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All-order colour structure and two-loop anomalous dimension of soft radiation in heavy-particle pair production at the LHC

M. Beneke[*] [†]
Institut für Theoretische Physik E, RWTH Aachen University,
D - 52056 Aachen, Germany

P. Falgari
IPPP, Department of Physics, University of Durham,
Durham DH1 3LE, England

C. Schwinn
Albert-Ludwigs Universität Freiburg,
Physikalisches Institut, D-79104 Freiburg, Germany

We consider the factorization and resummation of soft and Coulomb gluons for pair-production processes of heavy coloured particles at hadron colliders, and discuss: the construction of a colour basis that diagonalizes the leading soft function to all orders in perturbation theory; the determination of the two-loop soft anomalous dimension needed for NNLL resummations; and a general formula that provides all logarithmically enhanced $\mathcal{O}(\alpha_s^2)$ correction requiring as process-independent input only the one-loop hard matching coefficients.

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[*]Speaker.

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1. Introduction

Like in the well-studied Drell-Yan process, the partonic cross sections $q\bar{q}, qg, gg \to HH' + X$ of heavy-particle pair production contain higher-order terms

$$\left[\alpha_s \ln^2 \beta\right]^n, \qquad \beta^2 = 1 - z = 1 - M_{HH'}^2/\hat{s} \tag{1.1}$$

in their perturbative expansion, which should be summed to all orders, if it can be argued that the hadronic cross section is dominated numerically by these threshold logarithms. For two coloured particles in the final state with some fixed invariant mass $M_{HH'}$ complications arise relative to the Drell-Yan process due to colour-exchange and kinematics-dependent anomalous dimensions similar to di-jet production [1]. For the total partonic cross section the only parametrically enhanced logarithms arise from the true production threshold, $M_{HH'} = M_H + M_{H'}$ and the kinematical dependence disappears. On the other hand, the particles are non-relativistic in this region and even more strongly enhanced terms $(\alpha_s/\beta)^n$ appear due to the Coulomb force. Although resummation for total partonic cross sections has been performed in the past [2], the issue of factorization of soft and Coulomb gluons, and their simultaneous resummation has not been addressed with rigour until recently. In this proceedings article we discuss the factorization formula applicable to this situation, the diagonal colour basis for the leading soft function relevant to the heavy-particle pair production process, and the two-loop anomalous dimension for soft radiation. For details we refer to [3]. See the talk by P. Falgari [4] for a discussion of resummation of the squark anti-squark production cross section at the LHC.

To define the LL, NLL, etc. approximations of the resummed cross section in the presence of Coulomb effects, we note that near threshold the usual expansion, where $\alpha_s \ln \beta$ in the exponent of (1.2) below counts as order one, is combined with an expansion in β, such that α_s/β also counts as order one. This leads to a parametric representation of the expansion of the cross section in the form

$$\begin{aligned}\hat{\sigma}_{pp'} \;=\; & \hat{\sigma}^{(0)} \sum_{k=0} \left(\frac{\alpha_s}{\beta}\right)^k \exp\Big[\underbrace{\ln\beta\, g_0(\alpha_s \ln\beta)}_{\text{(LL)}} + \underbrace{g_1(\alpha_s \ln\beta)}_{\text{(NLL)}} + \underbrace{\alpha_s g_2(\alpha_s \ln\beta)}_{\text{(NNLL)}} + \ldots\Big] \\ & \times \big\{1\,\text{(LL,NLL)}; \alpha_s, \beta\,\text{(NNLL)}; \alpha_s^2, \alpha_s\beta, \beta^2\,\text{(NNNLL)}; \ldots\big\},\end{aligned} \tag{1.2}$$

which reproduces the standard structure [2] away from threshold for $k = 0$ and no expansion in β. This implies that at $\mathcal{O}(\alpha_s^2)$ relative to the Born cross section the terms $\alpha_s^2 \times \{1/\beta; \ln^{2,1}\beta; \beta \times \ln^{4,3}\beta\}$ are NNLL. Note that $\alpha_s^2 \ln^{2,1}\beta$ terms may arise from the product of a Coulomb-enhanced one-loop correction α_s/β and a β-suppressed soft emission $\alpha_s \beta \ln^{2,1}\beta$. This highlights the subtle point that one must consider approximations one order beyond the standard eikonal approximation to capture all NNLL terms.

2. Factorization of soft and Coulomb gluons

The factorization of soft and Coulomb gluons is a non-trivial issue, since the non-relativistic energy of the heavy particles is of the same order as the soft gluon momentum. Hence, Coulomb exchanges are not part of the hard process and take place (diagrammatically) "in between" the soft gluon emissions. In [3] it is shown that both effects are simultaneously resummed by means of the formula

$$\hat{\sigma}(\beta,\mu) = \sum_a \sum_{i,i'} H^a_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J^a_{R_\alpha}(E-\frac{\omega}{2}) W^{a,R_\alpha}_{ii'}(\omega,\mu), \tag{2.1}$$

which contains a multiplicative short-distance coefficient $H^a_{ii'}$ in each colour (in higher orders also spin) configuration, and a convolution of soft functions $W^{a,R_\alpha}_{ii'}$ with Coulomb functions $J^a_{R_\alpha}$. The factorization of soft gluons from collinear fields follows from a field redefinition with a light-like Wilson line in soft-collinear effective theory (SCET) in the usual way [5]. To prove the decoupling from non-relativistic fields we redefine $\psi_a(x) = S^{(R)}_v(x^0)_{ab}\,\psi^{(0)}_b(x)$ with a time-like Wilson line

$$S^{(R)}_v(x) = \overline{\mathrm{P}}\exp\left[-ig_s\int_0^\infty dt\; v\cdot A^c_s(x+vt)\mathbf{T}^{(R)c}\right]. \tag{2.2}$$

This has the effect of turning D^0_s into ∂^0 in the leading-order PNRQCD Lagrangian

$$\begin{aligned}\mathscr{L}_{\text{PNRQCD}} \;=\; & \psi^\dagger\left(iD^0_s + \frac{\vec{\partial}^2}{2m_H} + \frac{i\Gamma_H}{2}\right)\psi + \psi'^\dagger\left(iD^0_s + \frac{\vec{\partial}^2}{2m_{H'}} + \frac{i\Gamma_{H'}}{2}\right)\psi' \\ & + \int d^3\vec{r}\,\left[\psi^\dagger\mathbf{T}^{(R)a}\psi\right](\vec{r})\left(\frac{\alpha_s}{r}\right)\left[\psi'^\dagger\mathbf{T}^{(R')a}\psi'\right](0).\end{aligned} \tag{2.3}$$

The key observation is that S_v drops out from the Coulomb interaction expressed in terms of the new fields, since $S^{(R)\dagger}_v\mathbf{T}^{(R)a}S^{(R)}_v = [S_{\text{ad}}]^{ab}\,\mathbf{T}^{(R)b}$ in any representation R and since S_{ad} in the adjoint representation is real and independent of $\vec{r}$. For the latter point it is important that the soft gluon field in D^0_s depends only on x^0 [6], while the Coulomb interaction is non-local but instantaneous. This proves decoupling of soft gluon and Coulomb resummation, since soft gluons disappear from the leading-order Lagrangians for the other fields. They do not disappear from higher-order terms in the SCET and PNRQCD Lagrangians, but these sub-leading interactions can be treated as perturbations in β, resulting in the sum over a in (2.1).

3. All-order diagonal colour basis for the leading soft function

The soft function relevant to the leading power in the β expansion ($a=1$ in (2.1)) is given by the Fourier transform of $\hat{W}^{R_\alpha}_{ii'}(z,\mu) = P^{R_\alpha}_{\{k\}}c^{(i)}_{\{a\}}\hat{W}^{\{k\}}_{\{ab\}}(z,\mu)c^{(i')*}_{\{b\}}$ where

$$\hat{W}^{\{k\}}_{\{ab\}}(z,\mu) = \langle 0|\overline{\mathrm{T}}[S_{v,b_4k_2}S_{v,b_3k_1}S^\dagger_{\bar{n},jb_2}S^\dagger_{n,ib_1}](z)\mathrm{T}[S_{n,a_1i}S_{\bar{n},a_2j}S^\dagger_{v,k_3a_3}S^\dagger_{v,k_4a_4}](0)|0\rangle, \tag{3.1}$$

and $\{a\} = a_1a_2a_3a_4$ denotes a colour multi-index for the $2\to 2$ production process. $\hat{W}^{R_\alpha}_{ii'}(z,\mu)$ is obtained from (3.1) by projecting it with $P^{R_\alpha}_{\{k\}}$ on an irreducible representation R_α in the product $R\otimes R' = \sum_\beta R_\beta$ of final state representations, and on the elements $c^{(i)}_{\{a\}}$ of a suitable colour basis. The number of basis elements ($i,i' = 1\ldots n$) is constrained by colour conservation. Further decomposing the product of initial state colour representations into irreducible ones according to $r\otimes r' = \sum_\alpha r_\alpha$, the number of basis elements equals the number of pairs $P_i = (r_\alpha, R_\beta)$ of equivalent representations r_α and R_β. For example, in case of a $8\otimes 8\to 8\otimes 8$ process, $P_i \in$

$\{(1,1),\ (8_S,8_S),\ (8_A,8_S),\ (8_A,8_A),\ (8_S,8_A),\ (10,10),\ (\overline{10},\overline{10}),\ (27,27)\}$, so the basis is eight-dimensional. One then finds [3] that $W^{R_\alpha}_{ii'}(\omega,\mu)$ in (2.1) is diagonal to all orders in perturbation theory in the colour basis

$$c^{(i)}_{\{a\}} = \frac{1}{\sqrt{\dim(r_\alpha)}} C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4} \tag{3.2}$$

constructed from the Clebsch-Gordan coefficients that couple the initial and final colour representations to the equivalent pairs P_i. This result follows from colour conservation; the fact that the Coulomb interaction is diagonal in the irreducible colour representations; and the ability to combine the two final-state Wilson lines into a single one using $C^{R_\alpha}_{\alpha a_1 a_2} S^{(R)}_{v,a_1 b_1} S^{(R')}_{v,a_2 b_2} = S^{(R_\alpha)}_{v,\alpha\beta} C^{R_\alpha}_{\beta,b_1 b_2}$, since the Wilson lines for a heavy-particle pair produced directly at threshold carry the same velocity vector v. Finally, due to Bose symmetry of the soft function there is no interference of the production from a symmetric and antisymmetric colour octet, which excludes the only possible off-diagonal terms after applying the arguments above.

4. Two-loop anomalous dimension for heavy-particle pair production

Once the leading soft function is diagonal and reduced to the soft function of an effective $2 \to 1$ process with a coloured final-state particle in representation R_α, the soft gluon part of the resummation can be done in SCET just as for Drell-Yan production [7]. For NNLL resummation, one only needs in addition the two-loop anomalous dimension and the one-loop finite term of the heavy-particle soft functions. Since (3.1) is essentially the “square” of the soft function relevant to the effective $2 \to 1$ amplitude, the two-loop anomalous dimension satisfies Casimir scaling and can be extracted from [8]. In particular, potential three-particle colour correlations vanish trivially after the reduction to the $2 \to 1$ process. The result can be converted to the anomalous dimension required in the Mellin-space resummation formalism [2], which requires the one-loop calculation of the soft function, and one finds [3]

$$D^{(1)R_\alpha}_{HH'} = -C_{R_\alpha} C_A \left(\frac{460}{9} - \frac{4\pi^2}{3} + 8\zeta_3\right) + \frac{176}{9} C_{R_\alpha} T_F n_f, \tag{4.1}$$

which has been confirmed independently in [9]. The process-independent ingredients for NNLL resummation of threshold logarithms in arbitrary production processes of heavy coloured particles at hadron colliders are now in place. The resummation of the squark anti-squark production cross section at the LHC at NLL using the formalism outlined above is discussed in [4].

5. Threshold enhancements at NNLO

When resummation is not required one can use the fixed-order expansion of (2.1) to calculate the velocity-enhanced terms at NNLO. However, to obtain all $\alpha_s^2 \ln^{2,1}\beta$ correction terms, one must include logarithms from sub-leading heavy-quark potentials and sub-leading soft gluon couplings [3]. These additional terms have been worked out explicitly in [10]. The contributions from sub-leading soft gluon couplings could be a source of velocity-enhanced three-particle colour correlations, which have been calculated in [11] and found to be non-zero at the amplitude level. The result of [10] implies that there are no contributions to the $\ln\beta$ terms from such three-particle correlations in both, the virtual and real corrections to the total cross section. This holds independent of particular colour representations.

Here we provide the velocity-enhanced terms at NNLO for the production of a pair of heavy particles with equal mass m in the scattering of massless partons in colour representations r and r', respectively, under the only assumption that the Born cross section admits an S-wave term proportional to β. The heavy-particle pair is in colour representation R_α and a definite spin state (singlet or triplet for spin-1/2 fermions). Denoting by $\sigma_X^{(2)}$ the NNLO correction relative to the Born cross section, the threshold expansion reads [10]

$$
\begin{aligned}
\sigma_X^{(2)} = {}& \frac{4\pi^4 D_{R_\alpha}^2}{3\beta^2} + \frac{\pi^2 D_{R_\alpha}}{\beta}\left\{(-8)\,(C_r+C_{r'})\left[\ln^2\left(\frac{2m\beta^2}{\mu}\right) - \frac{\pi^2}{8}\right] + 2\,(\beta_0+4C_{R_\alpha})\ln\left(\frac{2m\beta^2}{\mu}\right)\right. \\
&\left. \qquad -8C_{R_\alpha} - 2a_1 - 4\,\mathrm{Re}\,[C_X^{(1)}] + 2\beta_0\ln\left(\frac{2m}{\mu}\right)\right\} \\
&+128\,(C_r+C_{r'})^2\ln^4\beta + 64\,(C_r+C_{r'})\left\{4\,(C_r+C_{r'})\,(L_8-2) - \frac{\beta_0}{3} - 2C_{R_\alpha}\right\}\ln^3\beta \\
&+\left\{\frac{8}{3}\,(C_r+C_{r'})^2\left[72L_8^2 - 288L_8 + 576 - 35\pi^2\right] + \frac{16}{9}\,(C_r+C_{r'})\left[18\,\mathrm{Re}\,[C_X^{(1)}]\right.\right. \\
&\quad \left. + 18\beta_0\,(-L_8+2) + 36C_{R_\alpha}\,(-3L_8+7) + C_A(67-3\pi^2) - 20n_l T_f\right] \\
&\quad \left. + 16C_{R_\alpha}\,(\beta_0+2C_{R_\alpha})\right\}\ln^2\beta \\
&+\left\{8\,(C_r+C_{r'})^2\left[8L_8^3 - 48L_8^2 + \left(192 - \frac{35\pi^2}{3}\right)L_8 - 384 + \frac{70\pi^2}{3} + 112\zeta_3\right]\right. \\
&\quad + 2\,(C_r+C_{r'})\left[-16\,\mathrm{Re}\,[C_X^{(1)}]\,(-L_8+2) + \beta_0\left(-8L_8^2 + 32L_8 - 64 + \frac{11\pi^2}{3}\right)\right. \\
&\qquad + 2C_{R_\alpha}\left(-24L_8^2 + 112L_8 - 224 + \frac{35\pi^2}{3}\right) \\
&\qquad \left. + C_A\left(\frac{8}{3}\left(\frac{67}{3} - \pi^2\right)L_8 - \frac{4024}{27} + \frac{59\pi^2}{9} + 28\zeta_3\right) + \frac{4n_l T_f}{9}\left(-40L_8 + \frac{296}{3} - \pi^2\right)\right] \\
&\quad + 4C_{R_\alpha}\left[-4\,\mathrm{Re}\,[C_X^{(1)}] - 4\,(\beta_0+2C_{R_\alpha})\,(-L_8+3) + C_A\left(-\frac{98}{9} + \frac{2\pi^2}{3} - 4\zeta_3\right)\right. \\
&\qquad \left.\left. + \frac{40}{9}n_l T_f\right] + 16\pi^2 D_{R_\alpha}\left[C_A - 2D_{R_\alpha}(1+v_{\mathrm{spin}})\right]\right\}\ln\beta + \mathcal{O}(1)\,. \qquad (5.1)
\end{aligned}
$$

Here C_r, $C_{r'}$ and C_{R_α} denote the quadratic Casimir operators of the colour representations, $\beta_0 = \frac{11}{3}C_A - \frac{4}{3}n_l T_f$ is the one-loop beta-function coefficient, and $L_8 = \ln(8m/\mu)$. The quantities D_{R_α}, $a_1 = \frac{31}{9}C_A - \frac{20}{9}n_l T_f$ and v_{spin} are connected with the heavy-quark potentials such that $v_{\mathrm{spin}} = 0$ and $-2/3$ for a pair of spin-1/2 fermions in a spin-singlet and spin-triplet state, respectively, and D_{R_α} refers to the strength of the Coulomb potential in representation R_α ($D_{R_\alpha} = -C_F$ for the singlet and $D_{R_\alpha} = -(C_F - C_A/2)$ for the octet representation). The only process-specific input is $\mathrm{Re}\,[C_X^{(1)}]$, equal to one half of the the one-loop short-distance coefficient $H_X(M,\mu)$ in (2.1), when the heavy-particle pair is in colour and spin state X. Alternative to a direct computation, it can also be deduced from the constant term in the threshold limit of the NLO production cross section $\sigma_X^{(1)}$ in colour and spin channel X by comparing the expansion of $\sigma_X^{(1)}$ to the formula

$$\sigma_X^{(1)} = -\frac{2\pi^2 D_{R_\alpha}}{\beta} + 4\,(C_r + C_{r'})\left[\ln^2\left(\frac{8m\beta^2}{\mu}\right) + 8 - \frac{11\pi^2}{24}\right] \\ -4\,(C_{R_\alpha} + 4\,(C_r + C_{r'}))\,\ln\left(\frac{8m\beta^2}{\mu}\right) + 12 C_{R_\alpha} + 2\,\mathrm{Re}\,[C_X^{(1)}] + \mathscr{O}(\beta)\,. \qquad (5.2)$$

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