0.1 | -1.61 | 28.12 | -1.65 | 28.08 | 2.40 | 543.3 |
| 0 | -1.69 | 28.04 | -1.69 | 28.04 | 2.375 | 542.5 |

$$
\begin{aligned}
P_{1}-P_{2} & =1.15 \mathrm{in} \cdot \mathrm{Hg} \cdot \\
P_{2} & =10.33 \mathrm{in} \cdot \mathrm{Hg} \cdot \text { Gage } \\
P_{3} & =-1.56 \mathrm{in} \cdot \mathrm{Hg} \cdot \text { Gage } \\
P_{4} & =-1.05 \mathrm{in} \cdot \mathrm{Hg} \cdot \text { Gage } \\
P_{5} & =-0.45 \mathrm{in} \cdot \mathrm{Hg} \cdot \text { Gage } \\
P_{6} & =2.72 \mathrm{in} \cdot \mathrm{Hg}_{8} \cdot \text { Gage }
\end{aligned}
$$

$$
P_{5}=-0.45 \text { in. He. Gage Iron - Constantan Thermo- }
$$

Reference $0^{\circ} \mathrm{F}$.

TABLE V
$X=8.1$ INCHES

| $\begin{gathered} r \\ \text { Inches } \end{gathered}$ | $M^{\prime \prime}$ | $\begin{gathered} \mathrm{P} \\ \mathrm{Hg} . \mathrm{Abs} \end{gathered}$ | 11 | $\begin{gathered} \mathrm{T}_{\mathrm{O}} \\ \text { Deg. } \mathrm{R} . \end{gathered}$ | $\stackrel{T}{\mathrm{~T}} \mathrm{Deg}_{\mathrm{g}}^{\mathrm{R}} .$ | $\begin{array}{r} \mathrm{v} \\ \mathrm{ft} . / \mathrm{sec} . \end{array}$ | $\begin{aligned} & \mathrm{v} / \mathrm{r} \\ & . / \mathrm{in} . \mathrm{sec} \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | . 335 | 30.42 | . 320 | 563.5 | 551 | 368.6 | 410 |
| 0.8 | . 333 | 30.07 | . 316 | 561 | 550 | 362.2 | 453 |
| 0.7 | . 313 | 29.60 | . 298 | 559 | 549 | 341.7 | 488 |
| 0.6 | .275 | 29.17 | .270 | 556 | 548 | 310.6 | 517.5 |
| 0.5 | . 238 | 28.82 | . 229 | 554 | 548 | 263.3 | 526 |
| 0.4 | . 199 | 28.58 | . 185 | 551.5 | 548 | 213 | 532.5 |
| 0.3 | . 135 | 28.32 | . 135 | 549 | 546 | 155 | 517 |
| 0.2 | . 089 | 28.15 | . 089 | 545 | 544.5 | 101.8 | 509 |
| 0.1 | . 01 | 28.08 | . 01 | 543.3 | 543.3 | 11.42 | 114.2 |
| 0 | 0 | 28.04 | 0 | 542.5 | 542.5 | 0 | ind. |

TABLE VI
$x=8.1$ INCHES

| $\begin{gathered} \mathrm{r} \\ \text { Inches } \end{gathered}$ | $\mathrm{Pr}_{\mathrm{R}} / \mathrm{P}_{\mathrm{C}}$ | $\mathrm{P} / \mathrm{P}_{\mathrm{C}}$ Theory | $\begin{aligned} & P / P_{C} \\ & \text { Test } \\ & \hline \end{aligned}$ | ${ }^{{ }^{\mathrm{T}_{\mathrm{R}} / \mathrm{T}_{\mathrm{T}_{\mathrm{O}}}}}$ | $\mathrm{T}_{\mathrm{o}} / \mathrm{T}_{\mathrm{oc}}$ <br> Theory | $T_{0} / T_{o c}$ Test |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | 1.0849 | 1.0849 | 1.0849 | 1.0387 | 1.0387 | 1.0387 |
| 0.8 | " | 1.0665 | 1.0724 | " | 1.03055 | 1.0341 |
| 0.7 | " | 1.0505 | 1.0556 | " | 1.0234 | 1.0304 |
| 0.6 | " | 1.03695 | 1.0403 | " | 1.0172 | 1.0249 |
| 0.5 | " | 1.0255 | 1.0278 | " | 1.01196 | 1.0212 |
| 0.4 | " | 1.0162 | 1.0193 | " | 1.00764 | 1.0166 |
| 0.3 | " | 1.009 | 1.010 | " | 1.0043 | 1.012 |
| 0.2 | " | 1.004 | 1.004 | " | 1.00191 | 1.0046 |
| 0.1 | " | 1.001 | 1.001 | " | 1.00048 | 1.0015 |
| 0 | " | 1.000 | 1.000 | " | 1.000 | 1.000 |

TABLE VII
DATA OBTAINED AT $X=14.1$ INCHES
$\underset{r}{ } P_{0 \prime \prime} H g$. Po"Hg. P'"Hg. P'" Hg. $T_{0}^{\prime} \quad T_{0}{ }^{\prime}$ Inches Gage Abs. Gage Abs. Millie volts Deg. R. $\begin{array}{llllll}0.9 & 1.97 & 31.63 & 0.44 & 30.10 & 2.91\end{array}$

| 0.8 | 1.76 | 31.42 | 0.16 | 29.82 | 2.89 | 560.3 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0.7 | 1.38 | 31.04 | -0.22 | 29.44 | 2.84 | 558.6 |
| 0.6 | 0.9530 .61 | -0.55 | 29.11 | 2.82 | 557.9 |  |
| 0.5 | 0.40 | 30.06 | -0.86 | 28.80 | 2.76 | 555.7 |

$\begin{array}{lllllll}0.4 & -0.20 & 29.46 & -1.20 & 28.46 & 2.71 & 554\end{array}$
$\begin{array}{lllllll}0.3 & -0.81 & 28.85 & -1.45 & 28.21 & 2.64 & 551.8\end{array}$
$\begin{array}{lllllll}0.2 & -1.33 & 28.33 & -1.65 & 28.01 & 2.62 & 551\end{array}$
$\begin{array}{lllllll}0.1 & -1.68 & 27.98 & -1.80 & 27.86 & 2.60 & 550.4\end{array}$
$\begin{array}{lllllll}0 & -1.83 & 27.83 & -1.83 & 27.83 & 2.59 & 550\end{array}$
$P_{1}-P_{2}=1.15$ in. Hg. Barometer $=29.66$ in. Hg. $\mathrm{P}_{2}=10.33$ in. Hg. Gage $\mathrm{T}_{2}=106^{\circ} \mathrm{F}$. $P_{3}=-1.56$ in. Hg. Gage Room Temp. $=76^{\circ} \mathrm{F}$. $P_{4}=-1.05$ in. Hg. Gage $W=0.0665 \mathrm{lb} . / \mathrm{sec}$. $P_{5}=-0.45$ in. Hg. Gage Iron-Constantan Thermocouple, $P_{6}=2.72$ in. Hg. Gage Reference $0^{\circ} \mathrm{F}$.

## TABLE VIII

$$
x=14.1 \text { INCHES }
$$

| $\begin{gathered} \mathrm{r} \\ \text { Inches } \end{gathered}$ | $1{ }^{1}$ | $\begin{gathered} P \\ \text { "Hg. Abs. } \end{gathered}$ | M | $\begin{gathered} \mathrm{T}_{0} \\ \mathrm{Deg} . \mathrm{R}_{0} \end{gathered}$ | $\stackrel{T}{\mathrm{~T}} \mathrm{~B} . \mathrm{R} .$ | $\stackrel{V}{\text { ft. }}$ | $\begin{aligned} & \mathrm{v} / \mathrm{r} \\ & \mathrm{ft} . \operatorname{in} . \sec . \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | . 270 | 30.20 | . 262 | 563 | 554.5 | 303.8 | 337.4 |
| 0.8 | . 274 | 29.94 | . 266 | 562 | 554 | 306.5 | 383.5 |
| 0.7 | . 274 | 29.58 | . 266 | 560 | 551.5 | 306.2 | 438 |
| 0.6 | . 270 | 29.20 | . 262 | 559.5 | 551 | 302.2 | 504 |
| 0.5 | . 249 | 28.86 | . 242 | 558 | 551 | 279 | 558 |
| 0.4 | . 223 | 28.56 | . 212 | 555 | 550 | 245 | 612 |
| 0.3 | . 176 | 28.27 | . 162 | 535.5 | 550 | 187 | 624 |
| 0.2 | . 112 | 28.01 | . 112 | 551.5 | 550 | 129 | 645 |
| 0.1 | . 045 | 27.86 | . 045 | 550.7 | 550.7 | 51.8 | 518 |
| 0 | 0 | 27.83 | 0 | 550 | 550 | 0 | ind. |

## TABLE IX

## $\mathrm{X}=14.1$ ILCEES

| $\stackrel{r}{\text { Inches }}$ | $\mathrm{P}_{\mathrm{R} / \mathrm{P}_{\mathrm{C}}}$ | ${ }^{\mathrm{P} / \mathrm{P}} \mathrm{P}_{\mathrm{C}}^{\mathrm{O}} \mathrm{ry}$ | $\begin{aligned} & \mathrm{P} / \mathrm{P}_{\mathrm{C}} \\ & \text { Test } \end{aligned}$ | $\mathrm{T}_{\mathrm{O}_{\mathrm{Y}} / \mathrm{T}_{\mathrm{O}_{\mathrm{C}}}}$ | To/m Theory | $\begin{aligned} & \mathrm{To} / \mathrm{T}_{\mathrm{ol}} \\ & \text { Test } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.9 | 1.0852 | 1.0852 | 1.0852 | 1.024 | 1.024 | 1.024 |
| 0.8 | " | 1.0669 | 1.076 | " | 1.01893 | $1.0 \% 18$ |
| 0.7 | " | 1.0509 | 1.063 | " | 1.0145 | 1.0182 |
| 0.6 | " | 1.0372 | 1.049 | " | 1.01068 | 1.0173 |
| 0.5 | " | 1.0256 | 1.037 | " | 1.0074 | 1.0145 |
| 0.4 | " | 1.0163 | 1.026 | " | 1.00474 | 1.0091 |
| 0.3 | " | 1.009 | 1.016 | " | 1.00867 | 1.00636 |
| 0.2 | " | 1.003 | 1.006 | " | 1.00118 | 1.0027 |
| 0.1 | " | 1.001 | 1.001 | " | 1.0004 | 1.0013 |
| 0 | " | 1.000 | 1.000 | " | 1.000 | 1.000 |

TABLE X

## DATA OBTAIMD AT $X=2.5$ INCHES SIX INCH LONG PIPE

| $\begin{gathered} \mathrm{r} \\ \text { Inches } \end{gathered}$ | ${\stackrel{P}{\mathrm{O}^{( }}{ }^{\mathrm{Hg} .} \mathrm{Gage}}^{2}$ | "Hg. Gage | $\begin{gathered} \text { To } \circ^{\prime} \\ \text { Milli volts } \end{gathered}$ | $\begin{gathered} \mathrm{T}_{0}^{\prime} \\ \mathrm{Deg} . \mathrm{R} . \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: |
| 0.9 | 4.97 | 0.66 | 3.02 | 564.7 |
| 0.8 | 3.77 | 0.11 | 2.92 | 561.3 |
| 0.7 | 2.47 | -0.55 | 2.78 | 556.4 |
| 0.6 | 1.39 | -1.13 | 2.64 | 551.7 |
| 0.5 | 0.29 | -1. 69 | 2.55 | 548.6 |
| 0.4 | -0.90 | -2.20 | 2.42 | 544 |
| 0.3 | -1.84 | -2.63 | 2.31 | 540.3 |
| 0.2 | -2.45 | -2.83 | 2.19 | 536.1 |
| 0.1 | -2.85 | -3.01 | 2.12 | 533.6 |
| 0 | -3.06 | -3.07 | 2.08 | 532.1 |
| $P_{1}-1$ | 1.21 in. 10.68 in. -1.82 in. -1.19 in. -0.53 in. 2.78 in. | . g. Gage g. Gage g. Gage g. Gage g. Gage |  |  |
| Barometer $=30.27 \mathrm{in} . \mathrm{Hz}$. |  |  |  |  |
| Iron - | stantan | mocouple | Reference $0^{\circ}$ |  |

TABLE XI
DATA OBTAINED AT $\mathrm{X}=1.4$ INCHES

$P_{1}-P_{2}=2.10$ in. Hg .
$P_{2}=21.0$ in. Hg. Gage
$P_{3}=-3.17$ in. Hg. Gage
$P_{4}=-2.13$ in. Hg. Gage
$P_{5}=-0.81$ in. Hg. Gage
$P_{6}=5.31$ in. Hg. Gage
Barometer $=29.74$ in. If.
$T_{0_{2}}=111.5^{\circ} \mathrm{F}$.
Iron - Constantan Thermocouple: Reference $0^{\circ} \mathrm{F}$.


## er




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|  |  |  |  |  | 3 ${ }^{\text {a }}$ |  |  | W |  | ： | ． |  |  | ＋ |  |  |  |
|  |  |  |  |  | \％ |  |  | Fide |  |  |  |  |  |  |  |  |  |
|  |  | ${ }^{3}$ |  |  |  |  |  | ${ }^{1 / 3}$ |  |  |  |  |  |  |  |  |  |
|  |  |  | W |  |  |  | ［ ${ }^{\text {P }}$ | 模 | 5 | 1 |  | ${ }_{\text {krit }}$ |  |  | ${ }^{\text {s }}$ |  |  |
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FIGURE I
SCHEMATIC LAY-OUT OF EXPERIMENTAL SET-UP


# FIGURE 2 VORTEX VALVE 



24 ADVUSTAELE VANES sET AT 10* WITH TANGENT TO PIPE WALL 2

NOTE: PREsgune lines ARE LED OUT THROUGH TWE ADAPTET PLATE FROM ALL STATIG PRESSURE TAPS.

TAO SOLDERED
in stud


VANE DETAIL


STUD DETAIL



## (a)

## PPRESS BINDER

BGS 2507

MADE BY

## PRODUCTS. INC.



