MOGPS Supplementary Material

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1 Test functions

1.1 Poloni function

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} \left[1 + \left(A_1 - B_1\left(x_1, x_2\right)\right)^2 + \left(A_2 - B_2\left(x_1, x_2\right)\right)^2\right] \\ \left(x_1 + 3\right)^2 + \left(x_2 + 1\right)^2 \end{pmatrix} \tag{1}$$

where

$$\begin{aligned} A_1 &= 0.5\sin(1) - 2\cos(1) + \sin(2) - 1.5\cos(2) \\ A_2 &= 1.5\sin(1) - \cos(1) + 2\sin(2) - 0.5\cos(2) \\ B_1\left(x_1, x_2\right) &= 0.5\sin\left(x_1\right) - 2\cos\left(x_1\right) + \sin\left(x_2\right) - 1.5\cos\left(x_2\right) \\ B_2\left(x_1, x_2\right) &= 1.5\sin\left(x_1\right) - \cos\left(x_1\right) + 2\sin\left(x_2\right) - 0.5\cos\left(x_2\right) \end{aligned}$$

$$\text{s.t.} \begin{bmatrix} -\pi & -\pi \end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix} \pi & \pi \end{bmatrix}^T .$$

See Poloni et al. [1].

1.2 Kursawe function

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} \sum_{i=1}^{2} \left[-10\exp\left(-0.2\sqrt{x_i^2 + x_{i+1}^2}\right)\right] \\ \sum_{i=1}^{3} \left[\left|x_i\right|^{0.8} + 5\sin\left(x_i^3\right)\right] \end{pmatrix} \tag{2}$$

$$\text{s.t.} \begin{bmatrix} -5 & -5 & -5 \end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix} 5 & 5 & 5 \end{bmatrix}^T .$$

See Kursawe [2].

1.3 Viennet function 1

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} 0.5\left(x_1^2 + x_2^2\right) + \sin\left(x_1^2 + x_2^2\right) \\ \dfrac{\left(3x_1 - 2x_2 + 4\right)^2}{8} + \dfrac{\left(x_1 - x_2 + 1\right)^2}{27} + 15 \\ \dfrac{1}{x_1^2 + x_2^2 + 1} - 1.1\exp\left[-\left(x_1^2 + x_2^2\right)\right] \end{pmatrix} \tag{3}$$

$$\text{s.t.} \begin{bmatrix}-3 & -3\end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix}3 & 3\end{bmatrix}^T.$$

See Viennet et al. [3].

1.4 Viennet function 2

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} x_1^2 + (x_2 - 1)^2 \\ x_1^2 + (x_2 + 1)^2 + 1 \\ (x_1 - 1)^2 + x_2^2 + 2 \end{pmatrix} \tag{4}$$

$$\text{s.t.} \begin{bmatrix}-2 & -2\end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix}2 & 2\end{bmatrix}^T.$$

See Viennet et al. [3].

1.5 Schaffer function 1

$$\boldsymbol{f}(x) = \begin{pmatrix} x^2 \\ (x-2)^2 \end{pmatrix} \tag{5}$$

$$\text{s.t.} - 10 \leq x \leq 10.$$

See Schaffer [4].

1.6 Schaffer function 2

$$\boldsymbol{f}(x) = \begin{pmatrix} \begin{cases} -x, & \text{if } x \leq 1 \\ x - 2, & \text{if } 1 < x \leq 3 \\ 4 - x, & \text{if } 3 < x \leq 4 \\ x - 4, & \text{if } x > 4 \end{cases} \\ (x-5)^2 \end{pmatrix} \tag{6}$$

$$\text{s.t.} - 5 \leq x \leq 10.$$

See Schaffer [4].

1.7 Fonseca-Fleming function

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} 1 - \exp\left[-(x_1 - 1)^2 - (x_2 + 1)^2\right] \\ 1 - \exp\left[-(x_1 + 1)^2 - (x_2 - 1)^2\right] \end{pmatrix} \tag{7}$$

$$\text{s.t.} \begin{bmatrix}-4 & -4\end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix}4 & 4\end{bmatrix}^T.$$

See Fonseca and Fleming [5].

1.8 Four bar truss

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} L\left(2x_1 + \sqrt{2}x_2 + \sqrt{x_3} + x_4\right) \\ \frac{FL}{E}\left(\frac{2}{x_1} + \frac{2\sqrt{2}}{x_2} - \frac{2\sqrt{2}}{x_3} + \frac{2}{x_4}\right) \end{pmatrix} \tag{8}$$

where

$$\begin{aligned} F &= 10 \\ E &= 2 \times 10^5 \\ L &= 200 \end{aligned}$$

$$\text{s.t.} \begin{bmatrix} 1 & \sqrt{2} & \sqrt{2} & 1 \end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix} 3 & 3 & 3 & 3 \end{bmatrix}^T.$$

See Cheng and Li [6].

1.9 ZDT1

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} x_1 \\ g\left(1 - \sqrt{\frac{x_1}{g}}\right) \end{pmatrix} \tag{9}$$

where

$$g = 1 + 9\sum_{i=2}^{N} x_i$$

$$\text{s.t.} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T.$$

See Zitzler et al. [7].

1.10 ZDT2

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} x_1 \\ g\left(1 - \left(\frac{x_1}{g}\right)^2\right) \end{pmatrix} \tag{10}$$

where

$$g = 1 + 9\sum_{i=2}^{N} x_i$$

$$\text{s.t.} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T.$$

See Zitzler et al. [7].

1.11 ZDT3

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} x_1 \\ g\left(1 - \sqrt{\frac{x_1}{g}} - \frac{x_1}{g}\sin(10\pi x_1)\right) \end{pmatrix} \tag{11}$$

where

$$g = 1 + 9\sum_{i=2}^{N} x_i$$

$$\text{s.t.} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T.$$

See Zitzler et al. [7].

1.12 ZDT6

$$\boldsymbol{f}(\boldsymbol{x}) = \begin{pmatrix} 1 - e^{-4x_1}\sin^6 6\pi x_1 \\ g\left(1 - \left(\frac{1 - e^{-4x_1}\sin^6 6\pi x_1}{g}\right)^2\right) \end{pmatrix} \tag{12}$$

where

$$g = 1 + 9\left(\sum_{i=2}^{N} x_i\right)^{0.25}$$

$$\text{s.t.} \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T \leq \boldsymbol{x} \leq \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T.$$

See Zitzler et al. [7].

2 Results

2.1 Poloni function

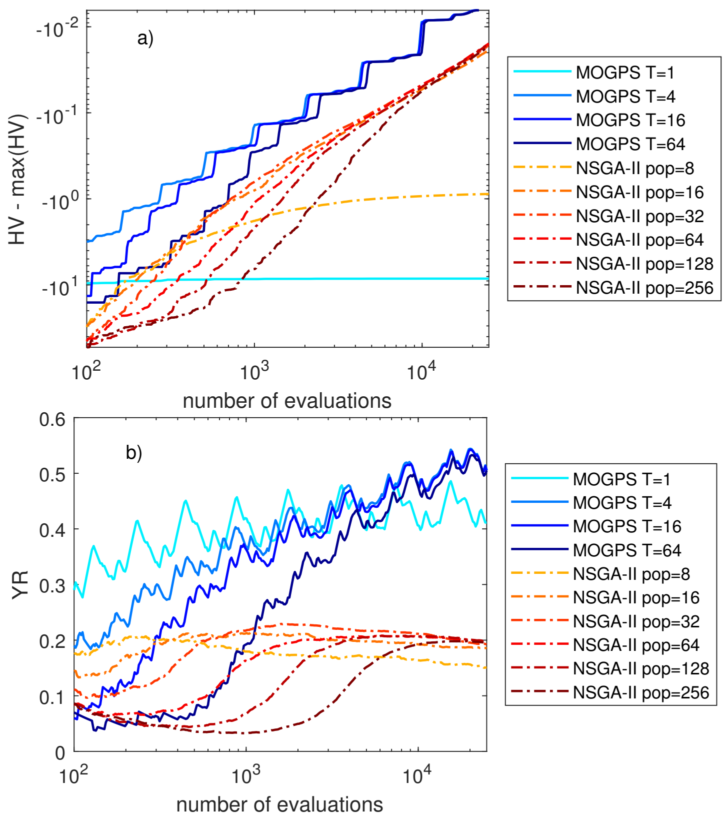

Figure 1: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the Poloni test function for different parameter sets. The reference point for the hypervolume metric is set to (20|30).

2.2 Kursawe function

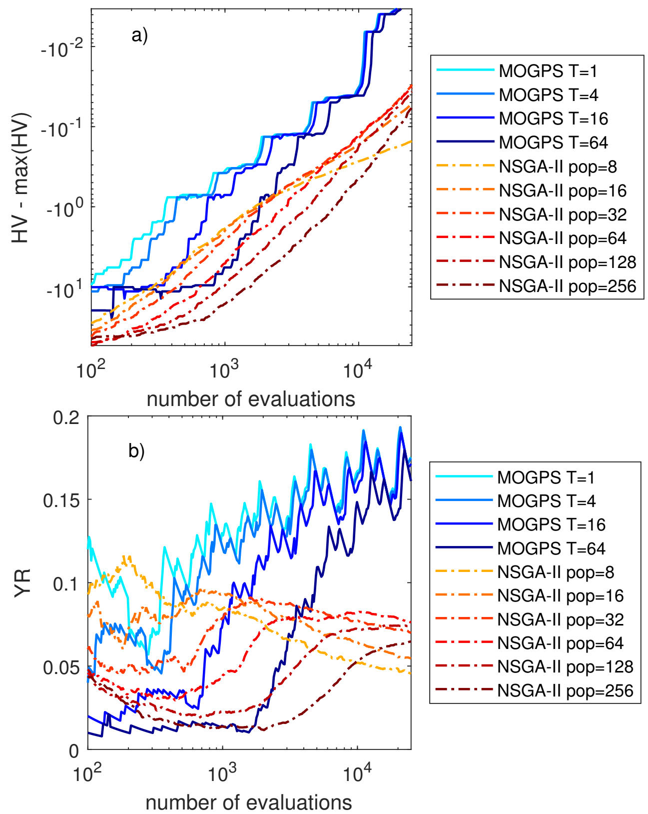

Figure 2: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the Kursawe test function for different parameter sets. The reference point for the hypervolume metric is set to (-15|5).

2.3 Viennet function 1

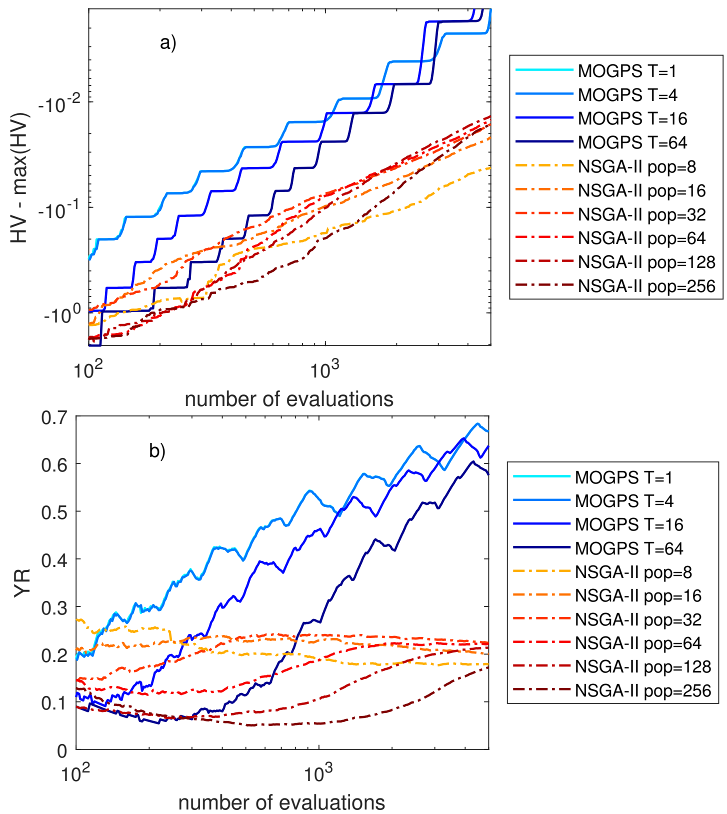

Figure 3: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the Viennet test function 1 for different parameter sets. The reference point for the hypervolume metric is set to (10|17.5|0.2).

2.4 Viennet function 2

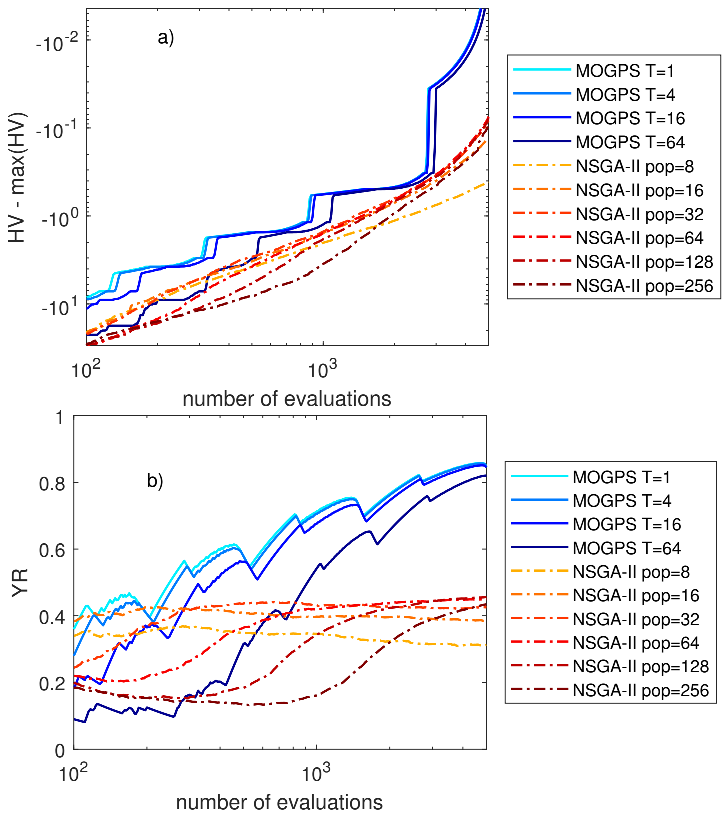

Figure 4: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the Viennet test function 2 for different parameter sets. The reference point for the hypervolume metric is set to (10|10|10).

2.5 Schaffer function 1

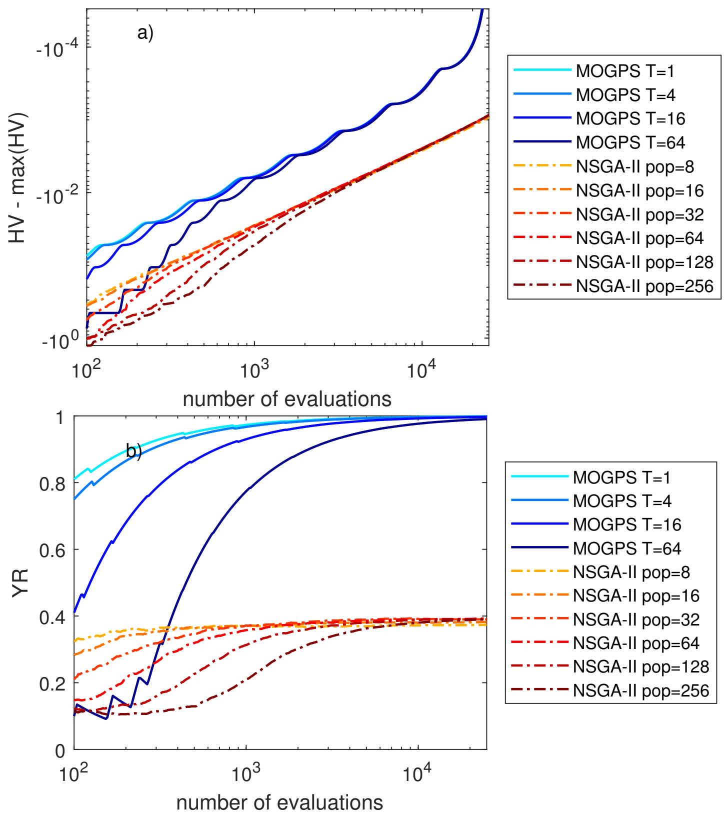

Figure 5: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the Schaffer test function 1 for different parameter sets. The reference point for the hypervolume metric is set to (5|5).

2.6 Schaffer function 2

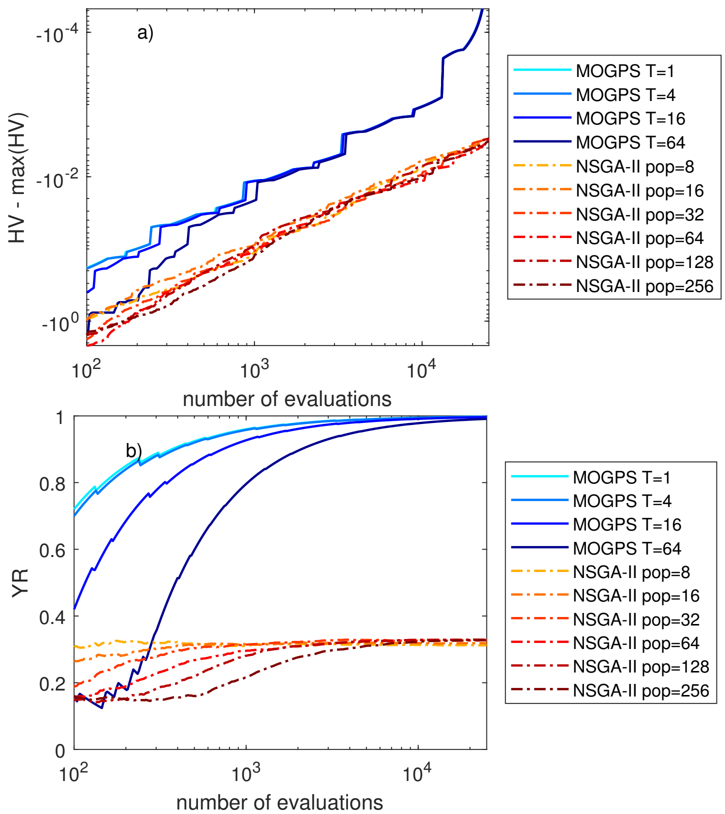

Figure 6: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the Schaffer test function 2 for different parameter sets. The reference point for the hypervolume metric is set to (2|20).

2.7 Fonseca-Fleming function

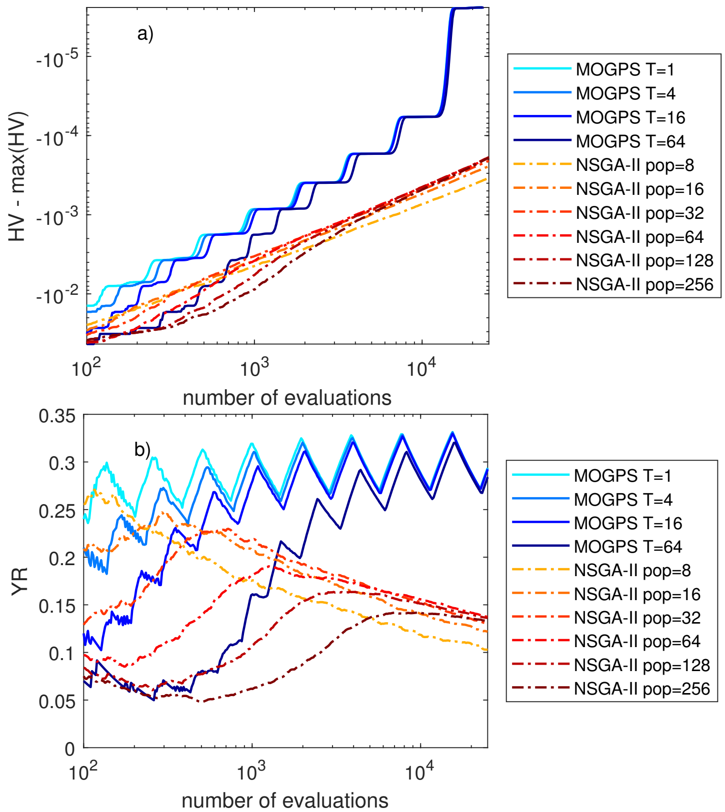

Figure 7: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the Fonseca-Fleming test function for different parameter sets. The reference point for the hypervolume metric is set to (1|1).

2.8 Four bar truss

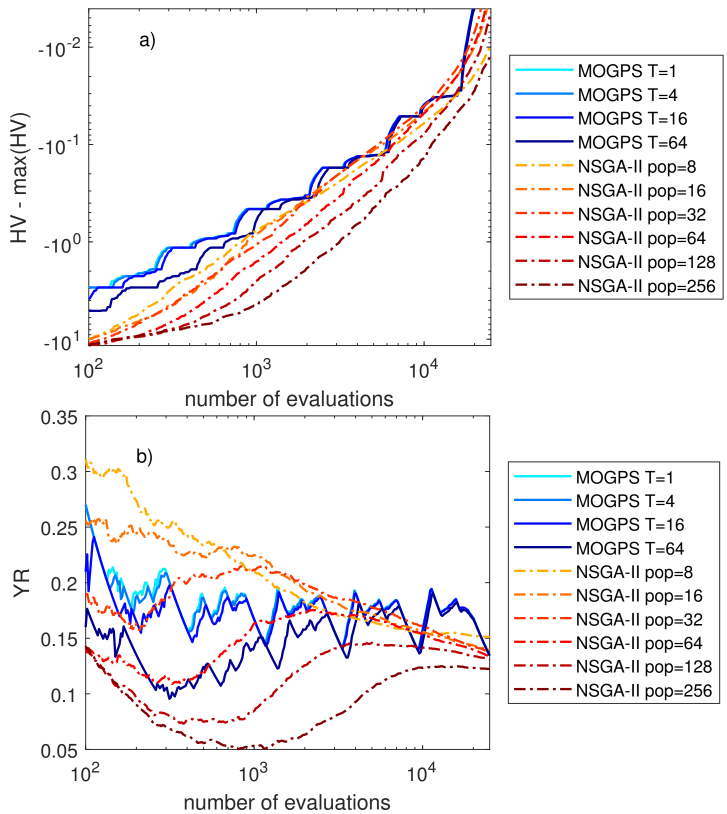

Figure 8: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the four bar truss test function for different parameter sets. The reference point for the hypervolume metric is set to (3000|0.05).

2.9 ZDT1

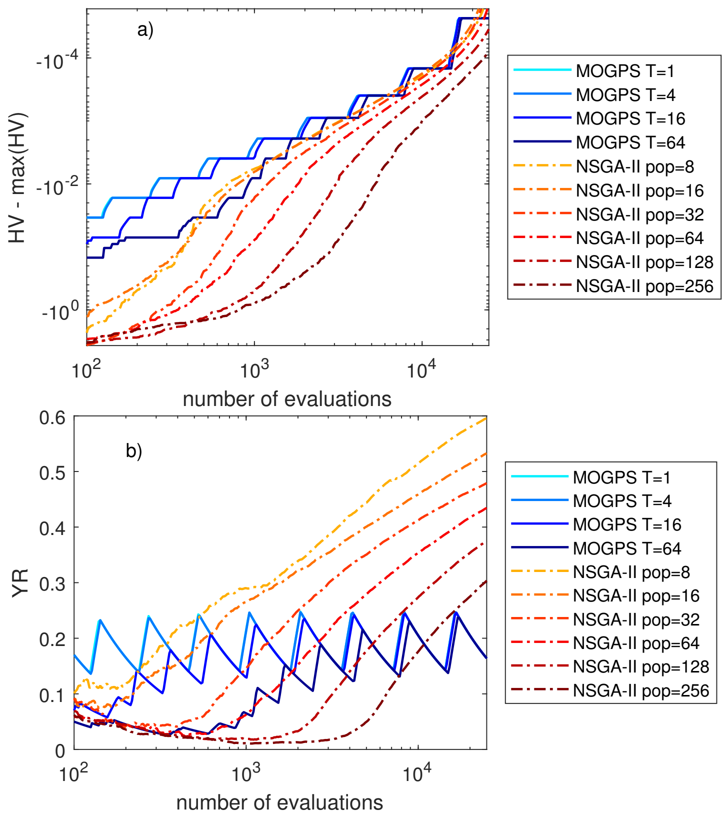

Figure 9: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the ZDT1 test function for different parameter sets. The reference point for the hypervolume metric is set to (2|2).

2.10 ZDT2

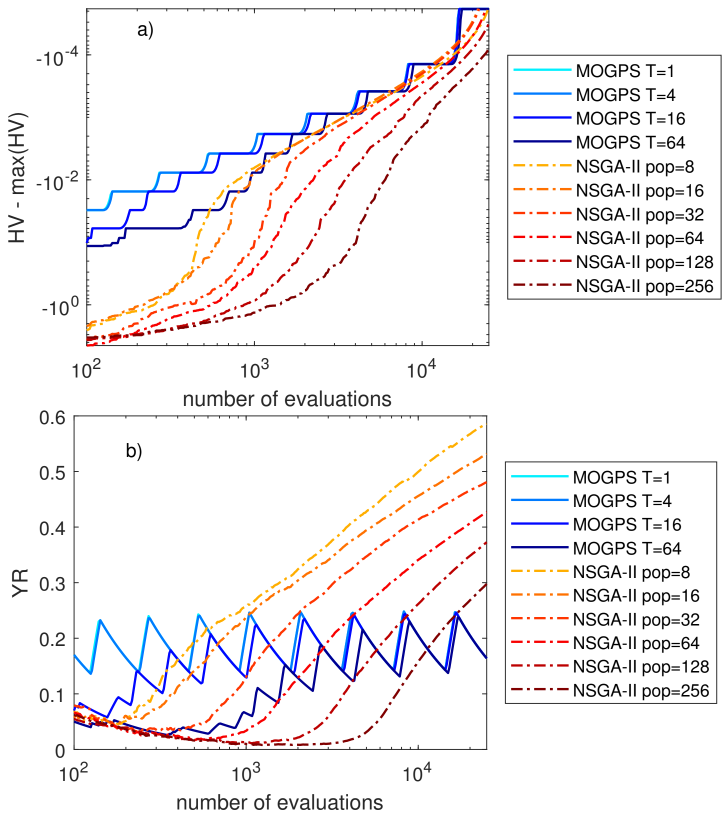

Figure 10: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the ZDT2 test function for different parameter sets. The reference point for the hypervolume metric is set to (2|2).

2.11 ZDT3

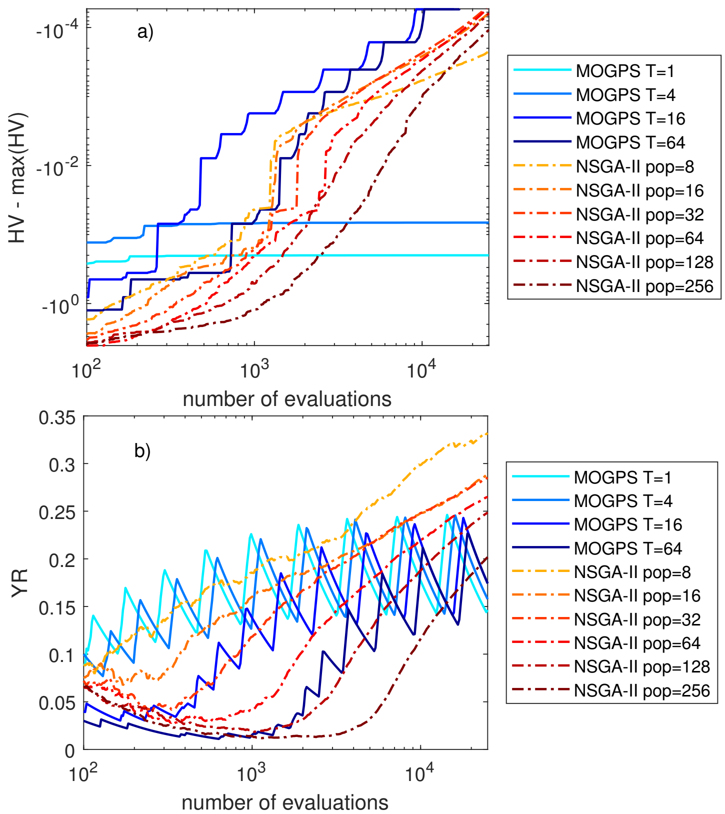

Figure 11: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the ZDT3 test function for different parameter sets. The reference point for the hypervolume metric is set to (2|2).

2.12 ZDT6

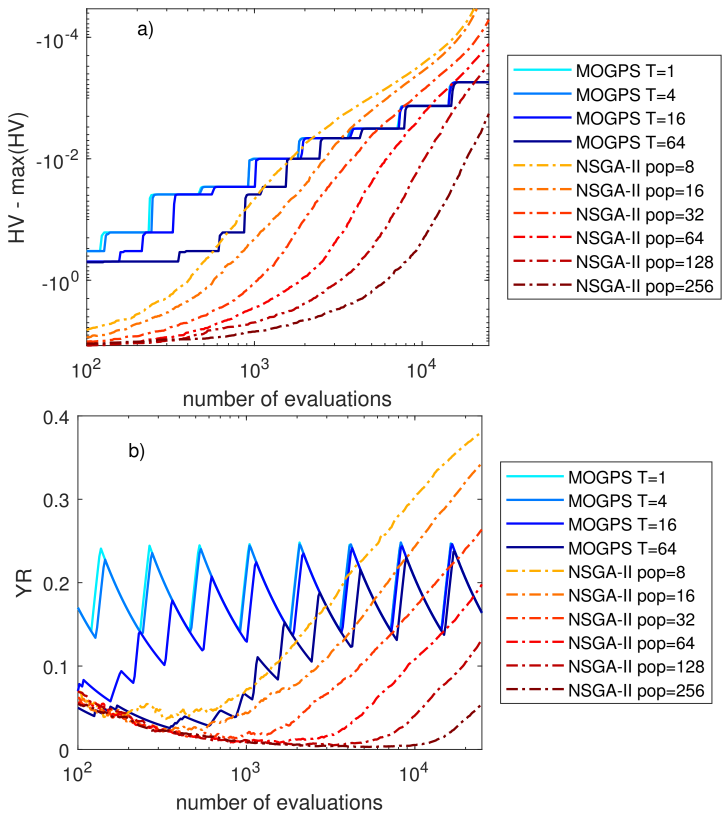

Figure 12: Evolution of a) the hypervolume metric and b) the yield ratio over the number of objective function evaluations on the ZDT6 test function for different parameter sets. The reference point for the hypervolume metric is set to (2|2).

References

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