Gaussian phase autocorrelation as an accurate compensator for FFT-based atmospheric phase screen simulations

SPIE.

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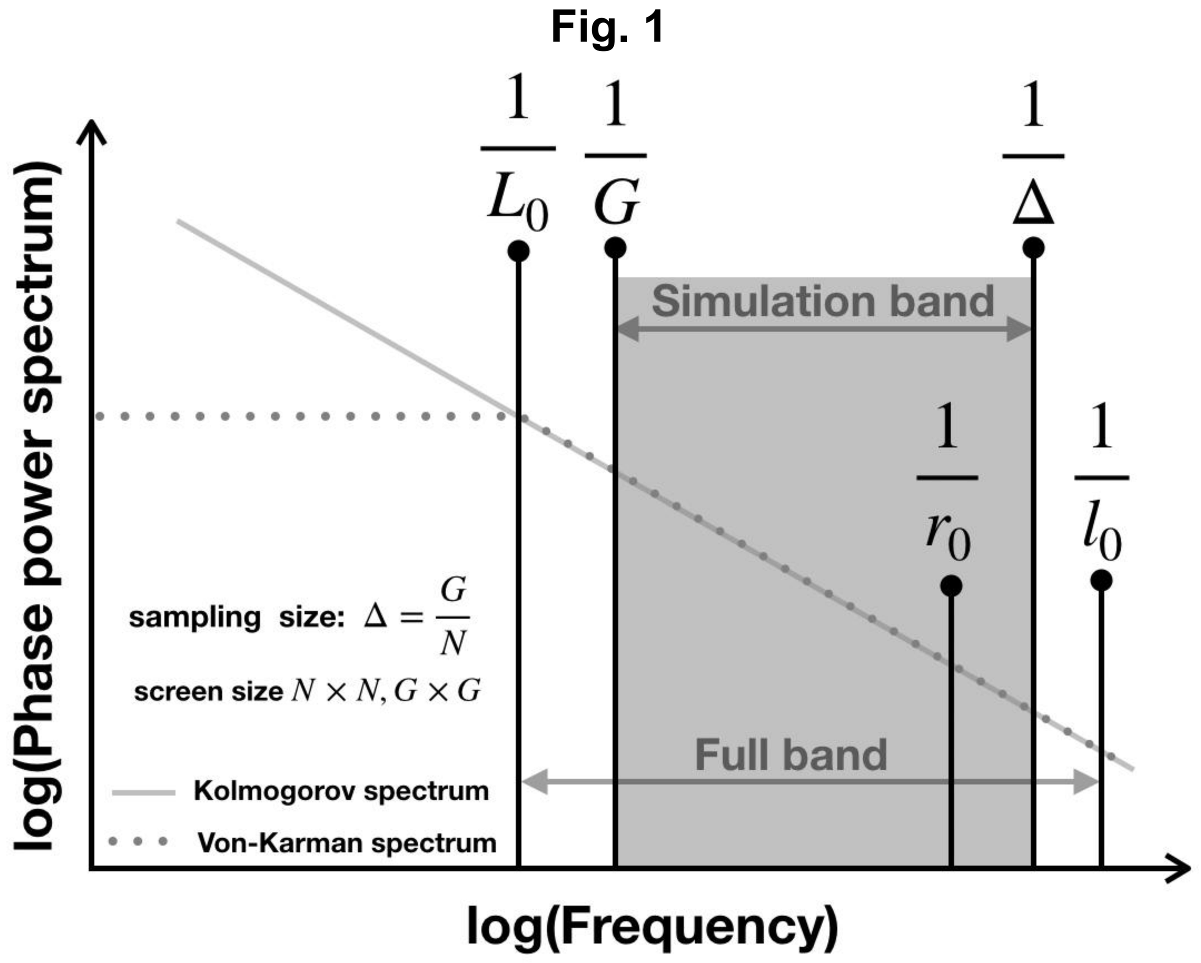
INTRODUCTION

For a variety of purposes such as the design and development of adaptive optics systems, speckle imaging techniques, atmospheric propagation studies etc., it is essential to simulate a good atmospheric phase screen model. Methods based on Zernike polynomial expansions[2], FFT-based methods[3–9], Optimization method[11] etc. have been in use for this purpose. FFT-based methods are computer memory size friendly and widely accepted. Because of undersampling at low and high frequency region in the power spectrum, it provide a limitation in resolution of phase power spectrum. Explanation[1] shown in Fig. 1

Fig. 1



The simulation band $\left(\frac{1}{G} - \frac{1}{\Delta}\right)$ is actually smaller than full band $\left(\frac{1}{L_0} - \frac{1}{l_0}\right)$.

For simulations of imaging with small apertures relative to the outer scale, we need a screen of small size, but cutting out small screens from a larger screen is not the right solution to this problem.

In this work, we present a method to deal with small G/L_0 phase screen simulation using the FFT-based method, inspired by Jingsong Xiang's[9] work on phase screen simulation.

PROPOSED METHOD

Step:1

Obtaining Phase Autocorrelation Matrix using Phase Power Spectrum

$$D_\phi(m,n) = 2(B_\phi(0,0) - B_\phi(m,n)) \ldots (1)$$

$$B_\phi^{FFT}(m,n) = \sum_{m'=-N_x/2}^{N_x/2-1} \sum_{n'=-N_y/2}^{N_y/2-1} f_{FFT}^2(m',n')\, e^{i2\pi\left(\frac{m'm}{N_x}+\frac{n'n}{N_y}\right)} \ldots (2)$$

$$B_\phi^{SUB}(m,n) = \sum_{p=1}^{N_p} \sum_{m'=-3}^{2} \sum_{n'=-3}^{2} f_{SUB}^2(m',n')\, e^{i2\pi 3^{-p}\left(\frac{(m'+0.5)m}{N_x}+\frac{(n'+0.5)n}{N_y}\right)} \ldots (3)$$

$$B_\phi(m,n) = B_\phi^{FFT}(m,n) + B_\phi^{SUB}(m,n) \ldots (4)$$

Step:2

Compensation for residual error

Using eq (1) : $D_{error}(m,n) = D_{theory}(m,n) - D_\phi(m,n) \ldots (5)$

Error matrix cannot be just added to the D_ϕ matrix to compensate for remaining error.

REASON:

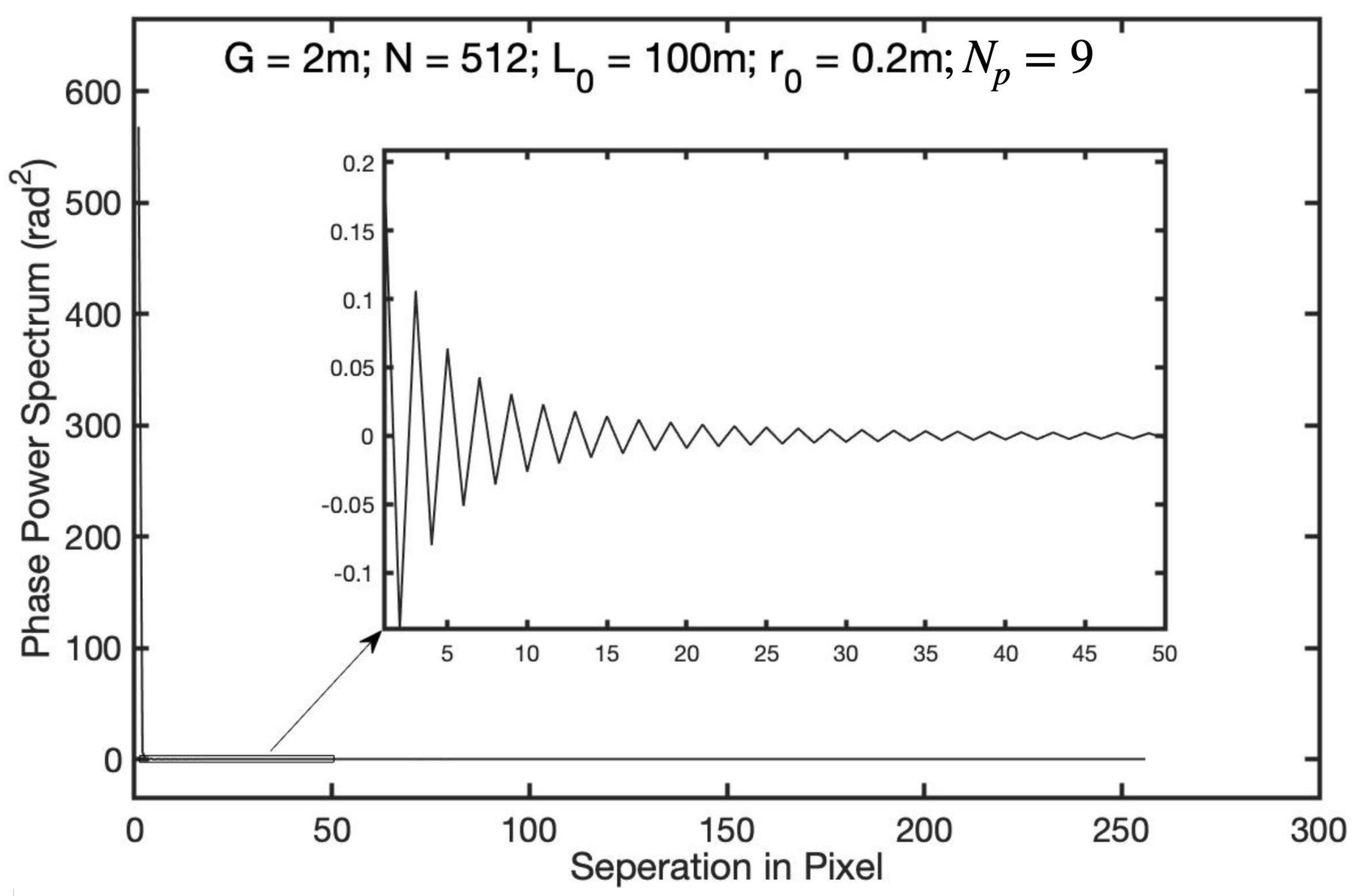
If we take the Fourier transform of this polynomial equation, the resultant curve will be completely different with a different order of moments, just like Gibbs phenomena. This introduces unwanted error in the final result.

Using MATLAB, we find the best fit of $D_{error}(m,n)$ using cftool to obtain coefficients of the required Gaussian function (with 95% confidence bounds)

$$B_{tot}(m,n) = B_\phi(m,n) + B_{gauss}(m,n) \ldots (6)$$

Step:3

Phase Screen Simulation using B_{tot} matrix



Remove Negative elements

Step: 3.1 $B_{tilt}(r) = B_{tilt}(0) - r^2\sigma_{tilt}^2/2 \ldots (7)$

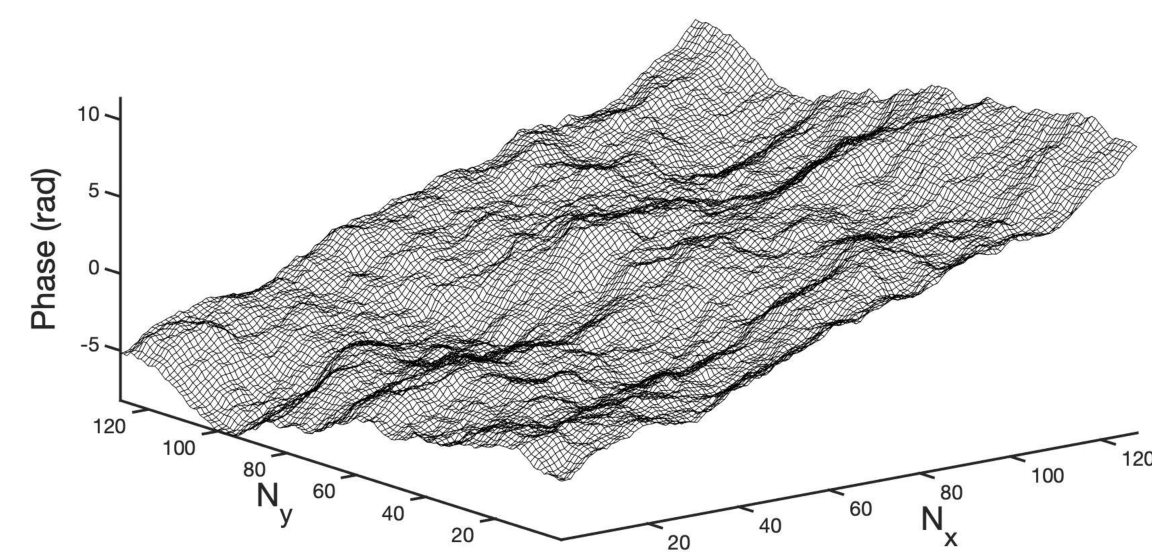
$$, \sigma_{tilt}^2 = \frac{B_{tot}(G/2+\Delta) - B_{tot}(G/2)}{\Delta(G-\Delta)/2}$$

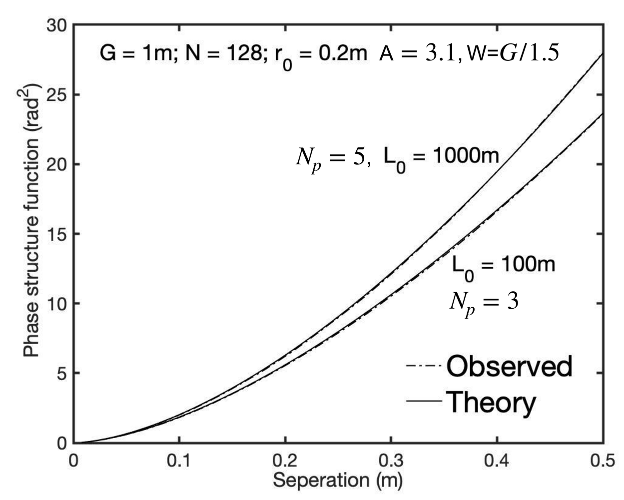
Step: 3.2 $B_{high}(r) = B_{tot}(r) - B_{tilt}(r) \ldots (8)$

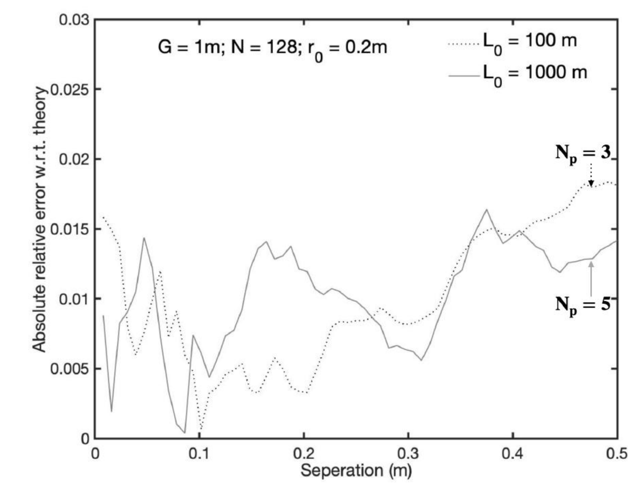
Step: 3.3 Obtain Phase Screen from $B_{tilt}(r)$ and $B_{high}(r)$ using below

$$\phi(m\Delta, n\Delta) = \sum_{m'=-N/2}^{N/2-1} \sum_{n'=-N/2}^{N/2-1} \left[R_a(m',n') + iR_b(m',n')\right] f(m'\Delta', n'\Delta') \exp\left[i2\pi(m'm+n'n)/N\right]$$

RESULTS







CONCLUSIONS

In this paper, we put forward a new method to compensate for the residual error in the Low and/or High-frequency region of FFT simulated phase screens after compensating with the modified subharmonic method. This method provides very accurate phase screen structure for even G/L_0 ratios as small as 1/1000. No Patch Normalization factor is needed, no need to calculate subharmonic weight coefficient and weights to compensate for high-frequency components, as done by Sedmak. Finally, the accuracy of this method from low-frequency to the high-frequency range is better than 1.8% for G/L_0 as low as 1/1000.

REFERENCES

1. Sedmak, G., Private Comm. ,2019
2. Roddier N., SPIE Vol.1237,668-679
3. Herman, B.J. and Strugala, L.A. SPIE vol. 1221 183-192 (1990)
4. E. M. Johnsson and D. T. Gravel, Proc. SPIE 2200, 372-383 (1994)
5. Sedmak,G. App. Opt. 37, 4605-4613 (1998)
6. Lane et al., Wave in Random Media Vol 2 , No. 3 209-224 (1992).
7. Sedmak,G. App. Opt. 43(23), 4527-4538 (2004)
8. McGlamery, B. L. , proc. SPIE 74 225-233 (1976)
9. Xiang, J. S. Optical eng. 53 (2014)
10. Xiang, J., Opt. Express 20(1) 681-687 (2012)
11. Zhang D., et al. , Optik 178, 1023-1028 (2019)