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The influence of visual-spatial skills on the association between processing of non-symbolic numerical magnitude and number word sequence skills

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Abstract

Non-symbolic numerical magnitude processing skills are assumed to be fundamental to mathematical learning. Recent findings suggest that visual-spatial skills account for associations between children's performance in visually presented non-symbolic numerical magnitude comparison tasks and their performance in visually presented arithmetic tasks. The aim of the present study was to examine whether associations between children's performance in visually presented tasks assessing non-symbolic numerical magnitude processing skills and in tasks assessing early mathematical skills, which do not involve visual stimulation, may also be mediated by visual-spatial skills. This line of reasoning is based on the assumption that children make use of mental visualization processes when working on tasks assessing early mathematical skills, like the knowledge of the sequence of number words, even when these tasks do not involve visual stimulation. We assessed four- to six-year old children's performance in a non-symbolic numerical magnitude comparison task, in tasks concerning the knowledge of the sequence of number words and in a developmental test to assess visual-spatial skills. Children's non-symbolic numerical magnitude processing skills were found to be associated with their number word sequence skills. This association was fully mediated by inter-individual differences in visual-spatial skills. The effect size of this mediation effect was small. We assume that the ability to construct mental visualizations constitutes the key factor underlying this mediation effect.

Keywords: approximate number system; numerical magnitude processing; number word sequence skills; visual perception; mental visualization

Introduction

Human beings are assumed to possess an evolutionarily ancient, innate system dedicated to extracting and representing approximate numerical magnitude information. This so-called approximate number system (ANS; see Piazza, 2010, for an overview) enables us to discriminate between sets of different numerical quantities, a crucial ability for everyday life. We are faster and more accurate in comparing two visually presented dot arrays with respect to their quantity the more the ratio deviates from one (e.g., van Oeffelen and Vos, 1982). The ability to discriminate between sets of different numerical quantities has been observed in preverbal infants (e.g., Izard et al., 2009), and it undergoes a progressive refinement throughout development (Halberda, Ly, Wilmer, Naiman, & Germine, 2012; Piazza, 2010). Besides this developmental variation, individuals of the same age have also been shown to vary in their ability to discriminate between sets of different numerical quantities and it has been hypothesized that these inter-individual differences are linked to mathematical skills (e.g., Halberda, Mazzocco, & Feigenson, 2008). Recent meta-analyses have lent support to this notion by showing a significant association between non-symbolic numerical magnitude processing skills and symbolic math performance (Chen & Li, 2014; Fazio, Bailey, Thompson, & Siegler, 2014; Schneider et al., 2017). According to Chen and Li (2014), the association remains significant when considering potential moderators like general cognitive abilities, and it does not differ significantly between children and adults. On the other hand, Fazio et al. (2014) reported higher correlations for children compared to adults and Schneider et al. (2017) also detected a small moderating effect of age.

Visual perception skills have recently been suggested to account for the association between non-symbolic numerical magnitude processing skills and arithmetic performance (see Zhou, Wei, Zhang, Cui, & Chen, 2015). Indeed, according to the triple-code model (Dehaene, 1992; Dehaene & Cohen, 1995) processing of numerical magnitude information is assumed to involve a visual-spatial representation, i.e. an internal number line, with quantities being represented as variable distributions of activation. It is further assumed that these numerical magnitude representations rely on neural circuitries initially developed for coding internal representations of space and provide the basis for mental arithmetic (“neuronal recycling hypothesis“, Dehaene, 2005; Dehaene & Cohen, 2007).

Moreover, researchers have demonstrated that the performance in visually presented tasks assessing non-symbolic numerical magnitude processing skills is influenced by the visual properties of the stimuli (e.g., the aggregate surface of the items and the density or interitem spacing; cf. Gebuis & Reynvoet, 2012). Gebuis, Kadosh and Gevers (2016) consequently suggested that performance in visually presented non-symbolic numerical magnitude comparison tasks depends on the ability to integrate different visual cues. Cognitive control skills are also assumed to play a role in flexibly adapting the weights of different visual cues as well as inhibiting responses to salient visual cues which are incongruent with the numerical dimension. For example, the processing of the comparison pair with larger aggregate surface of the items in one array but larger number of items in the other array (arrays on the right and left side of Fig. 1a, respectively) should require the inhibition of a right hand response as well as the integration of information about the aggregate surface of the items and the interitem spacing. Gebuis and colleagues (2016) thus suggested that the association between non-symbolic numerical magnitude processing skills and arithmetic performance might be mediated by the ability to combine different sensory cues and by cognitive control skills.

To test their so-called visual perception hypothesis, Zhou and colleagues (2015) asked Chinese third- to fifth-grade children to work on different tasks including non-symbolic numerical magnitude comparison and arithmetic, visual-spatial tasks like figure matching, visual tracing, and mental rotation as well as other cognitive control tasks concerning verbal working memory, choice reaction time, and reasoning abilities. While the association between non-symbolic numerical magnitude processing skills and arithmetic performance was still significant after controlling for visual tracing, mental rotation, verbal working memory, choice reaction time, and reasoning abilities, it was no longer significant after controlling for figure matching skills. In the figure matching task, abstract line figures were presented and children were asked to decide whether a target figure (presented on the left hand side of a computer screen) also appeared in a row of another three figures (presented on the right hand side of a computer screen). According to Zhou and colleagues (2015), this kind of visual perceptual processing might be critical for both non-symbolic numerical magnitude processing and arithmetic performance and thus account for their association. In particular, Zhou and colleagues (2015) suggest that the speed of visual perception might be crucial because their stimuli in the figure matching task as well as in the non-symbolic numerical magnitude comparison task were presented briefly (400 ms and 200 ms, respectively) and their visually presented arithmetic tasks were also assessed under time pressure. However, they also point out that several studies revealed an association between non-symbolic numerical magnitude processing and early mathematical skills, for example knowledge of the sequence of number words, which are assessed without time constraints and without visual stimulation (e.g., Halberda et al., 2008; Mussolin, Nys, & Leybaert, 2012; Wagner Fuhs & McNeil, 2013). Zhou and colleagues (2015) assume that uncontrolled factors such as attention, working memory, processing speed, executive functions, or general intelligence rather than visual perception may underlie this association. Developmental theories, however, suggest that children construct a visual-spatial representation of numerical information, i.e. a mental number line, before they enter school (e.g., Resnick, 1983). At the beginning of its development, the mental number line is assumed to be an ordinal representation of the sequence of number words, which, for example, enables children to identify preceding and succeeding numbers of given ones (e.g., Fritz, Ehlert, & Balzer, 2013). In line with this assumption, it has been demonstrated recently that number word sequence skills of six- to ten-year old children are associated with their spatial visualization skills (Zhang et al., 2014). Linn and Petersen (1985) distinguished three types of spatial skills, i.e. spatial perception (determining spatial relationships with respect to one's own orientation), mental rotation (mentally rotating a two or three dimensional figure), and spatial visualization (processing complicated, multistep manipulations of spatially presented information). While spatial perception and mental rotation typically require only a single solution strategy, spatial visualization requires multiple solution strategies and a flexible adaptation of a repertoire of solution procedures (Linn & Petersen, 1985). For example, spatial visualization skills are assessed by tasks like the Figure-Ground task in which respondents are asked to find given shapes in a complex, confusing background. The findings by Zhang and colleagues (2014) suggest that children make use of spatial visualization processes when being confronted with tasks assessing their knowledge of the sequence of number words even without visual stimulation. Theoretically, spatial visualization skills may also be involved in non-symbolic numerical magnitude processing in order to capture and process the respective stimuli, which is assumed to require the integration and the flexible adaptation of the weights of their different visual-spatial cues (see Gebuis et al., 2016). If so, the association between non-symbolic numerical magnitude processing and early mathematical skills like knowledge of the sequence of number words may also be mediated by spatial visualization skills.

In the present study, we addressed this issue by assessing four- to six-year old children's performance in a non-symbolic numerical magnitude comparison task, in tasks concerning the knowledge of the sequence of number words, and in a developmental test of visual perception (DTVP-3, Hammill, Pearson, & Voress, 2014). The visual properties of the stimuli used in the non-symbolic numerical magnitude comparison task were systematically varied, allowing us to examine whether the association between non-symbolic numerical magnitude processing skills and number word sequence skills might differ for trials in the non-symbolic numerical magnitude comparison task in which specific visual cues were congruent or incongruent with the numerical dimension. The number word sequence tasks did not include any visual cues, and performance in these tasks has been shown to predict mathematical school achievement (Krajewski & Schneider, 2009). Likewise, performance in the DTVP-3 was shown to be associated with mathematical school achievement (Hammill et al., 2014). The DTVP-3 includes five subtests: 1) Eye-Hand Coordination, 2) Copying, 3) Figure-Ground, 4) Visual Closure, and 5) Form Constancy. Spatial visualization skills, as conceptualized by Linn and Peterson (1985), can be assumed to be captured by the Figure-Ground subtest which requires the identification of figures hidden in a complex, confusing background. Accordingly, we examined whether the association between non-symbolic numerical magnitude processing and number word sequence knowledge may be mediated by children’s performance in the Figure-Ground subtest. By running supplementary analyses, we additionally investigated whether performance in another subtest of the DTVP-3 may significantly mediate the association between performance in a non-symbolic numerical magnitude comparison task and performance in number word sequence tasks.

Method

Participants

A total of 156 typically developing children (80 girls) from different kindergartens in the region of

Frankfurt am Main, Germany, participated in this study. Parental informed and written consent was obtained for each child. Children additionally provided verbal assent to participate in this study and were compensated for participation (e.g. by receiving a pencil). The children's age ranged from 4 years and 2 months to 6 years and 11 months (mean age = 5 years and 5 months, SD = 10 months). None of the children attended school.

Material and Procedure

All children started with the Developmental Test of Visual Perception (DTVP-3, Hammill et al. 2014), then proceeded with the non-symbolic numerical magnitude comparison task, and finally worked on the number word sequence tasks. The different tasks were carried out individually.

Developmental Test of Visual Perception (DTVP-3)

The DTVP-3 is a standardized test of visual perception for children aged 4 to 12. In the present study, we used a German adaptation of the DTVP-3 (see Frostigs Entwicklungstest der visuellen Wahrnehmung-3, FEW-3, Büttner, Dacheneder, Müller, Schneider, & Hasselhorn, in preparation). The stimulus material and the instructions are identical for both tests but the ceiling rule differs. While the ceiling for the DTVP-3 is a score of zero after three consecutive items, the ceiling for the German adaptation is a score of zero after four consecutive items. The five subtests of the DTVP-3 will be described in the following.

Eye-Hand Coordination. Children are asked to draw a line within given visual boundaries and without lifting the pencil. The subtest is composed of five items that increase in difficulty from straight short lines to curved lines. For items 1 and 2, 1 point is awarded for predefined segments in which the child's line has not extended beyond the boundaries of the segment. In items 3, 4, and 5, the child can be awarded 0 to 4 points. The maximum score is 192 and there is no ceiling in this subtest. Consequently, all children are asked to work on all the five items. Internal consistency in the sample of this study was $\alpha = .77$.

Copying. Children are shown different figures and asked to draw them on a piece of paper. The 18 different figures increase in difficulty. The child can receive 0 to 3 points for each item (maximum score is 54). Internal consistency for this subtest in the sample was α = .88.

Figure-Ground. Children are shown different figures and are asked to find as many of the figures as possible on a page where some of the figures are hidden in a complex, confusing background. The subtest comprises 23 items with increasing difficulty. The child receives 1 point for each shape correctly identified for those items not receiving a 0. An item is scored 0 if the child selected a shape that is not in the stimulus drawing. The maximum score is 69. Internal consistency in the sample was α = .77.

Visual Closure. Children are shown a figure and asked to select the exact figure from a series of figures that have been incompletely drawn. The subtest comprises 26 items with increasing difficulty (maximum score is 26). Internal consistency in the sample was α = .78.

Form Constancy. Children are shown a stimulus figure and asked to find it in a series of figures. The targeted figure has a different size, position, and/or shade, and it may be hidden in a distracting background. The subtest comprises 24 items with increasing difficulty and the maximum score is 51. Internal consistency for this subtest in the sample was α = .69.

On every subtest testing begins with item 1. Scoring for subtest Eye-Hand Coordination and Copying is a standardized and stepwise procedure of using a scoring template for measuring angles and lines but also comparing children's free hand drawings with stimulus drawings of the manual. In order to guarantee objective scoring, all free-hand drawings of children were scored independently by two research assistants. In case of disagreements a third and last resort decided the case. As the subtests have different score ranges (maximum scores for subtest 1 to 5 are 192, 54, 69, 26 and 51), raw scores were transformed into percentages (of the respective maximum score) in order to yield the same scale for all subtests. The respective scores were not normally distributed (Shapiro-Wilk test: Eye-Hand Coordination: $p < .001$, Copying: $p = .002$, Figure-Ground: $p = .002$, Visual Closure: $p = .017$).

Non-symbolic numerical magnitude comparison task

Sets of black dots were presented in two white squares on the left and the right hand side of the screen on a computer running Presentation® software (Neurobehavioral Systems, Inc.). From a viewing distance of about 60 cm, each of the white squares had a visual angle of 9.91° (104 mm). On each trial, one of the white squares contained 32 dots (reference numerosities) and the other one 14, 20, 26, 38, 44 or 50 dots (deviants). This resulted in six different comparison pairs. Each of the six comparison pairs appeared eight times, four times with the reference numerosity on the left and four times on the right hand side. Every single comparison pair had a unique configuration of dots. The dot sets were created using a Matlab script of Gebuis and Reynvoet (2011) varying different visual properties of the stimuli (i.e., area extended [convex hull], total surface [the aggregate surface of all dots in one array], density [area extended/total surface], item size [average diameter of the dots presented in one array], and total circumference [circumference of all dots in one array, taken together]) so that no single visual cue was informative about numerical magnitude across all trials (see Fig. 1 for exemplary depictions). Each of the five different visual cue conditions involved trials in which the respective visual cue was congruent or incongruent with the numerical dimension. Children were asked to indicate, without using counting strategies, the side of the larger numerical magnitude by answering with the left index finger when it was larger on the left hand side and by using the right index finger when it was larger on the right hand side. Responses were given by pressing the left and right CTRL-buttons of the computer keyboard. Reaction times (RT) and errors (ER) were recorded, and the instruction stressed both speed and accuracy. The order of trials was pseudo-randomized to avoid consecutive identical comparison pairs. The experiment started with six warm-up trials (data not recorded), followed by in total 48 experimental trials (6 comparison pairs × 8 repetitions). The experimenter pressed a button to start a trial, whereupon a black screen was presented for 1000 ms. After the black screen had vanished, the target appeared until a response was given, but only up to a maximum duration of 6000 ms. If no response was given, a trial was classified as erroneous. No feedback was given regarding the correctness of responses. Mean RT and mean ER were used as individual markers of the ANS (see e.g., Inglis & Gilmore, 2014, for a discussion on different indices of the ANS). Correct responses were used for computing mean response times. Response times below 200 ms were excluded from further analysis. This trimming resulted in .12% of response exclusions.

While response times were not normally distributed (Shapiro-Wilk test: $p < .001$), ER was normally distributed (Shapiro-Wilk test: $p = .45$). To estimate the reliability of the non-symbolic numerical magnitude comparison task, the Spearman's rank correlation coefficient between RT in odd and even trials ($r = .86$, $p < .001$) and ER in odd and even trials ($r = .68$, $p < .001$) was computed.

Number word sequence tasks

The number word sequence tasks assessed how well children knew the precise number word sequence forward and backward and whether they could identify individual elements within the sequence. Children were asked to recite the number word sequence forwards up from 1 to 31 (scoring a 1 for correctly reciting up to 10, scoring a 2 for reciting up to 20, and a 3 for reciting up to 30) and backwards from 5 to 1 (scoring a 2 for correctly reciting the complete sequence and a 1 if at least two numbers were recited in the correct order). The children were also asked to name subsequent numbers after a designated one (namely 5, 9, and 18; maximum score = 3) as well as numbers preceding a given one (namely 4, 8, and 12; maximum score = 3). Total scores ranging from 0 to 11 were used to estimate children's knowledge of the sequence of number words. These scores were not normally distributed (Shapiro-Wilk test: $p < .001$). To estimate the reliability of the number word sequence tasks, internal consistency was computed ($\alpha = .87$).

Statistical Analyses

To assess effects of ratio between the two to-be-compared numerical magnitudes in the non-symbolic numerical magnitude comparison task, we collapsed trials with deviants smaller than the reference (14, 20, 26) and trials with deviants larger than the reference (38, 44, 50) into three levels of ratio (14 or 50 vs. 32, 20 or 44 vs. 32, 26 or 38 vs. 32) and looked for linear trends separately for RT and ER.

Further analyses were carried out with Mplus (Version 7.2; Muthén & Muthén, 2015) using robust maximum-likelihood estimators to account for the fact that some measures were not normally distributed. To verify associations between the different variables, we employed correlational as well as partial correlation analyses (controlling for age). To check whether the association between non-symbolic numerical magnitude processing skills and number word sequence skills may differ for trials in the non-symbolic numerical magnitude comparison task, in which specific visual cues were congruent or incongruent with the numerical dimension, partial correlation analyses (controlling for age) were conducted separately for the five different visual cue conditions (area extended, total surface, density, item size, total circumference) and for congruent and incongruent trials. The respective correlation coefficients were compared directly by using Wald tests. Additionally, ER in congruent and incongruent trials of the non-symbolic numerical magnitude comparison task were compared separately for each of the five different visual cue conditions.

We used mediation analyses to test whether the association between non-symbolic numerical magnitude processing and number word sequence skills is mediated by spatial visualization skills. Children's spatial visualization skills were assessed by their performance in the Figure-Ground subtest of the DTVP-3. On the one hand, mediation analysis allows to investigate direct associations used in this study to examine the relation between non-symbolic numerical magnitude processing and number word sequence skills, while holding constant the spatial visualization skills. On the other hand, mediation analysis provides estimates of the statistical significance of indirect associations, used in this study to evaluate whether spatial visualization skills mediate the association between non-symbolic numerical magnitude processing and number word sequence skills. In subsequent mediation analyses, four separate models were tested each including performance in one of the four other subtests of the DTVP-3 as a potential mediator. Age was used as control variable in all mediation models. To assess the statistical significance of indirect associations, a bootstrapping method with bias-corrected confidence estimates was used. Confidence intervals (95%) for the indirect associations were obtained using 5000 bootstrap samples. If a confidence interval does not include zero, the indirect effect is deemed statistically different from zero representing evidence for a mediating effect (e.g., Hayes & Preacher, 2014).

Results

Demonstrating the signature of the ANS, ER in the non-symbolic numerical magnitude comparison task decreased the more the ratio between the two to-be-compared numerosities deviated from one (26 or 38 vs. 32: ER = 45%, 20 or 44 vs. 32: ER = 36%, 14 or 50 vs. 32: ER = 30%; $F(1, 155) = 167.34$, $p < .001$, $\eta_p^2 = .52$). This represents a large effect according to Cohen (1988). By contrast, RT in the non-symbolic numerical magnitude comparison task increased the more the ratio deviated from one (26 or 38 vs. 32: RT = 1642 ms, 20 or 44 vs. 32: RT = 1728 ms, 14 or 50 vs. 32: RT = 1743 ms; $F(1, 155) = 11.50$, $p = .001$, $\eta_p^2 = .07$). There was, however, no indication of a speed-accuracy-trade-off ($r = .12$, $p = .15$). When using individual level median RT instead of mean RT, there was no significant effect of ratio (Friedman's ANOVA: $\chi\ 2(2) = 3.73$, $p = .16$) suggesting that the reversed ratio effect based on mean RT is mainly due to outlier trials with relatively long response times. As the signature of the ANS was only detected on the basis of ER but not on the basis of RT, ER in the non-symbolic numerical magnitude comparison task was used as an indicator of non-symbolic numerical magnitude processing skills in subsequent analyses. In this regard, it has recently been demonstrated that accuracy/ER based measures are more informative about the underlying ANS acuity than RT based measures (see Dietrich et al., 2016). In addition, recent meta-analyses revealed higher correlations between non-symbolic numerical magnitude processing skills and symbolic math performance for overall accuracy/ER compared to overall RT in a non-symbolic numerical magnitude processing task (Fazio et al., 2014; Schneider et al., 2017).

Descriptive statistics and correlations between the different variables are shown in Table 1 (see also Fig. A.1). Most importantly, ER in the non-symbolic numerical magnitude comparison task, performance in the number word sequence tasks, and performance in the Figure-Ground subtest (spatial visualization skills) were found to be significantly interrelated even after controlling for age.

Comparison of ER in congruent and incongruent trials of the non-symbolic numerical magnitude comparison task revealed significant differences for each of the five different visual cue conditions (see Table A.1). Observed correlations between number word sequence skills and performance in trials of the non-symbolic numerical magnitude comparison task, in which different visual cues were either congruent or incongruent with the numerical dimension are as follows: area extended congruent $r = -.14$, $p = .08$ [two-sided], area extended incongruent: $r = -.11$, $p = .19$ [two-sided]; total surface congruent $r = -.07$, $p = .39$ [two-sided], total surface incongruent $r = -.15$, $p = .06$ [two-sided]; density congruent $r = -.15$, $p = .06$ [two-sided], density incongruent: $r = -.07$, $p = .36$

[two-sided]; item size congruent $r = -.07$, $p = .39$ [two-sided], item size incongruent $r = -.15$, $p = .06$ [two-sided]; total circumference congruent $r = -.08$, $p = .32$ [two-sided], total circumference incongruent $r = -.16$; $p = .04$ [two-sided]. Comparison of the respective correlations did not reveal any significant differences (area extended congruent vs. incongruent: $r = -.14$ vs. $r = -.11$; $p = .60$ [two-sided]; total surface congruent vs. incongruent: $r = -.07$ vs. $r = -.15$; $p = .57$ [two-sided]; density congruent vs. incongruent: $r = -.15$ vs. $r = -.07$; $p = .57$ [two-sided]; item size: congruent vs. incongruent: $r = -.07$ vs. $r = -.15$; $p = .57$ [two-sided]; total circumference congruent vs. incongruent: $r = -.08$ vs. $r = -.16$; $p = .42$ [two-sided]).

The mediation model revealed that the association between non-symbolic numerical magnitude processing and number word sequence skills (see Table 1) was no longer significant after controlling for spatial visualization skills and it was significantly mediated by spatial visualization skills (see Fig. 2/Table 2). Using the standards of Cohen (1988), as proposed by Shrout and Bolger (2002), the mediation (indirect) effect can be considered to be small (see standardized coefficients in Table 2). Moreover, age had a significant partial effect on number word sequence skills (unstandardized coefficient = .17, $z = 5.98$, $p < .001$, standardized coefficient = .48). Subsequent mediation analyses revealed that performance in the Copying subtest of the DTVP-3 significantly mediated the association between non-symbolic numerical magnitude processing and number word sequence skills. This mediation (indirect) effect can also be considered to be small (see Table 2). Performance in the other subtests of the DTVP-3 did not significantly mediate the association between non-symbolic numerical magnitude processing and number word sequence skills (see Table 2). Please note that there is an ongoing debate about effect sizes for mediation (indirect) effects (see e.g., Lachowicz, Preacher, & Kelley, 2018). To meet previously identified limitations, Lachowicz et al., (2018) developed the upsilon effect size. The calculation of upsilon under consideration of a covariate (age in our case) is not yet provided according to our state of knowledge. A calculation without age as covariate revealed upsilon values, which according to Lachowicz et al. (2018) show small effects in all our mediation models. Similarly, based on the standardized coefficients of the mediation (indirect) effects (another effect size measure under debate, see Lachowicz et al., 2018), all the effects can be considered to be small using the standards of Cohen (1988), as proposed by Shrout and Bolger (2002). We thus decided to use the standardized coefficients of the mediation (indirect) effects as effect size measures.

Discussion

We assessed four- to six-year old children's performance in a non-symbolic numerical magnitude comparison task, in tasks concerning the knowledge of the sequence of number words and in a task to assess spatial visualization skills. In line with previous findings, non-symbolic numerical magnitude processing skills were found to be associated with number word sequence skills. After controlling for age, this association was rather low ($r = .20$, $p = .02$) but still significant and comparable to previous meta-analytic findings ($r = .20$, see e.g., Chen & Li, 2014). Both number word sequence skills and non-symbolic numerical magnitude processing skills were significantly associated with children's spatial visualization skills, even after controlling for age. Thus, children with better spatial visualization skills tended to show better performance in comparing two visually presented dot arrays with respect to their quantity as well as in tasks assessing early mathematical skills which do not involve any visual stimulation, like knowing the sequence of number words.

According to the mediation analysis, the association between non-symbolic numerical magnitude processing skills and number word sequence skills was fully mediated by spatial visualization skills. This mediation (indirect) effect can be considered to be small. Nevertheless, it might be taken as evidence for the visual perception hypothesis postulated by Zhou and colleagues (2015), according to which visual perception skills account for the association between non-symbolic numerical magnitude processing skills and mathematical performance. Zhou and colleagues (2015), however, suggested that the speed of visual perception constitutes the key underlying factor. As we assessed spatial visualization skills and number word sequence skills without time constraints, it seems unlikely that inter-individual differences in the speed of visual perception account for the mediation effect we were able to substantiate.

Supplementary mediation analyses revealed that performance in the Copying subtest of the DTVP-3 significantly mediated the association between non-symbolic numerical magnitude processing skills and number word sequence skills. This mediation (indirect) effect can also be considered to be small. While the Copying subtest requires children to draw given figures on a piece of paper, the Figure-Ground subtest requires them to identify given figures hidden in a complex, confusing background. In contrast, in the other subtests of DTVP-3, it is either not necessary to process predefined figures (Eye-Hand Coordination) or it is necessary to compare predefined figures with other modified figures (Visual Closure and Form Constancy). Thus, only in the Copying and in the Figure-Ground subtest given and target figures should exactly correspond to each other. Capturing the given figures should thus be a common requirement of these two tasks and it can be assumed that children mentally visualized the figures in order to accomplish the two tasks. In this regard, it has been reported that better visual object imagery abilities (i.e., mental representations of the visual appearance of individual objects, in terms of form, color, shape, brightness, texture and size) facilitate perceptual processing of visual objects (Vannucci, Mazzoni, Chiorri, & Cioli, 2008). Moreover, mental visualization processes may also play a role in non-symbolic numerical magnitude processing tasks as well as in number word sequence tasks. Comparing two visually presented dot arrays with respect to their quantity may involve a mental visualization of at least one of the respective dot arrays, and accomplishing number word sequence tasks may involve visualizations in form of a mental number line. Inter-individual differences in the ability to construct and to shortly maintain mental visualizations may thus account for the mediation effect found in the present study. Findings from previous studies indeed suggest that the ability to maintain visual-spatial information in working memory plays an important role in mathematical learning (see e.g., Raghubar, Barnes, & Hecht, 2010 for an overview). Preschoolers' performance in the number word sequence tasks used in the present study was also found to be associated with their performance in visual-spatial working memory tasks (see Krajewski & Schneider, 2009). Findings from Mussolin and colleagues (2012), however, revealed that the association between non-symbolic numerical magnitude processing skills and early mathematical skills like number word and Arabic number knowledge seems to persist after controlling for visual-spatial working memory capacities. To assess children's visual-spatial working memory capacities, Mussolin and colleagues (2012) used the Corsi block task that requires participants to observe an experimenter tap a sequence of blocks and to repeat the sequence correctly. It is assumed that this task captures spatial dynamic rather than visual static memory capacities (see e.g., Baddeley, 2003). Future studies are therefore needed to examine the influence of visual static memory capacities on the association between processing of non-symbolic numerical magnitude and early mathematical skills as well as to disentangle the influence of visual static memory capacities and mental visualization abilities. As the different stimuli we used to assess children's visual-spatial skills remained visible until a response was given, the ability to construct mental visualizations of the stimuli should have been more important for performance than the ability to maintain these mental visualizations. We consequently assume that the association between non-symbolic numerical magnitude processing skills and early mathematical skills like knowledge of the sequence of number words is mediated by inter-individual differences in the ability to construct rather than to maintain mental visualizations.

A limitation of our study is that other factors that may account for the association between non-symbolic numerical magnitude processing skills and number word sequence skills were not assessed. For example, one may argue that inter-individual differences in intelligence rather than in mental visualization abilities may account for the mediation effect detected in the present study. Indeed, preschoolers' performance in the number word sequence tasks used in the present study has not only been found to be associated with their performance in visual-spatial working memory tasks but also with their performance in a non-verbal intelligence test (see Krajewski & Schneider, 2009). However, intelligence was assessed in various studies looking at the association between non-symbolic numerical magnitude processing skills and mathematical performance and, to our knowledge, none of these studies revealed that the association was fully mediated by inter-individual differences in intelligence (see e.g., Chen & Li, 2014 for an overview). This suggests that the mediation effect detected in the present study can hardly be entirely attributed to inter-individual differences in intelligence. One may, however, argue that it is a facet of intelligence mediating the association between non-symbolic numerical magnitude processing skills and number word sequence skills, namely, spatial visualization skills. Indeed, spatial visualization skills have been regarded as an important component of intelligence (see e.g., Linn & Petersen, 1985). In the light of this view, the findings of the present study allowed to identify an element of intelligence which plays an important role in non-symbolic numerical magnitude processing as well as in processing of the sequence of number words.

Moreover, cognitive control skills may mediate the association between non-symbolic numerical magnitude processing skills and mathematical performance (see e.g., Gebuis et al., 2016). Cognitive control skills are assumed to be needed in non-symbolic numerical magnitude processing tasks in order to flexibly adapt the weights of different visual cues as well as to inhibit responses to salient visual cues which are incongruent with the numerical dimension. In line with this view, associations between non-symbolic numerical magnitude processing skills and early mathematical skills in preschool children from low-income homes were demonstrated to be limited to trials of a non-symbolic numerical magnitude comparison task in which the size of the area occupied by the stimuli conflicts with the number of elements (i.e., more numerous stimuli occupy a smaller area) and the correlation became non-significant when controlling for inhibitory cognitive control skills (Wagner Fuhs & McNeil, 2013). These findings could, however, not be replicated in a recent study investigating preschool children from middle class families (Keller & Libertus, 2015). In the present study, the comparison of the correlations between number word sequence skills and performance in trials of the non-symbolic numerical magnitude comparison task, in which different visual cues were either congruent or incongruent with the numerical dimension, did not reveal any significant differences, also suggesting that cognitive control skills did not substantially influence the association. As, however, earlier studies revealed conflicting results (e.g., Keller & Libertus, 2015; Wagner Fuhs & McNeil, 2013), future studies are needed to further explore this issue.

In previous studies examining children's approximate number system, short presentation times were used in the non-symbolic numerical magnitude comparison task in order to prevent children from using counting strategies (see e.g., Libertus et al., 2011/2013; Mazzocco et al., 2011). In the present study, the sets of different numerical quantities appeared up to a maximum duration of 6000 ms. Indeed, Libertus et al., 2011/2013 as well as Mazzocco et al., (2011) used shorter presentation times but they also used smaller set sizes (Libertus et al: 4 to 15; Mazzocco et al: 1 to 14) which can definitely be assumed to trigger the use of counting strategies. Nevertheless, we cannot rule out that some of our participants attempted to count some of the stimuli. However, children were instructed not to count the dots and the number of dots (14 to 50) combined with the restricted response time (6000 ms) should have prevented these kinds of attempts. Moreover, the mean reaction time of our participants was relatively fast (1713 ms) indicating that they generally identified the side of the larger numerical magnitude without using counting strategies.

To conclude, our findings revealed that inter-individual differences in visual-spatial skills might fully mediate the association between children's performance in visually presented tasks assessing non-symbolic numerical magnitude processing skills and in tasks assessing number word sequence skills which do not involve visual stimulation. We assume that the ability to construct mental visualizations constitutes the key factor underlying this mediation effect. Further evidence is needed to substantiate this conclusion. If valid, it may provide important implications for supporting early math learning.

Figure Captions

Figure 1 Exemplary depiction of stimuli in a non-symbolic numerical magnitude comparison task. a) Comparison pair with larger aggregate surface of the items in one array but larger number of items in the other array (aggregate surface area incongruent with numerical dimension). b) Comparison pair with larger aggregate surface and larger number of items in one array (aggregate surface area congruent with numerical dimension).

Figure 2 Mediation model testing whether performance in the Figure-Ground subtest of the Developmental Test of Visual Perception (spatial visualization) mediates the association between non-symbolic numerical magnitude processing (comparison) and number word sequence skills (number word sequence).

Figure A.1 Correlations (a) between number word sequence skills (number word sequence) and non-symbolic numerical magnitude processing skills (comparison), (b) between number word sequence skills and performance in the Figure-Ground subtest of the Developmental Test of Visual Perception (spatial visualization), and (c) between non-symbolic numerical magnitude processing skills and performance in the Figure-Ground subtest of the Developmental Test of Visual Perception.

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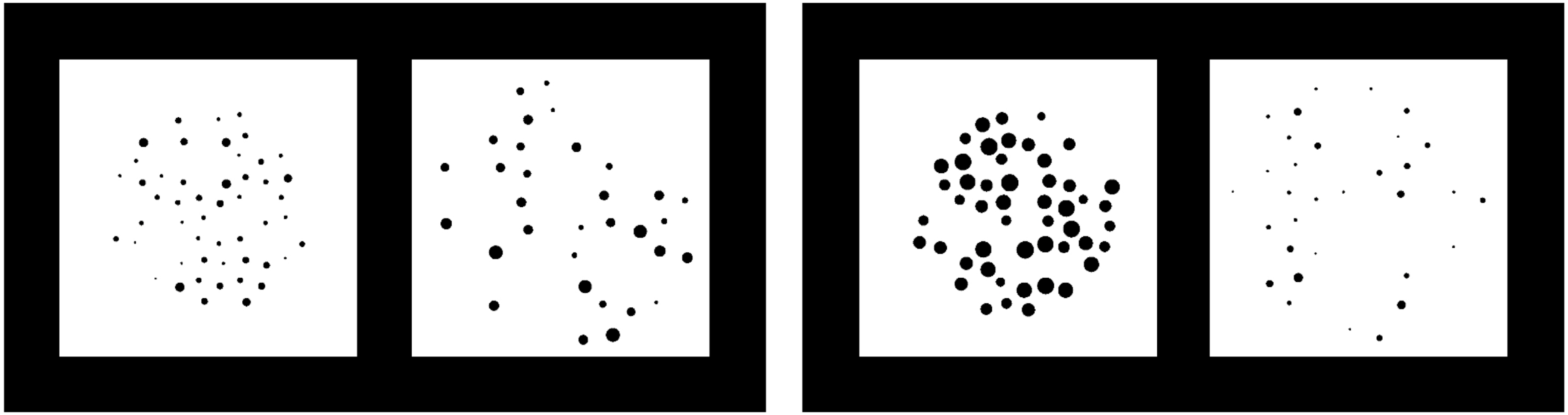
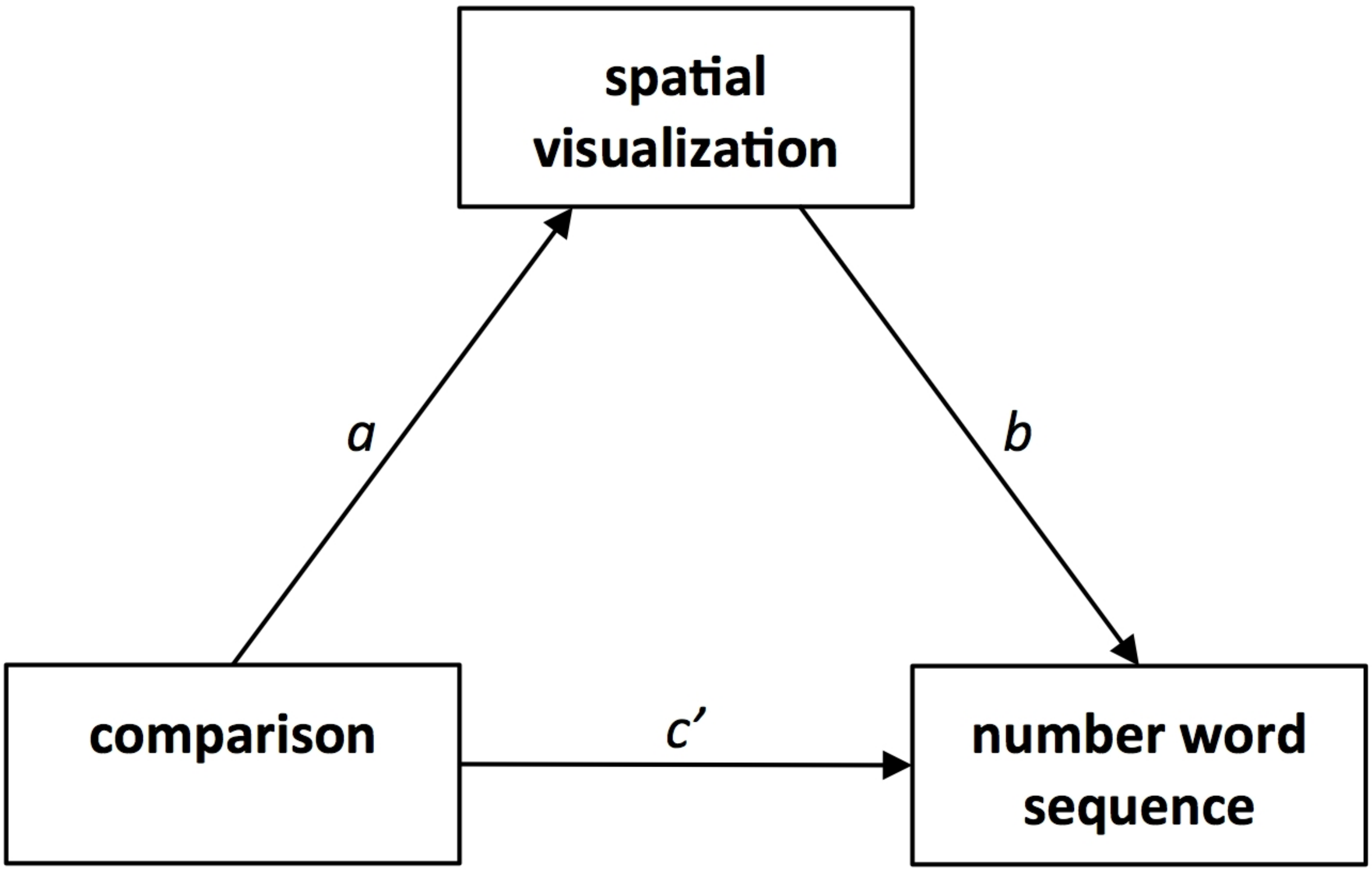


a

b



direct effect (*c′*) = -3.55 (standardized = -.11), p = .07;
indirect effect (ab) = -1.70 (standardized = -.06),
confidence interval (95%) = -.17 to -3.79
(control variable: age)

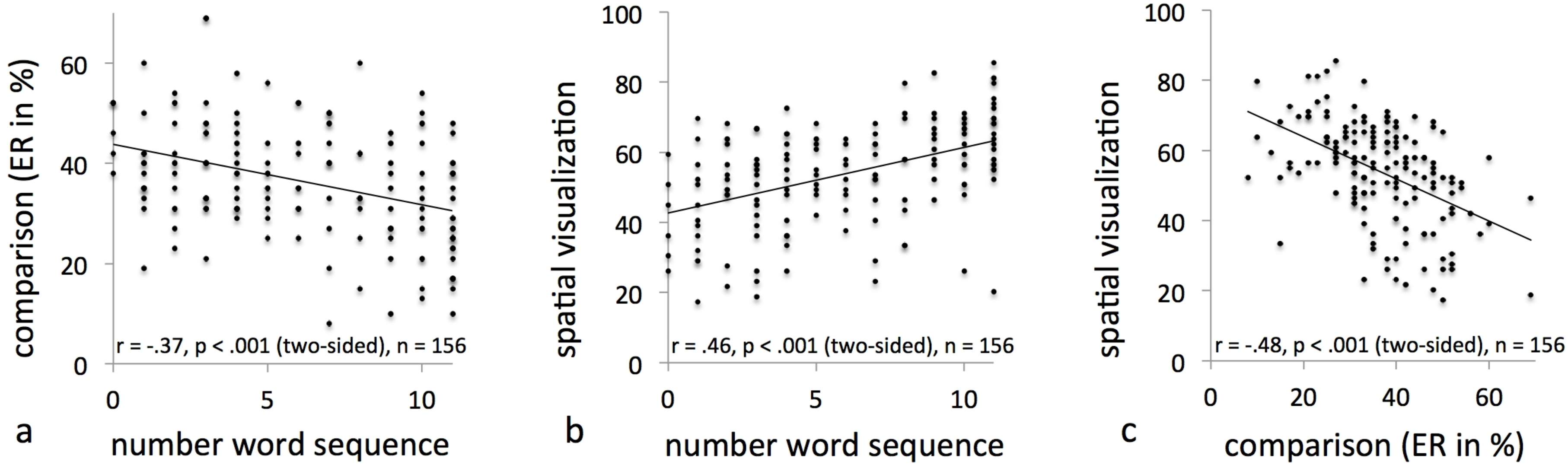

comparison (ER in %)
r = -.37, p < .001 (two-sided), n = 156
number word sequence
a
spatial visualization
r = .46, p < .001 (two-sided), n = 156
number word sequence
b
spatial visualization
r = -.48, p < .001 (two-sided), n = 156
comparison (ER in %)
c

Table 1

Correlation coefficients (below the diagonal) and partial correlation coefficients (controlling for age, above the diagonal) between performance in the different subtests of the Developmental Test of Visual Perception (Eye-Hand Coordination, Copying, Figure-Ground, Visual Closure, Form Constancy), performance in the non-symbolic numerical magnitude comparison task (comparison [ER in %]), performance in the number word sequence tasks (number word sequence), and age (in months).

	1.	2.	3.	4.	5.	6.	7.	*M*	*SD*
1. Eye-Hand Coordination	-	.41**	.14	.15	.10	-.18*	.15	70	14
2. Copying	.64**	-	.39**	.43**	.18*	-.30**	.27*	29	13
3. Figure-Ground	.40**	.58**	-	.37**	.19*	-.36**	.23*	54	14
4. Visual Closure	.38**	.59**	.51**	-	.20*	-.10	.41**	35	13
5. Form Constancy	.26*	.32**	.31**	.30**	-	-.26*	.15	56	11
6. comparison (ER in %)	-.37**	-.45**	-.48**	-.26*	-.34*	-	-.20*	37	11
7. number word sequence	.46**	.55**	.46**	.57**	.29**	-.37**	-	6	4
8. age (in months)	.61**	.65**	.50**	.46**	.30**	-.38**	.61**	65	10

* $p < .05$, ** $p < .001$ (two-sided); $n = 156$

Table 2

Results of mediation models including performance in one of the five subtests of the DTVP-3 (Eye-Hand Coordination, Copying, Figure-Ground, Visual Closure, Form Constancy) as a potential mediator of the association between non-symbolic numerical magnitude processing and number word sequence skills. Age was used as control variable in all mediation models.

	direct effect				indirect effect		
	unstand. coef.	*stand. coef.*	*z*	*p (two-sided)*	*unstad. coef.*	*stand. coef.*	*conf. interval (95%)*
Eye-H. Coord.	-4.67	-.15	-2.48	.01	-.58	-.02	.02 to -1.80
Copying	-3.43	-.11	-1.68	.09	-1.82	-.06	-.63 to -3.81
Figure-Ground	-3.55	-.11	-1.80	.07	-1.70	-.06	-.17 to -3.79
Visual Closure	-4.17	-.13	-2.30	.02	-1.08	-.04	.32 to -2.79
Form Constancy	-4.54	-.15	-2.29	.02	-.72	-.02	.31 to -2.62

n = 156; un/stand. coef. = un/standardized coefficient; conf. = confidence

Table A.1

Comparison of error rates (in %) in congruent and incongruent trials of the non-symbolic numerical magnitude comparison task separately for each of the five different visual cue conditions (area extended, total surface, density, item size, total circumference).

	incongruent			congruent			*p* (two-sided)
	M	*SD*	*SE*	*M*	*SD*	*SE*	
area extended	49.08	14.64	1.17	24.18	18.19	1.45	$p < .001$
total surface	47.46	18.57	1.48	25.76	19.58	1.56	$p < .001$
density	27.36	19.03	1.52	46.44	18.81	1.50	$p < .001$
item size	47.46	18.57	1.48	25.76	19.58	1.56	$p < .001$
total circumference	52.71	19.24	1.54	25.92	17.01	1.36	$p < .001$