

**VERTICAL CRUSTAL MOVEMENTS BASED ON  
PRECISE LEVELLINGS IN ESTONIA**

MAAKOORE VERTIKAALLIIKUMISED EESTIS  
TÄPPISNIVELLEERIMISTE ANDMETEL

**TARMO KALL**

A Thesis  
for applying for the degree of Doctor of Philosophy in Geodesy

Väitekirj  
filosoofiadoktori kraadi taotlemiseks geodeesia erialal

Tartu 2016



**Eesti Maaülikooli doktoritööd**

**Doctoral Theses of the  
Estonian University of Life Sciences**





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Estonian University of Life Sciences

According to verdict No 6-14/16-7 of February 29, 2016, the Doctoral Committee of Engineering Science of the Estonian University of Life Sciences has accepted the thesis for the defence of the degree of Doctor of Philosophy in Geodesy.

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Defence of the thesis: Estonian University of Life Sciences, room 2B5,  
Kreutzwaldi 5, Tartu on April 29, 2016 at 13:30.

English language revision by Mare-Anne Laane.

Publication of the thesis is supported by Estonian University of Life Sciences.

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ISSN 2382-7076  
ISBN 978-9949-536-84-9 (trükis)  
ISBN 978-9949-536-85-6 (pdf)

*To the memory of my father, Jaan Kall*

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## LIST OF PUBLICATIONS

The present thesis is based on the following papers, which are referred to by Roman numerals. The papers are reproduced with the kind permission of the publishers.

- I** Kall, T. and Jürgenson, H., 2008. Postglacial land uplift in Estonia based on geodetic measurements on Põltsamaa-Lelle levelling line, *In* Cygas, D. and Froehner, K.D. eds., *The 7th International Conference “Environmental Engineering”, May 22–23, 2008, Selected Papers*, Vilnius Gediminas Technical University Press “Technika”, Vilnius, pp. 1325–1333.
- II** Kall, T., Oja, T., and Tänavsuu, K., 2014. Postglacial land uplift in Estonia based on four precise levelings. *Tectonophysics*, **610**, pp. 25–38, doi: 10.1016/j.tecto.2013.10.002.
- III** Kall, T., Liibus, A., Wan, J., and Raamat, R., 2016. Vertical crustal movements in Estonia determined from precise levellings and observations of the level of Lake Peipsi. *Estonian Journal of Earth Sciences*, **65**(1), pp. 27–47, doi: 10.3176/earth.2016.03.

Contribution of the authors to the papers was as follows:

	<b>I</b>	<b>II</b>	<b>III</b>
Data preparation	<b>TK</b>	<b>TK, KT</b>	<b>TK, RR, AL</b>
Study design and methods	<b>TK</b>	<b>TK, TO</b>	<b>TK, AL</b>
Data analysis	<b>TK</b>	<b>TK, TO, KT</b>	<b>TK, RR, AL, JW</b>
Preparation of the manuscript	<b>TK, HJ</b>	<b>TK, TO</b>	<b>TK, AL, JW</b>

AL – Aive Liibus, HJ – Harli Jürgenson, JW – Junkun Wan, KT – Kalmer Tänavsuu, RR – Rivo Raamat, **TK** – Tarmo Kall, TO – Tõnis Oja

## LIST OF SYMBOLS AND UNITS

$a$	Secular (eustatic) sea level rise
$b$	Rise of the geoid, either in mm/yr or as unitless scale factor
$c$	Misclosure (height differences, vertical velocity differences) in the levelling loop
$d$	Discrepancy of fore and back height difference in the levelling section
$e$	Observation residual
$\bar{e}$	Standardized residual
$\dot{h}$	Vertical displacement difference, relative vertical displacement
$\dot{H}$	Vertical displacement, change in height
$l$	Length of the levelling section
$L$	Length of the levelling line
$m^2$	Variance of height difference
$n$	Number of the quantities (observations, benchmarks, levelling loops)
$p$	Calculated probability
$P$	Perimeter of the levelling loop
$r$	Pearson's correlation coefficient
$S_0$	<i>A posteriori</i> standard deviation of unit weight
$S_0^2$	<i>A posteriori</i> variance of unit weight
$SD$	Standard deviation
$t$	Time epoch
$v_{app}$	Apparent land uplift, uplift relative to the mean sea level
$v_{lev}$	Levelled land uplift, uplift relative to the geoid

$v_{abs}$	Absolute land uplift, uplift relative to the geocentre
$w$	Observation weight
$\alpha$	Significance level
$\Delta h$	Height difference, relative height
$\Delta v$	Vertical velocity difference, relative vertical velocity
$\Delta t$	Time difference, time interval
$\eta$	Levelling random error
$\mu$	Mean standard error of the relative vertical velocities
$\sigma$	Levelling systematic error
$\sigma_0^2$	<i>A priori</i> variance of unit weight, 1
$\tau$	<i>A priori</i> levelling standard error
<b>A</b>	Design matrix of the levelling observations
<b>e</b>	Error vector of the levelling observations, vector of the residuals
<b>T</b>	Diagonal matrix of the levelling epochs
<b>v</b>	Vector of the vertical velocities of the benchmarks
<b>W</b>	Weight matrix
<b>y</b>	Vector of the levelling observations
<b><math>\Sigma</math></b>	Covariance matrix of the observations
dam	Decametre, unit of length in the metric system equal to ten metres
Gal	Unit of acceleration, used especially in gravity measurements. One Gal equals a change in the rate of motion of one centimetre per second per second
ka	Kiloyear, a time unit defined as the period of one thousand years
Pa · s	Pascal-second, a unit of dynamic viscosity

## LIST OF ABBREVIATIONS

ANOVA	Analysis of Variance, statistical method
BIFROST	Baseline Inferences for Fennoscandian Rebound, Sea-level, and Tectonics; network of the GPS stations to measure crustal deformation in Fennoscandia for geodynamic, sea-level, and tectonic studies
BIQUE	Best Invariant Quadratic Unbiased Estimator, method of variance component estimation, statistical method
BM	Benchmark, point of the levelling network
CHAMP	CHAllenging Minisatellite Payload, German satellite mission for geoscientific and atmospheric research
CLN	Common Levelling Network
CORS	Continuously Operating GPS or GNSS Reference Station
DORIS	Doppler Orbitography and Radiopositioning Integrated by Satellite, French satellite system
EST2013LU	Land uplift model of Estonia based on the levelling network of the fundamental benchmarks (LNF)
EST2015LU	Land uplift model of Estonia based on the common levelling network (CLN)
fBM	Fundamental (deep seated) benchmark, point of the levelling network
GIA	Glacial Isostatic Adjustment
GNSS	Global Navigation Satellite System
GOCE	Gravity field and steady-state Ocean Circulation Explorer, European Space Agency's satellite mission
GPS	Global Positioning System
GRACE	Gravity Recovery And Climate Experiment, satellites for measurements of Earth's gravity field anomalies

IAUE	Iterated Almost Unbiased Estimation, method of variance component estimation, statistical method
LNF	Levelling Network of Fundamental benchmarks
LU	Land Uplift
NKG2005LU	Fennoscandian land uplift model based on GPS, tide gauge and levelling data and geophysical model by Lambeck et al. (1998)
RMS	Root Mean Square
VCM	Vertical Crustal Movement
VV	Vertical Velocity
VLBI	Very Long Baseline Interferometry

## INTRODUCTION

*“Nothing in this world is a gift.  
Whatever must be learned must be  
learned the hard way.”*

**Carlos Castaneda**

1925–1998

The sea level of the Gulf of Bothnia and the Baltic Sea has been observed to be decreasing through the centuries. This phenomenon was first mentioned in 1491 when the citizens of the city of Östhammar wrote a petition to the regent and archbishop of Sweden requesting to move the city to a new location, since it was not accessible any more even with the fishing boats (Ekman 1991). Rise of the land from the sea has been described also in the book of sermons by bishop Ericus of Turku (Finland). He described various natural phenomena, especially appearance of the new rocks from the sea as the signs of the approach of Armageddon (Zhelmin 1958).

It is a general consensus now that continuous land uplift (LU) in Fennoscandia and North Europe is caused by the effect known as postglacial rebound. Postglacial rebound, sometimes called glacial isostatic adjustment (GIA), is a complex geophysical phenomenon comprising several processes like vertical and horizontal crustal movements, rise of the geoid, change of the gravity field, change of the sea level, geocentre variations, etc. (Ekman and Mäkinen 1996; Peltier 1999; Sella et al. 2007; Steffen et al. 2008; Rangelova and Sideris 2008; Lidberg et al. 2010; Klemann and Martinec 2011). These processes are reactions of Earth to the changes in the distribution of glacial and oceanic surface masses, i.e. results from the load and unload of heavy ice sheets to the Earth’s crust and accompanying suppression and rise of the crust and decrease and rise of the sea level in the last ice-age 13,000–60,000 years ago (Quaternary 2015). Due to the viscous properties of Earth’s mantle, these processes are still observable today.

Global warming observed in the last decades has caused glaciers (in the mountains, Greenland, Antarctica) melting and sea-level rising. Large social-economic hazards (shoreline changes, flooding of agricultural



lands with salty sea water, pollution of drinking water with salty sea water, redistribution of sediments along sandy coasts, erosion, residences of coastal cities remaining under water and related migration of population, etc.) are associated with the global sea-level rise (Church and White 2006, 2011; Nicholls and Cazenave 2010; Rignot et al. 2011). However, geodetic measurements of the sea-level (tide gauge, satellite altimetry, GRACE) contain signals of the ongoing process of GIA. Traditionally, the GIA signal has been removed from the sea-level observations to obtain the change in the sea-level. Therefore, knowledge of the effect of GIA is important to determine the global sea-level rise and thus to monitor and understand the global warming and climate change. The observed GIA signal has been used also for the determination of ice sheet history and the modelling of the Earth's mantle viscosity. Since Earth's crust is constantly moving due to the GIA, it is important to know it to update the geodetic datums, both vertical and horizontal.

Fennoscandian postglacial rebound is a phenomenon that has been studied in detail. Different kinds of observation data (levelling, sea- and lake-level records, GNSS (Global Navigation Satellite System) measurements, satellite- and terrestrial gravity measurements, geophysical and geological data) have been used for the modelling of the rebound process or constraint GIA models (Ekman 1996; Lambeck, Smither, and Ekman 1998; Lidberg et al. 2007; Steffen and Kaufmann 2005; Steffen, Gitlein, et al. 2009; Vestøl 2006). However, the repeated levellings have remained one of the most precise methods for the determination of the contemporary rates of postglacial rebound and other vertical crustal movements (VCMs) of the Earth, related to the other types of loading (oceans, atmosphere), tectonic structures, earthquakes, volcanism, etc. (Cross et al. 1987).

As postglacial rebound affects Estonian territory, national scientists have also studied this phenomenon extensively using levelling, gravity, sea-level and GNSS observations (Zhelnin 1960, 1964, 1966; Yakubovski 1973; Vallner and Zhelnin 1975; Vallner et al. 1988; Randjärv 1968, 1993; Sildvee et al. 1973; Sildvee and Müdel 1978, 1980; Pobedonostsev 1975; Jevrejeva et al. 2002; Oja et al. 2012, 2014). However, previous studies have been based on the First (1933–1943) and the Second (1948–1969), or the First and the Third (1970–1996) levelling of the Estonian levelling network. Recently, the Fourth levelling campaign of the Estonian levelling network was performed (2001–2012), but these

levelling data have not yet been used in the VCM determination. Thus, since the Estonian levelling network has been levelled at least four times by now, some lines even up to nine times, it gives an excellent opportunity to estimate vertical velocities (VVs) of the geodetic points of the levelling network, the benchmarks (BMs), and change of the VVs in time using different levelling combinations.

Therefore, this thesis studies VCMs in Estonia based on the precise levelling observations of the Estonian levelling network, including the latest ones from 2001 to 2011. The results of the study have been published in international peer-reviewed journals and conference proceedings. However, the dissertation includes also additional analyses, explanations and details that were not included in the publications due to the limits set to the volume of the manuscript by publishers.

The first task of the present thesis was to determine postglacial LU in Estonia. For detecting a long-term VV signal, the latest releveling data (2001–2010) of the Estonian levelling network as well as previous precise levelling data from 1933 to 1996 were used. In order to detect a postglacial rebound signal from the repeated levellings, the BMs have to be stable. They should not be influenced by the near-surface processes like volume change in soil affected by variable groundwater level, frost heave, etc. Deep seated, so-called fundamental BMs (fBMs) should be less prone to such disruptive processes. Therefore, the levelling network of fBMs (LNF) was compiled by collecting all available levelling observations from 1933 to 2010 between the selected fBMs. Kinematic adjustment of the LNF's observations revealed velocities of the fBMs. Using the VVs of the fBMs, a LU model for Estonia was created (Paper **II**).

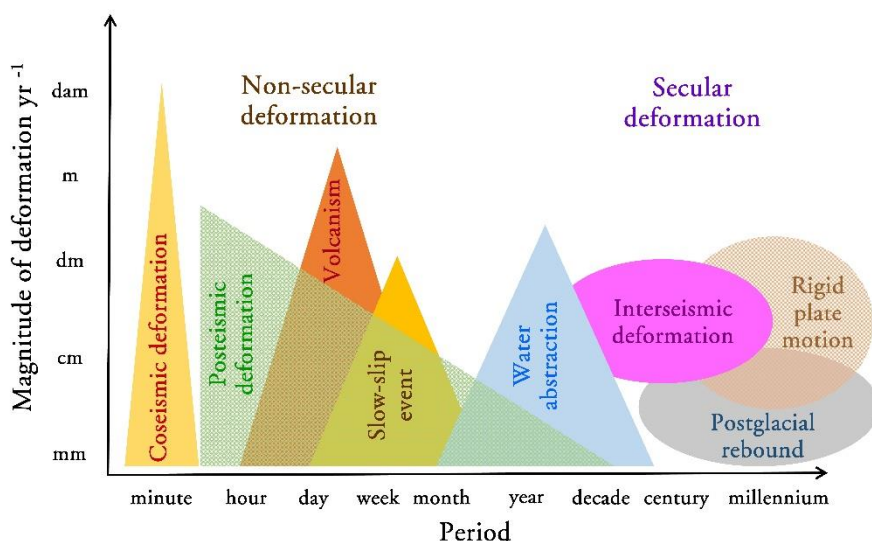
The second task of this study was to analyse the change of the BMs' VVs of the Estonian levelling network in time. A network (so-called common levelling network (CLN)) was created by using common BMs and observations from all four precise levelling campaigns of the Estonian levelling network from 1933 to 2011. VVs from different levelling combinations were calculated and analysed. Since the CLN has a denser set of BMs than the LNF, the model of the VCMs for Estonia revealed also signals not related to the postglacial rebound, but of other origin (e.g. different compaction of the sediments, groundwater pumping, mining, BMs' instability, etc. (Paper **III**)).

Thirdly, VCMs along the Põltsamaa-Lelle geodynamic line (in the central part of Estonia) were studied. A statement about the highest VCM gradient in Estonia across the Pärnu-Narva fault zone (Orviku 1960) was set up in the study by Vallner et al. (1988). Põltsamaa-Lelle levelling line crosses this geological fault as well as other faults of the basement or sedimentary rocks. A large number of the levelling observations on the Põltsamaa-Lelle line (11 levellings) make this line perfect to check the hypothesis that the gradient of the VCM is the largest across the Pärnu-Narva geological fault (Paper I).

# 1. REVIEW OF THE LITERATURE

## 1.1. Classification of the Earth's crustal movements

Movements of the Earth's crust could be expressed as horizontal, vertical or combined deformations. They can be divided regarding to the time period of their impact on secular, periodic and episodic movements. Main forces causing deformations, their duration and magnitude are presented in Figure 1 (Cross et al. 1987; Stanaway et al. 2012).



**Figure 1.** Spatial and temporal extent of the deformations of the Earth's crust (Stanaway et al. 2012).

Tectonic processes like thermal convections in the mantle cause mainly horizontal movements of the Earth's plates but VCMs up to 0.5 mm/yr are also observed. Coseismic and postseismic deformations and volcanism are closely related to the plate tectonics. They are mainly episodic, but postseismic deformations can continue for decades.

Human induced withdrawal of oil, gas or groundwater, mining, etc. can induce the subsidence of the ground. In Estonia this type of subsidence can be observed in Tallinn, Tartu, Pärnu, and also in the mining area in the NE of Estonia (Lutsar et al. 1973; Listra and Talviste 1988; Toomik and Liblik 1998; Mets et al. 2000; Kall and Torim 2003; Rüdja 2004;

Kalm 2007). Elsewhere, such cases are reported for example in New Jersey, Mexico City, Paris, etc. (Sun et al. 1999; Fruneau et al. 2005; Osmanoglu et al. 2011).

Tidal forces created by the gravitational counteraction of the Earth and other celestial bodies (Moon and Sun mainly) produce periodical (e.g. diurnal, 8.5 and 18.6 years, etc.) to permanent deformations of the Earth's crust. Tides of the oceans created by these gravitational forces produce secondary deformations of the Earth's crust due to the loading effect (so-called ocean loading). Other loadings, such as load of atmosphere (atmospheric loading) and hydrologic loading, produce temporal deformations of the Earth's crust, whereas loading of the glaciers (e.g. Antarctic and Greenland) produces secular deformations. Uplift of the crust released from the load of the glaciers in the last ice age (i.e. postglacial rebound) is the main research interest of this thesis.

In addition, deformation of the levelling network can be classified to (Hein 1986):

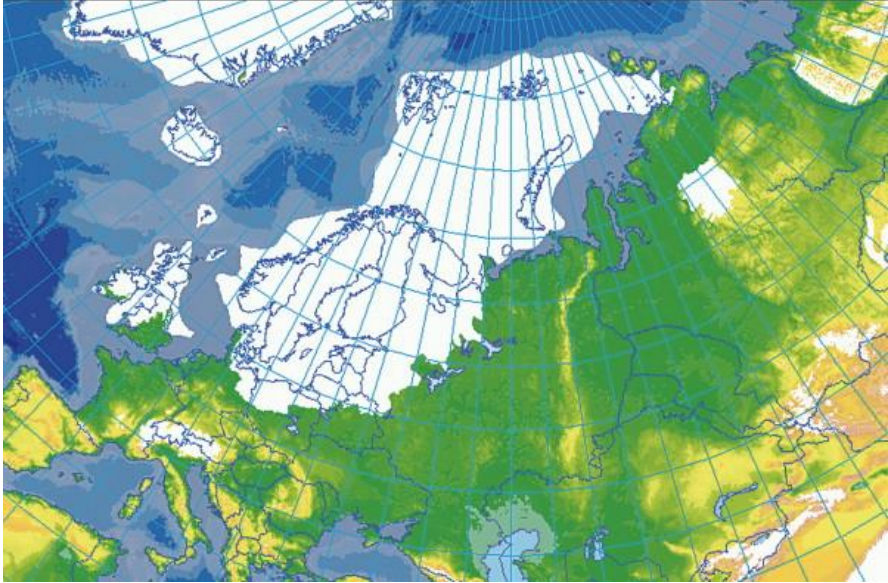
- Translation and rotation of the whole network in relation to the inertial reference frame. This type of deformation does not change the geometry of the network and takes place, for example, with plate tectonics.
- Steady deformations, which produce change of the network with constant size and direction. Usually this type of deformations takes place in smaller scales (small part of the network, short period).
- Uneven deformation, which depends on the spatial location of the BMs. For example, due to postglacial rebound, BMs of the Estonian levelling network in SE Estonia subside approximately  $-1$  mm/yr, while BMs in NW Estonia rise approximately  $2.5$  mm/yr (Vallner et al. 1988).
- Movement of a single BM, which is usually random and generally related to the local phenomena. For example, a single BM can move due to swelling and shrinking of the soil (influenced by the groundwater level) or due to construction works in a building where the BM is mounted.

LU rates depend on the reference surface that was used for their determination. According to the terminology by Ekman and Mäkinen (1996), LU rates can be determined relative to:

- i)* mean sea level. Obtained LU rates are “apparent”, with the notation  $v_{app}$ ;
- ii)* geoid. Obtained rates are “levelled”. To obtain levelled LU rates, secular sea-level rise  $a$  has to be added to the apparent LU rate  $v_{lev} = v_{app} + a$ . Estimates of sea-level rise are varying, but in Nordic countries the value 1.32 mm/yr is often used (Ågren and Svensson 2007; Vestøl 2006);
- iii)* geocentre. Obtained rates are “absolute”. Change of a geoid  $b$  has to be added to the levelled LU rate  $v_{abs} = v_{app} + a + b$ . Change of the geoid is approximately 6% of the absolute LU (Ekman and Mäkinen 1996; Vestøl 2006) for the period approximately 1891–1990.

## 1.2. Postglacial rebound of the Earth’s crust

In the history of Earth, colder periods have been found to vary with warmer periods. Cold periods are known as glacial periods or ice-ages while warmer periods are interglacials (Steffen and Wu 2011). In the last glacial maximum, approximately 19,000–22,000 years ago (Peltier 1999; Steffen and Wu 2011; Yokoyama et al. 2000), large areas in Fennoscandia and North Europe (Weichsel glaciation, Figure 2), North-America (Wisconsin glaciation) and Alps (Würm glaciation) were covered with thick glaciers (Quaternary 2015). In the last glacial maximum, North Europe, Fennoscandia and some other parts of Europe were covered with the Scandinavian Ice Sheet (Climate change 2015). The maximum thickness of this ice sheet has been estimated approximately from 2000 to 3200 m (Tushingham and Peltier 1991; Peltier 1994; Lambeck, Smither, and Johnston 1998; Lambeck et al. 2000; Siegert et al. 2001; Fleming and Lambeck 2004; Siegert and Dowdeswell 2004; Peltier 2004; Lambeck et al. 2010; Tarasov et al. 2012; Peltier et al. 2015; Abe-Ouchi et al. 2015).

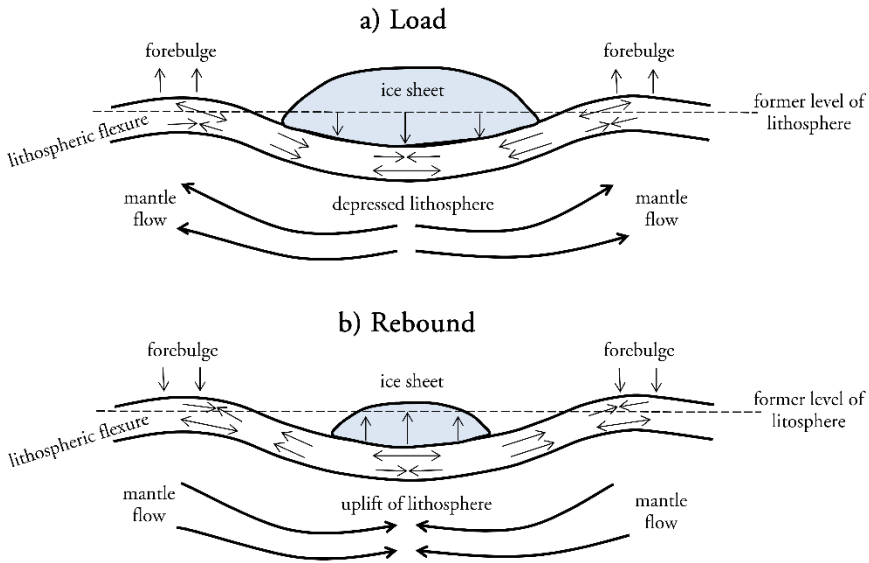


**Figure 2.** Extent of the late Weichsel glaciation in the last glacial maximum (white areas) (Ehlers et al. 2007).

Due to the growth of the ice sheets, global sea level decreased approximately 120–135 m compared to its present level and under the heavy load of the glaciers the Earth’s crust was suppressed by several hundred meters (Clark and Mix 2002; Lambeck et al. 2014; Steffen 2007; Fairbanks 1989; Fleming et al. 1998; Yokoyama et al. 2000). Beneath the depressed Earth’s crust, viscous mantle material was flowing away (Figure 3a). Due to the mantle flow, Earth’s crust raised outside the ice sheet margins (Figure 3a). This uplifted crust is called glacial forebulge or peripheral bulge (Steffen 2013b; McGuire 2012). Latest glaciation of Fennoscandia generated a forebulge approximately of 40–60 m in height (Fjeldskaar 1994a; Brevik and Jenssen 1994).

After deglaciation approximately 7,000–20,000 years ago, the volume of the continental glaciers decreased approximately 70%. Global sea level rose approximately 100 m in average (Milne et al. 2006; Farrell and Clark 1976) and the Earth’s crust rebounded (Figure 3b), trying to restore the

state of isostatic<sup>1</sup> equilibrium. Earth’s mantle began to slowly flow back, inducing forebulge areas to collapse and migrate towards the ice load (Figure 3b). Subsidence of these areas can still be observed at present, including the SE of Estonia (Sella et al. 2007; Engelhart et al. 2009; Vink et al. 2007; Rosentau et al. 2007).



**Figure 3.** Postglacial rebound of the Earth’s crust. a) During glaciation, after being loaded by the ice sheet, elastic lithosphere subsided causing viscous mantle to flow aside. Loaded by the ice sheet, depressed lithosphere forced the surrounding areas – “forebulge” – to uplift and migrate outward. b) During deglaciation, ice sheet loading slowly disappeared and lithosphere started to uplift trying to restore its previous state of isostatic equilibrium. Material of the viscous mantle slowly flows back to its initial position, inducing forebulge areas to collapse and migrate towards the ice loads. Drawing by the author based on McGuire (2012) and Steffen (2013b).

Initial rebound of the Earth’s crust after melting of the ice sheets was due to the elastic response of the crust much quicker than nowadays when we can observe mainly slow viscoelastic response of the Earth’s mantle. According to the viscous properties of the Earth, rebound takes place almost exponentially in time (Ranalli 1995; Turcotte and Schubert

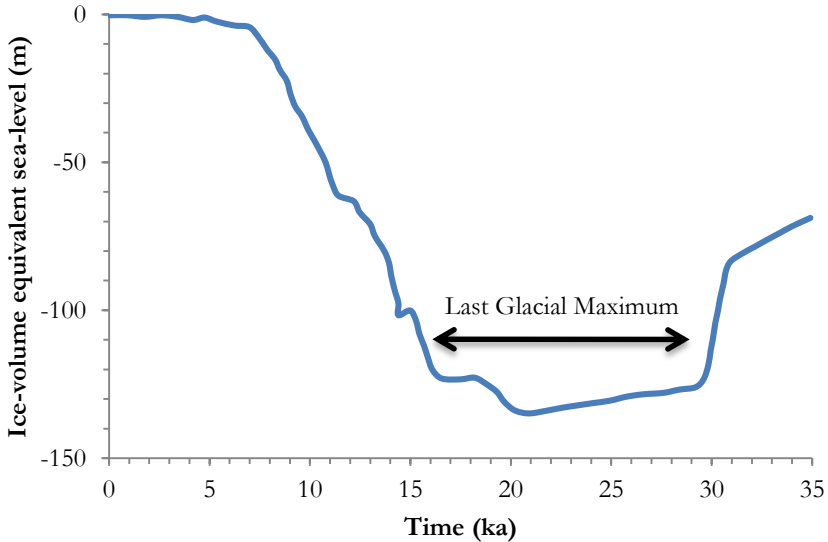
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<sup>1</sup> Isostasy is a term that describes gravitational balance between Earth’s lithosphere and asthenosphere, where plates are floating at height, which depends on the thickness and density of the plates (Isostasy 2015).



2002; Whitehouse 2009). In addition to the VCM, GIA induces also horizontal crustal movements, which can be predicted by GIA models and observed by CORS measurements (Johansson et al. 2002; Milne et al. 2004).

The change of the sea level during the last glaciation to the present time has been subject of discussions for many years. On a long-term scale, the main contributor to the global sea-level change is the growth and melting of ice sheets during glacial and interglacial periods. However, tectonic or GIA induced subsidence or uplift of the Earth's surface are also contributors on a regional and local scale (Church and Gregory 2001; Steffen and Wu 2011; Lambeck et al. 2014). Long-term relative sea level (with respect to the present sea level) is determined from radiocarbon dated samples of former sea-level indicators (corals, wood, pollen, shells, sediment cores) where the present and former sea-level locations are also determined (Church and Gregory 2001; Steffen and Wu 2011). For example, Lambeck et al. (2014) have calculated sea-level change for the past ~35,000 years (Figure 4) based on coral and sediment records from different locations over the world.



**Figure 4.** Change of the global sea level from ~35,000 years ago to the present according to Lambeck et al. (2014).

The rate of the sea-level change is not constant. In different phases of glaciation and deglaciation, it has changed. Although for the last two or three millennia, the sea level has remained quite stable, rise of the sea level has been observed in the past two centuries based on tide gauge records (Lambeck et al. 2014, 2004; Douglas 2001). There are various estimates of sea-level rise ranging from 1 to 3 mm/yr for different observation periods, mostly based on tide gauge observations but also on satellite altimetry (Douglas 2001; Gornitz 1995; Spada and Galassi 2012). For Fennoscandia, secular sea-level rise between 1.2 and 1.32 mm/yr has been determined based on tide gauge, continuously operating GPS or GNSS reference station (CORS), and levelling observations (Ekman 1993; Vestøl 2006; Richter et al. 2012).

As was mentioned above, since the Earth's mantle reacts to the retreat of the ice sheets as a viscous fluid (viscosity  $\sim 10^{21}$  Pa s), uplift of the crust at present is rather slow. Absolute LU determined by the CORS observations at the main areas of the postglacial rebound in Fennoscandia and Canada is up to 11 mm/yr in the Northern part of the Gulf of Bothnia and 13 mm/yr in the Southern part of Hudson Bay (Lidberg et al. 2010; Henton et al. 2006). Since starting of deglaciation, the Earth's crust at the centre of the Fennoscandian uplift has been raised approximately 650 meters (Fjeldskaar 1994a). Yet, in order to reach isostatic equilibrium, the crust should rise approximately 120–140 meters more in Fennoscandia (Steffen et al. 2006; Steffen and Wu 2011).

Redistribution of the Earth's interior masses (mantle flow) caused also secular change of the gravity field, which is still observable by absolute and relative gravity measurements (Timmen et al. 2012, 2006; Mäkinen et al. 2005; Ekman and Mäkinen 1996) and special satellite missions, especially by Gravity Recovery And Climate Experiment (GRACE) (Müller et al. 2005, 2007; Steffen, Müller, et al. 2009; van der Wal et al. 2008; Latychev, Mitrovica, Tamisiea, et al. 2005). Since GIA-induced gravity change is varying spatially, it is best to give estimates for gravity changes as the ratio between the gravity change and the vertical motion of the crust. For example, the ratio between the gravity change and the LU  $-0.204 \pm 0.058$   $\mu\text{Gal}/\text{mm}$  was determined based on relative gravity measurements from 1966 to 1993 on the Fennoscandian LU gravity line (Ekman and Mäkinen 1996). This value was recalculated based on additional gravity observations (from 1966 to 2003) and the ratio between  $-0.16$  and  $-0.20$   $\mu\text{Gal}/\text{mm}$  was obtained due to the use of different LU data (Mäkinen et al. 2005).

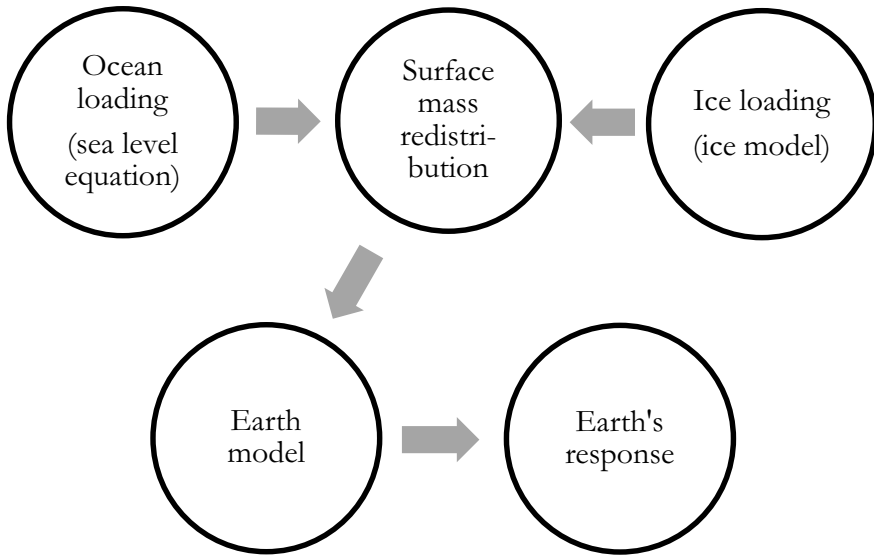
The ratios between the gravity change and the LU mentioned previously are in accordance with the weighted average ratio from the absolute gravity measurements  $-0.163 \pm 0.02 \mu\text{Gal}/\text{mm}$  (Timmen et al. 2012). The latter value is based on the 84 gravity determinations at 34 different stations from 2003 to 2008 in Fennoscandia. The ratios indicated fit also with the numerical prediction from the GIA model  $-0.163 \mu\text{Gal}/\text{mm}$  (Olsson et al. 2015). Based on the GRACE observations, secular gravity change from 1.2 to 1.5  $\mu\text{Gal}/\text{yr}$  has been calculated for the maximum area of the Fennoscandian LU in the Gulf of Bothnia at different computation centres (Steffen, Gitlein, et al. 2009). Average secular gravity change from GRACE corresponds approximately to the ratio  $-0.17 \mu\text{Gal}/\text{mm}$  between the gravity change and the LU. Again, this is in accordance with the corresponding values from the relative and absolute gravity measurements considering measurement uncertainties.

The mantle flow related to postglacial rebound causes also the secular rise of the geoid. The rise of the geoid can be calculated taking the time derivative of the Stokes formula (Sjöberg 1983; Kakkuri 1997). By using different LU and gravity change data, maximum geoid rises 0.68 mm/yr and 0.6 mm/yr were estimated by Sjöberg and Kakkuri respectively for the centre area of the Fennoscandian LU. Later, Sjöberg estimated the value  $0.76 \pm 0.20 \text{ mm}/\text{yr}$  for the maximum geoid rise at Fennoscandia and  $-0.16 \pm 0.04 \mu\text{Gal}/\text{mm}$  for the ratio between the gravity change and the LU (Sjöberg 1989). Probably the most known study of the geoid rise estimation was performed by Ekman and Mäkinen (1996). They found the value 0.6 mm/yr for the geoid rise in the centre of the Fennoscandian LU area, based on the LU map by Ekman (1996) and the maximum gravity change  $-0.2 \mu\text{Gal}/\text{mm}$  from the Fennoscandian 63° gravity line. The scale factor for the geoid rise in the Fennoscandian area 1.06 ( $5.7\% \pm 2.3\%$ ) of the absolute LU was found by Vestøl (2006). This corresponds to the geoid rise  $\sim 0.6 \text{ mm}/\text{yr}$  in the Fennoscandian LU centre.

### 1.3. Glacial isostatic adjustment models

GIA model predicts the Earth's response to the surface loading of an ocean and ice sheets (Whitehouse 2009), i.e. it can be used for example to compute sea-level change, Earth's crust uplift rates and values, remaining uplift, forebulge and its migration (Steffen 2013a). Knowledge about the Earth's structure and glacial history is needed for GIA model calculations (King et al. 2010). This is realized through the Earth model

and ice model (Kierulf et al. 2014; Steffen and Wu 2011). Construction of the ice model requires also knowledge about changes in the ocean mass during glaciation and deglaciation. Redistribution of ocean water is usually calculated with the sea-level equation (Farrell and Clark 1976). Calculated changes in the ocean mass and the ice model are employed as loads in the Earth model, which then determine the GIA model (Figure 5).



**Figure 5.** Input and result of a glacial isostatic adjustment model. Ice and ocean loadings are implemented to an Earth model. Specifics of the ice and Earth models decide the Earth’s response to the loading (Whitehouse 2009).

A simplest Earth model used in GIA modelling is spherically symmetric, i.e. one-dimensional (1D) model (Milne and Mitrovica 1998; Kaufmann and Lambeck 2002; Peltier 1974; Steffen and Kaufmann 2005; Fjeldskaar 1994b; Milne et al. 2004). Such models assume linear mantle rheology<sup>2</sup> and material flow in the radial direction only (Kierulf et al. 2014; Steffen and Wu 2011). In the last decades, more complex 2D and 3D Earth models (Sabadini et al. 1986; Gasperini and Sabadini 1989, 1990; Kaufmann et al. 1997; Kaufmann and Wu 2002b, 2002a; Paulson et al. 2005; Latychev, Mitrovica, Tromp, et al. 2005) have been

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<sup>2</sup> Rheology is a science studying the flow and deformation of materials (Ranalli 1995).

developed, allowing linear, nonlinear, composite or transient rheology (Steffen and Wu 2011). An Earth model usually consists of an elastic lithosphere layer with constant thickness and one up to 20 viscoelastic mantle layers. A uniform viscosity of each layer is usually presumed. Lithospheric thicknesses vary between 70 km and 200 km, while mantle viscosities range between  $10^{19}$  and  $10^{24}$  Pa s. Elastic properties of the lithosphere are obtained from seismological investigations (King et al. 2010). Mantle viscosities are usually taken from the inverse solution (knowing the uplift of lithosphere and the ice model) or from independent geophysical studies (Whitehouse 2009).

Ice model is also required as input for GIA model, as was mentioned above. An extent and thickness of the glaciers and their change during glacial history are determined with the ice model. The ice models are given with chosen time steps (usually between 500 and 10 000 years) and they cover typically the whole last glacial cycle (Whitehouse 2009; King et al. 2010). In regional ice models, glaciers cover a certain region such as Fennoscandia, North America, Greenland, British Isles, Antarctic (e.g. models by Nakada and Lambeck 1988; Lambeck 1993; Lambeck, Smither, and Johnston 1998; Siegert et al. 2001; Marshall et al. 2002; Siegert and Dowdeswell 2004; Fleming and Lambeck 2004; Lambeck et al. 2010; Tarasov et al. 2012). Global ice models cover whole glaciated areas in Late Pleistocene, e.g. ICE-3G, ICE-4G, ICE-5G and ICE-6G models (Tushingham and Peltier 1991; Peltier 1994, 2004; Peltier et al. 2015), RSES model (Lambeck et al. 2000) and GLAC-1b model (Tarasov 2013; Goslin et al. 2015).

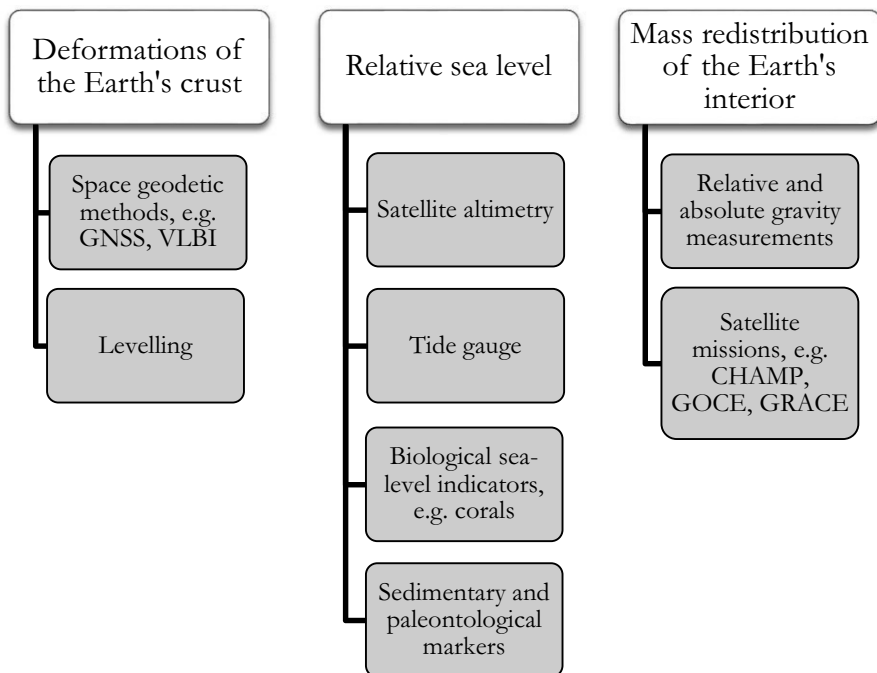
Two main approaches for creation of the ice models are:

- i)* Extent of the glaciers is determined by the extent of the geological markers (terminal moraine) at different epochs. Thickness of the glaciers is calculated based on the initial Earth model and LU velocities. Initial Earth model will be updated during the iterations so that it would fit with the relative sea level observations. Based on this approach, ICE-3G, ICE-4G, ICE-5G and RSES ice models were computed (King et al. 2010; Steffen and Wu 2011).
- ii)* The other approach uses thermodynamic models of glaciers. Geological observations are applied as constraints in fitting the

model. The Fennoscandian ice model by Näslund (2010) was compiled using that approach.

Different ice models give quite different values for the thickness of the ice, which shows considerable uncertainty regarding the ice models. For example, according to different ice models, maximum thicknesses of the Scandinavian and Laurentide ice sheets remain between 2000...3200 m and 3400...5200 m, respectively (Nakada and Lambeck 1988; Tushingham and Peltier 1991; Lambeck 1993; Peltier 1994; Lambeck, Smither, and Johnston 1998; Lambeck et al. 2000; Siegert et al. 2001; Marshall et al. 2002; Siegert and Dowdeswell 2004; Fleming and Lambeck 2004; Peltier 2004).

Different observational data are used to constrain GIA models (Figure 6).



**Figure 6.** Observation data used to constrain glacial isostatic adjustment models according to Whitehouse (2009) and Steffen and Wu (2011).

The longest time-period, dating back up to several thousand years, is covered by relative sea level observations. These are also the most valuable constraints of GIA models (Argus and Peltier 2010; Steffen and Wu 2011). Although precise geodetic observations (GNSS and DORIS satellite systems, VLBI, satellite laser ranging, satellite altimetry, levelling, tide gauge, gravity) comprise the shortest observation period (from decades to  $\sim 300$  years) compared to the relaxation time of the mantle, their dense spatial distribution and high accuracy reduces the bounds of admissible solutions in GIA models significantly (Argus and Peltier 2010; King et al. 2010). Estimates of the vertical motions of the sites constrain the ice sheet thickness and horizontal motions constrain the rheology of the lithosphere and mantle (Argus and Peltier 2010).

There are several GIA models that predict Earth's surface movements for Fennoscandia (Argus and Peltier 2010; Lambeck, Smither, and Johnston 1998; Milne et al. 2001, 2004; Steffen and Kaufmann 2005; Steffen et al. 2006; Wal et al. 2013; Whitehouse et al. 2006; Zhao et al. 2012). Predicted LUs in Fennoscandia by different GIA models are presented in Table 1.

**Table 1.** Absolute land uplift (LU) rates (mm/yr) in Fennoscandia and Estonia visually interpolated from different glacial isostatic adjustment (GIA) models and as observed from continuous GNSS.

GIA model	LU rate	
	Maximum	In Estonia
Lambeck et al. (1998)	10 <sup>(1)</sup>	1.2...3.7 <sup>(1)</sup>
Milne et al. (2001)	12	1...3
Milne et al. (2004)	12	0.5...3.5
Steffen and Kaufmann (2005)	12	2...4 <sup>(2)</sup>
U3L3_V1 (Steffen et al. 2006)	9	2...4 <sup>(2)</sup>
ICE-5G VM2 T90 Rot (Argus and Peltier 2010)	11	3 <sup>(2)</sup>
A2 (Zhao et al. 2012)	9	4.5...6
Dry UMT4 (Wal et al. 2013)	8	–
Observed from continuous GNSS (Lidberg et al. 2010)	11	2...4.8 <sup>(2)</sup>

<sup>(1)</sup> Apparent LU is transformed to the absolute by eustatic sea-level rise rate 1.2 mm/yr by Nakiboglu and Lambeck (1991).

<sup>(2)</sup> Only for the NW part of Estonia.

As can be seen from Table 1, different GIA models give quite different rates for LU. The study by Guo et al. (2012) where 14 GIA models from different authors were compared showed also that the models differ significantly from each other. The main reasons for the differences in the LU predictions from different GIA models (e.g. Table 1) are (Guo et al. 2012; King et al. 2010; Zhao et al. 2012; Steffen et al. 2006):

- different profiles of the Earth model's mantle viscosity;
- implementation of lateral heterogeneities in the Earth model's lithospheric thicknesses and mantle viscosities;
- different reference frames used in GIA models;
- use of different ice models and ice model's uncertainties;
- errors in GNSS velocities (observational noise, systematic errors of different reference frames, modelling of error sources, different data processing strategies);
- differences in solving the sea-level equation;
- differences in the methods computing Earth's response to the surface load.

Therefore, more dense and accurate constraints from geodetic observations in GIA areas remain one of the options to improve GIA models (King et al. 2010).

#### **1.4. Empirical land uplift maps and models for Fennoscandia and Estonia**

Empirical models of LU can be used as constraints in GIA modelling. They also have an important role to reduce geodetic observations to a certain epoch (Olsson et al. 2012). However, opposite to GIA models, empirical LU models can only describe recent movements (from decades to a couple of centuries back), which is a very short period compared to the relaxation time of the mantle.

Compilation of empirical LU models has long traditions in the Nordic countries. Several empirical LU maps/models have been constructed for this area. A comprehensive overview about the history of observations and compiling maps of LU in Fennoscandia is given in Ekman (1991;



1993; 2009). Main geodetic observations which can be used to determine the VVs and to compile empirical LU models are (Ekman 1996; Vestøl 2006; Steffen and Wu 2011):

- sea-level observations;
- lake-level observations;
- repeated levellings;
- repeated GNSS campaigns;
- CORS measurements.

LU determination by the abovementioned geodetic observations and their combinations is reviewed in the remaining part of this section.

### **Sea-level observations**

Regular sea-level observations in the Baltic Sea started in Stockholm in 1774 (Ekman 1988). Several tide gauges were mounted on the coast of Sweden and Finland in the middle of the 19<sup>th</sup> century. Regular observations at Kronstadt mareograph were started in 1806 (Bogdanov et al. 2000). The first mareograph in Estonia was built in Tallinn in 1842. Regular sea-level observations in other Estonian tide gauges were started in the 1880s (Jevrejeva et al. 2002). First LU maps for Fennoscandia were based also on the sea-level observations from tide gauges (Blomqvist and Renqvist 1914; Witting 1922; Bikis 1940; Witting 1943; Gutenberg 1941; Model 1950; Hela 1953; Bergsten 1954). A number of LU studies for Fennoscandia were based on tide gauge observations over the 20<sup>th</sup> century (Thomsen and Hansen 1970; Kääriäinen 1975; Sjöberg 1986; Vermeer et al. 1988; Davis et al. 1999; Ekman 1996).

### **Lake-level observations**

Lake-level observation have also been used for LU computations in the inner parts of Fennoscandia (Sieger 1893; Bergsten 1930; Sirèn 1951; Pässe 1990, 1998; Ekman 1996). Lake-level records can be used for LU determination when the water gauges are quite distant from each other in the same lake, i.e. on the coast of the large lakes (Kakkuri 1997). Tilt of the lake level can be determined by differentiating lake-level observations between two water gauges and calculating linear slope of

these differences, i.e. LU difference between two water gauges is obtained. By using water level records of Lake Peipsi from 1926 to 2006, LU values at the water gauges were calculated by Raamat (2009) in Estonia.

### **Repeated levellings**

The geodetic levellings in the Baltic Sea area were started from the second half of the 19<sup>th</sup> century: in Estonia in 1868, in Sweden in 1886, in Norway in 1877, and in Finland in 1892 (Torim 1993; Mäkinen and Saaranen 1998; Lysaker et al. 2006). Three levelling campaigns covering national levelling networks had been finished in Sweden and Finland and four in Estonia by 2015 (Mäkinen and Saaranen 1998; Svensson et al. 2006). There is no obvious network of repeated levellings in Norway. Therefore, many lines have been levelled only once, which makes the calculation of LU challenging (Vestøl 2006). Repeated levellings have been extensively used in Fennoscandian LU studies (Kääriäinen 1953, 1963; Simonsen 1968; Randjärv 1968; Bedsted Andersen et al. 1974; Ussisoo 1977; Bakkeliid 1979, 1986; Bjerhammar 1980; Suutarinen 1983; Kakkuri and Vermeer 1985; Vallner et al. 1988; Randjärv 1993; Mäkinen and Saaranen 1998; Vestøl 2006).

### **Repeated GNSS campaigns**

Before the era of CORS, repeated GPS campaigns were used to determine crustal deformations (including postglacial LU). However, in some special projects and in the areas of sparse CORS stations, it is still used (Anzidei et al. 1996; Clarke et al. 1998; Pan and Sjöberg 1999; Sjöberg et al. 2000, 2004; Dietrich et al. 2005; Oja et al. 2014). Long-term static sessions (at least 12 hours recommended) with two or more GPS receivers are performed and measurements are repeated on the same network with some time span.

### **CORS measurements**

Since the end of the 20<sup>th</sup> century, CORS became the main tool to detect and monitor crustal deformations, including GIA processes. In 1993, a special project called “Baseline Inferences from Fennoscandian Rebound Observations, Sealevel and Tectonics”, known as BIFROST, was initiated for monitoring GIA in Fennoscandia (Scherneck et al. 2001). Since then, a number of studies have been published analysing

vertical and horizontal movements resulting from GIA processes, constraining GIA models and inverting Earth models (Scherneck et al. 1998; Johansson et al. 2002; Scherneck et al. 2003; Milne et al. 2004; Bergstrand et al. 2005; Steffen and Kaufmann 2005; Steffen et al. 2006; Lidberg et al. 2007; Wang et al. 2008; Argus and Peltier 2010; Kollo and Vermeer 2010; Scherneck et al. 2010; Lidberg et al. 2010; Kierulf et al. 2014). Similar studies have been conducted in North America (Henton et al. 2006; Calais et al. 2006; Sella et al. 2007; Snay et al. 2007; Braun et al. 2008; Mazzotti et al. 2011; George et al. 2011; Zhao 2013).

## **Combination of observations for LU modelling**

Tide gauge observations, lake-level tilts and repeated levellings have been combined to draw up the LU maps (Kakkuri and Chen 1992; Kakkuri 1997; Ekman 1996). Further, a combination of CORS, tide gauge observations and repeated levellings was used by Danielsen (2001) and Vestøl (2006) to create the LU model for Fennoscandia. Afterwards, Ågren and Svensson (2007) extended Vestøl's model further to south using the geophysical model by Lambeck, Smither, and Ekman (1998). The LU model obtained is called NKG2005LU.

### **1.4.1. Studies of land uplift in Estonia**

In Estonia, LU studies started in the 1960s when the network of the First precise levelling (1933–1943) was repeated (1948–1969) and the first observation results became available (Zhel'nin 1958, 1960, 1964, 1966; Randjärv 1968; Vallner and Zhel'nin 1975; Vallner 1978). When the Third (1970–1996) levelling data became available, the values of VVs were re-estimated (Vallner et al. 1988; Randjärv 1993). It was concluded that in addition to general postglacial LU, several local VV anomalies related to the tectonic activity of the block structure of the basement (Pobul and Sildvee 1975) are taking place in Estonia. After completion of the Fourth levelling (2001–2012), all observations were re-analysed and new VVs for BMs were calculated (Papers **II** and **III**).

Sea-level observations have been also used to determine LU values in Estonia (Yakubovski 1973; Pobedonostsev 1975; Vallner et al. 1988; Jevrejeva et al. 2002) but quite controversial results were obtained (Paper **II**). Table 2 gives an overview of the apparent VVs according to the different LU maps for Fennoscandia and Estonia.

**Table 2.** Apparent land uplift (LU) rates visually interpolated from different maps/models and their uncertainties (mm/yr) in Fennoscandia and Estonia.

LU map/model	LU rates		Uncertainty of LU rates
	Maximum	In Estonia	
Ekman (1996)	9.0	0...2.5 <sup>(1)</sup>	±0.2...±0.5
Kakkuri and Chen (1992)	9.0	-1.0...2.5	–
Kakkuri (1997)	9.0	-1.0...2.0 <sup>(1)</sup>	±0.5
Danielsen (2001)	8.0	–	±0.2...±1.0
Vestol (2006)	8.0	0.5...2.0 <sup>(1)</sup>	±0.5
NKG2005LU (Ågren and Svensson 2007)	8.0	-0.5...2.0	±0.5 <sup>(3)</sup>
Johansson et al. (2002) <sup>(4)</sup>	9.0 <sup>(2)</sup>	0.3...3.0 <sup>(2)</sup>	±1.3 <sup>(5)</sup>
Lidberg et al. (2010)	9.0 <sup>(2)</sup>	0.5...3.0 <sup>(2)</sup>	±0.8 <sup>(6)</sup>
Kierulf et al. (2014)	8.0 <sup>(2)</sup>	0...2.5 <sup>(2)</sup>	±0.7 <sup>(7)</sup>
Yakubovski (1973)	–	-1.4...2.2 <sup>(1)</sup>	±0.3
Vallner et al. (1988)	–	-0.8...2.8	±0.3
Randjärv (1993)	–	-1.5...2.0	±0.3
Jevrejeva et al. (2001)	–	-1.2...1.8 <sup>(1)</sup>	±0.2...±0.4
EST2013LU (Paper II)	–	-0.7...2	±0.4
EST2015LU (Paper III)	–	-0.8...2.8	±0.2

<sup>(1)</sup> Only for the NW part of Estonia or Estonian coast.

<sup>(2)</sup> Absolute LU rates are transformed to the apparent ones by the formula  $v_{abs} = (a + v_{app}) \cdot b$ , where  $a = 1.32$  mm/yr and  $b = 1.06$  (Vestøl 2006).

<sup>(3)</sup> Root mean square (RMS) residuals between the model and the GPS and tide gauge observations.

<sup>(4)</sup> LU rates from the “standard solution”.

<sup>(5)</sup> RMS residuals between the GPS velocities and the glacial isostatic adjustment model.

<sup>(6)</sup> From the time series analysis, using CATS software based on power-law noise.

<sup>(7)</sup> From the time series analysis, using CATS software including a combination of white noise and flicker noise.

There are several explanations for VV discrepancies in Table 2. Firstly, since levelling is a relative method, only velocity differences are estimable. In order to obtain apparent VVs, relative VVs have to be tied at least to one known apparent VV. Therefore, LU maps based on levellings depend on the choice of the reference apparent VV. For example, Zhelnin (1958; 1960; 1964; 1966) used apparent VV value 2.5

mm/yr for Tallinn fBM FR241 (VV of Tallinn tide gauge 1.9 mm/yr based on Bikis (1940) was corrected for local subsidence based on levelling data). Vallner and Zhelnin (1975); Vallner (1978) and Vallner et al. (1988) also used the same value. In Vallner and Zhelnin (1975) and Vallner (1978), the value 1.7 mm/yr for Tallinn fBM FR241 was used, combining repeated levellings with Yakubovski's (1973) VVs for tide gauges in Kunda and Vormsi. Randjärv (1993) constrained his map with Yakubovski's VVs for tide gauges in Kronstadt, Salacgriva, Liepaja and Baltiisk, and Tallinn based on Vallner and Zhelnin (1975).

Furthermore, different calculation methods and a set of levelling observations have been used in different studies. For example, data from only two levelling campaigns (usually the First and the latest) were used in earlier studies (Vallner et al. 1988; Randjärv 1993) to calculate the VVs of the BMs, whereas all available levellings were used jointly in the kinematic adjustment for the present study (Paper **II** and **III**).

Secondly, possible reasons for discrepancies concerning LU maps based on the tide gauge observations are: *i*) standard periods for calculations were different; *ii*) a different methodology was used to fill observation gaps; *iii*) rod changes and adding zero corrections, levelling ties to the BMs, height system changes, additional corrections (hydrological, meteorological, etc.) were handled by different approaches (Paper **II**).

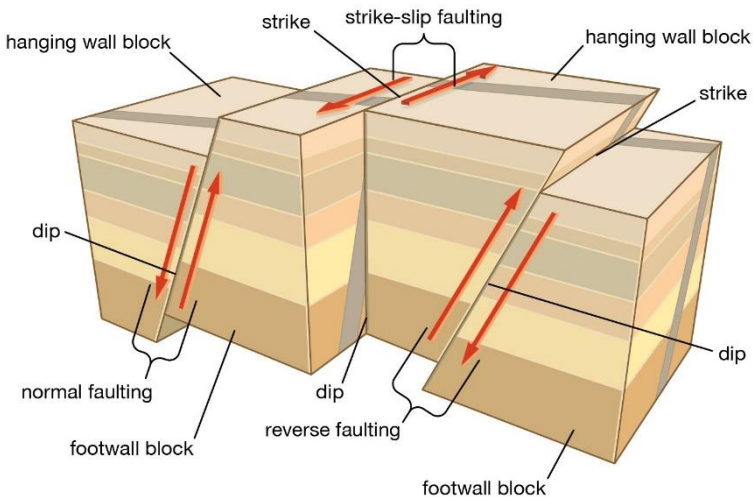
Thirdly, LU models based on GNSS velocities, in turn, depend on the following error sources: different processing strategies and software packages; different lengths of the observation time series; offsets in data; satellite orbit and antenna model errors; troposphere modelling errors; GNSS antenna phase centre variation and offset modelling errors; site dependent errors (e.g. snow and ice on the antenna radome); different reference frames can contaminate GNSS velocity estimates or increase the uncertainty of the estimated velocities (Johansson et al. 2002; King et al. 2010; Lidberg et al. 2010). For example, differences between reference frames ITRF2000 and ITRF2005 cause a difference in absolute VVs up to 1 mm/yr (Lidberg et al. 2010).

In addition, interpolation errors to draw up LU maps by hand or to produce gridded models of LU, errors in the estimates of the secular sea level and geoid change in order to transform absolute VVs to apparent ones can produce different VV estimates. Discrepancies between different LU maps for Estonia are presented in Paper **II**.

## 1.5. Tectonic movements of the Earth's crust

### 1.5.1. Tectonic faults

Tectonic<sup>3</sup> processes inside the Earth have caused several deformations of the Earth's crust, which in general, are called tectonic faults. Forces deforming crust can be either *i*) tensional forces; *ii*) compressional forces; or *iii*) shear forces. Tensional forces cause deformations in the upper crust called normal faults, compressional forces cause reverse or thrust faults, and shear forces strike-slip faults (Turcotte and Schubert 2002), (Figure 7).



**Figure 7.** Types of tectonic faults: normal fault; strike-slip fault; and reverse or thrust fault (Fault 2015).

Determination of a displacement direction in the faulting zone allows determination of the fault type, since the direction of a displacement along the fault is parallel with a dipping plane. With the normal and reverse faults, both horizontal and vertical displacements take place when with the strike-slip fault, only horizontal strain occurs (Turcotte and Schubert 2002).

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<sup>3</sup> Tectonics is a science studying deformation of the rocks forming Earth's crust and forces producing such deformations (Tectonics 2015).

Most known tectonic faults are the boundaries between the rigid lithospheric plates, i.e. tectonic plates. The recent plate tectonics model NNR-MORVEL56 gives a set of angular velocities for 56 tectonic plates, which altogether describe motions of 99.8% Earth's surface (Argus et al. 2011). But many regional tectonic faults also occur; for example, Pernicana normal fault in Italy (Obrizzo et al. 2001); tectonic fault in Apennine, Italy (D'Anastasio et al. 2006); tectonic faults at Betic Cordillera, Bajo Segura and San Miguel in Spain (Giménez et al. 2000, 2009).

Displacements of many tectonic faults are related to the earthquakes. In general, displacements in the fault zones occur during the earthquake, between the earthquakes, a fault usually is locked and stress is accumulating up to the level which causes rupture of the fault (Turcotte and Schubert 2002). However, stress accumulated with the constant slow motion of the blocks can also release an earthquake. It has been shown that an accumulated displacement of  $\sim 1$  meter responds to the earthquake with the magnitude  $M \approx 6$  (Wells and Coppersmith 1994).

Displacements and related earthquakes in the fault zones have been widely studied all over the world by various geodetic methods (length measurements, levelling, GNSS and DORIS satellite systems, VLBI), (Jian Lin and Stein 1989; Savage and Lisowski 1995a; Hodgkinson et al. 1996; 1995b; Giménez et al. 2000; Obrizzo et al. 2001; Langbein and Bock 2004; Titus et al. 2005; D'Anastasio et al. 2006; Williams et al. 2006; Giménez et al. 2009). Focus has been on the relationship between the earthquakes and the geological faults in Estonia also (Bulin 1978; Sildvee 1988; Nikonov and Sildvee 1991; Sildvee 1991; Sildvee and Vaher 1995; Nikonov 2002, 2011).

### **1.5.2. Tectonic faults and related vertical crustal movements in Estonia**

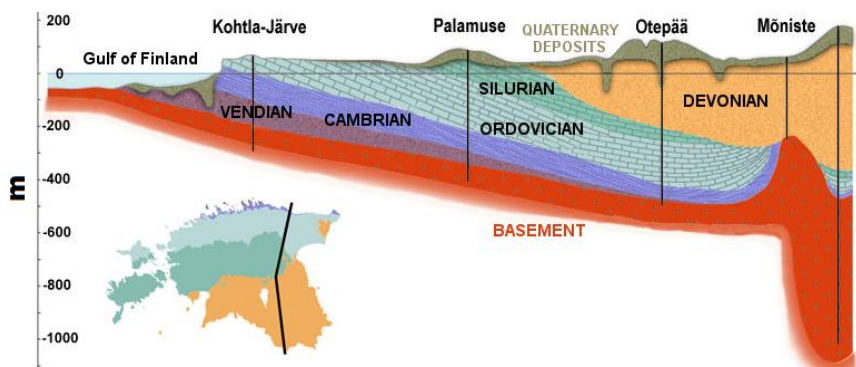
Researchers' interests in Estonia have focused on the reasons of VCMs beside postglacial rebound, which would explain irregularities of the VCMs detected from the repeated levellings since the beginning of the studies in the 1960s. Several studies cover correlation between the VCMs and tectonic structures (Zhelnin 1965; Heinsalu and Sildvee 1971; Sildvee 1973; Zhelnin et al. 1975; Sildvee and Miidel 1978, 1980; Vallner et al. 1988). In addition, studies about correlation between earthquakes

and tectonic structures in Estonia were mentioned in the previous section.

### 1.5.2.1. Geological overview of Estonia

Until the 1950s, geological structure of Estonia was treated as a very simple, almost a monoclinical structure. In the later studies, a typical tectonic structure with blocks and faults in the crystalline basement as well as in the sedimentary cover was discovered (Vaher et al. 1962; Pobul and Sildvee 1975; Puura 1979; Raukas and Teedumäe 1997).

The upper part of the crust of Estonia can be divided into three layers: *i)* Precambrian crystalline basement; *ii)* 100–800 m thick cover of Vendian, Cambrian, Ordovician, Silurian and Devonian sedimentary rocks; and *iii)* 0–200 m thick surface layer of Pleistocene Quaternary deposits (Figure 8).



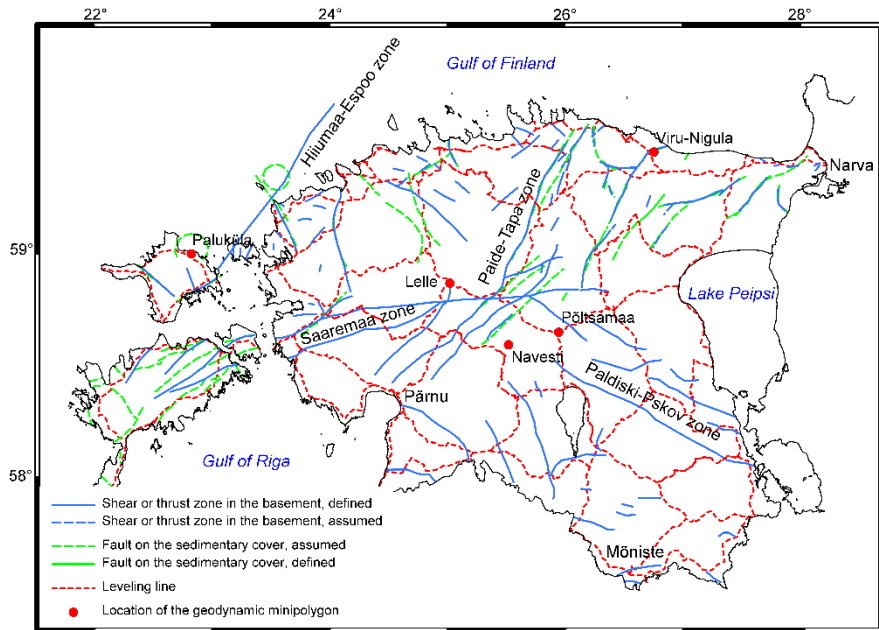
**Figure 8.** Geological profile of Estonia from north to south (Roosaare et al. 2000). Modified by the author.

The Precambrian basement includes two large units: the orogenic Svecofennian complex of plutonic and metamorphic rocks and the anorogenic complex of plutonic rapakivi granites and related rocks. According to the drilling data, the surface of the crystalline basement deepens quite smoothly from north to south with the tilt 2–4 m/km with some uplifts in the basement, for example at Mõniste (Figures 8 and 9) (Pirrus 2001; All et al. 2004; Soesoo et al. 2004; Raukas and Teedumäe 1997). Sedimentary rock cover mainly consists of sandstone, siltstone, limestone, dolomite, marl and clay (Sildvee and Vaher 1995).



Composition of Quaternary deposits varies greatly depending on the relief of the bedrock. Main material is moraine, second important materials are sand and clay, but gravel, mud, lime, peat, plateau gravel etc. are also present (Kajak 1999; Pirrus 2001).

Inside the Svecofennian block, several faults from different origin, age and size appear, locations of which are mainly determined by magnetometric, gravimetric and seismic observations. Main fault is the Paldiski-Pskov shear zone (Figure 9), which according to the gravimetric and magnetic modelling, is the reverse fault (Soesoo et al. 2004). Along this shear zone, one of the Estonian geodynamic minipolygons – the Põltsamaa-Lelle geodynamic line – is situated (Figure 9) where several geodetic measurements (levelling, gravimetry, GPS) have been conducted (Paper I).



**Figure 9.** Faults of the crystalline basement and sedimentary cover in Estonia. Data from the Geological Survey of Estonia ([http://kaart.egk.ee/geol\\_kaart/](http://kaart.egk.ee/geol_kaart/)), according to All et al. (2004) and Soesoo et al. (2004). Locations of the precise levelling lines of national height network and geodynamic minipolygons have been added.

Another remarkable tectonic fault of the crystalline basement is the Middle-Estonian or Saaremaa shear zone (Figure 9) running from the island of Saaremaa to the East. It is thought that a large vertical

displacement (several km) between the blocks exists there. This fault is intersecting with the Põltsamaa-Lelle geodynamic line also. Other considerable tectonic faults are the NW–SE directional Paide-Tapa transform fault zone (Figure 9, also cutting the Põltsamaa-Lelle geodynamic line) and the NE–SW directional Hiiumaa-Espoo shear zone (All et al. 2004; Soesoo et al. 2004).

Faults in the sedimentary rock cover are mostly 0.5–5 km wide and 5 to 100 km long and run mainly in the S–N or SW–NE direction (Figure 9). Vertical displacement between the blocks is from 5 to 50 m. Sedimentary cover is less disrupted by tectonic movements than the crystalline basement. Opposite the crystalline basement, many faults in the sedimentary cover are detected by drilling, mainly in the mining area at the NE part of Estonia (Lutt and Raukas 1993; Pirrus 2001; Sokman et al. 2008). One of the Estonian geodynamic minipolygons, the Viru-Nigula minipolygon (Figure 9), was established there by the geodesy group of the Institute of Physics and Astronomy of the Estonian Academy of Sciences to detect horizontal and vertical crustal movements related to the Aseri fault zone.

#### **1.5.2.2. Vertical crustal movements from repeated levellings associated with the tectonic structures in Estonia**

As was mentioned above, in several studies, correlation between the VCMs from the repeated levellings and the geological structures of Estonia has been established. No clear correlation between the VCMs and the block structure of the crystalline basement was found by Sildvee and Miidel (1978) and Sildvee and Miidel (1980). Five velocity planes of the VCMs (south-eastern, middle, north-eastern, north-western and western Saaremaa), moving parallel or under a small angle relative to each other, were determined by Vallner et al. (1988). They found also that the annual VV planes calculated coincide with the regional gravity structures and that the VCMs in Estonia follow block movements. Nevertheless, comparing LU maps for Estonia with the block structure of the crystalline basement, Vallner et al. came to the same conclusion as Sildvee and Miidel that no clear correlation exists between them. They found no accordance between the VCMs and the structure of the sedimentary cover either.

On the other hand, correlation between the VCMs and the location of tectonic fault zones has been found based on the profiles of accumulated VVs of BMs along the levelling lines (e.g. Figure 12) cutting fault zones. The correlation coefficient up to 0.7 between the locations of the rapid change of the VVs and geologic structures was found for some levelling lines (Sildvee 1973; Zhelnin et al. 1975). The correlation between VVs and tectonic structures appeared most clearly on the Pärnu-Narva fault zone (Orviku 1960) (Figure 9), where in a 20–40 km wide zone, the largest gradient of the VCMs ( $\sim 0.05$  mm/yr per km) in Estonia has been observed by Zhelnin (1966); Vallner and Zhelnin (1975) and Vallner et al. (1988). The stream beds of the rivers in the Pärnu River basin have been also influenced by vertical crustal movements in the Pärnu-Narva fault zone (Müidel 1966, 1991; Sildvee et al. 1973; Torim and Sildvee 2002).

Further, based on the profiles of accumulated VCMs, along the levelling lines, correlation between the rapid changes of VCMs, gravity changes and tectonic faults have also been found within the levelling lines intersecting the Paldiski-Pskov and the Saaremaa fault zone (Figure 9). In addition, based on the steep changes of the VVs along the levelling lines, Tapa basement block was clearly distinguished by Vallner et al. (1988). On the background of the general postglacial LU, differential block movement has been identified in Pärnu, Kilingi-Nõmme, Lelle, Tapa, Tamsalu, Tõrva, Saaremaa, Matsalu Bay, Põltsamaa, and Valga areas (Vallner and Zhelnin 1975; Vallner et al. 1988) (Figure 11). According to the authors, the VCMs detected in these areas coincide with the borders of the crystalline basement blocks and could not be explained just by levelling errors.

Based on the methodology described by Giménez et al. (2000), graphs of the cumulative VVs of the BMs along the levelling lines of the Estonian levelling network were analysed by Vilde (2013). Correlation between the VVs, gravity anomalies and tectonic faults was searched. In contrast to the previous similar studies, data from the Fourth precise levelling (2001–2012) were also included into the calculation of the profiles of cumulative VVs. Vilde concluded that there was clear correlation between the rapid changes of the VVs of the BMs and changes in the gravity anomalies on Põltsamaa–Abja-Paluoja, Koidula-Tartu, Haapsalu-Tallinn, and Pärnu-Haapsalu levelling lines. Also, correlation between the VVs of the BMs and the locations of the

tectonic faults were identified on Jõhvi-Tapa, Tallinn-Tapa and Põltsamaa-Lelle levelling lines (Vilde 2013) (Figure 11).

To summarise, in previous studies, however, no clear correlation between the locations of the rapid change of the VVs and geologic structures was found. Firstly, the reason is that levellings have been repeated too infrequently. Secondly, such correlation could be also fortuitous, caused by the levelling errors or BM instability (Zhelnin et al. 1975; Rüdja 2004). Note also that the crystalline basement in Estonia is covered with the sedimentary rocks and Quaternary sediments which redistribute the tension caused by the possible movements of the crystalline basement blocks (Raukas 1978; Rüdja 2004).

### **1.5.2.3. Studies on the geodynamic minipolygons**

Since previous studies showed a link between the anomalies of the VVs of the BMs, changes in gravity anomalies and location of the tectonic faults, three geodynamic minipolygons (in Navesti, Viru-Nigula and Paluküla (Figure 9)) were established in 1970–1974 by the geodesy group of the Institute of Physics and Astronomy of the Estonian Academy of Sciences to investigate this correlation in detail. For the same reason, repeated levellings were started on the Põltsamaa-Lelle levelling line (Figure 9) in 1963 by the Estonian Academy of Sciences. After Osmussaare earthquake (Nikonov 2002) in 1976, Nõva geodynamic minipolygon in NW Estonia was established in order to investigate the earthquake-related VCMs (Vallner et al. 1975; Torim 1978; Zhelnin and Torim 1981; Torim 1989; Torim and Sildvee 2002).

Four levellings were performed on the Paluküla polygon (1972, 1975, 1977 and 1982) but no significant VCMs were detected. In the Navesti polygon, eight repeated levellings (1971 (twice), 1973 (twice), 1976, 1980, 1983 and 1987) were carried out and VCMs up to 0.9 mm/yr were detected in the parts of the polygon where the riverbed of the Navesti River was deepened. Three precise levellings and seven length measurements were performed on the Viru-Nigula polygon between 1975 and 1988. Horizontal movements up to 1 mm/yr and VCMs up to 0.3 mm/yr were detected. Altogether 11 precise levellings (1936, 1961, 1964, 1965, 1966, 1969, 1972, 1980, 1982, 1987, and 2006) were performed on the Põltsamaa-Lelle geodynamic line and several changes of the VCMs along the line related to hydrologic reasons, changes on gravity anomaly and tectonic faults were detected. For vertical

displacements induced by the Osmussaare earthquake were estimated up to 2 mm on the Nõva polygon (Vallner et al. 1975; Torim 1978; Zhelmin and Torim 1981; Torim 1989; Torim and Sildvee 2002; Kall and Oja 2006).

## 2. AIM OF THE STUDY

The aim of this study was:

- To detect vertical crustal movements in Estonia from the precise levellings of the Estonian levelling network and find out possible changes of vertical crustal movements in time.

The following research questions and hypothesis were set up:

- i)* What is the long-term VV signal of the BMs of the Estonian levelling network considering latest re-levelling data from 2001 to 2011?
- ii)* What are the differences between adjustments of different levelling combinations and mathematical models parametrizing VVs of the BMs with and without heights?
- iii)* Are the VVs of the BMs constant in time?
- iv)* Hypothesis: the highest gradient of the VCMs in Estonia is across the Pärnu-Narva geological fault (hinge line).

In order to answer the research questions and test the hypothesis, the following tasks were fulfilled:

- Two networks for kinematic adjustment were formed: *i)* levelling network of fundamental benchmarks (LNF) to determine the postglacial rebound in Estonia; and *ii)* common levelling network (CLN) in order to estimate change of the VVs of the BMs in time.
- Levelling standard errors were estimated through *i)* random and systematic levelling errors; and *ii)* loops' misclosures. Next, these estimates were used to calculate the weights of height differences.
- The graphs of the cumulative VVs of the BMs along the levelling lines and time series of the height differences of the levelling sections were estimated in order to detect unstable BMs.
- The long-term VVs of the BMs on the Põltsamaa-Lelle levelling line were determined based on the time series of the observed

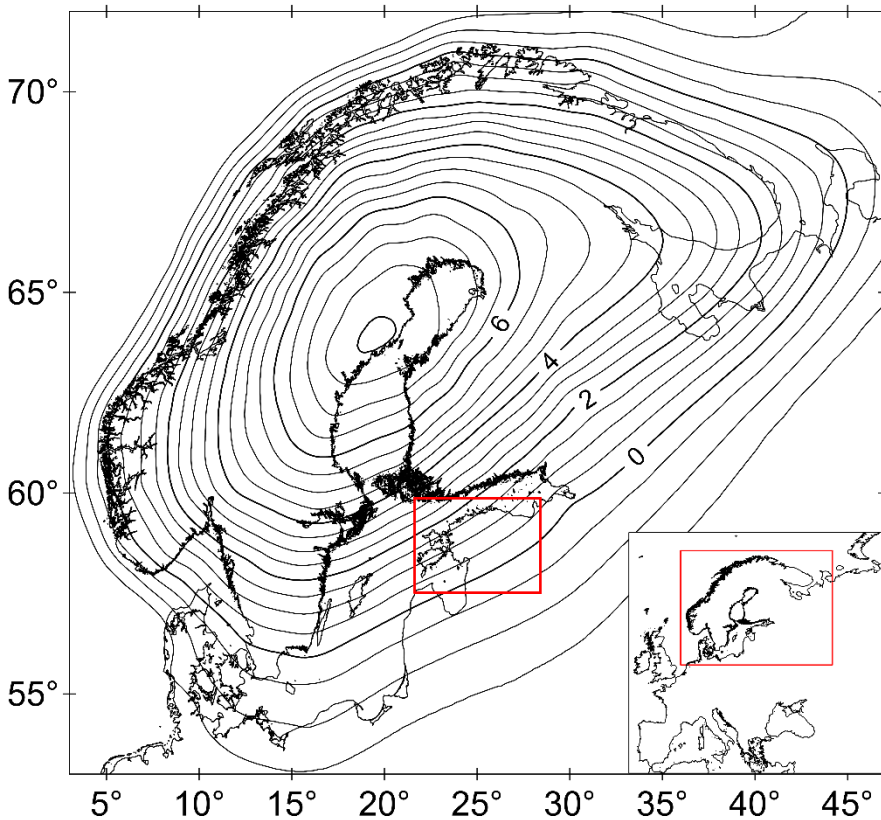
height differences. Obtained long-term VVs were compared with the values from the other results.

- Kinematic adjustments of the LNF and the CLN were performed in order to determine the VVs of the BMs. The outliers among the observations were detected and removed or downweighted.
- VVs of the BMs and accuracy estimates of the kinematic adjustment from different mathematical models and levelling combinations were compared.
- The change of the VVs of the BMs in time was evaluated by ANOVA test and multivariate analysis. Variance component estimation of the levelling campaigns used was performed.
- Land uplift models EST2013LU and EST2015LU were compiled based on the VVs of the BMs. Accuracy of the obtained models was estimated (model's residuals, cross-validation, comparison of gridding techniques). Models were compared with each other and with the other results based on the repeated levellings, tide gauge, and GNSS observations.

### 3. MATERIALS AND METHODS

#### 3.1. Levellings of the Estonian levelling network

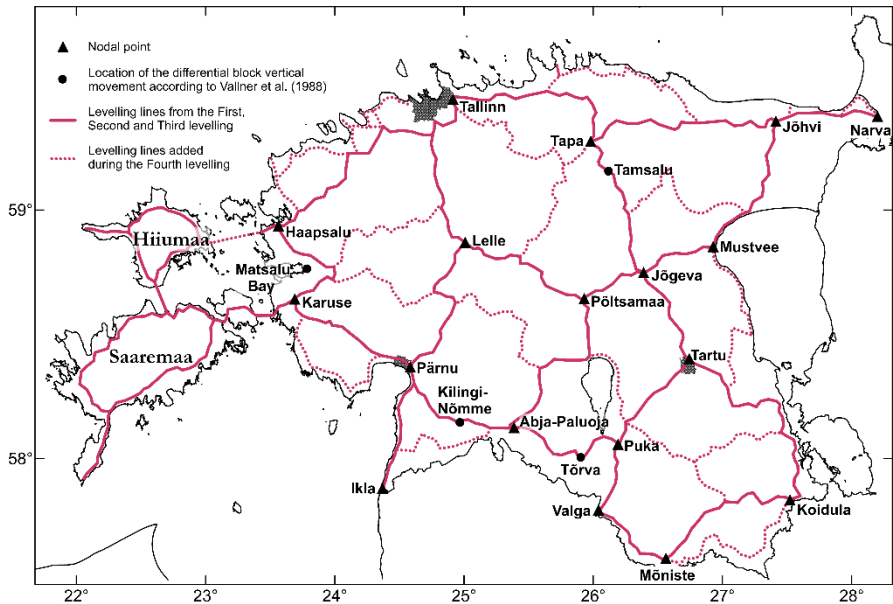
Estonia is located on the southern slope of the Fennoscandian (Baltic) Shield, in the north-western part of the East European Plain, on the eastern coast of the Baltic Sea. The country is located at the south-east edge area of the Fennoscandian postglacial rebound. According to the recent empirical rebound model NKG2005LU (Ågren and Svensson 2007), apparent LU rates relative to sea level remain within  $-0.5$  to  $2$  mm/yr in Estonia (Figure 10).



**Figure 10.** Location of the study area in relation to the region of the Fennoscandian postglacial rebound according to the apparent land uplift model NKG2005LU (Ågren and Svensson 2007). Isolines of the land uplift are in mm/yr.



Modern renovated precise Estonian levelling network (Figure 11) is based on the network established by the Cadastre Department of the Ministry of Agriculture of the Estonian Republic in 1933–1943. The establishment of height networks in countries around the Baltic Sea was initiated by the Baltic Geodetic Commission in 1932 (Torim 1993).



**Figure 11.** Configuration of the precise levelling network of Estonia in 2012. Solid lines are the levelling lines from the First, Second, and Third levelling; dotted levelling lines were added during the Fourth levelling.

The following chapters present an overview of levellings of the Estonian levelling network.

### 3.1.1. First levelling: 1933–1943

The first precise Estonian levelling network levellings were performed in 1933–1943. In 1943, the network on the mainland consisted of six loops (~1800 km levelling lines) with 1151 BMs. The network consisted mainly from wall BMs. In the nodal points of the lines, 23 fundamental BMs were seated at the depth of 2.5 m. An additional levelling loop was established on the island of Saaremaa, which was connected with the mainland by levelling on ice and short-term sea-level observations in 1940. The network was not established on the island of Hiiumaa (Tamm 1992; Torim 1993).

Levellings were classified as the high precision and precise levellings at that time. The six loops on the mainland were levelled with a high precision methodology. Zeiss III and Zeiss A optical levels with a micrometer were used together with 3-m invar rods with 5-mm graduation. Metal stakes or level rod turning plates (turtles) were applied as the rod bases at the station. Equal sight lengths max 50 m were determined with the metal wires or tapes (Torim 1993). Obtained average levelling standard errors were: random error  $\eta = \pm 0.32 \text{ mm}/\sqrt{\text{km}}$  and systematic error  $\sigma = \pm 0.03 \text{ mm/km}$ .

The levelling loop on the island of Saaremaa (Figure 11) was levelled with a precise methodology characterized by the levelling standard errors: random error  $\eta = \pm 1.00 \text{ mm}/\sqrt{\text{km}}$  and systematic error  $\sigma = \pm 0.20 \text{ mm/km}$ . Zeiss II optical levels together with the cm-graduated wooden rods were used. Again, equal sight lengths max 50–70 m were determined with the tape, metal wire or by an optical device (Torim 1993). Considering used equipment, methodology and obtained levelling standard errors, levellings on the island of Saaremaa correspond to the third order in today's terms (Vaníček et al. 1980).

### 3.1.2. Second levelling: 1948–1969

Major part of the Second levellings was done by the Estonian Academy of Sciences. The extent of the levelling lines was ~2100 km (Torim and Jürma 2011). Majority of the lines of the First levelling period were re-levelled, some lines more than once. This time levelling loop was established on the island of Hiiumaa also. In addition, four geodynamic minipolygons were established and levelled (Figure 9, Section 1.5.2.3). Connections between the network on the mainland and the loop on Saaremaa were determined by short-term sea-level observations (August 1963) and hydrostatic levelling in 1968. Between the islands of Saaremaa and Hiiumaa, short-term sea-level observations (August 1958) and long-term tide gauge observations (1945–1958) were used (Tamme 1972; Tamm 1992; Torim 2009).

Compared to the First levelling campaign, a slightly different levelling methodology was used. At the beginning of the Second levelling campaign, one-way levelling with two optical plumb level instruments (Zeiss A, Zeiss Ni 004, HA-1), together with the 5-mm graduation invar rods, was employed. Maximum sight length was 50 m. Later, the fore

and back levelling methodology and Zeiss automatic level Ni 007 instruments were adopted. Maximum sight length in that case was 40 m due to a lower magnification of the telescope of the level. Precision of the levellings done by the Estonian Academy of Sciences in 1950–1969 is characterized by a random error  $\eta = \pm 0.49 \text{ mm}/\sqrt{\text{km}}$  and a systematic error  $\sigma = \pm 0.07 \text{ mm/km}$  (Torim 2009).

Additionally, the Narva-Tallinn-Pärnu-Ikla line (505 km, Figure 11) was levelled by the General Committee of Geodesy and Cartography of the Soviet Union in 1948, in order to connect the Baltic States of the Soviet Union to the Kronstadt height system. Standard errors of these levellings were: random error  $\eta = \pm 0.56 \text{ mm}/\sqrt{\text{km}}$  and systematic error  $\sigma = \pm 0.06 \text{ mm/km}$  (Torim 2000).

### 3.1.3. Third levelling: 1970–1996

During the Third levelling campaign, majority of the lines of the First and the Second levelling campaigns were re-levelled by the Estonian Academy of Sciences (altogether ~1900 km). The fore and back levelling methodology along the same rod bases (50-cm long metal stakes) with the maximum sight length of 50 m was used. Zeiss automatic level instrument Ni 007 and invar rods with a 5-mm graduation were applied. Hydrostatic levelling was used in order to connect the levelling loop on the island of Saaremaa with the mainland in 1978. Additionally, short-term sea-level observations were performed in March 1979. To connect the islands of Saaremaa and Hiiumaa, short-term sea-level observations (summer 1971) and hydrostatic levellings in 1976 and 1977 were performed (Tamm 1988, 1992; Torim 2009). Precision of the levellings by the Estonian Academy of Sciences in 1970–1996 is characterized by a random error  $\eta = \pm 0.42 \text{ mm}/\sqrt{\text{km}}$  and a systematic error  $\sigma = \pm 0.05 \text{ mm/km}$  (Torim 2009).

Some lines of the levelling network were measured by the General Committee of Geodesy and Cartography of the Soviet Union as well. In 1970, the Narva-Tallinn-Pärnu-Ikla line was repeated ( $\eta = \pm 0.53 \text{ mm}/\sqrt{\text{km}}$ ,  $\sigma = \pm 0.06 \text{ mm/km}$ ). The General Committee of Geodesy and Cartography of the Soviet Union also performed additional levellings (~900 km) in 1981–1983 by repeating lines

previously levelled by the Estonian Academy of Sciences on the Estonian mainland. Standard errors of the levellings by the General Committee of Geodesy and Cartography of the Soviet Union in 1981–1983 were: random error  $\eta = \pm 0.71 \text{ mm}/\sqrt{\text{km}}$  and systematic error  $\sigma = \pm 0.10 \text{ mm}/\text{km}$  (Torim 2000). Basic configuration of the levelling network between 1933 and 1996 is presented in Figure 2 in Paper II.

#### 3.1.4. Fourth levelling: 2001–2012

The work with the latest re-levelling was started with inspection of the levelling lines in 2001. It appeared that 48% of the historical BMs were broken, could not be found or had been destroyed. Preserved or broken BMs were repaired and additional new BMs were installed to replace the destroyed or missing ones. Altogether 928 new BMs were installed (Torim and Jürma 2007). Compared to the historical levelling network, additional levelling lines were established in order to achieve smaller and more evenly distributed loops. New connections between the mainland and the islands of Saaremaa and Hiiumaa were established using hydrodynamic levelling in 2010 (Liibus et al. 2013). The renovated levelling network consists of 26 loops with the length of  $\sim 3700 \text{ km}$  (Figure 11; Figure 3 in Paper II). The last precise levelling was carried out using Trimble/Zeiss digital level DiNi 11 and 12 and NEDO 3-m invar rods with code bars calibrated twice per year at the Finnish Geodetic Institute. Lines were levelled fore and back in different halves of a day using the same rod bases (50-cm long metal stakes) with the maximum allowable sight length of 40 m. However, actual sight length of the measurements was  $\sim 30 \text{ m}$  on average. At the same time, meteorological data (temperature of the rods and air, air pressure, humidity, direction and speed of the wind) were recorded. Work ordered by Estonian Land Board was carried out by the corporation “Planserk” under the direction of Dr. Andres Rüdja. Mean standard errors of the Fourth levelling campaign were: random error  $\eta = \pm 0.18 \text{ mm}/\sqrt{\text{km}}$  and systematic error  $\sigma = \pm 0.04 \text{ mm}/\text{km}$  (Torim and Jürma 2011; Maaamet: Geoportaal: Kõrgusvõrk (Estonian Land Board: Geoportal: Levelling network) 2013).

According to the author’s knowledge, rod temperature and scale corrections have been added to the observations of the Second and the Third levelling (Dr. Ants Torim, personal communication). The height differences of the First and the Fourth levelling were without rod

temperature and scale corrections. The First levelling was conducted with uncalibrated levelling rods (Dr. Andres Rüdja, personal communication). Final rod calibration results for the Fourth levelling were not available to the author at the time of levelling data preparation. Regarding refraction corrections, their influence on the VV values is inconsiderable and has a random character in Estonia (Vallner 1965, 1978). Further, observations without gravity correction were used for all levelling campaigns. However, a small systematic error due to ignoring gravity anomalies is eliminated from the VV calculations when height differences are differentiated in the completely re-levelled network and re-levelled segments follow close paths (cf. Section 3.2.2), which was the case in the common levelling network (CLN).

### **3.2. Formation of the levelling network of the fundamental benchmarks and common levelling network**

From the levelling data described above, two levelling networks were established in order to fulfill the study tasks: *i)* the network of the BMs and height differences common to all four levellings, so-called common levelling network (CLN) presented in Paper **III**; *ii)* the network of the selected, presumably stable BMs (mostly fundamental BMs), which have been levelled at least twice, so-called levelling network of the fundamental BMs (LNF) (Paper **II**). Both networks have their own advantages and disadvantages:

- CLN gives a possibility to calculate levelling loops' misclosures (both height and VV differences) and estimate *a priori* levelling standard errors by height difference misclosures and *a priori* mean standard errors of the BMs' relative VVs by relative VV misclosures;
- CLN enables calculation of VVs: *i)* from two different mathematical models: "heights included" and "heights eliminated" model; and *ii)* from different combinations of the four levellings;
- CLN enables evaluation of the change of the BMs' VVs in time;
- Correlation between the repeated levellings can be determined in the CLN (multivariate analysis);

- Levelling data in addition to the four levellings cannot be used in the CLN; neither can potential stable BMs non-levelled in all four levellings be used;
- LNF allows inclusion of all levelling observations between selected BMs, even the height difference measured just once, therefore increasing the degrees of freedom and reliability of the network;
- Only stable (presumably) BMs can be selected to establish the LNF;
- Using the “heights eliminated” mathematical model in the kinematic adjustment of the LNF is impossible, since all height differences cannot be differentiated (design matrices of the levellings are different). Note that possible systematic levelling errors could be eliminated or reduced using the “heights eliminated” model (Cross et al. 1987; Mäkinen and Saaranen 1998).

Principles and the process of forming the LNF and the CLN are presented in Papers **II** and **III**. Some aspects of establishing the LNF and the CLN, which were not presented in the papers, are discussed in the following chapters.

### **3.2.1. Detection of unstable benchmarks using profiles of cumulative vertical displacements**

According to Brown and Oliver (1976), unstable BMs can be easily detected by their unusually large and singular movements. Unstable BMs contaminate the VV calculation based on the assumption of the constant velocity.

Vertical displacement of the BM B ( $\dot{H}_B$ ) in reference to BM A ( $\dot{H}_A$ ), the displacement of which is usually taken zero, can be calculated by differentiating levelled height differences between the same BMs (Vaniček and Krakiwsky 1986):

$$\dot{h}_{AB} = \Delta h(t_2) - \Delta h(t_1), \quad (1)$$

where  $\dot{h}_{AB}$  is the increment of the vertical displacement of BM B in relation to BM A;  $\Delta h(t_1)$  and  $\Delta h(t_2)$  are height differences between BMs A and B measured at the epochs  $t_1$  and  $t_2$ , respectively. Vertical displacement of BM B ( $\dot{H}_B$ ) is obtained from the relation:

$$\dot{H}_B = \dot{H}_A + \dot{h}_{AB}. \quad (2)$$

The systematic error  $\sigma$  (mm/km) of the levelling line with the length  $L$  was calculated using the accumulated height difference discrepancies  $\sum d$  of the fore and back levellings according to the Lallemand (1889) formula:

$$\sigma = \frac{\sum d}{L}. \quad (3)$$

According to the Lallemand (1889) formula, the levelling random error  $\eta$  (mm/ $\sqrt{\text{km}}$ ) for the line with length  $L$  is:

$$\eta = \frac{1}{3} \sqrt{\frac{\sum_{i=1}^n \frac{d_i^2}{l_i} - \frac{\sum d^2}{L}}{n}}, \quad (4)$$

where  $d_i$  (mm) is the discrepancy of fore and back levelling height difference in the section with the length  $l_i$  and  $n$  is the number of BMs in the levelling line.

The standard deviation  $SD$  of the height difference in section  $i$  with the length  $l_i$  is (Bomford 1980):

$$SD_{\Delta h_i} = \sqrt{\eta^2 l_i + \sigma^2 l_i^2}. \quad (5)$$

From the standard deviations of the height differences  $\Delta h(t_1)$  and  $\Delta h(t_2)$ , standard deviation of the increment  $\dot{h}_{AB}$  of the vertical displacement of BM B in relation to BM A can be calculated:

$$SD_{\dot{h}_{AB}} = \sqrt{SD_{\Delta h(t_1)}^2 + SD_{\Delta h(t_2)}^2} . \quad (6)$$

Standard deviation of the cumulative vertical displacement of BM B in relation to BM A is:

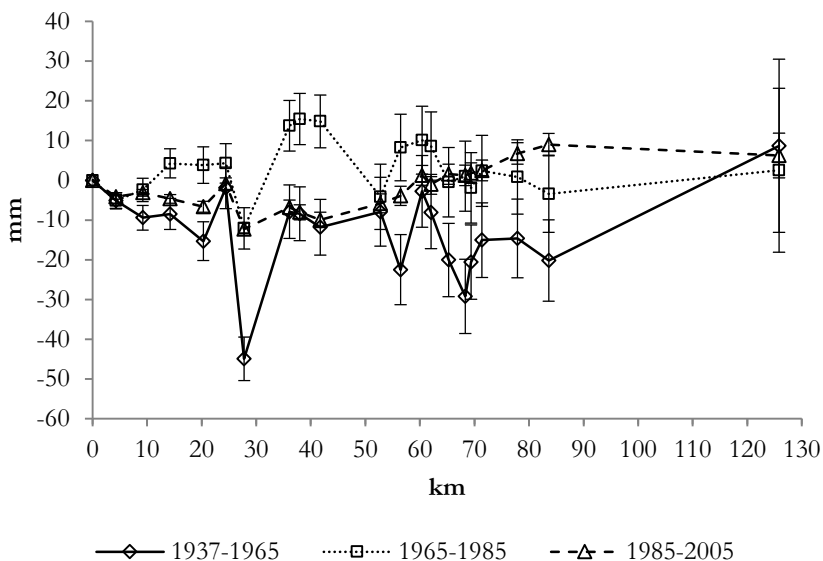
$$SD_{\dot{H}_B} = \sqrt{SD_{\dot{H}_A}^2 + SD_{\dot{h}_{AB}}^2} , \quad (7)$$

where standard deviation of the reference BM A  $SD_{\dot{H}_A}^2$  is usually taken zero. Vertical displacement of a BM B with respect to a BM A was considered statistically significant ( $\alpha = 0.05$ ) when  $\dot{H}_B \geq 2SD_{\dot{H}_B}$ .

To analyse BMs' movements, profiles of cumulated vertical displacements along the levelling lines with accumulated standard deviations of the displacements were compiled according to Vaníček and Krakiwsky (1986), e.g. Figure 12.

The BMs with significantly large displacements forming spikes on the profiles usually indicate local anomalies of vertical displacements (Giménez et al. 2000). Negative spikes (like large negative spike between the 25 and 30 km in Figure 12) are associated with the near surface processes like ground water lowering, compression of the sediments, subsidence of buildings, mining, heavy traffic (Ellenberg 1987; Giménez et al. 2000). Positive spikes may result from human activity related displacements like construction works, levelling errors, but also due to the near surface processes like frost heave (Giménez et al. 2000).



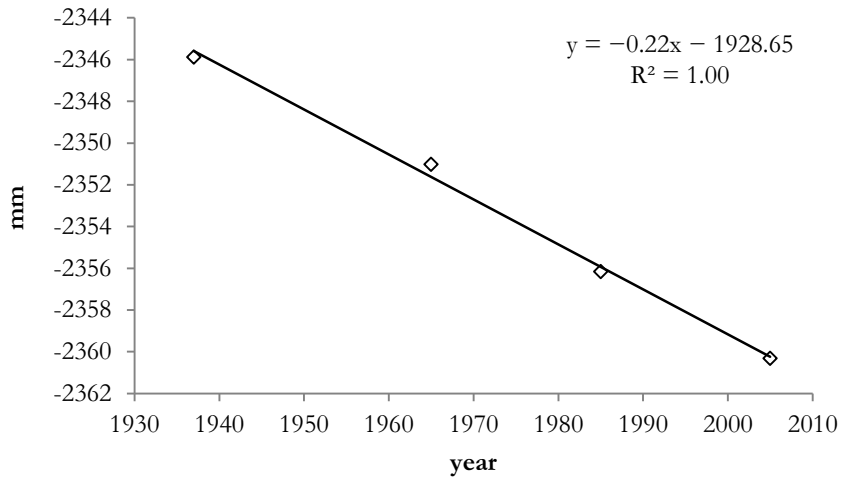


**Figure 12.** Profile of the cumulative vertical displacements along the Haapsalu-Tallinn levelling line between the levellings in 1937, 1965, 1985, and 2005. Error bars represent cumulative standard deviation of the displacement. Note a large negative spike between the 25 and 30 km, which indicates unstable wall benchmark mounted on the foundation of the small farmhouse.

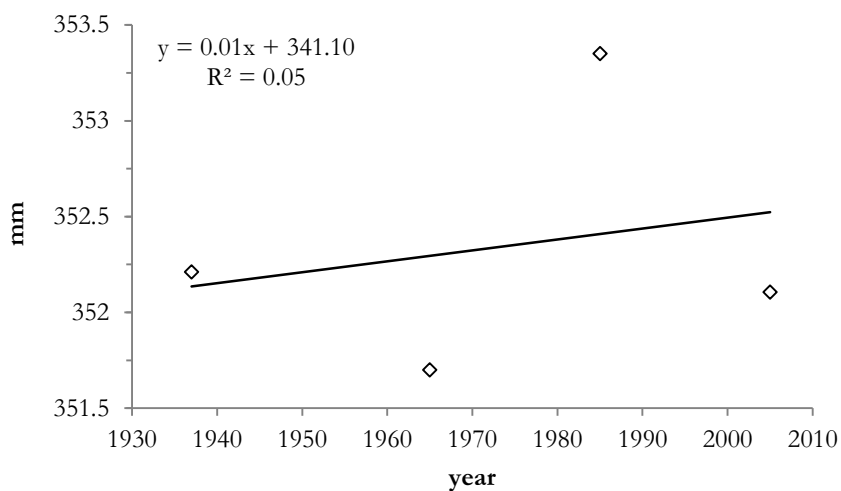
On the other hand, positive spikes can arise from halokinetic processes and heavy buildings (e.g. old churches) where sediments below the building are more compressed than those of its surroundings (Ellenberg 1987). Therefore, spikes on the profiles are not related to the postglacial rebound or tectonic causes and such BMs were eliminated from the networks whenever possible by summing up the adjacent height differences. However, studies have found that majority of BMs are not affected by abovementioned factors and are stable or relatively stable, based on the BMs height position on the plot of cumulative vertical displacements compared to adjoining areas (Lilienberg and Setunskaya 1969; Ellingwood and Holdahl 1972; Lilienberg et al. 1988).

In addition, a time series of height differences for each levelling section were composed. If constant VV for BMs is assumed, then the change of the height difference in the section with time should be linear (Figure 13). Change in the linear slope between two levelling epochs can be observed in the case of an uneven VV (Figure 14). Again, sections with irregular change of the height difference were excluded from the

networks if possible by summing up the adjacent height differences (as many as needed to change the trend line of the scatterplot as linear as possible).



**Figure 13.** Height difference change in time in the Haapsalu-Tallinn levelling line in a section between the BMs FRAM-20 and SR406.

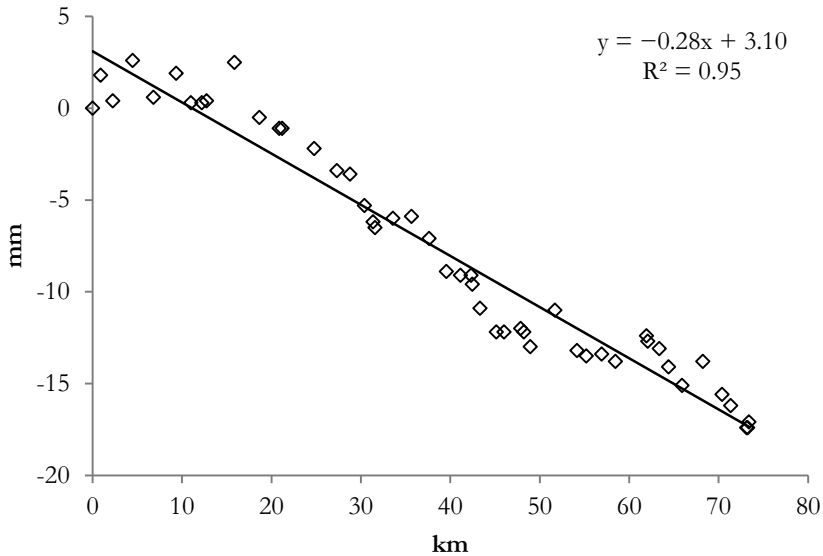


**Figure 14.** Height difference change in time in the Haapsalu-Tallinn levelling line in a section between the BMs PR388 and PR387. If possible, the sections with irregular height difference change were eliminated from the network by summing up the adjacent height differences.

### 3.2.2. Sources of systematic errors in levelling observations

Levelling errors can be divided to random and systematic errors and blunders (outliers) (Vaníček et al. 1980). Vaníček et al. claim also that blunders are generally easy to detect. Systematic errors can be related to the levelling instrument, levelling rods or levelling environment. Certain types of systematic errors can be eliminated or minimized by the following strict levelling procedures: *i)* line sections should be levelled twice – fore and back; *ii)* fore and back levellings should be performed in the different halves of a day; *iii)* sighting lengths and heights should be restricted; *iv)* differences between fore and back sight lengths should be restricted; *v)* there should be an even number of setups in the section. Other systematic errors can be controlled by adding certain corrections to the observations (rod scale and temperature correction, gravity correction). The third type of systematic errors is harder to control due to the complexity of the physical phenomena causing it (refraction error) or even existence unknown (Vaníček et al. 1980).

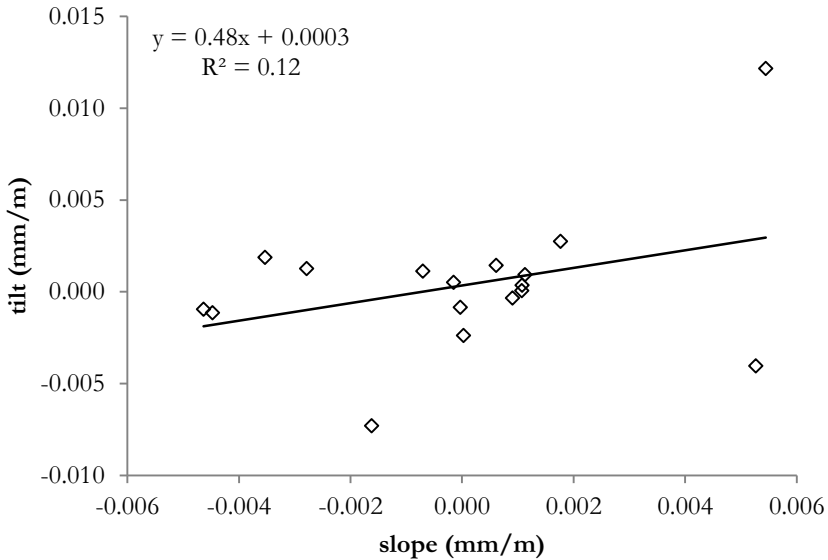
Since systematic errors tend to accumulate in proportion to distance and height or height gradient (Vaníček et al. 1980), some methods of their detection have been developed utilizing this tendency. Probably the systematic error estimation proposed by Lallemand (1889), Eq. (3) is most known, where accumulation of the fore and back levelling discrepancies along the length of the levelling line can also be graphically presented (Figure 15). The influence of systematic errors is deduced from the linear tendency of the cumulative discrepancies on the graph. Slope of the trend line can be used as the estimation of the systematic error and for correction of height differences with the systematic error correction (Kostoglodov et al. 2001). Also, some numerical tests (sign criterion test, criterion for zero mean of errors (Kubáčková et al. 1987)) have been employed to detect systematic errors among levelling observations (Paper I; Kostoglodov et al. 2001).



**Figure 15.** Accumulated discrepancies between the fore and back levelled height differences along the Põltsamaa-Lelle levelling line in 1964. Note that discrepancies start to accumulate from  $\sim 20$  km. Systematic error calculated by the Lallemand (1889) formula, Eq. (3), is  $-0.23$  mm/km.

Another method that employs the tendency of systematic errors to accumulate in proportion to height or height gradient (primarily rod calibration errors and residual refraction errors) has been proposed by Stein (1981). In this method, scatterplots of height difference change per section length (tilt) versus height difference per section length (slope) are compiled (Figure 16).

A linear trend on the plot, either positive or negative, indicates systematic correlation between the height and the observed change in height. For example, a change of the levelling rod length between the two levellings can cause such a correlation (Stein 1981; Bilham 2001). Several studies have employed Stein’s method to identify or remove systematic errors from levelling observations (Giménez et al. 1996; Mäkinen and Saaranen 1998; Giménez et al. 2000; Bilham 2001; Kostoglodov et al. 2001; D’Anastasio et al. 2006; Savage and Svarc 2010).



**Figure 16.** Change of the height difference between levellings in 1939 and 1958 versus height difference (both per section length) along the Mõniste-Petseri levelling line. Note that there is no significant correlation between slope and tilt and consequently, no slope dependent errors can be detected.

Finally, a levelling error, which can be considered as systematic also, results from neglecting the gravity anomalies (Holdahl 1978). Misclosure of the levelling loop is theoretically zero only when height differences are in geopotential units, i.e. variations of the gravity along the levelling lines have been taken into account. Magnitude of the influence of neglecting gravity corrections on the BMs' VV calculation is proportional to the following factors:

- i)* gravity anomaly is large and topography over the study area changes rapidly;
- ii)* number of single levellings, i.e. segments of the network not re-levelled were used for the VV calculations.

Influence of neglecting gravity corrections is inconsiderable for small study areas at minor variations of gravity anomalies (Fuhrmann et al. 2014). Moreover, the effect of ignoring gravity anomaly is eliminated from VV calculations when the height differences between the BMs are differentiated, i.e. the “heights eliminated” model is employed (Paper

**III**). The effect cancels only when re-levelled segments follow close paths, assuming that gravity over a study area has remained unchanged between the levelling periods (Brown and Oliver 1976; Vaníček 1976; Holdahl 1978; Vaníček et al. 1980; Carrera et al. 1991). However, it should be mentioned that temporal and secular (like rise of the geoid due to LU, cf. Section 1.2) variations of the geoid over a study area affect also VV calculations (Brown and Oliver 1976; Hein 1986). In the present study, it is assumed that the geoid remained stable for the entire levelling period used in the VV calculations. Nevertheless, such assumption should not affect the VV calculations, since the effect is below the observation noise (Brown and Oliver 1976).

### **3.3. Determination of vertical crustal movements from levelling data**

#### **3.3.1. Kinematic adjustment**

Kinematic adjustment methodology (Mäkinen and Saaranen 1998; Kooi et al. 1998; Kleijer et al. 2002; Fuhrmann et al. 2013) was chosen for the BMs' VV calculation to take into account all repeated levelling observations simultaneously. In addition, it provides a full variance-covariance matrix both for parameters and observations and therefore observation outliers detection methods can be employed. Two models of kinematic adjustment were used: *i*) the “heights included” model where BMs' heights as well as vertical velocities were parametrized as unknowns (Papers **II** and **III**); and *ii*) the “heights eliminated” model where unnecessary parameter for the present study – height – was eliminated by forming levelling observation differences and only BMs' VVs were parametrized as unknowns (Paper **III**).

Different estimates for the VVs from the “heights included” and the “heights eliminated” model might be expected. The main reason for that is the VV information contained in the loops' misclosures. By forming levelling observation differences, like it was done in the “heights eliminated” model, that information is lost. However, VV information in the loops' misclosures remains in the parameters of the “heights included” model (Cross et al. 1987). Further, it is possible that loops' misclosures contain some systematic errors. In that case, systematic errors remain in the parameters of the “heights included” model. However, systematic errors are eliminated when applying the “heights

eliminated” model. Therefore, systematic errors can also be the reason for the different VV estimates.

### 3.3.1.1. *A priori* errors of the levelling observations and vertical velocity determination

To calculate the variances of height differences, the following two approaches were employed:

- i)* random and systematic errors of each levelling line were calculated by fore and back height difference discrepancies and variances  $m^2$  of the height differences were calculated from the random and systematic errors and section lengths (Paper II);
- ii)* levelling standard errors for each levelling campaign were calculated from loops’ misclosures and variances  $m^2$  of height differences were calculated from levelling standard errors. The influence of the LU on the loops’ misclosures was evaluated also (Paper III).

Weights of the levelling observations were calculated as reciprocal values of the variances.

In the areas of postglacial LU, like Estonia, the loops’ misclosures of the single levelling campaign also contain the effect of the LU. The reason is that different lines in the levelling loop are levelled at different times (see maximum time differences of completing each loop of the CLN in Table 1 in Paper III). The shorter the time period for closing the entire loop, the smaller the LU effect is.

The evaluation of the influence of the LU on the loop’s misclosure is possible by transforming the measured height differences to the mean epoch of the levelling campaigns (1936.7, 1961.2, 1982.1 and 2006.9), i.e. by the time-homogenization. For time-homogenization, VVs of the BMs from the kinematic adjustment of the CLN with re-scaled weights were calculated. VVs to the BMs in the CLN were also interpolated from the Fennoscandian LU model NKG2005LU (Ågren and Svensson 2007) and from the gridded model based on Figure 2 by Vallner et al. (1988). Thereafter, loops’ misclosures before and after time homogenization

were compared to obtain the LU corrections. Also, the influence of LU corrections on the *a priori* levelling standard errors was estimated.

In addition to the accuracy estimates of the BMs' VVs from the kinematic adjustment, it can also be estimated *a priori* from relative VV misclosures in the levelling loops of the CLN. Relative VV between two BMs (difference between BMs' VVs,  $\Delta v$ ) was calculated by the formula:

$$\Delta v = \frac{\Delta h_2 - \Delta h_1}{\Delta t}, \quad (8)$$

where  $\Delta h_1$  is a height difference of the initial levelling;  $\Delta h_2$  is a height difference of the repeated levelling, and  $\Delta t$  is the time interval between the two levellings. Thus, at repeated levellings with the same network configuration, it is possible to form a network of relative VV between consecutive BMs. VV differences can be treated in the same way as height differences in the levelling network: *i*) sum of the VV differences in the loop reflects the misclosure (theoretically equal to zero); *ii*) VV differences can be adjusted to obtain the VVs of the BMs. Misclosures of the VV differences in the levelling loops of the CLN are presented in Table 3.

**Table 3.** Misclosures of the vertical velocity differences (mm/yr) in the levelling loops of the common levelling network (Figure 1 in Paper III).

Loop	Perimeter (km)	Levelling combination					
		1-2	2-3	3-4	1-3	1-4	2-4
I	336.6	-0.38	-0.90	0.83	-0.62	-0.14	-0.11
II	400.9	-1.22	0.11	0.46	-0.25	0.12	0.37
III	455.4	0.10	-3.02	1.70	-1.49	-0.22	-0.34
IV	396.7	0.84	-3.46	0.62	-0.92	-0.15	-0.46
V	337.0	-0.22	-0.62	1.65	-0.75	-0.10	-0.02
VI	387.8	-0.62	0.92	-1.50	0.44	0.12	0.39
VII	210.4	-2.07	-0.06	0.05	-1.25	-0.68	0.01
VIII	132.9	-	-1.95	0.98	-	-	-0.08

Based on the misclosures of the VV differences (Table 3), *a priori* standard error of relative VVs (mm/yr) for the 100 km long segment was calculated using the formula (Randjäv 1993):



$$\mu_{c_{\Delta v}} = \sqrt{\frac{1}{n} \sum \frac{c_{\Delta v}^2}{0.01P}}, \quad (9)$$

where  $c_{\Delta v}$  is the misclosure of the VV differences in the levelling loop;  $P$  is the perimeter of the loop in km, and  $n$  is the number of the loops. Standard errors of the annual VV determination based on the misclosures of the VV differences are presented in Table 4.

**Table 4.** *A priori* mean standard errors (mm/yr) of relative vertical velocity (VV) determination on the 100-km long segment based on the misclosures of the VV differences in Table 3.

Levelling combination					
1-2	2-3	3-4	1-3	1-4	2-4
±0.63	±1.03	±0.62	±0.51	±0.19	±0.14

Table 4 shows clearly that a shorter time period between levellings contributes to a larger standard error in the combination of two levellings (e.g. 2-3 vs 1-4).

### 3.3.1.2. Kinematic adjustment procedure

Weighted least squares kinematic adjustment of the LNF (Paper **II**) and CLN (Paper **III**) was performed to obtain estimates of the BMs' VVs. The adjustment was performed iteratively in the following steps:

- i*) Initial adjustment of the dataset with *a priori* weights;
- ii*) Estimation of the adjustment results;
- iii*) Detection of outliers among the observations;
- iv*) Removing or downweighting of the outlying observation;
- v*) After removing or downweighting outliers, adjustment was continued from the step *i*): readjustment of the dataset was carried out until no outliers were detected;
- vi*) Variance component estimation was performed after outlier detection.

### 3.3.1.3. Estimation of adjustment results

Results of the adjustments were evaluated mainly through two statistics: *a) a posteriori* variance of unit weight  $S_0^2$  (Eq. (11) in Paper II; Eq. (7) in Paper III); *b) a posteriori* mean standard error of the relative VVs,  $\mu$  (Eq. (13) in Paper II; Eq. (8) in Paper III). The variance of unit weight was compared with the *a priori* value 1, using a  $\chi^2$ -test. Furthermore,  $S_0^2$  and  $\mu$  from the adjustment of the “heights included” and the “heights eliminated” model in the CLN were compared using the F-test. VVs from the “heights included” and the “heights eliminated” model were compared using the t-test. In addition, standard deviations for all parametrized variables and for observations were evaluated as well (Papers II and III).

### 3.3.1.4. Detection and handling of outliers

To detect the outliers, statistical hypothesis testing based on the “data snooping” technique by Baarda (1968) described in Paper II was used. The data snooping method is based on the assumption that there is only one outlier in the set of observations. Therefore, after least squares adjustment, observation with the largest standardized residual  $\bar{e}_i$  was removed from the dataset. After that the least squares adjustment was repeated and observation with the largest  $\bar{e}_i$  was removed again. This procedure was continued iteratively until no more outliers were detected.

Detected outliers were not removed from the dataset, but were downweighted instead to calculate the final VVs for the EST2015LU model in Paper III. The reason is that VVs for the EST2013LU model were calculated with a smaller number of observations (observations of the First levelling were left out) and exclusion of more observations was not eligible. For weighting the outlying observation down, the “*Danish method*” was used (Krarup et al. 1980; Caspary 2000; Knight and Wang 2009; Banaś 2012).

“*Danish method*” is considered to be a robust method, i.e. less sensitive to outliers. It is entirely a heuristic method which is not based on any theoretical foundation (Juhl 1984). The idea is that the indicator of the observation quality is a residual of the observation. Therefore, after least

squares adjustment, *a priori* weights of the observations are replaced with the new ones. Then new least squares adjustment is performed and from new residuals, new weights are computed. This iterative process is repeated until no new outliers are detected. The following algorithm was used in the present study:

$$(w_i)_{k+1} = \begin{cases} (w_i)_k & \text{if } (\bar{e}_i)_k < 3 \\ (w_i)_k \cdot \exp\left(-\frac{|(e_i)_k| \cdot \sqrt{(w_i)_k}}{2(S_0)_k}\right) & \text{if } (\bar{e}_i)_k \geq 3 \end{cases}, \quad (10)$$

where  $k$  is the number of iteration;  $w$  is the weight of the observation  $i$ ;  $e$  is the residual of observation  $i$ ;  $\bar{e}_i$  is the standardized residual of observation  $i$ ; and  $S_0$  is the standard deviation of unit weight.

### 3.3.1.5. Estimation of the variance components

The size of the variance of unit weight  $S_0^2$  obtained after the kinematic adjustment reflects possible outliers in observations or how the different levelling data and *a priori* levelling error estimates fit together with the model of constant VVs. The larger the  $S_0^2$  from the *a priori* value  $\sigma_0^2 = 1$ , the poorer the fit is. After removing detected outliers from the observations and following kinematic adjustment, the *a priori* standard error estimates of the used levellings could be re-scaled uniformly by the *a posteriori* variance of the unit weight  $S_0^2$ . However, instead of the uniform rescaling, different variance factors  $S_0^2$  were used for different levelling groups ( $k = 1, 2, \dots, m$ ):

$$\Sigma = \text{diag}(S_{0_1}^2 \mathbf{W}_1^{-1} \quad S_{0_2}^2 \mathbf{W}_2^{-1} \quad \dots \quad S_{0_m}^2 \mathbf{W}_m^{-1}), \quad (11)$$

where  $\mathbf{W}_k$  is the weight matrix of the levelling group  $k = 1, 2, \dots, m$ , and  $\Sigma$  is the covariance matrix of the observations.

There are several methods for variance component estimation. They differ in the estimation principle as well as in the assumption of data

distribution. Most methods use a model where linear dependency between the unknown variance matrix of the observations and the known cofactor matrix is assumed (Teunissen and Amiri-Simkooei 2007). The best known variance component estimation methods are Helmert (Helmert 1872; Welsch 1978), Bique (Koch 1978; Welsch 1984; Koch 2010), Minque (Rao 1971), Förstner (Förstner 1979), and IAUE (Lucas 1985). The methods are well described in the geodetic literature (Amiri-Simkooei 2007; Bähr et al. 2007; Bossler 1984; Crocetto et al. 2000; Hsu 1999; Wang et al. 2009; Grodecki 1997; Grafarend et al. 1980), so they are not duplicated here. Every method has its strengths and weaknesses. Some methods produce negative variance components, some take large computational resources, some produce estimates not completely unbiased.

To estimate variances of levellings used for VV determination in the present study, observations were divided into groups. Eight groups were used for the LNF on the basis of different levelling campaigns and conductors. Observations of the CLN were divided into four groups on the basis of four levelling campaigns. The IAUE and Helmert methods were used for the variance component estimation in the LNF and Helmert, Bique and Förstner methods in the CLN. After the variance component estimation, *a priori* levelling standard errors were re-scaled and kinematic adjustment was repeated with the re-scaled weights.

### 3.3.2. Testing the change of benchmarks' vertical velocities in time

A hypothesis that BMs' VVs are constant in time, i.e. VVs have remained unchanged between the levelling periods, was set up. In order to test the hypothesis, an ANOVA test was conducted. Based on the *i*) differences between VVs from the different levelling combinations; *ii*) the sum of the squared observation residuals; and *iii*) degrees of freedom of the kinematic adjustment, the significance of the VV changes between mean levelling epochs was evaluated (Kakkuri and Vermeer 1985; Mäkinen and Saaranen 1998), i.e. the hypothesis that the VVs of the BMs were constant in time ( $\mathbf{v}_{2-3} - \mathbf{v}_{1-2} = \mathbf{v}_{3-4} - \mathbf{v}_{2-3} = \mathbf{v}_{3-4} - \mathbf{v}_{1-2} = \mathbf{0}$ ) was tested. In order to take into account the correlation between the VVs from different levelling combinations, the "heights eliminated" model (Eq. (5) in Paper III) was rewritten such that it was possible to obtain three VVs ( $\mathbf{v}_{1-2}$ ,  $\mathbf{v}_{2-3}$ ,  $\mathbf{v}_{3-4}$ ; VVs between the First and the Second, the

Second and the Third, the Third and the Fourth mean levelling epochs, respectively) for each BM from the single adjustment (Eq. (9) in Paper III).

By representing changes of VVs ( $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$ ;  $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$ ;  $\mathbf{v}_{3-4} - \mathbf{v}_{1-2}$ ) graphically, local velocity anomalies can be detected in places where VV differences are abruptly changing. In fact, local VV anomalies can be detected already before the kinematic adjustment if raw VV differences on the left side of Eqs. (12)–(14) are normed with their *a priori* standard errors. Large values of normed VV change indicate also local VV anomalies (Mäkinen and Saaranen 1998). Raw VV changes can be obtained by subtracting rows in Eq. (5) in Paper III:

$$\begin{aligned} & (\mathbf{T}_3 - \mathbf{T}_2)^{-1}(\mathbf{y}_3 - \mathbf{y}_2) - (\mathbf{T}_2 - \mathbf{T}_1)^{-1}(\mathbf{y}_2 - \mathbf{y}_1) \\ &= \mathbf{A}(\mathbf{v}_{2-3} - \mathbf{v}_{1-2}) + (\mathbf{T}_3 - \mathbf{T}_2)^{-1}(\mathbf{e}_3 - \mathbf{e}_2) - (\mathbf{T}_2 - \mathbf{T}_1)^{-1}(\mathbf{e}_2 - \mathbf{e}_1), \end{aligned} \quad (12)$$

$$\begin{aligned} & (\mathbf{T}_4 - \mathbf{T}_3)^{-1}(\mathbf{y}_4 - \mathbf{y}_3) - (\mathbf{T}_3 - \mathbf{T}_2)^{-1}(\mathbf{y}_3 - \mathbf{y}_2) \\ &= \mathbf{A}(\mathbf{v}_{3-4} - \mathbf{v}_{2-3}) + (\mathbf{T}_4 - \mathbf{T}_3)^{-1}(\mathbf{e}_4 - \mathbf{e}_3) - (\mathbf{T}_3 - \mathbf{T}_2)^{-1}(\mathbf{e}_3 - \mathbf{e}_2), \end{aligned} \quad (13)$$

$$\begin{aligned} & (\mathbf{T}_4 - \mathbf{T}_3)^{-1}(\mathbf{y}_4 - \mathbf{y}_3) - (\mathbf{T}_2 - \mathbf{T}_1)^{-1}(\mathbf{y}_2 - \mathbf{y}_1) \\ &= \mathbf{A}(\mathbf{v}_{3-4} - \mathbf{v}_{1-2}) + (\mathbf{T}_4 - \mathbf{T}_3)^{-1}(\mathbf{e}_4 - \mathbf{e}_3) - (\mathbf{T}_2 - \mathbf{T}_1)^{-1}(\mathbf{e}_2 - \mathbf{e}_1), \end{aligned} \quad (14)$$

where,  $\mathbf{v}_k$  – vector of the unknown VVs of the BMs in the levelling campaign  $k = 1, 2, 3, 4$ ;  $\mathbf{y}$  – vector of the levelling observations in the campaign  $k$ ;  $\mathbf{e}$  – error vector of the levelling observations in the campaign  $k$ ; and  $\mathbf{T}$  – diagonal matrix of the levelling epochs in the campaign  $k$ .

### 3.3.3. Detecting correlation between the vertical velocities and velocity changes

As was pointed out by Mäkinen and Saaranen (1998), conclusions regarding changes of the BMs' VVs depend on the between-epoch correlation of repeated levellings. A correlation can be a real relationship (e.g. the same equipment sharing the same possible systematic errors was used for re-levellings of the lines) or just accidental, fortuitous. *A priori*,

it was assumed that repeated levellings of the same levelling line have been performed independent of each other. In order to find out whether correlations existed between repeated levellings, multivariate analysis of the four levellings was carried out according to Mäkinen and Saaranen (1998). The calculation algorithm is presented in Paper III.

### 3.3.4. Detection of height-dependent errors in the levellings

Systematic levelling errors (refraction and rod calibration errors), which accumulate in proportion to distance and height, according to Section 3.2.2, may contaminate the measurements. Accumulation of systematic errors in proportion to height occurs mainly on mountainous areas (Tanaka et al. 1984). Accumulated systematic errors can appear as the large *a posteriori* variance of unit weight of kinematic adjustment in the combination of three or four levellings. They do not reveal themselves in combination of only two levellings because VV values exceed these errors in that case (Mäkinen and Saaranen 1998).

Although Estonia is a relatively flat country, presence of the height-dependent errors among the observations of the CLN were tested graphically. The raw VV changes from the left sides of Eqs. (12)–(14) were normed by the *a priori* levelling standard errors according to the section lengths. Graphs with the normed VV changes on the vertical axis and height differences on the horizontal axis were compiled for the VV pairs  $\mathbf{v}_{1-2}$  to  $\mathbf{v}_{2-3}$ ,  $\mathbf{v}_{2-3}$  to  $\mathbf{v}_{3-4}$ , and  $\mathbf{v}_{1-2}$  to  $\mathbf{v}_{3-4}$ . Linear correlation between the normed VV changes and height differences indicates the presence of the height-dependent errors.

### 3.3.5. Modelling of vertical crustal movements

For VCM surface modelling, VVs of the BMs obtained from the kinematic adjustment with the re-scaled weights based on variance component estimation and removed or downweighted outliers were used. Two VCM models for Estonia were compiled: *i*) based on the VVs of the BMs of the LNF (Paper II) named as EST2013LU, and *ii*) based on the VVs of the BMs from the solution of the so-called expanded CLN (the Second, Third and Fourth levelling of the CLN, plus some additional BMs and levelling lines common only to the Second, Third and Fourth levelling), (Paper III). The last model named as EST2015LU

was compiled by including the VVs of the water gauges of Lake Peipsi determined by Raamat (2009).

The modelled surfaces of the VCMs for Estonia were compiled using software *Surfer* 11.6 by *Golden Software*. The software uses a “gridding” technique to create modelled surfaces from a dataset of XYZ coordinates. A user defines the area where the grid will be created, chooses the grid spacing (1/2 of the average spacing between the closest points pairs (Hengl 2006) was used in the current study), an interpolation method and several other parameters (e.g. tension factor, variogram model, power and order of polynomial) depending on the selected interpolation method. In the present study, the following interpolation methods with different input parameters were tested: *i*) kriging; *ii*) minimum curvature; *iii*) local polynomial; and *iv*) natural neighbor. In Paper **II**, minimum curvature and kriging methods were used. The final model EST2013LU was compiled using the kriging method. In Paper **III**, kriging, minimum curvature, local polynomial and natural neighbor methods were tested. The final model EST2015LU was compiled using the minimum curvature method. Before the final surface creation, several filtering techniques were applied:

- BMs with local VV anomalies were filtered out by comparing initial VV surface with the reference surface, based on the NKG2005LU model (Paper **II**), which was an iterative procedure, until a visually smooth surface was obtained.
- BMs deviating from the neighboring ones more than set up tolerance were eliminated from the dataset visually (Papers **II** and **III**).
- After removing the outlying BMs, the gridded surface was filtered with the low pass filter “moving average” or nonlinear filter “threshold averaging” built in *Surfer* (Papers **II** and **III**).

The final gridded models EST2013LU and EST2015LU were tested by calculating residuals at the observation points and using a cross validation technique. In addition, profiles of the BMs raw VVs between the First and the Second (1–2), the Second and the Third (2–3), and the Third and the Fourth (3–4) levelling along the Koidula-Tartu-Jõgeva-Põltsamaa-Lelle-Tallinn levelling line were compared with the modelled surfaces. Compiled VCM models were also compared with the previous LU models for Estonia and Fennoscandia. For that, VCM maps by

Vallner et al. (1988) and Randjärv (1993) were gridded and tested. A map by Ekman (1996) was gridded by the Nordic Geodetic Commission. The gridded LU model NKG2005LU was obtained also from the same institution. Further, VVs interpolated from the EST2013LU and the EST2015LU model were compared with the VVs from Estonian CORS by Oja et al. (2014) and coastal tide gauges by Yakubovski (1973); Pobedonostsev (1975) and Jevrejeva et al. (2002). Comparison was made also with the VVs of six tide gauges, which were recalculated and corrected with the satellite altimetry data by Liibusk and Wan (Paper III).

### **3.3.6. Determination of vertical crustal movements along the Põltsamaa-Lelle levelling line**

The Põltsamaa-Lelle levelling line crosses several fault zones (Figure 9). One of the zones is the Pärnu-Narva fault zone, which marks the hinge line based on the change in the shoreline gradient and the shoreline tilting of the Baltic Ice Lake (Orviku 1960; Pärna 1962; Miidel 1966; Vallner et al. 1988; Miidel 1995; Rosentau et al. 2008; Vassiljev et al. 2012). According to Vallner et al. (1988), the gradient of the VCM is greatest (1–2 mm) across the Pärnu-Narva line compared to the other parts of Estonia. Since the Põltsamaa-Lelle levelling line crosses that fault and it has been levelled significantly more times (in 1936, 1961, 1964, 1965, 1966, 1969, 1972, 1980, 1982, 1987, and 2006) than other lines of the Estonian levelling network, this line is particularly suitable to test the hypothesis that the VCM gradient is greatest across the Pärnu-Narva fault zone.

The levellings were tested against systematic errors using the sign criterion and the test of zero mean of the discrepancies between the fore and back levellings. The levelling random and systematic errors were calculated according to Eqs. (1) and (2) in Paper I. Weights of the height differences were calculated as a reciprocal values of their variances (Eq. (5)). Weighted regression analysis of the height differences of the BMs common to the abovementioned levellings was performed. Slopes of the regression lines (e.g. Figure 14) gave differences of VVs between the BMs (i.e., relative VVs between consecutive BMs). VV of the Lelle fBM relative to the Põltsamaa fBM was determined from the sum of the VV differences of the consecutive BMs. Uncertainties of the BMs relative VVs were obtained from the standard errors of the slopes of the



regression lines. The long-term (1936–2006) gradient of the VCM across the Pärnu-Narva hinge line was calculated from the relative VV between Põltsamaa and Lelle. The obtained gradient was tested against the gradients from the other LU models along the same levelling line and with the overall VCM gradient of Estonia (Paper I).

The height differences in each levelling section were tested against outliers using standardized residuals of the weighted regression model. These outliers represent anomalies in the long-term VCMs. An attempt was made also to calculate long-term relative VVs of the BMs based on the slope of the linear regression line when the outliers were removed.

In addition, profiles of the BMs' cumulative VVs relative to the Põltsamaa fBM between consecutive levelling periods were compiled to detect the short-term VCMs. The spikes in the profiles were interpreted according to Section 3.2.1. In addition to spikes, steps and slope changes on the profiles were analysed. These are the features that can be related to geological or tectonic reasons, i.e. different compaction of recent sediments, regional tectonic tilt, or active tectonic structures (Giménez et al. 2000). Statistical significance of short-term VVs relative to the Põltsamaa fBM were evaluated by standard deviation of the cumulative relative VV (Holdahl 1978):

$$SD_{VV} = \sqrt{\frac{(\eta_1^2 L + \sigma_1^2 L^2) + (\eta_2^2 L + \sigma_2^2 L^2)}{(t_2 - t_1)^2}}, \quad (15)$$

where  $\eta_1$ ,  $\sigma_1$  and  $\eta_2$ ,  $\sigma_2$  are the *a priori* random and systematic levelling errors of the levellings 1 and 2, respectively, based on Eqs. (1) and (2) in Paper I.  $L$  is the distance of the BM from the Põltsamaa fBM, and  $t_1$  and  $t_2$  are the epochs of levellings 1 and 2.

## 4. RESULTS

### 4.1. Results of the adjustment of the levelling network of the fundamental benchmarks

The main estimated parameters of the kinematic adjustment of the LNF and the resulting VVs of the BMs are presented in Table 5. Results of the variance component estimation by the Helmert method are provided in Table 6. The main effect of the removing outliers was the change of the VVs of two BMs in the Pärnu area by  $\sim 1$  mm/yr. Change of the VVs of other BMs was negligible:  $\pm 0.06$  mm/yr on average. Higher effect came from variance component estimation: systematic change of the BMs' VVs by  $-0.25 \pm 0.15$  mm/yr and standard deviations by  $-0.23 \pm 0.11$  mm/yr on average was observed (Paper II).

**Table 5.** Main parameters and results of the kinematic adjustment of the levelling network of fundamental benchmarks.

Parameter	Value
Number of benchmarks (with heights/with vertical velocities)	106/84
Number of observations, first iteration	361
Number of loops/length of loops (km)	15/2222
Variance of unit weight $S_0^2$ , first iteration	$\pm 18.52$
Number of removed outliers	11
Variance of unit weight $S_0^2$ after the outliers were removed	$\pm 3.60$
Variance of unit weight $S_0^2$ after re-adjustment with re-scaled weights	$\pm 1.00$
Average standard deviation of adjusted vertical velocities (mm/yr)	$\pm 0.52$
Mean standard error of adjusted vertical velocity differences (mm/yr)	$\pm 0.53$
Effect of removing outliers on the vertical velocities (mm/yr)	$\pm 0.06$
Change of the vertical velocities after variance component estimation (mm/yr)	$-0.25 \pm 0.15$
Change of the standard deviation of the vertical velocities after variance component estimation (mm/yr)	$-0.23 \pm 0.11$

Levellings of the LNF were divided into eight groups according to the performer and the levelling campaign. From the variance component estimation, it appeared that observations of the First levelling (groups I

and II, Table 6) should appear weighted down the most (approximately three times). The variance factors  $\sim 4$  were obtained for the Second and the Third levelling (groups IV and V). Negative variance factor was obtained for group VIII, probably due to a small number of observations in this group.

**Table 6.** Results of variance component estimation by the Helmert method.

Levelling group*, number of observations	<i>A priori</i> levelling standard error (mm/ $\sqrt{\text{km}}$ )	Variance factor	Re-scaled levelling standard error (mm/ $\sqrt{\text{km}}$ )
I, n=33	$\pm 0.33$	9.68705	$\pm 1.03$
II, n=9	$\pm 0.70$	10.6897	$\pm 2.28$
III, n=34	$\pm 0.57$	1.04389	$\pm 0.59$
IV, n=69	$\pm 0.49$	3.89271	$\pm 0.96$
V, n=87	$\pm 0.47$	4.55516	$\pm 1.01$
VI, n=22	$\pm 0.66$	2.33447	$\pm 1.01$
VII, n=91	$\pm 0.18$	0.70177	$\pm 0.15$
VIII, n=5	$\pm 2.25$	-0.00519	-

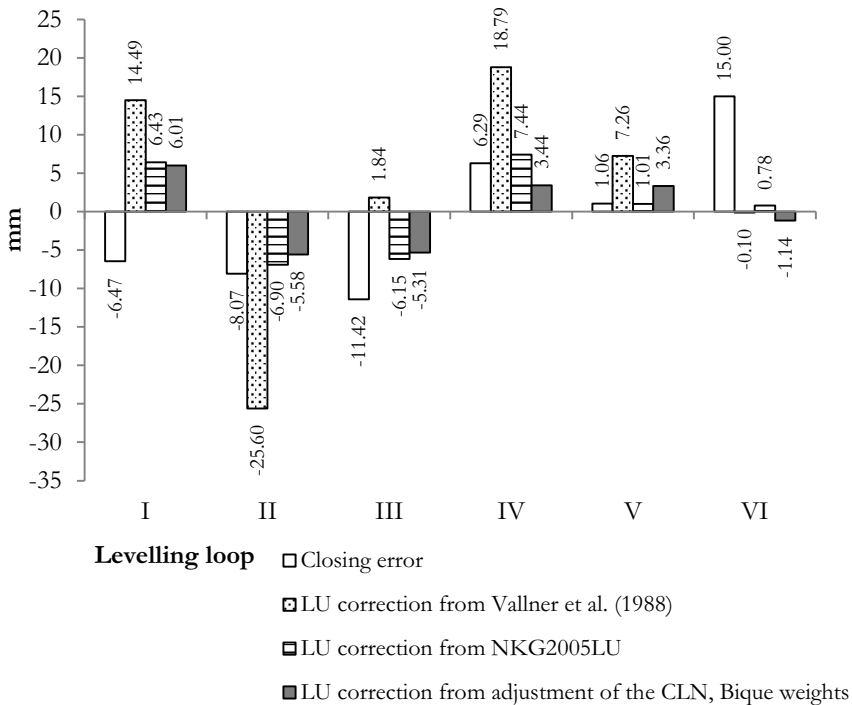
\* Group I – precise levelling by Cadastre Department of the Ministry of Agriculture of Estonia on the mainland of Estonia (1933–1943); II – levelling (2nd order) by the same institution on the island of Saaremaa (1939–1940); III – precise levellings by the General Committee of Geodesy and Cartography of the Soviet Union in 1948 and 1970; IV – precise levellings by the Estonian Academy of Sciences in 1950–1969; V – precise levellings by the Estonian Academy of Sciences in 1970–1996; VI - 2nd order levellings by the General Committee of Geodesy and Cartography of the Soviet Union in 1981–1983; VII – precise levelling coordinated by Estonian Land Board in 2001–2012; VIII – hydrostatic and hydrodynamic levellings across the straits of the Väinameri Sea in 1968, 1974 and 2010.

#### 4.2. Results of the adjustment of the common levelling network

As was pointed out in Section 3.2, adjustment of the CLN gives more opportunities to analyse VCMs, including change of the BMs' VVs between the levelling periods. In the following sections, the results presented in Paper III are summarized. However, some additional details are highlighted here also.

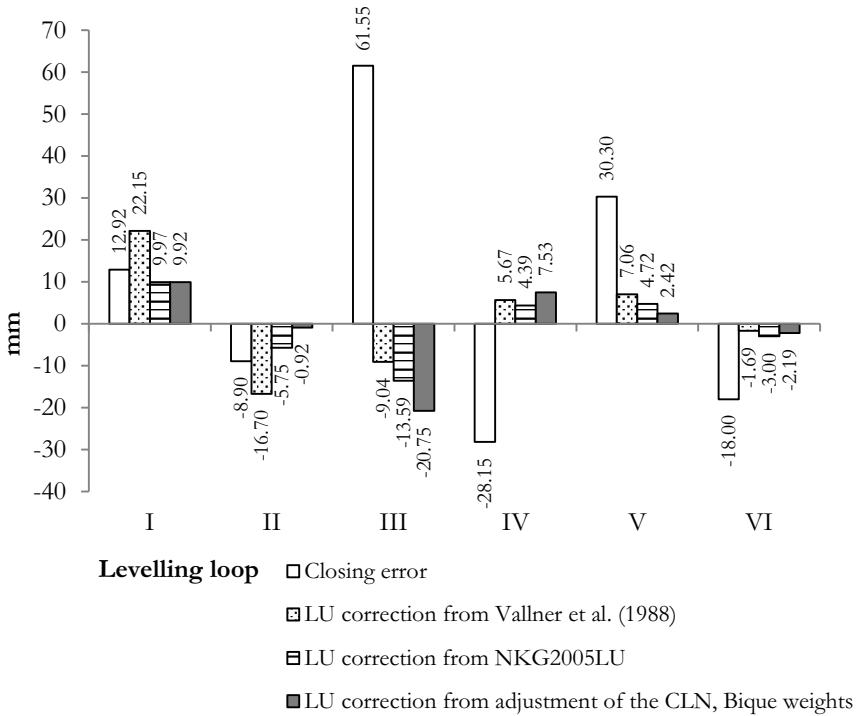
### 4.2.1. The influence of land uplift on the misclosures of the levelling loops

Since the time period for closing of the levelling loops was longest for the Second and the Third levelling (max 25 years), LU corrections were also largest for these levellings. The levelling loops' misclosures for the Second and the Third levelling and the effect of LU on the misclosures are presented in Figures 17 and 18. Corrections were largest for loops I, III and IV (Figure 1 in Paper III) of the Second and the Third levelling, where in some cases, correction exceeded the misclosure itself. The size of the correction was related to the VV information used. The largest corrections were obtained using the VVs from Vallner et al. (1988).



**Figure 17.** Loops' misclosures of the Second levelling and the contribution of land uplift (LU) contained in the misclosures. LU corrections were calculated using the vertical velocities of the benchmarks from three different sources. The effect of LU was negligible for loops VII and VIII because the levellings were conducted during one year.

The levelling standard error estimates based on the loops' misclosures and the effect of the LU correction on the standard errors are presented in Table 7. The influence of the LU correction on the levelling standard error was largest for the Second levelling (an order of magnitude of  $\pm 0.2 \text{ mm}/\sqrt{\text{km}}$  on average). The effect of LU corrections on the First and the Fourth levelling was smallest (influence on the levelling standard errors was an order of magnitude of  $\pm 0.01 \text{ mm}/\sqrt{\text{km}}$ ). Again, this was correlated with the time period for closing of the loops which in case of the First and the Fourth levelling was 1–7 years.



**Figure 18.** Loops' misclosures of the Third levelling and the contribution of land uplift (LU) contained in the misclosures. LU corrections were calculated using the vertical velocities of benchmarks from three different sources. The effect of LU was negligible for loops VII and VIII because the levellings were conducted during one year.

According to Mäkinen and Saaranen (1998), the influence of VVs on the loops' misclosures should be evaluated with caution. When levelling standard errors  $\tau$  (Eq. (1) in Paper III) are calculated based on time-homogenized observations, it is impossible to estimate which VV information is the most suitable, since time-homogenization does not

always make misclosures smaller. Unbiased estimation of levelling variance is not achieved with a simple square function using time-homogenized height differences because of the internal-loop correlation accompanying time-homogenization.

However, though the influence of LU on the loops' misclosures shown in Table 7 is high, differences between the kinematic adjustment using the “heights included” or the “heights eliminated” model are inconsiderable in almost all levelling combinations (cf. Section 4.2.2). Recall that the “heights included” and the “heights eliminated” model handle VV information in the loops' misclosures differently. Therefore, it is theoretically possible to obtain different results from these two models, since differentiation of the observations (“heights eliminated” model) eliminates also VV information contained in loops' misclosures. Misclosures without LU correction were used in the present study for the calculation of the *a priori* levelling standard errors and weights of the observations of the CLN.

**Table 7.** The estimates of the levelling standard error  $\tau$  in  $\text{mm}/\sqrt{\text{km}}$  from the loops' misclosures without land uplift (LU) correction and with LU correction using different vertical velocity information for time-homogenization of the observations.

Source for the LU correction	Levelling campaign			
	1.	2.	3.	4.
Without LU correction	$\pm 1.387$	$\pm 0.423$	$\pm 1.576$	$\pm 0.191$
Corrections from Vallner et al. (1988)	$\pm 1.396$	$\pm 0.848$	$\pm 1.698$	$\pm 0.207$
Corrections from the NKG2005LU (Ågren and Svensson 2007)	$\pm 1.392$	$\pm 0.564$	$\pm 1.526$	$\pm 0.201$
Corrections from the adjustment of the CLN, combination 1–2–3–4, Bique weights	$\pm 1.396$	$\pm 0.506$	$\pm 1.416$	$\pm 0.198$

#### 4.2.2. Accuracy and vertical velocity estimates from the kinematic adjustment depending on the levelling combination and type of the mathematical model used

Least squares adjustment of the single levelling campaigns was carried out first to test the observation weights calculated from *a priori* levelling

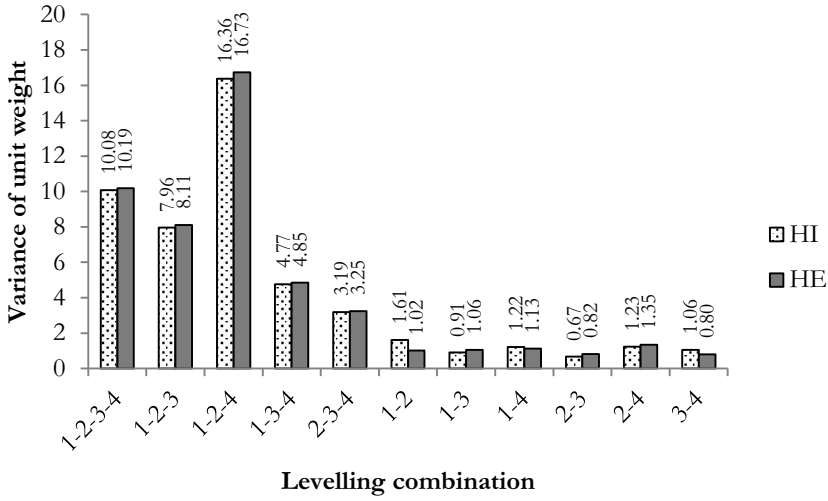
standard errors. Goodness-of-fit test ( $\chi^2$ -test) for estimating *a posteriori* variance of unit weight  $S_0^2$  was performed to see if it differs from the *a priori* value 1. Results of the test (Table 8) indicate that *a priori* chosen levelling standard errors fit well with the adjustment model of the single levelling campaign where only BMs' heights are unknown parameters.

**Table 8.** *A posteriori* variances of unit weight  $S_0^2$  from the weighted least squares adjustment of the individual levelling campaigns and the results of the  $\chi^2$ -test that  $S_0^2 > 1$ .

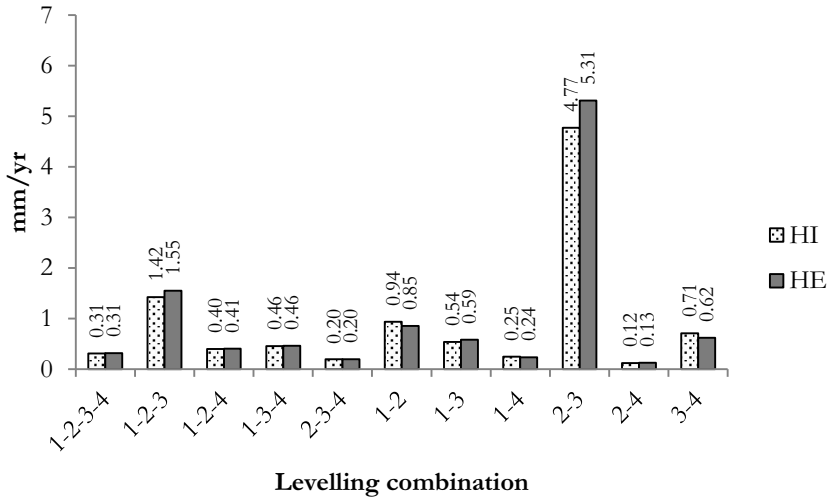
Levelling campaign	<i>A priori</i> levelling standard error $\tau$ (mm/ $\sqrt{\text{km}}$ )	<i>A posteriori</i> variance of unit weight $S_0^2$	p-value of the hypothesis that $S_0^2 > 1$
1.	$\pm 1.387$	0.9960	0.4318
2.	$\pm 0.423$	1.1470	0.3277
3.	$\pm 1.576$	0.8949	0.5196
4.	$\pm 0.191$	0.8968	0.5180

Next, kinematic adjustment of the observations of the CLN was done. Here, *a posteriori* variance of unit weight depends on: *i*) the relationships between the weights of the observations of separate levelling campaigns; *ii*) the fit of the observations with the adjustment model of constant VVs; and *iii*) the existence of outliers among the observations. Adjustment was carried out using the “heights included” and the “heights eliminated” mathematical model in eleven different levelling combinations (Paper III). Results of the adjustments were evaluated through two statistical parameters: *a*) *a posteriori* variance of unit weight  $S_0^2$  (Figure 19), and *b*) *a posteriori* mean standard error of adjusted relative VVs  $\mu$  (Figure 20).

Comparing Figure 20 with Table 4, we see that the *a priori* and *a posteriori* estimates for VV standard errors do not differ significantly, except for the levelling combination 2–3 (Figure 21). The difference is most probably caused by the hydrostatic levelling height differences across the Vänameri, since *a priori* error estimates did not take them into account but *a posteriori* estimates did.

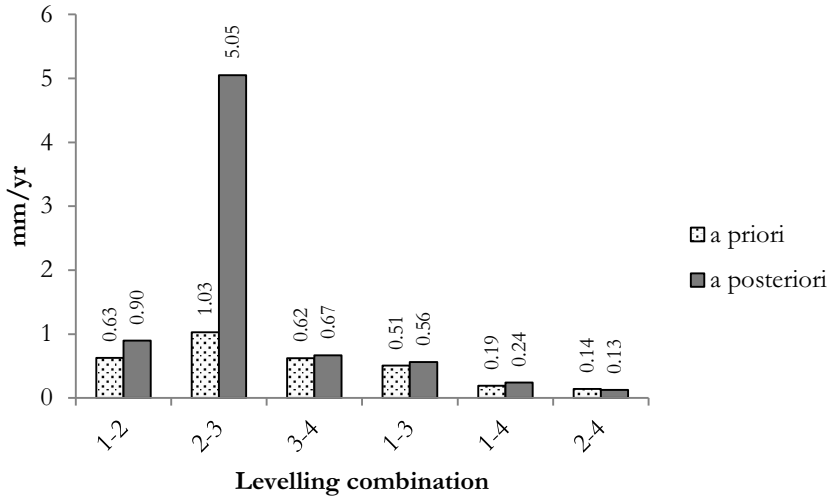


**Figure 19.** *A posteriori* variance of unit weight  $S_0^2$  from the kinematic adjustment of the common levelling network by the “heights included” (HI) and the “heights eliminated” (HE) model. Differences of  $S_0^2$  between the two models were statistically insignificant in all levelling combinations.



**Figure 20.** *A posteriori* mean standard error  $\mu$  of the adjusted relative vertical velocities from the kinematic adjustment of the common levelling network by the “heights included” (HI) and the “heights eliminated” (HE) model. Differences of  $\mu$  between the two models were statistically insignificant in all levelling combinations.





**Figure 21.** *A priori* and *a posteriori* mean standard errors of the relative vertical velocities determined from the loops’ misclosures of the VV differences (Table 4) and from kinematic adjustment of the common levelling network (Figure 20), respectively.

Next, differences between VV estimates from the “heights included” and the “heights eliminated” model were analysed using the t-test (Table 9).

**Table 9.** Root mean square (RMS) difference of vertical velocity estimates from the “heights included” and the “heights eliminated” model and standard deviation of the differences. The rejection criterion of the hypothesis that RMS difference = 0 was  $t_{crit} = 1.967$  ( $\alpha = 0.05$ ). Differences that are statistically significant are underlined.

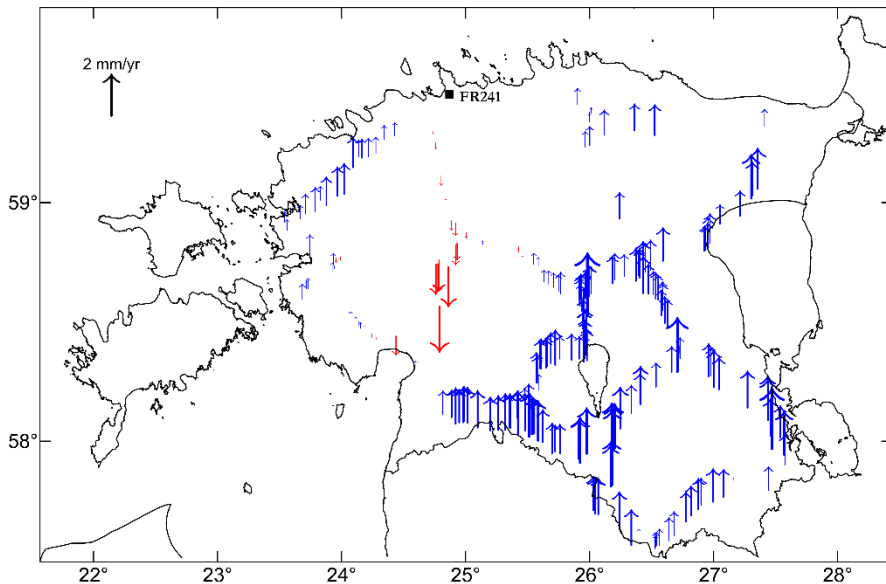
Levelling combination	RMS difference (mm/yr)	<i>SD</i>	t-statistic
1-2-3-4	$\pm 0.01$	$\pm 0.143$	1.047
1-2-3	$\pm \underline{0.44}$	$\pm 0.721$	9.134
1-2-4	$\pm 0.01$	$\pm 0.144$	1.039
1-3-4	$\pm 0.01$	$\pm 0.299$	0.501
2-3-4	$\pm 0.01$	$\pm 0.158$	0.947
1-2	$\pm \underline{1.00}$	$\pm 1.134$	13.198
1-3	$\pm 0.01$	$\pm 0.772$	0.194
1-4	$\pm 0.01$	$\pm 0.319$	0.469
2-3	$\pm 0.22$	$\pm 8.528$	0.412
2-4	$\pm 0.01$	$\pm 0.159$	0.941
3-4	$\pm 0.12$	$\pm 1.000$	1.796

For most levelling combinations there were no significant differences between VV estimates between the two models. Indeed, VV estimates from the two solutions should coincide if the levelling loops were closed in a short time period and the reference epoch for the adjustment was chosen so that the levellings were performed symmetrically with respect to it (Cross et al. 1987). Significant VV differences between the “heights included” and the “heights eliminated” model were obtained only for the combinations 1–2–3 and 1–2. Large VV differences (although not statistically significant) were also obtained from the combinations 2–3 and 3–4.

In order to elucidate the reason for the VV differences between the two models in the combinations 1–2–3 and 1–2 (Table 9), the kinematic adjustment was performed for every loop of the CLN separately using both the “heights included” and the “heights eliminated” model. Additionally, *i*) different adjustment reference epochs were tested; and *ii*) the time-homogenized observations (cf. Section 3.3.3) were used in the kinematic adjustment of the combinations 1–2–3 and 1–2. From the tested techniques, only time-homogenization of the observations eliminated differences between VVs from the “heights included” and the “heights eliminated” model in the levelling combinations 1–2–3 and 1–2.

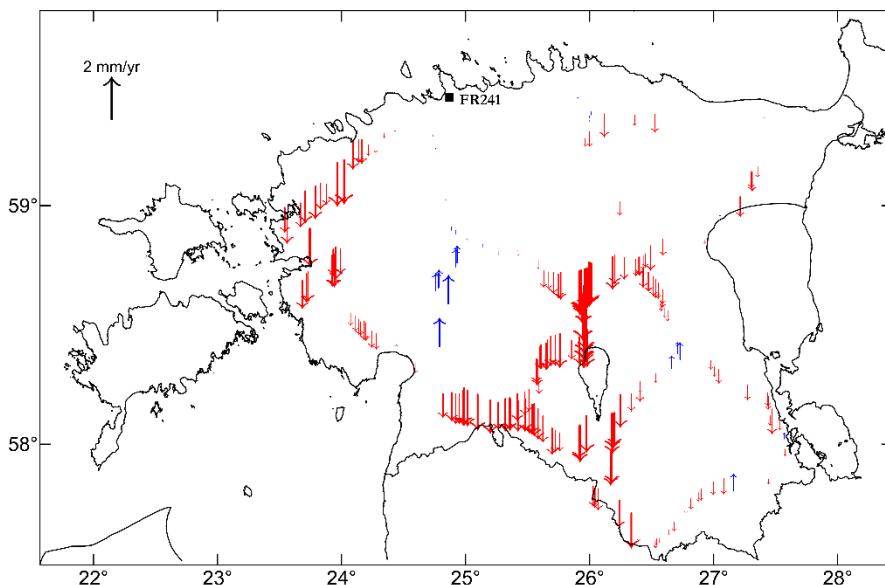
#### **4.2.3. Change of the vertical velocities of the benchmarks in time**

The VVs of the BMs from the different levelling combinations were calculated from Eq. (9) in Paper **III**. Then, the differences of the VVs were evaluated by an ANOVA test (Section 3.3.2; Paper **III**). Results of the tests showed a significant change in VVs between the Second and the Third levelling period (i.e.,  $\mathbf{v}_{2-3} - \mathbf{v}_{1-2} \neq 0$  and  $\mathbf{v}_{3-4} - \mathbf{v}_{2-3} \neq 0$ ; Figures 22 and 23). No significant change between VVs from the levelling combinations 1–2 and 3–4 was found (i.e.,  $\mathbf{v}_{3-4} - \mathbf{v}_{1-2} \neq 0$ ). The ANOVA confirmed no significant changes in VVs between the levelling combinations 1–2 and 2–4, and 1–3 and 3–4 either.



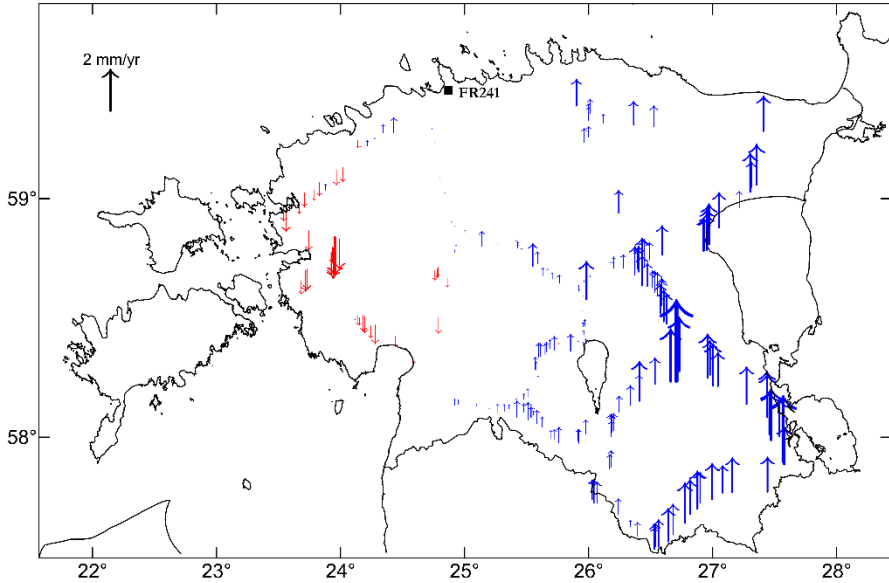
**Figure 22.** Change of the vertical velocities (VVs)  $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$  of the benchmarks solved from Eq. (9) in Paper **III**. Arrows up and down mark the positive and negative change, respectively.  $\mathbf{v}_{1-2}$  and  $\mathbf{v}_{2-3}$  are the average VVs between the First and the Second, the Second and the Third levelling, respectively. Both VVs are relative to the fundamental benchmark FR241 in Tallinn.

However, the performed ANOVA test (Table 3 in Paper **III**) is sensitive to the weight ratios between separate levellings. In order to verify that the differences between the VVs  $\mathbf{v}_{3-4}$  and  $\mathbf{v}_{1-2}$ ;  $\mathbf{v}_{2-4}$  and  $\mathbf{v}_{1-2}$ ;  $\mathbf{v}_{3-4}$  and  $\mathbf{v}_{1-3}$  are insignificant, a new ANOVA test was carried out. The BMs' VVs were now taken from the model independent of the *a priori* chosen weights (Eq. (11) in Paper **III**). The results of the new ANOVA test showed that contrary to previous results, all VV differences between the abovementioned combinations were statistically significant. That includes also the change between VVs from the levellings 1–2 and 3–4 (Figure 24) previously determined as insignificant. The least change of the BMs' VVs was between VV pairs 1–2 and 2–4. Interestingly, the same levelling combination (1–2–4) gave the largest  $S_0^2$  in the kinematic adjustment (Figure 19; Table 2 in Paper **III**). This suggests that in addition to the observations misfitting with the model of the constant VVs, large  $S_0^2$  in this combination is also related to the issue of ratios of *a priori* weights of these levellings being incorrect.



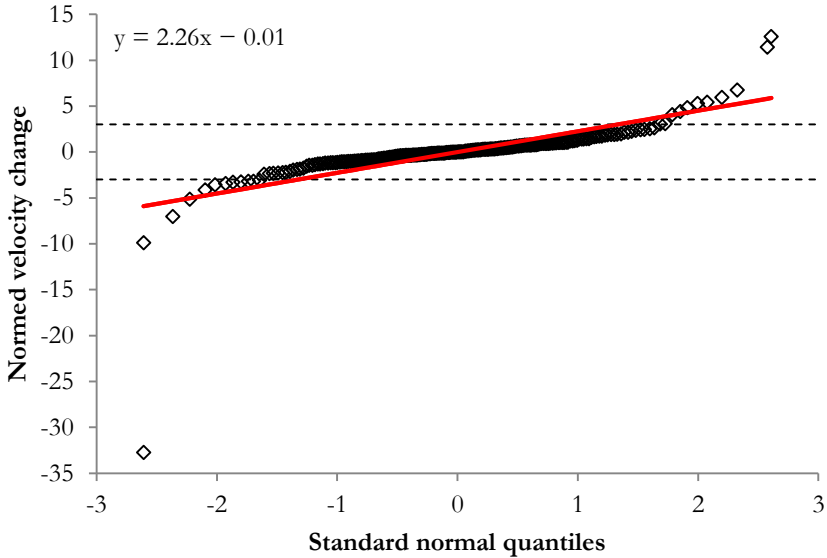
**Figure 23.** Change of the vertical velocities (VVs)  $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$  of the benchmarks solved from Eq. (9) in Paper III. Arrows up and down mark the positive and negative change, respectively.  $\mathbf{v}_{2-3}$  and  $\mathbf{v}_{3-4}$  are the average VVs between the Second and the Third, the Third and the Fourth levelling, respectively. Both VVs are relative to the fundamental benchmark FR241 in Tallinn.

Although the ANOVA test for VV change was performed before the outlier detection test, outliers did not affect the results significantly. Change of the BMs' VVs after removing outliers was small ( $-0.04 \pm 0.14$  mm/yr) compared to the VVs before the outlier test. This change can be considered insignificant relative to the standard deviation of the VVs ( $\pm 0.6$  mm/yr). An ANOVA test was also carried out with the dataset where detected outliers were not removed but were weighted down instead of using the “Danish method” (Krarup et al. 1980; Kubik et al. 1984). Conclusions about the VV change of the BMs from this test were the same. From Figures 22–24 it can be seen that the change of the VVs is not associated only with a few outlying BMs but with a larger group of BMs.



**Figure 24.** Change of the vertical velocities (VVs)  $\mathbf{v}_{3-4} - \mathbf{v}_{1-2}$  of the benchmarks solved from Eq. (11) in Paper III. Arrows up and down mark the positive and negative change, respectively.  $\mathbf{v}_{1-2}$  and  $\mathbf{v}_{3-4}$  are the average VVs between the First and the Second, the Third and the Fourth levelling, respectively. Both VVs are relative to the fundamental benchmark FR241 in Tallinn.

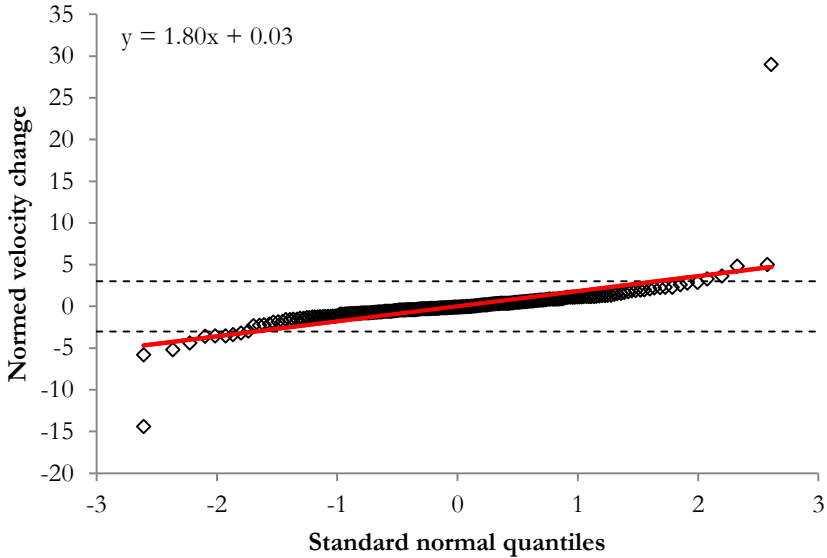
In Figures 22–24, the local VV anomalies, by abruptly changing arrow heights, can be noticed. These anomalies can be detected already before the adjustment by norming changes in raw VVs on the left side of Eqs. (12)–(14) with their *a priori* standard errors and marking the large values (Figures 25–27). Outliers (normed VV change is  $> 3$ ) in Figures 25–27 coincide well with the VV anomalies in Figures 22–24. Naturally, outliers in Figures 25–27 depend also on a priori standard error estimates.



**Figure 25.** Normal probability plot of the raw vertical velocity (VV) changes from the First and the Second to the Second and the Third levelling on the line sections of the common levelling network. VV changes ( $n=220$ ) on the left side of Eq. (12) were normed by their *a priori* standard errors (Table 8) according to the section lengths. Points with ordinate values higher than  $\pm 3$  (dashed lines) can be considered as outliers. The total number of outliers was 23.

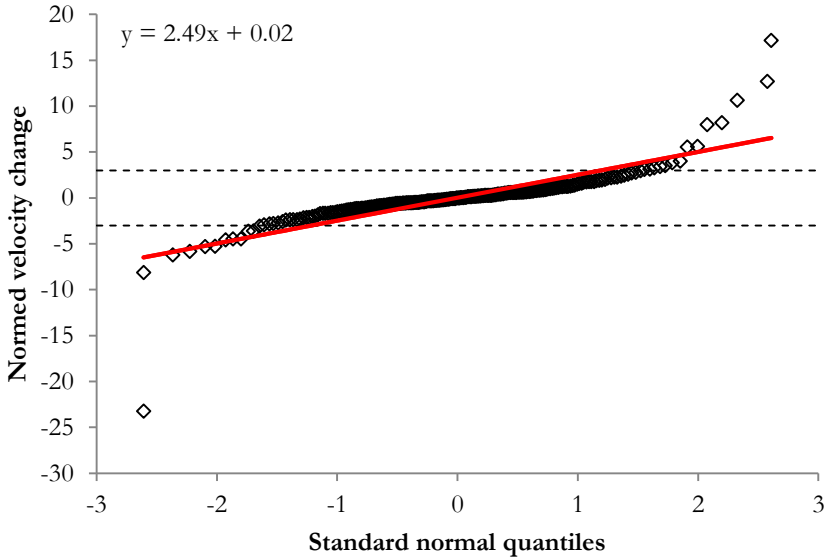
For example, when raw VV changes  $\mathbf{v}_{3-4} - \mathbf{v}_{1-2}$  were normed with standard errors from the variance component estimation (cf. Section 4.2.6), then only four normed VV changes were larger than 3 compared to 26 in Figure 27. In addition, the points are located better along the straight line, i.e. the empirical distribution of the raw VV changes fits better with a normal distribution.

When empirical distribution in a standard normal quantile plot follows normal distribution (solid straight line in Figures 25–27), then a large variance of unit weight from the kinematic adjustment was most probably caused by incorrect weights rather than by a change of VVs in time.



**Figure 26.** Normal probability plot of the raw vertical velocity (VV) changes from the Second and the Third to the Third and the Fourth levelling on the line sections of the common levelling network. The VV changes ( $n=220$ ) on the left side of Eq. (13) were normed by their *a priori* standard errors (Table 8) according to the section lengths. Points with ordinate values higher than  $\pm 3$  (dashed lines) can be considered as outliers. The total number of the outliers was 14.

Most empirical points in Figures 25–27 are located on the straight line, i.e. they are following a normal distribution (approximately  $N(0, 2.26)$  (Figure 25),  $N(0, 1.80)$  (Figure 26) and  $N(0, 2.49)$  (Figure 27)). The standard deviations 2.26, 1.80 and 2.49 are close to the square root of the variances of unit weight from the kinematic adjustment of the levelling combinations 1–2–3, 2–3–4 and 1–2–3–4 by the “heights eliminated” model ( $\sqrt{8.11} = 2.85$ ,  $\sqrt{3.25} = 1.80$ ,  $\sqrt{10.19} = 3.19$ , Figure 19), as they should. Thus, even local VV anomalies could be explained by the underestimated levelling errors, supposing that the *a priori* levelling standard errors based on loops’ misclosures were too small (Mäkinen and Saaranen 1998).



**Figure 27.** Normal probability plot of the raw vertical velocity (VV) changes from the First and the Second to the Third and the Fourth levelling on the line sections of the common levelling network. VV changes ( $n=220$ ) on the left side of Eq. (14) were normed by their *a priori* standard errors (Table 8) according to the section lengths. Points with ordinate values higher than  $\pm 3$  (dashed lines) can be considered as outliers. The total number of the outliers was 26.

#### 4.2.4. Correlation between the vertical velocities and velocity changes

According to Mäkinen and Saaranen (1998), conclusions regarding VV changes (Figures 22–24) depend on between-epoch correlation of repeated levellings. Correlation can be a real relationship or just accidental, fortuitous. However, between-epoch correlation influences deformation analyses independent of whether the correlation is fortuitous or genuine. In order to determine whether the correlation exists between different levelling campaigns, multivariate analysis was conducted according to the algorithm presented in Paper III. Significant correlation ( $p < 0.05$ ) was found between the Second and the Third levelling (Table 10). However, the results of this test are approximate due to the time-homogenization of the levellings.



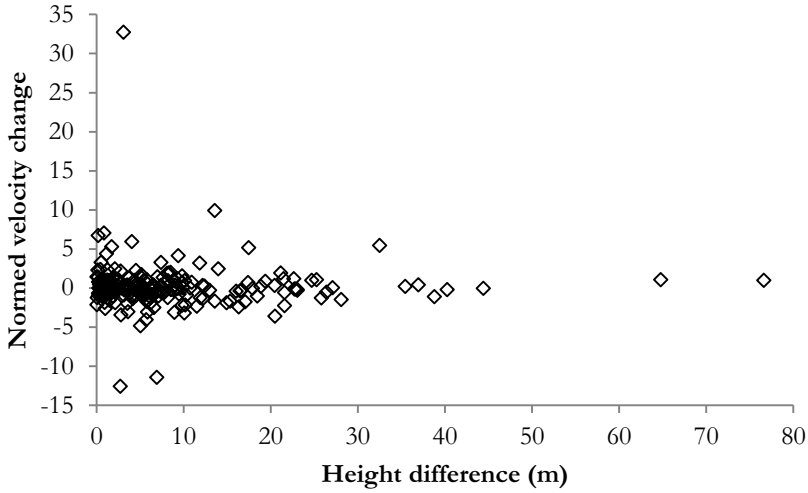
**Table 10.** Between-epoch correlation matrix of the repeated levelling campaigns based on multivariate analysis (Paper III).

Levelling campaign	1.	2.	3.	4.
1.	1.000			
2.	-0.046	1.000		
3.	-0.209	0.691	1.000	
4.	0.180	-0.200	0.427	1.000

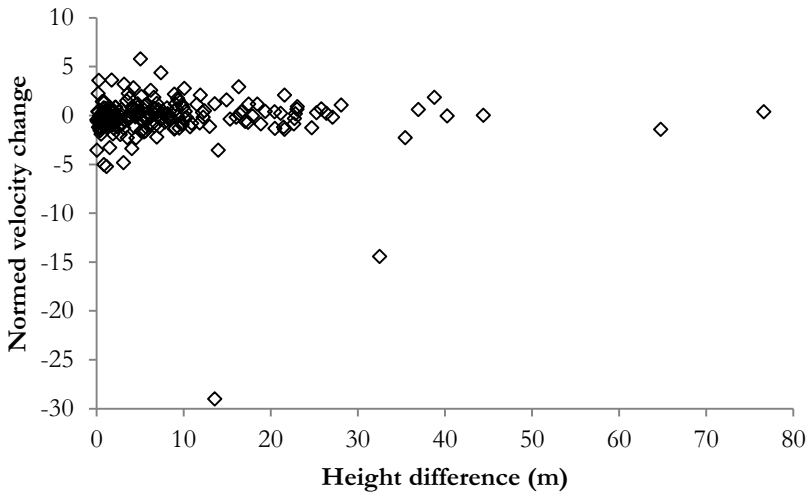
#### 4.2.5. Height-dependent errors in the levellings

From the previous analyses it appeared that the static adjustment of the individual levelling campaigns fitted well with *a priori* weights (Table 8), but the kinematic adjustment of the combination of the three and four levellings gave large variances of unit weight (Figure 19). Possible explanations for this include: *i*) ratios of the weights between the levellings being non-compatible in terms of kinematic adjustment; and *ii*) uneven VV of the BMs between levelling periods ascertained in Section 4.2.3. However, change of the BMs velocities can appear also from the *i*) between-epoch correlation between the repeated levellings (Section 4.2.4); and *ii*) from levelling errors. Levelling errors that do not show themselves in the misclosures but can be seen from the large variance of unit weight in the kinematic adjustment of three and four levelling combinations can be systematic errors (cf. Section 3.2.2).

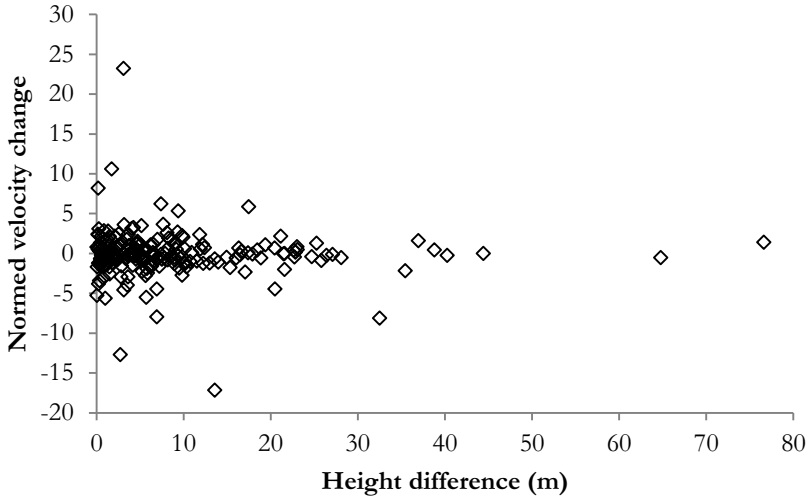
As was mentioned in Section 3.2.2, systematic errors tend to increase in proportion to height or height gradient along the levelling route. Therefore, it is reasonable to test whether the detected VV changes correlate with the topography along the levelling line. A test of the height-dependent errors was performed according to the methodology described in Section 3.3.4, i.e. graphs of the normed VV change on the vertical axis and height differences on the horizontal axis were compiled (Figures 28–30).



**Figure 28.** Relationship between normed raw vertical velocity (VV) changes from the First and the Second to the Second and the Third levelling (i.e., left side of Eq. (12) normed by their *a priori* standard errors (Table 8) according to the section lengths) and surface topography. The VV changes are taken in the direction of positive height differences.



**Figure 29.** Relationship between normed raw vertical velocity (VV) changes from the Second and the Third to the Third and the Fourth levelling (i.e., left side of Eq. (13) normed by their *a priori* standard errors (Table 8) according to the section lengths) and surface topography. The VV changes are taken in the direction of positive height differences.



**Figure 30.** Relationship between normed raw vertical velocity (VV) changes from the First and the Second to the Third and the Fourth levelling (i.e., left side of Eq. (14) normed by their *a priori* standard errors (Table 8) according to the section lengths) and surface topography. The VV changes are taken in the direction of positive height differences.

Let us assume that VV changes determined in Section 4.2.3 were not real but only apparent (caused by the levelling errors) and the size of levelling errors is related to the ground topography (the greater the height difference the larger the error is). Then linear trend between the VV changes and height differences should appear on the graphs. However, based on the composed graphs (Figures 28–30) and conducted regression analyses, VV changes do not correlate with topography. Even if such errors exist in observations, they are overwhelmed by the BMs motions or other levelling errors.

#### 4.2.6. Results of the variance component estimation

The variance component estimation of the four levelling campaigns of the CLN was performed using the Helmert, Bique, and Förstner methods. After convergence to unity, the methods led to identical variance factors. The only difference was in the number of the iterations necessary (Helmert and Bique = 14 iterations, Förstner = 40 iterations). Variance factors and re-scaled levelling standard errors obtained after the variance component estimation are presented in Table 11. Like in

the LNF (Table 6), observations of the First levelling campaign were also weighted down most in the CLN.

**Table 11.** Results of the variance component estimation by the Helmert, Bique, and Förstner methods. All methods ultimately led to the same result, except for the number of the iterations (Helmert and Bique = 14 iterations, Förstner = 40 iterations).

Levelling	<i>A priori</i> levelling standard error (mm/ $\sqrt{\text{km}}$ )	Variance factor	Re-scaled levelling standard error (mm/ $\sqrt{\text{km}}$ )
1	$\pm 1.387$	3.19138	$\pm 2.478$
2	$\pm 0.423$	0.93683	$\pm 0.409$
3	$\pm 1.576$	0.95863	$\pm 1.543$
4	$\pm 0.191$	1.22798	$\pm 0.212$

Change of the BMs' VVs after variance component estimation was  $0.05 \pm 0.03$  mm/yr and change of the standard deviations of the BMs' VVs  $-0.03 \pm 0.01$  mm/yr.

### 4.3. Vertical crustal movements along the Põltsamaa-Lelle levelling line

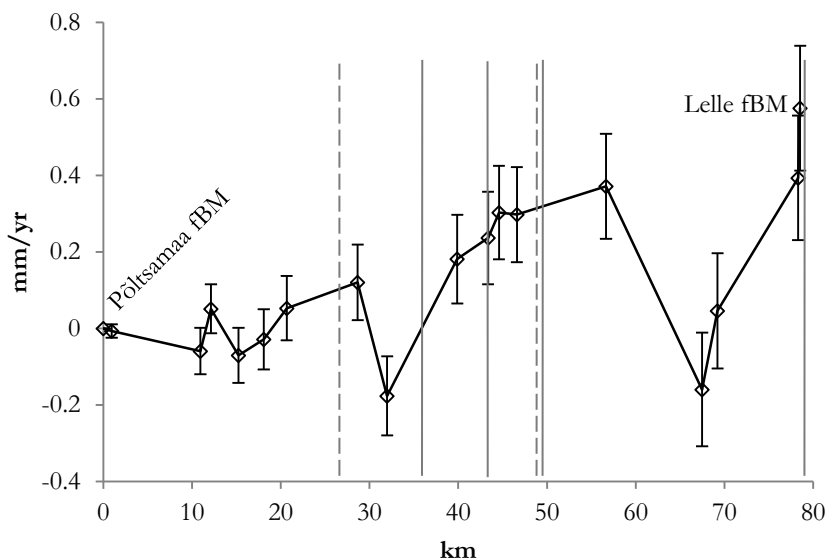
Relative VVs and their standard errors between the common BMs of the levellings used were found from the slope of the weighted linear regression line (cf. Section 3.3.2). The VV of the Lelle fBM relative to the Põltsamaa fBM  $0.53 \pm 0.15$  mm/yr was calculated from the sum of the relative VVs in levelling sections (Figure 4 in Paper I; Figure 31). From this relative VV gradient of VCM  $\sim 0.001$  degrees was found. This gradient coincides well with the average gradient 0.0005 degrees of the EST2013LU model (Figure 7 in Paper II) and 0.0008 degrees of the EST2015LU model (Figure 4 in Paper III).

In addition, outliers among height differences were pointed out using standardized residuals of the regression model. These anomalies were related to different factors: levelling errors, changes in groundwater level, subsidence of buildings, anthropogenic and tectonic reasons, etc. (Paper I). Relative VV of the Lelle fBM relative to the Põltsamaa fBM based on the slope of the linear regression line when the outliers were removed was even smaller (0.30 mm/yr) than in the previous case.

Compared to historical VCM maps of Estonia (Vallner et al. 1988; Randjärv 1993), the result obtained is significantly smaller. Values of

relative vertical movement by previous authors remain between 0.9...1.3 mm/yr (Paper I). The discrepancy can be explained with different calculation methods and use of levelling data from different years. At the same time, the result 0.53 mm/yr (and 0.30 mm/yr) coincides well with the values from the maps of Fennoscandian postglacial rebound (Ekman 1996; Vestøl 2006; Ågren and Svensson 2007), which remain between 0.3...0.7 mm/yr. Consequently, the hypothesis about the largest gradient of vertical movements in Estonia along the Põltsamaa-Lelle levelling line (cf. Sections 2 and 3.3.2) was not confirmed in the present study.

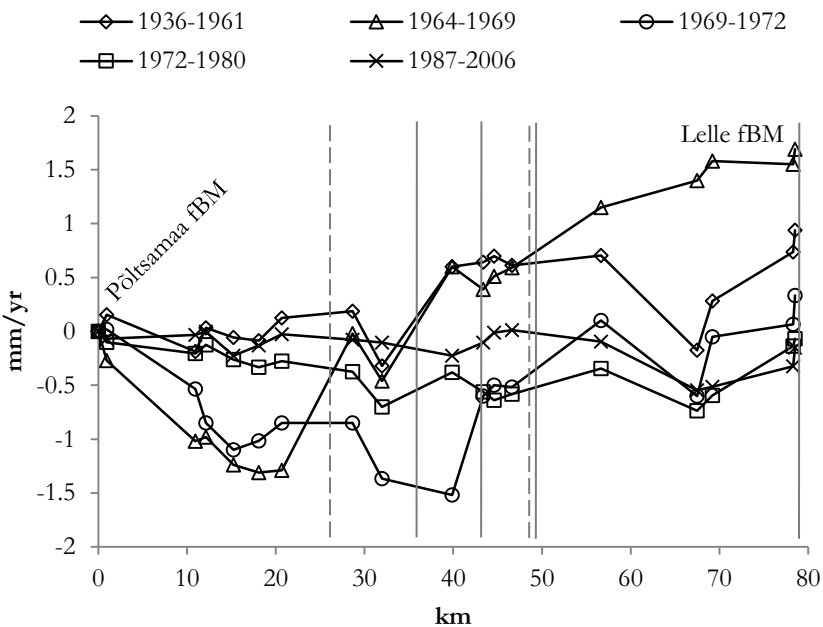
The long-term relative VVs of the other BMs relative to the Põltsamaa fBM were determined as well (Figure 31). In addition to the general trend of postglacial LU, one positive and two negative spikes can be observed. Positive spike at 12 km is probably related to different compaction of sediments, since the BM is placed at the old Pilstvere church. Probably its subsurface is more compacted than that of its vicinity BMs placed at farm buildings.



**Figure 31.** Long-term vertical velocities of the benchmarks along the Põltsamaa-Lelle levelling line from the kinematic adjustment of levellings from 1936, 1961, 1964, 1969, 1972, 1980, 1987, and 2006. Vertical dashed lines mark locations of the assumed faults of the sedimentary cover. According to Figure 9, vertical dotted lines mark locations of the defined shear or thrust zones of the basement.

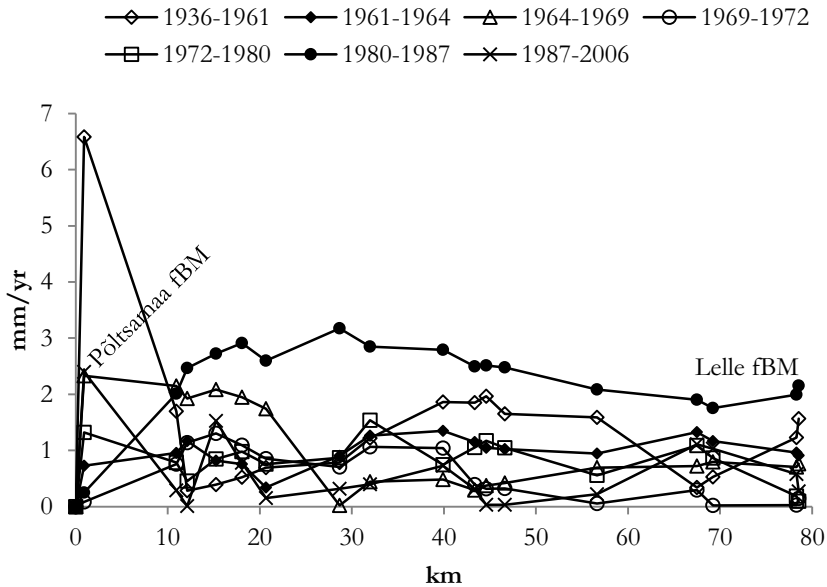
These typical features of many old churches have been described also by Ellenberg (1987). Negative spikes at 32 and 68 km are most likely to be related to local subsidence of the buildings due to the compaction of the sediments they are placed in (former Oisu dairy and dwelling house in Kärü). However, they may be related also to the movement of tectonic structures, since negative spikes can precede some step-like features related to the tectonic movements (Giménez et al. 2000). Such features can be found at 15–29 and 40–57 km in Figure 31. The Põltsamaa-Lelle levelling line crosses several geologic faults of the crystalline basement or sedimentary rock cover (Figure 9).

In addition, short-term vertical movements were also analysed. Graphs of the cumulative vertical movements of the BMs between consecutive levelling epochs were compiled according to Section 3.2.1 (Figures 32 and 34).



**Figure 32.** Short-term vertical velocities (VV) of the benchmarks along the Põltsamaa-Lelle levelling line. VVs in periods 1961–1964 and 1980–1987 were removed from the graph due to unrealistic values, which are probably related to the systematic errors in levellings (cf. Figure 34). Vertical dashed lines mark the locations of the assumed faults of the sedimentary cover. Vertical dotted lines mark the locations of the defined shear or thrust zones of the basement (cf. Figure 9).

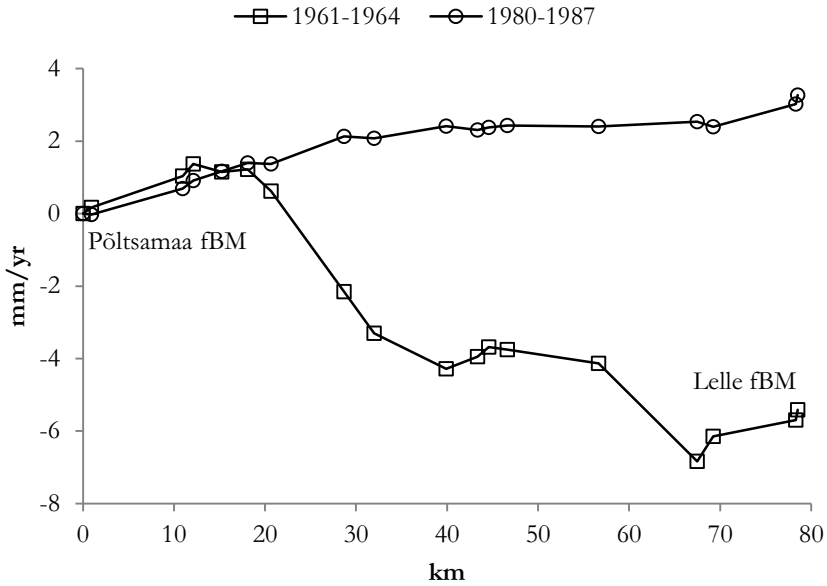
As can be seen, the vertical movements in the Põltsamaa-Lelle line have a fluctuating character, alternating once with the uplift and then the subsidence. Some step-like vertical movements preceded or followed by the spike can be seen in Figure 32. As was mentioned above, such features have been found to be related to geological or tectonic reasons like different compaction of recent sediments, regional tectonic tilt, or active tectonic structures (Giménez et al. 2000). However, most of the short-time VVs of the BMs relative to the Põltsamaa fBM, except for VVs between 1936 and 1961 and 1980 and 1987, are statistically insignificant, since the standard deviation of the VVs based on Eq. (15) exceeds the velocities (Figure 33).



**Figure 33.** Short-term vertical velocities of the benchmarks normed by their standard deviations (Eq. (15)) along the Põltsamaa-Lelle levelling line relative to the Põltsamaa fundamental benchmark. Vertical velocity of the benchmark is statistically significant if its normed velocity exceeds 2 (95% confidence level).

In any case, the short-term VVs detected from the levelling data should be interpreted with caution, since levelling errors could dominate over small VVs and may lead to false conclusions. In the author’s opinion, that was the case with the VVs from the periods 1961–1964 and 1980–1987 (Figure 34). VV values from 1.37 to  $-6.83$  mm/yr for the period 1961–1964 and from  $-0.02$  to 3.27 mm/yr for the period 1980–1987 of

the BMs were obtained. Most of the subsidence occurred in the section of 20–40 km in 1961–1964. The reason of this subsidence may very well be systematic errors in levelling in 1964. Note that accumulation of fore and back levelling discrepancies in that section was approximately  $-8$  mm (Figure 15).



**Figure 34.** Short-term vertical velocities of the benchmarks along the Põltsamaa-Lelle levelling line between 1961–1964 and 1980–1987.

On the other hand, Zhelnin (1966) explained the large subsidence of the BMs along the Põltsamaa-Lelle levelling line in 1961–1964 by the drought in 1964 when the ground water level was extraordinarily low. Zhelnin proposed that the drought had no influence on the first 20 km of the levelling line because limestone lies only at the depth of 1.3 m. For the next levelling sections, the subsidence of the BMs appears, as the thickness of the Quaternary sediments increases.

Relative subsidence of the BMs on the first 20–30 km is visible almost in all levelling combinations in Figure 32. Furthermore, it can be seen also in the profile of the long-term vertical movements (Figure 31), although these are statistically insignificant. It could be related to the differential VCM of different geologic structures. Statistically significant



short-term VVs in Figure 33 are most likely related to the variation of local hydrological conditions, different compaction of sediments, or levelling errors.

#### 4.4. Modelled surfaces of vertical crustal movements from the levelling data in Estonia

As was mentioned in Section 3.3.1.9, two VCM models were compiled for Estonia: EST2013LU (Paper II) and EST2015LU (Paper III). Statistics of the kinematic adjustment of the expanded CLN, VVs of the BMs which were used for compiling the EST2015LU model, are presented in Table 12.

**Table 12.** Main numerical results of the kinematic adjustment of the expanded common levelling network. The effect of the variance component estimation on the VVs of the BMs was negligible, since the *a priori* and *a posteriori* variance factors did not differ significantly (Paper III).

Parameter	Value
Variance of unit weight $S_0^2$ , first iteration	$\pm 5.21$
Number of detected outliers	41
Variance of unit weight $S_0^2$ after the outliers were weighted down	$\pm 0.86$
Average standard deviation of the adjusted vertical velocities	$\pm 0.11$ mm/yr
Mean standard error of the adjusted vertical velocity differences	$\pm 0.12$ mm/yr
The effect of the outliers on the vertical velocities	$\pm 0.04$ mm/yr

Different filtering techniques were implemented in the gridding process for removing outliers (cf. Section 3.3.1.9). For the EST2013LU model (Paper II), 8% of the BMs were removed from the LNF dataset. After that, the kriging method was used for gridding. The final grid was smoothed with the moving average filter. For the EST2015LU model (Paper III), 19% of the BMs were removed from the CLN dataset. Then, the minimum curvature method was used for gridding. The final grid was smoothed with the threshold average filter. Different gridding techniques gave quite similar results. For example, difference between the VV surfaces created by minimum curvature and kriging methods was  $\pm 0.07$  mm/yr on average (Paper II). Four gridding methods based on the BMs' VVs from the kinematic adjustment of the CLN are compared

in Table 13. As a result, no significant differences were found between different gridding techniques.

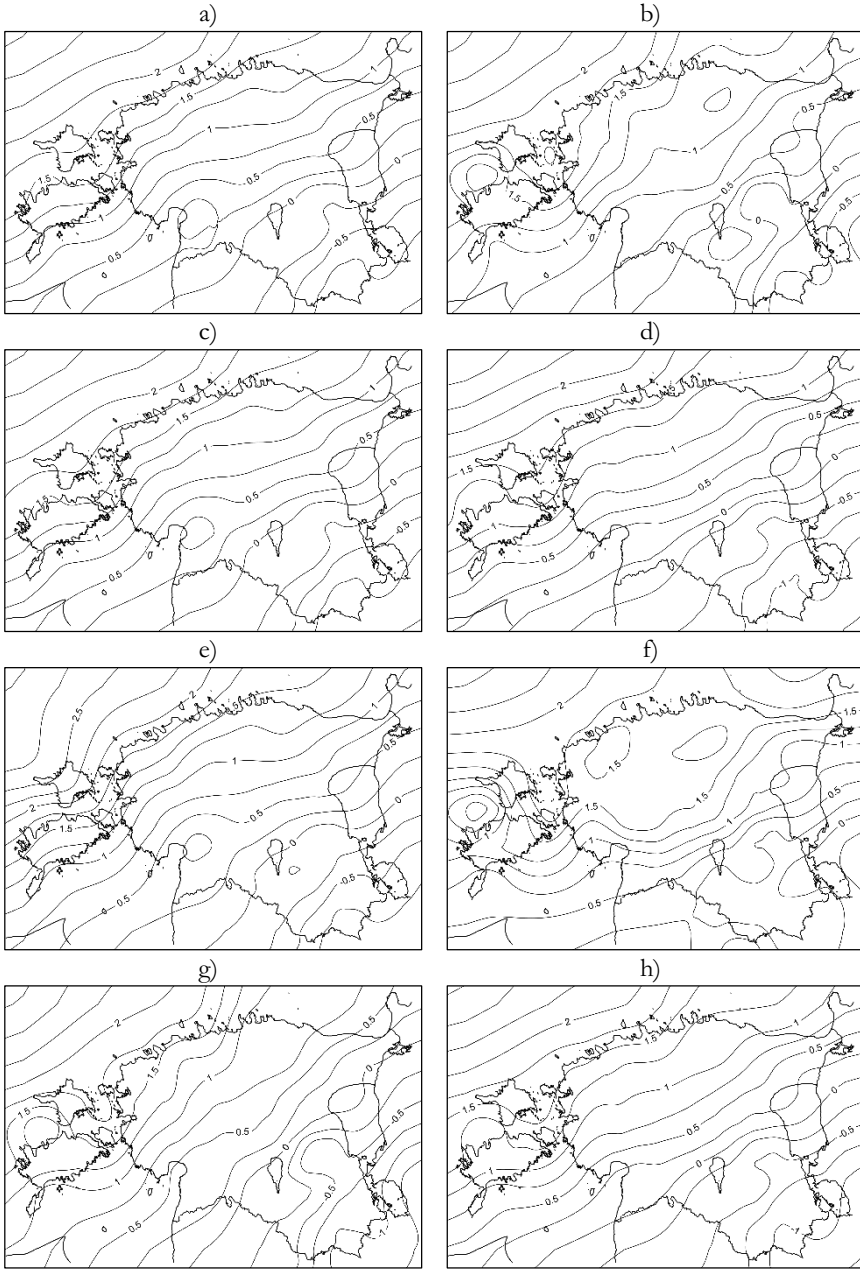
**Table 13.** Statistics of gridded model evaluation.

Method of gridding	Model residuals (mm/yr)				Cross validation residuals (mm/yr)		
	Mean	Min	Max	RMS	Mean	RMS	Average absolute deviation
Minimum curvature	0.002	-2.686	0.724	±0.239	-0.011	±0.376	±0.193
Kriging	-0.003	-2.732	0.667	±0.248	-0.001	±0.304	±0.161
Natural neighbor	-0.006	-2.966	0.545	±0.253	0.012	±0.344	±0.181
Local polynomial	-0.007	-2.774	0.617	±0.248	0.001	±0.300	±0.162

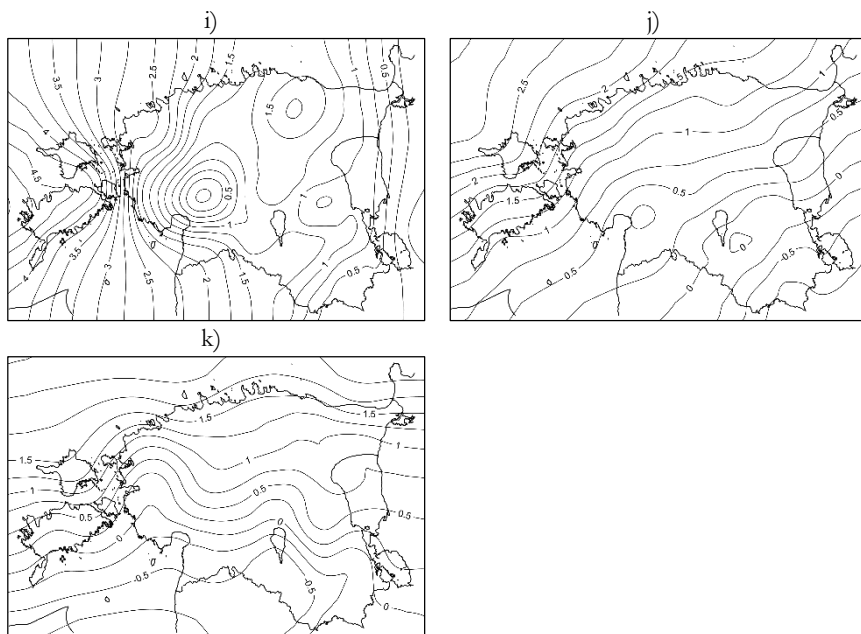
The CLN allows calculation of the VVs of the BMs from different levelling combinations. From the comparison of the EST2013LU and the EST2015LU model with the previous Fennoscandian or Estonian LU maps it was found that the VV isolines are more declined in an east–west direction than in historical maps. In the further investigation, gridded surfaces of the LU were created based on the VVs of the different levelling combinations from the adjustment of the CLN (Figure 35 a...g). A minimum curvature gridding technique was used. Obtained grids were smoothed with threshold average filter.

Combinations of three and four levellings (Figure 35 a...e) gave relatively similar surfaces regarding the direction as well as VV values of isolines. In contrast to the other combinations, VV isolines of the combination 1–2–3 are more declined in a SW–NE direction and values of the isolines shifted 0.25 mm/yr in a NW–SE direction.

From surfaces based on only two levellings, combinations 1–2 and 2–3 were found to differ most (Figure 35 f and i). VV anomalies in the form of closed curves and large VV values are noticeable. The combination 3–4 (Figure 35 g) is also distinguishable since VV isolines of it are ordinated in a NW–SE direction. VV isolines of the model of the levelling combinations 1–3 are in a more conventional SW–NE direction, whereas isolines of the combinations 1–4, 2–4, 3–4 are more W–E ordinated.



**Figure 35.** Isolines of the vertical velocities (VVs) based on different levelling combinations of the common levelling network: a) 1–2–3–4; b) 1–2–3; c) 1–2–4; d) 1–3–4; e) 2–3–4; f) 1–2; g) 1–3; h) 1–4. VVs are given relative to the fundamental benchmark FR241 in Tallinn (apparent VV 1.7 mm/yr). The minimum curvature method in software *Surfer* was used for gridding. Threshold average filter was applied for smoothing.



**Figure 35 (continued).** Isolines of the vertical velocities (VVs) based on different levelling combinations of the common levelling network: i) 2–3; j) 2–4; k) 3–4. VVs are given relative to the fundamental benchmark FR241 in Tallinn (apparent VV 1.7 mm/yr). The minimum curvature method in software *Surfer* was used for gridding. Threshold average filter was applied for smoothing.

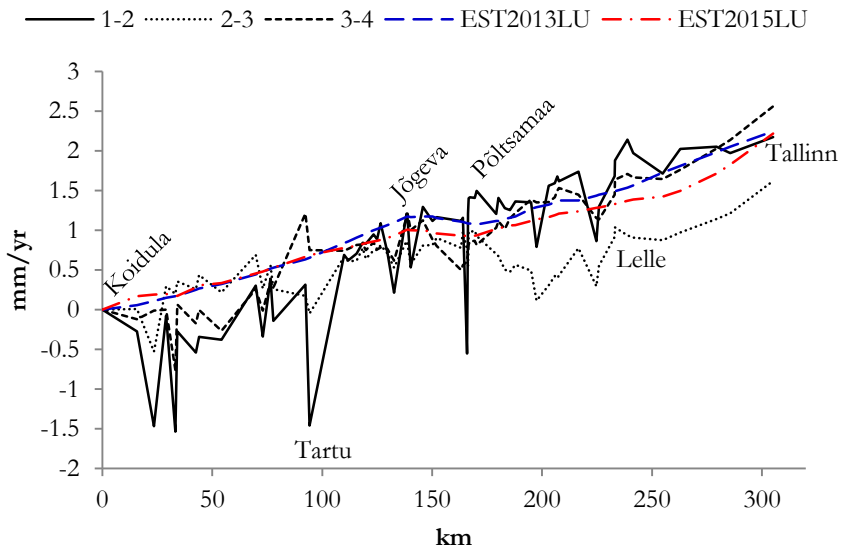
The VV values of the isolines as well as their directions in the models of two levelling combinations are clearly different. Differences between the models based on VVs with a shorter time period between levellings (1–2, 2–3 and 3–4) could be explained by this time element where possible systematic levelling errors in the dataset come to the fore. Models from the combinations with a longer time period between the levellings were more similar to each other, although the direction of the isolines in the 2–4 levellings model was more declined in a W–E direction than in the combinations 1–3 and 1–4.

To summarise, it can be concluded from the results of the analysis of the VCM models using all possible levelling combinations that data from the First and the Third levelling influenced isolines decline in a SW–NE direction (modelled surfaces based on VVs of the combinations 1–2–3 and 1–3), whereas data from the Second and especially Fourth levelling influenced isolines decline in a more W–E direction (combinations 1–2–4, 2–3–4 and 2–4). The larger weights of the Second and particularly

the Fourth levelling had a higher influence on the VV isolines of the EST2015LU model (Paper III) as well on the EST2013LU model (Paper II), resulting in their decline in a more W–E direction than in earlier maps.

#### 4.4.1. Accuracy of the EST2013LU and the EST2015LU model

Accuracy of the EST2013LU and the EST2015LU model was estimated *i)* by calculating residuals from the models at the observation points; *ii)* by the models' cross validation residuals; and *iii)* by calculating differences between raw (unadjusted) VVs of the BMs along the Koidula-Tartu-Jõgeva-Põltsamaa-Lelle-Tallinn levelling line with the VVs interpolated from the models. Results of the estimation are presented in Figure 36 and Table 14.



**Figure 36.** Profiles of the raw (unadjusted) vertical velocities (VVs) between the First and the Second (1–2), the Second and the Third (2–3), and the Third and the Fourth (3–4) levelling along the Koidula-Tartu-Jõgeva-Põltsamaa-Lelle-Tallinn levelling line. The profiles of the EST2013LU (Paper II) and the EST2015LU (Paper III) model are indicated for comparison. VVs are given relative to Koidula.

**Table 14.** Accuracy estimates (mm/yr) of the gridded surfaces of the EST2013LU and the EST2015LU model.

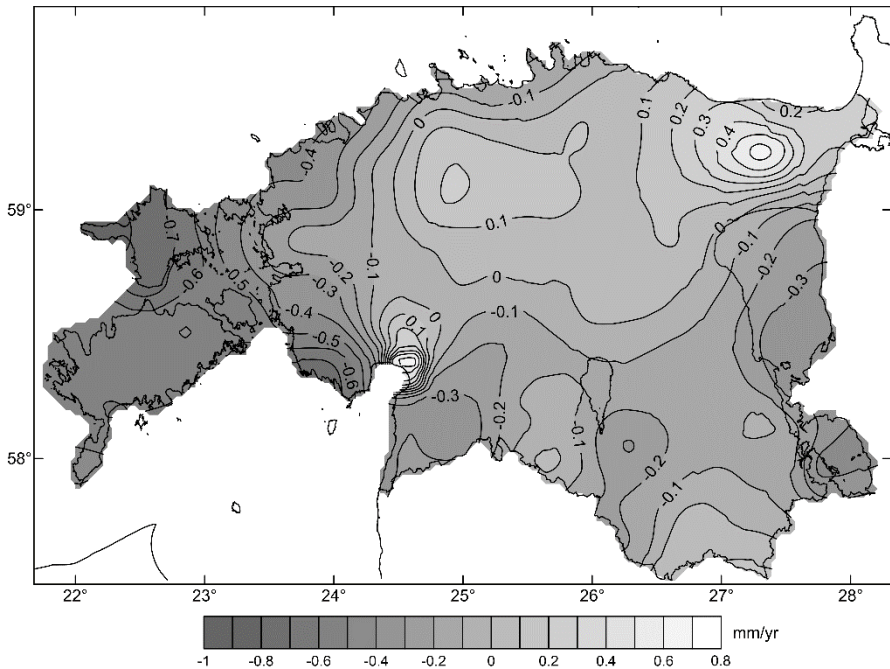
Statistic		EST2013LU	EST2015LU
Residuals of the model	Min	-0.41	-3.07
	Max	0.54	0.95
	Mean	0.01	-0.06
	RMS	$\pm 0.17$	$\pm 0.34$
Cross validation residuals	Min	-0.48	-1.98
	Max	0.52	2.21
	Mean	0.01	-0.02
	RMS	$\pm 0.16$	$\pm 0.30$
Differences from the raw vertical velocities 1-2	Min	-0.60	-0.77
	Max	2.11	2.14
	Mean	0.17	0.06
	RMS	$\pm 0.56$	$\pm 0.59$
Differences from the raw vertical velocities 2-3	Min	-0.25	-0.23
	Max	1.18	1.02
	Mean	0.42	0.31
	RMS	$\pm 0.54$	$\pm 0.43$
Differences from the raw vertical velocities 3-4	Min	-0.58	-0.54
	Max	0.93	0.94
	Mean	0.14	0.03
	RMS	$\pm 0.28$	$\pm 0.27$

Mean accuracy estimate of the EST2013LU and the EST2015LU model is  $\pm 0.39$  mm/yr and  $\pm 0.24$  mm/yr, respectively. These estimates are based on the mean standard error of the adjusted VV differences, average standard deviation of the adjusted VVs (Tables 5 and 12) and the RMS of the models' residuals and cross validation residuals (Table 14).

#### 4.4.2. Differences between the EST2013LU and the EST2015LU model

The differences between the EST2013LU and the EST2015LU model are presented in Figure 37. To compare these two models, it is required to take into account that they are based on the different reference BM and VV. The apparent VV of 1.9 mm/yr at the BM "SR Luidja" on the island of Hiiumaa (Figure 5 in Paper II) was used in the EST2013LU to

fix the VV solution. This value was obtained from the Ristna tide gauge apparent VV 2.1 mm/yr combined with the precise levellings between the tide gauge and the BM. The EST2015LU model is based on the VV of the fBM FR241 in Tallinn (apparent VV 1.7 mm/yr was taken according to Randjärv (1993), Davis et al. (1999) and Paper II). Due to the different configuration of the levelling networks and distribution of the residuals in the kinematic adjustment, it produced the largest differences between the two models in the islands of Hiiumaa and Saaremaa ( $-0.4 \dots -0.8$  mm/yr). However, overall the two models fit well with each other:  $\text{RMS} = \pm 0.30$  mm/yr. In most parts of the mainland, differences do not exceed  $\pm 0.1$  mm/yr. The largest differences in the mainland locate in Pärnu and NE of Estonia (up to 0.6 mm/yr). These differences are related to the local subsidence due to groundwater pumping (Pärnu) and oil shale mining (NE Estonia) (Listra and Talviste 1988; Mets et al. 2000; Rüdja 2004). These VV anomalies were highlighted in the EST2015LU model due to a denser set of BMs. Differences over Lake Peipsi are related to the use of the VVs of the water gauges of Lake Peipsi in Paper III.



**Figure 37.** Differences between the EST2013LU and the EST2015LU model (EST2013LU minus EST2015LU). Isoline interval is 0.1 mm/yr.

#### 4.4.3. Verification of the EST2013LU and the EST2015LU model with results from other studies

The EST2013LU and the EST2015LU model were compared with previous VCM models, CORS and tide gauge VVs (Tables 15 and 16). Main difference of the EST2013LU and the EST2015LU model relative to the NKG2005LU model (Ågren and Svensson 2007) is in the different direction of the VV isolines. In the models proposed in the present study, the isolines are inclined more towards the west–east than in the NKG2005LU model. Also, the isolines are shifted relative to each other, partially due to the different VV value at the reference point of the kinematic adjustment. In general, the fit with the NKG2005LU model is good, since differences remain mostly within the limits of the combined uncertainty of the models ( $\pm 0.6$  mm/yr).

**Table 15.** Vertical velocity differences (mm/yr) between the EST2013LU model and earlier results.

Difference	Min	Max	Average	<i>SD</i>	RMS
EST2013LU–Vallner et. al. 1988	–1.79	0.80	–0.43	$\pm 0.65$	$\pm 0.78$
EST2013LU–Randjärv 1993	–0.96	1.41	0.10	$\pm 0.57$	$\pm 0.57$
EST2013LU–Ekman 1996	–0.90	0.71	–0.37	$\pm 0.44$	$\pm 0.57$
EST2013LU–NKG2005LU (Ågren and Svensson 2007)	–0.39	0.57	0.03	$\pm 0.23$	$\pm 0.23$
EST2013LU–tide gauge and satellite altimetry (Paper III)	–0.66	0.80	0.00	$\pm 0.58$	$\pm 0.53$
EST2013LU–CORS (Oja et al. 2014)	–0.57	0.48	0.00	$\pm 0.37$	$\pm 0.34$
EST2013LU–Yakubovski 1973	–1.25	1.58	–0.05	$\pm 0.95$	$\pm 0.89$
EST2013LU–Pobedonostsev 1975	–0.66	1.90	0.96	$\pm 0.88$	$\pm 1.28$
EST2013LU–Jevrejeva et al. 2002	–0.09	1.74	0.81	$\pm 0.52$	$\pm 0.96$

Differences between the EST2013LU model and the older LU maps for Estonia and Fennoscandia (Vallner et al. 1988; Randjärv 1993; Ekman 1996) are larger than with the NKG2005LU model. Still, most of the differences are within the limits of the combined uncertainties of these models.



The largest discrepancies appeared from the comparison of EST2013LU surface with the VVs from Estonian tide gauge observations (Yakubovski 1973; Pobedonostsev 1975; Jevrejeva et al. 2002). At some stations, VVs coincide within the limits of uncertainties, but for most of the stations, differences exceed the uncertainties by 2–3 times. At the same time, discrepancies between the VVs obtained from the adjustment and the tide gauge solutions are of the same order as the inconsistency between the different tide gauge solutions.

**Table 16.** Vertical velocity differences (mm/yr) between the EST2015LU model and earlier results.

Difference	Min	Max	Average	<i>SD</i>	RMS
EST2015LU–Vallner et. al. 1988	–2.20	1.13	–0.29	±0.60	±0.67
EST2015LU–Randjärv 1993	–1.47	1.33	0.24	±0.51	±0.57
EST2015LU–Ekman 1996	–1.48	0.78	–0.24	±0.34	±0.42
EST2015LU–NKG2005LU (Ågren and Svensson 2007)	–1.01	1.00	0.16	±0.24	±0.29
EST2015LU–tide gauge and satellite altimetry (Paper III)	–0.75	1.53	0.00	±0.84	±0.76
EST2015LU–CORS (Oja et al. 2014)	–0.54	0.47	0.00	±0.36	±0.34
EST2015LU–Yakubovski 1973	–0.56	1.02	0.21	±0.57	±0.58
EST2015LU–Jevrejeva et al. 2002	0.42	2.48	1.15	±0.45	±1.24

The EST2015LU model fits to the historical tide gauge results (Yakubovski 1973) much better than the EST2013LU (RMS = ±0.58 mm/yr, compared to the EST2013LU’s ±0.89 mm/yr; Tables 15 and 16). The EST2015LU model fits also well to the results of Jevrejeva et al. (2002), considering the value of standard deviation (±0.45 mm/yr; Table 16). However, the large RMS value (±1.24 mm/yr) and the average value (1.15 mm/yr) of the differences suggest that the EST2015LU surface is systematically shifted relative to the tide gauge VVs by Jevrejeva et al. (2002). The same effect (larger RMS, smaller standard deviation) was observed also with the EST2013LU model compared to the results by Jevrejeva et al. (2002) (RMS = ±0.96 mm/yr; *SD* = ±0.52 mm/yr; Table 15).

The residual differences of the EST2013LU and the EST2015LU model with the Estonian CORS VVs (Table 17; Oja et al. 2014) were also small (RMS =  $\pm 0.34$  mm/yr; Tables 15 and 16). However, residual differences with the recalculated tide gauge VVs corrected with the satellite altimetry data (Table 17; Paper III) remain large (RMS =  $\pm 0.53$  and  $\pm 0.76$  mm/yr). Fit of the tide gauge and satellite altimetry VVs to the EST2015LU model is notably better when the Pärnu tide gauge was removed from the comparison (RMS =  $\pm 0.48$ ;  $SD = \pm 0.42$  mm/yr; Paper III).

**Table 17.** Absolute ( $v_{abs}$ , according to Liibusk and Wan (Paper III) and Oja et al. (2014)) and apparent ( $v_{app}$ , interpolated from the EST2013LU and EST2015LU models) vertical velocities (mm/yr) at the Estonian CORS and tide gauges. Locations of the stations are presented in Figure 6 in Paper III.

	Site	$v_{abs}$	$v_{app}$	
			EST2013LU	EST2015LU
CORS	AUDR	3.29	0.26	0.52
	KURE	3.43	0.67	1.27
	SUUR	4.61	1.67	1.96
	TOIL	3.18	1.21	1.05
	TORA	2.13	-0.05	0.12
	MISS	1.78	-0.66	-0.75
	KARG	4.26	1.77	2.45
Tide gauge	Ristna	3.7	1.94	2.75
	Virtsu	3.2	0.77	1.25
	Pärnu	2.8	0.18	-0.38
	Paldiski	3.3	1.60	1.98
	Suurpea	2.9	1.73	1.88
	Narva	2.4	1.10	0.85

Finally, fit of the EST203LU and the EST2015LU model to the historical VCM maps of Estonia compiled based on repeated levellings (Vallner et al. 1988; Randjärv 1993) was within  $\pm 0.65$  mm/yr on average. Taking into account the uncertainties of the models (on average  $\pm 0.2 \dots 0.4$  mm/yr), the fit is within the limits of uncertainties of the models. The main discrepancy between the models obtained in the present study and the historical VCM maps of Estonia is again in the different direction of the VV isolines.

## 5. DISCUSSION

VCMs based on the repeated levellings in Estonia were studied in this research. Firstly, the LNF was formed by selecting the stable fBMs of the Estonian levelling network and all available levelling observations from 1933 to 2010 between them. This enabled detection of the fBMs' long-term VV signal (Paper II). Long-term VV of the fBMs was expected to reflect primarily a postglacial rebound signal. Secondly, the CLN was formed based on the common BMs from four repeated levellings and respective levelling data from 1933 to 2011 (Paper III). The CLN was used to evaluate a possible change of the BMs' VVs in time. A denser set of BMs in the CLN made it possible to detect local VV anomalies as well. Thirdly, VCMs along the Põltsamaa-Lelle levelling line were analysed in order to test the hypothesis about the largest VCM gradient across the Pärnu-Narva geologic fault (Paper I).

### 5.1. Comparison of the proposed models of vertical crustal movements with the results of previous studies

Numerous previous studies have reported different values for apparent LU rates, ranging from  $-1.5$  to  $3$  mm/yr in SE and NW of Estonia, respectively. These rates were generally confirmed in the present study. The LU rates from  $-0.7$  to  $2.0$  mm/yr were determined based on the EST2013LU model (Paper II). The model EST2015LU based on a denser set of the BMs supported these rates with LU rates ranging from  $-0.8$  to  $2.8$  mm/yr (Paper III).

The main discrepancy between the compiled models and previous Estonian VCM maps is in the direction of the VV isolines. In the EST2013LU and the EST2015LU model, those are declined more in the west–east direction than in previous maps (e.g. maps by Vallner et al. (1988) and Randjärv (1993)). The main contributor to that direction of the isolines was the influence of the Second and the Fourth levelling campaign (Section 4.3). All four and the last three levellings were used to compile the EST2013LU and the EST2015LU model, respectively. However, data from only two levelling campaigns, usually the First and the latest, were used in the calculations of earlier Estonian VCM maps. Therefore, VV isolines of the historical VCM maps of Estonia coincide more with the isolines of the models calculated based on the First and the Second or the First and the Third levelling (Figure 35 f and g).

Differences between the maps/models are also related to the use of the different reference BMs and VVs for the calculation of the apparent VVs from the relative ones (cf. Sections 1.4.1 and 4.4.2). In general, fit of the two models obtained in the current study to the earlier VCM maps of Estonia does not exceed  $\pm 0.65$  mm/yr, which is within the limits of combined uncertainties of the models (Tables 15 and 16).

Similarly with the earlier Estonian VCM maps, the direction of the VV isolines of the EST2013LU and the EST2015LU model does not coincide exactly with the isolines of the recent LU maps for Fennoscandia, such as the map by Ekman (1996) and the NKG2005LU model (Ågren and Svensson 2007) (cf. Figure 8 in Paper II). Ekman used maps by Vallner et al. (1988) and Randjärv (1968) to draw the isolines in his map over Estonian mainland. Thus, the differences with Ekman's model are partly related to the differences with the abovementioned Estonian VCM maps. Differences with the NKG2005LU model can be explained by the fact that across Estonia this model is basically the GIA model by Lambeck, Smither, and Ekman (1998). Across Estonia, it is a geophysical model only, i.e. neither geodetic nor sea level data have been used there as additional constraints. Insufficient constraints for Estonia in the NKG2005LU model could explain the differences between the models. However, the overall fit between the models obtained in the present study and the map by Ekman (1996) and the NKG2005LU model is good, remaining within  $\pm 0.50$  mm/yr and  $\pm 0.26$  mm/yr, respectively.

Such west–east direction of the VV isolines in the EST2013LU and the EST2015LU model in central and east Estonia was not supported by the results from other observations like GNSS, tide gauge, tide gauge and satellite altimetry (Oja et al. 2014; Jevrejeva et al. 2002). This disagreement, at least partly, is related to a sparse set of CORS and tide gauges also. However, the west–east direction of the VV isolines in the EST2013LU and the EST2015LU model was supported by the VVs of the water gauges of Lake Peipsi determined by Raamat (2009). These VVs of the water gauges were incorporated also into the EST2015LU model (Paper III). Such direction of the VV isolines over Lake Peipsi and SE Estonia was supported also by the isolines of the map by Randjärv (1993) who incorporated also BMs' VVs from Russia and Latvia. To add here, curvilinear shape of the VV isolines in South-Estonia was supported by the recent map of Latvian VCM (Celms 2014).

One limitation of this study was the use of the levelling data without gravity corrections and partly without the rod calibration corrections (the First and the Fourth levelling data, cf. Section 3.1.4). Disagreement between the VVs from the CORS, the tide gauge and the levelling data may result from systematic errors either in the levelling or the tide gauge and CORS data. However, a small systematic error introduced by ignoring gravity corrections should be cancelled from VV calculations when the height differences between the BMs are differentiated, i.e. the “heights eliminated” model is employed, as was the case in the CLN (Paper III) (cf. Section 3.2.2). There were no significant differences between the VV and accuracy estimates from the “heights included” and the “heights eliminated” model. Therefore, levelling observations contained no systematic errors to be removed from the dataset by differentiating the observations. Moreover, no height-dependent errors were found from the dataset. However, using raw height differences (without rod calibration and gravity corrections) should not influence the calculated VVs. It only slightly increases the noise level (Fuhrmann et al. 2014; Olav Vestøl, personal communication).

At the same time, discrepancies appear between the GNSS VVs from different determinations (Tables 1 and 2) due to the reasons described in Section 1.4). Similarly, VV solutions from the repeated GNSS observations and CORS solution for Estonia differ systematically (Oja et al. 2014). Best fit was found between the CORS solution and the NKG2005LU model. Despite the different direction of the VV isolines between the levelling and GNSS solutions, overall fit of the EST2013LU and the EST2015LU model to the GNSS VVs (Table 17; Oja et al. 2014) is good ( $\pm 0.34$  mm/yr, Tables 15 and 16), remaining below combined uncertainty of the models and the GNSS VVs. To confirm the west–east direction of the VV isolines of the EST2013LU and the EST2015LU model, it is required to determine more independent results (like GNSS VVs) from central and east Estonia or to incorporate repeated levellings from Russian and Latvian side to the present calculations.

The largest discrepancies ( $\pm 1.02$  mm/yr in average, Tables 15 and 16) appeared from the comparison of EST2013LU and EST2015LU models with the VVs from Estonian most relevant tide gauge solutions (Yakubovski 1973; Pobedonostsev 1975; Jevrejeva et al. 2002). At most of the stations, differences exceed uncertainties by two or three times. At the same time, differences between the tide gauge and the

EST2013LU and EST2015LU VVs are in the same order as the differences among the previous tide gauge solutions (Table 2 in Paper II). Differences have a systematic nature. For example, RMS difference between the models obtained in this study and the tide gauge VVs from the solution of Jevrejeva et al. (2002) was  $\pm 1.11$  mm/yr on average. At the same time, the standard deviation of the differences was more than two times lower,  $\pm 0.49$  mm/yr.

Therefore, new VVs of Estonian coastal tide gauges were calculated by Liibusk and Wan (Table 17; Paper III), employing different methodologies (shorter sea level observation period, tide gauge data were corrected with satellite altimetry data). Comparing VVs from the EST2013LU and the EST2015LU model with that new tide gauge and satellite altimetry solution, the RMS of the residual VVs decreased almost two times compared to previous tide gauge solutions (Tables 15 and 16). The fit of the EST2015LU model to the tide gauge and satellite altimetry VVs was much better without site Pärnu (outlier due to the subsidence of Pärnu). Reasons for the differences between the levelling and tide gauge VV solutions should be further investigated, especially for the effects of possible systematic errors in the data.

Finally, gridded surfaces of the EST2013LU and the EST2015LU model were compared with the raw VVs of the BMs from levelling combinations 1–2, 2–3, and 3–4. A comparison was made along the Koidula-Tartu-Jõgeva-Põltsamaa-Lelle-Tallinn levelling line relative to Koidula (Table 14 and Figure 36). The profiles of the EST2013LU and the EST2015LU model coincide with the profiles of the cumulative VVs between the Third and the Fourth levelling best. Clearly, this is related to the higher weights of the Fourth levelling in the kinematic adjustment. However, neither does the fit to the VVs of the BMs from the other levelling combinations exceed the combined uncertainty of the LU models and BMs' VVs themselves, remaining within the limits  $\pm 0.6$  mm/yr.

## 5.2. Comparison of the EST2013LU and the EST2015LU model

Another question is a considerable VV difference (up to 0.8 mm/yr) between the EST2013LU and the EST2015LU model (Figure 37) on the islands of Saaremaa and Hiiumaa. Essentially, this difference is caused by the use of different reference BMs and VVs in the kinematic adjustment of the LNF and the CLN (cf. Section 4.4.2).

However, detailed analysis revealed that the VV differences depend also on the used network and the levelling data. For example, the VVs of the BMs were not influenced by the reference BM in the CLN (e.g. fixing apparent VV 1.7 mm/yr for Tallinn produced VV 2.5 mm/yr on the island of Hiiumaa and vice versa). However, such one-to-one relationship was not observed in the LNF. When apparent VV 1.7 mm/yr was fixed for the BM in Tallinn, VV 2.4 mm/yr was obtained on the island of Hiiumaa. But the opposite calculation in the LNF, e.g. fixing VV 2.4 mm/yr for the island of Hiiumaa, produced VV 2.2 mm/yr for Tallinn instead of 1.7 mm/yr, i.e. a difference  $\sim 0.5$  mm/yr was obtained. This difference is also observable between the EST2013LU and the EST2015LU model on the islands of Saaremaa and Hiiumaa.

When the VVs of the BMs from different solutions of the LNF (reference VV 2.4 mm/yr on the island of Hiiumaa or 1.7 mm/yr in Tallinn) were represented on the map of Estonia, it appeared that a sudden jump between the VVs (0.4...0.5 mm/yr) appeared on the island of Saaremaa and continued approximately with the same size on the mainland. Therefore, it can be assumed that the reason for the abovementioned VV differences is the height differences used for crossing the strait between the islands of Saaremaa and Hiiumaa in the LNF. In addition, the largest residual differences of the EST2015LU model with both the GNSS and tide gauge and satellite altimetry VVs occurred in the island of Hiiumaa. This suggests that height connections between the islands of Saaremaa and Hiiumaa should be re-examined or repeated in the future. In the future, more than just one reliable tide gauge or GNSS VV should be used in order to fix the VV solution from the levelling data.

### **5.3. Local vertical velocity anomalies based on the EST2015LU model**

Levellings from the Second, Third and Fourth levelling campaigns were used to compile the EST2015LU model. Decision to omit the First levelling data was made based on the *a posteriori*  $S_0^2$  estimates (Figure 19) and variance component estimation, where observations from the First levelling campaign were heavily weighted down (Table 11). That allowed formation of a denser network of BMs (so-called expanded CLN) than the CLN of the four levelling campaigns.

Analysis of the EST2015LU model confirmed some local VV anomalies revealed also in previous studies. One is the area surrounding Pärnu (Figure 4 in Paper **III**) where local subsidence due to a decrease in groundwater level has been identified (Listra and Talviste 1988; Mets et al. 2000). Another notable anomaly is related to the oil shale mining area in NE Estonia (Rüdja 2004). Thirdly, anomalies in the form of rapid change of the direction of the VV isolines (U-shape) near Kilingi-Nõmme and Puka (Figure 11) can be noticed. Kilingi-Nõmme anomaly is based on the number of BMs, including fBM in Abja-Paluoja and Mõisaküla, the VVs of which are relatively larger than in adjacent BMs. Puka anomaly is based on the four BMs, including the Puka fBM. It is difficult to find an explanation to these anomalies. There are Proterozoic faults in the crystalline basement crossing in the Puka area (Figure 9). Also, quite large variations of gravity anomaly have been established in this area (Ellmann et al. 2011). Both the presence of a tectonic fault as well as the variation of gravity anomaly can be related to the rapid change of the BMs' VVs along the levelling line (Giménez et al. 2000). Locations of these anomalies coincide also with the locations of the differentiated movements of separate blocks of the basement in Tõrva and Kilingi-Nõmme according to Vallner and Zhelnin (1975) and Vallner et al. (1988) (Figure 11). However, in the author's opinion, the vertical movement anomalies in Puka and Kilingi-Nõmme are most likely to be related to the different compaction of the sediments.

As was described in Section 4.2.3, the local VV anomalies can be detected from Figures 22–24 in the places where arrow heights are abruptly changing compared with the neighboring ones. In fact, BMs with abruptly changing VVs in Figure 23 mainly coincide with the BMs, which were visually removed during the modelling (Figure 4 in Paper **III**). These BMs were considered as unstable.

#### **5.4. Differences between adjustments of levelling combinations and “heights included” and “heights eliminated” models**

From the performed  $\chi^2$ -test, it was determined that the variances of the unit weight  $S_0^2$  in the kinematic adjustment of two levelling combinations did not differ significantly from *a priori* value 1 whereas  $S_0^2$  from three and four levelling combinations did (Figure 19; Table 2 in Paper **III**). This indicates either that *i*) the model with constant VVs (Eqs. (4) and (5) in Paper **III**) of the BMs between the levelling periods



does not fit with the observation data; *ii*) the ratios of weights of the different levellings were incompatible; or *iii*) the levellings contain errors. Outliers had a great impact on the variance of unit weight. For example, during the process of data snooping in the kinematic adjustment of the combination 1–2–3–4, variance of unit weight decreased from 10.80 to 1.79, i.e.  $\sim 6$  times. Reasons related to the change of the BMs' VVs in time and levelling errors are discussed in Section 5.5.

Further, larger values of  $S_0^2$  occurred in the combinations where the First and the Second levelling were involved (Figure 19). This was probably related to the ratios of the weights between the First and the Second levelling being incorrect. *A priori* estimates of the standard errors of the First levelling were approximately three times smaller than the estimates from variance component estimation (Table 11).  $S_0^2$  for the combination 1–2 was smaller in the “heights eliminated” model than in the “heights included” model. Differences of  $S_0^2$  between the “heights included” and the “heights eliminated” model can be related to the differentiation of the observations in loops with the same loop's misclosure sign. Signs of the loops' misclosures for the First and the Second levelling coincided completely in all loops (Table 1 in Paper III). However, smaller  $S_0^2$  in the “heights eliminated” model could also be related to the fact that the solution of the “heights eliminated” model of two levelling combinations is independent of the weights (Mäkinen and Saaranen 1998).

The mean standard error of the relative VVs ( $\mu$ ) was strongly influenced by the Fourth levelling (Figure 20). Including the Fourth levelling in the adjustment combination reduced the standard error significantly. The reason probably was that the weights of the Fourth levelling were approximately two times higher than those of the others. Additionally, leaving out the middle levelling from a combination of three (e.g. 2–3–4 compared with 2–4) influenced only slightly the error estimates. Mäkinen and Saaranen (1998) explained this effect by a simple linear regression where omitting an observation in the middle of abscissa values has no significant influence on the value of the slope.

The mean standard error of the relative VVs was also influenced by the time period between the levellings. A longer time period between the levellings contributed to a smaller standard error (e.g. combinations 1–3

compared with 1–2 or 2–3, 2–4 compared with 2–3 or 3–4). The effect of a longer time period could also be explained by the abovementioned regression analogy. The large standard error in the combination 2–3 was related to the loops on the islands of Saaremaa and Hiiumaa and height differences connecting the mainland to Saaremaa, and Saaremaa to Hiiumaa. Adjustment of the same levelling combination for the mainland part of the CLN plus the loop on the island of Saaremaa gave nearly four times smaller mean standard error. The repeated hydrostatic levellings connecting the islands of Saaremaa and Hiiumaa have been performed with a very short time interval between the levellings. Additionally, quite large discrepancy between the two height differences was obtained. When the adjustment was performed only with the mainland part of the CLN, the mean standard error of the VV differences reduced further two times. This indicates that the height connections of the island of Saaremaa with the mainland or the levelling loop on Saaremaa influence the error estimates in the levelling combination 2–3. (Paper **III**).

The effect of eliminating heights from the solution was insignificant for all levelling combinations. Statistically, neither variances of unit weight nor standard errors of the VVs from the “heights included” and the “heights eliminated” model differed significantly whichever levelling combination was used (in all cases of  $S_{0_1}^2 / S_{0_2}^2$  and  $\mu_1^2 / \mu_2^2$ , the ratio did not differ significantly ( $p > 0.05$ ) from *a priori* value 1, Table 2 in Paper **III**). In addition,  $S_0^2$  from three and four levelling combinations was smaller in the “heights included” model, whereas combinations of two levellings gave smaller  $S_0^2$  in the “heights eliminated” model in half of the cases. The ratio  $S_{0_1}^2 / S_{0_2}^2$  was most likely related to the ratio of the weights between the levellings. The solution for two levelling combinations in the “heights eliminated” model is independent of weights, whereas the solution from the “heights included” model is the best linear unbiased estimator (BLUE) only when the ratio of weights from the two levellings is correct (Mäkinen and Saaranen 1998). An increase of  $S_0^2$  in the levelling combinations 1–3, 2–3 and 2–4 for the “heights eliminated” model was most probably related to a correct ratio of observation weights in the “heights included” model. This difference, however, only concerned the error estimates. The VVs from the two models were identical.

In Section 4.2.2, different tests were pointed out in order to elucidate the reasons for VV differences between the “heights included” and the “heights eliminated” model in the levelling combinations 1–2 and 1–2–3. Only time-homogenization eliminated differences between the VVs of the “heights included” and the “heights eliminated” model in previously problematic combinations. Therefore, it is probable that the differences were caused by the longer time period of closing of the loops during the Second and the Third levelling compared to the the First and the Fourth (Table 1 in Paper **III**). The long duration of completing the loops can also explain why the VVs between the two models of the abovementioned problematic levelling combinations (1–2–3, 1–2, 2–3 and 3–4) did not differ for loop numbers I, VI and VII (Figure 1 in Paper **III**) even before time-homogenization. The time period of closing of these loops was smaller compared to the other loops within the same levelling combination.

In summary, from the comparison of two mathematical models of kinematic adjustment it was clarified that neither error estimates nor VV estimates differ significantly between the “heights included” and the “heights eliminated” model. Therefore, it may be concluded that the levelling observations used contained no such systematic errors that could be removed from the dataset by differentiating the observations.

### **5.5. Change of the benchmarks’ vertical velocities in time**

Another research question raised in Chapter 2 was: do the BMs’ VVs change in time? As was pointed out in Section 5.4, variances of unit weight  $S_0^2$  of three and four levelling combinations were large and it may be related to the change of the BMs’ VVs, incorrect weights, or levelling errors.

In order to test the hypothesis about the VV change between the levelling periods, VVs from different levelling combinations were compared with an ANOVA test (Section 4.2.3). Results indicated that a significant change of the VVs between the levelling periods had taken place, even if the internal correlation between the VVs and the ratios of the weights was taken into account. The smallest change in the BMs’ VVs was observed between the VV pairs 1–2 and 2–4. Interestingly, the same levelling combination (1–2–4) gave the largest  $S_0^2$  in the kinematic adjustment (Figure 19). This suggests that large  $S_0^2$  in three and four

levelling combinations in addition to the uneven VVs of the BMs between the levelling periods is also related to the incorrect weights of the levellings.

Unfortunately, results of the ANOVA tests are inconclusive, since a significant between-epoch correlation was found between the Second and the Third levelling campaign ( $r \approx 0.7$ ,  $p < 0.05$ ; Section 4.2.4). The between-epoch correlation influences the deformation analyses, independent of whether the correlation is fortuitous or genuine (Mäkinen and Saaranen 1998). The significant VV changes presented in Section 4.2.4 can be related to the correlation between the Second and the Third levelling. The same conclusion was made by Mäkinen (2000) for Finland, where a significant VV change was found between the First to the Second and the Second to the Third levelling. At the same time, a significant negative correlation was found between the First and the Third levelling in Finland. In the author's opinion, a negative correlation ( $r \approx -0.6$ ,  $p < 0.05$ ) between the changes of the VVs  $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$  and  $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$  (Figures 22 and 23) suggests also that some levelling data are contaminated with systematic errors. Therefore, change of the VVs between the levelling periods may only be apparent.

As was explained above, the large variance factor of three and four levelling combinations as well as the detected VV change between the levelling periods can be explained by the levelling errors supposing that the levelling errors are significantly larger than it was assumed *a priori*. For example, a variance of the unit weight  $S_0^2 = 10.078$  was obtained from the kinematic adjustment of the “heights included” model of the levelling combination 1–2–3–4 (Figure 19). The detected VV change over time (Section 4.2.3; Tables 3 and 4 in Paper III) could also be explained by levelling errors when we assume that the levelling error is  $S_0 = \sqrt{10.078} = 3.2$  larger than indicated by the loops' misclosures. Then *a priori* levelling standard error estimates of all four levellings (Table 8) could be re-scaled uniformly by 3.2 times. Instead of that, the variance component estimation using different methods (Helmert, Bique, Förstner, IAUE) was performed, that as a result, placed most of the error on the First levelling.

Then, an attempt was made to find out the nature of these levelling errors that do not reveal themselves in the loops' misclosures, but

become apparent in a combination of three or four levellings (large  $S_0^2$  in the kinematic adjustment). They cannot be detected based on only two levellings as the VV values exceed these errors in that case (Mäkinen and Saaranen 1998). A logical explanation would be to look for the height-dependent systematic errors (refraction, rod scale errors). Theoretical relationships between systematic errors in the levellings and the topography of a levelling route have been shown by Stein (1981), Stein et al. (1986) and Craymer and Vaníček (1995). An attempt has been made to determine if these levelling errors depend on the topography along the levelling route, which however, turned out to be negative. Height-dependent errors were not detected in the present study (Section 4.2.5).

However, results of the height-dependent error test do not mean that levellings contain no systematic errors. Change of the topography along the levelling routes is quite small in Estonia in order to definitely confirm that. In fact, in Section 3.1 some limitations of the used data were pointed out (concerning rod and gravity corrections) but a certain methodology (differentiating the levelling observations) was used to assure that they cannot contaminate VV determination. In most levelling combinations, no significant differences existed between the VV estimates from the “heights included” and the “heights eliminated” model. Therefore, it can be concluded that levellings contained no such systematic errors that would be removed by differentiating the observations.

In summary, no definite answer can be provided whether the large variance of unit weight in combination of three or four levellings was caused by the uneven VVs of the BMs, improperly chosen weights, or systematic levelling errors. Most probably it is caused by the combination of all aforementioned factors. Further research on this topic is needed.

### **5.6. Gradient of vertical crustal movement along the Põltsamaa-Lelle levelling line**

A hypothesis was set up in Section 2 based on the conclusions from Vallner and Zhelnin (1975), Vallner (1978) and Vallner et al. (1988): the highest gradient of the VCM in Estonia is across the Pärnu-Narva geological fault (hinge line) (Orviku 1960). The Põltsamaa-Lelle levelling

line is particularly suitable to test this hypothesis, since it has been levelled more times than any other line in the Estonian levelling network and it crosses the abovementioned fault as well as many other faults (Figure 9). The long-term gradient  $0.53 \pm 0.15$  mm/yr was obtained based on 11 repeated levellings along the Põltsamaa-Lelle levelling line. The gradient obtained is not larger than an average LU gradient for the whole of Estonia based on the EST2013LU and the EST2015LU model. This result is approximately two times smaller than the gradient obtained in the abovementioned earlier studies. However, it coincides well with the gradients obtained from Fennoscandian LU maps (Ekman 1996; Vestøl 2006).

Detailed analysis revealed quite large variations of small-term vertical movements along the Põltsamaa-Lelle line (Figures 32 and 34). Discrepancy between the gradient obtained in this study and from earlier Estonian LU maps is most likely the result from the levellings used in previous calculations. As was mentioned above, data from only two levelling campaigns (usually the first and the latest in order to obtain longer time-period between levellings and minimize the effect of levelling errors onto VV calculations) were used in the calculations of the earlier VCM maps of Estonia. Therefore, VV values were found dependent directly on the choice of these two levellings. This can be interpreted as a simple linear regression of the two height differences between the same BMs with a time interval between the measurements in abscissa (Mäkinen and Saaranen 1998; Mäkinen 2000). It is possible to draw a straight line right through the two data points.

For example, Randjärv (1993) obtained apparent VVs 0.34 mm/yr for the Põltsamaa fBM and 1.22 mm/yr for the Lelle fBM from an adjustment which gives a relative VV 0.88 mm/yr between these BMs. His calculations were based on the levellings of 1936 and 1961. Raw (unadjusted) VV difference between the Põltsamaa and Lelle fBMs based on these two levellings is 0.94 mm/yr. In the present study, all performed levellings were used to obtain the VV difference between the two BMs on the Põltsamaa-Lelle levelling line. Therefore, reliability of the obtained value is increasing dramatically according to the theory of Baarda (1968). This increase of the reliability is independent of the improvement in precision that comes from the increased time span between the two levellings (Mäkinen 2000). In summary, the hypothesis about the highest VCM gradient in Estonia across the Pärnu-Narva geological fault (hinge-line) was not confirmed in the present study.

## 6. CONCLUSIONS

Focus in the dissertation was on the calculations of the VCMs in Estonia based on four precise levelling campaigns from 1933 to 2011. The VVs of the BMs were found from the joint kinematic adjustment of four or three levelling campaigns. Two different mathematical models, “heights included” and “heights eliminated”, were used in the adjustment. The methodology in the present study allowed detecting outliers among observations, evaluating the results of the measurements and adjustment statistically, and estimating the change of the BMs’ VVs in time. For modelling of the VCMs, different options of the computer software *Surfer* were used. Results of the modelling were verified by finding residuals between the modelled surface and adjusted VVs at the observation points, using the cross validation technique, and by comparing the models with the results from other geodetic measurements (CORS, tide gauge, other LU models). The main findings of the study are:

- Two LU models were calculated based on the repeated levellings. The EST2013LU model (Paper **II**) based on the sparser set of presumably stable fBMs (77 fBMs) revealed a general long-term trend of the postglacial uplift in Estonia. The second EST2015LU model (Paper **III**) is based on the common BMs of the Second, the Third and the Fourth levelling plus the VVs of the water gauges of Lake Peipsi. Since there were almost 300 observation points for compiling the second model, in addition to the postglacial rebound signal, some LU anomalies were also revealed. Average uncertainty estimates of the EST2013LU and the EST2015LU model were  $\pm 0.39$  mm/yr and  $\pm 0.24$  mm/yr, respectively.
- Territory of Estonia is subject to ongoing postglacial rebound from  $-0.8$  to  $2.8$  mm/yr. However, the direction of the VV isolines of the models calculated in the present study are more declined in the west–east direction than in previous studies. The main contributor to this direction of the isolines is the Second and especially the Fourth levelling and higher weights of these levellings compared to the weights of the First and the Third levelling. This direction of the isolines is currently supported by the vertical movements of the water gauges of Lake Peipsi determined by Raamat (2009) and the VCM map by Randj arv

(1993) who used the BMs' VVs from Russia and Latvia also. Curvilinear shape of the VV isolines of the EST2015LU model in South Estonia is supported also by the Latvian VCM map based on the recent repeated levelling of Latvia (Celms 2014). To confirm the west–east direction of the VV isolines of the EST2013LU and the EST2015LU model, it is required to determine more independent results (like GNSS VVs) from central and east Estonia and to incorporate repeated levellings from Russian and Latvian side to the present calculations.

- As was mentioned above, data from the Second and especially the Fourth levelling influence the VV isolines of LU models obtained in the present study to decline in a more W–E direction compared to the earlier maps/models. However, data from the First and the Third levelling influence the VV isolines to decline in a SW–NE direction. These relationships between the levelling combinations and direction of the VV isolines explain also the differences between the LU models obtained in the present study and earlier LU maps for Estonia, which were based only on two levellings. Different gridding techniques give quite similar results for the LU maps. Removing BMs from the model influences gridding results most.
- In addition to the general trend of the postglacial uplift, local VV anomalies are presented. Main VV anomalies according to the EST2015LU model are located in Pärnu and NE of Estonia. These VV anomalies are related to the groundwater pumping (Pärnu) and oil shale mining (NE of Estonia).
- Fit of the EST2013LU and the EST2015LU model to the most recent Fennoscandian LU maps by Ekman (1996) and the NKG2005LU model (Ågren and Svensson 2007) was good. Differences with the NKG2005LU model remained within  $\pm 0.26$  mm/yr on average. Discrepancies with the LU map by Ekman (1996) remained within  $\pm 0.50$  mm/yr on average. Fit of the models obtained in the present study to the historical VCM maps of Estonia compiled based on repeated levellings (Vallner et al. 1988; Randjärv 1993) was within  $\pm 0.65$  mm/yr on average. Taking into account the uncertainties of the models (on average  $\pm 0.2\dots 0.5$  mm/yr), this is within the limits of uncertainties of the models. Historical VCM maps of Estonia based on repeated levellings fit also well to each other (within  $\pm 0.5$  mm/yr).



- Fit of the EST2013LU and the EST2015LU model to the Estonian CORS VVs (Oja et al. 2014) was good. Residual differences remained within  $\pm 0.34$  mm/yr on average. Largest discrepancies with the models calculated in the present study were obtained with the VVs of the coastal tide gauges. Differences with the historical results (Yakubovski 1973; Pobedonostsev 1975; Jevrejeva et al. 2002) remained within  $\pm 1.02$  mm/yr on average. Differences were systematic in nature. In Paper III, VVs of the six tide gauges were recalculated and corrected with satellite altimetry results. Residual differences with these VVs were  $\pm 0.66$  mm/yr on average, i.e. differences decreased  $\sim 1.6$  times. Reasons for large discrepancies with the tide gauge results need to be further clarified.
- There is a significant change in the BMs' VVs from the period between the First and the Second levelling to the period between the Second and the Third levelling as well as from the period between the Second and the Third levelling to the period between the Third and the Fourth levelling. A significant change in the VVs was also detected from the period between the First and the Second levelling to the period between the Third and the Fourth levelling. However, in the author's opinion, this change in the VVs is apparent rather than real, depending either on the estimation of the *a priori* levelling errors, levelling errors, or correlation between the levellings. For example, it was found from the multivariate analysis of the levellings that there is a strong correlation between the Second and the Third levelling. Significant negative correlation was found also between the changes of the VVs  $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$  and  $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$ . Check of the height-dependent systematic levelling errors, however, revealed no such errors. However, the detected VV change can be explained also by an underestimation of the *a priori* levelling errors determined from the loops' misclosures or fore and back levelling discrepancies. Variance component estimation weighted *a priori* standard error of the First levelling down most, approximately 3.2 times.
- Eliminating heights from the adjustment model by calculating differences between the levellings, and hopefully removing some unmodelled levelling errors from the calculation of the VVs did not change the estimated VVs. Moreover, error estimates from

the two adjustment models had no significant difference. The only differences between the “heights included” and the “heights eliminated” model were different VV estimates in the levelling combinations 1–2 and 1–2–3. These were related to the long time period of closing the levelling loops in the Second and the Third levelling. The influence of the LU on the loops’ misclosures was also largest for the Second and the Third levelling.

- Values of the BMs’ VVs depend largely on the levelling combination chosen for the calculations. Combinations of the three and four levellings and combinations of the two levellings with the larger time difference between them gave relatively similar results.
- In order to verify and increase the reliability of the BMs’ VVs from the levellings on the islands of Saaremaa and Hiiumaa, height connections between the mainland and Saaremaa, Saaremaa and Hiiumaa, and also between Hiiumaa and the mainland should be re-examined and also repeated in the future.
- The hypothesis about the largest gradient of the VCM in Estonia across the Pärnu-Narva geologic fault was not proven in the present study. The determined gradient of the VCM along the Põltsamaa-Lelle levelling line  $0.53 \pm 0.15$  mm/year is not larger than an average LU gradient obtained for the whole of Estonia from the EST2013LU and the EST2015LU model. It also coincides well with respective values interpolated from the Fennoscandian LU maps. An earlier conclusion about the largest gradient was largely based on the selected two levellings used in the VV calculations. The long-term VCM gradient detected from three, four or more levellings provided no support to such conclusion.

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## SUMMARY IN ESTONIAN

### Sissejuhatus

Maapinna tõus merepinna suhtes (näiv maatõus) on Põhja-Euroopas ja Eestis olnud juba pikka aega tuntud fenomen. Rannajoone muutumist võis märgata isegi inimese eluea jooksul. Vanim säilinud dokument, mis kirjeldab Botnia lahe lõunaosa veetaseme alanemist maapinna suhtes, on Rootsi linna Östhammari elanike palvekiri Rootsi peapiiskopile ja regendile 1491. a, kus paluti viia linn teise kohta, kuna sellele ei olnud võimalik enam isegi kalapaatidega juurde pääseda. Arusaadavalt arvati algul, et fenomeni näol on tegemist veetaseme alanemisega maapinna suhtes ning efekti hakati lähemalt uurima alles 18. sajandil algul (Ekman 1991).

Tänapäeval on teada, et maapinna tõus Fennoskandias on seotud Maa ülemiste kihtide vastumõjuga viimase jääaja (Weichseli jääaeg) jäämasside koormusele (jäakilbi maksimaalne paksus Fennoskandias ~19 000–22 000 a tagasi oli erinevatel hinnangutel 2000–3200 m) (Tushingham ja Peltier 1991; Peltier 1994; Lambeck, Smither ja Johnston 1998; Lambeck jt 2000; Siegert jt 2001; Fleming ja Lambeck 2004; Siegert ja Dowdeswell 2004; Peltier 2004; Lambeck jt 2010; Tarasov jt 2012; Peltier jt 2015; Abe-Ouchi jt 2015). Pärast jää sulamist püüab Maa taastada jääajaeelset tasakaaluasendit (tuntud kui GIA – glatsioisostaatiline tasakaalustumine) (Peltier 1999; Steffen ja Wu 2011), mis on siiani vaadeldav. Tõus on maksimaalne Botnia lahe põhjaosas, kus maapind kerkib merepinna suhtes kuni 10 mm/a (Ekman 1996; Lambeck, Smither ja Ekman 1998; Vestøl 2006; Ågren ja Svensson 2007). Kuid GIA ei tekita üksnes maatõusu. See on keerukas geofüüsikaline fenomen, mis põhjustab lisaks veel ka maakoore horisontaalset liikumist, geoidi muutust, gravitatsioonivälja muutust, meretaseme muutust, Maa raskuskeskme ümberpaiknemist (Ekman ja Mäkinen 1996; Peltier 1999; Sella jt 2007; Steffen jt 2008; Rangelova ja Sideris 2008; Lidberg jt 2010; Klemann ja Martinec 2011).

Jääajajärgne maatõus on oluliselt mõjutanud nii maastiku kujunemist kui rannaalade olustikku Eestis. Viimastel aastakümnetel jälgitav kliima soojenemine on aga põhjustanud jääliustike (Gröönimaa, Antarktika) sulamise ja globaalse meretaseme tõusu, mistõttu on absoluutse maatõusu (Maa raskuskeskme suhtes) kiirus vähenenud või meretõusule isegi alla jäänud. Sellega on seotud suured sotsiaal-majanduslikud riskid

rannaaladele (põllumajandusmaade ja põhjavee reostumine soolase mereveega, rannaalade erosioon, elupaikade vee alla jäämine jms) (Church ja White 2006, 2011; Nicholls ja Cazenave 2010; Rignot jt 2011). Kuna geodeetiliste mõõtmistega tuvastatav meretaseme tõus sisaldab endas jääajajärgse maatõusu signaali, tuleb see signaal meretaseme tõusu teadasaamiseks andmetest eemaldada. Seetõttu on maatõusu täpsete väärtuste teadmine oluline ka globaalse meretaseme tõusu hindamiseks ja kliimamuutuste monitooringuks. Kuna maakoore on GIA tõttu pidevas muutumises, siis on selle teadmine oluline ka geodeetiliste võrkude ajakohastamisel. Lisaks kasutatakse GIA-signaali jääliustike ja Maa vahevöö modelleerimisel.

Eestis intensiivistusid maakoore vertikaalliikumiste uuringud 1950. aastate lõpus, kui järk-järgult ilmusid esimesed kordusnivelleerimise andmed. Kordusnivelleerimisi alustas ENSV Teaduste Akadeemia Füüsika ja Astronoomia Instituudi (Astrofüüsika ja Atmosfäärifüüsika Instituut) geodeesia töörühm 1950. alguses. Enne seda teati maatõusu kiirusi üksnes Eesti ranniku kohta, mis arvatati ranniku veemõõdujaamade vaatlusandmete põhjal (nt Witting 1922). Esimese (1933–1943) ja teise (1948–1969) nivelleerimise põhjal arvutasid maatõusu kiirused Eestis näiteks Zhelnin (1958, 1960, 1964, 1966), Randjärv (1968), Vallner (1978) ning Vallner ja Zhelnin (1975). Kui kättesaadavaks said ka kolmanda nivelleerimise (1970–1996) tulemused täpsustati maatõusu kiirusi (Randjärv 1993; Vallner jt 1988). Varasemalt on maakoore vertikaalliikumisi Eestis määratud veel ka ranniku veemõõdujaamade vaatlusandmete põhjal (Yakubovski 1973; Pobedonostsev 1975; Vallner jt 1988; Jevrejeva jt 2002) ja GNSS-mõõtmiste ning GIA-modelleerimiste põhjal (Oja jt 2012; Oja jt 2014).

Varasemates uurimistöodes (nt Randjärv 1993; Vallner jt 1988) on reeperite kiiruste arvutamiseks kasutatud kahe nivelleerimise kõrguskasve. Selles uurimistöös on maatõusu kiirused arvutatud Eesti kõikide nivelleerimiste, k.a. neljanda nivelleerimise (2001–2012), andmete põhjal. See võimaldas tunduvalt tõsta arvutatud kiiruste usaldusväärsust.

## Uurimistöö eesmärk

Uurimistöö eesmärk oli leida maakoore vertikaallikumiste kiirused Eestis nelja kordusnivelleerimise andmete põhjal ja välja selgitada reeperite kiiruste muutumine ajas. Eesmärgi täitmiseks püstitati järgnevad uurimisküsimused ja hüpotees:

- i)* Millised on Eesti kõrgusvõrgu reeperite pikaajalise liikumise kiirused?
- ii)* Millised on erinevate nivelleerimiskombinatsioonide ja reeperite „kõrgustega“ ning „kõrgusteta“ liikumiskiirusi parametreerivate matemaatiliste mudelite kinemaatilise tasanduse erinevused?
- iii)* Kas reeperite liikumiskiirused on ajas konstantsed?
- iv)* Hüpotees: suurim vertikaallikumiste gradient Eestis on risti Pärnu-Narva geoloogilise rikkega, nn šarniirrikkega.

## Materjalid ja meetodika

Uurimistöö läbiviimiseks on kasutatud Eesti kõrgusvõrgu kõrgtäpsete nivelleerimiste esimese (1933–1943), teise (1948–1969, kolmanda (1970–1996) ja neljanda (2001–2011) kordusnivelleerimise andmeid. Mõõtmistulemused olid ilma gravimeetrilise parandita. Lattide skaala- ja temperatuuriparandid olid sisse viidud vaid teise ja kolmanda nivelleerimise mõõtmistulemustesse. Nivelleerimisandmete põhjal moodustati kaks võrku:

- i)* nn fundamentaalreeperite võrk (LNF), kuhu kaasati, nagu nimetus ütleb, peamiselt kõrgusvõrgu fundamentaalreeperid. Need eeldati olevat stabiilsemad kui ülejäänud reeperid (alluma vähem külmakergetele, pinnase hüdrogeoloogiliste tingimuste muutusele jms). Sellesse võrku kaasati kõik kättesaadavad kõrguskasvud nende reeperite vahel, k.a. sellised, mida oli nivelleeritud vaid ühel korral (Artikkel II);
- ii)* nn ühiste reeperite võrk (CLN), kuhu kaasati vaid need reeperid, mis olid ühised kõigi nelja nivelleerimise jaoks. Sellesse võrku kaasati vaid need kõrguskasvud, mida oli nivelleeritud vähemalt neljal korral. Kui valitud reeperite vahel oli kõrguskasve mõõdetud rohkem kui neli korda, tehti valik reeperite

vertikaalliikumiste graafikute ja kõrguskasvude aegridade põhjal (Artikkel **III**).

Nende kahe võrgu andmete töötluste põhjal oli võimalik vastata kolmele esimesele uurimisküsimusele. Uurimistöös püstitatud hüpoteesi testiti Põltsamaa-Lelle nivelleerimiskäigu kordusnivelleerimiste põhjal (Artikkel **I**).

Andmete eeltöötluste käigus arvatati nivelleerimiste juhuslikud ja süstemaatilised vead ning polügoonide sulgemisvead. Kuna nivelleerimispolügoonid olid moodustatud erinevatel aegadel nivelleeritud käikudest, siis hinnati ka maatõusu mõju polügoonide sulgemisvigadele. Koostati vertikaalliikumiste profiilid käikude kaupa ja sektsioonide kõrguskasvude aegridade graafikud. Neid kasutati ebastabiilsete reeperite tuvastamiseks. Vertikaalliikumiste kiiruste *a priori* veahinnangute leidmiseks arvatati ka kiiruste sulgemisvead polügoonides (CLN).

Süstemaatiliste vigade tuvastamiseks andmestikus kasutati märgitesti, nullvigade testi ja Lallemand'i (1889) graafilist analüüsi (Artikkel **I**). Kõrgusest sõltuvate vigade tuvastamiseks moodustati nivelleerimiskäikude kaupa punktdiagrammid, kus x-teljel olid sektsioonide kõrguskasvud ja y-teljel *a priori* veahinnangute põhjal normeeritud vertikaalliikumiste kiiruste muutused (Artikkel **III**).

Kiiruste arvutamiseks kasutati kaalutud kinemaatilist tasandust vähimruutude meetodil. Tasanduse parameetriteks „kõrgustega“ matemaatilises mudelis olid reeperite kõrgused valitud referentsepoohhil (2000) ja vertikaalliikumise kiirused (LNF ja CLN võrk). „Kõrgusteta“ mudelis, mida saab rakendada vaid ühiste reeperite võrgus, olid parameetriteks vaid reeperite liikumiskiirused. Kaalud kõrguskasvudele leiti nende dispersioonide kaudu. Dispersioonid leiti omakorda kas *i*) käikude juhuslike ja süstemaatiliste vigade põhjal (LNF) või *ii*) sulgemisvigade abil arvatud nivelleerimiskampaania standardvea põhjal (CLN).

Tasandusprotseduuri käigus:

- i) tuvastati jämedad vead mõõtmistulemustest „*data snooping*“ meetodit kasutades (Baarda 1968);
- ii) eemaldati mõõtmistulemustest jämedad vead (EST2013LU mudeli koostamisel) või kaaluti need alla „Taani“ meetodit (Krarup 1980) kasutades (EST2015LU mudeli koostamisel);
- iii) hinnati tasandusjärgse kaaluühiku dispersiooni  $S_0^2$  erinevust *a priori* väärtusest  $\chi^2$ -testi abil;
- iv) leiti mõõtmistulemustele uued kaalud dispersioonikomponentide hindamise kaudu Helmerti, IAUE, Bique ja Förstneri meetoditest;
- v) hinnati leitud parameetrite täpsust relatiivsete kiiruste keskmise standardvea ( $\mu$ ) ja kiiruste keskmise standardhälbe põhjal.

Teoreetiliselt võivad „kõrgustega“ ja „kõrgusteta“ matemaatiline mudel anda samas geodeetilises võrgus (CLN) reeperitele erinevad vertikaalliikumise kiirused. Põhjus on selles, et nimetatud mudelid käsitlevad nivelleerimispolügoonide sulgemisvigades peituvat kiirusinformatsiooni erinevalt. Lisaks võivad reeperite kiirused kahest mudelist erineda selle tõttu, et kordusmõõtmiste diferentseerimisel („kõrgusteta“ matemaatiline mudel) on teoreetiliselt võimalik mõõtmisandmetest eemaldada süstemaatilised vead, mis on ühised mõlemale nivelleerimisele (näiteks gravimeetrilise parandi andmata jätmise). Diferentseerimata kordusmõõtmistesse („kõrgustega“ matemaatiline mudel) jäävad aga sellised vead edasi. Samuti võivad erineda „kõrgustega“ ja „kõrgusteta“ matemaatilise mudeli tasanduse täpsushinnangud (Mäkinen ja Saarinen 1998). Seetõttu on kahe mudeli kiiruste erinevusi võrreldud t-testiga ja täpsushinnangute erinevusi F-testiga.

Reeperite vertikaalliikumiste kiiruste muutumist ajas on hinnatud ANOVA-testiga (Kakkuri ja Vermeer 1985; Mäkinen ja Saarinen 1998). Erinevate nivelleerimisperioodide kiiruste võrdlemisel tuleb arvestada sellega, et leitud kiirused on omavahel korreleeritud (näiteks esimese ja teise ning teise ja kolmanda nivelleerimisperioodi kiiruste võrdlemisel tuleb arvestada, et mõlemate kiiruste leidmisel on kasutatud teist nivelleerimist). Korrelatsiooni arvestamiseks modifitseeriti „kõrgusteta“

matemaatilist mudelit nii, et reeperitele oli võimalik korraga ühest tasandusest leida kolm kiirust: perioodide 1–2, 2–3 ja 3–4 vahel. Lisaks analüüsiti kiiruste muutumist ka graafiliselt tõenäosuspaberi abil. Kuna järeldused kiiruste muutumise osas sõltuvad andmetevahelisest korrelatsioonist, siis hinnati ka kordusnivelleerimiste vahelist korrelatsiooni multivariatilise analüüsi põhjal.

Maakoore vertikaalliikumiste mudelid koostati programmi Surfer abil. Testiti erinevaid võrgustamise algoritme: *kriging*, *minimum curvature* ehk *spline*, *local polynomial* ja *natural neighbor*. Võrgu sammu valimisel lähtuti lähimate punktipaaride keskmisest vahekaugusest. Mudelpindade silumiseks võrreldi mudelpindu referentspinnaga (NKG2005LU), eemaldati anomaalseid reepereid lähimate reeperite kiiruste võrdlemise teel ja rakendati programmis Surfer olemasolevaid interpoleeritud võrgust erindite eemaldamise filtreid nagu libisev keskmine (*moving average*) või lävendi keskmine (*threshold average*).

Vallner jt (1988) töös püstitatud väidet, et suurim vertikaalliikumiste gradient Eestis on risti Pärnu-Narva geoloogilise rikkega, kontrolliti Põltsamaa-Lelle nivelleerimiskäigu mõõtmiste põhjal. Antud nivelleerimiskäik on selleks eriti sobiv, kuna ristub eelnimetatud geoloogilise rikkega. Samuti on seda mõõdetud võrreldes teiste nivelleerimiskäikudega tunduvalt rohkem – 11 korda. Reeperite pikaajalised liikumiskiirused leiti 11 nivelleerimise kõrguskasvude kaalutud regressioonianalüüsi kaudu. Saadud pikaajalist trendi võrreldi antud töös saadud vertikaalliikumiste mudelite keskmiste gradientidega ja teistelt maakoore vertikaalliikumiste kaartidelt või -mudelilt interpoleeritud väärtustega.

## **Tulemused ja arutelu**

### *Fundamentaalse reeperite võrgu tasanduse tulemused*

Tasanduse esimeses lähenduses koosnes võrk 361 kõrguskasvust ja 106 reeperist. 11 vaatlust tuvastati erinditena ja eemaldati tasandusest. Viimases lähenduses saadi tasanduse kaaluühiku dispersiooniks 3,60. Tulemusena leiti vertikaalliikumise kiirused 84 reeperile, mille keskmine standardhälve oli  $\pm 0,5$  mm/a. Dispersioonikomponentide hindamiseks jagati vaatlused 8 gruppi, lähtudes kordusnivelleerimise perioodidest ja tööde läbiviijatetest. Enim, keskmiselt 3,2 korda, kaaluti alla esimese



nivelleerimise vaatlusandmed. 2,2 korda kaaluti alla kolmanda nivelleerimise ja 2 korda teise nivelleerimise vaatlusandmed.

### *Ühiste reeperite võrgu tasanduse tulemused*

Ühine võrk võimaldas tasandusi teostada 11 erinevas nivelleerimisandmete kombinatsioonis ja kahte matemaatilist mudelit – „kõrgustega“ ja „kõrgusteta“ – kasutades. Võrku kuulus kokku 249 reeperit ja 1011 kõrguskasvu. Teise ja kolmanda nivelleerimise polügoonide moodustamiseks kasutati küllaltki erinevatel aegadel nivelleeritud käike (max ajavahe oli 25 a teise nivelleerimise III polügooni moodustamisel). See tähendab, et eeldatavalt sisaldasid polügoonide sulgemisvead olulisel määral kiirusinformatsiooni. Eespool selgitati, et teoreetiliselt võivad selle tõttu reeperite kiiruste väärtused „kõrgustega“ ja „kõrgusteta“ mudelist tulla erinevad. Kuna kõrguskasvude kaalud arvutati polügoonide sulgemisvigade kaudu, siis hinnati ka, kui suur oli maatõusu mõju polügoonide sulgemisvigadele ja nende kaudu arvutatud nivelleerimise standardvigadele. Selgus, et maatõusu mõju teise ja kolmanda nivelleerimise polügoonide sulgemisvigadele ulatus kuni 25 mm-ni ja nivelleerimise standardvigadele kuni  $\pm 0,2 \text{ mm}/\sqrt{\text{km}}$  -ni.

Erinevate nivelleerimiskombinatsioonide tasanduse täpsushinnangute võrdlusest selgus, et suurimad kaaluühiku dispersioonid saadi kolme ja nelja nivelleerimise kombinatsioonidest. Kahe nivelleerimise kombinatsioonide tasandusjärgsed kaaluühiku dispersioonid ei erinenud statistiliselt oluliselt *a priori* väärtusest 1. Selle põhjused võivad olla kas: *i*) konstantseid reeperite vertikaalliikumise kiirusi eeldav mudel ei sobi vaatlusandmetega kokku, *ii*) erinevate nivelleerimiskampaaniate kaalud ei ole omavahel õigetes proportsioonides või *iii*) nivelleerimisandmed sisaldavad vigu. Näiteks avaldas jämedate vigade eemaldamine kaaluühiku dispersioonile märgatavat mõju, kahanedes ~6 korda. Suurimad kaaluühiku dispersioonid ilmselt olid kombinatsioonides, kus kasutati esimest ja teist nivelleerimist. Ilmselt oli see tingitud nende nivelleerimiste *a priori* kaalude ebaõigetest proportsioonidest. Ka dispersioonikomponentide hindamine kaalus esimese nivelleerimise vaatlused kõige enam alla. „Kõrgustega“ ja „kõrgusteta“ mudelite kaaluühiku dispersioonid ei erinenud omavahel statistiliselt oluliselt.

Tasandusjärgsete relatiivsete vertikaalliikumise kiiruste keskmised standardvead sõltusid enim sellest, kas neljas nivelleerimine oli

kombinatsiooni kaasatud või mitte. Neljanda nivelleerimise kaasamine vähendas keskmist standardviga tunduvalt. See oli tõenäoliselt seotud neljanda nivelleerimise tunduvalt suuremate kaaludega, võrreldes ülejäänutega. Kolme nivelleerimise kombinatsioonist keskmise väljajätmine ei mõjutanud oluliselt relatiivsete kiiruste keskmist standardviga (nt kombinatsioon 2–3–4 võrreldes kombinatsiooniga 2–4). Mäkinen ja Saaranen (1998) on seletanud seda efekti lineaarse regressioonsirge analoogiaga, kus keskmise punkti väljajätmine ei muuda oluliselt sirge tõusu väärtust. Samuti võib välja tuua, et pikem ajaperiood nivelleerimiste vahel kahe nivelleerimise kombinatsioonis vähendas keskmist standardviga oluliselt (nt kombinatsioon 2–3 võrreldes kombinatsiooniga 2–4).

Tasandusjärgne relatiivsete vertikaalliikumise kiiruste keskmine standardviga oli suurim teise ja kolmanda nivelleerimise kombinatsioonis. See oli tõenäoliselt seotud Saaremaa ja Hiiumaa nivelleerimispolügoonidega ja mandri ning Saaremaa ja Saaremaa ning Hiiumaa vaheliste veeületuse kõrguskasvudega. Kui tasandus antud kombinatsioonis viidi läbi ainult mandri pluss Saaremaa polügooni nivelleerimistega, saadi tulemuseks ligi neli korda väiksem keskmine standardviga. Samuti oli ligi viis korda väiksem tasanduse-eelselt relatiivsete vertikaalliikumise kiiruste sulgemisvigade põhjal, kus veeületuse kõrguskasve ei kasutatud, arvatud standardviga. See viitab, et Saaremaa ja Hiiumaa vahelised veeületuse kõrguskasvud või Hiiumaa polügooni kõrguskasvud mõjutasid oluliselt kiiruste täpsushinnanguid. Samuti ei erinenud statistiliselt oluliselt, nagu kaaluühikute dispersioonide puhulgi, relatiivsete vertikaalliikumiste kiiruste keskmised standardvead „kõrgustega“ ja „kõrgusteta“ mudelite vahel.

„Kõrgustega“ ja „kõrgusteta“ matemaatilistest mudelistest saadud reeperite vertikaalliikumise kiiruste võrdlemisel selgus, et enamiku nivelleerimiskombinatsioonide puhul ei olnud kiiruste erinevus statistiliselt oluline. Oluline erinevus tuvastati vaid esimese ja teise ning esimese, teise ja kolmanda nivelleerimise kombinatsioonides. Kõrguskasvude transformeerimine nivelleerimiskampaania keskmisele vaatlusepohhile aga likvideeris kiiruste erinevused „kõrgustega“ ja „kõrgusteta“ mudelite vahel. Seetõttu on põhjust arvata, et kiiruste erinevus oli tingitud teise ja kolmanda nivelleerimise polügoonide sulgemise pikast ajavahemikust võrreldes esimese ja teise nivelleerimisega. Sellele viitab ka see, et kombinatsioonidest 1–2 ja 1–2–3 arvatud kiirused „kõrgustega“ ja „kõrgusteta“ mudelite vahel

nivelleerimispolügoonides I, VI ja VII ei erinenud ka juba enne transformeerimist keskmisele vaatlusepohhile. Nendes polügoonides oli ka teisel ja kolmandal nivelleerimisel polügoonide sulgemise aeg lühike võrreldes ülejäänutega.

Vertikaalliikumise kiiruste vahelist korrelatsiooni arvestava „kõrgusteta“ mudelist leitud reeperite kiiruste ANOVA-test näitas, et esimese ja teise, teise ja kolmanda ning kolmanda ja neljanda nivelleerimisperioodi reeperite kiirused on statistiliselt oluliselt erinevad, st vertikaalliikumise kiirused on ajas muutunud. Kiiruste muutumine võib aga olla ka kunstlik, statistiline, mis on põhjustatud kas korrelatsioonist nivelleerimisandmete vahel, eeldatavate *a priori* nivelleerimisvigade ülehindamisest või siis nivelleerimisvigadest endast. Nivelleerimisperioodide kõrguskasvude multivariatiiivne analüüs tuvastaski statistiliselt olulise korrelatsiooni ( $r \approx 0,7$ ) teise ja kolmanda nivelleerimisperioodi andmestiku vahel. Korrelatsioon andmestiku sees aga mõjutab oluliselt järeldusi kiiruste muutumise osas, pannes kahtluse alla, kas kiiruste muutus on olnud ikka tõeline (Mäkinen ja Saaranen 1998; Mäkinen 2000). Ka negatiivne korrelatsioon ( $r \approx -0,6$ ) erinevate nivelleerimisperioodide kiiruste muutuste vahel ( $\mathbf{v}_{2-3} - \mathbf{v}_{1-2}$  ja  $\mathbf{v}_{3-4} - \mathbf{v}_{2-3}$  vahel) viitab, et mingi nivelleerimisperioodi andmestikus võivad esineda süstemaatilised vead.

Järeldust kiiruste muutumise osas mõjutavad ka nivelleerimisvigade *a priori* hinnangud. Kui oletada, et nivelleerimisvead on eeldatust suuremad, siis kiiruste muutuse järeldus ei pruugi enam olla statistiliselt oluline, kuna sel juhul nivelleerimisvead võivad ületada muutust. Nelja nivelleerimise ühises kinemaatilises tasanduses saadud kaaluühiku dispersioon näitab, et nivelleerimisvead on  $\sim 3,2$  korda suuremad, kui sulgemisvigade põhjal *a priori* eeldati. Dispersioonikomponentide hindamine aga asetab suurema osa nivelleerimisveast, nagu fundamentaalreeperite võrguski, esimese nivelleerimisperioodi mõõtmistele.

Kiiruste muutumise järeldus aga võib olla ka otseselt seotud vigadega andmestikus. Nivelleerimisvigade tõttu avalduv kiiruste muutus ei ole tõeline, vaid näiline. Nivelleerimisvead, mis ei avaldu sulgemisvigades, aga ilmnevad kolme või nelja nivelleerimise kombinatsiooni kinemaatilises tasanduses suurte kaaluühiku dispersioonidena, võivad olla süstemaatilised vead (Mäkinen ja Saaranen 1998). Loogiline oleks otsida seetõttu seost kiiruste muutuse ja maapinna topograafia vahel,

kuna teatud süstemaatilised vead (lati parandid, refraktsioon) on maapinna kõrguste muutumisega korrelatsioonis (Vaníček jt 1980). Kõrguskasvude ja normeeritud kiiruste muutuse vahel aga ei tuvastatud mingit seost. Seetõttu ei andnud uurimistöö kindlat vastust, kas kolme ja nelja nivelleerimise kombinatsiooni tasandusest saadud suur kaaluühiku dispersioon on põhjustatud reeperite ebaühtlasest vertikaalliikumisest, ebaõigelt valitud kaaludest või nivelleerimisvigadest.

### *Maakoore vertikaalliikumiste mudelite loomine*

Tasanduse tulemusel saadud reeperite vertikaalliikumise kiirusi kasutades on loodud kaks maakoore vertikaalliikumiste mudelit:

- i)* EST2013LU mudel põhineb fundamentaalreeperite võrgu esimese, teise, kolmanda ja neljanda nivelleerimise tasandusest saadud kiirustel. Mudel loodi *kriging* meetodit kasutades.
- ii)* EST2015LU mudel põhineb nn laiendatud ühiste reeperite võrgu teise, kolmanda ja neljanda nivelleerimise tasandusest saadud kiirustel. Mudel loodi *minimum curvature* meetodit kasutades. Otsus mitte kasutada esimese nivelleerimisperioodi kõrguskasve selle mudeli loomisel põhines faktil, et teise, kolmanda ja neljanda nivelleerimise tasandusjärgne kaaluühiku dispersioon oli kõikidest kolme ja nelja nivelleerimise kombinatsioonidest väikseim. Samuti sellel, et dispersioonikomponentide hindamine kaalus esimese nivelleerimise mõõtmised kõige enam alla. See võimaldas ka luua nn laiendatud ühiste reeperite võrgu, kuna teise, kolmanda ja neljanda nivelleerimise ühiseid reepereid on tunduvalt rohkem kui algses esimese, teise, kolmanda ja neljanda nivelleerimise ühiste reeperite võrgus.

Paljudest varasematest uuringutest on maakoore vertikaalliikumiste kiirusteks Eestis määratud  $-1,5...3$  mm/a (Zhelnin 1966; Vallner ja Zhelnin 1975; Vallner jt 1988; Randjärv 1993; Ekman 1996; Ågren ja Svensson 2007). Selles uuringus leidsid need väärtused üldjoontes kinnitust. Maakoore vertikaalliikumiste kiirused Eestis mudeli EST2013LU põhjal ulatusid  $-0,7...2,0$  mm/a ja EST2015LU põhjal  $-0,8...2,8$  mm/a.

Mudelite EST2013LU ja EST2015LU võrdluses varasemate Eesti ja Fennoskandia maatõusu mudelitega tuvastati, et saadud mudelite vertikaallikumiste isojooned on rohkem ida-lääne suunas kaldu kui varsematel mudelitel. Et välja selgitada, millised nivelleerimised seda eelkõige põhjustavad, loodi vertikaallikumiste mudelid ka CLN teiste nivelleerimiskombinatsioonide tasandusest saadud kiirusi kasutades. Mudelite võrdlemisel selgus, et teine ja neljas nivelleerimine mõjutavad isojooni enam kalduma ida-lääne suunas, samas kui esimene ja kolmas nivelleerimine mõjutavad isojooni kalduma konventsionaalses kirde-edela suunas. Oma osa sellisel isojoonte suunal on ka selles, et teine ja eriti neljas nivelleerimine olid tasanduses tunduvalt suuremate kaaludega, kui esimene ja kolmas nivelleerimine.

EST2013LU ja EST2015LU mudelite omavahelises võrdluses selgus, et mandriosas langevad mudelid suures osas kokku (keskmine erinevus ei ületanud  $\pm 0,1$  mm/a). Suurimad erinevused mandriosas (kuni  $\pm 0,6$  mm/a) puudutasid Pärnu ja Kirde-Eesti piirkondi, EST2015LU mudelil esitatud lokaalsete kiirusanomaaliate tõttu. Need on oletatavalt seotud põhjavee tarbimisega (Pärnu) ja põlevkivi kaevandamisega (Kirde-Eesti) (Listra ja Talviste 1988; Mets jt 2000; Rüdja 2004). Erinevused Peipsi järvel on tingitud Peipsi veemõõdujaamade kiiruste kasutamisest EST2015LU mudelis. Saare- ja Hiiumaal ulatusid kahe mudeli erinevused kuni  $-0,8$  mm/a. Ruutkeskmine erinevus kogu Eesti ulatuses jäi siiski  $\pm 0,30$  mm/a piiridesse, mis ei ületa mudelite kombineeritud täpsushinnangut. Kahe mudeli erinevused on peamiselt põhjustatud erinevate lähtepunktide ja -kiiruste kasutamisest. Põhjalikum analüüs selgitas aga, et järsk hüpe kiiruste erinevuses toimus Saaremaal ja püsis enam-vähem konstantsena kogu mandriosa ulatuses. See viitas, et erinevus kahe mudeli kiirustes on põhjustatud eelkõige Saaremaa ja Hiiumaa vahelise veeületuse kõrguskasvudest, mistõttu seda ühendust tuleks tulevikus detailsemalt analüüsida ja võimalusel sidet korrata.

Mudelite täpsust hinnati mudelpinna hälvete arvutamisega vaatluspunktide tasandatud kiiruste suhtes, ristvalideerimise meetodil ning tasandamata kiiruste ja mudelpinnast interpoleeritud kiiruste erinevuste põhjal profiilil Koidula-Tartu-Jõgeva-Põltsamaa-Lelle-Tallinn. EST2013LU ja EST2015LU mudelpindade täpsuseks hinnati vastavalt  $\pm 0,39$  ja  $\pm 0,24$  mm/a. Hinnangud on arvutatud tasandatud kiiruste ja mudelpinnast interpoleeritud kiiruste erinevuste, ristvalideerimise hälvete ja kiiruste tasandusjärgsete vigade hinnangute põhjal (Tabel 5 ja Tabel 12).

Mudeleid võrreldi teiste maatõusu mudelitega ning mudelistest interpoleeritud kiirusi sõltumatutest mõõtmistest leitud vertikaalliikumiste kiirustega (GNSS, ranniku veemõõdejaamad). Võrdluse tulemused on toodud Tabelites 15 ja 16. Peamine erinevus selles töös leitud mudelite ja varasemate mudelite vahel on erinev kiiruste isojoonte suund. Antud töös saadud mudelite isojoonte suunda toetavad Peipsi järve veemõõdujaamade kiirused (Raamat 2009) ning Randjärve (1993) maakoore vertikaalliikumiste kaardi isojoonte suunad üle Peipsi järve ja Lõuna-Eestis. Erinevus Randjärve ja teiste varasemate Eesti autorite kaartide vahel on see, et Randjärv kasutas oma kaardi loomisel ka nivelleerimisandmeid Venemaalt ja Lätist. EST2015LU mudeli isojoonte kõverdumist Lõuna-Eestis toetab ka uusimate nivelleerimisandmete põhjal loodud Läti maakoore vertikaalliikumiste kaart (Celms 2014).

EST2013LU ja EST2015LU mudelite võrdluses Eesti varasemate peamiste maatõusu mudelitega (Vallner jt 1988; Randjärv 1993) jäid erinevused keskmiselt  $\pm 0,65$  mm/a piiridesse, mis siiski ei ületa võrreldud mudelite kombineeritud täpsushinnangut (95%). Arvatavasti on erinevused põhjustatud kasutatud nivelleerimisandmete erinevusest ja ka lähtepunktide kiiruste erinevusest. Kuna varasemad Eesti autorite kaardid on koostatud vaid kahte nivelleerimist kasutades, sõltub leitud kiirus otseselt tehtud valikust (nt kombinatsioon esimene-teine nivelleerimine vs esimene-kolmas nivelleerimine). Selles uurimistöös kasutati kolme või nelja nivelleerimise kombinatsiooni, kus kiirused nii otseselt ei sõltu sellest valikust kui kahe nivelleerimise puhul. Seetõttu sarnaneb ka siinses uurimistöös esimese ja kolmanda nivelleerimise põhjal leitud maatõusu mudel (Joonis 36 g) rohkem Eesti varasemate maatõusu mudelitega. Samas on varasemad autorid kasutanud ka kas *i*) antud uurimistöoga võrreldes erinevat lähtepunkti või *ii*) teistsugust kiirust lähtepunktis.

Võrdlus varasemate Fennoskandia maatõusu mudelitega (Ekmani 1996. a kaart, mudel NKG2005LU (Ågren and Svensson 2007)) näitas, et erinevused jäävad keskmiselt  $\pm 0,40$  mm/a piiridesse. Erinevused selles uurimistöös saadud mudelitega olid suuremad võrdluses Ekmani (1996) mudeliga, keskmiselt  $\pm 0,50$  mm/a. See on seletatav sellega, et Ekman kasutas oma mudeli isojoonte tõmbamiseks Eesti sisemaal Vallner jt (1988) ja Randjärv (1968) andmeid. Sobivus NKG2005LU mudeliga oli aga väga hea, keskmiselt  $\pm 0,26$  mm/a, eriti kui arvestada, et NKG2005LU mudel on Eesti aladel põhimõtteliselt Lambeck, Smither

ja Ekmani (1998) geofüüsikaline mudel, mille loomiseks ei kasutatud geodeetilisi andmeid Eestist. Selle uurimistöö mudelite ja NKG2005LU isojoonte suundade erinevuse üks võimalik põhjendus ongi Eesti andmete puudumine NKG2005LU mudeli koostamisel.

Mudelite sobivus GNSS-püsijaamade kiirustega on hea, jäädes keskmiselt  $\pm 0,34$  mm/a piiridesse. Suurimad erinevused mudelitest interpoleeritud kiirustega saadi võrdluses Eesti ranniku veemõõdujaamade kiirustega. Keskmine erinevus varasematest uuringutest saadud kiirustega oli  $\pm 1,02$  mm/a. Erinevus Artiklis III ümberarvutatud ja satelliitltimeetria andmetega parandatud kiirustega aga ligikaudu kaks korda väiksem:  $\pm 0,66$  mm/a. Erinevused veemõõdujaamade kiirustega on aga pigem süstemaatilist laadi. Sellele viitab näiteks, et ruutkeskmine erinevus selle töö mudelite ja Jevrejeva jt (2002) veemõõdujaamade kiiruste vahel on  $\pm 1,11$  mm/a, samas kui erinevuste standardhälve on keskmiselt  $\pm 0,49$  ehk üle kahe korra väiksem. Erinevusi veemõõdujaamade kiirustega tuleks edaspidi täiendavalt uurida, eriti võimalike süstemaatiliste vigade osas andmestikus.

#### *Põltsamaa-Lelle nivelleerimiskäik*

Üheteistkümne nivelleerimise kõrguskasvude kaalutud lineaarsest regressioonanalüüsist määrati Lelle fundamentaalreeperi pikaajaliseks kiiruseks Põltsamaa fundamentaalreeperi suhtes  $0,53 \pm 0,15$  mm/a. Võrreldes varasematelt Eesti vertikaalliikumiste kaartidelt interpoleeritud kiirustega (0,9...1,3 mm/a) on see ligikaudu kaks korda väiksem. Samas ühtib see hästi nii viimatistelt, kui ka varasematelt Fennoskandia maatõusu kaartidelt interpoleeritud väärtustega, mis jäävad vahemikku 0,3...0,7 mm/a. Artiklis I saadud Põltsamaa-Lelle nivelleerimiskäigu vertikaalliikumiste gradient ( $\sim 0,001$  kraadi) ühtib hästi EST2013LU ja EST2015LU keskmise gradiendiga (vastavalt 0,0005 ja 0,0008 kraadi). Seetõttu ei leidnud hüpoteesi, et antud nivelleerimiskäigul on vertikaalliikumiste gradient võrreldes Eesti muude piirkondadega suurim, selles uurimistöös tõestust. Varasem väide suurima gradiendi kohta antud nivelleerimiskäigul (Vallner ja Zhelnin 1975; Vallner jt 1988) põhines suuresti kasutatud kahest nivelleerimisest. Pikaajalise vertikaalliikumise väärtused Põltsamaa-Lelle trassil ei kinnitanud seda väidet.

Lühiajaliste vertikaalliikumiste analüüs Põltsamaa-Lelle nivelleerimiskäigul tuvastas suure kiiruste varieeruvuse. Lühiajalised kiiruste muutused on enamasti tingitud Kvaternaari setete erinevast tihenemisest näiteks hüdroloogiliste tingimuste erinevuse tõttu või ka nivelleerimisvigadest. Lühiajaliste kiiruste analüüsil tuleb alati arvestada võimalusega, et nivelleerimisvead domineerivad kiiruste üle, mistõttu nende interpreteerimisel tuleks olla ettevaatlik.

## Järeldused

Doktoritöös käsitleti maakoore vertikaalliikumiste arvutamist Eestis nelja kordusnivelleerimise andmete (1933–2011) põhjal. Reeperite kiirused leiti nivelleerimiste ühisest kaalutud kinemaatilisest tasandusest, kasutades kahte matemaatilist mudelit: nn „kõrgustega“ ja „kõrgusteta“ mudelit. Kasutatud meetodika võimaldas leida vaatlusandmete hulgast erindid, hinnata mõõtmis- ja tasandustulemusi statistiliselt ning hinnata reeperite kiiruste muutumist nivelleerimisperioodide vahel. Maakoore vertikaalliikumiste modelleerimiseks kasutati tarkvara Surfer erinevaid võimalusi. Mudelite täpsust hinnati mudelist interpoleeritud ja reeperite tasandatud kiiruste vaheliste erinevuste leidmise, ristvalideerimise ja sõltumatute mõõtmistulemustega (GNSS-püsijaamad, veemõõdujaamad, teised mudelid) võrdlemise teel. Doktoritöö peamised tulemused olid järgmised:

- Arvutati kaks maakoore vertikaalliikumiste mudelit: EST2013LU (Artikkel II) ja EST2015LU (Artikkel III). Mudelite keskmiseks täpsushinnanguks saadi vastavalt  $\pm 0,39$  mm/a ja  $\pm 0,24$  mm/a. Lisaks hinnati maakoore vertikaalliikumiste gradienti 58 km pikkusel Põltsamaa-Lelle nivelleerimiskäigul, milleks saadi  $\sim 0,001$  kraadi ( $0,53 \pm 0,15$  mm/a) (Artikkel I).
- Eesti territooriumil jätkub postglatsiaalne maatõus  $-0,8$  kuni  $2,8$  mm/a. Võrreldes varasemate uuringute tulemustega on aga doktoritöös leitud mudelite vertikaalliikumiste isojooned rohkem kallutatud lääne–ida suunas. Eelkõige mõjutavad isojoonte sellist suunda teise ja neljanda nivelleerimise andmed ning nende suurem kaal võrreldes esimese ja kolmanda nivelleerimise kaaludega. Sellist isojoone suunda toetavad hetkel Peipsi järve veetaseme mõõtmistest arvatud veemõõdujaamade kiirused (Raamat 2009) ja Randjärve (1993)



maakoore vertikaalliikumiste kaart, kus isojoonte interpoleerimiseks on kasutatud ka Venemaa ja Läti reeperite kiirusi. EST2015LU isojoonte suunda Lõuna-Eestis toetab ka Läti viimaste nivelleerimisandmete põhjal koostatud vertikaalliikumiste kaart (Celms 2014). Isojoonte suuna täpsemaks verifitseerimiseks oleks vajalik mudelisse kaasata nivelleerimisandmeid Venemaalt ja Lätist ning leida Ida-Eesti GNSS-püsijaamade vertikaalliikumise kiirused.

- Lisaks üldisele postglatsiaalse maatõusu trendile esineb lokaalseid kiiruste anomaaliaid. Eriti tulevad need esile Pärnu ümbruses (põhjavee tarbimisest tingitud vajumine) ja Kirde-Eestis (põlevkivi kaevandamisest tingitud jääkvajumine).
- Doktoritöös leitud mudelite sobivus viimaste Fennoskandia maatõusu kaartide ja mudelitega oli hea. Erinevus Ekmani (1996) kaardiga jäi keskmiselt  $\pm 0,50$  mm/a piiridesse ning erinevus NKG2005LU mudeliga (Ågren ja Svensson 2007) keskmiselt  $\pm 0,26$  mm/a piiridesse. Erinevus varasemate Eesti maakoore vertikaalliikumiste kaartidega (Vallner jt 1988; Randjärv 1993) jäi keskmiselt  $\pm 0,65$  mm/a piiridesse. Võttes arvesse nende mudelite täpsust (keskmiselt  $\pm 0,2$  kuni  $\pm 0,5$  mm/a) jäävad erinevused mudelite kombineeritud täpsushinnangu piiridesse. Varasemad Eesti vertikaalliikumiste kaardid sobituvad ka omavahel üsna hästi. Keskmiselt jäävad erinevused  $\pm 0,5$  mm/a piiridesse.
- Sobivus GNSS-püsijaamade kiirustega (Oja jt 2014) oli samuti hea, jäädes keskmiselt  $\pm 0,34$  mm/a piiridesse. Suurimad erinevused doktoritöö vertikaalliikumiste mudelitega tulid ilmsiks võrdluses Eesti ranniku veemõõdujaamade kiirustega. Keskmise kiiruste erinevus varasemate uuringute veemõõdujaamade kiirustega oli  $\pm 1,02$  mm/a. Erinevused olid enamikus süstemaatilist laadi. Artiklis III ümberarvutatud ja satelliitaltimeetria andmetega korrigeeritud veemõõdujaamade kiiruste ja doktoritöö mudelitest interpoleeritud kiiruste keskmiseks erinevuseks saadi  $\pm 0,66$  mm/a. Suured erinevused veemõõdujaamade kiirustega vajavad edaspidist täiendavat uurimist.
- Reeperite kiirused on nelja nivelleerimisperioodi vahel oluliselt muutunud. Autori hinnangul on see muutus aga mitte reaalne kiiruste muutumine, vaid näiline, mis on tingitud kas *a priori*

vigade alahindamisest, nivelleerimisvigadest või korrelatsioonist kordusnivelleerimiste vahel. Nii näiteks tuvastati multivariatiiivsel analüüsil oluline korrelatsioon teise ja kolmanda nivelleerimise vahel ning negatiivne korrelatsioon kiiruste muutuste vahel. Süstemaatilisi maapinna kõrgusest sõltuvaid nivelleerimisvigu andmestikust aga ei tuvastatud. Dispersioonikomponentide hindamisel leiti, et esimese nivelleerimise vead on keskmiselt 3,2 korda suuremad kui edasi-tagasi nivelleerimise kõrguskasvude erinevuse või polügoonide sulgemisvigade põhjal *a priori* eeldati.

- Mõõtmisandmete diferentseerimine („kõrgusteta“ matemaatiline mudel) ja sellega loodetavalt ühiste nivelleerimisvigade eemaldamine andmestikust ei muutunud kiiruste väärtusi võrreldes „kõrgustega“ mudeliga. Samuti ei erinenud oluliselt kahe mudeli tasanduse täpsushinnangud. Ainuke erinevus kahe mudeli testimisel oli esimese ja teise ning esimese, teise ja kolmanda nivelleerimise kombinatsioonist saadud kiiruste oluline erinevus. Arvatavasti oli see erinevus põhjustatud teise ja kolmanda nivelleerimise polügoonide sulgemise pikast ajavahest. Pärast kõrguskasvude transformeerimist nivelleerimisperioodi keskmisele epohhile kiiruste erinevused kombinatsioonidest 1–2 ja 1–2–3 kahe mudeli vahel elimineerusid. Ka maatõusu mõju polügoonide sulgemisvigadele oli suurim teise ja kolmanda nivelleerimisperioodi puhul, nagu eeldati.
- Reeperite kiirused sõltuvad suuresti kasutatud nivelleerimisandmete kombinatsioonist. Kolme ja nelja nivelleerimise kombinatsioonid ja need kahe nivelleerimise kombinatsioonid, mis on teostatud pikema ajavahemiku tagant, andsid suhteliselt sarnased kiirused.
- Saaremaa ja Hiiumaa reeperite kiiruste usaldusväärsuse tõstmiseks ja verifitseerimiseks oleks tarvis mandri ja Saaremaa ning Saaremaa ja Hiiumaa vahelised veeületuse kõrguskasvud üle kontrollida. Võimalusel tuleks veeületuse kõrguskasvude määramist tulevikus uuesti korrata.
- Hüpotees Eesti suurimast maakoore vertikaalliikumiste gradiendist Pärnu-Narva joonel ei leidnud kinnitust. Põltsamaa-Lelle vahelise pikaajalise vertikaalliikumise gradient ei erinenud oluliselt doktoritöö mudelite keskmisest gradiendist ning samuti varasematest Fennoskandia maatõusu mudelitest interpoleeritud

gradientidest Põltsamaa-Lelle joonel. Varasem järeldus suurima gradiendi kohta sõltus otseselt valitud kahest nivelleerimisest, mida kasutati kiiruste arvutamisel. Pikaajaline, kolme, nelja ja enama nivelleerimise kombinatsioonist arvatatud vertikaalliikumise gradient ei toetanud seda järeldust.

## ACKNOWLEDGEMENTS

This work was completed at the Department of Geomatics of the Estonian University of Life Sciences. For two decades, the department has been providing inspirational opportunities and atmosphere. I would like to thank my colleagues all through these years. My great thanks are due to the head of the department, associate professor Siim Maasikamäe, for his versatile help and support.

My sincere thanks are due to Dr. Ants Torim. Our discussions and his profound knowledge about Estonian levellings were invaluable. I am sincerely grateful to Helju Jürma and the whole Department of Geodesy of Estonian Land Board. Without their help with the levelling data it would have been hard to accomplish this work.

I am sincerely thankful to my supervisor professor Jüri Randjärv for his guidance throughout all my studies from 1989 when I was still a freshman. He is the person who directed me to the field of science and guided through the master's and doctoral studies.

My special thanks are due to my other supervisor, associate professor Aive Liibusk for her friendship and persistent encouragement and support. I am ineffably thankful for her help not only in making my dissertation manuscript readable, but in organizing the defence of the thesis as well.

I am thankful to all my co-authors for their contribution to the published papers. My special thanks go to Tõnis Oja for his long-term co-operation on several papers and conference presentations. His ideas and thorough comments were very inspiring and helped to improve the papers immensely.

The study was financially supported by the Estonian Science Foundation grant ETF 8749. I am thankful to my fellow researchers during the grants of the Estonian Science Foundation, associate professor Harli Jürgenson, Dr. Andres Rüdja, professor Artu Ellmann, for their support and constructive criticism concerning my work. I am also grateful to my students for their help in data collection and analysis.

I greatly appreciate Marguerite Oetjen's help preparing me for the defence of the thesis. I also thank my friends Erik and Mihkel for their encouragement and support.

And last but not least, I thank from all of my heart my family: my wife Katri, my daughter Karina, my mother and my late father, my brother and his family, and my mother-in-law. There are not enough words to describe my gratitude to them.



**I**

**PUBLICATIONS**

**Kall, T.** and Jürgenson, H., 2008.

POSTGLACIAL LAND UPLIFT IN ESTONIA BASED ON  
GEODETIC MEASUREMENTS ON PÕLTSAMAA-LELLE  
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Reprinted from: Environmental Engineering. The 8th International  
Conference: Selected Papers. Vilnius Gediminas Technical University  
Press “Technika”, pp. 1325–1333.



## POSTGLACIAL LAND UPLIFT IN ESTONIA BASED ON GEODETIC MEASUREMENTS ON PÕLTSAMAA-LELLE LEVELLING LINE

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**Abstract.** Postglacial rebound in well-known and widely studied phenomena in Fennoscandia and also in Estonia. In order to study this phenomena and local crustal movements in the Põltsamaa-Lelle region, the 74 km long experimental line was established along the levelling line of national levelling network. During 1961-1987 regular high precision levellings with interval 3-8 years were performed on that line.

Levelling line is very suitable for studying postglacial rebound because its direction is perpendicular to land uplift isolines and also it is sufficient for studying vertical movements related to tectonic faults because several of them are crossing with the line.

In 2006 new levelling measurements were made on Põltsamaa-Lelle line in connection with renovation works of Estonian levelling network. Also there is high precision GPS measurements performed on that line in 1997 related to establishing national geodetic network and repeated in 2005 and 2007 according to study the crustal deformations on the line.

Using additional data, specified value of the land uplift on Põltsamaa-Lelle levelling line was determined.

**Keywords:** postglacial land uplift, levelling, GPS, tectonic activity.

### 1. Introduction.

Subject of this study is determination of postglacial rebound on 74 km long Põltsamaa-Lelle levelling line (Fig. 1a) using high precision levelling and GPS measurement data. Using this levelling line in the present case is convenient because the direction of levelling line is perpendicular to isolines of postglacial rebound.

Previous investigations of postglacial rebound of Earth's crust and local tectonic movements using high precision levelling data and compilations of maps of vertical movements of Earth's crust for Estonia or for the whole Baltic were carried out by Zhelnin [29], Vallner and Zhelnin [33], Vallner, Sildvee and Torim [32], Randjärv [23, 24], Torim [31], using sea level records by Jakubovski [9] and Jevrejeva et al [10]. Maps of Fennoscandian postglacial rebound (including also the territory of Estonia) were compiled by Ekman [2], Lambeck et al [18], and Vestøl [35]. Using only GPS-data, the Fennoscandian postglacial rebound were determined by BIFROST project [11]. Vertical movements of Earth's crust and changes of gravity on Põltsamaa-Lelle levelling line were investigated by Kall and Oja [13] and Oja [21, 22]. On similar geodynamic line in Finland the changes of gravity and vertical movements of Earth's crust were studied by Mäkinen [20] and Ekman and Mäkinen [3]. Compared to earlier similar investigations we had in the present study the opportunity to use additional high precision levelling data

from 2006 and also GPS measurements made in 1997, 2005 and 2007 on the four Estonian geodetic reference network points along Põltsamaa-Lelle levelling line.

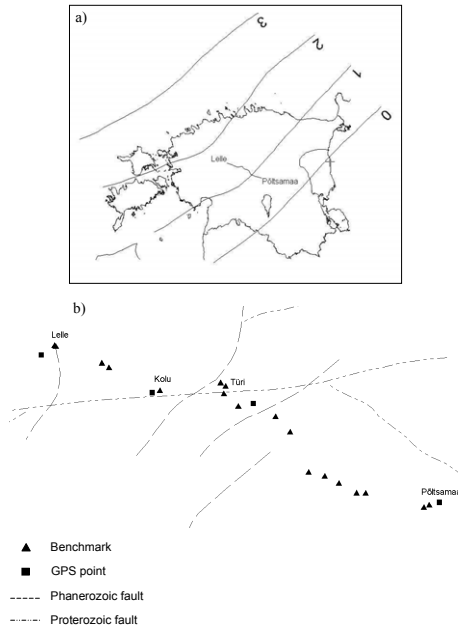
In order to determine the long term vertical movement of Earth's crust we used height differences of common sections to all levelling years. All levelling data were checked for presence of random and systematic errors and if needed systematic errors were removed. Also the weights for height differences were calculated. For determination of vertical movements the linear fitting between height differences and levelling years was calculated. Slope of this fitting gave the value of long term vertical movement by sections. By sum up of those slopes the velocities of vertical movement of benchmarks in relative to Põltsamaa fundamental benchmark were obtained.

From GPS-measurements carried out in 1997, 2005 and 2007 the vertical movement between Põltsamaa and Lelle was determined in order to check the vertical movement value obtained from levellings.

### 2. Geological basement.

Crystalline basement of the levelling line is formed mainly by amphibole gneiss, amphibolite and pyroxene gneiss, which belongs to complex of metamorphic rocks of South-Estonia and West-Estonia [14]. Depth of the sedimentary rocks reaches up to 320-360 meters and are formed mainly by Ordovician and Silurian carbonate

rocks and quaternary sediments. The Quaternary sediments, which depth reaches up to 0.5-10 meters, are formed mainly by till, clay, sand and loam [12]. Crystalline basement crosses by several faults (Fig. 1b), from which largest are Tapa-Pärnu, Saaremaa-Mustvee and Paldiski-Pihkva faults or fault zones. Locations of the faults were determined mainly by magnetometric and gravimetric mapping data. In addition, there are faults also in sedimentary rocks. Basement is especially divided around the towns Võhma and Türi, where several faults of different direction are intersecting. [30].



**Fig. 1.** a) Location of the Põltsamaa-Lelle levelling line in Estonia and land uplift isolines mm/year according to Ekman [2]. b) Põltsamaa-Lelle levelling line and locations of faults according to Koppelmaa [14].

### 3. Levellings.

Levellings were started in 1936 and repeated in 1961, 1964, 1969, 1972, 1980, 1982, 1987 and 2006. Line was partly levelled in 1965 and 1966.

Levelling random and systematic errors were calculated using formulas:

$$\eta^2 = 1/4n \cdot \sum d^2 / r \quad (1)$$

$$\sigma^2 = 1/\sum L \cdot \sum s^2 / L, \quad (2)$$

where  $d$  is discrepancy between the fore and back levelled height difference in section,  $r$  is length of the section,  $n$  is number of sections,  $s = \sum d$ ,  $L = \sum r$  [4]. Levelling precisions are presented in Table 1.

**Table 1.** Precision of the levellings on Põltsamaa-Lelle levelling line.

Levelling year, order of levelling	Random error $\eta$ (mm/km)	Systematic error $\sigma$ (mm/km)
1936 (II)	0.31	0.02
1961 (II)	0.49	0.18
1964 (II)	0.45	0.12
1965 (II)	0.77	0.19
1966 (II)	0.79	0.08
1969 (II)	0.11	0.01
1972 (II)	0.46	0.02
1980 (II)	0.36	0.01
1982 (II)	0.55	0.09
1987 (II)	0.52	0.12
2006 (I)	0.17	0.04

### 3.1. Detecting of systematic errors.

It is known, that levelling systematic errors accumulate with proportional to length of levelling line and height or height gradient [34]. Different authors [8, 5, 16, 25, and 28] were deal with detecting systematic errors in levelling data and removing them from data.

There are several sources of systematic errors in levelling, most of them is caused by refraction, less influence have neglecting of the corrections of gravimetric anomalies and astronomic corrections, neglecting of the rod scale and temperature corrections, neglecting of the level's collimation error and neglecting of the correction of normal heights [7]. Best way to avoid systematic errors is careful observance of levelling methodology.

Main statistical criterions to detect systematic errors in geodetic data are sign criterion and criterion of zero mean of errors [16].

Sign criterion is based on fact that according to theory of probability there is equal number of errors with different signs. Sign criterion of systematic errors is calculated using formula:

$$R = |2k - n|, \quad (3)$$

where  $k$  is the number of errors with positive signs and  $n$  is total number of errors. Zero hypothesis ( $H_0: \Sigma \sigma = 0$ ) is rejected with the level of significance  $\alpha = 0.05$ , if  $R > 2\sqrt{n}$ . Problem is, that sign criterion is not enough efficient to detect systematic errors because it do not take into account size of errors [16]. Therefore the more advanced method is criterion of zero mean of errors, according to which the arithmetic mean of random errors with normal distribution is zero. Whether calculated arithmetic mean differs statistically significantly from zero or not is checked by t-test. If differs, there is reason to suspect that sample contains systematic errors. Zero hypothesis  $H_0: \Sigma \sigma / n = 0$  is rejected with the level of significance  $\alpha = 0.05$ , if  $t > t_{\alpha/2, \nu}$ , where  $\nu$  is the degrees of freedom [36].

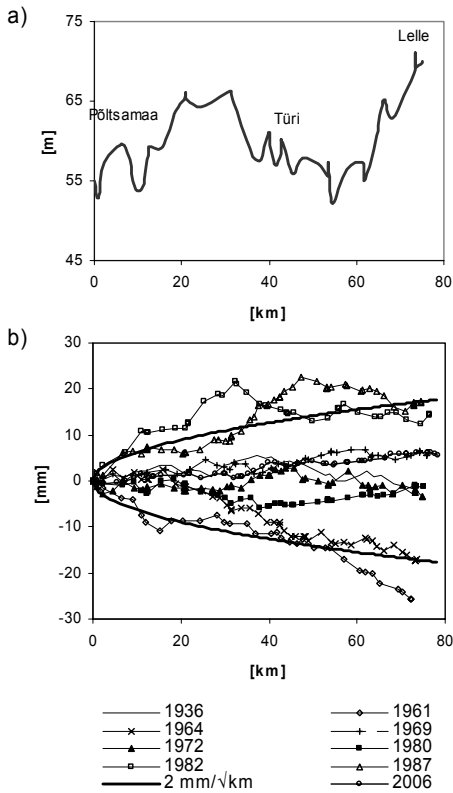


Fig. 2. a) Topography along levelling line. b) Sum of fore and back levelling discrepancies and admissible random error.

Criterion of zero mean can be used only if the distance between benchmarks is more or less equal through the levelling line. If it is not a case, the most useful way to evaluate the presence of systematic errors is the graphical method proposed by Lallemand [17], which is based on presumption of accumulation of systematic errors. In this case, the graphs of levelling errors will be compiled, where on X-axis there is length of levelling line and on Y-axis there is cumulative discrepancies of fore and back levelled height differences. If cumulative discrepancies show linear tendency on graph, there is influence of systematic errors. Slope of that linear line can be used as the estimate of systematic errors [15].

If the criterion of zero mean of errors, sign criterion and compiled graphs of cumulative discrepancies of fore and back levelled height differences (Fig. 2b) did confirm

the presence of systematic errors in levelling, the linear trend line was found between the cumulative discrepancies and length of the line in order to remove systematic errors from data. Correction to height difference was obtained by slope  $b$  of linear line and by length  $L_{ij}$  between benchmarks  $i$  and  $j$  using formula:

$$\varepsilon_{ij} = bL_{ij}. \quad (4)$$

Correction was disjointed from measured height difference and corrected height difference was obtained:

$$h'_{ij} = h_{ij} - \varepsilon_{ij}. \quad (5)$$

According to tests the levellings of 1961, 1964 and 1987 did not answer the requirements of sign criterion or zero mean of errors criterion.

In order to detect systematic errors, many authors [1, 6, 19, and 26] were based on connection between the topography of levelling line and refraction or rod scale error. In the present article, in order to find systematic errors caused by refraction or rod scale error, correlation between topography and change of height difference normalized by section length was looked for. Corresponding graphs were also compiled. Biggest correlation coefficients were obtained for 1980-1982 and 1982-1987 levellings,  $r=0.22$  and  $r=0.24$  correspondingly, but they were statistically insignificant. Still it is pointing to possible levelling errors in 1982 levelling; it is also can be observed from graph of cumulative discrepancies (Fig. 2b), especially in the beginning of the line.

#### 4. Determination of postglacial land uplift on Põltsamaa-Lelle line

In order to find the long-term change of measured height differences, the weighted linear fit between height differences and levelling epochs of common sections was found using SAS software. Reciprocal value of section levelling variance was taken as weight of section's height difference. Change of height difference  $\Delta h$ , obtained from slope of linear fit line, its standard deviation  $S$  and statistics  $p$  and  $R^2$  are given in Table 2.

At first, all available height differences of section were used in order to find long-term change. Since some of the determined changes were statistically insignificant, the second fit was made after removing outliers in order to get best and statistically significant fit. Height difference considered outlier when its standardized residual was bigger than two. Changes of height differences from second fit are given in Table 2 in parenthesis.

**Table 2.** Slopes of linear fit lines presented on Fig. 3 and fit statistics.  
In parenthesis – value after removing the outliers from fit.

Section		Slope of linear fit $\Delta h$ (mm/year)	Standard deviation of slope S (mm/year)	p	R <sup>2</sup>
From	To				
FR EV	PR 263	-0.01 (-0.07)	0.02 (0.01)	0.6065 (0.0001)	0.04 (0.96)
PR 263	SR 268	-0.05 (-0.25)	0.06 (0.05)	0.3903 (0.0031)	0.11 (0.79)
SR 268	SR 269	0.10	0.02	0.0003	0.86
SR 269	SR 270	-0.12	0.01	0.0001	0.95
SR 270	SR 272	0.05	0.02	0.0191	0.57
SR 272	SR 273	0.08	0.01	0.0002	0.88
SR 273	SR 276	0.07 (0.06)	0.03 (0.02)	0.0407 (0.0227)	0.39 (0.55)
SR 276	SR 278	-0.29	0.05	0.0002	0.80
SR 278	SR 280	0.32	0.08	0.0034	0.68
SR 280	SR 282	0.06	0.01	0.0003	0.82
SR 282	SR 283	0.07	0.01	0.0001	0.94
SR 283	SR 284	0.00 (0.03)	0.01 (0.003)	0.7386 (0.0001)	0.01 (0.95)
SR 284	SR 289	0.07	0.03	0.0297	0.51
SR 289	SR 293	-0.53	0.07	0.0001	0.89
SR 293	SR 294	0.20	0.05	0.0047	0.70
SR 294	SR 298	0.34	0.02	0.0001	0.99
SR 298	FR EV	0.18	0.00	0.0001	1.00

Same method was used for finding of the vertical movements of benchmarks using height differences from which the systematic error component was removed according to description given in section 3.1. Obtained results were 0.04 mm/year bigger then vertical movements received from non-corrected height differences.

Long-term changes of height differences grouped by sections are presented in Fig. 3. Most anomalous changes of height differences of sections based on Fig. 3 and causes of outliers will be discussed next.

From graphs (Fig. 3) the height difference and corresponding levelling year which do not coincide well with linear fit can be observed. Difference from linear trend can be statistically significant or not. Reasons for statistically significant outliers can be levelling mistakes, change of groundwater level, subsidence of building, anthropogenic, tectonic reasons etc. Presuming that change of height difference is caused by postglacial rebound then change of all height differences should be with positive linear trend.

In section 0-0.9 km (Fig. 3a) there is positive linear trend in change of height differences in 1936-1961 and in 1961-1964 (slope in 1961-1964 indeed not statistically significant) was noticed which turns in to

negative or stable in later periods. It is possible, that there is error in 1936 levelling.

In section 0.9-11 km (Fig. 3b) there is overall negative trend, which replaced with positive trend (8.4 mm) in 1982-1987. In 1987-2006 the height difference was remain unchanged. It is possible, that there is active fault in 1982-1987 or levelling error in 1982 which is visible also in cumulative error graph (Fig. 2b). But even if we exclude levelling in 1982 there is significant displacement (5.0 mm) also in 1980-1987.

Overall trend in section 12.2-15.7 km (Fig. 3d) is negative, except for outlier in 1987. It is possible that reason of that outlier is active fault between 1982 and 1987 or error in 1987 levelling.

In section 15.7-18.5 km (Fig. 3e) there is slight negative trend is noticeable in 1936-1982 followed by positive trend in 1982-2006 (statistically significant slope is in 1987-2006). Maybe it is active fault since 1982?

In section 20.1-28.8 km (Fig. 3g) there is small positive linear trend where biggest outliers are height differences measured in 1964 and 1965. It is possible that we dealing here with active fault between 1961 and 1966, since in this section also geologically determined fault is located (Fig. 1b).

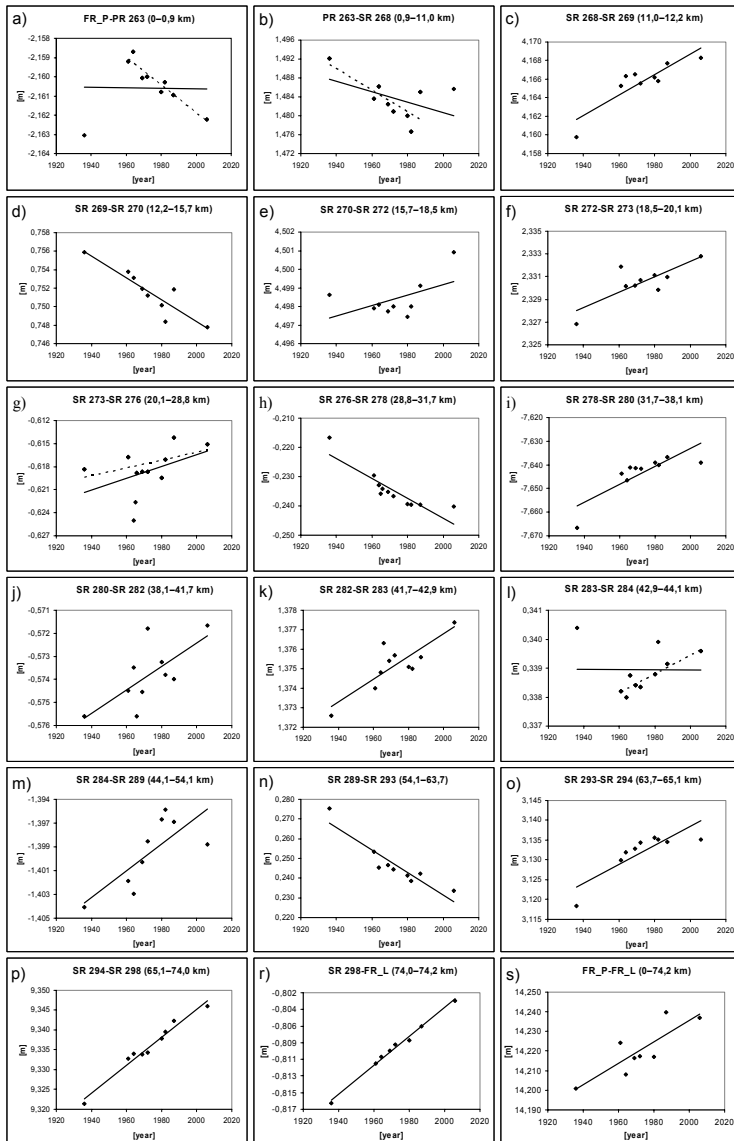


Fig. 3. Weighted linear fit of height differences a) — using all available height differences, b) - - - - after removing the outliers.

Opposite movements in adjacent sections (Fig. 3h and 3i) are probably caused by subsidence of building (Oisu dairy) in which the benchmark SR278 is located.

Beside long-term positive trend in section 38.1-41.7 km (Fig. 3j), the height differences measured in 1965 and 1972 can be isolated as outliers. From analysis of height differences it became obvious that rest of height differences, measured in 1961-1987 do not differ

significantly from previously measured height difference. Since one benchmark (SR282) of that section is located in town Türi, it can be possible that the reason for such wobbly variation is the change of groundwater level in town. Since basement on the levelling line is especially divided just near to town Türi, it is possible that reason because height differences measured in 1965 and 1972 differs from others is tectonic activity in those years.

In section 42.9-44.1 km (Fig. 3l) there is long term positive trend in 1961-2006 can be observed (outlier distinguished with eyes do not differ statistically from previous nor next measured height difference). Reason, because height difference measured in 1936 do not fit in to this trend, is possible subsidence of benchmark installed in 1936 or levelling error in that year.

Positive trend in change of the height differences in section 44.1-54.1 km (Fig. 3m) on starting epochs is replaced with small negative trend in 1982-2006 a. Tectonic fault determined with geologic measurements (Fig. 1b) might been active in period 1964-1982, when the positive change of height differences was fastest.

Reverse changes of height differences in adjacent sections on Fig. 3n and 3o again leads to conclusion, that reason is subsidence of the building (benchmark SR293 is located in the dwelling house).

The last section, 74.0-74.2 km (Fig. 3r), is particularly interesting. There is the best linear fit in whole levelling line. Long-term change of height difference is 0.18 mm/year and that is only for section length 200 meters. Considering period 1936-2006 we get 12.6 mm change of height difference in absolute scale, which is 63 mm/km. Either we have here unstable benchmark or active tectonic fault? Benchmarks of this section should be in stable condition, one of them is wall mark in Lelle Orthodox Church and other is fundamental benchmark. In Fig. 1b we can see that near of this section the fault is located.

In order to get relative vertical movements of benchmarks in relation to Põltsamaa fundamental benchmark, the consecutive sum of slopes of linear fit was found:

$$v_i = \sum_{i=1}^n \Delta h_i \quad (6)$$

Standard deviation of the relative vertical movement was found by formula:

$$S_{v_i} = \sqrt{\sum_{i=1}^n S_i^2} \quad (7)$$

where  $S_i$  is the standard deviation of linear fit line. Vertical movement of the benchmarks in relation to Põltsamaa fundamental benchmark is estimated as statistically significant on the 95% confidence level if signal-to-noise ratio was:

$$S/N = v_i/S_{v_i} > 2 \quad (8)$$

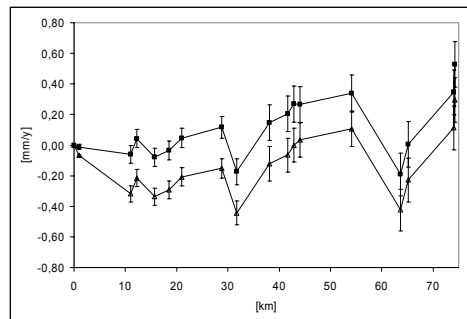
Results are presented in Table 3. Graph of the benchmark's relative vertical movements based on Table 3 is presented on Fig. 4.

Based on Fig. 3s and 4 and Table 3 the postglacial land uplift of the whole leveling line can be brought out which is 0.53 mm/year (0.30 mm/year when outliers were removed). If we compare this result with recent maps of vertical movements of Estonia [24, 31 and 32] it appears that obtained result is significantly smaller then presumed so far.

**Table 3.** Vertical movement of the benchmarks in relation to Põltsamaa fundamental benchmark. Values presented in parenthesis were obtained from change of height differences after removal of outliers.

Benchmark's No.	Vertical movement $v$ (mm/year)	Standard deviation $S$ (mm/year)	S/N ratio
FR_P	0.00 (0.00)	0.00 (0.00)	
PR 263	-0.01 (-0.07)	0.02 (0.01)	-0.6 (-11.3)
SR 268	-0.06 (-0.32)	0.06 (0.05)	-1.1 (-6.0)
SR 269	0.04 (-0.22)	0.06 (0.06)	0.7 (-3.9)
SR 270	-0.08 (-0.34)	0.06 (0.06)	-1.3 (-5.9)
SR 272	-0.03 (-0.29)	0.06 (0.06)	-0.5 (-5.0)
SR 273	0.05 (-0.21)	0.06 (0.06)	0.7 (-3.5)
SR 276	0.12 (-0.15)	0.07 (0.06)	1.7 (-2.4)
SR 278	-0.17 (-0.44)	0.09 (0.08)	-2.0 (-5.6)
SR 280	0.15 (-0.12)	0.12 (0.11)	1.3 (-1.1)
SR 282	0.20 (-0.07)	0.12 (0.11)	1.7 (-0.6)
SR 283	0.27 (0.00)	0.12 (0.11)	2.3 (0.0)
SR 284	0.27 (0.03)	0.12 (0.11)	2.3 (0.3)
SR 289	0.34 (0.11)	0.12 (0.12)	2.8 (0.9)
SR 293	-0.19 (-0.42)	0.14 (0.14)	-1.4 (-3.1)
SR 294	0.00 (-0.23)	0.15 (0.14)	0.0 (-1.6)
SR 298	0.35 (0.11)	0.15 (0.14)	2.3 (0.8)
FR_L	0.53 (0.30)	0.15 (0.14)	3.6 (2.0)

Values of relative vertical movement obtained from previous authors range between 0.9-1.3 mm/year (Table 4). Discrepancy can be explained with different calculation methods and use of not identical levelling data. In same time, obtained result 0.53 mm/year coincides better or even well with the values from maps of Fennoscandian postglacial rebound [2, 18 and 35] which range between 0.3-0.7 mm/year (Table 4)



**Fig. 4.** Long-term average vertical movement of Earth's crust on Põltsamaa-Lelle line. ■ – vertical movements calculated by change of all available height differences in section, ▲ – vertical movements calculated by change of height differences in section after removal of outliers.

**Table 4.** Relative vertical movement  $\Delta v$  mm/year between the Põltsamaa and Lelle interpolated from vertical movement maps and difference from value 0.53 mm/year obtained in the current study.

Model	$\Delta v$	$\Delta v - 0.53$
Vallner et al 1988	1.31	0.78
Randjärv 1993	0.90	0.37
Torim 2004	1.09	0.56
Ekman 1996	0.72	0.19
Lambeck 1998	0.37	-0.16
Vestol 2006	0.43	-0.10

## 5. GPS measurements.

Along to Põltsamaa-Lelle levelling line, the four Estonian geodetic reference network second order points (Põltsamaa97, Taikse97, Kolu97 and Põllu97) are located (Fig. 1b). On these points the GPS measurements were performed in 1997, 2005 and 2007.

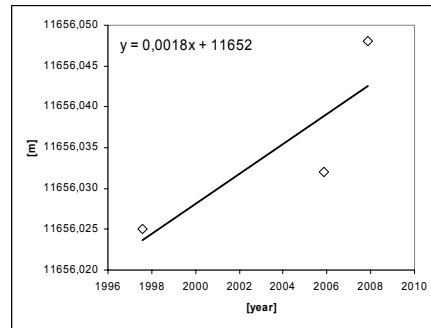
In 1997 the measurement on those points was made in relation to establishing Estonian geodetic reference network and was performed by private company AS Planserik by order of Estonian Land Board. Profound analysis about construction of points, measurements and data processing of Estonian geodetic reference network is presented by Rüdja [27].

In 2005 and 2007 the repeated measurements on those points were performed by Department of Geomatics of Estonian University of Life Sciences. In 2005 four Trimble 5800/R8 receivers were used. In 2007 three Trimble 5800/R8 and one Trimble 5700 (with Zephyr antenna) receivers were used. Length of sessions prolong 16-18 hours in both repeated measurement, data recording interval was 30 second in 2005 and 15 seconds in 2007. Trimble Total Control software was used for data processing. In data processing NGS relative antenna calibration data, IGS final orbit data and final ionospheric TEC grid data were used. For modeling of troposphere, the Saastamoinen's model was used. Fixed Lc vectors were used as final solution.

In order to check the postglacial land uplift value obtained from the levellings the change of ellipsoidal height difference between Põltsamaa and Lelle was calculated (Fig. 5).

Obtained change of ellipsoidal height difference 1.8 mm/year is absolute change. In order to compare it with value received from levellings it must be converted in to apparent change of height difference. For that, from value 1.8 mm/year the value of eustatic sea level rise 1.2 mm/year and change of geoid (approximately 6%) must be disjointed. After converting the value 0.67 mm/year was obtained as apparent relative vertical movement which coincides with the value 0.53 mm/year obtained from levellings.

It is necessary to repeat GPS measurement in order to increase the reliability of results obtained so far.



**Fig. 5.** Change of ellipsoidal height difference between Põltsamaa and Lelle.

## 6. Conclusions.

For determination of vertical movements of Earth's crust in Estonia there has been problem with lack of steady benchmarks which would be attached in to basement. Depth of Quaternary sediments often does not allow this and therefore on interpretation of vertical movements it is necessary to take into account the possible benchmark instability.

In current study the main attention was attended to investigation of long-term change of benchmarks heights. For that, common height differences of all available levelling data were used and trend line between change of height differences and time of levelling was determined. In almost all sections there was strong correlation between change of height difference and time. The local causes (subsidence of buildings, different compaction of sediments), possible levelling errors, tectonic activity and postglacial rebound were brought out as main reasons for change of height differences.

Main factor of long-term vertical movements in Estonia as also in Fennoscandia is postglacial land uplift. In present article on 74 km long levelling line between fundamental benchmarks of Põltsamaa and Lelle the average postglacial land uplift value  $+0.53 \pm 0.15$  mm/year was determined using all available levelling data. Obtained value is approximately two times smaller than respective value from recent maps of vertical movements in Estonia but coincides well within the error limit with value interpolated from Fennoscandian postglacial land uplift maps and with value obtained from repeated GPS measurements.

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**Kall, T.,** Oja, T., and Tänavsuu, K., 2014.

POSTGLACIAL LAND UPLIFT IN ESTONIA BASED ON  
FOUR PRECISE LEVELLINGS

*Tectonophysics*, **610**, pp. 25–38. doi:10.1016/j.tecto.2013.10.002



## Postglacial land uplift in Estonia based on four precise levelings

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### ARTICLE INFO

#### Article history:

Received 15 November 2012

Received in revised form 2 October 2013

Accepted 4 October 2013

Available online 18 October 2013

#### Keywords:

Precise leveling

Kinematic adjustment

Land uplift

Estonia

Variance estimation

### ABSTRACT

The vertical velocities of the fundamental benchmarks of Estonian 1st order leveling network were estimated, based on the four precise leveling campaigns from 1933 to 2010. The kinematic least squares adjustment of the network was used, where heights and velocities were introduced as unknown parameters. For detection of outliers, Baarda's data snooping method was applied. Estimation of variance components by the Helmert's and the IAUE methods provided realistic weights in the network adjustment and revealed also that the observation errors of the first three levelings are up to 3 times larger than was assumed a priori.

To obtain apparent uplift rates and fix velocity in kinematic adjustment, the velocity value +2.1 mm/yr of the Ristna tide gauge on island Hiiumaa was transferred to the nearest stable benchmark by using precise levelings from tide gauge to the national height network. Average standard deviation of velocities at benchmarks was estimated to be  $\pm 0.5$  mm/yr.

Based on apparent vertical velocities of the benchmarks, an interpolated land uplift surface was created using "kriging" and "minimum curvature" gridding methods. Although the two methods gave similar surfaces (RMS of differences was below  $\pm 0.1$  mm/yr), the kriging solution was used in comparison with the results of the earlier studies. The uncertainty of new velocity surface was estimated to be  $\pm 0.5$  to  $\pm 0.7$  mm/yr.

The new land uplift model fits well with the Fennoscandian land uplift model NKG2005LU. The RMS of the differences was  $\pm 0.2$  mm/yr. Good agreement was also confirmed with the earlier land uplift maps of Estonia (RMS of differences  $\pm 0.6$  mm/yr). However, clear disagreement was noticed when tide gauge observations from several studies were used in comparison (RMS of differences up to  $\pm 1.0$  mm/yr). Apparently, different velocity solutions of Estonian tide gauges are systematically biased; the reasons for these biases need further investigation.

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### 1. Introduction

Fennoscandian postglacial rebound (PGR) is profoundly studied phenomena. Different kinds of observation data (leveling, sea- and lake-level records, GNSS measurements, satellite- and terrestrial gravity measurements, geophysical and geological data) have been used for modeling the rebound process (Ekman, 1996; Lambeck et al., 1998; Lidberg et al., 2007; Steffen and Kaufmann, 2005; Steffen et al., 2009; Vestøl, 2006). Estonia is located at the south-east edge area of the rebound (Fig. 1). According to the recent empirical rebound model NKG2005LU (Ågren and Svensson, 2007), apparent land uplift rates relative to sea level remain within  $-0.5$  to  $+2$  mm/yr in Estonia. The NKG2005LU model was compiled using the geophysical model by Lambeck et al. (1998) and the empirical model by Vestøl (2006). The two models were merged so that transition from one to another would be as smooth as possible. Some imperfections and artifacts of Vestøl's model (roughness of the surface, "cylinders" around GNSS stations) were smoothed by filtering (Ågren and Svensson, 2007).

The model by Lambeck et al. (1998) was constructed using relative sea level (RSL) indicators, the global and refined Fennoscandian ice sheet models and the three-layered Earth model. It was further constrained to Ekman's (1996) tide gauge (TG) uplift values. However, no observation data from Estonia were used as constraints for the geophysical modeling although the model covers the Baltic States, including Estonia.

The TG uplift values by Ekman (1996), vertical velocities of the permanent GNSS network (BIFROST) in ITRF2000 by Lidberg (2004), also in Lidberg et al. (2007) and repeated leveling data from Norway, Sweden and Finland from 1890 to 2004 were used by Vestøl (2006) to calculate the empirical model for land uplift rates. Least squares collocation (LSC) with a 5th degree polynomial trend surface was used to solve land uplift values and other parameters including eustatic rise of the sea level and rise of the geoid. On average, reliability of the uplift values better than  $\pm 0.5$  mm/yr was evaluated by Vestøl. His model has been extended quite far in the south-east direction (it covers one third of Estonian territory). However, the surface over Estonia is an extrapolation since no observation data (from TGs, leveling networks, GNSS stations) were used for the modeling in south-eastern areas.

The fourth leveling campaign of Estonia, started in 2001, is almost complete and provides the opportunity to calculate the new vertical

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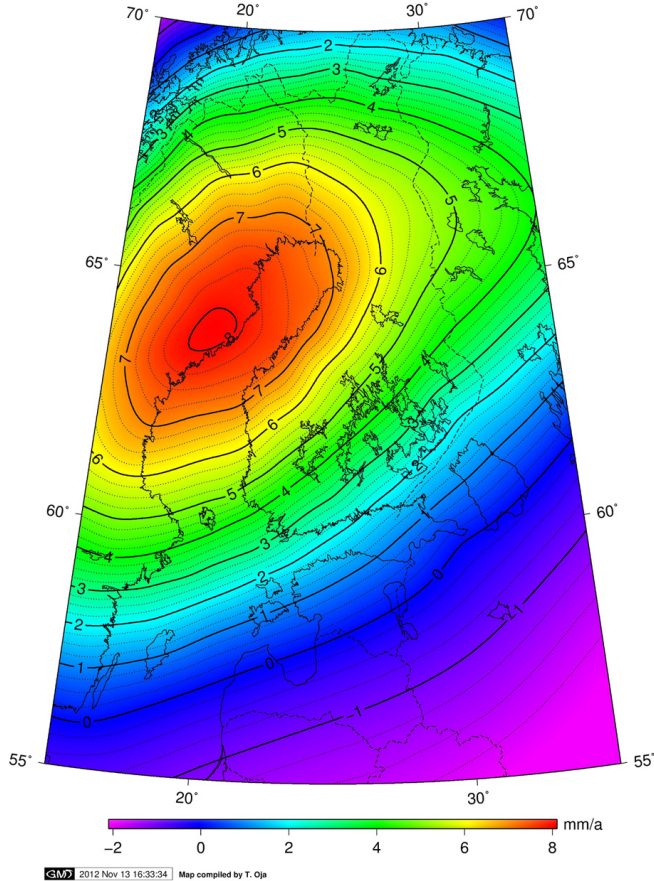


Fig. 1. Fennoscandian apparent land uplift according to the NKG2005LU (Ågren and Svensson, 2007).

velocity values for the benchmarks (BMs) of the height network. The aim of this study was to calculate the new vertical velocities to the stable BMs of the height network using all four high precision leveling data from period 1933–2010 and kinematic adjustment method which for the first time was applied to find the land uplift velocities in Estonia. The new local empirical uplift model (later named as EST2013LU) has been derived which then was used to evaluate the quality of NKG2005LU model over Estonia. Note that NKG2005LU at the south-eastern rim of the PGR area is represented by a geophysical model only, i.e. neither geodetic nor sea level data have been used there as additional constraints. Additionally, several sets of uplift rates or modeled velocity surfaces of the earlier studies (see Section 2.1 below) were used for the evaluation of the EST2013LU and NKG2005LU models.

Sections 2 and 3 give an overview of previous studies of vertical crustal movements in Estonia, underlying leveling observations and the used BMs. Section 4 provides an overview of the methodology of the kinematic adjustment. In Section 5 the results from the adjustment are described and new vertical velocities of the BMs with uncertainty estimates are presented. It provides an assessment of the quality of

leveling observations based on variance component estimation. In Sections 6 and 7 the new local land uplift surface is compiled based on estimated velocity rates which are compared with the results of the previous studies. Finally, the results are discussed and the problems requiring further investigation are brought out.

## 2. History of land uplift maps in Estonia

This section reviews the earlier studies of land uplift in Estonia. It includes i) the fit of the earlier results with each other and with the NKG2005LU model, and ii) a discussion about proper reference to the apparent velocity value for calculation of the land uplift solution from the leveling data.

### 2.1. Land uplift maps based on repeated levelings

Several studies to determine land uplift from precise leveling data have been carried out in Estonia (Randj arv, 1993; Torim, 2004; Vallner and Zhelnin, 1975; Vallner et al., 1988; Zhelnin, 1966). Data from only two leveling campaigns (usually the first and the latest)

were used in these studies to calculate the vertical movement of the BMs. Velocity of the height difference change between two BMs (difference of vertical velocity,  $\Delta v$ ) was calculated by the formula:

$$\Delta v = \frac{\Delta h_2 - \Delta h_1}{\Delta t} \quad (1)$$

where  $\Delta h_1$  is a height difference of the initial leveling,  $\Delta h_2$  is a height difference of the repeated leveling and  $\Delta t$  is the time interval between the two levelings.

Thus, when repeated leveling has been performed using the same network configuration, it is possible to form a network of vertical velocity differences between consecutive BMs which then can be adjusted in a similar way to the adjustment of height differences. The abovementioned studies provide insufficient information about methods used in the adjustment of velocity differences: it is known that most of the adjustment computations were done manually with simplified methods. For example, Randjärv (1993) used the so-called Popov's iterative method for adjustment. The Popov method is a graphical adjustment method of a geodetic network, where the closing errors of the loops are divided between the measurements iteratively, in proportion to the so-called "red numbers". For example, in adjustment of the leveling loop, the ratio of the length of the leveling section to the loop's perimeter is taken as the "red number" (Randjärv, 2007).

For the weighting of velocity differences in adjustment, the variance of velocity difference  $m_{\Delta v}^2$  was calculated. When information about leveling standard errors was available, the following formula was used:

$$m_{\Delta v}^2 = \frac{(m_1^2 + m_2^2) \cdot L}{\Delta t^2} \quad (2)$$

where  $m_1^2$  is the variance ( $\text{mm}^2/\text{km}$ ) of the initial leveling,  $m_2^2$  is the variance of the repeated leveling and  $L$  is the distance (km) between the two BMs. Here, the variances of the two levelings were calculated using forward and backward discrepancies of the height differences in the leveling line (Eqs. (7), (8), (9)) or by loop closing errors. However, formulas without direct information about leveling uncertainties were also applied. For example, the following formula for finding the variance of a velocity difference of a 100 km leveling line segment was employed by Vallner et al. (1988):

$$m_{\Delta v}^2 = k \cdot \left(\frac{30}{\Delta t}\right)^2 \cdot \frac{L}{100} \quad (3)$$

where 30 is the average time difference between the two levelings in years and  $k$  is a coefficient (1, 2 or 3), depending on the accuracy class of the levelings. Because of the simplified methods used in the adjustment of velocity differences, no complete statistical information (like variance–covariance matrix of parameters and observations) was obtained. Also, no statistical tests for evaluating adjustment results or detecting observation outliers were performed. According to the land uplift maps by Estonian authors (Table 1), apparent land uplift rates

remain within  $-1$  to  $+3$  mm/yr. Estimated uncertainties of the relative vertical velocities range from  $\pm 0.2$  to  $\pm 0.4$  mm/yr (Randjärv, 1993; Vallner and Zhelnin, 1975; Vallner et al., 1988). These estimates were obtained from closing errors of velocity differences in leveling loops.

In the present study, a number of historical Estonian land uplift maps on paper by different authors were also digitized and gridded. The comparison of gridded surfaces and the NKG2005LU model revealed a quite good fit between the modeled surfaces (Table 1). The average RMS value of differences between the gridded surfaces is  $\pm 0.4$  mm/yr. These discrepancies result from the use of different reference values of the apparent uplift, moreover the leveling data and methodology used by the different authors were not the same. For example, when Vallner et al. (1988) and Torim (2004) fixed their velocities to the apparent land uplift value  $+2.4$  mm/yr at Tallinn TG, then Vallner and Zhelnin (1975) and Randjärv (1993) constrained their relative velocities to the value  $+1.7$  mm/yr at Tallinn, according to Yakubovski (1973) TG velocities at Kunda and Vormsi and repeated levelings from Kunda and Vormsi to Tallinn. RMS of differences between Estonian land uplift maps and land uplift model NKG2005LU is  $\pm 0.7$  mm/yr. The reasons for discrepancies are same as explained before. In addition, the inaccuracy of the NKG2005LU model over Estonia must also be considered here since no observations from Estonia were used in modeling as mentioned before. Residuals of the NKG2005LU model with the GPS and TG observations were up to  $\pm 0.53$  mm/yr on average (Ågren and Svensson, 2007). If this number is assumed to present uncertainty of NKG2005LU in Estonia then even these older studies based on repeated leveling indicate good performance of the NKG2005LU model over the study area.

However, it should be kept in mind that the spatially distributed, realistic uncertainty of NKG2005LU was not rigorously estimated by Ågren and Svensson (2007). In areas without geodetic and sea level data it would be impossible anyway since the geophysical model by Lambeck et al. (1998) included no uncertainty information for vertical velocities. Therefore, the only way to assess the quality of NKG2005LU model in Estonia was to compare it with historical Estonian land uplift maps (done in this section), and geodetic datasets like the uplift rates from repeated levelings (will be done in following sections).

## 2.2. Land uplift maps based on tide gauge observations

The apparent land uplift rates using the Estonian coastal TG (Fig. 2) records have been determined in several studies (Jevrejeva et al., 2002; Pobedonostsev, 1975; Vallner et al., 1988; Yakubovski, 1973). Table 2 gives an overview of the discrepancies among apparent velocities at Estonian TGs. The NKG2005LU model fits best with TG velocities determined by Yakubovski (1973). Yakubovski's solution is also the only one which can be considered to agree with the NKG2005LU model within the limits of expanded uncertainty. There is also a large mismatch between different velocity solutions of Estonian TGs themselves. The average RMS of the discrepancies is  $\pm 1.1$  mm/yr

**Table 1**  
Mean and RMS of differences (mm/yr) between Estonian land uplift maps and NKG2005LU.

	Zhelnin (1966)	Vallner and Zhelnin (1975)	Vallner et al. (1988)	Torim (2004)	Randjärv (1993)	NKG2005LU
Zhelnin (1966)	0	$0.4 \pm 0.46$	$0.2 \pm 0.34$	$0.2 \pm 0.33$	$0.7 \pm 0.70$	$-0.7 \pm 0.94$
Vallner and Zhelnin (1975)	-	0	$-0.2 \pm 0.38$	$-0.2 \pm 0.33$	$0.4 \pm 0.40$	$-0.4 \pm 0.61$
Vallner et al. (1988)	-	-	0	$-0.1 \pm 0.17$	$0.5 \pm 0.51$	$-0.6 \pm 0.77$
Torim (2004)	-	-	-	0	$0.5 \pm 0.52$	$-0.5 \pm 0.69$
Randjärv (1993)	-	-	-	-	0	$0.2 \pm 0.53$
NKG2005LU	-	-	-	-	-	0

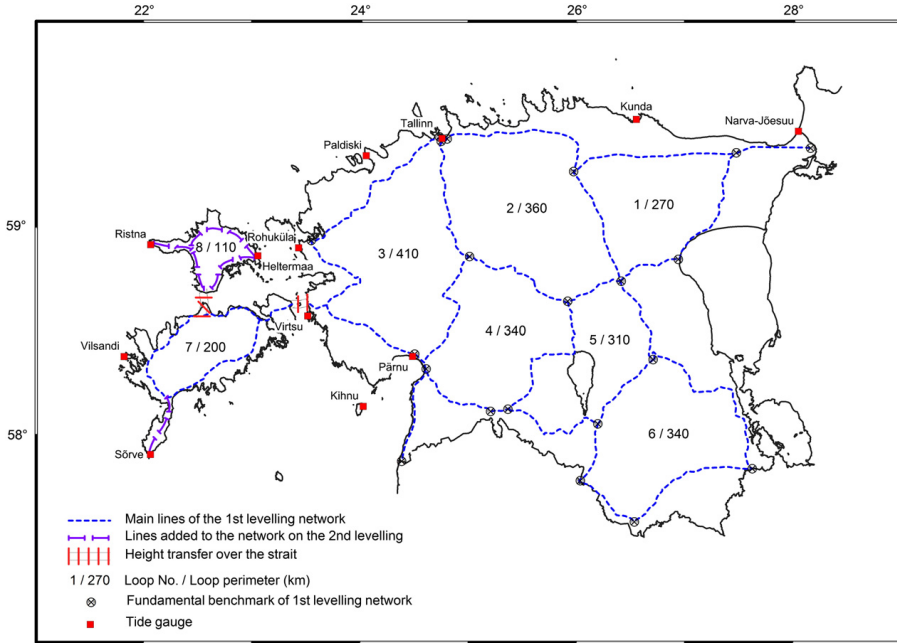


Fig. 2. The lines of precise leveling network in Estonia which have been leveled at least three times from 1933 to 1996.

which clearly exceeds the uncertainty estimates of TG velocities ( $\pm 0.2... \pm 0.4$  mm/yr). Possible reasons for these discrepancies are:

- Standard periods for calculations were different;
- A different methodology was used to fill observation gaps;
- Readings taken at different times, rod changes and adding zero corrections, leveling ties to the BMs, height system changes, and additional corrections (hydrological, meteorological, etc.) were handled differently.

Selection of the reference TG for the kinematic adjustment, therefore, was not an easy task. The velocities from the three sources (Jevrejeva et al., 2002; Pobedonostsev, 1975; Yakubovski, 1973) differ least at Pärnu, Ristna, Rohuküla and Virtsu TGs (the differences are within  $\pm 0.3$  mm/yr). Of these stations, fit with the Ekman's (1996) and NKG2005LU uplift models is best at Ristna (within  $\pm 0.3$  mm/yr). The apparent velocity value of Ristna TG based on Jevrejeva et al. (2002); Pobedonostsev (1975) and Yakubovski (1973) ranges from +1.8 to +2.4 mm/yr. Accordingly, the average velocity of the Ristna

TG +2.1 mm/yr with the RMS of differences between the three solutions  $\pm 0.26$  mm/yr can be concluded for the approximate period of 1950–1980. This average apparent velocity of the Ristna TG was transferred to the nearest stable BM “SR Luidja” using leveling connections between the TG and the national height network. This BM, with the velocity rate of 1.9 mm/yr, was fixed in the kinematic adjustment of this study.

### 3. Precise levelings in Estonia

#### 3.1. First leveling: 1933–1943

The first precise leveling network, covering all of Estonia, was established by the Cadastre Department of the Ministry of Agriculture of the Estonian Republic in 1933–1943. The establishment of height networks in countries around the Baltic Sea was initiated by the Baltic Geodetic Commission in 1932 (Torim, 1993). In 1943, the network on the mainland consisted of six loops (1800 km leveling lines) with 1151 BMs; 23 nodal points of the network were fundamental BMs. An

Table 2  
RMS of differences (mm/yr) between Estonian tide gauge apparent velocities and NKG2005LU.

	Yakubovski (1973) (1889–1970)	Tamm, as cited in Vallner et al. (1988) (1968–1983)	Pobedonostsev (1975) (1930–1972)	Jevrejeva et al. (2002) (1892–1991)	NKG2005LU (1892–1991)
Yakubovski (1973)	0	$\pm 1.06$	$\pm 1.12$	$\pm 1.19$	$\pm 0.83$
Tamm, as cited in Vallner et al. (1988)	–	0	$\pm 1.87$	$\pm 1.66$	$\pm 1.20$
Pobedonostsev (1975)	–	–	0	$\pm 0.64$	$\pm 1.30$
Jevrejeva et al. (2002)	–	–	–	0	$\pm 0.95$
NKG2005LU	–	–	–	–	0



additional leveling loop was established on the largest island of Saaremaa (2673 km<sup>2</sup>) which was connected to the mainland by leveling on ice. No network was established on Hiiumaa (989 km<sup>2</sup>), the second largest island.

### 3.2. Second and third levelings: 1948–1969 and 1970–1996

The second and the third levelings took place in 1948–1969 and 1970–1996, respectively. The major part of these levelings was done by the geodesy group of the Institute of Physics and Astronomy of the Estonian Academy of Sciences (EAS). The extent of the leveling lines was 2100 km in the second period and 1884 km in the third period (Torim and Jürma, 2011). Many lines were re-leveled more than once. In addition, new leveling loop was established on the island of Hiiumaa during the second leveling. Connections between the network on the mainland and loops on the islands were determined by hydrostatic leveling. The network configuration of the 1st, 2nd and 3rd leveling lines is presented in Fig. 2.

Some lines of the leveling network were measured by the General Committee of Geodesy and Cartography (GCCG) of the Soviet Union, as well. In 1948 and 1970, the Narva–Tallinn–Pärnu–Ikla line (505 km) was leveled in order to connect the Baltic States of the Soviet Union to the Kronstadt height system. The GCCG also performed additional levelings (921 km) in 1981–1983 by repeating lines previously leveled by the EAS on the Estonian mainland (Torim, 2000).

### 3.3. Fourth leveling: 2001–2012

The work with the latest leveling started with inspection of old leveling lines which was done from 2001 to 2007. It appeared that

48% of the BMs were broken, could not be found or had been destroyed. Preserved or broken BMs were repaired, and additional new BMs were installed to replace the destroyed or missing ones. Altogether 928 new BMs were installed (Torim and Jürma, 2007). Additional leveling lines were established in order to achieve smaller and more evenly distributed loops, compared to the historical leveling network (Fig. 2). New connections between the mainland and the islands of Saaremaa and Hiiumaa were established using hydrodynamic leveling (Liibus, 2013). The renovated leveling network consists of 26 loops with the length of 3700 km (Fig. 3). Note that the first three precise levelings were performed with traditional equipment (high precision optical levels and invar rods with 0.5 cm graduation). The last precise leveling was carried out with high precision digital instruments (Trimble/Zeiss digital level DiNi 11 and 12 and NEDO 3 m invar rods with a code bar). Instruments were calibrated twice per year at the Finnish Geodetic Institute (Torim and Jürma, 2011).

The mean standard errors of all four leveling campaigns based on the forward and backward discrepancies of the height differences in the leveling line and loop closing errors are presented in Table 3. The standard errors of the first, second and third levelings are quite similar. The clearly smaller standard error of the final, fourth leveling is obviously the effect of modern leveling equipment and methodology used in 2001–2010.

## 4. Materials and methods

This section provides an overview about the selection of BMs and observations for the formation of the network of kinematic adjustment. The adjustment method, evaluation of the results and outlier detection used in the study are explained in detail.

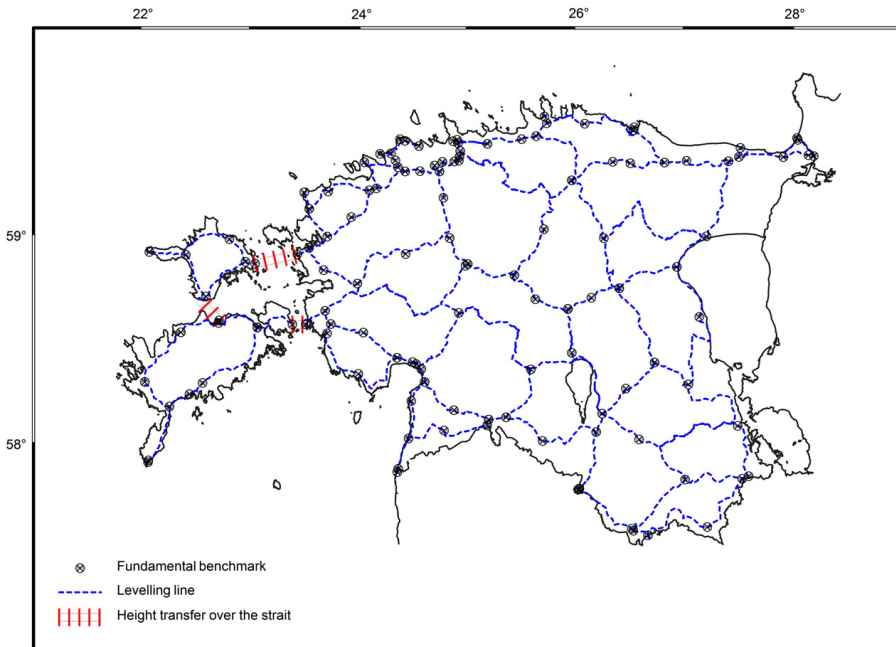


Fig. 3. Renovated precise leveling network of Estonia in 2012.

**Table 3**  
Uncertainty estimates of precise levelings (Torim, 2000; Torim and Jürma, 2011).

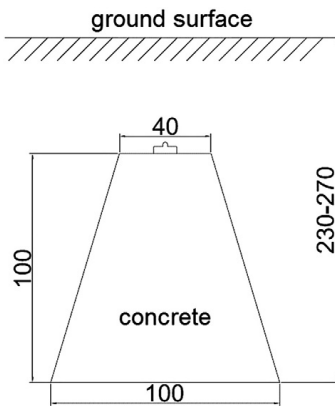
Leveling period	Extent of leveling lines (km)	Random standard error (mm/√km)	Systematic standard error (mm/km)	Leveling standard error estimated from loops misclosures (mm/√km)
1933–1943	1727	±0.318	±0.034	±0.44
1950–1969	2101	±0.490	±0.070	±0.47
1970–1996	1884	±0.475	±0.049	±0.54
2001–2010	3435	±0.168	±0.037	±0.28

#### 4.1. Used benchmarks and leveling data

Most of the deep seated, so called fundamental BMs (fBMs) (Fig. 4) mounted before the first leveling have been preserved to the present day and are in good condition. These fBMs are primarily used in the present study for calculation of vertical velocities. All previously published studies about vertical crustal movements in Estonia (e.g. Randjärv, 1993; Vallner et al., 1988) have also relied on fBMs. The stability of the fBMs has been previously studied by aforementioned authors. For example, geotechnical studies at the locations of BMs used in their study were carried out by Vallner et al. (1988) in collaboration with geologists.

The stability of the ground has been observed already during the mounting of the fBMs in 1930s. It was possible to install BMs into the bedrock (limestone, sandstone) in north and west Estonia and West-Estonian islands, however, in other areas it was difficult to mount the fBMs into bedrock because of thick sedimentary cover (greatest thickness up to 200 m is found in SE-Estonia and at ancient buried valleys). Unstable soils, such as clay, quicksand, and swampy areas, were avoided on mounting of fBMs in sediments. Instead, BMs have been placed in sand distribution areas.

In some areas without fBMs nearby it was necessary to include wall BMs into the network of kinematic adjustment: in such cases, BMs in the old stone buildings (churches, dairies, etc.), built on stable ground, were only used. Vertical motions of BMs, which are placed on Quaternary deposits, can be influenced by near-surface processes like volume change in soil affected by variable groundwater level, frost heave etc. (Demoulin and Collignon, 2000; Kall and Torim, 2003; Wyatt, 1989; Zerbini et al., 2001). Deep-seated fBMs are less prone to such disruptive processes.



**Fig. 4.** Fundamental benchmark mounted during the 1st leveling campaign. Dimensions are in cm.

In light of the latest levelings, graphical analysis of the stability of BMs was performed. The height difference scatter plots for each leveling section and relative vertical movement scatter plots were drawn for every leveling line. Stable BMs with steadily changing height differences were identified in the scatter plots of height differences. BMs were regarded stable if there are no peaks, changes of slope or sign of the velocities on the scatter plots of the relative vertical movements (Giménez et al., 2000). To facilitate detection of the anomalously moving BMs, additional BMs near (2–5 km) suspicious BMs were added to the network. The final dataset and the selected BMs are shown in Fig. 5. All available precise leveling data observed in 1933–2010 between selected BMs were used in the adjustment.

#### 4.2. Kinematic adjustment model

The proper methodology for vertical velocity calculation should deal with all repeated leveling observations and be able to detect observation outliers. One method that satisfies these criteria is the so-called kinematic adjustment (Kleijer et al., 2002; Mäkinen and Saaranen, 1998), in which both heights and vertical velocities of BMs are parameterized in the observation equation. The observation equation for observed height difference  $y_i$  or  $H_{jk}^i$  between BMs  $j$  and  $k$  at the epoch  $t_i$  is

$$y_i = H_{jk}^i = H_{jk}^0 + v_{jk}(t_i - t_0) + e_{jk} = H_j^0 - H_k^0 + (t_i - t_0)v_j - (t_i - t_0)v_k + e_{jk} \quad (4)$$

where

$H_{jk}^0$  height difference between BMs  $j$  and  $k$  at reference epoch  $t_0$ ,  
 $v_{jk}$  velocity of height difference  $H_{jk}^0$ ,  
 $H_j^0$  and  $H_k^0$  heights of BMs  $j$  and  $k$  at epoch  $t_0$  (unknowns),  
 $v_j$  and  $v_k$  velocities of BMs  $j$  and  $k$ , (unknowns),  
 $e_{jk}$  error in the observation.

In matrix notation this becomes,

$$y = [A_H, TA_H] \begin{bmatrix} H^0 \\ v \end{bmatrix} + [e] = B \begin{bmatrix} H^0 \\ v \end{bmatrix} + [e] \quad (5)$$

where

$A_H$  design matrix of leveling network,  
 $H_0$  vector of unknown heights at reference epoch  $t_0$ ,  
 $v$  vector of unknown velocities,  
 $y$  vector of observations in leveling,  
 $e$  residual vector,  
 $T = \text{diag}(t_1 - t_0, \dots, t_m - t_0)$   
 diagonal matrix of epochs of observations.

The least squares solution for the heights and velocities is given as:

$$\begin{bmatrix} H^0 \\ v \end{bmatrix} = [B^T W B]^{-1} B^T W y \quad (6)$$

where  $W$  is the weight matrix of leveling data.

It is important to note that only height and velocity differences are estimable from leveling data. In order to obtain apparent velocities (relative to sea level) at all BMs, at least one BM with known apparent velocity value has to be fixed in the adjustment.

The average uncertainties of the leveling campaigns from Table 3 were not used for establishing the weights of observations since many lines have been leveled more than four times (in some cases up to 9 times, see Kall and Jurgenson (2008) for example). Instead, the weights of the height differences were calculated by random standard error  $\eta$  (mm/√km) and systematic standard error  $\sigma$  (mm/km) of

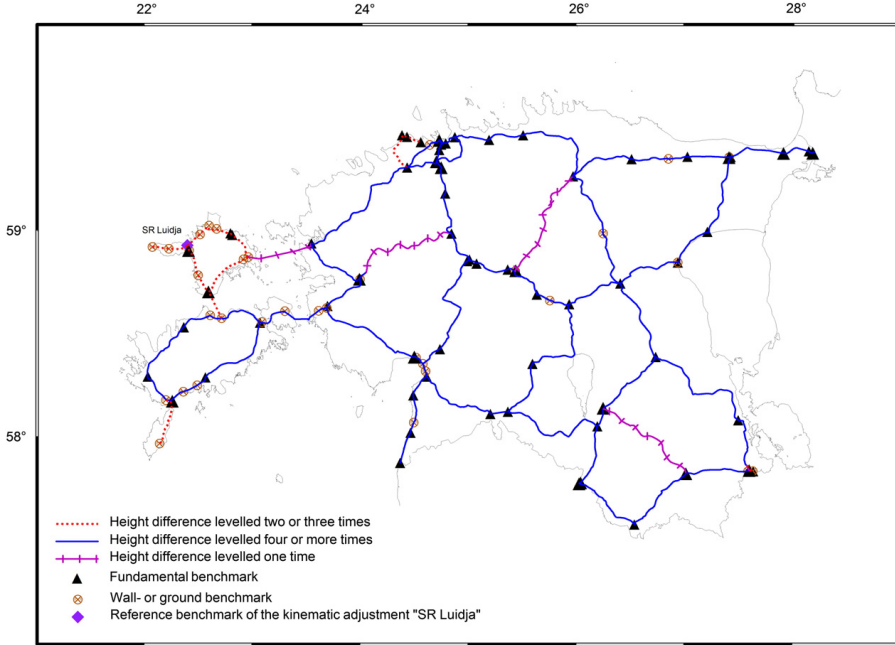


Fig. 5. Height differences and benchmarks used in the kinematic adjustment. Network consists of 2222 km of leveling lines, 15 loops and 361 height differences between 106 benchmarks, among which 84 benchmarks have been leveled at least twice.

the specific leveling line, evaluated using the formulas (Jordan et al., 1956):

$$\eta_l^2 = \frac{1}{4n} \cdot \sum_r \frac{d^2}{r} \tag{7}$$

$$\sigma^2 = \frac{1}{4L} \cdot \sum_L \frac{d^2}{L} \tag{8}$$

where  $d$  is the discrepancy of height difference (the difference between forward and backward leveling between each BM);  $r$  is the distance between two BMs (length of the section);  $n$  is the number of the sections and  $L$  is length of the leveling line.

Variance  $m_i^2$  of each height difference (with section length  $r$ ) was then calculated by adding up random- and systematic errors of the corresponding leveling line:

$$m_i^2 = \eta_l^2 r + \sigma^2 r^2 \tag{9}$$

which resulted in weight of the observation  $i$ :

$$w_i = \frac{S_0^2}{m_i^2} \tag{10}$$

where  $S_0^2 = 1 \text{ mm}^2$  is the a priori variance of unit weight.

In order to avoid rank deficiency in the design matrix, the height and the velocity of at least one BM need to be fixed (Kleijer et al., 2002). To avoid unnecessary complications in the calculations, height and the velocity to the same BM (“SR Luidja”, Fig. 5) were fixed in the current adjustment. Moreover, at least two observations at one BM from different time periods are needed for calculation of the velocities.

#### 4.3. Evaluation of adjustment results

For evaluation of the adjustment results, the a posteriori variance of unit weight (also called global variance factor) was first calculated as:

$$S_0^2 = \frac{e^T W e}{df} \tag{11}$$

where  $W$  is weight matrix,  $df$  is the degrees of freedom of the network and:

$$e = B \begin{bmatrix} H^0 \\ v \end{bmatrix} - y \tag{12}$$

is the vector of the residuals obtained from the adjustment. The terms in Eq. (12) were introduced earlier.

The variance of unit weight was compared with the a priori value 1, using a  $\chi^2$ -test at 0.05 level of significance (also known as “goodness of fit” or “global variance” test). Failure on this test indicates outliers in observations or an incorrect weight model.

The estimated mean standard error  $\mu$  of the adjusted velocity values was calculated using the velocity part of diagonal elements of parameter cofactor matrix  $Q_{\alpha\alpha} = [B^T W B]^{-1}$ :

$$\mu = \sqrt{\frac{\text{tr}[B^T W B]^{-1}}{n}} \cdot S_0 \tag{13}$$

where  $n$  is the number of velocity components.

Standard deviation  $S_i$  of velocity of the BM  $i$  was calculated as:

$$S_i = S_0 \sqrt{q_{v_i, v_i}} \tag{14}$$

where  $q_{v_i}$  is the corresponding diagonal element of the velocity component of the matrix  $Q_{v_i}$  and  $S_0$  is standard deviation of unit weight (Eq. (11)).

For detection of outliers, statistical hypothesis testing based on the “data snooping” technique by Baarda (1968) was used. In this test, the normalized residual  $\bar{e}_i$  of the observation  $i$  is computed as:

$$\bar{e}_i = \frac{|e_i|}{\sqrt{q_{ii}}} \quad (15)$$

where  $e_i$  is the observation residual (Eq. (12)) and  $q_{ii}$  is the diagonal element of cofactor matrix  $Q_{vv}$  of the residuals calculated as:

$$Q_{vv} = W^{-1} - B(B^T W B)^{-1} B^T. \quad (16)$$

If the normalized residual rejection criterion  $S_0 \times 3.29$  was exceeded, then the corresponding observation is considered as an outlier with 99.9% probability (Ghilani and Wolf, 2006). The method is based on the assumption that there is only one outlier in the set of observations. Therefore, only one observation with the largest normalized residual should be removed from the set before readjustment. After readjustment, the global variance factor was evaluated again, and the data snooping procedure was repeated until no outliers were detected.

## 5. Results

### 5.1. Outlier detection

In order to examine the effect of the weights, an adjustment with and without a weight matrix (actually the weight matrix is then the unit/identity matrix,  $W = I$ ) was carried out. In the case of the unweighted adjustment, the parameters were estimated using the formula:

$$\begin{bmatrix} H^0 \\ v \end{bmatrix} = [B^T B]^{-1} B^T y. \quad (17)$$

Standard deviation of unit weight  $S_0 = \pm 9.19$  was obtained in the adjustment without the weights and  $S_0 = \pm 4.30$  in the adjustment with the weights. However, even  $S_0$  from the weighted adjustment differs significantly from the a priori value 1 ( $\alpha = 0.05$ ) as was verified by the  $\chi^2$ -test.

Outlier detection by data snooping was carried out in order to explore possible reasons for large value of  $S_0$  of the weighted adjustment. The eight observations, indicated as outliers by this test were located in the city of Pärnu or surroundings; it is known that land in the Pärnu area on the SW coast of Estonia subsides. Three unevenly oscillating BMs in this area were removed from the adjustment by summing up height differences along the leveling line with unstable BMs to preserve the loop configuration. Altogether 11 observations were rejected from the adjustment, after which  $S_0 = \pm 1.89$  was obtained, which still differs from unity. It is possible that it is caused by nonlinear change of some BM's velocities. This nonlinearity was also observed from scatter plots of common height differences, composed for all levelings. Apparently, these uneven movements are induced by the local effects, including the local subsidence and rebound in urbanized areas, unstable wall BMs, etc. In the present study, the exact locations and causes of the nonlinearly changing velocities were not examined more closely. There are some studies done in Estonia in this matter (Kall and Torim, 2003; Lutsar, 1965).

The cause of the large variance factor could also be that chosen weights are too high, i.e. the leveling errors are actually larger than selected a priori estimates. The four outliers that remained in the dataset exceeded the rejection criterion only a little. One deviating observation was on the island of Hiiumaa, where due to fewer observation campaigns, the leveling dataset is smaller than for the rest

of the network. Based on that, it was decided not to remove more observations from the adjustment. Since large standard deviation of unit weight can be caused also by the incorrect a priori weights, evaluation of the stochastic model was carried out next.

### 5.2. Variance component estimation

There are different methods for estimation of variance–covariance components (Helmert, MINQUE, BIQUE, MLA, LSE, IAUE etc.), some of them have been used in land uplift calculations (Mäkinen and Saarinen, 1998; Vestøl, 2006). Some authors have claimed that under certain general restrictions (normality of observations, initial approximate values of the variance–covariance components are proportional to their true values), different statistical methods of variance component estimation will lead to similar numerical results (Crocetto et al., 2000; Fotopoulos et al., 2005; Grodecki, 1997; Yu, 1996). The Helmert method (Grafarend et al., 1980; Wang et al., 2009) and the Iterated Almost Unbiased Estimation (IAUE) method (Bossler, 1984; Lucas, 1985) were used in the present study.

The former provides unbiased and invariant estimates but is very time-consuming, requires lot of computational resources and sometimes produces negative estimates. Advantages of the IAUE method are that it does not produce negative variance component estimates, demands less computational resources and converges much more quickly than other approaches. However, the estimator from IAUE method is not always unbiased (Amiri-Simkooei, 2007; Bähr et al., 2007; Egeltoft, 1992; Fotopoulos et al., 2005).

The observations were divided into eight groups for the evaluation (Table 4). The first iteration was carried out with the a priori weights, assuming that variance factors (VFs) of the groups are 1. Initial estimates for the group's VFs  $f_i$  were obtained. Then the weight matrix of every group was divided by the new VF of the group from the iteration, leading to an improved weight matrix which was used as an input for the next iteration. Iterations were repeated until the adjustment solution converged, i.e. group's VFs converged to unity. The final estimate of variance of unit weight for the group after iterations was:

$$\hat{S}_0^2 = \prod_{i=0}^m f_i^{-2}. \quad (18)$$

The final leveling standard error ( $\text{mm}/\sqrt{\text{km}}$ ) of the group was estimated as:

$$\hat{m} = \sqrt{\hat{S}_0^2 \cdot m^2} \quad (19)$$

where  $m$  is the initial estimate of mean leveling standard error of the group, based on discrepancies of forward and backward height differences of the leveling.

The variance of unit weight of the whole network, as well as the group VFs, approached unity already more or less after the 5th iteration. The VF of the group VIII converged later than VFs of the other groups; moreover it did not converge to unity. The IAUE estimation produced an exceptionally small VF for this group and the Helmert method gave a negative estimate. Tests with the different a priori weight models for this group lead to similar VF. Apparently, the reason for such behavior in this group is the small number of observations, because in general, the larger number of the redundant observations is leading to faster convergence of the estimated variance to unity (Hsu, 1999). Non-stabilizing redundancy number of group VIII was also observed during the iterations by the IAUE method, as the value of redundancy decreased about two times following each iteration. Most likely, the obtained estimate for the standard error of this group (Table 4) is biased due to the small redundancy number and is not a realistic value. Group VIII contain data from hydrostatic leveling and theoretically, this leveling method cannot achieve such small uncertainty estimates.

**Table 4**

Group's variance factors  $f^2$ , final estimates of the variances of the unit weight  $S_0^2$  and leveling errors  $m$  using Helmert and IAUE methods. Group I – precise leveling of Cadastre Department of Ministry of Agriculture of Estonia on mainland Estonia (1933–1943); II – leveling (2nd order) by the same institution in island Saaremaa (1939–1940); III – precise levelings of Soviet Union General Committee of Geodesy and Cartography (GCGC) in 1948 and 1970; IV – precise levelings of Estonian Academy of Sciences (EAS) in 1950–1969; V – precise levelings of EAS in 1970–1996; VI – 2nd order levelings of Soviet Union GCGC in 1981–1983; VII – precise leveling coordinated by Estonian Land Board in 2001–2012; VIII – hydrostatic and hydrodynamic levelings over the straits of Väinameri Sea in 1968, 1974 and 2010. A priori standard error estimates of the levelings are based on random and systematic errors of the leveling lines. A priori error estimate of group VIII is based on doctoral research of Aive Liibusk (2013).

Group/no. of observations	Helmert, estimates after 1st iteration		Helmert, 18th iteration		IAUE, 23th iteration		Leveling standard error (mm/√km)	
	$f^2$	$S_0^2$	$f^2$	$S_0^2$	$f^2$	$S_0^2$	A priori	After Helmert estimation
I, n = 33	6.13404	9.68705	1.00002	9.66244	1.00008	0.33	1.03	
II, n = 9	9.61900	10.68970	1.00000	10.28762	1.00000	0.70	2.28	
III, n = 34	1.82471	1.04389	0.99998	1.04508	0.99994	0.57	0.59	
IV, n = 69	3.83580	3.89271	1.00000	3.89902	1.00000	0.49	0.96	
V, n = 87	4.31584	4.55116	1.00000	4.55927	0.99999	0.47	1.01	
VI, n = 22	2.07743	2.33447	1.00000	2.34825	0.99999	0.66	1.01	
VII, n = 91	1.34398	0.70177	1.00000	0.68177	1.00000	0.18	0.15	
VIII, n = 5	0.69534	–0.00519	1.00000	0.00000	0.21878	2.25	–	
Whole network, n = 350	3.60094	1.00000	–	1.00000	–	–	–	

Statistically significant ( $\alpha = 0.05$ ) differences between the a priori variances and Helmert or IAUE estimates occurred in the groups I, II, IV and V. Estimated variances of unit weight were largest in groups I and II (Table 4). In the other groups (III, VI and VII), the differences between a priori and estimated variances of unit weight were not significant. A reason for the differences may be that used a priori estimates were not based on network geometry, but only on backward and forward leveling discrepancies of individual leveling lines. Estimates from adjustment are based also on network closing errors, i.e. a priori estimates for groups I, II, IV and V were poor approximations as far as the Helmert or IAUE method is concerned. The final adjustment was done with the Helmert weights for groups I–VII, for the group VIII a priori weights were used instead. The mean standard error of velocity differences  $\pm 0.53$  mm/yr and the standard deviation of unit weight  $\pm 1.00$  were obtained. Compared with the first iteration results, BM velocities decreased by  $-0.29$  mm/yr and the standard deviations of velocities by  $-0.25$  mm/yr on average.

## 6. Land uplift model

As only velocity differences are estimable in Eq. (4), they should be referred to at least one known velocity. As explained in Section 2.2, there are large discrepancies between estimated velocities of the Estonian TGs. Eventually, it was decided to choose the apparent velocity  $+2.1$  mm/yr at the Ristna TG, as a reference value since the discrepancy between different studies was smallest there (see explanation in Section 2.2). Apparent velocities with uncertainties of the BMs relative to the Ristna TG from the final adjustment with weights from Helmert estimation (see Section 5.2) are presented in Table 5.

For the compilation of the apparent land uplift model of Estonia, software “Surfer” by Golden Software was used. Two different gridding methods, the “kriging” and the “minimum curvature” (MC) (see Cressie (1993) and Smith and Wessel (1990) for an overview of the methods) were tested. Both methods, MC and kriging are widely used in earth sciences including land uplift modeling (Chen et al., 2011; Dehghani et al., 2009; Lenõtre et al., 1999; Mäkinen and Saaranen, 1998; Perrone et al., 2013). The extent of the modeling area in both cases was  $57.58^\circ$ – $59.46^\circ$  in the north direction and  $22.03^\circ$ – $28.15^\circ$  eastward. Grid resolution chosen was  $5 \times 5$  km.

The purpose of the modeling was to create the surface which would primarily reflect the PGR signal in Estonia. It was decided that the modeled surface should not deviate from the velocities at the observation points more than the mean standard error of the velocities,  $\pm 0.5$  mm/yr, estimated before. The apparent velocity values of 84 BMs were used as input data in surface computation.

Modeling was started with the MC method. To suppress the disturbing effect of the local velocity anomalies, values 0.25 and 0.5

were chosen for the internal tension factor and boundary tension factor, respectively, by the trial and error method.

After a first iteration, it appeared that the modeled surface fits in well with the values at the observation points (min, max, and RMS of the residuals were  $-0.25$ ,  $+0.20$  and  $\pm 0.07$  mm/yr), however several local velocity anomalies were identified in the Pärnu and Tallinn regions (ground subsidence due to groundwater withdrawal), north-east Estonia (subsidence within the mining area), and Hiiumaa (which has a few fundamental BMs, therefore, the velocities are more likely to be affected by near-surface processes). To reduce the effect of the local velocity anomalies, the modeled surface was compared with the so-called expected velocity surface for which the NKG2005LU model was taken. The velocity gradients for both surfaces were calculated. The general direction of the land uplift gradients in Estonian territory according to the NKG2005LU is from northwest to southeast, with the value of  $-0.87$  mm/per 100-km on average. The differences between the expected and observed velocity gradients indicate regions of local velocity anomalies in relation to PGR land uplift. These regions did match with the above-mentioned areas of the local velocity anomalies (Tallinn, Pärnu, etc.). The BMs in which velocities differed from the nearby BMs and affected the gradient difference model at most, were removed from observation points before the next computation step. The total number of removed BMs was 26 (31%). The resulting model is shown in Fig. 6.

Compilation of the kriging model was started again with the whole dataset (84 apparent velocities). Since weights of the variables in kriging depend on how they are correlated in space, then modeling results also depend on how the correlations in input parameters have been estimated. To describe the spatial correlation, the experimental variogram was created. To select parameters of the experimental variogram there are a few rules of thumb: the number of data pairs in the lag should be a minimum of 30; the average nearest neighbor distance can be taken for length of the lag; and the number of lags multiplied by the lag distance should be equal to half the maximum distance between the dataset points (Journel and Huijbregts, 1978). Based on these rules, the average nearest neighbor distance ( $\sim 9$  km) and a half maximum distance between dataset (130 km), the number of lags of the variogram should be 14 to 15.

The velocity field in a small area such as Estonia has the property of being directionally dependent (i.e. anisotropic) due to the effect of PGR. To determine the direction and ratio of the anisotropy, a planar surface ( $z(x, y) = A + Bx + Cy$ ) was fitted to observed velocity values. The general direction of the land uplift contour lines (azimuth angle about  $70^\circ$ ) and the anisotropy ratio (about 1.6) was detected from this surface. To detect the anisotropy of the variogram, it was compiled in different directions, choosing a certain angular tolerance within which the pairs of points were used to form a variogram. Angular tolerance should be

**Table 5**

Apparent velocities of the benchmarks and their standard deviations (both in mm/yr) relative to the Ristna tide gauge (+2.1 mm/yr). F – fundamental benchmark, W – wall benchmark.

Location	Type of the benchmark	Latitude	Longitude	Apparent velocity	Standard deviation of the velocity
Tallinn	F	59°25'	24°47'	1.72	0.49
Tallinn	F	59°25'	24°45'	1.66	0.50
Tapa	F	59°16'	25°58'	1.24	0.49
Jõhvi	F	59°21'	27°25'	0.46	0.67
Mustvee	F	58°51'	26°56'	0.71	0.48
Jõgeva	F	58°45'	26°24'	0.58	0.49
Tartu	F	58°23'	26°44'	-0.02	0.49
Petseri	F	57°50'	27°38'	-0.61	0.54
Mõniste	F	57°35'	26°32'	-0.69	0.50
Valga	F	57°47'	26°02'	-0.35	0.48
Puka	F	58°03'	26°11'	-0.11	0.49
Abja	F	58°08'	25°22'	0.01	0.48
Mõisaküla	F	58°07'	25°12'	0.01	0.48
Põltsamaa	F	58°39'	25°56'	0.47	0.48
Lelle	F	58°52'	25°00'	0.95	0.48
Pärnu	F	58°24'	24°30'	0.00	0.53
Haapsalu	F	58°56'	23°32'	1.11	0.49
Petseri	W	57°50'	27°35'	-0.53	0.54
Petseri	W	57°50'	27°38'	-0.77	0.54
Karuse	W	58°38'	23°41'	0.52	0.48
Orissaare	W	58°34'	23°05'	1.10	0.48
Kuressaare	W	58°15'	22°29'	0.07	0.51
Meiste	W	58°36'	22°36'	1.09	0.48
Ikla	F	57°53'	24°22'	-0.22	0.48
Uulu	F	58°18'	24°36'	0.01	0.48
Sonda	W	59°21'	26°51'	1.51	0.56
Püssi	F	59°22'	27°01'	1.29	0.50
Põllu	F	59°27'	24°52'	1.69	0.49
Koogi	F	59°26'	25°11'	1.86	0.49
Kahala	F	59°28'	25°30'	1.82	0.49
Keila	F	59°18'	24°26'	1.35	0.50
Orissaare	F	58°33'	23°04'	1.10	0.48
Tahula	F	58°17'	22°34'	0.76	0.49
Lümanda	F	58°18'	22°02'	0.93	0.49
Lümanda	F	58°18'	22°02'	0.93	0.49
Võhma	F	58°32'	22°21'	1.14	0.49
Võhma	F	58°32'	22°22'	1.14	0.49
Põllu	F	58°51'	24°59'	0.93	0.48
Sindi	F	58°26'	24°44'	0.07	0.51
Pühalepa	W	58°52'	22°57'	1.71	0.28
Vilupi	W	58°52'	22°55'	1.34	0.27
Nurste	W	58°47'	22°30'	1.59	0.26
Malvaste	W	59°02'	22°36'	1.49	0.24
Kabala	F	58°42'	25°38'	0.63	0.49
Lokuta	F	58°49'	25°22'	0.48	0.50
Käru	F	58°51'	25°04'	1.05	0.50
Jõgeva	F	58°45'	26°24'	0.56	0.49
Tapa	F	59°16'	25°58'	0.81	0.49
Räpina	F	58°05'	27°30'	-0.58	0.82
Paluküla	F	58°59'	22°47'	2.29	0.32
Paluküla	F	58°59'	22°48'	2.24	0.32
Tallinn	F	59°24'	24°44'	1.27	0.51
Pärnu	W	58°19'	24°36'	-0.18	0.47
Muraste	F	59°27'	24°26'	1.70	1.15
Suurupi	F	59°28'	24°23'	1.53	1.13
Tabasalu	F	59°26'	24°33'	1.97	1.20
Kohula	F	59°11'	24°47'	1.29	0.48
Rapla	F	58°59'	24°50'	1.14	0.48
Karuse	F	58°38'	23°41'	0.74	0.49
Viljandi	F	58°21'	25°35'	0.48	0.54
Kauksi	F	59°00'	27°12'	0.88	0.56
Narva	F	59°23'	28°09'	0.98	0.51
Vaeküla	F	59°21'	26°30'	1.42	0.49
Võiste	F	58°12'	24°29'	0.06	0.48
Häädemeeste	F	58°01'	24°28'	-0.12	0.48
Orissaare	F	58°33'	23°04'	1.11	0.48
Triigi	W	58°35'	22°43'	1.15	0.48
Hellamaa	W	58°37'	23°18'	1.05	0.49
Hanila	W	58°37'	23°36'	0.70	0.48
Tallinn	W	59°25'	24°38'	1.36	0.52
Puski	W	58°54'	22°25'	1.95	0.14
Tareste	W	59°01'	22°40'	1.41	0.26

**Table 5 (continued)**

Location	Type of the benchmark	Latitude	Longitude	Apparent velocity	Standard deviation of the velocity
Mändjala	F	58°13'	22°21'	0.44	0.49
Tiirimetsa	W	58°11'	22°12'	0.75	0.49
Mäebe	F	57°58'	22°08'	0.31	0.53
Reigi	W	58°59'	22°31'	2.05	0.35
Ristna	W	58°55'	22°04'	2.40	0.40
Ristna	W	58°55'	22°13'	1.92	0.41
Türi	W	58°48'	25°26'	0.73	0.49
Pilistvere	W	58°40'	25°45'	0.53	0.48
Pärnu	W	58°19'	24°36'	-0.23	0.47
Häädemeeste	W	58°04'	24°29'	-0.12	0.48

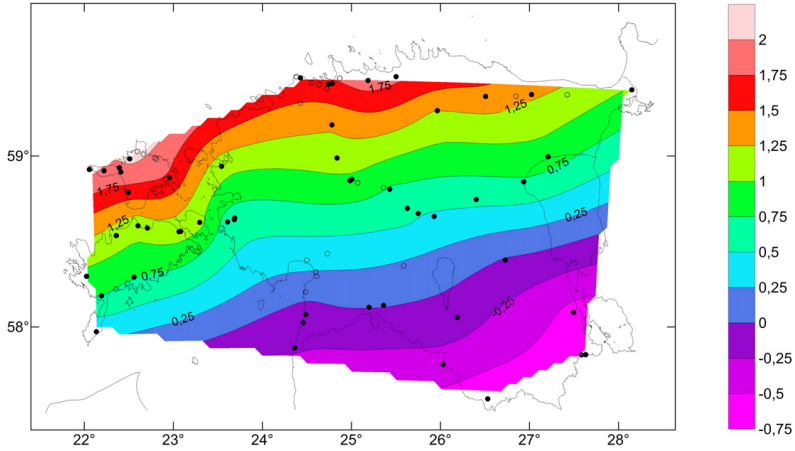
chosen so that there will be a sufficient number of data pairs (min 30) to form a variogram. Small angular tolerance limits the number of points coming from other directions, but in the case of a sparse network, there may not be a sufficient number of data pairs. The result may be an incorrect variogram. On the other hand, large angular tolerance creates a sufficient number of data pairs and a variogram which is easy to model, but also makes heavier to detect the anisotropy.

A good rule for finding a balanced solution for a variogram is to start with smaller angular tolerance and gradually increase it until a sufficient number of data pairs for modeling is obtained. As a result, a value of 34° was selected for the angular tolerance. Next, the separate variograms for four main directions of land uplift in Estonia were compiled (SW–NE, SE–NW, N–S and E–W) in order to detect the exact direction of the lowest variability of apparent vertical movements. The smallest variation appeared in the SW–NE direction (azimuth angle of 68°), corresponding to the anisotropy direction detected on the aforementioned planar model. For compilation of the final experimental variogram, the angular tolerance of 34°, the anisotropy azimuth angle of 68°, the number of lags of 14 and the lag distance of 9.3 km were used. In the case of such parameters the number of data pairs in lags varied from 26 to 101.

For modeling of the experimental variogram, linear function with the slope  $1.108e - 006$  (mm/yr)<sup>2</sup>/m and nugget effect value  $0.04$  (mm/yr)<sup>2</sup> was used. This variogram was then used in gridding with the ordinary kriging method to produce velocity surface which was found to be smoother than the MC solution. Once again, areas where the neighboring BMs have relatively different velocities (especially at Pärnu and Tallinn, as well as in NE Estonia) were distinguished on the kriging model as well. Seven anomalous BMs (8%) in these areas were removed from the dataset, after that a new gridding was performed. The resulting model is shown in Fig. 7.

Comparing two modeled surfaces (Figs. 6 and 7), it can be seen that the kriging produces a smoother surface than MC, although the number of the removed BMs is considerably smaller. The direction of the vertical movement contour lines on both models is generally the same and discrepancies between the surfaces range from  $-0.18$  to  $+0.22$  mm/y with the mean value  $-0.01$  mm/yr and  $RMS \pm 0.07$  mm/yr. Therefore, the statistically insignificant difference between two models can be concluded here considering the average uncertainty  $\pm 0.5$  mm/yr of velocities estimated before. The biggest discrepancies between the two models are in the Hiiumaa Island and the Tallinn and Pärnu areas. Note that the MC model has been compiled with fewer BMs in these areas.

Accuracy of both models has also been evaluated with the cross validation technique. The MC and the kriging models' cross validation estimates for the velocities RMS, standard deviation, and absolute average deviation between the first and last iteration did not differ significantly. However, cross validation estimates of the residuals of the last iteration for kriging as well as for MC methods were vastly improved, since largely deviating BMs have been removed from the final iteration.



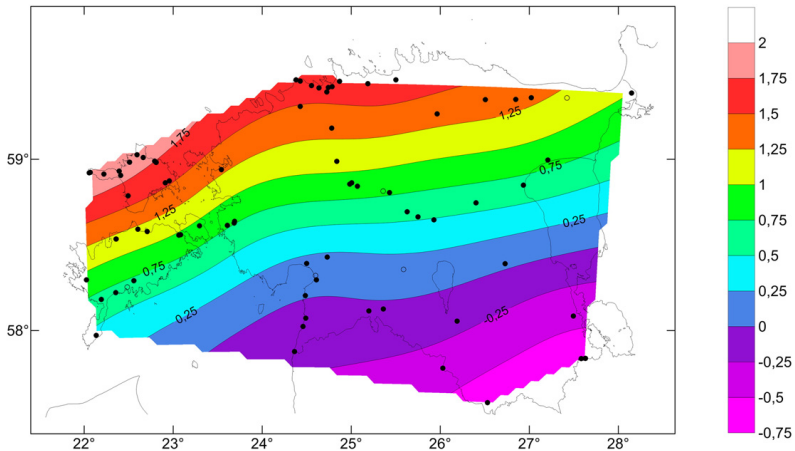
**Fig. 6.** Modeled apparent land uplift in mm/yr based on minimum curvature gridding method. Velocities are computed using repeated levelings, Helmert weights and apparent velocity of benchmark “SR Luidja” fixed at +1.9 mm/yr (see explanation in text). Contour interval is 0.25 mm/yr. Statistics of model residuals: min –0.12, max 0.11, mean 0.00 and RMS  $\pm 0.03$  mm/yr. Model accuracy based on the standard deviation of cross validation is  $\pm 0.76$  mm/yr. Filled circles are benchmarks used in modeling. Open circles are benchmarks that were removed from modeling in the process of iteration.

**7. Comparison with results from other studies**

The model derived by the kriging technique was chosen as reference in following comparison because it produced a smoother surface and the number of removed deviant BMs was smaller. Moreover, no significant difference between the MC and kriging velocity surfaces was noticed. For the sake of comfortable referencing the kriging model was named as EST2013LU. The modeled surfaces of EST2013LU (Fig. 7) and NKG2005LU (Fig. 1) were compared at first. The main discrepancy between the two models is a different direction of the velocity contour lines (Fig. 8). In the EST2013LU model, those are more inclined towards the west–east than in the NKG2005LU model. Also, the contour lines are

shifted relative to each other ( $-0.15$  mm/yr), partially due to the different velocity value at the reference point “SR Luidja”.

In general, the coincidence between the models is good. Differences remain mostly within the limits of the mean standard error for the velocities (Fig. 9). This shows good quality of the NKG2005LU model even in areas where it was compiled without any constraining observations. The differences, however, are of a systematic nature (growing in SW–NE direction) and are caused by different direction of velocity contour lines of the two models. The reasons for these discrepancies require further detailed investigation, in particular with respect to systematic errors in leveling data. Compared with the other well-known land uplift map of Fennoscandia by Ekman (1996), the



**Fig. 7.** The modeled surface of apparent land uplift EST2013LU in mm/yr gridded by kriging method. Velocities are computed using repeated levelings, Helmert weights and apparent velocity of benchmark “SR Luidja” +1.9 mm/yr. Contour interval is 0.25 mm/yr. Statistics of model residuals: min –0.41, max 0.54, mean 0.01 and RMS  $\pm 0.17$  mm/yr. Model accuracy based on the cross validation standard deviation is  $\pm 0.74$  mm/yr. Filled circles are benchmarks used in modeling. Open circles are benchmarks that were removed from modeling in the process of iteration.

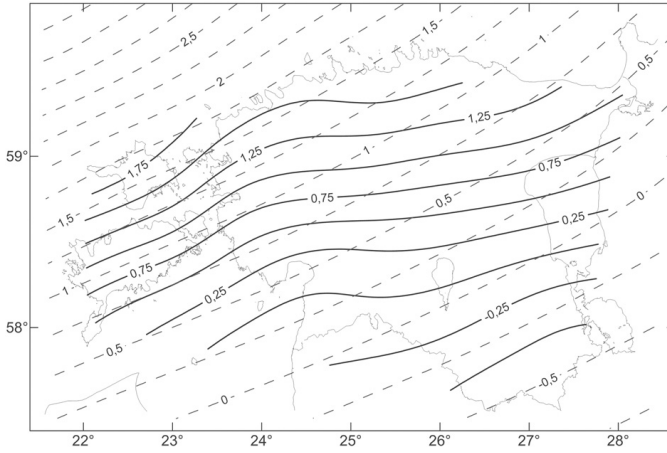


Fig. 8. Contour lines of apparent vertical velocity of EST2013LU model (solid lines) and NKG2005LU model (dash lines). Contour interval on both models is 0.25 mm/yr.

differences are slightly larger (Table 6). The apparent velocity difference at the reference point of adjustment is higher compared to the NKG2005LU model and the differences are increasing in SW–NE direction.

The differences of the EST2013LU model with some of the latest local land uplift maps by Estonian scientists (Randj arv, 1993; Vallner et al., 1988) are larger than with the regional Fennoscandian models (Table 6). Still, considering accuracies of these local solutions, the differences are insignificant. The maps with differences (not presented in the current paper) showed that contour lines of the EST2013LU surface are again more inclined in the east–west direction than in these local models.

In order to compare the EST2013LU model with data from Estonian TGs, velocities from the model were interpolated to the TGs. The differences between the interpolated values and the most relevant

determinations of the Estonian TG velocities (Jevrejeva et al., 2002; Pobedonostsev, 1975; Yakubovski, 1973) (Fig. 10) are of the same order as the differences among the three TG solutions (Table 2). If the velocity at the P aru TG (which is obviously influenced by local subsidence) is not taken into account, then the best fit with the model is with Yakubovski's TG solution (RMS of differences is  $\pm 0.76$  mm/yr).

## 8. Discussion and conclusions

For the first time in Estonia, the kinematic adjustment model and all available precise leveling observations from 1933 to 2010 have been used for estimation of the vertical velocities of the BMs. Earlier maps of vertical crustal movements in Estonia were compiled using data only from two – the first and the last – leveling campaigns. The used

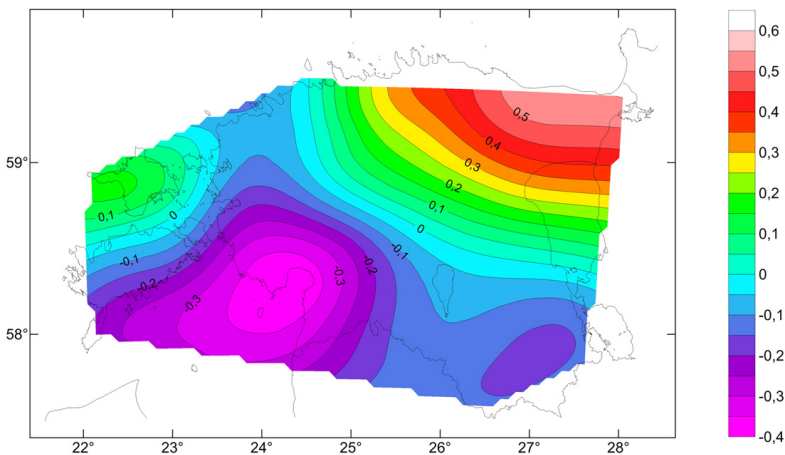


Fig. 9. Differences between EST2013LU model and the NKG2005LU model (EST2013LU minus NKG2005LU). Contour interval is 0.05 mm/yr. Statistics of the differences: min  $-0.39$  mm/yr, max  $0.55$  mm/yr, mean  $-0.02$  mm/yr, RMS  $\pm 0.23$  mm/yr.



**Table 6**  
Differences (mm/yr) between the EST2013LU model and earlier results.

Difference	Min	Max	Average	RMS
EST2013LU–Vallner et al. (1988)	-1.79	0.74	-0.59	±0.58
EST2013LU–Randjärv (1993)	-1.08	1.43	0.03	±0.63
EST2013LU–Tide gauge	-1.25	1.90	0.66	±1.05
EST2013LU–NKG2005LU	-0.39	0.55	0.02	±0.23
EST2013LU–Ekman (1996)	-0.91	0.67	-0.44	±0.43

methodology allowed us to re-weight the dataset, statistically evaluate the results in detail and detect possible outliers among observations.

More or less the same uncertainty values were a priori assumed for all precise leveling data. Only data from 4th leveling indicated two times lower uncertainties. The weight evaluation by variance component estimation using IAUE and Helmert methods proved that height differences of the 1st leveling should be weighted down most (approximately three times). The cause of that needs further investigation. One possible reason – not examined in the present study – is systematic errors in leveling data (rod scaling errors, refraction, etc.). For example, in the first precise leveling of Estonia, uncalibrated invar rods were used, thus the corresponding corrections to observations were not introduced. Also, preliminary observation data of the 4th leveling without rod calibration and refraction corrections were used in the present study. That can also affect the adjustment results – probably heights more than velocities – but since 4th leveling observations have relatively higher weights than other levelings, they can also have a larger influence on velocities. In general, refraction corrections are negligible in Estonia (Vallner, 1978).

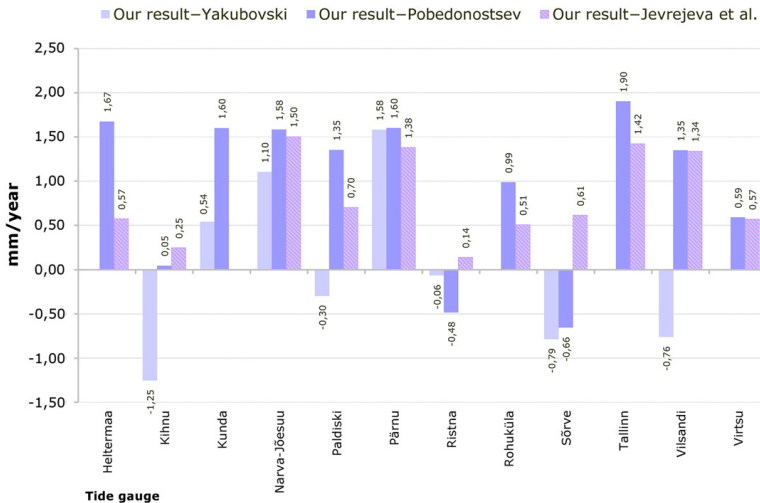
As levelings provide only velocity differences, relative velocities have to be tied to at least one BM with known apparent velocity. The apparent velocity of +1.9 mm/yr at the BM “SR Luidja” on Hiiumaa Island was used to fix the velocity solution in the present study. This value was obtained from Ristna TG velocity +2.1 mm/yr combined with the precise levelings between the TG and the BM. The apparent velocities of the BMs from the kinematic adjustment with the re-scaled weights were used for the compilation of land uplift surface in Estonia. Software

“Surfer” gridding techniques MC and kriging were used. These two different methods led to quite similar results since the RMS of differences between the gridded surfaces was less than ±0.1 mm/yr. The largest discrepancies were noticed in locally unstable areas (Hiiumaa, Pärnu, Tallinn) where a number of BMs were removed as outliers during the computation of velocity surface with MC. Moreover, the statistics from MC and kriging in combination with cross validation did not differ significantly. The uncertainty of kriging solution which was selected as the best model of apparent velocity surface was estimated to be ±0.56 mm/yr based on the uncertainties of velocities from kinematic adjustment and the residuals between the adjusted velocities and modeled surface. Uncertainty ±0.74 mm/yr of the same model was estimated from the cross validation. This model obtained by the kriging method (named as EST2013LU) was chosen for comparison with other results as it gave a smoother surface and the number of removed deviant BMs was smaller.

The land uplift model EST2013LU fits well with regional land uplift model NKG2005LU. The RMS of differences is ±0.23 mm/yr, which is apparently inside the limits of the combined uncertainty of the models. Velocity difference between two models at the location of reference BM “SR Luidja” is ~0.15 mm/yr. The main disagreement between the models is slightly different direction of contour lines of the velocities. Reasons for that should be further investigated, especially for the effects of possible systematic errors in the leveling data.

The new model EST2013LU also fits within the limits of uncertainty with previous land uplift maps of Estonia but does not support some anomalously shaped contour lines presented in historical maps. As in the case of the NKG2005LU model, the direction of velocity contour lines of the EST2013LU are slightly different towards west–east compared to the historical land uplift maps of Estonia. The bias with the map of the Vallner et al. (1988) results also from the velocity difference (~0.53 mm/yr) at the location of the reference BM “SR Luidja”.

Finally, the largest discrepancies appeared from the comparison of EST2013LU surface with the velocities from Estonian TG observations. At some stations, velocities coincide within the limits of uncertainties, but for most of the stations, discrepancies exceed the uncertainties by 2–3 times. At the same time, discrepancies between velocities obtained



**Fig. 10.** Velocities of TGs interpolated from EST2013LU model compared to tide gauge solutions by Yakubovski (1973), Pobedonostsev (1975) and Jevrejeva et al. (2002). Locations of tide gauges are presented in Fig. 2.

from the adjustment and TG solutions are of the same order as the inconsistency between the different TG solutions. Apparently, there are still unresolved problems with Estonian TG observations therefore the apparent velocities based on the data of TGs are not a suitable for the evaluation of velocity models.

### Acknowledgments

This study was supported by the Estonian Science Foundation grant ETF 8749: “Determination of height reference frame on the Estonian coastal sea using water level monitoring and laser scanning data”.

We thank Prof. M. Kasser and Dr. J. Ågren for the useful comments and suggestions which helped to improve the original manuscript considerably.

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**Kall, T.,** Liibusk, A., Wan, J., and Raamat, R., 2016.

VERTICAL CRUSTAL MOVEMENTS IN ESTONIA  
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*Estonian Journal of Earth Sciences*, **65**(1), pp. 27–47, doi:  
10.3176/earth.2016.03.

## Vertical crustal movements in Estonia determined from precise levellings and observations of the level of Lake Peipsi

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Received 19 June 2015, accepted 2 October 2015

**Abstract.** The aim of this study was to evaluate vertical velocities of the benchmarks and their change over time based on the four precise levellings of the Estonian levelling network from 1933 to 2011, with the mean epochs being 1936.7, 1961.2, 1982.1 and 2006.9. The vertical velocities of the benchmarks were estimated using two mathematical models. Both models gave similar results for almost all levelling combinations. Significant discrepancies between the velocities from two models were found only in two combinations where levelling loops' closing time was long compared to the time between the mean epochs of levellings. From the analysis of post-adjustment variances of unit weight and the ANOVA test, a significant change in the benchmark velocities between mean epochs of the levellings was detected. However, due to correlation between the second and third levellings it remained unresolved whether the velocity change was a real change or fortuitous when relying only on this correlation. The detected velocity change could also be explained by the levelling error. Iterated variance component estimation assigned most of the error to the first levelling. In addition, level records of Lake Peipsi from 1921 to 2006 were used for the first time to calculate lake tilt between water gauges. Velocities of the benchmarks from the combination of the last three levellings and water gauges of Lake Peipsi were used to compile the map of the vertical crustal movements (EST2015LU). The main feature of the compiled map was the SE–NW directional postglacial land uplift. However, compared to earlier maps for the region, our isolines declined more in the W–E direction, due to the larger influence of the fourth levelling and velocities from lake tilts. Overall fit of the compiled map with the velocities of continuously operating Global Navigation Satellite System reference stations and coastal tide gauges was  $\pm 0.4$  to  $\pm 0.5$  mm yr<sup>-1</sup>.

**Key words:** vertical crustal movement, levelling, lake tilts, land uplift.

**Abbreviations:** BM – benchmark, CLN – common levelling network, CORS – continuously operating GNSS reference station, GNSS – Global Navigation Satellite System, LSQ – least squares, RMS – root mean square, SA – satellite altimetry, TG – tide gauge, VCM – vertical crustal movement, VV – vertical velocity, WG – water gauge.

### INTRODUCTION

Repeated levelling has remained one of the most precise methods for the determination of vertical movements of the Earth's crust. Many different methods for the calculation of vertical crustal movements (VCM) from repeated levelling data have been developed (Holdahl 1978; Carrera & Vaníček 1986; Hein 1986). Most of the methods were developed between the 1960s and 1980s, when a number of countries finished repeated levellings of their levelling networks. Until now, most of the Estonian levelling network has been levelled at least four times. This provides an excellent opportunity to estimate changes in VCMs over time using different levelling combinations.

The first precise levelling covering all of Estonia was completed between 1933 and 1943. In 1943 the

network consisted of six loops on the mainland and one loop on the Island of Saaremaa, altogether approximately 2000 km of levelling lines and 1300 benchmarks (BMs); 23 nodal points of the network were deep-seated 'fundamental benchmarks'. The second and third levellings were carried out during 1948–1969 and 1970–1996, respectively. The fourth levelling was performed between 2001 and 2012 using digital levelling methodology and shorter sight lengths. More details about the repeated levellings in Estonia are provided by Kall et al. (2014).

The first maps of VCMs of inland Estonia were compiled from the results of the first two levellings (Zhelnin 1958, 1960, 1964, 1966; Randjärv 1968; Vallner & Zhelnin 1975; Vallner 1978). Values of VCMs in Estonia were previously known only from coastal tide gauge (TG) observations (e.g., Witting 1922). Results

from the levellings confirmed that a general trend of land uplift in Estonia occurred in a SE–NW direction and correlated well with VCMs for southern Finland calculated from repeated levellings by Kääriäinen (1953, 1963). When the third levelling data for Estonia became available, the values of vertical velocities (VVs) of the BMs were re-estimated (Vallner et al. 1988; Randjärv 1993). It was concluded that in addition to general postglacial land uplift, differentiated block movements took place. The accuracy of the velocities was estimated to be on average between  $\pm 0.2$  and  $\pm 0.4 \text{ mm yr}^{-1}$  (Vallner & Zhelmin 1975; Vallner et al. 1988; Randjärv 1993).

Water level observations on the coast of Lake Peipsi (Fig. 1), Estonia's largest (3555 km<sup>2</sup>) and the fifth largest lake in Europe, have been conducted by the Estonian Weather Service since 1921. The main purpose of the observations is water management: regulation of outflows, amelioration, building of hydraulic structures, etc. In the present study, water level observations were used for the first time for geodetic purposes to calculate land uplift.

The aim of this study was to estimate VCMs from different levelling combinations as well as from lake level recordings and to evaluate VV changes over time. This paper is organized as follows. The first two sections provide an overview of the levelling data used and uncertainties regarding them. An overview of the calculations used to estimate VVs of the BMs from levelling data is given. The differences between the obtained solutions are presented and analysed. The next two sections discuss the change in VVs over time and evaluate levelling errors based on variance component estimation. Next, an overview of VV calculation based on lake level observations is provided. The last two sections are devoted to the compilation and evaluation of the VCM models for Estonia.

## LEVELLING OBSERVATIONS AND STATIC ADJUSTMENT OF THE NETWORK

To determine temporal changes in VVs of the BMs, a common levelling network (CLN) based on the levelling lines and BMs common to all four levellings (1933–1943, 1948–1969, 1970–1996 and 2001–2012; mean epochs 1936.7, 1961.2, 1982.1 and 2006.9, respectively) was created (Fig. 1).

Compared to the so-called maximum network, where all levelling lines of a network can be used, including lines levelled just one time, the CLN has some advantages: (i) estimation of the VVs from two different mathematical models (Eqs (4) and (5)), (ii) estimation of

the change in the VVs of BMs over time (Eqs (9)–(11)), (iii) determination of the between-epoch correlation of the repeated levellings (Eqs (12)–(20)). Levelling observations without gravity correction were used in our study. However, the small systematic error introduced by ignoring gravity anomalies will be cancelled from VV calculations when height differences are differentiated in the completely re-levelled network and re-levelled segments follow close paths (Brown & Oliver 1976; Vaniček 1976; Holdahl 1978; Vaniček et al. 1980; Carrera et al. 1991). This was the case in our CLN.

In order to be a 'true' CLN, height differences between BMs and the nodal points of the network should be common to all levellings (Mäkinen & Saaranen 1998). It was necessary sometimes to sum adjacent height differences from different observation epochs to achieve closed network loops. In such cases the observation epoch of the summarized height difference was obtained as the weighted average where the section length was used as weight. The effect of epoch averages on adjustment results was negligible, since variation between the observation epochs of the adjacent height differences was small compared to the time-lag between the repeated levelling campaigns.

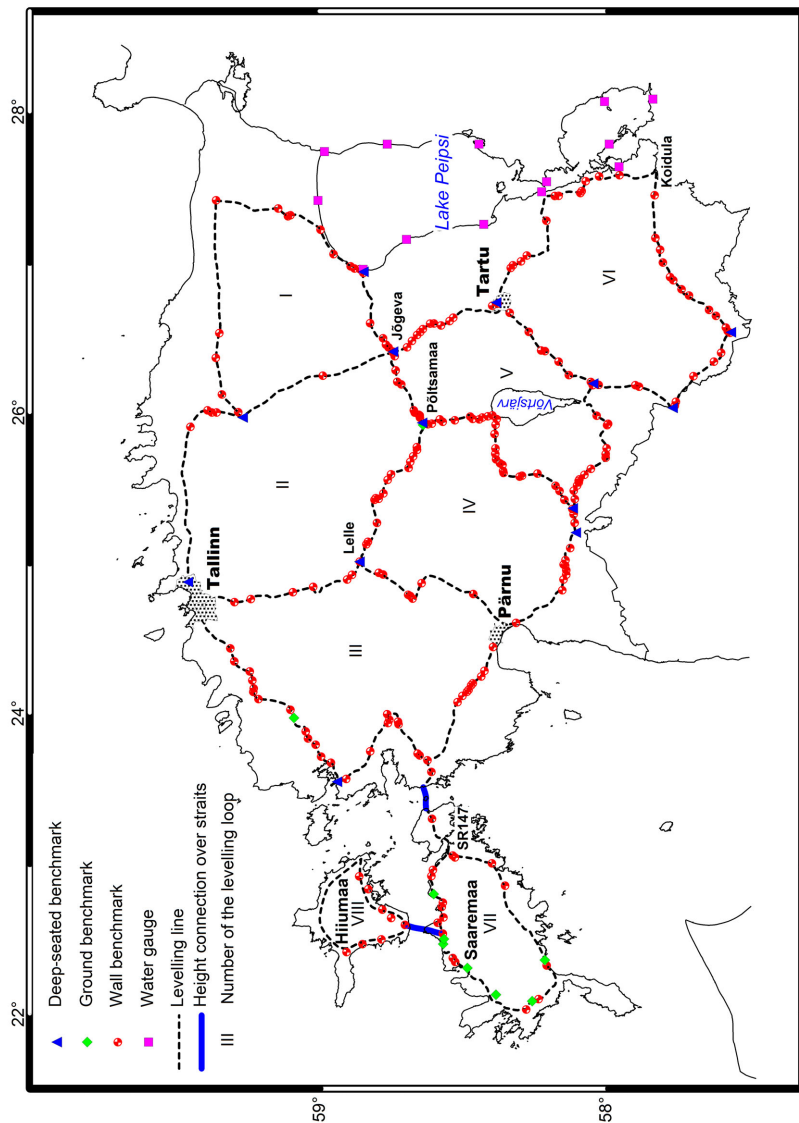
For each levelling line in the network, comparative graphs of the relative cumulative VVs of the BMs along the levelling line and a time series of section height differences were composed. Based on the graphs, as many as possible anomalously behaving BMs (peaks, slope or sign changes of the VVs on the graph (Giménez et al. 2000)) were removed from the network. Generally, such BMs had been mounted in unstable buildings, on unstable ground, or were located in areas of known local VV anomalies (cities of Tallinn, Pärnu and Tartu, and East Estonia, Fig. 1). Velocity anomalies in those areas have been associated with fluctuations in groundwater level, intensive groundwater consumption or oil shale mining (Lutsar et al. 1973; Mets et al. 2000; Kall & Torim 2003; Rüdja 2004; Kalm 2007).

To determine the weights  $w$  of the height differences, misclosures of the levelling loops  $c$  were used (Table 1).

*A priori* levelling standard error estimates were calculated for every levelling campaign using the equation

$$\tau = \sqrt{\frac{1}{n} \cdot \frac{\sum_{i=1}^n c_i^2}{P_i}}, \quad (1)$$

where  $n$  is the number of loops,  $c_i$  is the misclosure of the levelling loop (mm) and  $P_i$  is the perimeter of the levelling loop (km). *A priori* levelling standard error estimates  $\tau$  based on the loops' misclosures in Table 1 are presented in Table 5.



**Fig. 1.** The common levelling network based on the four precise levellings of Estonia with the mean epochs 1936.7, 1961.2, 1982.1 and 2006.9 and water gauges on the coast of Lake Peipsi used in the current study. Since the network of the first levelling campaign (1933–1943) did not extend to the Island of Hiiumaa, the loop there was based on the common benchmarks (BMs) of the last three levellings. The total length of the levelling lines was 2190 km (2000 km in the first levelling), the number of BMs was 249 (241) and the number of sections was 255 (246). The number of WCGs was 13.

**Table 1.** Misclosures of the levelling loops of the common levelling network. A loop was not established on the Island of Hiiumaa (loop VIII, Fig. 1) during the first levelling campaign

Levelling loop	Loop perimeter $P$ (km)	Loop misclosure $c$ (mm)/max time difference of the closing of the loop (years)			
		Levelling campaign			
		1	2	3	4
I	336.60	-7.14/1	-6.47/13	12.92/18	3.97/0
II	400.94	-0.16/6	-8.07/21	-8.90/22	2.66/7
III	455.41	-17.07/3	-11.42/25	*61.55/16	-2.06/1
IV	396.73	13.28/2	6.29/13	-28.15/12	-4.91/1
V	336.96	0.15/3	1.06/14	30.30/10	5.87/1
VI	387.75	2.20/4	15.00/3	-18.00/7	-3.54/3
VII	210.35	**50.70/1	1.02/0	-0.03/0	1.63/0
VIII	132.89	-	4.72/0	-26.52/0	1.18/2

\* The large closing error might be related to the contribution of land uplift, considering the loop’s closing time and the fact that this loop is located in the area of the highest rate of land uplift in Estonia. For example, after implementing land uplift correction, the closing error of this loop was reduced down to ~32 mm, depending on the VVs used for corrections.

\*\* The levelling loop on the Island of Saaremaa (loop VII) was levelled with third order precision (Vaniček et al. 1980). Therefore, the misclosure of this loop is relatively bigger in terms of Vaniček et al. (1980) compared to other loops within the same levelling campaign.

Variances of height differences between BMs were calculated by using the formula

$$m_i^2 = \tau_i^2 L, \tag{2}$$

where  $L$  is the length of the section (km).

Variances of the observations were used to compose the covariance matrix  $\Sigma$  of the observations. Weight matrix  $\mathbf{W}$  is related to observations error vector  $\mathbf{e}$ , covariance matrix of observations  $\Sigma$  and cofactor matrix  $\mathbf{Q}$  of observations as follows (Ghilani & Wolf 2006, pp. 159–172):

$$\Sigma = E(\mathbf{e}\mathbf{e}^T) = \sigma_0^2 \mathbf{W}^{-1} = \sigma_0^2 \mathbf{Q}, \tag{3}$$

where  $\sigma_0^2$  is the *a priori* variance of unit weight, 1.

Least squares (LSQ) adjustment (with minimum constraints) of the observations of each levelling campaign was then performed separately and *a posteriori* variances of unit weight was estimated using a  $\chi^2$ -test. Post-adjustment variance of unit weight estimations did not differ significantly from the *a priori* value of 1 ( $\alpha = 0.05$ ). Consequently, the weights used in the adjustments were internally correct as far as the individual levelling campaign adjustments were concerned. In the case of the kinematic adjustment of two or more levelling combinations, *a posteriori* variance of unit

weight depends also on the relationships between the weights of the observations of separate levelling campaigns.

### MATHEMATICAL MODELS OF THE KINEMATIC ADJUSTMENT OF THE LEVELLING NETWORK

According to the Gauss–Markov model  $\mathbf{Y} = \mathbf{A}\mathbf{X} + \mathbf{e}$ , an observation equation that relates observations (height differences and corresponding levelling epochs) with the heights and VVs of BMs (‘heights included’ model) in the case of four levellings, is

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{pmatrix} = \begin{pmatrix} \mathbf{A}_1 & \mathbf{T}_1 \mathbf{A}_1 \\ \mathbf{A}_2 & \mathbf{T}_2 \mathbf{A}_2 \\ \mathbf{A}_3 & \mathbf{T}_3 \mathbf{A}_3 \\ \mathbf{A}_4 & \mathbf{T}_4 \mathbf{A}_4 \end{pmatrix} \begin{pmatrix} \mathbf{H} \\ \mathbf{v} \end{pmatrix} + \begin{pmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \\ \mathbf{e}_4 \end{pmatrix}, \tag{4}$$

where  $\mathbf{A}$  is the design matrix of the levelling network;  $\mathbf{H}$  is the vector of the unknown heights of the BMs at the arbitrary chosen reference epoch  $t_0$ ;  $\mathbf{v}$  is the vector of the unknown VVs of the BMs;  $\mathbf{y}$  is the vector of the levelling observations (height differences);  $\mathbf{e}$  is



the error vector of the levelling observations and  $\mathbf{T} = \text{diag}(t_1 - t_0, \dots, t_m - t_0)$  is the diagonal matrix of the levelling epochs.

All four levellings in the CLN had the same design matrix, i.e.  $\mathbf{A}_1 = \mathbf{A}_2 = \mathbf{A}_3 = \mathbf{A}_4 = \mathbf{A}$ . In such a network it is possible to eliminate unnecessary parameters by calculating the differences between the observed height differences of the same BMs. As a result, a ‘heights eliminated’ model, where only VVs are parameterized, is obtained (Mäkinen & Saaranen 1998; Mäkinen 2002):

$$\begin{pmatrix} (\mathbf{y}_2 - \mathbf{y}_1) \\ (\mathbf{y}_3 - \mathbf{y}_2) \\ (\mathbf{y}_4 - \mathbf{y}_3) \end{pmatrix} = \begin{pmatrix} (\mathbf{T}_2 - \mathbf{T}_1)\mathbf{A} \\ (\mathbf{T}_3 - \mathbf{T}_2)\mathbf{A} \\ (\mathbf{T}_4 - \mathbf{T}_3)\mathbf{A} \end{pmatrix} \mathbf{v} + \begin{pmatrix} (\mathbf{e}_2 - \mathbf{e}_1) \\ (\mathbf{e}_3 - \mathbf{e}_2) \\ (\mathbf{e}_4 - \mathbf{e}_3) \end{pmatrix}. \quad (5)$$

By differentiating observations, the new observations, which are basically differences of VVs between the BMs, are obtained. They are correlated through the second and third levelling datasets:

$$\begin{pmatrix} (\mathbf{e}_2 - \mathbf{e}_1) \\ (\mathbf{e}_3 - \mathbf{e}_2) \\ (\mathbf{e}_4 - \mathbf{e}_3) \end{pmatrix} = \sigma_0^2 \begin{pmatrix} \mathbf{Q}_1 + \mathbf{Q}_2 & -\mathbf{Q}_2 & 0 \\ -\mathbf{Q}_2 & \mathbf{Q}_2 + \mathbf{Q}_3 & -\mathbf{Q}_3 \\ 0 & -\mathbf{Q}_3 & \mathbf{Q}_3 + \mathbf{Q}_4 \end{pmatrix}. \quad (6)$$

Gauss–Markov estimates of the VVs from the ‘heights eliminated’ model are the best linear unbiased estimators. However, compared to the ‘heights included’ model it provides only linear unbiased estimators, since the velocity information contained in the loops’ misclosures is lost when levelling observation differences are formed (Cross et al. 1987; Mäkinen 2002). Different estimates for the VVs from the ‘heights included’ and ‘heights eliminated’ models might therefore be expected. It is theoretically possible that if loop misclosure contains some systematic errors, they remain in the parameters of the ‘heights included’ model. However, such systematic errors are eliminated when applying the ‘heights eliminated’ model. Therefore, systematic errors can also be the reason for the different VV estimates. Mäkinen & Saaranen (1998) have shown that VVs from the two different models do not differ in practice. Only the error estimates of the parameters can be different.

### Results of the common levelling network adjustment

As shown in Table 2, it was possible to form eleven combinations of the four levellings to calculate VVs of the CLN BMs (Fig. 1). In addition, VVs were calculated using both the ‘heights included’ and ‘heights eliminated’ models. Since details of all adjustments would take

**Table 2.** The main statistics of the kinematic adjustment of the common levelling network, grouped by the adjustment model and levelling combinations: degrees of freedom  $df$ , variances of unit weight  $S_0^2$  and the corresponding  $p$ -values of the null-hypothesis  $S_0^2 = \sigma_0^2$ , comparison of variances of unit weight  $S_0^2$  and mean standard errors  $\mu$  of the adjusted vertical velocity differences from the ‘heights included’ and ‘heights eliminated’ models, with  $p$ -values for the null-hypotheses  $S_0^2/S_0^2 = 1$  and  $\mu_1^2/\mu_2^2 = 1$ . Null hypotheses were rejected if  $p < 0.05$

Statistic	Levelling combination											
	1–2–3–4	1–2–3	1–2–4	1–3–4	2–3–4	1–2	1–3	1–4	2–3	2–4	3–4	
‘heights included’	$df$	504	258	258	258	269	12	12	12	14	14	14
	$S_0^2$	10.078	7.959	16.364	4.768	3.192	1.614	0.905	1.222	0.672	1.227	1.055
	$p$ -value	0.000	0.000	0.000	0.000	0.000	0.080	0.541	0.198	0.804	0.192	0.322
	$\mu_1$	0.312	1.424	0.400	0.459	0.196	0.939	0.538	0.245	4.774	0.123	0.711
‘heights eliminated’	$df$	498	252	252	252	262	6	6	6	7	7	7
	$S_0^2$	10.185	8.108	16.726	4.848	3.248	1.025	1.060	1.129	0.824	1.345	0.799
	$p$ -value	0.000	0.000	0.000	0.000	0.000	0.292	0.384	0.238	0.567	0.224	0.471
	$\mu_2$	0.314	1.549	0.405	0.463	0.198	0.853	0.585	0.236	5.310	0.128	0.622
$F = S_0^2/S_0^2$	1.011	1.019	1.022	1.017	1.018	1.575	1.171	1.083	1.226	1.097	1.321	
$p$ -value	0.453	0.441	0.431	0.447	0.443	0.323	0.383	0.499	0.352	0.419	0.384	
$F = \mu_1^2/\mu_2^2$	1.006	1.088	1.012	1.009	1.010	1.101	1.088	1.040	1.112	1.048	1.144	
$p$ -value	0.473	0.251	0.463	0.471	0.469	0.493	0.422	0.519	0.408	0.446	0.460	

up much space, only discrepancies in VVs between the two models and two error estimates of the kinematic adjustments in the form of (i) *a posteriori* variance of unit weight  $S_0^2$  and (ii) mean standard error of the adjusted VV differences  $\mu$  are presented. Variance of unit weight  $S_0^2$  was calculated by the formula

$$S_0^2 = \frac{\mathbf{e}^T \mathbf{W} \mathbf{e}}{df}, \quad (7)$$

where  $\mathbf{W}$  is the weight matrix,  $df$  is the degrees of freedom of the network and  $\mathbf{e}$  is the vector of the residuals obtained from the adjustment. The mean standard error of the adjusted VV differences,  $\mu$ , was obtained by the formula

$$\mu = \sqrt{\frac{tr[\mathbf{Q}_{xx}]}{n}} \cdot S_0, \quad (8)$$

where  $\mathbf{Q}_{xx}$  is the parameter cofactor matrix and  $n$  is the number of velocity components.

A  $\chi^2$ -test was performed to test the closeness of  $S_0^2$  to the *a priori* value  $\sigma_0^2 = 1$ . The comparison of  $S_0^2$  and  $\mu$  from the ‘heights included’ and ‘heights eliminated’ models was performed using an *F*-test. The results of statistical evaluation of the post-adjustment statistics are presented in Table 2.

*Dependence of the standard error of the vertical velocity differences on the levelling combination used*

According to the  $\chi^2$ -test, *a posteriori* variances of unit weight ( $S_0^2$ ) from models containing three and four levelling combinations differed significantly from an *a priori* value of 1 ( $p < 0.05$ ), independently of the model used (Table 2). The fact that the  $S_0^2$  of two levelling combinations did not differ from unity but all three and four levelling combinations did indicated that the VVs of the BMs between levelling periods were uneven, observations contained errors or weight matrices of the observations did not fit together.

The mean standard errors ( $\mu$ ) of the calculated VV differences were strongly influenced by including the fourth levelling into the adjustment. This was probably related to the weights of the fourth levelling being approximately two times higher than the others. Additionally, leaving out the middle levelling from a combination of three did not influence error estimates significantly (e.g., 2–3–4 compared with 2–4). Mäkinen & Saaranen (1998) explained this effect using simple linear regression as an example, where leaving out an observation in the middle of abscissa values does not significantly influence the value of the slope.

The standard error of the VV differences was also influenced by the time period between levellings. A longer time period between levelling central epochs contributed to smaller standard errors (e.g., combinations 1–3 compared with 1–2 or 2–3, 2–4 compared with 2–3 or 3–4). The large standard error in the combination 2–3 was related to the very short time period (1 year) between the repeated hydrostatic levellings connecting the Islands of Saaremaa and Hiiumaa (Fig. 1), plus the large discrepancy (10.2 mm) between the height differences. The adjustment of the same levelling combination for the mainland part of the CLN plus loop on the Island of Saaremaa gave an approximately four times smaller mean standard error. When adjustment was performed only with the mainland part of the CLN, the mean standard error of the VV differences reduced further two times. This indicates that the height connections of the Island of Saaremaa with the mainland or the levelling loop on Saaremaa also affect error estimates in the levelling combination 2–3.

*Dependence of the standard error of the vertical velocity differences on the type of the model used*

Differences of  $S_0^2$  between the ‘heights included’ and ‘heights eliminated’ models can be related to (i) differentiating the observations in loops with the same loops’ misclosure signs and (ii) solution of the ‘heights eliminated’ model of two levelling combinations being independent of weights (Mäkinen & Saaranen 1998). However, this was not the case in our CLN, where differences of  $S_0^2$  between the ‘heights included’ and ‘heights eliminated’ models were insignificant ( $p < 0.05$ ) in all levelling combinations (Table 2). Differences between mean standard error  $\mu$  from the ‘heights included’ and ‘heights eliminated’ models were also insignificant. Differences between the ratios  $S_0^2/S_0^2$  of different levelling combinations were most likely caused by the differences between the ratios of observation weights for the levellings involved. The solution for two levelling combinations in the ‘heights eliminated’ model is not influenced by this weight ratio, whereas the solution from the ‘heights included’ model is the best linear unbiased estimator only when the ratio of weights from the two levellings is correct (Mäkinen & Saaranen 1998). This difference, however, only concerns the error estimates; the VVs from the two models were identical for the majority of levelling combinations.

*Dependence of vertical velocity estimates on the model used*

As mentioned above, for most levelling combinations no significant differences existed between VVs between

the ‘heights included’ and ‘heights eliminated’ models. Indeed, velocity estimates from the two solutions should coincide if the levelling loops were closed in a short time period and the reference epoch for the adjustment  $t_0$  was chosen so that the levellings were performed symmetrically with respect to it (Cross et al. 1987). Significant VV differences between the ‘heights included’ and ‘heights eliminated’ models were obtained only for the combinations 1–2–3 and 1–2. Large VV differences (although not statistically significant) were also obtained from the combinations 2–3 and 3–4. The transformation of the observations to the mean for each levelling period epoch eliminated the differences between the VVs of the ‘heights included’ and ‘heights eliminated’ models, showing that differences were caused by a longer time period of the closing of the loops during the second and third levellings compared to the first and fourth levellings (Table 1).

### CHANGE IN THE VERTICAL VELOCITIES OVER TIME

Based on the differences between VVs from the different levelling combinations, the sum of the squared observation residuals and degrees of freedom of the kinematic adjustment, it is possible to evaluate the significance of the VV changes between mean levelling epochs using an ANOVA test (Kakkuri & Vermeer 1985; Mäkinen & Saaranen 1998).

To reckon with a correlation between the VVs (for example, velocities from the combinations 1–2 and 2–3 are correlated through the observations of the second levelling), the ‘heights eliminated’ model Eq. (5) was rewritten so that it was possible to obtain three VVs for each BM from a single adjustment:

$$\begin{pmatrix} (y_2 - y_1) \\ (y_3 - y_2) \\ (y_4 - y_3) \end{pmatrix} = \begin{pmatrix} (T_2 - T_1)A \\ (T_3 - T_2)A \\ (T_4 - T_3)A \end{pmatrix} \begin{pmatrix} v_{1-2} \\ v_{2-3} \\ v_{3-4} \end{pmatrix} + \begin{pmatrix} (e_2 - e_1) \\ (e_3 - e_2) \\ (e_4 - e_3) \end{pmatrix}, \quad (9)$$

where  $v_{1-2}$ ,  $v_{2-3}$ ,  $v_{3-4}$  are the VVs of the BMs between the first and second, second and third, and third and fourth mean levelling epochs, respectively. The remaining terms were introduced earlier.

The model (Eq. (9)) established the reference for the evaluation of VV change between the mean levelling epochs where the correlation through the second and third levellings is automatically taken into account. In order to test the hypothesis that the VVs of the BMs were constant in time (i.e.,  $v_{2-3} - v_{1-2} = v_{3-4} - v_{2-3} = v_{3-4} - v_{1-2} = 0$ ), VV differences ( $v_k - v_j$ ) were found. The mean square of VV differences is

$$S_{dv_j}^2 = \frac{\sum_1^n (v_k - v_j)^2}{n}, \quad (10)$$

where  $j = 1-2$ ,  $2-3$  and  $k = 2-3$ ,  $3-4$  are levelling combinations and  $n$  is the number of BMs. Based on the mean square values for both the VV differences from Eq. (10) and the residuals from Eq. (9) found in column 4 in Table 3, we obtained the  $F$ -statistic as the quotient between the two, as listed in column five. By comparing this value with the critical value  $F_{crit}$ , we found the  $p$ -values listed in column six.

The difference between the VVs from levellings 1–2 and 3–4 was not statistically significant, whereas VVs from the 1–2 and 2–3 levellings, as well as the 2–3 and 3–4 levellings, were significantly different ( $p < 0.05$ , Table 3). The results suggest that there was a significant change in VVs between the second and third levellings. The ANOVA confirmed no significant changes in VV between the levelling pairs 1–2 and 2–4, and 1–3 and 3–4. Similar ANOVA results were also obtained for the VVs from the adjustment of the whole CLN, i.e., height differences from the straits between the mainland and islands and loops on the islands did not influence the results of the analysis.

Even though the performed ANOVA test (Table 3) is invariant to the overall scaling of the *a priori* weights, it is sensitive to the ratios of the weights of separate levellings. In order to verify that the difference between the VVs  $v_{3-4}$  and  $v_{1-2}$  was not statistically significant, we did a new test using a model less sensitive to the *a priori* weight ratio. The model for the evaluation of

**Table 3.** The results of the ANOVA testing the hypothesis that differences of the vertical velocities (VVs) from Eq. (9) were zero, i.e., the VVs of the BMs were constant in time. The results are given only for the mainland part of the common levelling network

Source	Sum of squares	Degrees of freedom	Mean square	$F$ -statistic	$p$ -value
$v_{2-3} - v_{1-2}$	376.759	215	1.752		
Residual	8.073	15	0.538		
ANOVA				3.256	0.0060
$v_{3-4} - v_{2-3}$	295.796	215	1.376		
Residual	8.073	15	0.538		
ANOVA				2.556	0.0200
$v_{3-4} - v_{1-2}$	135.642	215	0.631		
Residual	8.073	15	0.538		
ANOVA				1.172	0.3800

VV differences  $v_{3-4} - v_{1-2}$  was obtained by subtracting the first from the third row in Eq. (9) (Mäkinen & Saaranen 1998):

$$\begin{aligned} (T_4 - T_3)^{-1}(y_4 - y_3) - (T_2 - T_1)^{-1}(y_2 - y_1) &= A(v_{3-4} - v_{1-2}) \\ + (T_4 - T_3)^{-1}(e_4 - e_3) - (T_2 - T_1)^{-1}(e_2 - e_1). \end{aligned} \quad (11)$$

Based on the VV differences from the solution of Eq. (11), the hypothesis that  $v_{3-4} - v_{1-2} = 0$  was tested using an ANOVA (see Table 4 for the results).

Though the alternative hypothesis  $v_{3-4} - v_{1-2} \neq 0$  was not proven previously (see Table 3), in Table 4 the difference between the VVs  $v_{3-4}$  and  $v_{1-2}$  was also statistically significant ( $p < 0.05$ ). The failure to reject the null hypothesis in Table 3 was probably related to the non-compatible weight ratios of the levellings used. Compared to the results in Table 3, the mean square of the residuals in Table 4 is now essentially an uncertainty estimate related to the misclosure of the VV differences on the left side of Eq. (11).

Although the ANOVA test for velocity change was performed before the outlier detection test, in our opinion, it did not affect results significantly. Change in

the VVs of the BMs after removing outliers was small ( $-0.04 \pm 0.14 \text{ mm yr}^{-1}$  on average) compared to the VVs before the outlier test. This change can be considered insignificant relative to the standard deviation of the VVs from Eq. (9) ( $\pm 0.6 \text{ mm yr}^{-1}$ ). We also performed an ANOVA test in which detected outliers were not removed from the dataset but were weighted down using the ‘Danish method’ (Krarup et al. 1980). Conclusions about the change in the VVs of the BMs from this test were the same.

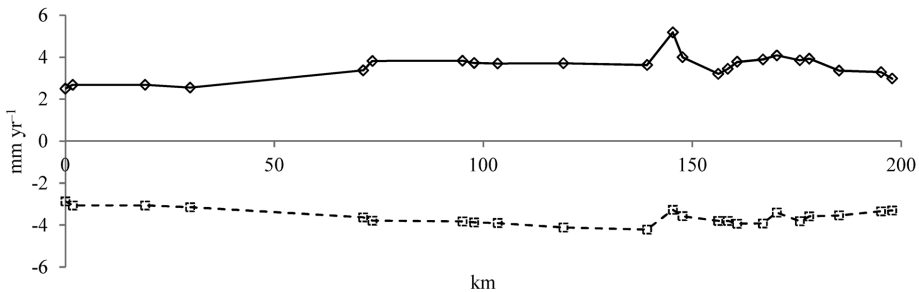
As an example of changes in VVs, the profile of changes  $v_{2-3} - v_{1-2}$  and  $v_{3-4} - v_{2-3}$  in VVs solved from Eq. (9) for levelling loop VII in Saaremaa (Fig. 1) is presented in Fig. 2. The VV changes are relative to BM FR241 in Tallinn. The changes in VVs are far away from zero as was *a priori* hypothesized. Also, changes in VVs from different combinations are not close to each other. Opposite signs of the VV changes  $v_{2-3} - v_{1-2}$  and  $v_{3-4} - v_{2-3}$  suggest systematic levelling errors, most likely in the second or third levelling.

**Correlations between the vertical velocities and velocity changes**

**Table 4.** Results of the ANOVA testing the hypothesis that differences of the vertical velocities  $v_{3-4} - v_{1-2}$  from Eq. (11) equalled zero. The test included only the mainland part of the common levelling network

Source	Sum of squares	Degrees of freedom	Mean square	F-statistic	p-value
$v_{3-4} - v_{1-2}$	261.415	215	1.216		
Residual	0.474	5	0.095		
ANOVA				12.838	0.0040

Conclusions about the change in VVs with time (for our results see Tables 3 and 4) depend on the correlation between the levellings (Mäkinen & Saaranen 1998). The correlation can be a real relationship (for example, the same equipment was used in different levelling campaigns, thus, these levellings may share the same source of possible systematic errors) or just accidental, fortuitous. *A priori*, it was assumed that repeated levellings of the same levelling line have been performed independently from each other. In order to find out whether correlation existed between repeated levellings,



**Fig. 2.** Profile of changes  $v_{2-3} - v_{1-2}$  (solid line) and  $v_{3-4} - v_{2-3}$  (dashed line) in apparent VVs of the benchmarks (BMs) solved from Eq. (9) along levelling loop VII in Saaremaa starting from the wall BM SR147 (Fig. 1). Here  $v_{1-2}$ ,  $v_{2-3}$  and  $v_{3-4}$  are the average VVs between levellings 1 and 2, 2 and 3, and 3 and 4, respectively. Changes in the VVs are relative to BM FR241 in Tallinn.

multivariate analysis of the four levellings was carried out according to the algorithm by Mäkinen & Saaranen (1998).

- (i) The height differences were transformed to a central epoch  $\bar{t}_k$  of its levelling campaign  $k = 1, 2, 3, 4$  using VVs  $\bar{\mathbf{h}}$  from the kinematic adjustment of four levellings after the removal of the outliers and re-scaled weights:

$$\bar{\mathbf{y}}_k = \mathbf{y}_k + [(\bar{t}_k - t_0)\mathbf{I} - \mathbf{T}_k]\mathbf{A}\bar{\mathbf{h}}. \quad (12)$$

- (ii) The same lengths for all levellings were used:

$$\mathbf{G} = \text{diag}[\dots, (d_1(i, j)d_2(i, j)d_3(i, j)d_4(i, j))^{\frac{1}{4}}, \dots]. \quad (13)$$

- (iii) From the solution of the multivariate regression equation

$$(\bar{\mathbf{y}}_1 \ \bar{\mathbf{y}}_2 \ \bar{\mathbf{y}}_3 \ \bar{\mathbf{y}}_4) = \mathbf{A}(\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2 \ \bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4) + (\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3 \ \mathbf{e}_4) \quad (14)$$

a between-epochs variance matrix  $\mathbf{D}$  and the corresponding correlation matrix  $\mathbf{\Pi}$  can be obtained.

- (iv)  $\mathbf{D}$  was estimated by the covariances between the residuals:

$$(\bar{\mathbf{h}}_1 \ \bar{\mathbf{h}}_2 \ \bar{\mathbf{h}}_3 \ \bar{\mathbf{h}}_4) = (\mathbf{A}^T \mathbf{G}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{G}^{-1} (\bar{\mathbf{y}}_1 \ \bar{\mathbf{y}}_2 \ \bar{\mathbf{y}}_3 \ \bar{\mathbf{y}}_4). \quad (15)$$

$$(\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3 \ \mathbf{e}_4) = (\mathbf{I} - \mathbf{A}(\mathbf{A}^T \mathbf{G}^{-1} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{G}^{-1}) (\bar{\mathbf{y}}_1 \ \bar{\mathbf{y}}_2 \ \bar{\mathbf{y}}_3 \ \bar{\mathbf{y}}_4), \quad (16)$$

$$\mathbf{R}_0 = (\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3 \ \mathbf{e}_4)^T \mathbf{G}^{-1} (\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{e}_3 \ \mathbf{e}_4), \quad (17)$$

$$\mathbf{D} = n^{-1} \mathbf{R}_0. \quad (18)$$

- (v) To obtain the correlation matrix  $\mathbf{\Pi}$ , the vector of the standard deviations of the VVs  $\mathbf{S}$  was calculated from the variance matrix  $\mathbf{D}$ :

$$\mathbf{S} = \sqrt{\text{diag}(\mathbf{D})}, \quad (19)$$

and the correlation matrix equalled

$$\mathbf{\Pi} = \mathbf{S}^{-1} \mathbf{D} \mathbf{S}^{-1}. \quad (20)$$

In all formulas  $\bar{\mathbf{y}}_k$  is the vector of the time-homogenized height differences of the levelling campaign  $k = 1, 2, 3, 4$  at the central epoch  $\bar{t}_k$  of the levelling  $k$ ;  $t_0$  is the reference epoch of the kinematic adjustment from which the VVs  $\bar{\mathbf{h}}$  were obtained;  $d_k(i, j)$  is the levelling distance between the BMs  $i$  and  $j$ ;  $\bar{\mathbf{h}}_k$  is the vector of the BMs' heights of the levelling  $k$  at  $\bar{t}_k$ ;  $\mathbf{A}$  is the design matrix of the levellings;  $\mathbf{e}_k$  is the error vector of the levelling  $k$ ;  $n$  is the number of levelling loops.

Numerically:

$$\mathbf{D} = \begin{pmatrix} 1.925 & -0.037 & -0.430 & 0.043 \\ -0.037 & 0.330 & 0.590 & -0.020 \\ -0.430 & 0.590 & 2.203 & 0.110 \\ 0.043 & -0.020 & 0.110 & 0.030 \end{pmatrix} \text{mm}^2 \text{km}^{-1} \quad (21)$$

and

$$\mathbf{\Pi} = \begin{pmatrix} 1 & -0.046 & -0.209 & 0.180 \\ -0.046 & 1 & 0.691 & -0.200 \\ -0.209 & 0.691 & 1 & 0.427 \\ 0.180 & -0.200 & 0.427 & 1 \end{pmatrix}. \quad (22)$$

The strong correlation between the second and the third levelling (0.691) was significant ( $p < 0.05$ ). The second and third levellings were done using the same levelling methodology, and partly with the same team and equipment. Therefore, these levellings may share the same instrumental and methodological errors. The significant VV changes presented in Tables 3 and 4 can be related to the correlation between the second and the third levelling. At the same time it should be remembered that the result of this test is approximate due to the time-homogenization.

## ESTIMATION OF THE VARIANCE COMPONENTS

After the kinematic adjustment of the levelling combination 1–2–3–4, a variance of unit weight  $S_0^2 = 10.078$  was obtained from the 'heights included' model (Table 2). The variance of unit weight reflected how the different levelling data and levelling error estimates fit together with the model with constant VVs. The larger the  $S_0^2$ , the poorer the fit. The detected VV change over time (Tables 3 and 4) could also be explained by levelling errors, if it is assumed that the levelling error is  $S_0 = \sqrt{10.078} = 3.2$  larger than indicated by loop misclosures. Then the *a priori* levelling standard error estimates from Eq. (1) could be re-scaled uniformly by 3.2 times for all four levellings. But instead of the uniform rescaling, different variance factors  $S_0^2$  were used for different levelling groups ( $k = 1, 2, 3, 4$ ):

$$\mathbf{\Sigma} = \text{diag}(S_0^2 \mathbf{W}_1^{-1} \ S_0^2 \mathbf{W}_2^{-1} \ S_0^2 \mathbf{W}_3^{-1} \ S_0^2 \mathbf{W}_4^{-1}), \quad (23)$$

where  $\mathbf{W}_k$  is the weight matrix of the levelling group  $k$  ( $k = 1, 2, 3, 4$ ) and  $\mathbf{\Sigma}$  is the covariance matrix of the observations.

There are many different methods for estimating  $S_0^2$ . The best known are the Helmert (Helmert 1872; Welsch 1978), Bique (Koch 1978, 2010; Welsch 1984),

Minque (Rao 1971), Förstner (Förstner 1979) and IAUE (Lucas 1985) methods. An overview of different variance component estimation techniques can also be found in Amiri-Simkooei (2007) and Bähr et al. (2007).

Before the evaluation of the variance components, outliers were removed from the dataset using a ‘data snooping’ method (Baarda 1968; also described in Kall et al. 2014). The observation was rejected from the dataset if its standardized residual was larger than  $S_0 \times 3.29$  ( $\alpha = 0.001$ ). The method is based on the assumption that there is only one outlier in the set of the observations. Therefore only one observation with the biggest standardized residual was removed from the set, after what adjustment was repeated. This was done iteratively until no outliers were detected. Altogether 25 iterations were performed. Most of the detected outliers belonged to the first levelling (altogether 17 outliers from 24). Four outliers belonged to the second levelling, two outliers to the third and one outlier to the fourth levelling. The outliers greatly influenced the *a posteriori* variance of unit weight, as was also brought out by Kall et al. (2014). From first to 25th iteration  $S_0^2$  decreased from 10.08 to 1.79, i.e. about 6 times.

After removing the outliers from the dataset the variance component estimation of the four levelling campaigns of the CLN was performed using the Helmert, Bique and Förstner methods. After convergence to unity, the methods led to identical variance factors. The only difference was in the number of the iterations necessary (Helmert and Bique = 14 iterations, Förstner = 40 iterations). Variance factors and re-scaled levelling standard errors obtained after the variance component estimation are presented in Table 5.

The obtained variance factors (Table 5) were not homogeneous when compared to each other or with respect to the *a priori* levelling standard errors. Neither were the variance factors uniform with respect to the levelling standard errors obtained from Eq. (21)

**Table 5.** Results of the variance component estimation using the Helmert, Bique and Förstner methods. All methods ultimately led to the same result, except for the number of the iterations (Helmert and Bique = 14 iterations, Förstner = 40 iterations)

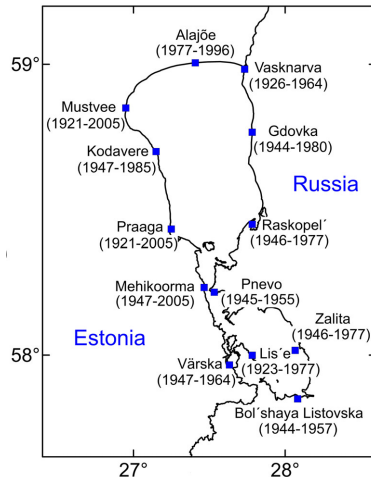
Levelling	<i>A priori</i> levelling standard error $\tau$ (mm km <sup>-0.5</sup> )	Variance factor	Re-scaled levelling standard error (mm km <sup>-0.5</sup> )
1	±1.387	3.19138	±2.478
2	±0.423	0.93683	±0.409
3	±1.576	0.95863	±1.543
4	±0.191	1.22798	±0.212

(diagonal elements of the variance matrix 1.387<sup>2</sup>, 0.574<sup>2</sup>, 1.484<sup>2</sup> and 0.173<sup>2</sup>). These could serve for the best estimates of the levelling standard errors, adjusting every levelling campaign separately and considering only the interior fit of the height differences. The methodology for the estimation of the variance components in a constant VV model does not only consider the interior fit of the height differences but also their location in the time scale (Mäkinen & Saaranen 1998).

**TILT OF LAKE PEIPSI**

Lake tilts between water gauge (WG) pairs from monthly mean water level records relative to the Baltic Height System 1977 of Lake Peipsi (Fig. 1) were determined by Raamat (2009) and utilized in the present study. Lake level recordings from 1921 to 2006 from the Estonian as well from the Russian side (Fig. 3) were used for tilt calculation. The shortest observation time series was 10.08 years (Pnevo) and the longest, 83.75 years (Mustvee). The longest gap in the time series was 2 years in Lis’e, Raskopel’ and Zalita. The value of the missing month in the time series was calculated as the average of the same month from the previous and following years to avoid the influence of the seasonal variation in the water level.

Next, 69 combinations of the WG pairs sharing a common observation period were formed and water



**Fig. 3.** Water gauges around Lake Peipsi (Fig. 1) and water level observation periods.

level observations were differentiated between them. The shortest and longest common time series between the WG pairs were ~3 years (Gdovka–Alajõe) and 75.42 (Mustvee–Praagaa) years, respectively. The slope of the linear trend of the water level differences between the WG pairs reflects the tilt of the lake level, i.e., VV difference between WGs (Mainville & Craymer 2005; Bruxer & Southam 2008). Closing errors of the VV differences in triangles of WGs did not exceed  $\pm 0.56 \text{ mm yr}^{-1}$  and the standard error of the VV differences from the closing errors was  $\pm 0.23 \text{ mm yr}^{-1}$ .

Strong storms, especially in the autumn–winter period, have significant influence on the water level of Lake Peipsi (Tavast 2009). Therefore, the water level at the WGs can be quite different, which also causes outliers in water level differences between the WG pairs. Outliers in the water level differences were detected and removed in the first iteration visually, trying to raise the value of the determination coefficient  $R^2$  of the trendline. In the second iteration, observations with standard residuals of the regression analysis larger than two ( $r_i > 2$ ) were removed. The high and low water level cycle of Lake Peipsi is approximately 11 years (Jaani 1973); therefore, the sinusoidal change in water level was also taken into account when removing the outliers.

After removing the outliers, regression analysis of the differentiated water level observations of all WG pairs was repeated. Insignificant slopes ( $\alpha = 0.05$ ), a total of 19, mostly time series with a length of less than 10 years, were removed from the following calculation. From the significant slopes (a total of 50), 17 slopes with  $R^2 > 0.61$  were selected as VV differences for the weighted LSQ adjustment. Similarly to trivial and non-trivial baselines in GPS network processing, correlation exists between WG differences. From 17 selected slopes, five were correlated, i.e., WG differences which use the same observation data form a closed loop. However, these WG differences are only partly correlated, since common observation periods in different WG pairs only partly overlapped. Moreover, values of VVs in LSQ adjustment are not influenced by the correlation between WG pairs. Only the accuracy of VVs is over-estimated.

The LSQ adjustment method was used in pairwise analysis of the WGs. The following observation equation, which accounts for the discrepancy between pairs of the WGs by introducing a residual error, was applied to each pair (Mainville & Craymer 2005):

$$\Delta v_{ij}^{\text{obs}} + r_{ij} = v_i - v_j, \quad (24)$$

where  $\Delta v_{ij}^{\text{obs}}$  is the average VV difference of point  $j$  relative to point  $i$ , from the slope of the linear trend of the water level differences between the WG pairs  $i$  and  $j$ .

The other variables are the output from the LSQ adjustment:  $v_i$  and  $v_j$  are the VVs of WGs  $i$  and  $j$ ;  $r_{ij}$  is the residual error in the observed average VV difference  $\Delta v_{ij}^{\text{obs}}$ . The  $\Delta v_{ij}^{\text{obs}}$  were weighted according to the standard errors of the regression slopes. The VV of the southernmost WG (Bol'shaya Listovska) was fixed to zero to calculate the relative VVs of the WGs. The results of the LSQ adjustment are presented in Table 6.

## MODELLING A SURFACE OF VERTICAL CRUSTAL MOVEMENTS FOR ESTONIA

Based on the estimations of  $S_0^2$  and  $\mu$  (Table 2) and the variance component estimation (Table 5), where data of the first levelling were heavily weighted down, we opted to skip that levelling but otherwise follow Eq. (4) when calculating final velocities for the VCM model. Using only the three latest levellings also made it possible to enclose more BMs in the network, since the second, third and fourth levellings have more common BMs than the CLN of the four levellings. Variance component estimation based on the Bique method using the second, third and fourth levellings gave the levelling standard errors of 0.453, 1.475 and 0.209  $\text{mm km}^{-0.5}$ , respectively, which only slightly differ from the *a priori* estimates (Table 5).

Levelling is a relative method where only VV differences are estimable. In order to obtain apparent (related to sea level) VVs, relative VVs have to be tied at least with one known apparent velocity. We decided to relate our apparent VVs to the deep-seated BM FR241 in Tallinn with the value of  $+1.7 \text{ mm yr}^{-1}$ . The BM FR241 is placed in limestone and has been used in many previous studies as a stable BM for relating relative VV differences from levellings to the apparent ones. For example, Zhelmin (1958, 1960, 1964, 1966) used the apparent VV value of  $+2.5 \text{ mm yr}^{-1}$  (velocity  $+1.9 \text{ mm yr}^{-1}$  from Bikis (1940) corrected for local subsidence). Vallner & Zhelmin (1975), Vallner (1978) and Vallner et al. (1988) also used same value. Vallner & Zhelmin (1975) and Vallner (1978) used the value of  $+1.7 \text{ mm yr}^{-1}$  for Tallinn, combining repeated levellings with Yakubovski's (1973) VVs for TGs in Kunda and Vormsi for the period 1889–1970. Randj r (1993) also constrained his map with Yakubovski's (1973) VVs for TGs in Kronstadt, Salacgriva, Liepaja and Baltijsk, and Tallinn from Vallner & Zhelmin (1975). The apparent VV of  $+1.7 \text{ mm yr}^{-1}$  for the Tallinn TG was confirmed also by Davis et al. (1999) based on sea-level observations from 1928 to 1938 from Permanent Service for Mean Sea Level (<http://www.psmsl.org/data/>). The same apparent VV of  $+1.7 \text{ mm yr}^{-1}$  for FR241 was found also by Kall et al. (2014), where VVs were constrained to the apparent

**Table 6.** Vertical velocities (VVs) and their standard deviations (both  $\text{mm yr}^{-1}$ ) at the water gauges (WGs) of Lake Peipsi from the weighted least squares (LSQ) adjustment of the lake tilts according to Raamat (2009). Relative VVs are given with respect to the Bol'shaya Listovska WG whose VV was fixed to zero in the LSQ adjustment. Apparent VVs of the WGs were calculated relative to the Mustvee WG with an apparent VV  $+0.73 \text{ mm yr}^{-1}$ . This is the average apparent VV of the five benchmarks (BMs) of the CLN near the Mustvee WG. Apparent VVs of the BMs were obtained through the kinematic adjustment of the CLN relative to BM FR241 in Tallinn with an apparent VV  $+1.7 \text{ mm yr}^{-1}$ .

Water gauge	Latitude (B°)	Longitude (L°)	Relative VV	Standard deviation of the relative VV	Apparent VV
Bol'shaya Listovska	57.8493	28.0825	0	0	-0.28
Gdovka	58.7663	27.7828	0.97	$\pm 0.01$	0.69
Lis'e	57.9994	27.7826	0.23	$\pm 0.01$	-0.05
Pnevo	58.2161	27.5326	0.52	$\pm 0.01$	0.24
Raskopel'	58.4495	27.7828	0.60	$\pm 0.01$	0.32
Zalita	58.0160	28.0660	0.21	$\pm 0.01$	-0.07
Alajõe	59.0050	27.4075	1.17	$\pm 0.02$	0.89
Kodavere	58.6996	27.1493	0.80	$\pm 0.01$	0.52
Mehikoorma	58.2328	27.4660	0.36	$\pm 0.01$	0.08
Mustvee	58.8496	26.9493	1.01	$\pm 0.01$	0.73
Praaga	58.4329	27.2493	0.58	$\pm 0.01$	0.30
Värska	57.9661	27.6325	0.22	$\pm 0.02$	-0.06
Vasknarva	58.9830	27.7327	1.17	$\pm 0.01$	0.89

velocity of Ristna TG  $+2.1 \text{ mm yr}^{-1}$ . Although we found new VVs for six coastal TGs, the Tallinn TG was not among them, since it has been sinking during most of our calculation period (1960–2010) (see Lutsar 1965; Kall & Torim 2003). There were also large gaps in the time series for that TG in our calculation period.

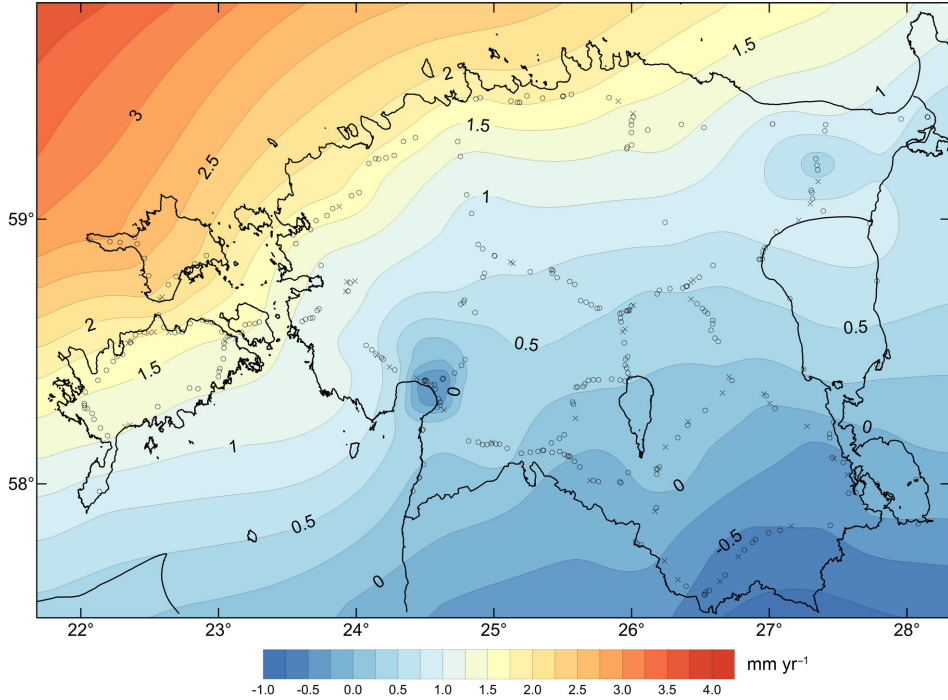
Outliers among the observations were detected iteratively using the 'data snooping' method by Baarda (1968) with the significance level  $\alpha = 0.001$ . The detected outliers (altogether 41 outliers from 10 iterations) were weighted down using the 'Danish method' according to Caspary (2000). Then, the apparent VVs of the BMs relative to deep-seated BM FR241 in Tallinn were found from the LSQ kinematic adjustment with the Bique weights. Post-adjustment statistics  $S_0^2 = 0.86$  and  $\mu = \pm 0.12 \text{ mm yr}^{-1}$  were obtained.

No direct levelling connections between the WGs of Lake Peipsi and the BMs of the CLN were available for common processing. Therefore, relative VVs of the WGs from the LSQ adjustment of the lake level tilts (fourth column in Table 6) were indirectly connected with the apparent VVs from the kinematic adjustment of the levellings. For that purpose average apparent VV ( $+0.73 \text{ mm yr}^{-1}$ ) of the five BMs close to the Mustvee WG (within a radius of 5 km) was assigned to this WG. The VVs of other WGs were shifted relative to it (final column in Table 6).

For the creation of the modelled surface of the VCMs, gridding with the Surfer software (Golden Software Inc.) was used. Several gridding methods were tested. The 'minimum curvature' method was chosen based on the residuals of the gridded surface at the observation points, a cross-validation technique and visual appearance. The VCM surface was created for the area  $57.45^\circ\text{--}59.82^\circ\text{N}$  and  $21.67^\circ\text{--}28.64^\circ\text{E}$ , with a grid spacing of  $2 \times 2 \text{ km}$  in accordance with the average distance between the closest BM pairs. For removing outliers, BMs deviating more than  $\pm 0.3 \text{ mm yr}^{-1}$  from the neighbouring ones were visually eliminated from the dataset (altogether 53 BMs from 336), after which the gridded surface was filtered based on the 'threshold averaging' method available in Surfer. The obtained VCM model (Fig. 4) was named EST2015LU for referencing purposes.

The main feature of the compiled map (Fig. 4) is the SE–NW directional postglacial land uplift. However, there are also two conspicuous VCM anomalies, one of which is the area surrounding Pärnu (see Fig. 1). The geological setting of Pärnu is characterized by a Proterozoic crystalline basement covered by Silurian limestone and Devonian sandstone. The surface of sedimentary rocks lies at a depth from  $-10$  to  $-15 \text{ m}$  (Tavast & Raukas 1982). These Palaeozoic rocks are covered with Quaternary sediments. The lower layer is loamy till of Late Weichselian age. The loamy till is





**Fig. 4.** The model of vertical crustal movements EST2015LU based on 283 apparent vertical velocities (VVs) of the benchmarks (BMs) from the kinematic adjustment of the second, third and fourth levellings of the common levelling network (Fig. 1) relative to BM FR241 in Tallinn with the apparent VV of  $+1.7 \text{ mm yr}^{-1}$ . The apparent VVs of 12 water gauges (WGs) of Lake Peipsi (Table 6) were also used. The contour interval is  $0.25 \text{ mm yr}^{-1}$ . Average standard errors of the VVs were  $\pm 0.12 \text{ mm yr}^{-1}$  (BMs) and  $\pm 0.01 \text{ mm yr}^{-1}$  (WGs). Statistics of model residuals were min  $-3.07 \text{ mm yr}^{-1}$  (BM in Pärnu), max  $+0.95 \text{ mm yr}^{-1}$  (also BM in Pärnu), mean  $-0.06 \text{ mm yr}^{-1}$  and root mean square (RMS)  $\pm 0.34 \text{ mm yr}^{-1}$ . The model's cross-validation residual RMS error was  $\pm 0.30 \text{ mm yr}^{-1}$ . The circles indicate the location of BMs and WGs used in the modelling (a total of 295); crosses (x) indicate the location of BMs visually removed before modelling (53).

covered with varved clay or silt with an average thickness of  $\sim 10 \text{ m}$ . Clay is lacking in only a few areas. Its thickness changes rapidly and is related to irregular topography of the underlying till. The varved clay is covered with a 2–3 m thick layer of Holocene marine and aeolian fine sand and silt which also may be locally absent (Kohv 2011; Talviste et al. 2012; Hang & Kohv 2013). Local subsidence in Pärnu has been related to a decrease in groundwater level. This causes a change in pore pressure, which in turn depends on the thickness of the clay (Listra & Talviste 1988; Mets et al. 2000). The groundwater level in Pärnu has been continuously decreasing since the 1960s. Groundwater pumping achieved its maximum in 1988–1990, causing a lowering of the piezometric level by approximately  $-10$  to  $-12 \text{ m}$ . In

1990–2000 the piezometric level gradually recovered (mean rise  $+5 \text{ m}$ , max  $12 \text{ m}$ ). The piezometric level in Pärnu has been stable since 2001 at an altitude from 0 to  $1.5 \text{ m}$  with seasonal variation within  $\pm 1.5 \text{ m}$  (Kohv & Hang 2013). Between 1960 and 1988 the subsidence of the BMs up to  $-20 \text{ mm yr}^{-1}$  was observed (Listra & Talviste 1988). According to recent results, the subsidence of the BMs has stabilized and in one area where the clay is thinner (6 m in average) has even been replaced by uplift (Miller 2013). Our adjustment results show that the sinking of the BMs in Pärnu reach up to  $-3.5 \text{ mm yr}^{-1}$ . The largest residuals of the calculated VVs compared to the smoothed model EST2015LU (Fig. 4) also occur in the Pärnu area. The subsidence of the BMs in Pärnu gradually decreases from the city centre to the suburbs.

A second VCM anomaly in Fig. 4 is in the NE of Estonia. This area is geologically well studied due to the oil shale deposit which has been mined in Estonia for over 90 years. Land surface subsidence occurs above the underground mines when mined-out cavities collapse. The first spontaneous collapse of the underground mine and subsidence of the surface occurred in 1964. Until 2003, 73 collapses have been registered (Pastarus & Sabanov 2005). As a result of these collapses, the surface may subside from 0.7 to 1.5 m, depending on the type of mining (Toomik & Liblik 1998). The residual subsidence of the BMs in that area is most likely related to the mining of the oil shale (Rüdja 2004). According to our study, the residual subsidence of the BMs in relation to the surrounding area is approximately  $-0.7 \text{ mm yr}^{-1}$ .

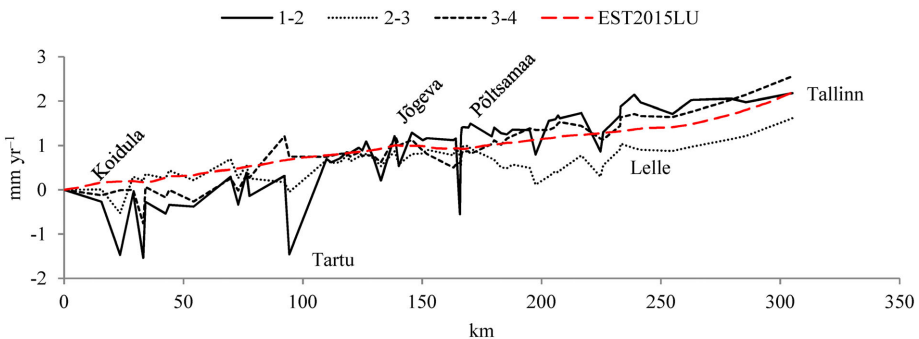
In Fig. 4, the isolines of VCMs in central Estonia decline more in an east–west direction than in previous land uplift maps of Fennoscandia or Estonia. The declination of the isolines was also noticed in the land uplift model EST2013LU (Kall et al. 2014). However, this direction of the isolines is supported by the VVs of the WGs of Lake Peipsi. In addition, the isolines over Lake Peipsi and SE Estonia on the EST2015LU model (Fig. 4) greatly resemble the isolines of the VCM map by Randj r v (1993), who incorporated levelling data from the Russian side and Latvia into his calculations. Other Estonian VCM maps (e.g., Vallner et al. (1988) and Torim (2004)), where isolines in SE Estonia do not form such shapes, have been compiled based only on Estonian levelling data.

Surfaces of the VCMs from the other levelling combinations were constructed as well. From the comparison of all VCM models it was concluded that

data from the first and third levellings influenced the isolines of the VCMs to decline in a SW–NE direction (VCM models based on the VVs of the combinations 1–2–3 and 1–3), whereas data from the second and especially fourth levellings influenced the isolines to decline in a more W–E direction (combinations 1–2–4, 2–3–4 and 2–4). The larger weights of the second and particularly fourth levellings had a larger influence on the isolines of the EST2015LU model, resulting in their declining in a more W–E direction than in earlier maps.

It is obvious that some information about the VVs of the BMs has been lost during the modelling process. In order to highlight that information, the profile of cumulative raw relative VVs of the BMs from SE Estonia (Koidula) to North Estonia (Tallinn) was compiled (Fig. 5). The VVs relative to Koidula were interpolated from the EST2015LU model along this line and added to the profile. The profile of the EST2015LU model follows the profiles of the cumulative VVs between the second and third and the third and fourth levellings most noticeably. Clearly this is related to the fact that the EST2015LU model was based on the second, third and fourth levellings. The fact that the EST2015LU profile follows the cumulative VV profile of the third and fourth levellings better is related to the higher weights of the fourth levelling in the kinematic adjustment.

In addition to the general postglacial tilt, several spikes can be noticed on the profiles. Some spikes are common to all three VV combinations, differing only in the magnitude of the spikes. However, there are also spikes that have happened only in one combination (e.g., 1–2). Spikes in general indicate local anomalies of VCMs. Negative spikes are associated with near-surface



**Fig. 5.** Profile of the raw (unadjusted) vertical velocities of the benchmarks between the first and second (1–2), second and third (2–3) and third and fourth (3–4) levellings along the Koidula–Tartu–J geva–P ltsamaa–Lelle–Tallinn levelling line (Fig. 1) relative to Koidula. The profile of the EST2015LU model (Fig. 4) along this line relative to Koidula was added for comparison.

processes like groundwater lowering, compression of the sediments, subsidence of buildings, mining, heavy traffic, etc. Positive spikes may result from human activity-related displacements like construction work or levelling errors, but also from near-surface processes like frost heave (Ellenberg 1987; Giménez et al. 2000). Therefore, spikes on the profiles are not related to postglacial rebound or tectonic causes. In Estonia, where most of the country is covered with Quaternary sediments thicker than 5 m, most of the local VCM anomalies are related to the compaction or expansion of sediments.

Steps and changes in the slope on the profiles can be related to geological or tectonic phenomena, such as different compaction of recent sediments, regional tectonic tilt or active tectonic structures (Giménez et al. 2000). Based on this methodology, Vilde (2013) analysed the graphs of the cumulative VVs of four precise levellings of the Estonian levelling network. He concluded that there was a connection between the steps on the cumulative VV profiles and locations of the tectonic faults on the Jõhvi–Tapa, Tallinn–Tapa and Põltsamaa–Lelle levelling lines.

Step-like features in Fig. 5 can be noticed on kilometres 40 and 70 between Koidula and Tartu, as well as between Põltsamaa and Lelle (170–230 km). The Põltsamaa–Lelle line crosses several tectonic faults of the crystalline basement and sedimentary rocks. A relation between VCMs along this line and tectonic faults has been identified in previous studies (Sildvee 1973; Vallner & Zhelmin 1975; Vallner et al. 1988; Kall & Oja 2006; Kall & Jürgenson 2008). Changes in the slope of VVs can also be observed in both abovementioned lines at least in one levelling combination. Most likely it is related to different compaction of the sediments. For example, the Koidula–Tartu levelling line runs intermittently on till and sand of different thicknesses. The relationship between changes in the slope and different compaction of sediments has been shown in a previous study by Zhelmin (1966). For example, he explained the great subsidence of the BMs along the Põltsamaa–Lelle levelling line in 1961–1964 by the drought in 1964, when the groundwater level was extraordinarily low. According to Zhelmin (1966), the drought had no influence for the first 20 km of the levelling line because sedimentary rocks only lie at a depth of 1.3 m. The subsidence of the BMs appeared for further sections of the line because the thickness of the Quaternary sediments also increased.

Several attempts have been made to relate VCMs from the repeated levellings to the block structure of the crystalline basement and sedimentary cover of Estonia. However, no firm connection between the VCMs and block structure of the crystalline basement has been found (Sildvee & Miidel 1978, 1980). Five velocity planes of

VCMs for Estonia (southeastern, middle, northeastern, northwestern and western Saaremaa) were calculated by Vallner et al. (1988). These planes move parallel or under a small angle relative to each other and coincide with the regional gravity structures. Vallner et al. (1988) concluded that VCMs follow block movements in Estonia. Nevertheless, based on the comparison between the VCM map and the block structure of the crystalline basement (Pobul & Sildvee 1975), Vallner et al. (1988) came to the same conclusion as the former authors that there is no clear correlation between them. They found no accordance between the VCMs and structure of the sedimentary cover either.

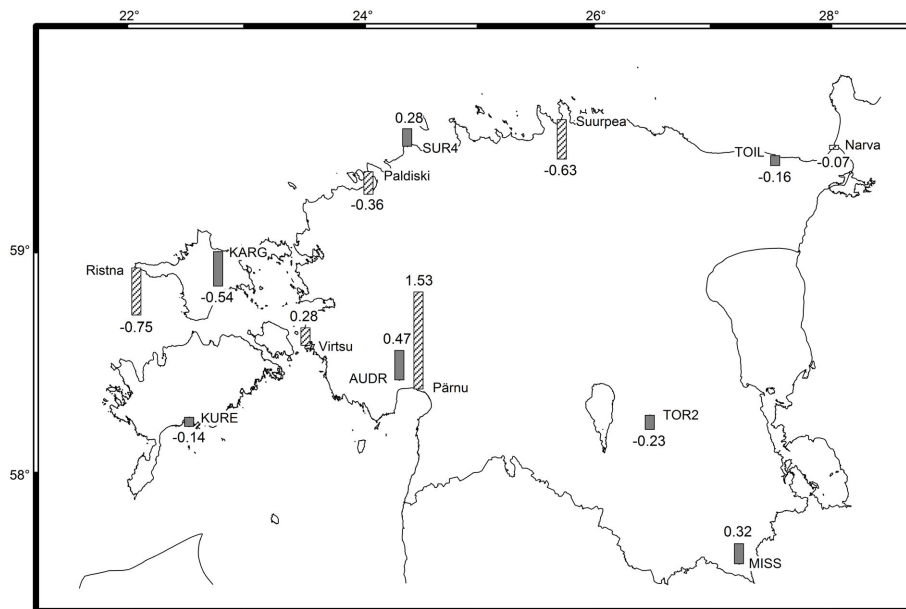
#### COMPARISON WITH VERTICAL VELOCITIES FROM THE CONTINUOUSLY OPERATING GNSS REFERENCE STATIONS AND COMBINATION OF TIDE GAUGE OBSERVATIONS AND SATELLITE ALTIMETRY

The EST2015LU model was compared with the VVs of the seven continuously operating Global Navigation Satellite System (GNSS) reference stations (CORS) in Estonia as reported by Oja et al. (2014). The absolute VVs of the CORS ( $v_{\text{ABS}}$ ) were calculated in an ITRF2005 reference frame and based on a 6.5-year time period (November 2007 to May 2014). From the EST2015LU model, apparent VVs ( $v_{\text{APP}}$ ) related to sea level were interpolated to the CORS. In order to compare absolute VVs with apparent ones, the following equation was employed:

$$v_{\text{ABS}} = (a + v_{\text{APP}}) \cdot b, \quad (25)$$

where  $a$  is secular sea level rise (eustatic rise) and  $b$  is a dimensionless variable describing the change in the geoid. In the present study  $b$  was fixed to the value 1.06 (Vestøl 2006), which corresponds to about 6% ( $\sim 0.6 \text{ mm yr}^{-1}$ ) of the Fennoscandian land uplift maximum ( $\sim 10 \text{ mm yr}^{-1}$ ) at the Gulf of Bothnia. A similar value for the geoid change was also found by Tamisiea et al. (2002) and Ekman & Mäkinen (1996),  $0.5 \text{ mm yr}^{-1}$  and  $0.6 \text{ mm yr}^{-1}$ , respectively.

The value of  $a$  was found from the VV difference  $v_{\text{ABS}} - v_{\text{APP}}$  of the six CORS from a LSQ solution that minimized the velocity residuals. Using this method, the value of  $a$  was  $2.11 \pm 0.14 \text{ mm yr}^{-1}$ . A similar value was also obtained by Oja et al. (2014), using the same methodology of employing  $v_{\text{APP}}$  values interpolated to CORS from different land uplift models. Residual VVs of the CORS obtained from Eq. (25) are presented in Fig. 6.



**Fig. 6.** Residual vertical velocities (VVs, in  $\text{mm yr}^{-1}$ ) (observed–interpolated) from the comparison of the GNSS-based (grey bars) and tide gauge and satellite altimetry (TG & SA)-based absolute VVs (patterned bars) with the apparent VVs interpolated from the EST2015LU model (Fig. 4). The root mean square values of the residual VVs were  $\pm 0.34 \text{ mm yr}^{-1}$  (GNSS) and  $\pm 0.76 \text{ mm yr}^{-1}$  (TG & SA;  $\pm 0.48 \text{ mm yr}^{-1}$  without Pärnu).

The comparison of the EST2015LU model with GNSS data gave the RMS of the residual VVs  $\pm 0.34 \text{ mm yr}^{-1}$ . The largest residuals occur in the Pärnu area and on the Island of Hiiumaa (AUDR and KARG sites; Fig. 6). The larger difference in the Pärnu area could be explained by the fact that levelling-based VVs cover the period when this area was subsiding, while GNSS observations were performed when subsidence had stopped. The difference on the Island of Hiiumaa may result from errors in levelling-based VVs, since Hiiumaa is connected to the CLN only from the one side with the hydrostatic or hydrodynamic levelling between Saaremaa and Hiiumaa (Fig. 1). The differences also result in part from the choice of the reference apparent VV for the kinematic adjustment of the CLN.

Because of systematic differences, historical VVs for Estonian TGs (Yakubovski 1973; Pobedonostsev 1975; Jevrejeva et al. 2002) cannot be used for the evaluation of land uplift models (Kall et al. 2014). For the present study new VVs for Estonian coastal TGs were calculated using a different methodology. The sea level observation

period was shorter (1960–2010) than in previous studies and TG data were corrected using satellite altimetry (SA) data (1992–2013). The methodology used was based on the studies by Kuo et al. (2004, 2008). Absolute VVs ( $v_{\text{ABS}}$ ) were found for six TGs. Apparent VVs ( $v_{\text{APP}}$ ) were interpolated to TGs from the EST2015LU model in Fig. 4. Absolute VVs were compared with apparent VVs as well as with GNSS velocities. The value for sea level rise  $a = 1.49 \pm 0.34 \text{ mm yr}^{-1}$  was obtained. Residual VVs of the TGs based on Eq. (25) are presented in Fig. 6.

The comparison of VVs from the EST2015LU model with the TG & SA data yielded the RMS of the residual VVs  $\pm 0.76 \text{ mm yr}^{-1}$ . This is slightly better than that found by Kall et al. (2014). Like with GNSS data, the largest residuals occur in Pärnu and on the Island of Hiiumaa but also in North Estonia (Pärnu, Ristna and Suurpea sites; Fig. 6). The reasons for these biases are most likely the same as previously discussed. The fit with the VVs of TGs is much better without Pärnu. Residual VV RMS without Pärnu was  $\pm 0.48 \text{ mm yr}^{-1}$ .

This discrepancy is within the limits of uncertainty, considering the accuracy of the VV determination from levellings and TG & SA.

At the same time, the comparison between the CORS and TG absolute VVs showed that differences were even larger than with the EST2015LU model (RMS =  $\pm 0.8$  mm yr<sup>-1</sup>). The VVs of GNSS were systematically larger than those of the TGs. In our opinion, the main reason for the difference is related to the reference frames used. For TG & SA VVs the frame is defined by the orbit of the TOPEX/POSEIDON mission. The CORS VVs are aligned to ITRF2005. Santamaría-Gómez et al. (2014) have pointed out that the VV differences between the estimates from the Kuo et al. (2004) method and BIFROST GPS VVs in the Baltic Sea remain between  $-2.0$  and  $0.5$  mm yr<sup>-1</sup>: on the latitude of Estonia the differences are between  $-0.8$  and  $0.5$  mm yr<sup>-1</sup>. However, the errors of our method and GNSS may also be similar to the order of  $\pm 0.8$  mm yr<sup>-1</sup> as we are using an indirect method to estimate VVs with the geophysical assumption that TG and SA observed the identical geocentric sea level rise. Another assumption is that only linear vertical motion exists, as the time span of CORS is much smaller than that for TG.

## SUMMARY AND CONCLUSIONS

The aim of this study was to evaluate VCMs in Estonia over time, based on a CLN created from a dataset of four precise levellings, and from water level observations of Lake Peipsi. The difference between *a posteriori* variances of unit weight between three or four and two levelling combinations indicates that the VVs of the BMs between the levelling periods have been uneven, observations contained errors, or weight matrices of the observations did not fit together in the kinematic adjustment of more than two levelling combinations. Estimates of the mean standard error of VV differences were mostly dependent on whether the fourth levelling was involved in the adjustment. The mean standard error of the VV differences was also influenced by the time period between levellings. The larger the time period, the smaller the standard error was.

Neither variances of unit weight nor mean standard errors of the VV differences differed significantly between the models parameterizing VVs with and without heights, regardless of levelling combinations used. In most levelling combinations, except 1–2–3 and 1–2, there were also no significant differences between the VVs from the ‘heights included’ and ‘heights eliminated’ models. Therefore, levelling observations did not contain systematic errors which would be removed from the dataset by differencing the observations.

A significant difference between VVs  $v_{1-2}$ ,  $v_{2-3}$ ,  $v_{3-4}$  was found. This means that VVs of the BMs changed over time. The change in VV can either be a real change or fortuitous, caused by levelling errors or correlation between the levellings. Since a strong significant correlation (0.691,  $p < 0.05$ ) between the second and third levellings was detected, it remained unresolved whether the VV change was real or dependent on the between-epoch correlation of the second and third levellings. Between-epoch correlation influences deformation analyses, independently of whether the correlation is fortuitous or genuine (Mäkinen & Saarinen 1998).

The detected VV change over time could also be explained by levelling errors, if it is assumed that the levelling error was 3.2 times larger than indicated by loops’ misclosures. The increase could be divided uniformly to the four levellings but iterated variance component estimation put most of the error on the first levelling.

Apparent VVs from the kinematic adjustment of the combination of the second, third and fourth levellings, as the mathematically best fitting observations with re-scaled weights along with the apparent VVs of the WGs of Lake Peipsi (Table 6) were used to draw the VCM model EST2015LU (Fig. 4). Compared to earlier VCM maps for Estonia and Fennoscandia, the isolines of EST2015LU over Lake Peipsi and SE Estonia resemble most the VCM map by Randj arv (1993). The EST2015LU model also highlighted two VCM anomalies. The first, in P arnu, is related to the compaction of the sediments due to groundwater withdrawal. The second one is located in NE Estonia and is probably related to oil shale mining in that area.

The comparison of absolute and apparent VVs of seven CORS, based on Eq. (25), yielded an average residual VV of  $\pm 0.34$  mm yr<sup>-1</sup>. Taking the average standard deviation of the VVs and the errors of EST2015LU into account, the fit between the VVs from the EST2015LU model and CORS can be considered good.

Absolute VVs for six TGs based on sea level observations from 1960 to 2010 were calculated and corrected by using SA data (1992–2013). From the comparison of the TG & SA VVs with the ones interpolated from the EST2015LU model, an average residual VV of  $\pm 0.76$  mm yr<sup>-1</sup> was found. Larger residuals on the Island of Hiiumaa (with both CORS and TG & SA VVs) suggest that height connections between the Islands of Saaremaa and Hiiumaa should be re-examined. At the same time, the differences between the CORS and TG VVs were even greater (RMS =  $\pm 0.8$  mm yr<sup>-1</sup>), whereas VVs from the CORS were systematically larger. The main reason for the CORS and TG & SA VV difference is related to the reference frames used.

**Acknowledgements.** The authors thank the Estonian Land Board for providing levelling data and the Estonian Weather Service for providing water level observations of Lake Peipsi. J. G. Garcia and an anonymous referee are thanked for their constructive comments on the manuscript.

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## Maapinna vertikaallikumised Eestis, tuginedes täppisnivelleerimistele ja Peipsi järve veetaseme vaatlustele

Tarmo Kall, Aive Liibus, Junkun Wan ja Rivo Raamat

Eesti kõrgusvõrk on praeguseks ajaks kõrgtäpsete nivelleerimistega mõõdetud juba neli korda (nivelleerimiskampaaniate keskmised epohhid 1936,7, 1961,2, 1982,1 ja 2006,9 aastat). Käesoleva uurimuse eesmärk on hinnata Eesti kõrgusvõrgu reeperite vertikaallikumise kiirusi ja nende muutust ajas nelja nivelleerimise (periood 1933–2011) andmete põhjal. Vertikaallikumise kiiruste arvutamiseks kasutati vähimruutude meetodil parameetrilist kinemaatilist tasandamist. Rakendati kaht matemaatilist mudelit, kus esimeses olid parameetriteks reeperite kõrgused ja kiirused ning teises ainult kiirused. Viimased leiti nivelleerimiste üheteistkümnest kombinatsioonist. Erinevate matemaatiliste mudelite kiirused erinesid ainult kahe nivelleerimise kombinatsiooni puhul. Mõlemal juhul oli kaasatud teine või kolmas nivelleerimiskampaania, mille polügoonide sulgemise ajavahemik on samas suurusjärgus või ületab nivelleerimiskampaaniate keskmiste epohhide vahe. Tasandusjärgsete kaaluühiku dispersioonide ja ANOVA-testi põhjal tuvastati keskmiste nivelleerimiseepohhide vahel oluline reeperite kiiruste muutus. Kuna aga multivariativse analüüsi põhjal tuvastati teise ja kolmanda nivelleerimiskampaania vahel oluline korrelatsioon, jääb selgusetuks, kas reeperite kiiruste muutus on olnud tõeline, füüsikalistel põhjustel toimunud, või juhuslik, üksnes matemaatiline, tuginedes tuvastatud korrelatsioonile. Leitud kiiruste muutust võib põhjendada ka nivelleerimisvigadega. Läbiviidud dispersiooni-komponentide hindamine asetas suurema osa nivelleerimisveast esimesele nivelleerimiskampaaniale. Esmakordselt on vertikaallikumiste määramiseks kasutatud ka Peipsi järve veetaseme andmeid perioodist 1921–2006. Tuginedes viimase kolme nivelleerimise põhjal saadud reeperite vertikaallikumise kiirustele ja Peipsi järve kallete põhjal leitud veemõõdupostide kiirustele, koostati Eesti maapinna vertikaallikumiste mudel EST2015LU. Mudeli peamine tunnusjoon on kagu-loodesuunaline jääajajärgne maadõus. Kuid antud mudeli vertikaallikumiste isobaasid on rohkem lääne-ida suunas kallutatud kui varasemate sama piirkonna kohta koostatud vertikaallikumise skeemide või mudelite omad. See on tingitud peamiselt neljanda nivelleerimise suuremast mõjust, mida toetavad ka Peipsi veemõõdujaamade kiirused. Saadud vertikaallikumiste mudeli sobivus GPS-püsijaamade ja ranniku veemõõdujaamade kiirustega oli suurusjärgus  $\pm 0,4$  kuni  $\pm 0,5$  mm/a.

# CURRICULUM VITAE

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## Education

1978–1989 Vastseliina Secondary School  
1989–1993 Estonian Agricultural University, diploma in land surveying  
1993–1997 Estonian Agricultural University, MSc, geodesy (*cum laudè*)  
2002–2016 Estonian University of Life Sciences, PhD, geodesy

## Professional employment

1994–1995 Estonian Agricultural University, junior researcher  
1995–1998 Estonian Agricultural University, assistant  
Since 1998 Estonian University of Life Sciences, lecturer

## Field of research

Natural Sciences and Engineering, Geosciences, Geodesy (land uplift, levelling, GNSS)

## **Projects**

- 2004–2007 Grant ETF5731: *Influence of the Gravimetric Anomalies on the Geoid of the Gulfs of Riga and Finland*. Senior personnel.  
(Principal investigator: Harli Jürgenson)
- 2008–2010 Grant ETF7356: *Application of space technologies to improve geoid and gravity field models over Estonia*. Senior personnel.  
(Principal investigator: Artu Ellmann)
- 2011–2013 Grant ETF8749: *Determination of height reference frame on the Estonian coastal sea using water level monitoring and laser scanning data*. Senior personnel.  
(Principal investigator: Harli Jürgenson)
- 2016 Grant 8P160012MIGO: *Impact of the sea level rise on the Baltic Sea shoreline changes and real estate boundaries*. Senior personnel.  
(Principal investigator: Aive Liibus)

## **Awards**

- 2014 Raefond Prize of the Estonian University of Life Sciences
- 2013 Innovative lecturer of the Estonian University of Life Sciences
- 2005 Kristjan Jaak stipendiate
- 1997 Estonian Academy of Sciences' student research competition, II Prize

## **Dissertations supervised**

The total number of supervised dissertations (Master's degree) – 8

## Publications

The total number of publications – 25

Most relevant publications:

1.1. – 3 publications

Kall, T. et al., 2016. Vertical crustal movements in Estonia determined from precise levellings and observations of the level of Lake Peipsi. *Estonian Journal of Earth Sciences*, **65**(1), pp. 27–47, doi: 10.3176/earth.2016.03.

Kall, T., Oja, T. & Tänavsuu, K., 2014. Postglacial land uplift in Estonia based on four precise levelings. *Tectonophysics*, **610**, pp.25–38.

Kall, T. & Torim, A., 2003. Vertical movements on the territory of Tallinn. *Journal of Geodynamics*, **35**(4–5), pp.511–519.

1.2. – 1 publication

Jürgenson, H. & Kall, T., 2004. The difference between the N60 and BK77 height systems. *Nordic journal of surveying and real estate research*, **1**(1), pp.72–85.

3.1. – 3 publications

Liiusk, A. et al., 2014. Correcting Tide Gauge Series Due to Land Uplift and Differences between National Height Systems of the Baltic Sea Countries. In *IEEE/OES Baltic 2014 International Symposium: 2014 IEEE/OES Baltic International Symposium “Measuring and Modeling of Multi-Scale Interactions in the Marine Environment”*: Tallinn, Estonia, May 26–29, 2014. IEEE, (IEEE Conference Proceedings), pp. 1–8.

Kall, T. & Jürgenson, H., 2008. Postglacial land uplift in Estonia based on geodetic measurements on Põltsamaa-Lelle levelling line. In D. Cygas & K. D. Froehner, eds. *The 7th International Conference “Environmental Engineering”, May 22–23, 2008, Selected Papers*. Vilnius: Vilnius Gediminas Technical University Press “Technika”, pp. 1325–1333.

Kall, T. & Oja, T., 2006. Geodetic and geophysical repeated measurements in geodynamic monitoring networks of Estonia. In F. Sanso & A. J. Gil, eds. *Geodetic Deformation Monitoring: From Geophysical to Engineering Roles*. International Association of Geodesy Symposia. IAG Symposium on Geodetic Deformation Monitoring - From Geophysical to Engineering Roles. Jaen, Spain: SPRINGER-VERLAG BERLIN, pp. 222–230.

## **Professional trainings**

1. Static and Kinematic GPS-based Positioning Using GIPSY/OASIS, 16–18 September, 2014, University of Beira Interior, Covilha, Portugal.
2. International ESA Summer School on GNSS, 1–11 September, 2010, Slettestrand, Denmark
3. IAG School on Reference Frames, 7–12 June, 2010. Mytilene, Greece.
4. Glacial Isostatic Adjustment (GIA) Modelling Training School, 1–5 June 2009, COST Action ES0701, Gävle, Sweden.
5. Graduate School Course in Geomatics “*Photogrammetric and Geodetic Technologies Applied to Other Fields of Research*”, 15–18 September 2008, Helsinki University of Technology, Finland

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## Isikuandmed

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## Teenistuskäik

1994–1995 Eesti Põllumajandusülikool, nooremteadur  
1995–1998 Eesti Põllumajandusülikool, assistent  
Alates 1998 Eesti Maaülikool, lektor

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Loodusteadused ja tehnika, Maateadused, Geodeesia (maakoore vertikaalliikumised, nivelleerimised, GNSS)

## Projektid

- 2004–2007 Grant ETF5731: *Gravimeetriliste anomaaliate mõju geoidile Riia- ja Soome label*. Põhitäitja.  
(Vastutav täitja: Harli Jürgenson)
- 2008–2010 Grant ETF7356: *Kosmosetehnoloogia rakendused geoidi ja gravitatsioonivälja täpsustamiseks Eesti alal*. Põhitäitja.  
(Vastutav täitja: Artu Ellmann)
- 2011–2013 Grant ETF8749: *Eesti rannikumere kõrgusraamistiku määramine veetaseme monitooringu ja laserskanneerimise andmetest*. Põhitäitja.  
(Vastutav täitja: Harli Jürgenson)
- 2016 Grant 8P160012MIGO: *Läänemere veetaseme tõusu mõju rannajoone muutustele ja kinnistupiiridele*. Põhitäitja.  
(Vastutav täitja: Aive Liibusk)

## Tunnustused

- 2014 Eesti Maaülikooli raefondi preemia
- 2013 Eesti Maaülikooli innovatiivne õppejõud
- 2005 Kristjan Jaagu stipendium
- 1997 Eesti Teaduste Akadeemia üliõpilastööde konkurss, II preemia

## Juhendatud väitekirjad

Juhendatud väitekirjade (magistrikraad) üldarv – 8

## Publikatsioonid

Teaduslike publikatsioonide üldarv – 25

Tähtsaimad publikatsioonid:

1.1. – 3 publikatsiooni

Kall, T. et al., 2016. Vertical crustal movements in Estonia determined from precise levellings and observations of the level of Lake Peipsi. *Estonian Journal of Earth Sciences*, **65**(1), pp. 27–47, doi: 10.3176/earth.2016.03.

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1.2. – 1 publikatsioon

Jürgenson, H. & Kall, T., 2004. The difference between the N60 and BK77 height systems. *Nordic journal of surveying and real estate research*, **1**(1), pp.72–85.

3.1. – 3 publikatsiooni

Liibusk, A. et al., 2014. Correcting Tide Gauge Series Due to Land Uplift and Differences between National Height Systems of the Baltic Sea Countries. In *IEEE/OES Baltic 2014 International Symposium: 2014 IEEE/OES Baltic International Symposium “Measuring and Modeling of Multi-Scale Interactions in the Marine Environment”*: Tallinn, Estonia, May 26–29, 2014. *IEEE, (IEEE Conference Proceedings)*, pp. 1–8.

Kall, T. & Jürgenson, H., 2008. Postglacial land uplift in Estonia based on geodetic measurements on Põltsamaa-Lelle levelling line. In D. Cygas & K. D. Froehner, eds. *The 7th International Conference “Environmental Engineering”, May 22–23, 2008, Selected Papers*. Vilnius: Vilnius Gediminas Technical University Press “Technika”, pp. 1325–1333.

Kall, T. & Oja, T., 2006. Geodetic and geophysical repeated measurements in geodynamic monitoring networks of Estonia. In F. Sanso & A. J. Gil, eds. *Geodetic Deformation Monitoring: From Geophysical to Engineering Roles*. International Association of Geodesy Symposia. IAG Symposium on Geodetic Deformation Monitoring - From Geophysical to Engineering Roles. Jaen, Spain: SPRINGER-VERLAG BERLIN, pp. 222–230.



## **Erialased täienduskoollitused**

1. Static and Kinematic GPS-based Positioning Using GIPSY/OASIS, 16–18 September, 2014, University of Beira Interior, Covilha, Portugal.
2. International ESA Summer School on GNSS, 1–11 September, 2010, Slettestrand, Denmark
3. IAG School on Reference Frames, 7–12 June, 2010. Mytilene, Greece.
4. Glacial Isostatic Adjustment (GIA) Modelling Training School, 1–5 June 2009, COST Action ES0701, Gävle, Sweden.
5. Graduate School Course in Geomatics “*Photogrammetric and Geodetic Technologies Applied to Other Fields of Research*”, 15–18 September 2008, Helsinki University of Technology, Finland





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FERTILIZATION IN A CROP ROTATION EXPERIMENT  
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Dotsent **Endla Reintam**, emeriitprofessor **Anne Luik**

11. veebruar 2016

### KRISTI PRAAKLE

*CAMPYLOBACTER SPP.* AND *LISTERIA MONOCYTOGENES* IN  
POULTRY PRODUCTS IN ESTONIA

*CAMPYLOBACTER SPP.* JA *LISTERIA MONOCYTOGENES*  
LINNULIHATOODETES EESTIS

Professor **Mati Roasto**, professor **Marja-Liisa Hänninen** (Helsingi Ülikool),  
professor **Hannu Korkeala** (Helsingi Ülikool)

4. märts 2016

ISSN 2382-7076

ISBN 978-9949-536-84-9 (trükis)

ISBN 978-9949-536-85-6 (pdf)

