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Internet service provider network evolution in the presence of changing environmental conditions

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THESIS

INTERNET SERVICE PROVIDER NETWORK EVOLUTION IN THE PRESENCE OF CHANGING ENVIRONMENTAL CONDITIONS

by

Aaron Sanchez

March 2010

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INTERNET SERVICE PROVIDER NETWORK EVOLUTION IN THE PRESENCE OF CHANGING ENVIRONMENTAL CONDITIONS

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ABSTRACT

Internet Service Providers (ISPs) offer access to the Internet and other network resources to their customers. The ISP marketplace is extremely competitive, requiring ISPs to provide their services with limited resources. In this thesis, we use constrained optimization to reflect the tensions between changing customer demands and infrastructure costs that the ISP faces in its investment decisions. Specifically, we model the traffic routing decisions, the investment in augmenting capacity decisions, and the investment in building network infrastructure decisions made by ISPs. We develop three models: a traffic engineering model, a network provisioning model, and a multi-period network provisioning model. To develop our experiments, we use a real ISP, the Abilene network. We focus primarily on explaining the factors that lead to changes in network performance and extract investment policies for ISPs to maximize the effectiveness of limited resources.

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EXECUTIVE SUMMARY

Internet Service Providers (ISPs) offer access to the Internet and to other network resources to their customers. The ISP marketplace is extremely competitive, requiring ISPs to provide their services with limited resources. We develop models that highlight tensions between changing customer demand and infrastructure costs. The insights from these models can help to improve ISP decision making, as well as help to explain ongoing network growth.

We use constrained optimization to model decisions made by ISPs in the face of changing environmental conditions. Specifically, we examine the traffic routing decisions, the investment in augmenting capacity decisions, and building network infrastructure decisions made by ISPs. Using these models, we conduct numerical experiments to compare performance and topologies that result from ISP decisions.

We develop three ISP decision models: a traffic engineering model, a network provisioning model and a multi-period network provisioning model. These models capture the decisions ISPs make to satisfy demand for network traffic from customers. The traffic engineering model assumes the ISP can only use the current network to meet demand, while the network provisioning models assume the ISP has a budget to invest in additional arc capacity and/or to build new arcs. We demonstrate each model with simple examples that highlight tensions faced by ISPs.

We use a real ISP, the Abilene network, as a case study for our decision models. We introduce hypothetical changes in demand to Abilene, solve the resulting traffic engineering model, and note the effects on arc utilization. Using an assumed budget and the network provisioning model, we propose an "optimal" Abilene built from scratch. We compare the performance of the "optimal" Abilene design with the Abilene network to elicit the factors network designers consider when building network topologies.

In our numerical experiments, we consider the performance and network topology of three network design techniques on a set of 20 synthetic demand matrices. We develop a heuristic design technique that is reactive to changing customer demands and augments capacity where arcs reach saturation. We develop a myopic design technique

that has knowledge of current customer demands and invest optimally for the current period. We develop an omniscient design technique that has knowledge of customer demands in all time periods and invests optimally across all time periods. The results show that ISPs can improve performance either by building network infrastructure or by improving knowledge of future customer demands.

We conduct a sensitivity analysis by varying budget and costs. Intuitively, an increase in budget allows an ISP to increase performance; likewise, a decrease in costs allows an ISP to increase performance. The results highlight an ISPs ability to improve performance with either increased resources or reducing infrastructure costs. The network topologies result in mesh designs as ISPs invest to improve performance.

We vary the investment horizon in our experiments and analyze both the omniscient and myopic design techniques. For the omniscient design technique, we show that reducing an ISP's investment horizon deteriorates performance. If an ISP has knowledge of future customer demands, it is better off investing resources as soon as possible to improve performance in current and future periods. These results highlight the value of knowing future customer demands.

For the myopic design technique, we show that increasing an ISP's investment horizons can improve performance. Increasing investment horizons can provide two benefits to an ISP operating under myopic design conditions. First, the ISP is able to realize customer demands before investing in augmenting capacity and/or building new arcs. Second, the ISP has access more resources to invest in these realized customer demands. Although increasing an ISP's investment horizon offers these benefits, the ISP must find a balance between realizing customer demands and investing in these demands. The results show that choosing long investment horizons decreases performance.

ISPs can approach omniscient performance by choosing an investment horizon that balances when and how much to invest. The best myopic strategy seems to be one that waits long enough between investments to observe real changes in customer demands, but not so long that the network performs too poorly in the meantime. These findings are useful because ISPs typically do not know future customer demands and can implement policies that mimic the myopic technique discussed in this thesis.

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I humbly thank **God**, **Jesus Christ,** and the **Holy Spirit** for serenity, and persistence in writing this thesis and throughout my life …

"I can do all things through Christ who strengthens me."

Philippians 4:13

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I. INTRODUCTION

A. BACKGROUND

During the last two decades, the Internet has grown from a small research network to a global communication infrastructure that is critical for both civilian and military systems. This growing importance has created considerable interest in the Internet's structural features (e.g., efficiency or vulnerability) and the way in which these features evolve over time. Both technologists and policy makers would like to know, *What will the future network look like?* and, *What are the drivers of this change?* Part of the answer can be traced to the owners and operators of the Internet itself.

The public Internet of today is a loose federation of more than 10,000 independent, but interconnected, communication networks, each known as an *Autonomous System* (AS). Each AS operates a network comprised of *routers* and communication *links*. A router is a hardware device that inspects incoming data packets and forwards each one over an outbound link toward its respective destination. The *Internet Protocol* (*IP*) specifies the rules used by the routers for forwarding packets.

The owner and operator of each AS is an *Internet Service Provider (ISP)*—a business entity that invests limited resources in the provisioning and management of the network in order to achieve stated performance objectives. An ISP can manage one or more independent networks, but without loss of generality, we will assume that there is a one-to-one correspondence between an AS and its ISP, often referring simply to the ISP.

Each AS faces a demand for traffic across its network, typically characterized by a matrix of origin-destination (O-D) pairs, called a *demand matrix*. An explicit objective of the ISP is to satisfy the demand for traffic using limited network resources (e.g., link bandwidths), and often with the objective of minimizing its operating cost. This is challenging because the ISP does not know the demand matrix in advance (although it can measure the current traffic on its network at any point), and this demand fluctuates over time.

The ISP has two primary mechanisms that it can use to influence whether or not its network will satisfy its demand matrix. The first is *traffic engineering*—the ISP can configure the routers and their protocols to control the pathways used by the data packets as they cross the network. The second is *network provisioning*—the ISP can invest to increase the quantity, capacity, and/or location of physical hardware in the network. *Network provisioning* is sometimes also called *network augmentation* or *network design*, depending on whether the network of interest already exists or is built from scratch.

This thesis focuses on the drivers of ISP network evolution, as reflected by the traffic engineering and network provisioning decisions of the ISP in the face of changing traffic demand. Specifically, we formulate and solve constrained optimization problems representing these decision problems over a multi-period time horizon. These problems are important because the ISP marketplace is hypercompetitive, thus requiring ISPs to do more with fewer resources. In addition, constantly changing demands require ISPs to adapt their network infrastructure in an ongoing manner. We develop models that highlight tensions between changing customer demand and infrastructure costs. The insights from these models help to improve ISP decision making, as well as help to explain ongoing network growth.

A secondary objective of this thesis is to explore the relationship between traffic demand, operational constraints, and the structure of the network topologies that arise from this design process. There has been considerable academic attention on the large-scale structure of Internet topologies, but most of this work has been descriptive, rather than explanatory. Our aim is to look at ISP network evolution form a "first principles" perspective and characterize the network organization that results from functional requirements.

B. LITERATURE REVIEW OF PREVIOUS WORK

Our modeling and analysis, to explain the traffic engineering and network provisioning decisions made by ISPs over multi-period time horizons, draws from previous work in the literature. We highlight several contributions and include references for a more in-depth coverage of the topics.

1. Traffic Engineering

In practice, IP routing controls the flow of traffic based on a network's topology and the configuration of protocol parameters. The protocols obtain a set of traffic routing paths without regard to the traffic demands or the utilization of the arcs in the network. It is the ISP's responsibility to set protocol parameters such that the ISP's customers receive an acceptable level of service. Rexford (2006) offers a model that finds a set of protocol parameters using optimization. The model uses existing network topology and traffic demands as inputs to find an optimal set of protocol parameters based on performance objectives (Rexford, 2006).

The traffic engineering solutions we obtain using constrained optimization result from traffic demand as input and a specified performance measure (e.g., minimize cost). These solutions represent idealized traffic engineering, which might not be realized in practice by actual routing protocols, but they are useful in identifying an upper bound on ISP performance.

2. Network Provisioning and Design

Modeling an ISP's design and provisioning decisions using constrained optimization is not new. Forsgren and Prytz (2006), and references therein, cover a wide range of network design problems and their solutions using optimization models. The performance criteria used in their models include throughput, redundancy, and cost, with some discussion of the issues that arise when choosing one performance criterion versus another.

In this thesis, we focus exclusively on cost as the performance measure that drives decisions made by ISPs in the provisioning of their networks.

3. Network Topology

Because of the tremendous growth of the Internet over the last 20 years, considerable effort has gone to understanding its overall connectivity. Much of this effort has been statistical in nature, with researchers emphasizing features such as the distributions and correlations of node connectivity.

In contrast to the purely statistical characterizations of Internet connectivity, Li, Alderson, Doyle, and Willinger (2004) develop a “first principles approach to understanding Internet topology at the router level” (p. 1). The researchers develop a model that incorporates functional requirements, physical constraints, and economic constraints, which they claim are insightful and reflective of network engineering. The physical constraints include the number of link connections possible using current router technologies. The economic constraints include the cost of infrastructure such as creating new link connections between routers. They generate simply toy networks according to these constraints and compare their performance to networks generated to match observed statistics. The research shows that networks generated solely to match connectivity statistics do not perform well enough to be realistic representations of the Internet.

Alderson, Li, Willinger, and Doyle (2005) use data from real ISP networks to provide experimental confirmation of this theoretical result. They include data from the Corporation for Education Network Initiatives (CENIC)—a regional network from California that “acts as an ISP for the state’s colleges and universities” (Alderson et al., 2005, p. 1212), and data from the Abilene Network—“the national Internet backbone for higher education” (p. 1211). Their research validates the claim that “technological and economic forces are relevant and real to the real Internet” (p. 1211).

Alderson, Chang, Roughan, Uhlig, and Willinger (2006) discuss the relationship between Internet topology and its associated traffic. A basic observation is that the architecture of the Internet leads to many different vantage points from which to measure and model Internet topology. Specifically, the TCP/IP protocol stack has a physical layer, a data link layer, a network layer, a transport layer, and an application layer. As a result, the researchers stress that “the meaning of network ‘topology’ and ‘traffic’ depends directly on one’s choice of focus” (p. 570).

Recent work done at the Naval Postgraduate School considers other features of network topology that are relevant to this study of ISP network evolution.

Barkley (2008) identifies the router or link attacks that cause the worst possible disruption of traffic flow to an IP-based network. He formulates optimization models that solve the traffic-engineering problem for maximum flow across the network. Barkley then formulates an optimization model that simulates the attack of an intelligent adversary who wants to cause the worst possible disruption of traffic flow on the network. That is, the optimization problem minimizes the maximum flow. He then uses these models to analyze the Abilene network for worst-case disruptions and reports the results.

Derosier (2008) compares heuristic and optimization-based approaches to the design of ISP network topologies. He first analyses an existing U.S. National Tier-1 ISP and observes key design principles. Second, he develops heuristic and optimization models to generate realistic ISP networks based on observations of the Tier-1 ISP. Finally, he compares the performance of the networks created using the heuristic models and the optimization models, based on cost and throughput.

4. Traffic Matrices

Realistic traffic matrices are critical for assessing how ISPs react to changes in their environment. Since ISPs do not make their traffic matrices available for public use, creation of realistic traffic matrices is required for analysis. There is extensive study in the field of generating traffic matrices.

Zhang, Roughan, Duffield, and Greenberg (2003) propose a method for accurate computation of traffic matrices using link load measurements. Their work uses these link load measurements, which are readily available in IP networks using Simple Network Management Protocol (SNMP), along with network routing configuration information. *Tomogravity* is the name of their method, which has two steps. The first step is to use a gravity model along with the link load data to generate an initial solution. The second step minimizes the problem size by using knowledge of the topology configuration and network routing and refines the initial solution using quadratic programming.

Zhang, Roughan, Lund, and Donoho (2003) propose a different method for estimating traffic matrices using information theory. Realizing that most inference problems are ill-posed because they involve many more unknowns than known data; they use "regularization" on the ill-posed problem via an entropy penalization approach. Finally, the method uses convex optimization based algorithms to solve for traffic matrices.

Roughan (2005) introduces a simplification of an approach to the synthesis of traffic matrices using *gravity models*. Roughan shows that using gravity models to synthesize Internet traffic matrices is reasonable. He uses an exponential distribution to generate the necessary independent and identically distributed random variables. He then generates the traffic matrix using the gravity model. Gravity models received their name from Newton's law of gravitation. The idea is that the demand for network traffic between two cities, in an ISP network, is proportional to the product of their populations.

In this thesis, we generate synthetic matrices for traffic demand according to a gravity model. Henceforth, we refer to these matrices as *demand matrices*.

5. Network Growth

McPherson (2009) contrasts networks that grow randomly with those that result from design. Specifically, she considers probabilistic graph formation models, including classic Erdös-Rényi models, geometric random graphs, and preferential attachment models. She compares their behavior with optimization-based models that deliberately grow network structure to achieve a stated performance objective. McPherson uses the minimization spanning tree problem as the basis for this comparison.

This thesis extends this type of analysis to the design of ISP networks.

C. OVERVIEW OF WORK IN THIS THESIS AND CHAPTER OUTLINE

In Chapter II, we formulate three optimization models including the traffic engineering model, network provisioning model, and a multi-period network provisioning model. Using a few simple examples, we demonstrate the tensions faced by ISPs in traffic routing and network design. In Chapter III, we use an existing ISP, Abilene, as a case study. In Chapter IV, we perform numerical experiments that compare the performance of three design techniques. In Chapter V, we discuss conclusions and offer topics for further research.

II. INTERNET SERVICE PROVIDER DECISION MODELS

A. MODELING AN ISP NETWORK

The primary driver of ISP traffic is the demand from its customers. The fundamental problem faced by an ISP is to find a way to route traffic through its network that satisfies all demands. These traffic routes must also meet operational constraints such as the capacity available to carry the traffic on each arc.

ISPs typically face two objectives when routing traffic: *minimize cost* and *minimize arc congestion*. Throughout this thesis, we focus on routing traffic to minimize cost, as defined below. This *performance measure* is the objective of all network models in this thesis. We consider the way in which ISPs use traffic engineering or network provisioning to meet this performance measure.

B. NOTATION

Throughout this thesis, we follow the notation and conventions in Ahuja, Magnanti, and Orlin (1993). Let N be a set of nodes, each representing a router in an ISP network. We define a *path* as a sequence of nodes in a network without repetition. Here, a node can correspond to the origin, intermediate point, or destination along a particular path. Let $n = 1, 2, \ldots, |N|$ index set N. Aliases for n include i, j, s, and t. The set $A \subset N \, x \, N$ represents the set of arcs that connect nodes in a network. We use (i, j) to denote an arc from node i to node j.

Let X_{ijt} represent the quantity of traffic bound for destination t that traverses arc (i, j), and let u_{ij} represent the upper bound (or *capacity*) on traffic over arc (i, j). We define the *utilization* of arc (i, j) to be $\sum_t X_{ijt} / u_{ij}$, the fraction of its capacity in use. We say that arcs with utilization near 1.0 are "congested" and arcs with utilization equal to 1.0 are "saturated."

C. DEMAND MATRICES

We generate synthetic demand matrices according to a *gravity model*. Let b_{st} be the demand for traffic from origin *s* bound for destination *t*. Let *p(i)* be the population of city *i* in millions of people. Let γ_i be the ISP market penetration at city *i*. Then the population of ISP customers is $\gamma_i\ p(i)$. A general formulation for the demand between two cities is $b_{st} = \gamma_s\ p(s) \gamma_t\ p(t)$, where γ_s is the market penetration at city *s*, γ_t is the market penetration at city *t*, *p(s)* is the population at origin city *s*, and *p(t)* is the population at destination city *t*. Using the population of six cities as input into the gravity model, an example O-D demand matrix is in Table 1.

City	Population (millions)
N1	1.50
N2	0.25
N3	0.75
N4	0.75
N5	1.25
N6	2.50

	N1	N2	N3	N4	N5	N6
N1	-0.83	0.04	0.11	0.11	0.19	0.38
N2	0.04	-0.17	0.02	0.02	0.03	0.06
N3	0.11	0.02	-0.47	0.06	0.09	0.19
N4	0.11	0.02	0.06	-0.47	0.09	0.19
N5	0.19	0.03	0.09	0.09	-0.72	0.31
N6	0.38	0.06	0.19	0.19	0.31	-1.12

Legend:
B(s,t) B(s,t) = Demand for Traffic Gbps

Table 1. Six cities and their populations (left), example O-D demand matrix using gravity model and $\gamma_i = 0.32$ (right). For example, in the O-D demand matrix (right), the entry in row N1, column N2 = 0.04 is the demand from N1 to N2. Likewise, the entry in row N1, column N1 = -0.83 is the negative sum of the demand for traffic from all other cities to N1.

D. TRAFFIC ENGINEERING TO MINIMIZE COST

The ISP seeks traffic flows X_{ijt} that minimize cost and satisfies the demand for network traffic. Let c_{ij} represent the cost to route a unit of traffic across arc *(i, j)*.

Let d_{ij} represent the distance of arc *(i, j)*. Let the elastic variable W_{st} represent the quantity of traffic originating at *s* that cannot reach its destination *t* due to insufficient arc capacity on the network. We say that such traffic has been "dropped" by the network. We assume the ISP incurs a penalty ρ per unit of undelivered traffic W_{st} and that this penalty is the same for all traffic. To ensure that the network incurs this penalty only when no alternate path exists, we set $\rho = |N|C$, where $C = \max_{(i,j)\in A} c_{ij}$. Therefore, the penalty per unit of dropped traffic is more costly than the per-unit cost of routing traffic along the most expensive path in the network. Note that when $c_{ij} = 1$, the operating cost reduces to the number of arc segments, or "hops" traversed by all traffic. In contrast, when $c_{ij} = d_{ij}$ the operating cost is Gb-mile travel distance.

We assume the ISP knows the demand matrix that it faces. Using the available network topology, the ISP seeks the flows that minimize cost. We formulate the corresponding traffic engineering model as follows.

Sets

N	Nodes
A	Directed Arcs $A \subset N \, x \, N$

Index use [~cardinality]

$n \in N$	Nodes, an ordinal set (alias *i, j, s, t*) [~tens]
$(i, j) \in A$	Directed arcs from i to j arc *(i, j)* [~hundreds]

Parameters [units]

b_{st}	Demand (<0) or supply (≥ 0) from origin *s* bound for destination *t*, where $b_{tt} = -\sum_{s \neq t} b_{st}, \; \forall t \in N$ [Gbps]
c_{ij}	Cost, to send a unit of traffic over arc $(i, j) \in A$ [cost]
u_{ij}	Upper bound capacity on traffic over directed arc $(i, j) \in A$ [Gbps]

ρ Penalty, per unit of traffic that does not reach its destination [cost/Gbps]

<u>Decision Variables</u> [units]

X_{ijt} Quantity of traffic bound for destination *t* on arc $(i, j) \in A$ [Gbps]

W_{st} Quantity of O-D traffic from *s* to *t* that is dropped [Gbps]

<u>Formulation</u> **TE**

$$\min_{X,W} \quad Z_{\mathbf{TE}} = \sum_{(i,j)\in A}\left(\sum_{t\in N} c_{ij} X_{ijt}\right) + \rho \sum_{s,t\in N} W_{st} \tag{TE0}$$

$$s.t. \quad \sum_{j:(s,j)\in A} X_{sjt} - \sum_{i:(i,s)\in A} X_{ist} + W_{st} = b_{st}, \quad \forall s,t \in N, s \neq t \tag{TE1}$$

$$\sum_{j:(t,j)\in A} X_{tjt} - \sum_{i:(i,t)\in A} X_{itt} - \sum_{s\neq t} W_{st} = b_{tt}, \forall t \in N \tag{TE2}$$

$$\sum_{t\in N} X_{ijt} \leq u_{ij}, \quad \forall (i, j) \in A \tag{TE3}$$

$$X_{ijt} \geq 0, \quad \forall (i, j) \in A, \forall t \in N \tag{TE4}$$

$$W_{st} \geq 0, \quad \forall (s,t) \in N \tag{TE5}$$

<u>Discussion</u>

The objective function (TE0) includes the cost of routing traffic over the network and incurs a penalty ρ for each unit of dropped traffic W_{st}. The constraints (TE1) and (TE2) ensure a balance of traffic flow at every node, whether it is a destination or not. The constraint (TE3) ensures that the traffic along arc *(i, j)* does not exceed the available capacity. The constraint (TE4) ensures network flows are nonnegative. The constraint (TE5) ensures that any dropped traffic is nonnegative.

TE Example

With an example, we demonstrate the formulation and solution of model **TE**. Consider a simple ISP network with 6 nodes and 16 arcs, each arc with a capacity of 1 unit. Assume $c_{ij} = 1$. This simple ISP network appears in Figure 1.

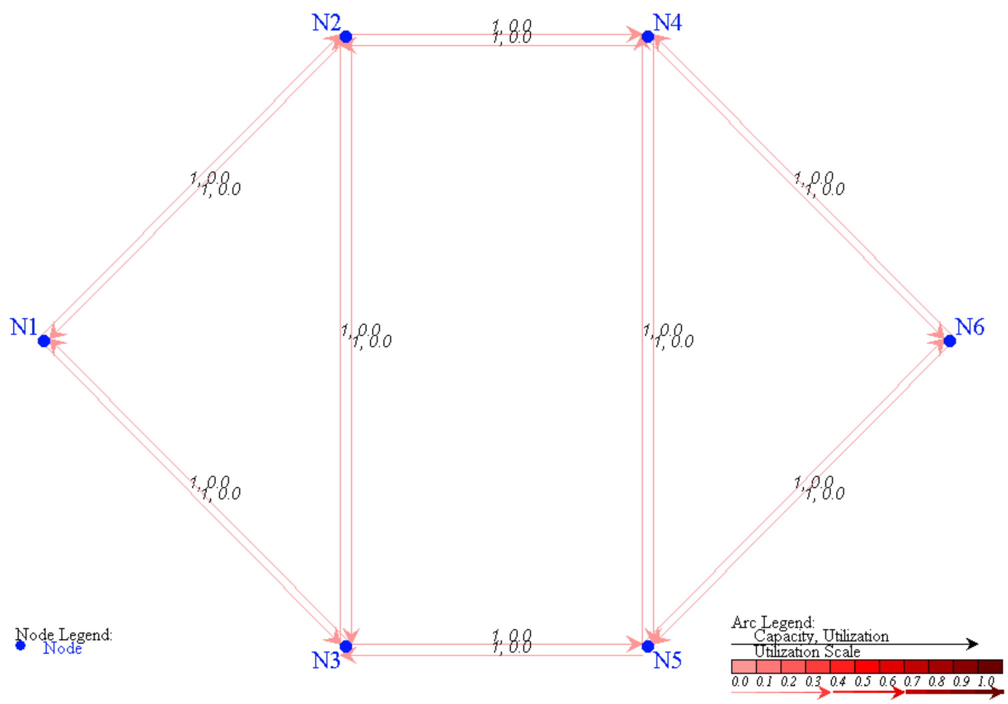


Figure 1. Simple ISP network

Assume a simplified demand of 1 unit from node N3 to node N4 and 1 unit from node N1 to node N6. The O-D demand matrix appears in Table 2.

	N1	N2	N3	N4	N5	N6
N1						1.00
N2						
N3				1.00		
N4				-1.00		
N5						
N6						-1.00

Legend:

B(s,t)	B(s,t) = Demand for Traffic Gbps

Table 2. Simple ISP O-D demand matrix

Using the known demand matrix, the ISP solves **TE** with $Z_{\mathbf{TE}}$ = 5. The flows include 1 unit from node N1 to node N6, along the path N1-N3-N5-N6, and 1 unit from node N3 to node N4 along the path N3-N2-N4. These flows appear in Figure 2.

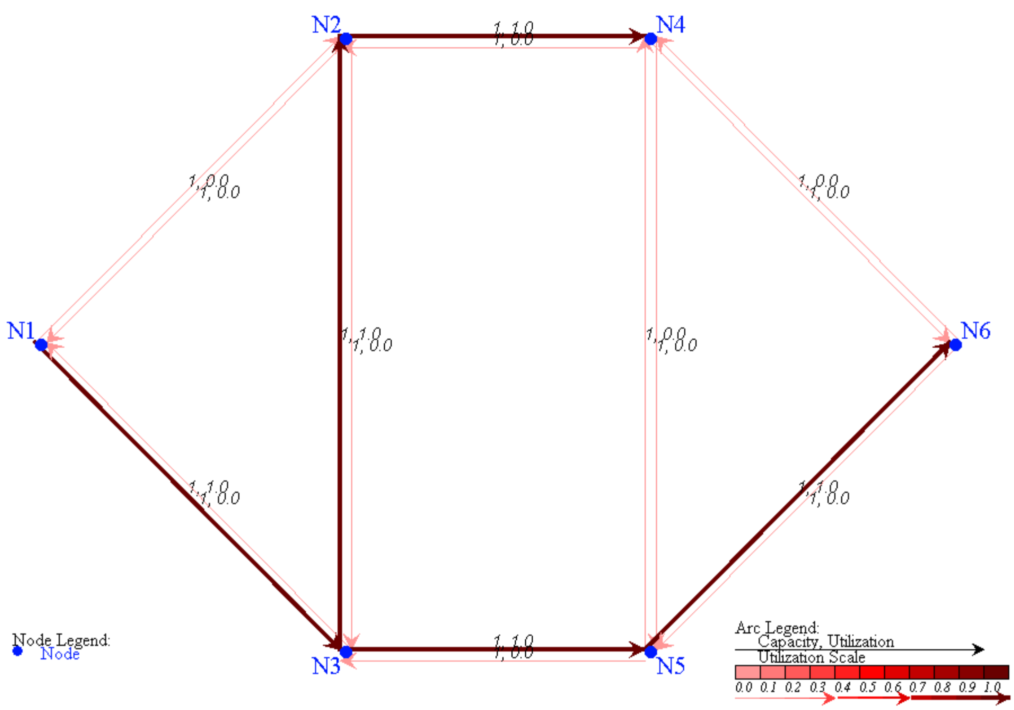


Figure 2. Simple ISP network with solution to **TE,** $Z_{\mathbf{TE}}$ = 5.

E. NETWORK PROVISIONING TO MINIMIZE COST

In this problem, the ISP seeks traffic flows X_{ijt} that minimize cost and satisfies the demand for network traffic; however, the ISP has a limited budget to invest in additional capacity for the existing arcs and/or to build additional arcs. We assume that building a new arc incurs a fixed cost, plus a variable cost that increases with the capacity of the arc, both of which increase with the physical length of the arc. In contrast, adding capacity to an existing arc incurs only the variable cost.

We assume the ISP knows the demand matrix and uses the available budget for additional capacity to provision on existing arcs and/or to build new arcs in the network. The ISP seeks the best provisioning of resources and routing of traffic to minimize cost. This formulation builds on the traffic engineering model with additional parameters, variables, and constraints. We formulate the network provisioning model as follows.

Additional Parameters [units]

d_{ij}	Distance of arc $(i, j) \in A$ [miles]
α	Per distance cost of building a new arc [cost/mile]
β	Per unit distance cost of provisioning capacity over an arc [cost/Gbps-mile]
maxcapacity	The maximum allowable capacity to provision on any new arc $(i, j) \in A$ [Gbps]
budget	Total available budget [cost]

Additional Decision Variables [units]

V_{ij} Binary Variable {0, 1}, 1 if arc $(i, j) \in A$ is built, and 0 otherwise [binary]

R_{ij} Binary Variable {0, 1}, 1 if arc $(i, j) \in A$ exists, and 0 otherwise [binary]

Y_{ij} Quantity of additional capacity to provision on arc $(i, j) \in A$ [Gbps]

Formulation **NP**

$$\min_{R,V,Y,X,W} \quad Z_{\mathbf{NP}} = \sum_{(i,j)\in A} \left(\sum_{t \in N} c_{ij} X_{ijt} \right) + \rho \sum_{s,t \in N} W_{st} \qquad \text{(NP0)}$$

$$\begin{aligned}
s.t. \quad & \text{(TE1)}, \text{(TE2)}, \text{(TE4)}, \text{(TE5)} & & \\
& \sum_{t \in N} X_{ijt} \le u_{ij} + Y_{ij}, & \forall (i, j) \in A, & \quad \text{(NP1)} \\
& \alpha \sum_{(i,j)\in A} d_{ij} V_{ij} + \beta \sum_{(i,j)\in A} d_{ij} Y_{ij} \le budget & & \quad \text{(NP2)} \\
& Y_{ij} \le R_{ij} \; maxcapacity, & \forall (i, j) \in A & \quad \text{(NP3)} \\
& V_{ij} \le R_{ij}, & \forall (i, j) \in A & \quad \text{(NP4)} \\
& R_{ij} \le u_{ij} + V_{ij}, & \forall (i, j) \in A & \quad \text{(NP5)} \\
& \sum_{j} R_{ij} \ge 2, & \forall i \in N & \quad \text{(NP6)} \\
& V_{ij} = V_{ji}, & \forall (i, j) \in A & \quad \text{(NP7)} \\
& R_{ij} = R_{ji}, & \forall (i, j) \in A & \quad \text{(NP8)} \\
& V_{ij} = \{0,1\}, & \forall (i, j) \in A & \quad \text{(NP9)} \\
& R_{ij} = \{0,1\}, & \forall (i, j) \in A & \quad \text{(NP10)} \\
& Y_{ij} \ge 0, & \forall (i, j) \in A & \quad \text{(NP11)}
\end{aligned}$$

Discussion

As with formulation **TE**, the objective function (NP0) includes the cost of routing traffic over the network and incurs a penalty ρ for each unit of dropped traffic W_{st}. We again have constraints (TE1) and (TE2) to maintain balance of flow and constraints, along with (TE4) and (TE5) to prevent nonnegative flows and dropped traffic. The constraint (NP1) ensures traffic along arc *(i, j)* does not exceed the available capacity which now includes newly added capacity Y_{ij}. The constraint (NP2) ensures the total cost of additional network provisioning does not exceed the available budget. The constraint (NP3) ensures additional capacity provisioned on arc *(i, j)* does not exceed a maximum allowable capacity limit. The constraint (NP4) ensures that if arc *(i, j)* is built, then it is available. The constraint (NP5) ensures that if arc *(i, j)* does not already exist (i.e., $u_{ij} = 0$) and it is not built (i.e., $V_{ij} = 0$), then it is not available. In addition, we prevent the unnecessary construction of an arc *(i, j)* that already exists by setting $V_{ij} = 0$ if $u_{ij} > 0$. The constraint (NP6) ensures that for each node an arc exists to at least two other nodes. The constraints (NP7) and (NP8) ensure symmetry between built arcs. The constraints (NP9) and (NP10) ensure V_{ij} and R_{ij} are binary variables. The constraint (NP11) ensures additional arc capacity on any arc in the network is nonnegative.

NP Example

With a simple example, we demonstrate the formulation and solution to model **NP**. Consider a simple ISP network with 6 nodes and 16 arcs, each arc with a capacity of 1 unit. Assume the ISP has the option to build and provision arcs (N2, N5), (N5, N2), (N3, N4), and/or arc (N4, N3) which appear as dashed lines. Assume c_{ij} = 1 and d_{ij} = 1 and that the ISP faces the demand matrix in Table 2. This simple ISP network is in Figure 3.

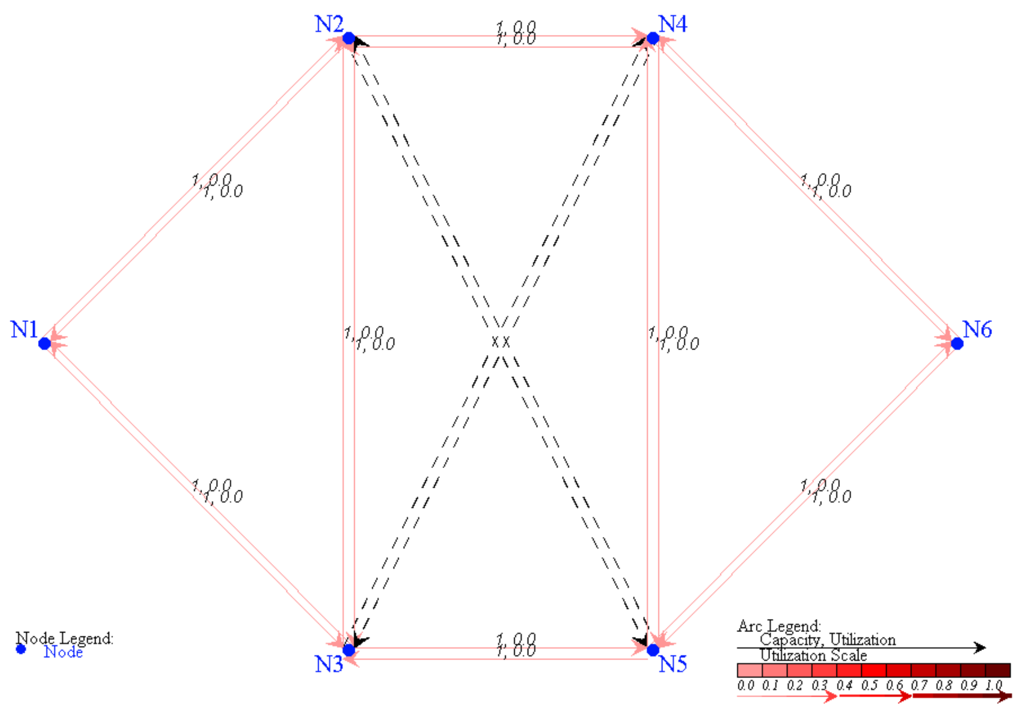


Figure 3. Simple ISP network with 4 potential arcs.

Assume the ISP has a *budget* of 3 units to augment capacity and/or build arcs and that parameter α = 1 and β = 1. The ISP solves **NP** using the *budget* and the demand matrix in Table 2. After solving **NP**, Z_{NP} = 4. The ISP invests 2 units to build arc (N3, N4) and (N4, N3), and 1 unit to provision 1 unit of capacity on arc (N3, N4). The flows include 1 unit from node N1 to node N6, along the path N1-N2-N4-N6, and 1 unit from node N3 to node N4 along the newly built arc (N3, N4). The resulting network is in Figure 4.

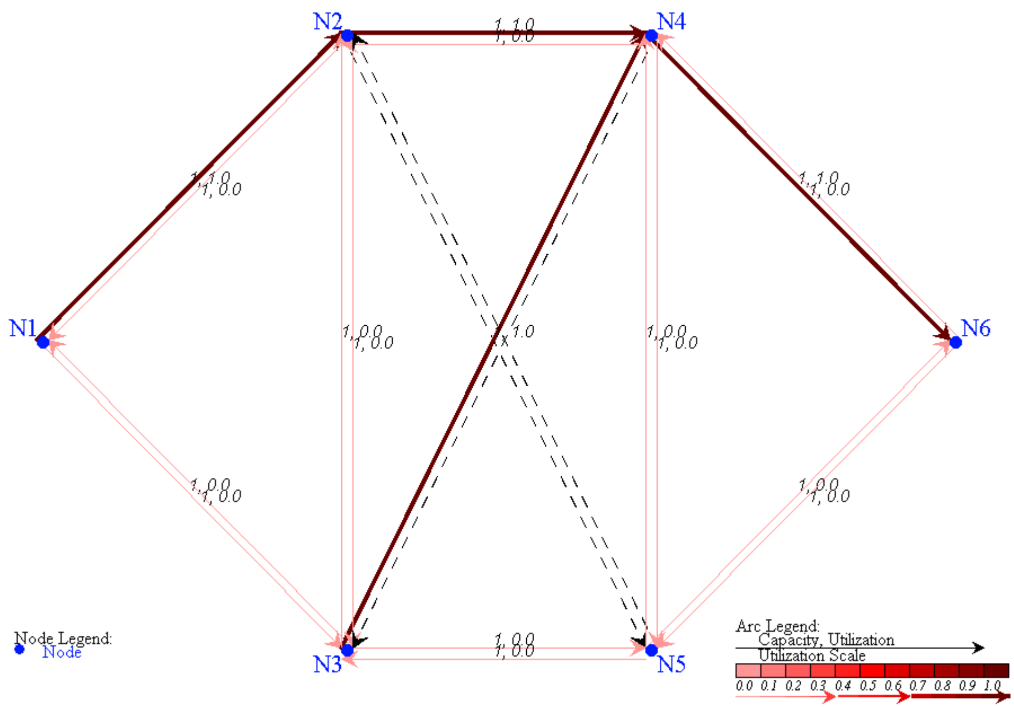


Figure 4. Simple ISP network with additional arc (N3, N4) built and provisioned with 1 unit of capacity; $Z_{\mathbf{NP}} = 4$.

F. MULTI-PERIOD NETWORK PROVISIONING TO MINIMIZE COST

We now consider the network provisioning model extended for multiple time periods and where the ISP has an operating *budget* in each time period to invest in additional capacity for the existing arcs and/or to build additional arcs. By construction, the ISP cannot provision an arc unless it is built. We assume that an arc, once built and/or provisioned with new capacity, is available for all future time periods. We let T represent the set of periods over which we are solving the network design model, and use $\tau \in T$ to denote each time period.

As before, we assume the ISP knows the values in the demand matrix for all periods. The formulation is similar to the network provisioning model with the addition of a set of periods and additional parameters, variables, and constraints. We formulate the multi-period network design model for finding traffic flows that meet demand at minimum cost.

Additional Sets

T Periods

Additional Index use [~cardinality]

$\tau \in T$ Periods [~tens] alias τ'

Additional Parameters [units]

b_{st}^{τ} Demand (<0) or supply (≥ 0) from origin s bound for destination t in period τ where $b_{tt}^{t} = -\sum_{s \neq t} b_{st}^{t}, \; \forall t \in N$ [Gbps]

$budget^{\tau}$ Total available budget in period τ [cost]

Decision Variables [units]

B^{τ} The quantity of budget to spend in period τ [cost]

R_{ij}^{τ} Binary Variable {0, 1}, 1 if arc $(i, j) \in A$ exists in period τ 0 otherwise [binary]

V_{ij}^{τ} Binary Variable {0, 1}, 1 if arc $(i, j) \in A$ is built in period τ 0 otherwise [binary]

Y_{ij}^{τ} Quantity of additional capacity to provision on arc $(i, j) \in A$ in period τ [Gbps]

X_{ijt}^{τ} Quantity of traffic bound for destination t on arc $(i, j) \in A$ in period τ [Gbps]

W_{st}^{τ} Quantity of O-D traffic that is dropped in period τ [Gbps]

<u>Formulation</u> **MPNP**

$$\min_{B,R,V,Y,X,W} \qquad Z_{\mathbf{MPNP}} = \sum_{\tau} \left(\sum_{(i,j)\in A} \left(\sum_{t\in N} c_{ij} X^{\tau}_{ijt} \right) + \rho \sum_{s,t\in N} W^{\tau}_{st} \right) \qquad \text{(MPNP0)}$$

$$s.t. \quad \sum_{j:(s,j)\in A} X^{\tau}_{sjt} - \sum_{i:(i,s)\in A} X^{\tau}_{ist} + W^{\tau}_{st} = b^{\tau}_{st}, \quad \forall \tau \in T, \forall s,t \in N (s \neq t) \qquad \text{(MPNP1)}$$

$$\sum_{j:(t,j)\in A} X^{\tau}_{tjt} - \sum_{i:(i,t)\in A} X^{\tau}_{itt} - \sum_{s\neq t} W^{\tau}_{st} = b^{\tau}_{tt}, \forall \tau \in T, \forall t \in N \qquad \text{(MPNP2)}$$

$$\sum_{t\in N} X^{\tau}_{ijt} \leq u_{ij} + \sum_{\tau' \leq \tau} Y^{\tau'}_{ij}, \qquad \forall \tau \in T, \forall (i,j) \in A, \qquad \text{(MPNP3)}$$

$$\alpha \sum_{(i,j)\in A} d_{ij} V^{\tau}_{ij} + \beta \sum_{(i,j)\in A} d_{ij} Y^{\tau}_{ij} \leq B^{\tau}, \quad \forall \tau \in T \qquad \text{(MPNP4)}$$

$$\sum_{\tau} B^{\tau} \leq \sum_{\tau} budget^{\tau} \qquad \text{(MPNP5)}$$

$$\sum_{\tau} V^{\tau}_{ij} \leq 1, \qquad \forall (i,j) \in A, when\ u_{ij} = 0 \qquad \text{(MPNP6)}$$

$$V^{\tau}_{ij} = 0, \qquad \forall \tau \in T, \forall (i,j) \in A, when\ u_{ij} > 0 \qquad \text{(MPNP7)}$$

$$R^{\tau}_{ij} \leq R^{\tau+1}_{ij}, \qquad \forall \tau \in T, \forall (i,j) \in A \qquad \text{(MPNP8)}$$

$$V^{\tau}_{ij} \leq R^{\tau}_{ij}, \qquad \forall \tau \in T, \forall (i,j) \in A \qquad \text{(MPNP9)}$$

$$R^{\tau}_{ij} \leq R^{0}_{ij} + \sum_{\tau' \leq \tau} V^{\tau'}_{ij}, \qquad \forall \tau \in T, \forall (i,j) \in A, \qquad \text{(MPNP10)}$$

$$Y^{\tau}_{ij} \leq R^{\tau}_{ij}\ maxcapacity, \qquad \forall \tau \in T, \forall (i,j) \in A \qquad \text{(MPNP11)}$$

$$u^{0}_{ij} \leq R^{0}_{ij}\ maxcapacity, \qquad \forall (i,j) \in A \qquad \text{(MPNP12)}$$

$$R^{0}_{ij} \leq u^{0}_{ij} + V^{0}_{ij}, \qquad \forall (i,j) \in A \qquad \text{(MPNP13)}$$

$$\sum_{j} R^{\tau}_{ij} \geq 2, \qquad \forall \tau \in T, \forall i \in N \qquad \text{(MPNP14)}$$

$$R^{\tau}_{ij} = R^{\tau}_{ji}, \qquad \forall \tau \in T, \forall (i,j) \in A \qquad \text{(MPNP15)}$$

$$V^{\tau}_{ij} = V^{\tau}_{ji}, \qquad \forall \tau \in T, \forall (i,j) \in A \qquad \text{(MPNP16)}$$

$$V^{\tau}_{ij} = \{0,1\}, \qquad \forall \tau \in T, \forall (i,j) \in A \qquad \text{(MPNP17)}$$

$$R^{\tau}_{ij} = \{0,1\}, \qquad \forall \tau \in T, \forall (i,j) \in A \qquad \text{(MPNP18)}$$

$$W^{\tau}_{st} \geq 0, \qquad \forall (s,t) \in N, \forall \tau \in T \qquad \text{(MPNP19)}$$

$$X^{\tau}_{ijt} \geq 0, \qquad \forall (i,j) \in A, \forall t \in N, \forall \tau \in T \qquad \text{(MPNP20)}$$

$$Y^{\tau}_{ij} \geq 0, \qquad \forall (i,j) \in A, \forall \tau \in T \qquad \text{(MPNP21)}$$

Discussion

The objective function (MPNP0) minimizes the cost of routing traffic over the network over all periods, again with a penalty ρ for each unit of dropped traffic W_{st}^{τ}. The constraints (MPNP1) and (MPNP2) ensure a balance of traffic at every node in each period. The constraint (MPNP3) ensures the traffic along arc *(i, j)* does not exceed the available capacity in period τ. The constraints (MPNP4) and (MPNP5) ensure that the cost of additional capacity, provisioned on arc *(i, j)*, and the cost of additional arcs, does not exceed the available budget. The constraint (MPNP6) ensures that if an arc does not exist, then it can build at most once. The constraint (MPNP7) ensures that if the arc exists, then it cannot be built in any future period. The constraints (MPNP8), (MPNP9), and (MPNP10) ensure that if we build an arc in period τ, then it exists in all future periods. The constraint (MPNP11) ensures that the additional capacity added to arc *(i, j)* in period τ does not exceed a maximum allowable capacity limit. The constraint (MPNP12) ensures that if an arc already exists before the first period, then that arc will exist for all periods. The constraint (MPNP13) ensures that we can build an arc in future periods if it does not exist before period one. The constraint (MPNP14) ensures that for each node, an arc exists to at least two other nodes. The constraints (MPNP15) and (MPNP16) ensure that symmetry exists between built arcs. The constraints (MPNP17) and (MPNP18) ensure that V_{ij}^{τ} and R_{ij}^{τ} are binary variables. The constraint (MPNP19) ensures that dropped flows are non-negative. The constraint (MPNP20) ensures no negative traffic on any arc in the network. The constraint (MPNP21) ensures no negative additional arc capacity on any arc in the network.

MPNP Example: The Value of Knowing Future Demands

In this example, we demonstrate use of the **MPNP** model and the value of knowing future demand. Again, consider the ISP network in Figure 3. As before, we assume that the ISP is considering building and provisioning arcs (N2, N5), (N5, N2), (N3, N4), and/or arc (N4, N3), which appear as dashed lines.

We consider two periods, $\tau = 1$ and $\tau = 2$. We assume the ISP has a *budget*, $B = 3$, to invest in additional arc capacity and/or building new arcs and that parameters $\alpha = 1$ and $\beta = 1$.

Without Knowledge of Future Demands

For $\tau = 1$, the ISP solves **NP** using B and the demand matrix in Table 2. The ISP invests 2 units to build arc (N3, N4) and (N4, N3), and 1 unit to increase the capacity of arc (N3, N4) to 1 unit. $Z_{\mathbf{NP}} = 4$. As before, the resulting network is Figure 4.

At $\tau = 2$, assume the demand for network traffic changes, increasing demand from node N2 to node N4 by 2 units. The $\tau = 1$ and $\tau = 2$ demand matrices appear in Table 3.

	N1	N2	N3	N4	N5	N6
N1						1.00
N2						
N3				1.00		
N4				-1.00		
N5						
N6						-1.00

Legend:
B(s,t) — B(s,t) = Demand for Traffic Gbps

	N1	N2	N3	N4	N5	N6
N1						1.00
N2				2.00		
N3				1.00		
N4				-3.00		
N5						
N6						-1.00

Legend:
B(s,t) — B(s,t) = Demand for Traffic Gbps

Table 3. Demand matrix at $\tau = 2$ (left), demand matrix at $\tau = 2$ (right).

Since the ISP used its entire *budget B* in τ = 1, the ISP solves **TE** with new demands, $Z_{\mathbf{TE}}$ = 11 at τ = 2. The flows include 1 unit from node N2 to node N4, along the arc (N2, N4), 1 unit from node N2 to node N4 along path N2-N3-N5-N4, and 1 unit from node N3 to node N4 along arc (N3, N4). The network at τ = 2 is in Figure 5. Note that one unit of traffic destined for node N6 is dropped, as illustrated in Table 4.

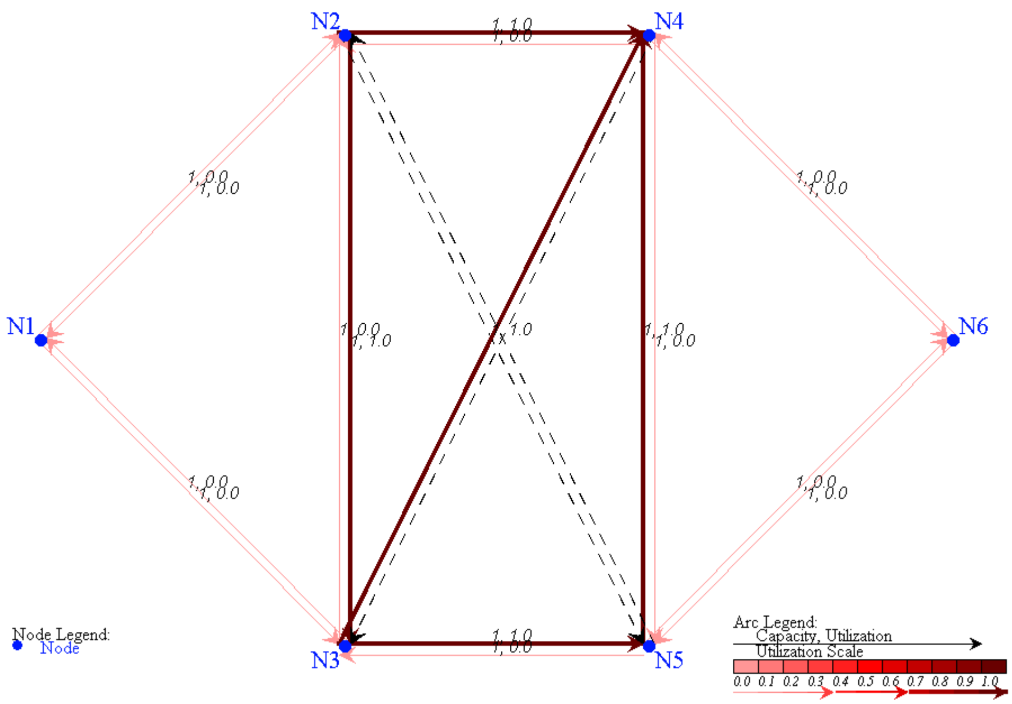
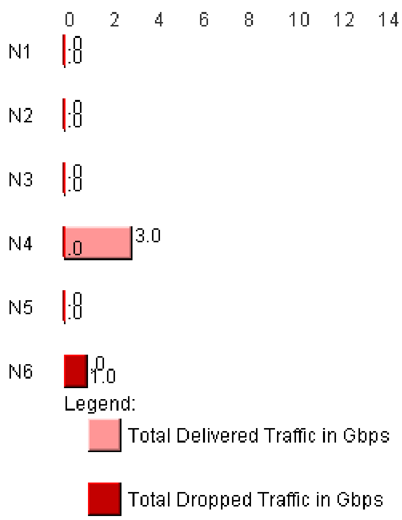


Figure 5. Without knowledge of future demand, $Z_{\mathbf{NP}}$ = 11 at τ =2.



	N1	N2	N3	N4	N5	N6
N1						1.00 1.00
N2				2.00		
N3				1.00		
N4				-3.00		
N5						
N6						-1.00

Legend:

B(s,t) — B(s,t) = Demand for Traffic Gbps

B(s,t) W(s,t) (shaded) — B(s,t) = Demand for Traffic Gbps; W(s,t) = Dropped Traffic Gbps

Table 4. Delivered and dropped traffic (left), demand matrix at $\tau = 2$ (right). For example, in the demand matrix on the right, the shaded cell at row N1, column N6, represents dropped demand. The top number represents the demand, while the bottom number represents the quantity of that demand that is dropped.

With Knowledge of Future Demands

Again, consider the ISP network in Figure 3. Assume the ISP knows the demand matrices for both $\tau = 1$ and $\tau = 2$ from the outset. The ISP solves **MPNP** for $\tau = 1$ and $\tau = 2$ with B and both demand matrices, and obtains, $Z_{\mathbf{MPNP}} = 12$; 5 at $\tau = 1$ and 7 at $\tau = 2$. The flows at $\tau = 2$ include 1 unit from node N1 to node N6, along the path N1-N3-N5-N6, 1 unit from node N3 to node N4 along path N3-N2-N4 and 2 units on arc (N2, N4). All demand is satisfied. The ISP network at $\tau = 2$ is in Figure 6. Note the ISP invests B to augment the capacity on arc (N2, N4) to 3 units.

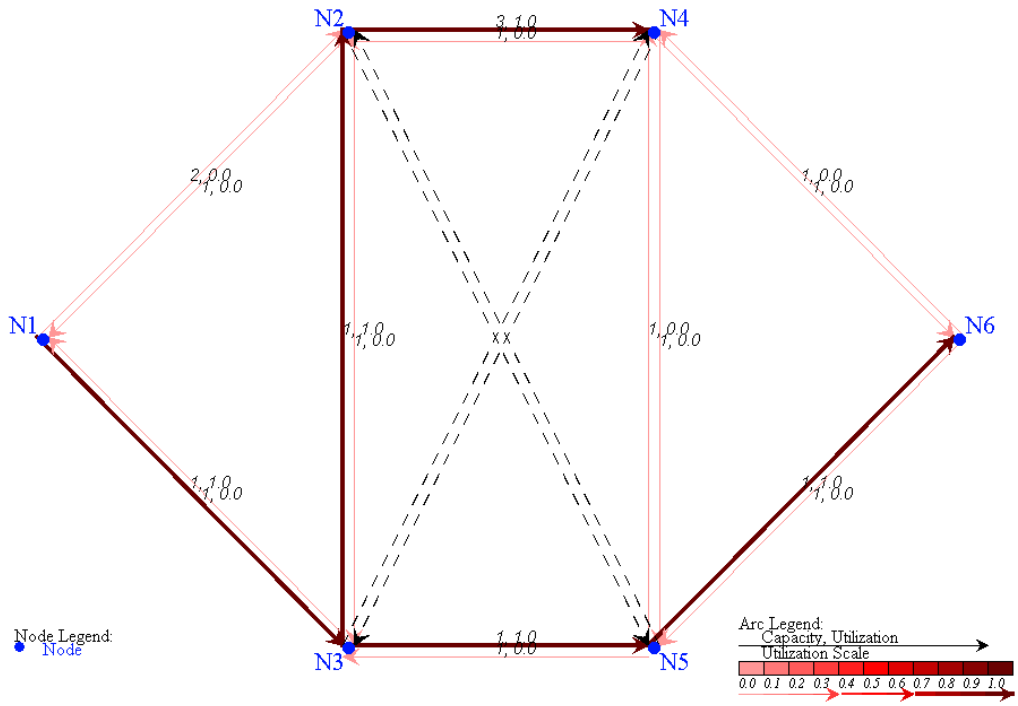


Figure 6. With knowledge of future demand, $Z_{\mathbf{MPNP}} = 12$.

Without knowledge of future demands the ISP minimizes costs at $\tau = 1$ at the expense of future costs and dropped traffic at $\tau = 2$. The ISP solved **NP** at $\tau = 1$ and **TE** at $\tau = 2$ for a $Z_{\mathbf{NP}} = 4$ and a $Z_{\mathbf{TE}} = 11$ for a total $Z_{\mathbf{NP}} + Z_{\mathbf{TE}} = 15$.

With knowledge of future demand, the ISP delays myopic gains in favor of optimal provisioning for both $\tau = 1$ and $\tau = 2$. The ISP solved **MPNP** at $\tau = 1$ and $\tau = 2$ for a $Z_{\mathbf{MPNP}} = 12$.

The value of knowledge in this example is 4 units. With knowledge of future demands, the optimal solution is for the ISP to incur a greater expense in $\tau = 1$ in order to achieve a greater savings in $\tau = 2$. These findings appear in Figure 7.

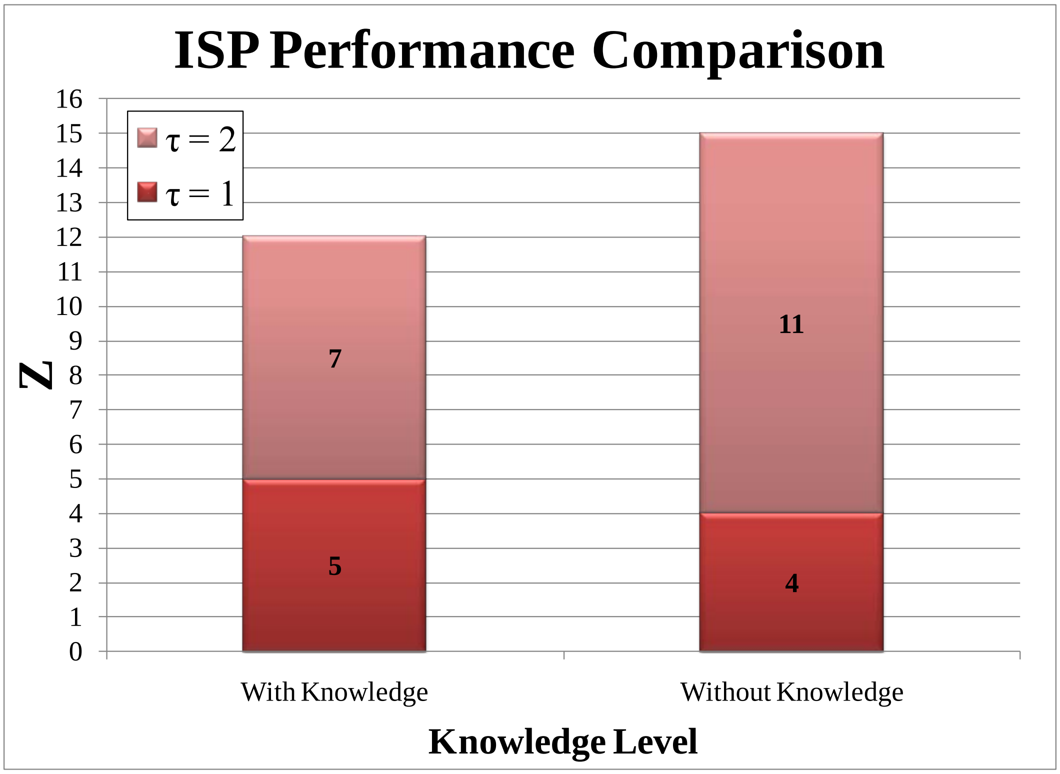


Figure 7. A comparison of performance with and without knowledge of future demand.

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III. CASE STUDY: ABILENE

A. THE ABILENE NETWORK

Abilene is a high-speed research and education network providing IP services to universities throughout the United States (Qwest, 2010). Internet2, Cisco, Nortel Networks, Juniper Networks, Qwest Communications, and Indiana University created Abilene in a joint effort (Qwest, 2010). The network is used by hundreds of universities in the United States for services such as “tele-immersion, virtual laboratories, distance learning, distributed performing arts, tele-medicine, grid computing and digital libraries” (Qwest, 2010).

Figure 8 illustrates the topology of Abilene (Qwest, 2010). Abilene has *Points-of-Presence* (POPs) in 11 cities throughout the U.S. connected via 28 arcs, each with 10 Gbps of capacity.

Figure 8. Abilene topology (From Qwest, 2010).

Although the topology of Abilene is well documented, the demand matrices are unknown. We generate the synthetic O-D demand matrix in Table 6 using the gravity model in Chapter II and the population of each POP in Table 5.

POP	Population (millions)	Customer Population (millions)
Atlanta (ATL)	0.50	0.16
Chicago (CHI)	2.90	0.93
Denver (DEN)	0.60	0.19
Houston (HOU)	2.20	0.70
Indianapolis (IND)	0.80	0.26
Kansas City (KSC)	0.50	0.16
Los Angeles (LAX)	3.80	1.22
New York (NYC)	8.40	2.69
Sunnyvale (SUN)	0.10	0.03
Seattle (SEA)	0.60	0.19
Washington D. C. (WDC)	0.60	0.19

Table 5. Population and assumed customer population of 11 Abilene POPs; population data from U.S. Census Bureau (2010).

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL	-1.03	0.14	0.03	0.11	0.04	0.02	0.19	0.42	0.01	0.03	0.03
CHI	0.14	-5.25	0.17	0.64	0.23	0.14	1.10	2.44	0.03	0.17	0.17
DEN	0.03	0.17	-1.22	0.13	0.05	0.03	0.23	0.50	0.01	0.04	0.04
HOU	0.11	0.64	0.13	-4.14	0.18	0.11	0.84	1.85	0.02	0.13	0.13
IND	0.04	0.23	0.05	0.18	-1.62	0.04	0.30	0.67	0.01	0.05	0.05
KSC	0.02	0.14	0.03	0.11	0.04	-1.03	0.19	0.42	0.01	0.03	0.03
LAX	0.19	1.10	0.23	0.84	0.30	0.19	-6.54	3.19	0.04	0.23	0.23
NYC	0.42	2.44	0.50	1.85	0.67	0.42	3.19	-10.58	0.08	0.50	0.50
SUN	0.01	0.03	0.01	0.02	0.01	0.01	0.04	0.08	-0.21	0.01	0.01
SEA	0.03	0.17	0.04	0.13	0.05	0.03	0.23	0.50	0.01	-1.22	0.04
WDC	0.03	0.17	0.04	0.13	0.05	0.03	0.23	0.50	0.01	0.04	-1.22

Legend:
B(s,t) — B(s,t) = Demand for Traffic Gbps

Table 6. Abilene O-D demand matrix.

B. TRAFFIC ENGINEERING ON EXISTING NETWORK

Using the demand matrix in Table 6 and the known topology of Abilene, we solve model **TE** and obtain a $Z_{\mathbf{TE}}$ of 91. The resulting traffic flows appear in Figure 9. The delivered traffic and arc capacities appear in Table 7.

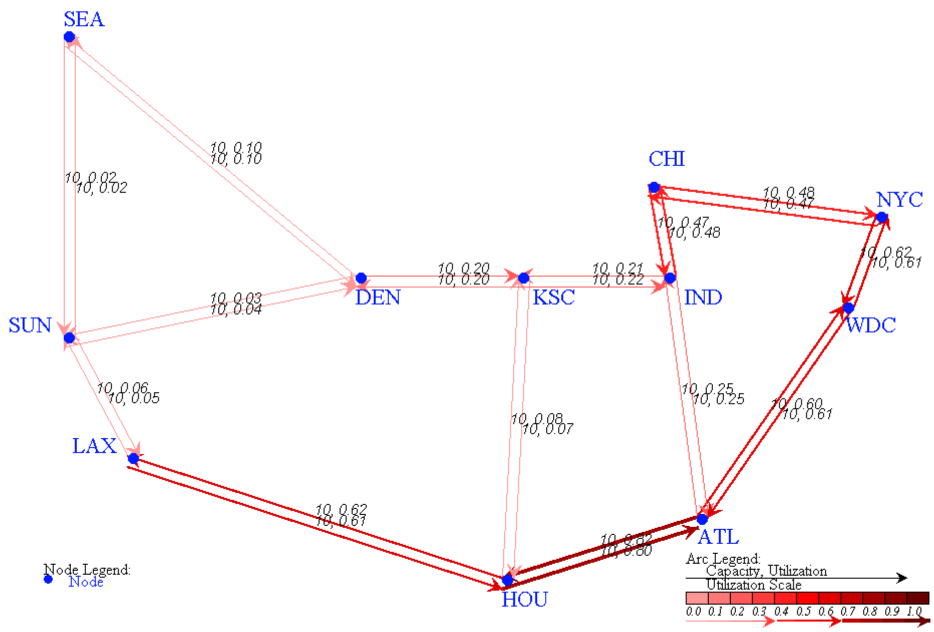
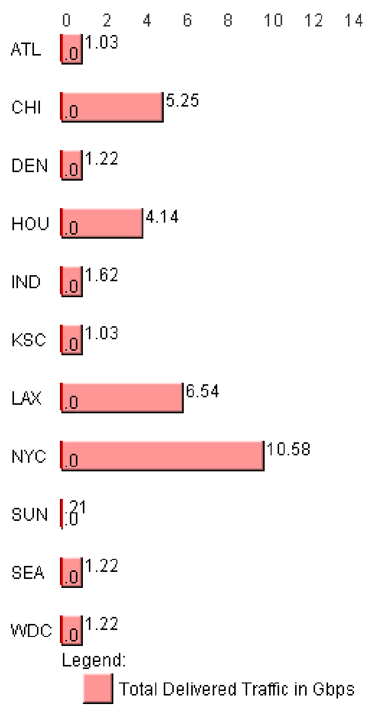


Figure 9. Z_{TE} = 91 for Abilene under assumed demand in Table 6.



	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL				8.13 10.00	2.45 10.00						6.02 10.00
CHI					4.79 10.00			4.78 10.00			
DEN						2.03 10.00			0.37 10.00	0.99 10.00	
HOU	8.05 10.00					0.75 10.00	6.08 10.00				
IND	2.53 10.00	4.79 10.00				2.16 10.00					
KSC			2.03 10.00	0.67 10.00	2.24 10.00						
LAX				6.08 10.00					0.54 10.00		
NYC		4.78 10.00									6.13 10.00
SUN			0.37 10.00				0.54 10.00			0.24 10.00	
SEA			0.99 10.00						0.24 10.00		
WDC	6.02 10.00							6.13 10.00			

Legend:
X(s,t) / U(s,t)
X(s,t) = Arc Flow, X(origin,destination) Gbps
U(s,t) = Arc Capacity, U(origin,destination) Gbps
Utilization Scale
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Table 7. POP delivered traffic (left), arc flow and capacity matrix (right). For example, in the arc flow and arc capacity matrix on the right, cells can contain two numbers. The top number is the total traffic flow, the bottom number is the available capacity.

C. EXAMPLES: HYPOTHETICAL CHANGES IN DEMAND

1. Increase Demand: LAX and NYC

We consider the case where demands increase between LAX and NYC by 10 Gbps. The new demand matrix is in Table 8. We solve **TE** with the new demand matrix and note the utilization on the arcs in the network. The resulting traffic flows are in Table 8. Note the dropped traffic between NYC and LAX in Table 8.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL	-1.03	0.14	0.03	0.11	0.04	0.02	0.19	0.42	0.01	0.03	0.03
CHI	0.14	-5.25	0.17	0.64	0.23	0.14	1.10	2.44	0.03	0.17	0.17
DEN	0.03	0.17	-1.22	0.13	0.05	0.03	0.23	0.50	0.01	0.04	0.04
HOU	0.11	0.64	0.13	-4.14	0.18	0.11	0.84	1.85	0.02	0.13	0.13
IND	0.04	0.23	0.05	0.18	-1.62	0.04	0.30	0.67	0.01	0.05	0.05
KSC	0.02	0.14	0.03	0.11	0.04	-1.03	0.19	0.42	0.01	0.03	0.03
LAX	0.19	1.10	0.23	0.84	0.30	0.19	-16.54	13.19 0.92	0.04	0.23	0.23
NYC	0.42	2.44	0.50	1.85	0.67	0.42	13.19 0.92	-20.58	0.08	0.50	0.50
SUN	0.01	0.03	0.01	0.02	0.01	0.01	0.04	0.08	-0.21	0.01	0.01
SEA	0.03	0.17	0.04	0.13	0.05	0.03	0.23	0.50	0.01	-1.22	0.04
WDC	0.03	0.17	0.04	0.13	0.05	0.03	0.23	0.50	0.01	0.04	-1.22

Legend:

B(s,t)	B(s,t) = Demand for Traffic Gbps
B(s,t) W(s,t)	B(s,t) = Demand for Traffic Gbps W(s,t) = Dropped Traffic Gbps

Table 8. Demand between LAX and NYC increases by 10 Gbps from Table 6.

Notice the saturated arcs Figure 10. In particular, note arc (WDC, NYC), arc (NYC, WDC), arc (IND, CHI), and arc (CHI, IND). These arcs create a *cut*—a partition of the node set N into two parts because no additional traffic can flow between the nodes in either partition. In Abilene, this cut puts SEA, SUN, LAX, DEN, KSC, HOU, IND, ATL, and WDC, in one partition, CHI and NYC in the other. The *cut-set*—the set of arcs that partition the nodes in a network into two disjoint sets, includes the four saturated arcs. The dropped traffic is a result of the cut in Abilene.

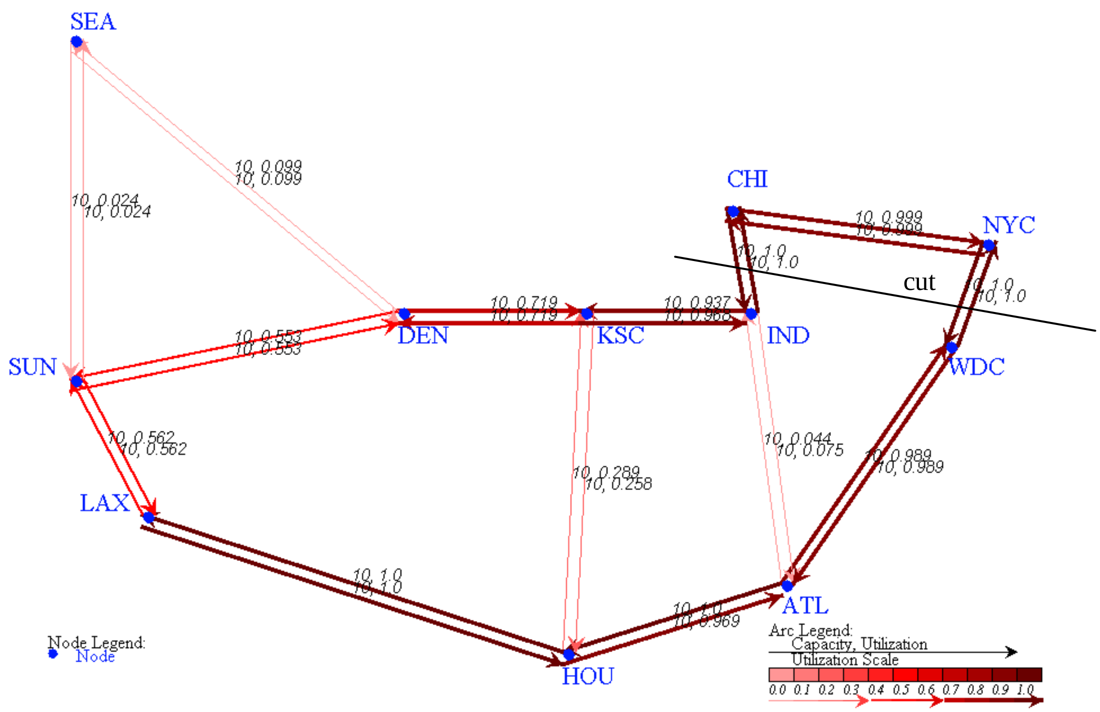


Figure 10. Saturated arcs result in a cut on Abilene.

In this example, we demonstrate one possible result of increasing demands between two POPs on a network. Without an intelligent ISP augmenting capacity on congested arcs, they become saturated resulting in a cut. The cut prevents nodes in the network from communicating causing traffic to drop. An ISP must monitor and augment capacity on congested arcs to prevent dropping traffic.

2. Increase Demand: HOU and IND

We consider the case where demands increase between HOU and IND by 10 Gbps. The new demand matrix is in Table 9. We solve **TE** with the new demand matrix and note the utilization on the arcs in the network. The resulting traffic flows are in Figure 11. Note the dropped traffic between NYC and SUN and between NYC and SEA in Table 9.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL	-1.03	0.14	0.03	0.11	0.04	0.02	0.19	0.42	0.01	0.03	0.03
CHI	0.14	-5.25	0.17	0.64	0.23	0.14	1.10	2.44	0.03	0.17	0.17
DEN	0.03	0.17	-1.22	0.13	0.05	0.03	0.23	0.50	0.01	0.04	0.04
HOU	0.11	0.64	0.13	-14.14	10.18	0.11	0.84	1.85	0.02	0.13	0.13
IND	0.04	0.23	0.05	10.18	-11.62	0.04	0.30	0.67	0.01	0.05	0.05
KSC	0.02	0.14	0.03	0.11	0.04	-1.03	0.19	0.42	0.01	0.03	0.03
LAX	0.19	1.10	0.23	0.84	0.30	0.19	-6.54	3.19	0.04	0.23	0.23
NYC	0.42	2.44	0.50	1.85	0.67	0.42	3.19	-10.58	0.08 0.08	0.50 0.21	0.50
SUN	0.01	0.03	0.01	0.02	0.01	0.01	0.04	0.08 0.08	-0.21	0.01	0.01
SEA	0.03	0.17	0.04	0.13	0.05	0.03	0.23	0.50 0.21	0.01	-1.22	0.04
WDC	0.03	0.17	0.04	0.13	0.05	0.03	0.23	0.50	0.01	0.04	-1.22

Legend:

B(s,t) — B(s,t) = Demand for Traffic Gbps

B(s,t) W(s,t) (shaded) — B(s,t) = Demand for Traffic Gbps; W(s,t) = Dropped Traffic Gbps

Table 9. Demand between HOU and IND increases by 10 Gbps from Table 6.

Notice the saturated arcs Figure 11. In particular, note arc (KSC, IND), arc (IND, KSC), arc (HOU, ATL), and arc (ATL, KSC). These arcs create a cut in Abilene with SEA, SUN, LAX, DEN, KSC, and HOU in one partition, CHI, IND, ATL, NYC, and WDC in the other. The cut-set includes the four saturated arcs. The dropped traffic is a result of the cut in Abilene.

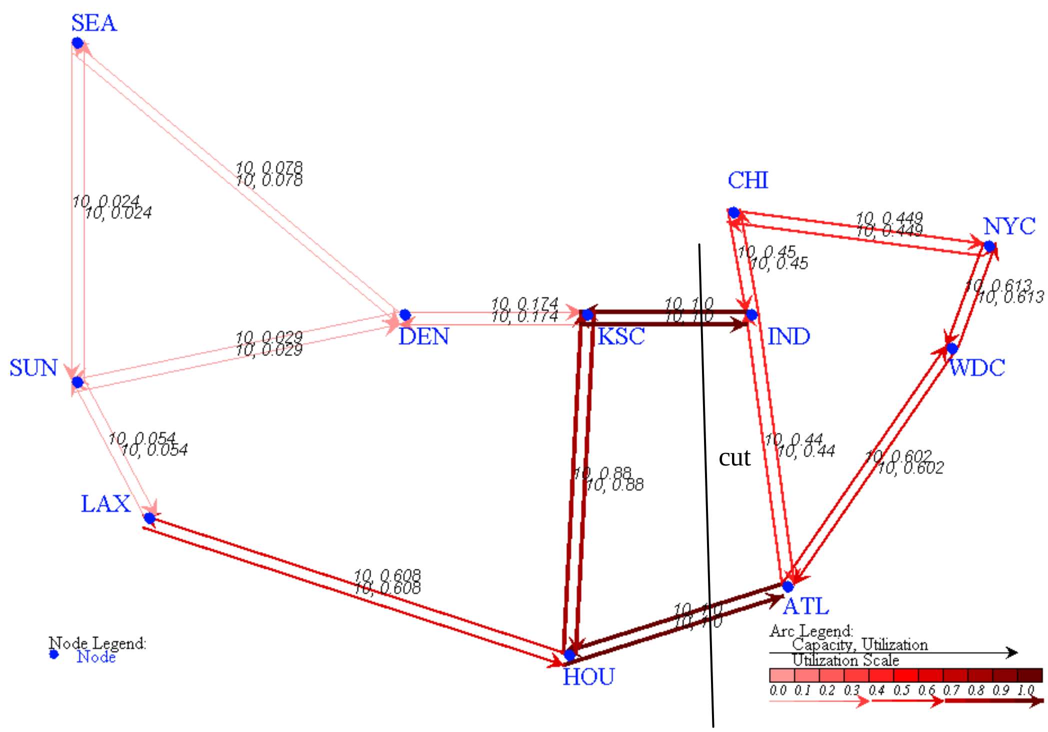


Figure 11. Saturated arcs result in a cut on Abilene.

Because the network does not have residual capacity to route additional flow from HOU to IND, it would seem that an increase in demand between these POPs would cause traffic to drop between these two POPs; however, this is not the case.

We increase the demand between HOU and IND by 5 Gbps. The new demand matrix is in Table 10. We solve **TE** with the new demand matrix. Note that no traffic is dropping between HOU and IND; however, traffic is dropping between other POPs that reside on either side of the cut in Table 10.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL	-1.03	0.14	0.03	0.11	0.04	0.02	0.19	0.42	0.01	0.03 0.03	0.03
CHI	0.14	-5.25	0.17	0.64	0.23	0.14	1.10 1.10	2.44	0.03 0.03	0.17 0.17	0.17
DEN	0.03	0.17	-1.22	0.13	0.05	0.03	0.23	0.50 0.50	0.01	0.04	0.04 0.04
HOU	0.11	0.64	0.13	-19.14	15.18	0.11	0.84	1.85	0.02	0.13	0.13
IND	0.04	0.23	0.05	15.18	-16.62	0.04	0.30	0.67	0.01	0.05	0.05
KSC	0.02	0.14	0.03	0.11	0.04	-1.03	0.19	0.42	0.01	0.03	0.03
LAX	0.19	1.10 1.10	0.23	0.84	0.30	0.19	-6.54	3.19 2.79	0.04	0.23	0.23
NYC	0.42	2.44	0.50 0.50	1.85	0.67	0.42	3.19 2.79	-10.58	0.08 0.08	0.50 0.50	0.50
SUN	0.01	0.03 0.03	0.01	0.02	0.01	0.01	0.04	0.08 0.08	-0.21	0.01	0.01 0.01
SEA	0.03 0.03	0.17 0.17	0.04	0.13	0.05	0.03	0.23	0.50 0.50	0.01	-1.22	0.04 0.04
WDC	0.03	0.17	0.04 0.04	0.13	0.05	0.03	0.23	0.50	0.01 0.01	0.04 0.04	-1.22

Legend:

B(s,t) — B(s,t) = Demand for Traffic Gbps

B(s,t) W(s,t) — B(s,t) = Demand for Traffic Gbps; W(s,t) = Dropped Traffic Gbps

Table 10. Demand between HOU and IND increases by 5 Gbps from Table 9.

An increase in traffic between POPs on either side of the cut causes traffic to drop; however, it is less expensive to route traffic across shorter paths and drop traffic across longer paths.

For example, because of the distance, routing 1 Gbps of traffic between LAX and NYC is more expensive than routing 1 Gbps of traffic between HOU and IND. Thus, if 1 Gbps of traffic must be dropped, the minimum cost solution drops the most expensive flows first.

This objective of minimum cost prefers to drop longer flows in exchange for shorter less expensive flows that traverse the cut. Once all of these more expensive flows drop any increase in demand between HOU and IND will begin to drop.

Now consider an increase in the demand for traffic between IND and HOU of 5 Gbps. The new demand matrix is in Table 11. We solve **TE** with the new demand matrix. Note that traffic is now dropping between HOU and IND in Table 11.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL	-1.03	0.14	0.03 0.03	0.11	0.04	0.02	0.19 0.18	0.42	0.01 0.01	0.03 0.03	0.03
CHI	0.14	-5.25	0.17 0.17	0.64 0.64	0.23	0.14	1.10 1.10	2.44	0.03 0.03	0.17 0.17	0.17
DEN	0.03 0.03	0.17 0.17	-1.22	0.13	0.05	0.03	0.23	0.50 0.50	0.01	0.04	0.04 0.04
HOU	0.11	0.64 0.64	0.13	-24.14	20.18 0.86	0.11	0.84	1.85 1.85	0.02	0.13	0.13
IND	0.04	0.23	0.05	20.18 0.68	-21.62	0.04	0.30 0.30	0.67	0.01 0.01	0.05 0.05	0.05
KSC	0.02	0.14	0.03	0.11	0.04	-1.03	0.19	0.42 0.42	0.01	0.03	0.03 0.03
LAX	0.19	1.10 1.10	0.23	0.84	0.30 0.30	0.19	-6.54	3.19 3.19	0.04	0.23	0.23 0.23
NYC	0.42	2.44	0.50 0.50	1.85 1.85	0.67	0.42 0.42	3.19 3.19	-10.58	0.08 0.08	0.50 0.50	0.50
SUN	0.01 0.01	0.03 0.03	0.01	0.02	0.01 0.01	0.01	0.04	0.08 0.08	-0.21	0.01	0.01 0.01
SEA	0.03 0.03	0.17 0.17	0.04	0.13	0.05 0.05	0.03	0.23	0.50 0.50	0.01	-1.22	0.04 0.04
WDC	0.03	0.17	0.04 0.04	0.13	0.05	0.03 0.03	0.23 0.23	0.50	0.01 0.01	0.04 0.04	-1.22

Legend:

B(s,t)	B(s,t) = Demand for Traffic Gbps
B(s,t) W(s,t)	B(s,t) = Demand for Traffic Gbps W(s,t) = Dropped Traffic Gbps

Table 11. Demand between HOU and IND increases by 5 Gbps from Table 10.

Thus, we observe that traffic routing is not intuitive. The objective of the ISP will drive the routes chosen. In this example, minimizing cost drives traffic routing to prefer shorter routes.

D. THE "OPTIMAL" ABILENE NETWORK

1. "Clean Slate" Network Design

We consider a simple question: If an ISP could build a network from scratch to serve the Abilene POPs, what would it look like?

We start with the eleven cities corresponding to Abilene POPs in the United States. These cities and their populations appear in Table 5.

We consider all (11) (10) = 110 possible arcs between the eleven POPs; they appear as dashed lines in Figure 12. We generate a synthetic demand matrix based on the gravity model discussed in Chapter II. This O-D demand matrix is in Table 6.

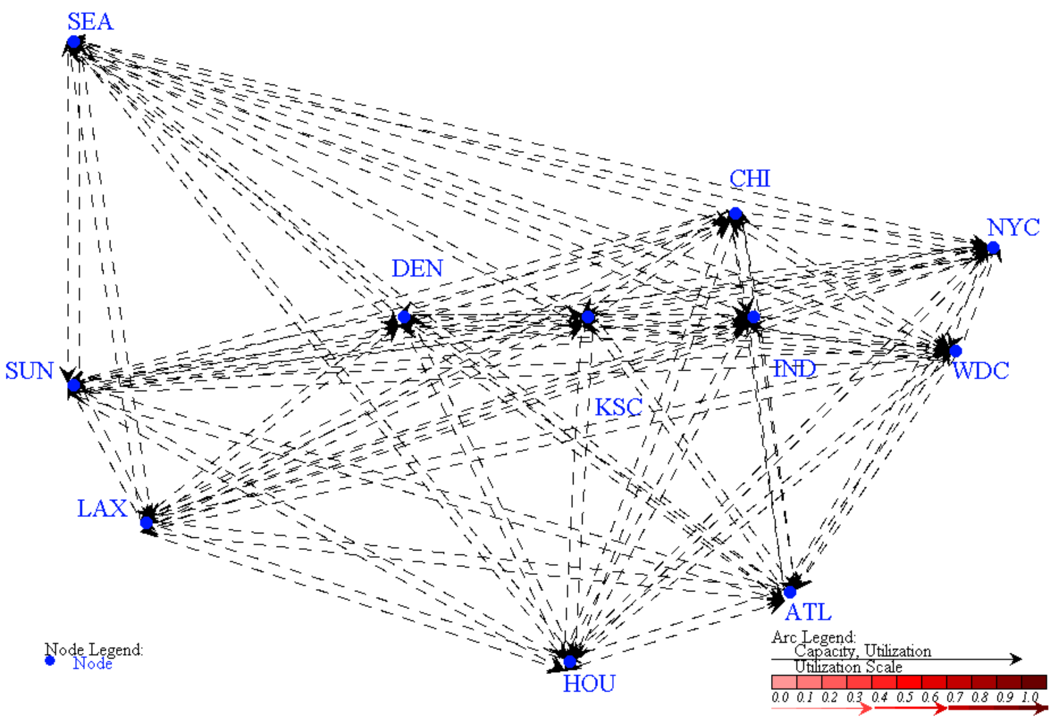


Figure 12. 110 possible arcs in the design of an "optimal" Abilene.

We infer a *budget* from the existing Abilene topology, which includes 28 arcs with 10 Gbps of capacity provisioned on each arc. Let $\alpha = 10$ and $\beta = 1$.

We calculate an inferred *budget B* from the following formula: $budget = \alpha \sum_{(i,j)\in A} d_{ij} + \beta \sum_{(i,j)\in A} d_{ij}\, u_{ij} \;, \forall (i, j) \in Abilene$.

We endow the ISP with an initial *budget B* = 304,466 to design and provision the network in a way that minimizes $Z_{\mathbf{NP}}$.

The network designer faces a choice when building a network: build direct connections between O-D pairs (and incur fixed link costs), or build fewer arcs but increase the distance over which traffic travels. The relative merit of each depends on parameters α and β. Without loss of generality, suppose $\beta = 1$. Small values of α mean that the fixed costs of building an arc are relatively small. In contrast, when α is large, the fixed arc costs are expensive. We use the **NP** model, *budget B*, the values of parameter α and β, and the demand matrix from Table 6.

Consider first the case when $\alpha = 0$. We solve model **NP** with B = 304,466, and the demand matrix from Table 6. The resulting network design appears in Figure 13. From this, we observe that when $\alpha = 0$, the "optimal" design builds direct connections as long as there is sufficient *budget*.

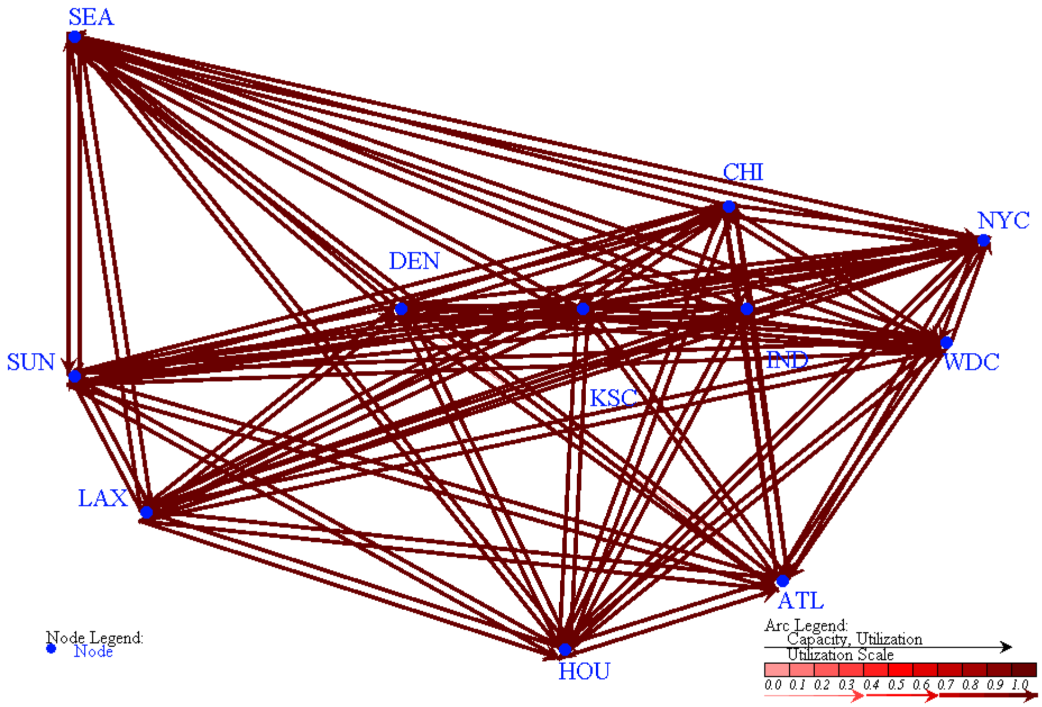
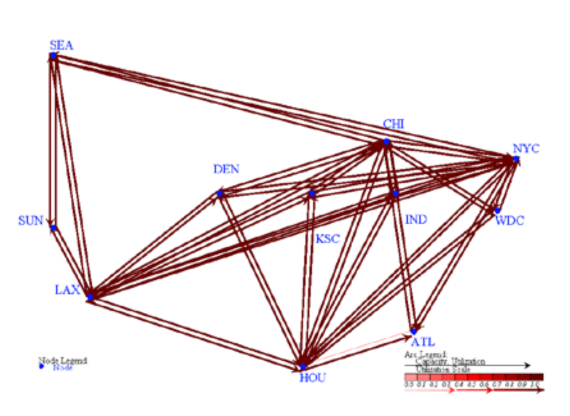


Figure 13. The ISP builds 110 arcs with $\alpha = 0$ and routes traffic at $Z_{\mathbf{NP}} = 34$.

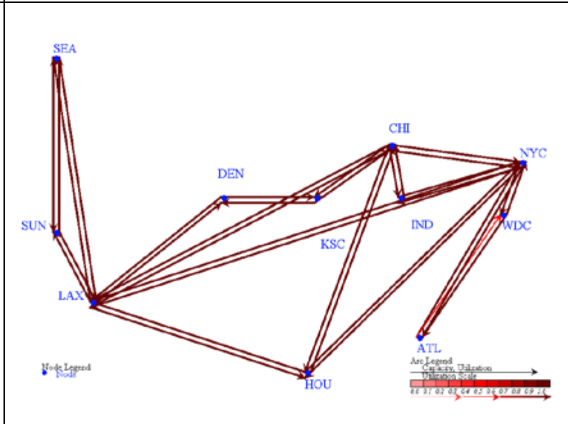
Next, we solve **NP** with α = 5, 10, 15, and 20 and note the results. By increasing the value of α, building additional arcs becomes more expensive. For example, in order for the ISP to use arc (HOU, ATL), d_{ij} = 608 miles, the ISP will have to pay an upfront cost of α 608 prior to provisioning any capacity on that arc.

We solve **NP** with B = 304,466, each value of α, and the demand matrix in Table 6. The resulting topology of four "optimal" network designs appear in Figure 14.

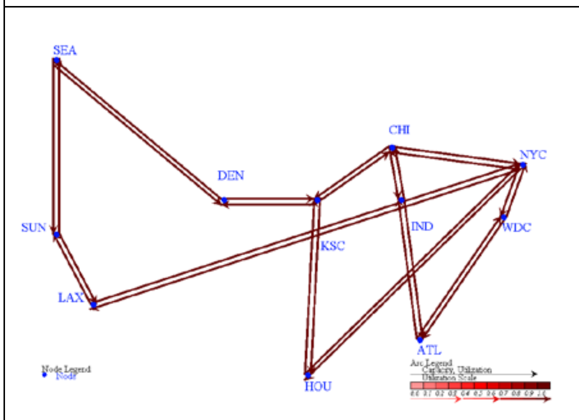
(a) α = 5, Z_{NP} = 37, Arcs Built = 58



(b) α = 10, Z_{NP} = 44 **,** Arcs Built = 34



(c) α = 15, Z_{NP} = 57 , Arcs Built = 26



(d) α = 20, Z_{NP} = 79, Arcs Built = 24

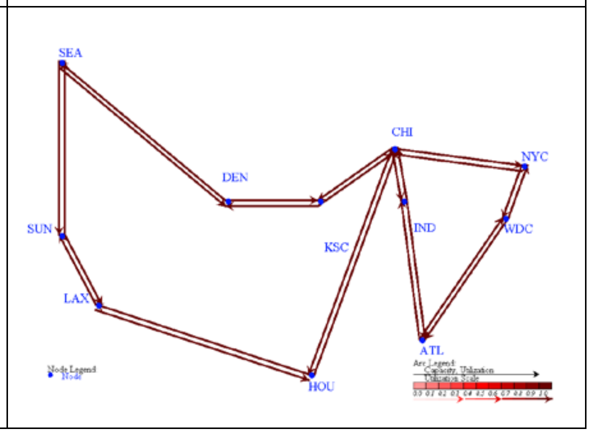


Figure 14. Topology of four "optimal" Abilene network designs.

Even when facing the same demand matrix, different costs associated with building network infrastructure can drive ISPs to design different network topologies. This result appears in Figure 14. As costs, controlled in this example by α, to build arcs increase, ISPs build fewer arcs and route more traffic on these arcs. At the same time, the Z_{NP} increases. These results appear in Figure 15.

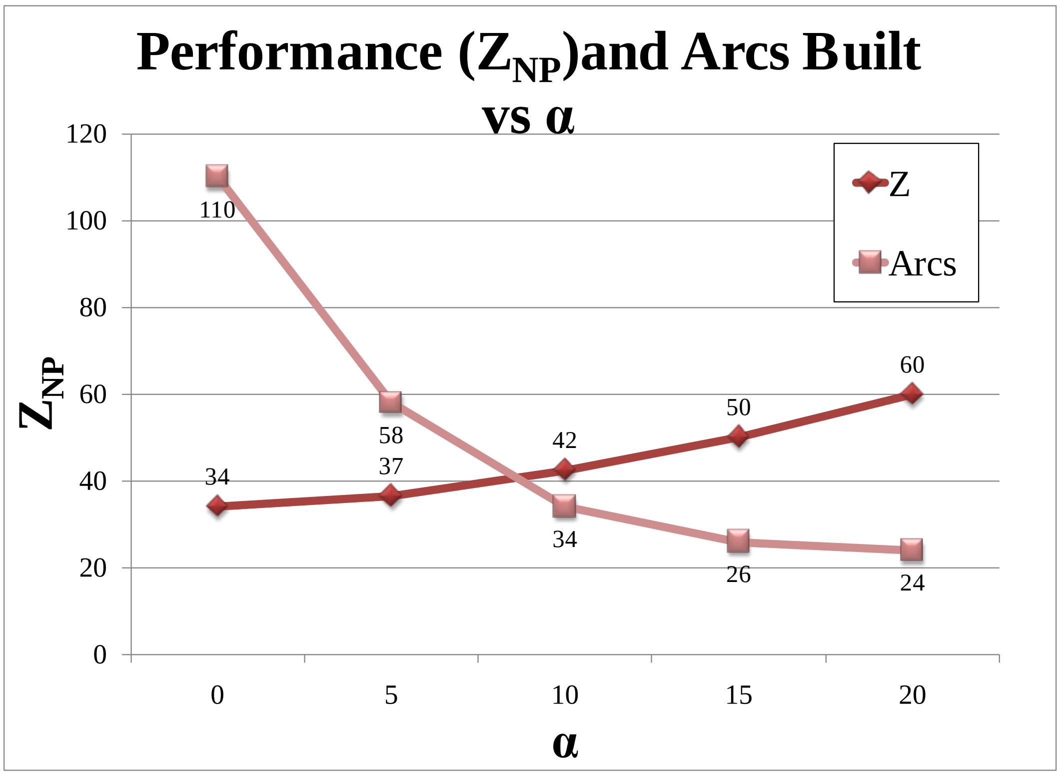


Figure 15. "Optimal" Abilene performance, Z_{NP}, and number of arcs built versus an increasing parameter α.

2. “Optimal” Abilene versus Abilene

We solve model **NP** with α = 15, an initial *budget B* = 304,466 and the O-D demand matrix in Table 6. The resulting “optimal” Abilene design appears in Figure 16.

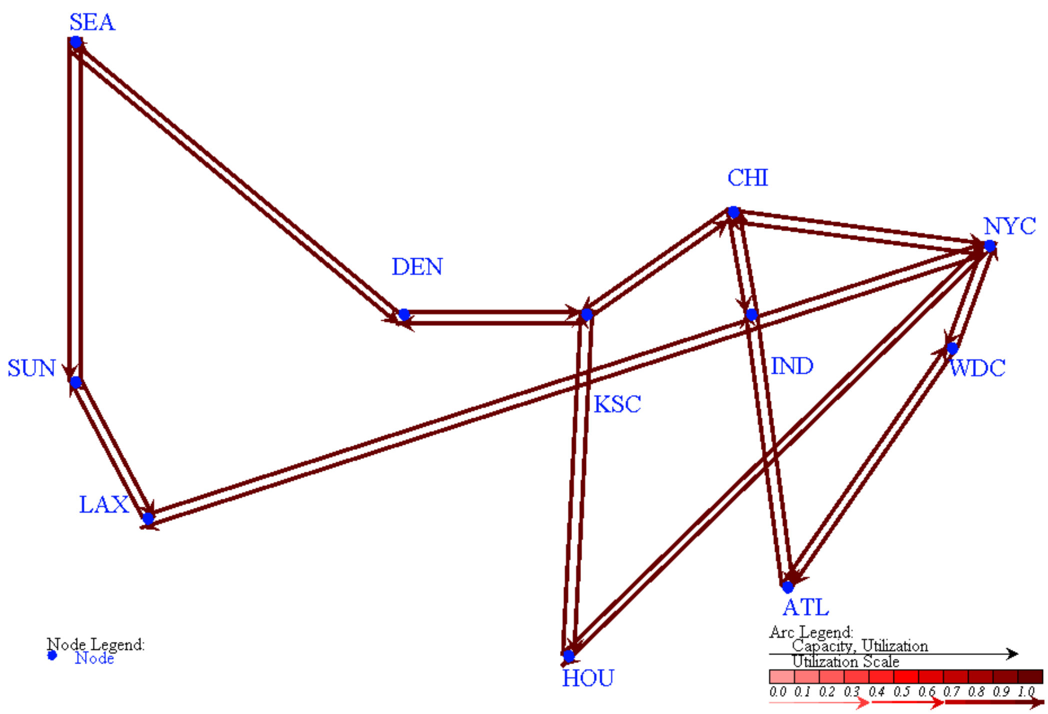


Figure 16. “Optimal” Abilene, Z_{NP} = 57.

We compare the “optimal” Abilene design with Abilene. The “optimal” Abilene design has 26 arcs while Abilene has 28. The “optimal” Abilene design performs at nearly half the Z_{NP} of Abilene. All arcs in the “optimal” Abilene are saturated while the maximum arc utilization on Abilene is 81%. A comparison of the “optimal” and real Abilene designs is in Figure 17.

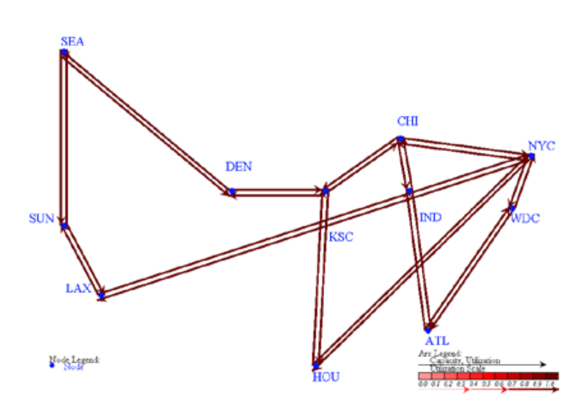
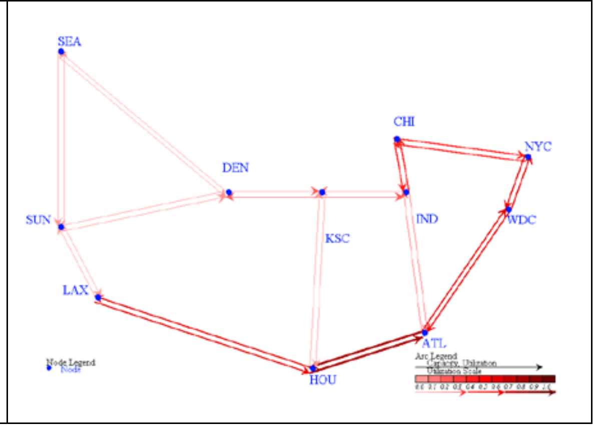
“Optimal” Abilene	**Abilene**
	

Figure 17. Comparison of network topologies, “optimal” Abilene (left), Abilene (right).

While the “optimal” Abilene outperforms the real Abilene, the resulting topology is “optimal” only for that demand. In the “optimal” design, all arcs are saturated, thus, an increase in demand will cause the “optimal” design to drop traffic. In contrast, Abilene is well equipped to handle an increase in demand. The average arc utilization on the real Abilene design is only 32%, thus the arcs on the real Abilene design are capable of handling an increase in demand. The penalty the real Abilene design incurs is poor performance relative to the “optimal” design. A comparison of Abilene with four “optimal” Abilene designs with α = 0, 5, 10, 15, and 20 appears in Table 12.

	Abilene	"Optimal" α = 0	"Optimal" α = 5	"Optimal" α = 10	"Optimal" α = 15	"Optimal" α = 20
Number of Arcs	**28**	110	58	34	**26**	24
Performance (Z_{NP})	**91**	34	37	44	**57**	79
Maximum Arc Utilization	**0.81**	1.00	1.00	1.00	**1.00**	1.00
Minimum Arc Utilization	**0.02**	0.0001	0.20	0.50	**1.00**	1.00
Average Arc Utilization	**0.32**	0.99	0.99	0.99	**1.00**	1.00

Table 12. Arc utilization and performance comparison of "optimal" Abilene designs and the real Abilene design.

3. Proportional Changes in Demand

We demonstrate that with an "optimal" design, proportional changes in the demand matrix lead to proportional provisioning of additional arc capacity.

Assume the customer population of each of the eleven Abilene POPs increases by 10%. Using these increased customer populations as input to the gravity model discussed in Chapter II, we generate a new demand matrix in Table 13.

Note a proportional increase in customer population of 10% leads to an increase in demand matrix values of exactly 21%.

$$b_{st} = \gamma_s\, p(s)\, \gamma_t\, p(t) \rightarrow 1.1\, \gamma_s\, p(s)\, 1.1\, \gamma_t\, p(t) = 1.21\, \gamma_s\, p(s)\, \gamma_t\, p(t) = \mathbf{1.21}\, b_{st}$$

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL	-1.24	0.18	0.04	0.13	0.05	0.03	0.23	0.51	0.01	0.04	0.04
CHI	0.18	-6.35	0.21	0.77	0.28	0.18	1.33	2.95	0.04	0.21	0.21
DEN	0.04	0.21	-1.48	0.16	0.06	0.04	0.28	0.61	0.01	0.04	0.04
HOU	0.13	0.77	0.16	-5.00	0.21	0.13	1.01	2.24	0.03	0.16	0.16
IND	0.05	0.28	0.06	0.21	-1.96	0.05	0.37	0.81	0.01	0.06	0.06
KSC	0.03	0.18	0.04	0.13	0.05	-1.24	0.23	0.51	0.01	0.04	0.04
LAX	0.23	1.33	0.28	1.01	0.37	0.23	-7.91	3.86	0.05	0.28	0.28
NYC	0.51	2.95	0.61	2.24	0.81	0.51	3.86	-12.81	0.10	0.61	0.61
SUN	0.01	0.04	0.01	0.03	0.01	0.01	0.05	0.10	-0.25	0.01	0.01
SEA	0.04	0.21	0.04	0.16	0.06	0.04	0.28	0.61	0.01	-1.48	0.04
WDC	0.04	0.21	0.04	0.16	0.06	0.04	0.28	0.61	0.01	0.04	-1.48

Legend:

B(s,t)	B(s,t) = Demand for Traffic Gbps

Table 13. O-D demand matrix generated by increasing the customer population of all eleven Abilene POPs by 10%, these values represent an increase in b_{st} of 21% from the values in Table 6.

We start with the "optimal" Abilene design Figure 16. We solve model **NP** using a B = 20,000 and the increased O-D demand matrix in Table 13. We note where additional arc capacity is provisioned. The solution to **NP** appears in Figure 18.

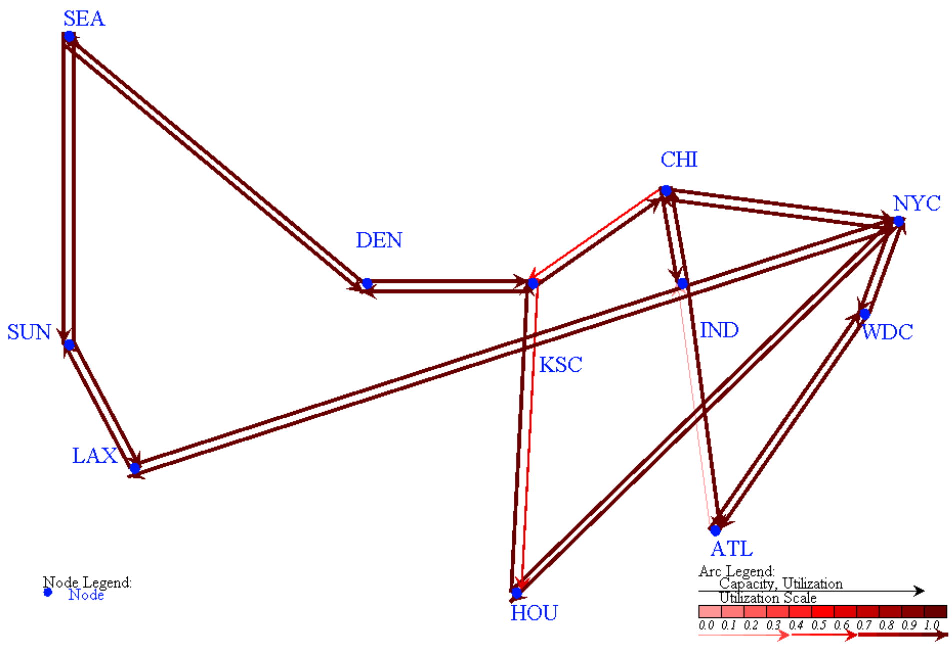


Figure 18. The solution to **NP** with a proportionally increased O-D demand matrix.

We note that the capacity provisioned on most arcs increases. A comparison of the additional capacity provisioned on each arc appears is in Table 14.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL					0.28 0.28						0.84 0.84
CHI					1.75 1.75	1.65 1.65		6.09 6.09			
DEN						1.36 1.36				0.66 0.66	
HOU						1.19 1.19		3.78 3.78			
IND	0.31 0.31	1.72 1.72									
KSC		1.87 1.87	1.33 1.33	1.00 1.00							
LAX								6.74 6.74	1.23 1.23		
NYC		5.90 5.90		3.97 3.97			6.77 6.77				1.88 1.88
SUN							1.20 1.20			1.05 1.05	
SEA			0.69 0.69						1.02 1.02		
WDC	0.81 0.81							1.91 1.91			

Legend:
X(s,t) U(s,t) — X(s,t) = Arc Flow, X(origin,destination) Gbps; U(s,t) = Arc Capacity, U(origin,destination) Gbps
Utilization Scale
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL					0.40 18.56						0.98 0.98
CHI					2.14 2.14	0.88 1.65		7.90 7.90			
DEN						1.68 1.68				0.85 0.85	
HOU						1.88 1.88		4.78 4.78			
IND	0.40 0.40	2.14 2.14									
KSC		2.08 2.08	1.68 1.68	0.68 1.00							
LAX								8.16 8.16	1.46 1.46		
NYC		6.70 6.70		5.98 5.98			8.16 8.16				2.27 2.27
SUN							1.46 1.46			1.24 1.24	
SEA			0.85 0.85						1.24 1.24		
WDC	0.98 0.98							2.27 2.27			

Legend:
X(s,t) U(s,t) — X(s,t) = Arc Flow, X(origin,destination) Gbps; U(s,t) = Arc Capacity, U(origin,destination) Gbps
Utilization Scale
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Table 14. Capacity matrix for original O-D demand matrix (left), capacity matrix for increased O-D demand matrix (right).

By comparing the two arc capacity matrices, we can see that a proportional increase in demand leads to a near proportional distribution of arc capacity over the existing arcs. No new arcs are created; however, capacity is increased on existing arcs to meet new demand.

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IV. NUMERICAL EXPERIMENTS

A. MULTI-PERIOD CHANGES IN DEMAND

We consider the evolution of an ISP's network over multiple time periods. We compare three distinct network design techniques utilizing the network models discussed in Chapter II. The three design techniques include a heuristic technique, a myopic technique, and an omniscient technique.

1. Baseline Abilene Network

We use the existing Abilene Network discussed in Chapter III as the baseline for each network design experiment. This network has 28 arcs, each provisioned with 10 Gbps of capacity, that connect eleven cities throughout the United States. An illustration of this baseline network is in Chapter III, Figure 9.

2. Demand Matrices

We consider a horizon of five years, each divided into quarters, for a total of T = 20 time periods. Let B^τ denote the O-D demand matrix in period $\tau = 1, 2, \dots, 20$. We generate 20 synthetic demand matrices (B^1, B^2, … , B^{20}), one for each period in the experiment. Specifically, we model incremental changes in the customer population of each POP from one period to the next and generate the next quarter's O-D demand matrix using the updated customer populations as input to the gravity model discussed in Chapter II. A side-by-side comparison of the assumed customer population of Abilene POPs at $\tau = 1$ and $\tau = 20$ appears in Table 15.

Point of Presence	Population (millions)	Assumed Customer Population (millions) τ =1	Assumed Customer Population (millions) τ = 20
Atlanta (ATL)	0.50	0.16	0.25
Chicago (CHI)	2.90	0.93	1.46
Denver (DEN)	0.60	0.19	0.31
Houston (HOU)	2.20	0.7	1.19
Indianapolis (IND)	0.80	0.26	0.43
Kansas City (KSC)	0.50	0.16	0.25
Los Angeles (LAX)	3.80	1.22	1.81
New York (NYC)	8.40	2.69	4.31
Sunnyvale (SUN)	0.10	0.03	0.05
Seattle (SEA)	0.60	0.19	0.29
Washington D. C. (WDC)	0.60	0.19	0.34

Table 15. Assumed customer population of Abilene POPs at τ =1 and τ = 20; population data from U.S. Census Bureau (2010).

3. Budget Resources on Costs

We assume a yearly *budget* of 25% of the replacement cost for the baseline infrastructure of Abilene. We infer the cost to build and provision each arc using the cost equation from Chapter III. We divide the yearly *budget* into a *per period budget* (*PPB*). We endow the ISP with a *PPB* of 20,000.

4. Heuristic Technique

We first consider a heuristic design technique that is purely reactive. In each time period, the ISP solves model **TE** to obtain the minimum cost routes, and then it uses its available budget to increase the capacity of arcs that are saturated by the min-cost flows. The rationale for this technique is that increasing the capacity of an arc that is not saturated will not reduce $Z_{\mathbf{TE}}$. However, increasing the capacity of an arc that was saturated by the min-cost flows could reduce $Z_{\mathbf{TE}}$.

The heuristic rule for allocating budget to provision arcs is as follows. Using the current quarter's O-D demand matrix as input, we solve model **TE** and observe the resulting network traffic. The heuristic then identifies any arcs that are saturated, and it invests the *PPB* in an equal amount of additional capacity to provision on the saturated arcs. For example, if there are 4 saturated arcs, the heuristic invests the *PPB* to increase the capacity on these arcs by an equal amount. If there are no saturated arcs, the heuristic does not invest the *PPB*. After provisioning additional capacity, the heuristic resolves model **TE** for new flows at $Z_{\mathbf{TE}}$. The heuristic technique assumes saturated arcs need additional capacity for future quarters.

5. Myopic Technique

We next consider a myopic network design technique in which the ISP invests its available budget optimally for the current demand. However, the ISP has no knowledge of future demands. Given the current O-D demand matrix, we solve model **NP** to optimally provision existing arcs and/or build new arcs in the network in order to route traffic from its origin to its destination in the least expensive manner. Thus, the ISP designs the network optimally for the current demand, but it cannot anticipate future demand.

6. Omniscient Technique

Finally, we consider a network design technique that is omniscient, in the sense that we assume the ISP has knowledge of current demand and all future demands. Moreover, we assume that the ISP can invest its budget for all T = 20 periods upfront.

Specifically, given demand matrices for the current period and all future periods, we solve model **MPNP** to optimally provision network arcs to minimize the aggregate routing cost over all time periods. The omniscient design provides a lower bound on cost minimization, since it represents the best an ISP can do.

7. Performance Evaluation of Three Different Network "Design" Techniques Over 20 Periods

Our design techniques result in three different network topologies at $\tau = 20$. They appear in Figure 19. Table 16 shows the arc capacities and min-cost flows for each topology.

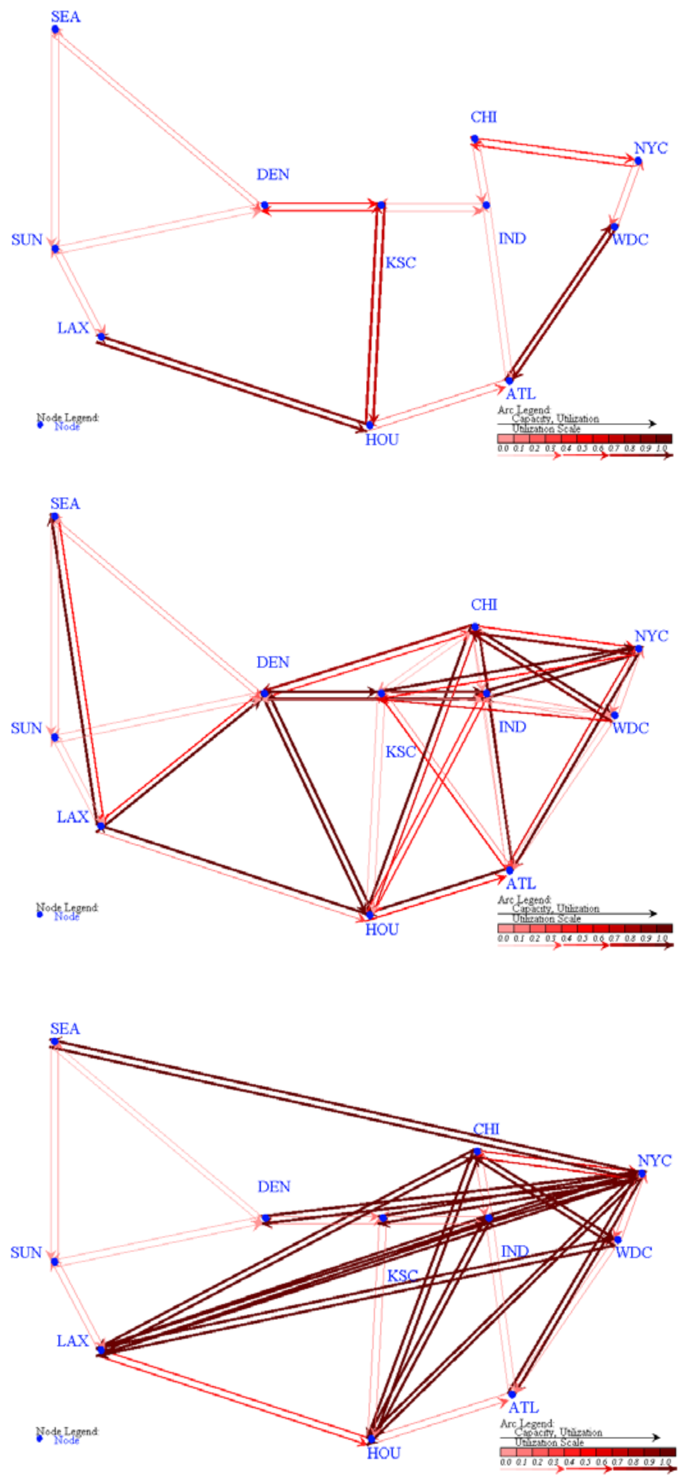


Figure 19. Heuristic design (top), myopic design (center), omniscient design (bottom),

topologies at τ = 20.

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL				15.20 42.87	0.62 10.00						15.53 16.02
CHI					12.45 80.38			12.56 26.17			
DEN						5.23 10.00			0.89 10.00	2.51 10.00	
HOU	15.20 42.87					7.64 10.00	14.95 16.02				
IND	0.62 10.00	12.45 80.38				11.28 61.01					
KSC			5.30 10.00	7.57 10.00	11.28 61.01						
LAX				15.02 16.02					1.27 10.00		
NYC		12.56 26.17									15.93 65.40
SUN			0.89 10.00				1.34 10.00			0.54 10.00	
SEA			2.44 10.00						0.61 10.00		
WDC	15.53 16.02							15.93 65.40			

Legend:

X(s,t)
U(s,t) — X(s,t) = Arc Flow, X(origin,destination) Gbps; U(s,t) = Arc Capacity, U(origin,destination) Gbps

Utilization Scale

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL		0.44 160.97		14.74 14.74	0.11 10.00	0.06 0.11		6.17 8.90			0.08 10.00
CHI	0.36 0.36		4.09 4.94	4.37 4.37	0.63 10.00	0.37 3.88		6.30 11.52			1.76 1.76
DEN		4.45 9.52		0.78 0.78	1.22 1.22	10.76 10.76	1.79 3.34		0.36 10.00	2.16 10.00	
HOU	5.85 10.00	2.14 3.14	0.45 0.45		0.51 1.27	0.75 10.00	14.04 14.04				
IND	0.27 10.00	0.63 10.00	1.06 1.06	0.51 1.09		0.11 10.00		1.86 1.86			0.15 0.96
KSC	0.06 0.19	0.37 3.97	0.61 10.00	0.30 10.00	0.11 10.00			11.68 11.68			0.09 0.93
LAX			12.38 12.38	3.04 10.00					0.15 10.00	0.88 0.88	
NYC	13.95 13.95	9.12 10.00			1.86 1.86	1.08 1.85					1.48 10.00
SUN			0.42 10.00				0.09 10.00			0.01 10.00	
SEA			2.51 10.00				0.53 0.92		0.01 10.00		
WDC	1.11 10.00	0.73 0.78			0.15 0.99	0.09 0.14		1.48 10.00			

Legend:

X(s,t)
U(s,t) — X(s,t) = Arc Flow, X(origin,destination) Gbps; U(s,t) = Arc Capacity, U(origin,destination) Gbps

Utilization Scale

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

	ATL	CHI	DEN	HOU	IND	KSC	LAX	NYC	SUN	SEA	WDC
ATL				2.54 10.00	0.74 10.00			1.09 1.09			1.77 10.00
CHI				1.88 1.88	1.97 10.00		2.49 2.49	6.83 10.00			0.50 0.50
DEN						1.49 10.00		1.63 1.63	0.85 10.00	0.29 10.00	
HOU	2.65 10.00	1.74 1.74			0.46 0.46	1.35 10.00	4.47 10.00	4.75 4.75			
IND	0.62 10.00	1.98 10.00		0.46 0.46		1.40 10.00	0.71 0.71	1.77 1.77			
KSC			1.41 10.00	1.54 10.00	1.29 10.00			1.08 1.08			
LAX		2.63 2.63		4.25 10.00	0.71 0.71			7.18 7.18	1.69 10.00		0.52 0.52
NYC	1.09 1.09	6.82 10.00	1.63 1.63	4.75 4.75	1.77 1.77	1.08 1.08	7.18 7.18			1.84 1.84	2.63 10.00
SUN			0.86 10.00				1.61 10.00			1.00 10.00	
SEA			0.36 10.00					1.84 1.84	0.93 10.00		
WDC	1.78 10.00	0.50 0.50					0.52 0.52	2.62 10.00			

Legend:

X(s,t)
U(s,t) — X(s,t) = Arc Flow, X(origin,destination) Gbps; U(s,t) = Arc Capacity, U(origin,destination) Gbps

Utilization Scale

0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0

Table 16. Arc capacities and network flows at τ = 20 for heuristic design (top left), myopic design (top right), and omniscient design (bottom center).

The heuristic design technique maintains the original topology of the baseline Abilene network. The myopic design technique builds 32 additional arcs to the baseline

Abilene network. The omniscient design technique builds 26 additional arcs. The omniscient design performs at nearly half the cost of the myopic design technique and nearly a third of the cost of the heuristic design techniques; however, the arc utilization on each network is comparable. A comparison of arc utilization is in Table 17. The cumulative performance of the three design techniques appears in Figure 20.

	Heuristic Design	Myopic Design	Omniscient Design
Number of Arcs	28	60	54
τ = 20 Maximum Arc Utilization	0.97	1.00	1.00
τ =20 Minimum Arc Utilization	0.05	0.00	0.06
τ = 20 Average Arc Utilization	0.37	0.47	0.59
Cumulative Performance (Z) at τ = 20	3,064	2,091	1,326

Table 17. Arc utilization of min-cost flows at τ = 20 on networks resulting from each of the three network design techniques.

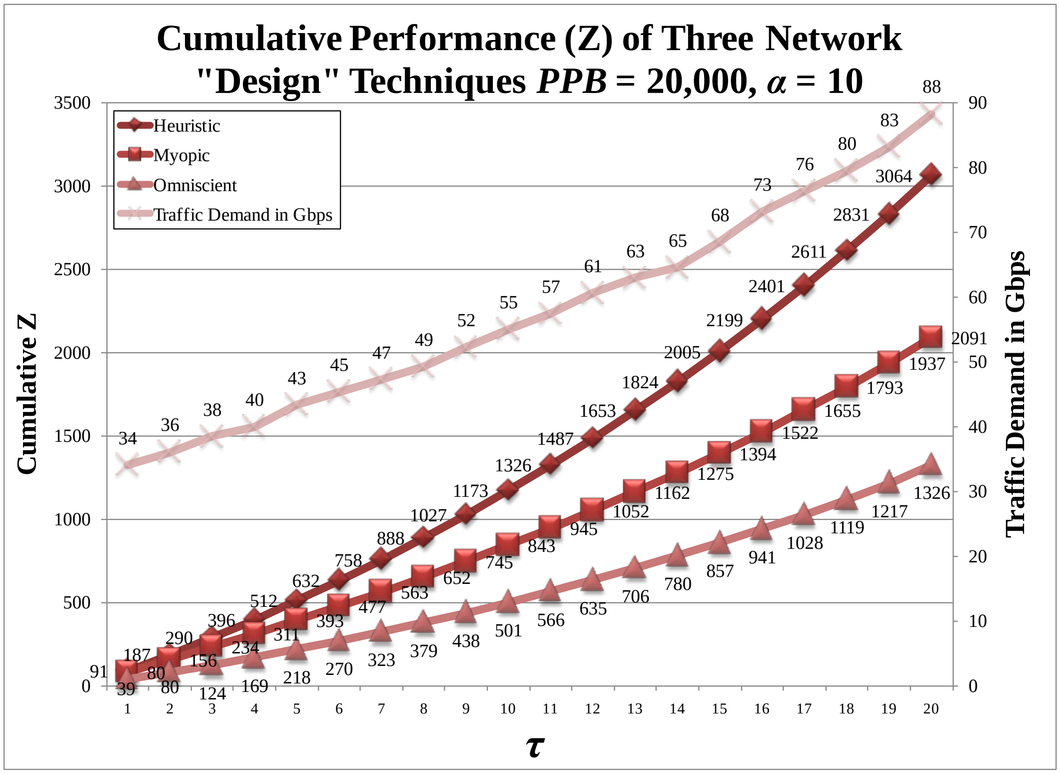


Figure 20. Cumulative performance of three network design techniques. For example, the value at $\tau = 10$ on the heuristic line = 1326 is the cumulative performance (minimum cost) of the heuristic design technique over 10 time periods. The value at $\tau = 10$ on the traffic demand in Gbps line = 55 is the total demand for traffic at time period 10.

In this experiment, the three design techniques result in different network topologies and therefore different performance values. The omniscient design significantly outperforms the heuristic and myopic designs. The omniscient design benefits from having knowledge of all demand matrices in all periods and having access to the aggregate budget for all periods. Thus, it is able to build and provision arcs optimally over all periods. The myopic technique can build arcs and provision them optimally; however, it knows only the current demand matrix and can spend only a single *PPB*. Thus, it is at a disadvantage relative to the omniscient design technique. The heuristic design technique only adds capacity to existing arcs to improve performance and is unable to benefit from building new arcs. These results highlight the importance of building new network infrastructure and/or gaining knowledge of future customer demands.

B. SENSITIVITY ANALYSIS

We consider the sensitivity of the previous experiment's results to changes in the per period budget (*PPB)* and the fixed cost of building new arcs (α). Either increasing the *PPB* or decreasing α will allow the ISP to build more arcs. We consider how these changes effect performance and network topology.

1. Varying PPB

First, we rerun the experiment using three *PPB* values: 8,000, 20,000, and 45,000. The corresponding performance of the three design techniques appear in Figure 21. Figure 22 shows a comparison of the resulting topologies.

We observe that the performance of the heuristic is unaffected by an increase in *PPB*. The cumulative cost of network flows over all T = 20 time periods is $Z_{\mathbf{HEUR}}$ = 3,064, and this does not change with increased budget. This result indicates that the objective value $Z_{\mathbf{TE}}$ achieved with *PPB* = 8,000 cannot be improved without building additional arcs. The ISP will only waste these additional resources unless it builds new arcs. In contrast, the performance of the myopic and omniscient techniques improves as *PPB* increases. Table 18 summarizes these costs. The performance of the myopic technique approaches that of the heuristic technique as *PPB* decreases, and it approaches that of the omniscient technique as *PPB* increases. This result highlights the intuitive effect *budget* has on the ability of an ISP to improve performance. If an ISP has more resources, shown here by a larger *PPB*, then the ISP is better able to improve performance. If an ISP has fewer resources, shown here by a smaller *PPB*, then the ISP is less able to improve performance. This result also highlights the value of information. If the ISP has knowledge of future demand, such as in the omniscient technique, then it can improve performance by investing where demands increase.

	***PPB* = 8,000**	***PPB* = 20,000**	***PPB* = 45,000**
Heuristic	3,064	3,064	3,064
Myopic	2,817	2,091	1,395
Omniscient	1,693	1,326	1,162

Table 18. Cumulative cost of min-cost network flows for all $T = 20$ time periods under three design techniques and with different budgets.

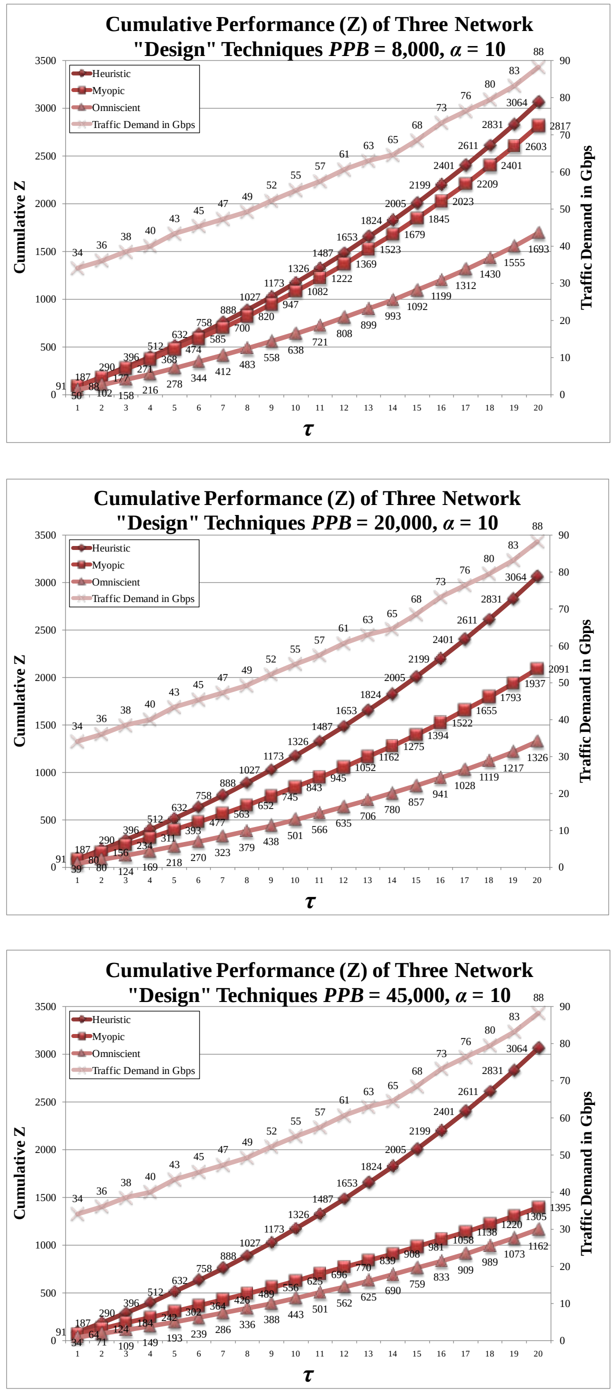


Figure 21. Cumulative performance of network design technique sensitivity to changes in *PPB*: *PPB* = 8,000 (top), *PPB* = 20,000 (center), *PPB* = 45,000 (bottom). In all three cases, the performance of the heuristic design technique is the same. With increased budget, the myopic design performs closer to the omniscient design.

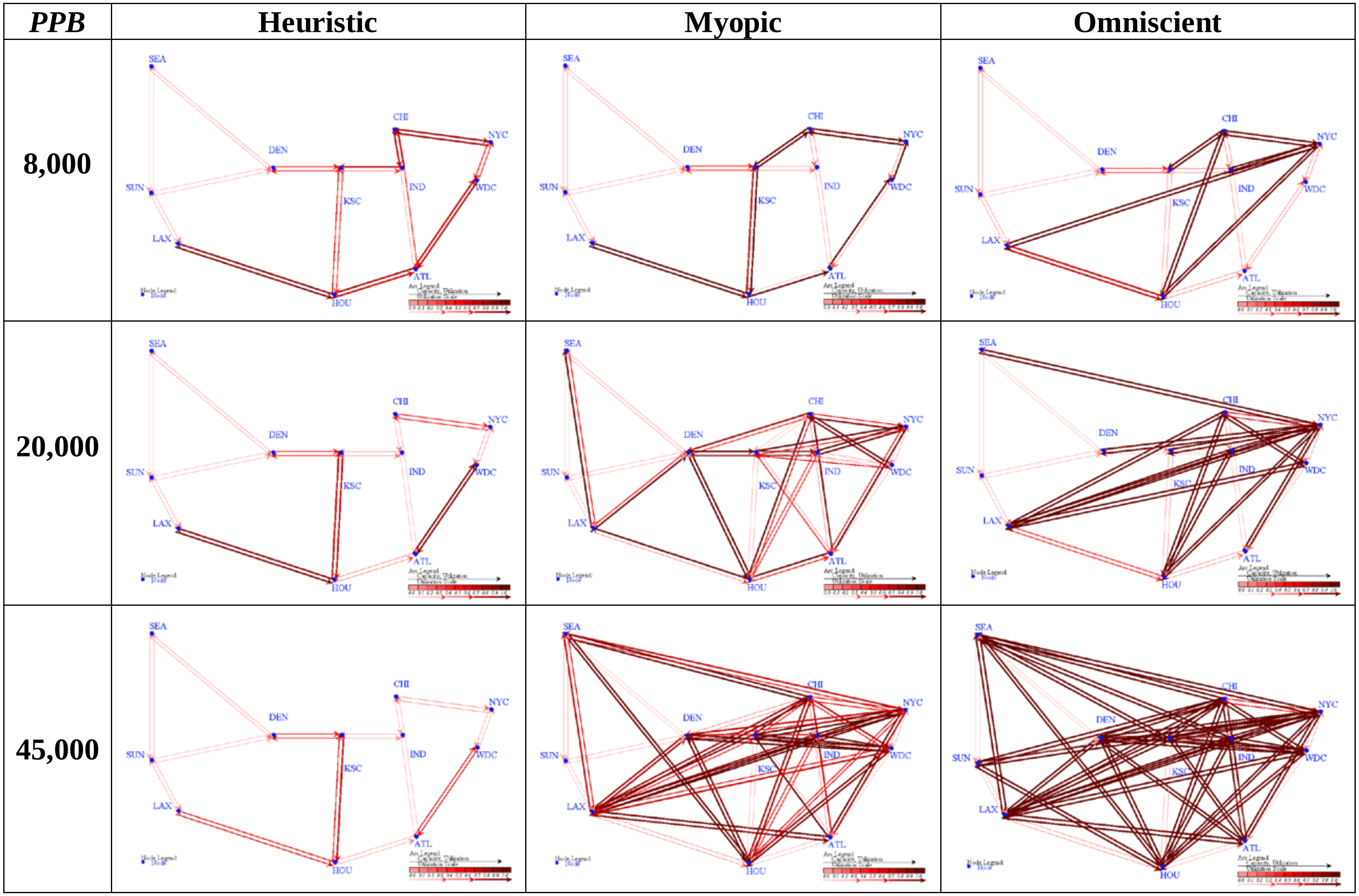


Figure 22. Comparison of resulting topologies at $\tau = 20$ with *PPB* = 8,000, 20,000, and 45,000. With increased budget, both the myopic and omniscient designs yield topologies that are more "mesh-like."

Note in Figure 22 that the utilization of the arcs on the heuristic design decreases with increased *PPB*, while the Z_{TE} from Figure 21 remains constant. This implies that spending resources over *PPB* = 8,000 will only decrease the utilization of arcs on the network but will not improve performance. The ISP would be better off not spending additional resources in arc capacity. Note in Figure 22 that the myopic and omniscient design improve their performance by building additional arcs. As the *PPB* increases, the resulting topology looks more like a mesh design.

2. Varying α

We rerun the experiment with different fixed costs for building new arcs, specifically α = 20, 10, and 5. The corresponding performance of the three design techniques appear in Figure 23. Figure 24 shows a comparison of the resulting topologies.

Note, that the performance of the heuristic is unaffected by an increase in α. This result is intuitive since the heuristic design technique does not allow the ISP to build additional arcs. The myopic and omniscient techniques both improve performance as α decrease. Table 19 summarizes these costs. The performance of the myopic technique approaches heuristic performance as α increases and approaches omniscient performance as α decreases. This result highlights the effect costs have on the ability of an ISP to improve performance because they can build more arcs with the same budget. If costs decrease, shown here by a lower α, then ISPs are better able to improve performance; if costs increase, shown here by a higher α, then ISPs are less able to improve performance.

	α = 20	α = 10	α = 5
Heuristic	3,064	3,064	3,064
Myopic	2,801	2,091	1,568
Omniscient	1,495	1,326	1,178

Table 19. Cumulative cost of min-cost flows for all T = 20 time periods under three different design techniques and with different fixed costs for new arc construction.

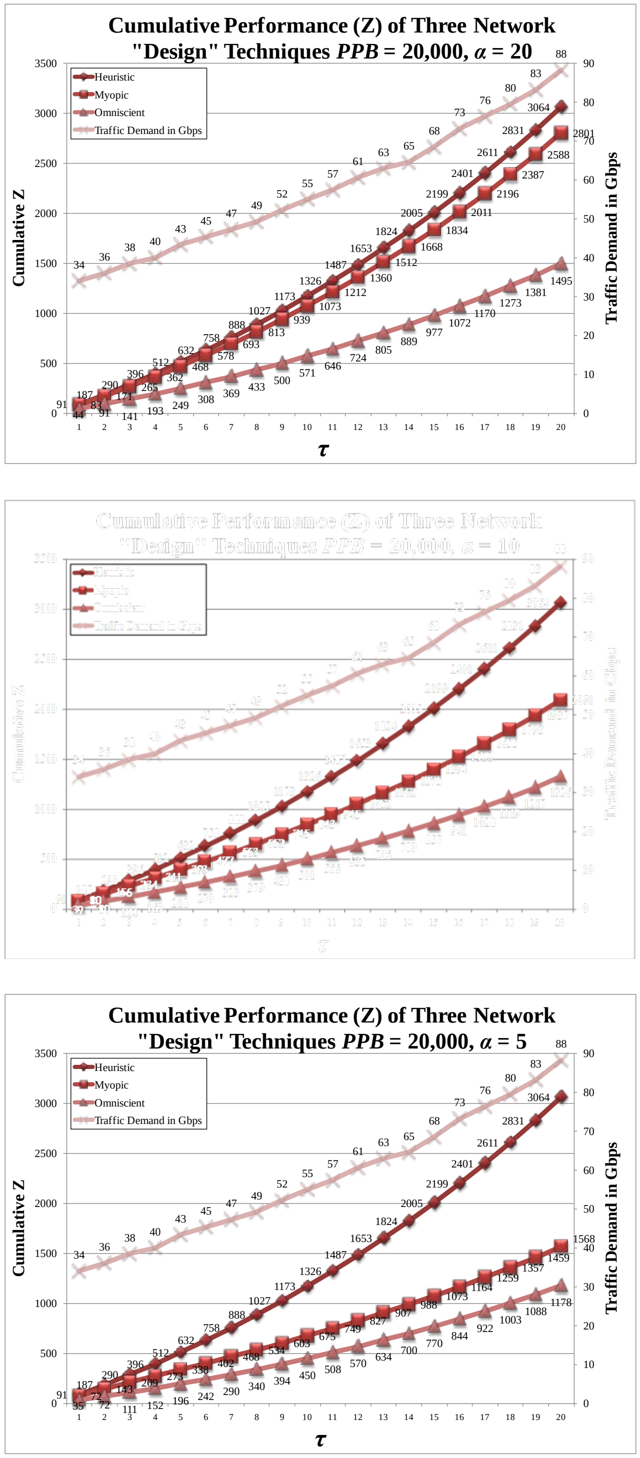


Figure 23. Cumulative performance of network design technique sensitivity to changes in α: $\alpha = 20$ (top), $\alpha = 10$ (center), $\alpha = 5$ (bottom). In all three cases, the performance of the heuristic design technique is the same. With decreased costs, the myopic design performs more like the omniscient design.

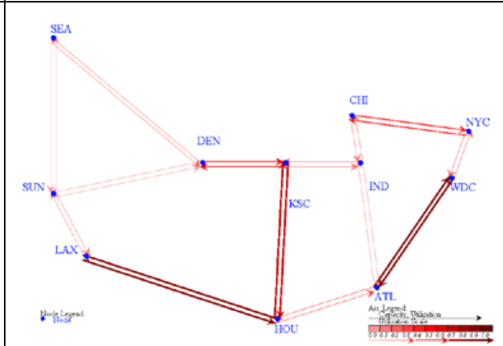
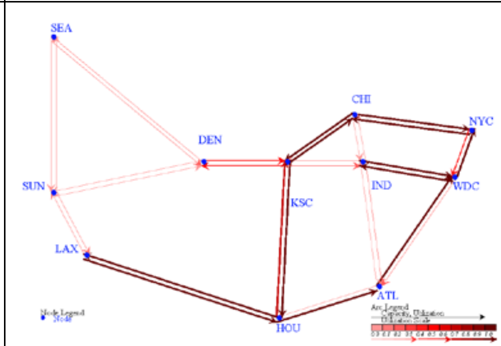
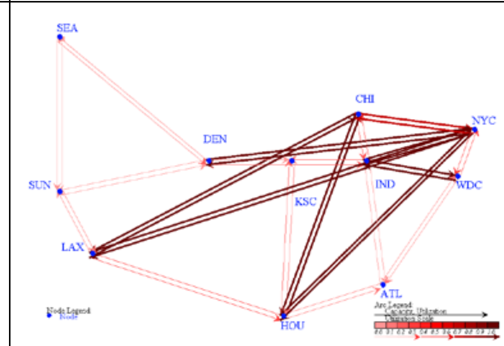
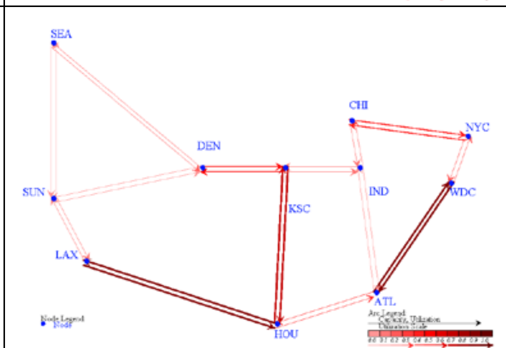
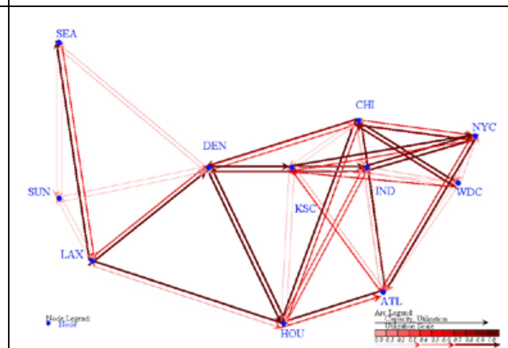
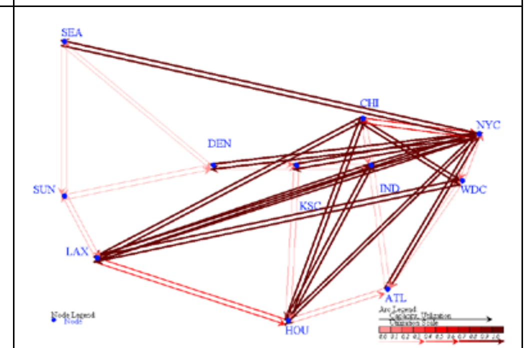
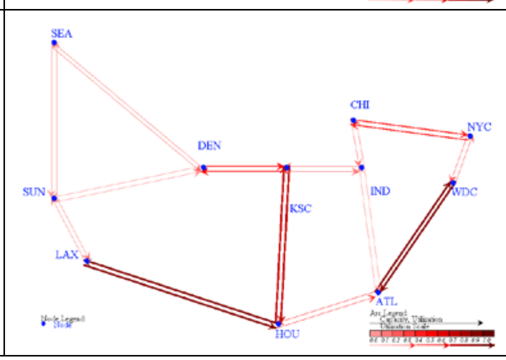
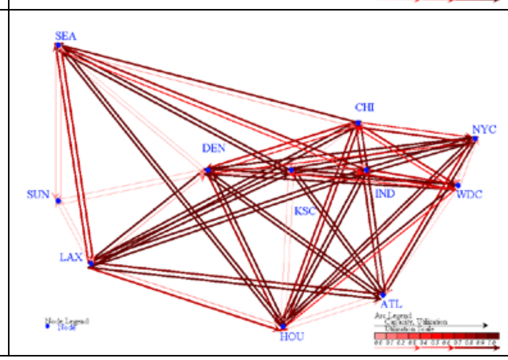
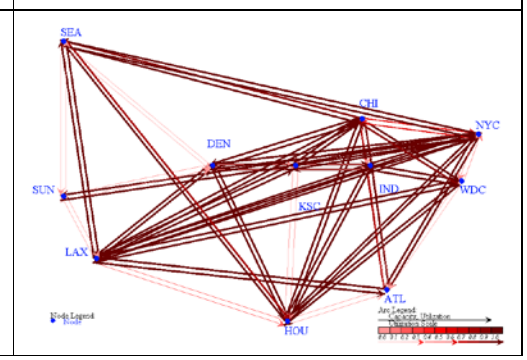
α	Heuristic	Myopic	Omniscient
20			
10			
5			

Figure 24. Comparison of resulting topologies at $\tau = 20$ with $\alpha = 20$, 10, and 5. With decreased costs, both the myopic and omniscient design yield topologies that are more "mesh-like."

Note in Figure 24 the utilization of the arcs on the network that results from heuristic design remains constant; this is intuitive since the *PPB* is fixed. Note in Figure 24 that as α decreases the myopic and omniscient techniques improve performance by building and provisioning additional arcs with the available *PPB*. As α decreases, the resulting topology looks more like a mesh design.

This sensitivity analysis has demonstrated the effects of *budget* and costs on ISP performance and topology. Intuitively, an increase in *budget* allows an ISP to improve performance; likewise, a decrease in costs allows an ISP to improve performance. As the *budget* of an ISP increases and/or costs decrease, to improve performance, the topology of an ISP approaches a mesh design.

C. INVESTMENT HORIZONS

We next consider the following question: *When should the ISP invest in new infrastructure, and how much should it invest?* We define an *investment horizon* to be the interval between successive investments in network provisioning. Keeping the total investment the same, we consider the investment horizons in Table 20.

Amount Invested in each Time Period	Length of Investment Horizon: 1	2	4	5	10	20
τ = 1	*PPB*	*2 x PPB*	*4 x PPB*	*5 x PPB*	*10 x PPB*	*20 x PPB*
τ = 2	*PPB*	0	0	0	0	0
τ = 3	*PPB*	*2 x PPB*	0	0	0	0
τ = 4	*PPB*	0	0	0	0	0
τ = 5	*PPB*	*2 x PPB*	*4 x PPB*	0	0	0
τ = 6	*PPB*	0	0	*5 x PPB*	0	0
τ = 7	*PPB*	*2 x PPB*	0	0	0	0
τ = 8	*PPB*	0	0	0	0	0
τ = 9	*PPB*	*2 x PPB*	*4 x PPB*	0	0	0
τ = 10	*PPB*	0	0	0	0	0
τ = 11	*PPB*	*2 x PPB*	0	*5 x PPB*	*10 x PPB*	0
τ = 12	*PPB*	0	0	0	0	0
τ = 13	*PPB*	*2 x PPB*	*4 x PPB*	0	0	0
τ = 14	*PPB*	0	0	0	0	0
τ = 15	*PPB*	*2 x PPB*	0	0	0	0
τ = 16	*PPB*	0	0	*5 x PPB*	0	0
τ = 17	*PPB*	*2 x PPB*	*4 x PPB*	0	0	0
τ = 18	*PPB*	0	0	0	0	0
τ = 19	*PPB*	*2 x PPB*	0	0	0	0
τ = 20	*PPB*	0	0	0	0	0

Table 20. Amount invested in period τ given a specific investment horizon length

1. Varying Investment Horizons with the Omniscient Technique

In previous experiments, the omniscient design technique benefits from having an investment horizon equal to 20 periods. In this experiment, we restrict this constraint. Specifically, we ask how the omniscient design technique will perform with reduced investment horizons. We rerun the experiment for the omniscient design technique with the investment horizons in Table 20 and analyze the changes in performance and network topology.

We plot the performance of the omniscient design technique with varying investment horizons in Figure 25. Note the performance of the omniscient design technique with a 20-period investment horizon is equivalent to the performance of the omniscient design technique from previous experiments. The cumulative performance versus period τ appears in Figure 26. The cumulative performance versus varying investment horizons appears in Figure 27. Figure 28 shows a comparison of the resulting topologies at $\tau = 20$.

We observe in Figure 25 and in Figure 27 that reducing the investment horizon deteriorates performance. This is further evident in Figure 26 by plotting cumulative performance.

The results show that if an ISP has knowledge of future customer demands, as it does in the omniscient design technique, then the ISP can improve performance by investing resources as soon as possible in augmenting arc capacity and/or building new arcs. Any delay in investing resources, demonstrated here by decreasing the investment horizons, can only deteriorate performance.

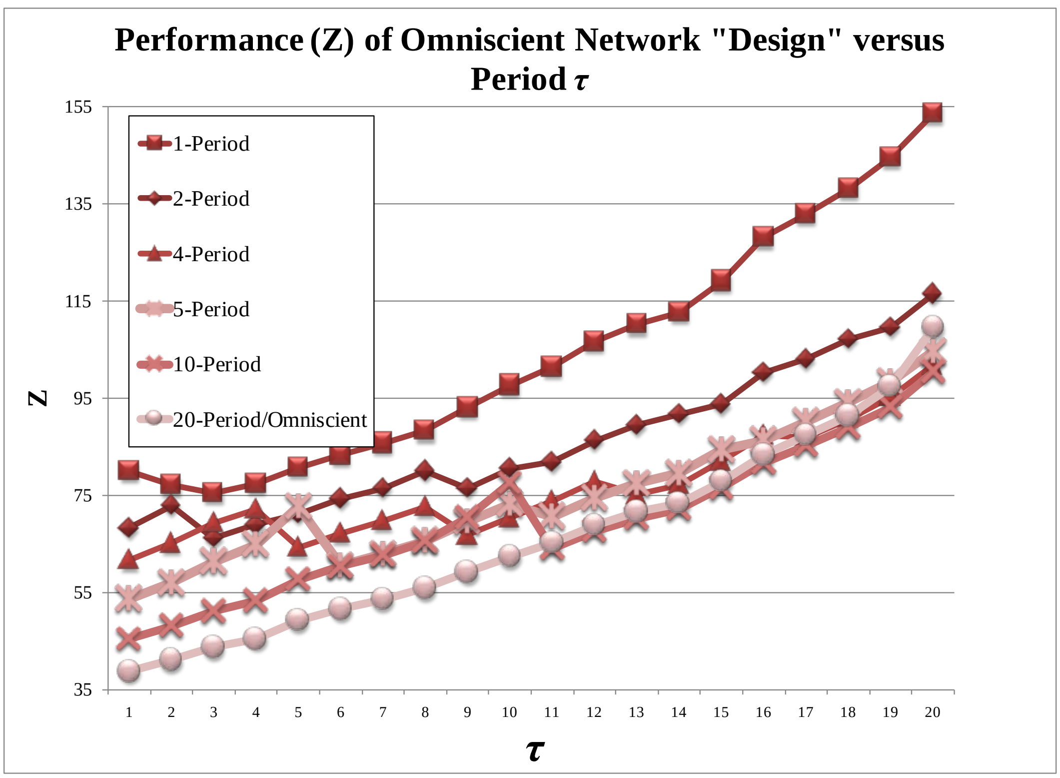


Figure 25. Performance of omniscient network design with varying investment horizons versus period τ.

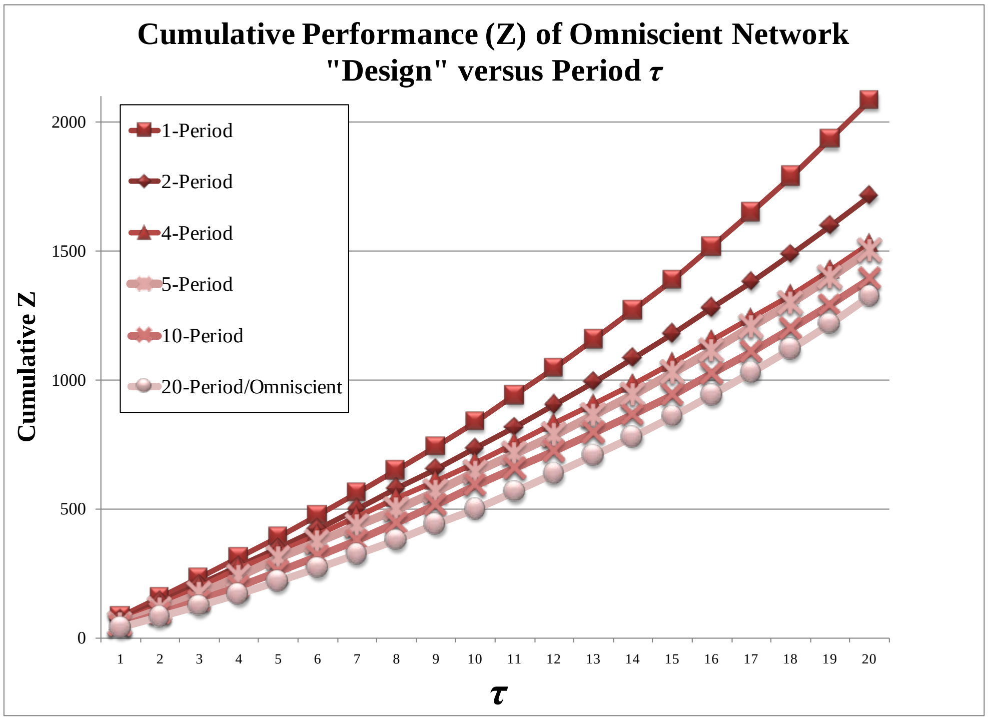


Figure 26. Cumulative performance of omniscient network design with varying investment horizons versus period τ.

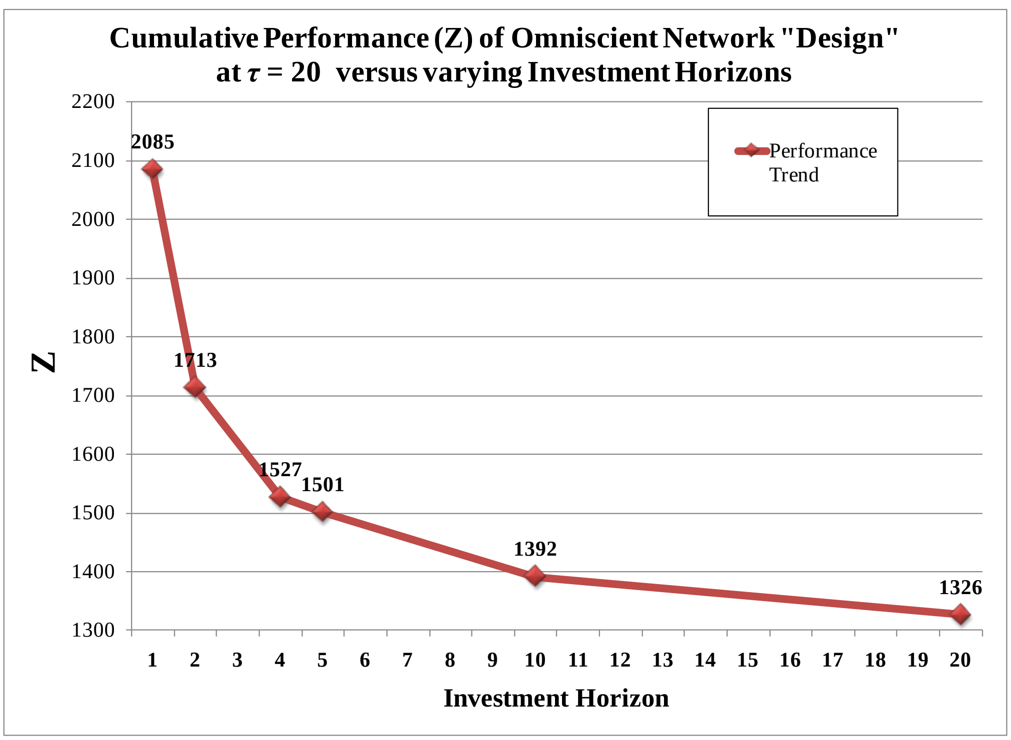
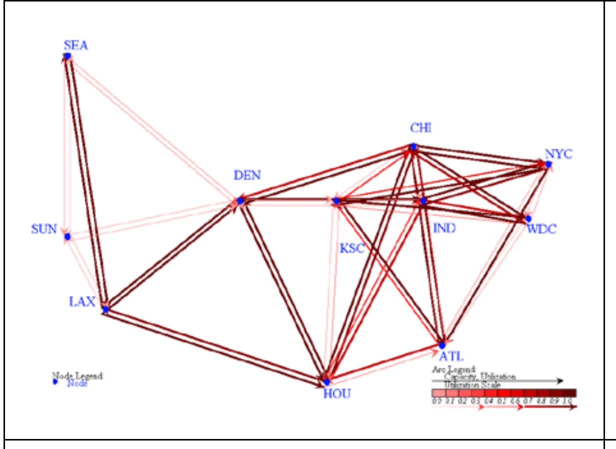
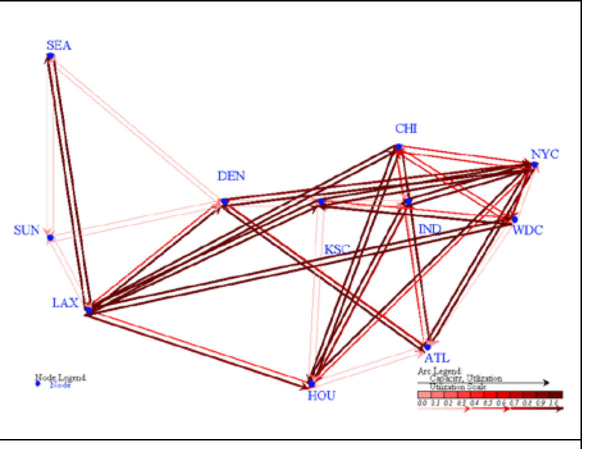


Figure 27. Cumulative performance of omniscient network design versus varying investment horizons.

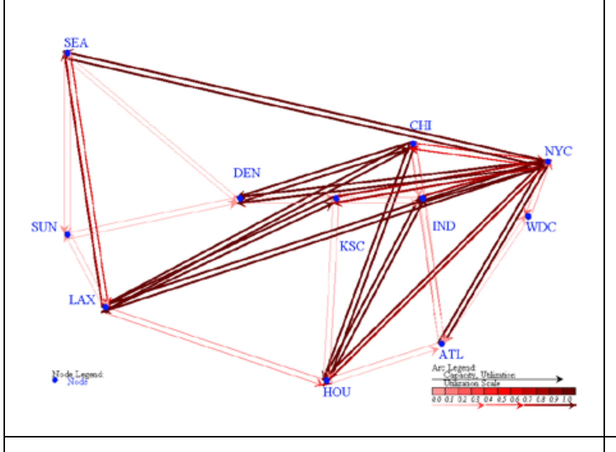
Singe-Period



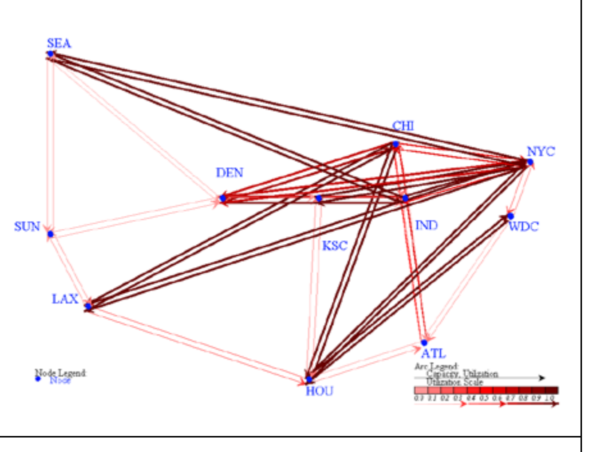
2-Period



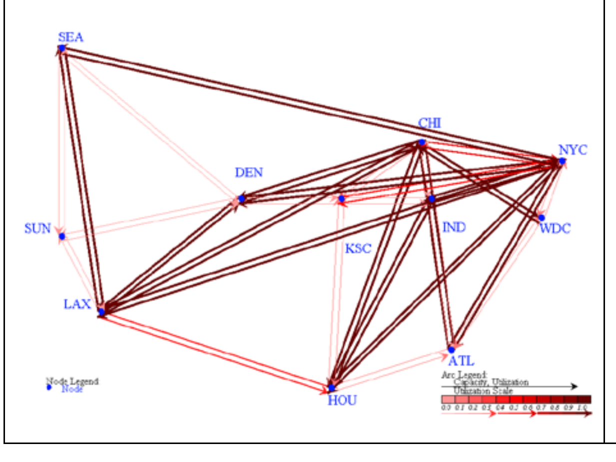
4-Period



5-Period



10-Period



20-Period/Omniscient

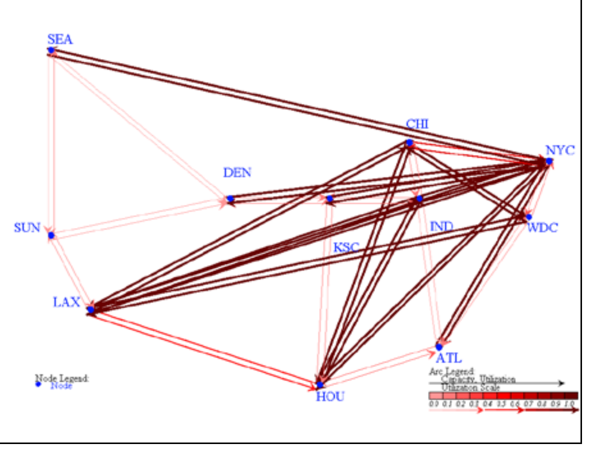


Figure 28. Omniscient network design topologies at $\tau = 20$ from varying investment horizons. The longer the investment horizon, the better the ISP is able to invest optimally for future demand.

We observe in Figure 28 that as investment horizons increase from a single-period to 20-periods, the ISP builds longer arcs. The single-period topology has many short saturated arcs. As investment horizons increase, the ISP has access to more resources and builds longer arcs. This result highlights the value of having access to resources with knowledge of future customer demands. If an ISP has knowledge of future customer demands, as it does in the omniscient design technique, then it can improve performance by provisioning and/or building arcs to satisfy optimally current and future demands.

2. Varying Investment Horizons with the Myopic Technique

In previous experiments, the myopic design technique can spend its *PPB* one period at a time. In this experiment, we relax that constraint. Specifically, we ask whether the ISP can do better by spending larger amounts, but less frequently. We rerun the experiment for the myopic design technique with the investment horizons specified in Table 20 and examine the effects on performance and topology.

We plot the performance of the myopic design technique with varying investment horizons in Figure 29. Note the performance of the myopic design technique with a single-period investment horizon is equivalent to the performance of the myopic design technique from previous experiments. The cumulative performance for each time period appears in Figure 30. The cumulative performance for each investment horizon appears in Figure 31. Figure 32 shows a comparison of the resulting topologies at $\tau = 20$.

We observe in Figure 31 that increasing the investment horizon initially improves performance but then it deteriorates as investment horizons become too long. The performance continuously improves as the investment horizon increases from 1-period to 4-periods; however, when the investment horizon reaches 5-periods, performance begins to deteriorate. From this point on, increasing the investment horizon will result in further deteriorating performance.

Increasing the investment horizon can benefit the ISP in two ways. First, because a longer period of time has elapsed between investments, the ISP gains knowledge of customer demands before investing. Second, the ISP has a larger quantity of resources to invest in meeting customer demands. For example, with a 4-period investment horizon, the ISP invests 4 *PPB* in period 1 and then becomes aware of customer demands in time periods 2, 3, and 4 before investing again. This delay in investment allows the ISP to gain knowledge of where customer demands have increased. The ISP also has more *PPB* in time period 5. With an investment horizon of 4-periods, the ISP has access to 4 *PPB*.

There is a limit to the benefits of increasing an ISP's investment horizon. For example, with a 20-period investment horizon, although the ISP has access to 20 *PPB* in period 1, it only has knowledge of customer demands in period 1 and can only investment optimal for that period. After period 1, performance deteriorates quickly relative to the other investment horizons. This is evident in Figure 29. The ISP must find a balance between realizing customer demands and investing in these realized demands.

The myopic design technique with 4-period and 5-period investment horizons performs well relative to the omniscient technique. The myopic design technique with 4 and 5-period investment horizons significantly outperforms the myopic design from previous experiments. These results indicate that without knowledge of future demand, as is the case in the myopic technique, an ISP can improve performance by choosing a proper investment horizon. These results are important because ISPs typically do not have knowledge of future customer demands.

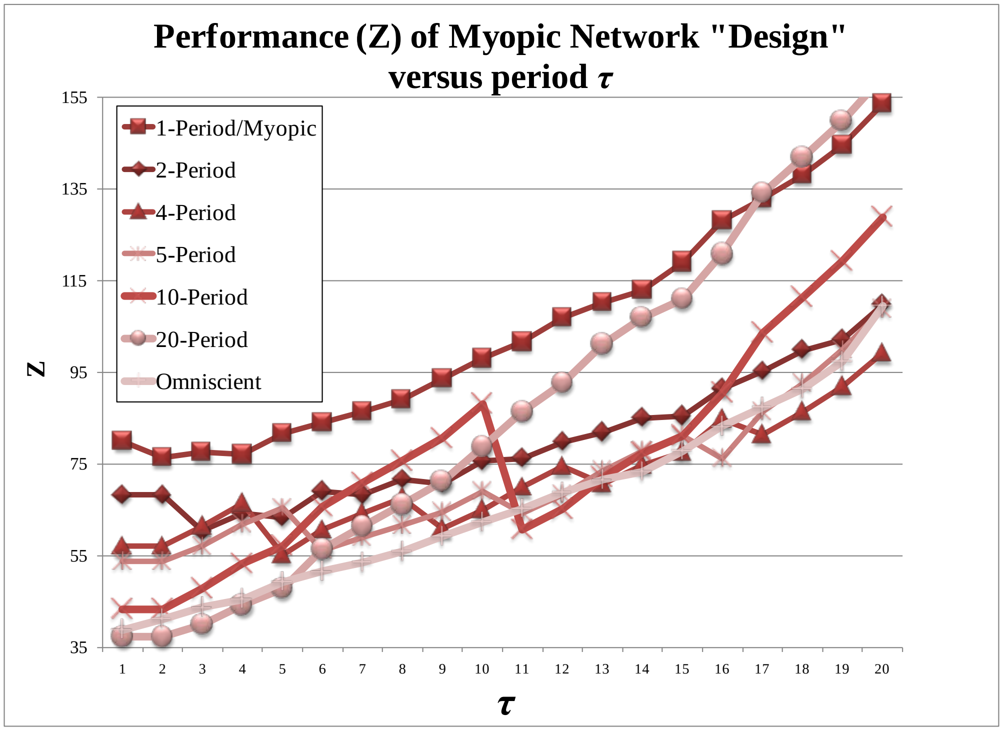


Figure 29. Performance of myopic network design with varying investment horizons versus period τ.

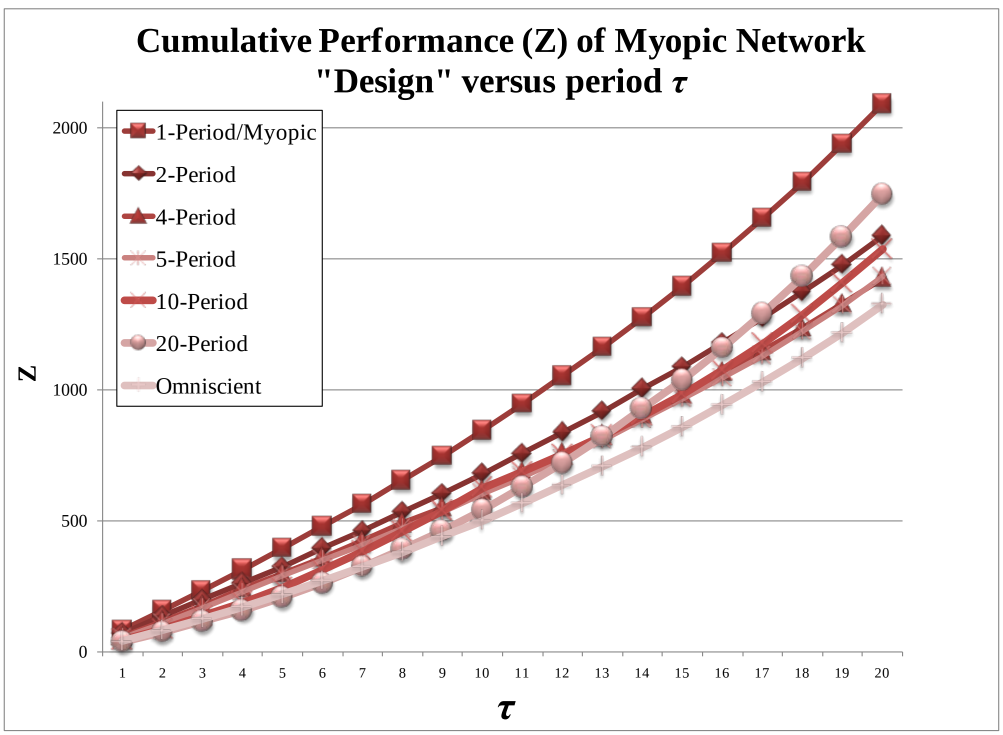


Figure 30. Cumulative performance of myopic network design with varying investment

horizons versus period τ.

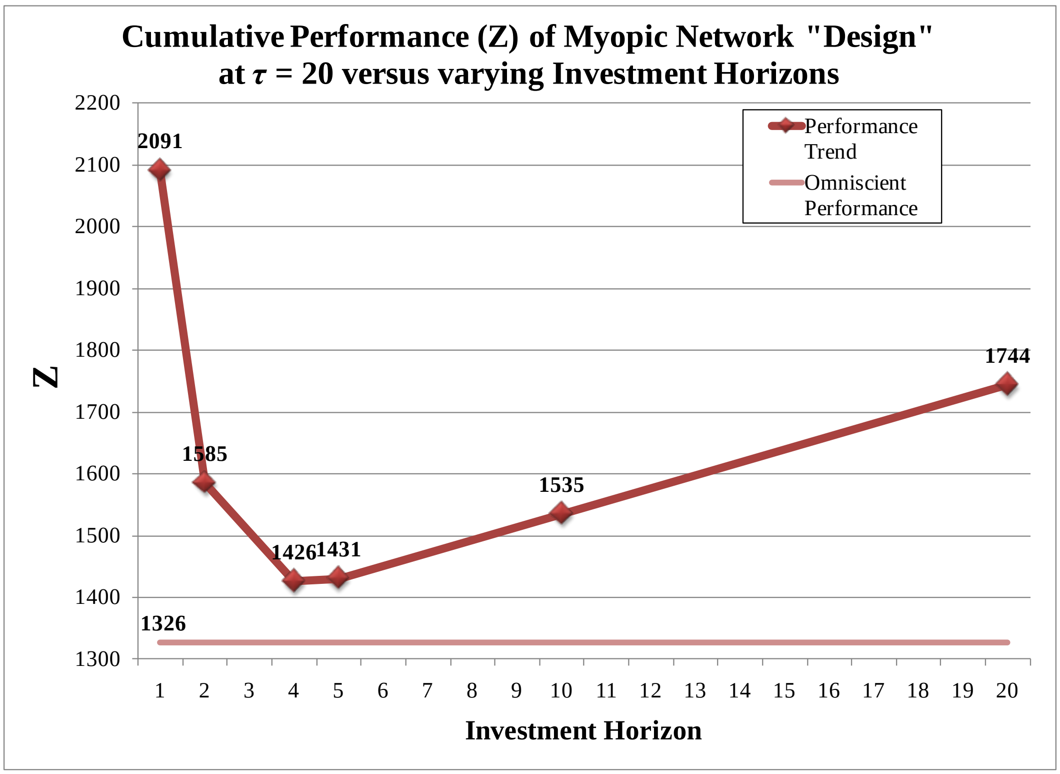
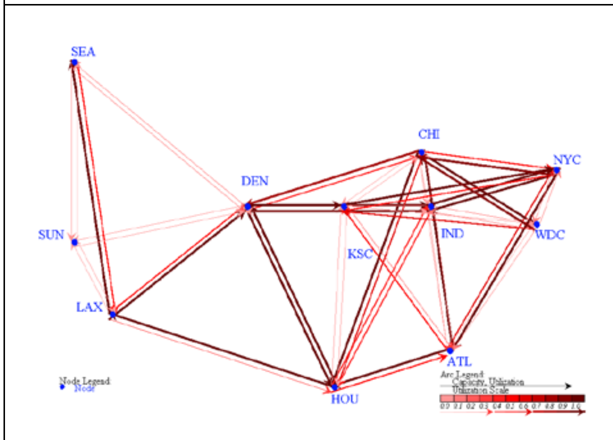
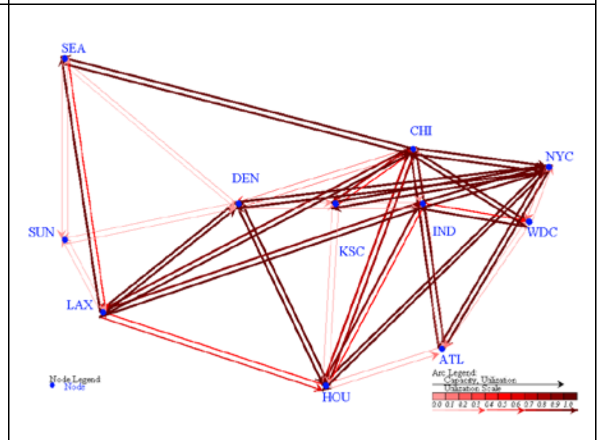


Figure 31. Cumulative performance of myopic network design versus varying investment horizons.

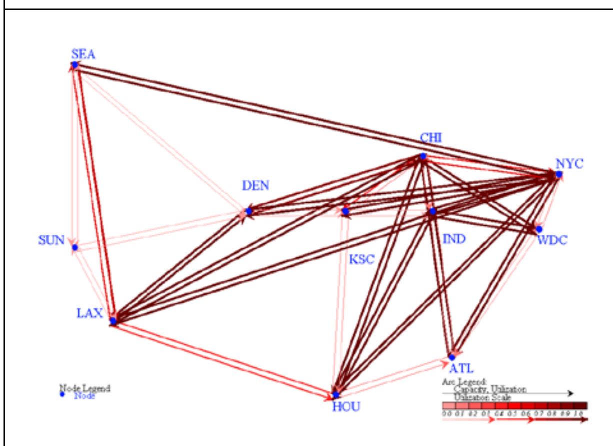
Single-Period/Myopic



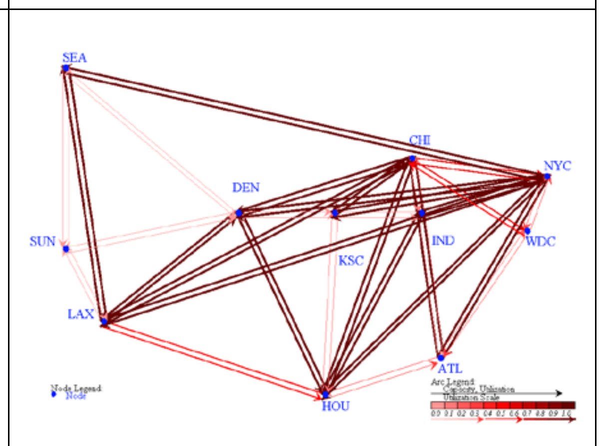
2-Period



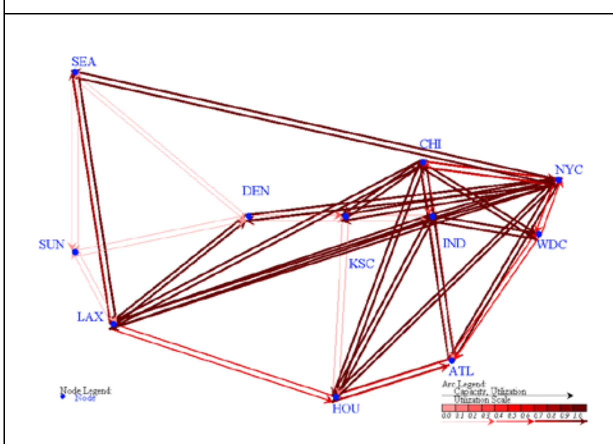
4-Period



5-Period



10-Period



20-Period

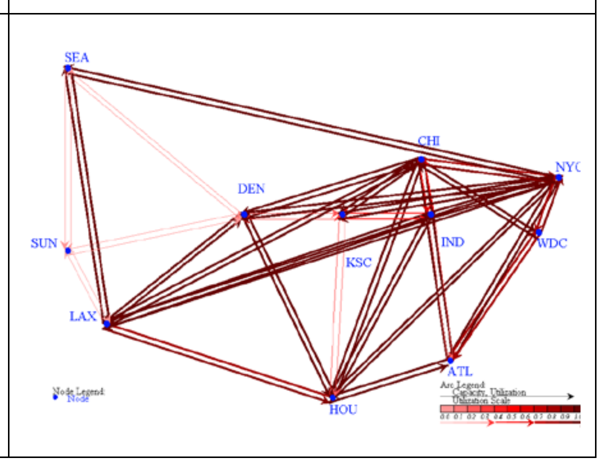


Figure 32. Myopic network design topologies at $\tau = 20$ from varying investment horizons. With periodic investment, the ISP balances uncertainty in demand with a larger investment budget.

In Figure 32, we observe that there is no significant change in the topology of a myopic design technique with investment horizons.

We vary the investment horizon in our experiments and analyze the effects on performance and topology. For the omniscient design technique, we observe that performance can only deteriorate by decreasing the investment horizon. These results support the value of knowing future customer demands. If ISPs can gain knowledge of where customer demands for their services will increase, they can improve future performance by investing resources as soon as possible.

For the myopic design technique, we observe that ISPs can improve performance by selecting an investment horizon that allows spending resources after realizing customer demands. This allows the ISP to achieve performance that approaches omniscient performance. These findings are useful because ISPs typically do not have knowledge of future customer demands; however, they can implement policies that mimic the myopic technique discussed here.

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V. CONCLUSIONS AND FUTURE WORK

In this thesis, we model ISP decision making via a traffic engineering model, a network provisioning model, and a multi-period network provision model. These models capture ISP decision making in the presence of changing environmental conditions such as customer population growth, changing infrastructure costs, and ISP budget constraints.

We show that as customer demand increases, ISPs must increase the capacity on available arcs or build new arcs to handle the increase in customer demand. If ISPs fail to make these changes, demand will increase to the point where the current network infrastructure can no longer handle demand. We also show that traffic routing is not intuitive. Different objectives lead to different "optimal" routes. In this thesis, we assume the ISP wants to minimize operating cost, and this drives traffic routing to prefer shorter traffic routes.

Using an assumed demand matrix inferred from customer populations, we develop an "optimally designed" Abilene and compare it to the real Abilene network. While the "optimal" Abilene outperforms the real Abilene, the resulting topology is "optimal" only for that demand. This exercise helps to focus on the factors that influence performance such as infrastructure cost.

We consider the performance and resulting network topologies of three network design techniques on a set of 20 synthetic demand matrices. We present a heuristic design technique that is reactive to changing customer demands and augments capacity where arcs reach saturation. We develop a myopic design technique that only has knowledge of customer demands and access to the ISP's budget in the current time period. We develop an omniscient design technique that has knowledge of customer demands and access to the ISP's budget in all time periods. The performance and topologies of the three design techniques are considerably different. The results show that ISPs can improve performance either by building network infrastructure or by improving knowledge of customer demands.

We conduct a sensitivity analysis by varying *budget* and costs. Intuitively, an increase in *budget* or a decrease in costs allows an ISP to improve performance by augmenting capacity and/or building more arcs. With greater *budget*, network topologies look more like mesh designs as ISPs invest to improve performance.

We vary the investment horizons in our experiment and focus on the omniscient and myopic design techniques. For the omniscient design technique, we show that reducing the investment horizon deteriorates performance. If an ISP has knowledge of future customer demands, then it is better off investing resources as soon as possible to improve performance in current and future periods. These results highlight the value of knowing future customer demands.

For the myopic design technique, we show that increasing the investment horizon can improve performance and provides two benefits to an ISP. First, the ISP is able to realize customer demands before investing in augmenting capacity and/or building new arcs. Second, the ISP is able to access more resources to invest in these realized customer demands. Although increasing the investment horizon yields benefits, the ISP must find a balance between realizing customer demands and investing in these realized demands. The results show that choosing long investment horizons can diminish the benefits. ISPs can approach omniscient performance by choosing proper investment horizons. These findings are useful because ISPs typically do not know future customer demands; however, they can implement policies that mimic the myopic technique discussed here and choose an investment horizon that improves performance.

The results of this thesis leave many opportunities for future research. The potential benefits of finding a balance between how much to spend and when to spend are attractive for ISPs, since they typically do not have knowledge of future customer demands. Further research will need to focus on identifying these spending policies.

The resulting topologies from the experiments suggest that mesh designs provide good performance in the presence of changing demand. In these experiments, how would mesh designs perform compared to other network topologies such as ring, bus, star, or hierarchical designs? Research focused on network performance across various topologies would yield interesting results and could lead ISPs to focus their network design to a specific topology.

The experiments in this thesis used a single set of 20 synthetic demand matrices. But these demand matrices represent only a single trajectory of evolving demand. Would these design techniques perform equally well under different demand growth? This would help to answer the question how sensitive are the design techniques to variability in changing demand.

In this thesis, we minimize the cost of delivering traffic, but other ISP objectives, such as minimizing network congestion, may also be important. Performing this analysis for different objectives might show other topologies or design techniques to be better.

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APPENDIX

Synthetic Demand Matrices used in Numerical Experiments

For example, the row origin ATL, destination ATL, B^1 = -1.03 is the total demand from all other POPs to ATL in period 1. Likewise, the row origin ATL, destination CHI, B^3 = 0.16 is the demand for traffic from ATL to CHI in period 3.

Origin	Destination	B^1	B^2	B^3	B^4	B^5	B^6	B^7	B^8	B^9	B^{10}
ATL	ATL	-1.03	-1.07	-1.14	-1.17	-1.27	-1.35	-1.39	-1.47	-1.58	-1.64
ATL	CHI	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.23	0.24
ATL	DEN	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
ATL	HOU	0.11	0.11	0.12	0.12	0.13	0.15	0.15	0.16	0.17	0.17
ATL	IND	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06	0.06	0.06
ATL	KSC	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04
ATL	LAX	0.19	0.19	0.20	0.21	0.23	0.24	0.24	0.26	0.28	0.29
ATL	NYC	0.42	0.45	0.48	0.49	0.53	0.57	0.59	0.61	0.66	0.69
ATL	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ATL	SEA	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.05
ATL	WDC	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05
CHI	ATL	0.14	0.15	0.16	0.17	0.18	0.19	0.20	0.21	0.23	0.24
CHI	CHI	-5.25	-5.60	-6.06	-6.29	-6.82	-7.11	-7.39	-7.79	-8.26	-8.68
CHI	DEN	0.17	0.18	0.19	0.20	0.22	0.22	0.23	0.24	0.25	0.27
CHI	HOU	0.64	0.68	0.71	0.75	0.82	0.87	0.91	0.97	1.01	1.03
CHI	IND	0.23	0.25	0.27	0.28	0.30	0.31	0.33	0.34	0.37	0.38
CHI	KSC	0.14	0.15	0.16	0.17	0.18	0.18	0.19	0.20	0.22	0.23
CHI	LAX	1.10	1.14	1.23	1.27	1.38	1.41	1.47	1.57	1.67	1.76
CHI	NYC	2.44	2.64	2.91	3.00	3.26	3.41	3.55	3.70	3.94	4.16
CHI	SUN	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
CHI	SEA	0.17	0.18	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.27
CHI	WDC	0.17	0.19	0.19	0.21	0.23	0.23	0.25	0.26	0.28	0.29
DEN	ATL	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
DEN	CHI	0.17	0.18	0.19	0.20	0.22	0.22	0.23	0.24	0.25	0.27
DEN	DEN	-1.22	-1.28	-1.34	-1.38	-1.50	-1.56	-1.60	-1.65	-1.71	-1.85
DEN	HOU	0.13	0.14	0.14	0.15	0.16	0.17	0.17	0.18	0.18	0.19
DEN	IND	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
DEN	KSC	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04
DEN	LAX	0.23	0.23	0.24	0.25	0.27	0.27	0.28	0.29	0.30	0.33
DEN	NYC	0.50	0.54	0.57	0.58	0.64	0.66	0.68	0.69	0.72	0.78
DEN	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
DEN	SEA	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05
DEN	WDC	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06
HOU	ATL	0.11	0.11	0.12	0.12	0.13	0.15	0.15	0.16	0.17	0.17
HOU	CHI	0.64	0.68	0.71	0.75	0.82	0.87	0.91	0.97	1.01	1.03
HOU	DEN	0.13	0.14	0.14	0.15	0.16	0.17	0.17	0.18	0.18	0.19
HOU	HOU	-4.14	-4.39	-4.58	-4.82	-5.21	-5.58	-5.87	-6.12	-6.38	-6.61
HOU	IND	0.18	0.19	0.19	0.21	0.22	0.24	0.25	0.26	0.27	0.28
HOU	KSC	0.11	0.11	0.12	0.12	0.13	0.14	0.14	0.15	0.16	0.17
HOU	LAX	0.84	0.86	0.89	0.93	1.01	1.07	1.12	1.19	1.23	1.28
HOU	NYC	1.85	1.99	2.10	2.20	2.39	2.58	2.71	2.79	2.92	3.04
HOU	SUN	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03
HOU	SEA	0.13	0.14	0.15	0.16	0.16	0.18	0.18	0.19	0.19	0.20
HOU	WDC	0.13	0.14	0.14	0.15	0.17	0.18	0.19	0.20	0.21	0.21

Origin	Destination	B^1	B^2	B^3	B^4	B^5	B^6	B^7	B^8	B^9	B^{10}
IND	ATL	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06	0.06	0.06
IND	CHI	0.23	0.25	0.27	0.28	0.30	0.31	0.33	0.34	0.37	0.38
IND	DEN	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
IND	HOU	0.18	0.19	0.19	0.21	0.22	0.24	0.25	0.26	0.27	0.28
IND	IND	-1.62	-1.74	-1.84	-1.93	-2.02	-2.16	-2.25	-2.34	-2.50	-2.63
IND	KSC	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06
IND	LAX	0.30	0.32	0.33	0.35	0.37	0.38	0.40	0.42	0.45	0.48
IND	NYC	0.67	0.73	0.79	0.82	0.86	0.93	0.97	0.99	1.07	1.13
IND	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
IND	SEA	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.07
IND	WDC	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08
KSC	ATL	0.02	0.03	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04
KSC	CHI	0.14	0.15	0.16	0.17	0.18	0.18	0.19	0.20	0.22	0.23
KSC	DEN	0.03	0.03	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04
KSC	HOU	0.11	0.11	0.12	0.12	0.13	0.14	0.14	0.15	0.16	0.17
KSC	IND	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06	0.06
KSC	KSC	-1.02	-1.07	-1.11	-1.16	-1.23	-1.26	-1.32	-1.40	-1.50	-1.61
KSC	LAX	0.19	0.19	0.20	0.21	0.22	0.22	0.23	0.25	0.27	0.29
KSC	NYC	0.42	0.44	0.47	0.48	0.52	0.53	0.56	0.58	0.63	0.68
KSC	SUN	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
KSC	SEA	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
KSC	WDC	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05
LAX	ATL	0.19	0.19	0.20	0.21	0.23	0.24	0.24	0.26	0.28	0.29
LAX	CHI	1.10	1.14	1.23	1.27	1.38	1.41	1.47	1.57	1.67	1.76
LAX	DEN	0.23	0.23	0.24	0.25	0.27	0.27	0.28	0.29	0.30	0.33
LAX	HOU	0.84	0.86	0.89	0.93	1.01	1.07	1.12	1.19	1.23	1.28
LAX	IND	0.30	0.32	0.33	0.35	0.37	0.38	0.40	0.42	0.45	0.48
LAX	KSC	0.19	0.19	0.20	0.21	0.22	0.22	0.23	0.25	0.27	0.29
LAX	LAX	-6.54	-6.80	-7.23	-7.49	-8.12	-8.39	-8.74	-9.20	-9.76	-10.36
LAX	NYC	3.19	3.36	3.61	3.71	4.04	4.18	4.36	4.54	4.84	5.17
LAX	SUN	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05
LAX	SEA	0.23	0.23	0.25	0.26	0.28	0.29	0.29	0.30	0.32	0.34
LAX	WDC	0.23	0.24	0.24	0.26	0.28	0.29	0.30	0.32	0.35	0.36
NYC	ATL	0.42	0.45	0.48	0.49	0.53	0.57	0.59	0.61	0.66	0.69
NYC	CHI	2.44	2.64	2.91	3.00	3.26	3.41	3.55	3.70	3.94	4.16
NYC	DEN	0.50	0.54	0.57	0.58	0.64	0.66	0.68	0.69	0.72	0.78
NYC	HOU	1.85	1.99	2.10	2.20	2.39	2.58	2.71	2.79	2.92	3.04
NYC	IND	0.67	0.73	0.79	0.82	0.86	0.93	0.97	0.99	1.07	1.13
NYC	KSC	0.42	0.44	0.47	0.48	0.52	0.53	0.56	0.58	0.63	0.68
NYC	LAX	3.19	3.36	3.61	3.71	4.04	4.18	4.36	4.54	4.84	5.17
NYC	NYC	-10.58	-11.33	-12.19	-12.61	-13.67	-14.34	-14.96	-15.50	-16.48	-17.45
NYC	SUN	0.08	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.12	0.13
NYC	SEA	0.50	0.54	0.60	0.62	0.65	0.69	0.71	0.72	0.75	0.80
NYC	WDC	0.50	0.54	0.57	0.61	0.66	0.69	0.73	0.76	0.82	0.86

Origin	Destination	B^1	B^2	B^3	B^4	B^5	B^6	B^7	B^8	B^9	B^{10}
SUN	ATL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	CHI	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
SUN	DEN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	HOU	0.02	0.02	0.02	0.02	0.03	0.03	0.03	0.03	0.03	0.03
SUN	IND	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	KSC	0.00	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	LAX	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05
SUN	NYC	0.08	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.12	0.13
SUN	SUN	-0.21	-0.22	-0.23	-0.23	-0.25	-0.27	-0.27	-0.28	-0.29	-0.31
SUN	SEA	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	WDC	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SEA	ATL	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.05
SEA	CHI	0.17	0.18	0.20	0.21	0.22	0.23	0.24	0.25	0.26	0.27
SEA	DEN	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05
SEA	HOU	0.13	0.14	0.15	0.16	0.16	0.18	0.18	0.19	0.19	0.20
SEA	IND	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07	0.07
SEA	KSC	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.04	0.04
SEA	LAX	0.23	0.23	0.25	0.26	0.28	0.29	0.29	0.30	0.32	0.34
SEA	NYC	0.50	0.54	0.60	0.62	0.65	0.69	0.71	0.72	0.75	0.80
SEA	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SEA	SEA	-1.22	-1.30	-1.40	-1.47	-1.55	-1.62	-1.67	-1.71	-1.78	-1.89
SEA	WDC	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06
WDC	ATL	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05
WDC	CHI	0.17	0.19	0.19	0.21	0.23	0.23	0.25	0.26	0.28	0.29
WDC	DEN	0.04	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06
WDC	HOU	0.13	0.14	0.14	0.15	0.17	0.18	0.19	0.20	0.21	0.21
WDC	IND	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08
WDC	KSC	0.03	0.03	0.03	0.03	0.04	0.04	0.04	0.04	0.05	0.05
WDC	LAX	0.23	0.24	0.24	0.26	0.28	0.29	0.30	0.32	0.35	0.36
WDC	NYC	0.50	0.54	0.57	0.61	0.66	0.69	0.73	0.76	0.82	0.86
WDC	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
WDC	SEA	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.06
WDC	WDC	-1.22	-1.30	-1.35	-1.44	-1.57	-1.63	-1.73	-1.81	-1.95	-2.03

Origin	Destination	B^{11}	B^{12}	B^{13}	B^{14}	B^{15}	B^{16}	B^{17}	B^{18}	B^{19}	B^{20}
ATL	ATL	-1.72	-1.77	-1.82	-1.85	-2.00	-2.11	-2.22	-2.35	-2.47	-2.58
ATL	CHI	0.25	0.25	0.26	0.26	0.29	0.29	0.31	0.33	0.35	0.36
ATL	DEN	0.05	0.05	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.08
ATL	HOU	0.18	0.19	0.20	0.20	0.22	0.23	0.25	0.26	0.28	0.29
ATL	IND	0.07	0.07	0.07	0.08	0.08	0.09	0.09	0.10	0.10	0.11
ATL	KSC	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06	0.06
ATL	LAX	0.31	0.32	0.32	0.33	0.34	0.37	0.38	0.40	0.42	0.45
ATL	NYC	0.72	0.74	0.76	0.77	0.83	0.89	0.93	0.98	1.02	1.06
ATL	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
ATL	SEA	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
ATL	WDC	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08	0.08
CHI	ATL	0.25	0.25	0.26	0.26	0.29	0.29	0.31	0.33	0.35	0.36
CHI	CHI	-8.95	-9.38	-9.80	-10.07	-10.75	-11.21	-11.80	-12.31	-12.90	-13.49
CHI	DEN	0.28	0.30	0.31	0.33	0.35	0.37	0.40	0.42	0.43	0.46
CHI	HOU	1.08	1.14	1.23	1.26	1.35	1.41	1.50	1.57	1.63	1.73
CHI	IND	0.40	0.41	0.44	0.47	0.50	0.52	0.54	0.58	0.60	0.63
CHI	KSC	0.25	0.25	0.27	0.27	0.29	0.29	0.31	0.32	0.35	0.37
CHI	LAX	1.84	1.90	1.96	2.02	2.09	2.19	2.28	2.37	2.50	2.64
CHI	NYC	4.22	4.46	4.62	4.72	5.09	5.34	5.58	5.81	6.07	6.30
CHI	SUN	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.07
CHI	SEA	0.28	0.29	0.31	0.32	0.34	0.35	0.37	0.38	0.41	0.43
CHI	WDC	0.31	0.33	0.34	0.36	0.39	0.39	0.42	0.45	0.47	0.50
DEN	ATL	0.05	0.05	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.08
DEN	CHI	0.28	0.30	0.31	0.33	0.35	0.37	0.40	0.42	0.43	0.46
DEN	DEN	-1.97	-2.11	-2.18	-2.26	-2.44	-2.64	-2.82	-2.94	-3.04	-3.24
DEN	HOU	0.21	0.23	0.24	0.25	0.27	0.30	0.32	0.33	0.34	0.37
DEN	IND	0.08	0.08	0.09	0.09	0.10	0.11	0.11	0.12	0.13	0.13
DEN	KSC	0.05	0.05	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.08
DEN	LAX	0.36	0.38	0.39	0.40	0.42	0.46	0.48	0.50	0.52	0.56
DEN	NYC	0.82	0.89	0.91	0.94	1.02	1.12	1.18	1.23	1.27	1.34
DEN	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
DEN	SEA	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08	0.09	0.09
DEN	WDC	0.06	0.07	0.07	0.07	0.08	0.08	0.09	0.10	0.10	0.11
HOU	ATL	0.18	0.19	0.20	0.20	0.22	0.23	0.25	0.26	0.28	0.29
HOU	CHI	1.08	1.14	1.23	1.26	1.35	1.41	1.50	1.57	1.63	1.73
HOU	DEN	0.21	0.23	0.24	0.25	0.27	0.30	0.32	0.33	0.34	0.37
HOU	HOU	-6.94	-7.43	-7.89	-8.06	-8.55	-9.24	-9.72	-10.12	-10.50	-11.28
HOU	IND	0.30	0.32	0.35	0.36	0.39	0.41	0.43	0.46	0.48	0.51
HOU	KSC	0.18	0.19	0.21	0.21	0.22	0.23	0.25	0.26	0.27	0.30
HOU	LAX	1.37	1.45	1.52	1.56	1.60	1.75	1.82	1.89	1.97	2.14
HOU	NYC	3.15	3.40	3.59	3.65	3.89	4.27	4.45	4.62	4.78	5.11
HOU	SUN	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06
HOU	SEA	0.21	0.22	0.24	0.25	0.26	0.28	0.30	0.31	0.32	0.35
HOU	WDC	0.23	0.25	0.27	0.28	0.30	0.31	0.34	0.36	0.37	0.41

Origin	Destination	B^{11}	B^{12}	B^{13}	B^{14}	B^{15}	B^{16}	B^{17}	B^{18}	B^{19}	B^{20}
IND	ATL	0.07	0.07	0.07	0.08	0.08	0.09	0.09	0.10	0.10	0.11
IND	CHI	0.40	0.41	0.44	0.47	0.50	0.52	0.54	0.58	0.60	0.63
IND	DEN	0.08	0.08	0.09	0.09	0.10	0.11	0.11	0.12	0.13	0.13
IND	HOU	0.30	0.32	0.35	0.36	0.39	0.41	0.43	0.46	0.48	0.51
IND	IND	-2.77	-2.89	-3.07	-3.21	-3.43	-3.64	-3.78	-4.02	-4.18	-4.43
IND	KSC	0.07	0.07	0.07	0.08	0.08	0.08	0.09	0.09	0.10	0.11
IND	LAX	0.51	0.53	0.55	0.58	0.60	0.64	0.66	0.69	0.73	0.78
IND	NYC	1.17	1.23	1.30	1.35	1.45	1.56	1.60	1.70	1.77	1.86
IND	SUN	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02
IND	SEA	0.08	0.08	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.13
IND	WDC	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.13	0.14	0.15
KSC	ATL	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06	0.06
KSC	CHI	0.25	0.25	0.27	0.27	0.29	0.29	0.31	0.32	0.35	0.37
KSC	DEN	0.05	0.05	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.08
KSC	HOU	0.18	0.19	0.21	0.21	0.22	0.23	0.25	0.26	0.27	0.30
KSC	IND	0.07	0.07	0.07	0.08	0.08	0.08	0.09	0.09	0.10	0.11
KSC	KSC	-1.72	-1.80	-1.86	-1.92	-1.99	-2.08	-2.22	-2.27	-2.43	-2.62
KSC	LAX	0.31	0.32	0.33	0.34	0.34	0.36	0.38	0.38	0.42	0.45
KSC	NYC	0.72	0.75	0.77	0.79	0.83	0.88	0.93	0.94	1.01	1.08
KSC	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
KSC	SEA	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
KSC	WDC	0.05	0.06	0.06	0.06	0.06	0.06	0.07	0.07	0.08	0.09
LAX	ATL	0.31	0.32	0.32	0.33	0.34	0.37	0.38	0.40	0.42	0.45
LAX	CHI	1.84	1.90	1.96	2.02	2.09	2.19	2.28	2.37	2.50	2.64
LAX	DEN	0.36	0.38	0.39	0.40	0.42	0.46	0.48	0.50	0.52	0.56
LAX	HOU	1.37	1.45	1.52	1.56	1.60	1.75	1.82	1.89	1.97	2.14
LAX	IND	0.51	0.53	0.55	0.58	0.60	0.64	0.66	0.69	0.73	0.78
LAX	KSC	0.31	0.32	0.33	0.34	0.34	0.36	0.38	0.38	0.42	0.45
LAX	LAX	-10.84	-11.39	-11.67	-11.94	-12.38	-13.42	-13.82	-14.30	-15.03	-16.06
LAX	NYC	5.35	5.65	5.72	5.82	6.04	6.65	6.77	6.98	7.32	7.78
LAX	SUN	0.06	0.06	0.06	0.07	0.07	0.07	0.08	0.08	0.08	0.09
LAX	SEA	0.35	0.37	0.39	0.40	0.41	0.43	0.45	0.46	0.49	0.53
LAX	WDC	0.39	0.42	0.42	0.44	0.46	0.49	0.51	0.55	0.57	0.62
NYC	ATL	0.72	0.74	0.76	0.77	0.83	0.89	0.93	0.98	1.02	1.06
NYC	CHI	4.22	4.46	4.62	4.72	5.09	5.34	5.58	5.81	6.07	6.30
NYC	DEN	0.82	0.89	0.91	0.94	1.02	1.12	1.18	1.23	1.27	1.34
NYC	HOU	3.15	3.40	3.59	3.65	3.89	4.27	4.45	4.62	4.78	5.11
NYC	IND	1.17	1.23	1.30	1.35	1.45	1.56	1.60	1.70	1.77	1.86
NYC	KSC	0.72	0.75	0.77	0.79	0.83	0.88	0.93	0.94	1.01	1.08
NYC	LAX	5.35	5.65	5.72	5.82	6.04	6.65	6.77	6.98	7.32	7.78
NYC	NYC	-17.99	-19.11	-19.73	-20.15	-21.45	-23.12	-23.99	-24.93	-26.01	-27.51
NYC	SUN	0.13	0.14	0.15	0.15	0.17	0.18	0.19	0.20	0.20	0.21
NYC	SEA	0.81	0.86	0.91	0.93	1.00	1.05	1.10	1.13	1.19	1.27
NYC	WDC	0.91	0.98	1.00	1.03	1.13	1.19	1.26	1.34	1.38	1.48

Origin	Destination	B^{11}	B^{12}	B^{13}	B^{14}	B^{15}	B^{16}	B^{17}	B^{18}	B^{19}	B^{20}
SUN	ATL	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	CHI	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.07
SUN	DEN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
SUN	HOU	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.05	0.05	0.06
SUN	IND	0.01	0.01	0.01	0.02	0.02	0.02	0.02	0.02	0.02	0.02
SUN	KSC	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	LAX	0.06	0.06	0.06	0.07	0.07	0.07	0.08	0.08	0.08	0.09
SUN	NYC	0.13	0.14	0.15	0.15	0.17	0.18	0.19	0.20	0.20	0.21
SUN	SUN	-0.32	-0.34	-0.36	-0.38	-0.41	-0.44	-0.46	-0.48	-0.49	-0.52
SUN	SEA	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SUN	WDC	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02
SEA	ATL	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
SEA	CHI	0.28	0.29	0.31	0.32	0.34	0.35	0.37	0.38	0.41	0.43
SEA	DEN	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08	0.09	0.09
SEA	HOU	0.21	0.22	0.24	0.25	0.26	0.28	0.30	0.31	0.32	0.35
SEA	IND	0.08	0.08	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.13
SEA	KSC	0.05	0.05	0.05	0.05	0.06	0.06	0.06	0.06	0.07	0.07
SEA	LAX	0.35	0.37	0.39	0.40	0.41	0.43	0.45	0.46	0.49	0.53
SEA	NYC	0.81	0.86	0.91	0.93	1.00	1.05	1.10	1.13	1.19	1.27
SEA	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
SEA	SEA	-1.94	-2.05	-2.18	-2.25	-2.38	-2.49	-2.61	-2.71	-2.87	-3.07
SEA	WDC	0.06	0.06	0.07	0.07	0.08	0.08	0.08	0.09	0.09	0.10
WDC	ATL	0.05	0.05	0.06	0.06	0.06	0.07	0.07	0.08	0.08	0.08
WDC	CHI	0.31	0.33	0.34	0.36	0.39	0.39	0.42	0.45	0.47	0.50
WDC	DEN	0.06	0.07	0.07	0.07	0.08	0.08	0.09	0.10	0.10	0.11
WDC	HOU	0.23	0.25	0.27	0.28	0.30	0.31	0.34	0.36	0.37	0.41
WDC	IND	0.09	0.09	0.10	0.10	0.11	0.11	0.12	0.13	0.14	0.15
WDC	KSC	0.05	0.06	0.06	0.06	0.06	0.06	0.07	0.07	0.08	0.09
WDC	LAX	0.39	0.42	0.42	0.44	0.46	0.49	0.51	0.55	0.57	0.62
WDC	NYC	0.91	0.98	1.00	1.03	1.13	1.19	1.26	1.34	1.38	1.48
WDC	SUN	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.02
WDC	SEA	0.06	0.06	0.07	0.07	0.08	0.08	0.08	0.09	0.09	0.10
WDC	WDC	-2.17	-2.32	-2.39	-2.48	-2.69	-2.80	-2.99	-3.18	-3.31	-3.57

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