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A ROBUST APPROACH TO THE ORDER DETECTION FOR THE DAMPED SINUSOIDS BASED ON THE SHIFT-INVARIANCE PROPERTY

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ABSTRACT

In this paper, the problem of robust order detection for the damped sinusoids in impulsive noise environments is addressed. First of all, a series of candidate models are assumed. It is a challenging issue to extract the potential signal subspaces for these candidate models in a computationally efficient way. To alleviate this difficulty, the successive robust low-rank decomposition (SRLRD) procedure is devised. Correspondingly, the values of the subspace-based automatic model order selection (SAMOS) criterion are calculated. As a result, the signal order estimate is determined by the candidate model with the minimum criterion value. The consistency and superiority of the proposed estimator is validated by the simulation results in comparison with the other existing schemes.

***Index Terms*—** Robust order detection, damped sinusoids, successive robust low-rank decomposition, subspace-based automatic model order selection, impulsive noise.

1. INTRODUCTION

Spectral analysis of sinusoidal signals [1] has been a classical but ever active topic in the signal processing community, finding its applications in a wide range of areas. For example, in music and voiced speech signal processing, the measured signals from the microphone array can be modeled as a two-dimensional (2-D) harmonic signal [2], which is characterized by the temporal fundamental frequencies and directions-of-arrival (DoAs). The accurate acquisition of these parameters is crucial to the signal enhancement and source localization. In the biomedical engineering, the free induction decay (FID) signal, which is measured using the spectroscopic methods such as the nuclear magnetic resonance (NMR) and nuclear quadrupole resonance (NQR), may be modeled as a sum of exponentially damped sinusoids well [3]. With the exact quantification of these damped sinusoids, it is advantageous to extract the useful biomedical information for diagnosis. Before estimating the sinusoidal parameters in a parametric way, it is essential to detect the signal order, that is the number of the signal's sinusoids [4].

The work of spectral analysis includes two aspects: 1) the detection of the signal order, and 2) the estimation of the frequencies and/or damping factors. In this paper, we focus on the order detection for the damped sinusoidal signals. During the past decades, there has been published numerous literature on this problem. In [5], the various information theoretic criteria are proposed for the determination of the number of the undamped sinusoids embedded in the white-Gaussian noise, such as the minimum description length (MDL) criterion, direct Kullback-Leibler (KL) approach, cross-validatory KL approach based on the Akaike information criterion (AIC), generalized cross-validatory KL approach based on the generalized information criterion (GIC) and Bayesian approach based on the Bayesian information criterion (BIC). For the damped sinusoids, there have been proposed several methods based on the rank determination of the data matrix, including the order estimators of MUltiple SIgnal Classification (MUSIC) [6], ESTimation ERror (ESTER) [7], Subspace-based Automatic Model Order Selection (SAMOS) [8], etc. In most of the above work, the background noise is assumed as white Gaussian.

In the practical applications, the impulsive noise is an important class of observation noise [9]. It follows a non-Gaussian and heavy-tailed distribution, and occurs randomly with a value several times larger than the standard deviation of the background noise. Therefore, the conventional order detection methods, which are designed for the white-Gaussian noise, are not directly applicable to the impulsive noise.

Recently, there have been proposed several approaches to the signal order detection in impulsive noise environments, including [10–12], etc. Since the information theoretic criteria, such as the BIC [10], AIC, and MDL [11], are derived under the assumption of large data length, they are not applicable to the damped sinusoids [8, 13]. In [12], the minimum covariance determinant (MCD) and MM estimators are employed in combination with the bootstrap technique, respectively, in order to detect the source number in impulsive noise environments. However, their asymptotic consistency with respect to the signal-to-noise ratio (SNR) is not guaranteed [12]. Thus, it is meaningful to devise a robust approach to the order detection for the damped sinusoids, which is resistant to impulsive noise.

In this work, we try to address this issue by estimating the rank of the signal subspace. In detail, a series of candidate models are assumed firstly, which are differentiated in terms of the signal order. Then, the potential signal subspace is extracted for each candidate model, and the detection criterion value is calculated. Finally, the signal order, which corresponds to the minimum value of the detection criterion, is selected. In the conventional order detection [6–8], the signal subspace is extracted just by the singular value decomposition (SVD), which is not resistant to impulsive noise.

To overcome this difficulty, the successive robust low-rank decomposition (SRLRD) procedure is devised. For each candidate model, the potential signal subspace can be extracted from the impulsive noise environment by the robust low-rank decomposition (RLRD), that is by minimizing the ℓ_p-norm ($1 \leq p < 2$) of the projection error. Nevertheless, it is computationally heavy to conduct the RLRD, especially for all the candidate models. To keep the computational burden into a reasonable extent, the SRLRD is proposed as a relaxed scheme, where the potential signal subspaces are extracted column by column. For the correct model, this is equivalent to recovering the signal subspace in a greedy way, and is expected to be accurate when the SNR is sufficiently high. Therefore, it is feasible to find the order estimate by imposing the rank-determination criterion on the output of the SRLRD. Here, the SAMOS is adopted as the rank-determination criterion due to its performance gain over the other schemes.

The rest of this paper is organized as follows. The signal model and problem formulation are introduced in Section 2. Then, our robust signal order detection approach is designed in Section 3. The explanation of its asymptotic consistency is also provided. The simulation results are presented in Section 4 to evaluate the performance of the proposed detection approach. Finally, the conclusion is drawn in Section 5.

2. PROBLEM FORMULATION

Consider the damped sinusoids, which are corrupted by impulsive noise, as follows:

$$x(n) = s(n) + v(n) = \sum_{m=1}^{M} \rho_m z_m^n + v(n), \qquad (1)$$

for $n = 1, \cdots, N$, where $z_m = e^{-\alpha_m + j\omega_m}$ represents the mth sinusoidal pole, with $\omega_m \in [0,\ 2\pi)$, $\alpha_m > 0$ and $\rho_m \in \mathbb{C}$ being the frequency, damping factor and complex-valued amplitude of z_m, respectively; and M represents the signal order, that is the number of the sinusoidal poles. Here, $v(n)$ is the additive impulsive noise, which is assumed as independent and identically distributed (i.i.d.).

The purpose of this paper is to detect the order of the damped sinusoids $s(n)$ from the observation $x(n)$, that is M, in impulsive noise environments. How to overcome the outliers is the critical issue in the order detection.

3. ALGORITHM DEVELOPMENT

Suppose that there exist L candidate models, which are indexed as $l = 1, 2, \cdots, L$, respectively, and correspond to the different signal orders. In the lth candidate model, there exist l sinusoidal poles. In this work, it is aimed to select the correct model from these candidate models in the impulsive noise environment.

In our order detection procedure, the data matrix is constructed firstly. With the use of the SRLRD, the potential signal subspaces are then extracted for the candidate models indexed by $l = 1, 2, \cdots, L$. Correspondingly, the values of the SAMOS criterion are calculated. As a result, the candidate model with the minimum criterion value is selected as the correct model. The detail is illustrated as follows.

3.1. Extraction of the Signal Subspace with the SRLRD

First of all, we construct the data matrix for the observation $x(n)$ as follows:

$$\mathbf{X} = \mathbf{S} + \mathbf{Q}, \qquad (2)$$

where $\mathbf{X} \in \mathbb{C}^{P \times P'}$ ($P' \triangleq N - P + 1$) is the Hankel matrix with the (i, j)th element $[\mathbf{X}]_{i,j} = x(i + j - 1)$, $i = 1, \cdots, P, j = 1, \cdots, P'$, and the row number P satisfying $M < P < N + 1 - M$. The matrices $\mathbf{S}$ and $\mathbf{Q}$ are the noise-free part and disturbance of $\mathbf{X}$, respectively.

Now let us focus on the correct model, with $l = M$. On one hand, with the Vandermonde decomposition, $\mathbf{S}$ of (2) is expressed as [14]:

$$\mathbf{S} = \mathbf{A}_M \mathbf{\Gamma}_M \mathbf{H}_M^T, \qquad (3)$$

where

$$\begin{aligned} \mathbf{\Gamma}_M &= \operatorname{diag}\left(\begin{bmatrix} \rho_1 & \rho_2 & \cdots & \rho_M \end{bmatrix}^T \right), \\ \mathbf{A}_M &= \begin{bmatrix} \mathbf{a}(z_1) & \mathbf{a}(z_2) & \cdots & \mathbf{a}(z_M) \end{bmatrix}, \\ \mathbf{H}_M &= \begin{bmatrix} \mathbf{h}(z_1) & \mathbf{h}(z_2) & \cdots & \mathbf{h}(z_M) \end{bmatrix}, \end{aligned}$$

with $\operatorname{diag}(\boldsymbol{\rho})$ denoting the diagonal matrix with the elements of $\boldsymbol{\rho}$ on the main diagonal, and $\mathbf{a}(z_m) = \begin{bmatrix} z_m & \cdots & z_m^P \end{bmatrix}^T$, $\mathbf{h}(z_m) = \begin{bmatrix} 1 & z_m & \cdots & z_m^{N-P} \end{bmatrix}^T$, $m = 1, \cdots, M$. On the other hand, $\mathbf{S}$ is decomposed using SVD as:

$$\begin{aligned} \mathbf{S} &= \mathbf{U} \mathbf{\Lambda} \mathbf{V}^H \\ &= \begin{bmatrix} \mathbf{U}_s & \mathbf{U}_n \end{bmatrix} \begin{bmatrix} \mathbf{\Lambda}_s & \mathbf{0} \\ \mathbf{0} & \mathbf{\Lambda}_n \end{bmatrix} \begin{bmatrix} \mathbf{V}_s & \mathbf{V}_n \end{bmatrix}^H \\ &= \mathbf{U}_s \mathbf{\Lambda}_s \mathbf{V}_s^H, \end{aligned} \qquad (4)$$

where $\mathbf{U}_s \in \mathbb{C}^{P\times M}$, $\mathbf{V}_s \in \mathbb{C}^{P'\times M}$, $\mathbf{\Lambda}_s$ is an $M\times M$ diagonal matrix, and $\mathbf{U}_n \in \mathbb{C}^{P\times(P-M)}$, $\mathbf{V}_n \in \mathbb{C}^{P'\times(P'-M)}$, $\mathbf{\Lambda}_n$ is the $(P-M)\times(P'-M)$ zero matrix.

By comparing (3) and (4), it is seen that the columns of $\mathbf{A}_M$ span the same space as those of $\mathbf{U}_s$. Therefore, we denote the signal subspace by $\mathbb{U}_s = \mathrm{span}(\mathbf{A}_M)$. Obviously, the rank of $\mathbb{U}_s$, denoted by $\mathrm{rank}(\mathbb{U}_s)$, is equal to M. Since the target of the SAMOS criterion is to determine the signal order from the rank of $\mathbb{U}_s$, it is essential to extract the signal subspace accurately in impulsive noise environments.

In this work, it is proposed to find the signal subspace with the use of the SRLRD. Since $\mathrm{rank}(\mathbb{U}_s) = M$, $\mathbf{S}$ can be decomposed as the product of two low-rank matrices:

$$\mathbf{S} = \mathbf{A}'_M \mathbf{B}'^T_M, \tag{5}$$

where $\mathbf{A}'_M \in \mathbb{C}^{P\times M}$ and $\mathbf{B}'_M \in \mathbb{C}^{P'\times M}$ are of the full column rank. By comparing (3) and (5), it is seen that $\mathbf{A}_M$ and $\mathbf{A}'_M$ span the same space, i.e.,

$$\mathbb{U}_s = \mathrm{span}(\mathbf{A}_M) = \mathrm{span}(\mathbf{A}'_M). \tag{6}$$

Therefore, we can extract the signal subspace $\mathbb{U}_s$ from $\mathbf{X}$ by estimating $(\mathbf{A}'_M, \mathbf{B}'_M)$ as follows:

$$(\hat{\mathbf{A}}'_M, \hat{\mathbf{B}}'_M) = \arg\min_{(\mathbf{A}'_M, \mathbf{B}'_M)} \left\|\mathbf{X} - \mathbf{A}'_M \mathbf{B}'^T_M\right\|_p^p, \tag{7}$$

where $\hat{\mathbf{A}}'_M$ and $\hat{\mathbf{B}}'_M$ stand for the estimates of $\mathbf{A}'_M$ and $\mathbf{B}'_M$, respectively. Here, $\|\cdot\|_p$ denotes the ℓ_p-norm ($1 \le p < 2$) of one matrix, which is more resistant to outliers than the conventional ℓ_2-norm.

Since the optimization of (7) is nonconvex, it is computationally prohibitive to search for the globally optimal solution. Instead, we utilize the alternating optimization method (AOM) [15] as a relaxed scheme. In detail, (7) is solved in an iterative way. At the kth iteration, $\hat{\mathbf{A}}'_M$ and $\hat{\mathbf{B}}'_M$ are updated by solving the following two subproblems:

$$\mathbf{A}'^{(k)}_M = \arg\min_{\mathbf{A}'_M} \left\|\mathbf{X} - \mathbf{A}'_M \mathbf{B}'^{(k-1)T}_M\right\|_p^p, \tag{8}$$

$$\mathbf{B}'^{(k)}_M = \arg\min_{\mathbf{B}'_M} \left\|\mathbf{X} - \mathbf{A}'^{(k)}_M \mathbf{B}'^T_M\right\|_p^p. \tag{9}$$

Since (8) and (9) are both convex, the globally optimal solutions to them can be obtained by various gradient-based algorithms [16], respectively.

It should be addressed that, the two convex subproblems (8) and (9) are of the dimensions PM and $P'M$, respectively. If we follow the procedure of the SAMOS in [8], and extract the potential signal subspace according to (7) for each candidate model $l = 1, \cdots, L$, the computational cost will still be huge. To alleviate this issue, the SRLRD procedure is devised as a relaxed scheme, where the potential signal subspaces are extracted column by column. For the correct model, that is $l = M$, (7) is rewritten as:

$$(\hat{\mathbf{A}}'_M, \hat{\mathbf{B}}'_M) = \arg\min_{(\mathbf{A}'_M, \mathbf{B}'_M)} \left\|\mathbf{X} - \sum_{m=1}^{M} \mathbf{a}'_m \mathbf{b}'^T_m\right\|_p^p, \tag{10}$$

where $\{\mathbf{a}'_m\}_{m=1}^M$ and $\{\mathbf{b}'_m\}_{m=1}^M$ stand for the M columns of $\mathbf{A}'_M$ and $\mathbf{B}'_M$, respectively. In the SRLRD procedure, the columns $\left\{(\mathbf{a}'_m, \mathbf{b}'_m)\right\}_{m=1}^M$ are estimated in a successive way:

$$(\hat{\mathbf{a}}'_m, \hat{\mathbf{b}}'_m) = \arg\min_{(\mathbf{a}'_m, \mathbf{b}'_m)} \left\|\mathbf{X} - \sum_{i=1}^{m-1} \hat{\mathbf{a}}'_i \hat{\mathbf{b}}'^T_i - \mathbf{a}'_m \mathbf{b}'^T_m\right\|_p^p, \tag{11}$$

for $m = 1, 2, \cdots, M$. When handling each pair of $(\mathbf{a}'_m, \mathbf{b}'_m)$, $m = 1, \cdots, M$, we fix the values of the previous columns as the estimated: $(\mathbf{a}'_i, \mathbf{b}'_i) = (\hat{\mathbf{a}}'_i, \hat{\mathbf{b}}'_i)$, $i = 1, \cdots, m-1$. In essence, the similar idea is also found in the greedy algorithm [17], which is extensively applied in the sparse signal reconstruction, with the reliability evidenced by the numerous computational results.

3.2. Order Detection with the SAMOS Criterion

Based on the extraction of the signal subspace, it is possible to detect the signal order with the SAMOS criterion. With the use of the SVD, we take the first M left singular vectors of $\mathbf{A}'_M$, which is denoted by $\mathbf{U}_s^{(M)} = \begin{bmatrix}\mathbf{u}_1^{(M)} & \cdots & \mathbf{u}_M^{(M)}\end{bmatrix}$. Since $\mathbf{U}_s^{(M)}$ spans the same space as $\mathbf{A}_M$, it bears the shift-invariance property of [8]:

$$\mathbf{U}_{s\uparrow}^{(M)} = \mathbf{U}_{s\downarrow}^{(M)} \left(\mathbf{\Phi}\mathbf{D}\mathbf{\Phi}^{-1}\right), \tag{12}$$

where $\mathbf{D} = \mathrm{diag}([z_1, \cdots, z_M]^T)$, $\mathbf{\Phi}$ is a unitary matrix; and the subscripts $\uparrow$ and $\downarrow$ denote the first and last row-deleting operators, respectively. Consequently, the matrix $\mathbf{U}_{tb}^{(M)} \triangleq \begin{bmatrix}\mathbf{U}_{s\uparrow}^{(M)} & \mathbf{U}_{s\downarrow}^{(M)}\end{bmatrix}$ has the rank of M.

For the candidate models with $l < M$, the SRLRD procedure will halt just when the estimation of $\{\mathbf{a}'_m\}_{m=1}^l$ is finished. According to (6), $\mathbf{a}'_m \in \mathrm{span}(\mathbf{A}_M)$, $m = 1, \cdots, l$. For the candidate models with $l > M$, the SRLRD will continue until $\{\mathbf{a}'_m\}_{m=1}^l$ are all estimated. Thus, the estimation of $\{\mathbf{a}'_m\}_{m=M+1}^l$ is subjected to the projection error between $\mathbf{X}$ and $\hat{\mathbf{A}}'_M$. In these two cases, the matrix $\mathbf{U}_s^{(l)}$, which consists of the first l left singular vectors of $\mathbf{A}'_l = \begin{bmatrix}\mathbf{a}'_1 & \cdots & \mathbf{a}'_l\end{bmatrix}$, no longer bears the shift-invariance property as (12), and the rank of $\mathbf{U}_{tb}^{(l)}$ becomes larger than l.

Note that the noiseless data matrix $\mathbf{S}$ of (2) is not available in practice. Thus, we estimate the matrix $\mathbf{U}_{tb}^{(l)}$ with the observed data matrix $\mathbf{X}$ of (2), which is denoted by $\hat{\mathbf{U}}_{tb}^{(l)}$. Based on the principle of the SAMOS criterion, it is proposed to determine the number of the sinusoidal poles by

$$\hat{M} = \arg\min_{l\in\{1,\cdots,L\}} d(l), \tag{13}$$

where L is confined to an integer number less than $\min\{(P-1)/2, N-P+1\}$, and

$$d(l) = \frac{1}{l}\sum_{i=l+1}^{2l} \hat{\gamma}_i, \tag{14}$$

with $\hat{\gamma}_i$ being the ith largest singular value of $\hat{\mathbf{U}}_{tb}^{(l)}$. Here, the sinusoidal order detection criterion of (13) is termed as the robust SAMOS (R-SAMOS).

3.3. Consistency of the R-SAMOS Detection Criterion

When $\mathrm{SNR} \to \infty$, the data matrix $\mathbf{X}$ of (2) converges to $\mathbf{S}$. Accordingly, the signal subspace, which is estimated by the observation $\mathbf{X}$, should bear the shift-invariance property of (12). Furthermore, the matrix $\hat{\mathbf{U}}_{tb}^{(M)}$ has the rank of M, and the detection metric value, $d(M)$ of (14), is equal to zero. For the candidate models with $l \neq M$, the shift-invariance property normally vanishes from the potential signal subspaces. As a result, $d(l) > 0$ for $l \neq M$ [8]. Therefore, by means of the R-SAMOS detector, the correct signal order can be found with the probability of one when the SNR is sufficiently high.

It should be addressed that, there exists the threshold behavior of the signal order detection at certain SNR. This means that, the probability of correct detection (PCD) is equal to one above this threshold SNR; and the PCD falls down quickly below the threshold SNR. This originates from the phenomenon of the subspace swapping in the low SNR regime [18].

4. SIMULATION RESULTS

In this section, we show the performance of the proposed order detection approach with respect to SNR. Here, the SNR is explicitly defined as the average signal power divided by the noise variance. The performance is evaluated in terms of $\mathrm{PCD} = S_0/S$, with S_0 and S being the number of correct detection trials and the total number of trials, respectively. All the results provided are the averages of 500 independent runs, which are conducted on a PC with an Intel(R) Core(TM) i7-8750H CPU @ 2.20 GHz, with 16.0 GB of installed memory.

In this study, we consider the two-tone damped sinusoidal signal: $s(n) = A_1 e^{(-\alpha_1 + j\omega_1)n + j\phi_1} + A_2 e^{(-\alpha_2 + j\omega_2)n + j\phi_2}$, where $\omega_1 = 0.3$, $\alpha_1 = 0.01$, $A_1 = 1$, $\phi_1 = 1$, and $\omega_2 = 0.7$, $\alpha_2 = 0.02$, $A_2 = 1$, $\phi_2 = 2$. As for the noise part, we consider two common impulsive noise models, that is the Gaussian mixture model (GMM) and generalized Gaussian distribution (GGD):

GMM: The PDF of the circular GMM noise, v, is:

$$p_v(v) = \sum_{i=1}^{2} \frac{\rho_i}{\pi\sigma_i^2} \exp\left(-\frac{|v|^2}{\sigma_i^2}\right), \tag{15}$$

which consists of two terms of Gaussian distributions with the different variances σ_i^2, $i = 1, 2$. Here, it is set that $c_1 = 0.1$, $c_2 = 0.9$, and $\sigma_1^2 = 100\sigma_2^2$, which means that the outliers come present with the probability of 10% and with the variance 100 times that of the background noise. Correspondingly, the total variance of v is $\sigma_v^2 = 10.9\sigma_2^2$.

GGD: The PDF of the circular GGD noise, v, is:

$$p_v(v) = \frac{\beta\Gamma(4/\beta)}{2\pi\sigma_v^2\Gamma^2(2/\beta)} \exp\left(-\frac{|v|^\beta}{c\sigma_v^\beta}\right), \tag{16}$$

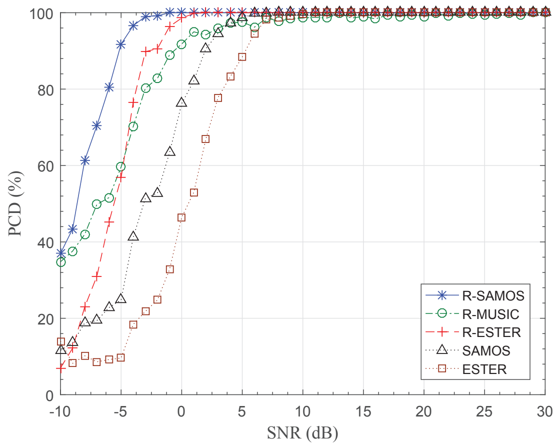
where σ_v^2 is the variance of the GGD, $\Gamma(\cdot)$ is the Gamma function, $c = \sqrt[\beta]{\Gamma(2/\beta)/\Gamma(4/\beta)}$, and $\beta > 0$ is the shape parameter of the GGD. Here, it is set that $\beta = 0.5$.

It is shown in [19] that the best shift-invariance property of the signal subspace is attained when the data matrix is as square as possible. Therefore, in the construction of the data matrix $\mathbf{X}$, the row number P is set as $\lfloor N/2 \rfloor$, with $\lfloor u \rfloor$ denoting the largest integer smaller than u. In the extraction of the signal subspace, the p of the ℓ_p-norm in (7) is set as $p = 1.1$. The setting of the row number and p value is found empirically to result in good performance. For comparison, we combine the detection criteria of MUSIC [6] and ESTER [7] with the SRLRD, which are named as the robust MUSIC (R-MUSIC) and ESTER (R-ESTER), respectively. To show the robustness of the devised SRLRD, we also provide the results of the SAMOS [8] and ESTER [7] criteria, where the potential signal subspaces are extracted just by the conventional SVD. Without loss of generality, the maximum possible signal order is set as $L = 5$.

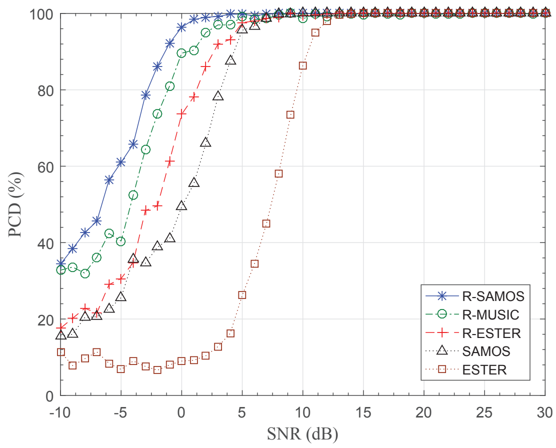
Fig. 1 shows the PCDs of the R-SAMOS criterion in the GMM noise environment, as well as those of the R-MUSIC, R-ESTER, SAMOS, and ESTER criteria. According to Fig. 1(a), for the data length $N = 50$, the R-SAMOS criterion attains the almost perfect detection (APD), that is $\mathrm{PCD} \geq 95\%$, when $\mathrm{SNR} \geq -4$ dB; while the PCDs of the R-MUSIC and R-ESTER criteria become higher than 95% when $\mathrm{SNR} \geq 3$ dB and $\mathrm{SNR} \geq -1$ dB, respectively. This means that the R-SAMOS criterion bears the threshold SNR advantage over the R-MUSIC and R-ESTER by at least 3 dB, which corresponds to the power saving of 50%. In particular, the R-SAMOS criterion attains the perfect detection (PD), that is $\mathrm{PCD} = 100\%$, when $\mathrm{SNR} \geq -1$ dB. Whereas, the PCD of the R-MUSIC fluctuates slightly below 1 in this SNR range, and the R-ESTER criterion achieves the PD when $\mathrm{SNR} \geq 2$ dB. For $N = 100$, the R-SAMOS criterion achieves the APD when $\mathrm{SNR} \geq 0$ dB. Compared with the R-MUSIC and R-ESTER criteria, the R-SAMOS criterion bears the threshold SNR advantage by at least 2 dB, which corresponds to the power saving of 37%.

Fig. 2 shows the PCDs in the GGD noise environment. For $N = 50$, the R-SAMOS, R-MUSIC, and R-ESTER criteria attain the APD when $\mathrm{SNR} \geq 0$ dB, $\mathrm{SNR} \geq 2$ dB, and $\mathrm{SNR} \geq 2$ dB, respectively. This means that the R-SAMOS criterion bears the threshold SNR advantage by 2 dB, or equivalently, provides the power saving of 37%. Similar to the case in Fig. 1(a), the R-SAMOS criterion achieves the PD when $\mathrm{SNR} \geq 6$ dB. Whereas, the PCD of the R-MUSIC criterion fluctuates slightly in this SNR range, and the R-ESTER criterion achieves the PD when $\mathrm{SNR} \geq 8$ dB. For $N = 100$, the R-SAMOS, R-MUSIC, and R-ESTER criteria attain the APD when $\mathrm{SNR} \geq 3$ dB, $\mathrm{SNR} \geq 3$ dB, and $\mathrm{SNR} \geq 8$ dB, respectively. It is also noted that the R-SAMOS criterion achieves the PD when $\mathrm{SNR} \geq 12$ dB, whereas the PCD of the R-MUSIC criterion fluctuates slightly in this SNR range.

According to Figs. 1 and 2, the threshold SNRs of the SAMOS and ESTER criteria are normally larger than those of the R-SAMOS criterion by $4 - 11$ dB and $3 - 11$ dB, for $N = 50$ and $N = 100$, respectively. Therefore, it is concluded that the proposed SRLRD brings the robustness to the signal subspace extraction in comparison with the SVD.

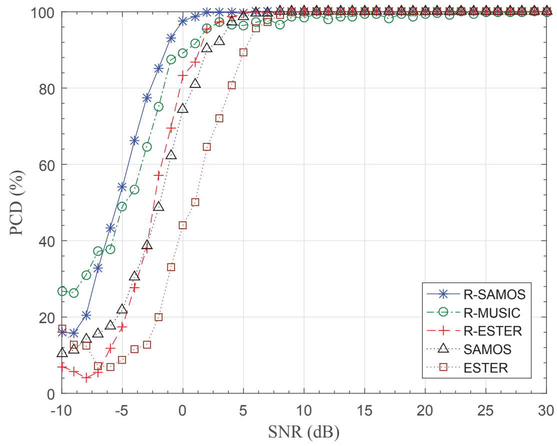


(a)

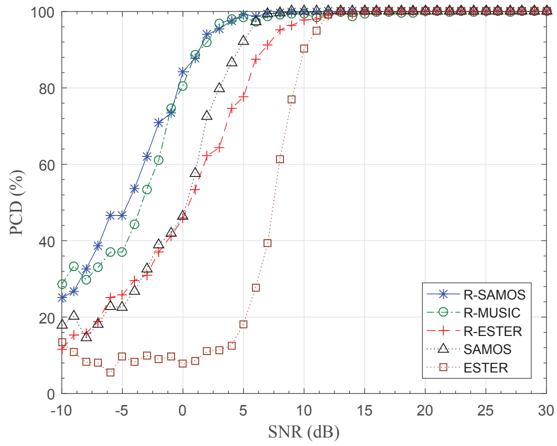


(b)

Fig. 1: PCDs for the two-tone damped sinusoidal signals in the GMM noise environment, with: (a) $N = 50$ and (b) $N = 100$.



(a)



(b)

Fig. 2: PCDs for the two-tone damped sinusoidal signals in the GGD noise environment, with: (a) $N = 50$ and (b) $N = 100$.

5. CONCLUSION

In this work, the R-SAMOS criterion is developed for the order detection of the damped sinusoids in impulsive noise environments. To keep the computational complexity into a reasonable extent, the SRLRD procedure is devised to extract the potential signal subspaces for a series of candidate models. Further on, the values of the SAMOS detection criterion are calculated for these candidate models. As a result, the candidate model with the minimum criterion value is selected as the correct model. The simulation results demonstrate the consistency of the R-SAMOS criterion when the SNR is sufficiently large, and its performance advantage over the other signal order detection schemes.

Future works include the extension of the R-SAMOS criterion to the more general scenarios such as the multi-channel and multi-dimensional sinusoidal signals, and its application in source localization [2], biomedical signal analysis [3], wireless communication [20], and so on.

6. ACKNOWLEDGMENT

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