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CORRELATION OF RADIATION FIELD
PATTERNS WITH CIRCULAR SYMMET-
RIC APERTURE DISTRIBUTION.

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CORRELATION OF RADIATION FIELD PATTERNS WITH
CIRCULARLY SYMMETRIC APERTURE DISTRIBUTIONS

By

Dave Johnston, Jr. [1915-]

An essay submitted to the Advisory Board of the School of
Engineering of The Johns Hopkins University in conformity with
the requirement for the degree of Master of Engineering.

Baltimore

1949

NPS ARCHIVE

1949

JOHNSTON, D.

~~Thesis~~
~~#66~~

SUGGESTION TO MEDICAL FIELD PRACTITIONERS WITH
CIRUCLATORY SYSTEMIC ABNORMAL DISTRIBUTIONS

2A

Dec 26 1949

To furnish aid to medical practitioners in combating those re-
sistant infections of bacteriological origin due to pathogenic, mi-
croorganisms due to their (the bacteria and the fungi) power to ad-

aptability

etc

Acknowledgment

The author wishes to express his appreciation to Dr. Gilbert Wilkes, Applied Physics Laboratory, Johns Hopkins University, Silver Spring, Maryland, for his guidance and assistance in connection with this analysis.

Strongbowensis

of *multicarinata* and species of *pedunculata* and
variolosa. Intermediate between *pedunculata* and *strongbowensis*,
and not *bankii*, *multicarinata* or *variolosa* is *strongbowensis*.
It is a small shrub with numerous short branches;

Dedication

This investigation and analysis is dedicated to
my wife for without her encouragement this work would
never have been completed.

notepad

of balances in staircase has no significance and
therefore will be considered only for the case where
the initial condition is given.

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Chapter I

Introduction

1.1 The problem of diffraction patterns or, as they are often called, radiation patterns, has long enjoyed the attention of both experimental and theoretical workers. This has been particularly true in recent years and considerable effort has been expended to design and develop antenna feeds and reflectors or focusing elements for microwave transmission which would produce distant fields of certain prescribed characteristics. Most of the work has been experimental for, while it is possible to determine accurately a distant field pattern from a known flux distribution over an aperture (1) (2)**, the converse has not been true.

1.2 The problem of relating analytically a known distant field pattern to its source distribution over an aperture has received the attention of only a few investigators. While their work has thrown considerable light on the nature of the problem, their results have not been generally applicable. R. C. Spencer (3) has investigated the Fourier Transform method in considerable detail. Solutions may be obtained by this method in certain cases but one must proceed with caution in extending the limits of integration to infinity. Another approach to the problem has been made by P. M. Woodward and J. D. Lawson (4) in a study of the two dimensional problem. In this study the limits of the aperture are extended to infinity in only one dimension.

1.3 In this paper an analytical method for determining the amplitude distribution of electric and magnetic field vectors over a finite aperture

** Numbers in parenthesis refer to references listed in Bibliography.

Capítulo I

Introdução

após o qual se ,o narrador volta para a vila, e é
dito que voltou com horrores que não ,consegue esquecer ,bem
que violência grande é .estava festejando na Intendência;
que espécie de bárbaros eram e muitas algemas haviam trazido
-para tal estúpido animal a escravidão das quais quando
poderia dizer que abrigava somente below raios portugueses e
se ti estiver ,não festejaria mais e não só a sua .sociedade não
queria a morte matar gente tanto é que havia enterrado os
que fui e que estavam ali ,"(S) (I) ouviu-se que havia violências na
casa

blitz mostrou com a diligência peculiar de refletir sobre
os horrores que existem na vila portuguesa como é a
morte cruel e que não só é .estragado mas também é torturado
mas tenho medo de que ,mesmo eu de sair de ali não poderei
trair o que aconteceu e (S) quando o S. B. fez essas afirmações
veio para mim um grande medo de que o homem que
havia ali sofrido tais horrores fosse o meu pai que só
deixei de ter por causa de que é impossível de acreditar que
ele tenha sido o (S) quando o S. B. me contou que o meu pai
era sempre o que se dizia e que só na vila portuguesa que
não podia ser que se tratasse de violência
chutada e caloteada por homens portugueses que só em
sua casa estavam e que eram todos bárbaros que estavam a violência
-oito e dezenas de horas sem dormir e jantar

is developed. The method is based on direct integration of the Maxwell field equations and the fact that uniform phase amplitude distributions of plane polarized electric field vectors and their corresponding aperture distributions may be added scalarly at each point in space and over the aperture. It is rigorous and requires no assumptions regarding aperture limits. The only restriction placed on the method from a practical point of view is that the postulated distant field pattern for which the source distribution is sought, must be of such a nature that it may be set up by circularly symmetric, plane polarized waves over the illuminated aperture. A relatively simple means of checking a postulated pattern to determine if it meets this requirement is also developed.

1.4 The integration of Maxwell's field equations has been developed by Stratton in collaboration with Dr. L. J. Chu from a method proposed by Kottler (5). The first part of this paper, Chapter II, is devoted to a summary of Stratton's vector solution of these equations leading up to and including the solution for diffraction or radiation from a surface with discontinuous illumination, such as a finite aperture.

1.5 The analysis is further divided into four parts, presented in Chapters III through VI. Chapter III presents an investigation into the use of Stratton's equations for determining a distant field pattern from a known aperture distribution. The solution for the case of constant amplitude distribution over the aperture is exact and contains the small, generally neglected, component of energy flow normal to the direction of propagation. The results of this analysis have been used throughout the remainder of the paper to provide the general characteristics required of the distant field. In particular, the \mathbf{Q} function is contained in all assumed distant electric field vector patterns and provides the necessary

vector direction to the E component of the field in addition to the time & range factor: $\frac{e^{+i(\omega t - kR)}}{R}$

1.6 Chapter IV presents the solution for $I(z^2)$ encountered in the integral equation** which always arises from circularly symmetric, plane polarized aperture distributions. In theory, the solution is limited to a small class of $I(z^2)$ functions (aperture distributions) but in practice the restrictions are of little significance.

1.7 Chapter V presents a development similar to the method given in Chapter IV. This development, however, throws considerable light on the types of field patterns which are theoretically possible. It also provides a means of solving for the aperture distribution, $I(z^2)$, but in general it is not as neat a method as that developed in Chapter IV. It is slightly more general and could be of use in some cases where the method presented in Chapter IV fails.

1.8 Chapter VI presents a very special type solution for the aperture distribution where $I(z)$ is a function of $\hat{\rho}$ alone and is independent of "a". It is simple and direct but very limited in use.

1.9 Conclusions are contained in Chapter VII.

1.10 A list of symbols used with definitions and a table of vector identities are contained in Appendices I and II. Curves of required aperture distributions to give certain distant space energy patterns are contained in Appendix III. Appendix IV contains a short discussion on one special property of determinants.

$$E(x) = G a^2 \int_0^r J_0(xz) z I(z^2) dz$$

$\frac{d}{dt} \text{exit rate of pollutants at height } z \text{ due to fragmentation} = \text{exit rate of pollutants due to}$
 $(\alpha_f - f_w) + \frac{\partial}{\partial z} \text{leakage rate}$

$$E \circ ({}^s E) \circ E ({}^s x) = {}^s s_i x = (x) E$$

Chapter II

Direct Integration of Maxwell's Field Equations

2.1 The solution to Maxwell's equations given here is the solution developed by J. A. Stratton (1) and is contained in detail in his Electromagnetic Theory, sections 8.14 and 8.15.^{**} The following development has been modified slightly to better serve the purpose of this paper.

2.2 We shall postulate that at every point in space the electric and magnetic field vectors are subject to Maxwell's field equations. Further, let us assume that the field equations contain the time only as a factor $e^{+i\omega t}$ and write the field equations in the form:

$$\nabla \times \underline{E} - i\omega\mu \underline{H} = -\underline{J}^* \quad (2.01)$$

$$\nabla \times \underline{H} + i\omega \epsilon \underline{E} = \underline{J} \quad (2.02)$$

$$\nabla \cdot \underline{H} = \frac{1}{\mu} \rho^* \quad (2.03) (6)$$

$$\nabla \cdot \underline{E} = \frac{1}{\epsilon} \rho \quad (2.04)$$

where \underline{J}^* and ρ^* are fictitious densities of "magnetic current" and "magnetic charge" which to the best of our knowledge have no physical existence. Both the real and fictitious currents and charges are related by the equations of continuity

$$\nabla \cdot \underline{J} - i\omega \rho = 0 \quad (2.05)$$

$$\nabla \cdot \underline{J}^* - i\omega \rho^* = 0 \quad (2.06)$$

^{**} Stratton uses the expression e^{+ikR} for a positive traveling wave. This is a matter of controversy and the writer, in conformity with engineering practices, prefers to use e^{-ikR} .

Capitole II

environnements de l'écoulement des fluides

notamment est de ceux ayant plusieurs éléments de rotation soit 1.5
soit au statique et hydrostatique et basé (1) sur la loi de Bernoulli
et (2) sur la loi de conservation de l'énergie équation de Bernoulli
, et que soit le cas pour les deux types de fluides solides et liquides
notamment aux ensembles de forces qui agissent sur eux soit 2.5
l'ensemble d'effets de forces qui agissent sur eux soit l'ensemble des
forces qui agissent sur eux soit l'ensemble des forces qui agissent sur eux soit 3.5
l'ensemble d'effets de forces qui agissent sur eux soit l'ensemble des forces qui agissent sur eux soit 4.5

(10.1)

$$*\underline{I} = \underline{H} \omega i - \underline{\Sigma} \times \underline{\nabla}$$

(10.2)

$$\underline{I} = \underline{\Sigma} \underline{\nabla} \omega i + \underline{H} \times \underline{\nabla}$$

(3) (10.3)

$$*\underline{q} \frac{1}{\Delta} = \underline{H} \cdot \underline{\nabla}$$

(10.3)

$$\underline{q} \frac{1}{\Delta} = \underline{\Sigma} \cdot \underline{\nabla}$$

base "équation fondamentale" de mécanique classique soit $*\underline{q} = \underline{H}$ où \underline{H} est le
moment cinétique et \underline{q} est la force et base "équation fondamentale"
de mécanique quantique soit $\underline{q} = \underline{H}$ où \underline{H} est l'opérateur de
moment et \underline{q} est l'opérateur de position

(20.1)

$$\underline{o} = \underline{q} \omega i - \underline{I} \cdot \underline{\nabla}$$

(20.2)

$$\underline{o} = *\underline{q} \omega i - *\underline{I} \cdot \underline{\nabla}$$

et formule fondamentale soit $\underline{X} \frac{d}{dt} + \underline{\nabla} \times \underline{A} = \underline{q} \times \underline{B} + \underline{J} \times \underline{B}$
et formule fondamentale soit $\underline{X} \frac{d}{dt} + \underline{\nabla} \times \underline{A} = \underline{q} \times \underline{B} + \underline{J} \times \underline{B}$

which may be readily shown to be satisfied by the field equations by taking the divergence of (2.01) and (2.02).

2.3 The vectors \underline{E} and \underline{H} satisfy**

$$\nabla \times (\nabla \times \underline{E}) - k^2 \underline{E} = i \omega \mu \underline{J} - \nabla \times \underline{J}^* \quad (2.07)$$

$$\nabla \times (\nabla \times \underline{H}) - k^2 \underline{H} = i \omega \epsilon \underline{J}^* + \nabla \times \underline{J} \quad (2.08)$$

where $k^2 = \omega^2 \epsilon \mu$ since

$$\nabla \times \underline{E} - i \omega \mu \underline{H} = - \underline{J}^* \quad (2.01)$$

$$\nabla \times (\nabla \times \underline{E}) - i \omega \mu \nabla \times \underline{H} = - \nabla \times \underline{J}^*$$

$$\nabla \times (\nabla \times \underline{E}) - i \omega \mu (\underline{J} - i \omega \epsilon \underline{E}) = - \nabla \times \underline{J}^*$$

$$\nabla \times (\nabla \times \underline{E}) - \omega^2 \epsilon \mu \underline{E} = i \omega \mu \underline{J} - \nabla \times \underline{J}^* \quad (2.09)$$

and $\nabla \times \underline{H} + i \omega \epsilon \underline{E} = \underline{J} \quad (2.02)$

$$\nabla \times (\nabla \times \underline{H}) + i \omega \epsilon \nabla \times \underline{E} = \nabla \times \underline{J}$$

$$\nabla \times (\nabla \times \underline{H}) + i \omega \epsilon (- \underline{J}^* + i \omega \mu \underline{H}) = \nabla \times \underline{J}$$

$$\nabla \times (\nabla \times \underline{H}) - \omega^2 \epsilon \mu \underline{H} = i \omega \epsilon \underline{J}^* + \nabla \times \underline{J} \quad (2.10)$$

2.4 A direct proof of the desired result can be obtained by applying the vector analogue of Green's Theorem to the field equations. Let V be a closed region of space bounded by a regular surface S , and let \underline{P} and \underline{Q} be two vector functions of position which together with their first and second derivatives are continuous throughout V and on the surface S , then, applying the divergence theorem to the vector $\underline{P} \times (\nabla \times \underline{Q})$

$$\int_V \nabla \cdot [\underline{P} \times (\nabla \times \underline{Q})] dV = \int_S [\underline{P} \times (\nabla \times \underline{Q})] \cdot \underline{n} dA \quad (2.11)$$

** A table of vector identities is contained in Appendix II.

(SO.2) has (SO.2) to correspond with article
"Regulation II has 2 categories of" 3.8

$$(70.5) \quad {}^*T \times \nabla - T_{\nabla} \omega = \exists^s g - (\exists \times \nabla) \times \nabla$$

$$(20.c) \quad \nabla \times (\nabla \times \mathbf{B}) + \mu_0 \epsilon_0 \mathbf{E} = \mathbf{H}^{\text{ext}} - (\mathbf{H} \times \nabla) \times \mathbf{B}$$

Corla 143 " $\omega =$ " 6300 rpm

$$(10.8) \quad {}^*I = H_2(3) - \exists x \Delta$$

$$^*\underline{I} \times \underline{\nabla} - = \underline{H} \times \underline{\nabla} \text{ u w i } - (\underline{\exists} \times \underline{\nabla}) \times \underline{\nabla}$$

$$*\underline{I} \times \nabla = (\exists s \in \omega : \underline{I}) \wedge \omega s - (\exists x \nabla) \times \underline{\nabla}$$

$$(\mathbf{B}, \mathbf{S})^* \nabla \times \nabla - \nabla \times (\mathbf{B} \cdot \mathbf{S}) = \mathbf{E} \times \mathbf{B}^5 \mathbf{S} - (\mathbf{E} \times \nabla) \times \nabla$$

$$(\mathbf{E}, \mathbf{B}) = \frac{1}{c} \mathbf{E}_0 \sin(\omega t) + \frac{1}{c} \mathbf{B}_0 \cos(\omega t)$$

$$\underline{J} \times \underline{\nabla} = \underline{E} \times \underline{\nabla} - 3\omega i + (\underline{H} \times \underline{\nabla}) \times \underline{\nabla}$$

$$\overline{G} \times \nabla = (\underline{H} \times \nabla; +^* \underline{I}^-) \circ \nabla + (\underline{H} \times \nabla) \times \nabla$$

$$(0.1.1) \quad \underline{I} \times \nabla + \underline{I} \circ \underline{w} \circ \underline{s} = \underline{H} \circ \underline{A} \circ \underline{s}^5 w - (\underline{H} \times \nabla) \times \underline{\nabla}$$

Within the bounds of one district several sets of living fossils
and fossil groups have been found at different stages of evolution, and
at various points of geological time. In other words, it is
true that some of the rocks may be older than others, and
it appears that no two sets of fossils are of equal age.

$$(\Delta \times \nabla) = \frac{1}{2} \left(\nabla \times (\nabla \times \Delta) + \Delta \times (\nabla \times \nabla) \right)$$

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The integrand of the volume integral may be expanded to

$$\int_V [\nabla \times \underline{P} \cdot \nabla \times \underline{Q} - \underline{P} \cdot \nabla \times (\nabla \times \underline{Q})] dV = \int_S [\underline{P} \times (\nabla \times \underline{Q}) \cdot \underline{n} da \quad (2.12)$$

By simply interchanging \underline{P} and \underline{Q}

$$\int_V [\nabla \times \underline{Q} \cdot \nabla \times \underline{P} - \underline{Q} \cdot \nabla \times (\nabla \times \underline{P})] dV = \int_S [\underline{Q} \times (\nabla \times \underline{P}) \cdot \underline{n} da \quad (2.13)$$

and subtracting (2.13) from (2.12) we have:

$$\begin{aligned} & \int_V [\underline{Q} \cdot \nabla \times (\nabla \times \underline{P}) - \underline{P} \cdot \nabla \times (\nabla \times \underline{Q})] dV \\ &= \int_S [\underline{P} \times (\nabla \times \underline{Q}) - \underline{Q} \times (\nabla \times \underline{P})] \cdot \underline{n} da \end{aligned} \quad (2.14)$$

2.5 In equation (2.14)

let $\underline{P} \equiv \underline{\alpha}$

$\underline{Q} \equiv \phi \underline{n}$ where \underline{n} is a unit vector in an arbitrary direction and

$$\phi = \frac{e^{-ikr}}{r} ; \quad \nabla \phi = -\phi \left(ik + \frac{1}{r} \right) \underline{r}_0 \quad (2.15)$$

$$\nabla \cdot \nabla \phi = -k^2 \phi \quad (2.16)$$

We have the following identities:

$$(1) \quad \nabla \times \underline{Q} = \nabla \times \phi \underline{\alpha} = \nabla \phi \times \underline{\alpha} = \phi \nabla \times \underline{\alpha}$$

$$\therefore \nabla \times \underline{Q} = \nabla \phi \times \underline{\alpha} \quad \text{since } \nabla \times \underline{\alpha} = 0 \quad (2.17)$$

$$\begin{aligned} (2) \quad \nabla \times \nabla \times \underline{Q} &= \nabla \phi (\nabla \cdot \underline{\alpha}) - \underline{\alpha} (\nabla \cdot \nabla \phi) + (\underline{\alpha} \cdot \nabla) \nabla \phi - (\nabla \phi \cdot \nabla) \underline{\alpha} \\ &= \underline{\alpha} k^2 \phi + (\underline{\alpha} \cdot \nabla) \nabla \phi \end{aligned}$$

$$\nabla \cdot \underline{\alpha} = 0 \text{ and } (\nabla \phi \cdot \nabla) \underline{\alpha} = 0 \text{ since } \underline{\alpha} \text{ is a constant}$$

$$\begin{aligned} (\underline{\alpha} \cdot \nabla) \nabla \phi &= \nabla (\underline{\alpha} \cdot \nabla \phi) - (\nabla \phi \cdot \nabla) \underline{\alpha} + \underline{\alpha} \times (\nabla \times \nabla \phi) \\ &+ \nabla \phi \times (\nabla \times \underline{\alpha}) = \nabla (\underline{\alpha} \cdot \nabla \phi) \quad \text{since} \end{aligned}$$

as components of the Legendre tensor will be integrated out

$$(E.I.S) \quad \text{and } \underline{\underline{\omega}} \cdot (\underline{\underline{\theta}} \times \nabla) \times \underline{\underline{\nabla}} = \omega \left[(\underline{\underline{\theta}} \times \nabla) \times \nabla \cdot \underline{\underline{\theta}} - \underline{\underline{\theta}} \times \nabla \cdot \underline{\underline{\nabla}} \right] \quad \text{2. term I: magnetostatic quantity}$$

$$(E.I.S) \quad \text{and } \underline{\underline{\omega}} \cdot (\underline{\underline{\theta}} \times \nabla) \times \underline{\underline{\theta}} = \omega \left[(\underline{\underline{\theta}} \times \nabla) \times \nabla \cdot \underline{\underline{\theta}} - \underline{\underline{\theta}} \times \nabla \cdot \underline{\underline{\theta}} \right] \quad \text{2. term for (E.I.S) term (E.I.S) magnetostatic term}$$

$$\omega \left[(\underline{\underline{\theta}} \times \nabla) \times \nabla \cdot \underline{\underline{\theta}} - (\underline{\underline{\theta}} \times \nabla) \times \nabla \cdot \underline{\underline{\theta}} \right]$$

$$(M.I.S) \quad \text{and } \underline{\underline{\omega}} \cdot [(\underline{\underline{\theta}} \times \nabla) \times \underline{\underline{\theta}} - (\underline{\underline{\theta}} \times \nabla) \times \underline{\underline{\theta}}] =$$

(M.I.S) no longer in 2.2

$$\underline{\underline{\omega}} \equiv \underline{\underline{\theta}}$$

we can neglect the magnetic field in the air gap: $\Phi \equiv 0$

$$(E.I.S) \quad \underline{\underline{\omega}} \left(\frac{1}{r} + \frac{1}{R} \right) \Phi - \underline{\underline{\theta}} \nabla \cdot \underline{\underline{\theta}} = \underline{\underline{\theta}} \nabla \cdot \underline{\underline{\theta}} \quad ; \quad \frac{\underline{\underline{\theta}} \nabla \cdot \underline{\underline{\theta}}}{\underline{\underline{\theta}}} = \Phi$$

$$(E.I.S) \quad \underline{\underline{\theta}}^2 \underline{\underline{\theta}} = \underline{\underline{\theta}} \nabla \cdot \underline{\underline{\theta}} \cdot \nabla$$

so the magnetic field is zero in the air gap

$$\underline{\underline{\theta}} \times \nabla \Phi = \underline{\underline{\theta}} \times \underline{\underline{\theta}} \nabla = \underline{\underline{\theta}} \underline{\underline{\theta}} \times \nabla = \underline{\underline{\theta}} \times \nabla \quad (1)$$

$$(E.I.S) \quad 0 = \underline{\underline{\theta}} \times \nabla \cdot \underline{\underline{\theta}} \quad \text{since } \underline{\underline{\theta}} \times \underline{\underline{\theta}} \nabla = \underline{\underline{\theta}} \times \nabla \quad ;$$

$$\underline{\underline{\theta}} (\nabla \cdot \underline{\underline{\theta}}) - \underline{\underline{\theta}} \nabla (\nabla \cdot \underline{\underline{\theta}}) + (\underline{\underline{\theta}} \nabla \cdot \nabla) \underline{\underline{\theta}} - (\underline{\underline{\theta}} \cdot \nabla) \underline{\underline{\theta}} \nabla = \underline{\underline{\theta}} \times \nabla \times \nabla \quad (2)$$

$$\underline{\underline{\theta}} \nabla (\nabla \cdot \underline{\underline{\theta}}) + \underline{\underline{\theta}}^2 \underline{\underline{\theta}} =$$

$$0 = \underline{\underline{\theta}} (\nabla \cdot \underline{\underline{\theta}}) \quad \text{since } 0 = \underline{\underline{\theta}} \cdot \nabla$$

$$(\underline{\underline{\theta}} \nabla \times \nabla) \times \underline{\underline{\theta}} + \underline{\underline{\theta}} (\nabla \cdot \underline{\underline{\theta}}) - (\underline{\underline{\theta}} \nabla \cdot \underline{\underline{\theta}}) \nabla = \underline{\underline{\theta}} \nabla (\nabla \cdot \underline{\underline{\theta}})$$

$$+ \nabla \underline{\underline{\theta}} \times (\underline{\underline{\theta}} \cdot \nabla) = (\underline{\underline{\theta}} \times \nabla) \nabla +$$

$$(\nabla \phi \cdot \underline{\alpha}) \underline{\alpha} = 0, \quad \underline{\nabla} \times \underline{\nabla} \phi = 0, \quad \underline{\nabla} \times \underline{\alpha} = 0$$

$$\therefore \underline{\nabla} \times (\underline{\nabla} \times \underline{Q}) = \underline{\alpha} k^2 \phi + \underline{\nabla} (\underline{\alpha} \cdot \underline{\nabla} \phi) \quad (2.18)$$

$$(3) \quad (\underline{\nabla} \times \underline{P}) = (\underline{\nabla} \times \underline{E}) \quad (2.19)$$

$$(4) \quad \underline{\nabla} \times (\underline{\nabla} \times \underline{P}) = \underline{\nabla} \times (\underline{\nabla} \times \underline{E}) = k^2 \underline{E} + i \omega \mu \underline{J} - \underline{\nabla} \times \underline{J}^* \quad (2.20)$$

Substituting equations (2.17), (2.18), (2.19) and (2.20) in equation (2.14) we have:

$$\begin{aligned} & \int_V \left\{ \phi \underline{\alpha} \cdot \left[k^2 \underline{E} + i \omega \mu \underline{J} - \underline{\nabla} \times \underline{J}^* \right] - \underline{E} \cdot \left[\underline{\alpha} k^2 \phi + \underline{\nabla} (\underline{\alpha} \cdot \underline{\nabla} \phi) \right] \right\} dV \\ &= \int_S [\underline{E} \times (\underline{\nabla} \phi \times \underline{\alpha}) - \phi \underline{\alpha} \times (\underline{\nabla} \times \underline{E})] \cdot \underline{n} dA \end{aligned} \quad (2.21)$$

The integral over the volume may be written

$$\begin{aligned} & \underline{\alpha} \cdot \int_V (i \omega \mu \underline{J} \phi - \underline{\nabla} \times \underline{J}^* \phi) dV - \int_V \underline{E} \cdot \underline{\nabla} (\underline{\alpha} \cdot \underline{\nabla} \phi) dV \\ &= \underline{\alpha} \cdot \int_V (i \omega \mu \underline{J} \phi - \underline{\nabla} \times \underline{J}^* \phi) dV + \int_V (\underline{\alpha} \cdot \underline{\nabla} \phi) (\underline{\nabla} \cdot \underline{E}) dV \\ & - \int_V \underline{\nabla} \cdot [(\underline{\alpha} \cdot \underline{\nabla} \phi) \underline{E}] dV = \underline{\alpha} \cdot \int_V [i \omega \mu \underline{J} \phi - \underline{\nabla} \times \underline{J}^* \phi \\ & + \underline{\nabla} \phi (\underline{\nabla} \cdot \underline{E})] dV - \underline{\alpha} \cdot \int_S (\underline{E} \cdot \underline{n}) \underline{\nabla} \phi dA \end{aligned} \quad (2.22)$$

The integral over the surface in equation (2.20) may be written

$$\begin{aligned} & \int_S \left\{ [(\underline{\alpha} \cdot \underline{E})(\underline{\nabla} \phi \cdot \underline{n}) - (\underline{E} \cdot \underline{\nabla} \phi)(\underline{\alpha} \cdot \underline{n})] - \phi [(\underline{\alpha} \cdot \underline{E})(\underline{\nabla} \cdot \underline{n}) \right. \\ & \left. + (\underline{\alpha} \cdot \underline{\nabla})(\underline{E} \cdot \underline{n})] \right\} dA \\ &= \underline{\alpha} \cdot \int_S \left\{ (\underline{n} \cdot \underline{\nabla} \phi) \underline{E} - (\underline{E} \cdot \underline{\nabla} \phi) \underline{n} - \phi [(\underline{n} \cdot \underline{\nabla}) \underline{E} - (\underline{E} \cdot \underline{n}) \underline{\nabla}] \right\} dA \\ &= \underline{\alpha} \cdot \int_S \left\{ \underline{\nabla} \phi \times (\underline{E} \times \underline{n}) + \phi [\underline{n} \times (\underline{\nabla} \times \underline{E})] \right\} dA \\ &= \underline{\alpha} \cdot \int_S (\underline{n} \times \underline{E}) \times \underline{\nabla} \phi + \phi [\underline{n} \times (i \omega \mu \underline{H} - \underline{J}^*)] dA \end{aligned} \quad (2.23)$$

$$0 = \underline{\Phi} \times \underline{\nabla}, \quad 0 = \underline{\Phi} \nabla \times \underline{\nabla}, \quad 0 = \underline{\Phi} (\underline{\nabla} \cdot \underline{\Phi} \nabla)$$

$$(2.18) \quad (\underline{\Phi} \nabla \cdot \underline{\Phi}) \underline{\nabla} + \underline{\Phi}^s \nabla \underline{\Phi} = (\underline{\Phi} \times \underline{\nabla}) \times \underline{\nabla} \quad \therefore$$

$$(2.19) \quad (\underline{\Xi} \times \underline{\nabla}) = (\underline{\Phi} \times \underline{\nabla}) \quad (2)$$

$$(2.20) \quad * \underline{\Gamma} \times \underline{\nabla} - \underline{\Gamma} \mu \omega i + \underline{\Xi}^s \nabla = (\underline{\Xi} \times \underline{\nabla}) \times \underline{\nabla} = (\underline{\Phi} \times \underline{\nabla}) \times \underline{\nabla} \quad (2)$$

at (2.20) from (2.18), (2.19), (2.14) are same quantities

at (2.20) we have

$$\text{LHS} \left\{ \left[(\underline{\Phi} \nabla \cdot \underline{\Phi}) \underline{\nabla} + \underline{\Phi}^s \nabla \underline{\Phi} \right] \cdot \underline{\Xi} - \left[* \underline{\Gamma} \times \underline{\nabla} - \underline{\Gamma} \mu \omega i + \underline{\Xi}^s \nabla \right] \cdot (\underline{\Phi} \times \underline{\nabla}) \right\}$$

$$(2.21) \quad \text{LHS } \underline{\Xi} \cdot \left[(\underline{\Xi} \times \underline{\nabla}) \times \underline{\Phi} - (\underline{\Phi} \times \underline{\Phi} \nabla) \times \underline{\Xi} \right] =$$

the terms in the above equation will be same

$$\text{LHS } (\underline{\Phi} \nabla \cdot \underline{\Phi}) \underline{\nabla} \cdot \underline{\Xi} \left\} - \text{LHS } (\underline{\Phi} * \underline{\Gamma} \times \underline{\nabla} - \underline{\Phi} \underline{\Gamma} \mu \omega i) \right\} \cdot \underline{\Phi}$$

$$\text{LHS } (\underline{\Xi} \cdot \underline{\nabla}) (\underline{\Phi} \nabla \cdot \underline{\Phi}) \left\} + \text{LHS } (\underline{\Phi} * \underline{\Gamma} \times \underline{\nabla} - \underline{\Phi} \underline{\Gamma} \mu \omega i) \right\} \cdot \underline{\Phi} =$$

$$\underline{\Phi} * \underline{\Gamma} \times \underline{\nabla} - \underline{\Phi} \underline{\Gamma} \mu \omega i \left\} \cdot \underline{\Phi} = \text{LHS } [\underline{\Xi} (\underline{\Phi} \nabla \cdot \underline{\Phi})] \cdot \underline{\nabla} \left\} -$$

$$(2.22) \quad \text{LHS } \underline{\Phi} \nabla (\underline{\Xi} \cdot \underline{\Xi}) \cdot \underline{\Phi} - \text{LHS } [(\underline{\Xi} \cdot \underline{\nabla}) \underline{\Phi} \nabla +$$

$$(\underline{\Xi} \cdot \underline{\nabla}) (\underline{\Xi} \cdot \underline{\Phi})] \underline{\Phi} - \left\{ (\underline{\Xi} \cdot \underline{\Phi}) (\underline{\Phi} \nabla \cdot \underline{\Xi}) - (\underline{\Xi} \cdot \underline{\Phi} \nabla) (\underline{\Xi} \cdot \underline{\Phi}) \right\} \underline{\Phi}$$

$$\text{LHS } \left\{ [\underline{\Xi} \cdot \underline{\Xi}] (\underline{\nabla} \cdot \underline{\Phi}) + \right.$$

$$\text{LHS } \left\{ [\underline{\nabla} (\underline{\Xi} \cdot \underline{\Xi}) - \underline{\Xi} (\underline{\nabla} \cdot \underline{\Xi})] \underline{\Phi} - \underline{\Xi} (\underline{\Phi} \nabla \cdot \underline{\Xi}) - \underline{\Xi} (\underline{\Phi} \nabla \cdot \underline{\Xi}) \right\} \cdot \underline{\Phi} =$$

$$\text{LHS } \left\{ [(\underline{\Xi} \times \underline{\nabla}) \times \underline{\Xi}] \underline{\Phi} + (\underline{\Xi} \times \underline{\Xi}) \times \underline{\Phi} \nabla \right\} \cdot \underline{\Phi} =$$

$$(2.23) \quad \text{LHS } \left\{ [(* \underline{\Gamma} - \underline{\Gamma} \mu \omega i) \times \underline{\Xi}] \underline{\Phi} + \underline{\Phi} \nabla \times (\underline{\Xi} \times \underline{\Xi}) \right\} \cdot \underline{\Phi} =$$

Combining equations (2.22) and (2.23) and since \underline{n} is arbitrary

$$\int_V (i\omega \mu \underline{J}\phi - \nabla \times \underline{J}^* \phi + \nabla \phi \frac{1}{\epsilon} P) dv = \int_S [i\omega \mu (\underline{n} \times \underline{H}) \phi + (\underline{n} \times \underline{E}) \times \nabla \phi + (\underline{n} \cdot \underline{E}) \nabla \phi - \underline{n} \times \underline{J}^*] da \quad (2.24)$$

The identity

$$\begin{aligned} \int_V \nabla \times \phi \underline{J}^* dv &= \int_S \underline{n} \times \underline{J}^* \phi da \\ \int_V [(\nabla \times \underline{J}^*) \phi - \underline{J}^* \times \nabla \phi] dv &= \int_S \underline{n} \times \underline{J}^* \phi da \quad \text{or} \\ \int_V \nabla \times \underline{J}^* \phi dv &= \int_S \underline{n} \times \underline{J}^* \phi da + \int_V \underline{J}^* \times \nabla \phi dv \end{aligned}$$

reduces equation (2.24) to

$$\begin{aligned} \int_V (i\omega \mu \underline{J}\phi - \underline{J}^* \times \nabla \phi + \frac{1}{\epsilon} P \nabla \phi) dv &= \int_S [i\omega \mu (\underline{n} \times \underline{H}) \phi \\ + (\underline{n} \times \underline{E}) \times \nabla \phi + (\underline{n} \cdot \underline{E}) \nabla \phi] da \end{aligned} \quad (2.25)$$

2.6 The validity of this relation has been established for regions within which both $\underline{E} \equiv \underline{E}$ and $\underline{Q} \equiv \phi$ \underline{n} are continuous and possess continuous first and second derivatives.. \underline{Q} however, has a singularity at $r = 0$ and consequently this point must be excluded. Let x' , y' , z' be the coordinate of an interior point and let a sphere of radius r_1 be circumscribed about the point x' , y' , z' its normal, \underline{n} , directed out of V and consequently radially toward the center. The area of the sphere vanishes with the radius as $4\pi r_1^2$ and since $\nabla \phi = -\phi(i\mathbf{k} + \frac{1}{r}\mathbf{n}) \underline{r}_0 = \phi(i\mathbf{k} + \frac{1}{r}\mathbf{n}) \underline{m}$ and $(\underline{n} \times \underline{E}) \times \underline{m} + (\underline{n} \cdot \underline{E}) \underline{m} = \underline{E}$ the contribution of the spherical surface to the right hand side of equation (2.25) reduces to $4\pi E(x', y', z')$

The value of \underline{E} at any interior point of V is, therefore:

$$\begin{aligned} E(x', y', z') &= \frac{1}{4\pi} \int_V (i\omega \mu \underline{J}\phi - \underline{J}^* \times \nabla \phi + \frac{1}{\epsilon} P \nabla \phi) dv \\ &- \frac{1}{4\pi} \int_S [i\omega \mu (\underline{n} \times \underline{H}) \phi + (\underline{n} \times \underline{E}) \times \nabla \phi + (\underline{n} \cdot \underline{E}) \nabla \phi] da \end{aligned} \quad (2.26)$$

equation of motion has (35.4) has (35.5) another equation

$$(35.5) \quad \left. \begin{aligned} \phi(\underline{H} \times \underline{m}) \underline{\mu} \omega i \end{aligned} \right] = \text{rot} \left(\underline{q} \frac{1}{3} \underline{\phi} \nabla + \underline{\phi}^* \underline{I} \times \underline{\nabla} - \underline{\phi} \underline{I} \underline{\mu} \omega i \right)$$

$$\text{rot} \left[{}^* \underline{I} \times \underline{m} - \underline{\phi} \nabla (\underline{E} \cdot \underline{m}) + \underline{\phi} \nabla \times (\underline{E} \times \underline{m}) + \right]$$

transformed into

$$\text{rot} \left. \underline{\phi}^* \underline{I} \times \underline{m} \right] = \text{rot} \left. \underline{\phi}^* \underline{I} \underline{\phi} \times \underline{\nabla} \right]$$

$$\text{so } \text{rot} \left. \underline{\phi}^* \underline{I} \times \underline{m} \right] = \text{rot} \left. \left[\underline{\phi} \nabla \times {}^* \underline{I} - \underline{\phi} ({}^* \underline{I} \times \underline{\nabla}) \right] \right]$$

$$\text{rot} \left. \underline{\phi} \nabla \times {}^* \underline{I} \right] + \text{rot} \left. \underline{\phi}^* \underline{I} \times \underline{m} \right] = \text{rot} \left. \underline{\phi}^* \underline{I} \times \underline{\nabla} \right]$$

or (35.5) becomes another

$$(35.5) \quad \left. \begin{aligned} \phi(\underline{H} \times \underline{m}) \underline{\mu} \omega i \end{aligned} \right] = \text{rot} \left(\underline{\phi} \nabla \underline{q} \frac{1}{3} + \underline{\phi} \nabla \times {}^* \underline{I} - \underline{\phi} \underline{I} \underline{\mu} \omega i \right)$$

$$\text{rot} \left[\underline{\phi} \nabla (\underline{E} \cdot \underline{m}) + \underline{\phi} \nabla \times (\underline{E} \times \underline{m}) + \right]$$

now we have two equations of motion which is equivalent to equation (35.5)

we can express current density in terms of components of electric field and magnetic field as $\underline{J} = J_x \underline{i} + J_y \underline{j} + J_z \underline{k}$

it is possible to find a relation between current density and electric field

and J_x, J_y, J_z from definition of current density $J_x = \epsilon_0 E_x$

it is sufficient to establish a relation between current density and electric field

to do this we consider $\underline{J} = J_x \underline{i} + J_y \underline{j} + J_z \underline{k}$ and $\underline{E} = E_x \underline{i} + E_y \underline{j} + E_z \underline{k}$

and $\underline{B} = B_x \underline{i} + B_y \underline{j} + B_z \underline{k}$ and $\underline{H} = H_x \underline{i} + H_y \underline{j} + H_z \underline{k}$

$$\underline{J} = \underline{m} (\underline{E} \cdot \underline{m}) + \underline{m} \times (\underline{E} \times \underline{m})$$

$$\underline{J} = \underline{m} (J_x \underline{i} + J_y \underline{j} + J_z \underline{k}) = \underline{m} (E_x \underline{i} + E_y \underline{j} + E_z \underline{k})$$

is it possible to find a relation between current density and magnetic field

$(\delta, f, g, h) \underline{B} \nabla \underline{F}$ at equation (35.5) will be able to find a relation between current density and magnetic field

so we have $\underline{J} = J_x \underline{i} + J_y \underline{j} + J_z \underline{k}$ and $\underline{B} = B_x \underline{i} + B_y \underline{j} + B_z \underline{k}$

$$\text{rot} \left. \left(\underline{\phi} \nabla \underline{q} \frac{1}{3} + \underline{\phi} \nabla \times {}^* \underline{I} - \underline{\phi} \underline{I} \underline{\mu} \omega i \right) \right] \frac{1}{\pi \mu} = (\delta, f, g, h) \underline{B}$$

$$(35.5) \quad \text{rot} \left[\underline{\phi} \nabla (\underline{E} \cdot \underline{m}) + \underline{\phi} \nabla \times (\underline{E} \times \underline{m}) + \phi(\underline{H} \times \underline{m}) \underline{\mu} \omega i \right] \frac{1}{\pi \mu} -$$

An obvious interchange of vectors leads to the corresponding expression for \underline{H} .

$$\begin{aligned} \underline{H}(x',y',z') = & \frac{i}{4\pi} \int_V (i\omega \epsilon \underline{J}^* \phi + \underline{J} \times \underline{\nabla} \phi + \frac{1}{\mu} \rho^* \underline{\nabla} \phi) dV \\ & + \frac{i}{4\pi} \int_S [i\omega \epsilon (\underline{n} \times \underline{E}) \phi - (\underline{n} \times \underline{H}) \times \underline{\nabla} \phi - (\underline{n} \cdot \underline{H}) \underline{\nabla} \phi] da \quad (2.27) \end{aligned}$$

2.7 If all currents and charges can be enclosed within a sphere of finite radius, the field is regular at infinity and either side of S may be chosen as its interior.

2.8 Let us suppose now that the charge and current densities are confined to a thin layer at the surface S . As the depth of the layer diminishes, the densities may be increased so that in the limit the volume densities are replaced by surface densities. If the region V contains no charge or current within its interior or on its boundary S , the field at an interior point is

$$\underline{E}(x',y',z') = -\frac{i}{4\pi} \int_S [i\omega \epsilon (\underline{n} \times \underline{H}) \phi + (\underline{n} \times \underline{E}) \times \underline{\nabla} \phi + (\underline{n} \cdot \underline{E}) \underline{\nabla} \phi] da$$

and since either side of S may be chosen as its interior

$$\underline{E}(x) = -\frac{i}{4\pi} \int_S [i\omega \epsilon (\underline{n} \times \underline{H}) \phi + (\underline{n} \times \underline{E}) \times \underline{\nabla} \phi + (\underline{n} \cdot \underline{E}) \underline{\nabla} \phi] da \quad (2.28)$$

where $\underline{E}(x)$ is the electric field vector distribution out of the aperture.

2.9 In the case of radiation from an aperture the aperture will be defined as a finite circular area in space of radius "a" over which the source distribution of electromagnetic energy exists. The aperture is shielded in the direction of the negative normal by a perfectly absorbing screen of radius "a" through which electromagnetic energy may not pass. The area over the face of a parabolic reflector closely approximates this definition of an aperture.

2.10 Equation (2.26) holds true only if \underline{E} and \underline{H} are continuous and have continuous first derivatives at all points of S . It cannot, there-

referente la discussione dell'effetto magnetico su un campo magnetico assorbito da

• E tot

$$\text{vib} \left[\Phi \nabla^* q \frac{1}{\lambda} + \Phi \nabla \times \underline{H} + \Phi^* \underline{H} \times \underline{\omega} \right] \Big|_{\pi \frac{1}{4}} = (\underline{f}, \underline{f}, \underline{x}) H$$

$$(36.5) \quad \text{vib} \left[\Phi \nabla (H \cdot \underline{\omega}) - \Phi \nabla \times (H \times \underline{\omega}) - \Phi (\underline{E} \times \underline{\omega}) \times \underline{\omega} \right] \Big|_{\pi \frac{1}{4}} +$$

To consider a similar discussion of the magnetic field effect on the $\pi \frac{1}{4}$ term

then \underline{E} to this result has resulted in terms of the form of the effect of

the rotation of the medium on the motion of

the rotation of the medium on the motion of the medium on the motion of the medium

then the effect of the rotation of the medium on the motion of the medium

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at large rotation on

$$\text{vib} \left[\Phi \nabla (E \cdot \underline{\omega}) + \Phi \nabla \times (E \times \underline{\omega}) + \Phi (H \times \underline{\omega}) \times \underline{\omega} \right] \Big|_{\pi \frac{1}{4}} = (\underline{f}, \underline{f}, \underline{x}) \underline{E}$$

contrary to the results of the $\pi \frac{1}{4}$ term in this result

$$(36.6) \quad \text{vib} \left[\Phi \nabla (E \cdot \underline{\omega}) + \Phi \nabla \times (E \times \underline{\omega}) + \Phi (H \times \underline{\omega}) \times \underline{\omega} \right] \Big|_{\pi \frac{1}{4}} = (\underline{x}, \underline{x}) \underline{E}$$

contrary to the results of the $\pi \frac{1}{4}$ term in this result

then the effect of the rotation of the medium on the motion of the medium

then the effect of the rotation of the medium on the motion of the medium

then the effect of the rotation of the medium on the motion of the medium

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then the effect of the rotation of the medium on the motion of the medium

fore, be applied directly to the problem of radiation from an aperture. To obtain the required extension to such cases consider the closed surface S (closed at infinity) to be divided into two zones S_1 and S_2 by a closed contour C , as in Fig. 1.

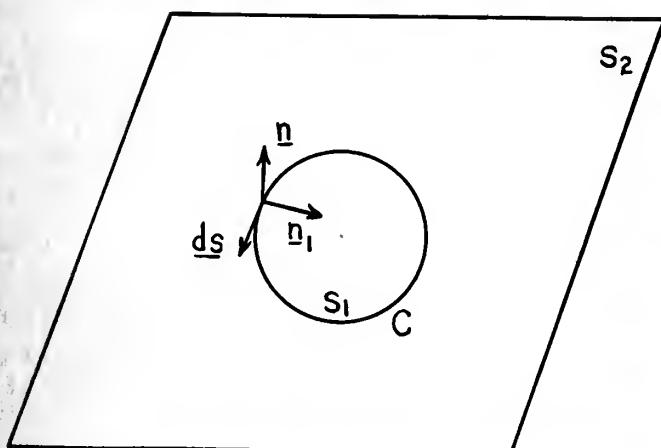


Figure 1

The vectors E and H and their first derivatives are continuous over S_1 and satisfy the field equations. The same is true for S_2 . However, the components of E and H which are tangential to the surface are subject to a discontinuous

change in passing across C . The occurrence of such discontinuities can be reconciled with the field equations only by the further assumption of a line distribution of charges or currents about the contour C . This line distribution of sources contributes to the field, and only when it is taken into account do the resultant expressions for E and H satisfy Maxwell's equations.

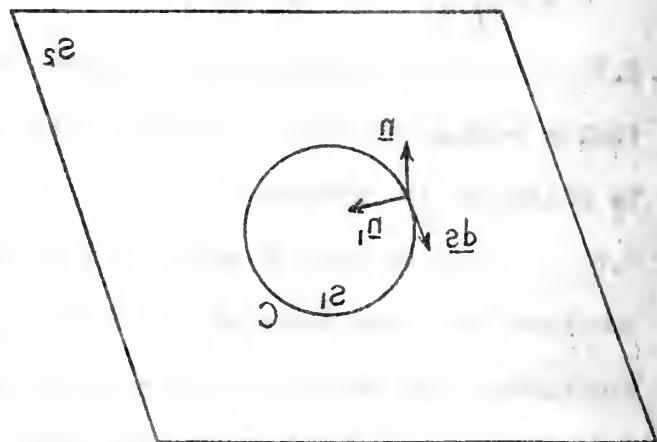
2.11 A method of determining a contour distribution consistent with the requirements of the problem was proposed by Kottler (5). A discontinuity in the tangential components of E and H in passing on the surface from zone S_1 to zone S_2 implies an abrupt change in the surface current density. The termination of a line of current, in turn, can be accounted for according to the equation of continuity by an accumulation of charge on the contour. Let ds be an element of length along the contour in the positive direction as determined by the positive normal n in Fig. 1. Let n_1 be a unit vector lying in the surface, normal to both n and ds and directed into zone 1. Designate the line densities of electric and mag-

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L'art et la culture dans le monde contemporain

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Editorial

ed nro collaudato si deve lo stesso est .3 questo giacere al quando
a lo collaudato restare ed se vno esempio belli ed che bastano
qui altri .3 questo ed tuona essere lo segnale lo collaudato dall'
si si soto vno buo ,bello ed di condizione scorsa lo collaudato
venerdì 11 luglio 1900

„softwars e' l'hardware

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-Eñoneth A .(d) roifell yd berogotc eñu mifdori eñ To efundoripet oñ
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sñt .I .ñt ak g Iñteri evitivoe oñt yd berogotc en ziftoñtib evitivoe
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-qñt han oñtñs oñt yd berogotc enñt oñt ejñemisno .I añt añt berogotc

netic charge by τ and τ^* . The equations (2.05) and 2.06), when applied to surface currents become

$$\underline{m}_1 \cdot (\underline{K}_1 - \underline{K}_2) = i\omega \tau; \quad \underline{m}_1 \cdot (\underline{K}_1^* - \underline{K}_2^*) = i\omega \tau^*$$

where K = electric current surface density

K^* = magnetic current surface density

$$\underline{K} = -\underline{n} \times \underline{H}; \quad \underline{K}^* = \underline{n} \times \underline{E}$$

Hence:

$$i\omega \tau = \underline{m}_1 \cdot (\underline{m} \times \underline{H}_2 - \underline{m} \times \underline{H}_1) = (\underline{H}_2 - \underline{H}_1) \cdot (\underline{m}_1 \times \underline{m})$$

$$i\omega \tau^* = \underline{m}_1 \cdot (\underline{m} \times \underline{E}_2 - \underline{m} \times \underline{E}_1) = -(\underline{E}_2 - \underline{E}_1) \cdot (\underline{m}_1 \times \underline{m})$$

$$\underline{m}_1 \times \underline{m} = \underline{ds} \quad (\text{Figure 1})$$

2.12 For radiation from an aperture S_2 represents an opaque screen and over it \underline{E}_2 and \underline{H}_2 are everywhere zero. Therefore, the field at any point on the shadow side is from equation (2.26)

$$\begin{aligned} \underline{E}(x) = & -\frac{1}{i\omega \epsilon} \frac{1}{4\pi} \oint_C \underline{\nabla} \phi (\underline{H}_1 \cdot \underline{ds}) - \frac{1}{4\pi} \int_S [i\omega \mu (\underline{m} \times \underline{H}_1) \phi \\ & + (\underline{m} \times \underline{E}_1) \times \underline{\nabla} \phi + (\underline{m} \cdot \underline{E}_1) \underline{\nabla} \phi] da \end{aligned} \quad (2.29)$$

and for the magnetic field vector distribution:

$$\begin{aligned} \underline{H}(x) = & \frac{1}{i\omega \mu} \frac{1}{4\pi} \oint_C \underline{\nabla} \phi \underline{E}_1 \cdot \underline{ds} + \frac{1}{4\pi} \int_S [i\omega \epsilon (\underline{m} \times \underline{E}_1) \phi \\ & - (\underline{m} \times \underline{H}_1) \times \underline{\nabla} \phi - (\underline{m} \cdot \underline{H}_1) \underline{\nabla} \phi] da \end{aligned} \quad (2.30)$$

Equations (2.29) and (2.30) will be used as the initial relations in the analytical developments which follow.

Defining matrix $(\underline{H}, \underline{v})$ as follows with \underline{v} being the vector of the second column of \underline{A} and \underline{H} being the vector of the first column of \underline{A}

$${}^* \nabla \omega_i = (\underline{\underline{H}} - {}^* \underline{\underline{A}}) \cdot \underline{v} \quad ; \quad \nabla \omega_i = (\underline{\underline{A}} - \underline{\underline{H}}) \cdot \underline{v}$$

Vertical column fractions of $\underline{\underline{H}}$ = 1 steady

Vertical column fractions of $\underline{\underline{A}}$ = ${}^* \underline{\underline{A}}$

$$\underline{\underline{A}} \times \underline{\underline{H}} = \underline{\underline{A}}$$

$$\underline{\underline{A}} \times \underline{\underline{A}} = \underline{\underline{A}}$$

Result

$$(\underline{v} \times \underline{v}) \cdot (\underline{\underline{H}} - {}^* \underline{\underline{A}}) = (\underline{\underline{H}} \times \underline{v} - {}^* \underline{\underline{A}} \times \underline{v}) \cdot \underline{v} = \nabla \omega_i$$

$$(\underline{v} \times \underline{v}) \cdot (\underline{\underline{A}} - \underline{\underline{H}}) = (\underline{\underline{A}} \times \underline{v} - \underline{\underline{H}} \times \underline{v}) \cdot \underline{v} = {}^* \nabla \omega_i$$

$$(1 \text{ steady}) \quad \underline{\underline{v}} = \underline{v} \times \underline{v}$$

It is clear that the components of \underline{v} are zero in most modelling now SI.0

that is built only "horizontal". Then steady-state can be set if two sum

(SI.0) \underline{H} remains zero at this we have not the

$$\Phi((\underline{\underline{H}} \times \underline{v}) \omega_i) \Big|_{\underline{v} = 0} = -(\underline{\underline{A}} \cdot \underline{\underline{H}}) \Phi \nabla \Big|_{\underline{v} = 0} = (\underline{\underline{A}} \cdot \underline{\underline{H}})$$

(SI.0)

$$\text{and } [\Phi \nabla (\underline{\underline{A}} \cdot \underline{v}) + \Phi \nabla \times (\underline{\underline{A}} \times \underline{v}) +$$

horizontal component of vertical flow after this we have

$$\Phi((\underline{\underline{A}} \times \underline{v}) \omega_i) \Big|_{\underline{v} = 0} = \underline{\underline{A}} \cdot \underline{\underline{A}} \Phi \nabla \Big|_{\underline{v} = 0} = (\underline{\underline{A}} \cdot \underline{\underline{A}})$$

(SI.0)

$$\text{and } [\Phi \nabla (\underline{\underline{H}} \cdot \underline{v}) - \Phi \nabla \times (\underline{\underline{H}} \times \underline{v}) -$$

with the additional condition that the flow of (SI.0) \rightarrow (SI.0) and (SI.0)

not to build up a flow from (SI.0) to (SI.0)

Chapter III

Determination of the Distant Field

3.1 It has been shown by Stratton and Chu (Chapter I of this paper) that the electric field vector resulting from radiation through an aperture may be expressed as

$$4\pi \underline{E}(x) = -\frac{i}{\omega \epsilon} \oint_C \underline{\nabla} \phi (\underline{H}_1 \cdot d\underline{s}) - \int_S [i\omega \mu (\underline{n} \times \underline{H}_1) \phi + (\underline{n} \times \underline{E}_1) \times \underline{\nabla} \phi + (\underline{n} \cdot \underline{E}_1) \underline{\nabla} \phi] d\underline{a} \quad (3.01)^*$$

The subscript 1 in this expression refers to area 1 (area over the aperture). Since \underline{E} and \underline{H} are zero throughout area 2 (area in plane of aperture outside the aperture) \underline{E}_1 and \underline{H}_1 will simply be written as \underline{E} and \underline{H} . In addition, the development will be confined to circularly symmetric, plane polarized \underline{E} and \underline{H} distributions so that $\underline{E} = \underline{E}(\hat{\rho}, a)$; $\underline{H} = \underline{H}(\hat{\rho}, a)$.

3.2 The purpose of solving equation (3.01) is to determine the vector and range characteristics of $\underline{E}(x)$ resulting from \underline{E} and \underline{H} distributions as assumed above.. This end will be met most easily by assuming a constant amplitude distribution over the aperture and this will be done.

At present, however, for development purposes let

$$\underline{E} = \underline{I}(\hat{\rho}, a) \underline{i} \quad \text{and}$$

$$\underline{H} = \underline{M}(\hat{\rho}, a) \underline{j}$$

and let the aperture lie in the \hat{x} , \hat{y} , plane such that $\underline{n} = \underline{k}$, Fig. 2, page .

*The time factor $e^{+i\omega t}$ is understood to be present in this and similar expressions. It will later be included in the \underline{G} and \underline{F} functions. \underline{E} and \underline{H} are in time phase over the aperture.

III *Cont.*

black contacts set to no transmission

Chapitre I On the basis of the above analysis we can conclude that the main factor influencing the choice of the model is the type of the target market.

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मैंने अपनी प्राचीनता को बहुत बढ़ावा दी है।

$$** \text{ (10.3)} \quad \text{def} \left[\underline{\phi\nabla} (\underline{A} \cdot \underline{m}) + \underline{\phi\nabla} \times (\underline{A} \times \underline{m}) + \right]$$

to a meeting planned for the 1st, November, 1940, at the

$$b_{NG} \quad i(\hat{e}, \hat{g}) I = \pi$$

$$1 + (-6 - 9) \Sigma = T$$

Figure 2 illustrates the relationship between the number of observations and the estimated probability of detection.

三

With all thanks and best regards from all three I make off
-and I hope you will be well and happy always

3.3 In equation (3.01) consider

$$\oint_C \nabla \phi (\underline{H} \cdot d\underline{s})$$

Note that for $\underline{H} = M(\hat{\rho}, a)$ \underline{j} and $d\underline{s}$ in the \hat{x}, \hat{y} , plane $\underline{H} \cdot d\underline{s}$ will cancel in pairs about the contour C. Hence,

$$\oint_C \nabla \phi (\underline{H} \cdot d\underline{s}) = 0 \quad \text{and}$$

$$4\pi E(x) = - \int_S [i\omega\mu(\underline{n} \times \underline{H})\phi + (\underline{n} \times \underline{E}) \times \nabla \phi + (\underline{n} \cdot \underline{E}) \nabla \phi] da \quad (3.02)$$

Performing the indicated vector multiplication:

$$i\omega\mu(\underline{n} \times \underline{H})\phi = i\omega\mu [\underline{k} \times M(\hat{\rho}, a) \underline{j}] = -i\omega\mu M(\hat{\rho}, a) \underline{i}$$

$$\text{and since } \underline{H} = \frac{\underline{E}}{\eta} \quad , \quad \eta = \sqrt{\frac{\mu}{\epsilon}} \quad , \quad c = \sqrt{\frac{1}{\mu\epsilon}} = f\lambda, \quad \underline{k} = \frac{2\pi}{\lambda}$$

we may write

$$i\omega\mu(\underline{n} \times \underline{H})\phi = -i\omega\mu M(\hat{\rho}, a) \underline{i} = -\frac{i\omega\mu}{\eta} I(\hat{\rho}, a) \underline{i} \quad (3.03)$$

$$= -\frac{i\omega}{\sqrt{\mu\epsilon}} I(\hat{\rho}, a) \underline{i} = -i\underline{k} I(\hat{\rho}, a) \underline{i}$$

$$\begin{aligned} \text{and } (\underline{n} \times \underline{E}) \times \nabla \phi &= [\underline{k} \times I(\hat{\rho}, a) \underline{i}] \times \nabla \phi \\ &= I(\hat{\rho}, a) [\underline{j} \times \nabla \phi] \end{aligned}$$

tablasas (10.2) maitaups al

3.3

$$(\underline{\omega} \cdot \underline{H}) \underline{\phi} \nabla \phi$$

Iesnos IIIw ab . H emalq , \hat{x} , \hat{y} odi nt sh has \hat{z} (ω, \hat{q}) $M = H$ tot fadz ezoN
, sonell . O iiesnos odi jweda ering nt

$$\text{ban } 0 = (\underline{\omega} \cdot \underline{H}) \underline{\phi} \nabla \phi$$

$$(30.2) \text{ uab } \left[\underline{\phi} \nabla (\underline{E} \cdot \underline{m}) + \underline{\phi} \nabla \times (\underline{E} \times \underline{m}) + \underline{\phi} (\underline{H} \times \underline{m}) \text{ uwi } \right] = (\underline{x}) \underline{E} \pi +$$

:maitaups rojew heftaibni est jwimotibni

$$\underline{i}(\omega, \hat{q}) M \text{ uwi} - = \left[\underline{i}(\omega, \hat{q}) M \times \underline{s} \right] \text{ uwi} = \underline{\phi} (\underline{H} \times \underline{m}) \text{ uwi}$$

$$\frac{\pi s}{\pi} = \underline{s} , \underline{A} = \frac{1}{3\pi} \underline{V} = \underline{o} , \frac{\underline{m}}{3} \underline{V} = \underline{r} , \frac{\underline{E}}{\pi} = \underline{H} \text{ conte has}$$

efirw yam en

$$(30.2) \underline{i}(\omega, \hat{q}) I \frac{\text{uwi}}{\pi} - = \underline{i}(\omega, \hat{q}) M \text{ uwi} - = \underline{\phi} (\underline{H} \times \underline{m}) \text{ uwi}$$

$$\underline{i}(\omega, \hat{q}) I \underline{s} - = \underline{i}(\omega, \hat{q}) I \frac{\underline{m}}{3\pi} - =$$

$$\underline{\phi} \nabla \times \left[\underline{i}(\omega, \hat{q}) I \times \underline{s} \right] = \underline{\phi} \nabla \times (\underline{E} \times \underline{m}) \text{ bns}$$

$$[\underline{\phi} \nabla \times \underline{i}] (\omega, \hat{q}) I =$$

$$\nabla \phi = \frac{\partial \phi}{\partial r} \underline{r}_o = -\frac{e^{-ikr}}{r} (ik + \frac{1}{ikr}) \underline{r}_o = -ik\phi \left(1 + \frac{1}{ikr}\right) \underline{r}_o$$

$$(\underline{m} \times \underline{E}) \times \nabla \phi = -ik\phi I(\hat{P}, a) \left[1 + \frac{1}{ikr}\right] [\underline{k} \times \underline{r}_o]$$

$$\underline{r}_o = \sin \theta \cos \varphi \underline{i} + \sin \theta \sin \varphi \underline{j} + \cos \theta \underline{k}$$

$$\underline{k} \times \underline{r}_o = \cos \theta \underline{i} - \sin \theta \cos \varphi \underline{k} \quad \text{and}$$

$$(\underline{m} \times \underline{E}) \times \nabla \phi = -ik\phi I(\hat{P}, a) \left(1 + \frac{1}{ikr}\right) (\cos \theta \underline{i} - \sin \theta \cos \varphi \underline{k}) \quad (3.04)$$

$$(\underline{m} \cdot \underline{E}) \nabla \phi = I(\hat{P}, a) \left[\underline{k} \cdot \underline{i} \right] \nabla \phi = 0 \quad (3.05)$$

Substituting equations (3.03), (3.04) and (3.05) in equation (3.02)

we have:

$$4\pi E(x) = ik \int_S \phi I(\hat{P}, a) \left[\underline{i} + \left(1 + \frac{1}{ikr}\right) (\cos \theta \underline{i} - \sin \theta \cos \varphi \underline{k}) \right] da \quad (3.06)$$

For the distant field, $r \ggg 1$ and $\left(1 + \frac{1}{ikr}\right) \rightarrow 1$

$$\text{hence } 4\pi E(x) = ik \left[(1 + \cos \theta) \underline{i} - \sin \theta \cos \varphi \underline{k} \right] \int_S \phi I(\hat{P}, a) da \quad (3.07)$$

$$= \frac{2\pi i}{\lambda} \left[(1 + \cos \theta) \underline{i} - \sin \theta \cos \varphi \underline{k} \right] \int_S \frac{e^{-ikr}}{r} I(\hat{P}, a) da \quad (3.08)$$

3.4 From the geometric relations derived in Fig. 2

$$da = \hat{P} d\hat{P} d\hat{\phi}$$

$$r = R - \hat{P} \sin \theta \cos(\hat{\phi} - \varphi). \text{ In this expression } R \ggg \hat{P} \sin \theta \cos(\hat{\phi} - \varphi)$$

$$\underline{\underline{u}} \cdot \left(\frac{1}{\sqrt{3}} + i \right) \underline{\underline{s}} = \underline{\underline{u}} \cdot \left(\frac{1}{\sqrt{3}} + i \right) \stackrel{\text{def}}{=} -\frac{1}{\sqrt{3}} \underline{\underline{e}}_z = \underline{\underline{u}} \cdot \frac{\partial}{\partial z} = \Phi \nabla$$

$$\left[\underline{\underline{u}} \times \underline{\underline{t}} \right] \left[\left(\frac{1}{\sqrt{3}} + i \right) \underline{\underline{u}} \cdot \underline{\underline{t}} \right] \underline{\underline{s}} = \Phi \nabla \times (\underline{\underline{u}} \times \underline{\underline{u}})$$

$$(\underline{\underline{u}} \cdot \underline{\underline{u}}) + i \underline{\underline{u}} \times \underline{\underline{u}} = \underline{\underline{u}} \cdot \underline{\underline{u}} + i \underline{\underline{u}} \times \underline{\underline{u}} = \underline{\underline{u}}$$

$$\underline{\underline{u}} \cdot \underline{\underline{u}} = \cos \theta \cos \theta \sin \theta + i \sin \theta \cos \theta \sin \theta = \underline{\underline{u}} \cdot \underline{\underline{u}}$$

$$(\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} + i \underline{\underline{u}} \times \underline{\underline{u}} = \Phi \nabla \times (\underline{\underline{u}} \times \underline{\underline{u}})$$

$$u = \Phi \nabla \left[\underline{\underline{u}} \cdot \underline{\underline{u}} \right] (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} = \Phi \nabla (\underline{\underline{u}} \cdot \underline{\underline{u}})$$

$$u = (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} + i \left[\underline{\underline{u}} \times \underline{\underline{u}} \right] \underline{\underline{u}} = (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} + i \underline{\underline{u}} \times \underline{\underline{u}}$$

$$u = (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} + i \left[\underline{\underline{u}} \times \underline{\underline{u}} - (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} \right] \underline{\underline{u}} = (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} + i \underline{\underline{u}} \times \underline{\underline{u}}$$

$$u = (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} + i \left[\left[\underline{\underline{u}} \times \underline{\underline{u}} - (\underline{\underline{u}} \cdot \underline{\underline{u}}) \underline{\underline{u}} \right] \frac{\partial}{\partial u} \right] \underline{\underline{u}} =$$

$$\hat{u} + \hat{q}(\hat{u}) = u$$

$$(p - q) \cos \theta \sin \theta \{ \cos^2 \theta \cos^2 \alpha + \sin^2 \theta \cos^2 \theta \sin^2 \alpha - \{ 1 \cos^2 \theta \sin^2 \theta - \theta \sin^2 \alpha \}$$

and we may substitute $r = R$ in the denominator of equation (3.08).

$$\text{This gives } E(x) = \frac{2\pi i}{4\pi\lambda} \left\{ [1 + \cos] \underline{i} - \sin \theta \cos \varphi \underline{k} \right\} \left\{ \right.$$

$$\left. \frac{e^{-ikR}}{R} \int_S I(\hat{P}, a) e^{ik\hat{P} \sin \theta \cos(\hat{\phi} - \varphi)} \right\} \hat{P} d\hat{P} d\hat{\phi}$$

$$= \frac{G}{2\pi} \int_{\hat{P}=0}^a \int_0^{2\pi} I(\hat{P}, a) e^{ik\hat{P} \sin \theta \cos(\hat{\phi} - \varphi)} \hat{P} d\hat{P} d\hat{\phi} \quad (3.09)$$

$$\text{where } G = i \frac{\pi e^{+i(wt - kR)}}{\lambda R} \left[(1 + \cos \theta) \underline{i} - \sin \theta \cos \varphi \underline{k} \right] \quad (3.10)$$

3.5 Now, since φ is independent of $\hat{\phi}$ we may replace $d\hat{\phi}$ by

$d(\hat{\phi} - \varphi)$ in equation (3.09). Also let $ik\hat{P} \sin \theta = z$

Equation (3.09) becomes

$$E(x) = \frac{G}{2\pi} \int_{\hat{P}=0}^a I(\hat{P}, a) \hat{P} d\hat{P} \int_0^{2\pi} e^{iz \cos(\hat{\phi} - \varphi)} d(\hat{\phi} - \varphi)$$

From Jahnke and Emde (7), page 149

$$J_m(z) = \frac{i^{-m}}{2\pi} \int_0^{2\pi} e^{iz \cos \varphi} e^{im\varphi} d\varphi \quad \text{OR}$$

$$2\pi J_0(z) = \int_0^{2\pi} e^{iz \cos \varphi} d\varphi$$

$$\begin{aligned} \therefore E(x) &= \frac{G}{2\pi} \int_{\hat{P}=0}^a I(\hat{P}, a) \hat{P} 2\pi J_0(k\hat{P} \sin \theta) d\hat{P} \\ &= G \int_0^a I(\hat{P}, a) \hat{P} J_0(k\hat{P} \sin \theta) d\hat{P} \end{aligned} \quad (3.11)$$

3.6 To determine the general form of $E(x)$ let $I(\hat{P}, a) = \bar{I}$

i.e. constant amplitude distribution over the aperture then

$$\begin{aligned} E(x) &= G \bar{I} \int_0^a \hat{P} J_0(k\hat{P} \sin \theta) d\hat{P} \\ &= G \bar{I} \int_0^a \frac{k\hat{P} \sin \theta}{k^2 \sin^2 \theta} J_0(k\hat{P} \sin \theta) d(k\hat{P} \sin \theta) \end{aligned}$$

$$\left\{ \left[\pm q_{200} \theta \sin \alpha - i [200+1] \right] \right\} \frac{i \pi s}{\lambda \pi \Delta} = (3) \exists$$

$$\left(\frac{\partial}{\partial \theta} \left[\frac{\partial}{\partial \theta} \left\{ (q-\hat{q})_{200} \theta \sin \alpha \right\} \right] \right) \frac{\partial \alpha}{\partial \theta}$$

$$\left(\frac{\partial}{\partial \theta} \left[\frac{\partial}{\partial \theta} \left\{ (q-\hat{q})_{200} \theta \sin \alpha \right\} \right] \right) \frac{\partial \alpha}{\partial \theta} =$$

$$\left[\left(q_{200} \theta \sin \alpha - i [200+1] \right) \right] \frac{(q\hat{q}-300)}{\pi \Delta} =$$

$$q_{200} \theta \sin \alpha - i [200+1] =$$

$$S = \theta \sin^2(\theta) = (q-\hat{q}) \Delta$$

$$(q-\hat{q}) \Delta (q-\hat{q})_{200} \sin^2 \theta \left\{ \left(\frac{\partial \alpha}{\partial \theta} \right) \right\} \frac{\partial \alpha}{\partial \theta} = (3) \exists$$

$$q_{200} \theta \sin^2 \theta + 200 \sin^2 \theta \int \frac{\partial \alpha}{\partial \theta} d\theta = (S) \sin \theta$$

$$q_{200} \theta \sin^2 \theta \int \frac{\partial \alpha}{\partial \theta} d\theta = (S) \sin \theta$$

$$\left(q_{200} \theta \sin \alpha \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \frac{\partial \alpha}{\partial \theta} = (3) \exists$$

$$\left(q_{200} \theta \sin \alpha \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \frac{\partial \alpha}{\partial \theta} =$$

$$\bar{T} = (S, \bar{A}) I$$

$$\left(q_{200} \theta \sin \alpha \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \bar{T} \bar{A} = (3) \exists$$

$$\left(q_{200} \theta \sin \alpha \right) \left(\frac{\partial \alpha}{\partial \theta} \right) \bar{T} \bar{A} =$$

$$\underline{E}(x) = \frac{G \bar{I} k \hat{P} \sin \theta J_1(k \hat{P} \sin \theta)}{(k \sin \theta)^2} \Bigg|_{\begin{array}{l} \hat{P}=a \\ \hat{P}=0 \end{array}} \quad (7)$$

$$= \frac{G \bar{I} a J_1(k a \sin \theta)}{k \sin \theta} = \frac{G \bar{I} a^2}{k a \sin \theta} J_1(k a \sin \theta)$$

let $x = k a \sin \theta$ and

$$\underline{E}(x) = \frac{G \bar{I} a^2}{x} J_1(x) \quad (3.12)$$

3.7 Similarly, using equation (2.30) we may solve for the magnetic field vector distribution in distant space.

From Stratton $4\pi \underline{H}(x) = \frac{i}{i\omega\mu} \oint_C \nabla \phi \underline{E} \cdot d\underline{s} + \int_S [i\omega \epsilon (\underline{n} \times \underline{E}) \phi - (\underline{n} \times \underline{H}) \times \nabla \phi - (\underline{n} \cdot \underline{H}) \nabla \phi] da$ (3.13)

Again, since $\underline{E} = I(\hat{P}, a) \underline{i}$ and $d\underline{s}$ lies in the x, y , plane

$$\oint_C \nabla \phi \underline{E} \cdot d\underline{s} = 0 \quad \text{and}$$

$$4\pi \underline{H}(x) = \int_S [i\omega \epsilon (\underline{n} \times \underline{E}) \phi - (\underline{n} \times \underline{H}) \times \nabla \phi - (\underline{n} \cdot \underline{H}) \nabla \phi] da \quad (3.14)$$

$$i\omega \epsilon (\underline{n} \times \underline{E}) \phi = i\omega \epsilon \eta M(\hat{P}, a) \phi \left(1 + \frac{1}{ikr}\right) \underline{i} \times \underline{r}_0$$

$$= ik M(\hat{P}, a) \phi \left(1 + \frac{1}{ikr}\right) [\sin \theta \sin \varphi \underline{k} - \cos \theta \underline{z}]$$

$$(\underline{n} \cdot \underline{H}) \nabla \phi = 0, \quad \text{Again for } r \ggg 1, \quad \left(1 + \frac{1}{ikr}\right) \rightarrow 1$$

$$4\pi \underline{H}(x) = \int_S ik \phi M(\hat{P}, a) [\underline{j} + \cos \theta \underline{j} - \sin \theta \sin \varphi \underline{k}] da$$

$$H(x) = \frac{ik}{4\pi} \int_S \frac{M(\hat{P}, a) e^{-ikr}}{r} [(1 + \cos \theta) \underline{j} - \sin \theta \sin \varphi \underline{k}] da$$

$$= \frac{2\pi i}{4\pi \lambda} \frac{e^{-ikR}}{R} [(1 + \cos \theta) \underline{j} - \sin \theta \sin \varphi \underline{k}] \int_S M(\hat{P}, a) e^{ik\hat{P} \sin \theta \cos(\hat{\phi} - \phi)} \hat{P} d\hat{P} d\hat{\phi}$$

$$= \frac{E}{2\pi} \int_S M(\hat{P}, a) e^{ik\hat{P} \sin \theta \cos(\hat{\phi} - \phi)} \hat{P} d\hat{P} d\hat{\phi} \quad (3.15)$$

$$\text{where } \underline{E} = \frac{i\pi e^{+i(\omega t - kR)}}{\lambda R} [(1 + \cos \theta) \underline{j} - \sin \theta \sin \varphi \underline{k}] \quad (3.16)$$

(r)

$$\theta = \hat{\varphi} \quad \left| \frac{(\theta \sin \hat{\varphi}) \bar{J}_0 \theta \sin \hat{\varphi} \bar{I}_0}{\sin (\theta \sin \hat{\varphi})} = (x) \right. \exists$$

$$\left(\frac{\theta \sin \hat{\varphi}}{\sin \theta \sin \hat{\varphi}} \bar{J}_0 \bar{I}_0 \right) = \left(\frac{\theta \sin \hat{\varphi}}{\sin \theta \sin \hat{\varphi}} \bar{J}_0 \bar{I}_0 \right) =$$

(SI.5)

$$\text{bns } \theta \sin \hat{\varphi} = x \quad \text{tef} \\ \left(\frac{x}{x} \bar{J}_0 \bar{I}_0 \right) = (x) \exists$$

oitemyam eft tot avele van en (SI.5) volgtwaer gelaan, vrystelling 7.5

soekj jinselik ni volgtwaer tot osoen hieft

$$- \Phi(\underline{E} \times \underline{m}) \underline{z} \omega_i \Big] \Big] + \underline{e} \underline{J}_0 \cdot \underline{E} \underline{\Phi} \nabla \Bigg\} \frac{1}{4\pi i} = (x) H \pi \neq \text{notwendig wort}$$

(SI.6)

$$\text{vab} \left[\underline{\Phi} \nabla (H \cdot m) - \underline{\Phi} \nabla \times (H \times m) \right]$$

ensoek, $\underline{e}, \underline{x}, \underline{z}$ eft al self nh was $\underline{z}(\underline{e}, \underline{g}) = \underline{I}$ souke, vrysta

$$\text{bns} \quad 0 \equiv \underline{e} \underline{J}_0 \cdot \underline{E} \underline{\Phi} \nabla \Bigg\}$$

(SI.7)

$$\text{vab} \left[\underline{\Phi} \nabla (H \cdot m) - \underline{\Phi} \nabla \times (H \times m) - \Phi(\underline{E} \times \underline{m}) \underline{z} \omega_i \right] \Big] = (x) H \pi \neq$$

$$\underline{m} \times \underline{z} \left(\frac{1}{4\pi i} + 1 \right) \Phi(\underline{e}, \underline{g}) M \times \underline{z} \omega_i = \Phi(\underline{E} \times \underline{m}) \underline{z} \omega_i$$

$$\left[\frac{1}{4} \theta \cos - \frac{1}{2} \Phi \sin \theta \sin \right] \left(\frac{1}{4\pi i} + 1 \right) \Phi(\underline{e}, \underline{g}) M \& i =$$

$$1 \leftarrow \left(\frac{1}{4\pi i} + 1 \right), i << \& \text{ tot nispa}, c = \underline{\Phi} \nabla (H \cdot m)$$

$$\text{vab} \left[\frac{1}{4} \Phi \sin \theta \sin - \frac{1}{2} \theta \cos + \frac{1}{2} \right] (\underline{e}, \underline{g}) M \& i = (x) H \pi \neq$$

$$\text{vab} \left[\frac{1}{2} \Phi \sin \theta \sin - \frac{1}{2} (\theta \cos + 1) \right] \frac{\frac{1}{4\pi i} - \frac{1}{2} (\underline{e}, \underline{g}) M}{\pi} \frac{i}{\pi \neq} = (x) H$$

$$\hat{e} \hat{g} \hat{e} \hat{g} \hat{e} \hat{g} (\Phi - \hat{\Phi}) \cos \theta \sin \hat{g} \& i \left[(\underline{e}, \underline{g}) M \right] \left[\frac{1}{4} \Phi \sin \theta \sin - \frac{1}{2} (\theta \cos + 1) \right] \frac{\frac{1}{4\pi i} - \frac{1}{2}}{\pi} \frac{i \pi \&}{\pi \neq} =$$

(SI.8)

$$\hat{e} \hat{g} \hat{e} \hat{g} \hat{e} \hat{g} (\Phi - \hat{\Phi}) \cos \theta \sin \hat{g} \& i \left[(\underline{e}, \underline{g}) M \right] \frac{\pi}{\pi \neq} =$$

(SI.9)

$$\left[\frac{1}{2} \Phi \sin \theta \sin - \frac{1}{2} (\theta \cos + 1) \right] \frac{(\pi \& - \pi \&) i + \pi \& i}{\pi \&} = \underline{z} \text{ osoek}$$

Equation (3.15) is identical in form to equation (3.09) and

$$\underline{H}(x) = \underline{F} \int_0^a M(\hat{P}, a) \hat{P} J_0(k \hat{P} \sin \theta) d\hat{P} \quad (3.17)$$

$$\text{For } M(\hat{P}, a) = \bar{M} ; \underline{H}(x) = \underline{F} \bar{M} a^2 \frac{J_1(x)}{x} \quad (3.18)$$

3.8 The solutions for $\underline{E}(x)$ and $\underline{H}(x)$ as they appear in equations (3.12) and (3.18) are the most useful forms in this analysis but to verify that they are correct in practice one must use the expression for the energy of the distant field since this energy is all that may be measured.

$$3.9 \quad P = \frac{1}{2} \underline{E} \times \bar{\underline{H}} \quad (3.19)$$

where $\bar{\underline{H}}$ indicates complex conjugate of \underline{H}

$$\bar{\underline{H}} = -\frac{i k e}{R \lambda} e^{-i(\omega t - k R)} \bar{M} \left[(1 + \cos \theta) \underline{j} - \sin \theta \sin \varphi \underline{k} \right] \quad (3.20)$$

$$\begin{aligned} \underline{P} &= -\frac{1}{2} \frac{(i)^2 k^2}{4R^2 \lambda^2} \bar{I} \bar{M} \left[a^2 \frac{J_1(x)}{x} \right]^2 \left[\left\{ (1 + \cos \theta) \underline{i} - \right. \right. \\ &\quad \left. \left. \sin \theta \cos \varphi \underline{k} \right\} \times \left\{ (1 + \cos \theta) \underline{j} - \sin \theta \sin \varphi \underline{k} \right\} \right. \\ &= \frac{1}{2} \frac{4\pi^2}{\lambda^2} \frac{\bar{I} \bar{M}}{4R^2} \left[a^2 \frac{J_1(x)}{x} \right]^2 \left[(1 + \cos \theta) (\sin \theta \cos \varphi) \underline{i} \right. \\ &\quad \left. + (1 + \cos \theta) (\sin \theta \sin \varphi) \underline{j} + (1 + \cos \theta)^2 \underline{k} \right] \end{aligned}$$

$$\underline{P} = \frac{1}{2} \frac{\pi^2 \eta M^2}{\lambda^2 R^2} \left[a^2 \frac{J_1(x)}{x} \right]^2 \left[(1 + \cos \theta) \right] \left[\sin \theta \cos \varphi \underline{i} + \sin \theta \sin \varphi \underline{j} + \cos \theta \underline{k} + \underline{k} \right]$$

$$\text{but } \sin \theta \cos \varphi \underline{i} + \sin \theta \sin \varphi \underline{j} + \cos \theta \underline{k} = \underline{k}_0$$

$$\text{and } \underline{k} = \cos \theta \underline{k}_0 - \sin \theta \underline{\theta}_0$$

$$\begin{aligned} \underline{P} &= \frac{\eta}{2} \left[\frac{\pi \bar{M}}{\lambda R} a^2 \frac{J_1(x)}{x} \right]^2 \left[1 + \cos \theta \right] \left[\underline{k}_0 + \cos \theta \underline{k}_0 - \sin \theta \underline{\theta}_0 \right] \\ &= \frac{\eta}{2} \left[\frac{\pi \bar{M}}{\lambda R} a^2 \frac{J_1(x)}{x} \right]^2 (1 + \cos \theta)^2 \underline{k}_0 - \frac{\eta}{2} \left[\frac{\pi \bar{M}}{\lambda R} a^2 \frac{J_1(x)}{x} \right]^2 \sin \theta (1 + \cos \theta) \underline{\theta}_0 \end{aligned}$$

In the region where θ is small equation (3.21) reduced to

$$\underline{P} = \frac{\eta}{2} \left[\frac{\pi \bar{M}}{\lambda R} a^2 \frac{J_1(x)}{x} \right]^2 (1 + \cos \theta)^2 \underline{k}_0$$

ben (20.5) notitje of niet al leesbaar al (21.5) notitje

$$(21.5) \quad \hat{q} \lambda (\theta \sin \hat{q} \hat{s}) \hat{x} \hat{q} (\hat{w}, \hat{q}) M^{\text{d}} \right] \underline{E} = \underline{(x) H}$$

$$(21.5) \quad \underline{(x) \bar{H}} - \underline{\delta \cdot \bar{M}} \underline{E} = \underline{(x) H}, \quad \bar{M} = (\hat{w}, \hat{q}) M$$

enigepte al negeve veld en $(x)H$ has $(x)H$ tot amplitude en
-veld of dat aangeeft dat al niet meer dan dat (21.5) has (21.5)
bedoelt dat een form van de veld die niet meer dan dat (21.5)
heeft dat dat (21.5) niet meer dan dat kan doemmeren
(1)

$$(21.5) \quad \underline{H} \times \underline{E} \underline{B} = \underline{F} \quad \text{e.d.}$$

H is de veld van deelbare magneten \underline{B} omtrek

$$(21.5) \quad \left[\frac{1}{2} \varphi \sin \theta \sin - \frac{1}{2} (\theta 20 + 1) \right] \underline{M} \frac{(\hat{w} - \hat{s} \hat{w})}{\hat{s} \hat{w}} - \underline{H}$$

$$- \underline{i} (\theta 20 + 1) \left[\left(\frac{(x) \bar{H}}{x} - \underline{\delta} \underline{M} \right) \bar{M} \right] \frac{\hat{s} \hat{w} (i)}{\hat{s} \hat{w} \hat{q} \hat{s}} \underline{B} = \underline{F}$$

$$\left\{ \frac{1}{2} \varphi \sin \theta \sin - \frac{1}{2} (\theta 20 + 1) \right\} \times \left\{ \frac{1}{2} \varphi \cos \theta \cos \theta \sin \right.$$

$$i (\varphi \cos \theta \sin) (\theta \cos + 1) \left[\left(\frac{(x) \bar{H}}{x} - \underline{\delta} \underline{M} \right) \bar{M} \right] \frac{\hat{s} \hat{w} \hat{q} \hat{s}}{\hat{s} \hat{w} \hat{q} \hat{s}} \underline{B} =$$

$$\left[\frac{1}{2} (\theta 20 + 1) + i (\varphi \sin \theta \sin) (\theta \cos + 1) + \right]$$

$$\left[\frac{1}{2} + \frac{1}{2} \theta 20 + i \varphi \sin \theta \sin + \frac{1}{2} \varphi \cos \theta \sin \right] \left[(\theta \cos + 1) \right] \left[\left(\frac{(x) \bar{H}}{x} - \underline{\delta} \underline{M} \right) \bar{M} \right] \frac{\hat{s} \hat{w} \hat{q} \hat{s} \pi}{\hat{s} \hat{w} \hat{q} \hat{s}} \underline{B} = \underline{F}$$

$$\underline{B} = \frac{1}{2} \theta 20 + i \varphi \sin \theta \sin + \frac{1}{2} \varphi \cos \theta \sin \quad \text{ind}$$

$$\theta \sin \theta \sin - \underline{\delta} \theta \cos = \frac{1}{2} \sin$$

$$\left[\underline{\theta} \theta \sin - \underline{\delta} \theta \cos + \frac{1}{2} \right] \left[\theta \cos + 1 \right] \left[\left(\frac{(x) \bar{H}}{x} - \underline{\delta} \underline{M} \right) \bar{M} \right] \frac{\pi}{\hat{s} \hat{w}} \underline{B} = \underline{F}$$

$$\theta (\theta \cos + 1) \theta \sin \left[\left(\frac{(x) \bar{H}}{x} - \underline{\delta} \underline{M} \right) \bar{M} \right] \frac{\pi}{\hat{s} \hat{w}} - \underline{\delta} \left[\theta \cos + 1 \right] \left[\left(\frac{(x) \bar{H}}{x} - \underline{\delta} \underline{M} \right) \bar{M} \right] \frac{\pi}{\hat{s} \hat{w}} =$$

at hetzelfde (21.5) notitje \underline{B} is θ omtrek niet meer dan dat

$$\underline{B} = (\theta \cos + 1) \left[\left(\frac{(x) \bar{H}}{x} - \underline{\delta} \underline{M} \right) \bar{M} \right] \frac{\pi}{\hat{s} \hat{w}} \underline{B} = \underline{F}$$

since $\sin \theta \ll (1 + \cos \theta)^{\frac{1}{2}}$ for $\theta < 20^\circ$. This expression for the energy flow agrees with the generally accepted solution given by Schelkunoff (2) for transmission through a circular aperture.

3.10 The remainder of this paper will be confined to relations between the electric field vector distribution and the corresponding aperture distribution. There is no loss in generality in doing this since the energy in distant space is proportional to the square of the amplitude of the electric field vector. Specifically;

$$\underline{E} = \eta \underline{H} \text{ and } \underline{P} = \frac{1}{2} \underline{E} \times \widetilde{\underline{H}} ;$$

$$\therefore \Phi \propto (\underline{E})^2$$

volt ω is the self inductance of the coil. $\theta > \theta_{\text{crit}} (\theta_{\text{crit}} + 1) > 0$ is the condition for (2) to be satisfied as shown below. The voltage across the capacitor is given by the equation

$$V_C = \frac{1}{C} \int i dt = \frac{1}{C} \int \frac{1}{R} (V - L \frac{di}{dt}) dt = \frac{1}{RC} (Vt - L \ln(1 + \frac{V}{L} t))$$
 which is a hyperbolic function of time.

Chapter IV

Solution for Aperture Distribution

4.1 Equation (3.11), page . $\underline{E}(x) = G \int_{\rho=0}^a J_0(k \hat{\rho} \sin \theta) I(\hat{\rho}, a) \hat{\rho} d\hat{\rho}$

is the integral equation that will always arise from a circularly symmetric plane polarized amplitude distribution over an aperture. Its solution for $\underline{E}(x)$ if $I(\hat{\rho}, a)$ is known is in general not difficult and may be accomplished by numerical integration if other means fail. However, the solution for $I(\hat{\rho}, a)$ when $\underline{E}(x)$ is known presents a different problem.

4.2 One method of attack has been to use Fourier transforms. This requires the assumption that the limits of integration may be extended from 0 to ∞ and considerable error may be introduced since the function $I(\hat{\rho}, a)$ may make a large contribution to the integral between a and ∞ .

If we write:

$$\begin{aligned}\underline{E}(x) &= G \int_0^\infty J_0(k \hat{\rho} \sin \theta) I(\hat{\rho}, a) \hat{\rho} d\hat{\rho} \\ &- G \left\{ \int_a^\infty J_0(k \hat{\rho} \sin \theta) I(\hat{\rho}, a) \hat{\rho} d\hat{\rho} \right\}\end{aligned}\quad (4.01)$$

then the error introduced in solving by Fourier transforms or Fourier - Bessel transforms is equal to the value of the second integral. Obviously, for certain $I(\hat{\rho}, a)$ functions this error will be large and only in special cases will it be zero. In practice, it is impossible to have $I(\hat{\rho}, a)$ equal to zero at $\hat{\rho} = a$ and hence an error will always be introduced if this method is used. If the unit step function is included as a factor in the aperture distribution and Fourier - Bessel transforms are used, the extension of the limits to infinity introduces no error but this method leads to rather formidable equations except in special cases.

4.3 The method developed here requires no simplifying assumptions regarding limits of integration nor aperture size. The equations are

$$\hat{q}_x \hat{q}_y (\hat{x} \hat{y}) I(\theta_{\text{ref}} \hat{q}_x) J \left[\begin{matrix} 0 \\ 0 = q \end{matrix} \right] =$$

$$\left(\hat{q}_x \hat{q}_y (\hat{x} \hat{y}) I(\theta_{\text{ref}} \hat{q}_x) J \left[\begin{matrix} 0 \\ 0 = q \end{matrix} \right] \right)^{\infty}$$

\hat{q}

relatively simple and the integrations are direct.

4.4 Returning to equation (3.11) $E(x) = G \int_0^a I(\hat{\rho}, a) \hat{\rho} J_0(k \hat{\rho} \sin \theta) d\hat{\rho}$

Assume that $I(\hat{\rho}, a)$ may be written as $I[(\hat{\rho}/a)^2] = I(z^2)$ (4.02)

This assumption imposes no further restrictions on $I(\hat{\rho}, a)$ since it has already been specified that the aperture distribution be circularly symmetric.

In equation (2.11): let $z = \hat{\rho}/a$

then $\hat{\rho} = az$

$d\hat{\rho} = a dz$

and $k \hat{\rho} \sin \theta = ka \sin \theta \frac{\hat{\rho}}{a} = xz$ since $x = ka \sin \theta$

The limits of integration become:

for $\hat{\rho} = 0$ $z = 0$

$\hat{\rho} = a$ $z = 1$

Hence: $E(x) = G \int_0^1 I(z^2) az J_0(xz) a dz$

$$= G a^2 \int_0^1 z I(z^2) J_0(xz) dz$$

(4.03)

for $I(z^2) = \sum_{k=0}^{\infty} I_{2k} z^{2k}$; $zI(z^2) = \sum_{k=0}^{\infty} I_{2k} z^{2k+1}$

and $E(x) = G a^2 \int_0^1 J_0(xz) \sum_{k=0}^{\infty} I_{2k} z^{2k+1} dz$

(4.04)

$$= G a^2 \sum_{k=0}^{\infty} I_{2k} \int_0^1 J_0(xz) z^{2k+1} dz$$

(4.05)

$$\hat{q} \cdot \theta = \hat{q} \cdot \hat{q} \left(\frac{\hat{q}}{|\hat{q}|} \right)^2 = \hat{q} \cdot \hat{q} = |\hat{q}|^2$$

$$(S^z)I = \left[S \left(\frac{\hat{q}}{|\hat{q}|} \right) \right] I$$

$$S \cdot \hat{q} = \hat{q}$$

Since $\hat{q} \neq 0$, we have $S \cdot \hat{q} = \hat{q}$

$$S \cdot \hat{q} = \hat{q}$$

$$S \cdot q = \hat{q}$$

$$S \cdot q = \hat{q}$$

$$\sin \theta = x \quad S \cdot q = \hat{q} \quad \sin \theta = \sin \hat{q}$$

$$\theta = \pi \quad \theta = \hat{q} \text{ rad}$$

$$I = \pi \quad I = \hat{q}$$

$$\sin (\pi) I = \sin (\hat{q}) I$$

$$\sin (\pi) I = (S^z) I \quad \sin \hat{q} I = (S^z) I$$

$$1 + \sum_{n=1}^{\infty} (-1)^n I = (S^z) I \quad 1 + \sum_{n=1}^{\infty} (-1)^n I = (S^z) I \text{ rad}$$

$$S^z = \sum_{n=1}^{\infty} (-1)^n (S^z) I = (S^z) I \text{ rad}$$

$$S^z = \sum_{n=1}^{\infty} (-1)^n (S^z) I = (S^z) I \text{ rad}$$

Taking the summation sign outside the integration is permissible here since only distributions which are possible in practice will be considered; to be possible

$$\sum_{k=0}^{\infty} I_{2k} z^{2k+1}$$

must be finite at $z=1$, hence $\sum_{k=0}^{\infty} I_{2k}$ must be finite.

4.5 Consider: $\int_0^1 J_0(xz) z^{2k+1} dz = \frac{1}{x^2} \int_0^1 xz J_0(xz) z^{2k} d(xz)$

integrating by parts where $\int u dv = uv - \int v du$

$$\text{let } u = z^{2k}, \quad dv = xz J_0(xz) d(xz)$$

$$\text{then } \int_0^1 J_0(xz) z^{2k+1} dz = \frac{1}{x^2} \left\{ xz J_0(xz) z^{2k} \Big|_{z=0}^{z=1} \right.$$

$$\left. - 2k \int_0^1 xz J_0(xz) z^{2k-1} dz = \frac{J_0(x)}{x} - \frac{2k}{x^2} \int_0^1 xz J_0(xz) z^{2k-1} dz \right.$$

Continued integration by parts gives

$$\begin{aligned} & \frac{J_0(x)}{x} - \frac{2k}{x^4} \int_0^1 (xz)^2 J_0(xz) z^{2k-2} dz = \frac{J_0(x)}{x} - \\ & \frac{2k}{x^4} \left\{ (xz)^2 J_1(xz) z^{2k-2} \Big|_{z=0}^{z=1} - (2k-2) \int_0^1 (xz)^2 J_1(xz) z^{2k-3} dz \right. \\ & = \frac{J_0(x)}{x} - 2k \frac{J_1(x)}{x^2} + 2k(2k-2) \frac{1}{x^4} \int_0^1 (xz)^2 J_1(xz) z^{2k-3} dz \end{aligned}$$

Further integration gives the series:

$$\begin{aligned} & \frac{J_0(x)}{x} - 2k \frac{J_1(x)}{x^2} - 2k(2k-2) \frac{J_2(x)}{x^3} - 2k(2k-2)(2k-4) \frac{J_3(x)}{x^4} + \dots \\ & = \frac{1}{2} \left[\frac{J_0(x)}{x/2} - k \frac{J_1(x)}{(x/2)^2} + k(k-1) \frac{J_2(x)}{(x/2)^3} - k(k-1)(k-2) \frac{J_3(x)}{(x/2)^4} + \dots \right] \\ & = \frac{1}{2} \left[\frac{J_0(x)}{1!} - \frac{k J_1(x)}{2!} + \frac{k(k-1) J_2(x)}{3!} - \frac{k(k-1)(k-2) J_3(x)}{4!} + \dots \right] \end{aligned}$$

Explaination of solution is as follows:

From the definition of binomial expansion we have

$$\sum_{n=0}^{\infty} \binom{n}{k} x^n = (1+x)^n \quad \text{for } n \in \mathbb{N}$$

(1)

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k \quad \text{for } n \in \mathbb{N}$$

(2)

Comparing (1) and (2), we get

$$\sum_{k=0}^n \binom{n}{k} x^k = (1+x)^n \quad \text{for } n \in \mathbb{N}$$

$$\sum_{k=0}^n \binom{n}{k} x^k = (1+x)^n \quad \text{for } n \in \mathbb{N}$$

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k \quad \text{for } n \in \mathbb{N}$$

Continuation of explanation of solution

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k \quad \text{for } n \in \mathbb{N}$$

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k \quad \text{for } n \in \mathbb{N}$$

$$(1+x)^n = \sum_{k=0}^n \binom{n}{k} x^k \quad \text{for } n \in \mathbb{N}$$

Continuation of explanation of solution

$$\dots + \frac{(x)^n}{n!} \binom{n}{k} (1-x)^{n-k} - \frac{(x)^n}{n!} \binom{n}{k} (1-x)^{n-k} =$$

$$\dots + \frac{(x)^n}{n!} \binom{n}{k} (1-x)^{n-k} - \frac{(x)^n}{n!} \binom{n}{k} (1-x)^{n-k} =$$

$$\dots + \frac{(x)^n}{n!} \binom{n}{k} (1-x)^{n-k} - \frac{(x)^n}{n!} \binom{n}{k} (1-x)^{n-k} =$$

Using this result we may write equation (4.05) as

$$E(x) = \frac{Ga^2}{2} \left\{ I_0 \Lambda_1(x) + \sum_{k=0}^{\infty} \sum_{j=1}^{k+1} (-1)^{j+1} I_{2k} \left[\frac{k(k-1)(k-2)\dots(k-j+2)}{j!} \Lambda_j(x) \right] \right\} \quad (4.06)$$

or

$$\begin{aligned} E(x) &= \frac{Ga^2}{2} \left\{ \Lambda_1(x) [I_0 + I_2 + I_4 + I_6 + \dots] \right. \\ &\quad - \frac{\Lambda_2(x)}{2!} [0 + I_2 + 2I_4 + 3I_6 + 4I_8 + 5I_{10} + \dots] \\ &\quad + \frac{\Lambda_3(x)}{3!} [0 + 0 + 2I_4 + 3 \cdot 2I_6 + 4 \cdot 3I_8 + 5 \cdot 4 \cdot 3I_{10} + \dots] \\ &\quad - \frac{\Lambda_4(x)}{4!} [0 + 0 + 0 + 3 \cdot 2I_6 + 4 \cdot 3 \cdot 2I_8 + 5 \cdot 4 \cdot 3 \cdot I_{10} + \dots] \dots \dots \\ &\quad (-1)^{p+1} \frac{\Lambda_p(x)}{p!} [(p-1)! I_{2p-2} + \frac{p!}{1!} I_{2p} + \frac{(p+1)!}{2!} I_{2p+2} \\ &\quad + \frac{(p+2)!}{3!} I_{2p+4} + \dots] \end{aligned}$$

$$E(x) = \frac{Ga^2}{2} \sum_{p=1}^{\infty} (-1)^{p+1} \frac{\Lambda_p(x)}{p!} \sum_{k=0}^{\infty} \frac{(p-1+k)!}{k!} I_{(2p-2+2k)} \quad (4.07)$$

4.6 Now: If the desired pattern is expressed in terms of $\Lambda_p(x)$ and if we consider the $\Lambda_p(x)$ as the independent variable we can evaluate coefficients of the $\Lambda_p(x)$ and solve for the I_{2k} . The requirement that the desired pattern be expressed in terms of $\Lambda_p(x)$ is not a restriction imposed by the method so much as a restriction imposed by nature. The reason for this is discussed in section 5.3 page .

4.7 The following two examples solved for distant space patterns for which the corresponding aperture distributions are known show in detail

as (20.+) weitere stimmen es fuer einst genau

$$(20.+) \quad \left\{ \frac{(x)_{\alpha} \Lambda (x+\beta-\delta) \cdots (x-\delta)(1-\delta)}{\Gamma(\beta)} \right\}_{\alpha=0}^{\infty} \sum_{k=0}^{1+\delta} (-1)^k \sum_{i=0}^{\infty} \frac{x^i}{i!} (x)_{\alpha} \Lambda = (x) \exists$$

$$\left[\cdots \cdots \cdots +_0 I + _1 I + _2 I + _3 I \right] (x) \Lambda = (x) \exists$$

$$\left[\cdots \cdots +_0 I z + _1 I + +_2 I \bar{z} + _3 I z + _4 I + 0 \right] (x) \Lambda =$$

$$\left[\cdots \cdots +_0 I + z + _1 \bar{z} + +_2 I z + _3 I \bar{z} + _4 I z + 0 + 0 \right] (x) \Lambda =$$

$$\cdots \cdots \left[\cdots +_0 I \cdot z + \bar{z} + _1 I z \cdot \bar{z} + +_2 I z \cdot z + _3 I z \cdot 0 + 0 + 0 \right] (x) \Lambda =$$

$$+_{0+q} I \frac{1}{\Gamma(1+q)} + q_{0+q} I \frac{1}{\Gamma(1+q)} + \cdots +_{0+q} I \frac{1}{\Gamma(1+q)} (x) \Lambda =$$

$$\cdots \cdots +_{0+q} I \frac{1}{\Gamma(1+q)} +$$

$$(20.+) \quad \left\{ \frac{(x)_{\alpha} \Lambda}{\Gamma(\beta)} \right\}_{\alpha=0}^{\infty} \sum_{k=0}^{1+\delta} (-1)^k \sum_{i=0}^{\infty} \frac{x^i}{i!} (x) \Lambda = (x) \exists$$

aber nur die ersten δ Glieder der Reihe stimmen mit den entsprechenden Gliedern von $(x) \Lambda$ überein, während die weiteren Glieder von $(x) \Lambda$ nicht stimmen. Es folgt, dass $(x) \Lambda$ nicht einst genau ist.

Wir schließen aus, dass $(x) \Lambda$ nicht einst genau ist.

Wir schließen aus, dass $(x) \Lambda$ nicht einst genau ist.

the method of solving for $I(z^2)$.

Example 1: Let $\underline{E}(x) = \underline{G} \bar{I} a^2 \frac{\underline{J}_1(x)}{x}$ for which $I(z^2) = \bar{I}$ (2)
that is constant amplitude.

$$\underline{E}(x) = \underline{G} \bar{I} a^2 \frac{\underline{J}_1(x)}{x} = \frac{1}{2} \underline{G} \bar{I} a^2 \underline{J}_1(x).$$

Equating coefficients:

$$\frac{1}{2} \underline{G} \bar{I} a^2 = \frac{\underline{G} a^2}{2} [I_0 + I_2 + I_4 + I_6 + \dots]$$

$$0 = -\frac{\underline{G} a^2}{2 \cdot 2!} [0 + I_2 + 2I_4 + 3I_6 + \dots]$$

$$0 = \frac{\underline{G} a^2}{2 \cdot 3!} [0 + 0 + 2I_4 + 3 \cdot 2I_6 + \dots]$$

$$0 = \dots \dots$$

$$0 = (-1)^{m-1} \frac{\underline{G} a^2}{2 \cdot m!} [0 + 0 + 0 + 0 + (m-1)! I_{2m-2} + \dots]$$

$$0 = \dots \dots$$

From the rules for evaluating determinants it is apparent that $I_{2k} = 0$ for $k \geq 1$

$$\therefore \frac{1}{2} \underline{G} \bar{I} a^2 = \frac{\underline{G} a^2}{2} I_0$$

$$I_0 = \bar{I}; \quad I(z^2) = \bar{I} \text{ and}$$

$$\underline{I}(\hat{P}, a) = \bar{I} \underline{i} \text{ which is known to be correct}$$

Example 2: Let $\underline{E}(x) = 2 \bar{I} \underline{G} a^4 \frac{\underline{J}_2(x)}{x^2} = \bar{I} \frac{\underline{G} a^4}{4} \underline{J}_2(x)$

$$\text{for which } I(z^2) = \bar{I} a^2 (1 - z^2)^{***}$$

Equating coefficients:

$$0 = \frac{\underline{G} a^2}{2} [I_0 + I_2 + I_4 + I_6 + \dots]$$

$$\frac{\bar{I} \underline{G} a^4}{4} = -\frac{\underline{G} a^2}{2 \cdot 2!} [0 + I_2 + 2I_4 + 3I_6 + \dots]$$

$$0 = \frac{\underline{G} a^2}{2 \cdot 3!} [0 + 0 + 2I_4 + 3 \cdot 2I_6 + \dots]$$

** Appendix IV

*** This may be readily shown using equation (3.11) p. .

$$(S) \quad \overline{I} = S + \frac{1}{2} I + \frac{1}{3} I^2 + \dots$$

$$(x) \left(\overline{I} - S \right) \underline{\underline{I}} = \frac{(x) \overline{I} - S \underline{\underline{I}}}{x} = (x) \underline{\underline{I}}$$

$$\dots + s_1 I + s_2 I + s_3 I + s_4 I \left[\frac{s_5 \underline{\underline{I}}}{x} \right] = S \underline{\underline{I}} \underline{\underline{I}}$$

$$\dots + s_1 I s_2 + s_2 I s_3 + s_3 I s_4 \left[\frac{s_5 \underline{\underline{I}}}{x^2} \right] = 0$$

$$\dots + s_1 I s_2 s_3 + s_2 I s_3 s_4 \left[\frac{s_5 \underline{\underline{I}}}{x^3} \right] = 0$$

$$\dots + s_1 s_2 s_3 I \left[\frac{s_4 s_5 \underline{\underline{I}}}{x^4} \right] = 0$$

$$1 \leq s_i \text{ and } 0 = s_5 I$$

$$I \left[\frac{s_4 s_5 \underline{\underline{I}}}{x^4} \right] = S \underline{\underline{I}} \underline{\underline{I}} \quad \therefore$$

$$\text{and } I = (\overline{S}) I \quad (\overline{I} = I)$$

$$\text{so now we want to write } \overline{I} I = (\overline{S}) I$$

$$(x) \left(\overline{I} - \frac{S \underline{\underline{I}}}{x} \right) I = (x) \overline{I} I - S \underline{\underline{I}} I =$$

$$\therefore (x-1) S \underline{\underline{I}} = (x) I$$

$$\dots + s_1 I + s_2 I + s_3 I + s_4 I \left[\frac{s_5 \underline{\underline{I}}}{x} \right] = 0$$

$$\dots + s_1 I s_2 + s_2 I s_3 + s_3 I s_4 \left[\frac{s_5 \underline{\underline{I}}}{x^2} \right] = \frac{s_5 \underline{\underline{I}}}{x}$$

$$\dots + s_1 s_2 s_3 + s_2 s_3 s_4 \left[\frac{s_5 \underline{\underline{I}}}{x^3} \right] = 0$$

Again $I_{2k} \equiv 0$ for $k > 2$ and we have:

$$\left. \begin{aligned} 0 &= I_0 + I_2 \\ -\bar{I}\alpha^2 &= 0 + I_2 \end{aligned} \right\} \quad \begin{aligned} I_2 &= -\bar{I}\alpha^2 \\ I_0 &= \bar{I}\alpha^2 \end{aligned}$$

$$I(z^2) = \bar{I}\alpha^2(1-z^2) \text{ and}$$

$$I(\hat{P}, \alpha) = \bar{I}\alpha^2(\bar{I} - (\hat{P}/\alpha)^2) \quad [i]$$

4.9 For a general solution assume that $\underline{E}(x) = \bar{I}G \Lambda_m(x)$

and equate coefficients of $\Lambda_m(x)$ in equation (4.07)

$$\bar{I}G = \frac{\underline{G}\alpha^2}{2} \frac{(-1)^{m+1}}{m!} \sum_{k=0}^{\infty} \frac{(m-1+k)!}{k!} I_{(2m-2+2k)}$$

but

$$I_{(2m-2+k)} \equiv 0 \text{ for } k > 0 \quad ** \quad \text{and}$$

$$\bar{I} = \frac{\alpha^2}{2} \frac{(-1)^{m+1}}{m!} (m-1)! I_{(2m-2)}$$

$$= \frac{\alpha^2}{2} \frac{(-1)^{m+1}}{m} I_{(2m-2)}$$

$$\therefore I_{(2m-2)} = \frac{2m}{\alpha^2} \bar{I} (-1)^{m+1} \quad (4.08)$$

However, I_{2k} for $2k < 2m-2$ are not equal to zero **. If we equate coefficients of $\Lambda_{(m-j)}$ ($j = 1, 2, 3, \dots, m-1$) in equation (4.07) for $\underline{E}(x) = \bar{I}G \Lambda_m(x)$; then $p = m-j$ in equation (4.07) and we have

$$0 = \frac{\underline{G}\alpha^2}{2} \frac{(-1)^{m-j+1}}{(m-j)!} \sum_{k=0}^{j-1} \frac{(m-j-1+k)!}{k!} I_{(2m-2j-2+2k)} \quad \text{or}$$

$$\sum_{k=0}^{\infty} \frac{(m-j-1+k)!}{k!} I_{(2m-2j-2+2k)} = 0 \quad (4.09)$$

$$j = 1, 2, 3, \dots, m-1$$

Given ω bns $s \in \text{rot } 0 \in \omega I$

$$sI - zI \quad (\quad sI + zI = 0)$$

$$sI = zI \quad (\quad sI + z = sI -$$

$$\text{bns } (sI - z) sI = (sI) I$$

$$\vdash [(sI - z) sI] = (sI) I$$

$$(sI) m \wedge \emptyset I =$$

$$(sI) m \wedge$$

$$(sI + s - zm) I \frac{1}{(s + s - zm)} \stackrel{s \neq 0}{\rightarrow} \frac{1 + zm}{1 - zm} (1) \frac{sI}{s} = \emptyset I$$

$$\text{bns } ** \quad 0 < s \text{ rot } 0 \in (s + s - zm) I$$

$$(s - zm) I \vdash (1 - zm) \frac{1 + zm}{1 - zm} (1) \frac{sI}{s} = \emptyset I$$

$$(s - zm) I \frac{1 + zm}{1 - zm} (1) \frac{sI}{s} =$$

$$\stackrel{1 + zm}{1 - zm} (1) \frac{sI}{s} = (s - zm) I \therefore$$

$$= i \quad \text{and}$$

$$0 \quad (sI + s - zm) I \frac{1}{(s + s - zm)} \stackrel{s \neq 0}{\rightarrow} \frac{1 + zm}{1 - zm} (1) \frac{sI}{s} = 0$$

$$0 = (sI + s - zm) I \frac{1}{(s + s - zm)} \stackrel{s \neq 0}{\rightarrow}$$

$$m \in \{ \dots, \frac{1}{2}, 0 \} = \mathbb{Z}$$

Equation (4.09) provides a ready means of solving for the I_{2k} for $2k < 2m-2$.

$$\text{For } j = 1 \quad \frac{(m-2)!}{0!} I_{(2m-4)} + \frac{(m-1)!}{1!} I_{(2m-2)} = 0$$

Substituting from equation (4.08) for $I_{(2m-2)}$ in the above expression,

$$I_{(2m-4)} = -\frac{2\bar{I}}{\alpha^2} (-1)^{m+1} \frac{m(m-1)}{1!} \quad (4.10)$$

For $j = 2$ we may solve for $I_{(2m-6)}$ and

$$I_{(2m-6)} = \frac{2\bar{I}}{\alpha^2} (-1)^{m+1} \frac{m(m-1)(m-2)}{2!} \quad (4.11)$$

The general expression for I_{2k} , $2k < 2m-2$, is

$$I_{[2m-2(j+1)]} = (-1)^j \frac{2\bar{I}}{\alpha^2} (-1)^{m+1} \frac{m(m-1)(m-2)\dots(m-j)}{j!} \quad (4.12)$$

and $I(Z^2)$ for $\underline{E}(x) = I \underline{G} A_m(x)$ is

$$I(Z^2) = \sum_{j=0}^{m-1} I_{[2m-2(j+1)]} Z^{[2m-2(j+1)]} \quad (4.13)$$

$$= \frac{2\bar{I}}{\alpha^2} (-1)^{m+1} \sum_{j=0}^{m-1} (-1)^j \frac{m(m-1)(m-2)\dots(m-j)}{j!} Z^{2m-2(j+1)} \quad (4.14)$$

If (4.14) is rearranged so that low powers of Z appear in the first of the summation then

$$\begin{aligned} I(Z^2) &= \frac{2\bar{I}}{\alpha^2} (-1)^{m+1} \left\{ (-1)^{m-1} \frac{m!}{(m-1)!} \frac{Z^0}{0!} + (-1)^{m-2} \frac{m!}{(m-2)!} \frac{Z^2}{1!} \right. \\ &\quad \left. + (-1)^{m-3} \frac{m!}{(m-3)!} \frac{Z^4}{2!} + \dots \right\} \\ &= \frac{2\bar{I}}{\alpha^2} \left\{ \frac{mZ^0}{0!} - \frac{m(m-1)Z^2}{1!} + \frac{m(m-1)(m-2)Z^4}{2!} + \dots \right\} \\ &= \frac{2\bar{I}}{\alpha^2} \sum_{j=0}^{m-1} \frac{m(m-1)(m-2)\dots(m-j)}{j!} Z^{2j} \quad (4.15) \end{aligned}$$

which is readily recognized as the binomial expansion of $\frac{2\bar{I}}{\alpha^2} m (1-Z^2)^{m-1}$

Therefore, the required aperture distribution to give a distant space

$$0 = (\alpha - \alpha m, \alpha) \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] + (\beta - \beta m, \beta) \Gamma \left[\frac{1 + \beta m}{\beta} \right]$$

$$(1+1) \frac{(\alpha - \alpha m)^{\alpha}}{\alpha} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] + (\beta - \beta m) \Gamma \left[\frac{1 + \beta m}{\beta} \right] = (\beta - \beta m) \Gamma$$

$$\frac{(\alpha - \alpha m)(1 - \alpha m)}{\alpha} \alpha m^{\alpha} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] + (\beta - \beta m) \Gamma \left[\frac{1 + \beta m}{\beta} \right] = (\beta - \beta m) \Gamma$$

$$(\beta - \beta m) \dots (\alpha - \alpha m) \frac{(1 - \alpha m)^{\alpha}}{\alpha} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] + (\beta - \beta m) \Gamma \left[\frac{1 + \beta m}{\beta} \right] = [(1 + \frac{1}{\alpha}) \alpha - \alpha m, \alpha] \Gamma$$

$$[(1 + \frac{1}{\alpha}) \alpha - \alpha m, \alpha] \frac{[(1 + \frac{1}{\alpha}) \alpha - \alpha m]^{\alpha}}{\alpha} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] = (\alpha \Gamma)$$

$$(1 + \frac{1}{\alpha}) \alpha - \alpha m \Gamma \left(\frac{(\alpha - \alpha m) \dots (\alpha - \alpha m) (1 - \alpha m)^{\alpha}}{\alpha} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] \right) + (\beta - \beta m) \Gamma \left[\frac{1 + \beta m}{\beta} \right] =$$

$$\frac{1}{\alpha} \left[\frac{(\alpha - \alpha m)^{\alpha - m}}{\alpha - m} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] + \frac{1}{\alpha} \frac{(\alpha - \alpha m)^{\alpha - m}}{\alpha - m} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] \right] + (\beta - \beta m) \Gamma \left[\frac{1 + \beta m}{\beta} \right] = (\alpha \Gamma)$$

$$\left\{ \dots + \frac{1}{\alpha} \frac{(\alpha - \alpha m)^{\alpha - m}}{\alpha - m} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] \right\} +$$

$$\left\{ \dots + \frac{1}{\alpha} \frac{(\alpha - \alpha m)(\alpha - \alpha m - 1) \dots (\alpha - \alpha m - m + 1) \alpha^{-m}}{\alpha - m} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] \right\} +$$

$$\left\{ \dots + \frac{1}{\alpha} \frac{(\alpha - \alpha m)(\alpha - \alpha m - 1) \dots (\alpha - \alpha m - m + 1) \alpha^{-m}}{\alpha - m} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] \right\} +$$

$$\left\{ \dots + \frac{1}{\alpha} \frac{(\alpha - \alpha m)(\alpha - \alpha m - 1) \dots (\alpha - \alpha m - m + 1) \alpha^{-m}}{\alpha - m} \Gamma \left[\frac{1 + \alpha m}{\alpha} \right] \right\}$$

electric field vector distribution of $E(x) = I \cdot L_m(x)$ is

$$I(Z^2) = \frac{2\bar{I}}{\alpha^2} m(1-Z^2)^{m-1} \quad \text{or}$$

$$I(Z^2) \sim m(1-Z^2)^{m-1} \quad (4.16)$$

4.10 Several interesting facts may be deduced from equation (4.16). Only when $L_1(x)$ is present in this distribution in distant space will the aperture distribution have a value at the edge of the aperture where $Z=1$. Since in practice there will always be some energy at the edge of the aperture we may conclude that $E(x)$ will always have $L_1(x)$ in it. This is important since it tends to introduce side lobes at low values of x . See Fig. 3, Appendix III. To radiate a lobeless pattern this tendency of $E(x)$ to go to zero at the zeros of $L_1(x)$ and further, to go negative (i.e. phase reversal) between alternate zeros of $L_1(x)$ must be overcome by higher $L_n(x)$ functions. But the introduction of higher $L_n(x)$ functions tends to broaden the pattern. This forces a compromise between beam width characteristics and sidelobe characteristics. The general tendency, as has been known from experiment, is that to radiate a narrow pattern means the introduction of sidelobes and vice versa, the reduction or elimination of sidelobes tends to broaden the pattern. Theoretically, from an examination of equation (4.16), it is seen that a lobeless pattern may be radiated without infinite energies over the aperture. All that is necessary is to choose $L_m(x)$ such that the first zero of $L_m(x)$ occurs at $x = ka \sin \theta = ka$ (i.e. at $\theta = 90^\circ$). Note here, that since $k = \frac{2\pi}{\lambda}$, and that λ is inversely proportional to the frequency the value of x increases as the frequency of the radiated energy increases and further, the order of $L_m(x)$ must be increased. Since $I(Z^2) \sim n(1-Z^2)^{m-1}$, the energy at the center of the aperture must be more and more sharply peaked to radiate a lobeless pattern as the frequency of the radiated energy is increased.

$$(\text{iii}) \quad \text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$$

$$\Rightarrow \text{rank } (sI - A_{\infty}) \text{ rank } \frac{I}{sI} = (sI)I$$

$$\Rightarrow \text{rank } (sI - A_{\infty}) \text{ rank } \sim (sI)I$$

(iii)

Since $\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$,
 $A_{\infty} = sI$. So A_{∞} is a scalar multiple of identity matrix.

Now A_{∞} is a scalar multiple of identity matrix. So $A_{\infty} = sI$.
 $s = 0$ or $s = \infty$.

If $s = 0$, then $A_{\infty} = 0$. Then $A_{\infty}^T = 0$ and $\text{adj } A_{\infty} = 0$.
 $\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$.

If $s = \infty$, then $A_{\infty} = \infty I$. Then $A_{\infty}^T = \infty I$ and $\text{adj } A_{\infty} = \infty I$.
 $\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$.

So $A_{\infty} = sI$ is a scalar multiple of identity matrix. Hence A_{∞} is a scalar multiple of identity matrix.

Now $A_{\infty} = sI$. Then $A_{\infty}^T = sI$ and $\text{adj } A_{\infty} = sI$.
 $\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$.

So $A_{\infty} = sI$ is a scalar multiple of identity matrix. Hence A_{∞} is a scalar multiple of identity matrix.

Now $A_{\infty} = sI$. Then $A_{\infty}^T = sI$ and $\text{adj } A_{\infty} = sI$.
 $\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$.

So $A_{\infty} = sI$ is a scalar multiple of identity matrix. Hence A_{∞} is a scalar multiple of identity matrix.

Now $A_{\infty} = sI$. Then $A_{\infty}^T = sI$ and $\text{adj } A_{\infty} = sI$.
 $\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$.

$$\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$$

$$\text{rank } A_{\infty} = \text{rank } A_{\infty}^T = \text{rank } (\text{adj } A_{\infty}) = \text{rank } (\text{adj } A_{\infty}^T) = \text{rank } I$$

The same is true as the dimensions of the aperture are increased. Fig. 4, Appendix III, shows the required aperture distribution to give a lobeless pattern for a frequency of radiation and aperture size such that $ka = 10$. Theoretically, a distant space distribution of $\Lambda_0(x)$ will give the desired lobeless pattern, however, in order that the practical requirement that the energy not be equal to zero at the edge of the aperture be met, a distribution of $\Lambda_0(x) + \Lambda_g(x)$ has been chosen. Note the shoulders that appear in this pattern at $\theta = 50^\circ$ and $\theta = 90^\circ$. These can be reduced only by increasing the proportion of $\Lambda_g(x)$ over the amount of $\Lambda_0(x)$. Increasing the proportion of $\Lambda_g(x)$ requires a reduction of the relative amount of energy at the edge of the aperture compared to the amount of energy at the center. In the limit, complete elimination of these shoulders may be obtained by reducing the energy at the edge of the aperture to zero and radiating a pure $\Lambda_g(x)$ pattern.

4.11 Figs. 5 and 5a, Appendix III, show clearly the effect on the required aperture distribution of narrowing the radiated pattern by choosing a distant space distribution containing a higher order $\Lambda(x)$ function which is subtractive. In Fig. 5, showing the distant field, a comparison is made between a Λ_6 pattern and a $2\Lambda_6 - \Lambda_{10}$ pattern. The half power point of the former occurs at $X = 3.1$ and that of the latter occurs at $X = 2.6$. A reduction of some 16% is obtained. Fig. 5a shows the comparison of aperture distributions corresponding to the two above fields. It will be seen that the distribution corresponding to the narrower pattern is hollow in the center of the aperture. Such distributions are always observed in center fed paraboloids and are caused by the feed support. The hollowing out of energy at the center has narrowed the distant space pattern, at the same time it has increased the amount of energy in the sidelobes. It appears, however, that in theory at least, a postulated field pattern could

and the general government will be compelled to make up the deficit.
 This is the only way to keep the currency circulating. In fact, the IIIrd proposal
 is far from being the only solution adopted by the Bank and the Treasury
 institution and most of the other Δ institutions supported by the government. The government
 will take some heavy burdens for the sake of the economy, and the right of
 the people to live will be maintained. The government will also have to give up
 some rights and enter into some obligations. $\Delta + \Delta = \Delta$
 and after several discussions, $\Delta = \Delta$ will be adopted.

The report says, Δ is chosen and decided. Δ
 The reason of this is that you and I are in agreement with Δ . The government must
 help the economy. The American and the British government had the same view. We, Japanese
 also have similar views. We must support the economy, and there are many such
 reasons. And then we can easily carry forward our policy of market
 economy. Δ is chosen and decided. Δ

Now, as far as the IIIrd proposal goes, Δ is chosen and decided. Δ
 The reason of this is that it is a new way, which has been adopted by the American, British
 and French governments. It is a good way to support the economy. And the
 central bank is a good place to do this. $\Delta - \Delta$
 The reason of this is that it is a good way to support the economy. And the
 central bank is a good place to do this. Δ
 The reason of this is that it is a good way to support the economy. And the
 central bank is a good place to do this. Δ

Now, as far as the IIIrd proposal goes, Δ is chosen and decided. Δ
 The reason of this is that it is a good way to support the economy. And the
 central bank is a good place to do this. Δ

Now, as far as the IIIrd proposal goes, Δ is chosen and decided. Δ

be approximated as closely as desired by a Fourier-Bessel series of Λ functions and then the required aperture distribution could be readily obtained using equation (4.16) page 26. An attempt at such an analysis is beyond the scope of this paper due to time limitations.

4.12 Two additional postulated distant space distributions with their corresponding aperture distributions are plotted in Fig. 6 and 7. In Fig. 6 curve 1 is the distant space distribution for

$$\phi \sim (\Lambda_1 + 2\Lambda_3 + \Lambda_5 + \Lambda_7 - \Lambda_2 - \Lambda_4 - \Lambda_6)^2$$

It shows the result that may be obtained from a combination of Λ functions.

The resultant pattern is lobeless to $ka \sin \theta = 10$. Its half power point occurs at $X = 4.2$. Curve 2 of this same Figure shows the effect of subtracting Λ_{10} from the distant space electric field distribution which gave rise to curve 1. The half power point now occurs at $X = 3.2$, a reduction of nearly 24%. Note the large increase of energy in the first side lobe which begins at $X = 3.6$. The pattern has been narrowed but at the expense of poor sidelobe structure. Fig. 6a shows the aperture distribution corresponding with curve 1 of Fig. 6; Fig. 6b shows the aperture distribution corresponding with curve 2 of Fig. 6.

Figure 7 is the aperture distribution for a pattern having no center lobe. Fig. 7a is the aperture distribution corresponding with this space pattern.

4.13 In addition to these patterns, Fig. 3 is a plot of the Λ functions for order 1 through 12 plus 16 and 20. A table of values for all Λ_p functions, 1 through 20, for values of argument zero to 10 is tabulated on the pages following Fig. 3. These values have been taken from "Table of Spherical Bessel Functions", Volume II, 1947, prepared by the Mathematical Tables Project, National Bureau of Standards. The page following contains powers of $(1-Z^2)$ evaluated at intervals of one tenth for $0 \leq Z \leq 1.0$ and is included for ready reference.

Δ is a linear operator defined by $\Delta f = \sum_{i=1}^n \frac{\partial}{\partial x_i} f$.
 - an operator that represents a total derivative with respect to each coordinate.
 - Δ is a linear operator. $\Delta^2 = (\sum_{i=1}^n \frac{\partial}{\partial x_i})^2 = \sum_{i=1}^n \frac{\partial^2}{\partial x_i^2}$
 - Δ is a differential operator with constant coefficients.

$$(\Delta - p_1 x_1 - p_2 x_2 - \dots - p_n x_n + q) u \sim \Phi$$

definition Δ is called a second order linear homogeneous partial differential operator with constant coefficients and $\Delta u = 0$ is called
 - the homogeneous equation associated with Δ .
 - Δ is a second order linear homogeneous partial differential operator with constant coefficients and $\Delta u = \Phi$ is called
 - the non-homogeneous equation associated with Δ .
 - Δ is a second order linear homogeneous partial differential operator with constant coefficients and $\Delta u = \Phi$ is called
 - the non-homogeneous equation associated with Δ .
 - Δ is a second order linear homogeneous partial differential operator with constant coefficients and $\Delta u = \Phi$ is called
 - the non-homogeneous equation associated with Δ .

definition Δ is called a second order linear homogeneous partial differential operator with constant coefficients and $\Delta u = 0$ is called

Δ

definition Δ is called a second order linear homogeneous partial differential operator with constant coefficients and $\Delta u = \Phi$ is called

definition Δ is called a second order linear homogeneous partial differential operator with constant coefficients and $\Delta u = \Phi$ is called

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3.

Chapter V

Theoretically Possible Patterns

5.1 Starting again with equation (4.05) page :

$$\underline{E}(x) = G a^2 \sum_{k=0}^{\infty} I_{2k} \int_0^1 J_0(xz) z^{2k+1} dz \quad (4.05)$$

Since $J_0(xz) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! m!} \left(\frac{x}{2}\right)^{2m} z^{2m}$ (7)

$$\underline{E}(x) = G a^2 \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \frac{I_{2k} (-1)^m}{m! m!} \left(\frac{x}{2}\right)^{2m} \int_0^1 z^{2k+2m+1} dz$$

$$= G a^2 \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \frac{I_{2k} (-1)^m}{m! m!} \left(\frac{x}{2}\right)^{2m} \frac{z^{2k+2m+2}}{2k! 2m+2} \Big|_{z=0}^{z=1}$$

$$= \frac{G a^2}{2} \sum_{k=0}^{\infty} \sum_{m=0}^{\infty} \frac{I_{2k} (-1)^m}{m! m! (k+m+1)} \left(\frac{x}{2}\right)^{2m} \quad (5.01)$$

If now $\underline{E}(x)$ is expressed in series form in powers of $(x/2)^{2n}$ $n = 0, 1, 2, 3..$ we may equate coefficients of $(x/2)^{2n}$ and solve for the I_{2k} .

5.2 This method permits a theoretical solution whenever the field distribution may be expressed as a power series and is more general than the method given in Chapter II. However, the real worth of this method is that it provides a rather easy means of investigating a proposed pattern to determine if it is theoretically possible. This may be shown as follows.

5.3 Let $G a^2 A$ be the coefficient of $(x/2)^{2n}$ of the proposed pattern, then equating coefficients from equation (5.01)

$$A = \frac{1}{2} \sum_{k=0}^{\infty} I_{2k} \frac{(-1)^m}{m! m!} \frac{1}{(k+m+1)}$$

$$\frac{2m! m!}{(-1)^m} A = \sum_{k=0}^{\infty} \frac{I_{2k}}{k+m+1} = \frac{1}{m+1} \sum_{k=0}^{\infty} \frac{I_{2k}}{1+k/m+1} \quad \text{or}$$

$$2(m+1)! m! (-1)^m A = \sum_{k=0}^{\infty} \frac{I_{2k}}{1+k/m+1} \quad (5.02)$$

$\frac{1}{(1+uv+s)} \sum_{n=0}^{\infty} s^n u^n v^n$

differentiate with respect to s and then put $s=0$

then we get $\frac{d}{ds} \left[\frac{1}{(1+uv+s)} \sum_{n=0}^{\infty} s^n u^n v^n \right] = \frac{1}{(1+uv)^2}$

$$\frac{1}{(1+uv)^2} \sum_{n=0}^{\infty} (uv)^n \frac{d}{ds} \left[\sum_{n=0}^{\infty} s^n u^n v^n \right] = (uv) \frac{d}{ds} \sum_{n=0}^{\infty} s^n u^n v^n$$

$$(uv) \frac{d}{ds} \left[\frac{1}{(1+uv)} \sum_{n=0}^{\infty} s^n u^n v^n \right] = (uv) \frac{d}{ds} \sum_{n=0}^{\infty} s^n u^n v^n$$

$$\frac{1}{(1+uv)^2} \sum_{n=0}^{\infty} (uv)^n \frac{d}{ds} \left[\frac{1}{(1+uv)} \sum_{n=0}^{\infty} s^n u^n v^n \right] = (uv) \frac{d}{ds} \sum_{n=0}^{\infty} s^n u^n v^n$$

$$\frac{1}{(1+uv)^3} \sum_{n=0}^{\infty} (uv)^n \frac{d}{ds} \left[\frac{(uv)^n}{(1+uv)} \sum_{n=0}^{\infty} s^n u^n v^n \right] = (uv) \frac{d}{ds} \sum_{n=0}^{\infty} s^n u^n v^n$$

$$(uv) \frac{d}{ds} \left[\frac{1}{(1+uv)^2} \sum_{n=0}^{\infty} (uv)^n \sum_{n=0}^{\infty} s^n u^n v^n \right] = (uv) \frac{d}{ds} \sum_{n=0}^{\infty} s^n u^n v^n$$

(1)

$$\frac{1}{(1+uv+s)} \frac{d}{ds} \left[\frac{1}{(1+uv)} \sum_{n=0}^{\infty} s^n u^n v^n \right] = A$$

$$\frac{1}{(1+uv+s)} \frac{d}{ds} \left[\frac{1}{(1+uv)} \sum_{n=0}^{\infty} s^n u^n v^n \right] = A \frac{1}{(1+uv)} \frac{d}{ds} \sum_{n=0}^{\infty} s^n u^n v^n$$

$$\frac{d}{ds} \left[\frac{1}{(1+uv+s)} \sum_{n=0}^{\infty} s^n u^n v^n \right] = A \frac{1}{(1+uv)} \frac{d}{ds} \sum_{n=0}^{\infty} s^n u^n v^n$$

If we consider a finite number of I_{2k} , and the I_{2k} are bounded, which they must be for the aperture distribution to be possible in practice, then as m increases towards infinity we may write equation (5.02) as

$$\frac{2(m+1)!}{m \rightarrow \infty} (m)! (-1)^m A = \sum_{k=0}^B I_{2k} \text{ where } B \text{ is finite} \quad (5.03)$$

In this expression for the I_{2k} their sum will remain finite only if A is of such form that it cancels the $(m+1)! m! (-1)^m$ on the left. This requires that $E(x)$ be a sum of Bessel Functions or similar functions, or combination thereof, and only in special cases will the distant space distribution not be periodic.

5.4 The requirement that $E(x)$ be expressible in terms of Bessel Functions is not in itself sufficient to establish that such a pattern is theoretically possible. Only a sum of single Bessel or similar functions will satisfy the requirements that the coefficients of A in equation (5.03) cancel the factorial expression $(m+1)! m! (-1)^m$

5.6 As an example consider the field vector pattern

$$E(x) = \bar{I} G a^2 \frac{J(x)}{x} e^{-(\frac{\alpha x}{2})^2} \quad (5.04)$$

Such a pattern would permit the reduction of side lobes to as small a value as desired if such a pattern were possible. It is known to be impossible in practice and it may be shown to be impossible in theory by applying the method given above. To do this

let

$$\begin{aligned} J_1(x) &= \sum_{m=0}^{\infty} \frac{(-1)^m}{2m!(m+1)!} \left(\frac{x}{2}\right)^{2m} \\ e^{-(\frac{\alpha x}{2})^2} &= \sum_{j=0}^{\infty} \frac{(-1)^j \alpha^{2j}}{j!} \left(\frac{x}{2}\right)^{2j} \\ E(x) &= \frac{\bar{I} G a^2}{2} \sum_{m=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-1)^{m+j} \alpha^{2j}}{j! m! (m+1)!} \left(\frac{x}{2}\right)^{2(m+j)} \end{aligned} \quad (5.05)$$

let $m+j=n$ and equate coefficients of $(x/2)^{2n}$

equation converges for $|z| < \infty$ and $\Re s > 0$. By putting $s = m + it$, we get the following result.

$$\text{Residue at } s = m + it: \sum_{n=0}^{\infty} \frac{(-1)^n (m+n)! (m+n)!}{(n+m)! n!} e^{(m+n)t}$$

Final result: $\Gamma(s) = \sum_{n=0}^{\infty} \frac{(-1)^n (m+n)! (m+n)!}{(n+m)! n!} e^{(m+n)t} \prod_{k=1}^m \frac{1}{k - s}$

Integration path: γ is a contour consisting of the real axis from $-R$ to R and a vertical line from R to $R + i\pi$.

Contour integration: $\int_{\gamma} f(z) dz = \int_{\gamma} \frac{e^{zt}}{z^m} dz = \int_{\gamma} \frac{e^{zt}}{(z-m)(z-m+1)\dots(z-m+m)} dz$

Residues: $\text{Res}(f, m) = \lim_{z \rightarrow m} (z-m) f(z) = \lim_{z \rightarrow m} \frac{e^{zt}}{(z-m+1)\dots(z-m+m)}$

$= \frac{e^{mt}}{(m+1)\dots(m+m)} = \frac{e^{mt}}{m! (m+m)!} = \frac{e^{mt}}{m! (m+m)!}$

$\text{Res}(f, m+1) = \lim_{z \rightarrow m+1} (z-m-1) f(z) = \lim_{z \rightarrow m+1} \frac{e^{zt}}{(z-m)\dots(z-m+m+1)}$

$= \frac{e^{(m+1)t}}{(m+1)\dots(m+1+m)} = \frac{e^{(m+1)t}}{(m+1)! (m+1+m)!} = \frac{e^{(m+1)t}}{(m+1)! (m+1+m)!}$

$$m! (m+m)! = m! (m+m)!$$

$$\left(\frac{e^t}{t}\right)^m \cdot \frac{1}{m!} \frac{d^m}{dt^m} \left(\frac{e^{zt}}{z^m}\right) = \frac{1}{m!} \frac{d^m}{dt^m} \left(\frac{e^{zt}}{t^m}\right)$$

Integration: $\int_{\gamma} f(z) dz = \int_{\gamma} \frac{e^{zt}}{z^m} dz = \int_{\gamma} \frac{e^{zt}}{(z-m)(z-m+1)\dots(z-m+m)} dz$

Residues: $\text{Res}(f, m) = \lim_{z \rightarrow m} (z-m) f(z) = \lim_{z \rightarrow m} \frac{e^{zt}}{(z-m+1)\dots(z-m+m)}$

$= \frac{e^{mt}}{(m+1)\dots(m+m)} = \frac{e^{mt}}{m! (m+m)!} = \frac{e^{mt}}{m! (m+m)!}$

$\text{Res}(f, m+1) = \lim_{z \rightarrow m+1} (z-m-1) f(z) = \lim_{z \rightarrow m+1} \frac{e^{zt}}{(z-m)\dots(z-m+m+1)}$

$= \frac{e^{(m+1)t}}{(m+1)\dots(m+1+m)} = \frac{e^{(m+1)t}}{(m+1)! (m+1+m)!} = \frac{e^{(m+1)t}}{(m+1)! (m+1+m)!}$

$m! (m+m)! = m! (m+m)!$

$$\left(\frac{e^t}{t}\right)^m \cdot \frac{1}{m!} \frac{d^m}{dt^m} \left(\frac{e^{zt}}{z^m}\right) = \frac{1}{m!} \frac{d^m}{dt^m} \left(\frac{e^{zt}}{t^m}\right)$$

$$\left(\frac{e^t}{t}\right)^{m+1} \cdot \frac{1}{(m+1)!} \frac{d^{m+1}}{dt^{m+1}} \left(\frac{e^{zt}}{z^{m+1}}\right) = \frac{1}{(m+1)!} \frac{d^{m+1}}{dt^{m+1}} \left(\frac{e^{zt}}{t^{m+1}}\right)$$

$$\left(\frac{e^t}{t}\right)^{m+2} \cdot \frac{1}{(m+2)!} \frac{d^{m+2}}{dt^{m+2}} \left(\frac{e^{zt}}{z^{m+2}}\right) = \frac{1}{(m+2)!} \frac{d^{m+2}}{dt^{m+2}} \left(\frac{e^{zt}}{t^{m+2}}\right)$$

$$m! (m+m)! = m! (m+m)!$$

$$\frac{\bar{I} Ga^2}{2} \frac{(-1)^m \alpha^{2j}}{j! m! (m+1)!} = \frac{Ga^2 (-1)^m}{m! m!} \sum_{k=0}^{\infty} \frac{I_{2k}}{2k+2m+2}$$

or for m very large $\bar{I} \frac{\alpha^{2j} (m+1) m!}{j! m! (m+1)!} = \sum_{k=0}^{\infty} I_{2k}$

let $m=0$ then $j=n$ and

$$I \alpha^{2n} (n+1)! = \sum_{k=0}^{\infty} I_{2k} \quad \text{and the } I_{2k} \rightarrow \infty \text{ as } n \rightarrow \infty$$

This is obvious for $\alpha > 1$. For $\alpha < 1$ let $\alpha = \frac{1}{\beta}$, $\beta > 1$.

and consider the ratio of the n^{th} and $(n+1)^{\text{th}}$ terms. We have

$$n^{\text{th}} \text{ term} = \bar{I} \frac{(n+1)!}{\beta^{2n}} \quad \text{and the } (n+1)^{\text{th}} \text{ terms} = \bar{I} \frac{(n+2)!}{\beta^{2(n+1)}}$$

$$\text{Their ratio will be } \frac{\bar{I} \frac{(n+2)!}{\beta^{2(n+1)}}}{\bar{I} \frac{(n+1)!}{\beta^{2n}}} = \frac{n+2}{\beta^2}$$

and as $n \rightarrow \infty$ this ratio becomes infinitely large, and the $I_{2k} \rightarrow \infty$.

Hence, we conclude that the pattern $E(x) = \bar{I} Ga^2 \frac{J_1(x)}{x} e^{-\left(\frac{\alpha x}{2}\right)^2}$
is impossible in theory as well as in practice.

5.6 In a similar manner it may be shown that products of Bessel Functions, such as $\frac{J_1(x)}{x} \frac{J_2(x)}{x^2}$ are theoretically impossible.

$$\text{Again use } \frac{J_1(x)}{x} = \sum_{m=0}^{\infty} \frac{(-1)^m}{2m! (m+1)!} \left(\frac{x}{2}\right)^{2m}$$

$$\text{and } \frac{J_2(x)}{x^2} = \sum_{j=0}^{\infty} \frac{(-1)^j}{4j! (j+2)!} \left(\frac{x}{2}\right)^{2j} \quad (5.06)$$

$$\text{then } \frac{J_1(x)}{x} \frac{J_2(x)}{x^2} = \sum_{j=0}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^{m+j}}{8m! j! (m+1)! (j+2)!} \left(\frac{x}{2}\right)^{2(m+j)} \quad (5.07)$$

and for $m+j=n$ we may equate the coefficients of $(x/2)^{2n}$.

This gives

$$\frac{\bar{I} Ga^2}{2} \frac{(-1)^n}{8m! j! (m+1)! (j+2)!} = \frac{Ga^2 (-1)^n}{m! m!} \sum_{k=0}^{\infty} \frac{I_{2k}}{2k+2m+2n}$$

$$\frac{g_s I}{s+vn} = \sum_{n=0}^{\infty} \frac{(1)^n}{(vn)!} I^n = \frac{s^2 \times v^n (1)}{(1+vn)!} \frac{s^{vn}}{v^n}$$

$$g_s I = \frac{vn (1+vn)^{s^2} I}{(1+vn)!} = \text{un real value}$$

$$= \text{real value}$$

$$\sum_{n=0}^{\infty} \frac{(1+vn)^{s^2} I}{(vn)!} = (1+vn)^{s^2} I$$

$$1 < \Re s, \quad \Re s = \infty, \quad 1 > \Re s, \quad 1 < \Re s$$

$$\frac{1}{(1+vn)^s} I = \quad + \quad \frac{1}{(1+vn)^s} I =$$

$$\frac{1}{(1+vn)^s} = \frac{v^n}{(1+vn)^s} I = \frac{1}{(1+vn)^s} I$$

$$\frac{1}{(1+vn)^s} = \frac{(vn)^s}{s!} \frac{v^n}{(vn)^s} I =$$

$$\frac{(x)_s}{s!} \frac{(x)_v}{v}$$

$$\sin x \frac{(x)_s}{s!} \frac{(x)_v}{v} = \frac{(x)_v}{v}$$

$$\frac{1}{(1+vn)^s} \frac{(x)_s}{s!} \frac{1}{(1+vn)^v} \frac{(x)_v}{v!} = \frac{(x)_v}{v}$$

$$\frac{1}{(1+vn)^s} \frac{(x)_s}{s!} \frac{1}{(1+vn)^v} \frac{(x)_v}{v!} = \frac{(x)_v}{v} \frac{(x)_v}{v}$$

$$\frac{1}{(1+vn)^s} \frac{(x)_s}{s!} \frac{1}{(1+vn)^v} \frac{(x)_v}{v!} = \frac{(x)_v}{v} \frac{(x)_v}{v}$$

$$\text{or } \frac{\bar{I}}{8} \frac{(mv+1)! mv!}{mv! j! (mv+1)! (j+2)!} = \sum_{k=0}^{\infty} \frac{I_{2k}}{1+k/m+1} \quad (5.08)$$

Let $j = m$, then $n = 2m$ and

$$\frac{\bar{I}}{8} \frac{(2mv+1)! (2mv)!}{mv! mv! (mv+1)! (mv+2)!} = \sum_{k=0}^{\infty} I_{2k} \quad (5.09)$$

$m \rightarrow \infty$ $m \rightarrow \infty$
An inspection of the ratio of n^{th} and the $(n+1)^{\text{th}}$ term shows that the

$I_{2k} \rightarrow \infty$ since

$$\frac{(mv+1)^{\text{th}}}{m^{\text{th}}} = \frac{(2mv+2)! (2mv+1)!}{mv! (mv+1)! (mv+1)! (mv+3)!} \frac{mv! mv! (mv+1)! (mv+2)!}{(2mv+1)! (2mv)!}$$

$$\underset{m \rightarrow \infty}{=} \frac{(2mv+2)(2mv+1)}{(mv+1)(mv+3)} = \frac{(2 + \frac{2}{m})(2 + \frac{1}{m})}{(1 + \frac{1}{m})(1 + \frac{3}{m})} = 4$$

and we conclude that the pattern $\underline{J}(x) \sim \frac{J_1(x)}{x} \frac{J_2(x)}{x^2}$ is impossible.

$$(6.2) \quad \frac{\zeta(s)}{1+vn} \sum_{n=0}^{\infty} = \frac{1}{(s+\gamma)} \frac{1}{(1+vn)} \frac{1}{\gamma!} \frac{1}{vn!} \frac{1}{s!}$$

$$(6.3) \quad \frac{\zeta(s)}{1+vn} \sum_{n=0}^{\infty} = \frac{1}{(s+vn)} \frac{1}{(1+vn)} \frac{1}{vn!} \frac{1}{vn!} \frac{1}{s!}$$

$$\frac{1}{(s+vn)!} \frac{1}{(1+vn)!} \frac{1}{vn!} \frac{1}{vn!} = \frac{1}{(s+vn)!} \frac{1}{(1+vn)!} \frac{1}{vn!} \frac{1}{vn!} = \frac{ds}{dvn} \frac{1}{vn}$$

$$\frac{(vn+s)(vn+s+1)}{(vn)^2 + 1} \frac{1}{vn!} = \frac{(1+vn+s)(1+vn+s+1)}{(1+vn)^2 + 1} \frac{1}{vn!}$$

$$\frac{(x)_s}{s!} \frac{(x)_s}{s!} \sim$$

Chapter VI

Solution for Aperture Distributions Which are Independent of Radius

6.1 A method of solving for $I(\hat{p}, a)$ in equation (3.11) page when $I(\hat{p}, a) = I(\hat{p})$, that is, I is not a function of "a" may be readily obtained as follows.

$$\underline{E}(x) = \underline{E}(ka \sin \theta) = G \int_0^a I(\hat{p}) J_0(k \hat{p} \sin \theta) \hat{p} d\hat{p} \quad (6.01)$$

$$\frac{d}{da} \underline{E}(x) = G \left\{ \int_0^a \frac{d}{da} [I(\hat{p}) J_0(k \hat{p} \sin \theta) \hat{p}] d\hat{p} + I(a) J_0(ka \sin \theta) a \right\} \quad (6.02)$$

Since the integrand of the integral is independent of "a" its derivative will be zero and hence the value of the integral will also be zero and

$$\frac{d}{da} \underline{E}(x) = G a I(a) J_0(ka \sin \theta)$$

$$\text{Hence, } I(a) = \frac{\frac{d}{da} \underline{E}(ka \sin \theta)}{G a J_0(ka \sin \theta)} \quad (6.03)$$

Example 1: for $\underline{E}(x) = G \bar{I} a^2 \frac{J_1(x)}{x}$ we have

$$I(a) = \frac{G \bar{I} \frac{d}{da} a^2 \frac{J_1(ka \sin \theta)}{ka \sin \theta}}{G a J_0(ka \sin \theta)} = \frac{\bar{I} a J_0(ka \sin \theta)}{a J_0(ka \sin \theta)} = \bar{I}$$

Hence,

$$I(\hat{p}) = \bar{I} \text{ and } I(\hat{p}, a) = \bar{I} i$$

Example 2: for $\underline{E}(x) = \bar{I} G a^4 \left(\frac{J_1(x)}{x} - \frac{2 J_2(x)}{x^2} \right)$ we have

$$\begin{aligned} \frac{d}{da} \underline{E}(ka \sin \theta) &= \frac{d}{da} \bar{I} G \left[a^2 \frac{(ka \sin \theta) J_0(ka \sin \theta)}{(k \sin \theta)^2} - \frac{2}{(k \sin \theta)^4} (ka \sin \theta)^2 J_2(ka \sin \theta) \right] \\ &= \bar{I} G \left[a^2 \frac{(ka \sin \theta) J_0(ka \sin \theta) (k \sin \theta)}{(k \sin \theta)^2} + 2 a^2 \frac{J_1(ka \sin \theta)}{k \sin \theta} \right. \\ &\quad \left. - \frac{2 (ka \sin \theta)^2}{(k \sin \theta)^4} J_2(ka \sin \theta) k \sin \theta \right] \end{aligned}$$

PROBLEMS

Explain the following terms:
 (a) $\lim_{n \rightarrow \infty} a_n = L$

(b) $\lim_{n \rightarrow \infty} a_n = L$, if for every $\epsilon > 0$, there exists a natural number N such that for all $n > N$, $|a_n - L| < \epsilon$.

$$(c) \lim_{n \rightarrow \infty} a_n = L \text{ if } \forall \epsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } |a_n - L| < \epsilon \text{ for all } n > N.$$

$$(d) \lim_{n \rightarrow \infty} a_n = L \text{ if } \forall \epsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } |a_n - L| < \epsilon \text{ for all } n > N.$$

Explain the following terms:
 (a) $\lim_{n \rightarrow \infty} a_n = L$

(b) $\lim_{n \rightarrow \infty} a_n = L$

$$\lim_{n \rightarrow \infty} a_n = L \text{ if } \forall \epsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } |a_n - L| < \epsilon \text{ for all } n > N.$$

$$\lim_{n \rightarrow \infty} a_n = L$$

$$\lim_{n \rightarrow \infty} a_n = L \text{ if } \forall \epsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } |a_n - L| < \epsilon \text{ for all } n > N.$$

$$\lim_{n \rightarrow \infty} a_n = L \text{ if } \forall \epsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } |a_n - L| < \epsilon \text{ for all } n > N.$$

$$\lim_{n \rightarrow \infty} a_n = L \text{ if } \forall \epsilon > 0, \exists N \in \mathbb{N} \text{ s.t. } |a_n - L| < \epsilon \text{ for all } n > N.$$

Explain the following terms:
 (a) $\lim_{n \rightarrow \infty} a_n = L$

(b) $\lim_{n \rightarrow \infty} a_n = L$

$\lim_{n \rightarrow \infty} a_n = L$

$$= \overline{I} G [a^3 J_0(ka \sin \theta)]$$

and

$$I(a) = \frac{\overline{I} G a^3 J_0(ka \sin \theta)}{G a J_0(ka \sin \theta)} = a^2$$

which may

be readily verified by direct integration of equation (6.01) for $I(\hat{\rho}) = \hat{\rho}^2$.

6.2 While this method is quite limited in practice it is easy to check a possible solution this way and if the expression for $I(a)$ in equation (6.03) results in a solution for I which is independent of θ the problem is readily solved. The space patterns which will give such a solution are those which arise from integration of equation (6.01).

$$\left[(\sin \omega t) \frac{d}{dt} \right] \underline{\mathbf{I}} =$$

$$\underline{\mathbf{I}}_D = \frac{(\sin \omega t) \frac{d}{dt} \underline{\mathbf{A}} \underline{\mathbf{I}}}{(\sin \omega t) \underline{\mathbf{A}} \underline{\mathbf{I}}} = (\omega) \underline{\mathbf{I}}$$

- $\hat{\underline{\mathbf{I}}} = \hat{\underline{\mathbf{I}}}' + (\omega) \underline{\mathbf{I}}$ where the second term is the steady-state component and
 $\underline{\mathbf{I}}'$ is the transient component due to initial conditions.
- At time $t=0$ the initial current in the primary coil is zero. At time $t=0$, the
ans 6 current in the primary coil is zero. At time $t=0$, the initial current in the primary
coil is zero. At time $t=0$, the initial current in the primary coil is zero.

Chapter VII

Conclusions

7.1 By following the accepted field equations of Maxwell and retaining actual limits of integration a method has been developed permitting correlation of the distant field with a circularly symmetric aperture distribution. This is in distinction to earlier methods based on Fourier transforms in which the limits of integration were extended to infinity, and the results of which were not in general realistic. The method developed here relies on the Λ functions and gives realistic expressions for the lobe structure of distant space patterns. Another valuable result of this work is the definition of the types of pattern that are physically possible, consistent with electromagnetic theory.

7.2 The method developed gives a close insight into the dependence of beam width, and therefore gain of the pattern, on the tolerance of lobe levels. As the pattern functions are made to converge to lobeless patterns the corresponding aperture distributions are found to converge toward a point source centrally located on the aperture. The relatively narrow and low lobe patterns obtained in better radar equipment are found to correspond to certain types of hollow aperture distributions which correspond closely to those obtained in this analysis.

7.3 All possible circularly symmetric amplitude distributions over an aperture set up distant field distributions that are expressible as sums of Λ functions. The absence of a Fourier - Bessel Λ Function method of developing pattern expressions is a distinct limitation to this approach to the problem.

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zakynthos, avond 12/12 al vroeg van vadum en
naar west houtwalen op 12/12 25 mds. zandvloer
zandvlakte, zand niet, laagd rijk moest en
laagd rijk dicht en heel har 2201 al
al, gral al achterhoek sand 2201 al vloedvaat
holten al vloed vloed de palmen en zand
zandvlakte dicht en holten op 13/12 nooit
oef har 2201 - 2202 laagd staande groen groen
2201 - 2202 zandvloed en lage sand

Appendix I
Table of Symbols (MKS units)

Symbol	Definition
a	radius of aperture
area 1	area within aperture
area 2	area outside aperture in plane of aperture
c	velocity of light $c = \frac{1}{\sqrt{\mu\epsilon}}$
C	contour bounding aperture
da	differential element of area 1. $da = \hat{\rho} d\hat{\rho} d\hat{\phi}$
ds	vector differential element of length about contour C.
<u>E(x',y',z')</u>	electric vector within volume V.
<u>E(x)</u>	electric field vector distribution in distant space
F	directrix, time and range factor of <u>E(x)</u>
f	frequency
G	directrix, time and range factor of <u>E(x)</u>
<u>H(x',y',z')</u>	magnetic vector within volume V
<u>H(x)</u>	magnetic field vector distribution in distant space
i	square root of minus 1
<u>i</u>	unit vector in \hat{x} direction
<u>I($\hat{\rho}$,a)</u>	electric field vector distribution over aperture
<u>I(z²)</u>	amplitude distribution of electric field vector over aperture
I	amplitude of <u>I($\hat{\rho}$,a)</u> for constant amplitude distribution
<u>j</u>	unit vector in \hat{y} direction
J _n	Bessel function of first kind, integral order n
<u>J, J*</u>	current density
k	phase constant
<u>k</u>	unit vector in z direction

7. *Introduction*

(*Continued from page 6*)

Definitions. *Definitions* are statements which define words or terms. They are used to give the meaning of words and to make clear the meaning of statements. Definitions are of two kinds: *simple definitions* and *extended definitions*.

Simple Definitions. A simple definition is one which defines a word by giving its meaning in other words. It is a statement which gives the meaning of a word in such a way that the meaning of the word can be easily understood. For example, if we say "A book is a collection of printed pages bound together," we are giving a simple definition of the word "book".

Extended Definitions. An extended definition is one which defines a word by giving its meaning in such a way that the meaning of the word can be easily understood. For example, if we say "A book is a collection of printed pages bound together, and it is used for reading or writing," we are giving an extended definition of the word "book".

Types of Definitions. Definitions can be classified into three types: *simple definitions*, *extended definitions*, and *definitive definitions*.

Simple Definitions. Simple definitions are those which define words by giving their meanings in other words. They are used to give the meaning of words and to make clear the meaning of statements. Definitions are of two kinds: *simple definitions* and *extended definitions*.

Extended Definitions. Extended definitions are those which define words by giving their meanings in such a way that the meaning of the word can be easily understood. For example, if we say "A book is a collection of printed pages bound together, and it is used for reading or writing," we are giving an extended definition of the word "book".

Definitive Definitions. Definitive definitions are those which define words by giving their meanings in such a way that the meaning of the word can be easily understood. For example, if we say "A book is a collection of printed pages bound together, and it is used for reading or writing," we are giving a definitive definition of the word "book".

Symbol

$\underline{K}, \underline{K}^*$	electric and magnetic current surface densities
$M(\hat{\rho}, a)$	magnetic field vector distribution over aperture
M	amplitude of $M(\hat{\rho}, a)$ for constant amplitude distribution
$M(z^2)$	amplitude distribution of magnetic field vector over aperture
\underline{n}	unit vector perpendicular to aperture in direction of radiation
R, θ, φ	space polar coordinate system $\underline{R}_0, \underline{\theta}_0, \underline{\varphi}_0$ - unit vectors
\underline{R}	position vector from center of aperture to point (R, θ, φ)
$\underline{\Sigma}$	position vector from point (\hat{x}, \hat{y}) in aperture to point (R, θ, φ)
\underline{z}_0	unit vector in R direction
x	$x \equiv ka \sin \theta$
$\hat{x}, \hat{y}, \hat{z}$	rectangular coordinate system $\underline{i}, \underline{j}, \underline{k}$ - unit vectors
z	$z \equiv \hat{\rho}/a$
β	angle between \underline{R} and $\hat{\rho}$
ϵ	permittivity of space
η	impedance of space $\eta = \sqrt{\frac{\mu}{\epsilon}}$ for conductance 0
$\underline{\theta}_0$	unit vector in θ direction
λ	free space wavelength
Λ_p	Lambda function of integral order p
μ	permeability of space
$\rho\rho^*$	charge density
$\hat{\rho}$	radius vector in aperture
Γ, Γ^*	line densities of electric and magnetic charge
$\hat{\varphi}$	angular measure in aperture
$\underline{\varphi}_0$	unit vector in φ direction

प्राचीन

संक्षिप्त रूप से विवरित करना चाहिए
विवरण यह करिता है कि विवरण
को निम्नलिखित ग्रन्थों में दिया गया

१२

१३

केवल विवरण के अन्तर्गत विवरण के अन्तर्गत

१४

विवरण के अन्तर्गत विवरण के अन्तर्गत विवरण

१५

विवरण के अन्तर्गत विवरण के अन्तर्गत विवरण

१६

(१०८) विवरण के अन्तर्गत विवरण

१७

(१०९) विवरण के अन्तर्गत विवरण

१८

विवरण के अन्तर्गत विवरण

१९

विवरण के अन्तर्गत

२०

विवरण के अन्तर्गत विवरण

२१

विवरण

२२

विवरण के अन्तर्गत

२३

विवरण के अन्तर्गत

२४

विवरण के अन्तर्गत

२५

विवरण के अन्तर्गत

२६

विवरण के अन्तर्गत

२७

विवरण के अन्तर्गत

२८

विवरण के अन्तर्गत

२९

विवरण के अन्तर्गत

३०

विवरण के अन्तर्गत

३१

विवरण के अन्तर्गत

३२

Symbol

Definition

 ϕ

$$\phi \equiv \frac{e^{-ikr}}{r}$$

 $\dot{\phi}$

energy at point in distant field

 χ angle between R and i ω angular velocity $\omega = 2\pi f$

0

Definition

$$\frac{d\theta}{dt} = \dot{\theta}$$

Angular
velocity

Angular
displacement

$$\theta = \theta(t) \quad \text{Angular displacement}$$

Appendix II
Vector Identities

- (1) $\underline{a} \times (\underline{b} \times \underline{c}) = (\underline{a} \cdot \underline{c})\underline{b} - (\underline{a} \cdot \underline{b})\underline{c}$
- (2) $\nabla(\phi + \psi) = \nabla\phi + \nabla\psi$
- (3) $\nabla(\phi\psi) = \phi \nabla\psi + \psi \nabla\phi$
- (4) $\nabla \cdot (\underline{a} + \underline{b}) = \nabla \cdot \underline{a} + \nabla \cdot \underline{b}$
- (5) $\nabla \times (\underline{a} + \underline{b}) = \nabla \times \underline{a} + \nabla \times \underline{b}$
- (6) $\nabla \cdot (\phi \underline{a}) = \underline{a} \cdot \nabla\phi + \phi \nabla \cdot \underline{a}$
- (7) $\nabla \times (\phi \underline{a}) = \nabla\phi \times \underline{a} + \phi \nabla \times \underline{a}$
- (8) $\nabla(\underline{a} \cdot \underline{b}) = (\underline{a} \cdot \nabla)\underline{b} + (\underline{b} \cdot \nabla)\underline{a} + \underline{a} \times (\nabla \times \underline{b}) + \underline{b} \times (\nabla \times \underline{a})$
- (9) $\nabla \cdot (\underline{a} \times \underline{b}) = \underline{b} \cdot \nabla \times \underline{a} - \underline{a} \cdot \nabla \times \underline{b}$
- (10) $\nabla \times (\underline{a} \times \underline{b}) = \underline{a} \nabla \cdot \underline{b} - \underline{b} \nabla \cdot \underline{a} + (\underline{b} \cdot \nabla)\underline{a} - (\underline{a} \cdot \nabla)\underline{b}$
- (11) $\nabla \times \nabla \times \underline{a} = \nabla \nabla \cdot \underline{a} - \nabla^2 \underline{a}$
- (12) $\nabla \times \nabla \phi \equiv 0$
- (13) $\nabla \cdot \nabla \times \underline{a} \equiv 0$

$$\begin{aligned}
 & \underline{\underline{C}}(\underline{\underline{A}} \cdot \underline{\underline{B}}) - \underline{\underline{B}}(\underline{\underline{C}} \cdot \underline{\underline{A}}) = (\underline{\underline{C}} \times \underline{\underline{A}}) \times \underline{\underline{B}} \quad (1) \\
 & \underline{\underline{A}} \nabla + \underline{\underline{B}} \nabla = (\underline{\underline{A}} + \underline{\underline{B}}) \nabla \quad (2) \\
 & \underline{\underline{A}} \nabla \underline{\underline{A}} + \underline{\underline{B}} \nabla \underline{\underline{B}} = (\underline{\underline{A}} \underline{\underline{A}}) \nabla \quad (3) \\
 & \underline{\underline{A}} \cdot \nabla + \underline{\underline{B}} \cdot \nabla = (\underline{\underline{A}} + \underline{\underline{B}}) \cdot \nabla \quad (4) \\
 & \underline{\underline{A}} \nabla \underline{\underline{B}} + \underline{\underline{B}} \nabla \underline{\underline{A}} = (\underline{\underline{A}} \underline{\underline{B}}) \times \nabla \quad (5) \\
 & \underline{\underline{A}} \cdot \nabla \underline{\underline{B}} + \underline{\underline{B}} \cdot \nabla \underline{\underline{A}} = (\underline{\underline{A}} \underline{\underline{B}}) \cdot \nabla \quad (6) \\
 & \underline{\underline{A}} \times \underline{\underline{B}} + \underline{\underline{B}} \times \underline{\underline{A}} = (\underline{\underline{A}} \underline{\underline{B}}) \times \underline{\underline{A}} \quad (7) \\
 & (\underline{\underline{A}} \times \underline{\underline{B}}) + (\underline{\underline{B}} \times \underline{\underline{A}}) + \underline{\underline{A}}(\nabla \cdot \underline{\underline{B}}) + \underline{\underline{B}}(\nabla \cdot \underline{\underline{A}}) = (\underline{\underline{A}} \underline{\underline{B}}) \nabla \quad (8) \\
 & \underline{\underline{A}} \times \underline{\underline{B}} - \underline{\underline{B}} \times \underline{\underline{A}} = (\underline{\underline{A}} \underline{\underline{B}}) \nabla \quad (9) \\
 & \underline{\underline{A}}(\nabla \times \underline{\underline{B}}) = \underline{\underline{A}} \underline{\underline{B}} + \underline{\underline{B}} \underline{\underline{A}} + (\underline{\underline{A}} \cdot \underline{\underline{B}}) \underline{\underline{B}} - (\underline{\underline{A}} \cdot \underline{\underline{B}}) \underline{\underline{A}} \quad (10) \\
 & \underline{\underline{A}} \times (\underline{\underline{B}} \times \underline{\underline{C}}) = \underline{\underline{A}} \underline{\underline{B}} \underline{\underline{C}} - \underline{\underline{A}} \underline{\underline{C}} \underline{\underline{B}} \quad (11) \\
 & \underline{\underline{A}} \underline{\underline{B}} = \underline{\underline{B}} \underline{\underline{A}} \quad (12) \\
 & \underline{\underline{A}} \underline{\underline{B}} \underline{\underline{C}} = \underline{\underline{B}} \underline{\underline{A}} \underline{\underline{C}} \quad (13)
 \end{aligned}$$

Curves and Tables

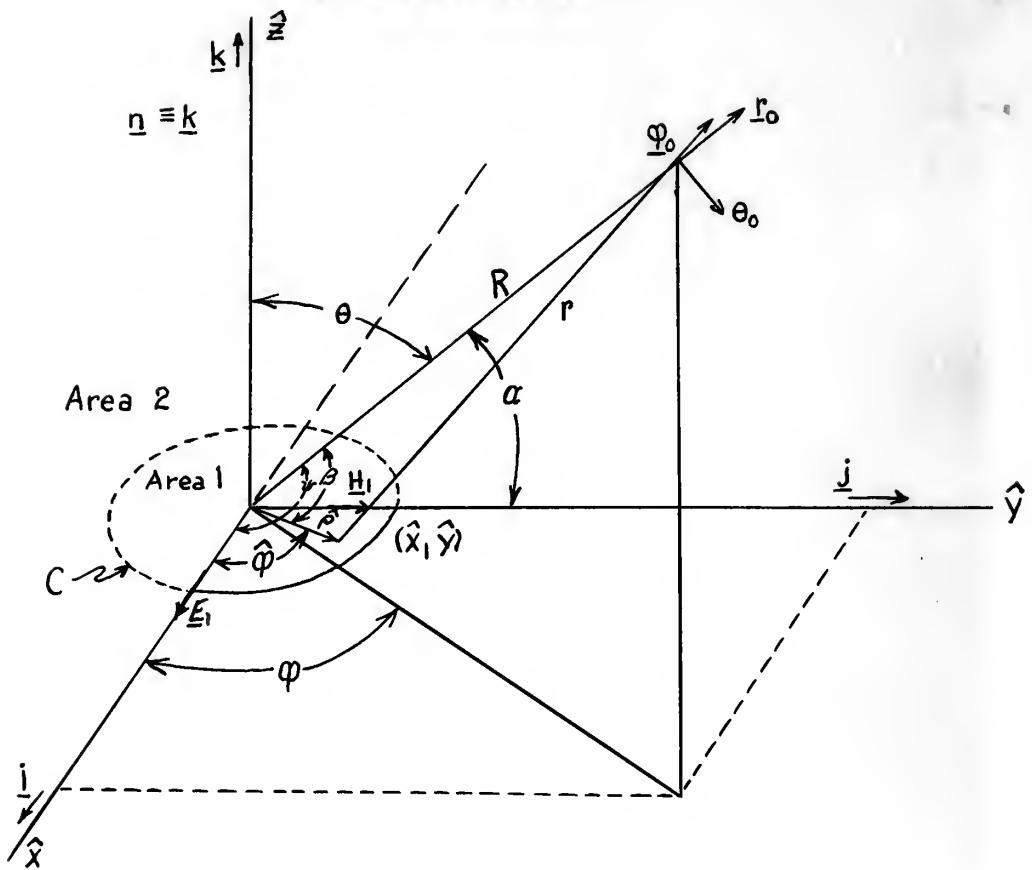


Figure 2

From Fig. 2

$$\frac{R_x}{R} = \cos \theta ; \quad \frac{R_y}{R} = \cos \alpha ; \quad \frac{R_z}{R} = \cos \varphi$$

$$R_y = R \cos \alpha = R \sin \theta \sin \varphi$$

$$R_x = R \cos \varphi = R \sin \theta \cos \varphi ; \quad \cos \varphi = \sin \theta \cos \varphi$$

$$R \cos \beta = R \sin \theta \cos(\hat{\phi} - \varphi) \quad \therefore \cos \beta = \sin \theta \cos(\hat{\phi} - \varphi)$$

$$r = \sqrt{R^2 + \hat{p}^2 - 2R\hat{p} \cos \beta} \quad \text{and for } R \gg \hat{p}$$

$$r = R \left(1 + \frac{\hat{p}^2}{R^2} - 2 \frac{\hat{p}}{R} \cos \beta \right)^{1/2} = R \left(1 - \frac{2\hat{p} \cos \beta}{R} \right)^{1/2} = R \left(1 - \frac{2\hat{p} \cos \beta}{2R} + \dots \right)$$

$$= R - \hat{p} \cos \beta = R - \hat{p} \sin \theta \cos(\hat{\phi} - \varphi)$$

Differential element of area in area 1 $\equiv da = \hat{p} d\hat{p} d\hat{\phi}$

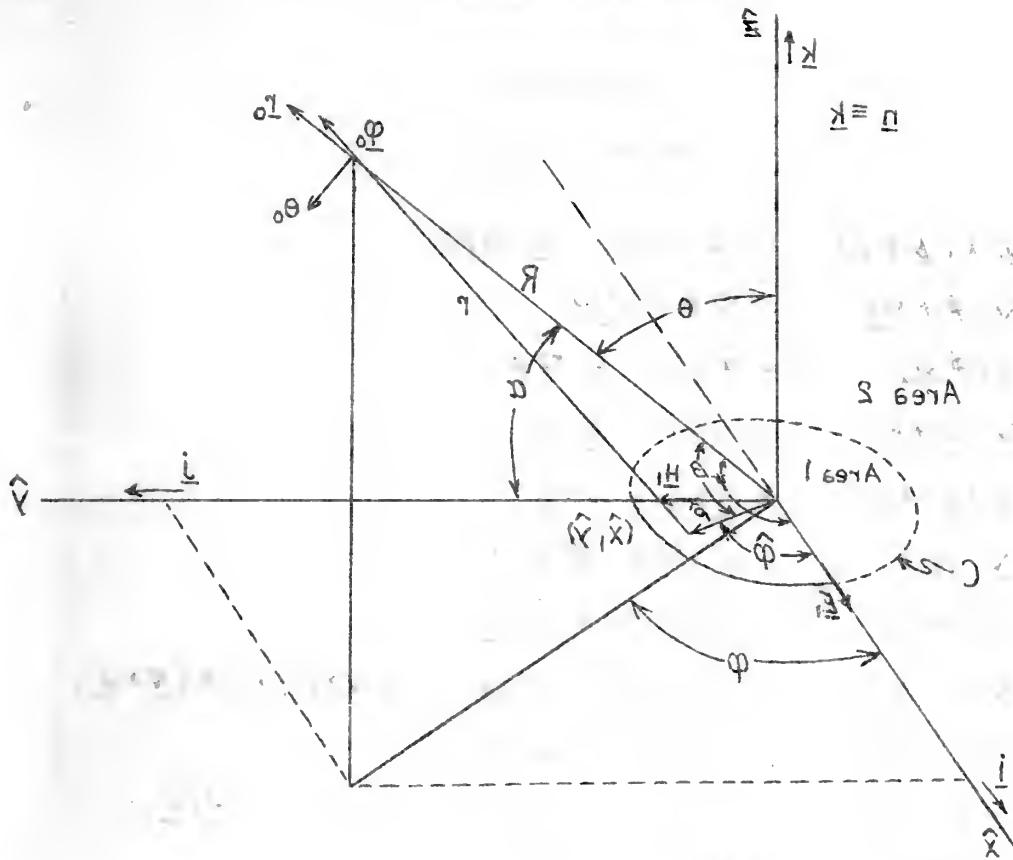


Figure 5

Figure 5

$$\frac{dx}{dt} = \cos \theta ; \quad \frac{dy}{dt} = \cos \alpha ; \quad \frac{dz}{dt} = \cos \phi$$

$$\dot{\theta} \sin \theta \sin \alpha \sin \phi = \dot{x} \cos \alpha$$

$$\dot{\theta} \sin \theta \sin \alpha \sin \phi = \dot{x} \cos \alpha \cos \phi$$

$$(\dot{\theta} - \dot{\phi}) \cos \theta \sin \alpha \sin \phi = \dot{x} \cos \alpha \cos (\phi - \theta)$$

$$\dot{\theta} \sin \theta \sin \alpha \sin \phi - \dot{\phi} \cos \theta \sin \alpha \sin \phi = \dot{x} \cos \alpha \cos (\phi - \theta)$$

$$(\dot{\theta} \sin \theta \sin \alpha - \dot{\phi} \cos \theta \sin \alpha) \sin \phi = \dot{x} \cos \alpha \cos (\phi - \theta)$$

$$(\dot{\theta} \sin \theta \sin \alpha - \dot{\phi} \cos \theta \sin \alpha) \cos \phi = \dot{x} \cos \alpha \sin (\phi - \theta)$$

FIGURE 3

$$\text{Bessel Function } \Delta\rho(X)$$

$$\Delta\rho(X) = \sum_{k=0}^{\infty} \frac{(-1)^k \rho!}{k! (\rho+k)!} \left(\frac{X}{2}\right)^{2k}$$

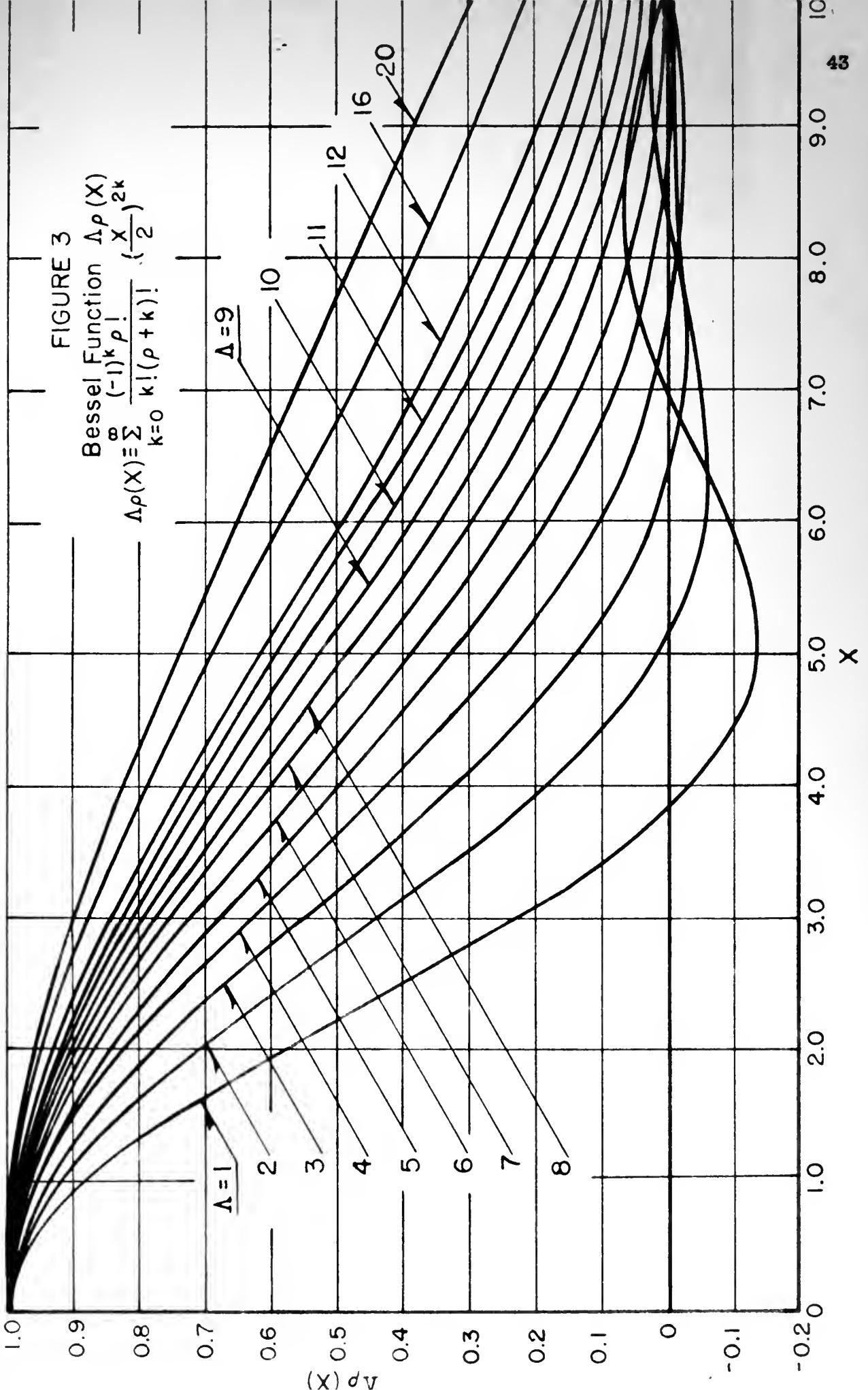




Table I. Values of $\lambda_p(\bar{x})$: $0 < p < 20$
Refer to Fig. 3

\bar{x}	λ_p	1	2	3	4	5	6	7	8	9	10	τ/ν
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.0
.5	96807	97933	98447	98756	98963	99111	99221	99308	99377	99433	99433	.5
1.0	88010	91923	93904	95103	95907	96484	96918	97257	97528	97751	97751	1.0
1.5	74392	82520	86704	89263	90993	92241	93184	93923	94517	95005		
2.0	57672	70567	77356	81590	84476	86575	88172	89428	90442	91278	91278	2.0
2.5	39768	57096	66540	72531	76684	79737	82079	83933	85438	86684		
3.0	22604	43208	54944	62594	67996	72021	75140	77629	79663	81357	81357	3.0
3.5	07850	29951	43300	52306	58813	63743	67611	70729	73296	75447		
4.0	03302	18206	32263	42169	49532	55224	59756	63452	66523	69117	69117	4.0
4.5	10269	08606	22371	32628	40520	46767	51831	56017	59535	62532		
5.0	13103	01490	14010	24037	32089	38648	44076	48634	52513	55851	55851	5.0
5.5	12416	03103	07389	16648	24486	31094	36697	41489	45625	49227		
6.0	09223	05397	02550	10597	17881	24280	29864	34741	39021	42796	42796	6.0
6.5	04734	05821	00618	05912	12363	18324	23703	28519	32823	36677		
7.0	00134	04291	02345	02524	07949	13285	18298	22913	27128	30965	30965	7.0
7.5	03607	03275	02936	00289	04587	09169	13685	17982	22003	26733		
8.0	05866	01413	02729	00988	02177	05934	09862	13248	17486	21030	21030	8.0
8.5	06426	00247	02053	01528	00581	03503	06794	10203	13591	16880		
9.0	05451	01431	01191	01554	00358	01772	04417	07316	10305	13289	13289	9.0
9.5	03395	02020	00366	01269	00801	00623	02649	05030	07598	10242		
10.0	00869	02037	00280	00843	00899	00067	01398	02281	05423	07710	07710	10.0



Table I (Continued)

χ^2/ν	11	12	13	14	15	16	17	18	19	20
0.0	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	0.0
0.5	99480	99520	99555	99584	99610	99633	99653	99672	99689	99703 0.5
1.0	97937	98094	98229	98346	98449	98540	98620	98692	98757	98816 1.0
1.5	95413	95759	96057	96315	96542	96742	96921	97081	97225	97355 1.5
2.0	91980	92576	93090	93538	93630	94278	94588	94866	95117	95345 2.0
2.5	87733	88629	89402	90077	90670	91197	91667	92090	92471	92818 2.5
3.0	82789	84016	85079	86010	86830	87560	88213	88801	89333	89816 3.0
3.5	77277	78852	80222	81425	82489	83439	84290	85058	85754	86388 3.5
4.0	71337	73259	74939	76419	77734	78910	79968	80925	81794	82588 4.0
4.5	65115	67366	69343	71095	72657	74059	75324	76471	77517	78473 4.5
5.0	58753	61299	63551	65555	67351	68970	70436	71769	72988	74106 5.0
5.5	52389	55184	57674	59903	61911	63729	65382	66892	68276	69550 5.5
6.0	46147	49136	51818	54237	56427	58421	60242	61911	63447	64866 6.0
6.5	40139	43259	46082	48647	50985	53124	55088	56897	58568	60116 6.5
7.0	34459	37643	40552	43215	45661	47912	49991	51914	53699	55358 7.0
7.5	29181	32363	35300	38014	40525	42852	45013	47024	48898	50649 7.5
8.0	24360	27477	30387	33102	35635	38001	40211	42280	44218	46037 8.0
8.5	20030	23024	25855	28525	31039	33405	35632	37729	39705	41568 8.5
9.0	16208	19029	21735	24317	26772	29103	31314	33410	35397	37280 9.0
9.5	12891	15501	18042	20498	22860	25124	27288	29355	31326	33206 9.5
10.0	10065	12435	14779	17079	19317	21483	23574	25585	27517	29372 10.0

TABLE II

Powers of $(1 - z^2)^n$ $0 < n < 20$

n	$(1 - z^2)$	$(1 - z^2)^2$	$(1 - z^2)^3$	$(1 - z^2)^4$	$(1 - z^2)^5$	$(1 - z^2)^6$
0	1.0	1.0	1.0	1.0	1.0	1.0
.1	.99	.9901	.970299	.96059601	.95099004	.94149014
.2	.96	.9216	.884736	.84924656	.81557269	.78275778
.3	.91	.8281	.755571	.68574961	.62403214	.56786925
.4	.84	.7056	.592704	.49787136	.41821194	.35129805
.5	.75	.5625	.421875	.31640625	.23750468	.17797851
.6	.64	.4096	.262144	.16777216	.10737418	.06871947
.7	.51	.2601	.132651	.06765201	.03450252	.01759628
.8	.36	.1296	.046656	.01679616	.00604661	.00217678
.9	.19	.0361	.006859	.00130321	.00024760	.00004704
1.0	0.0	0.0	0.0	0.0	0.0	0.0

n	$(1 - z^2)^7$	$(1 - z^2)^8$	$(1 - z^2)^9$	$(1 - z^2)^{10}$	$(1 - z^2)^{11}$	$(1 - z^2)^{12}$
0	1.0	1.0	1.0	1.0	1.0	1.0
.1	.95206534	.92274469	.91351724	.90458206	.89533824	.88638486
.2	.75144747	.72138957	.69253593	.66435262	.63825931	.61270973
.3	.51676101	.47025251	.42792978	.38941609	.35436864	.32247546
.4	.29509034	.24787588	.20821573	.17490121	.14691701	.12541028
.5	.13548288	.10011291	.07506468	.05631351	.04225513	.03167634
.6	.04398046	.02814749	.01801439	.01152920	.00757668	.00472235
.7	.00897410	.00457679	.00233416	.00119042	.00060711	.00030962
.8	.00078364	.00028211	.00010155	.00003655	.00001315	.00000473
.9	.00000093	.000000169	.000000032	.00000006	.00000001	.0.0
1.0	0.0	0.0	0.0	0.0	0.0	0.0

Continued following page

TABLE II

Differences in $(J - S_1)$, $(J - S_2)$, $(J - S_3)$ for $n > 20$

$\frac{e}{S} = \frac{1}{S_1} - \frac{1}{S_2}$	$\frac{e}{S} = \frac{1}{S_2} - \frac{1}{S_3}$	$\frac{e}{S} = \frac{1}{S_1} - \frac{1}{S_3}$
0.0	0.0	0.0
0.1	0.1	0.1
0.2	0.2	0.2
0.3	0.3	0.3
0.4	0.4	0.4
0.5	0.5	0.5
0.6	0.6	0.6
0.7	0.7	0.7
0.8	0.8	0.8
0.9	0.9	0.9
1.0	1.0	1.0
1.1	1.1	1.1
1.2	1.2	1.2
1.3	1.3	1.3
1.4	1.4	1.4
1.5	1.5	1.5
1.6	1.6	1.6
1.7	1.7	1.7
1.8	1.8	1.8
1.9	1.9	1.9
2.0	2.0	2.0
2.1	2.1	2.1
2.2	2.2	2.2
2.3	2.3	2.3
2.4	2.4	2.4
2.5	2.5	2.5
2.6	2.6	2.6
2.7	2.7	2.7
2.8	2.8	2.8
2.9	2.9	2.9
3.0	3.0	3.0
3.1	3.1	3.1
3.2	3.2	3.2
3.3	3.3	3.3
3.4	3.4	3.4
3.5	3.5	3.5
3.6	3.6	3.6
3.7	3.7	3.7
3.8	3.8	3.8
3.9	3.9	3.9
4.0	4.0	4.0
4.1	4.1	4.1
4.2	4.2	4.2
4.3	4.3	4.3
4.4	4.4	4.4
4.5	4.5	4.5
4.6	4.6	4.6
4.7	4.7	4.7
4.8	4.8	4.8
4.9	4.9	4.9
5.0	5.0	5.0
5.1	5.1	5.1
5.2	5.2	5.2
5.3	5.3	5.3
5.4	5.4	5.4
5.5	5.5	5.5
5.6	5.6	5.6
5.7	5.7	5.7
5.8	5.8	5.8
5.9	5.9	5.9
6.0	6.0	6.0
6.1	6.1	6.1
6.2	6.2	6.2
6.3	6.3	6.3
6.4	6.4	6.4
6.5	6.5	6.5
6.6	6.6	6.6
6.7	6.7	6.7
6.8	6.8	6.8
6.9	6.9	6.9
7.0	7.0	7.0
7.1	7.1	7.1
7.2	7.2	7.2
7.3	7.3	7.3
7.4	7.4	7.4
7.5	7.5	7.5
7.6	7.6	7.6
7.7	7.7	7.7
7.8	7.8	7.8
7.9	7.9	7.9
8.0	8.0	8.0
8.1	8.1	8.1
8.2	8.2	8.2
8.3	8.3	8.3
8.4	8.4	8.4
8.5	8.5	8.5
8.6	8.6	8.6
8.7	8.7	8.7
8.8	8.8	8.8
8.9	8.9	8.9
9.0	9.0	9.0
9.1	9.1	9.1
9.2	9.2	9.2
9.3	9.3	9.3
9.4	9.4	9.4
9.5	9.5	9.5
9.6	9.6	9.6
9.7	9.7	9.7
9.8	9.8	9.8
9.9	9.9	9.9
10.0	10.0	10.0
10.1	10.1	10.1
10.2	10.2	10.2
10.3	10.3	10.3
10.4	10.4	10.4
10.5	10.5	10.5
10.6	10.6	10.6
10.7	10.7	10.7
10.8	10.8	10.8
10.9	10.9	10.9
11.0	11.0	11.0
11.1	11.1	11.1
11.2	11.2	11.2
11.3	11.3	11.3
11.4	11.4	11.4
11.5	11.5	11.5
11.6	11.6	11.6
11.7	11.7	11.7
11.8	11.8	11.8
11.9	11.9	11.9
12.0	12.0	12.0
12.1	12.1	12.1
12.2	12.2	12.2
12.3	12.3	12.3
12.4	12.4	12.4
12.5	12.5	12.5
12.6	12.6	12.6
12.7	12.7	12.7
12.8	12.8	12.8
12.9	12.9	12.9
13.0	13.0	13.0
13.1	13.1	13.1
13.2	13.2	13.2
13.3	13.3	13.3
13.4	13.4	13.4
13.5	13.5	13.5
13.6	13.6	13.6
13.7	13.7	13.7
13.8	13.8	13.8
13.9	13.9	13.9
14.0	14.0	14.0
14.1	14.1	14.1
14.2	14.2	14.2
14.3	14.3	14.3
14.4	14.4	14.4
14.5	14.5	14.5
14.6	14.6	14.6
14.7	14.7	14.7
14.8	14.8	14.8
14.9	14.9	14.9
15.0	15.0	15.0
15.1	15.1	15.1
15.2	15.2	15.2
15.3	15.3	15.3
15.4	15.4	15.4
15.5	15.5	15.5
15.6	15.6	15.6
15.7	15.7	15.7
15.8	15.8	15.8
15.9	15.9	15.9
16.0	16.0	16.0
16.1	16.1	16.1
16.2	16.2	16.2
16.3	16.3	16.3
16.4	16.4	16.4
16.5	16.5	16.5
16.6	16.6	16.6
16.7	16.7	16.7
16.8	16.8	16.8
16.9	16.9	16.9
17.0	17.0	17.0
17.1	17.1	17.1
17.2	17.2	17.2
17.3	17.3	17.3
17.4	17.4	17.4
17.5	17.5	17.5
17.6	17.6	17.6
17.7	17.7	17.7
17.8	17.8	17.8
17.9	17.9	17.9
18.0	18.0	18.0
18.1	18.1	18.1
18.2	18.2	18.2
18.3	18.3	18.3
18.4	18.4	18.4
18.5	18.5	18.5
18.6	18.6	18.6
18.7	18.7	18.7
18.8	18.8	18.8
18.9	18.9	18.9
19.0	19.0	19.0
19.1	19.1	19.1
19.2	19.2	19.2
19.3	19.3	19.3
19.4	19.4	19.4
19.5	19.5	19.5
19.6	19.6	19.6
19.7	19.7	19.7
19.8	19.8	19.8
19.9	19.9	19.9
20.0	20.0	20.0

TABLE II (Continued)

	$(1 - z^2)^{13}$	$(1 - z^2)^{14}$	$(1 - z^2)^{15}$	$(1 - z^2)^{16}$	$(1 - z^2)^{17}$	$(1 - z^2)^{18}$	$(1 - z^2)^{19}$
0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
.1	.86874579 • 88820134	.86005833 • 54208634	.85145774 • 52040288	.84294317 • 49958676	.83451374 • 47960328	.82616859 • 46041914	
.2	.56467528 • 26704192	.54208634 • 24509014	.52040288 • 22113740	.49958676 • 20125503	.47960328 • 18512587	.46041914 • 16664272	
.3	.29245266 • 08707328	.28004192 • 07314575	.26704192 • 06144243	.24509014 • 05161164	.22113740 • 04355577	.20125503 • 03564171	
.4	.10366465 • 02375725	.10179179 • 01226344	.09900258 • 00751693	.09619344 • 00507056	.0932451 • 00000544	.09020769 • 00000027	
.5	.02375725 • 00193427	.02123793 • 00079227	.01922250 • 00004106	.01791795 • 00002094	.015790 • 00000021	.013790 • 00000007	
.6	.000000170 • 0.0	.000000061 • 0.0	.000000052 • 0.0	.00000004106 • 0.0	.000000021 • 0.0	.0000000170 • 0.0	
.7							
.8							
.9							
1.0							

(Continued) II

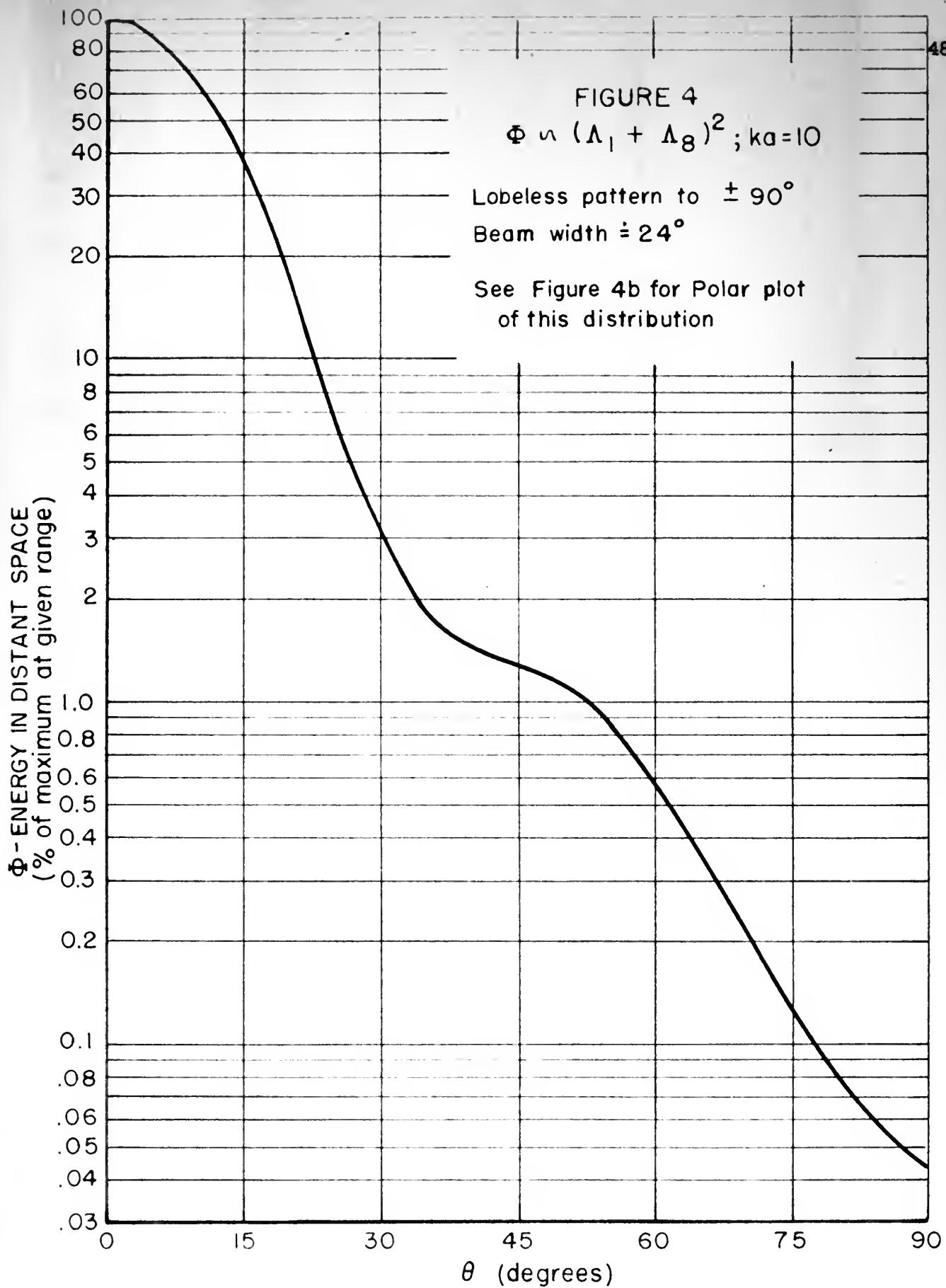


FIGURE 4

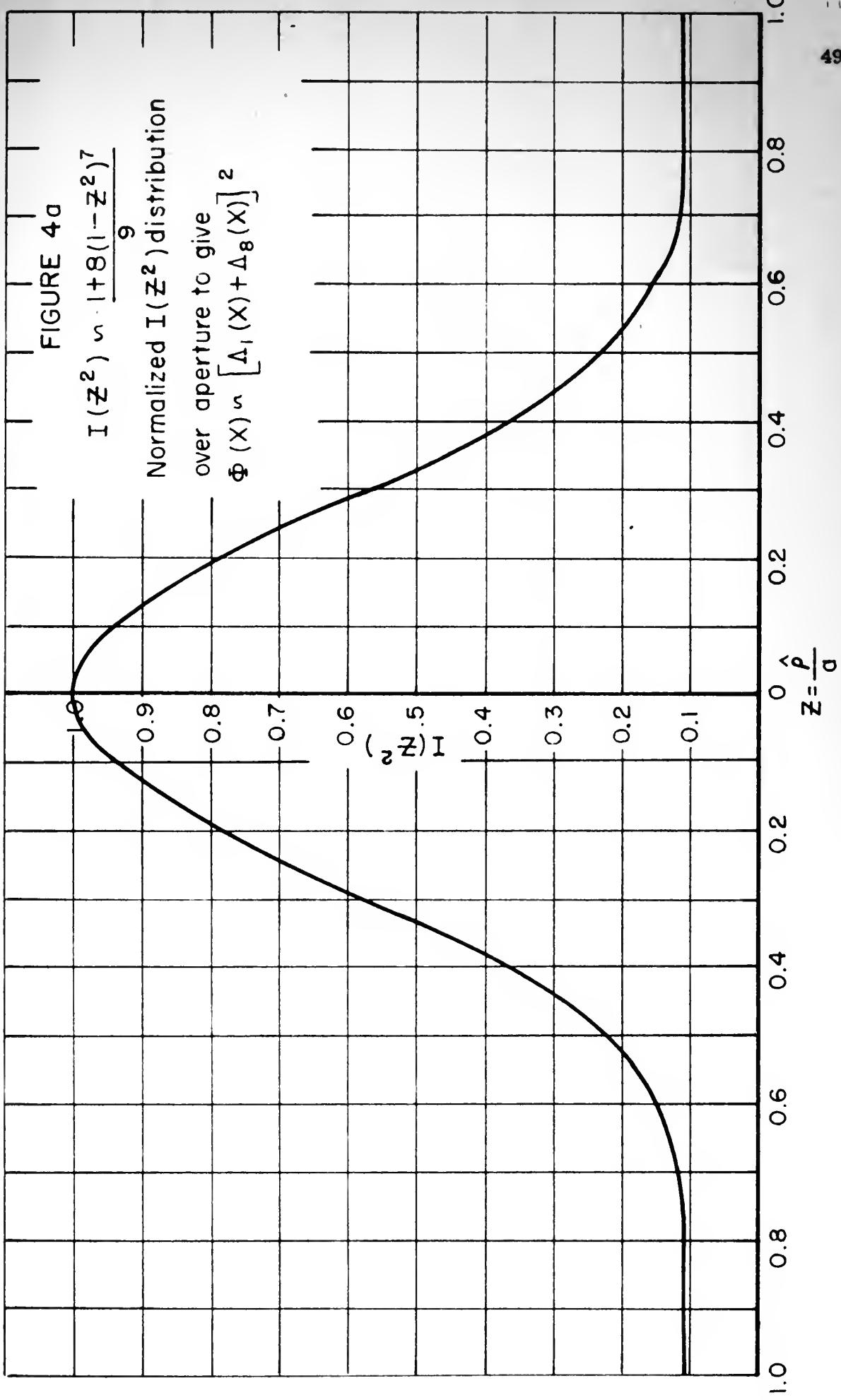
$$\Phi \propto (\Lambda_1 + \Lambda_2)^2; ka=10$$

Lobeless pattern to $\pm 90^\circ$ Beam width $\approx 24^\circ$ See Figure 4b for Polar plot
of this distribution

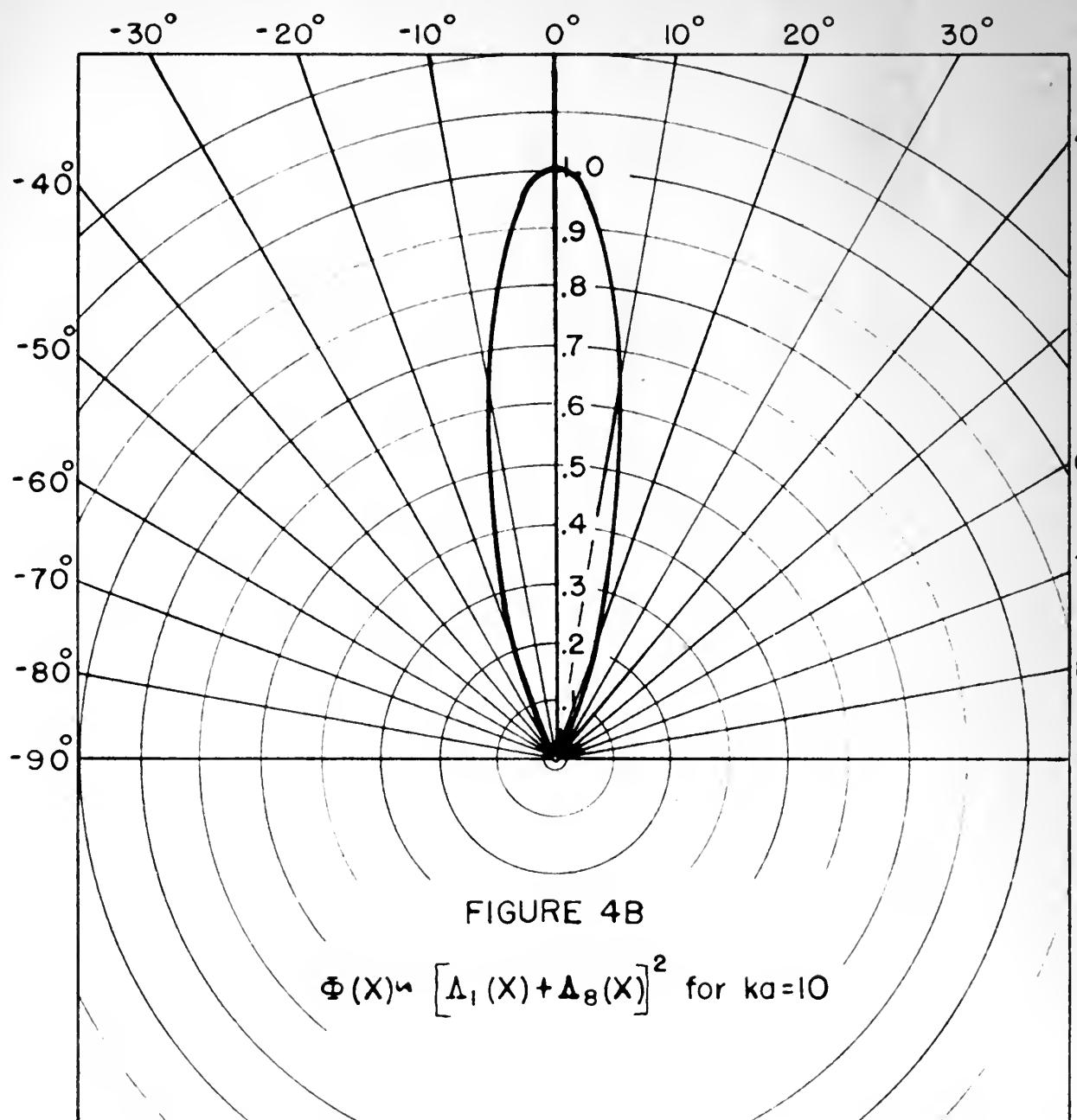
FIGURE 4a

$$I(z^2) \approx \frac{1+8(1-z^2)^7}{9}$$

Normalized $I(z^2)$ distribution
over aperture to give
 $\Phi(x) \approx [\Delta_1(x) + \Delta_8(x)]^2$







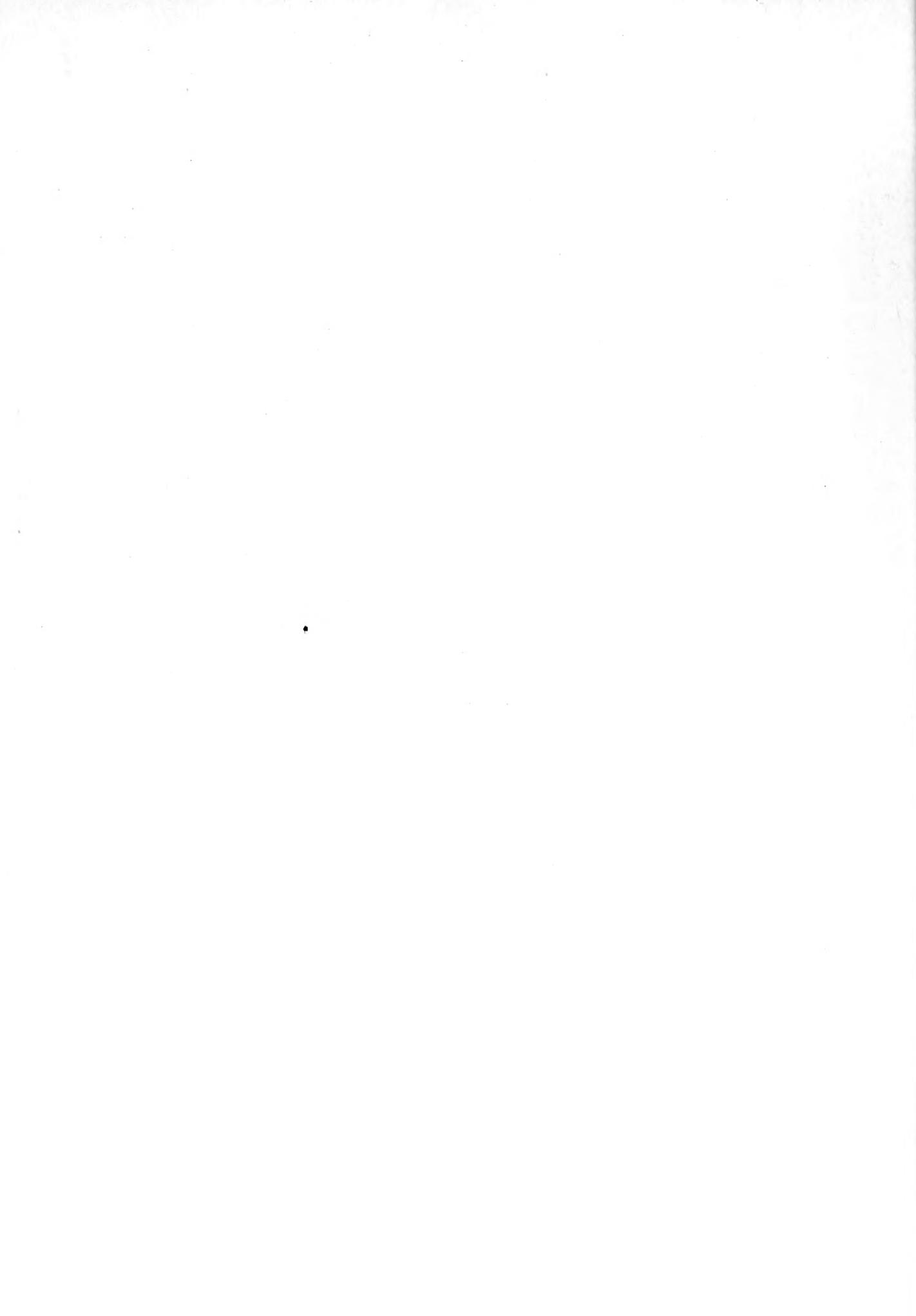


TABLE III

(Reference Figs. 4 and 4a)

Values of $\varphi(x) \sim [\lambda_1(x) + \lambda_2(x)]$ and corresponding $I(z^2)$

χ	0	.5	1.0	1.5	2.0	2.5	3.0
λ_1	1	.96907	.88010	.74392	.57672	.39768	.22604
λ_2	1	.99508	.97257	.95923	.89428	.83935	.77629
$E(x)$	2	1.96215	1.85267	1.68315	1.47100	1.25701	1.00233
$E_n(x)$	1	.98108	.92624	.84158	.73550	.61850	.50116
$\varphi(x)$	1	.962	.858	.710	.540	.384	.252
θ°	0	2.9	5.7	8.6	11.5	14.5	17.5

χ	.5	4.0	4.5	5.0	5.5	6.0	6.5
λ_1	.07850	-.02302	-.10269	-.12103	-.12416	-.09225	-.04734
λ_2	.70729	.63452	.56017	.48634	.41489	.34741	.28519
$E(x)$.78579	.60150	.45748	.36561	.29078	.25618	.22785
$E_n(x)$.89290	.50075	.22874	.17766	.14566	.12759	.11892
$\varphi(x)$.154	.0002	.0524	.0315	.0211	.0162	.0141
θ°	20.5	25.6	26.7	30.0	35.4	36.9	40.5

χ	7.0	7.5	8.0	8.5	9.0	9.5	10.0
λ_1	-.00124	+.03607	.05366	.06426	.05451	.03395	.00869
λ_2	.22913	.17682	.13748	.10203	.07315	.05281	.04150
$E(x)$.29779	.21589	.19614	.16629	.12766	.08425	.04212
$E_n(x)$.11590	.10794	.09859	.08314	.06385	.042075	.02075
$\varphi(x)$.0129	.0116	.0097	.0069	.00407	.00177	.00045
θ°	44.4	48.6	53.1	58.2	64.2	71.8	90.0

the following proposition holds.

TABLE III (Continued)

$I(\bar{z}) \backslash \bar{z}$	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
Constant	1.00	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.
$\delta(1-\bar{z}^2)^7$	8.00	7.448	6.716	4.144	2.560	1.072	.352	.072	.008	-	0
$I(\bar{z}^2)$	9.000	2.448	7.016	5.144	3.260	2.072	1.852	1.072	1.008	1.000	1.000
$I_{\nu\nu}(\bar{z}^2)$	1.000	.938	.790	.571	.373	.230	.152	.119	.112	.111	.111

Table III (continued)

$I(\bar{X}) \sim 18(1-\bar{X}_s)$

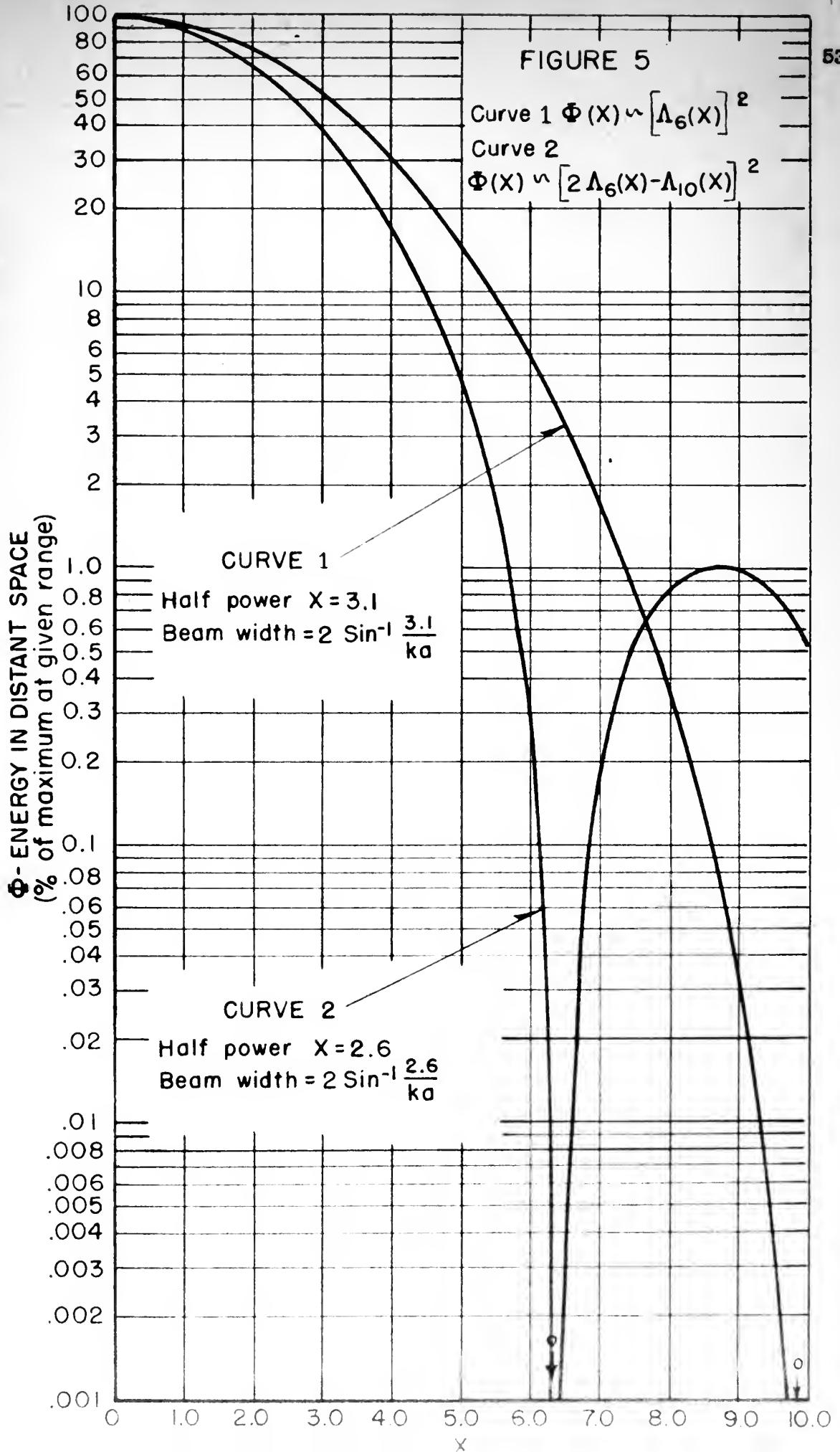
$I(\bar{X}) = 0.5 - 0.1 \cdot \bar{X} + 0.005 \cdot \bar{X}^2$

$I(\bar{X}) = 5$

\bar{X}	$I(\bar{X})$										
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.10	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.20	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.30	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.50	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.60	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.70	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.80	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.90	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

$I(\bar{X}) = 5$

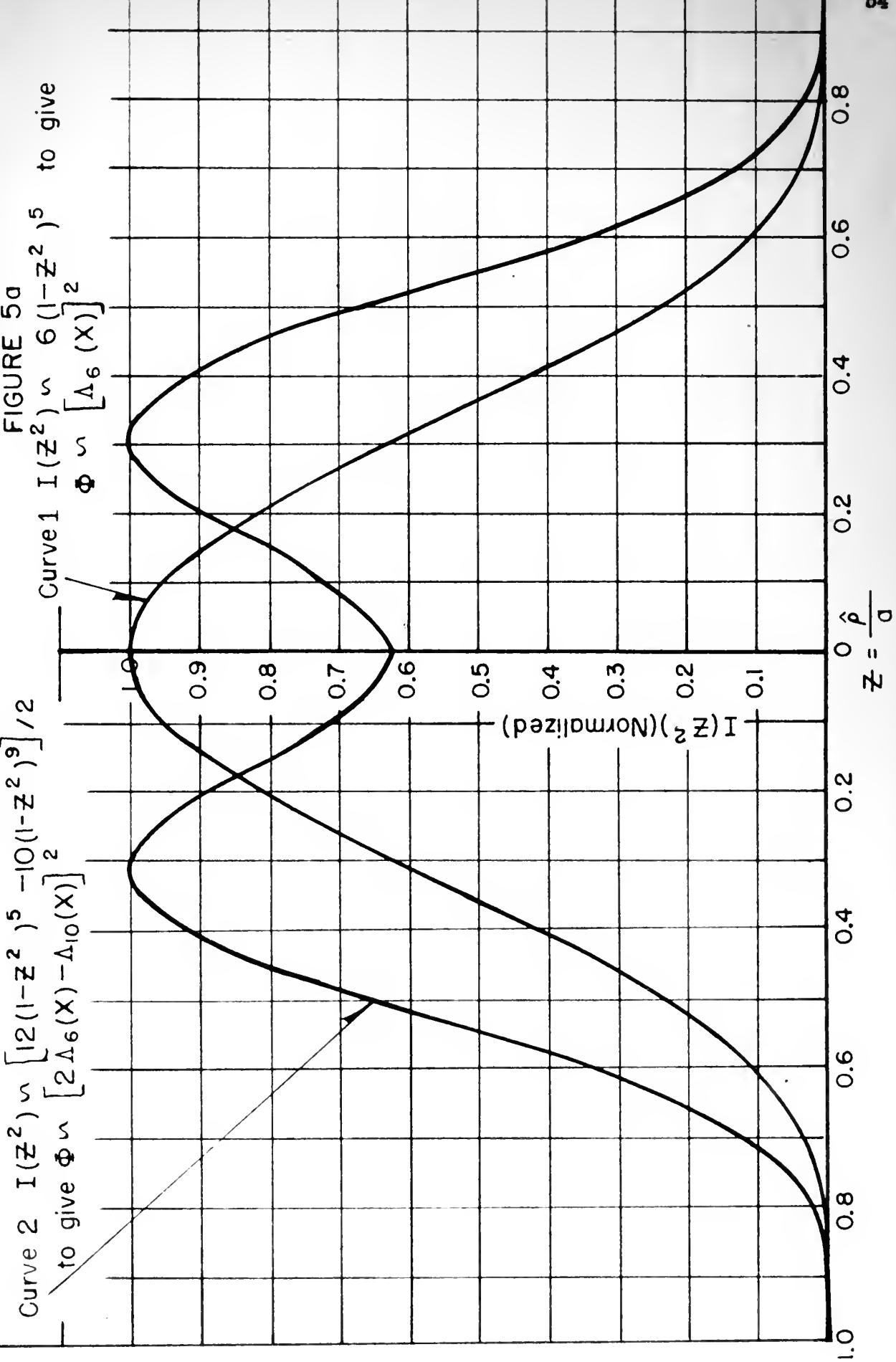
$I(\bar{X}) = 4$





Curve 2 $I(z^2) \propto [12(1-z^2)^5 - 10(1-z^2)^9]/2$
 to give $\Phi \propto [2\Lambda_6(X) - \Lambda_{10}(X)]^2$

Curve 1 $I(z^2) \propto 6(1-z^2)^5$
 to give $\Phi \propto [\Lambda_6(X)]^2$



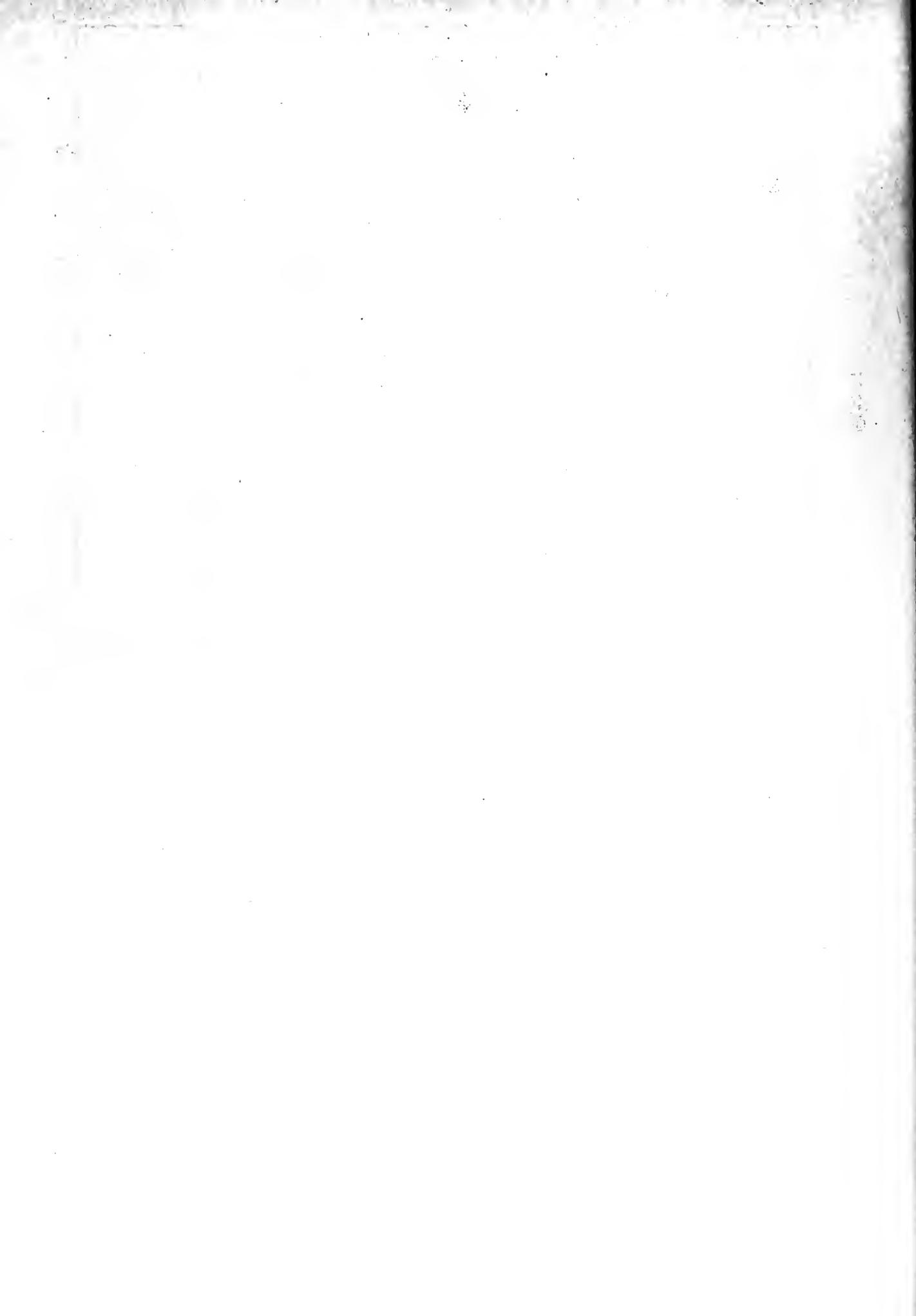


TABLE IV

(Reference FIG. 5)

a) Values of $\Phi(x) - (\sqrt{\Lambda}_6(x))^2$

$$\Phi(x) - [2\sqrt{\Lambda}_6(x) - \sqrt{\Lambda}_{10}(x)]^2$$

b) Values of $\Phi(x) - [2\sqrt{\Lambda}_6(x) - \sqrt{\Lambda}_{10}(x)]^2$

c) Values of $I(z^2)$ corresponding to (a)

d) Values of $I(z^2)$ corresponding to (b)

<u>a</u>		0	.5	1.0	1.5	2.0	2.5	3.0
$\sqrt{\Lambda}_6$	$\sqrt{\Lambda}_{10}$							
$\Phi(x)$	1.0	.99111	.96484	.92241	.86575	.79737	.72021	
	1.0	.990	.920	.850	.750	.636	.519	
$\sqrt{\Lambda}_6$	$\sqrt{\Lambda}_{10}$							
$\Phi(x)$	1.0	.63743	.56224	.46767	.38648	.31094	.24280	.18224
	.406	.205	.219	.149	.0922	.0590	.0236	
$\sqrt{\Lambda}_6$	$\sqrt{\Lambda}_{10}$							
$\Phi(x)$	7.0	7.5	8.0	8.5	9.0	9.5	10.0	
$\sqrt{\Lambda}_6$.15285	.09169	.05954	.03503	.01772	.00625	-.00067	
$\Phi(x)$.0176	.0084	.00351	.00123	.000514	.0000289		
<u>b</u>		0	.5	1.0	1.5	2.0	2.5	3.0
$2\sqrt{\Lambda}_6$	$\sqrt{\Lambda}_{10}$							
$\sqrt{\Lambda}_m(x)$	2.0	1.98222	1.92968	1.84482	1.73150	1.59474	1.44022	
	1.0	.99485	.97751	.95005	.91278	.86684	.81357	
$\Phi(x)$	2.0	.99789	.95217	.89477	.81872	.72790	.62695	
	1.0	.976	.908	.800	.670	.530	.394	

continued following page

VII.

(Jellulose 21. 2)

$$x^{\frac{1}{2}} \sim (\sqrt{x} - \sqrt{e}(x))^5$$

(4) For 1 to 1000, compare with 50 (a),

(5) A figure of 1 (), see table 1 (), to do (a)

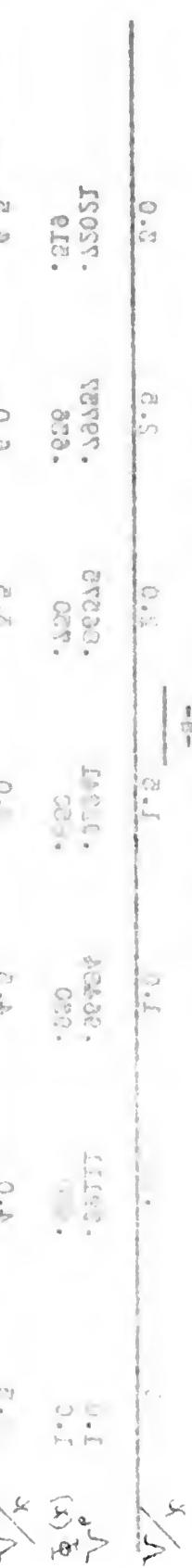


TABLE IV (Continued)

-b- (continued)

$\Lambda \backslash \chi$	3.5	4.0	4.5	5.0	5.5	6.0	6.5
$2\Lambda_6$	1.27486	1.10448	.93534	.77296	.62188	.48560	.36248
$-\Lambda_{10}$.75447	.69117	.62532	.55851	.49227	.42796	.36677
$E_m(\chi)$.52039	.41251	.31002	.21445	.12961	.05764	-.00429
$\tilde{E}(\chi)$.270	.171	.096	.046	.0168	.00331	.0000184
$\Lambda \backslash \chi$	7.0	7.5	8.0	8.5	9.0	9.5	10.0
$2\Lambda_6$.26570	.18733	.11868	.07066	.03544	.01246	-.00134
$-\Lambda_{10}$.50965	.25735	.21030	.16830	.13289	.10242	.07710
$E_m(\chi)$	-.04395	-.07395	-.09162	-.09874	-.09745	-.08996	-.07844
$\tilde{E}(\chi)$.00193	.00546	.0084	.00975	.0095	.0081	.00615
$Z \backslash I(Z^2) \sim 6(1-Z^2)^5$	<u>-c-</u>						
$I_m(Z^2)$	1	.951	.815	.625	.418	.237	.107
$I(Z^2) \sim 12(1-Z^2)^5$	0	.1	.2	.3	.4	.5	.6
$I_m(Z^2)$	0	.1	.2	.3	.4	.5	.6
$Z \backslash I(Z^2) \sim 12(1-Z^2)^5$	<u>-d-</u>						
$I(Z^2)$	0	.1	.2	.3	.4	.5	.6
$I_m(Z^2)$	0	.1	.2	.3	.4	.5	.6
$12(I-Z^2)^5$	12.000	11.41	9.78	7.50	5.025	2.84	1.284
$-10(I-Z^2)^9$	10.000	9.13	6.93	4.30	2.080	.75	.180
$I(Z^2)$	2.000	2.28	2.85	3.20	2.945	2.09	1.104
$I_m(Z^2)$	0.625	.712	.890	1.00	.92	.654	.345

TABLE II (Continued)

(continued)

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

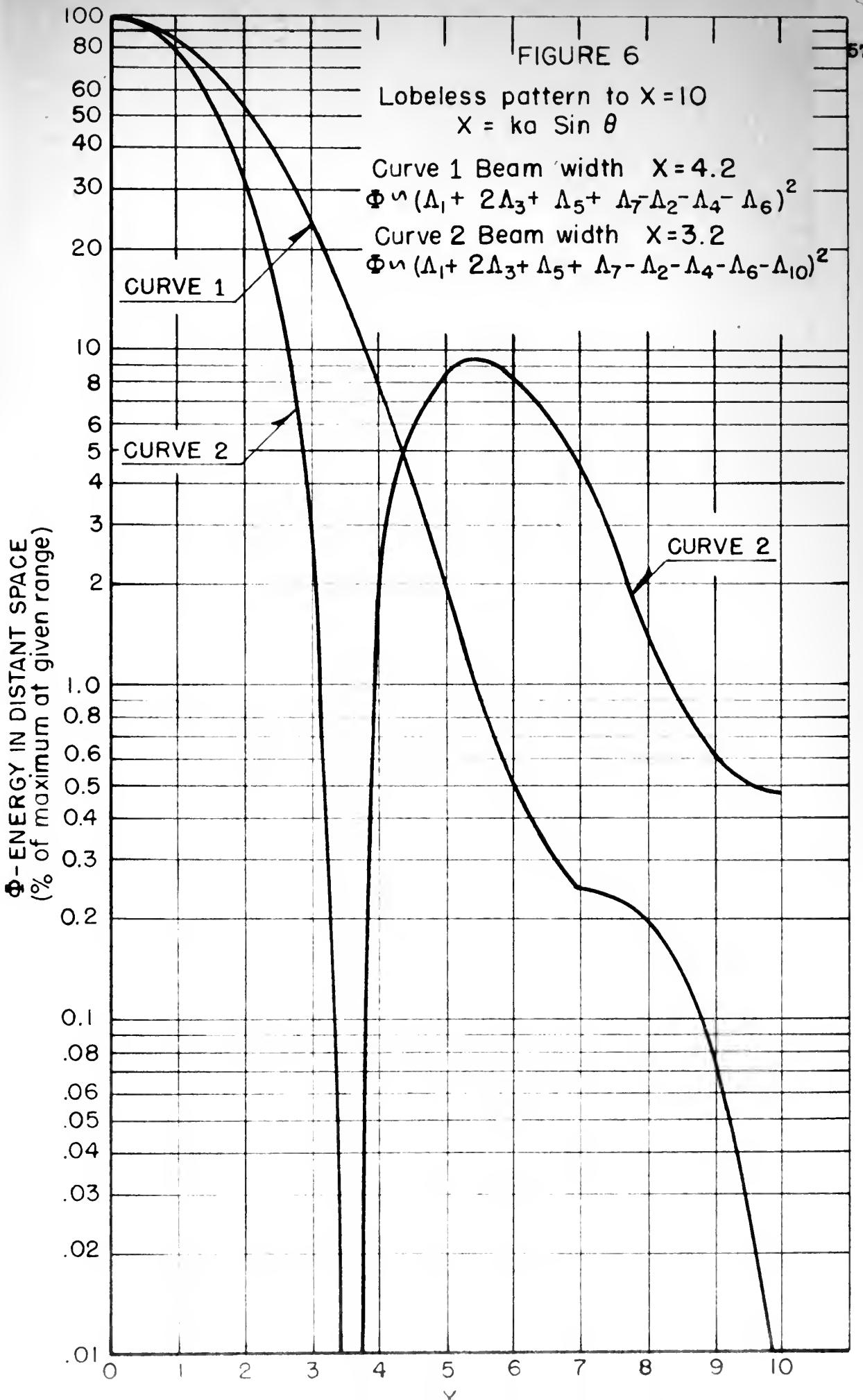
	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0

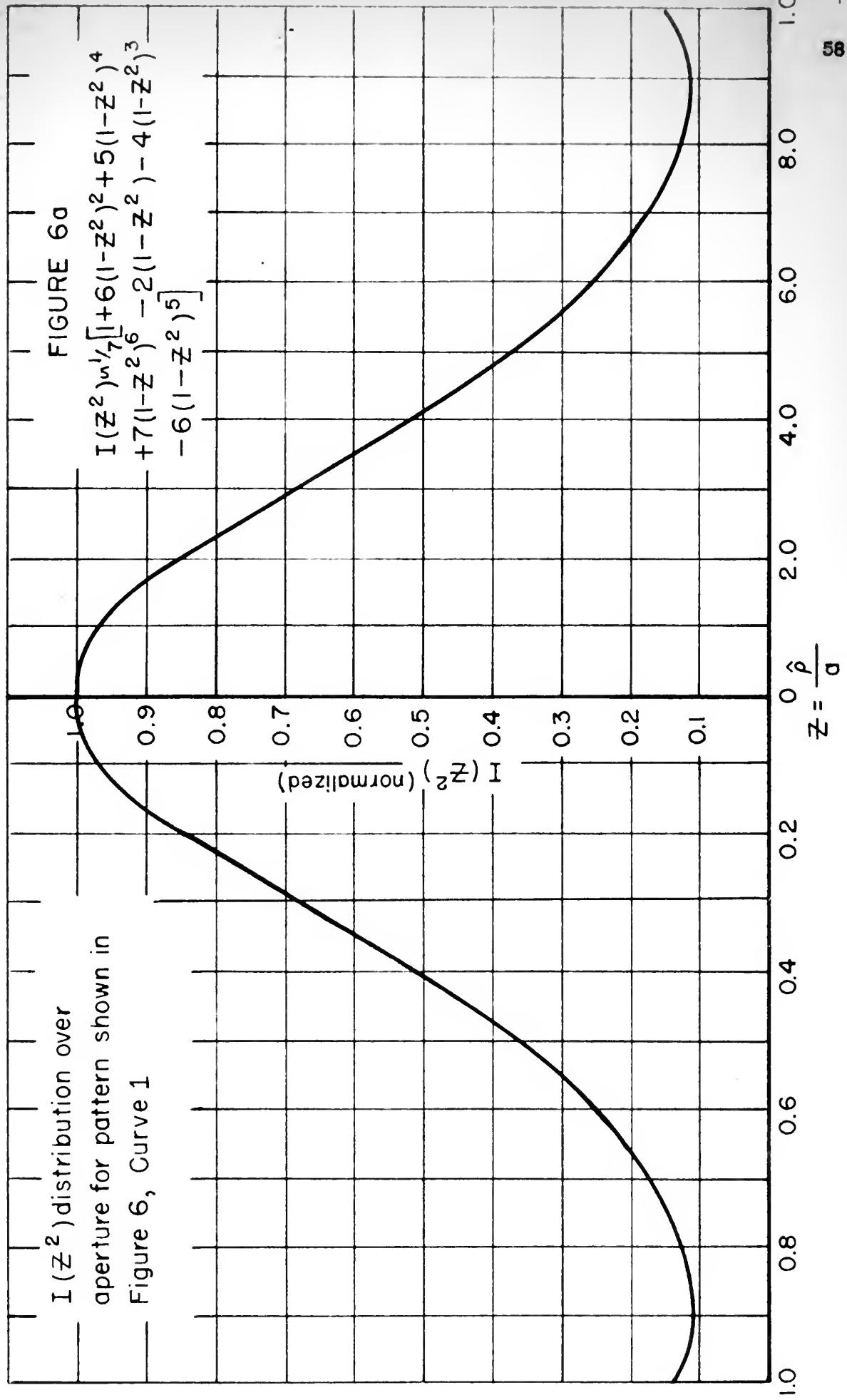




$I(z^2)$ distribution over
aperture for pattern shown in
Figure 6, Curve 1

FIGURE 6a

$$I(z^2)^{1/7} \left[1 + 6(1-z^2)^2 + 5(1-z^2)^4 \right. \\ \left. + 7(1-z^2)^6 - 2(1-z^2)^3 - 4(1-z^2)^5 \right. \\ \left. - 6(1-z^2)^7 \right]$$





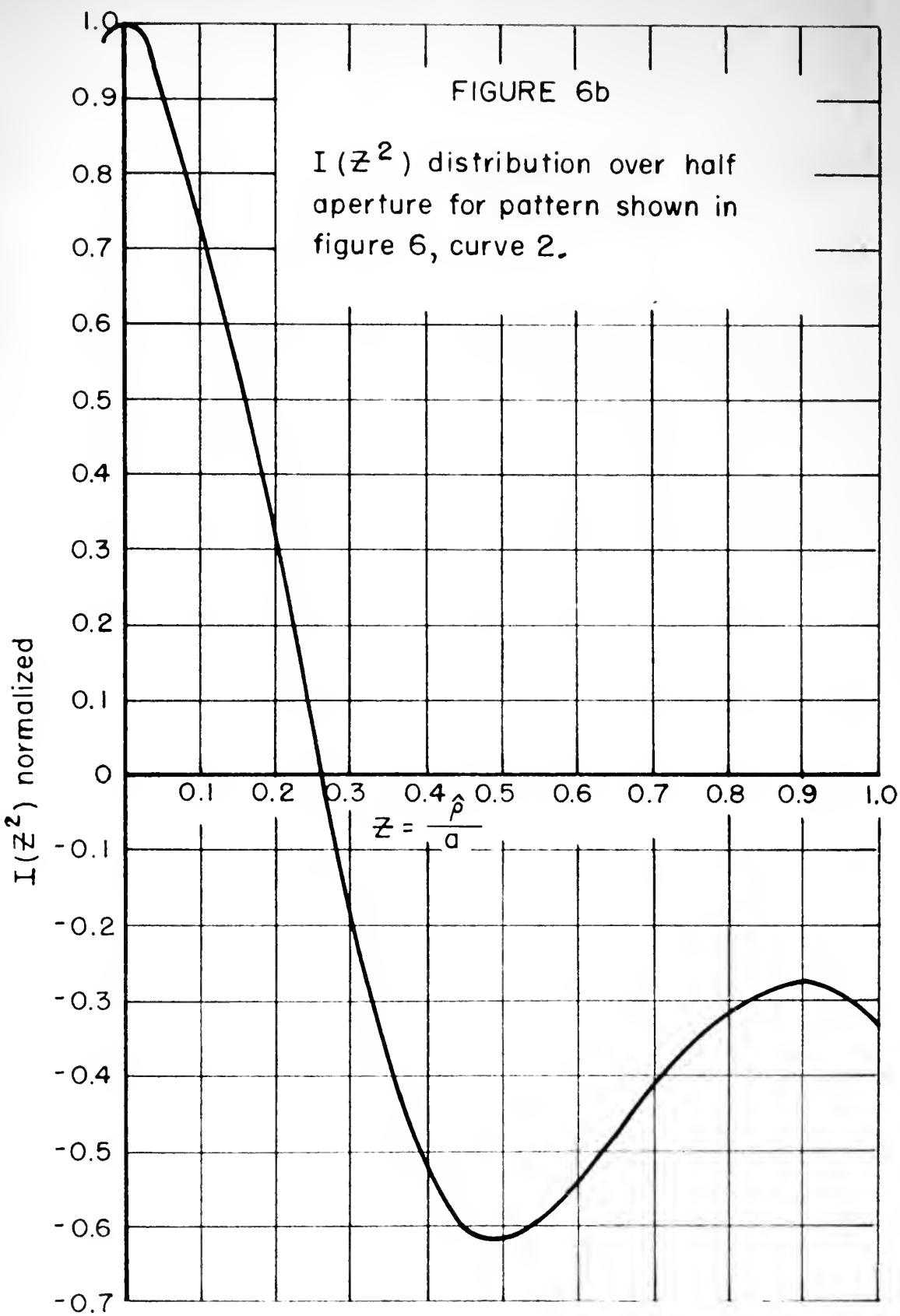




TABLE V

(Reference FIG. 6)

- a) Values for $\varPhi(x) \sim [\lambda_1(x) + 2\lambda_3(x) + \lambda_5(x) + \lambda_7(x) - \lambda_2(x) - \lambda_4(x) - \lambda_6(x)]^2$
- b) Values for $\varPhi(x) \sim [\lambda_1(x) + 2\lambda_3(x) + \lambda_5(x) + \lambda_7(x) - \lambda_2(x) - \lambda_4(x) - \lambda_6(x)]^2$
- c) Values for $I(z^2)$ corresponding to (a)
- d) Values for $I(z^2)$ corresponding to (b)

$$E(x) \sim \lambda_1 + 2\lambda_3 + \lambda_5 + \lambda_7 - \lambda_2 - \lambda_4 - \lambda_6$$

 \sqrt{x}

λ	0	• 5	1.0	1.5	2.0	2.5	3.0
1	1.00000	.96907	.88010	.74392	.57672	.39768	.22604
(2) 3	2.00000	1.93994	1.87808	1.75408	1.54732	1.33080	1.09888
5	1.00000	.98663	.95907	.90993	.84476	.76684	.67996
7	1.00000	.99221	.96916	.93194	.88172	.82079	.75140
Total	5.00000	1.91985	1.98645	4.31977	3.88052	3.31611	2.75628
- (2 ⁴⁺⁶)	3.00000	2.95800	2.83510	2.64024	2.38322	2.09264	1.77823
(x)	2.00000	1.96185	1.85133	1.67953	1.46520	1.22247	1.97825
$E(x)$							
n	1.00000	.98093	.92567	.83977	.75160	.61124	.48903
$E(x)$	1.00000	.960	.858	.704	.535	.375	.239
2	1.00000	.97933	.91923	.82520	.70567	.57096	.43208
4	1.00000	.98756	.95103	.89263	.81590	.72331	.62594
6	1.00000	.99111	.96484	.92241	.86575	.79737	.72021
Total	5.00000	2.65900	2.83510	2.64024	2.38732	2.09364	1.77825

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$$\begin{aligned} & \left[(\mathbf{x})_1 \wedge -(\mathbf{x})_2 \wedge -(\mathbf{x})_3 \wedge -(\mathbf{x})_4 \wedge -(\mathbf{x})_5 \wedge -(\mathbf{x})_6 \wedge -(\mathbf{x})_7 \right] \sim (\mathbf{x})_1 \wedge \\ & \left[(\mathbf{x})_1 \wedge -(\mathbf{x})_2 \wedge -(\mathbf{x})_3 \wedge -(\mathbf{x})_4 \wedge -(\mathbf{x})_5 \wedge -(\mathbf{x})_6 \wedge -(\mathbf{x})_7 \right] \sim (\mathbf{x})_1 \wedge \end{aligned}$$

$\exists x \sim (x) \exists$

TABLE V (Page 2)

-n- (continued)

$\Lambda \backslash \chi$	3.5	4.0	4.5	5.0	5.5	6.0	6.5
1	.7936	-.03562	-.10269	-.13103	-.12416	-.09223	-.04734
(2)	.66600	.64726	.44742	.29020	.14778	.05100	.01236
5	.58915	.49582	.40520	.32089	.24486	.17881	.12363
7	.67611	.52756	.51815	.44076	.36697	.29864	.23705
Total	.200374	.1.70612	.1.26806	.91082	.65545	.43622	.30096
$-(2+4+6)$.1.46000	.1.15599	.88001	.64175	.44639	.29490	.18415
E(X)	.74874	.71918	.39905	.26907	.18906	.14142	.11681
$\{ \frac{\partial}{\partial x} \}$							
2	.37437	.27457	.19403	.13454	.09453	.07071	.05841
4	.140	.0758	.0376	.0181	.00892	.00500	.00341
6	.29951	.19206	.08606	.01490	.0103	.05397	.05821
Total	.1.46000	.1.15599	.88001	.64175	.44639	.29490	.18415

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CONTINUATION OF THE BIBLIOGRAPHY

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TABLE V (page 3)

-a- (continued)

χ	7.0	7.5	8.0	8.5	9.0	9.5	1.0
1	-0.0134	+.03607	.03866	.06426	.06451	.07395	.00869
(2) 5	-0.04690	-0.05872	-0.05458	-0.04106	-0.02382	-0.00732	+.00560
5	-0.07949	-0.04587	-0.02177	-0.00581	-0.00358	-0.00801	-0.00899
7	.18298	.13685	.09862	.06794	.04117	.02649	.01398
Total	.21423	.16007	.12447	.09635	.07128	.04511	.01928
$\chi(X)$	-1.1513	.11513	.06185	.03534	.02222	.01649	.00127
$\chi(X)$.09905	.09824	.08913	.07473	.05479	.03137	.00801
$\Sigma(X)n$.04958	.04912	.04457	.05747	.02729	.01569	.00400
$\Pi(x)$.00246	.00231	.00199	.00140	.000744	.000246	.000016
2	-0.04291	-0.0275	-0.01412	+.00247	.01451	.02020	.02037
4	.02524	.00289	-.00988	-.01528	-.01554	-.01269	-.00843
6	.11285	.09169	.0594	.03503	.01722	.00923	-.00067
Total	.11513	.06185	.03534	.02222	.01649	.01374	.01127
$\chi_1(x)$	2.00000	1.96185	1.85135	1.67955	1.46320	1.22247	.97805
$\chi_2(x)$	1.00000	.99435	.97751	.95005	.91278	.86684	.81357

-b-

Table A (continued)

= (Gaussian)

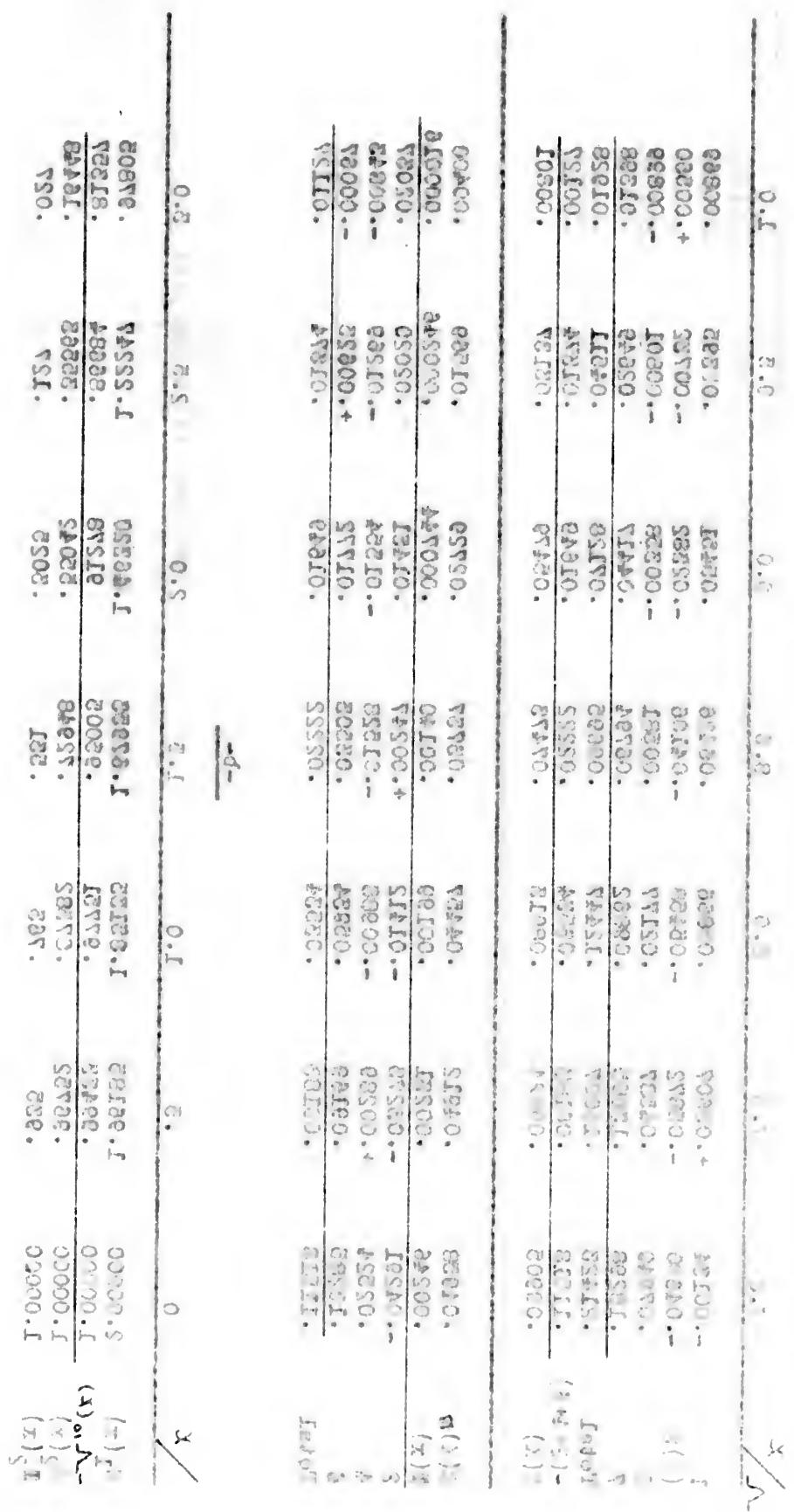


TABLE V (Part 4)

-b- (continued)

x	3.5	4.0	4.5	5.0	5.5	6.0	6.5
$\bar{F}_1(x)$.74974	.54913	.38805	.26907	.18906	.14142	.11681
$\bar{A}_{10}(x)$.75447	.69117	.62552	.55851	.49227	.42796	.36677
$\bar{F}_2(x)$	-.00573	-.14204	-.23727	-.25944	-.30321	-.28654	-.24996
$\bar{F}_2(x)$.0000329	.0202	.0561	.0836	.092	.082	.0625
x	7.0	7.5	8.0	8.5	9.0	9.5	10.0
$\bar{F}_1(x)$.09905	.09824	.08813	.07473	.05479	.03137	.00801
$\bar{A}_{10}(x)$.20965	.25753	.21020	.16860	.13289	.10242	.07710
$\bar{F}_2(x)$	-.21060	-.15909	-.12117	-.09407	-.07810	-.07106	-.06909
$\bar{F}_2(x)$.0145	.0255	.0147	.00885	.0061	.00505	.00478
z	0	.1	.2	.3	.4	.5	.6
Const.	1.000	1.000	1.000	1.000	1.000	1.000	1.000
$6(1-z^2)^2$	6.000	5.890	5.532	4.969	4.256	3.372	2.460
$5(1-z^2)^4$	5.000	4.800	4.425	3.435	2.490	1.580	.840
$7(1-z^2)^6$	7.000	6.587	5.491	3.985	2.457	1.246	.483
Total +	19.000	16.267	16.258	15.336	10.163	7.193	4.783
Total -	12.500	11.446	10.550	8.590	6.650	4.610	2.970
$I(z)$	7.200	6.821	5.908	4.796	3.623	2.598	1.813
$In(z)$	1.000	0.974	0.844	0.685	0.518	0.370	.259
$2(1-z^2)$	2.000	1.860	1.920	1.820	1.680	1.500	1.280
$4(1-z^2)^3$	4.000	3.680	3.540	3.020	2.372	1.688	1.048
$6(1-z^2)^5$	6.000	5.706	4.890	3.750	2.508	1.422	.642
Total +	12.000	11.446	10.350	8.590	6.560	4.610	2.970

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TABLE V (page 5)

(continued)

	.7	.8	.9	1.0
Const.	1.000	1.000	1.000	1.000
6(1- γ_2) ²	1.560	.780	.216	
5(1- γ_2) ⁴	.340	.025	.005	
7(1- γ_2) ⁶	.126	.014	.000	
Total +	.026	1.879	1.221	
Total -	1.762	.944	.408	1.000
$I_1(\gamma_2)$	1.264	.935	.815	1.000
$I_2(\gamma_2)$.161	.154	.116	.143
2(1- γ_2) ⁸	1.020	.720	.360	
4(1- γ_2) ¹⁰	.532	.183	.028	
6(1- γ_2) ¹²	.210	.066	.000	
Total +	1.762	.944	.408	0

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	.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
$I_1(\gamma_2)$	7.000	6.821	5.908	4.796	3.623	2.588	1.815	1.264	.935	.813	1.000
$I_0(1-\gamma_2)^9$	10.000	9.130	6.930	4.300	2.080	.750	.180	.020	—	—	—
Total +	-3.600	-2.309	-1.022	+.496	1.543	1.638	1.244	.935	.813	1.000	
$I_n(\gamma_2^2)$	-1.070	-2.730	-.541	+.165	+.514	+.613	.544	.415	.312	.271	.335

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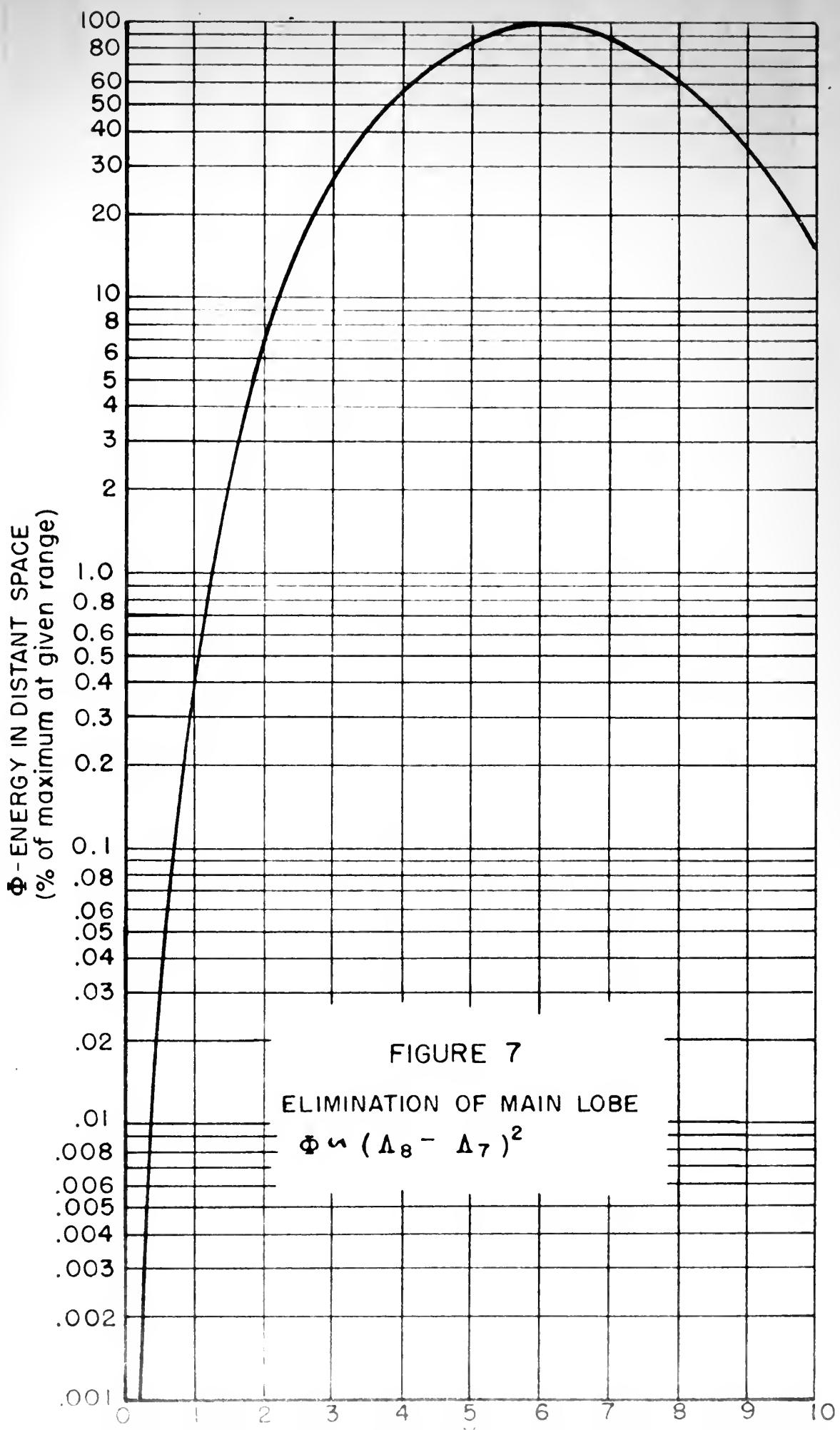
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Normalized $I(z^2)$ distribution
over aperture to eliminate main
lobe.

FIGURE 7a

$$I(z^2) \sim 8(1-z^2)^7 - 7(1-z^2)^6$$
$$\Phi(X) \sim [\Delta_8(X) - \Delta_7(X)]^2$$

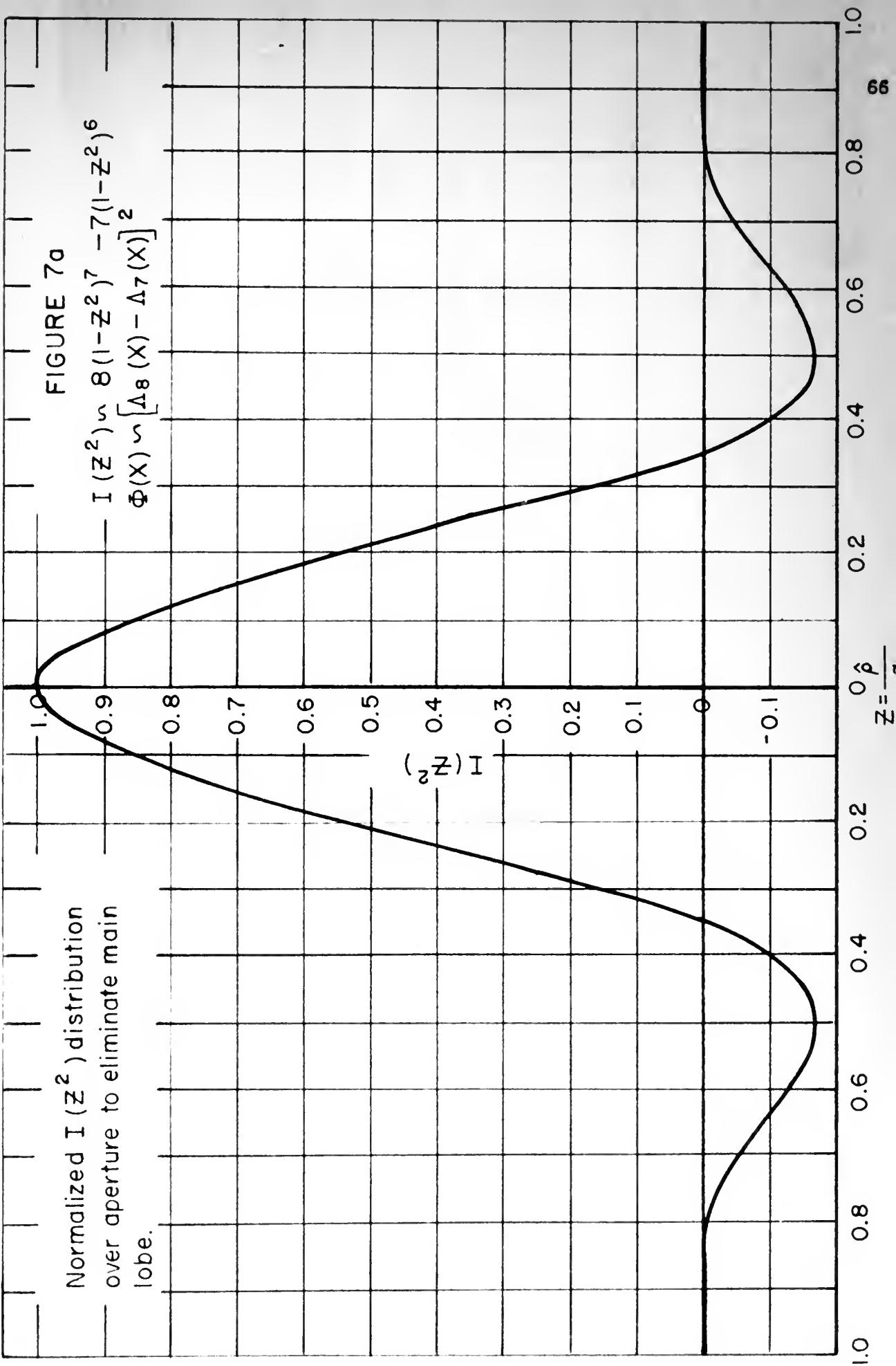




TABLE VI

(Reference Figs. 7)
 a) Values for $\Phi(x) - [\Lambda_8(x) - \Lambda_7(x)]^2$

b) Values for $I(7)$ corresponding to (a)

		$\frac{\Phi(x) - (\Lambda_8 - \Lambda_7)^2}{\Lambda}$					
		0	.5	1.0	1.5	2.0	2.5
Λ	8	1.00000	.99308	.97257	.95925	.93428	.88925
	7	1.00000	.99221	.96918	.93184	.89172	.82079
	$\Lambda(x)$	0	.00087	.00339	.00739	.01256	.01854
	$\Lambda_p(x)$	0	.01785	.06950	.15120	.25750	.38900
λ	8	0	.0000318	.00435	.0229	.066	.144
	7	0	.0000318	.00435	.0229	.066	.144
	$\lambda(x)$	0	.0000318	.00435	.0229	.066	.144
	$\lambda_p(x)$	0	.0000318	.00435	.0229	.066	.144
		4.0	4.5	5.0	5.5	6.0	6.5
Λ	8	.70729	.63452	.56017	.48634	.41489	.34741
	7	.67611	.59756	.51831	.44026	.36697	.29864
	$\Lambda(x)$	0.93118	.03696	.04186	.04538	.04792	.04877
	$\Lambda_p(x)$	•638	.757	.857	.935	.962	1.0000
λ	8	•407	.572	.735	.872	.923	1.0000
	7	•407	.572	.735	.872	.923	1.0000
	$\lambda(x)$	•407	.572	.735	.872	.923	1.0000
	$\lambda_p(x)$	•407	.572	.735	.872	.923	1.0000
		7.0	7.5	8.0	8.5	9.0	10.0
Λ	8	•22913	.17982	.15746	.10203	.07315	.05030
	7	•18298	.17685	.09862	.06794	.04417	.02649
	$\Lambda(x)$	•04615	.04297	.03886	.03409	.02898	.02381
	$\Lambda_p(x)$	•948	.981	.797	.699	.594	.489
λ	8	•900	.776	.654	.488	.353	.239
	7	•900	.776	.654	.488	.353	.239
	$\lambda(x)$	•900	.776	.654	.488	.353	.239
	$\lambda_p(x)$	•900	.776	.654	.488	.353	.239

Continued following page

TR 20

$$\int_{\alpha}^{\beta} (\bar{X} - \bar{Y})^2 d\bar{X} = \int_{\alpha}^{\beta} (\bar{X} - \bar{Y})^2 d\bar{Y}$$

(e) to calculate total $\text{L}(x)$ (second)

$$\int_{\alpha}^{\beta} (\bar{X} - \bar{Y})^2 d\bar{X} \sim \text{Chi}^2$$

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$\int_{\alpha}^{\beta} (\bar{X} - \bar{Y})^2 d\bar{X} = \int_{\alpha}^{\beta} (\bar{X} - \bar{Y})^2 d\bar{Y}$

TABLE VI (Continued)

Z	0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
$I(Z^2) - 8(1-Z^2)^7 - 7(1-Z^2)^6$											
$8(1-Z^2)$	8	7.448	6.016	4.144	2.560	1.072	.362	.072	.008	-	0
$7(1-Z^2)$	7	6.567	5.461	4.935	2.457	1.246	.485	.126	.014	-	0
$I_n(Z^2)$	1.000	.861	.585	.161	-.097	-.174	-.121	-.054	-.006	-	0

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$\left[\frac{1}{(S-5)} - \frac{1}{(S+5)} \right]$

S	$\frac{1}{(S-5)}$	$\frac{1}{(S+5)}$
0	0	0
5	0.200	0.050
10	0.050	0.020
15	0.020	0.010
20	0.010	0.005
25	0.005	0.002
30	0.002	0.001
35	0.001	0.0005
40	0.0005	0.0002
45	0.0002	0.0001
50	0.0001	0.00005

S	$\frac{1}{(S-5)}$	$\frac{1}{(S+5)}$
0	0	0
5	0.200	0.050
10	0.050	0.020
15	0.020	0.010
20	0.010	0.005
25	0.005	0.002
30	0.002	0.001
35	0.001	0.0005
40	0.0005	0.0002
45	0.0002	0.0001
50	0.0001	0.00005

Appendix IV

A Special Property of Determinants

Given a set of n simultaneous equations of the form obtained by equating coefficients of A_{mn} of equation (4.07) where $n - 1$ equations are equal to zero. We have an array of equations as follows:

$$(1) \quad 0 = a_0 I_0 + a_2 I_2 + a_4 I_4 + a_6 I_6 + \dots + a_{2(n-1)} I_{2(n-1)}$$

$$(2) \quad 0 = 0 + b_2 I_2 + b_4 I_4 + b_6 I_6 + \dots + b_{2(n-1)} I_{2(n-1)}$$

$$(3) \quad 0 = 0 + 0 + c_4 I_4 + c_6 I_6 + \dots + c_{2(n-1)} I_{2(n-1)}$$

.

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$$(j) \quad 0 = 0 + 0 + \dots + \chi_{2(j-1)} I_{2(j-1)} + \dots + \chi_{2(n-1)} I_{2(n-1)}$$

.

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$$(n) \quad 0 = 0 + 0 + \dots + \gamma_{2(n-1)} I_{2(n-1)}$$

Assume first that none of the I_{2k} is zero and that all coefficients exist. We may then solve for $I_{2k} \frac{D_{2k}}{D}$ where D is the determinant of the coefficients of the I_{2k} and D_{2k} is the determinant formed by replacing the I_{2k} column by the column on the left. Then all D_{2k} for $2k > 2(j - 1)$ will be zero and all I_{2k} for $2k > 2(j - 1)$ will be zero since the first column of D_{2k} will be a linear combination of columns 2 to $2(j - 1)$ and column $2k$. This reduces the number of equations in the original array to j and the j^{th} equation may then be solved for $I_{2(j-1)}$. The other equations may then be solved to give the remaining I_{2k} for $2k < 2(j - 1)$.

Theorem.

Under the above conditions

if $\liminf_{n \rightarrow \infty} \mu_n^2 < \infty$ then $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by applying ϵ -argument and dominated convergence theorem we haveand $\lim_{n \rightarrow \infty} \mathbb{E}[\mu_n^2] = \mu^2$ as $n \rightarrow \infty$ by dominated convergence theorem

$$(1-\alpha_n)I + (1-\alpha_n)\mu_n^2 + \alpha_n + \mu_n^2 + \mu_n^2 + \mu_n^2 = \mu^2 + \mu^2 + \mu^2 = 3\mu^2$$

$$(1-\alpha_n)I + (1-\alpha_n)\mu_n^2 + \alpha_n + \mu_n^2 + I + \mu_n^2 = 3\mu^2$$

$$(1-\alpha_n)I + (1-\alpha_n)\mu_n^2 + \mu_n^2 + \mu_n^2 + \mu_n^2 = 4\mu^2$$

$$(1-\alpha_n)I + (1-\alpha_n)\mu_n^2 + \mu_n^2 + \mu_n^2 + \mu_n^2 = 4\mu^2$$

$$(1-\alpha_n)I + (1-\alpha_n)\mu_n^2 + \mu_n^2 + \mu_n^2 + \mu_n^2 = 4\mu^2$$

from which we can see that $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$.Therefore $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theoremand we can get $\mu_n^2 \rightarrow \mu^2$ as $n \rightarrow \infty$ by dominated convergence theorem

As an example consider n equations for $n = 5$, $j = 3$.

$$\begin{aligned} 0 &= a_0 I_0 + a_2 I_2 + a_4 I_4 + a_6 I_6 + a_8 I_8 \\ 0 &= 0 + b_2 I_2 + b_4 I_4 + b_6 I_6 + b_8 I_8 \\ A &= 0 + 0 + c_4 I_4 + c_6 I_6 + c_8 I_8 \\ 0 &= 0 + 0 + 0 + d_6 I_6 + d_8 I_8 \\ 0 &= 0 + 0 + 0 + 0 + e_8 I_8 \end{aligned}$$

then $I_6 = \frac{D_6}{D}$. We assume that I_6 and I_8 exist and that $D \neq 0$.

In determinant form we have

$$\text{Column} \quad \begin{matrix} 0 & 2 & 4 & 6 & 8 \end{matrix}$$

$$D_6 = \begin{vmatrix} a_0 & a_2 & a_4 & 0 & a_8 \\ 0 & b_2 & b_4 & 0 & b_8 \\ 0 & 0 & c_4 & A & c_8 \\ 0 & 0 & 0 & 0 & d_8 \\ 0 & 0 & 0 & 0 & e_8 \end{vmatrix} = 0$$

However $D_6 = 0$ since the 0 column is identical with

$$\frac{a_0}{a_2 - \frac{a_4 b_2}{b_4}} \left[\text{Column } 2 - \frac{b_2}{b_4} \left\{ \text{Column } 4 - \frac{c_4}{A} \text{ Column } 6 \right\} \right]$$

Similarly $D_8 = 0$

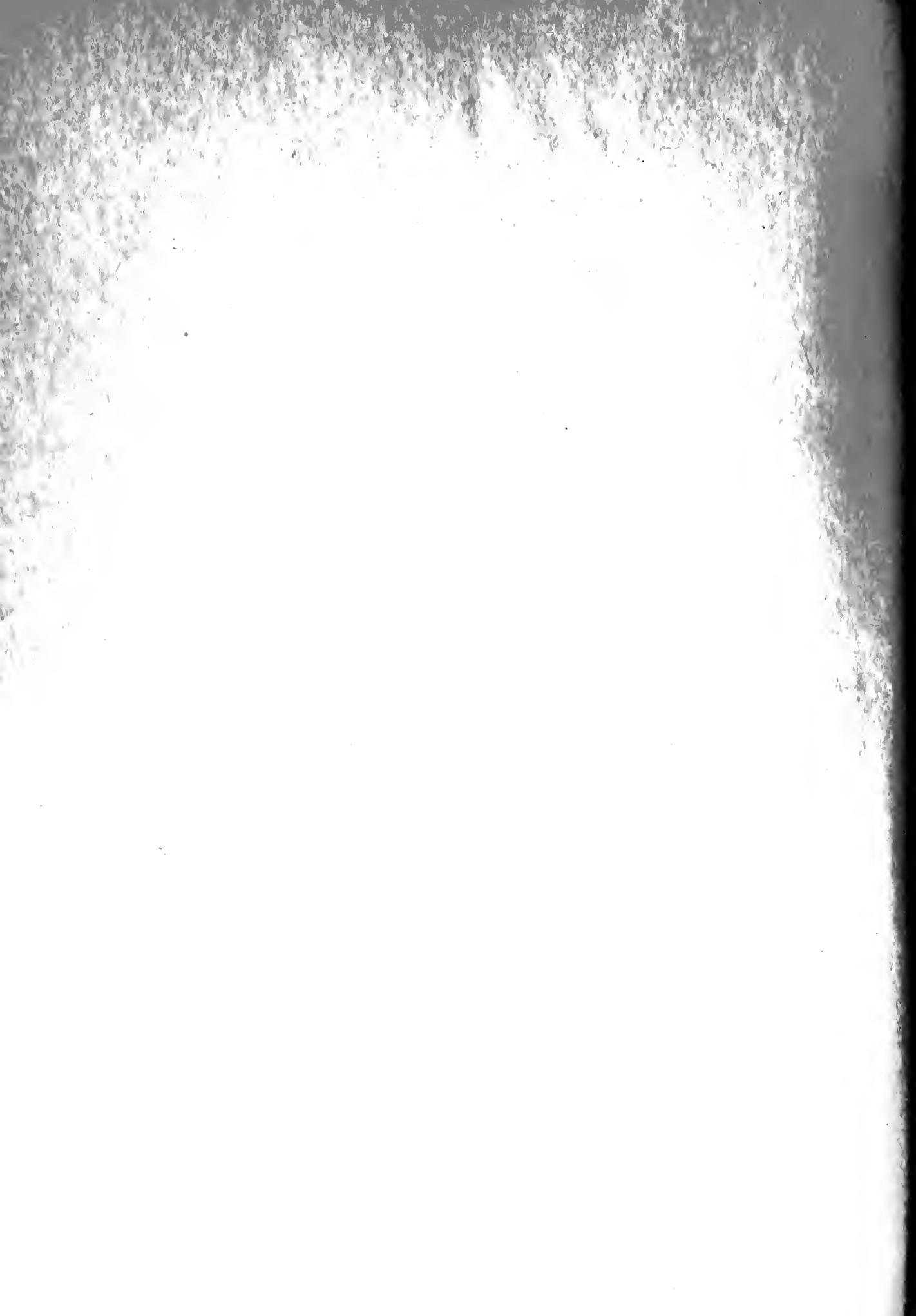
Hence, our original assumption was incorrect and we have left j non-trivial equations which may be easily solved.

$$\begin{aligned}
 & I_{11} + I_{12} + I_{13} + I_{14} + I_{15} = \\
 & I_{21} + I_{22} + I_{23} + I_{24} + 0 = \\
 & I_{31} + I_{32} + I_{33} + 0 + 0 = \\
 & I_{41} + I_{42} + 0 + 0 + 0 = \\
 & I_{51} + 0 + 0 + 0 + 0 =
 \end{aligned}$$

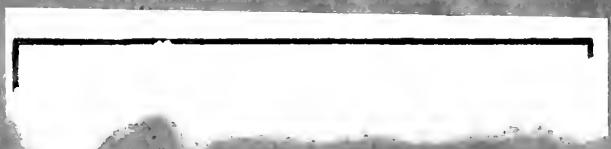
$\frac{\partial f}{\partial x_1} = 0$

$$\begin{matrix}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 0 \end{matrix} \\
 \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 \\
 0 & 0 & 0 & 0 & 1 & 0
 \end{bmatrix}
 \end{matrix}$$

$$\left[\begin{matrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{matrix} \right] \begin{matrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{matrix} = \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}$$









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Correlation of radiation field patterns



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