1948

Study of pulse-code modulation.

Sanger, Frank Moderno

Massachusetts Institute of Technology

STUDY OF PULSE-CODE MODULATION

FRANK M. SANGER, JR.

STUDY OF PULSE-CODE MODULATION

by

FRANK M. SANGER, JR.

Lieutenant Commander, United States Navy

B.S., United States Naval Academy
(1941)

SUBMITTED IN PARTIAL FULFILLMENT OF THE

REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

(1948)

CONTENTS

ABSTRACT

Pulse-code modulation (PCM) makes possible a transmission system in which the ultimate signal-to-noise ratio at the receiver is independent of the distance spanned, provided that noise peaks at the receiver remain below a sharply-defined threshold value of approximately half the signal strength. The advantages of such a system for long-distance communications are apparent. These advantages must be weighed, however, against the increased bandwidth required for transmission in any particular application.

The generation of a pulse-code signal involves three distinct processes: sampling of the intelligence signal, quantisation of the sample, and coding of the "size" thus determined. The combined processes of sampling and quantization give rise to a distortion signal resembling random noise in its frequency distribution throughout the spectrum.

In order to permit future studies of coding equipment of various types, a decoder has been designed and built to combine accuracy and flexibility at the expense of simplicity. A PCM system has been assembled, using this decoder in conjunction with a coder previously developed at the Research Laboratory of Electronics. Investigations of frequency response and distortion (with and without an interfering signal at the input to the decoder) have been carried out and the results compared with theoretical considerations.

PCM is a relatively new concept in the communications field, and affords ample opportunity for further investigation. The principle design problem to be met is simplification of equipment; application of PCM to most practical communications problems awaits development along these lines.

CHAPTER I

SOME ASPECTS OF PULSE-CODE MODULATION

Reasons for Interest in PCM

Pulse-code modulation (PCM) is a technique for the transmission of intelligence that virtually eliminates distance as a factor in quality of reception. Signal reproduction is uniform at all distances up to the point where noise peaks at the receiver input exceed a sharply defined threshold level of one-half the peak-to-peak signal amplitude. For many potential applications of PCM repeater equipment can be installed before this distance has been reached, and the signal rejuvenated without reduction in quality. Since there is no cumulative buildup of noise from one repeater station to the next, there is no limit on ultimate transmission path length. Distortion in the system can be reduced to any desired percentage by proper design of terminal equipment.

In addition to these unique qualities, PCM enjoys the advantages shared by all pulse modulation systems. Principle among these are power conservation (at the expense of increased transmission bandwidth) and adaptability to multiplexing by time division.

Summary of Processes Involved in PCM

In a PCM system the intelligence signal to be transmitted does not operate directly on the carrier wave (or subcarrier) as it does in other modulation methods. The signal amplitude is sampled at regular

intervals and the sampled amplitude caused to vary the width or amplitude of a pulse generated at the coder. Thus a width-modulated or amplitude-modulated pulse train is formed. The varying dimension of each pulse is than measured in discrete steps or quanta to give a numerical value for its "size". These values are then encoded and the resultant code signal is used to modulate the carrier wave. The nature of the transmitted signal is thus seen to be similar to that of a radiotelegraph signal. At the receiver the incoming signal is demodulated and decoded; the decoded "sizes" are used to generate another modulated pulse train differing from that formed at the transmitter only in the fact that the pulses are quantized in width or amplitude. This pulse train is than demodulated by means of a low-pass filter.

Sampling

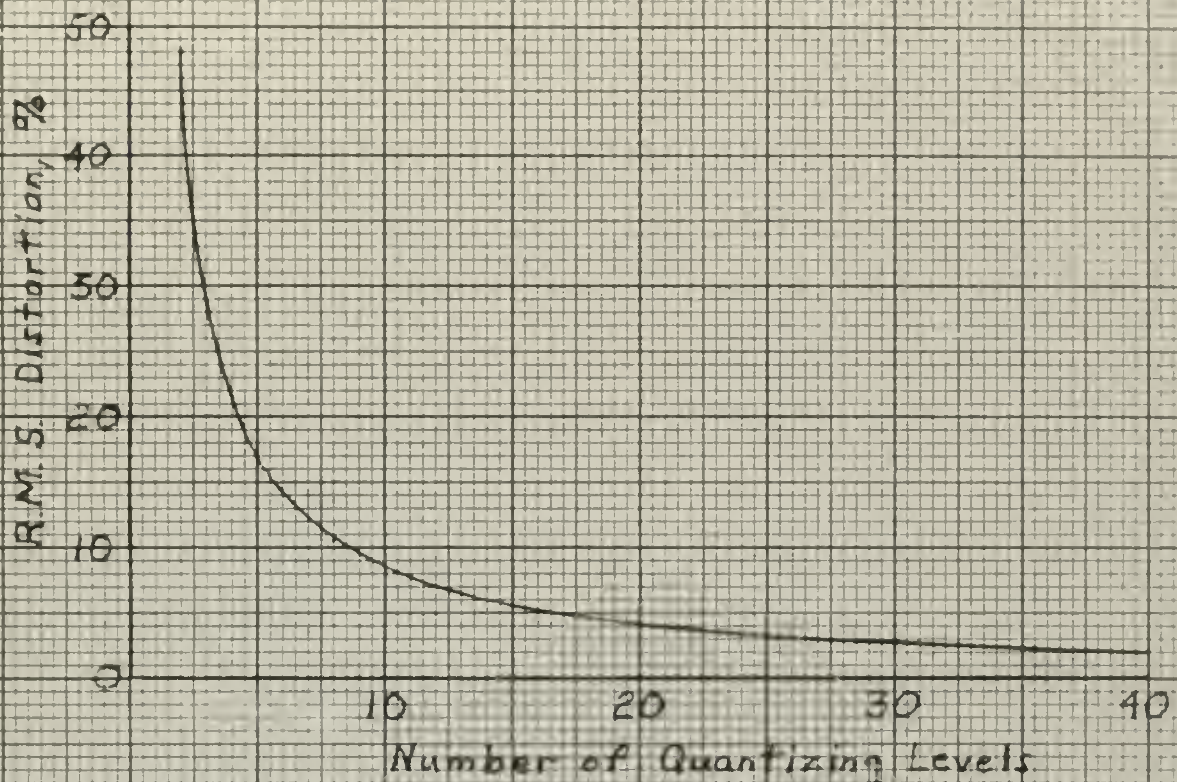
Sampling is a process common to all types of pulse modulation. Fourier analysis of a pulse train of repetition frequency p amplitude-modulated by a sinusoidal signal of frequency q reveals the presence of the frequencies q, np, and np $\pm$ q (n is an integer) in the resulting spectrum. Since demodulation is accomplished by means of a low-pass filter and since it is desired to have only the frequency q present in the output, it is apparent that the pulse repetition frequency (sampling rate) p must be at least twice the highest frequency component of the modulating signal. If this condition is met and if an ideal low-pass filter of cutoff frequency p/2 is used for demodulation, no distortion results from the sampling process when other factors do not come into consideration. For any practical output filter, p must of

course be somewhat higher than this value.

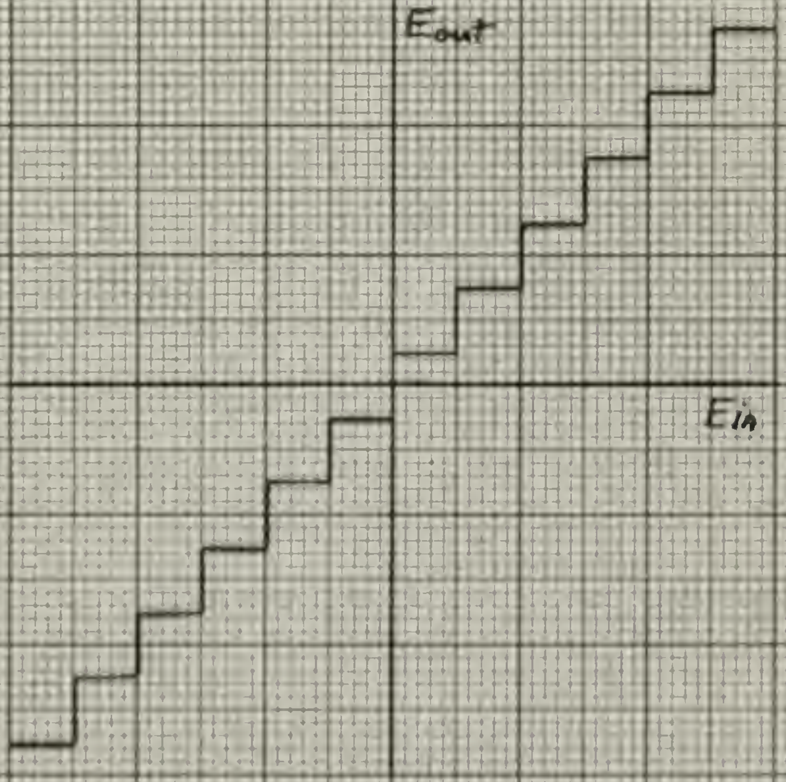
Similar analysis of a width-modulated pulse train reveals the presence of the frequencies q, np, and np $\pm$ mq (m is an integer, independent of n) as components of the spectrum. Since components of the type np - mq can fall into the pass band of any filter used, no exact criterion for sampling rate can be stated. Amplitudes of these difference components fall off fairly rapidly, however; for sampling rates four or five times the highest audio component spurious signals in the audio band are negligible, and quite satisfactory results are obtained when sampling rate is only twice the highest audio component.

Quantization

The process of quantization may be thought of as analagous to the representation of the value of π to any given number of significant figures. Quantization distortion results just as residual error results from the analogous process. This distortion has been investigated by several writers, and the theoretical results are shown in Figure I-1, together with the "staircase" transfer function that characterizes the quantization process. As the number of levels N used to cover the peak-to-peak amplitude of the signal is increased the curve becomes asymptotic to the hyperbola $D_{(rms)} = \frac{0.817}{N}$. Exact formulae for the cases involving smaller values of N vary somewhat with method of setting up the problem; observed values will vary appreciably with changes in input amplitude and d.c. level with respect to the treads or risers of the staircase, as may be seen from a comparison of the two cases shown

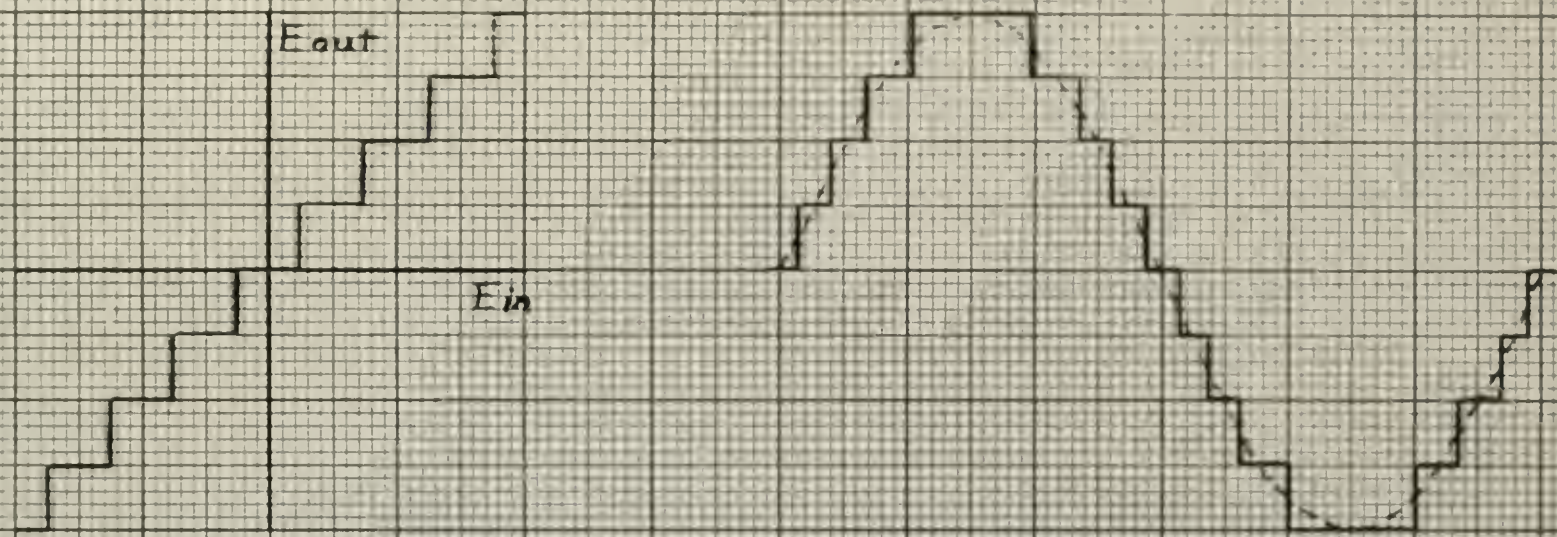


a. Distortion due to Quantization.



b. Transfer Characteristic for Quantization Process.

Figure I-1.



Transfer Characteristic Sine Wave Approximation

a. Zero level at mid-step

Transfer Characteristic Sine Wave Approximation

b. Zero level at mid-riser

Figure I-2. Quantization of a Sinusoidal Signal.

in Figure I-2.

The combined processes of sampling and quantization produce a myriad of components in the frequency spectrum of the resultant signal, quite similar in nature to random noise. Many of these components fall in the audio band passed by the output filter, producing the characteristic "quantization noise" noticeable in the decoded and filtered signal.

Coding

The binary system of numbers, or some modification thereof, has been used for coding in all PCM systems developed or suggested to date. In the binary system digits may take on only the values 0 or 1. Numbers are made up in the same manner as in the decimal system, but each digit is associated with a power of two instead of ten. Thus the number 109 in the decimal system ($1 \times 10^2 + 0 \times 10^0 + 9 \times 10^0$) becomes 1101101 in the binary system ($1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$). The presence or absence of a pulse at a given time in the operating cycle is used to indicate the digit value 1 or 0.

Other coding schemes might be employed, but the binary system lends itself admirably for two principle reasons. First, encoding and decoding at high speeds by means of fairly simple circuitry is possible. Second, the "on" or "off" nature of the digit pulses makes for a sharp noise threshold, since the receiving equipment need but distinguish between the presence or absence of a digit pulse.

Synchronization

Most PCM systems operate in such a manner that digit pulses are produced in continuous sequence (no interval between the groups making

up the code signals for successive samples). Some means of synchronizing the decoder with the coder must therefore be employed in order that the code groups will be properly sorted. In most of the experimental systems that have been developed to date a separately transmitted synchronizing signal is used to lock in the decoder timing circuits. This scheme is satisfactory for laboratory experimental work; the added transmission link can be avoided by the use of some method of marking one pulse of the series forming the complete cycle.

Bandwidth Requirements

Offhand, it would appear that the bandwidth requirements for PCM would be prohibitive; all pulse modulation systems are notorious consumers of bandwidth, and in PCM the number of pulses transmitted per unit time is multiplied by a factor of N, the number of binary digits used in the code. An N-fold increase in bandwidth requirement might be expected. Fortunately, this is not the case. Whereas in other pulse modulation schemes the information is carried by the shape of the pulses, in PCM pulse shape is not important. A bandwidth sufficient to allow the change from peak amplitude to zero amplitude in one digit repetition period fully meets the requirements for transmission of PCM.

Greig* has developed a simple formula relating transmission bandwidth and highest component frequency of the modulating signal:

$$F/f_h \cong 1.8 \ln\frac{80}{D}$$

Here D is the allowable quantization distortion in percent. This expression is derived from the hyperbolic approximation for quantization distortion, and hence gives accurate results only for low values of D. Single sideband transmission is assumed.

* Grieg, D. D., "Pulse Count Modulation System"; Elec. Com., 24, 3 (Sept., 1947), 84.

PCM Developments to Date

The three basic principles of pulse-code modulation were formulated in the 1920's by R. A. Heising and P. M. Rainey. In 1921 Rainey was issued a patent* covering a system for the transmission of facsimile over conventional telegraph facilities. In this patent he proposed quantization of the signal to be transmitted (ie., substitution of the nearest one of a finite number of discrete amplitude levels for the actual amplitude of the signal) and coding of the result to facilitate transmission. In 1928 Heising was issued a patent # covering a means of driving his "constant current" modulator. In this patent he proposed sampling of an audio signal at a supersonic frequency, and thus became the pioneer in the entire field of pulse modulation.

In 1938 A. H. Reeves@ combined the ideas of Heising and Rainey to produce the first pulse code modulation system. In his coder the audio signal is sampled to produce a length-modulated pulse train. Lengths of the individual pulses are then quantized by counting the number of oscillations from a fixed-frequency oscillator occurring during the pulse. A binary counter performs this function and its count for each pulse is transmitted in binary digits, using a separate transmitting frequency for each digit. In the decoder a similar counting circuit is set to the coder's count by the received signal, and counting is resumed. A pulse dependent in length on the time required

* Rainey, P.M., "Transmission System"; U. S. Patent 1,608,527 (1921).

\# Heising, R.A., "Transmission System"; U.S. Patent 1,655,543 (1928)

@ Reeves, A.H., "Transmission System"; U.S. Patent 2,272,070 (1942).

to reach saturation of the counter is generated for each group of digit pulses received, and the resultant length-modulated pulse train is demodulated by a low-pass filter.

Modifications of Reeves' basic system have been produced for experimental purposes at Federal Telecommunications Laboratories* and at Bell Telephone Laboratories.# In both systems time division of the digit pulses takes the place of the frequency division used by Reeves, and multiplexing of eight audio signals is accomplished by commutator circuits in the transmitter and receiver. Details of the decoding circuits are not available; it is known, however, that in the B.T.L. system decoding is inverse to coding (ie., a counting scheme), while in the F.T.L. system the individual digit pulses trigger circuits which produce pulses of length proportional to digit weight, and the outputs of these circuits are fed in parallel to the low-pass filter demodulator.

Another experimental system designed at B.T.L.* is of the " Feedback Subtraction " type. In the coder of this system the audio signal is used to modulate the amplitudes of the sampling pulses. The samples are then compared in amplitude with a reference step voltage which takes on successive values of 16, 8, 4, 2, and 1 units (where 31 is the maximum sample amplitude); whenever the sample exceeds the value of the reference voltage a digit pulse is transmitted and the sample is reduced in amplitude by an amount equal to the reference voltage. Thus

* Grieg, D.D., "Pulse Count Modulation System"; Elec. Com., 24, 3 (Sept., 1947), 84.

\# Black, H.S. and Edson, J.O., "PCM Equipment"; Elec. Eng., 66, 11 (Nov., 1947), 1123.

** Goodall, W.M., "Telephony by Pulse Code Modulation"; Bell Sys. Tech. Jnl., 26, 3 (July, 1947), 395.

it is seen that binary digits are transmitted in order of descending weight. In the decoder, operation is almost the same, except that in this case an initial full-magnitude (31 units) sample is cut down by the incoming pulses, each reducing the sample's amplitude by its binary weight.

A third (and by far the most elaborate) experimental system designed at B.T.L.# is of the " Instantaneous Comparison " type. A special coding tube of the cathode ray type is used in the transmitter. Amplitude-modulated pulse samples are applied to the vertical deflection plates of this tube and the electron beam is swept across the tube at the height determined by each sample. An aperture plate passes or blocks the beam; passage through openings in the aperture plate causes current flow in the anode circuit. The openings in the aperture plate are cut so as to produce the series of binary code digit pulses corresponding to the height of the beam. Decoding is accomplished by means of a simple ringing circuit pulsed with equal surges of current for each digit pulse received and sampled one pulse interval after the last digit; time constant and oscillation frequency of the circuit are such that at this instant the slope of the wave form is zero, and the component of amplitude contributed by each digit pulse corresponds to the pulse's binary weight. It is to be noted that this decoder requires that the digit pulses arrive in inverse order (ie., lowest weight first).

\# Meacham, L.A., and Peterson, E., "An Experimental Pulse Code Modulation System of Toll Quality"; Bell Sys. Tech. Jnl., 27, 1 (Jan., 1948), 1.

During the spring of this year a coder has been developed at the M.I.T. Research Laboratory of Electronics by Mr. B. D. Smith.* It is of a feedback type, but employs radically different methods from those of the B.T.L. feedback system. A resistance network fed by "flip-flop " circuits establishes the reference voltage, and the successive values of this voltage are compared with the sample. Each comparison results in a change of the reference voltage. When the sample exceeds the reference in amplitude a digit pulse is transmitted and the reference is increased by an amount corresponding to the weight of the next digit. When the reference exceeds the sample no digit pulse is transmitted, and the reference voltage is decreased by the amount corresponding to the weight of the next digit. Accurate coding is made possible in this circuit by the fact that reference levels are established entirely by the resistance network fed from stable voltage sources.

A decoder was developed at M.I.T. during the past spring by Mr. E. P. Westfall and Mr. A. M. Bettis. A ringing circuit pulsed once at the start of decoding is used to establish the binary digit weights; successive digit pulses in the received signal gate the output of the circuit and the gated outputs are combined for demodulation.

The M.I.T. coder and decoder have not been put together to form a complete system, and hence neither has been fully tested in operation and a working system for study of the properties of pulse code modulation has not materialized at the Institute. It is to be expected that

* Smith, B.D. jr., "Pulse-Code Modulation Method"; Thesis for S.M. Degree, M.I.T., June, 1948.

considerable work will be done in this field in the future, and it becomes apparent that the development of an accurate decoder, readily adaptable to use with various types of coders, is desirable.

CHAPTER II

DESCRIPTION OF THE DECODER

In the design of a decoding circuit for use in experimental work with coders of various types exacting requirements of accuracy and flexibility must be met. The decoder to be described has been designed with those requirements in mind, at the sacrifice of simplification.

Principle of Decoding

The method of decoding employed, originally proposed by B. D. Smith, makes use a simple resistance network shown in Figure II-1. The voltage at point "A" of this network is given by the expression:

$$V_a = \sum_{k=1}^{n} \frac{2^{n-k}}{2^n-1} V$$

In this expression k takes on the integer values corresponding to the numbers of the switches closed to the voltage source V. Thus the switches contribute to the voltage V_a in accordance with the binary digit system. In the decoder the incoming PCM digit pulses are caused to operate the corresponding switches and the voltage V_a is sampled after the arrival of each digit series.

Operation of the Decoder

The decoder is shown in block diagram in Figure II-2. Waveforms and timing relationships are shown in Figure II-3. Operation will be explained with reference to these figures.

Synchronizing pulses from the coder (Figure II-3(a)) are applied to the Synchronizing Multivibrator V1 which generates a rec-

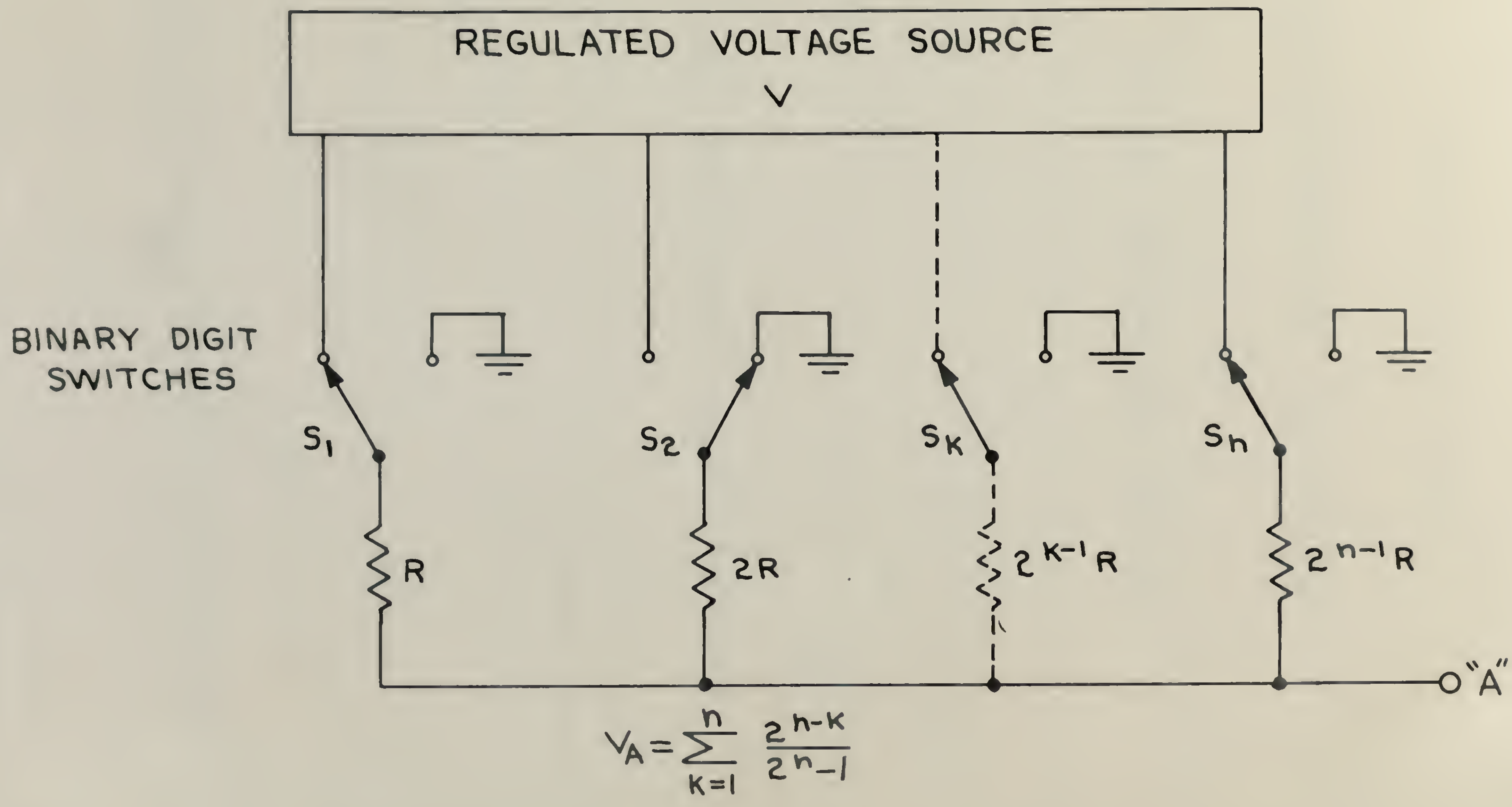


FIGURE II-1 METHOD OF DECODING

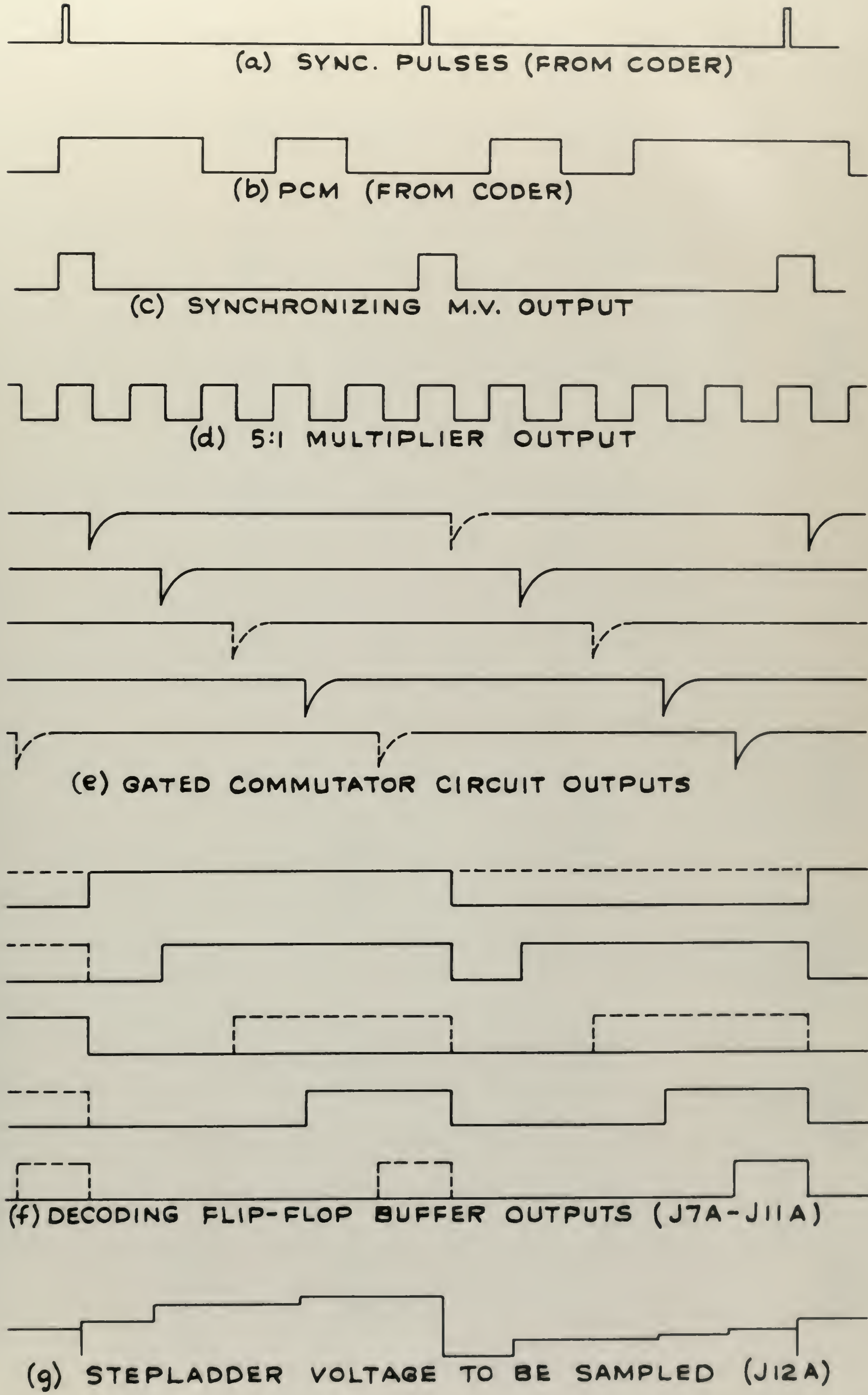


FIG. II-3 DECODER WAVEFORMS AND TIMING RELATIONSHIPS

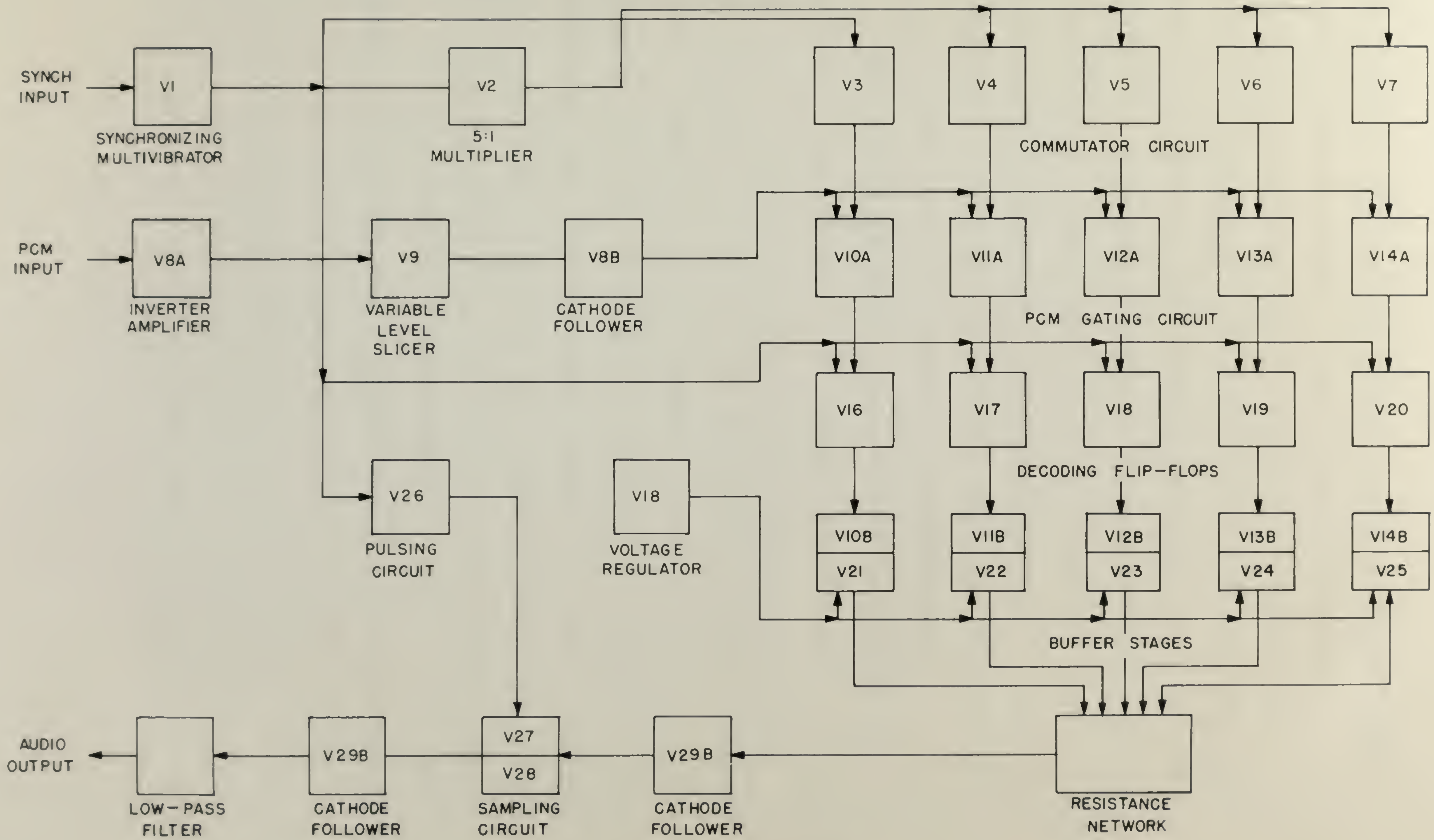


FIG. II-2 DECODER BLOCK DIAGRAM.

tangular pulse of variable width (Figure II-3(c)). This width adjustment is provided to permit proper synchronization with the coder output; the trailing edge of the pulses is caused to coincide with the midpoint of the first digit pulse from the coder. An output of this multivibrator is used to synchronize the 5:1 Frequency Multiplier V2. Adjustment is provided to permit synchronization of this multiplier at varying repetition rates. Its output (Figure II-3(d)), together with that of the Synchronizing Multivibrator, is applied to the Commutator Circuit, V3-V7, whose five stages produce pulses at the basic repetition rate, staggered in time relationship.

The incoming PCM signal (Figure II-3(b)) is inverted and amplified by V8A and passed to the Variable Level Slicer V9, which clips top and bottom of the signal to remove fluctuations due to noise. Its output is applied to the Cathode Follower V8B, an integral part of the PCM Gating Circuit V10A-V14A. In the presence of a PCM digit pulse each stage of the Gating Circuit passes the output of the corresponding stage of the Commutator Circuit. The gated outputs are shown as solid lines in Figure II-3(e); dotted lines indicate output pulses that are blocked off by the absence of a PCM enabling pulse.

The gated Commutator Circuit outputs are applied to the corresponding Decoding Flip-Flops, V16-V20, turning each of the five circuits "on" or leaving it "off" in accordance with the presence or absence of the corresponding digit pulse in the incoming PCM signal. At the end of the cycle of operation the flip-flops are turned off by the trailing edge of the Synchronizing Multivibrator output (Figure II-3(c)).

Outputs of the Decoding Flip-Flops are applied to the Buffer Stages V10B-V14B, whose plate excursions are limited by diode limiters V21-V25 to values of 0 and 75 volts. The Buffer Stages correspond to the switches of Figure 1. Their outputs (Figure 3(f)) are combined in the Decoding Resistance Network, whose output voltage is the "stepladder" of Figure 3(g). This voltage is sampled by the Sampling Circuit V28 and V29 at the time of the leading edge of the Synchronizing Multivibrator output (Figure 3(c)) and held on a storage condenser until the time of the next sample. Output of the sampling circuit is passed through a low-pass filter to regain the intelligence signal that originated at the coder.

Discussion of Circuitry

Circuit diagrams for the decoder are shown in Figures II-4a to II-4f. The Synchronizing Multivibrator V1 is a conventional "one-shot" circuit. By means of the pulse width control R1 the output pulse may be varied from about 5-20 microseconds in duration. The 5:1 Multiplier V2 is a conventional free-running multivibrator whose natural period can be varied from about 7-25 microseconds by means of the control R2, thus permitting synchronization with a variety of input signals. These circuits provide timing pulses for the entire decoder.

The Commutator Circuit V3-V7 is a ring counter quite similar to that used in the Army's Eniac computer. * In the Eniac circuit, a special pulsing circuit is provided to drive the ring at the cathodes; thus the additional loading of the grids brought about by the input circuit is avoided. Cathode drive was found, however, to be unsatisfactory for the present purpose because of the presence of a pulse

* Sharpless, T. K., "High-Speed N-Scale Counters"; Electronics 21, 3 (March, 1948), 122.

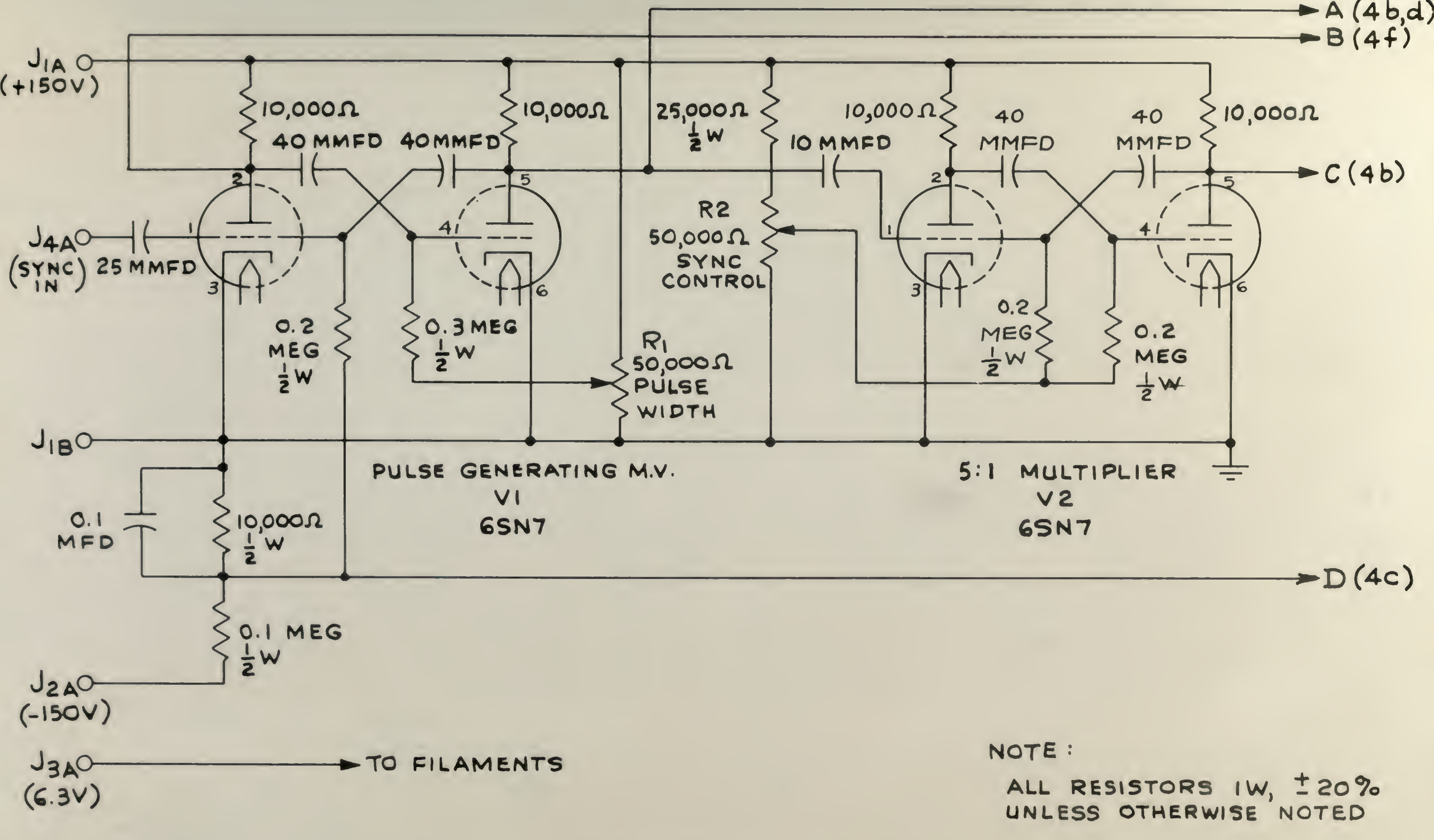
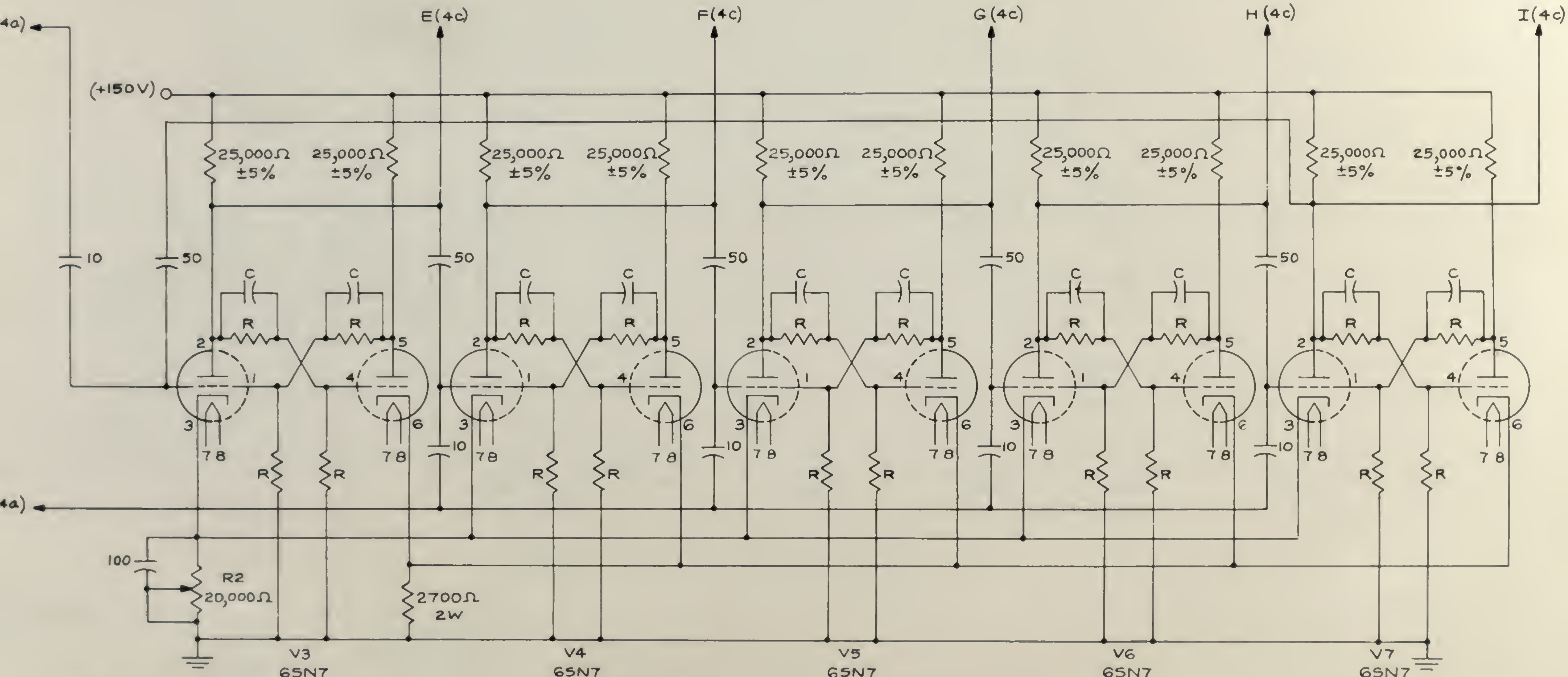


FIG. II-4a SYNCHRONIZING CIRCUITS



NOTE: $R = 50{,}000\Omega$, $\frac{1}{2}$W, AND ±5%
C=30 MMFD
ALL CAPACITOR VALUES ARE IN MMFD.

FIG. II-4b COMMUTATOR CIRCUIT

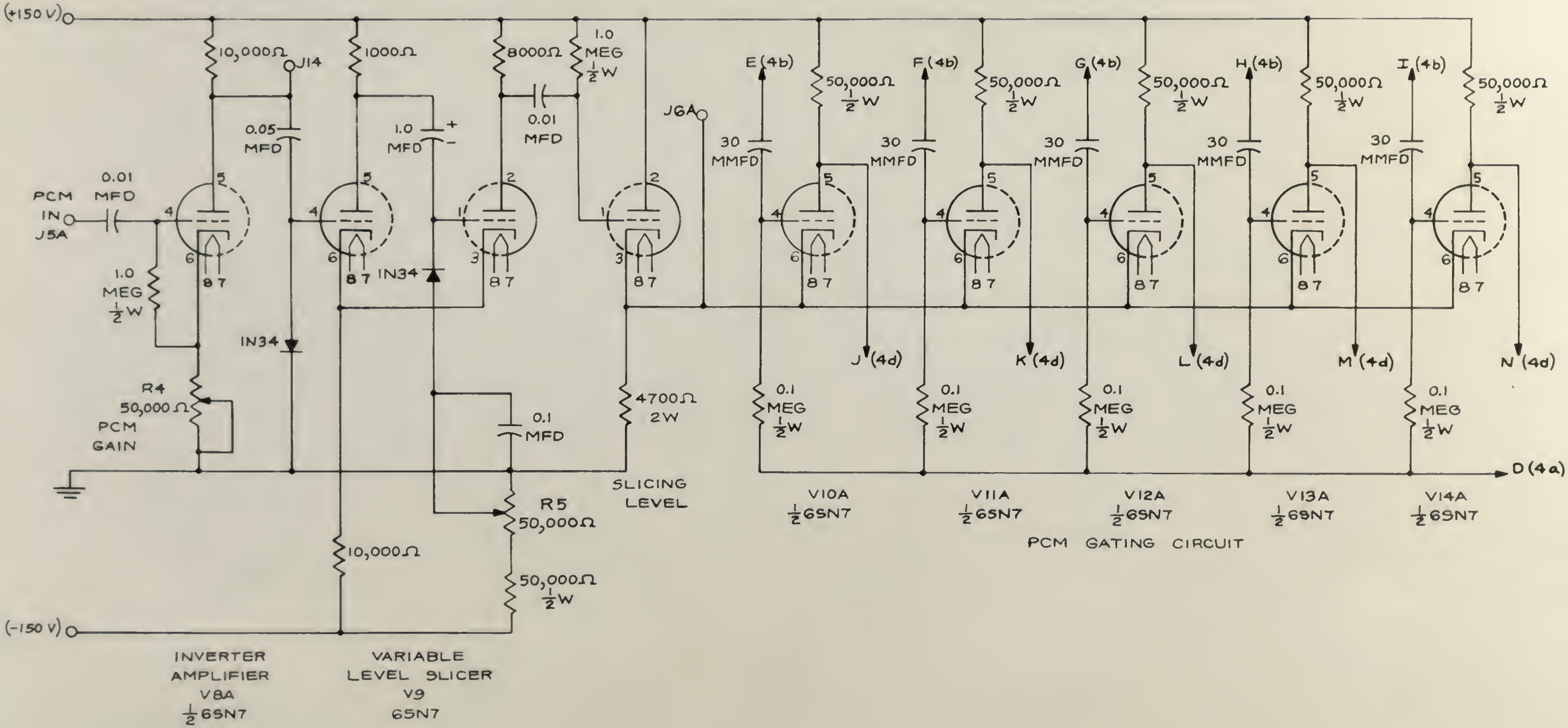
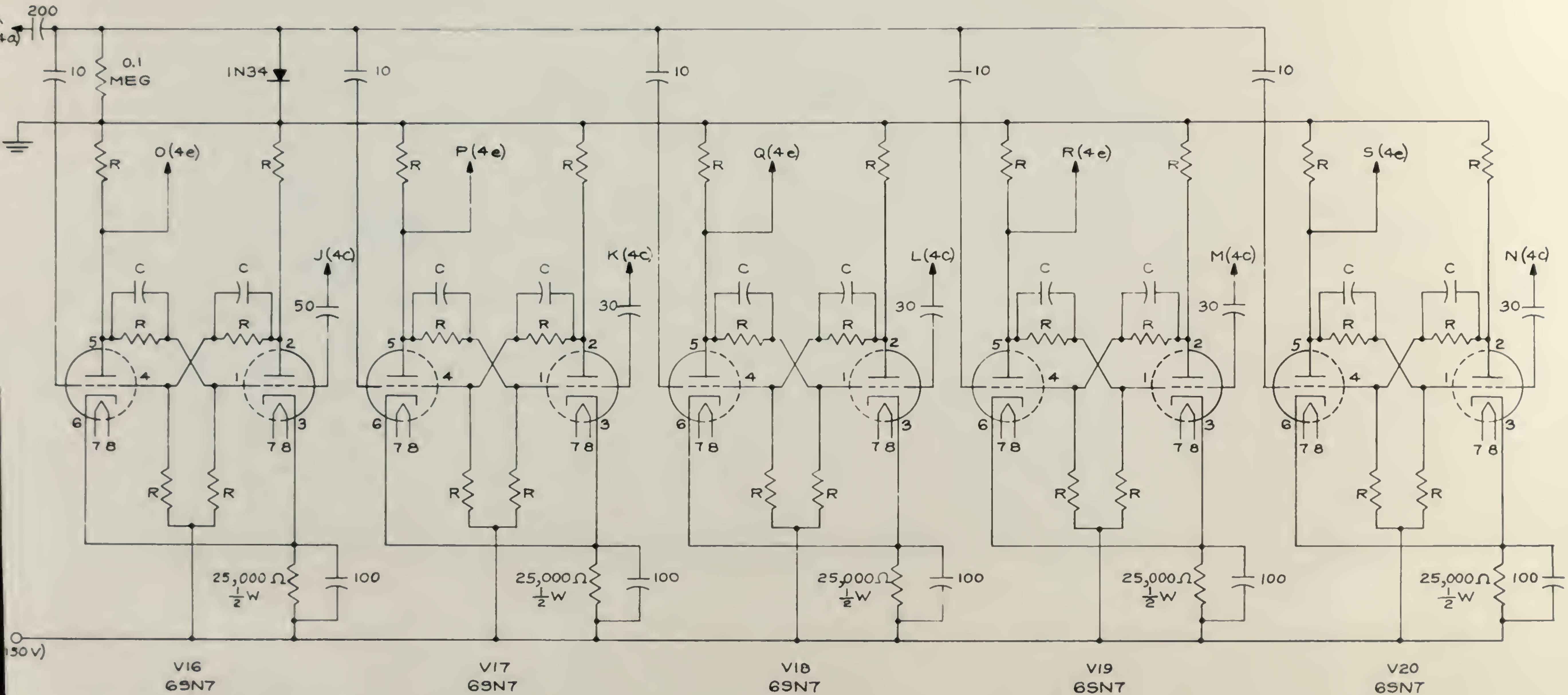


FIG II-4c PCM INPUT CIRCUIT



NOTE: $R = 50{,}000\,\Omega, \frac{1}{2}$ W, AND ±5%.
C = 30 MMFD
ALL CAPACITOR VALUES ARE IN MMFD.

FIG. II-4d DECODING FLIP-FLOPS

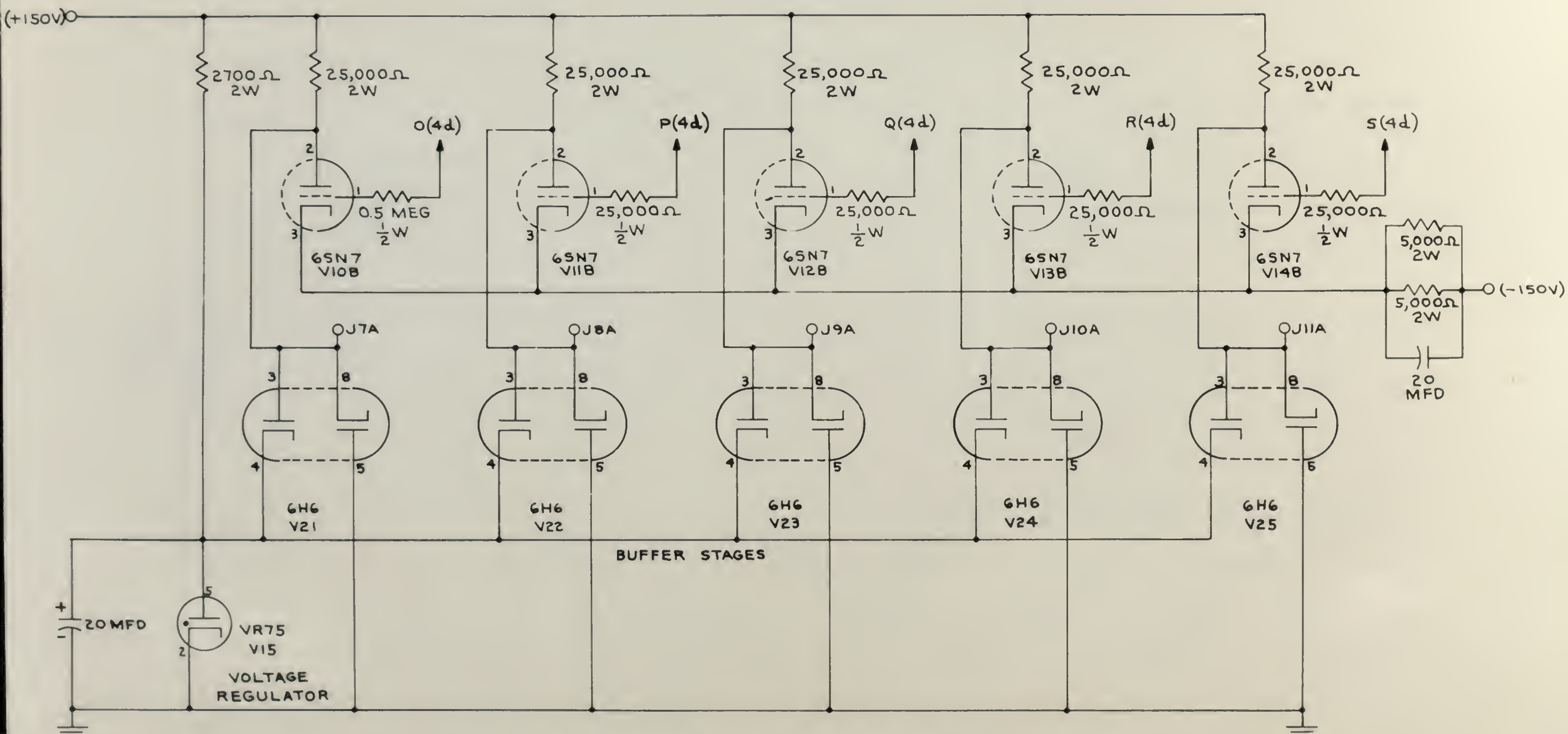
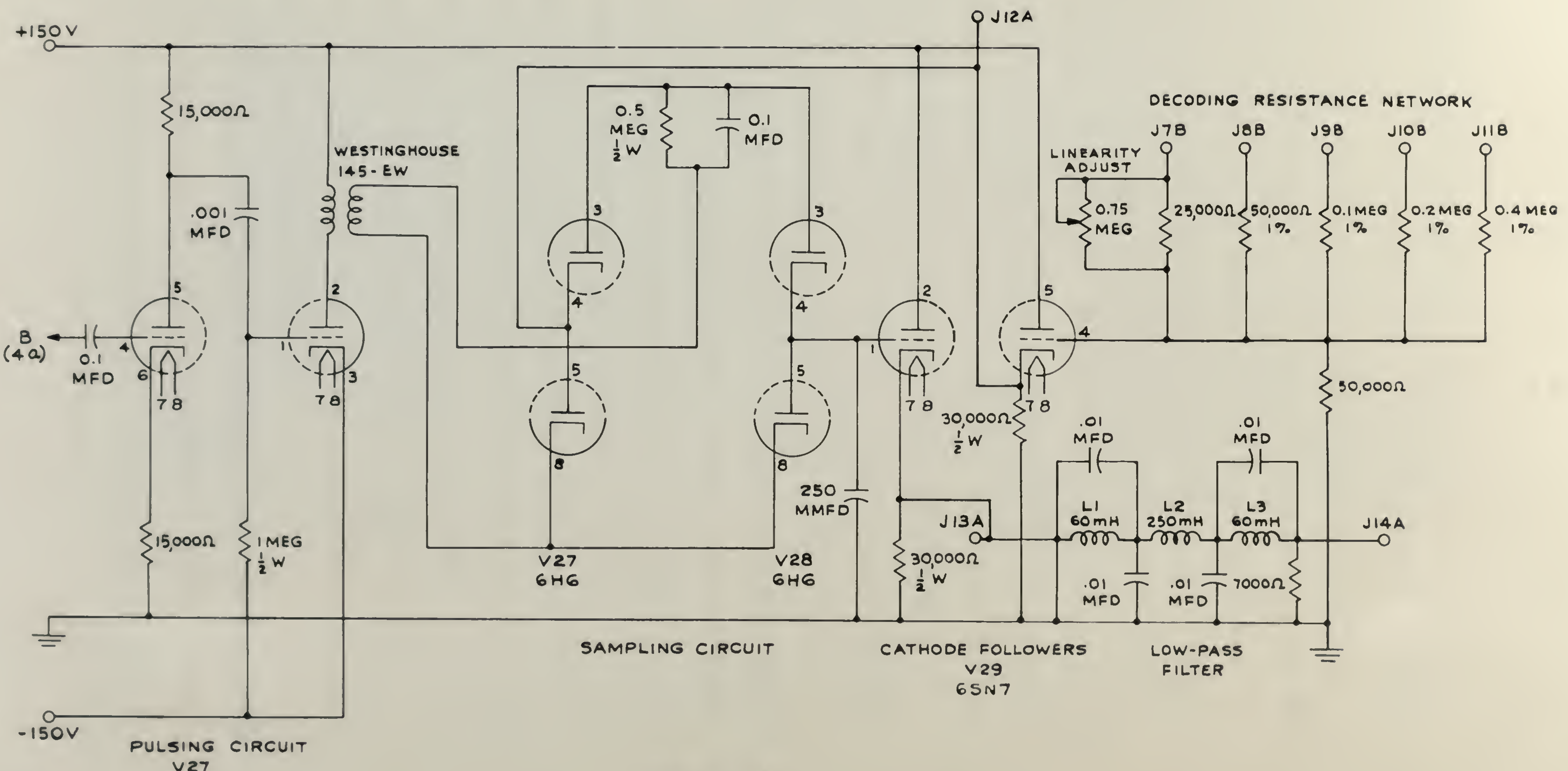


FIG. II-4e BUFFER STAGES AND VOLTAGE LIMITING CIRCUIT


+150V
15,000Ω
WESTINGHOUSE
145-EW
0.5
MEG
½ W
0.1
MFD
.001
MFD
B
(4a)
0.1
MFD
15,000Ω
1 MEG
½ W
-150V
PULSING CIRCUIT
V27
6SN7
V27
6HG
V28
6HG
250
MMFD
SAMPLING CIRCUIT
J12A
J13A
30,000Ω
½ W
CATHODE FOLLOWERS
V29
6SN7
DECODING RESISTANCE NETWORK
J7B
J8B
J9B
J10B
J11B
LINEARITY
ADJUST
0.75
MEG
25,000Ω
50,000Ω
1%
0.1 MEG
1%
0.2 MEG
1%
0.4 MEG
1%
50,000Ω
.01
MFD
L1
60mH
L2
250mH
L3
60mH
7000Ω
J14A
LOW-PASS
FILTER
FIG. II-4f

of considerable magnitude in the plate circuits of all stages at the time of occurrence of each driving pulse. The common cathode circuit used in the Eniac ring counter to avoid improper operation has been retained in the present counter. It is seen that the cathodes of the "A" halves of all stages are grounded through a common impedance, while the "B" halves are grounded through another impedance of lesser magnitude. In operation, one of the "A" halves and four of the "B" halves are conducting at any time. Negative pulses applied to the "A" halves bring about the change in state, cutting off the one "A" half that is conducting. Coupling from the "A" plate of each stage to the "A" grid of the following stage causes the stages to trigger in proper sequence. The time constant of this coupling circuit is five times that of the coupling circuit which transmits input pulses (of opposite polarity) to the same grid; this relationship insures that each stage will trigger on and off properly. Proper synchronization of the circuit is brought about by deriving the "off" trigger for the first stage from the Synchronizing Multivibrator; once turned on by the last stage the first stage can be turned off only by the pulse from this source. In operation the circuit starts with any one of the stages on, and counts successive pulses from the 5:1 Multiplier up to the point where the last stage is turned off; it remains in this condition until the arrival of a pulse at the first stage, and thereafter proceeds in proper synchronization.

The Inverter-Amplifier V8A is an ordinary amplifier stage with variable cathode degeneration to permit adjustment of gain. Figure II-5 shows the PCM input to this circuit for zero interference; Figure II-6 shows the input in the presence of an interfering pulse

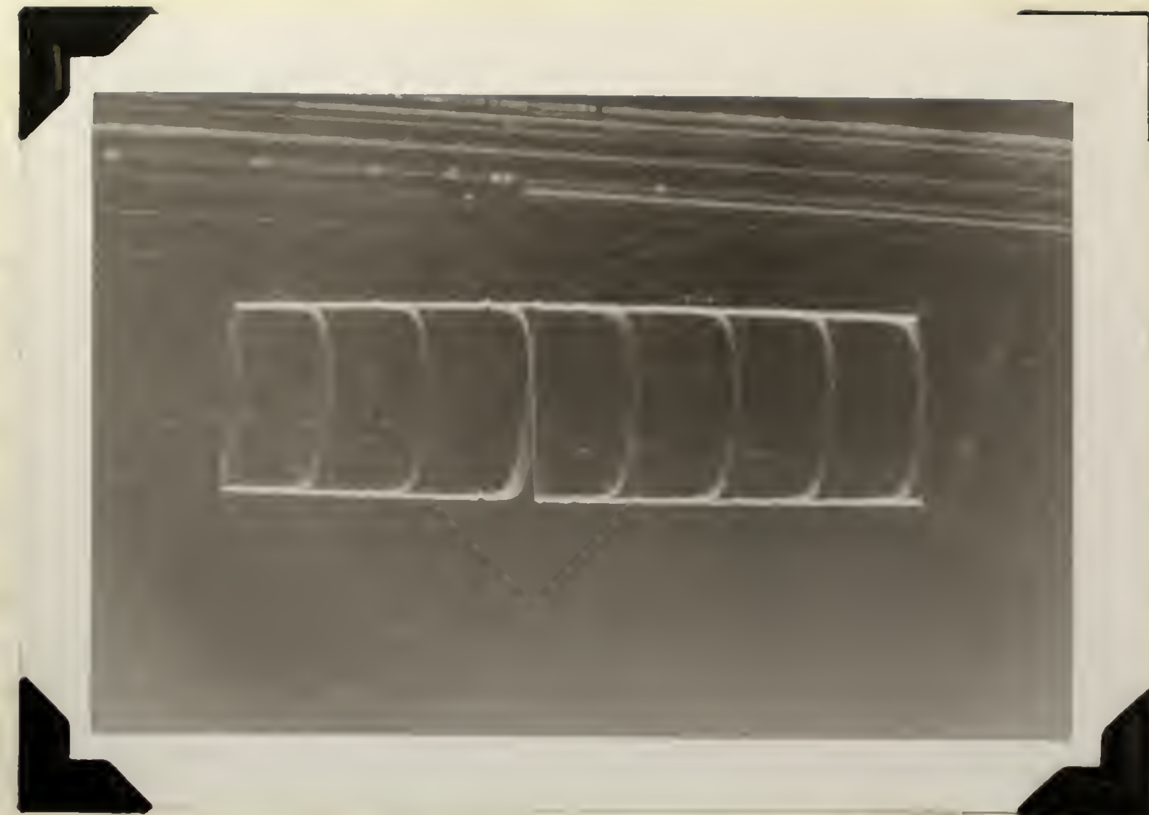


Figure II-5. PCM Input to Decoder without Interference.

Figure II-6. PCM Input to Decoder in Presence of Interference.

signal. The Variable Level Slicer V9 is of a type originally designed at Bell Telephone Laboratories. * Although the circuit resembles in configuration that of a cathode-coupled multivibrator, it functions in a different manner because of the choice of parameter values and the addition of the crystal diodes at the grids. In particular, the plate load resistor for the first triode is made small so as to give approximately unity gain around the feedback path when both triodes are conducting, and the plate-to-grid coupling condenser is made large enough to hold virtually constant potential during operation. The circuit trips whenever the input signal drops through a narrow potential range and trips back again when the signal rises through this same range. Potential level of this narrow slice may be varied by means of R5.

The cathode follower V8B produces a cathode potential varying from 0 volts in the presence of a digit pulse to about 40 volts in the absence of one. The cathodes of the PCM Gating Circuit V10A-V14A share the cathode load of the cathode follower. In the presence of a digit pulse the grids of these stages are slightly below cutoff, and a positive pulse applied to any of them by the Commutator Circuit is passed on to the corresponding Decoding Flip-Flop stage; in the absence of a digit pulse the grids of these stages are well below cutoff and pulses from the Commutator Circuit are blocked. Figure II-7 shows the waveform at the common cathode. This wave form is unaffected by the interference shown in Figure II-6.

The Decoding Flip-Flops V16-V20 are of conventional design. Each stage is triggered "on" by pulses passed by the associated PCM

* Meacham, L. A. and Peterson, E., "An Experimental Multichannel Pulse Code Modulation System of Toll Quality"; B.S. T.J. 27,1 (Jan.,1948),29.

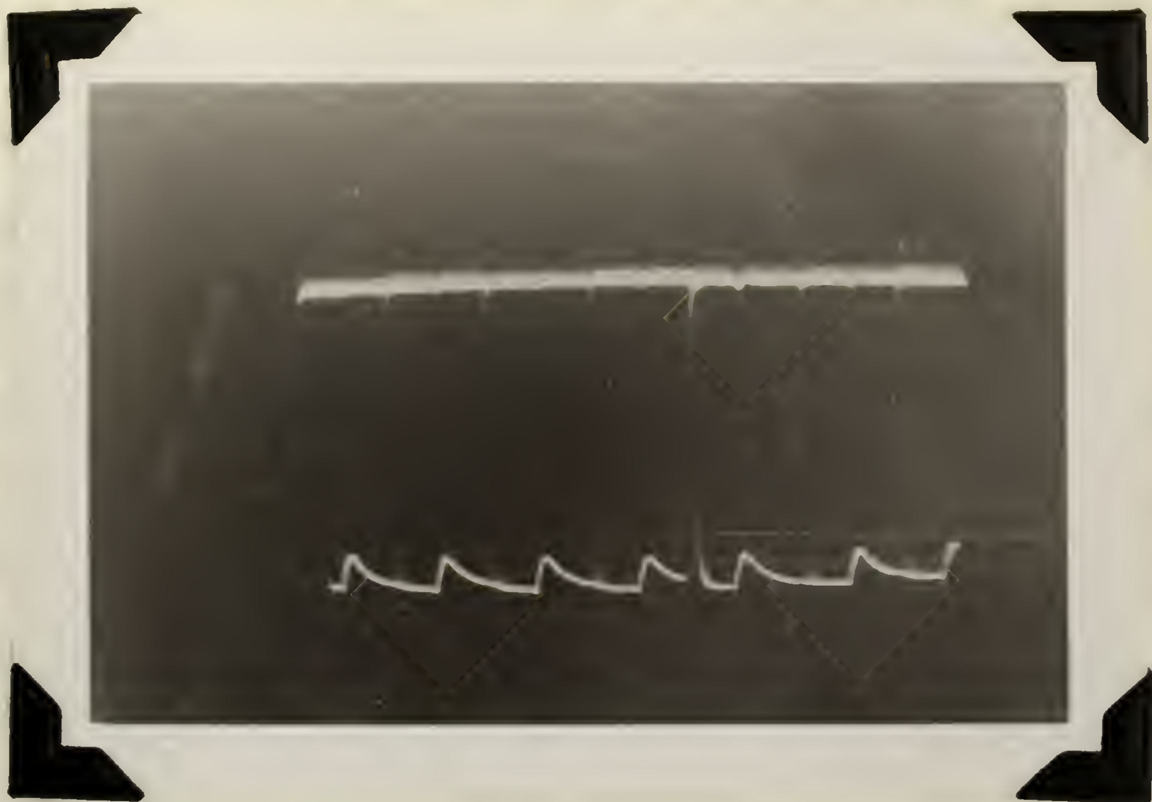


Figure II-7. Waveform at J6A. (PCM gating pulses and gated Commutator Circuit output pulses).

Figure II-8. "Staircase" Waveform at J12A. (Cathode Follower V8 removed to simulate all digit pulses present).

Gating Circuit stage, and "off" by the trailing edge of the Synchronizing Multivibrator output. Coupling to the first stage from its gate tube is tighter than the corresponding coupling for the other stages, because of the fact that an "on" pulse in this circuit must override an "off" pulse arriving at the other grid at the same time. A consideration of importance, particularly in the design of the first stage, is the on"" and "off" plate potential level, since resistance coupling to the buffer stages is highly desirable. The reason for this is apparent from a glance at the waveform for the first stage (Figure II-3(f)); if the first digit pulse is on (or off) for several successive cycles improper operation will result with condenser coupling. Furthermore, slight fluctuations of the flip-flop plate potentials at the time of the leading edge of the Synchronizing Multivibrator output pulse are amplified by the buffer stages when condenser coupling is used.

The Buffer Stages V10B-V14B are self-biased by a cathode R-C circuit, common to all stages, to a potential midway between the excursions of the Decoding Flip-Flop plates. The direct resistance coupling thus causes the buffers to shift from the cutoff condition to a heavily-conducting condition with change in the state of the associated flip-flop.

Limiting diodes V21-V25 hold the excursions of the buffer plates to 0 and 75 volts (established by the VR tube V15).

Connections from the buffer plates to the Decoding Resistance Network are made through jumpers. By removing jumpers the number of code digits used can be decreased for experimental purposes. By revers-

ing the jumpers the decoder can be readily adapted to use with a coder whose digit sequence is reversed. Resistors used in the decoding network are of the wire-wound precision type. For the first stage the plate resistance of the limiting diode becomes significant, and a compensating potentiometer R6 is shunted across the nominally 25K resistance. Figures II-8 through II-10 show various waveforms at the output of the cathode follower V27A.

The sampling Circuit V28-V29 is a wartime development of the Radiation Laboratory, M.I.T.* It was found to give considerably more accurate samples than other circuits tested. An important factor leading to this superiority is the reduction of capacity coupling between input and output of the circuit; in the parallel-triode circuit employed in the R.L.E. coder considerable distortion of the sample results from coupling through the inter-winding capacity of the pulse transformer. Figures II-11 and 12 show the output of the two circuits.

The decoded signal for 32-level PCM is shown in Figure II-13; the same signal after filtering is shown in Figure II-14. Figures II-15 through II-18 show the decoded signal for 16, 8, 4 and 2-level PCM, respectively. Failure of the sampling circuit holding condensers to fully charge is apparent in the 2-level and 4-level cases.

* Chance, Britton, "Some Precision Circuit Techniques Used in Wave-Form Generation and Time Measurement"; Rev. Sci. Inst. 17, 10 (Oct., 1946), 410.

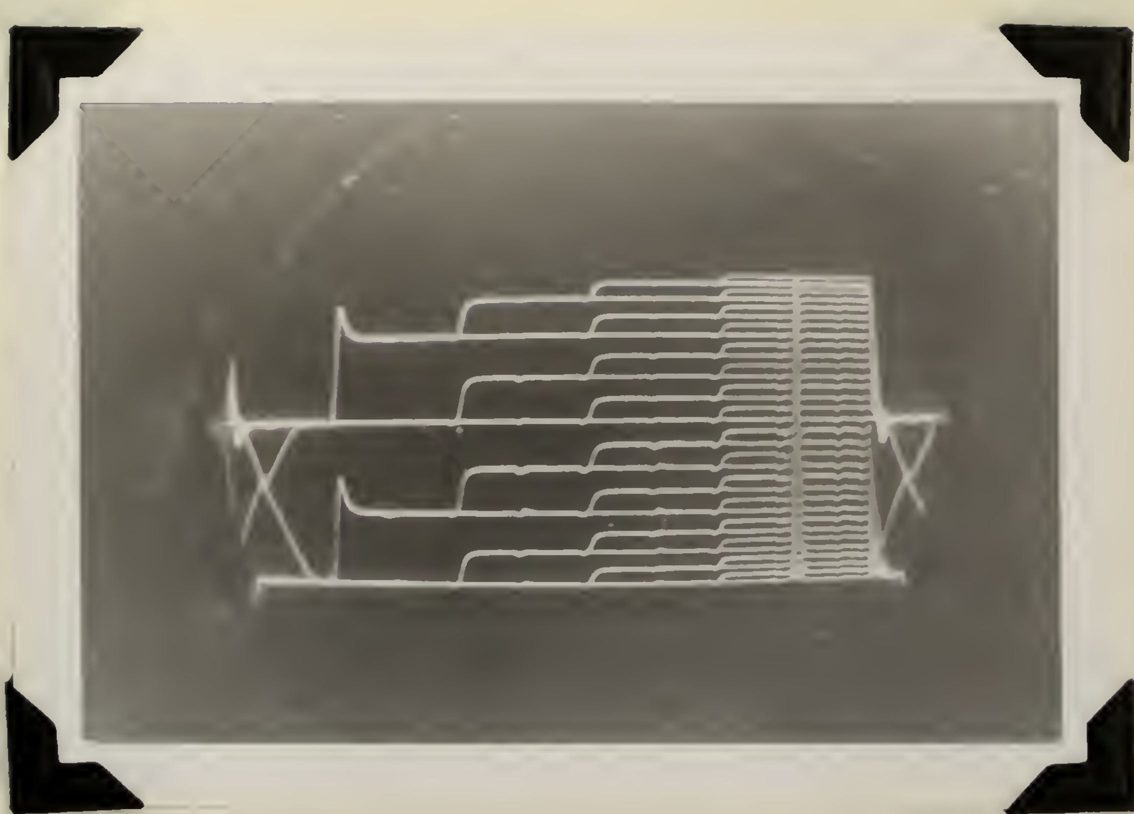


Figure II-9. "Staircase" Waveform at J12A. (Full load signal applied to coder).

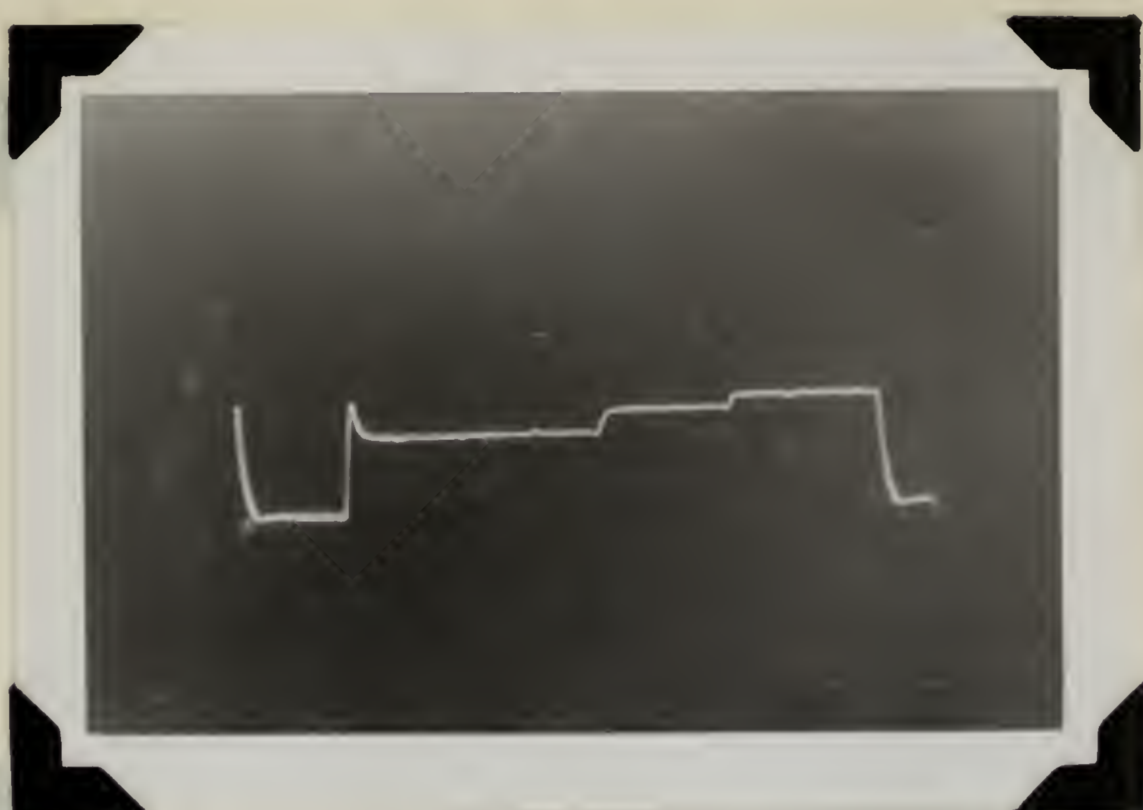


Figure II-10. "Staircase" Waveform at J12A. (Generation of 11-unit output level).

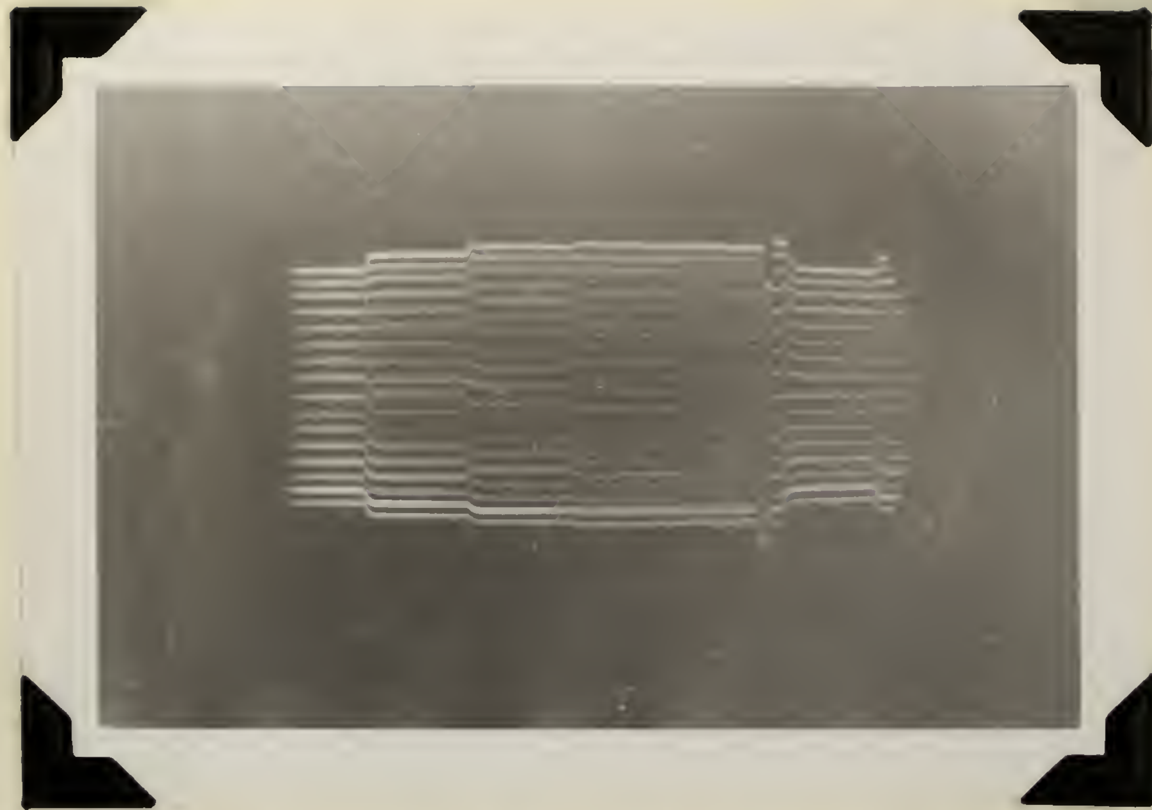


Figure II-11. Output of Parallel Triode Sampling Circuit.
(Sampling coder approximating voltage, 16 levels).

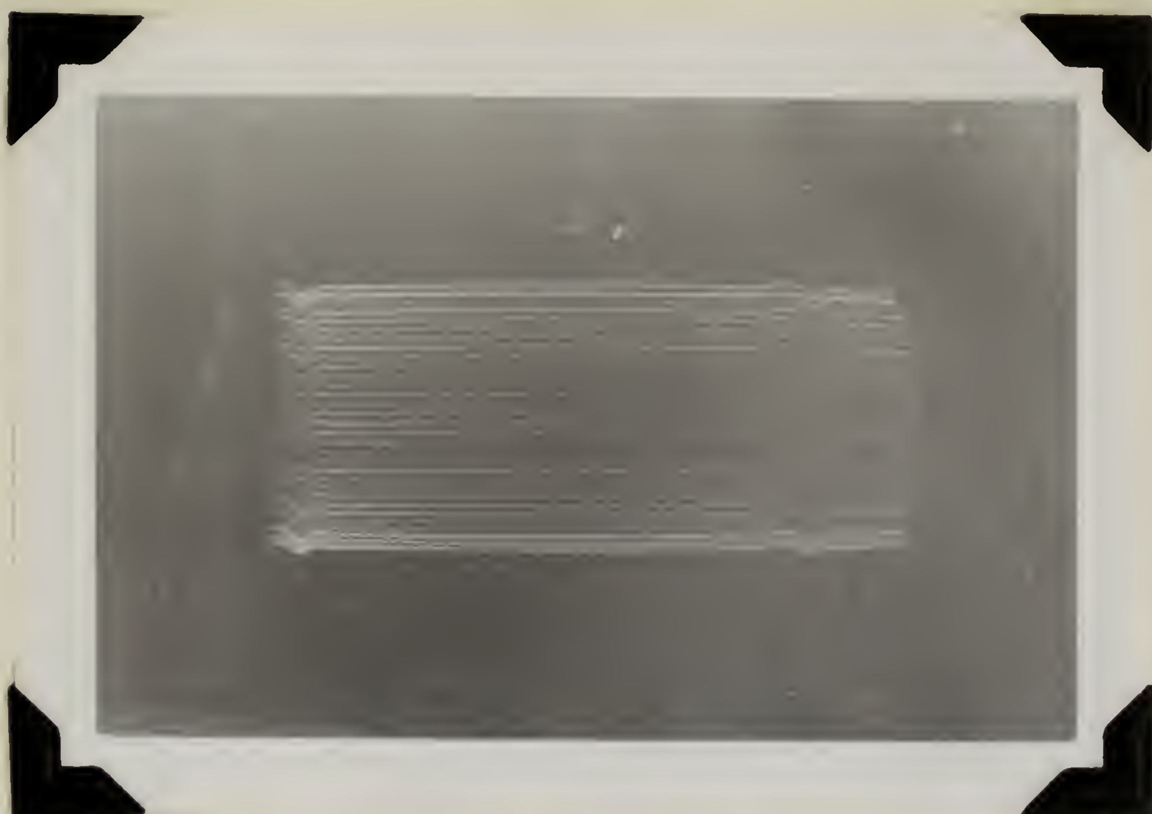


Figure II-12. Output of Four-Diode Sampling Circuit.
(Sampling decoder "staircase" voltage, 32 levels).

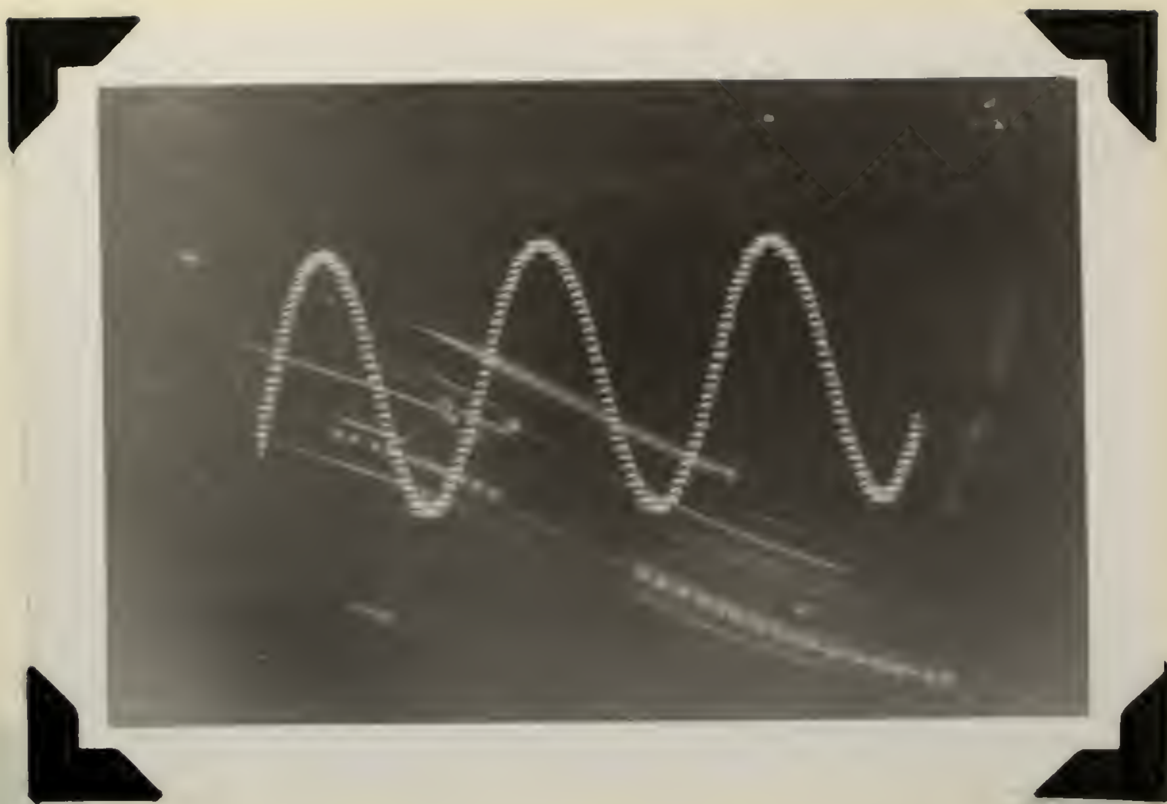


Figure II-13. 32-Level Decoder Output before Filtering.
(200 c.p.s. sine wave input to coder).

Figure II-14. 32-Level Decoder Output after Filtering.
(200 c.p.s. sine wave input to coder).

Figure II-15. 16-Level Decoder Output before Filtering.
(200 c.p.s. sine wave input to coder).

Figure II-16. 8-Level Decoder Output before Filtering.
(200 c.p.s. sine wave input to coder).

Figure II-17. 4-Level Decoder Output before Filtering.
(200 c.p.s. sine wave input to coder).

Figure II-18. 2-Level Decoder Output before Filtering.
(200 c.p.s. sine wave input to coder).

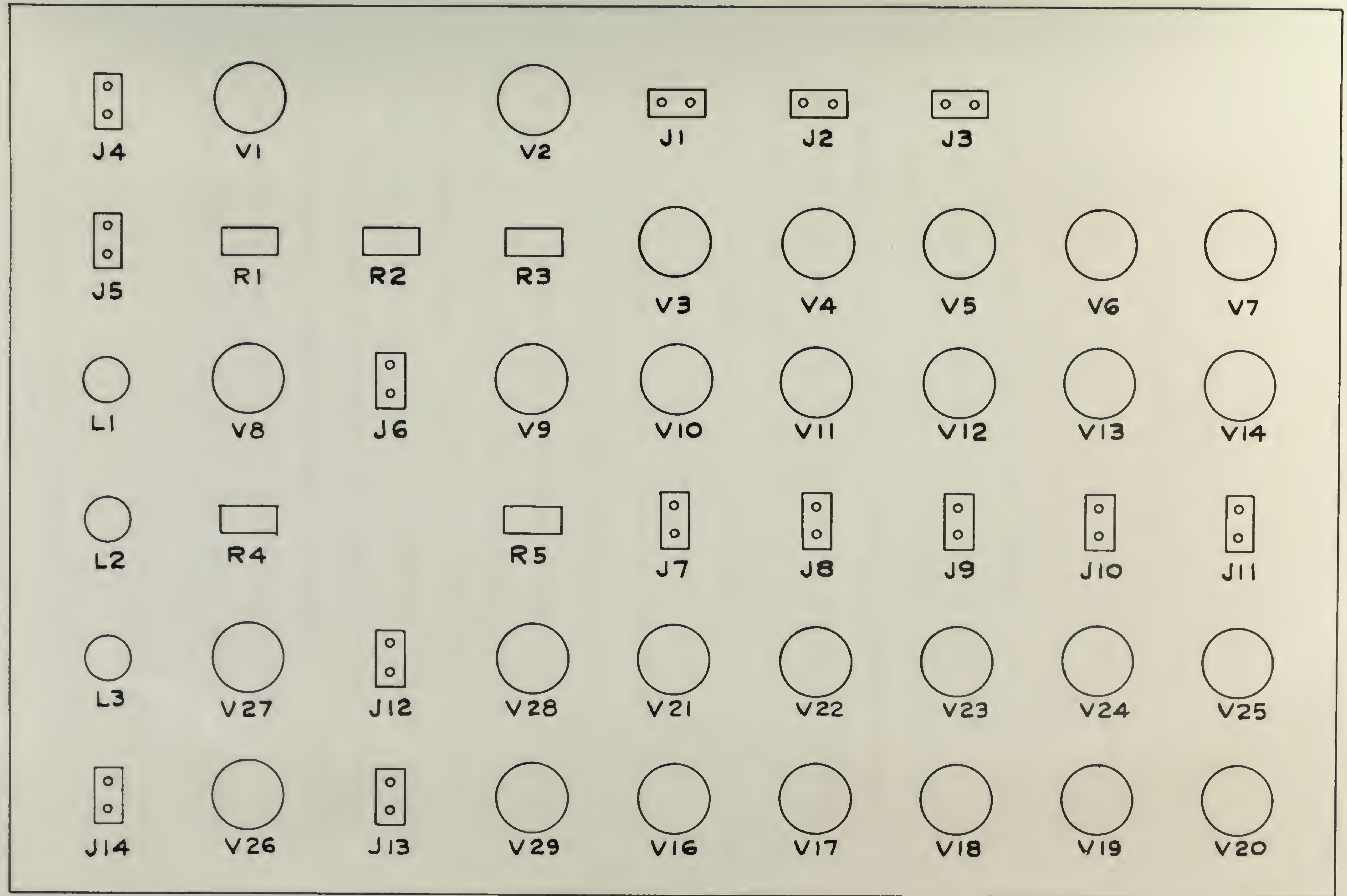


FIG. II-[redacted] 19 SUGGESTED LAYOUT FOR DECODER CHASSIS

Chapter III

Experimental Results

Upon the completion of design, construction and test of the decoder circuit, a PCM system was set up using the decoder in conjunction with a coder developed in the spring of this year by Blanchard D. Smith, Jr. at the M.I.T. Research Laboratory of Electronics.* No carrier signal was employed, since modulation and demodulation of the carrier in a PCM system involves no new techniques. The synchronizing input to the decoder was obtained from the trigger signal for the first "flip-flop" in the coder. Considerable difficulty was had in obtaining proper synchronization of coder and decoder, although in tests with a stable source of synchronizing pulses it had been found that the decoder remained properly "locked in" with a signal varying as much as 5% in frequency. This difficulty is ascribed to the use of a blocking oscillator as the source of basic repetition rate in the coder; this blocking oscillator is loaded by a step counter circuit which contributes to its frequency instability. A certain amount of success was had in overcoming this problem of synchronization when an external synchronizing signal was applied to the blocking oscillator, but best results were obtained during night-time and week-end tests when power supply fluctuations were at a minimum.

* Smith, B. D., Jr., "Pulse-Code Modulation Method"; Thesis for S.M. Degree, M.I.T., June 1948.

FREQUENCY RESPONSE

Frequency response of the system is dependent upon the response of the sampling circuits used at the coder input and at the decoder output. Both circuits are of the "boxcar" type, in which the input voltage is sampled at regular intervals and the sample amplitude held throughout the interval. For an ideal sampler of this type, the ratio of the fundamental component in the output to the input is given by the expression

$$\frac{E_o}{E_i} = \frac{\sin \pi \, f_a/f_r}{\pi \, f_a/f_r} .$$

In this expression f_a and f_r are the audio frequency and the sampling frequency, respectively. This function is plotted in Figure III-1, together with the measured response of the four-diode sampling circuit used in the decoder. Response of the parallel-triode circuit used at the coder input is also shown in this figure.

Overall frequency response is essentially the product of the responses of these two sampling circuits. Figure III-2 shown the theoretical ideal response (the square of the response for one ideal circuit) and the measured response for the actual system.

Neither of the sampling circuits functions in an entirely satisfactory manner. A considerable amount of capacity exists between input and output of the parallel triode circuit, and is partially responsible for its poorer frequency response. The four-diode circuit is better in this respect, since the plate-to-cathode capacity of the diodes is involved instead of the interwinding capacity of the pulse

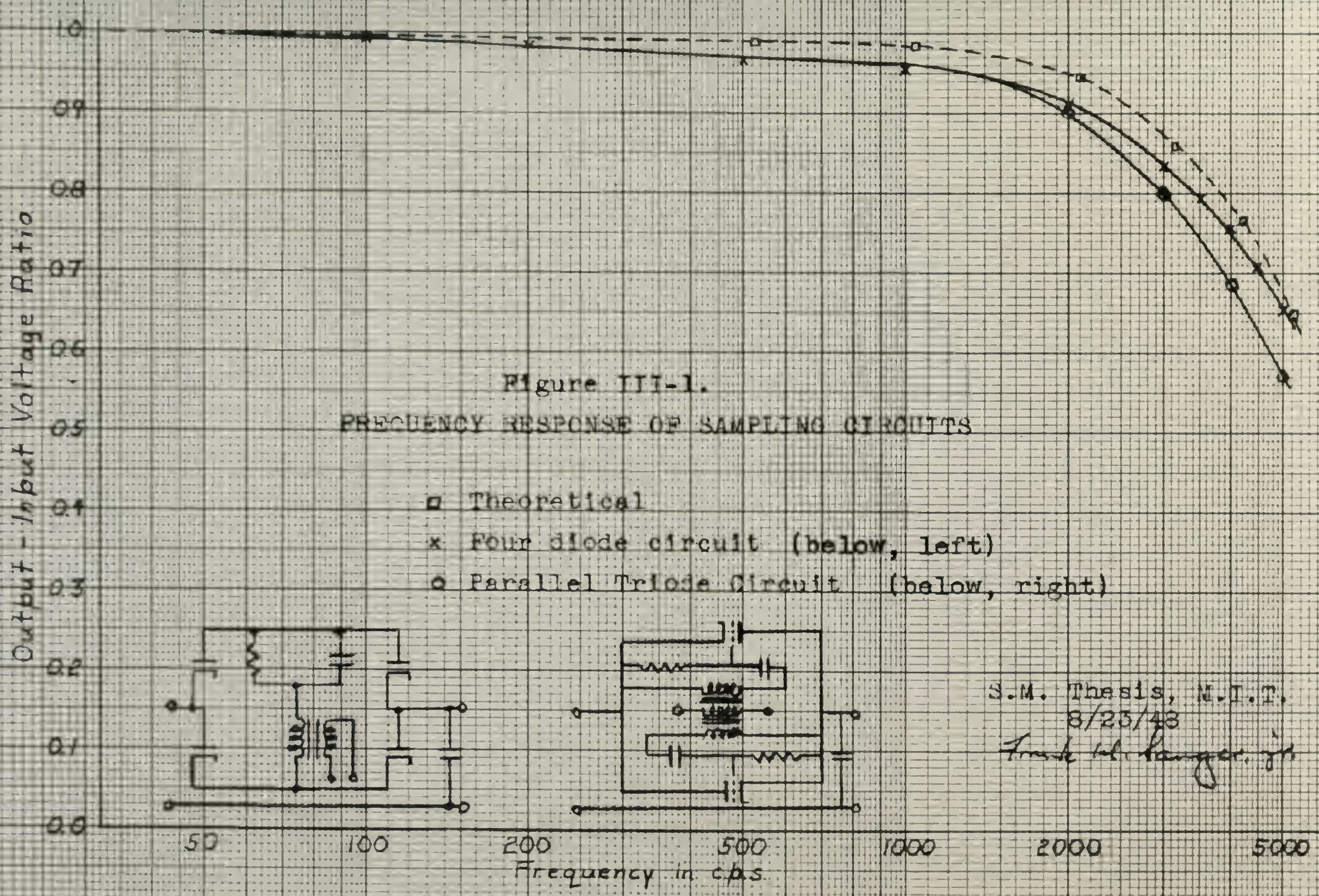
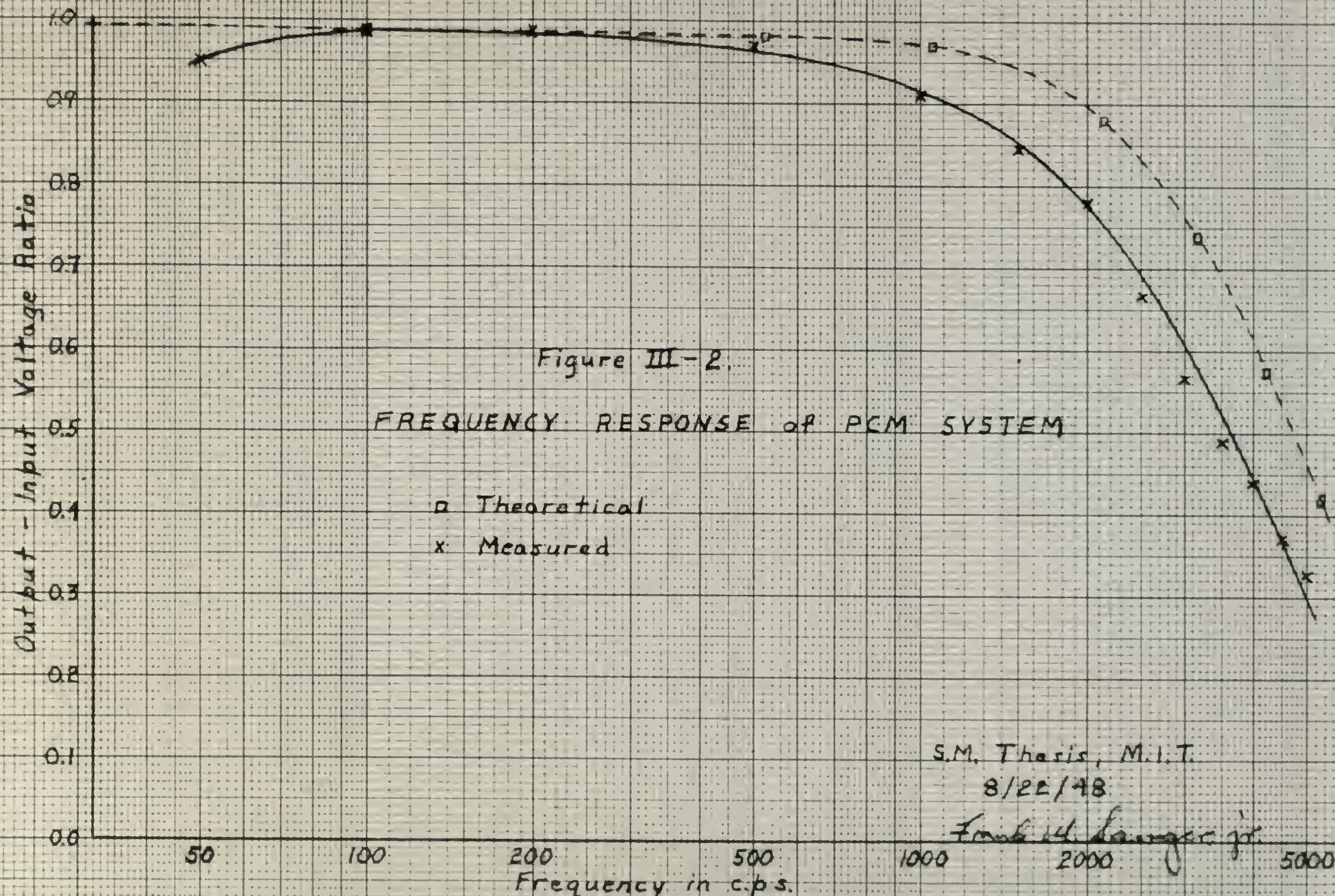


Figure III-1.
FREQUENCY RESPONSE OF SAMPLING CIRCUITS



Output - Input Voltage Ratio
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.8
0.1
0.0
50
100
200
500
1000
2000
5000
Frequency in c.p.s.
Figure III-2.
FREQUENCY RESPONSE of PCM SYSTEM
□ Theoretical
x Measured
S.M. Thesis, M.I.T.
8/22/48

transformer. Further improvement could be had by the use of crystal diodes in place of the 6H6 vacuum tubes. Another factor entering into the frequency response of both circuits is incomplete charging of the holding capacitor during the pulse. This is illustrated by the waveform for two-level decoder output (Figure II-18). Both circuits exhibit this fault; it is again less apparent in the four-diode circuit, and substitution of crystal diodes should again make for improvement.

Distortion

As has been previously pointed out, distortion due to quantization of the signal varies with orientation of the signal level with respect to the quantization steps when N, the number of such steps, is small, and approaches the hyperbola $D_{(rms)} \cong \frac{0.817}{N}$ when N is large (the hyperbolic approximation involves negligible error for N greater than 20). For a sinusoidal input signal, distortion components have been shown to be odd harmonics of the fundamental frequency. * Application of the sampling process to this distortion produces components scattered throughout the frequency spectrum. Bennett # has shown that the r.m.s. response of an ideal low-pass filter, of cutoff frequency equal to one-half the sampling rate, to a sampled signal of this type (assuming infinitesmal duration of samples) is equal to the r.m.s. value of the signal before sampling. Because of the finite frequency

* Clavier, A.G., Panter, P.F. and Greig, D. D., "PCM Distortion Analysis"; Elec. Eng., 66, 11 (Nov., 1947), 1120

\# Bennett, W. R., "Spectra of Quantized Signals"; Bell Sys. Tech. Jnl.. 27, 3 (July, 1948), 446.

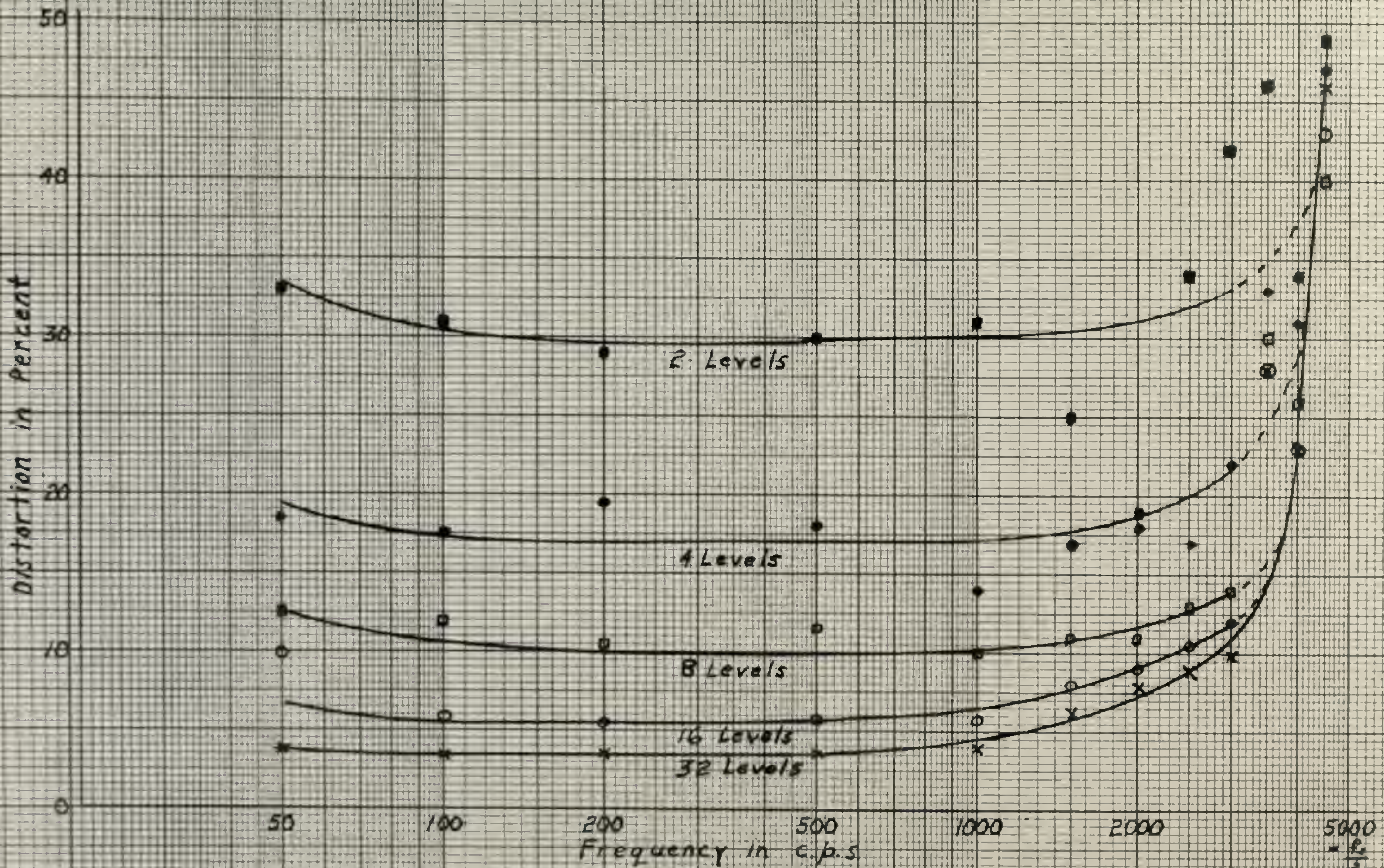
range required for transition from pass band to cutoff, a correction factor must be applied in determining the response of a practical filter. For the average filter this factor is approximately $\frac{1}{2}\sqrt{3}$. Theoretically, then, distortion in the filtered output is given by the expression $D_{(rms)} \doteq \dfrac{0.707}{N}$ when N is large.

Measurements of distortion in the system for various numbers of quantizing levels were made and are plotted in Figure III-3. The Hewlett-Packard Model 330B Distortion Analyzer was used for making these measurements. As Smith has pointed out, these measurements are made inaccurate by the fact that the analyzer does not actually measure r.m.s. distortion; instead, it measures the average value of the distortion signal and indicates r.m.s. value by scale conversion.

Of greater concern is the sharp increase in distortion, regardless of number of levels, at a frequency well below half the sampling rate. This breakdown is due largely to the previously-mentioned fault of the sampling circuits used at input and at output of the system; failure of the holding condensers to completely charge or discharge during the pulsing interval introduces large amounts of distortion.

Improvement of the sampling circuits will not eliminate increase in distortion with frequency. Even for the ideal "boxcar" sampling circuit fundamental output falls off with frequency while distortion components vary radically.

Audio signals (speech and music) were introduced into the system for qualitative test of performance. Results were similar to those reported by others, but demonstrated the poor distortion characteristics of the system at the higher frequencies. Background noise due to quantization was found to be objectionable to the ear for eight-level



PCM SYSTEM - DISTORTION vs. FREQUENCY
(As function of number of quantizing levels)

Figure III - 3

S.M. Thesis, M.I.T.
8/22/48
Frank W. Sanger jr.

quantization, and quite noticeable when thirty-two levels were used. The "square speech" resulting from two-level quantization was found to be intelligible as long as volume range of the speaker was not too great.

Distortion in the Presence of Interference

In order to confirm the interference-reduction characteristics of PCM the variable-amplitude output of a pulse generator was added to the PCM signal input to the decoder, using a simple resistance adding circuit. A common ground existed between the two input signals. Assuming an ideal slicing circuit designed to accept only signals of amplitude equal to or greater than the PCM signal, no distortion of the decoder output will exist until the interfering signal is equal in amplitude to the PCM signal. Distortion resulting after this threshold is reached is dependent on the duty cycle of the interfering pulse generator and not upon its frequency, provided that no harmonic relationship exists between its frequency and the digit repetition rate.

The introduction of a signal of this type at the decoder input can be treated as a problem in probability. Assuming an interference duty cycle (ratio of pulse duration to repetition period) R, it follows that the probability that any one of the Commutator Circuit outputs will be gated by the interference in any one cycle of decoder operation is also R. Provided that complete lack of synchronisation between interference and PCM input exists, the gating probabilities for the five Commutator Circuit outputs are independent of one another, and the Bernoulli Distribution Law * applies. For our purposes the law may be expressed as follows:

$$P_m(n) = \frac{m!}{n!\,(m-n)!} \quad R^n(1-R)^{m-n}.$$

* Goldman, Stanford, "Frequency Analysis, Modulation and Noise"; McGraw-Hill Co., New York, 1948, p. 290.

$P_m(n)$ is the probability that n Commutator Circuit outputs of a total of m will be gated in one cycle of decoder operation.

For the five-digit system under test, m = 5, and the expression becomes:

$$P(n) = \frac{120}{n!(5-n)!} \quad R^n(1-R)^{5-n}.$$

The computed values of P(n) for various values of R are given in Table III-1.

For each value of n there is equal probability of generating any one of a number of different amplitude levels. One digit pulse, for example, can generate amplitudes of 1,2,4,8 or 16 units; which one of these is produced depends on the position of the pulse. Table III-2 gives the amplitude levels associated with each value of n, and the number L of such levels.

The probability of generating any given amplitude level is the product of two factors: first, the probability of generating the number of digit pulses required to form the level, and second, the probability of forming the level in question when the required number of digit pulses are present. In equation form,

$$P(N) = \frac{P(n)}{L}.$$

The values of n and L corresponding to a given value of N are obtained from Table III-2.

The mean square value of the signal produced at the decoder output by the interference signal is given by the expression:

$$E'^2 = \sum_{N=1}^{31} P(N) \times N^2$$

TABLE III-1.

P(n) for Various Values of R.

R	n = 1	n = 2	n = 3	n = 4	n = 5
0.50	0.155	0.313	0.313	0.155	0.031
0.25	0.396	0.255	0.088	0.015	0.001
0.10	0.328	0.073	0.008	0.0005	-----
0.02	0.092	0.004	-----	------	-----

TABLE III-2.

Amplitude Levels for Various Values of N.

n	L	Associated Amplitude Levels	$\sum A^2$
1	5	1,2,4,8,16	341
2	10	3,5,6,9,10,12,17,18,20,24	1984
3	10	7,11,13,14,19,21,22,25,26,28	3906
4	5	15,23,27,29,30	3224
5	1	31	961

TABLE III-3.

R.M.S. Distortion for Various Values of R.

R	R.M.S. Distortion	% Total Output Signal (for Full-Load Sinusoid)
0.50	18.0 units	84%
0.25	11.1 "	72%
0.10	6.35 "	50%
0.02	2.71 "	24%

The R.M.S. function is tabulated in Table III-3. The table also includes a tabulation of this distortion as percentage of total output signal for full-load sinusoidal input.

The results of experimental measurements of distortion under the conditions described are shown in Figure III-4. The Hewlett-Packard Model 330B Distortion Analyzer was again used, and the accuracy of measured distortion values is again compromised by its method of measurement. Measurements were made at several frequencies both above and below the digit repetition frequency in order to confirm the fact that duty cycle and not frequency of the interfering signal is the factor determining the distortion.

Although the experimental results are of little value in a quantitative sense, they demonstrate the existence of the expected sharp threshold interference level and the decrease in distortion with decrease in interference duty cycle. The finite range of amplitude ratios required for the buildup to ultimate distortion level is ascribed to the fact that the Variable-Level Slicer Circuit operates over a narrow but finite range of input voltages instead of the infinitesmally narrow range assumed in the theoretical discussion. The change in threshold level with change in interference duty cycle is due to high-frequency attenuation in the input circuits, resulting in incomplete buildup of narrow pulses.

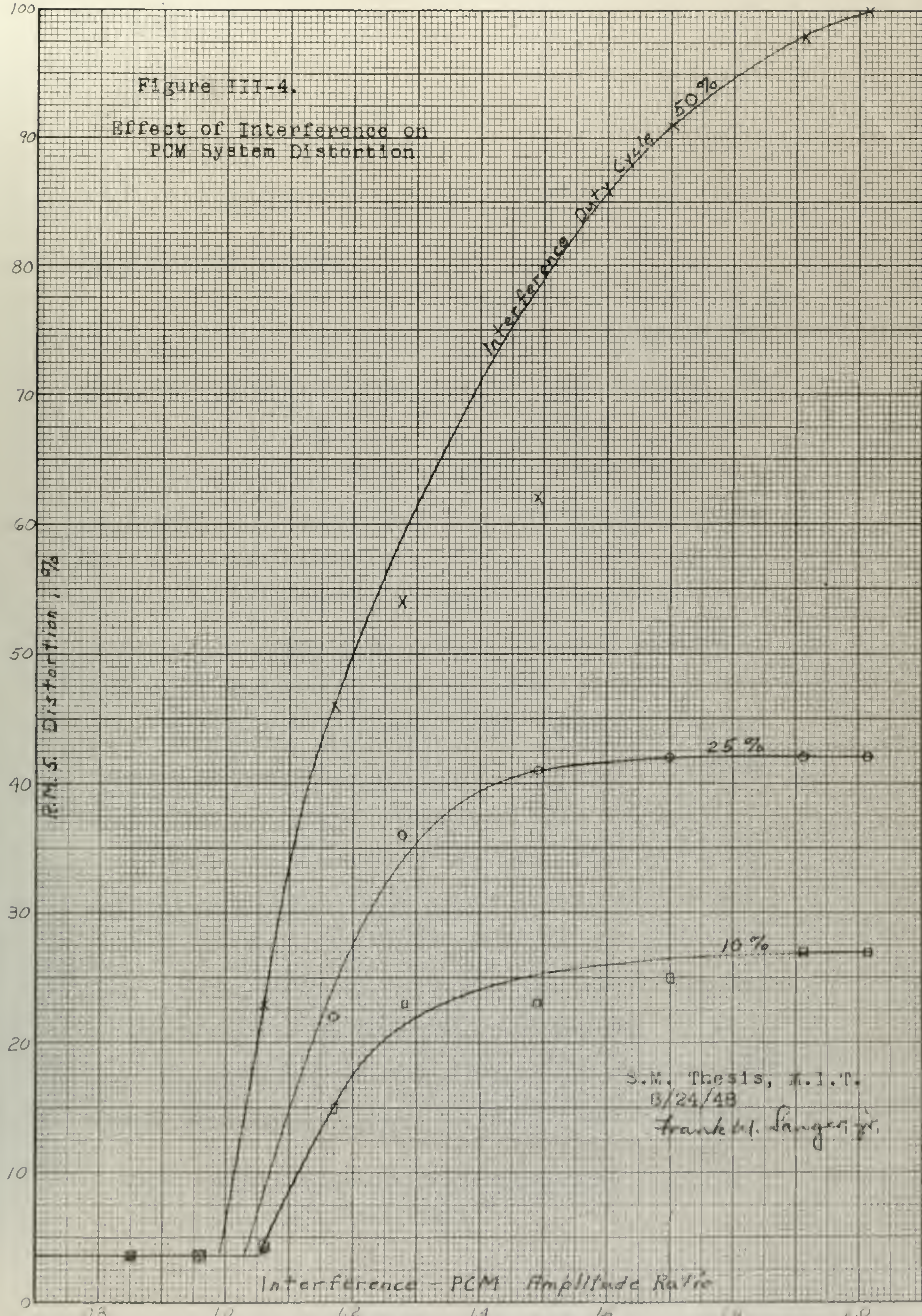


Figure III-4.
Effect of Interference on
PCM System Distortion
Interference Duty Cycle 50%
25%
10%
R.M.S. Distortion, %
Interference - PCM Amplitude Ratio
S.M. Thesis, M.I.T.
8/24/48
Frank W. Sanger, jr.
100
90
80
70
60
50
40
30
20
10
0
1.0
1.2
1.4

CHAPTER IV

Recommendations for Further Study

Improvements in Equipment

The most serious difficulty experienced in working with the PCM system has been found to lie in the coder timing circuits. * Figure IV-1, from the reference paper, shows these circuits. V1A is a free-running blocking oscillator, operating at a repetition rate of about 50 kcps. V1B is a blocking oscillator driven by the output of the step-charging circuit made up of C31, C32, V2, R91, R92 and C35. A staircase waveform is generated across C32, and is passed on to the cathode follower V3A. Cathode potential of this tube is applied to the grids of tubes V4-V7 (not shown) which form a commutator circuit. The cathodes of these tubes are biased successively more positive, so that consecutive risers of the staircase trigger the tubes in proper sequence. These tubes are dependent upon accurate step size for proper operation.

R92 is the adjustment for step size. This size must be such that the commutator stages trigger properly and at the same time must be such that the blocking oscillator V1B triggers on every fifth riser. Adjustment of R92 changes both the step size and the d-c level of the bottom step; this fact makes for a critically narrow range of settings where proper operation of V1B occurs. The commutator circuit bias voltages must be accurately

* Smith, B.D.jr., "Pulse Code Modulation Method"; Thesis for S.M. Degree, M.I.T., June, 1948.

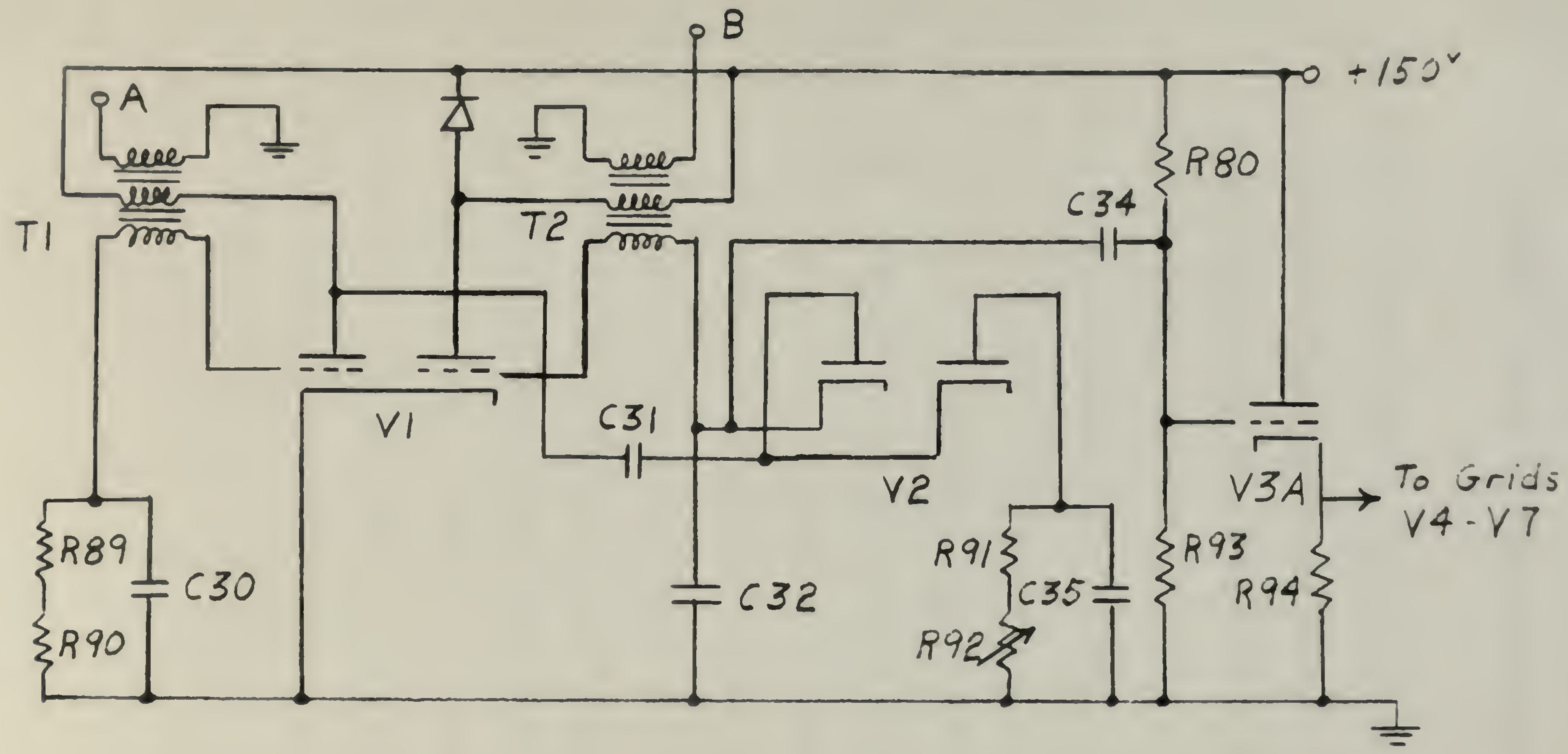


FIGURE IV-1

CODER TIMING CIRCUITS

adjusted so as to give proper operation within this narrow range of settings.

Under ideal conditions the strict requirements for the setting of R92 can be met, and satisfactory operation obtained. It is apparent, however, that the circuit is critically dependent on the maintenance of the proper waveform at the plate of the blocking oscillator V1A. Under normal daytime operating conditions, with accompanying fluctuations in line supply voltage, this waveform cannot be maintained, and erratic operation of the coder results. Changes in basic repetition rate of the blocking oscillator also enter into the problem; adjustment of R92 changes the loading on V1A and causes radical changes in frequency which the decoder (synchronized by the output of V1B) cannot follow.

Some improvement in stability was obtained by changing the values of some of the resistors in the commutator circuit bias network, and it is possible that further experimentation along this line may make for more improvement. Regulation of filament supply voltage would undoubtedly improve operation during line voltage fluctuations. It is felt, however, that complete redesign of timing and commutator circuits, with elimination of the blocking oscillator source of basic repetition rate, is desirable. Timing and commutator circuits similar to those used in the decoder are recommended.

Considerable improvement in high-frequency response of the system will result from correction of the faults in the sampling circuits. These faults have been discussed in a preceding section of

this paper, but will be briefly reviewed at this point. Falling off of the frequency response and distortion at high frequencies are due to incomplete charging of the holding condenser during the pulsing interval and to the existence of a direct capacity path between input and output. The four-diode circuit used in the decoder is somewhat better than the parallel triode circuit used in the coder as regards the first of these faults, and is far better as regards the second. Further improvement in both respects are to be expected from substitution of crystal diodes for the vacuum tubes used in the decoder. Lengthening of the driving pulse, if feasible, is also recommended. The limiting factor in any attempt to do this will be the pulse transformer used.

The input-to-output capacity of the sampling circuit used in the coder affects operation of the comparison circuit and ultimately changes the character of the outgoing PCM pulses. For ideal operation the inputs to the comparison circuit are constant in amplitude for the duration of each digit repetition period. If the constant-amplitude sample of the audio signal is greater in magnitude than the approximating voltage, a digit pulse is transmitted. Leading and trailing edges of the digit pulses coincide with the instants of change in the approximating voltage. The code signal transmitted and received at the decoder is a true representation of the sampled audio value.

The presence of input-to-output capacity in the sampling circuit causes distortion of the samples passed to the comparison circuit. Instead of being constant in amplitude, the sample is

varying throughout the digit repetition period. The comparison circuit may be forced to "change its mind" at some time during the period assigned to one of the digit pulses, with resultant foreshortening or late appearance of that pulse. In either case, improper decoding will occur if the decoder makes its test for the presence of the pulse before the change occurs.

A partial solution to the problem discussed above is to adjust the decoder so that the test is made as late as possible in the period assigned to each digit. When this scheme is attempted, however, a point is reached at which operation of the decoder suddenly fails completely. The Synchronizing Multivibrator is rather heavily loaded; at the point in question its output fails to reliably trigger the Decoding Flip-Flop stages. Minor modifications in design of the circuit to permit further lengthening of its output pulse are recommended if modifications to the coder sampling circuit do not solve the difficulty. It should be noted, however, that the circuit as herein described permits widening of the pulse considerably beyond the halfway point in the digit period (the proper time for testing for the presence of digit pulses rounded off by limited transmission bandwidth).

Continuation of Experimental Work

The study of distortion effects in PCM caused by interference from a pulse-type signal leads at once to the question of distortion due to other types of interference. Of particular interest is the question of random noise. Such interference might

arise from natural causes in a lengthy transmission path, or might originate (in military applications) in manmade jamming equipment. Such equipment provides a readily available means of testing the performance of a PCM system in the presence of noise-type interference. An analysis of the problem would involve, in addition to the work carried out in this paper for pulse-type interference, the use of probability functions for the noise signal itself. In making tests with this type signal, the slicing level should be set to half the PCM amplitude instead of equal to it as was done in the case of the pulse-type interference. The reason for this change is that the noise signal is equally distributed on both sides of the d-c level; proper slicing occurs when peaks of one polarity are as likely to introduce a pulse supposedly absent as are peaks of the opposite polarity to mask a pulse supposedly present.

An interesting, though perhaps not too instructive, study that might be made is an investigation of the possibilities of "ticker-tape" recordings. No particular difficulties involved in such a process are readily apparent, and the production of low-cost home and commercial recordings by use of PCM seems a distinct possibility.

The construction of a multiplexed PCM system is an obvious subject for further development at M.I.T. Considerable work has already been done along these lines by the telephone companies, but the field is new enough to permit room for many additional workers without too much duplication of effort. The decoder described in this paper was designed with multiplexing in mind, and should, with

the addition of output commutator circuits, handle without difficulty at least three channels using the sampling rate now employed.

The principle problem to be met in the PCM field is simplification of equipment. The complexity of most present-day apparatus is tremendous - the system at present under study is an excellent example, employing some sixty tubes, most of them dual triodes, to code and decode one channel of information. Because of this complexity the telephone companies, with adequate space for terminal equipment, have been the principle backers of PCM experimentation. Simplification of equipment would quickly open up many other fields for the use of PCM. An extreme example of this potential applicability is the field of guided missile telemetering. For long-range work, the noise-suppression characteristics of PCM make it ideally suitable in this field. On the other hand, the extreme space and weight limitations imposed on equipment mounted in the missile put present-day PCM coding equipment completely out of the picture.

Military communications is a field to which PCM is by nature well adapted. The development of some means of varying the code used for transmission would be a valuable contribution to the problem of communications security. The coding problem could be readily solved by the use of a coding tube of the type used in the Bell Telephone Laboratories' experimental system briefly described earlier in this paper. Any desired code could be obtained by proper punching of the aperture plate, and the code in use could be readily changed by changing tubes. Decoding of the signal is a far more complicated problem that remains to be solved.

Considerable activity is now going on in the PCM field, and rapid advances are likely in the not-too-distant future. It is hoped that the availability of a decoder adaptable to use with a variety of coders will be of material aid in future investigations at the Institute.