2002

BATTLESPACE/INFORMATION WAR (BAT/IW): a system-of-systems model of a strike operation

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NPS-OR-02-005

NAVAL POSTGRADUATE SCHOOL
Monterey, California

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by

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August 2002

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Prepared for: The MOVES Institute
Naval Postgraduate School
Monterey, California 93943

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MONTEREY, CA 93943-5000

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This report was prepared for the MOVES Institute, Naval Postgraduate School, Monterey, CA 93943, and funded by the Wayne E. Meyer Institute of Systems Engineering, Naval Postgraduate School, Monterey, CA 93943; Space-C2-Information Warfare, N6, 2000 Navy Pentagon, Washington, DC 20350-2000; and the MOVES Institute, Naval Postgraduate School, Monterey, CA 93943.

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REPORT DOCUMENTATION PAGE	Form approved OMB No 0704-0188

1. AGENCY USE ONLY (Leave blank)	**2. REPORT DATE** August 2002	**3. REPORT TYPE AND DATES COVERED** Technical Report

4. TITLE AND SUBTITLE
BATTLESPACE / INFORMATION WAR (BAT/IW): A System-of-Systems Model of a Strike Operation

5. FUNDING
N00014202WR20147

6. AUTHOR(S)
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7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Naval Postgraduate School
Monterey, CA 93943

8. PERFORMING ORGANIZATION REPORT NUMBER
NPS-OR-02-005

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)
The MOVES Institute
Spanagel Hall/Room 252
Naval Postgraduate School
Monterey, CA 93943

Wayne E. Meyer Institute of Systems Engineering
Bullard Hall
Naval Postgraduate School
Monterey, CA 93943

Space-C2-Information Warfare, N6
2000 Navy Pentagon
Washington, DC 20350-2000

10. SPONSORING/MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION/AVAILABILITY STATEMENT

12b. DISTRIBUTION CODE

13. ABSTRACT (Maximum 200 words.)
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The model is deterministic/expected value in nature, although it analytically represents time-dependent stochastic features such as system saturability. Model experimentation suggests the following results. Decreasing shooter latency can result in greater attrition than correspondingly increasing the probabilities of correct BDA or weapon assignment, although at the expense of a greater number of weapons fired per target killed. Erroneous BDA returns dead targets to the shooter targeting list. These dead targets not only result in wasted weapon expenditure but also take sensor/shooter resources away from legitimate live targets. Increasing the probability of correct BDA can result in a greater number of targets killed during a time period than increasing the probability of correct weapon assignment.

14. SUBJECT TERMS
deterministic network of queues with saturation; intelligence, surveillance, and reconnaissance; imperfect sensors; target losses

15. NUMBER OF PAGES
64

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	**18. SECURITY CLASSIFICATION OF THIS PAGE**	**19. SECURITY CLASSIFICATION OF ABSTRACT**	**20. LIMITATION OF ABSTRACT**
Unclassified	Unclassified	Unclassified	Unlimited

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Abstract

This paper presents a low-resolution, high-level modeling methodology for the analysis of the effectiveness of a Blue *system of systems* operating in a battlespace. The methodology enables quick turn around and efficient exploration of sensitivities of measures of Blue combat success to realistically imperfect Blue intelligence, surveillance and reconnaissance capabilities: limited and imperfect sensor surveillance and reconnaissance, particularly battle damage assessment (BDA), and finite, hence saturable, communications and weapons delivery capability. The model explicitly represents aircraft sorties, fires, sensor/shooter latencies, target losses, imperfect target type classification, imperfect weapon assignment, and BDA; various levels of the above imperfections can be applied, facilitating tradeoff studies.

The model is deterministic/expected value in nature, although it analytically represents time-dependent stochastic features such as system saturability. Model experimentation suggests the following results. Decreasing shooter latency can result in greater attrition than correspondingly increasing the probabilities of correct BDA or weapon assignment, although at the expense of a greater number of weapons fired per target killed. Erroneous BDA returns dead targets to the shooter targeting list. These dead targets not only result in wasted weapon expenditure but also take sensor/shooter resources away from legitimate live targets. Increasing the probability of correct BDA can result in a greater number of targets killed during a time period than increasing the probability of correct weapon assignment.

1. Introduction

This paper provides a low-resolution, high-level methodology "*scoping model*" for analysis of a Blue strike operation: missile-shooting and manned aircraft sortie response to a Red ground force that enters a region for hostile purposes (regional occupancy with territorial objective, staging for missile, e.g., SCUD, shots, etc.). For example, the beginning of a Major Regional Contingency (MRC). The model proposed facilitates efficient initial exploration of *sensitivities* of measures of Blue combat success to realistically imperfect Blue intelligence, surveillance and reconnaissance (ISR), and information warfare (IW) capabilities: limited and imperfect sensor surveillance and reconnaissance, particularly BDA, and finite, hence saturable, communications and weapons delivery capability. The effect of both Red and Blue (aircraft) decoys and false targets are implicitly present and readily analyzed. It is proposed that somewhat more detailed representation of Red air defense (AD) and Blue suppression of enemy air defense ((J)SEAD) be included in the present formulation. The ultimate aim of the model is to suggest the value of alternative operational architectures and investment programs.

The model realistically, but economically, assesses the capability of *limited Blue system-of-system capacities to provide needed services*: sensor regional coverage and potential target classification capability; communication, with delays (bandwidth limits); and shooter firing rate and lethality. Highly nonlinear unfavorable responses to deficiencies in such capabilities, and in their relative balance and mutual compatibility, are easily and quickly revealed by exploration of the model. These deficiencies can be potentially rectified by suitably improving, modifying, and *balancing* Sensor-Shooter capabilities. In particular the effect of *reducing bandwidth requirements*, e.g., by *storing* and occasionally updating slowly changing information appropriately, is implicitly reflected in the model: the above effect simply increases service rates (decreases latency). Sensitivity to such an architectural design possibility is traceable using the model; the effect may be considerable whenever the original system operates near a saturation level, wherein message traffic nearly reaches or exceeds available processing capacity (bandwidth). The model does not, however, address any downside issues associated with increased local information storage.

1.1 Model Features

In the models of the present paper realism and needed detail are introduced, but parsimoniously. In particular (see Figure 1),

- *Spatial considerations* are accounted for: the region $\Re$ is viewed as a collection of non-overlapping and inclusive subregions $\{\Re_i, i = 1, 2, \ldots, I\}$; for instance, choice of these allow for explicit range dependencies (introduction of range bands is a convenient simple device). The exact specification of the subregions is left to analyst discretion; it may be reasonable to let these be internally homogeneous in the sense of terrain, hence visibility. The number of subregions is arbitrary in principle, but limited by computational considerations. Some averaging within and between subregions is inevitable. If desired, and meaningful, a subregion can be an established route, e.g., road, from one point to another.
- *Red force type variability* is recognized: types are broad but flexible, being at present and *only for example*, Heavily Armored Vehicles (generically Tanks or Hard Targets), Light Armored Vehicles (Armored Personnel Carriers), Unarmored Vehicles (Trucks, etc.), Infantry, … generically classified as Soft Targets. The *state* of Red forces is taken to be characterized (at minimum) by the numbers of each of its force elements (see above) in each of the designated subregions at a particular time. If desired, the above state description can be extended or expanded (for instance, the numbers of Red targets in a particular formation, moving or still at a given time, can be recorded as state variables that evolve together as time elapses); the above is essential to a dynamic description of

system evolution. The state of Red is a (vector-valued) description that changes dynamically in time, responding to Red territorial objectives, and in reaction to actions by the Blue force.

- *Red force mobility/maneuver across subregions* (*e.g.*, *advance or retreat*) is modeled. This motion can well be a response to perceived Blue actions.
- *Blue Sensor and ISR ("Network-Centric") force capabilities and actions are modeled* (for operational effect, without engineering systems-level detail): the Sensor effort ("servicing capacity") allocated to subregion $\Re_i$ at any time determines the rate at which data on Red occupancy of that subregion can be obtained and transferred to a Blue candidate-for-targeting list; note that the model recognizes that it takes time for Blue Sensor/ISR assets to discover/process data on Red presence in a subregion, so if insufficient Blue assets are available there, an effective queue/waiting line develops made up of undetected Reds. The variation and adaptation of Blue Sensor assets across space (subregions) in response to Red movements and actions is a feature of modern Network-Centric warfare. It is also realistic to model the realistic errors, delays, and susceptibility to deception and jamming inherent in any information-gathering system; this is done in such a way as to allow assessment of the value of increased Sensor asset capability, and tactics for employment thereof.

The fact that Red forces (types and locations) are known by Blue only after delay and with error is represented by the Sensor/ISR submodel. In reality, Blue must act on the basis of an imperfect quantitative perception of the true Red state. The consequences of that imperfection are that his subsequent actions ("shooting," maneuvering) are affected. Our model indicates the value of altering the system components of our system-of-systems.

- *Blue Shooter's varied assets and capabilities are represented* in adjustable detail: surface-surface and air-surface missiles or bombs are viewed as tailored to, or optimized for, particular target types. For instance, a weapon suited to destruction of Heavy Armor can certainly kill Light Armor, but at greater than necessary expense per round, but a weapon optimized for Light Armor or Infantry will likely have small chance of killing Heavy Armor. The effect of misallocating weapons to targets, largely caused by ISR mistakes and delays, but also influenced by logistics (weapon supply), can be substantial, as measured by cost to Blue in own casualties, in dollars, and in campaign length. Of course the number, hence shooting ("service") rate of the Blue Shooter (missile firing tubes, plus aircraft sorties) influences the length of the target queue: the greater the gross Shooter rate, the shorter the delay in targeting a Red that has been detected and classified (although possibly incorrectly) by the Blue Sensor. As the rate of target addition to the Shooter Target List/Queue increases, the greater the (missile) Shooter's effective delay (latency) and the smaller the chance of a successful shot: this effect is *highly nonlinear* as potential target input approaches Shooter capacity, and thus is a source of important detrimental sensitivity. Certainly additions to the target list/queue that are the result of Sensor mistakes (initial misclassifications, or incorrect BDA leading to re-targeting of killed Red assets) can be seen to have cascading detrimental effects on system effectiveness, particularly when the overall system becomes heavily loaded, i.e., as increasingly many targets are placed on the Shooter's list. The model helps identify requirements for controlling such bottleneck situations.

- *The present model is deterministic/expected-value*, although it reflects time-dependent stochastic features such as system saturation analytically, using a mathematical device; cf. Filipiak (1988). The basic model formulation logic can be made to govern a discrete-event simulation if desired, or as the basis for probabilistic analysis; see Gaver and Jacobs (1999) for a simplified, but analytically treatable, version. The present modeling technique facilitates quick approximate investigations and model browsing for interesting effects.

2. Example Sensor-Shooter Architectures

Consider two specific examples of Sensor-Shooter architectures:

(A) Architecture 1: General Regional Coverage (GRC), Delayed Battle Damage Assessment (BDA), in which prosecuted (shot-at) targets are released back into their present (sub)region for eventual discovery and re-classification by a Regional Sensor System, or

(B) Architecture 2: Local, Quick Follow-up (LQF) *and* General Regional Coverage (GRC), Battle Damage Assessment (BDA), in which a Local Quick Follow-up sensor capability exists to immediately assess the results of a shot (*but* with potential error) and the just-targeted Red then re-targeted immediately if judged alive; if alive and judged dead it is released for eventual later rediscovery by the GRC (until such discovery it is free to engage in hostile acts). Tomahawk, Block 4, has some LQF capabilities.

Both Architectures are represented as generic. It will be the objective of subsequent work to characterize the operational capabilities of potential new systems and architectures by adjusting the parameters of our model. Then model experimentation can be used to suggest specific system investments to best add operational value.

3. Illustrative Invasion-Strike Scenario

Figure 1 depicts a region, $\Re$, into which Red units are envisioned to pour from the North. Their objective is to move Southwards, but perhaps to punctuate the journey with occasional hostile actions such as missile (e.g., SCUD) shots.

The Figure 1 legend describes illustrative prototypical Red units and their general states of perception by the Blue ISR. The Red units may be in any of the range bands shown: R_4 to R_1. The types shown here are (1) Heavy Armor Units (Hard Targets) both detected and undetected, and hence on a Blue virtual target list; (2) Light Armor and/or Infantry, undetected, but also detected and on the Blue virtual target list; (3) Decoys (deliberate) and False Targets (natural/environmental, e.g., civilian non-combatants), undetected and also detected and on the Blue virtual target list. These can be both undetected and detected, and placed on a target list.

Invasion-Strike Scenario

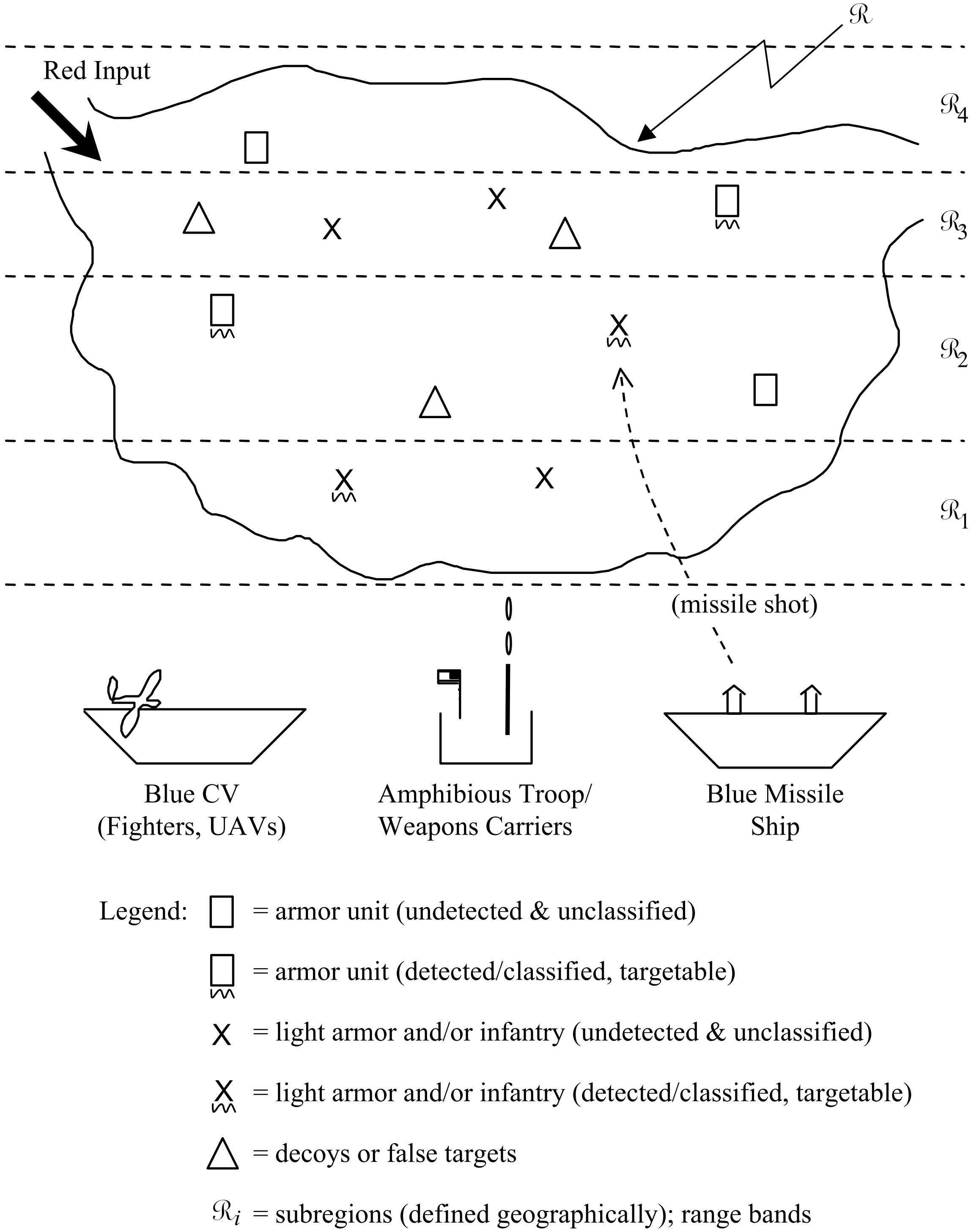
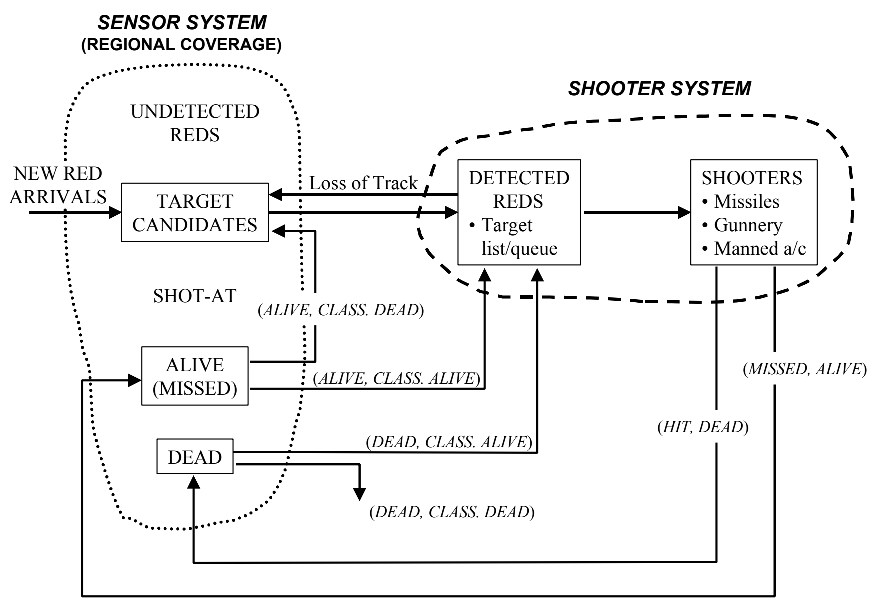


Figure 1

4. Flow Charts of Operational Architectures and Blue Perception

In Figures 2 and 3 we introduce flow charts to describe the general motion of Red units between states of concern to the Blue system of systems.

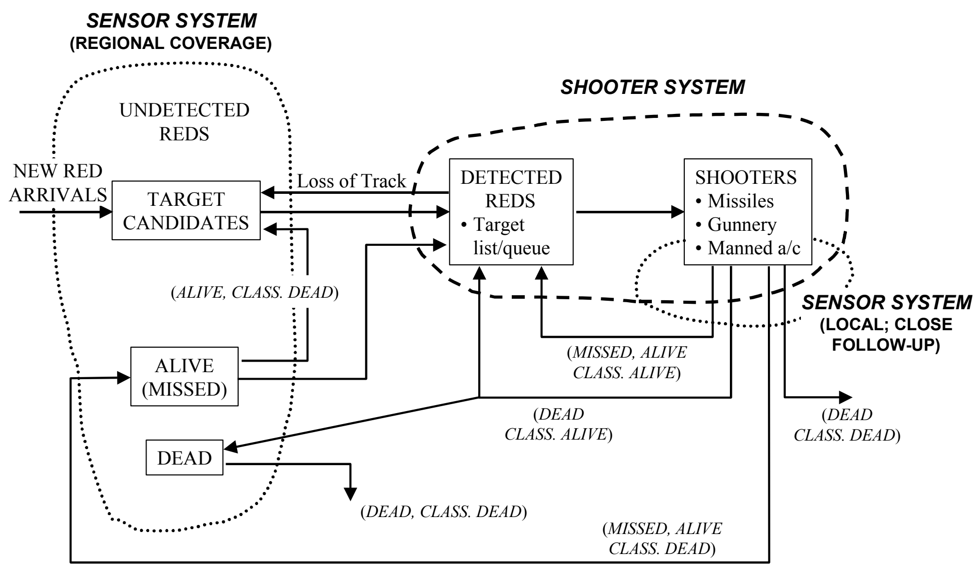
Figure 2 depicts the state transitions for units under Architecture 1, for which the sensor coverage is of the various regions. Note that this coverage may be made variable and adaptive, and that detections occur at effective (sweep) rates determined by Blue decision makers. Presumably the Blue allocation of detection effort should be made to conform to anticipated and predicted Red motion in the region; likewise that Red motion can react to perceived Blue sensor presence, which is of analyst-adjustable acuity. The model equations (see Appendix I) can be made to represent the various feedbacks that adjust Red and Blue behavior.



- *All* target classification and BDA decisions are error-prone or fallible to adjustable degree. Hence the DETECTED REDS target queue contains alive targets, and also dead (presumed alive) and dead (unclassified or BDAed)

Figure 2. Architecture 1:
General Regional Coverage Sensors (GRC) (Delayed Shooter BDA)
Manned a/c have immediate BDA as in Figure 3

Figure 3 represents state transitions of units governed by Architecture 2. Here we augment the GRC capability by an Immediate Shooter BDA capability associated with the missile shooters (only): when a Blue missile is fired at a Red target in $\Re_i$ the act is presumed to be immediately followed by a BDA sensor (possibly on the missile); if (1) a Blue miss or failure to kill is registered, an immediate return to the target list is scheduled, while (2) if a Blue kill is registered, the target is immediately *declared* dead and new targets are engaged. We model the effects of the several possible errors: in case (1) the mistake of registering a miss when a kill actually occurred results in at least one wasted shot, while in case (2) the error of mistakenly registering a kill means that the targeted Red is free to engage in further operations hostile to Blue.



- *All* target classification and BDA decisions are error-prone or fallible to adjustable degree.

Figure 3. Architecture 2:
GRC *and* Local Quick Follow-Up (LQF) (Immediate Shooter BDA)

5. Strike Dynamics

We present the details of the state variables, rate parameters, and dynamical equations in Appendix I. The equations are annotated term by term so that the contribution of each term is explained. In broad outline, Reds enter the region, possibly migrating Southwards, are detected by Blue sensors in each region (with delay), then placed on a target list (from which track losses can occur), are eventually shot at by either missiles (or gunnery) or manned aircraft. Those missed are subject to re-detection (BDA) and reclassification (without reference to individual past, except by regional location). The aim is to understand the effectiveness of Blue sensor-shooter assets of given force size, composition, and effectiveness against a composite Red force.

Our current formulation is the deterministic equivalent of a continuous-time multivariate stochastic Markov process. Such a stochastic process is one attractive modeling option, but has not been adopted for reasons of computational economy. Monte Carlo simulation has not been adopted here for the same reason. Ample precedent for the present style of modeling occurs in ecology, epidemiology, and population biology (see Murray (1989)), not to mention early military operations research; see Dockery and Woodcock (1993), Taylor (1980), and Anderson (1995). One difference in our model is the explicit representation of congestion, particularly possible saturation, of Blue's sensor, communication, and shooter resources. These latter resources are bound to be limited, so backlogs ("queues") effectively develop of Red units that await detection ("service"), and subsequently are placed virtually in line for shooter "service." However, that service can be much reduced in effective speed because of the presence of dead, misclassified targets, and decoys. Some of these are lost from track, and hence the target list. Classical queuing theory, see Kleinrock (1975), leads us to expect that some backlog will tend to develop at both the sensor and communication/shooter stages, even when demand for service does not exceed service rate at a stage; this is the result of short-term random fluctuations in the communications and shooter system. We choose to represent the latter effect approximately by modifying a simple mathematical device put forward by Agnew (1976) and Rider (1976), more recently discussed by Filipiak (1988). These papers introduce the non-linear H-function defined, along with parameters. The modification required for sensors has recognized that (a) the sensor "server" must itself be multiple (several sensor platforms potentially cover each subregion whose capacities are finite and saturable, *and* (b) must accommodate multiple Red target types and states;

the latter is accomplished by proportional processor sharing, and the former by increasing effective service (here sweep) rate. The modification for the communication/shooter system is the same, but modifies the processor sharing service intensity across the target list by priority weighting: w_{ik} represents the fraction of time made available to process (shoot) targets in region *i judged to be* of type *k*; c_{ijk} represents the sensor skill, it being the probability that a target of type *j* in region *i is believed to be, and treated as* of type *k*, where $c_{ijj} = 1$ represents perfect skill.

Each of the above parameters is treated in the present report as a static analyst-decision maker choice: a *constant*. However, the possibility, and attraction, is great to let these "parameters" be replaced by functions, and so be automatically adjusted, i.e., governed by feedback from Blue perception of Red states and inferred course of action.

6. Case Study 1: Constant Red Arrival Rate

We first consider an extremely simple scenario, wherein a constant number of Reds per day (500 Hard, 500 Soft) pour into the northernmost region, $\Re_4$. The basic sensor sweep rate is $\xi = 5000$. We present graphs of the model output, beginning with cumulative input and number undetected Figure 4(a) without Blue attrition. The subsequent figures appear in Appendix II. Note that in the non-attrited deterministic/fluid approximation the cumulative entry level of Red targets is strictly linear, and the cumulative number undetected quickly approaches linearity in time.

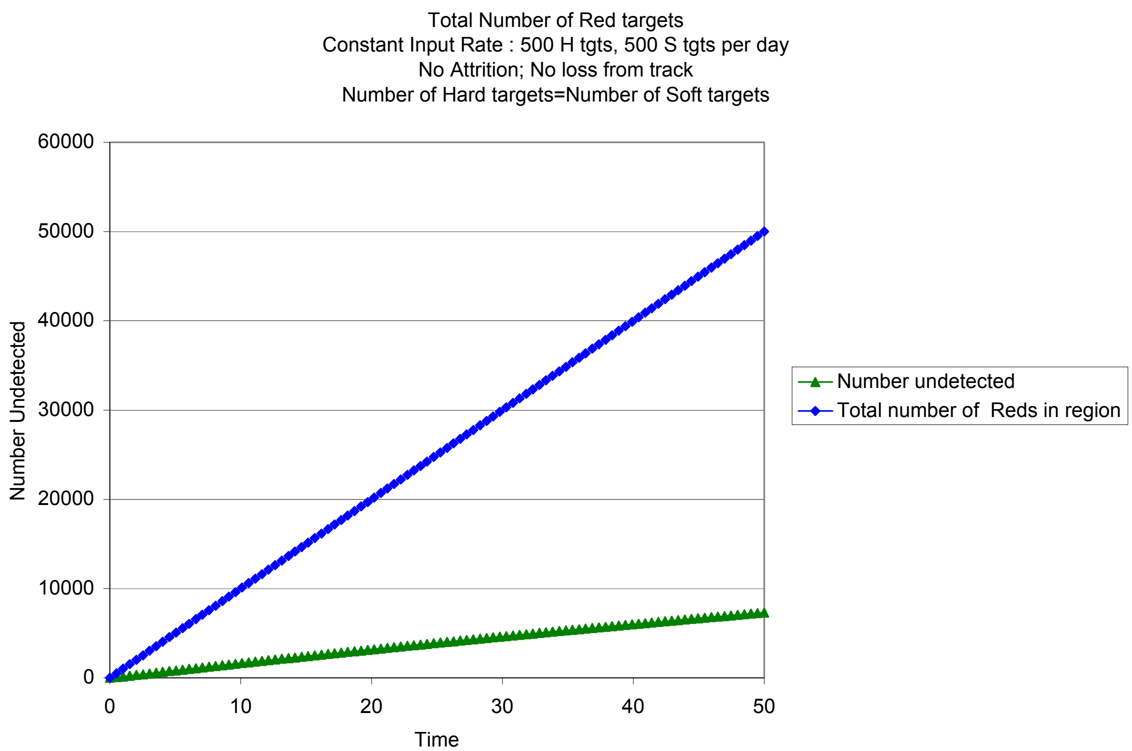


Figure 4(a)

Case Study 2: Accelerated Red Arrival Rate to a Maximum Number of Reds

In this case study Reds enter the region at an increasing rate until the number of Hard Reds that enter reaches 30,000 and the number of Soft Reds that enter reaches 10,000. The number of Hard Reds that enter the region by time t is $(100)t^2/2$ for $t \le 24.5$ and 30,000 for $t \ge 24.5$; the number of Soft Reds that enter the region by time t is $(100)t^2/2$ for $t \le 14.1$ and 10,000 for $t \ge 14.1$. Figure 5(a) displays the assumed arrival of Red targets into region $\Re$ and the number undetected for the basic sensor sweep rate $\xi = 9{,}000$.

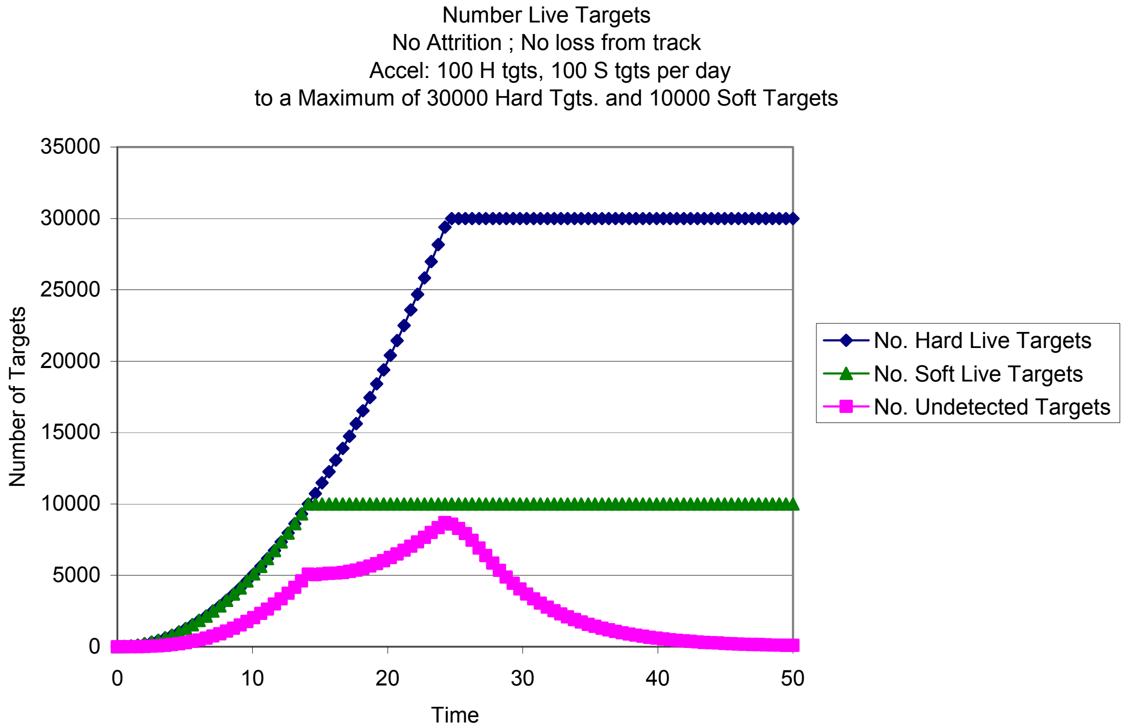

Number Live Targets
No Attrition ; No loss from track
Accel: 100 H tgts, 100 S tgts per day
to a Maximum of 30000 Hard Tgts. and 10000 Soft Targets
Number of Targets
35000
30000
25000
20000
15000
10000
5000
0
0
10
20
30
40
50
Time
No. Hard Live Targets
No. Soft Live Targets
No. Undetected Targets

Figure 5(a)

Tabled Results

The tables below and on the following page summarize various model combat outcomes of interest at $t = 25$ and $t = 50$ (days). One can see at a glance the effect of the presumed system architectures and parameters. More detailed graphical presentations are also possible. They are presented and discussed in Appendix II.

TABLE 6.1
Summary of Combat Outcomes
(Measures of Effectiveness)
Combat Duration = 25 days
Linear Input

	C=0.5; BDA=0.5		C=1.0; BDA=0.5		C=0.5; BDA=1.0		C=1.0; BDA=1.0	
	Arch. I	Arch II	Arch. I	Arch II	Arch. I	Arch II	Arch. I	Arch II
Number Undetected								
Latency= 1 hr	1.06E+04	8.87E+03	8.33E+03	5.17E+03	3.55E+03	3.51E+03	2.87E+02	2.73E+02
Latency= 0.5 hr	1.51E+03	3.66E+02	3.68E+02	2.31E+02	1.50E+02	1.29E+02	1.07E+01	6.40E+00
Number weapons expended								
Latency= 1 hr	5.23E+04	4.89E+04	5.43E+04	4.95E+04	3.92E+04	3.92E+04	3.47E+04	3.47E+04
Latency= 0.5 hr	9.05E+04	7.08E+04	8.38E+04	5.89E+04	4.70E+04	4.70E+04	3.54E+04	3.54E+04
Number Reds killed (out of 2.50E+04)								
Latency= 1 hr	1.42E+04	1.60E+04	1.65E+04	1.97E+04	2.12E+04	2.12E+04	2.46E+04	2.46E+04
Latency= 0.5 hr	2.34E+04	2.46E+04	2.46E+04	2.47E+04	2.48E+04	2.48E+04	2.50E+04	2.50E+04
Number weapons expended per target killed								
Latency= 1 hr	3.68E+00	3.04E+00	3.29E+00	2.51E+00	1.85E+00	1.85E+00	1.41E+00	1.41E+00
Latency= 0.5 hr	3.86E+00	2.88E+00	3.41E+00	2.38E+00	1.90E+00	1.90E+00	1.42E+00	1.42E+00
Number erroneous weapons expended								
Latency= 1 hr	3.27E+04	2.93E+04	1.55E+04	1.09E+04	1.93E+04	1.93E+04	0.00E+00	0.00E+00
Latency= 0.5 hr	5.66E+04	4.11E+04	2.44E+04	1.19E+04	2.32E+04	2.31E+04	0.00E+00	0.00E+00
Number Blue Aircraft Killed (out of 100)								
Latency= 1 hr	7.74E+00	6.33E+00	6.11E+00	3.60E+00	2.90E+00	2.89E+00	5.54E-01	5.54E-01
Latency= 0.5 hr	1.40E+00	6.01E-01	5.68E-01	3.93E-01	3.08E-01	3.07E-01	3.75E-02	3.25E-02
Number Red Weapons Expended								
Latency= 1 hr	2.69E+03	2.20E+03	2.12E+03	1.26E+03	1.00E+03	1.00E+03	1.89E+02	1.88E+02
Latency= 0.5 hr	4.81E+02	2.00E+02	1.93E+02	1.31E+02	1.05E+02	1.04E+02	1.28E+01	1.11E+01

Parametric Values
Constant Input Rate: 500 H tgts, 500 S tgts per day
pH|H=pH|S=pS|S=0.7; pS|H=0.1; 100 shooters
Time until tracked target loss=1 hr.
100 aircraft; 1 hr. on station; 11 hrs. off station

TABLE 6.2
Summary of Combat Outcomes
(Measures of Effectiveness)
Combat Duration = 50 days
Linear Input

	C=0.5; BDA=0.5		C=1.0; BDA=0.5		C=0.5; BDA=1.0		C=1.0; BDA=1.0	
	Arch. I	Arch II	Arch. I	Arch II	Arch. I	Arch II	Arch. I	Arch II
Number Undetected								
Latency= 1 hr	2.19E+04	1.88E+04	1.72E+04	1.17E+04	7.47E+03	7.42E+03	2.88E+02	2.74E+02
Latency= 0.5 hr	2.79E+03	3.67E+02	3.68E+02	2.32E+02	1.51E+02	1.29E+02	1.07E+01	6.41E+00
Number weapons expended								
Latency= 1 hr	1.03E+05	9.75E+04	1.07E+05	1.00E+05	7.82E+04	7.83E+04	7.00E+04	7.00E+04
Latency= 0.5 hr	1.83E+05	1.43E+05	1.69E+05	1.19E+05	9.45E+04	9.45E+04	7.08E+04	7.08E+04
Number Reds killed (out of 5.00E+04)								
Latency= 1 hr	2.79E+04	3.10E+04	3.26E+04	3.81E+04	4.22E+04	4.23E+04	4.96E+04	4.96E+04
Latency= 0.5 hr	4.71E+04	4.96E+04	4.96E+04	4.97E+04	4.98E+04	4.98E+04	5.00E+04	5.00E+04
Number weapons expended per target killed								
Latency= 1 hr	3.69E+00	3.14E+00	3.28E+00	2.62E+00	1.85E+00	1.85E+00	1.41E+00	1.41E+00
Latency= 0.5 hr	3.89E+00	2.88E+00	3.40E+00	2.39E+00	1.90E+00	1.90E+00	1.42E+00	1.42E+00
Number erroneous weapons expended								
Latency= 1 hr	6.43E+04	5.88E+04	3.07E+04	2.33E+04	3.85E+04	3.85E+04	0.00E+00	0.00E+00
Latency= 0.5 hr	1.14E+05	8.30E+04	4.94E+04	2.40E+04	4.65E+04	4.65E+04	0.00E+00	0.00E+00
Number Blue Aircraft Killed (out of 100)								
Latency= 1 hr	3.26E+01	2.75E+01	2.57E+01	1.66E+01	1.16E+01	1.16E+01	1.16E+00	1.15E+00
Latency= 0.5 hr	4.78E+00	1.23E+00	1.19E+00	8.07E-01	6.42E-01	6.38E-01	7.70E-02	6.70E-02
Number Red Weapons Expended								
Latency= 1 hr	1.09E+04	9.17E+03	8.56E+03	5.52E+03	3.88E+03	3.88E+03	3.85E+02	3.85E+02
Latency= 0.5 hr	1.59E+03	4.10E+02	3.96E+02	2.69E+02	2.14E+02	2.13E+02	2.56E+01	2.23E+01

Parametric Values
Constant Input Rate: 500 H tgts, 500 S tgts per day
pH|H=pH|S=pS|S=0.7; pS|H=0.1; 100 shooters
Time until tracked target is lost=1 hr.
100 aircraft; 1 hr. on station; 11 hrs. off station

Discussion of Tables 6.1 and 6.2

If target classification and BDA are perfect, then both Architectures result in the attrition of most of the Red targets for both shooter latencies. When the shooter latency time is 0.5 hour there is little difference in attrition for the two Architectures if either the probability of correct target classification or the probability of correct BDA is equal to 1. If the shooter latency is 1 hour, then more targets are attrited when one increases the probability of correct BDA from 0.5 to 1, than if the probability of correct classification is increased from 0.5 to 1.

More weapons are expended for the case of the probability of correct classification equal to 1 and probability of correct BDA is 0.5, than the case in which the probability of correct classification is equal to 0.5 and the probability of correct BDA is 1.

If the shooter latency is 1 hour and the probability of correct BDA and probability of correct classification is 0.5, then decreasing the shooter latency to 0.5 hour results in greater attrition than increasing either probability by itself. However, this improvement is at the expense of greater weapon expenditure per target killed.

TABLE 6.3
Summary of Combat Outcomes
(Measures of Effectiveness)
Combat Duration = 25 days
For accelerated input to a max of 3E4 Hard targets and 1E4 Soft targets

	C=0.5; BDA=0.5		C=1.0; BDA=0.5		C=0.5; BDA=1.0		C=1.0; BDA=1.0	
	Arch. I	**Arch II**	**Arch. I**	**Arch II**	**Arch. I**	**Arch II**	**Arch. I**	**Arch II**
Number Undetected								
Latency= 1 hr	2.37E+04	2.86E+04	2.37E+04	2.04E+04	2.13E+04	2.12E+04	1.43E+04	1.43E+04
Latency= 0.5 hr	1.89E+04	1.68E+04	1.37E+04	9.88E+03	1.13E+04	1.12E+04	4.63E+02	4.27E+02
Number weapons expended								
Latency= 1 hr	4.09E+04	4.30E+04	5.16E+04	4.45E+04	3.55E+04	3.55E+04	3.57E+04	3.57E+04
Latency= 0.5 hr	8.06E+04	7.16E+04	8.54E+04	7.31E+04	5.81E+04	5.82E+04	5.60E+04	5.60E+04
Number Reds killed (out of 4.00E+04)								
Latency= 1 hr	1.59E+04	1.14E+04	1.60E+04	1.93E+04	1.81E+04	1.81E+04	2.52E+04	2.52E+04
Latency= 0.5 hr	2.09E+04	2.29E+04	2.61E+04	2.99E+04	2.82E+04	2.82E+04	3.93E+04	3.93E+04
Number weapons expended per target killed								
Latency= 1 hr	2.57E+00	3.77E+00	3.23E+00	2.31E+00	1.96E+00	1.96E+00	1.46E+00	1.46E+00
Latency= 0.5 hr	3.86E+00	3.12E+00	3.27E+00	2.44E+00	2.06E+00	2.06E+00	1.42E+00	1.42E+00
Number erroneous weapons expended								
Latency= 1 hr	2.23E+04	2.65E+04	1.44E+04	8.59E+03	1.70E+04	1.70E+04	0.00E+00	0.00E+00
Latency= 0.5 hr	4.93E+04	4.13E+04	2.42E+04	1.53E+04	2.82E+04	2.82E+04	0.00E+00	0.00E+00
Number Blue Aircraft Killed (out of 100)								
Latency= 1 hr	1.28E+01	1.62E+01	1.33E+01	1.10E+01	1.14E+01	1.14E+01	8.16E+00	8.15E+00
Latency= 0.5 hr	9.92E+00	8.42E+00	7.58E+00	5.30E+00	5.67E+00	5.67E+00	1.34E+00	1.33E+00
Number Red Weapons Expended								
Latency= 1 hr	4.52E+03	5.68E+03	4.66E+03	3.87E+03	4.02E+03	4.02E+03	2.87E+03	3.00E+02
Latency= 0.5 hr	3.50E+03	2.98E+03	2.67E+03	1.87E+03	2.01E+03	2.01E+03	4.53E+02	1.37E+01

Parametric Values

Accelerated Input Rate: 100 H tgts, 100 S tgts per day
to maximum of 30K H tgts and 10K S tgts
pH|H=pS|S=0.7; pS|H=0.1; 100 shooters
Time until tracked target is lost=1 hr.
100 aircraft; 1 hr. on station; 11 hrs. off station

TABLE 6.4
Summary of Combat Outcomes
(Measures of Effectiveness)
Combat Duration = 50 days
For accelerated input to a max of 3E4 Hard targets and 1E4 Soft targets

	C=0.5; BDA=0.5		C=1.0; BDA=0.5		C=0.5; BDA=1.0		C=1.0; BDA=1.0	
	Arch. I	Arch II	Arch. I	Arch II	Arch. I	Arch II	Arch. I	Arch II
Number Undetected								
Latency= 1 hr	1.67E+04	1.90E+04	1.62E+04	1.22E+04	1.34E+04	1.34E+04	1.42E+03	1.39E+03
Latency= 0.5 hr	8.91E+03	6.38E+03	3.23E+02	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
Number weapons expended								
Latency= 1 hr	6.16E+04	8.30E+04	7.80E+04	6.96E+04	5.38E+04	5.38E+04	5.44E+04	5.44E+04
Latency= 0.5 hr	1.25E+05	1.14E+05	1.35E+05	1.02E+05	8.66E+04	8.66E+04	5.70E+04	5.70E+04
Number Reds killed (out of 4.00E+04)								
Latency= 1 hr	2.30E+04	2.10E+04	2.37E+04	2.77E+04	2.63E+04	2.63E+04	3.83E+04	3.83E+04
Latency= 0.5 hr	3.09E+04	3.35E+04	3.96E+04	4.00E+04	4.00E+04	4.00E+04	4.00E+04	4.00E+04
Number weapons expended per target killed								
Latency= 1 hr	2.68E+00	3.95E+00	3.29E+00	2.51E+00	2.05E+00	2.05E+00	1.42E+00	1.42E+00
Latency= 0.5 hr	4.04E+00	3.40E+00	3.41E+00	2.55E+00	2.17E+00	2.17E+00	1.43E+00	1.43E+00
Number erroneous weapons expended								
Latency= 1 hr	3.36E+04	4.73E+04	2.24E+04	1.16E+04	2.58E+04	2.58E+04	0.00E+00	0.00E+00
Latency= 0.5 hr	7.67E+04	6.68E+04	3.94E+04	2.24E+04	4.20E+04	4.20E+04	0.00E+00	0.00E+00
Number Blue Aircraft Killed (out of 100)								
Latency= 1 hr	4.40E+01	5.19E+01	4.38E+01	3.60E+01	3.83E+01	3.84E+01	2.06E+01	2.06E+01
Latency= 0.5 hr	3.11E+01	2.63E+01	1.83E+01	1.03E+01	1.31E+01	1.31E+01	1.37E+00	1.36E+00
Number Red Weapons Expended								
Latency= 1 hr	1.47E+04	1.73E+04	1.46E+04	1.20E+04	1.28E+04	1.28E+04	6.86E+03	6.86E+03
Latency= 0.5 hr	1.04E+04	8.76E+03	6.09E+03	3.42E+03	4.36E+03	4.36E+03	4.57E+02	4.55E+02

Parametric Values
Accelerated Input Rate: 100 H tgts, 100 S tgts per day
to maximum of 30K H tgts and 10K S tgts
pH|H=pS|S=0.7; pS|H=0.1; 100 shooters
Time until tracked target is lost=1 hr.
100 aircraft; 1 hr. on station; 11 hrs. off station

Discussion of Tables 6.3 and 6.4

If the shooter latency is 1 hour and the probabilities of correct classification and correct BDA are all 0.5, then decreasing the shooter latency to 0.5 hour results in greater attrition than increasing either the probabilities of correct classification or probabilities of correct BDA to 1. However, decreasing the latency to 0.5 hour results in a larger number of weapons required for every target killed, while increasing the probabilities of correct classification (respectively the probabilities of correct BDA) to 1 results in fewer weapons required for every target killed. Increasing the probabilities of correct BDA results in the smallest number of weapons per target killed.

If the probabilities of correct BDA are equal to 1, both Architectures result in the same number of kills and number of weapons per kill. The number of targets killed and number of weapons expended per kill are smallest when the probabilities of correct BDA are equal to 1.

Tables 6.5 – 6.7 present further results concerning the scenario with linear input.

TABLE 6.5
Summary of Combat Outcomes
Linear Input
C=0.5, BDA=0.5

	Sensor Rate=2500 Combat Duration=25 days 2.5E+04 Reds entered region		Sensor Rate=1250 Combat Duration=25 days 2.5E+04 Reds entered region		Sensor Rate=2500 Combat Duration=50 days 5.0E+04 Reds entered region		Sensor Rate=1250 Combat Duration=50 days 5.0E+04 Reds entered region	
	Arch 1	Arch 2	Arch 1	Arch 2	Arch 1	Arch 2	Arch 1	Arch 2
Number weapons expended								
Latency=1 hr (erroneous)	5.23E+04 (3.27E+04)	4.89E+04 (2.93E+04)	5.08E+04 (3.16E+04)	5.09E+04 (3.16E+04)	10.3E+04 (6.43E+04)	9.75E+04 (5.88E+04)	10.0E+04 (6.21E+04)	10.1E+04 (6.26E+04)
Latency=0.5 hr (erroneous)	9.05E+04 (5.66E+04)	6.93E+04 (4.03E+04)	8.00E+04 (4.98E+04)	8.59E+04 (5.27E+04)	18.3E+04 (11.4E+04)	14.3E+04 (8.30E+04)	16.2E+04 (10.1E+04)	17.5E+04 (10.8E+04)
Number Reds killed								
Latency=1 hr	1.42E+04	1.60E+04	1.54E+04	1.44E+04	2.79E+04	3.10E+04	3.00E+04	2.81E+04
Latency=0.5 hr	2.34E+04	2.41E+04	2.13E+04	2.38E+04	4.71E+04	4.96E+04	4.30E+04	4.76E+04
Number weapons expended per target killed								
Latency=1 hr	3.68E+00	3.06E+00	3.29E+00	3.53E+00	3.69E+00	3.08E+00	3.33E+00	3.59E+00
Latency=0.5 hr	3.87E+00	2.87E+00	3.76E+00	3.60E+00	3.89E+00	2.88E+00	3.77E+00	3.67E+00

Discussion of Table 6.5

Table 6.5 presents results for total number of weapons expended, the number of erroneous weapons expended, the number of Reds killed, and the number of weapons expended per target killed for two shooter latencies (1 hour and 0.5 hour) and two regional sensor rates (ξ=5000 and ξ=2500). The probabilities of correct classification and the probabilities of correct BDA are 0.5.

First consider a shooter latency of 1 hour. Note that Architecture 1 has more kills when the regional sensor rate is smaller (2500) than for the larger (5000) sensor rate. Recall that for Architecture 1 the regional sensor is providing all of the BDA. Dead targets that are (mis)classified as live by the regional sensor are returned to the targeting list for further action. The larger sensor rate is increasing the rate at which these misclassified dead targets are returned to the targeting list. These misclassified dead targets decrease the amount of effort the shooters devotes to the live targets on the list. Additionally, for Architecture 1 the slower sensor rate results in fewer expended weapons per target killed than the higher sensor rate. Recall that Architecture 2 has additional immediate BDA capability. In this case the larger regional sensor rate results in the removal of more misclassified dead targets resulting from the immediate BDA from the targeting list than that resulting from the slower regional sensor rate. Thus, Architecture 2 has more Red targets killed for the faster regional sensor rate than for the slower regional sensor rate. Further, the faster sensor rate results in fewer weapons expended per target killed than the slower sensor rate.

Next consider a shooter latency of 0.5 hour. The shorter shooter latency allows Architecture 1 to prosecute more misclassified dead targets in addition to the live targets on the targeting list. Thus, the number of targets killed for Architecture 1 is greater for the larger sensor rate than the smaller sensor rate. However, the number of weapons expended per killed target is also greater for the faster sensor rate than the slower sensor rate. The smaller shooter latency results in fewer misclassified dead targets being removed by the regional sensor from the target list in Architecture 2. However, Architecture 2 still has a slightly larger number of targets killed for the larger sensor rate. Further, the faster sensor rate results in fewer weapons expended per target killed than the slower sensor rate.

For Architecture 1, the larger sensor rate results in more weapons expended per target killed for a shooter latency of 0.5 hour as compared to that for a shooter latency of 1 hour. For Architecture 2, the larger sensor rate results in fewer weapons expended per target killed for a shooter latency of 0.5 hour as compared to that for a shooter latency of 1 hour. For Architecture 1, the smaller sensor rate results in more weapons expended per target killed for a shooter latency of 0.5 hour as compared to that for a shooter latency of 1 hour. For Architecture 2, the smaller sensor rate results in more weapons expended per target killed for a shooter latency of 0.5 hour as compared to that for a shooter latency of 1 hour.

TABLE 6.6
Summary of Combat Outcomes
Linear Input
C=1.0, BDA=0.5

	Sensor Rate=2500 Combat Duration=25 days 2.5E+04 Reds entered region		Sensor Rate=1250 Combat Duration=25 days 2.5E+04 Reds entered region		Sensor Rate=2500 Combat Duration=50 days 5.0E+04 Reds entered region		Sensor Rate=1250 Combat Duration=50 days 5.0E+04 Reds entered region	
	Arch 1	Arch 2	Arch 1	Arch 2	Arch 1	Arch 2	Arch 1	Arch 2
Number weapons expended								
Latency=1 hr (erroneous)	5.43E+04 (1.55E+04)	4.95E+04 (1.09E+04)	5.28E+04 (2.95E+04)	5.25E+04 (1.42E+04)	10.7E+04 (3.07E+04)	10.0E+04 (2.33E+04)	10.4E+04 (2.92E+04)	10.5E+04 (2.87E+04)
Latency=0.5 hr (erroneous)	8.38E+04 (2.44E+04)	5.89E+04 (1.19E+04)	8.26E+04 (2.40E+04)	7.06E+04 (1.78E+04)	16.9E+04 (4.94E+04)	11.9E+04 (2.40E+04)	16.8E+04 (4.90E+04)	14.2E+04 (3.56E+04)
Number Reds killed								
Latency=1 hr	1.65E+04	1.97E+04	1.65E+04	1.70E+04	3.26E+04	3.81E+04	3.26E+04	3.34E+04
Latency=0.5 hr	2.46E+04	2.47E+04	2.44E+04	2.47E+04	4.96E+04	4.97E+04	4.93E+04	4.96E+04
Number weapons expended per target killed								
Latency=1 hr	3.29E+00	2.50E+00	3.20E+00	3.09E+00	3.28E+00	2.62E+00	3.19E+00	3.14E+00
Latency=0.5 hr	3.41E+00	2.38E+00	3.39E+00	2.86E+00	3.41E+00	2.39E+00	3.41E+00	2.86E+00

Discussion of Table 6.6

Table 6.6 presents results for the case in which there is perfect target classification and the probabilities of correct BDA are 0.5. Perfect target classification implies that the most efficient weapon is expended on each target type, resulting in fewer weapons expended per target killed than if target classification were not perfect.

For Architecture 1, the regional sensor is performing all the BDA. If the regional sensor misclassifies a dead target as live, the dead target is placed on the targeting list for further prosecution. A larger regional sensor rate hastens the placement of misclassified dead targets to the targeting list. For a shooter latency of 1 hour, the number of targets killed using Architecture 1 for both regional sensor rates are equal. However, the number of weapons expended per target killed is larger for the higher regional sensor rate than for the smaller sensor rate. For the shorter shooter latency of 0.5 hours, the number of targets killed is slightly less for the higher regional sensor rate than the lower sensor rate. The number of weapons expended per target killed is less for the smaller sensor rate than the larger sensor rate.

Architecture 2 has immediate BDA capability in addition to the regional sensor BDA. The regional sensor can remove dead targets that have been misclassified by the immediate BDA capability from the targeting list. For the shooter latency of 1 hour, the number of targets killed using Architecture 2 is greater for the higher regional sensor rate than the lower one. The number of weapons expended per killed target for the higher sensor rate is less than that for the smaller sensor rate. For the shooter latency of 0.5 hours, the number of targets killed is about the same for both regional sensor rates. However, the number of weapons expended per target killed is less for the larger regional sensor rate than the smaller sensor rate.

For Architecture 1, the shorter shooter latency results in more weapons expended per target killed than the larger shooter latency. For Architecture 2, the shorter shooter latency results in more weapons expended per target killed than the larger shooter latency.

TABLE 6.7
Summary of Combat Outcomes
Linear Input
C=0.5, BDA=1.0

	Sensor Rate=2500 Combat Duration=25 days 2.5E+04 Reds entered region		Sensor Rate=1250 Combat Duration=25 days 2.5E+04 Reds entered region		Sensor Rate=2500 Combat Duration=50 days 5.0E+04 Reds entered region		Sensor Rate=1250 Combat Duration=50 days 5.0E+04 Reds entered region	
	Arch 1	Arch 2	Arch 1	Arch 2	Arch 1	Arch 2	Arch 1	Arch 2
Number weapons expended								
Latency=1 hr (erroneous)	3.92E+04 (1.93E+04)	3.92E+04 (1.93E+04)	3.88E+04 (1.91E+04)	3.90E+04 (1.91E+04)	7.82E+04 (3.85E+04)	7.83E+04 (3.85E+04)	7.74E+04 (3.81E+04)	7.77E+04 (3.82E+04)
Latency=0.5 hr (erroneous)	4.70E+04 (2.32E+04)	4.70E+04 (2.31E+04)	4.70E+04 (2.31E+04)	4.70E+04 (2.31E+04)	9.45E+04 (4.65E+04)	9.45E+04 (4.65E+04)	9.45E+04 (4.65E+04)	9.45E+04 (4.65E+04)
Number Reds killed								
Latency=1 hr	2.12E+04	2.12E+04	2.09E+04	2.10E+04	4.22E+04	4.23E+04	4.18E+04	4.20E+04
Latency=0.5 hr	2.48E+04	2.48E+04	2.48E+04	2.48E+04	4.98E+04	4.98E+04	4.98E+04	4.98E+04
Number weapons expended per target killed								
Latency=1 hr	1.85E+00	1.85E+00	1.86E+00	1.86E+00	1.85E+00	1.85E+00	1.85E+00	1.85E+00
Latency=0.5 hr	1.90E+00	1.90E+00	1.90E+00	1.90E+00	1.90E+00	1.89E+00	1.90E+00	1.90E+00

Discussion of Table 6.7

Table 6.7 presents results for the case in which the BDA is perfect and the probabilities of correct target classification are 0.5. For the particular scenario being considered, the main effect of target misclassification is to reduce the probability of killing a hard target when the hard target is misclassified as soft. Since the BDA is perfect, the perfect BDA will inform the shooters that a missed target is alive and that target will be re-targeted. The difference between Architectures 1 and 2 in this case is when the BDA information becomes available. Architecture 1 has all of its BDA done by the regional sensors. As a result, the outcome of the BDA is delayed. The additional immediate BDA capability of Architecture 2 implies that the result of BDA is known instantaneously. Thus, missed targets are always available for re-targeting with Architecture 2. Missed targets are available for re-targeting after a delay in Architecture 1.

7. Summary and Conclusions

This paper describes and exploits a low-resolution, high-level modeling methodology for the study of the effectiveness of a system of systems. The methodology facilitates quick turnaround efficient exploration of sensitivities of measures of Blue combat success to realistically imperfect Blue ISR and IW capabilities: limited and imperfect sensor surveillance and reconnaissance, particularly BDA, and finite, hence saturable, communications and weapons delivery capability. The model explicitly, but purposefully skeletally, represents aircraft sorties, fires, sensor/shooter latencies, target losses, imperfect target type classification, imperfect weapon assignment, and BDA. The model is deterministic/expected value or "fluid" in style, although it represents time-dependent, non-linear stochastic features such as system saturability by means of an analytical device.

The methodology is illustrated by the comparison of two sensor-shooter architectures in a strike scenario. In Architecture 1, battle damage assessment is performed by regional sensors, and is delayed. In Architecture 2, an additional capability of immediate BDA is added to that of regional sensors. Model experimentation suggests the following conclusions.

- Decreasing the Blue sensor/shooter latency can result in greater Red attrition than increasing the probabilities of correct target classification and BDA. However, the improvement is at the expense of greater weapon expenditure per target killed.
- One possible effect of erroneous BDA is to return dead targets to the shooter targeting list. These dead targets require shooter resources, and so decrease the amount of effort the shooters devote to live targets. Thus, dead targets that are returned to the targeting list not only result in wasted weapon expenditure but also in additional delay to prosecute live targets. The re-attacks may actually place manned aircraft at extra risk; see Table 6.2. A dead target returned to the targeting list through erroneous BDA has a similar effect to that of a decoy. If the shooter

latency is long, then increasing the probability of correct BDA can result in more targets being killed than increasing the probability of correct target classification.

- It is important to balance the resources of the sensors and the shooters. If the BDA is imperfect, then myopically increasing the regional sensor rate can undesirably increase the number of dead targets that are erroneously returned to the targeting list. If the shooter latency is large, then resources erroneously allocated to the dead targets on the targeting list result in more live targets being lost before a weapon reaches them. Thus, for Architecture 1, the effect of the increase in sensor detection rate can be a decrease in the number of targets killed during a time period; see Table 6.5.
- The effect of the additional immediate BDA capability of Architecture 2 depends on the regional sensor rate. The regional sensor can remove dead targets from the targeting list that have been placed there by erroneous instantaneous BDA. Thus, increasing the regional sensor rate can increase the number of targets killed for Architecture 2.

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APPENDIX I

In this appendix all mathematical development is recorded.

A1. State Variables

Below are listed the *state variables* that describe the system at any time t:

$R_{ij}(t)$ = Number of *detected*, hence targetable, *alive* Reds that are of type j in (sub)region i at time t. Note: j can denote a false target or decoy (which includes those previously targeted, missed and incorrectly left for dead).

$\overline{R}_{ij}(t)$ = Number of *alive* Reds *undetected* of type j that are in (sub)region i at time t. Note: these include some of those previously targeted.

$M_{ij}(t)$ = Number of *dead* Red targets of type j in (sub)region i unclassified at time t (required in Architecture Model 1; BDA classifies these (possibly incorrectly)).

$D_{ij}(t)$ = Number of *dead* Red targets of type j in (sub)region i that are still counted as alive at time t.

$K_{ij}(t)$ = Number of Red targets of type j in (sub)region i killed during $(0, t]$.

$S_i(\bullet,t)$ = Sensor effort "looking" at (sub)region i at time t. The $\bullet$ indicates that this may be modified by Blue in accordance with perceived Red state conditions.

$B_i(\bullet,t)$ = Number of Blue (missile) Shooters prosecuting targets in (sub)region i at time t. Again, the dot $\bullet$ signifies possible feedback-driven or scripted shooter allocations.

$A_i(\bullet,t)$ = Number of aircraft in region i at time t. Same comment concerning $\bullet$.

$\overline{A}(t)$ = Number of live aircraft at time t.

$W_{ij}(t)$ = Number of Weapons expended by *non-aircraft assets* in (sub)region i at targets perceived to be of type j during $[0, t]$.

$W_{ij}(a;t)$ = Number of Weapons expended by *aircraft assets* in (sub)region i at targets perceived to be of type j during $[0, t]$.

$W_{ij}(d;t)$ = Number of Weapons expended all assets in (sub)region i at dead targets of type j perceived to be alive during $[0, t]$.

$R_{i\bullet}(t) = \sum_j R_{ij}(t)$; total number of detected Red targets in (sub)region i at time t.

$\overline{R}_{i\bullet}(t) = \sum_j \overline{R}_{ij}(t)$; total number of undetected Red targets in (sub)region i at time t.

$D_{i\bullet}(t) = \sum_j D_{ij}(t)$; total number of dead Red targets in (sub)region i that are perceived to be alive at time t.

Next there follows a listing of current parameters that enter the equations for state development. These are initially constants, but may be made *time*, or *state* dependent; in the latter case dynamic adaptation can be modeled.

A2. Parameters

The following parameters are required to specify the dynamic evolution of the process of states:

ξ_i = (sweep) rate at which Red targets are processed by one sensor system in (sub)region i; this can be generalized to account for different sensor types, but has not been.

ν_{ij} = rate at which Red targets of type j are lost by the sensor system in (sub)region i.

α_{ij} = rate at which Red targets of type j in (sub)region i are active (shoot) and are detected by Blue.

$\alpha_{ij}(a)$ = probability a Red target of type j is detected while firing at a Blue aircraft

$p_{ikj}(a)$ = probability a target of type j in (sub)region i is killed by a Blue aircraft when the Blue aircraft is targeting it as a type k target.

$p_j(R,B)$ = probability a Red of type j kills a Blue aircraft.

$c_{ijk}(a;a)$ = probability an alive Red target of type j in (sub)region i is classified as a type k when it is being prosecuted by a Blue aircraft.

$c_{ijk}(d;a)$ = probability a dead Red target of type j in (sub)region i that is perceived to be alive is classified as a type k when it is being prosecuted by a Blue aircraft.

μ_i = rate at which a detected Red target in (sub)region i is prosecuted by a shooter.

w_{ik} = fraction of a Blue shooter's effort that is used to prosecute targets of type k in (sub)region i.

$w_{ik}(a)$ = fraction of a Blue aircraft's effort in (sub)region i that is used to prosecute targets of type k.

$c_{ijk}(a)$ = probability an alive Red target of type j in (sub)region i is classified as a type k when it is being prosecuted by a Blue shooter.

$c_{ijk}(d)$ = probability a dead Red target of type j in (sub)region i that is not perceived to be dead is classified as a type k when it is being prosecuted by a Blue shooter.

$m_{ij}(d)$ = probability a dead target of type j in (sub)region i is declared dead by the field sensor system.

$m_{ij}(a)$ = probability a live target of type j in (sub)region i is declared live by the field sensor system.

p_{ikj} = probability a target of type j in (sub)region i is killed by a Blue shooter when the Blue shooter is targeting it as a type k target.

$r_{ikj}(d|a)$ = probability a live target of type j in (sub)region i that has been prosecuted by Blue shooters as a type k and is still alive, but is classified as dead by the shooter sensor system; $r_{ikj}(a|a) = 1 - r_{ikj}(d|a)$.

$r_{ikj}(d|d)$ = probability a dead target of type j in (sub)region i that has been prosecuted by Blue shooters as a type k is classified as dead by the shooter sensor system; $r_{ikj}(a|d) = 1 - r_{ikj}(d|d)$.

$r_{ikj}(d|d;a)$ = probability a dead target of type j in (sub)region i that has been prosecuted by Blue aircraft as a type k is classified as dead by the aircraft; $r_{ikj}(a|d;a) = 1 - r_{ikj}(d|d;a)$.

$r_{ikj}(d|a;a)$ = probability a live target of type j in (sub)region i that has been prosecuted by Blue aircraft as a type k is still alive, but is classified as dead by the aircraft; $r_{ikj}(a|a;a) = 1 - r_{ikj}(d|a;a)$.

$1/\beta_i(a)$ = mean on-station time for an aircraft in (sub)region i.

$1/\rho_i(a)$ = mean time between an aircraft's departure from region i and its next arrival.

$\delta_{ij}(R,B)$ = rate of fire at a Blue aircraft by a Red of type j in (sub)region i.

$\delta_i(B,R)$ = rate of fire at a Red target by a Blue aircraft in (sub)region i.

λ_{ij} = arrival rate of targets of type j into (sub)region i from outside the region.

$\gamma_{i\ell j}$ = rate at which a target of type j in (sub)region i moves into (sub)region ℓ.

$H(x)$ = amount of effort each server expends when there are x customers waiting or being served; $H(x) = x/(1 + x)$. See Filipiak (1988), and Gaver, Jacobs, and Youngren (1998).

A.3 Architecture I (Deferred BDA): Model Defender Dynamics

$$\underbrace{\frac{d\overline{R}_{ij}(t)}{dt}}_{\substack{\text{Rate of change of}\\\text{undetected Red,}\\\text{type } j\text{, in } \Re_i}} = \underbrace{\lambda_{ij}(\bullet)}_{\substack{\text{Migration}\\\text{from exterior}\\\text{into } \Re_i}}$$

$$\underbrace{-\xi_i H\left(\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)+M_{i\bullet}(t)\right)\frac{S_i(t)\overline{R}_{ij}(t)m_{ij}(a)}{\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)+M_{i\bullet}(t)}}_{\text{Sensor detection of undetected Red, type } j\text{, in } \Re_i\text{. Detected unit declared alive.}}$$

$$+\underbrace{\nu_{ij}R_{ij}(t)}_{\substack{\text{Detected}\\\text{Reds lost}}}\underbrace{-\sum_{\ell\neq i}\gamma_{i\ell j}\overline{R}_{ij}(t)}_{\substack{\text{Migration}\\\text{undetected}\\\text{out of } \Re_i}}+\underbrace{\sum_{\ell\neq i}\gamma_{\ell ij}\left(R_{\ell j}(t)+\overline{R}_{\ell j}(t)\right)}_{\substack{\text{Migration into } \Re_i\\\text{(detected becomes undetected if}\\\text{leave } \Re_\ell \text{ in which detected)}}}$$

$$\underbrace{+\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k\frac{w_{ik}R_{ij}(t)c_{ijk}(a)\left(1-p_{ikj}\right)}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition": an alive Red target is missed}}$$

$$-\underbrace{\alpha_{ij}\overline{R}_{ij}(t)}_{\substack{\text{Detection from Red action}\\\text{(e.g., SCUD fired)}}}\quad-\underbrace{\alpha_{ij}(a)\delta_{ij}(R,B)I\left(A_i(t)>0\right)\overline{R}_{ij}(t)}_{\text{Detection from Red firing at aircraft}}$$

$$\underbrace{+\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{R_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(a;a)\left(1-p_{ikj}(a)\right)r_{ikj}(d|a;a)}_{\text{"Attrition": a miss by Blue aircraft and misclassification as dead}}$$

$$\underbrace{\frac{dR_{ij}(t)}{dt}}_{\substack{\text{Rate of change}\\ \text{of detected Red,}\\ \text{type } j\text{, in } \Re_i}} = \underbrace{\xi_i H\left(\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)+M_{i\bullet}(t)\right)\frac{S_i(t)\overline{R}_{ij}(t)m_{ij}(a)}{\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)+M_{i\bullet}(t)}}_{\text{Sensor detection Undetected Red, type } j}$$

$$-\underbrace{\nu_{ij}R_{ij}(t)}_{\text{Detected lost}} - \underbrace{\sum_{\ell\neq i}\gamma_{i\ell j}R_{ij}(t)}_{\text{Migration out of } \Re_i}$$

$$\text{Missiles}\begin{cases} -\underbrace{\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}R_{ij}(t)c_{ijk}(a)}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition" Red detected }\left(\text{dead disappear, alive join } \overline{R}_{ij}(t)\right)} + \underbrace{\alpha_{ij}\overline{R}_{ij}(t)}_{\substack{\text{Detection from}\\ \text{Red action}}} \end{cases}$$

$$\text{Aircraft}\begin{cases} +\underbrace{\alpha_{ij}(a)\delta_{ij}(R,B)I\left(A_i(t)>0\right)\overline{R}_{ij}(t)}_{\text{Detection from Red firing at aircraft}} \\ -\underbrace{\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{R_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(a;a)\left[p_{ikj}(a)+\left(1-p_{ikj}(a)\right)r_{ikj}(d|a;a)\right]}_{\text{"Attrition": a shot by Blue aircraft}} \end{cases}$$

$$\underbrace{\frac{dM_{ij}(t)}{dt}}_{\substack{\text{Rate change}\\ \text{unclassif. dead}\\ \text{Red, type } j\text{, in } \Re_i}} = \underbrace{\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}R_{ij}(t)c_{ijk}(a)p_{ikj}}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition": Alive Red, type } j\text{, killed in } \Re_i}$$

$$-\underbrace{\xi_i H\left(\overline{R}_{i\bullet}+R_{i\bullet}+D_{i\bullet}+M_{i\bullet}\right)S_i(t)\frac{M_{ij}(t)}{\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)+M_{i\bullet}(t)}}_{\substack{\text{Detection of unclassified dead Red}\\ \text{(with prob. } m_{ij}(d) \text{ declared dead and ignored after;}\\ \text{with prob. } 1-m_{ij}(d) \text{ declared alive and retargeted)}}}$$

$$+\underbrace{\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}D_{ij}(t)c_{ijk}(d)}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\substack{\text{Shot at ("Attrition" of) Red dead,}\\ \text{misclassif. as alive, and retargeted in } \Re_i}}$$

$$\underbrace{\frac{dD_{ij}(t)}{dt}}_{\substack{\text{Rate change dead}\\ \text{Red, type } j\text{, class.}\\ \text{as live in } \Re_i}} = \underbrace{\xi_i H\left(\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)+M_{i\bullet}(t)\right)S_i(t)\frac{M_{ij}(t)\left(1-m_{ij}(d)\right)}{\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)+M_{i\bullet}(t)}}_{\substack{\text{Sensor detection of unclassified dead Red,}\\ \text{type } j\text{, that classifies alive; } \Re_i}}$$

$$\text{Missile}\left\{-\underbrace{\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}D_{ij}(t)c_{ijk}(d)}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition" of dead, classified as alive Red,type } j\text{, in } \Re_i}\right.$$

$$\underbrace{-\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{D_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(d;a)r_{ijk}(d|d;a)}_{\text{Blue aircraft retargeting a dead Red and class. as dead}}$$

$$\underbrace{+\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{R_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(a;a)p_{ikj}(a)r_{ijk}(a|d;a)}_{\text{"Attrition": a hit by Blue aircraft and target is misclassified as alive}}$$

$$\frac{dA_i(t)}{dt} = \underbrace{-\beta_i(a)A_i(t)}_{\substack{\text{endurance}\\ \text{on site}}} \underbrace{-\sum_j \delta_{ij}(R,B)\left(R_{ij}(t)+\overline{R}_{ij}(t)\right)p_j(R,B)I\left(A_i(t)>0\right)}_{\text{Attrition of Blue by Red}}$$

$$\underbrace{+\rho_i(a)I\left(\left(\overline{A}(t)-A_i(t)\right)>0\right)\frac{R_{i\bullet}(t)}{1+R_{i\bullet}(t)}\left[\overline{A}(t)-A_i(t)\right]}_{\text{Arrival rate of aircraft to } \Re_i}$$

$$\frac{d\overline{A}(t)}{dt} = \underbrace{-\sum_i\sum_j \delta_{ij}(R,B)\left(R_{ij}(t)+\overline{R}_{ij}(t)\right)p_j(R,B)I\left(A_i(t)>0\right)}_{\text{Attrition of Blue aircraft by Red}}$$

$$\frac{dW_{ij}(t)}{dt} = \mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)w_{ij}\sum_k \frac{R_{ik}(t)c_{ikj}(a)+D_{ik}(t)c_{ikj}(d)}{R_{i\bullet}(t)+D_{i\bullet}(t)}$$

$$\frac{dW_{ij}(t;a)}{dt} = \delta_i(B,R)A_i(t)w_{ij}(a)\sum_k \frac{R_{ik}(t)c_{ikj}(a;a)+D_{ik}(t)c_{ikj}(d;a)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}$$

$$\underbrace{\frac{dW_{ij}(t;d)}{dt}}_{\substack{\text{Rate change wpns fired} \\ \text{at dead} \\ \text{Red, type } j\text{, class.} \\ \text{as live in } \Re_i}} = \underbrace{\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}D_{ij}(t)c_{ijk}(d)}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition" of dead, classified as alive Red, type } j\text{, in } \Re_i}$$

$$+ \underbrace{\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{D_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(d;a)}_{\text{Blue aircraft retargeting a dead Red and classified as dead}}$$

$$\underbrace{\frac{dK_{ij}(t)}{dt}}_{\substack{\text{Rate of change} \\ \text{of dead Red,} \\ \text{type } j\text{, in } \Re_i}} = \underbrace{\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}R_{ij}(t)c_{ijk}(a)p_{ikj}}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition" Red detected (dead disappear, alive join } \bar{R}_{ij}(t))}$$

$$+ \underbrace{\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{R_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(a;a)p_{ikj}(a)}_{\text{"Attrition" by Blue aircraft}}$$

A4. Architecture II (Immediate BDA): Model Defender Dynamics

$$\underbrace{\frac{d\overline{R}_{ij}(t)}{dt}}_{\substack{\text{Rate of change of}\\ \text{undetected Red,}\\ \text{type } j, \text{ in } \Re_i}} = \underbrace{\lambda_{ij}(\bullet)}_{\substack{\text{Migration}\\ \text{into } \Re_i}} + \underbrace{\sum_{\ell \neq i} \gamma_{\ell ij} \overline{R}_{\ell j}(t)}_{\substack{\text{Migration undet.}\\ \text{Red into } \Re_i}}$$

$$\underbrace{-\xi_i H\left(\overline{R}_{i\bullet}(t) + R_{i\bullet}(t) + D_{i\bullet}(t)\right) S_i(t) \frac{\overline{R}_{ij}(t) m_{ij}(a)}{\overline{R}_{i\bullet}(t) + R_{i\bullet}(t) + D_{i\bullet}(t)}}_{\text{Sensor detection undetected Red, type } j, \text{ in } \Re_i}$$

$$+ \underbrace{\nu_{ij} R_{ij}(t)}_{\text{Detected lost}} \underbrace{- \sum_{\ell \neq i} \gamma_{i\ell j} \overline{R}_{ij}(t)}_{\substack{\text{Migration}\\ \text{out of } \Re_i}} + \underbrace{\sum_{\ell \neq i} \gamma_{\ell ij} R_{\ell j}(t)}_{\substack{\text{Migration detected into } \Re_i\text{; becomes}\\ \text{undetected upon transit}}}$$

$$\underbrace{+\mu_i H\left(R_{i\bullet}(t) + D_{i\bullet}(t)\right) B_i(t) \sum_k \frac{w_{ik} R_{ij}(t) c_{ijk}(a)\left(1 - p_{ikj}\right) r_{ikj}(d|a)}{R_{i\bullet}(t) + D_{i\bullet}(t)}}_{\text{"Attrition": a miss; alive classif. dead}}$$

$$- \underbrace{\alpha_{ij} \overline{R}_{ij}(t)}_{\text{Detection from Red action}} - \underbrace{\alpha_{ij}(a) \delta_{ij}(R,B) I\left(A_i(t) > 0\right) \overline{R}_{ij}(t)}_{\text{Detection from Red firing at aircraft}}$$

$$\underbrace{+\delta_i(B,R) A_i(t) \sum_k w_{ik}(a) \frac{R_{ij}(t)}{0.01 + R_{i\bullet}(t) + D_{i\bullet}(t)} c_{ijk}(a;a)\left(1 - p_{ikj}(a)\right) r_{ikj}(d|a;a)}_{\text{"Attrition": a miss by Blue aircraft}}$$

Multi-Region, Multitype Targets

$$\underbrace{\frac{dR_{ij}(t)}{dt}}_{\substack{\text{Rate increase}\\\text{detected Red,}\\\text{type } j\text{, in } \Re_i}} = \underbrace{\xi_i H\left(\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)\right)S_i(t)\frac{\overline{R}_{ij}(t)m_{ij}(a)}{\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{Sensor detection of Red type } j \text{ in } \Re_i}$$

$$-\underbrace{\nu_{ij}R_{ij}(t)}_{\text{Lost/undetected}} - \underbrace{\sum_{\ell\neq i}\gamma_{i\ell j}R_{ij}(t)}_{\text{Migration out of } \Re_i}$$

$$\underbrace{-\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}R_{ij}(t)c_{ijk}(a)\left[p_{ikj}+(1-p_{ikj})r_{ikj}(d|a)\right]}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition" by Shooter (missiles/gunnery)}}$$

$$+\underbrace{\alpha_{ij}\overline{R}_{ij}(t)}_{\substack{\text{Detection from Red action}\\\text{(e.g. SCUD shot)}}} + \underbrace{\alpha_{ij}(a)\delta_{ij}(R,B)I\left(A_i(t)>0\right)\overline{R}_{ij}(t)}_{\text{Detection from Red firing at aircraft}}$$

$$\underbrace{-\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{R_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(a;a)\left[p_{ikj}(a)+(1-p_{ikj}(a))r_{ikj}(d|a;a)\right]}_{\text{"Attrition": a shot by Blue aircraft}}$$

$$\underbrace{\frac{dD_{ij}(t)}{dt}}_{\substack{\text{Rate increase}\\\text{Red dead}\\\text{misclassified}\\\text{as alive}}} = \underbrace{-\xi_i H\left(\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)\right)S_i(t)\frac{D_{ij}(t)m_{ij}(d)}{\overline{R}_{i\bullet}(t)+R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{Sensor detection; classified dead (slow BDA)}}$$

$$\underbrace{+\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}R_{ij}(t)c_{ijk}(a)p_{ikj}r_{ikj}(a|d)}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition" misclassified as dead}}$$

$$\underbrace{-\mu_i H\left(R_{i\bullet}(t)+D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}D_{ij}(t)c_{ijk}(d)r_{ikj}(d|d)}{R_{i\bullet}(t)+D_{i\bullet}(t)}}_{\text{"Attrition" delayed classif. dead as dead}}$$

$$\underbrace{+\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{R_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(a;a)p_{ikj}(a)r_{ijk}(a|d;a)}_{\text{"Attrition": a hit by Blue aircraft}}$$

$$\underbrace{-\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{D_{ij}(t)}{0.01+R_{i\bullet}(t)+D_{i\bullet}(t)}c_{ijk}(d;a)r_{ijk}(d|d;a)}_{\text{Blue aircraft retargeting a dead Red}}$$

$$\frac{dW_{ij}(t)}{dt} = \mu_i H\left(R_{i\bullet}(t) + D_{i\bullet}(t)\right)B_i(t)w_{ij}\sum_k \frac{R_{ik}(t)c_{ikj}(a) + D_{ik}(t)c_{ikj}(d)}{R_{i\bullet}(t) + D_{i\bullet}(t)}$$

$$\frac{dW_{ij}(t;a)}{dt} = \delta_i(B,R)A_i(t)w_{ij}(a)\sum_k \frac{R_{ik}(t)c_{ikj}(a;a) + D_{ik}(t)c_{ikj}(d;a)}{0.01 + R_{i\bullet}(t) + D_{i\bullet}(t)}$$

$$\frac{dA_i(t)}{dt} = \underbrace{-\beta_i(a)A_i(t)}_{\substack{\text{endurance}\\ \text{on site}}} - \underbrace{\sum_j \delta_{ij}(R,B)\left(R_{ij}(t) + \overline{R}_{ij}(t)\right)p_j(R,B)I\left(A_i(t) > 0\right)}_{\text{Attrition of Blue by Red}}$$

$$+ \underbrace{\rho_i(a)I\left(\overline{A}(t) - A_i(t)\right)\frac{R_{i\bullet}(t)}{1 + R_{i\bullet}(t)}\left[\overline{A}(t) - A_i(t)\right]}_{\text{Arrival rate of aircraft to } \Re_i}$$

$$\frac{d\overline{A}(t)}{dt} = \underbrace{-\sum_i \sum_j \delta_{ij}(R,B)\left(R_{ij}(t) + \overline{R}_{ij}(t)\right)p_j(R,B)I\left(A_i(t) > 0\right)}_{\text{Attrition of Blue aircraft by Red}}$$

$$\underbrace{\frac{dW_{ij}(t;d)}{dt}}_{\substack{\text{Rate increase}\\ \text{of wpns fired}\\ \text{at type } j\\ \text{Red dead}\\ \text{misclassified}\\ \text{as alive}}} = \underbrace{\mu_i H\left(R_{i\bullet}(t) + D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}D_{ij}(t)c_{ijk}(d)}{R_{i\bullet}(t) + D_{i\bullet}(t)}}_{\text{"Attrition" delayed classif. dead as dead}}$$

$$+ \underbrace{\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{D_{ij}(t)}{0.01 + R_{i\bullet}(t) + D_{i\bullet}(t)}c_{ijk}(d;a)}_{\text{Blue aircraft retargeting a dead Red}}$$

$$\underbrace{\frac{dK_{ij}(t)}{dt}}_{\substack{\text{Rate increase}\\ \text{dead Red,}\\ \text{type } j\text{, in } \Re_i}} = \underbrace{\mu_i H\left(R_{i\bullet}(t) + D_{i\bullet}(t)\right)B_i(t)\sum_k \frac{w_{ik}R_{ij}(t)c_{ijk}(a)p_{ikj}}{R_{i\bullet}(t) + D_{i\bullet}(t)}}_{\text{"Attrition" by Shooter (missiles/gunnery)}}$$

$$+ \underbrace{\delta_i(B,R)A_i(t)\sum_k w_{ik}(a)\frac{R_{ij}(t)}{0.01 + R_{i\bullet}(t) + D_{i\bullet}(t)}c_{ijk}(a;a)p_{ikj}(a)}_{\text{"Attrition": a shot by Blue aircraft}}$$

APPENDIX II

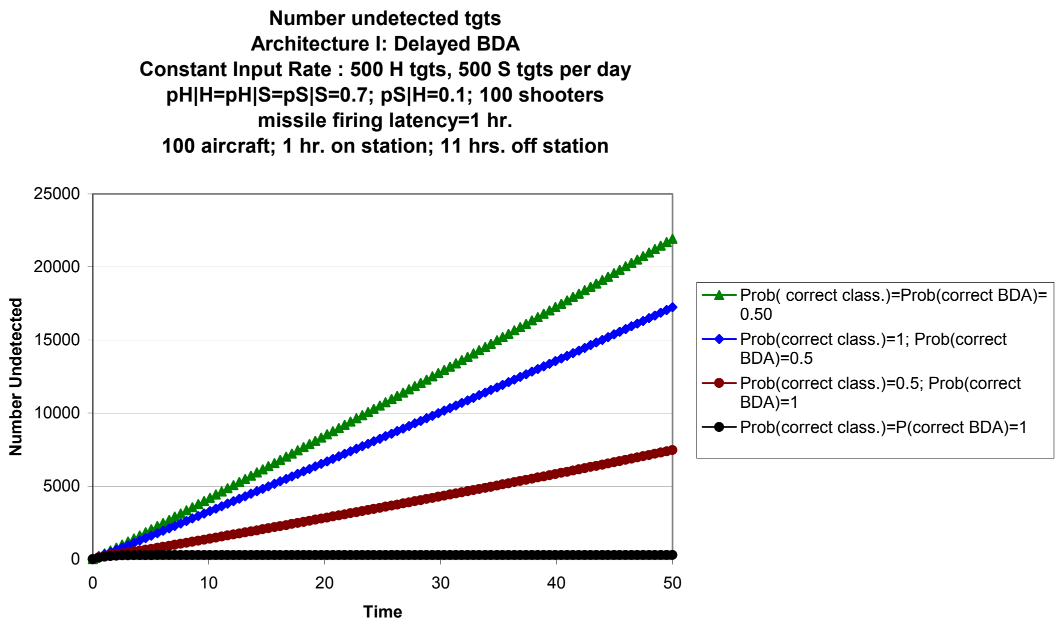

Figure 4(b)

4(b) With 100 (missile) shooters having basic latency of 1 hour, allowing attacks by 100 aircraft, and using Architecture 1, the number of undetected targets (b-1) remains about 0 if all classification and BDA capabilities are *perfect* (100%), but (b-2) continue to grow if classification and BDA skills both drop to 50%. In a tradeoff between (b-3) perfect classification and 50% BDA, or (b-4) 50% classification and perfect BDA, the latter is the more effective. But the system cannot control the number of undetected targets, which grows nearly linearly.

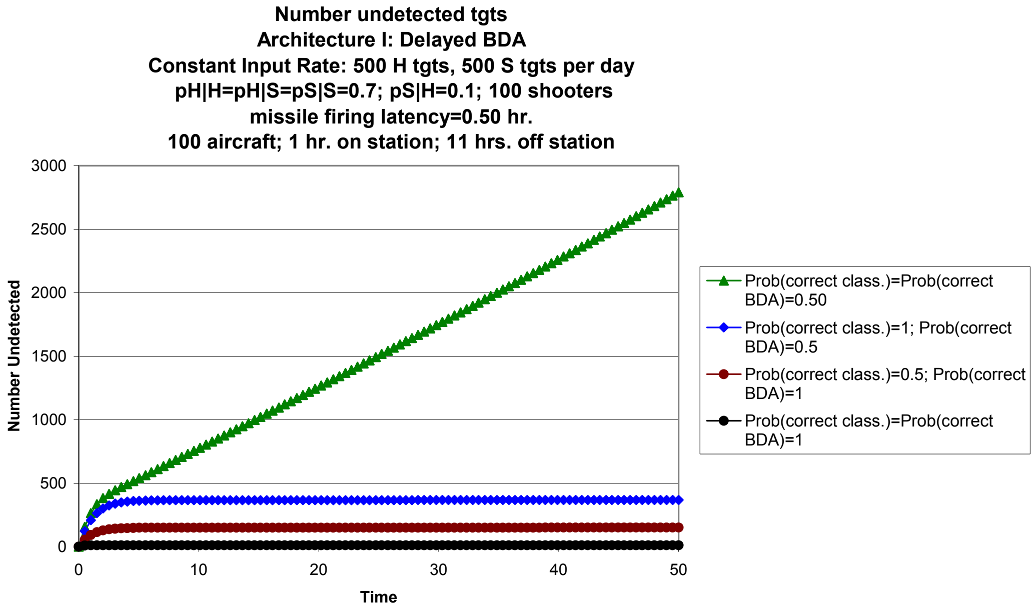

Figure 4(c)

4(c) Reducing latency to 0.5 hour from 1 hour improves/reduces the undetected target backlog considerably when classification and BDA are imperfect: (c-1) if classification and BDA skills are 50% then undetected backlog continues to increase, but at ~ 10% the rate obtained when latency is 1 hour; (c-2) if probability of correct classification is increased to 100%, but BDA is 50% then the backlog *quickly reaches a steady-state value* of around 400; if (c-3) classification skill if 50% while BDA is 100% that steady-state backlog is ~ 200, and (c-4) if both skills are 100% then there is virtually no backlog. Clearly the latency has a powerful effect on this measure of total system capability.

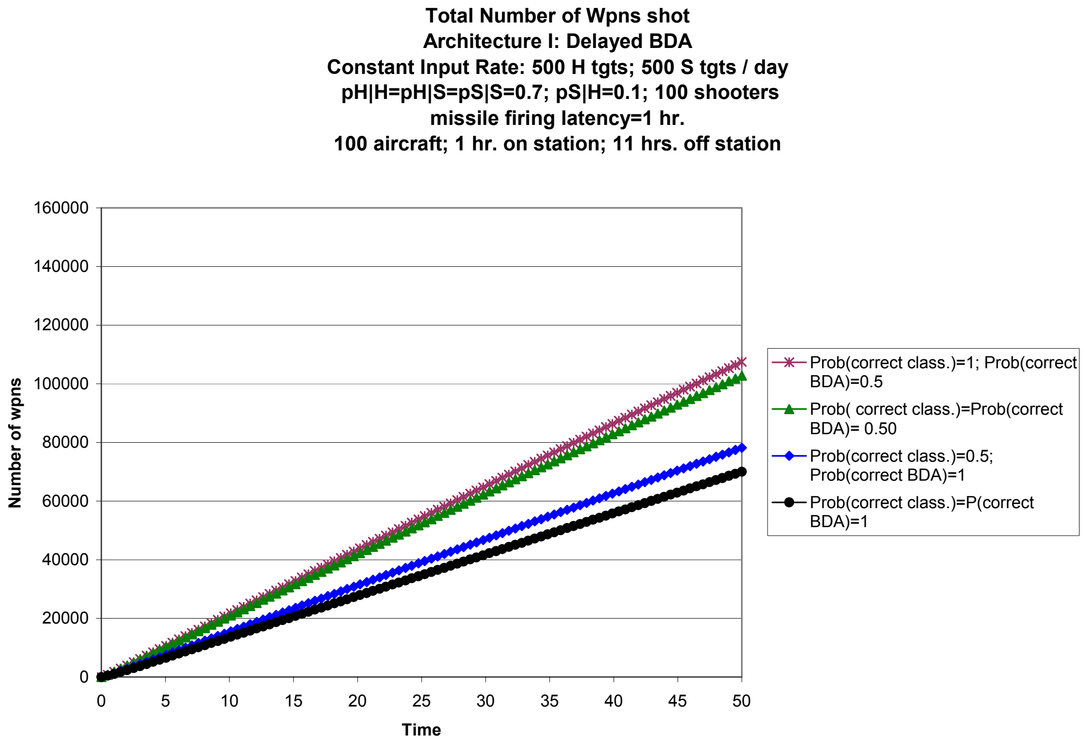

Figure 4(d)

4(d) If latency is 1 hour then in all cases the total number of weapons expended/shot increases nearly linearly in time, but at considerably different rates. (d-1) For perfect (100%) classification and BDA the number after 50 days is ~ 70,000. (d-2) For perfect (100%) classification and 50% BDA the number after 50 days is ~ 110,000, an increase of nearly 60%, while (d-3) if classification is 100% and BDA is 50% the corresponding number of shots is ~ 80,000. (d-4) If both classification and BDA are at the 50% level the number of shots is ~ 100,000. Consequently, if the system is initially operating at the classification 50%, BDA 50% level, by far the greatest payoff is increasing BDA.

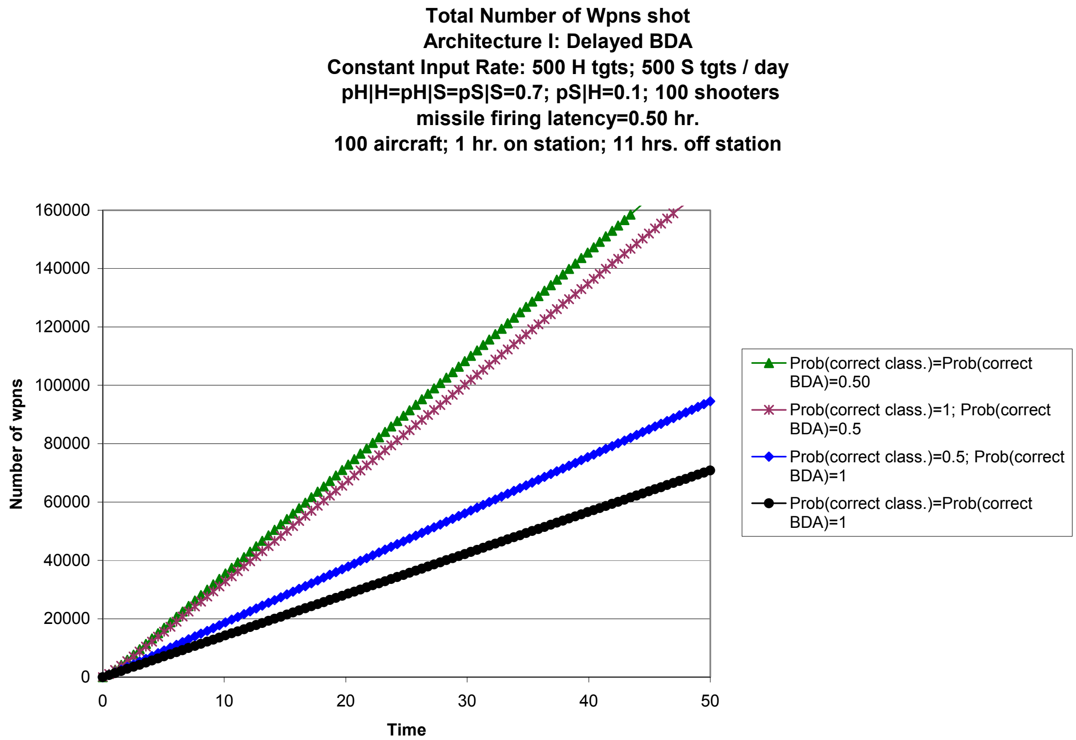

Figure 4(e)

4(e) Suppose latency is reduced to 0.5 hour. The pattern of (d) above is qualitatively followed in the same order, *but* the total number of weapons expended is much greater case by case. It remains preferable to improve BDA than to improve classification capability, starting them both at the 50% level.

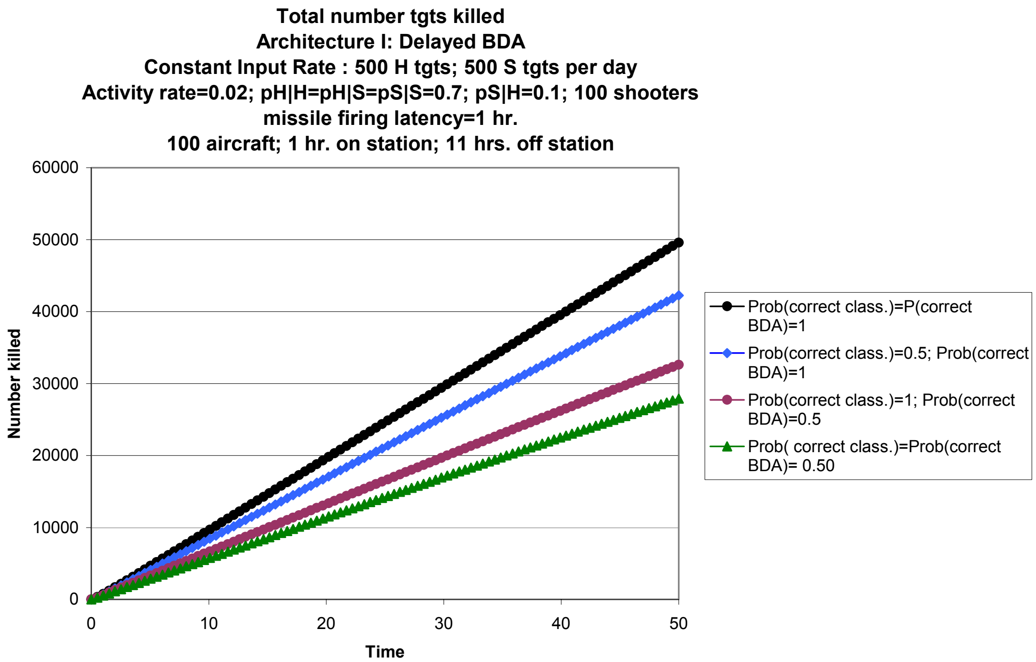

Figure 4(f)

4(f) Consider the cumulative number killed if latency is 1 hour. Here 50,000 Red targets are killed after 50 days if classification and BDA are both perfect (100%). If both are 50% capable this number drops to ~ 28,000; if classification improves to 100%, with BDA still 50% the number killed increases to ~ 33,000, while if classification remains at 50%, but BDA increases to perfection (100%), the number killed by time 50 increases to ~ 43,000. Once again the relative advantage of increasing BDA is suggested.

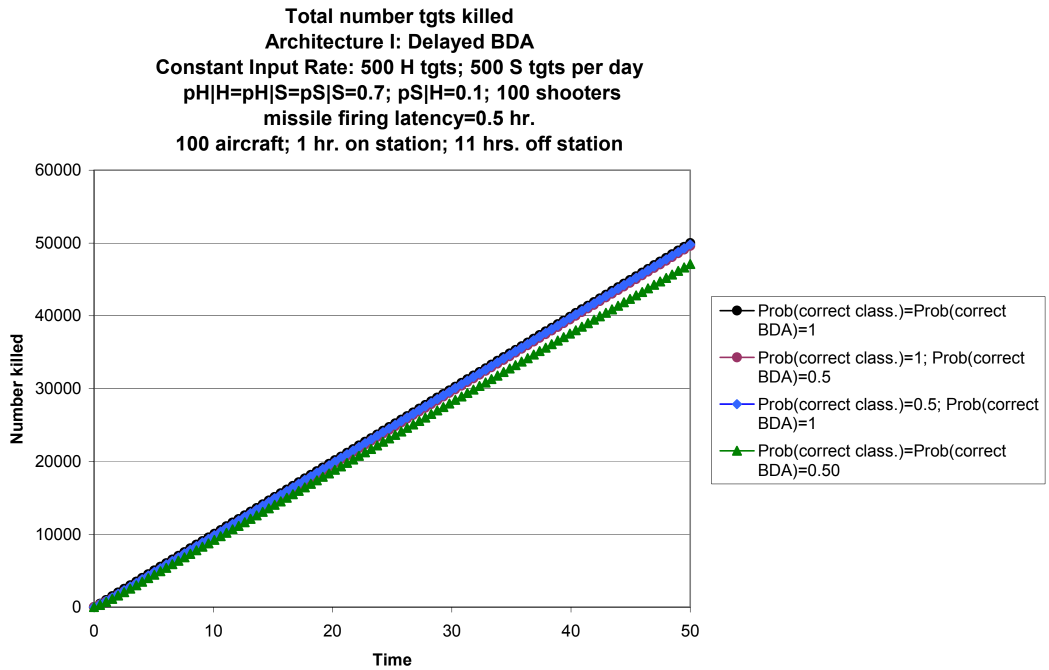

Figure 4(g)

4(g) Next let latency be reduced to 0.5 hour. Then all degrees of classification-BDA quality (examined) are essentially equivalent, achieving 50,000 kills after 50 days. Of course, the choice to improve BDA from 50% (along with 50% classification) obtains this kill rate far more economically than do the other choices.

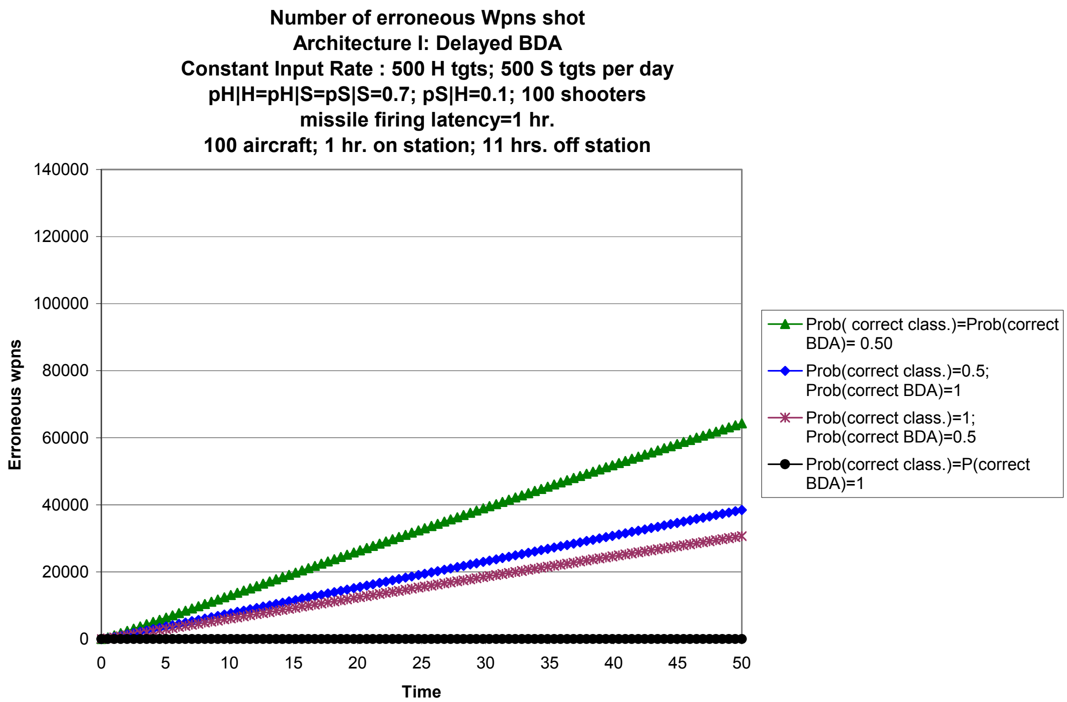

Figure 4(h)

4(h) For latency of 1 hour we see that the number of *erroneous* weapons fired increases almost linearly. *Erroneous* here includes Soft target weapons fired at Hard targets (minimal effect), Hard target weapons fired at Soft targets (maximal expense, *and* in some cases small, direct effect) and weapons fired at dead targets. (h-1) For perfect classification and BDA there are no errors; however, (h-2) for 50% capability for both classification and BDA the total number reaches ~ 60,000. (h-3) If probability of correct classification is 50%, but BDA is improved to perfect (100%), the expenditure drops to ~ 40,000, while (h-4) if classification increases to perfection (100%), and BDA remains at 50%, the erroneous shots expended fall to ~ 17,000. Thus, this measure is, in this case, actually improved more by increasing classification capability than by improving BDA.

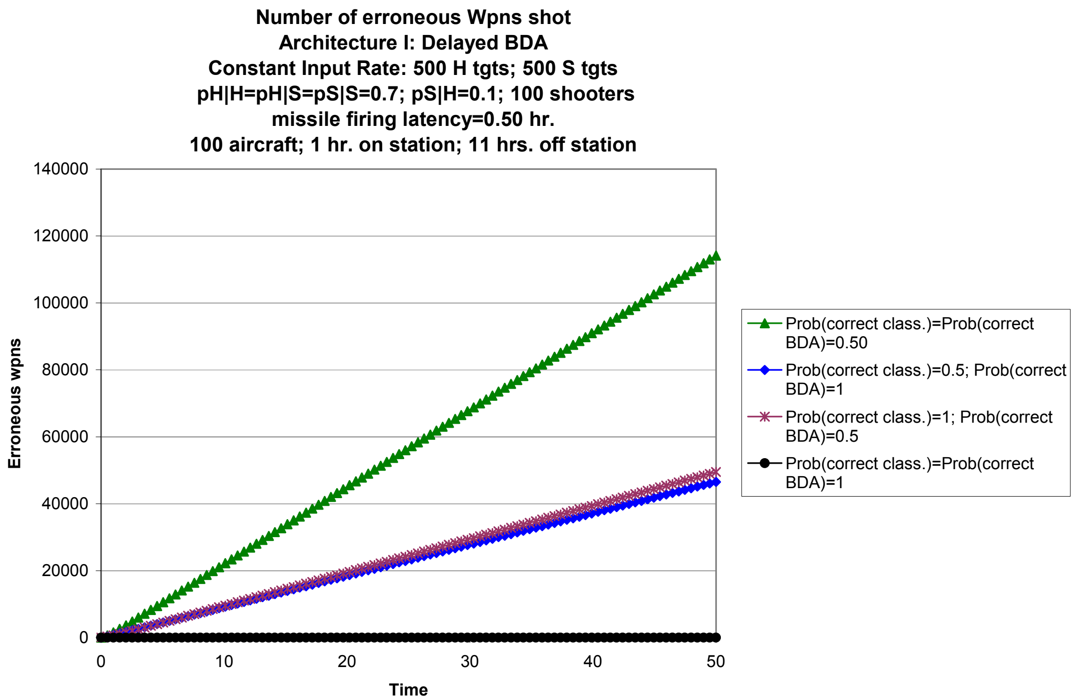

Figure 4(i)

4(i) For latency of 0.5 hour the number of erroneous weapons fired tends to increase linearly, but more rapidly, from case to case because of the increased raw shooting rate. In this case as well, it seems functionally better to improve classification than BDA.

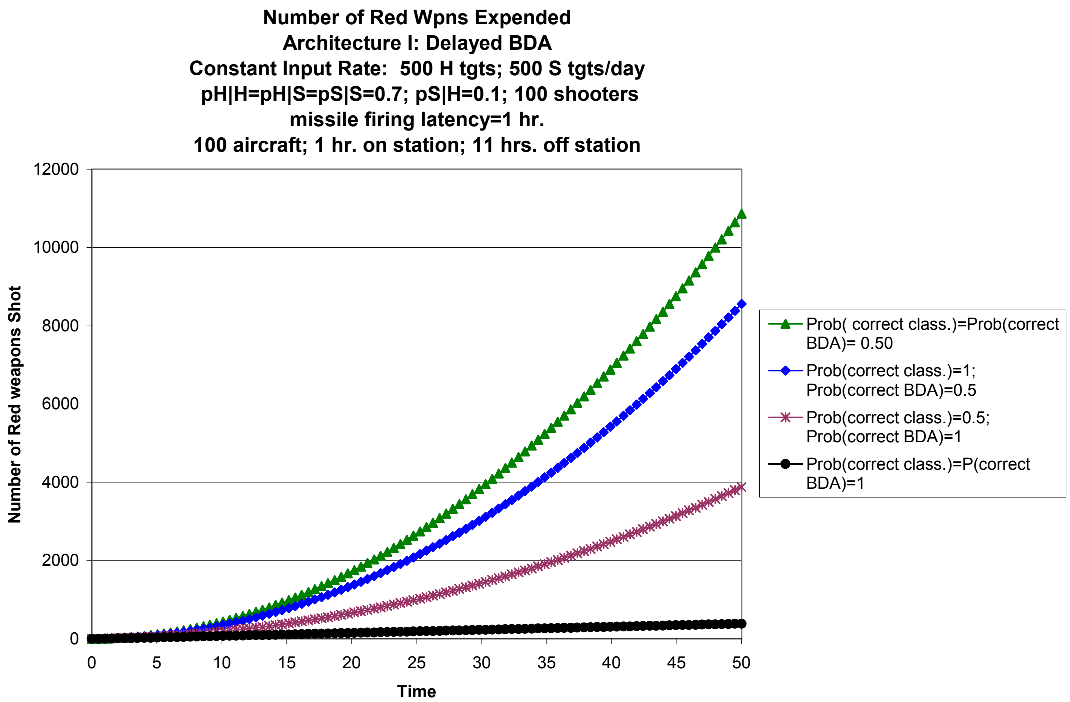

Figure 4(j)

4(j) With latency of 1 hour there is dramatic reduction in Red weapons fired by improving BDA (50% to 100%, classification at 50%) instead of improving classification (from 50% to 100%, BDA at 50%).

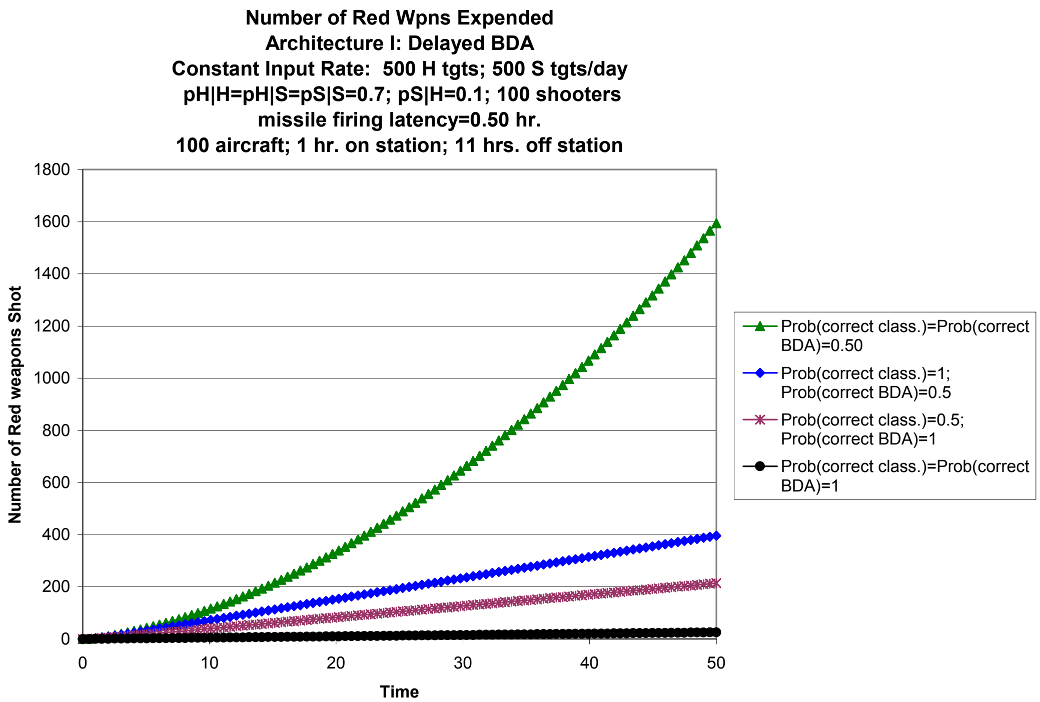

Figure 4(k)

4(k) With latency of 0.5 hour there is an effect in improving classification, but more of an effect from improving BDA quality.

Figure(s) 5: Architecture 2

The basic Figure 4(a) applies here first, showing the assumed arrival of Red targets into region $\Re$ and the number undetected.

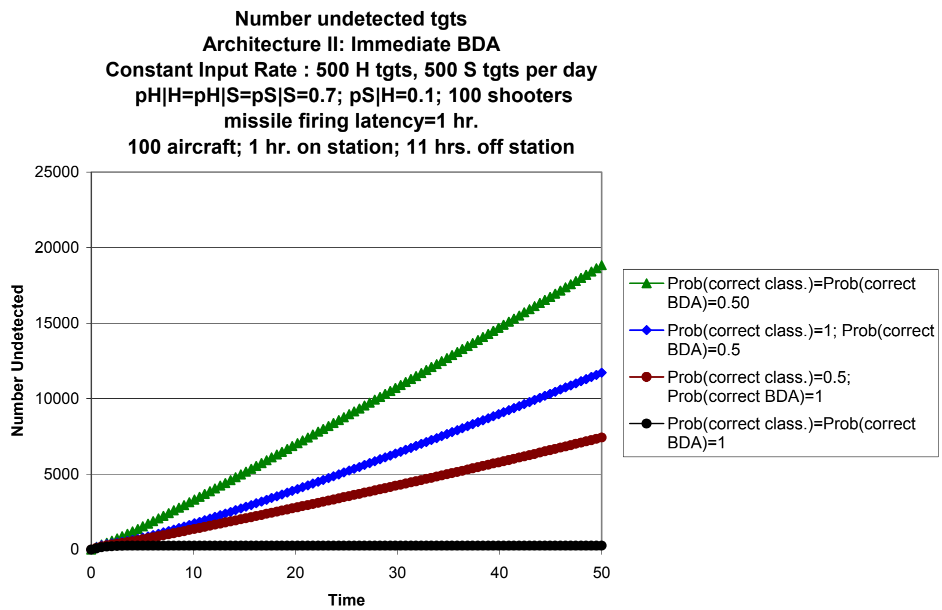

Figure 5(b)

5(b) Same general pattern as 4(b): using Architecture 2 with 1 hour latency the number of undetected targets (b-1) remains small if classification and BDA are perfect, but continue to grow (but at a slower rate than for Architecture 1) if classification and BDA skills are 50%. Again, (b-3) the number of undetected targets is reduced more by improving BDA than by improving classification.

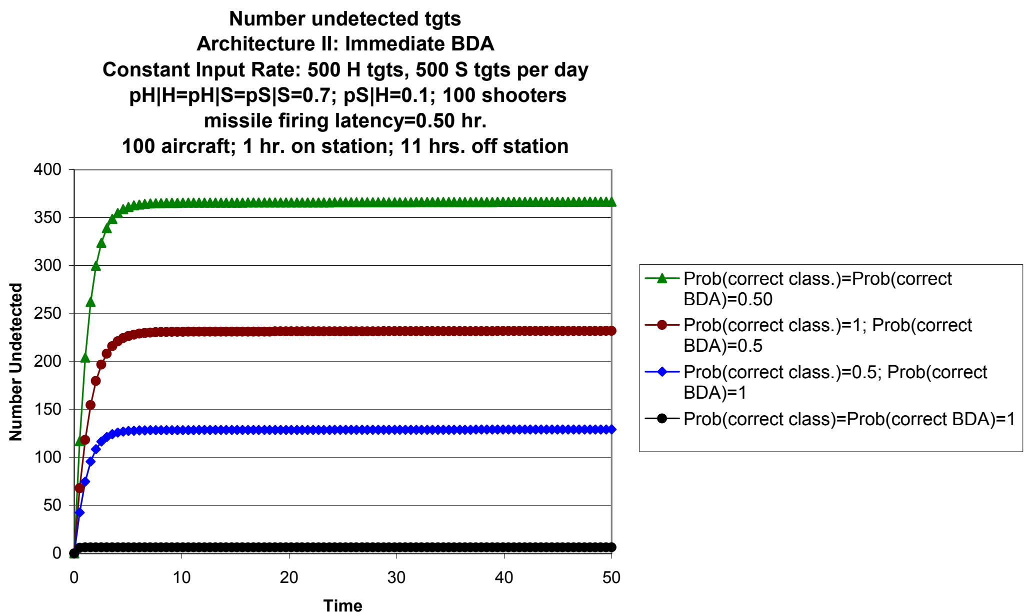

Figure 5(c)

5(c) Latency of 0.5 hour leads as before to steady-state backlogs of undetected targets: (c-1) perfect classification and BDA yield very small such backlogs; (c-2) 50% classification and BDA is essentially the same (about 370) as in Architecture 1; (c-3) improvement of BDA to perfect (100%) here reduces backlog to ~ 130, an improvement over Architecture 1.

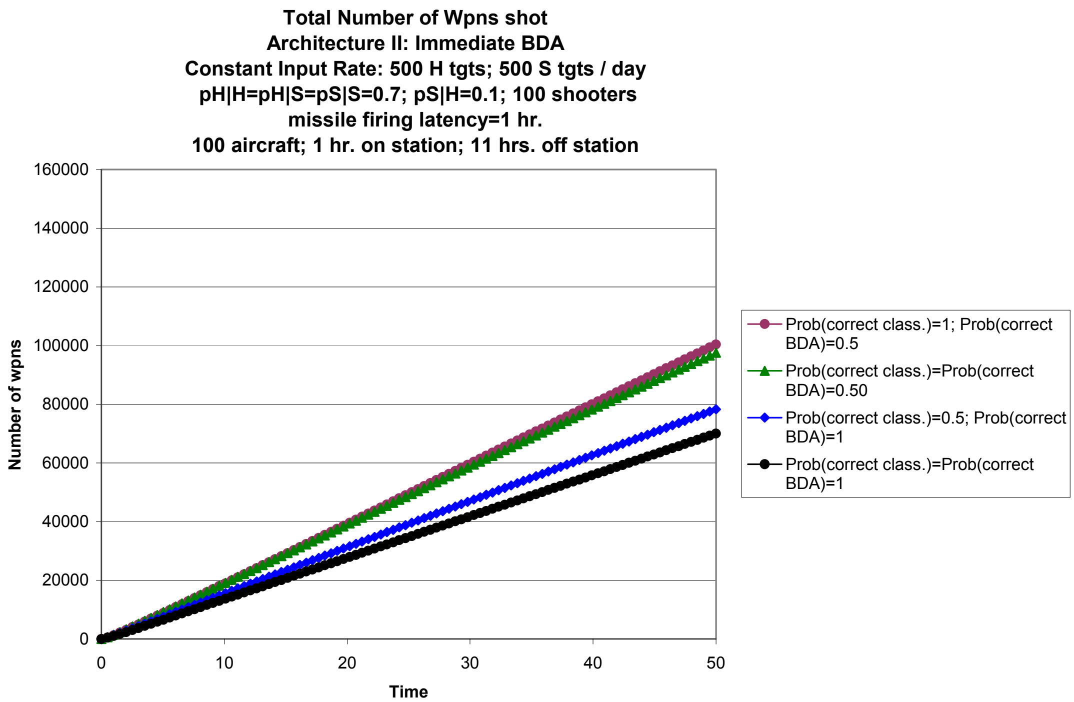

Figure 5(d)

5(d) When latency is 1 hour the total number of weapons shot is reduced quite marginally, to about 100,000 from Architecture 1 when classification and BDA are (d-1) both 50%, and (d-2) when classification is improved to 100%, BDA remaining at 100%. Improving BDA to 100% leads to improvement with classification at 50%, but gives nearly the same result as in Architecture 1.

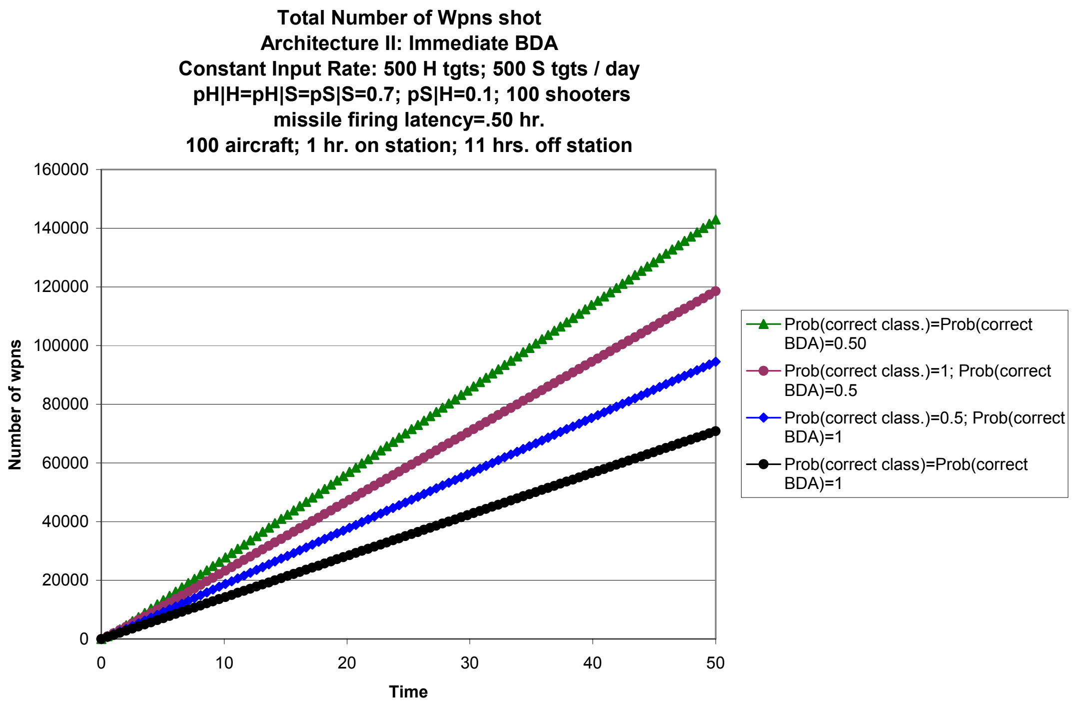

Figure 5(e)

5(e) If latency is reduced to 0.5 hour the number of weapons shot is noticeably reduced, case-by-case, from expenditures in Architecture 1.

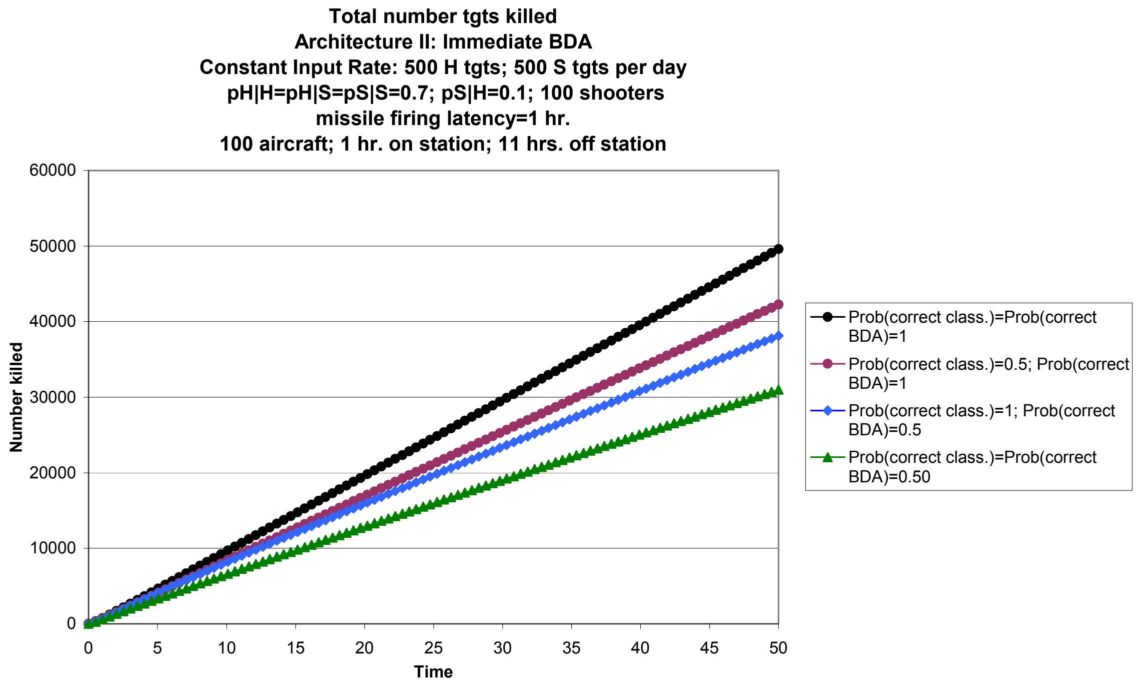

Figure 5(f)

5(f) When the latency is one hour the number of targets killed is ~ 50,000 when classification and BDA is perfect; this is the same as Architecture 1. When BDA is perfect, but classification is 50%, the result is the same as for Architecture 1. Architecture 2 results in slightly more kills than Architecture 1 in the other cases.

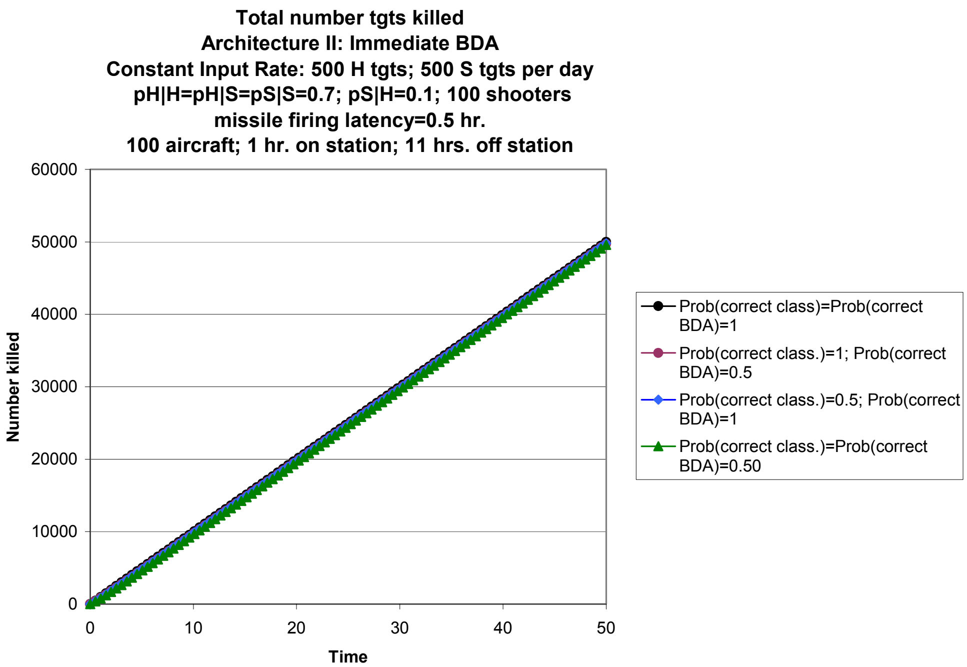

Figure 5(g)

5(g) Decreasing the latency to 0.5 hour results in all degrees of classification-BDA quality (examined) being equivalent, achieving 50,000 kills after 50 days.

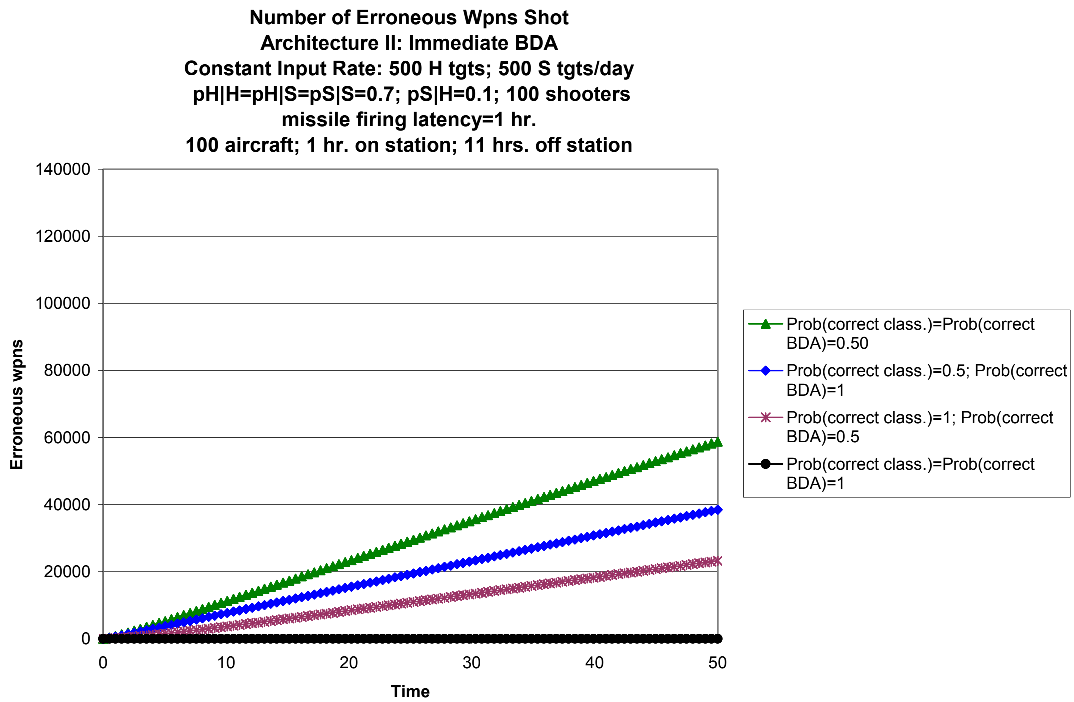

Figure 5(h)

5(h) For latency 1 hour: (h-1) the number of erroneous weapons shot increases nearly linearly to ~ 60,000 in 50 days, when classification and BDA qualities are both 50%, very little less than in Architecture 1. Increasing BDA alone to 100% drops the expenditure to ~ 40,000, but instead increasing classification alone to 100% reduces expenditures to slightly less than the corresponding Architecture 1 figure.

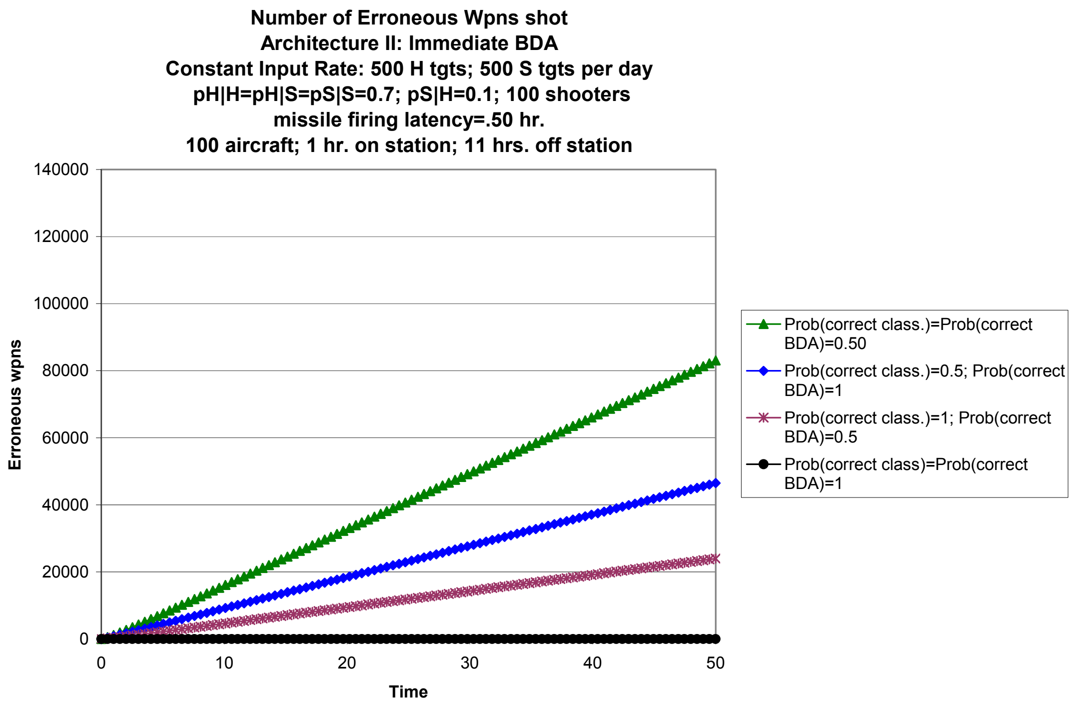

Figure 5(i)

5(i) Decreasing latency to 0.5 hour makes negligible difference in the cross-architecture erroneous weapon expenditure *except* when classification is 100%, and BDA quality 50%: then the shorter latency is associated with close to a 50% decrease in erroneous shots.

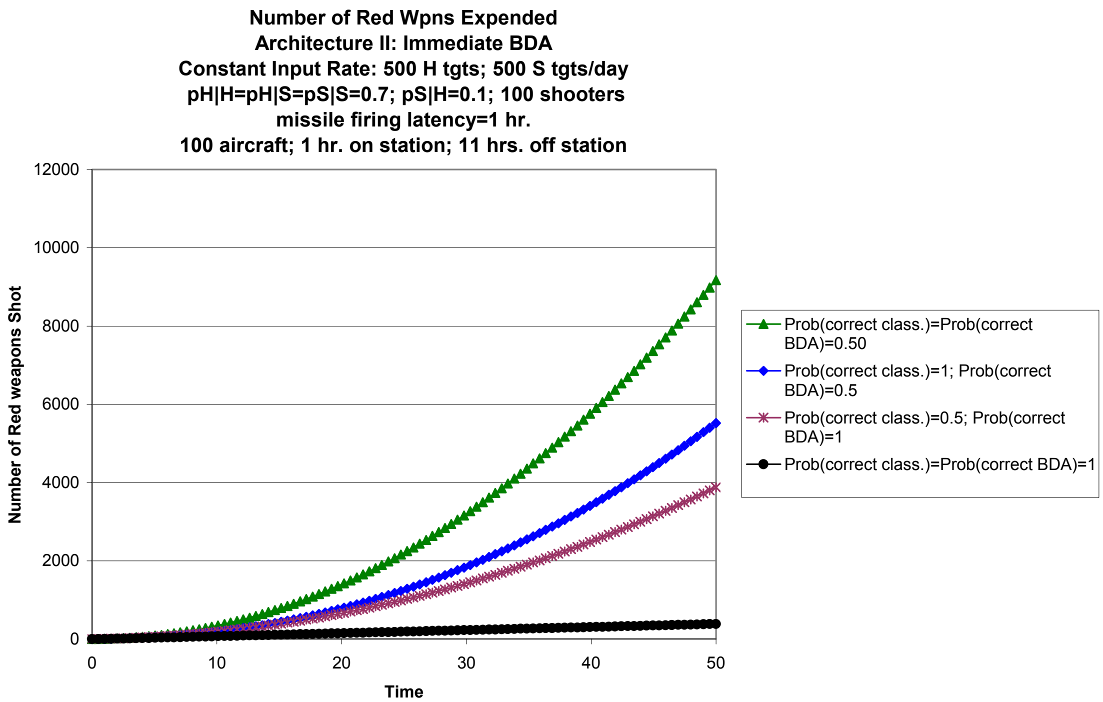

Figure 5(j)

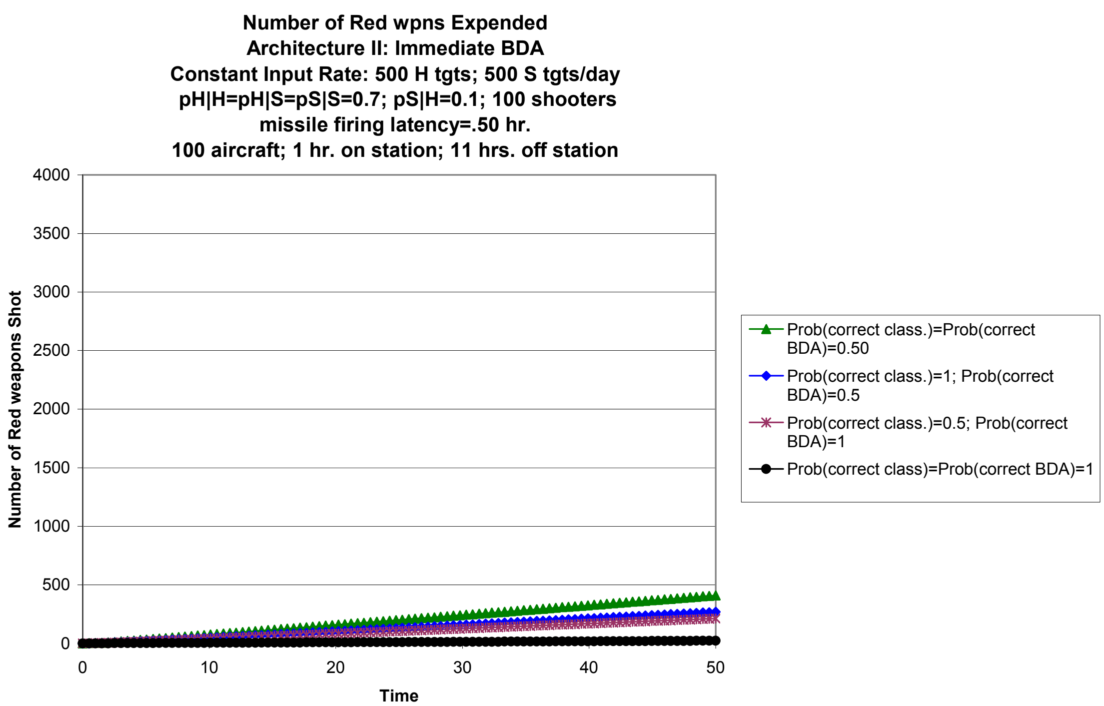

Figure 5(k)

APPENDIX III

Expected Number of Times Through the Shooting Server for Architecture 1

There are $j = 1, \ldots, J$ types of targets. When a live target of type j is acquired it is classified as a type k with probability $c_{jk}(a)$. The classification of the target influences the weapon that will be shot at it. The probability a shot kills a target of type j that is classified as type k is p_{jk}. After each shot, battle damage assessment (BDA) is performed. The probability a dead target of type j is correctly assessed as dead is $m_j(d)$. If a dead target is misclassified as live, it is returned to the shooter server. The probability the shooter server classifies a dead target of type j as a type k is $c_{jk}(d)$. Once a dead target is classified as dead, it is removed from the list of targets. The target is reclassified independently each time the target passes through the shooter server.

A target of type j can pass through the shooter server more than once while it is alive and may return to the shooter server when it is dead if it is misclassified as live.

Let

$N_A(j)$ = Number of times through the shooter server for a type j target while it is alive

$N_D(j)$ = Number of times through shooter server for a type j target while it is dead

$N_T(j)$ = Total number of times through the shooter server (both alive and dead) for a type j target

$$E\left[N_A(j)\right] = 1 + \sum_k c_{jk}(a)\left(1 - p_{jk}\right)E\left[N_A(j)\right] \qquad \text{(III.1)}$$

Solving,

$$\begin{aligned} E\left[N_A(j)\right] &= \frac{1}{1 - \sum_k c_{jk}(a)\left(1 - p_{jk}\right)} \\ &= \frac{1}{\sum_k c_{jk}(a) p_{jk}} \end{aligned} \qquad \text{(III.2)}$$

$$E\left[N_D(j)\right] = \left(1 - m_j(d)\right)E\left[N_D^0(j)\right] \qquad \text{(III.3)}$$

where

$$E\left[N_D^0(j)\right] = 1 + \sum_k c_{jk}(d)\left(1 - m_j(d)\right)E\left[N_D^0(j)\right] \tag{III.4}$$

Solving,

$$\begin{aligned} E\left[N_D^0(j)\right] &= \frac{1}{1 - \sum_k c_{jk}(d)\left[1 - m_j(d)\right]} \\ &= \frac{1}{m_j(d)} \end{aligned} \tag{III.5}$$

Thus,

$$\begin{aligned} E\left[N_T(j)\right] &= E\left[N_A(j)\right] + \left[1 - m_j(d)\right]E\left[N_D^0(j)\right] \\ &= \frac{1}{1 - \sum_k c_{jk}(a)\left(1 - p_{jk}\right)} + \left[1 - m_j(d)\right]\frac{1}{1 - \sum_k c_{jk}(d)\left[1 - m_j(d)\right]} \\ &= \frac{1}{\sum_k c_{jk}(a)p_{jk}} + \left[1 - m_j(d)\right]\frac{1}{m_j(d)} \end{aligned} \tag{III.6}$$

Example

Suppose there are two target types: Hard (H) and Soft (S). Let the probability of correct target classification be $c_{HH} = c_{SS} = c$. Let the probability of correct BDA of a dead target as dead be $m_j(d) = m$.

$$\begin{aligned} E\left[N_T(H)\right] &= \frac{1}{1 - \left[c\left(1 - p_{HH}\right) + (1-c)(1 - p_{HS})\right]} \\ &\quad + \left(1 - m\right)\frac{1}{1 - \left[1 - m\right]} \\ &= \frac{1}{cp_{HH} + (1-c)p_{HS}} + \frac{1-m}{m} \\ &= \frac{1}{f(c)} + \frac{1-m}{m} \end{aligned} \tag{III.7}$$

where

$$f(c) = cp_{HH} + (1-c)p_{HS} \tag{III.8}$$

Note that,

$$\frac{\partial}{\partial c}E\left[N_T\left(H\right)\right]=-\frac{p_{HH}-p_{HS}}{f\left(c\right)^2} \tag{III.9}$$

$$\frac{\partial}{\partial m}E\left[N_T\left(H\right)\right]=-\frac{1}{m^2} \tag{III.10}$$

Note that (III.9) and (III.10) are both nonpositive. If (III.9) > (III.10) then increasing the probability of correct BDA, *m*, will result in a greater decrease in the expected number of shots required to kill and correctly assess a dead target as dead than increasing the probability of correct classification, *c*.

The figures below display regions in which it is better to increase the probability of correct BDA assessment, *m*, than to increase the probability of correct target classification, *c*, and vice-versa. Values of the probability of correct target classification, *c*, are plotted on the y-axis and the values for the probability of correct BDA, *m*, are plotted on the x-axis. The plotted line are those values of (*m*, *c*) such that (III.9) is equal to (III.10). If the value of (*m*, *c*) is above the line, then increasing *m* will result in a greater decrease in the expected number of shots required to kill a hard target and correctly assess the dead target is dead than increasing the probability of correct target classification, *c*. If the value of (*m*, *c*) is below the line, then increasing the probability of correct target classification, *c*, will result in a larger decrease in the expected number of shots required to kill a target and correctly assess it is killed than increasing the probability of correct BDA.

For fixed value of *m*, let *c*(*m*) be that value of *c* for which (III.9) equals (III.10). If *c* > *c*(*m*), increasing the probability of correct BDA, *m*, results in a larger decrease in the expected number of shots required to kill and correctly assess that a dead target is dead than increasing the probability of correct classification *c*. Comparison of the four figures shows the following about the behavior of *c*(*m*). The value of *c*(*m*) is nondecreasing as the value of *m* increases. If the probability of kill is the same for a correctly classified target and an incorrectly classified target, then (III.9) equals 0 and it is always better to increase the probability of correct BDA, m. The largest value of m such that $c(m) = 0$ (increasing the probability of correct BDA is always better) decreases as the difference between the probability of kill for a correctly classified target and the probability of kill for an incorrectly classified target increases. The smallest value of m such that $c(m) = 1$ (increasing the probability of correct target classification is always better) increases as the difference between the probability of kill for a correctly classified target and the probability of kill for an incorrectly classified target increases. If for some m, $0 < c(m) < 1$, then increasing the difference between the probability of kill for a correctly classified target and an incorrectly classified target will decrease the value of $c(m)$. Hence, increasing the probability of kill for a correctly classified target while keeping the probability of kill for an incorrectly classified target the same may make it more advantageous to increase the probability of correct BDA.

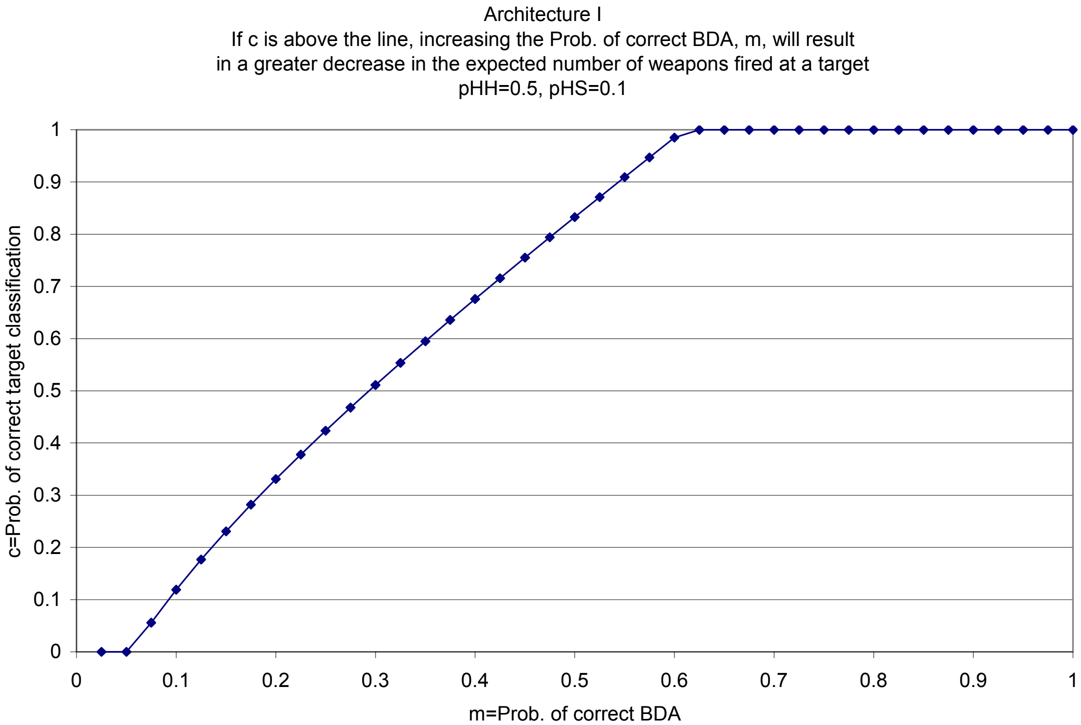

Figure III.1

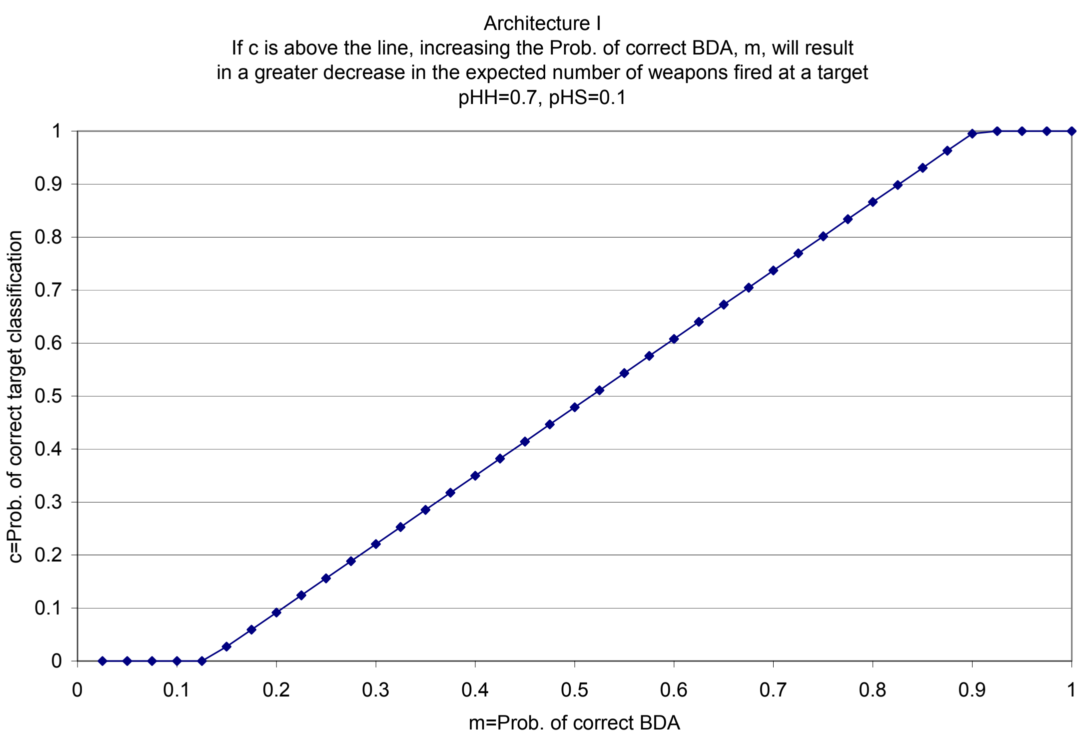

Figure III.2

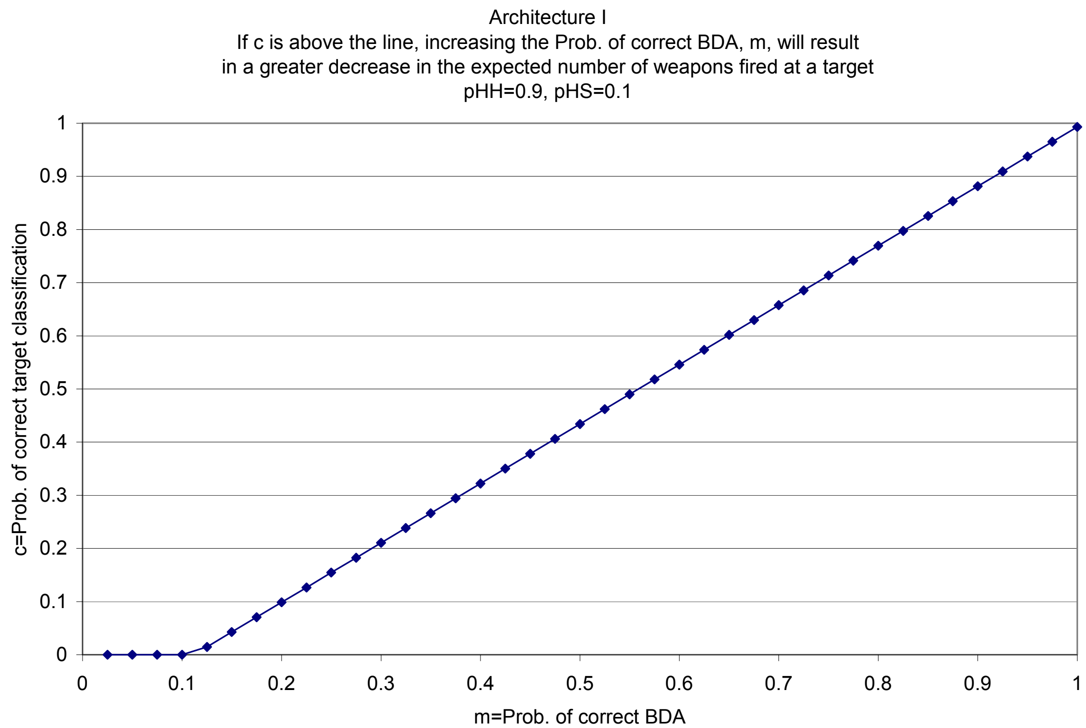

Figure III.3

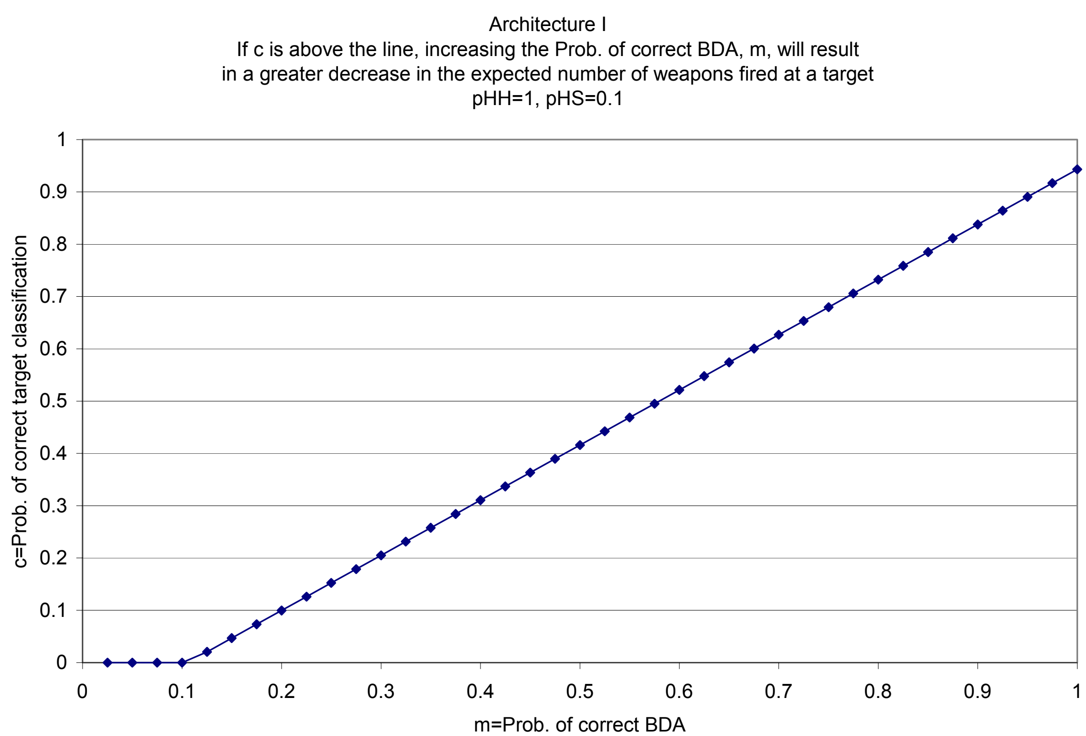

Figure III.4

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