A Power-Sensitive Servomechanism for the M.I.T. Network Analyzer

by

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May 20, 1949

Professor J. S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In accordance with the requirements for the Degree of Naval Engineer, we submit herewith a thesis entitled, "A Power Sensitive Servomechanism for the M.I.T. Network Analyzer."

Respectfully,

A POWER SENSITIVE SERVOMECHANISM
FOR THE
M.I.T. NETWORK ANALYZER

By

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of the Requirements for the
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I. Summary

Object

The object of this thesis was to design a servomechanism for maintaining a predetermined power output from a Network Analyzer phase-shifter as employed in power system studies.

Introduction

Since the power output of the phase-shifter when connected to a network, already supplied by other sources, is a function of its shaft angle, the power-sensitive servomechanism will maintain the predetermined power output by continuously controlling the shaft angle of the phase-shifter.

Procedure

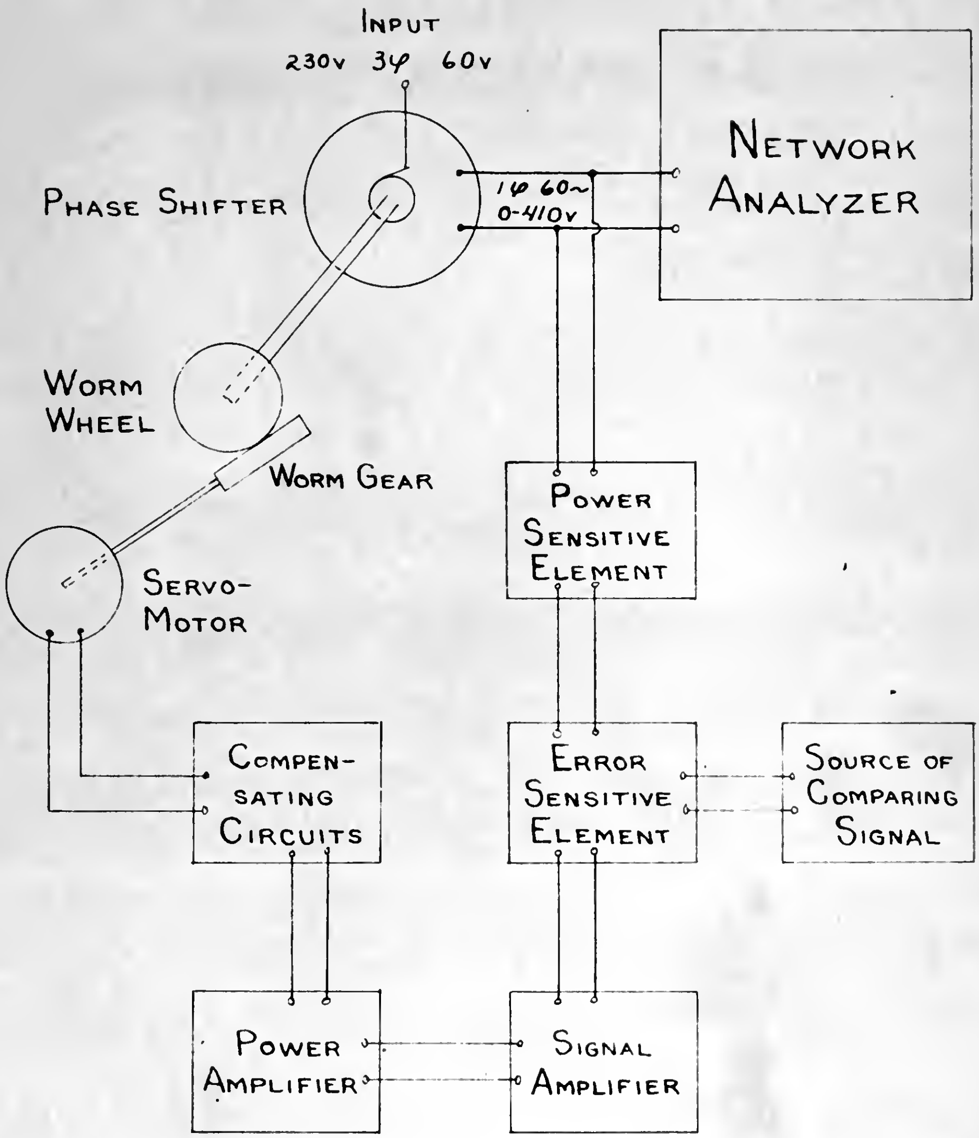
The block diagram of Figure I illustrates the approach that was made to the problem. After several commonly-used systems for accomplishing the purpose were studied, the armature-control direct-current motor system was chosen. Once the basic system was selected, the design of the individual components was undertaken.

The component parts that were built were as follows:

(1) A power-sensitive device consisting of an electronic wattmeter and a voltage amplifier.

(2) A comparing device made from a potentiometer.

FIGURE I
BLOCK DIAGRAM
OF
POWER SENSITIVE SERVOMECHANISM



(3) An electronic voltage and power amplifier; the error-sensitive feature of the voltage stage eliminated the need for an independent error-sensitive device.

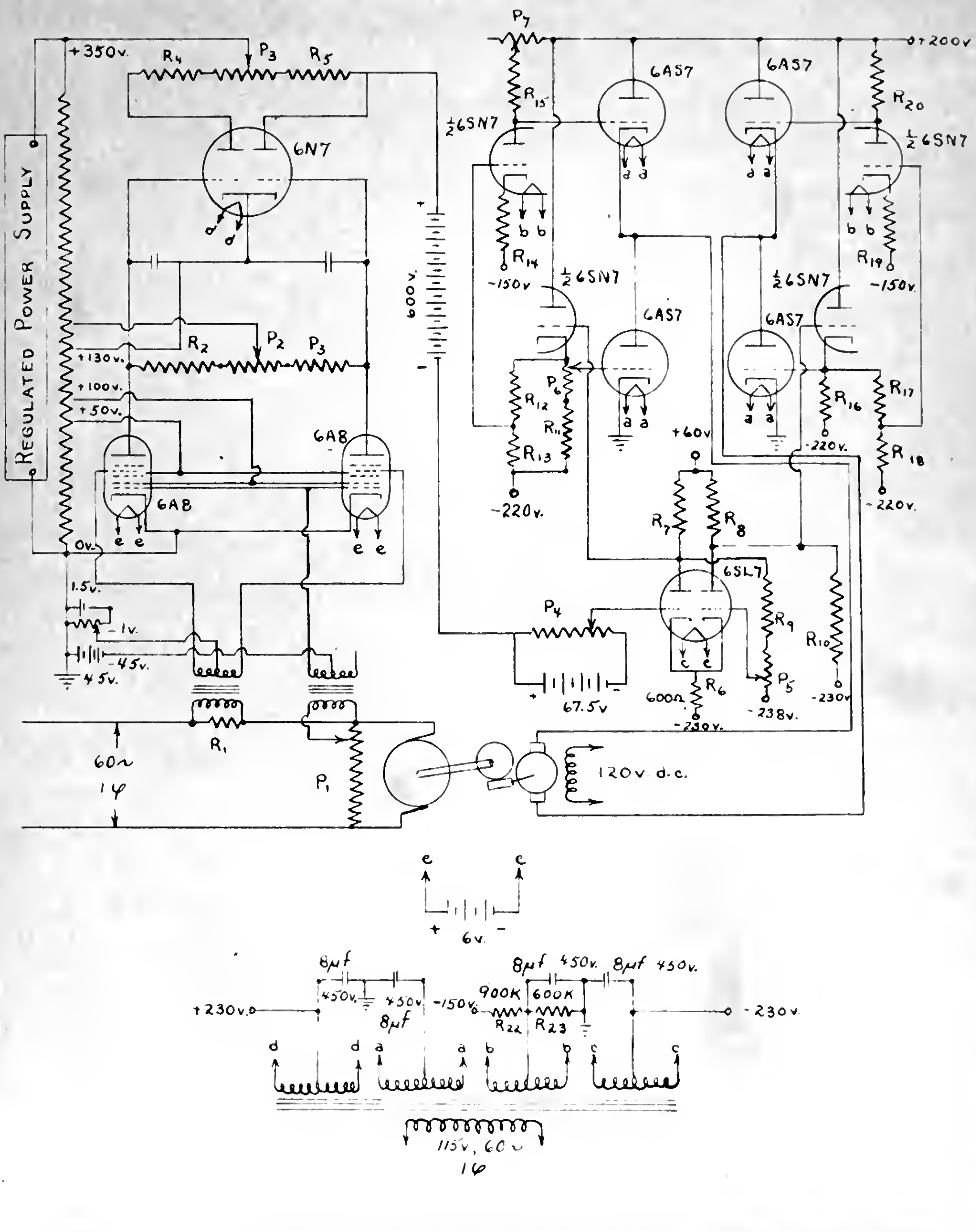
After these components had been developed, they were assembled into a closed loop, similar to that illustrated in Figure I. The circuit diagram of Figure II shows the details of the assembled servomechanism.

Results

Although the design cannot be considered sufficiently complete to permit construction of units for installation in the Network Analyzer, the progress that was made in the design definitely demonstrates that a power-sensitive servomechanism for the Network Analyzer is practical. The fact that the uncompensated system was able to maintain a relative power setting when operational tests were performed is the most important result.

Since the lack of time prevented extensive testing of the system, the system was not compensated. During operational tests the servomechanism showed a tendency to oscillate which indicated that compensation is needed. However, this "hunting" is indicative also of good response.

Figure II
Power Sensitive Servomechanism



Recommendations

To determine the performance of the uncompensated system so that compensating devices may be developed, frequency-response tests should be made.

In addition, although the design as shown in Figure II was able to justify the undertaking, several improvements should be made on its component parts before any permanent installation is planned.

II. Introduction

The M.I.T. Network Analyzer, located in Room 10-381, was built in 1929. The General Electric Company and the Institute cooperated in its development, design and construction. Although built primarily for educational purposes, careful consideration was given in its design to making it suitable for commercial engineering service. Since the time it was first placed into service, it has been used extensively as an aid in solving problems in both categories.

In the field of electric-power transmission and distribution, both theoretical and practical problems often become so complex that their analytic solution is nearly impossible. Electric light and power companies have vast networks of interconnecting power lines and loops. These networks usually include several generating stations and many sub-stations in order to supply loads adequately and maintain the system voltages at their proper values. Studies are being conducted much of the time to determine the most efficient use of existing equipment, how to expand to supply future loads, the effects of short circuits and other outages and other problems associated with system operating and planning. The use of a network analyzer for the solution of these problems avoids the nearly impossible task of hand calculation and analysis, and quickly

yields results which are within an accuracy of one to two per cent based on the data supplied for the problem. An important factor in a network-analyzer power-system problem solution is that the entire system or segment of the system under analysis may be studied as a whole rather than as a line-by-line or element-by-element process as is found in a hand-calculated solution.

The Network Analyzer consists of sufficient electrical parameters (adjustable units of capacitance, inductance and resistance), "phase shifters" and auto-transformers collected and arranged in such a manner that any average sized network can be represented in model size. Although the components are normally wired into a single-phase system, the Network Analyzer representation actually becomes the analog of the multi-phase system it is arranged to represent. In actual power networks, however, the generating units are rotating machines, but in the Network Analyzer, they are represented by static units known as "phase shifters".

A phase shifter, when used as a source of power for a network synthesized on the Network Analyzer, adequately represents an alternator supplying power to an actual network. This becomes apparent when one examines the general equations for power flow into a network (obtained from reference (5)). Consider that the single-phase network

of Figure III is made up of strictly constant parameters and a number of voltages whose magnitude and phase are known are impressed at n points. Assume that the positive direction of current flow is into the network. The vector diagram of voltages is shown in Figure IV, where the phase position of each voltage is indicated by its displacement angle with respect to a common axis of reference.

Inasmuch as the development of the expressions is included in the Appendix, only the results are furnished in the following. The expressions for the power and the reactive power, respectively, entering the network at point n of a total of s points of entry are:

$$P_n = \frac{|E_n|^2}{|Z_{nn}|} \sin \alpha_{nn} + \sum_{m=1}^{m=n-1} \frac{|E_m||E_n|}{|Z_{mn}|} \sin(\delta_{nm} - \alpha_{mn}) + \sum_{m=n+1}^{m=s} \frac{|E_m||E_n|}{|Z_{mn}|} \sin(\delta_{nm} - \alpha_{mn}) \qquad (1)$$

$$Q_n = -\frac{|E_n|^2}{|Z_{nn}|} \cos \alpha_{nn} + \sum_{m=1}^{m=n-1} \frac{|E_m||E_n|}{|Z_{mn}|} \cos(\delta_{nm} - \alpha_{mn}) + \sum_{m=n+1}^{m=s} \frac{|E_m||E_n|}{|Z_{mn}|} \cos(\delta_{nm} - \alpha_{mn}) \qquad (2)$$

FIGURE III
GENERAL n-TERMINAL NETWORK

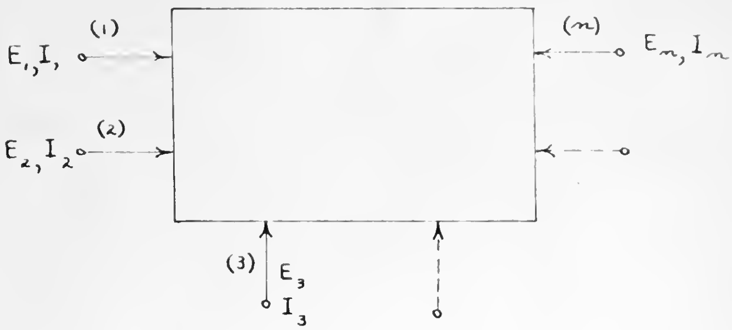
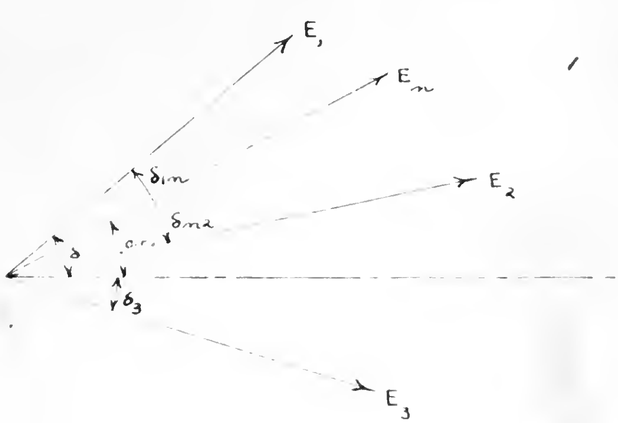


FIGURE IV
VECTOR DIAGRAM OF TERMINAL VOLTAGES
FOR ABOVE NETWORK



where:

(a) the impedances with equal subscripts are short-circuit driving-point impedances and those with unequal subscripts are short-circuit transfer impedances.

(b) the angle α is the complement of the impedance angle; i.e.

$$\theta_{12} = 90^\circ - \alpha_{12}$$

$$\theta_{n1} = 90^\circ - \alpha_{n1}$$

etc.

(c) in using these equations, it should be remembered that

$$\delta_{mn} = \delta_m - \delta_n$$

so that

$$\delta_{mn} = - \delta_{nm}$$

where as

$$\alpha_{mn} = + \alpha_{nm}$$

In view of the fact that equations (1) and (2) are so compact that they do not indicate readily the complex inter-relationship among the various machines that feed a network, a more comprehensive insight can be obtained by applying these equations to a network with a finite number of impressed voltages. A suitable case which il-

lustrates this inter-relationship adequately is one in which a network is supplied by three voltage sources, as shown in Figure V.

$$P_1 = \frac{|E_1|^2}{|Z_{11}|} \sin \alpha_{11} + \frac{|E_2||E_1|}{|Z_{21}|} \sin(\delta_{12} - \alpha_{21}) + \frac{|E_3||E_1|}{|Z_{31}|} \sin(\delta_{13} - \alpha_{31}) \tag{3}$$

$$P_2 = \frac{|E_2|^2}{|Z_{22}|} \sin \alpha_{22} + \frac{|E_1||E_2|}{|Z_{12}|} \sin(\delta_{21} - \alpha_{12}) + \frac{|E_3||E_2|}{|Z_{32}|} \sin(\delta_{23} - \alpha_{32}) \tag{4}$$

$$P_3 = \frac{|E_3|^2}{|Z_{33}|} \sin \alpha_{33} + \frac{|E_1||E_3|}{|Z_{13}|} \sin(\delta_{31} - \alpha_{13}) + \frac{|E_2||E_3|}{|Z_{23}|} \sin(\delta_{32} - \alpha_{23}) \tag{5}$$

In equations (3), (4) and (5), the magnitudes of the impressed voltages, the driving-point impedances, the transfer impedances, and the impedance angles for a particular network can be treated as constants under certain set conditions. Then the power flow into the network varies with the angles δ .

Dependent upon how the machines are represented, the angle delta (δ) can represent any one of several angular displacements. The entry points into the network can be considered as the terminals of the machines for load studies; as the excitation voltages of the machines where the syn-

FIGURE V
THREE TERMINAL NETWORK

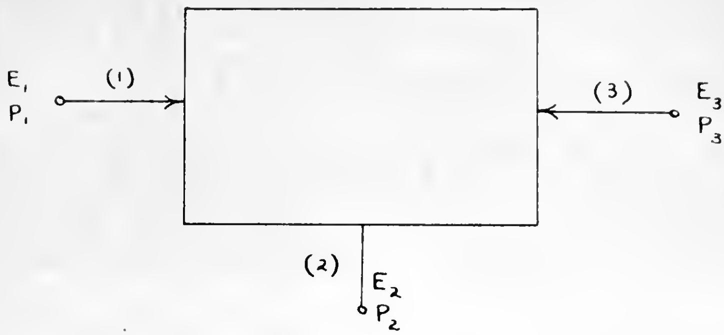


FIGURE VI
VECTOR DIAGRAM OF TERMINAL VOLTAGES
THREE-TERMINAL NETWORK ABOVE

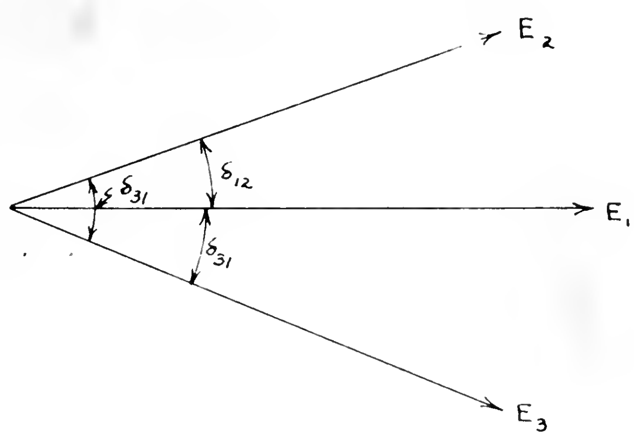
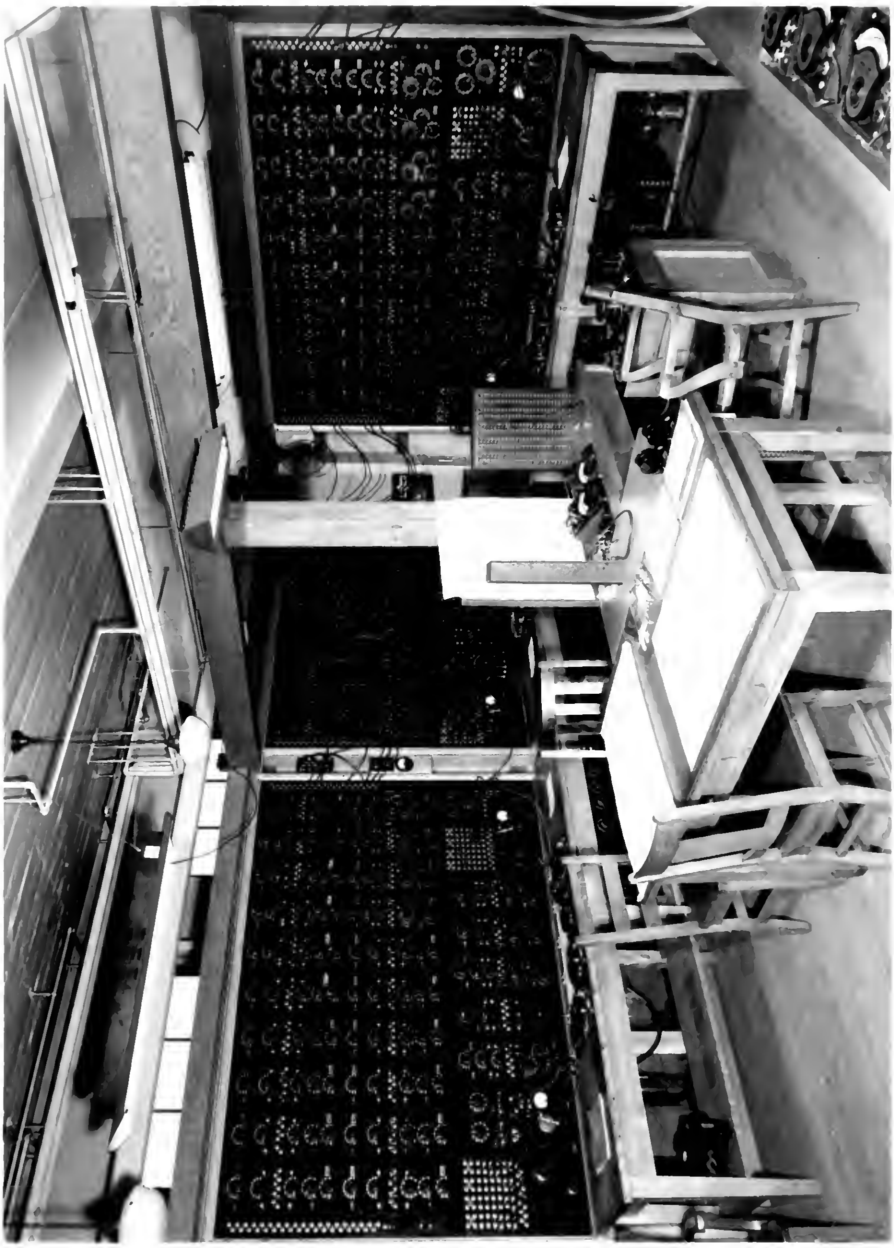


Figure VII

The Network Analyzer



chronous reactances are included in the network as for steady-state stability studies; or as the voltages behind transient reactances where the transient reactances are included in the network as for transient-stability studies. In the first case, the angles are those between machine terminal voltages; in the second case, the angles are those between machine rotors; and in the third case, the angles are not representative of an actual machine quantity although they are assumed to represent rotor angles.

In an actual power system, when a generator is connected to a network already supplied by other generators, its terminal voltage or rotor angle with respect to the other machines, is automatically regulated by other related factors, which in turn are controlled indirectly by the operator of the machine. For instance, the terminal voltage is maintained at a certain value by a regulator, and the power output of the prime mover (and therefore the generator) is determined by the governor setting. When these two variables are fixed for a machine in a system, that machine inherently finds the correct delta to satisfy expressions (1) and (2).

However, because the phase-shifter is a static element, the problem of finding the correct delta is more complicated. The phase-shifter, as pictured in Figure VIII, is built on a three-phase wound-rotor induction motor frame.

Figure VIII

Phase-Shifter

Normally Used
in Network Analyzer

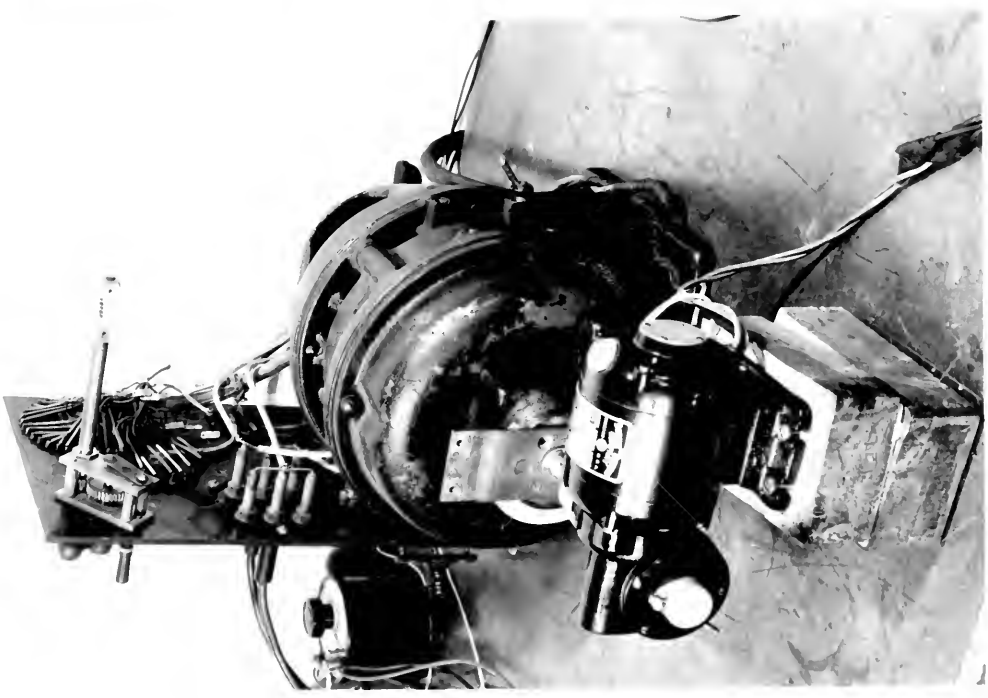
Figure IX

Test Phase-Shifter

Phase
Shifter

Servo
Motor

Control Panel
of Phase-Shifter



It has a single-phase stator and a three-phase rotor which can be rotated by means of a hand crank and gearing. Three-phase, 230-volt, 60-cycle per second power is supplied to the input terminals of the rotor. The stator has many taps on its single-phase winding so that a single-phase output voltage for the Network Analyzer of from zero to 400 volts may be obtained. Since the position of the rotor determines the position (in space and time) of the induced single-phase voltage with respect to the three-phase supply system, the output voltage of the phase-shifter can be continuously varied in both phase and magnitude. The phase shifter therefore differs from the alternator supplying a network in that its rotor angle is not obtained automatically; in other words, the voltage and angle must be set by the operator. The voltage can be set by selecting the proper tap, but the angle with respect to the other phase shifters must be found by cut and try until the stipulated power is obtained.*

When a problem is being set up (on the Network Analyzer) in which several power stations are involved, each station usually is scheduled to deliver a stipulated amount

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* At times, the phase shifters must be set for a stipulated reactive power, in which case both the voltage and the rotor position are set by cut and try.

of power to the network. If each phase shifter is set individually to the desired power, by the time the operators get around to setting the last phase shifter, those phase shifters which were set first require readjustment because of the effect of the latter stations on the network. The reasons for this effect are two-fold:

(1) Referring to equations (1) and (3) the power angle relationship between the machines δ_{12}, $\delta_{23} \cdots \delta_{mn}$ have all been altered.

(2) Inherent in each phase shifter are the resistance and reactance drops* which affect the angular displacement between the induced single-phase voltage and the terminal voltage. As the power output is changed due to the change in power-angle relationships, the current also changes. When the current changes, the resistance and reactance drops change; although this is a secondary effect, it contributes to the difficulty of finding the correct rotor position.

By making several rounds of adjustments and readjustments, the stipulated power output from each phase shifter finally is obtained. This is a tedious and time-consuming process, especially when the problem involves more stations than there are operators available.

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* See Appendix for vector diagram of Phase Shifter Voltages

In view of the foregoing, it becomes apparent that a device which would maintain a predetermined power output from a phase shifter, automatically compensating for the interactions between the power sources supplying the network when a problem is being set up, would have many advantages. The development of such a device not only would simplify the operation of the Network Analyzer as it is presently used, but might also lead to the employment of new techniques in transient analysis and other problems.

III. Procedure

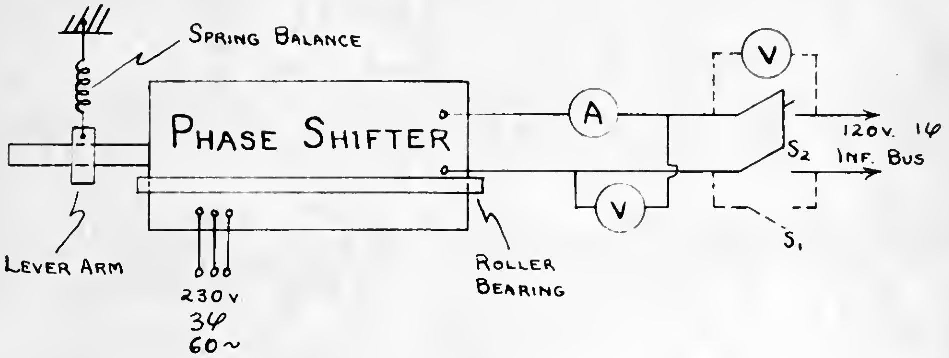
As established in the Introduction the power output of a phase-shifter is some function of rotor angle. In view of this fact, a power-sensitive servomechanism resembles a positional servomechanism, but has a varying reference position depending upon the network conditions. The initial consideration of the overall servomechanism system in block diagram form, Figure I , would lead one to believe that the design is simple. However, it becomes apparent that the design is far from simple when all the engineering aspects of this particular problem are taken into account.

In attacking the problem, the preliminary steps required the establishment of design specifications. These particulars included torque and speed of the servo motor, accuracy and stability of the overall system, loading effect of the measuring device on the phase shifter, reasonable overall size, and environment.

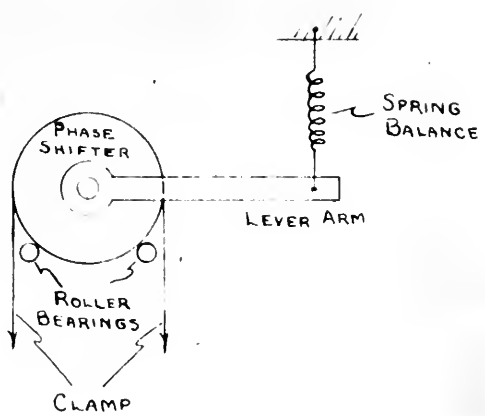
To determine the specifications for the servo-motor and consequently the rest of the system, a spare phase-shifter was removed from its mounting and set up as shown in Figure X . By bucking the output of the phase-shifter against a 110 volt, single-phase line, the torque and current as a function of rotor angle (with respect to electri-

FIGURE X

TEST SETUP TO OBTAIN TORQUE AS A FUNCTION OF ROTOR ANGLE OF A NETWORK ANALYZER PHASE-SHIFTER



END VIEW OF SETUP



cal zero) were obtained. The results are given in Table I. (see Appendix B). Unfortunately, as shown by Table I., torque as a function of rotor angle was found to be erratic in that it was grossly distorted by tooth effects, coulomb friction, and other idiosynchrasies inherent in the phase-shifter, rather than sinusoidal as would be expected.

Because the current base usually used for the Network Analyzer is one ampere, it was apparent from the data that a fractional horsepower motor would be adequate, approximately 1/20 horsepower at 40 revolutions per minute. As a check, and to obtain a realistic test, a 1/20 horsepower single-phase motor with a worm-gear head was used to drive the phase-shifter rotor while the phase-shifter was bucked against the line. From the performance of the motor and further study of Table I., the following specifications for the motor were drawn up:

(a) Normal output torque be 10 inch pounds; to allow for a margin of safety, the motor shall be capable of delivering 14 inch pounds.

(b) The output shaft speed be not more than 30 revolutions per minute.

(c) A worm gear drive is desirable for the following reasons:

(1) Reduces the heat dissipation problem. Often in network studies the power delivered by a phase-shifter may not be changed all day. By use of the worm gear, the motor would not be required to deliver full torque except when actually driving the rotor of the phase-shifter, due to the nearly irreversible characteristic of the worm gear drive.

(2) Reduces the effect on the motor of the erratic torque-angle characteristic of the phase-shifter. Although not apparent from Table I., abrupt minor reversals in torque occur as the rotor is turned; the nearly irreverseable characteristic of the worm gear drive should help overcome some of this effect.

(d) Type of servo-motor be direct-current, armature control for reasons discussed in the succeeding paragraphs. Study of the design led to the following overall specifications:

(1) Accuracy-error must be less than 1 per cent of the power desired. Error cannot be specified in rotor degrees due to the considerations given in the Introduction.

(2) Stability-be well damped, but with sufficient acceleration to maintain negligible error during the set-up of a problem.

(3) Loading effect of servomechanism measuring device on the phase shifter be as small as possible; should not be more than the wattmeters presently used.

(4) Environment-no extreme conditions, because the servomechanism will be installed as part of the Network Analyzer.

In deciding upon the type servo-motor, several systems were considered. Because of the environment, systems having hydraulic elements were eliminated. Therefore, an all-electric system seemed preferable and the following were considered.

(1) Ward-Leonard System

(2) Amplidyne System

(3) Relay or Contactor Type System

(4) Two-phase motor System

(5) Direct-current motor Systems

The Ward-Leonard System:

The Ward-Leonard system consists of a motor-generator which supplies the armature current of the servo-motor. Since voltage impressed on the field of the servo-motor is constant, the speed and direction of the motor is controlled by the armature current, which in turn is a function of the generator field current. The cost and space requirements

resulting from the three frames needed for this system seemed excessive for this application. In addition the time delays inherent in this system might cause oscillation between the various phase-shifters interconnected by the network.

The Amplidyne System:

The amplidyne system is similar to the Ward-Leonard system in that an amplidyne-generator is used to supply the armature current of the servo-motor. Although this system requires one less frame and offers greater dynamo-electrical amplification than the Ward-Leonard system, it is undesirable for similar reasons.

The Relay or Contactor Type System:

The servo-motor can be controlled by means of relays and contactors. Although such systems have the desirability of simplicity, no power consumption while in standby or static condition, and small control-power requirements; they also have the undesirable features of finite inherent error and a tendency to oscillate when attempting to hold a null position. In order to overcome these features, complicated electrical compensating circuits and mechanical dampers would be required. For this application it appeared that the lack of information available on these systems would be too great a handicap to overcome.

Two-phase Motor System

The two-phase motor system is based upon the variation in speed and direction of a two-phase motor when the voltage impressed upon one winding is fixed and the voltage impressed on the other winding is varied in phase and magnitude. Since alternating-current would be used throughout such a system, it would have the advantage of employing alternating-current amplifiers which offer ease of interstage coupling and are insensitive to drift. However, difficulties anticipated in stabilization and error-compensation appeared to outweigh these advantages.

Direct-Current Motor Systems:

(a) Field Control

When constant armature current is maintained, the speed and direction of the motor can be controlled by the differential field current supplied to the split field by an electronic amplifier. Although this system has the advantage of requiring only low-power tubes in the power amplifier, it has the disadvantages of requiring an auxiliary device to maintain constant armature current and requiring greater heat dissipation at standstill because full rated current is maintained in the armature at all times.

(b) Armature Control

When the field voltage is maintained constant, the speed and direction of the motor can be controlled by

varying the direction and magnitude of the armature current. Although larger and more powerful vacuum tubes are required in a power amplifier that supplies the armature current, there are several advantages in the use of this system for this application. Full rated torque is available at all speeds. Heat dissipation is minimized at standstill because the armature current is then zero. And, it is possible by properly designing the power amplifier to obtain a combination which would give a large torque for a small error signal.

The use of a direct-current system allows one to profit from not only the smooth control characteristics of direct-current motors but also the ease of stabilizing the overall system with electrical networks. In this application it was believed that the power required for the direct-current armature-control system could be supplied satisfactorily with vacuum tubes. Therefore this system was chosen.

Design of the overall system required the development of the following component parts:

(1) A power sensitive element

(2) The comparing device

(3) A voltage amplifier

(4) A power amplifier

(5) Compensating networks

Power-Sensitive Element:

A device that would produce a direct-voltage proportional to the real power in an alternating-current system was required as the power-sensitive element for the system. Real power consumed by an impedance element in the alternating-current system is given by

$$P = VI \cos \theta$$

where P = the real power

V = the effective voltage across that element of the system

I = the effective current through that element of the system

θ = the electrical angle between V and I

Thus the power sensitive element, in essence, is a wattmeter.

The only effective wattmeter available is the electro-mechanical type used in nearly every laboratory and by industry to indicate electrical power. An electro-mechanical wattmeter, due to its construction, has the disadvantages of having a large time constant, requiring several watts power from the circuit being measured, and producing only minute torque. If such a wattmeter were employed, additional equipage would be needed to transform the motion of the indicator into a direct-voltage signal to be used as the input to the servomechanism. Several ingenious means of varying elaborateness for accomplishing this transform-

ation can be visualized (See Appendix B). In view of the above-mentioned disadvantages and because of the additional complication of transformation, alternate means of obtaining a direct-voltage signal proportional to power output were sought. The most promising appeared to be the electronic wattmeter.

Although others have attempted to develop electronic wattmeters for various purposes, none could be found that were directly adaptable to this design. Churchill, in his thesis, reference (4), presented an electronic wattmeter which he had designed and tested; since in his design he had attempted to incorporate the feature of wide power and frequency ranges, it became quite complex. A simpler vacuum-tube wattmeter, employing multi-electrode tubes, was proposed by Pierce in reference (3); this circuit appeared to have the advantages of simplicity, quick response, and negligible loading effect on the system. Hence, Pierce's basic circuit, Figure XI, was modified, built and tested; the modified circuit is shown in Figure XII.

Inasmuch as the principles of operation are discussed in Appendix B, only a brief description of the circuit and the procedure followed in overcoming the difficulties encountered are presented here.

A push-pull type circuit employing two type 6A8GT vacuum tubes is the basic wattmeter element. The disposition of the electrodes of each 6A8GT pentagrid converter

Figure XI
Electronic Wattmeter - Original Circuit

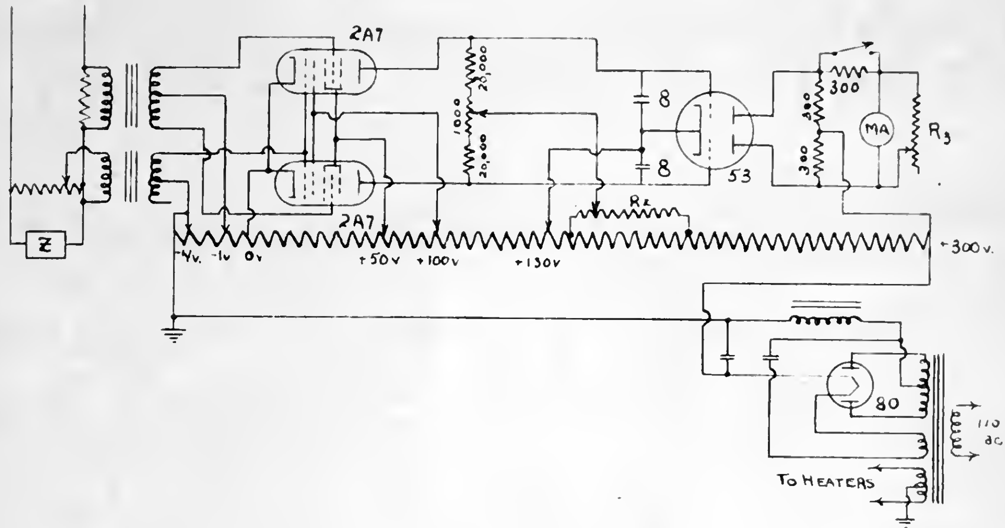
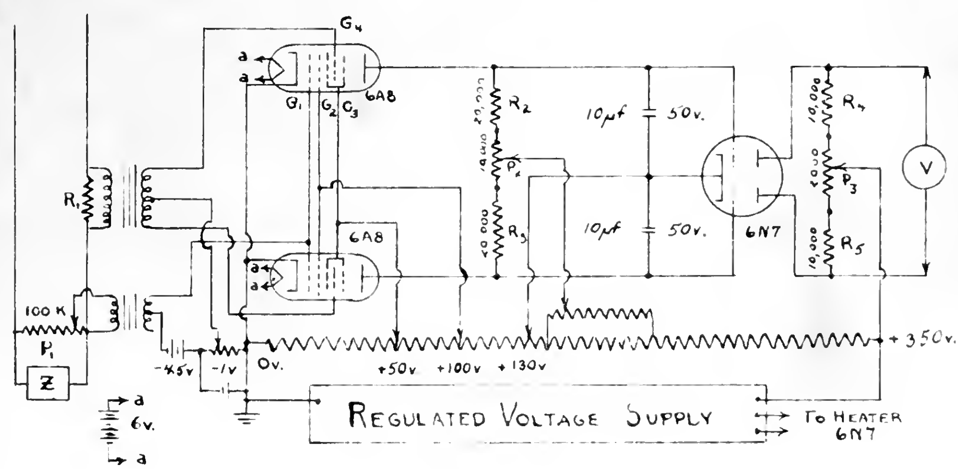


Figure XII
Electronic Wattmeter - Modified Circuit



can be seen in Figure XII. G_2 and G_3 are held positive at a fixed potential of 100 volts and 50 volts, respectively, above that of the cathode. G_1 and G_4 are biased to a negative potential of -4 volts and -1 volt, respectively, with respect to the cathode. An alternating voltage proportional to the line voltage is then applied to G_1 via a center-tapped transformer while an alternating voltage proportional to line current is applied to G_4 via another center-tapped transformer. A direct-current amplifier employing a high-mu, twin, power-triode (type 6N7) is used to amplify the signal so that the output signal from this device is large enough to be used diredtly in a comparing device.

A power sensitive device, similar in general arrangement to that of Figure XII (values of circuit elements not as shown in Figure XII) was built on a "breadboard" for experimentation and testing as an individual unit. The results of preliminary tests were disappointing because they revealed the following undesirable features:

(1) The output signal was too small.

(2) It was difficult to obtain initial balance in both the wattmeter element and the amplifier.

(3) The circuit was too sensitive to stray signals and noise; these stray signals seemed to result from pick-up by various circuit elements such as the

grid leads, the signal transformers and the biasing system; the noise came from the carbon resistors.

(4) The linear range of the device was not adequate.

(5) The drift in the device was so great that readings could not be reproduced.

The task of eliminating some of these faults and compensating for those which could not be eliminated required the use of cut and try methods. In some cases, a compromise had to be accepted because time did not allow further experimentation.

Output Signal:

The original circuit proposed by Pierce employed a milliammeter to indicate power. Inasmuch as the input to the power amplifier was to be an error voltage, it was desirable to have a voltage signal as the output from the power-sensitive device. To obtain a voltage signal, the ammeter was removed and larger resistors were installed in place of the 300 ohm resistors. In order to find the value of the resistors that produced the optimum signal under specified operating conditions, several sizes of resistors were tried; it became evident that the optimum signal was obtained when the resistors in the plate circuit of the type 6N7 tube were approximately 11,000 ohms. Therefore, two 10,000 ohm resistors and a balancing potentiometer of

2000 ohms were used.

Balance:

To allow for unbalance among vacuum tubes of the same type and the variation of the resistors (from their nominal value) used in the plate circuit, it is common practice to include a balancing potentiometer in electronic circuits that operate about a null position. The potentiometer originally used for balancing the type 6A8 tube circuit was found to be inadequate and had to be increased to 10,000 ohms.

Stray Pick-Up and Noise:

The stray pick-up in the grid leads was eliminated by replacing the ordinary leads with shielded leads.

The stray pick-up in the input transformers, which was due to the capacitance coupling between the primary and secondary of the individual transformers, was eliminated by grounding one side of the single-phase supply to the load; this procedure reduced the potential difference between the primary and secondary, since the secondary was at essentially ground potential. If it becomes necessary to have a potential difference between the primary and secondary of the input transformers, it will be necessary to use transformers that have electrostatic shields between the primary and secondary.

The stray pick-up in the biasing system of the type

6A8 tubes was the result of modifying Pierce's proposed circuit, and it appeared to originate entirely in the one-volt bias circuit. In attempting to use a bias battery which required a very low current drain, a high resistance potentiometer (one megohm) had been shunted across the 1.5 battery, inadvertently placing high resistance (about 0.6 megohms) between the grid and ground. When the bias battery was replaced by one which would allow the use of a 50-ohm potentiometer, this condition no longer existed and the stray pick-up was eliminated.

By replacing carbon resistors with wire-wound resistors, noise at low-power levels was eliminated.

Linear Range:

Although the main disadvantage in having only a small range of linearity in the power sensitive device is the added difficulty in calibrating the instrument, attempts were made to extend the linear range. The elimination of stray pick-up and drift extended the linear range considerably. Additional range was obtained by replacing the 8 microfarad electrolytic by-pass condensers with 10 microfarad Pyranol condensers. As a final precaution, the input signal was reduced to ascertain that the tubes would be operating in their more linear range.

Drift:

Many factors contribute to drift in uncompensated

direct-current electronic circuits similar to the power sensitive element. Voltage supplies may have poor regulation; heater voltages may vary; and resistors may change in value as their temperature changes. After a voltage supply with excellent regulation was substituted for the inferior plate supply that had been used, a six-volt battery with nearly constant voltage was used for heater supply, and the circuit was allowed to reach equilibrium temperature before a drift run was made, the drift over a reasonable length of time (several hours) was negligible. Long time drift due to changing tube characteristics can be compensated for by means of the balancing potentiometer insofar as the application of this wattmeter is concerned.

After the above-mentioned corrective measures had been accomplished, further tests were made. All of the preliminary tests had been performed with a pure resistance load; power was varied by varying the current as the voltage was held constant; then power was varied by changing the voltage impressed on a fixed resistance. As a final check on the performance of the power sensitive element, a constant power run was made; power was held constant while the load impedance angle was changed by altering the load network. The results of these tests may be found in the

APPENDIX C

~~Results~~ and are discussed in the Discussion of Results.

Here it is sufficient to state that a definite voltage signal is obtained from the power-sensitive device for a given power measurement, provided that the network parameters are not drastically altered. In other words, although this power-sensitive device produces a direct-voltage proportional to the power it measures, it is not entirely accurate when subject to a wide variation in power-factor angle. Despite this limitation it could be used as part of the servomechanism. However it could not be calibrated, and for each network set-up, the comparing signal would have to be set by checking the output of the phase-shifter with a wattmeter. The servomechanism would then maintain the power output of the phase-shifter at the desired value.

It is believed that a major reason for the foregoing inconsistency is the phase shift occurring in the input transformers. Therefore an electronic input circuit, which would eliminate the transformers, is described in the Recommendations. Another factor that may be contributing to the trouble is the bias used on the type 6A8 tubes; unfortunately, time did not permit investigation of the effects of varying the grid bias in order to determine the optimum operating point for these tubes.

Although the electronic wattmeter was far from perfect, the decision was made to procede with other components of the over-all design and, if time permitted, to return and continue investigating the wattmeter at a later date.

The Voltage and Power Amplifier:

One of the reasons for selecting a direct-current servo-motor with armature control was the fact that with proper amplifier design full-load torque would be available at all speeds with a small error voltage. Although this is a severe requirement and one not readily achieved, it was considered necessary because of the erratic torque requirements of the Network-Analyzer Phase-Shifter.

When the servo-motor (previously specified) was received and preliminary tests made, it was found that full-load torque required an armature current of approximately 250 milliamperes. This current requirement was attainable with a high-vacuum-tube power amplifier.

Another important requirement was that the motor had to be able to rotate in either direction. In other words, the power amplifier had to be able to supply current in either direction, depending on the polarity of the error

signal. This requirement could be met with a balanced bridge circuit.

Although the first amplifier* that was built and tested satisfied the latter requirement, it was unable to deliver sufficient current to accomplish the former. The main reason this amplifier failed to meet requirements was its poor voltage regulation due to the large fixed resistors in the bridge circuit. When this fact was established, the solution to the problem became obvious.

By replacing the fixed resistors of the bridge circuit with non-linear resistors, whose resistance would decrease as the current increased, the voltage regulation would be improved; then the circuit would meet the requirements. In view of the fact that a vacuum tube is an excellent non-linear resistance whose non-linearity may be controlled by the grid-cathode potential, a bridge circuit was developed utilizing four vacuum tubes. This circuit is shown in Figure XIII. The method of calculation used in developing this circuit can be found in Appendix D.

A comparatively new, low-impedance regulating tube was used. This tube, type 6AS7-G, was developed by RCA and is a low-mu, power twin-triode designed particularly for use as a series regulator tube in power supplies. Its

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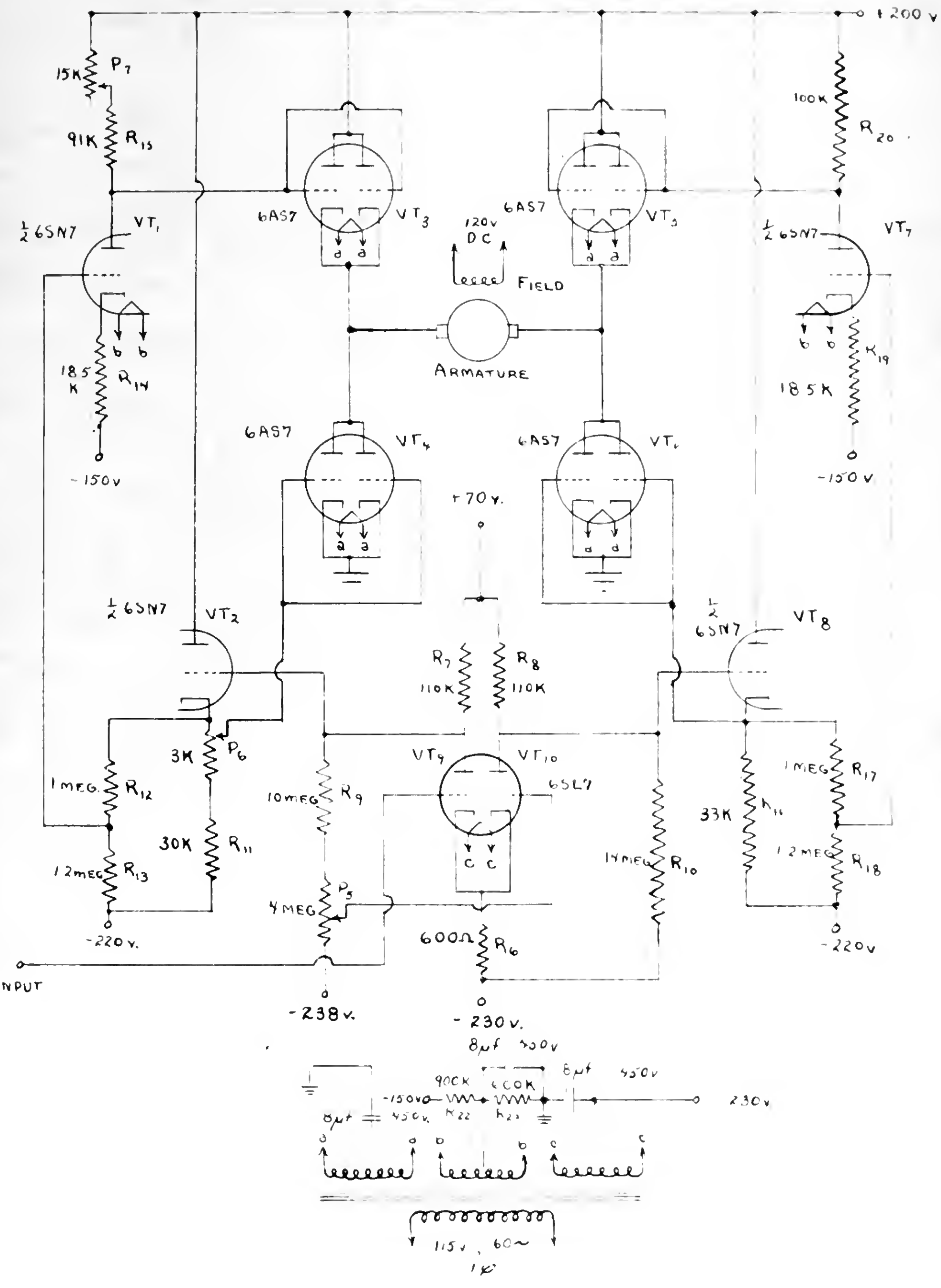
* See Appendix B for circuit and discussion.

Figure XIII
Voltage and Power Amplifier



low-mu, approximately 2.1, requires a rather large control voltage range from the voltage amplifier, but this is not serious. Since each unit of this dual-triode has a normal maximum plate current of 125 milliamperes with a potential drop of only 35 volts across the unit, it is perfectly suited for the application*. Only one tube is needed in each leg of the bridge when the two triode units in a tube are used in parallel.

The large grid swing of the 6AS7-G requires a rather elaborate grid control network. To obtain current flow in one direction, one pair of tubes (VT_3 and VT_6) are cut off when the other pair (VT_4 and VT_5) are passing the full load current; current flow in the opposite direction requires the inverse of this condition. The potential of the grids of VT_4 and VT_6 swing from zero volts at full load current to approximately minus 100 volts with respect to their cathodes (and ground) at cut-off; while the grid potentials of VT_3 and VT_5 swing from zero to minus 100 volts with respect to their cathodes or from approximately 100 volts

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* Precautions: (a) The 6AS7-G requires a 15 second warm-up period prior to applying a signal calling for full load current, otherwise the cathode will be damaged.

(b) The heater-cathode potential should be as small as possible. See Recommendations.

positive to 65 volts negative with respect to ground. The positive potential depends upon the potential drop across VT_4 or VT_6 at cut-off. As will be seen from the description of the remainder of the amplifier, the proper grid potential for full-load current is automatically developed by the circuit.

Proper grid potentials for the power amplifier stage are set by VT_1, VT_2, VT_7, and VT_8 and their associated resistor networks. Since VT_2 and VT_8 are cathode followers, the voltage gain in that stage of the amplifier is less than one. Similarly, the voltage gain of VT_1 and VT_7 together with the voltage-leveling resistors R_{12}, R_{13}, R_{17}, and R_{18} between their grids and the cathodes of VT_2 and VT_8 is approximately two. Therefore the voltage amplification had to be obtained in the stages preceding the grid-control stage.

Voltage amplification is obtained in the portion of the circuit containing VT_9 and VT_{10}. A voltage gain of approximately 44 is secured for small signals. Since this stage also acts as a phase inverter, it serves a dual purpose.

The phase inverter is an essential part of the circuit. When a signal is impressed on the grid of VT_2, the inverse of this signal must simultaneously be impressed on VT_8 for the bridge to function effectively. For example, if the

signal to VT_2 is six volts negative, the signal to VT_8 must be minus 110 volts because the zero signal for VT_2 and VT_8 is a potential of minus 60 on their respective grids. This feature of phase inversion is the crux of the design; by means of the grid-control network, the phase inverter effectively cuts off VT_3 and VT_6 and allows VT_4 and VT_5 to pass approximately 250 milliamperes through the armature of the servo-motor in one direction, or it cuts off VT_4 and VT_5 and allows VT_3 and VT_6 to pass approximately 250 milliamperes through the armature of the servo-motor in the opposite direction, depending on the polarity of the signal impressed on the input terminals of the voltage amplifying and inverting stage.

The foregoing arrangement satisfied the requirements set for the amplifier-motor combination. The current-forcing action of the power amplifier gave the good voltage regulation that was required. The other associated parts gave the torque-error voltage relationship that was desired. How this is accomplished can be seen by noting the voltages throughout the circuit under operating conditions.

When the signal to VT_2 is six volts negative, the signal to VT_1 is 110 volts negative and the signal to VT_3 is 65 volts negative, all with respect to ground. Since the cathode of VT_3 is at a positive potential of 35 volts with respect to ground due to the voltage drop across VT_4(recall

that a 6AS7-G tube has a potential drop of 35 volts when each triode unit is passing 125 milliamperes), the grid to cathode potential of VT_3 is 100 volts negative, the cut-off voltage of VT_3, and therefore no current flows through VT_3.

When the signal to VT_2 is six volts negative (as above), the signal to VT_4 is zero volts. Therefore VT_4 is conducting approximately 250 milliamperes.

As previously stated, VT_6 should be cut-off when VT_3 is cut-off and VT_5 should be conducting armature current when VT_4 is conducting current. The phase-inverter together with the grid-control network of VT_5 and VT_6 permits these tubes to function properly.

When the signal to VT_2 is six volts negative (as above), the signal to VT_8 is 110 volts negative because of the phase-inverter* action of VT_9 and VT_{10}. For this condition the signal to VT_6 is minus 100 volts, the cut-off grid potential. The cathode potential of VT_8 is approximately minus 100 volts and due to the voltage-divider action of R_{17} and R_{18}, the potential of the grid of VT_7 is approximately 150 volts negative. Therefore VT_7 is practically cut-off and its plate potential is approximately 140 volts positive (neglecting grid current of VT_5). Since the grid of VT_5 is

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* For details of the phase-inverter, refer to Appendix D.

connected to the plate of VT_7, the grid potential of VT_5 should be approximately 140 volts; however, this condition results in a large positive grid to cathode voltage on VT_5 and causes a large grid current to flow, which in turn affects the voltage at the plate of VT_7. In doing so, the grid to cathode potential is automatically reduced to a slightly negative value. Therefore VT_5 is allowed to conduct approximately 250 milliamperes.

Although the grid-control circuit and the associated phase-inverter circuit are somewhat complex, the overall voltage and power amplifier benefits from this complexity. If the input signal to the phase-inverter is larger than that required for full load armature current in either direction, no harm is done because the tubes cannot operate above their maximum ratings. The plate current of the 6AS7-G tubes is limited to 350 milliamperes by the grid current of VT_3 and VT_5 through the resistors R_{15}, P_7 and R_{20}, regardless of the size of the input signal. For an input signal greater than that required to produce full load current, either VT_1 or VT_7 is cut-off and the plate current in the other is limited by the respective plate resistor (R_{15} and P_7 or R_{17}) and grid-bias resistor (R_{14} or R_{19}). The maximum plate current through VT_2 and VT_8 is limited by the large biasing resistor required for proper circuit operation. The plate current in the phase-inverter stage is limited by the large load resistors. Many rigorous

operational tests were made on the amplifier and no harmful effects were observed.

Saturation occurs first in the power amplifier. Since saturation does not occur until the servo-motor is operating at maximum output, the operating range of the motor is fully utilized. In addition, saturation (and therefore the condition of maximum available torque from the servo-motor) occurs when a small signal is applied to the input terminals of the amplifier; hence, maximum torque is available for small error signals as well as for large error signals.

In spite of the fact that several supply voltages are required for the amplifier, only two of the supply voltages are critical. These are the two negative voltages to the phase-inverter stage. Even so, these voltages themselves are not critical, but the difference in potential between them is the critical factor. For the phase-inverter to function properly, a difference in potential of 8 volts must be maintained between the two negative voltages of the phase-inverter stage.

Figure XIV

Components

Filament
Trans-
formers

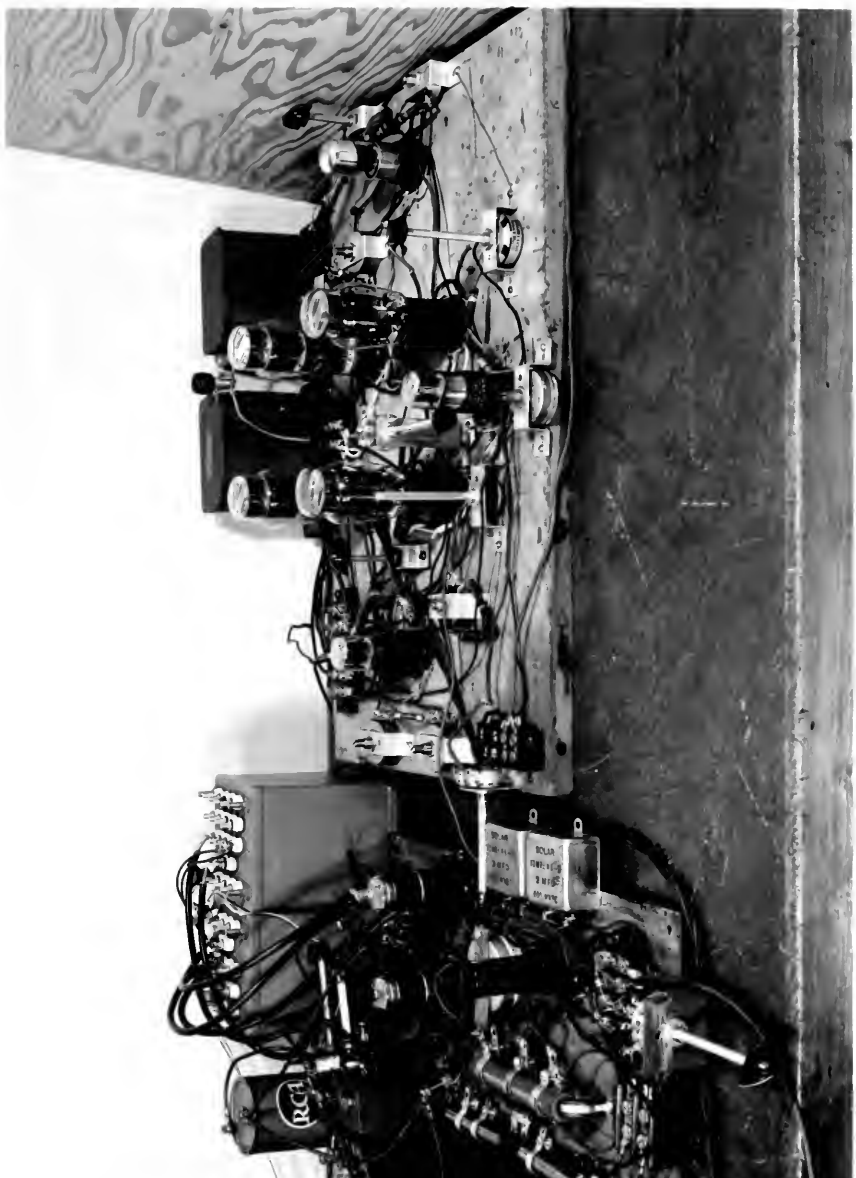
Power

Amplifier

Input
Trans-
formers

Power
Sensitive
Element

Biasing
System


RCA

Comparing Device:

For a servomechanism to know whether its output is in agreement with its input, the servomechanism must have some means of determining the error between the input and the output. The input signal to the power-sensitive servomechanism must be proportional to the power desired; the output signal must be proportional to the power delivered to the Network Analyzer by the phase-shifter involved; both signals must be compatible so that they can be combined to give an error signal. Inasmuch as the signal from the power-sensitive device is a direct voltage proportional to the power output of the phase-shifter, the simplest standard with which to compare this direct voltage is another direct voltage. This comparing voltage could be obtained from a battery or a well-regulated power supply; a linear potentiometer could be used to establish the level desired. If an accurate and reliable power-sensitive device had been developed, the potentiometer could be calibrated in watts. The error signal would than be the algebraic difference between the two direct voltages.

Since the electronic wattmeter was not perfected, but had to be used because time did not allow further work on this component, no attempt was made to build and calibrate an accurate comparing device.

Assembling the Components:

When all the components (except the compensating device) were developed and perfected as much as time permitted,

they were assembled into a unit. One of the problems encountered was the large difference in direct voltage level between the output of the power-sensitive element (+ 350 volts) and the input of the voltage and power amplifier (-230 volts). As an expedient solution rather than a recommended practice*, batteries were used for the voltage level reducer. Although this voltage leveling system was used only as an expedient in order to assemble the system and make a hasty test of its performance, the employment of batteries offered a means of combining a crude comparing device with the level changer. Two 300-volt Eveready Minimax batteries were connected in series; since these batteries each had a terminal voltage of 309 volts, a total of 618 volts were available. To buck out the excess voltage a 67.5-volt battery shunted by a high resistance potentiometer was used; therefore the voltage available for use as a comparing signal was 67.5 volts. (See Figure II for connections).

Two interesting conditions resulting from the use of batteries were noted when these components were connected together:

(1) The comparing device must be placed in the grid leg of the power amplifier. If the potentiometer is connected in the leg of the input to the power amplifier that included the cathode of the phase-inverter and the minus

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* See Conclusions and Recommendations.

230 volt power supply, a current flow of two to three milliamperes results; this current flow masks the error signal and the system will not function.

(2) Only one voltage leveling system is required. Apparently the common ground between the two components completes the circuit, and although the other leg can be connected together with a voltage leveling system, it is unnecessary because the overall system performs equally well in either case.

Closing the Loop:

After assembling the entire servomechanism, the loop was closed by bucking the output of the servo-controlled phase-shifter against another phase-shifter. Before the loop could be closed, the voltages of the two phase-shifters had to be matched in phase and magnitude to avoid excessive circulating current in the event that the servomechanism failed to operate.

Compensation Devices:

Compensation, as employed in servomechanism design, usually implies that additional equipage is added to the system to improve system performance. Therefore, the auxiliary devices needed for compensating a system cannot be specified until the response of the uncompensated servomechanism is known.

The generally accepted means of determining the accuracy and stability of a servomechanism (and the compensation

required by the system) is by frequency-response studies*. This approach is based on the steady-state response of a system to sinusoidal inputs. Such studies may be performed analytically or by a combination of analytical and experimental techniques.

In the purely analytical approach, each component of the servomechanism under study is represented by a mathematical transfer function*. The individual transfer functions are combined mathematically and converted into a non-dimensional expression, which is then plotted on a frequency basis. From the graph, one can estimate the performance of the uncompensated system and determine the need for compensation.

Often, existing equipment cannot be represented mathematically without unwarranted simplifying assumptions. In such cases, a frequency response study can be performed experimentally if the system arrangement is suitable. A sinusoidally varying input covering an appropriate frequency range can be introduced into the system and measurements of the phase and magnitude of the output with respect to the input can be obtained. From the graph of this data, one can estimate the performance of the servomechanism and determine the compensation required.

Since the power-sensitive servomechanism has several components which are difficult to represent mathematically, the latter method of obtaining its frequency response is

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* Refer to references (6), (8) and (11).

preferable. The loop can be broken at the point where the error signal is applied to the voltage and power amplifier. Then a sinusoidal input can be applied as a substitute for the error signal and suitable measurements can be obtained by means of strategically placed instruments.

Although a frequency-response study of the power-sensitive servomechanism was contemplated so that a complete report could be made on its performance and suitable compensation could be recommended, the lack of time prevented the performance of such extensive tests.

Tests Performed:

Although only operational tests were performed on the assembled servomechanism because time did not permit further investigation, several tests were made on the individual components. These tests were also of the performance type and extensive data was not recorded.

Figure II
Power Sensitive Servomechanism

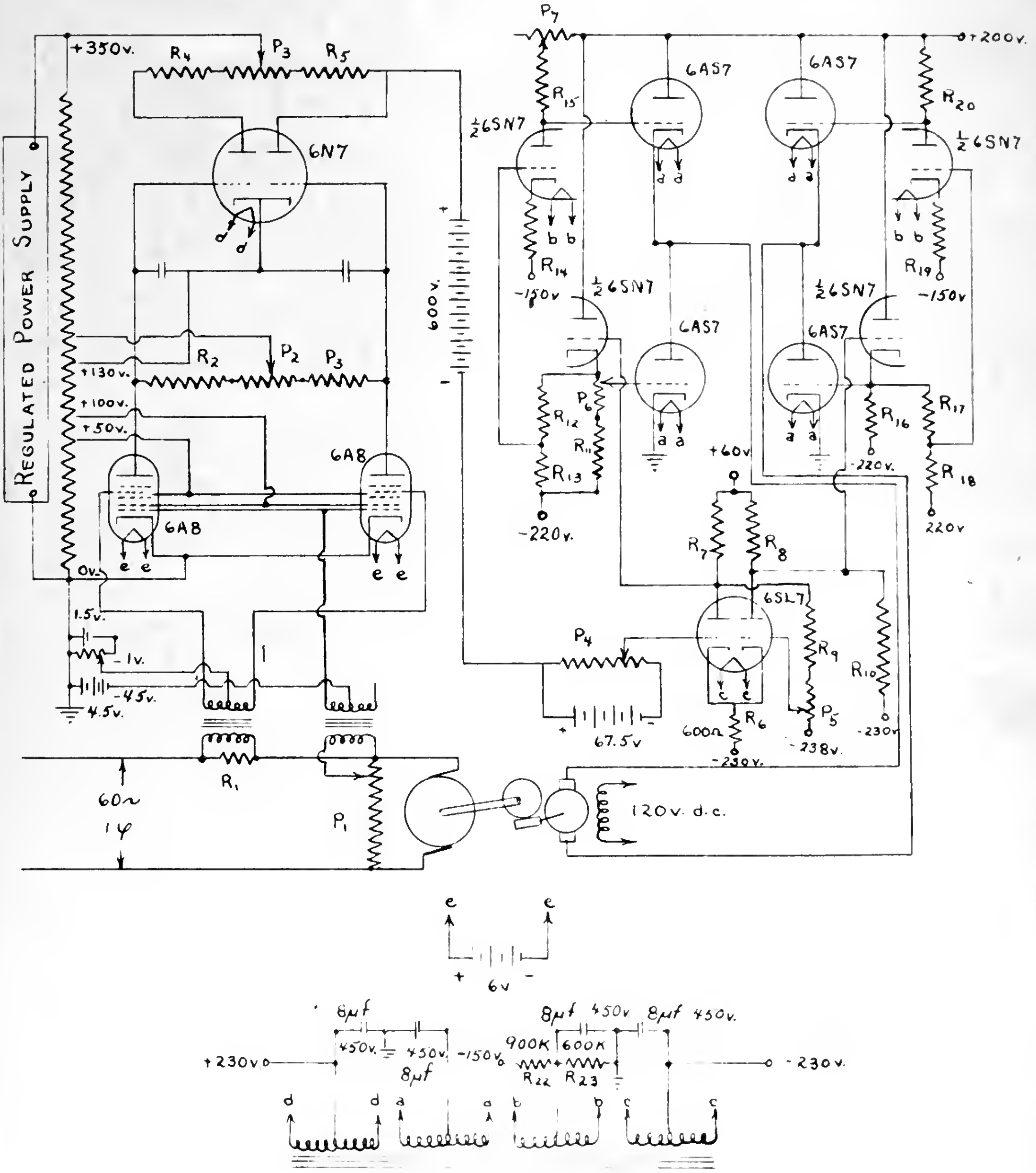


Figure XV

The Experimental Servomechanism

Power Supplies

Power
Amplifier

Comparing Device

Input Transformers

Voltage Leveling

Power Sensitive Element

Power Supply

Phase-Shifter

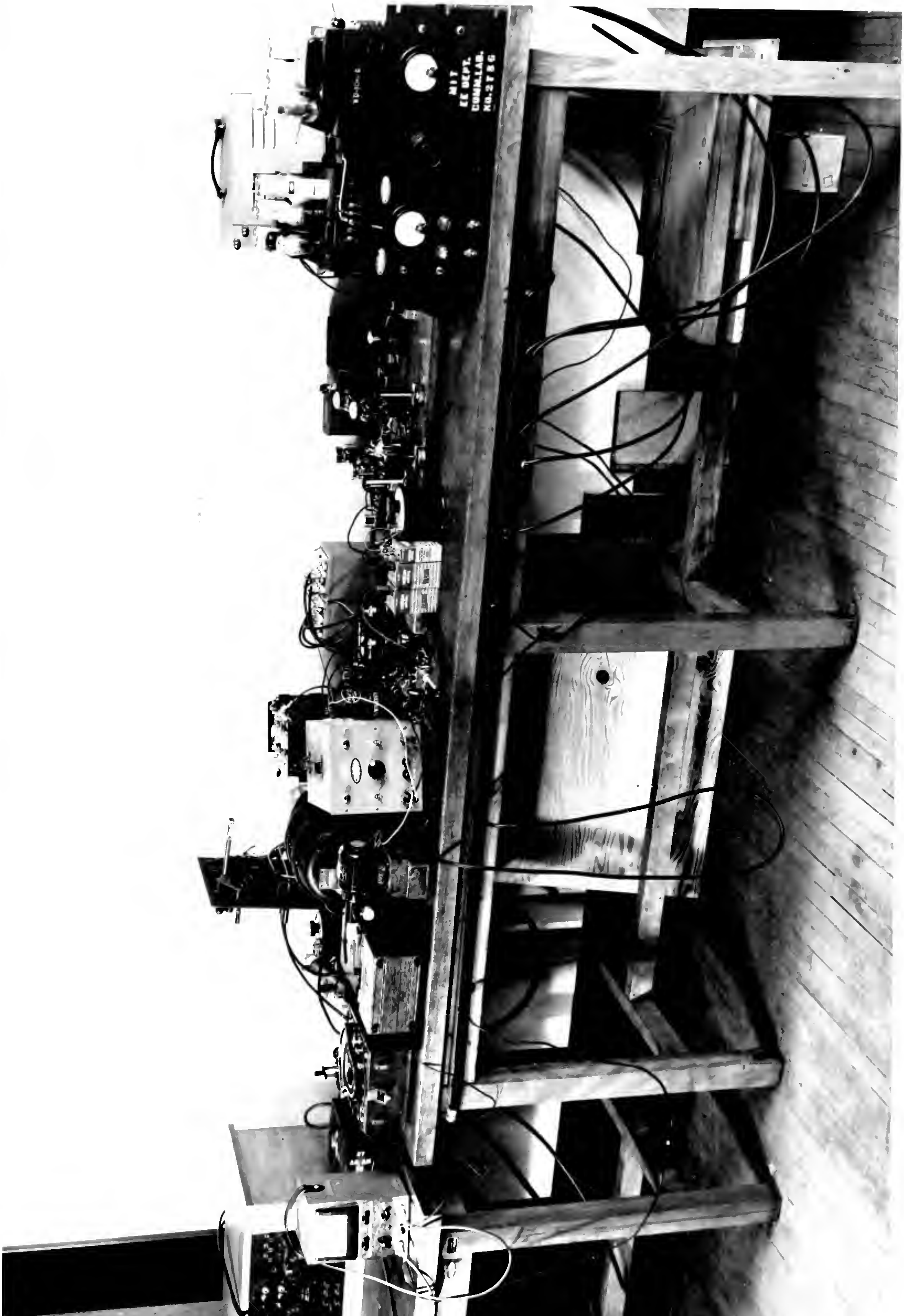
Servo-Motor

Voltage Pot.

Current Resistor

AC Meters

Phase-Shifter


MIT
EE DEPT.
COMM. LAB.

IV. Results

Although the design of the power-sensitive servomechanism as developed by this undertaking cannot be considered sufficiently complete to permit construction of units for installation in the Network Analyzer, the progress that was made in the development of the design definitely demonstrates that a power-sensitive servomechanism for the Network Analyzer is practical.

In spite of the fact that the lack of time did not permit extensive testing of the system that was built, when this system was assembled and the loop was closed, the servomechanism performed satisfactorily.

The fact that the uncompensated system was able to maintain a relative power setting is the most important result. When the angle of the hand-operated phase-shifter was varied, the servo-operated phase-shifter followed and maintained its approximate power setting. However, for certain power settings near zero-power-output, the servomechanism had a tendency to hunt. Although this oscillatory motion about a set-point indicated that compensation was needed, hunting in an uncompensated system is indicative of good response. Therefore the basic system is considered a success.

V. Discussion of Results

Although no quantitative results were obtained because of the lack of time, the qualitative results justify the undertaking; and therefore, the project can be considered successful.

Since the performance of the overall system is dependent on the performance of the components, each of the components that was finally assembled into the closed loop will be discussed briefly.

The power-sensitive element is the most important component of the servomechanism. Since this device measures the output of the system and produces the signal to be compared with the input signal, it must have an inherent accuracy greater than that required of the overall loop. The electronic wattmeter fulfilled this requirement only when the impedance to which the power flowed remained unchanged; in other words, the electronic wattmeter produced the same signal for a given power provided that the power-factor angle was not radically changed. Since most Network Analyzer problems involve fixed networks, this limitation would not disqualify the electronic wattmeter. However, in view of the variety of networks that are analyzed on the Network Analyzer, this limitation would prevent calibrating the signal from the electronic wattmeter on a permanent basis; the device would have to be re-calibrated for each problem. Except for this fact,

the electronic wattmeter appears to be ideally suited for the power-sensitive element.

Because of the fact that the comparing device is a simple direct-voltage source shunted by a potentiometer, its accuracy is limited only by the stability of the voltage source and fineness of the wire on the potentiometer*.

Since the voltage and power amplifier was able to provide the motor with sufficient power to enable the motor to rotate the phase-shifter when the error signal demanded a correction, the amplifier operated satisfactorily.

Although the motor and worm-gear drive was capable of rotating the phase-shifter under load conditions, the hunting that was observed is partially attributed to motor-worm-gear combination.

On the whole, the results indicate that a power-sensitive servomechanism employing these components is practical and can be adjusted to perform in an adequate manner provided that the following modifications be made:

(1) Perfect the electronic wattmeter or obtain a suitable substitute.

(2) Reduce the speed of the servo motor by either

(a) changing the motor
(b) increasing gear reduction

(3) Compensate the overall system.

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* See reference (8) pages 95 to 100.

VI. Recommendations

The recommendations can be classified into four categories:

A. improvements and modifications to the design that has been developed;

B. alternate equipment to perform the functions of certain individual components of the design;

C. alternate systems capable of being adapted to perform the function of a power-servomechanism;

D. future improvements to the Network Analyzer that might be possible if successful power-sensitive servomechanisms are installed to control the phase-shifters.

A. In order to design the compensating devices that appear to be required, a frequency-response test* should be performed on the existing power-sensitive servomechanism. Although the servomechanism should be serviceable as soon as the compensation has been accomplished, improved performance can be obtained from the overall system by making the following modifications to the components:

1. To the electronic wattmeter,

   (a) By experimentation, determine the proper grid-biases for the 6A8 pentagrid converters. Although the bias-voltages that were used allowed

* Refer to Procedure for details of test.

the wattmeter to function, these bias-voltages are not considered to be the proper value for optimum accuracy and response.

(b) Replace the present signal-transformers with transformers whose phase-shift is small enough to be negligible.

(c) Arrange the circuit of the wattmeter with respect to the general layout of the system so that the wattmeter reaches and remains at an equilibrium temperature at all times when the system is in use. By assigning an independent power supply to the wattmeter element and installing an isolating switch between the power sensitive device and the voltage-and-power amplifier, this section could be left on and it would be ready for use at all times.

(d) Investigate the possibility of using miniature tubes (6AS6* for the 6A8 and 12AX7 for the 6N7)

2. To the voltage and power amplifier,

(a) Investigate the possibility of improving the balancing of the output voltages of the phase-inverter.

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* See reference 7.

(c) Replace the input signal transformers with an electronic input circuit.

(i) For current input circuit see Figure XVI and Figure XXV.

(ii) For voltage input circuit use a cathode follower circuit. Then the amount of voltage signal to the electronic wattmeter may be set by the amount of the cathode resistor that is included in the grid circuit of the type 6A8 tubes.

2. For the comparing device,

(a) Use a Helipot* in place of the potentiometer.

(b) Use a well-regulated power supply in place of batteries as a direct voltage source.

3. For the coupling between the power-sensitive device and the voltage and power amplifier.

(a) Use a constant-current electronic coupling, similar to the circuit of Figure XVII, in place of the 600 volt batteries.

C. In view of the fact that, in the opinion of others who may wish to continue the work, the design presented may not be the optimum, a few alternate systems capable of being adapted to perform the function of a power-sens-

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* See references (7) and (8).

Figure XVI
Current Input Circuit

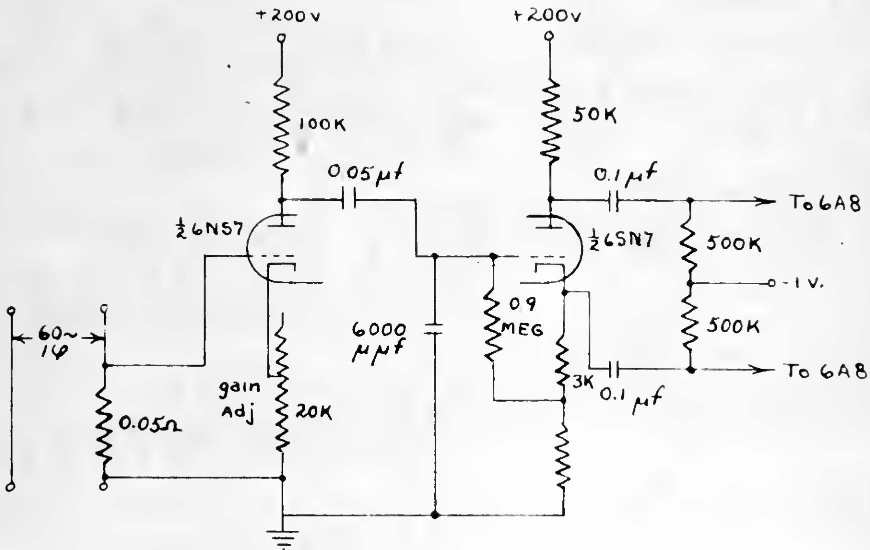
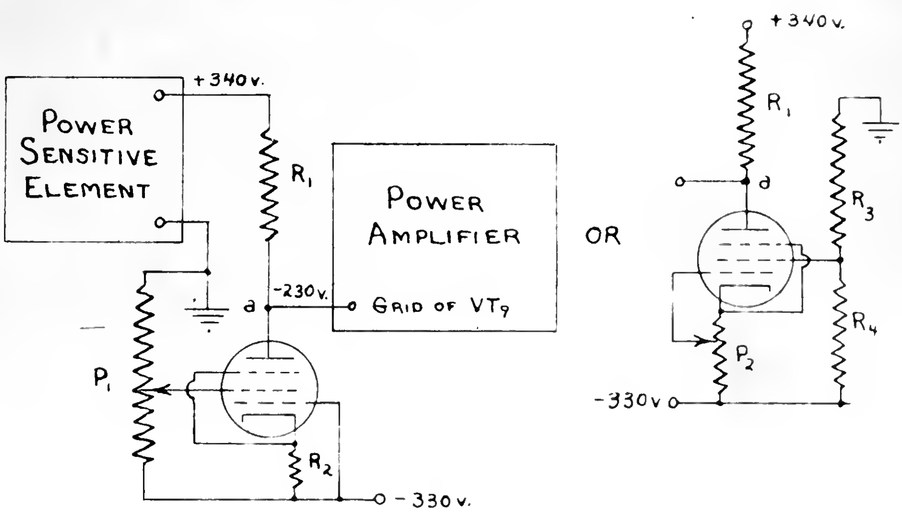


Figure XVII
Constant Current Coupling



itive servomechanism are presented.

1. Split-series motor system as described on page 427 of reference (7).
2. Magnetic amplifier systems as described in references (18), (19) and (20).
3. Two-phase motor systems as described on page 440 of reference (7) and also as described in reference (10). If two-phase systems are considered, the iron-vane bridge described in Appendix B offers an ideal method of control.
4. Relay or Contactor servo-systems as described on page 446 of reference (7). At present, considerable research is underway in this field and the results should be available in the near future.

D. If successful power-sensitive servomechanisms are installed to control the phase-shifters of the Network Analyzer, their installation not only would simplify the operation of the Network Analyzer as it is presently used, but might also lead to the employment of new techniques such as the following:

1. Automatic control of power, reactive power angle, and voltage for each unit with adjustments being made from a central operator's position.
2. Simplified representation of phase-shifting transformers by automatic control of phase-shifter output.

3. By incorporating automatic angle control for each phase-shifter, and with all phase-shifters controlled from a central point, transient stability problems can be solved utilizing computer techniques at the central point. The computer would solve the electro-mechanical differential equations by utilizing the network and the phase-shifters to give the relationship between power output and angle at each machine at every instant of time. The solution could appear in a set of curves on a recording instrument chart.

VII. Appendix

VII. Appendix

A. Supplementary Introduction

The general equations for power flow into a network were obtained from reference (5). The development of these equations, also obtained from pages 220 to 224 of reference (5), are presented here in order to justify their use in the Introduction of this thesis report.

Referring to the n - terminal network of Figure III composed of elements with strictly constant parameters, assume that the positive direction of current flow is into the network and that voltage of known phase and magnitude are impressed at the terminals. The vector diagram of voltages is shown in Figure IV where the phase position of each voltage vector is indicated by its displacement angle with respect to a common axis of reference.

By taking the product of the current and the conjugate of the voltage at any point of entry of the network, one may obtain the expression for power and reactive power. For example, at the points 1 and n:

$$\left.\begin{aligned} P_1 + jQ_1 &= \bar{E}_1 I_1 = \bar{E}_1 (I_{11} - I_{12} - I_{13} - \cdots - I_{1n}) \\ P_n + jQ_n &= \bar{E}_n I_n = \bar{E}_n (I_{nn} - I_{n1} - I_{n2} - I_{n3} \cdots) \end{aligned}\right\} \quad (6)$$

Figure III
General n-Terminal Network

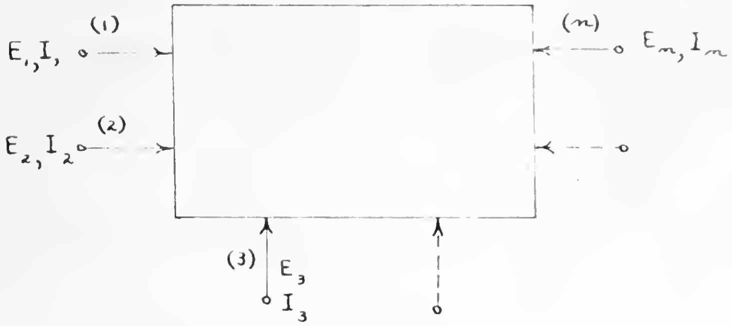
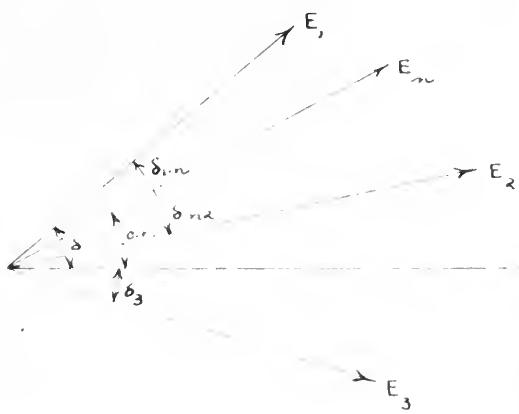


Figure IV
Vector Diagram of Terminal Voltages For Above Network



The component currents into which the total current at a point is split, are those that would flow, respectively, at that point in response to each voltage being applied alone with all the other terminal points of the network short-circuited. In the double-subscript notation employed the first symbol refers to the point to which the quantity belongs, and the second to the point of voltage application. Corresponding voltages and currents are related by impedances, for instance:

$$\left.\begin{array}{ll} I_{11} = \dfrac{E_1}{Z_{11}} & I_{12} = \dfrac{E_2}{Z_{12}} \\ & \\ I_{nn} = \dfrac{E_n}{Z_{nn}} & I_{n1} = \dfrac{E_1}{Z_{n1}} \end{array}\right\} \quad (7)$$

The impedances with equal subscripts are the short-circuit driving-point impedances and those with unequal subscripts are short-circuit transfer impedances.

Substituting equations (7) in equations (6) ~~and ( ) gives:~~

$$P_1 + jQ_1 = \overline{E}_1\left(\frac{E_1}{Z_{11}} - \frac{E_2}{Z_{12}} - \frac{E_3}{Z_{13}} - \cdots - \frac{E_n}{Z_{1n}}\right) \tag{10}$$

$$P_n + jQ_n = \overline{E}_n\left(\frac{E_n}{Z_{nn}} - \frac{E_1}{Z_{n1}} - \frac{E_2}{Z_{n2}} - \frac{E_3}{Z_{n3}} - \cdots\right) \tag{11}$$

$$P_1 + jQ_1 = |E_1|\angle\delta_1\left[\frac{|E_1|\angle\delta_1}{|Z_{11}|\angle\theta_{11}} - \frac{|E_2|\angle\delta_2}{|Z_{12}|\angle\theta_{12}} - \cdots - \frac{|E_n|\angle\delta_n}{|Z_{1n}|\angle\theta_{1n}}\right] \tag{12}$$

$$= \frac{|E_1|^2}{|Z_{11}|}\overline{\angle\theta_{11}} - \frac{|E_1||E_2|}{|Z_{12}|}\overline{\angle\delta_1 - \delta_2 + \theta_{12}} - \cdots - \frac{|E_1||E_n|}{Z_{1n}}\overline{\angle\delta_1 - \delta_2 + \theta_{1n}}$$

$$= \frac{|E_1|^2}{|Z_{11}|}(\cos\theta_{11} - j\sin\theta_{11}) - \frac{|E_1||E_2|}{|Z_{12}|}\left[\cos(\delta_1 - \delta_2 + \theta_{12}) - j\sin(\delta_1 - \delta_2 + \theta_{12})\right] - \cdots - \frac{|E_1||E_n|}{|Z_{1n}|}\left[\cos(\delta_1 - \delta_n + \theta_{1n}) - j\sin(\delta_1 - \delta_n + \theta_{1n})\right]$$

$$P_n + jQ_n = |E_n|\angle\delta_n\left(\frac{|E_n|\angle\delta_n}{|Z_{nn}|\angle\theta_{nn}} - \frac{|E_1|\angle\delta_1}{|Z_{n1}|\angle\delta_{n1}} - \frac{|E_2|\angle\delta_2}{|Z_{n2}|\angle\delta_{n2}} - \cdots\right) \tag{13}$$

$$= \frac{|E_n|^2}{|Z_{nn}|}\overline{\angle\theta_{nn}} - \frac{|E_n||E_1|}{|Z_{n1}|}\overline{\angle\delta_n - \delta_1 + \theta_{n1}} - \frac{|E_n||E_2|}{|Z|_{n2}}\overline{\angle\delta_n - \delta_2 + \theta_{n2}} - \cdots$$

$$= \frac{|E_n|^2}{|Z_{nn}|}(\cos\theta_{nn} - j\sin\theta_{nn}) - \frac{|E_n||E_1|}{|Z_{n1}|}\left[\cos(\delta_n - \delta_1 + \theta_{n1}) - j\sin(\delta_n - \delta_1 + \theta_{n1})\right] - \frac{|E_n||E_2|}{|Z_{n2}|}\left[\cos(\delta_n - \delta_2 + \theta_{n2}) - j\sin(\delta_n - \delta_2 + \theta_{n2})\right] - \cdots$$

$$P_1 + jQ_1 = \frac{|E_1|^2}{|Z_{11}|}\cos\theta_{11} - \frac{|E_1||E_2|}{|Z_{12}|}\cos(\delta_1 - \delta_2 + \theta_{12}) - \cdots \tag{14}$$

$$- \frac{|E_1||E_n|}{|Z_{1n}|}\cos(\delta_1 - \delta_n + \theta_{1n}) + j\left[-\frac{|E_1|^2}{|Z_{11}|}\sin\theta_{11} + \right.$$

$$\left.\frac{|E_1||E_2|}{|Z_{12}|}\sin(\delta_1 - \delta_2 + \theta_{12}) + \cdots + \frac{|E_1||E_n|}{|Z_{1n}|}\sin(\delta_1 - \delta_n + \theta_{1n})\right]$$

$$P_n + jQ_n = \frac{|E_n|^2}{|Z_{nn}|}\cos\theta_{nn} - \frac{|E_n||E_1|}{|Z_{n1}|}\cos(\delta_n - \delta_1 + \theta_{n1}) - \tag{15}$$

$$\frac{|E_n||E_2|}{|Z_{n2}|}\cos(\delta_n - \delta_2 + \theta_{n2}) - \cdots + j\left[-\frac{|E_n|^2}{|Z_{nn}|}\sin\theta_{nn} + \right.$$

$$\left.\frac{|E_n||E_1|}{|Z_{n1}|}\sin(\delta_n - \delta_1 + \theta_{n1}) + \frac{|E_n||E_2|}{|Z_{n2}|}\sin(\delta_n - \delta_2 + \theta_{n2}) + \cdots\right]$$

LET $\theta_{11} = 90^\circ - \alpha_{11}$, $\theta_{12} = 90^\circ - \alpha_{12}$ ... $\theta_{1n} = 90^\circ - \alpha_{1n}$

$\theta_{nn} = 90^\circ - \alpha_{nn}$, $\theta_{n1} = 90^\circ - \alpha_{n1}$, ETC.

THEN

$$P_1 + jQ_1 = \frac{|E_1|^2}{|Z_{11}|}\sin\alpha_{11} + \frac{|E_1||E_2|}{|Z_{12}|}\sin(\delta_1 - \delta_2 - \alpha_{12}) + \cdots + \tag{16}$$

$$\frac{|E_1||E_n|}{|Z_{1n}|}\sin(\delta_1 - \delta_n - \alpha_{1n}) + j\left[-\frac{|E_1|^2}{|Z_{11}|}\cos\alpha_{11} + \right.$$

$$\left.\frac{|E_1||E_2|}{|Z_{12}|}\cos(\delta_1 - \delta_2 - \alpha_{12}) + \cdots + \frac{|E_1||E_n|}{Z_{1n}}\cos(\delta_1 - \delta_n - \alpha_{1n})\right]$$

$$P_n + jQ_n = \frac{|E_n|^2}{|Z_{nn}|} \sin \alpha_{nn} + \frac{|E_n||E_1|}{|Z_{n1}|} \sin(\delta_n - \delta_1 - \alpha_{n1}) + \tag{17}$$

$$\frac{|E_n||E_1|}{|Z_{n2}|} \sin(\delta_n - \delta_2 - \alpha_{n2}) + \cdots + j\left[-\frac{|E_n|^2}{|Z_{nn}|} \cos \alpha_{nn} + \right.$$

$$\frac{|E_n||E_1|}{|Z_{n1}|} \cos(\delta_n - \delta_1 - \alpha_{n1}) + \frac{|E_n||E_1|}{|Z_{n2}|} \cos(\delta_n - \delta_2 - \alpha_{n2})$$

$$\left. + \cdots \right]$$

THEREFORE:

$$P_n = \frac{|E_n|^2}{|Z_{nn}|} \sin \alpha_{nn} + \sum_{m=1}^{m=n-1} \frac{|E_m||E_n|}{|Z_{mn}|} \sin(\delta_{mn} - \alpha_{mn}) \tag{18}$$

$$+ \sum_{m=n+1}^{m=s} \frac{|E_m||E_n|}{|Z_{mn}|} \sin(\delta_{nm} - \alpha_{mn})$$

$$Q_n = -\frac{|E_n|^2}{|Z_{nn}|} \cos \alpha_{nn} + \sum_{m=1}^{m=n-1} \frac{|E_m||E_n|}{|Z_{mn}|} \cos(\delta_{nm} - \alpha_{mn}) \tag{19}$$

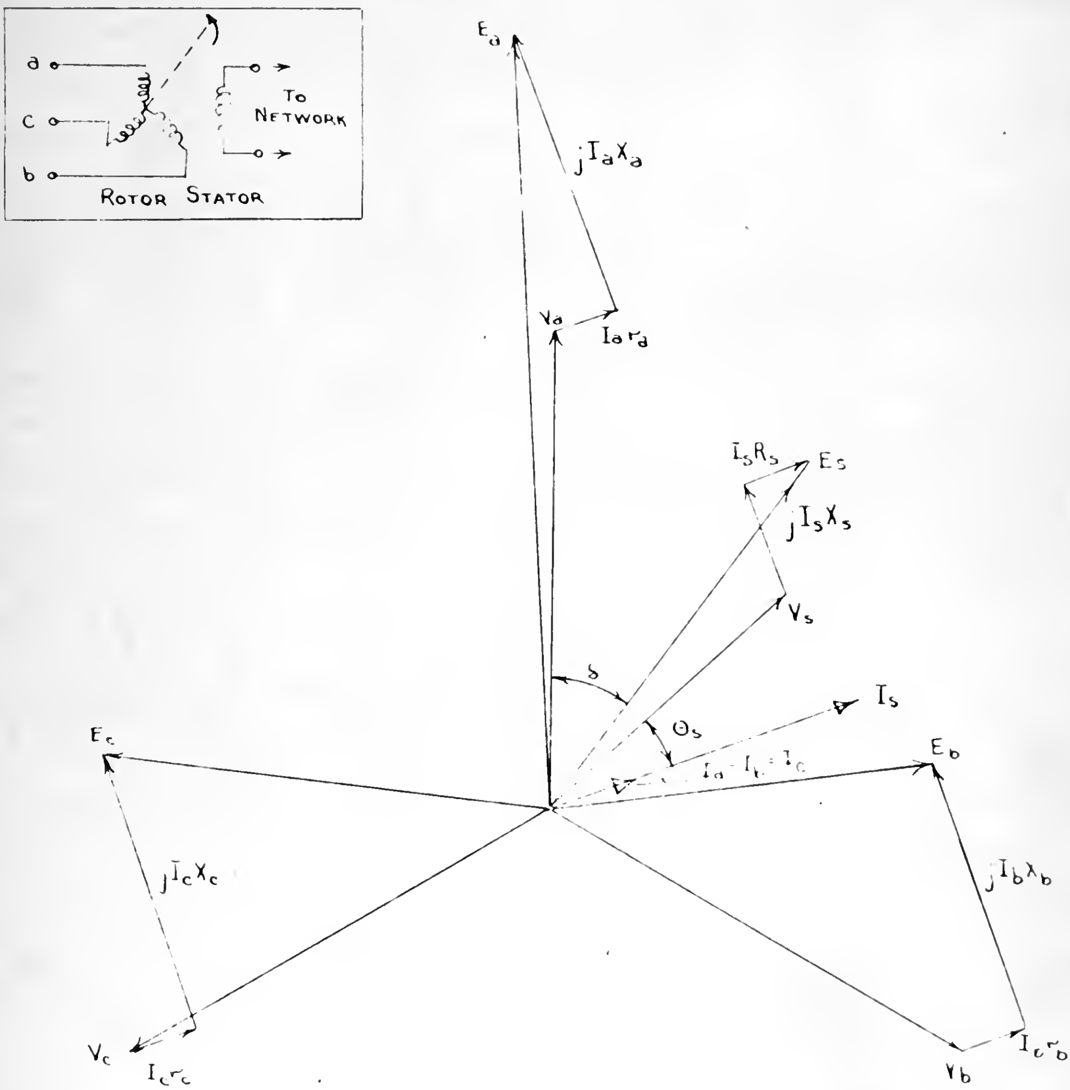
$$+ \sum_{m=n+1}^{m=s} \frac{|E_m||E_n|}{|Z_{mn}|} \cos(\delta_{nm} - \alpha_{mn})$$

In the body of the Introduction, mention was made of the effect of the internal resistance and reactance of the phase shifter when the operators are attempting to obtain a specified output from a machine. The analysis of a phase-shifter connected to a network or system is not simple. The vector diagram is more like that of a transformer or induction motor rather than that of an alternator. In addition, a correct and complete analysis would necessitate the use of symmetrical components since the device has a single-phase load. However, the vector diagram of the voltages and currents for a phase-shifter operating under typical conditions when connected to the Network Analyzer is shown in Figure XVIII. Although this is not a complete analysis, it is helpful in understanding the reason for undertaking the development of a Power-sensitive servomechanism for the Network Analyzer. In the vector diagram of Figure XVIII, the following symbols are used:

E_a, E_b, E_c = induced voltages in rotor polyphase windings
V_a, V_b, V_c = rotor terminal voltages (balanced three-phase)
I_a, I_b, I_c = rotor currents (neglecting magnetizing current)
E_s = induced single-phase stator voltage
I_s = stator single-phase current
x_a, x_b, x_c, x_s = leakage reactances
r_a, r_b, r_c, R_s = winding resistances
Θ_s = stator power factor angle
δ = mechanical angle, considering phase a terminal voltage as a reference. This angle can be altered by cranking the phase-shifter.

Figure XVIII

Vector Diagram of the Voltages and Currents of a Phase-Shifter in the Network Analyzer



B. Details of Procedure

The Power-Sensitive Device

Although an electronic wattmeter was employed in the assembled servomechanism, it did not have the accuracy or flexibility that is considered desirable. This fact gradually became apparent as one means after another was used in various attempts to improve its performance. Other alternate power-sensitive devices were investigated, and although no conclusive proof can be offered at this time, the authors feel that they have recommended logical solutions to this phase of the problem in the Recommendations. However, in order to make a record that may assist others who are interested in obtaining a signal proportional to power, the various devices that were investigated are listed in the following.

The electronic wattmeter was the first to be investigated because it seemed to offer many advantages. Among these advantages were compactness, no moving parts, low cost of construction using standard components, and ease of integrating this type of equipment into the other electronic equipment.

When a survey of the literature on electronic wattmeters was made, it indicated that the most promising type was the one proposed in reference (3) and partially investigated by Churchill in reference (4). Multi-electrode tubes

were used to measure alternating-current power and give an indication in direct-current.

It may be shown for some multi-electrode tubes that under certain conditions the application of alternating voltage to two controlling elements or grids results in a change in the direct current component of the plate current proportional to the product of the voltages at the grids and the cosine of their phase angle. Hence such tubes may be used in the construction of a vacuum tube wattmeter. The tubes recommended by Pierce in reference (3) were the type 2A7 and 6A7; these tubes are identical to the pentagrid converter (6A8) which was used.

The disposition of the electrodes of the tube used may be seen by referring to the diagram of Figure XII. The grids G_2 and G_3 are held positive at a fixed potential above that of the cathode, and G_1 and G_4 are biased to be negative with respect to the cathode. Alternating-current potentials proportional to voltage and current are then applied to G_1 and G_4. In such a case, the current passing through G_1 is dependent only on the potential of G_1, being independent of the potential of G_4. The proportion of this current reaching the plate is, however, dependent only on the potential of G_4. Therefore, if the characteristics are linear for both G_1 and G_4, one may expect that the plate current will be represented in the form

$$I_p = AE_{G1} + BE_{G1}E_{G4} + CE_{G4} + D \qquad (20)$$

where A, B, C and D are constants.

The first and third terms are of an alternating current nature and would not register on a direct-current ammeter in the plate circuit. The direct-current component of the product term is proportional to power, since E_{G1} and E_{G4} are made proportional to alternating voltage and current.

In other words, if a region can be located on the static characteristics of the tube employed such that within this region Ip versus E_{G1} for various values of E_{G4} can be represented by a family of straight lines which, if extended, pass through a common point, and if in this region I_p versus E_{G4} is, for some value of E_{G1} a straight line, then the plate current of the tube can be represented by equation and the tube can be used as a wattmeter. Unfortunately, there is no large region of linear variation of I_p versus E_{G1}, although something approximating that region may be found. In addition, curves such as these can hardly be accurate enough to give final evidence of the degree of linearity; and these curves may be different for individual tubes of the same type.

In view of the non-linearity of the characteristics of the tubes available, a push-pull type of circuit similar to that employed by Pierce in reference (3) was used.

Discussion of this circuit and its limitations are included in the main body of the thesis and will not be repeated here. However, because many difficulties were encountered several other power sensitive devices were investigated while work was continued on the electronic wattmeter. The authors feel that the electronic wattmeter should not be discarded before additional research proves it inadequate because it has many advantages in applications such as this servomechanism.

One of the alternate means of obtaining a signal proportional to power that was considered involves the use of Thyrite. Thyrite is the trade name of a nonlinear resistance material produced by the General Electric Company. This material is made of silicon carbide with a ceramic binder; a metal coating is sprayed on the surface to provide electrical contact. The instantaneous and steady-state volt-ampere characteristic of Thyrite is given by the equation.

$$I = KE^n \qquad (21)$$

The quantity K depends on the resistivity and dimensions of the particular Thyrite unit. The exponent n is at least 3.5 and can be as high as 7 in special cases. Voltage and current are almost exactly in phase, and the volt-ampere characteristic is symmetrical for both positive and nega-

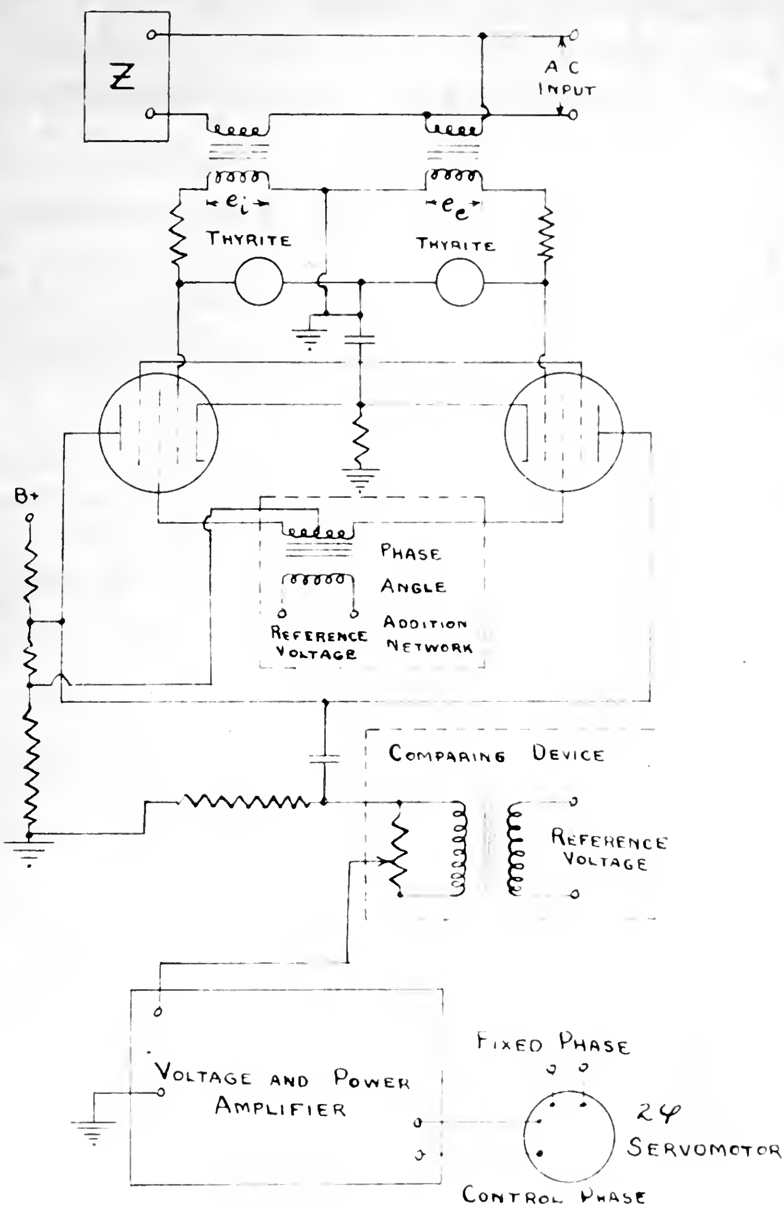
tive polarity.

A continuous rating of 1/4 watt per square inch is allowable in still air and a short time temperature rise of 80°C results from an input of 2000 watt-sec per cubic inch. The most serious limitation to the use of Thyrite for measurement elements is its high temperature coefficient; at constant voltage, current increases 1 per cent per degree centigrade. However, Thyrite may be operated at 110°C continuously and humidity has very little effect on properly impregnated units.

By taking advantage of the logarithmic characteristic of Thyrite, one should be able to obtain a voltage proportional to the logarithm of the line voltage and a voltage proportional to the logarithm of the line current. If some auxiliary means is used to account for the power factor angle, then the output signal of such a device could be made proportional to the power being measured by simple addition. It is suspected that a new type of wattmeter which has recently become available commercially uses this principle; however when the inventor was asked its components he was unwilling to disclose the information because the device was not fully covered by patents at that time. Figure XIX ~~in the Recommendations~~ suggests one method of employing Thyrite as a power sensitive element.

No physical research was done with Thyrite for the following reasons:

Figure XIX
Thyrite as a Power Sensitive Element
A Typical Circuit



(a) Inasmuch as only a small amount of information is available in regard to the use of Thyrite in such an application, investigation into its possibilities and development of a practical device appeared to be inadvisable because of the limited time available.

(b) At the time when this idea was being considered no adequate supply of Thyrite was available.

(c) The high temperature coefficient of Thyrite was viewed with apprehension.

Another alternate means that was considered for obtaining a direct-voltage signal proportional to power involved the use of a diode bridge. The principles upon which this approach was based is the multiplier-action of modulating circuits as discussed in references (15), (16), and (17). Inasmuch as a detailed analysis of modulator circuits is beyond the scope of this thesis, only the following brief explanation is offered to justify the attempt to use the circuit of Figure XXIV as a power-sensitive element.

The circuit of Figure XXIV can be represented as the block of non-linear impedance, Z_{12}, in Figure XX. Since the value of Z_{12} is dependent upon the function $f_2(t)$ entering the block from the bottom, one may write

$$Z_{12} = K\, f_2(t) \qquad (22)$$

However, Z_{12} is relatively unaffected by the magnitude of

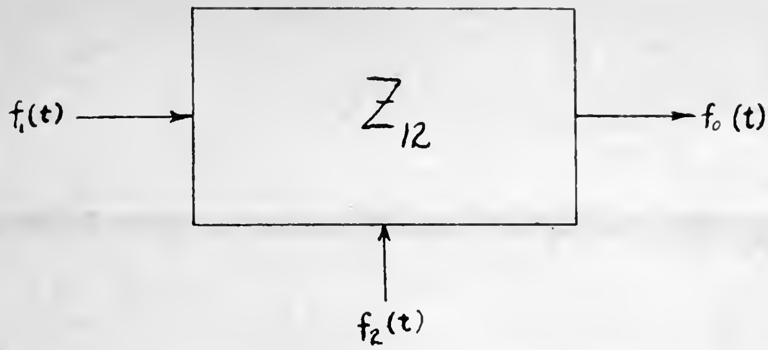


Figure XX.

$f_2(t)$. Now, if another function, $f_1(t)$, is entering the block from the left, and must pass through Z_{12} before leaving the block from the right side, then the function leaving the block may be expressed as

$$f_o(t) = f_1(t)\ Z_{12} = f_1(t)\ K\ f_2(t) \qquad (23)$$

In the circuit of Figure XXIV, these functions can be identified. The first function is a voltage signal proportional to line current

$$f_1(t) = A \cos (\omega t + \theta) \qquad (24)$$

The second function is a voltage signal proportional to line voltage, but because of the switching action of the non-linear impedance, it becomes a square wave in form and may be written

$$K\ f_2(t) = B(\cos \omega t + 1/3 \cos 3\omega t + 1/5 \cos 5\omega t \quad \ldots) \qquad (25)$$

Therefore the function leaving the block becomes

$$f_o(t) = AB \cos(\omega t + \theta)(\cos \omega t) + \frac{AB}{3} \cos(\omega t + \theta)(\cos 3\omega t) + \ldots \quad (26)$$

$$f_o(t) = \frac{AB}{2} \cos \theta + \frac{AB}{2} \cos(2\omega t + \theta) + \ldots \quad (27)$$

This expression indicates that if the undesirable components (all but the first term) can be eliminated, the device should give a direct-voltage proportional to power.

Type 1N34 crystal diodes were used in the first circuit that was built (see Figure XXIVA). When a test was made the output signals obtained were too small to measure. Therefore, 6H6 vacuum tube diodes were substituted for the crystal diodes (see Figure XXIVB) in order to obtain a larger output signal. The tests performed on this latter circuit demonstrated that the device was capable of producing a voltage-signal proportional to power. However, when waveforms of the signal output were observed, the presence of varying amounts of harmonics indicated that extensive filtering would be required before the signal could be used for the servo application. Since the signal that remained after these undesirable components were filtered-out was too weak to use without considerable amplification, this method of power measurement was abandoned.

In view of the fact that the novel ideas of electronic wattmeters, thyrite multipliers and diode multipliers were not developing into practical and reliable power-sensitive

elements, another approach to the problem of power measurement was required. Despite the disadvantages of the electrodynamic wattmeter mentioned in the Procedure, the dynamometer wattmeter appears to be the only practical means of measuring power that has proven itself reliable and accurate. If this type of wattmeter is to be employed in the power-sensitive element, however, auxiliary devices are needed to transform the angular displacement of its indicator into a direct-voltage signal.

Several schemes have been devised to accomplish this transformation. The most practical method was mentioned in the Recommendations. Other methods, some tried, some untried, are mentioned here.

Since the micro-torque* potentiometer requires only 0.003 inch-ounces of torque to overcome the friction and inertia of its rotating arm, one of these potentiometers was mounted on a dynamometer wattmeter and the rotating arm was attached to the indicator of the wattmeter with piano wire, as shown in Figure XXIIA. At first, this innovation appeared to be a simple method for obtaining a direct-voltage signal because a constant direct-voltage could be applied across the potentiometer and the center-tap would produce a voltage that varied linearly with the angular displacement of the meter. Unfortunately,

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* Manufactured by G. M. Giannini & Co., Pasadena, Cal.

even the slight torque required to overcome the friction of the potentiometer caused the meter movement to become "sticky" and inaccurate. Obviously, the torque required to move the potentiometer has to be very small with respect to the torque available from the wattmeter movement before this arrangement will perform satisfactorily; apparently, in spite of the fact that the micro-torque potentiometer required only 0.003 inch-ounces of torque, the torque available from the meter movement was inadequate.

As an outgrowth of the fact (established by attempting to use the micro-torque potentiometer) that the wattmeter could be used only if the meter indicator was not restrained by relatively large external torques, another means of obtaining a voltage signal proportional to angular displacement was tried. The general principle involved the use of an iron-vane meter* as the variable leg of a bridge circuit as shown in Figure XXI . The indicator of the iron-vane meter could be attached to the indicator of the wattmeter and allowed to follow the wattmeter movement. Since any movement of the iron vane upsets the bridge balance, an alternating voltage which is proportional to the angular displacement of the wattmeter, and there-

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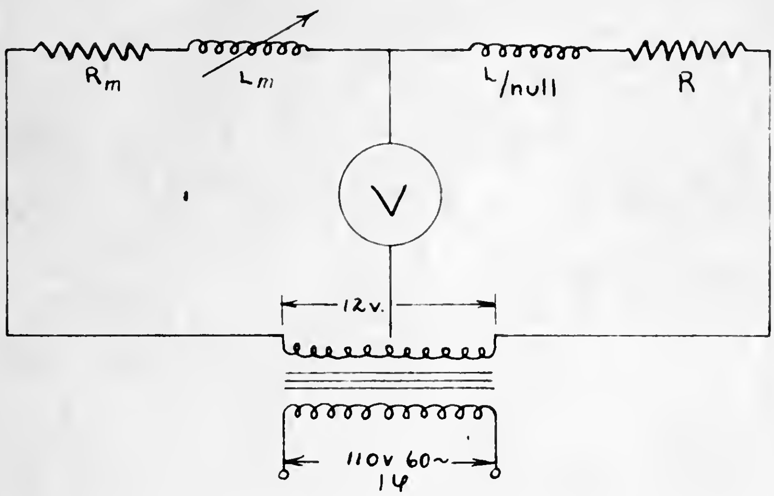
* Triplett a c milliammeter Model 237-R

FIGURE XXI
IRON VANE - ELECTRODYNAMIC WATTMETER
POWER SENSITIVE ELEMENT



Measured constants "Triplett" AC Milliammeter Model 237-R

for zero displacement L_m = 685 mh

R_m = 2,260 ohms

Best Null obtained in above circuit when

L = 713 mh

R = 2233 ohms

Notes: (a) Voltmeter indicator varied linearly with angular displacement of milliammeter indicator.

(b) This device could be used as a comparing device as well as a means of transforming angular displacement to voltage by calibrating either the balancing inductance or resistance.

fore proportional to the power can be obtained from the bridge. This alternating voltage can be amplified, rectified, and used for the existing servomechanism, or it could be used as an alternating signal for an alternating-current servomechanism. Due to the fact that employing this device required developing additional associated equipment before it could be incorporated into the components which had already been built, it was set aside for future study. However, in order to ascertain that an iron-vane meter would perform adequately in a bridge circuit of this type, a test was made (see Figure XXIIB and the results definitely proved that the output voltage of the bridge varied linearly with the angular displacement of the iron-vane meter.

Since all of the foregoing alternate methods of obtaining a direct-voltage signal proportional to power either proved unsuccessful or required the development of additional equipment, and since the electronic wattmeter was not entirely unsuccessful, the electronic wattmeter was used for the servomechanism. However, because the performance of the electronic wattmeter was not completely satisfactory, methods of improving its operation were constantly sought. Usually these improvements were incorporated into the device as they were discovered. One improvement was developed but was not installed because time did not permit revising the circuit to accommodate it; this

Figure XXII

Experiment Power-Sensitive Elements

A

Wattmeter
Micro-Torque
Potentiometer

B

Wattmeter
Iron-Vane

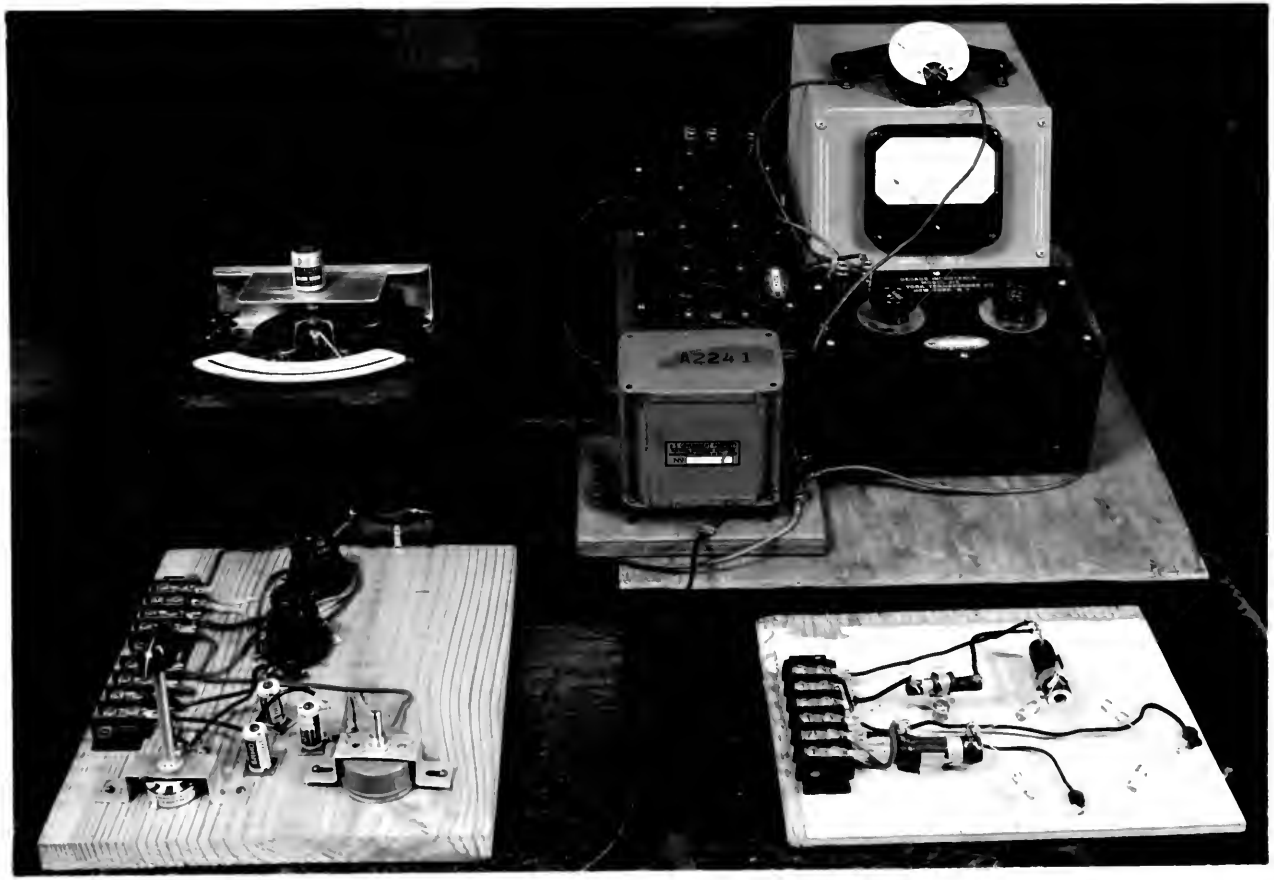
Piano
Wire

C

Vacuum-Tube
Diode Bridge

D

Crystal
Diode
Bridge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improvement is the electronic input circuit of Figure XVI and Figure XXV, which circuit would eliminate the need for input transformers. The advantages of substituting an electronic input circuit for the input transformers were enumerated in the Procedure.

The First Amplifier:

The Procedure mentioned the first voltage and power amplifier that was built but did not describe the circuit. To complete the record, the circuit of this amplifier is shown in Figure XXIII and is pictured in Figure XXV. When this circuit was initially built on the "breadboard", only four triode units (two "bottles") were placed in parallel in each of the bridge legs. Performance tests revealed that although the bridge was capable of reversing the motor and was sensitive to small input voltages, the bridge could not deliver sufficient current to enable the motor to deliver rated torque. Even when additional triode units were placed in parallel to increase the current output of the bridge, the bridge would not supply sufficient power to the motor. Apparently, the power was being dissipated in the bridge leg resistors, with the result that the voltage regulation was poor. The poor voltage regulation and motor performance are revealed by Table II:

Table I

Data Showing Torque vs Rotor Angle
For Network-Analyzer Phase-Shifter

Displacement from Neutral Point (mechanical degrees)	Three-Phase Voltage to Rotor (volts, a.c.)	Load Current (amps)	Torque (ft.lbs.)
0	77.0	0	-0.02
5	78.0	0.5	-0.15
10	78.0	0.5	-0.09
15	76.0	0.6	-0.03
20	77.0	0.7	0
25	77.0	0.9	0.05
30	77.0	1.0	1.00
35	77.0	1.1	0.26
40	77.0	1.5	0.44
45	77.0	1.7	0.50
50	76.5	2.0	0.64
55	76.5	2.45	0.62
60	76.0	2.6	0.20
65	75.2	2.5	-0.07
70	76.0	2.3	-0.13
75	76.0	2.1	0.01
80	76.0	2.05	0.18
85	76.0	2.10	0.07
90	76.0	2.2	0.70
95	75.0	2.35	0.95
100	75.0	2.71	1.40
105	74.0	3.11	1.55
110	72.8	3.67	1.90
135	77.0	- -	1.90
180	75.0	4.75	0.05

Figure XXIII
Original Power Amplifier

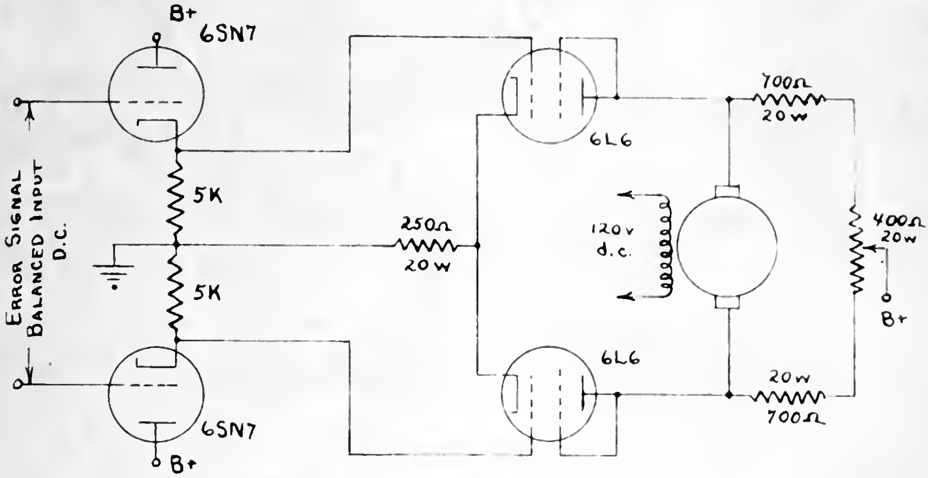


Figure XXIV
Diode Bridge Type Power Sensitive Elements

A. Crystal B. Vacuum-Tube

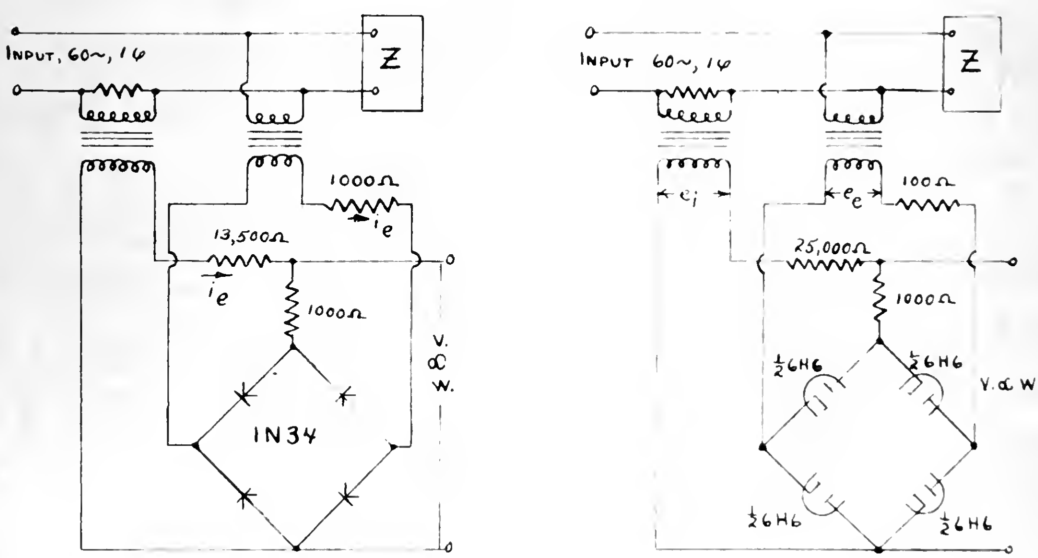


Figure XXV

System Elements

First
Power
Amplifier

Current
Input
to
Power
Sensitive
Element

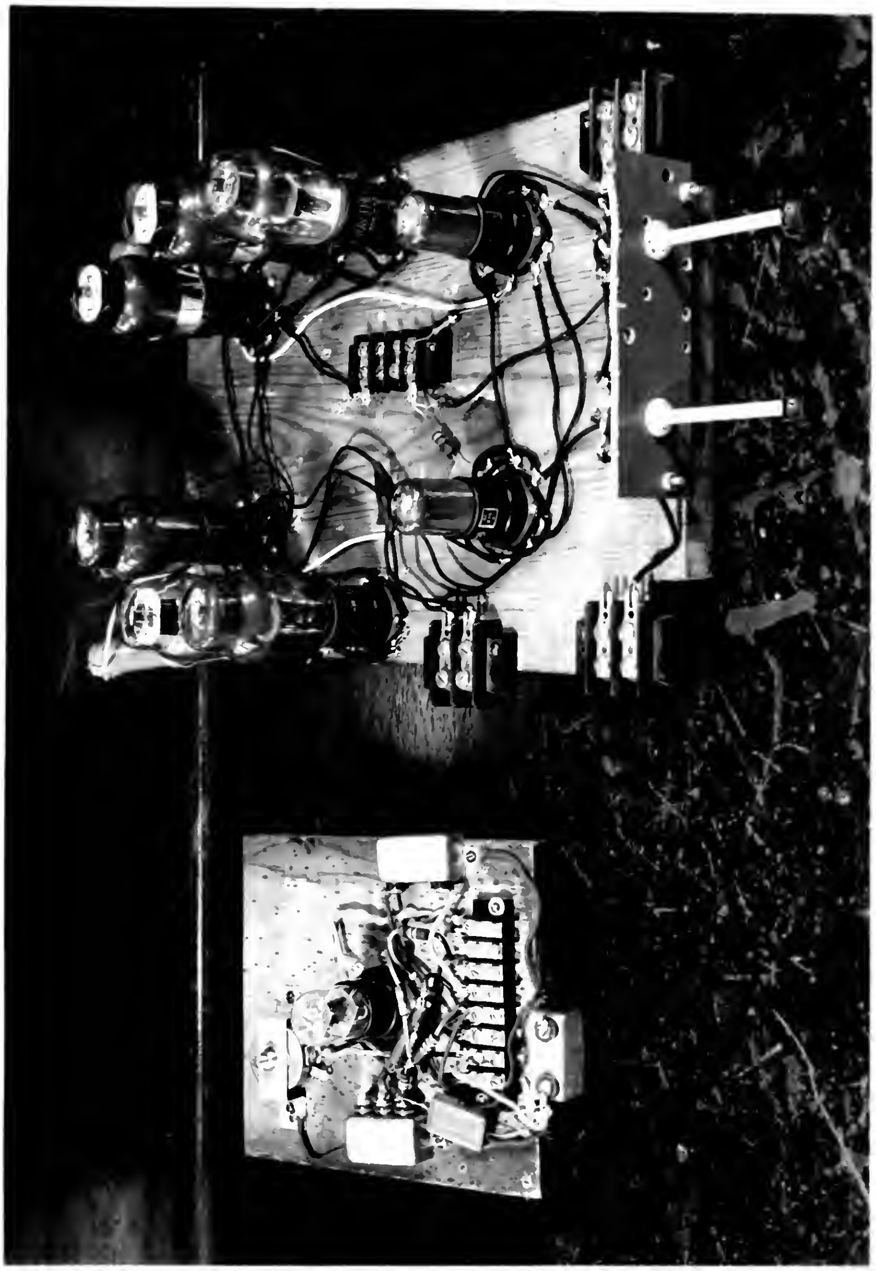


Table II

Signal	Condition	Voltage Across Motor	Motor Current	Torque
4.2v.	Motor running free	130v.	25ma.	0
4.2v.	Motor stalled	30v.	105ma.	65in.oz.

Since the resistors were the source of the trouble, they were replaced by vacuum-tubes and the circuit described in the Procedure was developed. In contrast to the first amplifier, the second amplifier had good voltage regulation and was able to deliver sufficient current to the motor to enable the motor to deliver better than rated-torque. Table III shows the performance of the second amplifier.

Table III

Signal	Condition	Voltage Across Motor	Motor Current	Torque
4.0v.	Motor running free	140v.	30ma.	0
4.0v.	Motor running	135v.	100ma.	60in.oz.
4.0v.	Motor stalled	45v.	300ma.	240in.oz.

Worm-gear drive:

Reasons for employing a worm-gear drive were given in the Procedure. Although the worm-gear drive is considered desirable for the power-sensitive servomechanism for the reasons given, the present general attitude toward the use of irreversible gear trains is completely negative. Reference (8) states that irreversible gear trains should be avoided except possible where the load inertia and load torques are small and that difficulty is caused

by the locking that takes place in many irreversible gears when subject to load torques. When practical servo men were questioned in regard to the use of the worm-gear drive, they summarized their objections by stating that worm-gear drives are undesirable for three reasons:

(1) the system will probably chatter,

(2) the drive has very low efficiency,

(3) gear tolerances are difficult to maintain.

Since very little information is available in the literature, no satisfactorily complete analysis of the problems encountered when the worm-gear drive is employed in servo-control could be found.

Figure XXVI
Test Setup of Power Sensitive Element

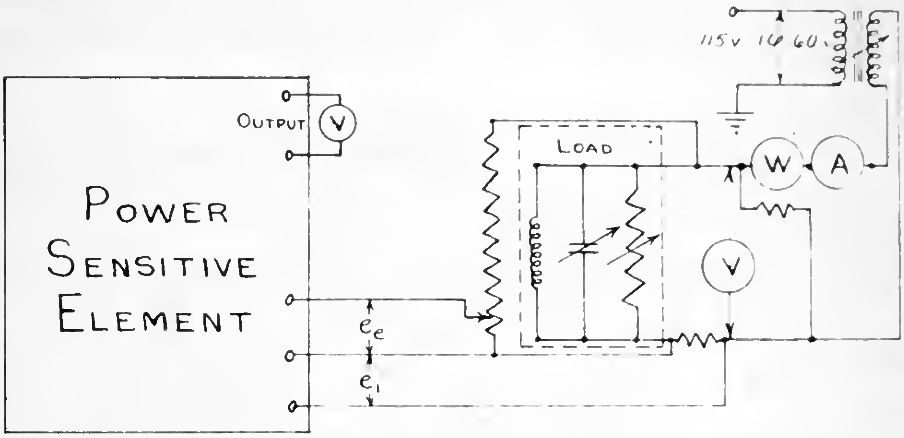


Figure XXVII
Test Setup of Power Amplifier

C. Typical Data and Curves

Table IV

Drift Run on Electronic Wattmeter

Run (3)

Time (minutes)	Power (watts ac)	Wattmeter Deflection (volts dc)	Wattmeter Power Supply (volts dc)
0	114	2.90	351
5	116	2.99	352
15	114.7	2.89	354
25	113.5	2.90	356
40	112.6	2.89	360

Table V

Constant Power Runs on Electronic Wattmeter

Run (6)

Power (watts ac)	Line Current (amps ac)	Line Voltage (volts ac)	Wattmeter Deflection (volts dc)	Remarks
100	0.6	148	5.7	R only
100	0.8	120	5.79	"
100	0.94	105	6.0	"
100	1.05	93	6.2	"
100	1.46	92	7.4	R, C & L
100	1.18	94	6.4	"
100	1.07	94	6.2	R & L

Run (7)

50	0.75	66	3.1	R & L
50	0.78	66	3.2	R, C & L
50	1.40	66	4.4	"
50	1.64	84	5.2	"
50	0.5	86	3.2	R only
50	0.65	67.5	3.1	"
50	0.70	68	3.1	"

Table VI

Bridge Type Circuit Utilizing Movable-Vane Meter

Run (1)

Vane Deflection (degrees)	Voltage (volts ac)
13	0.0050
17	0.0060
27	0.0082
38	0.0110
49	0.0138
57	0.0163
67	0.0187
75	0.0209
83	0.0226

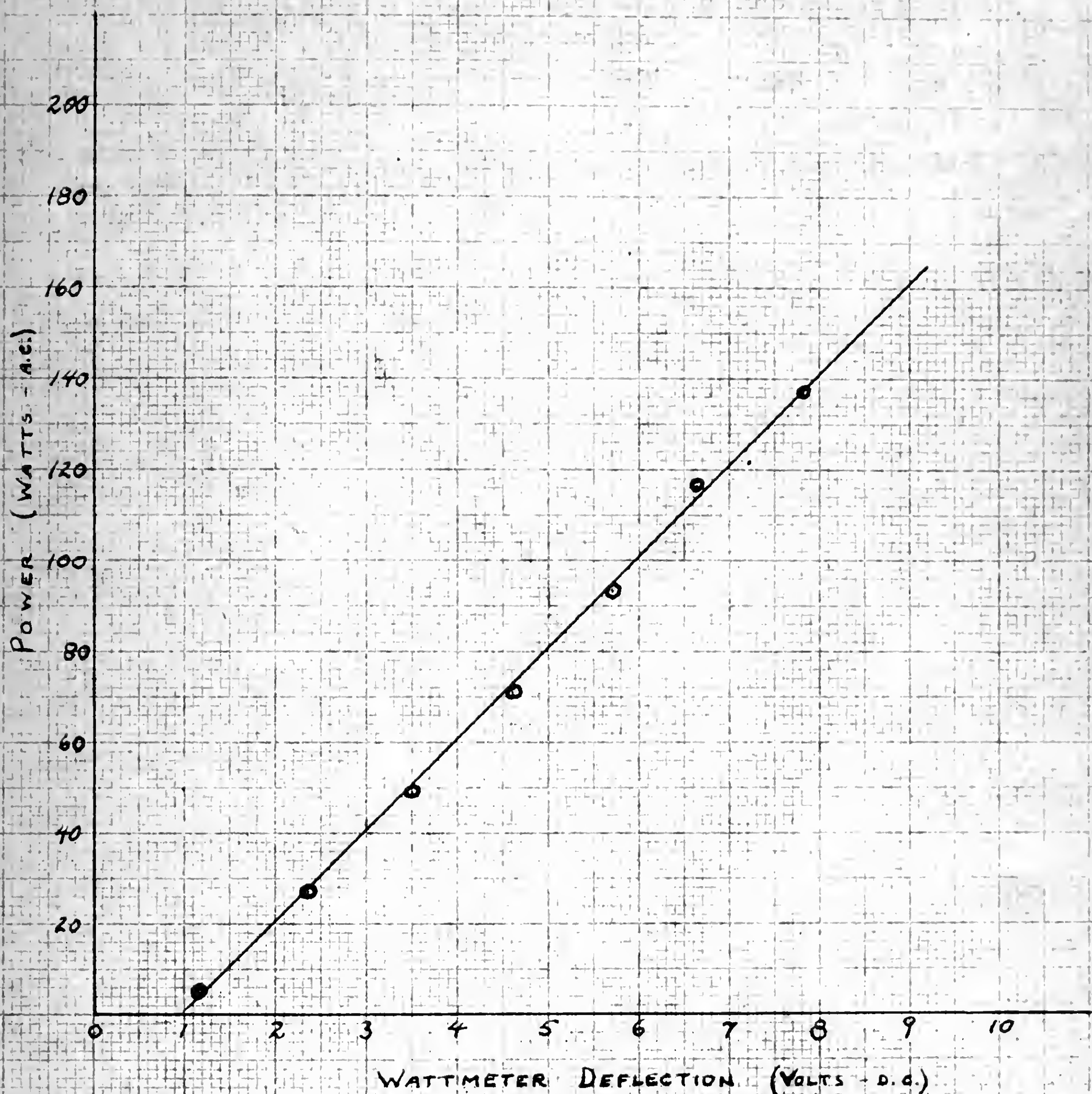
Run (3)

13.5	0.0050
21.0	0.0910
27.5	0.0118
34.0	0.0150
41.0	0.0183
48.0	0.0207
54.0	0.0225
60.5	0.0248
67.0	0.0267
72.5	0.0281
78.0	0.0299
84.0	0.0310

Note: Table I "Data Showing Torque vs Rotor Angle For Network-Analyzer Phase-Shifter" is found in Appendix B.

Tables II and III on power amplifier performance are also found in Appendix B.

FIGURE XXVIII



GRAPH I

A Typical Performance Curve

Power Sensitive Element

(Electronic Wattmeter)

A.C. Power as a Function of Wattmeter Deflection in Direct Voltage for a Resistance Load

W.L.G.
L.B.M.
4/8/49

D. Sample Calculations

Voltage and Power Amplifier

Since the first power amplifier that was built has been discarded, only the sample calculations for the successful amplifier are included here.

As explained in the Procedure, a bridge circuit, employing only vacuum tubes in the bridge legs, appeared preferable to other types. In addition, the type 6AS7-G tube seemed to be the outstanding tube for this application because of its high current-rating and low voltage drop.

The purpose of the following calculations was to determine the proper supply voltages and the values of the circuit parameters. Although this is an easy task for the bridge circuit, the determination of the voltages and resistances in the grid-control circuit is far from easy, especially since the number of voltage supplies was to be kept to a minimum.

In the bridge circuit of Figure XXIX, only the supply voltages had to be determined. It was desirable to maintain the motor armature as near ground potential as practicable in order to avoid possible harm to the motor insulation; therefore, the low potential side of the bridge is grounded. Since the motor is rated at 120 volts and the potential drop across each of the 6AS7-G tubes is 35 volts (when conducting), the supply voltage should be at least

Figure XXIX
Power Amplifier Development

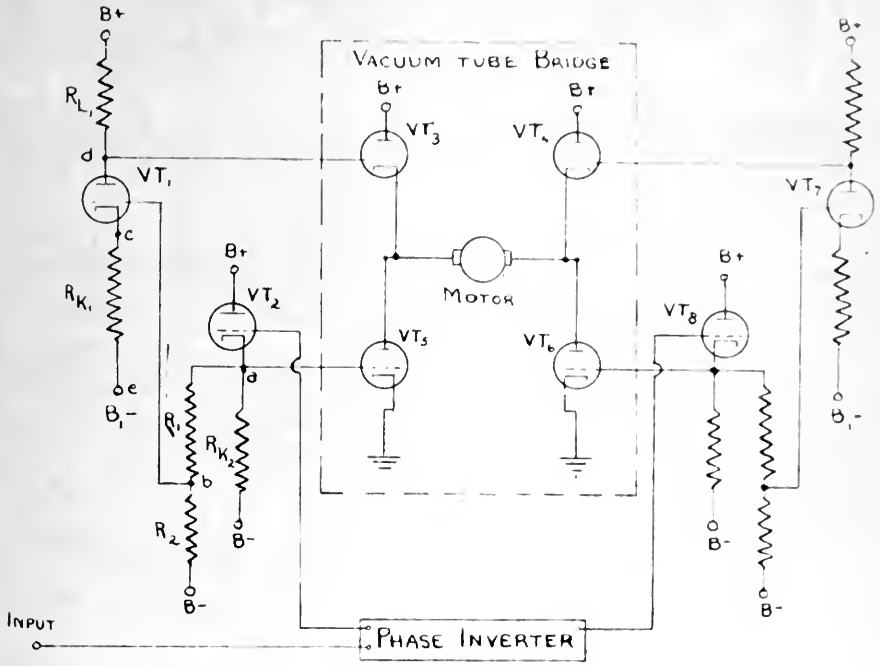
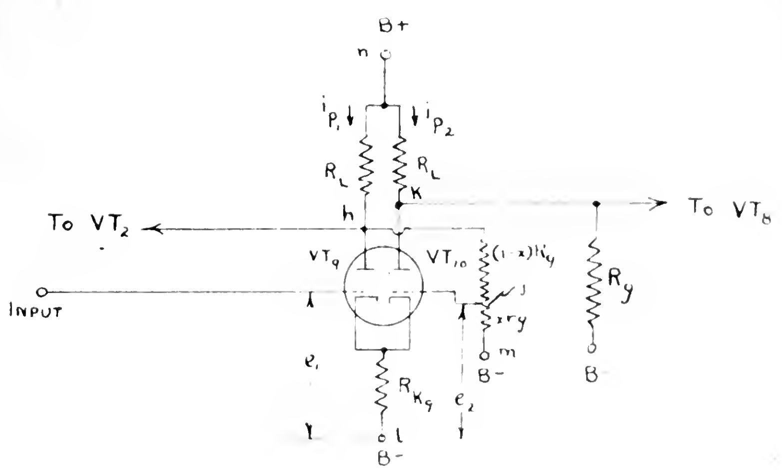


Figure XXX
Phase Inverter



190 volts. To allow for any slight variation in the power tubes, it is advisable to use a 200-volt power supply.

Inasmuch as the type 6AS7-G tube has a low mu (2.1), it requires a rather large control voltage. As Figure XXX reveals, a somewhat complicated grid-control network had to be developed in order to obtain the proper control voltages. A cut-and-try method is undoubtedly the most expedient means of determining the values of the various elements in this network.

To determine the grid swing of the power tubes, one must establish a load line on the tube characteristic sheet. The load line for the motor can be approximated by its armature resistance at standstill and a resistance equivalent to its back e.m.f. at full speed. Using this load line (400 ohms), one finds that the following grid to cathode potentials are required for operation:

(a) Zero volts for the tube to conduct rated current.

(b) Minus 100 volts for the tube to be cut-off.

Once the grid swing of the power tubes is established, it can be restated in terms of grid to ground voltage. For VT_4, the grid potential with respect to ground swings from zero to minus 100 volts. Since the cathode potential of VT_3 is the same as the plate potential of VT_4, the grid potential of VT_3 must swing from minus 65 volts to plus 100 volts.

To drive the grid potential of VT_3 over the large range of voltage required and in opposition to the grid of VT_4, the tubes VT_1 and VT_2 are utilized. The grid of VT_1 must be driven at a large negative voltage set by VT_2 through a voltage-leveling device. VT_1 should be cut-off when the grid to cathode potential of VT_3 is zero or, effectively, the grid of VT_3 is at the positive supply voltage of VT_1. However, the large grid current of VT_3 is used to automatically set the grid potential. On the other hand, VT_1 should be able to conduct enough current to lower its own plate potential to minus 65 volts, since that is the grid potential required to cut off VT_3. From these facts one can specify some of the supply voltages.

The upper supply voltage for both VT_1 and VT_2 should be 200 volts; in this manner the same voltage supply that is used for VT_3 can serve a multiple purpose. The lower supply voltage for VT_1 should be at least minus 130 volts to allow for a voltage drop of approximately 50 volts in VT_1. Other factors must be considered before one can specify the lower supply voltages for VT_2 and the voltage leveler.

Since VT_1 should be at cut-off when VT_4 is cut-off, the grid of VT_1 should be at least minus 150 volts when the grid of VT_4 is at minus 100 volts. Therefore, the lower supply voltage for the voltage leveler should be

approximately minus 220 volts to make V_b approximately midway between V_a and minus 220 volts. Now, one can use the same minus 220-volt supply as the lower voltage supply for VT_2, thereby eliminating the need for another supply.

The tubes, VT_1 and VT_2, in the grid-control network have not been specified because any medium-mu triode can be used. For this application, the type 6SN7 tube was selected for the following reasons:

(a) Both VT_1 and VT_2 may be enclosed in the same envelope since the 6SN7 is a twin-triode.

(b) The 6SN7 is a very reliable tube and is readily available.

The values of the resistors to be used in the circuit were calculated by means of the following equations:

$$V_a = -220 + i_{b2} R_{k2} = +200 - e_{b2} \qquad (28)$$

$$V_b = V_a - \left(\frac{V_a + 220}{R_1 + R_2}\right) R_1 \qquad (29)$$

$$V_c = -V_e + i_{b1} R_{k1} = +200 - i_{b1} R_{L1} - e_{b1} \qquad (30)$$

$$V_d = 200 - i_{b1} R_{L1} = -V_e + e_{b1} \qquad (31)$$

$$e_{c1} = V_b - V_c \qquad (32)$$

$$e_{c2} = V_s - V_a \quad (33)$$

$$e_{c3} = V_d - V_f \quad (34)$$

$$e_{c4} = V_a - 0 \quad (35)$$

The final values for the resistors were determined by a series of successive approximations. To illustrate the method employed, the final set of calculations are included here.

For motor rotation in the direction requiring VT_4 to conduct 250 milliamperes

$$e_{c4} = 0 \quad \text{and} \quad e_{b2} = 200\ v.$$

To be well within the ratings of the 6SN7, let

$$i_{p2} = 7\ ma.$$

For which $R_{k2} = \frac{V_{k2}}{i_{p2}} = \frac{220}{7 \times 10^{-3}} = 31.4 \times 10^{3}$ ohms.

And $e_{c2} = -6.2\ v.$

VT_3 should be cut-off; then $V_d = -65\ v.$

When $e_{c1} = 0$ and $e_{b1} = 35\ v.$, $i_{p1} = 2.7\ ma.$

For which $R_{L1} = \frac{E_{bb} - V_d}{i_{p1}} = \frac{200 - (-65)}{2.7 \times 10^{-3}} = 98 \times 10^{3}$ ohms.

Take $V_e = -150\ v.$

$$R_{K1} = \frac{V_c - V_e}{i_{p1}} = \frac{V_d - e_{p1} - V_e}{i_{p1}} = \frac{-65 - 35 - (-150)}{i_{p1}}$$

$$= \frac{50}{2.7 \times 10^{-3}} = 18.5 \times 10^{3} \text{ ohms.}$$

Determination of R_1 and R_2:

$$V_b = V_c = V_d - e_{p1} = -65 - 35 = -100 \text{ v.}$$

Take $R_1 = 1$ megohm so that current through R_1 and R_2 is negligible.

$$R_2 : R_1 = (-220 - V_b) : V_b$$

$$R_2 = \frac{R_1(-220 - (-100))}{(-100)} = \frac{1 \times 120}{100} = 1.2 \text{ megohm.}$$

To check that grid potentials will have the proper values for current flow in the opposite direction:

VT_4 should be cut-off, for which $e_{c4} = -100$ v.

and $V_a = -100$ v.

$$e_{p2} = E_{bb} - V_a = 200 - (-100) = 300 \text{ v.}$$

$$i_{p2} = 3.82 \text{ ma.}$$

$$i_{p2} R_{K2} = 3.82 \times 31.4 = 120 \text{ v.}$$

$$e_{c2} = -13 \text{ v.}$$

VT_3 should be conducting 250 ma.

$$V_b = V_a + \left(\frac{-220 - V_a}{R_1 + R_2}\right) R_1 = -100 + \frac{-220 - (-100}{2.2 \times 10^{6}} \times 10^{6}$$

$V_b = -100 - 54.5 = -154.5$ v.

$V_c = -150 + i_{p1} R_{k1}$

by cut-and-try find the value of i_{p1} that fulfills equations ( ) and ( ).

try 0.6 ma.

$V_c = -138.9$ v., $e_{c1} = -14.6$ v. giving $i_{p1} = 0.6$ ma.

Since this value of plate current checks, then

$V_d = 200 - (0.6 \times 10^{-3} \times 98 \times 10^{3}) = 141.3$ v.

However, the grid current of VT_3 reduces this potential to that of the cathode of VT_3 and the tube conducts 250 ma., as desired.

To develope the Phase-Inverter:

The swing of grid potential of VT_2 is -6 v. to -113 v. and the swing for VT_8 is then -113 v. to -6 v.

The voltage conditions that should fulfilled are:

at quiescent $V_n = V_k = -59.5$ v.

and $V_n = -6$ v. when $V_k = -113$ v.

and $V_n = -113$ v. when $V_k = -6$ v.

Use $R_L = 110{,}000$ ohms and $E_{bb} = 300$ v.

R_g should be such a value that current is negligible

Make R_g = 14 megohms

R_K should be such a value that the potential drop across VT_9 and VT_{10} change the same amount for equal and opposite signals.

R_K = 800 ohms. Then

$$E_{co} = -1.76\ v.$$

$$I_{bo} = 1.1\ ma.$$

$$E_{Bo} = 178\ v.$$

$$V_L = -(E_{co} + E_{Bo} - V_{ho}) = -(1.76 + 178 + 59.5) \cong -240\ v.$$

Using $V_L = -240\ v$, then $V_n = +60$

$$\frac{x R_g}{R_3} = \frac{1}{VG}$$

Since the 6SN7 is being operated in its linear region, $V\sigma$ can be determined from the Tube Characteristic Sheet by applying a signal of 1 v.

for $e_1 = 1\ v.$ assume $e_2 = -1\ v.$

$i_{p_1} = 1.5\ ma.$ and $i_{p_2} = 0.7\ ma.$

thus $e_K = (1.5 + 0.7)(800)(10^{-3}) = 1.76\ v.$ as it be for a balanced push-pull circuit.

$e_{p_1} = 134.5\ v.$ and $e_{p_2} = 222.5\ v.$

$e_{p_1} = -44\ v.$ $\qquad$ $e_{p_2} = +44\ v.$

So that $V\sigma = 44$ and $x R_g = \frac{1 \times 10^6}{44} = 0.316$ megohm.

and $V_m = V_L - i_{K_g} x R_g = -240 - 4.2 = -244.2\ v.$

General Notes on the Calculations for the Voltage and Power Amplifier

(a) To allow for variations in tube characteristics, xR_g should be variable. This feature will permit balancing the phase-inverter at zero signal; frequent rebalancing should not be necessary, but allowance must be made for any major change in tube characteristics resulting from "aging" or replacement.

(b) When the power supply for the electronic components of the servomechanism is designed, provision should be made for adjusting V_m slightly. However, the power supply should be stable and once an adjustment for V_m is made, the power supply should maintain the same potential between V_m and V_ℓ .

(c) Some of the values obtained in the foregoing calculations may not agree with those used in the actual circuit that was built. However, as is often the case with calculations for electronic circuits, the values obtained in these calculations are only approximate. In spite of this fact, the values are a good starting point. From this point, the optimum values can be obtained experimentally, as was done here.

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