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All-order colour structure in threshold resummation and squark-antisquark production at NLL

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We consider the resummation of soft and Coulomb gluons for pair-production processes of heavy coloured particles at hadron colliders, and discuss recent results on the construction of a basis in colour space that diagonalizes the soft function to all orders in perturbation theory and the determination of the two-loop soft anomalous dimension needed for NNLL resummations. We present results for the combined NLL resummation of soft gluon and Coulomb-gluon effects for squark-antisquark production at the LHC.

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1. Introduction

Perturbative corrections to the partonic cross sections for pair production of heavy coloured particles H,H' (top quarks, squarks, gluinos...) at hadron colliders contain terms of the form $[\alpha_s \ln^2\beta]^n$ ("threshold logarithms") and $(\alpha_s/\beta)^n$ ("Coulomb singularity") that are enhanced near the partonic threshold $\hat{s} \approx 4M^2$ but can be resummed to all orders in perturbation theory. (Here $\beta = (1-4M^2/\hat{s})^{1/2}$, M the average heavy-particle mass and $\hat{s}$ the partonic centre of mass-energy.) Resummation of threshold logarithms at next-to leading logarithmic accuracy (NLL) was achieved in [1] in the Mellin-moment formalism [2]. Coulomb resummation has been performed for top, squark and gluino pair production [3–5]. Here we report on work [6] towards threshold resummation at next-to-next-to leading logarithmic (NNLL) accuracy and the combination with Coulomb resummation. We present results for a combined momentum space [7] resummation of NLL threshold logarithms and leading Coulomb corrections for squark-antisquark production at the LHC.

2. Factorization, diagonalization and resummation

Near threshold, the partonic cross sections of the subprocesses $pp' \to HH'+X$ where $pp' \in \{qq, q\bar{q}, gg, gq, g\bar{q}\}$ satisfy the factorization formula [6] (with $E = \sqrt{\hat{s}}-2M$)

$$\hat{\sigma}_{pp'}(\hat{s},\mu) = \sum_{i,i'} H_{ii'}(M,\mu) \int d\omega \sum_{R_\alpha} J_{R_\alpha}(E-\tfrac{\omega}{2})\, W^{R_\alpha}_{ii'}(\omega,\mu) \tag{2.1}$$

that separates hard, soft and potential effects and includes the leading Coulomb singularities $(\alpha_s/\beta)^n$ to all orders.[1] The sum extends over the irreducible colour representations in the decomposition $R\otimes R' = \sum_\alpha R_\alpha$ of the final-state representations. The potential function J^{R_α} is proportional to the imaginary part of the non-relativistic zero-distance Coulomb Green function quoted e.g. in [4]. The soft function $W^{R_\alpha}_{ii'}$ is defined in terms of soft Wilson lines (see eq. (1.6) of [6]) and is related to the threshold logarithms while the $H_{ii'}$ encode the partonic hard-scattering processes. The indices i,i' denote the elements $c^{(i)}_{\{a\}}$ of a basis of colour structures. As we have shown in [6], the soft function is diagonal in the basis

$$c^{(i)}_{\{a\}} = \dim(r_\alpha)^{-\frac{1}{2}}\, C^{r_\alpha}_{\alpha a_1 a_2} C^{R_\beta *}_{\alpha a_3 a_4}, \tag{2.2}$$

where the C are Clebsch-Gordan coefficients combining the initial (final) state representations r, r' (R,R') to an irreducible representation r_α (R_β) and the index i labels pairs $P_i = (r_\alpha, R_\beta)$ of equivalent initial and final state representations. For quark-antiquark and gluon initiated squark-antisquark production the allowed pairs are $P_i \in \{(1,1),\ (8,8)\}$ and $P_i \in \{(1,1),\ (8_S,8),\ (8_A,8)\}$.

In the diagonal basis, the soft function in position space satisfies an evolution equation [6]

$$\frac{d}{d\ln\mu}\hat{W}^{R_\alpha}_{ii}(z_0) = 2\left((\Gamma^r_{\rm cusp}+\Gamma^{r'}_{\rm cusp})\ln\left(\frac{iz_0\mu e^{\gamma_E}}{2}\right) - (\gamma^{R_\alpha}_{H,s}+\gamma^r_s+\gamma^{r'}_s)\right)\hat{W}^{R_\alpha}_{ii}(z_0), \tag{2.3}$$

similar to that of the soft function in the Drell-Yan process [9]. Threshold logarithms can be resummed [9, 7] by evolving the soft function using (2.3) from a scale μ_s characteristic for the soft

[1] Eq. (2.1) applies to S-wave production of the heavy particles. Note that (as mentioned in [6]) this formula does not include $\mathcal{O}(\alpha_s^2 \log\beta)$ terms related to higher order potential effects and higher-dimensional soft functions that may account for the three parton correlations reported in [8]. However, these NNLL effects do not affect the soft function $W^{R_\alpha}_{ii'}$ discussed here.

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radiation to a scale μ_f where the parton distribution functions are evaluated. Using an analogous equation, the hard function is evolved from a scale $\mu_h \sim 2M$ to μ_f. The anomalous dimensions $\Gamma^r_{\rm cusp}$ and γ^r_s for a massless particle in the SU(3) representation r are known with the accuracy required for NNLL resummations. The soft anomalous dimension of the heavy particle system in the representation R_α, $\gamma^{R_\alpha}_{H,s}$, has been given at one-loop for some examples [1, 5]. The result up to two-loops, as required for NNLL resummations, was extracted in [6] from results of [10] and confirmed by [11] for top production (with C_{R_α} the quadratic Casimir invariant for the representation R_α):

$$\gamma^{R_\alpha}_{H,s} = \frac{\alpha_s}{4\pi}(-2C_{R_\alpha}) + \left(\frac{\alpha_s}{4\pi}\right)^2 C_{R_\alpha}\left[-C_A\left(\frac{98}{9}-\frac{2\pi^2}{3}+4\zeta_3\right)+\frac{40}{18}n_f\right]+\mathcal{O}(\alpha_s^3). \tag{2.4}$$

3. Combined soft and Coulomb resummation for squark-antisquark production

The factorization formula (2.1) allows a combined resummation of soft- and Coulomb effects using the momentum space solution [7] to the evolution equation (2.3) and the resummed Coulomb Green function J. We give here an analytical result for the partonic cross section that includes soft corrections resummed to NLL accuracy (the first term in square brackets), single Coulomb exchange interfering with resummed NLL soft radiation (the second term in square brackets) and higher-order Coulomb corrections $(\alpha_s/\beta)^n$ resummed without soft radiation (the last term):

$$\hat\sigma^{\rm NLL\otimes C}_{pp'} = \sum_{i,R_\alpha}\hat\sigma^{i,(0)}_{pp'}\left\{U_i^{R_\alpha}\left(\frac{Ee^{-\gamma_E}}{M}\right)^{2\eta}\left[\frac{\sqrt{\pi}}{2\Gamma(2\eta+\frac{3}{2})}+\frac{\kappa_{R_\alpha}}{\Gamma(2\eta+1)}\right]+2\kappa_{R_\alpha}{\rm Im}[\psi(1-i\kappa_{R_\alpha})]\right\}. \tag{3.1}$$

Here $\kappa_{R_\alpha} = (-C_{R_\alpha}/2)\alpha_s(\mu_C)\sqrt{2m_{\rm red}/E}$ with the reduced mass m_{red} of the heavy particle pair, $C_1 = -C_F$, $C_8 = 1/(2N_C)$, $\sigma^{i(0)}$ are the tree cross sections for the singlet and octet colour channels [12] at threshold, $\eta = 2a_\Gamma(\mu_s,\mu_f)$ and (γ_i^V and $\gamma^{\phi,r}$ are given in [6]):

$$U_i^{R_\alpha} = \exp[4S(\mu_h,\mu_s) - 2a_i^V(\mu_h,\mu_s) + 2a^{\phi,r}(\mu_s,\mu_f) + 2a^{\phi,r'}(\mu_s,\mu_f)]\left(\frac{4M^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_h,\mu_s)}$$

$$S(\mu_h,\mu_s) = -\int_{\alpha_s(\mu_h)}^{\alpha_s(\mu_s)} d\alpha_s \frac{\Gamma^r_{\rm cusp}(\alpha_s)+\Gamma^{r'}_{\rm cusp}(\alpha_s)}{2\beta(\alpha_s)}\int_{\alpha_s(\mu_h)}^{\alpha_s}\frac{d\alpha_s'}{\beta(\alpha_s')},$$

$$a_\Gamma(\mu_a,\mu_b) = -\int_{\alpha_s(\mu_a)}^{\alpha_s(\mu_b)} d\alpha_s \frac{\Gamma^r_{\rm cusp}(\alpha_s)+\Gamma^{r'}_{\rm cusp}(\alpha_s)}{2\beta(\alpha_s)}, \qquad a_i^V(\mu_a,\mu_b) = -\int_{\alpha_s(\mu_a)}^{\alpha_s(\mu_b)} d\alpha_s \frac{\gamma_i^V(\alpha_s)}{\beta(\alpha_s)}. \tag{3.2}$$

In order to calculate the squark-antisquark production cross section at the LHC, we match to the fixed-order NLO calculation [13] in the parameterization of [12] (see e.g. [5]) and convolute the partonic cross section (3.1) with the MSTW08NNLO parton-distribution functions. We choose μ_s as the scale $\tilde\mu_s$ that minimizes the fixed-order one-loop soft corrections [7], resulting in $\tilde\mu_s = 123-455$ GeV for $m_{\tilde q} = 200-2000$ GeV. In the quantity κ_{R_α} we set $\mu_C = 2M\max\{\beta,\alpha_s(\mu_C)\}$, motivated by the momentum transfer $|\vec k| \sim M\beta \sim M\alpha_s$ involved in the Coulomb corrections and the form of the known higher order corrections to the Coulomb Green function. In the results below, we identify the hard- and factorization scales, $\mu_h = \mu_f$. The black (solid) line in the left plot in figure 1 shows the corrections from soft and Coulomb-gluon effects as described by (3.1) relative to the fixed-order NLO cross section. The blue (long-dashed) line shows the NLL soft-gluon corrections alone (i.e. only the first term in the square bracket in (3.1)), in qualitative agreement with results from

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NLL resummation in Mellin space [5]. The red (dot-dashed) curve includes the effect of Coulomb-resummation added to the soft NLL corrections (without soft-Coulomb interference). Our choice for μ_C results in larger corrections than those found in [5] for $\mu_C = \mu_f$. Finally we compare to the NNLO$_{\text{approx}}$-results in eqs. (17-18) of [12] which include soft-Coulomb interference and the running coupling in the Coulomb potential at fixed order, together with NNLL soft corrections and further two-loop Coulomb corrections. For the magenta (dotted) curve the tree-level scaling functions $f^{(00)}$ at threshold have been used (as $\sigma^{i(0)}$ in (3.1)) whereas in the green (dashed) curve the full tree-level result is used as in [12]. The relative corrections obtained from (3.1) and the NNLO$^{\text{NR}}_{\text{approx}}$ result include different higher order effects and hence differ by $20-35\%$. The right plot in figure 1 shows the reduced factorization scale dependence of the NLL compared to the NLO cross section. The red band accounts for the variation of the soft scale between $\tilde{\mu}_s/2 < \mu_s < 2\tilde{\mu}_s$.

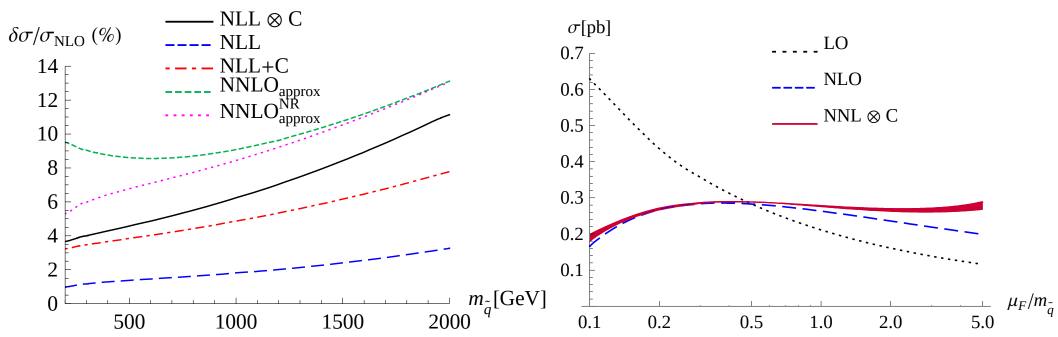

Figure 1: Squark-antisquark production cross section for an 14 TeV LHC and a gluino mass $m_{\tilde{g}} = 1.25 m_{\tilde{q}}$. Left: Relative corrections in various approximations. Right: factorization scale dependence for $m_{\tilde{q}} = 1$ TeV.

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