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### PS 2: Experience with the use of Phase-Shifting transformers

#### Flux distribution in transformer cores of Phase-Shifting transformers

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#### SUMMARY

Phase-shifting transformers (PSTs) are used to control the real power flow in transmission lines. A PST injects a voltage between its source and load side terminals that is in quadrature to the phase-to-ground voltage of the system. As a consequence the load current through the coil pairs of PSTs has a phase shift of close to  $\pm 90^\circ$  relative to the exciting voltage of the transformer or transformer pair, which is equivalent to a capacitive or inductive load of a regular transformer. Due to the superposition of exciting- and stray- flux significant local variations of the core fluxes may occur. The highest local fluxes in the core(s) of a PST are encountered when the PST is used to oppose the “natural” power flow through a transmission line, i.e. in “retard” operation. Even higher fluxes will occur during times of overload in the system. To avoid saturation of the transformer core possible load-dependent local flux variations have to be considered already at the design stage of the transformer.

This paper focuses on the determination of the fraction of load dependent flux that will enter the core and change the flux density distribution inside the transformer core.

The load dependent core flux variations in the series transformer of a two-core PST should be higher than in the exciter transformer, since it typically has the higher reactance. However, the method of calculation of the magnetic load of the transformer core is presented by the example of two exciter transformers with a five limb core, one having a simply concentric winding and one having a double concentric, low reactance winding arrangement, where the fraction of flux entering the core cannot be estimated by obvious assumptions.

The analysis is based on a lumped parameter model of the transformer consisting of winding resistances and inductances only. The winding resistances are determined analytically. The inductances are extracted systematically from a 2D Finite Element model of the transformer.

#### KEYWORDS

Phase-shifting transformer, retard operation, magnetic stray flux, capacitive loading of transformer, finite element model of transformer, lumped parameter model of transformer,

# 1 INTRODUCTION

Phase-shifting transformers (PSTs) are used to control the real power flow in transmission lines. A PST injects a controllable voltage  $\Delta V$  between its source and load side terminals that has a phase shift of about  $\pm 90^\circ$  to the phase-to-ground voltage of the system. Due to this  $90^\circ$  phase shift of  $\Delta V$  the PST controls real power driven through the lines, in contrast to the control of reactive power by in-phase voltage control of conventional power transformers.

As will be shown below this way of operation corresponds to a predominantly reactive loading of the transformer(s), inductive when the PST is used to boost power flow (advance operation) and capacitive when it is used to buck the natural power flow (retard operation).

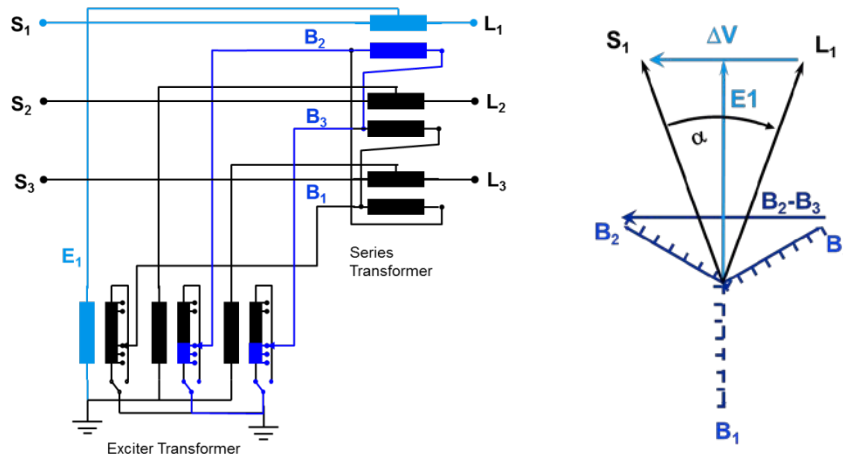
As in conventional transformers capacitive loading will lead to an increase of the flux density in parts of the core due to superposition of main flux and load dependent stray flux.

In order to avoid saturation in PSTs that are designed to operate in retard mode it is important to know which fraction of the stray flux will actually enter the transformer cores.

This paper focuses on the determination of the fraction of load dependent flux that will enter the core and change the flux density distribution inside the transformer core. The most commonly used two-core PST design concept is used to highlight the effects.

# 2 FLUX VARIATIONS WHEN LOADING PSTS

In a two core PST one transformer – the series transformer - is connected in series with the transmission line by its source and load side terminals S and L. The second transformer - the exciter transformer - is excited by the phase to ground voltages of the three-phase system. The series transformer is excited with a controllable voltage taken from the tap windings of the exciter transformer, see Figure 1. By taking the phase-to-phase voltage ( $B_2-B_3$ ,  $B_3-B_1$ ,  $B_1-B_2$ ) of two phases of the tap windings of the exciter transformer to excite the remaining phase of the series transformer one obtains the desired  $90^\circ$  phase shift for the voltage  $\Delta V$  injected between the S and L terminals.

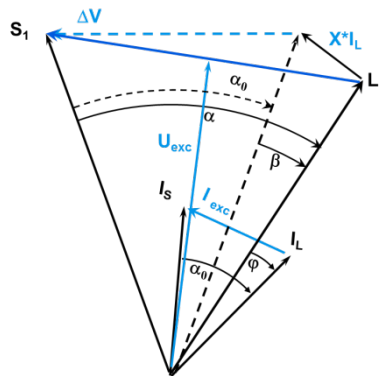


**Figure 1: Connection diagram and phasor diagram at no-load for a symmetrical two-core PST.**

The phase shift between voltages to ground at S and L at no-load and the phase shift between the S and L side load currents,  $\alpha_0$ , is given by the turn numbers. However, for a resistive-inductive load, load side current  $I_L$  lags the load side voltage  $L_1$  by  $\phi$ , see Figure 2.  $L_1$  lags the no-load load side voltage (dashed) by  $\beta$  due to the voltage drop  $X \cdot I_L$  at the PST's impedance. The exciter voltage  $U_{exc}$  is taken from the middle of the series winding between  $S_1$  and  $L_1$ . The vectorial difference between  $I_S$  and  $I_L$  is the current that flows into the exciter winding  $I_{exc}$ .  $I_{exc}$  leads  $U_{exc}$  by roughly  $90^\circ$ , which corresponds to a strongly capacitive load.

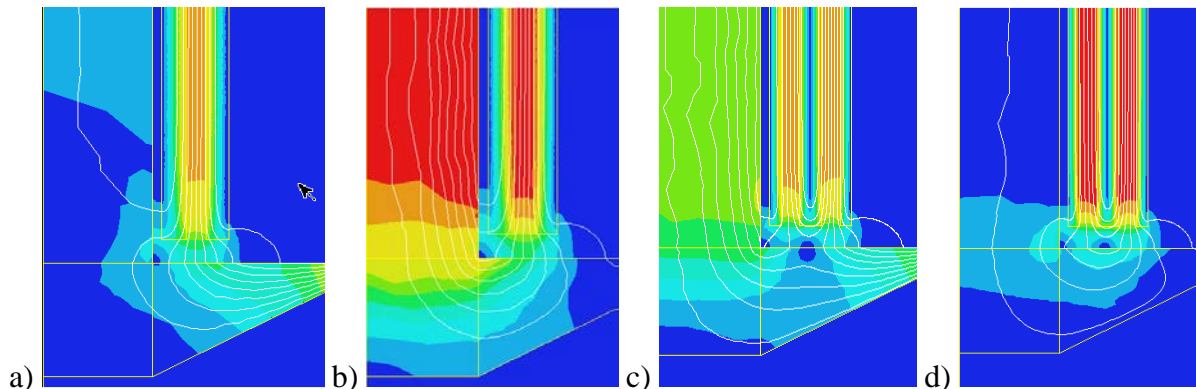
$U_{exc}$  leads the main limb induction of the exciter transformer by  $90^\circ$ , which means that the main limb flux is in phase opposition to the stray flux caused by  $I_{exc}$  inside the exciter winding. When the exciter

winding is the outermost winding and  $U_{exc}$  remains constant the sum of fluxes linked to the excitation has to remain constant as well. Consequently the flux in the main limb has to increase, see Figure 3b). When the exciter winding is closest to the core the main limb flux is fixed by  $U_{exc}$  and the stray flux lines have to close via the yokes, leading to an increased flux density there, see Figure 3a).



**Figure 2: Phasor diagram for one phase for a resistive-inductive load.**

It can be shown that the same effect occurs in the series transformer where the role of the exciter winding is taken by the excited winding (between  $B_2$  and  $B_3$  in Figure 1), the exciting voltage is  $B_2$ - $B_3$ . Figure 3 a) and b) illustrate the stray flux distributions for simply concentric winding arrangements discussed above. Figure 3 c) and d) show that it is more difficult to even qualitatively predict the stray flux distribution for double concentric winding arrangements which are frequently used in the exciter transformers of PSTs.



**Figure 3: Typical stray flux distribution for simply concentric, a) and b) and for double concentric winding arrangements c) and d). a): Voltage kept constant at inner winding, resistive current given for outer windings, at time of current max. b) Voltage constant at outer winding. c) Double concentric tap winding. Voltage kept constant at center (exciter) winding, resistive current given for innermost and outermost windings. d) Double concentric exciting winding. Voltage constant at inner and outer windings, resistive current given for middle winding.**

Usually the load dependent core flux variations are higher in the series transformer than in the exciter transformer, since it typically has the higher reactance. However, the method of calculation of the magnetic load of the transformer core is presented by the example of two exciter transformers with a five limb core, one having a simply concentric winding and one having a double concentric, low reactance winding arrangement, where the fraction of stray flux entering the core cannot be estimated by obvious assumptions.

## 2.1 Transformer model

The applied transformer model is based on lumped parameters. The working point is calculated by the differential equations

$$\mathbf{u}(t) = \mathbf{R} \cdot \mathbf{i}(t) + \frac{d}{dt} \Psi(t),$$

with the phase voltages  $\mathbf{u}(t)$ , the phase currents  $\mathbf{i}(t)$ , and the flux linkage matrix  $\Psi(t) = \mathbf{L} \cdot \mathbf{i}(t)$ . Any capacitive effects are neglected. By neglecting eddy current effects inside the winding, the winding resistances  $\mathbf{R}$  are calculated analytically by geometrical proportions.

The flux linkage matrix  $\Psi$  is determined by the inductance matrix  $\mathbf{L}$ , which is systematically extracted from a Finite Element (FE) model of the transformer. In this paper linear material properties of the transformer core are proposed. Therefore the inductance matrix does not depend on the working point. Once the inductance matrix is extracted, each arbitrary working point can be calculated with negligible computational cost and same accuracy compared to a FE computation.

The extraction of the inductances is based on a magnetic vector potential approach and can be applied to 2D and 3D FE models of the transformer [1]. In a 2D FE model the transformer is represented by its middle cross-sectional area (see Figure 6). Dependent on the depth ( $d_{2D}$ , see Figure 4), the main flux or the load dependent stray flux can be represented by a 2D FE model. The adoption can be derived by analytical considerations based on the magnetic energy at no load (nl), respectively at short circuit (sc). Figure 4 compares the magnetic energy determined by 3D and two 2D FE models of a transformer for different working points. The depth of the model FE2Dnl is adapted to represent the main flux:  $d_{2Dnl} = \frac{\pi \cdot d_{core}}{4}$ . The depth of the model FE2Dsc is adapted to represent the load dependent stray flux considering:  $d_{2Dsc} = \pi \cdot \frac{d_m}{2}$ , with  $d_m$  the mean diameter of the winding blocks.

For capacitive (cap), resistive (res), and inductive (ind) load at nominal current, Figure 4 illustrates that the magnetic energy inside the transformer core is considerably lower for the FE2Dsc model, however the stray flux and therefore the magnetic energy inside the air and the coil regions is in good agreement.

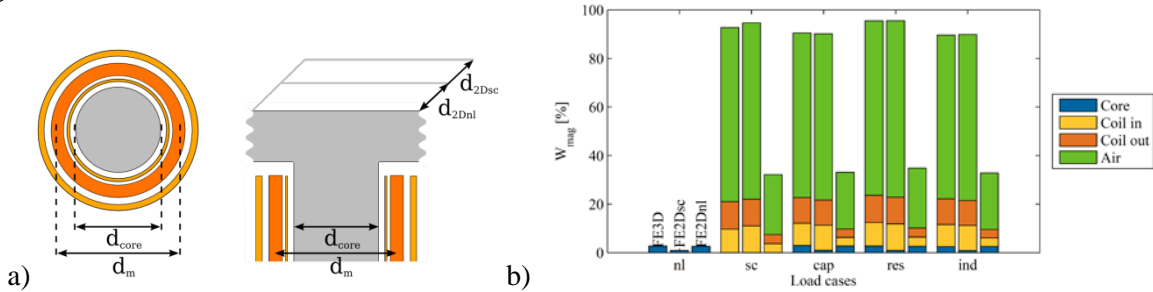


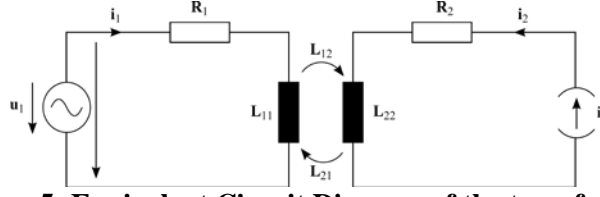
Figure 4: a) left: Top view on a limb and winding, right: Side view, with definition of depth  $d_{2D}$   
b) Comparison of the magnetic energy for different load cases and for 3D and 2D FE models

In a 2D FE model all 3D-effects, e.g. magnetic fluxes that close outside the cross-sectional area and influences of stray fluxes entering structural components, e.g. the transformer tank, cannot be considered. Therefore the determination of load dependent stray fluxes entering the core will be a worst case approximation. However, the extraction of the inductance matrix by means of a 2D FE model yields a significant lower modelling effort and computational cost which is desirable for an initial estimation of stray fluxes which enter the core, especially in an early design stage of the transformer.

To determine the working point, the transformer is excited by voltage sources applied to the primary windings and current sources applied to the secondary windings or vice versa, see Figure 5. Considering a steady state operation with  $\frac{d}{dt} = j\omega$ , the primary currents are calculated by:

$$\mathbf{i}_1 = (\mathbf{R}_1 + j\omega\mathbf{L}_{11})^{-1} \cdot (\mathbf{u}_1 - j\omega\mathbf{L}_{12}\mathbf{i}_2).$$

$\mathbf{L}_{11,22}$  is the self and mutual inductance matrix of the primary respectively the secondary windings and  $\mathbf{L}_{12} = \mathbf{L}_{21}$  is the mutual inductance matrix between the primary and the secondary winding system.



**Figure 5: Equivalent Circuit Diagram of the transformer.**

Subsequent to the determination of the primary currents the flux inside the core is calculated at certain positions ( $loc, i, i=1 \dots n$ ) of the yoke, main limb and side limb, see Figure 6:

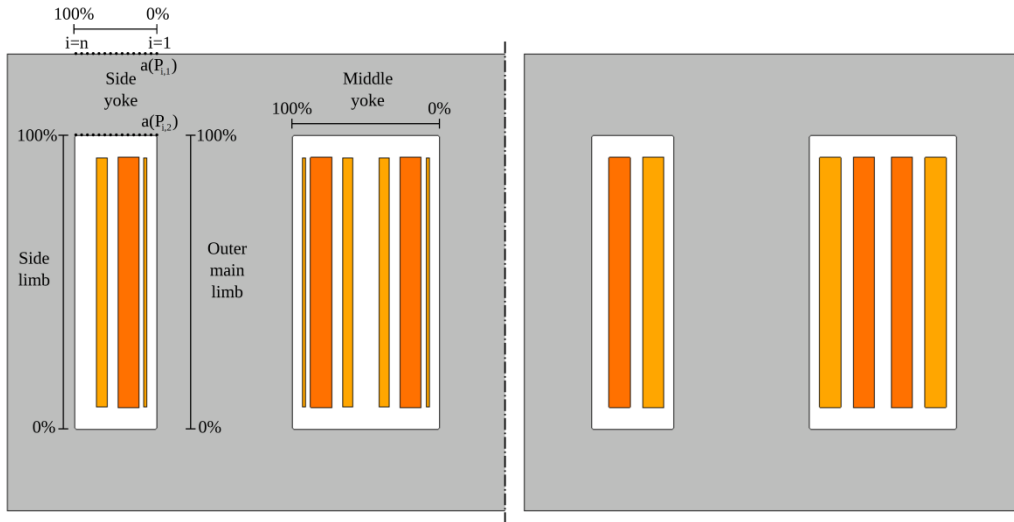
$$\varphi|_{loc,i} = \sum_{r=1}^{2m} \varphi_r|_{loc,i} \cdot i_r \quad \text{with} \quad \varphi_r|_{loc,i} = \oint \mathbf{a} dC_i \stackrel{\text{(for 2D FE)}}{=} (a(P_{i,1}) - a(P_{i,2})) \cdot d_{2D}.$$

The magnetic vector potential  $\mathbf{a}$ , for the excitation of each phase  $m$ , with the current shape function  $w$  is already determined for the extraction of the inductance matrix (for 2D FE  $w = \frac{N}{A_{coil}}$ , with  $N$  number of turns and  $A_{coil}$  cross-sectional area of the coil).

Towards the calculation of the flux distribution the mean flux-density distribution is calculated by

$$\overline{B}|_{loc,i} = \frac{\varphi|_{loc,i}}{A_{cs,i}},$$

with  $A_{cs,i}$ , the flux penetrating cross-section area of the transformer core.



**Figure 6: Section of the exciter transformer of a PST for two winding arrangements. Due to symmetry only the left halves of the core and windings are shown. Left: with double concentric windings, winding A, light orange, split 30%/70% (Lengths of limbs and yokes are given in percent for reference in the results), right: with simply concentric windings, winding A is the outer winding.**

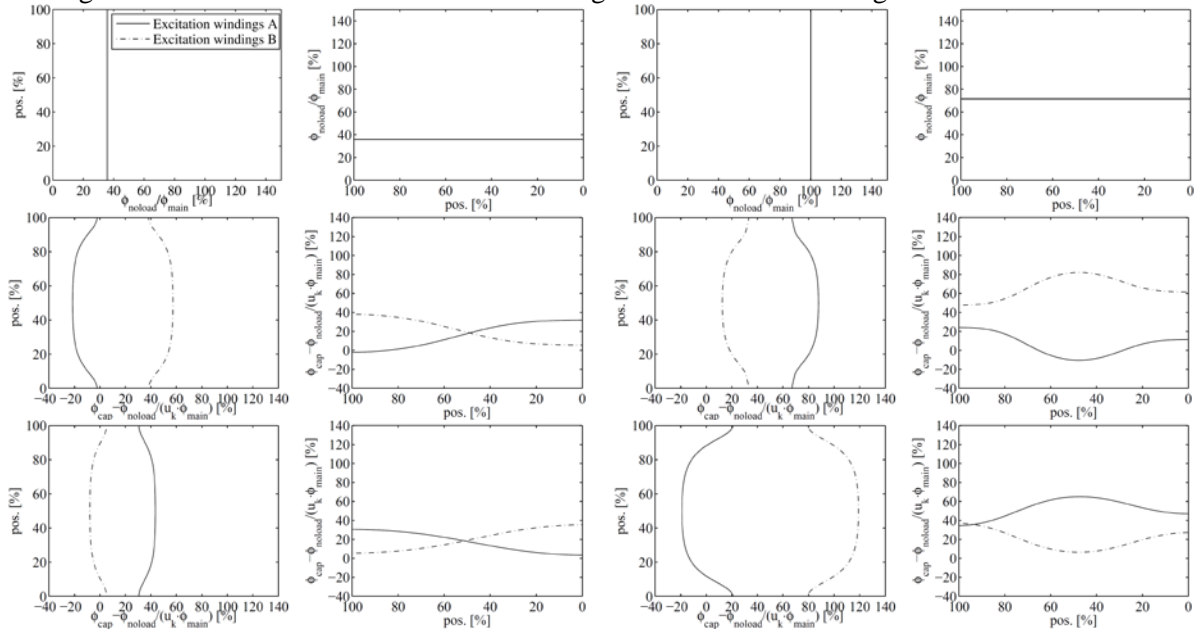
Below, the proposed transformer model is applied to identify local flux variations of two exciter transformers of a PST with double concentric and simply concentric windings as depicted in Figure 6.

## 2.2 Discussion of local flux variations of transformers due to capacitive load

The load of a PST's exciter transformer in retard operation corresponds to a capacitive load in regular transformers. For nominal current this load case is exemplarily given for phase L1 as:

$$\underline{u}_{L1,1} = U_{nom} \cdot e^{j0} \quad \underline{i}_{L1,2} = I_{2,nom} \cdot e^{-j90}.$$

The flux distribution for this working point is calculated for both exciter transformers depicted in Figure 6. For the exciter transformer with double concentric windings the inner and the outermost windings are in series connection and form winding A. The middle windings are labelled B. The



**Figure 7: Relative flux distributions vs. position along the core members, see Figure 6.**

**From left to right: side limb, vertical; side yoke, horizontal; main limb, vertical; middle yoke, horizontal.**

**Top row: for no-load, relative to rated flux in main limb,**

**Middle row: for capacitive load, flux variation related to reference stray flux ( $\Phi_{cap} - \Phi_{noload}$ ) / ( $u_k \cdot \Phi_{main}$ ), Double concentric.**

**Bottom row: for capacitive load, flux variation related to reference stray flux, Simply concentric.**

relative short circuit reactance, which is a measure for the stray flux, is  $u_k=8.2\%$ . For the exciter transformer with simply concentric winding arrangement the inner windings are labelled A and the outer windings are labelled B.

The relative short circuit reactance is  $u_k=22.2\%$ . Excitation is either from windings A or B, both cases are considered.

The top row of Figure 7 depicts the no-load flux related to the rated main flux. The flux is uniformly distributed along each section. Neither the way of excitation nor the winding arrangement influences the flux distribution. The flux density is at rated induction at the main limb and due to a yoke cross section  $A_{yoke} = 0.6 \cdot A_{main,limb}$  at its maximum at the middle yoke. Dependent on the rated core induction saturation may begin at the middle yoke. In this case, a fraction of the flux inside the middle yoke will be displaced to the side yoke and the side limb. The simulations presented here were performed using linear, non-saturating core material. In reference simulations it was found that saturation tends to produce a more equalized flux distribution. However, the focus of this work was to find the fraction of the stray flux that will enter the core. It was found that this fraction can be determined from linear simulations with only small error.

In the middle and bottom row of Figure 7 local flux variations due to the capacitive load are depicted for the double concentric winding (middle row) and the simply concentric winding arrangement (bottom row). The flux variation is related to a reference stray flux  $\phi_{stray,ref} = u_k \cdot \phi_{main}$ .

Due to the significant larger short circuit reactance the flux variations inside the core of the exciter transformer with simply concentric windings are much stronger. For both transformers it can be recognized, that dependent on the exciting voltage the flux is concentrated at the T-joints of the core or at the middle yoke, respectively the side yoke and side limb. Especially for transformers with a large short circuit reactance this effect is pronounced.

For the simply concentric winding arrangement one can see that only a small fraction of the stray flux enters the main limb, and up to 65% enters the main yoke when the innermost winding A is excited (Figure 6, bottom, right, full lines). When the outermost winding B is excited about 100% of the stray flux enters the main limb and the main yoke sees only about 20% (dashed lines). This is in agreement with the measurements shown in reference [2].

The flux distribution in double concentric winding arrangement, with its 30/70 split winding A excited resembles the one for the simply concentric arrangement with the outer winding B excited, but with less pronounced impact on the main limb and on the yoke. By contrast, when winding B in the middle is excited up to 80% of the stray flux enters the yoke.

As discussed in 2.1 in a 2D FE model all alternative magnetic flux paths that close outside the core e.g. the transformer tank, are considered. Therefore the determination of load dependent stray fluxes entering the core will be a worst case approximation.

### **3 CONCLUSION**

When PSTs are to be operated in the retard mode one needs to take into account the significant amount of stray flux that enters parts of the core and is in phase with the main flux. Knowledge of the fraction of stray flux entering the core is essential to avoid saturation effects in operation. A linear 2D FE model of the transformer is sufficient to determine this fraction of stray flux with sufficient accuracy.

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