ESSAYS ON MARKET MICROSTRUCTURE INVARIANCE

A thesis submitted to The University of Manchester for the degree of Doctor of Philosophy in the Faculty of Humanities

2018

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Final Word Count: 69,878

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ABSTRACT

The main intellectual contribution of this thesis consists of three self-contained essays. These essays are preceded by an introduction and literature review (chapters 1 and 2), and their results summarised in a concluding chapter 6. In the core essays, which constitute chapters 3-5 inclusive of the thesis, I test the empirical implications of two market microstructure invariance (MMI) principles first proposed by Kyle and Obizhaeva (2016b). The market setting I choose analyses trades in FTSE 100 index constituent stocks for the period between January 2007 and December 2009, a period which incorporates both the 2008-09 financial crisis and the introduction of alternative trading platforms for FTSE 100 stocks

The chapter 3 examines the MMI of bets, as applied to trades, in FTSE 100 index constituent stocks. To link bets and trades the thesis formulates an extended version of ITI model by Andersen et al. (2018) motivated by the MMI model (Kyle and Obizhaeva, 2016b). The model accounts for the level of trade intermediation and order shredding. I empirically test the model's trading activity prediction on trade data using panel estimation methodologies. I find that for highly capitalized stocks, trade counts yield the predicted 2/3 proportionality relationship to trading activity. Further investigations, using alternative notions of trading activity proposed by Clark (1973) and Ané and Geman (2000) reveal the predicted proportionality only for large trade-size stocks.

The chapter 4 develops the analysis of the earlier chapter by investigating the market microstructure invariance proposition in FTSE 100 constituent stocks at the level of individual stocks, using four different notions of trading activity. I find that the notion of trading activity proposed by Clark (1973) reveals the predicted 2/3 proportionality number for the majority of stocks. This result is consistent even when the first and last minutes of active trading in the market are excluded. Invariance models yield a 1/2 proportionality between the log values of trade counts and trading activity, whereas the intraday trading patterns in the specific market, the magnitude of trade size and its correlation with the volatility partly explain this value. Based on this, I show that analysis on a year by year, pre-crisis and in-crisis sample do not suggest a unified order flow composition across stocks.

The chapter 5 focuses on the MMI of transaction costs. I empirically test the respective predictions using three common proxies for transaction costs, namely quoted, effective, realized spreads on FTSE 100 stock trading data. As predicted by market microstructure invariance, I find that a -1/3 proportionality is present in average daily patterns in our sample for all three proxies of transaction costs, with larger trades having a negative impact on this proportionality when the underlying variables are estimated as intraday averages. My results suggest that market fragmentation does not impact the estimated invariance coefficients, though trading activity and volume traded on alternative platforms are negatively correlated with the percentage transaction costs on LSE per unit of volatility. Finally, the invariance prediction holds for a consolidated market, but the lower reduction in the realised spreads may suggest a greater impact of large trades in the alternative platforms or the fact that only few market makers benefit from an increase in trading activity.

DECLARATION

I, Efthymios Rizopoulos, declare that that no portion of the work referred to in the thesis has been submitted in support of an application for another degree or qualification of this or any other university or other institute of learning

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DEDICATION

Dedicated to the memory of my father

"You are gone but your belief in me has made this journey possible….."

ACKNOWLEDGEMENT

This thesis has been one of the most challenging yet rewarding achievements of my life. I would like to express my sincere and heartfelt gratitude to my supervisors, Professor Michael Bowe and Dr Sarah Zhang, for their patient guidance and encouragement. Their exceptional knowledge on market microstructure and their insightful critique of my work along the way has made the completion of this thesis possible. It was a real privilege and honour to cooperate with them and they have been inspiring examples of researchers for me.

I am also grateful to Professor Stuart Hyde for supervision in my early years of PhD. I would like to thank the members of the doctoral committee of the Alliance Manchester Business School who over time have provided constructive suggestions on the direction of the study. Special thanks to Dr Sungjun Cho, Dr Olga Kolokolova and Prof Alexandros Kostakis.

I am also grateful for invaluable discussions with Torben G. Andersen, Pete Kyle, and Anna Obizhaeva related to the work described in Chapter 3. I would also like to thank the participants at: the Kyle conference 2016 at the University of Maryland, the EFMA Meeting 2017, the INFINITI conference 2017, the Financial Econometrics and Empirical Asset Pricing Conference 2017 at Lancaster University and the FMA European Conference 2018, for their comments and suggestions regarding earlier versions of the chapters in this thesis. Financial support from the ESRC (AQM) programme is also gratefully acknowledged.

Four years at Alliance Manchester Business School were very special thanks to my fellow PhD students Yan Xu, Marcin Michalski and Chanaka Ganepola. Bearing and helping each other, sharing our knowledge and anxiety made this journey more colourful.

Finally, I would like to thank my family for their selfless support. Especially I would like to express my love and gratitude to my wife, Lyslee, for putting up with me in the years it took to complete this journey. Without her support both financially and emotionally, this dream would not come true.

CHAPTER 1

Introduction

1.1 Motivation

There is an on-going debate in the market microstructure literature addressing the optimal approach to model how order flow imbalances move prices and to develop more efficient measures of liquidity. Although the outcome of this debate is critical for our comprehension of how financial markets operate, developing models of security price dynamics and forecasting, as well as accurate liquidity estimators is indeed challenging. In recent years, although developments continue to occur in the literature, there is neither a consolidated framework to construct empirical measures for order flow imbalances, nor one which generates accurate forecasts regarding how price impact is differentiated across securities.

Various theoretical papers in market microstructure model trading are based on game theory and adopt a set of specific assumptions about both the characteristics and participants of the trading mechanism, as well as the information diffusion process. Some of these papers develop their theoretical models based on the concept of adverse selection, first discussed by Bagehot (Jack Treynor) (1971). Representative studies include but are not limited to Kyle (1985), Glosten and Milgrom (1985), Admati and Pfleiderer (1988) and Back and Baruch (2004). Other studies focus on inventory risks associated with trading such as Stoll (1978), Ho and Stoll (1981), Ho and Stoll (1983), O'Hara and Oldfield (1986), Grossman and Miller (1988) and Campbell and Kyle (1993). Several papers combine the idea of information asymmetry with order-processing costs or inventory risks, including Grossman and Stiglitz (1980), Easley and O'Hara (1987), Easley and O'Hara (1992) and Wang (1993). All these theoretical papers provide different perspectives as to how order flow imbalances move prices. However, drawing inferences regarding accurate empirical predictions from the respective models is difficult, and thus their empirical testing becomes challenging. As a result, the empirical proxies, for testing the relationship between price changes, order flow imbalances and their connection to stock characteristics are imperfect (e.g. Breen et al. (2002)).

Market microstructure invariance (MMI from here after) is a theory introduced by Kyle and Obizhaeva (2016b) that attempts to bridge the gap between theoretical market microstructure models and their empirical counterparts by imposing both “cross-sectional restrictions” and "time-series restrictions" that facilitate the empirical assessment of the former and also the implementation of liquidity measures that are contingent on order flow imbalances. Based on the idea of trading games, first described by Bagehot (Jack Treynor) (1971), Kyle and Obizhaeva (2016b) argue that trading a security is a game that involves a risk transfer from one market participant to another. Given that market participants trade many different securities, they play many different trading games simultaneously. These risk transfers are defined as "bets" which are *"decisions to obtain or relinquish a long-term position of a certain size in a certain security, with each decision distributed approximately independently of other similar decisions"* (Kyle and Obizhaeva, 2016b, p.1349). The trading games across securities are played in business time and the risk involved is asset-specific, with small or no correlation with market risk. MMI theory suggests that microstructure characteristics remain constant when examined from the perspective of an appropriate asset-specific business time clock that ticks at the arrival rate of bets in the market.

The idea of business-time is not novel. Mandelbrot and Taylor (1967) and Clark (1973) are among the first to discuss the notion of business time, a concept of time that is connected to the rate of arrival of trades or trading volume, in which price dynamics approximately follow a Brownian motion process. Other studies, such as Jones et al. (1994), Hasbrouck (1999) and Ané and Geman (2000) also investigate the nature of the appropriate business clock that describes what happens to the assets in their respective datasets. Based on the idea of "bets", MMI theory moves the discussion a step forward and links business time to the arrival rate of bets in the market, measuring trading activity as the product of expected dollar volume and returns volatility, and providing exact predictions of how microstructure characteristics relating to risk transfers vary with the specified trading activity. In a way, MMI theory attempts to apply appropriate scaling laws to different trading games played in different markets (e.g. an active and inactive market) so that they become fundamentally equivalent, and the same set of rules can be used to describe what actually happens in financial markets.

These precise predictions are implied by two core principles of MMI theory, the invariance of bets and the invariance of transaction costs that I empirically examine in this thesis. In a sense they are empirically testable formulas relating order frequency, order size and transaction costs to functions of observable volume and volatility. Investigating whether these predictions hold across different assets, time and markets is very important. If indeed they hold, they can serve as useful metrics for practitioners, as a benchmark for assessing arguments regarding high frequency trading, market collapses and liquidity estimation for researchers, and a "road-map" for regulators and policy makers. Until now, there are very few papers that examine the aforementioned predictions in the context of different securities and different markets. Kyle et al. (2012) examine invariance using daily data on news articles, Kyle et al. (2016) and Bae et al. (2016) investigate whether invariance holds for trades in US and Korean stocks, respectively. Andersen et al. (2018) test the invariance hypothesis in higher frequencies for trades in the S&P E-mini futures market by introducing Intraday Trading Invariance (ITI henceforth) as a purely empirical hypothesis motivated by MMI theory. Benzaquen et al. (2016) provide evidence for invariance in US stocks and futures contracts and propose an alternative definition of invariance and Kyle and Obizhaeva (2016a) examine invariance in data on trades for Russian stocks.

1.2 Research Focus and Contributions

Until now there is not any formal investigation of invariance in European markets and more specifically the UK equity market, which is one of Europe's largest and most liquid markets. Additionally, current empirical investigations of invariance principles use either daily frequency data or tick by tick data recorded in a way that does not accurately capture the evolution of trading in the selected market. This thesis aims to bridge this gap in the literature and contribute further to the empirical investigation of invariance, the validation of invariance principles, and the precision of empirical estimates. I also endeavour to provide potential explanations for the reasons why deviations from the invariance principles, if any, occur. At the same time, the thesis also contributes to the time deformation literature, which proposes that price changes follow different distributions in calendar and business time, and to the existing literature related to market microstructure invariance and related scaling laws. The thesis consists of three self-contained essays, which test the empirical implications of the two invariance principles as introduced by MMI theory, using trading data from FTSE 100 index stocks between January 2007 and December 2009. It is important to note that the dataset I employ records trades in the order in which they are executed. This enables me to draw inferences for invariance principles with respect to the actual trading process and characteristics of the market under investigation. Also, the choice of this sample period is deliberate, as it enables me to analyse whether either market fragmentation, arising from the introduction of alternative trading platforms, or the 2008-2009 financial crisis impact on the invariance properties of the FTSE 100 market. I now proceed to briefly summarise the research focus and specific contributions of each chapter.

The first essay investigates the invariance of bets and the consequent empirical prediction of an inherent proportionality between the number of bets and bet activity. Specifically, following

Andersen et al. (2018), it examines whether the number of trades concerning FSTE 100 is proportional to trading activity in the power of 2/3. The essay departs from Andersen et al. (2018) in the manner that connects the MMI theory for bets to transaction counts in smaller intervals. Based on the assumption that bets and their actual trades are linked in a nonlinear manner, I contribute by introducing an extended invariance relationship between the number of trades and trading activity, motivated by the MMI invariance relationship for bets. This relationship incorporates the potential effect of order shredding and intermediation structures. I argue that a bet arriving in the market may be shredded into multiple trades in a non-linear manner and intermediated more than once until it is fully executed as a trade, something may affect the estimated proportionality. This modification is important as it also enables the empirical specification to accommodate the specific way in which my specific transaction dataset records executed trades. I show that the stipulated 2/3 proportionality proposed by MMI theory and implied empirically be ITI theory is a special case of this modified invariance model, when assuming a linear relationship between bets and trades.

Based on the amended invariance model and using panel regression specifications incorporating two-way fixed effects, I conduct empirical tests for the invariance proportionality between the number of trades and trading activity against the null hypothesis of 2/3. I compare the findings from both the Kyle and Obizhaeva (2016b) and Andersen et al. (2016) invariance model specifications to two well-known alternatives in the literature, implied by Clark (1973) and Ané and Geman (2000), respectively. I express the latter in invariance terms so that their coefficient estimates are directly comparable with each other. I also examine whether the investigated proportionality is affected by stock characteristics such as market capitalization, trade size, trading volume and the number of trades. Generally, I find that the invariance proportionality results may be partly driven by the intraday dynamics of trading activity. When I apply the Kyle and Obizhaeva (2016b) and Andersen et al. (2016) invariance specifications the entire sample, I estimate the proportionality power coefficient to be closer to 0.6 rather than the expected value of 2/3. However, I do find that large and medium capitalization stocks (on average) exhibit the MMI predicted power of 2/3 proportionality between the number of trades and trading activity. Also, when controlling for trade size, large trade size stocks yield the predicted 2/3 proportionality when I use number of trades as a measure of trading activity, in the spirit of Ané and Geman (2000). I also find that order shredding can potentially explain deviations from the theoretical invariance proportionality as well as differences in the coefficient estimates between groups of stocks that are based on stock characteristics. I also argue that deviations from the predicted 2/3 proportionality may arise from a higher degree of trade intermediation, market fragmentation or the fundamental conceptual difference between trades and bets.

The second essay of this thesis contributes by examining the same empirical prediction of invariance of bets but at an individual stock level. I conduct regression for each stock against the null hypothesis of the invariance prediction of a 2/3 proportionality. The aim is both to investigate whether FTSE 100 stocks individually exhibit any common proportionality between trade counts and trading activity and also to identify whether this proportionality is similar to that suggested by ITI theory. In this respect, I test which notions of trading activity and their respective models specified in invariance terms, better describes the intraday patterns of LSE and microstructure properties of my dataset, and predicts more accurately any common invariance proportionality. The definitions of trading activity I use here are the same as in the first essay (i.e. those based on Clark (1973), Ané and Geman (2000), Kyle and Obizhaeva (2016b) and Andersen et al. (2016)).

I show that LSE trading venue is characterized by an L-shaped/reverse J-shaped pattern in realised volatility and a four-humped pattern in trading volume for the period under analysis. This contrasts with Werner and Kleidon (1996), Abhyankar et al. (1997) and Cai et al. (2004) who report a U-shape pattern in volatility and a two-humped pattern in trading volume, respectively. I find that the model which is based on notion of trading activity implied by Clark (1973) accurately confirms the expected 2/3 proportionality between the number of trades and trading activity for 70% of the stocks when averages across days are used as estimators for the underlying variables. This result is consistent even if the minutes that are characterised by extreme volatility and increases in trading volume, trade counts and trade size are excluded. However, the model fails to predict the specific proportionality for stocks that are characterised by high on average volatility. The use of intraday averages leads to lower estimates of proportionality for all notions of trading activity, likely due to measurement errors and sampling variation which bias the coefficients estimates.

Both models that are based on definitions of trading activity as suggested by in Kyle and Obizhaeva (2016b) and Andersen et al. (2016) predict 1/2 invariance proportionality for 86% of the stocks. These differences in the estimated proportionalities across models may be partly driven by intraday trading patterns, the magnitude of trade size which has a different impact on the models I investigate or the positive correlation between trade size and volatility in business time in the first/last 10 minutes. Based on the extended invariance model, I argue that the 1/2 proportionality value implies that bets in the futures markets investigate by Andersen et al. (2018) are larger than bets in stocks and thus are shredded into more pieces (assuming that on average bets shredded into same size trades in these two markets). I also discuss the view that orders are shredded more in the period between the first/last 10 minutes of active trading. Year by year analysis on the 1/2 proportionality indicates that there is no unified order flow pattern for all stocks, and that stock and/or industry specific characteristics are important for traders when choosing the trade size. The results for pre-crisis and in-crisis periods are similar, especially for the leading stocks in the FTSE 100 in terms of market capitalization. Local currency (GBP) trade size appears to be an important factor in explaining deviations from the 1/2 proportionality between the two periods.

The third essay in the thesis contributes to the literature by testing the empirical predictions linked to the second invariance principle, the invariance of transaction costs, on our sample of FTSE100 index stocks. This is the first comprehensive test of this principle in the literature. Specifically, I examine whether the percentage transaction costs per unit of volatility for FTSE 100 stock trades is proportional to trading activity in the power of -1/3. This is the first time that the specific prediction is investigated for trades in equity markets. Following Kyle and Obizhaeva (2016a), in order to connect bets to trades, I use their assertion that there exists a proportionality between the number of trades and bets, provided that tick and minimum trade size or other microstructure elements are distributed across stocks so that their impact on trading is identical. I complement the work of Kyle and Obizhaeva (2016a) which employs a database of Russian stocks, on the basis of which they test a different transaction cost invariance prediction, using different proxies for transaction costs. Specifically, I employ the effective and realized spreads in addition to the quoted bid-ask spread. Given the extent of market fragmentation for FTSE 100 equities which changes over the sample period, I also contribute to the literature by being the first study to research the extent to which the dispersion of trading activity and the volumes traded in alternative platforms affect the transaction costs and their stipulated proportionality with trading activity. Finally, I further explore this idea by constructing a hypothetical consolidated market framework, where a market participant has simultaneous access to different platforms.

The empirical results indicate that the invariance prediction of a -1/3 proportionality is present in average daily patterns in my sample for all three proxies of transaction costs I employ. All measured spreads decrease more during specific 5-minute time intervals across days than within specific days, providing that returns volatility is constant. I argue that larger trades appear to have a negative impact on proportionality, and this is more apparent within specific days rather than across the same 5-minutes intervals on different trading days. Moreover, I find that market fragmentation does not statistically affect the significance of the estimated invariance coefficients, though there exists a negative correlation between trading activity and volume traded on alternative platforms with the percentage transaction costs on the LSE per unit of volatility. Among the alternative platforms, trading activity on Chi-X has the highest negative correlation with LSE percentage transaction costs. Finally, when I treat the platforms as a hypothetical consolidated market, the results do not change significantly; however, the estimated invariance coefficients are lower when realized spreads are used as the proxy for transaction costs. Similar to Bessembinder and Kaufman (1997) and Boehmer and Boehmer (2003), I maintain that this is an indication of the greater impact of large trades on the alternative platforms, and/or similar to Degryse et al. (2015), that not all market makers take advantage of increases in liquidity generated by the increase in trading activity.

1.3 Thesis Outline

This thesis consists of three self-contained essays. Each essay has a separate introduction, data section, methodology, set of empirical results, conclusion and reference sections, as well as exclusive sections appropriate for the focus of each essay. The tables, figures and graphs are numbered independently for each essay, whereas the footnotes, equations and models are numbered in sequence from the beginning of the thesis. The same formatting rules apply for the titles and subtitles of each essay. Given that all the essays investigate MMI theory, a separate literature review chapter is included as chapter 2 in the thesis to set the context for the subsequent analysis.

The remainder of the thesis is structured as follows. Chapter 2 discusses the relevant literature that is linked to market microstructure invariance, as well as the invariance principles on which the essays are based. Chapter 3 presents the first essay, which investigates the empirical predictions of invariance of bets for trades, using panel regressions and different notions of trading activity. Chapter 4 continues by investigating the same invariance implication for individual stocks and its dependence on the intraday patterns of LSE, its differentiation during the respective years in the period under analysis, the impact of the 2008-2009 financial crisis, and the opening and closing minutes of trading. Chapter 5 investigates the empirical prediction of the invariance of transaction costs proposition and the potential impact of market fragmentation due to the introduction of alternative trading platforms on the proportionality between percentage transaction costs per unit of volatility and trading activity. Chapter 6 provides a brief conclusion of the main findings of the thesis.

Finally, I note that in the following essays I use “we” and “our” instead of “I” and “my”, respectively to reflect that each individual essay is linked to research papers which are co-authored with my supervisors Professor Michael Bowe and Dr. Sarah Zhang at Alliance Manchester Business School.

CHAPTER 2

Literature Review

2.1 Market microstructure invariance

Market microstructure invariance (MMI) is a theory, as well as a scaling and modelling principle, developed by Kyle and Obizhaeva (2016b). MMI is based on the idea of trading games between market participants which are intended to transfer risks, which they define as "bets", first described by Bagehot (Jack Treynor) (1971). These *"bets"* are *"decisions to obtain or relinquish a long-term position of a certain size in a certain security, with each decision distributed approximately independently of other similar decisions"* (Kyle and Obizhaeva, 2016b, p.1349). In reality, a bet can be executed either in its entirety or by sequentially placing orders in the course of one or more trading intervals. MMI theory maintains that microstructure characteristics which vary across stocks and time can remain constant when we examine them using a suitable asset-specific business-time scale. Based on this novel idea of bets, Kyle and Obizhaeva (2016b) argue that the arrival rate of bets in the market measures the rate at which business time changes for a specific asset (i.e. market velocity).

MMI theory consists of two principles, the invariance of bets and the invariance of transaction costs. Both principles generate empirically testable predictions, which are derived from simple expressions for order frequency, order size and transaction costs defined as functions of observable volume and volatility (the specific elements underlying each principle are explained in the respective chapters of the thesis). The invariance of bets argues that *"the distribution of the dollar risk transferred by a bet in units of business time is the same across asset* j *and time* t*, in the sense that there exists a random variable* $\tilde{I}$ *such that for any asset and time, the distribution of risk transfers* $\tilde{I}_{jt}$ *is market microstructure invariant"* (Kyle and Obizhaeva, 2016b, p.1352). Intuitively, $\tilde{I}_{jt}$ is the scaled size of the bet, which according to Kyle and Obizhaeva (2016) can be defined as the signed standard deviation of dollar mark-to-market gains or losses (i.e. the product of the currency value of risk transferred by the bet per unit of business time). The second principle, the invariance of transaction costs, states that *"the dollar expected transaction cost of executing a bet is the same function of the size of the bet when its size is measured as the dollar risk it transfers in units of business time, in the sense that there exists a function* $C_B(I)$ *such that for any asset* j *and time* t*, the dollar transaction-cost function* $C_{B,jt}(I)$ *is a market microstructure invariant"* (Kyle and Obizhaeva, 2016b, pp.1354-1355) . Intuitively, this means that the expected costs of executing two bets that lay in the same percentile of the bet size distribution, but having different currency sizes, must be the same in dollars, because *"they both transfer the same amount of risk per stock-specific unit of business time"* (Kyle and Obizhaeva, 2016b, p. 1355).

Figure 1-Market microstructure invariance principles in graphical representation

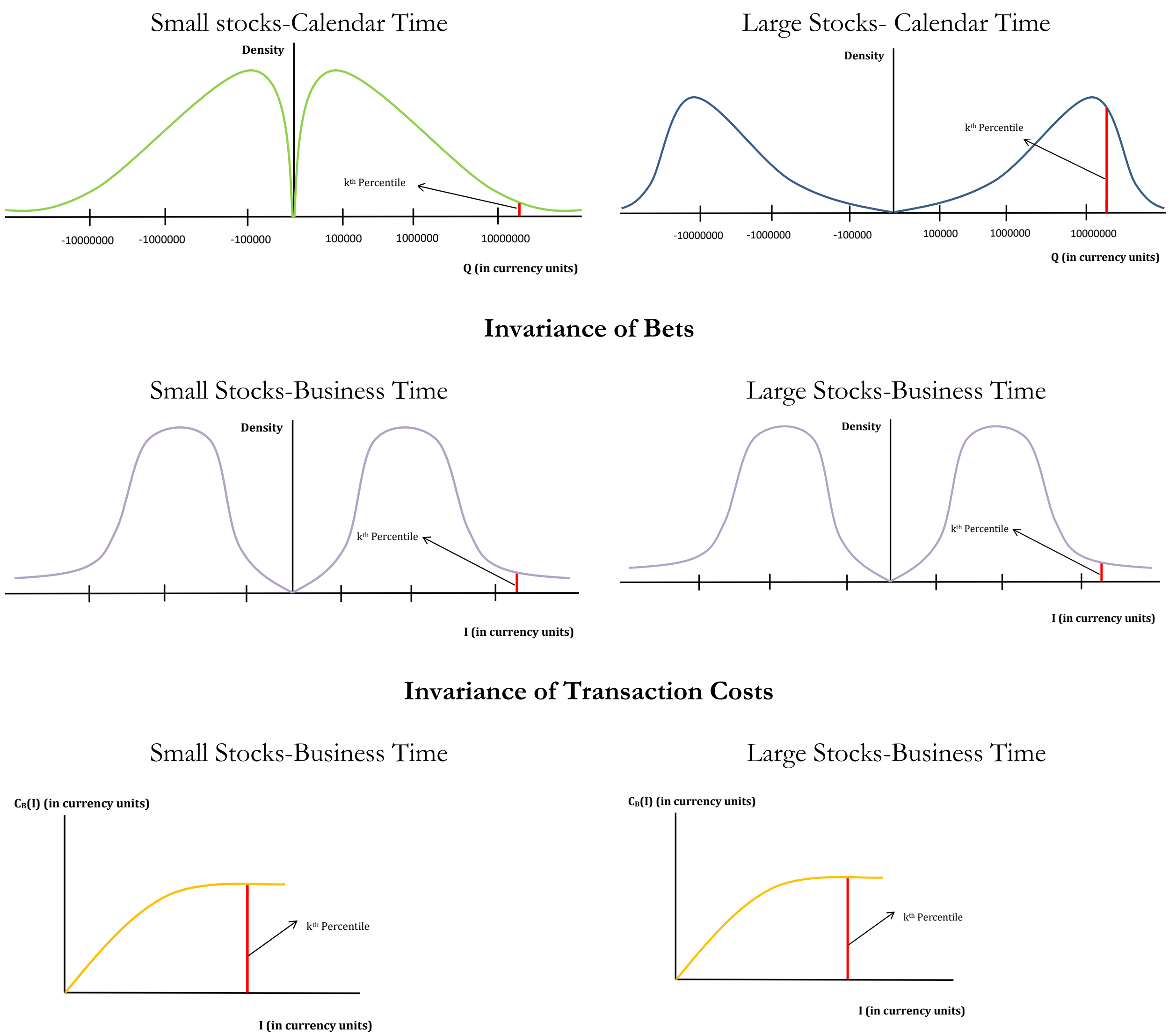

Figure 1 graphically illustrates the main idea of MMI theory and the invariance principles. Consider the example of two stocks, which I denote small and large. In calendar time, the distributions of bet sizes for small and large stocks are quite different, as depicted in the respective graphs. However, if we transform the bet size Q into I, based on the invariance of bets, the distributions become the same for these two stocks. Intuitively, bets that occupy the same k^{th} percentile of their respective bet-size distributions will have the same risk transfers (i.e.

same realized value of $\tilde{I}_{jt}$). In addition, based on the invariance of transaction costs, the transaction cost associated with executing the respective bets will be also the same.

The theoretical principles of MMI together with the consequent empirical predictions attempt to bridge a perceived gap in market microstructure between the theoretical models which address the manner in which order flow imbalances move prices and their subsequent empirical testing. These empirical predictions can also potentially serve as useful metrics for practitioners and researchers, as a benchmark for assessing arguments regarding high frequency trading, market collapses and liquidity estimation, and as a "road-map" for regulators and policy makers. In this sense, MMI theory is connected to the time deformation and microstructure literature that examines trading behaviour in financial markets and the connections between trading variables and transaction costs. In the following sections, I briefly summarize the most important papers which are representative of these specific areas.

2.2 Time deformation literature

The time deformation literature begins with the intuition that execution of individual trades leads to microscopic, normally and independently distributed, price fluctuations that are incremental to daily price changes. More specifically, every time new information becomes available, the variables of trading activity, most importantly the number of trades, the trading volume, the transaction rate[1], and the quote revision frequency shift with a consequent impact on prices. Engle (2000) explains that the arrival rate of information represents the speed at which business/economic time passes, and that a blunt measure of this rate is obtained when transaction times and prices are analysed simultaneously. Intuitively, given that price changes follow high-kurtosis distributions[2], the observed fat tails can be interpreted from the perspective of a business clock that ticks at different velocities compared to a real time "wall-clock" to allow for differentiation of trade execution speed across defined time periods (Kyle and Obizhaeva, 2011).

2.2.1 Measuring time deformation: Pareto distributions

Mandelbrot and Taylor (1967) are one of the first to examine the distribution of stock price changes (i.e. price volatility) by measuring time in volume of transactions. They argue that price changes have stable Pareto (power-law) distributions during defined calendar time intervals and

[1] Time between trades is defined as the reciprocal of the transaction rate
[2] Distributions with sharper peak around mean and fatter tails as compared to normal distribution

Gaussian distributions when these fixed intervals are measured in transactions time[3]. In a similar fashion to Mandelbrot and Taylor (1967), Gopikrishnan et al. (1998) analyse the probability distribution of stock price movements and deduce that they display asymptotic Pareto distributions with an exponent close to 3. Moving one step forward, Plerou et al. (2000)maintain that price fluctuations follow a complicated diffusion process. In this process, the diffusion constant is related to the number of transactions within a specific time interval and the variance of price fluctuations for all transactions. The number of transactions and the variance of price fluctuations between consecutive trades follow a power law distribution with a mean value of the exponent around 3. Provided that market impact is linear in trade size[4] and trades are i.i.d., the authors suggest that the Pareto distribution tails of price movements are attributable only to variance, whereas the number of transactions is responsible for any long-range correlations of volatility.

Gabaix et al. (2006b) identify a dependence of empirical price changes on the square-root price impact of i.i.d. trades, based on a specific version of a model introduced by Torre and Ferrari (1998)[5]. They maintain that the estimation of price impact and its connection to order size is problematic due to the joint endogeneity of order flow and returns[6]. Bouchaud et al. (2009) examine a different empirical predictions concerning volume and market impact and suggest that order flow is a *"highly persistent long-memory process[7]"* as large orders are split over long periods until fully executed in the presence of low revealed market liquidity. They assert that market impact may stem from random fluctuation in supply and demand, which may or may not be endogenous or informed, and that information from external news plays a secondary role in price formation.

2.2.2 Mixture of distribution Hypothesis

Other business time papers differentiate themselves from the approach which relies on Pareto distributions. Several papers investigate the "mixture" of distributions hypothesis (MDH) as a joint hypothesis linking returns and volume. Generally, these theories introduce an explicit way to model the impact of information on prices and volume by expressing the respective variables as a function of the arrival rate of information in the market. Clark (1973) suggests that the number of

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[3] Mandelbrot and Taylor (1967) reach this conclusion by introducing for the first time the concept of subordination (i.e. a time change stochastic process inside another stochastic process) when modelling financial returns

[4] Prices move upward or downward proportionally to the trade size

[5] Zhang (1999) and Gabaix et al.(2003) also propose a model with square-root price impact.

[6] Loeb (1983) allows for the exogeneity of order flow by using bids on various size blocks of stock. Both Torre (1997) and Grinold and Kahn (1999) mention square root price impact fits best this type of data.

[7] Autocorrelation in order flow decays very slowly

minor price innovations per day exhibits a log-normal distribution and that both volume and price innovations are triggered by the same information arrival process[8]. Assuming an identical distribution of distinctive price increments, he argues that return variance is proportional to trading volume when using the latter as an approximate transactions time clock. Both, Epps and Epps (1976) and Westerfield (1977) confirm this proportionality, though they underline that the theory of stable Paretian distributions proposed by Mandelbrot and Taylor (1967) cannot be ruled out.

Tauchen and Pitts (1983a) add that the positive relation between returns variance and trading volume is subject to a fixed number of traders[9], while Harris (1987) maintains that prices and volume unfold homogenous rates in business time[10]. Gallant et al. (1992), using a non-semiparametric specification, report a positive relationship between volatility and volume, both conditional on their past observations. Richardson and Smith (1994), introduce a direct test for the MDH and examine different distributional properties for the rate of information flow. They conclude that the bivariate distribution of price changes and volume is not as strong as previous studies suggest, and that the distributional properties of the information flow rate approach those of log-normal distribution. Andersen (1996) incorporates the market microstructure setting of Glosten and Milgrom (1985) into a modified version of the MDH, arguing that the full dynamic representation of the stochastic volatility process for the rate at which information arrives in the market performs better as compared to the standard MDH[11]. Bollerslev and Jubinski (1999), report that volume-volatility pairs for stocks which trade in the S&P 100 Composite index exhibit long-run hyperbolic rates of decay, consistent with a MDH in which the information arrival process has long-memory characteristics. Finally, Liesenfeld (2001) generalises the MDH standard model by allowing both the number of information arrivals and the sensitivity to new information to be dynamic over time. The revised model more accurately explains stock price fluctuations, whereas trading volume appears to be mainly affected by the information arrivals count.

[8] Clark (1973) implies that movements in returns are caused by a joint distribution between trading volume and prices conditional on current information

[9] This assumption is rational for mature markets. If the number of traders is evolving, then the average trading volume grows in a linear fashion with the number of traders.

[10] Harris (1986) shows that the relationship between returns variance and trading volume is also present in the cross-section of stocks.

[11] The main difference between the modified and standard version of MDH relates to the trading volume specification. The modified MDH includes a constant term that captures any noise or liquidity elements related to trading and changes the distribution to a conditional Poisson instead of a normal distribution.

2.2.3 Time deformation and market events

Hasbrouck (1999) approaches time deformation as a common feature embedded in the rate at which markets process events such as orders, quote or trade frequency occur. While time deformation defined in this way can only be estimated dependent upon a specified time horizon, the author finds a positive correlation in the long-term, although there is no persistent proportionality in the count intensities for different types of events. Jones et al. (1994) discover that the number of transaction per se, and not their size (i.e. volume), creates daily volatility. They state that the volume does not contain any extra information additional to that included in the trades count. In line with Jones et al. (1994), Ané and Geman (2000) report that the aggregate number of trades is a better business time clock than volume for generating independently and identically distributed Gaussian intraday returns. Dufour and Engle (2000), based on the VAR specification of Hasbrouck (1991), analyse the impact of time duration between successive transactions on the process of price formation. Their findings indicate that whenever these waiting times decline, trades have a greater impact on prices, the latter adjust faster to trade-related information, and the positive serial correlation of signed trades increases. Finally, they claim that active markets, where the increased participation of informed traders leads to high trading activity, are illiquid.

2.3 Transaction costs and market microstructure

2.3.1 On transaction costs

There is an extensive microstructure literature focusing on transaction costs, their measures, as well as their role in the trading process and their relation to market liquidity, price discovery and generally market quality. Foucault et al. (2013) argue that trading costs are an important dimension of market liquidity and can be decomposed into two main elements. The first, explicit element includes costs such as broker commissions, transaction taxes, trading fees related to the specific platform, and clearing and settlement fees. These are costs that the final investor has to pay to carry out the transaction and are easy to measure. In recent years, increased competition between intermediaries, and technological advances related to trading have decreased considerably such explicit transaction costs. The second element is the implicit transaction cost element, and refers to costs that stem from the illiquidity of the market, such as delay costs, impact costs and other costs that are occurring within the transaction process. This is the category which professionals analyse before placing an order and researchers primarily focus on when investigating market liquidity and other important market characteristics. However, accurately measuring this type of costs is not an easy task, due either to the lack of requisite data or imperfect transaction cost measurement and models.

The most commonly used measures of implicit trading costs are based on quotes posted by market makers in each trading venue. They are known as spread measures and include quoted, effective and realised spreads. The quoted spread is customarily defined as the difference between the best ask and bid price at a specific point in time. It measures the execution cost of an immediate round-trip transaction and is a good indicator of illiquidity for those trades that are small enough to be entirely executed at the best bid and ask quotes. Some papers prefer to use the half quoted spread when they focus on the execution costs per trade (e.g. Huang and Stoll, 1996, Bessembinder and Kaufman, 1997) or a normalised version by dividing the quoted spread by the midpoint. Microstructure literature documents that quoted spread reflects the available liquidity at a specific point in time for a round-trip transaction (Foucault et al., 2013), the market maker's compensation for offering immediacy (Fleming et al., 1996), order-processing costs (Stoll, 1985), inventory costs (Stoll, 1978; Amihud and Mendelson, 1980, Ho and Stoll, 1983) and losses to informed traders (Copeland and Galai, 1983, Glosten and Milgrom, 1985).

The effective spread is customarily defined as twice the absolute difference between the price at which a market order is executed and the midprice between the best ask and bid price at the exact point in time before trade execution. In a sense the effective spread is an approximation of the total price impact of a trade (or else slippage). It captures the price adjustment caused by trades with a size exceeding the maximum trade size which a market maker can accommodate without revising their quotes[12] (Fleming et al., 1996, Boehmer and Boehmer, 2003). Several papers prefer to use the effective half spread (e.g. Lee, 1993, Huang and Stoll, 1996) and account also for the direction of the order (positive for purchases and negative for sells) while others define the effective spread as a percentage of the midquote. As Foucault et al. (2013) highlight, the impact estimated by the effective spread is always positive, reflecting the limited liquidity in the market. Also the effective spread tends to increase with the trade size because larger market orders are often executed at less auspicious prices.

Both quoted and effective spreads assess the cost that a trader pays when placing a market order, and thus estimate transaction costs from the perspective of a liquidity demander. If the liquidity

[12] In other words, they account for order executions that occur deeper in the book (i.e. not at the best bid and ask prices)

supplier takes immediately the opposite side of the trade this cost can be considered to be their gain. However, in reality this interpretation is problematic as market orders may apply long-term pressure on security prices, which in turn move against the liquidity suppliers. Consequently, the liquidity supplier will not realise the effective or quoted spread but the difference between the execution price and the price at which the trade is liquidated (Huang and Stoll, 1996, Foucault et al., 2013). This difference (gain for liquidity provider and cost for liquidity taker) is less than what the effective and quoted spreads imply if, after the execution of the trade, prices move in the direction of the trade due to price adjustments (moving up for purchases or down for sells). The magnitude of this impact also indicates market liquidity and depth, and can contribute significantly to the cost that a market participant faces when trading. Also, it is well documented that these price adjustments are a result of adverse selection, as the market participants that take the other side of the trade will demand compensation to account for the likelihood of any private information being incorporated in the trade (see for example, Glosten and Milgrom, 1985, Kyle, 1985)

The realised spread is a measure of transaction costs that takes into account this price impact. It is customarily defined as twice the difference between the execution price and the midpoint between best ask and bid prices a certain time point after trade execution, after taking into account the direction of the trade (positive for purchases and negative for sells). The time instant in which the midpoint is calculated should be long enough after the initial transaction so that both bid and ask quotes have adapted to account for the price impact of the trade[13]. As with quoted and effective spreads, several papers prefer to use the realised half spread. The realised spread can also be considered as an approximation for the temporary price impact of a trade, interpreted as market-making profits net of adverse selection costs, but not order-processing, inventory and other costs associated with the trade (Bessembinder and Kaufman, 1997, Boehmer and Boehmer, 2003). In addition, (half) the difference between effective and realised spreads is used as a proxy for the permanent price impact and information content of a trade (Huang and Stoll, 1996, Boehmer and Boehmer, 2003). Apparently, the effective spread is equal to the realised spread plus the difference between the respective midpoints, that at a specific time point after the trade and that in the instant before the trade. If the change in the midpoints is positively correlated with the direction of the trade, the realised spread is always smaller than the effective spread. In a sense, if the effective spread is too small, then liquidity suppliers may experience losses. Thus, the effective

[13] In active markets this time instant is often set to five minutes after the initial transaction.

spread should be high enough to at least compensate liquidity suppliers for any price adjustments that move the market against them following the trade.

2.3.2 Alternative measures of transaction costs

Although these three measures of transaction costs are widely used by professionals and researchers to assess market liquidity, there are other alternatives measures and models. For example, in the absence of information regarding quotes, the closing or opening price, or the volume-weighted average price (VWAP) for trades that occur during the time interval of interest. Other measures are developed based on the idea of price impact, namely as the coefficient of a regression between the change of the midpoint during a specified interval and the signed trade imbalance between all executed buy and sell trades during this interval[14] (Foucault et al., 2013). Stoll (2000) uses a variation of this method, including in the regression the previous day's trade imbalance to determine whether prices will bounce back the following day. He states that the estimated coefficient is a measure of the sensitivity of quote changes over a specific time interval to the imbalance over that interval. He reports a positive price impact, statistically significant for 63.1% of the stocks in NYSE/AMSE and 71.2% in Nasdaq, an insignificant reversal and a variability of the coefficient across stocks (e.g. higher market capitalisation stocks have lower coefficients). When signed trades are not available, Hasbrouck (2007) suggests an alternative method by running a regression of the absolute value of price changes against the total trading volume over the same interval. The coefficient of that regression can be interpreted as the sensitivity of price changes to trading volume. This method is in the spirit of the well-known illiquidity ratio introduced by Amihud (2002), as the ratio of the absolute return of a stock to the trading volume over a specified period. An illiquid market for the specific stock will be characterized by a low Amihud ratio[15].

However, using trading volume or the ratio of trading volume to market capitalisation in order to estimate liquidity, and thus indirectly as a proxy for transaction costs, is not always optimal for every market. This is because such trading variables are likely to also increase at times when new information arrives in the market, a period that is also associated with higher volatility, and in turn with wider bid-ask spreads. Fleming (2003) reports that trading volume and trading frequency (i.e. number of trades executed) are poor proxies for liquidity for US Treasury securities. In that respect, some studies of transaction costs and liquidity propose non-trading measures. For

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[14] The reciprocal of this coefficient can be considered to be an indication of the market depth

[15] Thus higher transaction costs for trading the stock

example, Lesmond et al. (1999) and Bekaert et al. (2007) use the proportion of days with zero returns as a proxy of illiquidity, based on the idea that no trading may mean no changes in prices[16]. Lesmond (2005) and Bekaert et al. (2007) demonstrate that this estimator is positively correlated with bid-ask spreads and negatively correlated with trading volume.

Another way to measure transaction costs when quote prices are unavailable is to use only trade prices, and more specifically the covariance of returns. Roll (1984) is the first to propose such a measure for the effective bid-ask spread, based on the idea of bid-ask bounce and the consequent negative autocorrelation in trade-to-trade returns. Roll's measure, as it is known, is a function of the autocovariance of returns and is subject to distinct assumptions concerning the order arrival process. Any deviations from these assumptions will lead to a biased estimate of the effective bid-ask spread. Hasbrouck (2002) suggests two different approaches to estimating Roll's measure, using the GMM method or Bayesian techniques. Hasbrouck (2009) finds that the resulting estimator, based on daily closing prices, is a good proxy for more accurate measures that are based on higher frequencies[17]. In the spirit of Roll's measure, Holden (2009) develops a proxy for the effective spread, the "effective tick" based on the idea of observable prices clustering at specific ticks solely due to bid-ask spreads. Another estimator for the effective spread is suggested by Lesmond et al. (1999), and is known as the LOT measure. This maximum likelihood estimator is based on the assumption that informed trading occurs during non-zero-return days and is absent in zero-return days.

All the aforementioned measures of transaction costs are static as they do not take into account the possibility that larger orders often split into smaller trades, and are executed with a delay during the course of trading, sometimes over several days or even longer. If we consider the possibility of adverse price movements, this delay might be costly for the trader and can potentially lead to a cancelation of part of the initial order, which induces an extra opportunity costs. To account for this, Perold (1988) proposes a comprehensive measure for transaction costs that includes the opportunity cost due to delayed, partial or unexecuted orders, market impact, commission and other fees. This measure is known as "implementation shortfall" and consists of two components, the execution cost and the opportunity cost. Intuitively, this measure is the difference between the performance of an actual portfolio and a hypothetical "paper" portfolio

[16] If there is no trading the informed traders may have to pay a trade cost greater than the price change that their information suggests.

[17] Goyenko et al. (2009) comparing different effective and released spread and price impact measure also conclude that both monthly and annual low-frequency measures capture high-frequency measures of transaction costs

which includes securities whose trades are executed immediately and at the prevailing quote midpoint prices. Deciding upon a trading strategy that can minimise this shortfall is very important for market participants, and several papers try to provide a more comprehensive analysis of how this can be achieved (see for example Bertsimas and Lo (1998) and Kissell et al. (2003)).

2.3.3 Determinants of the bid-ask spread

The measures of transaction costs described in the previous sub-sections can also be considered to be measures of trading friction. In this sense, trading friction is either the time needed to execute a trade of a certain size (Lippman and McCall, 1986), or the compensation required for an immediate transaction (Demsetz, 1968). According to Stoll (2000) these two approaches converge, because the compensation for an immediate transaction is the payment to the counterparty to this transaction to convince them to buy or sell the asset immediately, and then unwind their position based on their strategy. Given that the bid-ask spread is an important measure of market friction[18] and an important component of liquidity, understanding the determinants of the magnitude of the spread is very important not only to researchers, but also to practitioners and policy makers.

Market microstructure literature explains that bid-ask spread mainly exist due to the costs of liquidity provision, such as order processing costs, adverse selection costs and inventory risk (Stoll, 1978, O'Hara, 1995, Foucault et al., 1997, Madhavan, 2000). Earlier market microstructure papers, such as Demsetz (1968), argue that the spread indicates the compensation for the suppliers of trading immediacy that includes the cost for any resources they use, the unwanted inventory risk and their market power. Christie and Schultz (1994) and Christie et al. (1994) provide empirical results regarding dealer's market power in NASDAQ, while other representative theoretical papers include Garman (1976), Stoll (1978), Amihud and Mendelson (1980), Cohen et al. (1981), Ho and Stoll (1981), Ho and Stoll (1983) and Laux (1995).

Other studies on market microstructure claim that the spread is the value of the information lost to traders who can time the market or possess superior information. So, based on this idea, the spread is not a measure of the cost of supplying immediacy, but rather an estimate of wealth redistribution from some traders to others. Copeland and Galai (1983) define the bid-ask spread as the difference between revenues that the market maker is expected to earn from liquidity-

[18] Stoll (2000) contends that both quoted and effective spreads are measures of total market friction

motivated traders and losses they except to incur from trading against information-motivated traders. They argue that the spread exists as a compensation for supplying quotes, and we can view this cost as equivalent to writing an option to those traders that are well informed and who are able to execute the option upon the arrival of new information in the market, and before the quotes are revised. They conclude that the bid-ask spread is a positive function of the price level and the return variance, a negative function of the estimates of market activity, depth, and continuity, and is also negatively correlated with the degree of competition. However, the most accepted view supporting the "informational" nature of the spread is the one that accepts the existence of adverse selection arising from information asymmetry. This stream of literature is based on the idea that well informed traders try to take advantage of any mispricing by market makers, and as a consequence market makers will incur loses whenever they trade with these types of traders. As a result, to avoid loses, the market makers must generate revenue from trading against other traders, which causes them to increase their bid-ask spread.[19]. This is a well-known example of the adverse selection phenomena, and was first noted by Bagehot (Jack Treynor) (1971). When informed traders buy or sell stocks based on their private information, their orders actually convey this information to the rest of the market. Other market participants, who are able to observe this order flow, will revise their expectations concerning the "true" value of the security (Foucault et al., 2013).

One of the earliest models for the spread that makes the aforementioned distinction is suggested by Glosten and Milgrom (1985). Their model concerns a pure dealership market in which all orders are market orders, and the specialist accommodates the arrival of two types of trader, namely informed traders and purely liquidity (or noise) traders. Their findings suggest that adverse selection alone can explain the existence of spread, whereas the magnitude of the spread is subject to how many informed or liquidity traders arrive in the market, the information quality possessed by informed traders, and the supply and demand elasticity among the liquidity traders. They also show that the spread declines on average throughout the day, due to the informativeness of trade prices. The paper by Kyle (1985) is another important study that analyses how trade prices react to order size (i.e. determinants of market depth) and in turn how this affects the transaction costs. This paper develops a model of trading in the presence of information asymmetry with a risk neutral informed trader, a random noise trader and a competitive risk neutral market maker. This model takes into account different properties of transaction costs, market tightness, depth, and resiliency. The model shows that the price impact of orders increases with their size, and it is

[19] Theoretically one can claim that bid-ask spread is also the result of market makers' being risk averse

inversely proportional to the amount of private information and proportional to the amount of noise trading. He concludes that in equilibrium, market depth remain constant over time, whereas all information is incorporated in the prices by the end of trading.

Other papers combine the idea of information asymmetry with the presence of order-processing costs or inventory risks, and/or relax the hypotheses concerning perfect competition, market orders, risk neutrality, and market structure. Easley and O'Hara (1987) propose an extension to Glosten and Milgrom's model by introducing the possibility of informed market participants trading multiple trade sizes, Easley and O'Hara (1992) include the possibility of no change in the value of the asset traded, Bloomfield et al. (2005) show by means of experiments that informed traders will use both market and limit orders if they are able to do so, and Calcagno and Lovo (2006) theoretically examine the competition among dealers in terms of its impact on the price. Both Bloomfield et al. (2005) and Calcagno and Lovo (2006) argue that it is the posted quotes and not only order flow that contain information. Hasbrouck (1988) states that the effect of inventory management means that the impact of trades on quote revisions is inconclusive, although large trades contain more information than small trades. Vayanos (1999) provides an analysis on the effect that this information may have on adverse selection[20].

Foucault et al. (2013) emphasise that the way to understand the significance of adverse selection, order-processing costs and inventory risk, which all affect the transaction costs, is to run regressions of price changes against order flow, in other words price impact regressions. The precise specification of these regressions is subject to assumptions regarding the nature of illiquidity and the order arrival process. For example if part of the order processing costs has a fixed component, then the market maker may want to offer better quotes for larger trades. De Jong et al. (1996) find that in Paris Bourse, the adverse selection component of the bid-ask spread increase slightly with the order size on average, but the order-processing component decreases. Huang and Stoll (1997) report for NYSE that the order-processing component accounts for the majority of the bid-ask spread, and that adverse selection component does not increase with the trade size in contrast to the predictions of Kyle (1985). Madhavan et al. (1997) note that on the NYSE, the adverse selection component exhibits a downward trend (it is high at the beginning of active trading and then decreases) in contrast to the operating cost component. They argue that this indicates an increase in the bargaining power of the market makers as trading approaches the closing period of the trading day. Based on the idea that inventory costs actually deplete over the

[20] Some other important studies include the papers by Admati and Pfleiderer (1988), Klemperer and Meyer (1989), Madhavan (1992), Glosten (1994), Biais et al. (1998) and Biais et al. (2000).

course of trading day, as market makers will have taken steps to reduce their risk to an acceptable level by gradually unwinding their positions, Hasbrouck (1991) measures the long-last price effect of the information content of trades, and states that the total price impact of a trade is higher in securities with lower market capitalization, suggesting that these stocks suffer more for information asymmetry. In the same spirit, Bouchaud et al. (2009) argue that under normal conditions, the main determinant of the spread in liquid, competitive and electronic markets (i.e. in markets where both order processing and inventory costs should be low) is the impact induced by adverse selection. They show that in electronic markets, both limit and market orders are approximately symmetrical (none of them is more favourable) and competition in the electronic market is enough to compress the spread towards its lowest value. Thus, in these markets, any changes in the spread can be attributed to the impact which is linearly related to the spread.

A central theme of the aforementioned literature is that it highlights two important issues. First, estimating a fully specified time deformation model for stock market dynamics is a challenging undertaking. A number of the relevant papers suggest using infinite variance distributions for price innovations; others use distributions with finite variance. The results of investigations into the nature of the distribution which best describes price changes are inconclusive. Also, studies based on the MDH suggest different ways of approximating the mixing variable in the relevant subordinate stochastic processes. While trading volume or the number of trades are the prevailing proxies, both are considered to be somewhat imperfect. The aforementioned papers analyse only volume or price changes, not both, in the context of business time. Thus, their simultaneous inclusion in a time series model is not undertaken. Second, developing models that analyse the determinants of transaction costs is also a demanding proposition. The papers mentioned above constitute a small, but I believe a representative fraction of microstructure papers that attempt this task. However, given the tendency towards an increase in trading activity since the advent of electronic markets, I believe that adverse selection considerations and the consequent impact they have on securities markets, appears to be the most logical route along which to develop models of transaction costs. The introduction of the notion of market microstructure invariance and the consequent empirical predictions for bets and transaction costs by Kyle and Obizhaeva (2016b) aims to resolve these two critical issues. The following chapters of the thesis provide empirical evidence regarding these two invariance principles for trades in FTSE 100 constituent stocks, examining the validity of the MMI principles and aiming to provide plausible explanations for any observed deviations.

CHAPTER 3

Market Microstructure Invariance in the FTSE 100

Abstract

We examine market microstructure invariance (MMI) trading relationships in FTSE 100 constituent stocks. We formulate an extended version of ITI model proposed by Andersen et al. (2018) as motivated by the original MMI model (Kyle and Obizhaeva, 2016b). The model proposes a non-linear relationship between bets with the introduction of an order shredding factor. We empirically test the model's trading activity prediction on trade data. We find that for highly capitalized stocks, trade counts yield the predicted 2/3 proportionality relationship to trading activity. Further investigation using alternative notions of trading activity proposed by Clark (1973) and Ané and Geman (2000) reveals the predicted proportionality for large trade-size stocks only. Any deviations from the stipulated 2/3 proportionality can potentially be explained by our extended invariance model.

3.1 Introduction

Market microstructure invariance (Kyle and Obizhaeva, 2016b; henceforth MMI) proposes that individual capital markets operate in a distinct business time[21] scale during which risk transfers occur. Rather than undertaking the customary analysis of the association between trading volume and business time, MMI investigates the relationship between the class of asset-specific risk transfers having little or no correlation with market risk (which MMI denotes as bets), and business time. In particular, Kyle and Obizhaeva (2016b) examine the relationship between trading activity, defined as the product of trading volume and return volatility in business time, and a variety of market microstructure characteristics, namely order size, order arrival rate, price impact, bid-ask spreads and price resilience. Intuitively, MMI proposes that such market microstructure characteristics remain approximately constant for all assets when estimated in business time. Their formulation leads to the MMI principle termed the "invariance of bets" which postulates that "the distribution of dollar risk transferred by a bet is the same when the dollar risk is measured in units of business time" (Kyle and Obizhaeva, 2016, p.1346). The empirical implication of this hypothesis is that the number of bets should be proportional to the two-thirds power of trading activity.

MMI is ultimately an empirical proposition, but bets are not directly observable. However, Andersen et al. (2018) find compelling results in line with a benchmark MMI invariance hypothesis, suitably amended to analyse trading data from the E-mini S&P 500 futures market, at both the intraday and daily level. We extend the analyses of both Kyle and Obizhaeva (2016) on MMI and Andersen et al. (2018) on intraday trading invariance, investigating their implications in an equity market context, specifically for FTSE 100 constituent stocks trading on the London Stock Exchange (LSE). The chapter contributes to the literature as follows. First, we introduce an extension to the model in Andersen et al. (2018) which is motivated by the original MMI model specification of the invariance of bets. Our modification assumes that the number of bets may relate to the number of trades in a non-linear manner, including order shredding as one component. Importantly, it also enables us to adapt to the specific manner in which our

[21] Business time is also referred to as operational time, economic time or information time. According to Hasbrouck (1999), "time deformation" invokes a differentiation between business time, in which a particular system develops, and calendar time in which someone observes it. The concept of using alternative time references when estimating trading activity is a salient feature of the empirical market microstructure literature which aggregates data over real time spans that incorporate shifting intervals of business time. The estimated returns over these periods are expected to follow combinations of business time distributions. Bochner (1955) proposes that time deformation can be specified in terms of subordinated stochastic processes, whereas Clark (1973) and Tauchen and Pitts (1983) express the relationship between business and calendar time as a function of latent or observed variables.

transaction dataset records executed trades and account for the potential impact of fragmentation and order slicing. Second, using panel regression specifications, we employ this extended invariance model to conduct the initial empirical test for individual traded equities of an invariance relationship between the number of trades and trading activity. Our tests analyse the entire subset of 70 equities which remain constituents of the FTSE 100 stocks trading on the LSE during our sample period from 2007 to 2009. We compare the findings from both the Kyle and Obizhaeva (2016b) and Andersen et al. (2016) invariance specifications to two well-known alternatives in the literature, implied by Clark (1973) and Ané and Geman (2000), respectively. Both Clark (1973) and Ané and Geman (2000) are advocates of the "mixtures of distributions hypothesis" (MDH) which *"assumes that events important to the pricing of a security occur at a random, not uniform, rate through time"* (Harris, 1987, p.127). While Clark (1973) utilizes trading volume as their notion of trading activity, Ané and Geman (2000) propose the number of trades as a *"better stochastic clock than […] volume"*.

Our principal empirical findings are as follows. We confirm the intraday trading invariance relationship as predicted by Andersen et al. (2018) in our sample of FTSE 100 stocks for some sub samples. Specifically, highly capitalized in our sample stocks on average exhibit the MMI predicted power of 2/3 proportionality between the number of trades and trading activity. This finding appears to be robust to both stock and time fixed effects. Large trade size stocks yield the predicted 2/3 proportionality if we apply number of trades as a measure of trading activity, in the spirit of Ané and Geman (2000). In particular for both MMI models by Kyle and Obizhaeva (2016b) and Andersen et al. (2016), we find the proportionality power coefficient to be closer to 0.6 rather than the expected value of 2/3. Results from tests of both the MDH hypotheses proposed by Clark (1973) and Ané and Geman (2000) diverge from the predicted 2/3 proportionality. They also demonstrate significant variation in the relationship between the number of trades and the adapted versions of trading activity when the stocks are clustered according to various inherent characteristics, such as market capitalisation, number of trades, trading volume and trade size. In contrast, when we apply the trading activity measure proposed by Kyle and Obizhaeva (2016) we find generally smaller variation in the invariance relationship across these differing stock clusters. Moreover, any variation that we do observe can be explained using the extended invariance model that we introduce.

Our results are in contrast to Benzaquen et al. (2016) who are able to confirm the empirical predictions of MMI for both U.S. futures and U.S. stock markets. As their analysed markets and time period differs from ours, the results might be different. Furthermore, as we confirm the MMI predictions for highly capitalized stocks, this might point towards cross-sectional differences of chosen stocks and markets. Finally, as Benzaquen et al. (2016) do not provide further details of their data sources, their sample might not suffer from the data reporting issues as ours (i.e. that liquidity demanding orders might be represented as multiple trades due to matching against several passive limit orders), which we discussed above. The differences in results illustrate the need to study MMI further on different markets and for different datasets and, where appropriate, to provide adjustments for institutional, instrument and data characteristics as we do in this chapter.

Finally, this chapter contributes to the existing time deformation literature, which proposes that price changes follow different distributions in calendar and business time. This literature analyses which family of distributions best describes variations in returns and their relationship with other variables of trading activity, especially trading volume (measured either in trade counts or the number of securities traded). Certain authors such as Mandelbrot and Taylor (1967) and Bouchaud et al. (2009) argue that price changes follow a (stable) Pareto-type distribution in calendar time and alternative distributions in business time. MMI relates closely to this body of literature, which links order arrival rates to business time rather than trading volume. Alternative perspectives assume that *"price changes are sampled from a set of distributions that are characterized by different variances"* (Karpoff, 1987, p.115), i.e. a *"mixture of distributions hypothesis"* (MDH). Proponents of this view include Clark (1973), Epps and Epps (1976), Tauchen and Pitts (1983b), Harris (1987), Gallant et al. (1992) and Andersen (1996), among others. In particular, Clark (1973) advocates a link between business time and trading volume. In contrast, due to the documented connection with price volatility, Jones et al. (1994), Ané and Geman (2000), Dufour and Engle (2000) maintain that the number of trades rather than trading volume provide a better proxy for business time. This chapter extends the scope of this existing analysis by exploring the simultaneous relationship between return volatility and measures of trading activity, as formulated by MMI (Kyle and Obizhaeva, 2016b) and Andersen et al. (2016).

The chapter proceeds as follows. Section 3.2 formulates the generalized theoretical invariance model. Section 3.3 outlines the methodology and the main and alternative empirical hypotheses. Section 3.4 highlights the characteristics and descriptive statistics of our dataset. Section 3.5 presents and discusses the empirical results. Section 3.6 concludes.

3.2 Model

3.2.1 The Kyle and Obizhaeva (2016) benchmark model

MMI complements existing theoretical models of market microstructure grounded in the notion that order flow imbalances generate price fluctuations, and which proceed to develop measures of market depth or liquidity. MMI postulates that "the distribution of the dollar risk transferred by a bet is the same when the dollar risk is measured in units of business time" (Kyle and Obizhaeva, 2016p, p.1346)[22]. Currently, there is neither a consolidated framework to construct empirical measures for order flow imbalances, nor one which provides forecasts relating to the differentiation of price impact across stocks. As a result, existing empirical proxies for testing the relationship between price changes, order flow imbalances and their connection to stock characteristics are imperfect (e.g. Breen et al. (2002)). The MMI attempts to bridge the gap between theoretical market microstructure models and their empirical counterparts by imposing "cross-sectional restrictions" and "time series restrictions" that facilitate both the empirical assessment of the former and the implementation of liquidity measures that are contingent on order flow imbalances.

The formulation of MMI by Kyle and Obizhaeva (2016b) assumes that over short calendar time intervals, the arrival rate of bets (asset-specific risk transfers) in the market is random. Denote the expected arrival rate of bets in asset j at time t by γ_{jt}. This bet arrival rate γ_{jt} measures market velocity, and together with the distribution of bet size, varies with trading activity. We represent (signed) bet size by the probability distribution of a random variable $\tilde{Q}_{jt}$ (number of shares), taking a positive (negative) sign for purchases (sales), where $E\{\tilde{Q}\} \approx 0$. Kyle and Obizhaeva (2016b) specify that that the average unit of bet volume $\bar{V}_{jt} := \gamma_{jt} \cdot E\left\{\left|\tilde{Q}_{jt}\right|\right\}$ results in ζ_{jt} units of total trading volume V (which is $\zeta_{jt} - 1$ units of intermediation volume per unit of bet volume). It follows that the value of ζ_{jt}, termed the volume multiplier, captures the number of times that a bet is intermediated until the order conveying the bet is fully executed. Consequently, the expected total market trading volume in asset *j* at time *t* is given by:

$$V_{jt} := \frac{\zeta_{jt}}{2} \cdot \gamma_{jt} \cdot E\left\{\left|\tilde{Q}_{jt}\right|\right\} \quad \textbf{(1)}$$

[22] MMI proposes two invariance principles: the "invariance of bets" and the "invariance of transactions costs" (Kyle and Obizhaeva, 2016b). We only consider the invariance of bets in this chapter.

where $E\left\{\left|\tilde{Q}_{jt}\right|\right\}$ is the average bet size. We divide the volume multiplier by two to ensure that a buy-bet matched with a sell-bet, counts as one unit of volume. On this basis, the "expected bet volume" in shares per unit of time, $\bar{V}_{jt}$, is:

$$\bar{V}_{jt} := \gamma_{jt} \cdot E\left\{\left|\tilde{Q}_{jt}\right|\right\} = \frac{2}{\zeta_{jt}} \cdot V_{jt} \tag{2}$$

If the volume multiplier, ζ_{jt} is known, then using equation (2) generates the unobservable bet volume, $\bar{V}_{jt}$ from knowledge of trading volume, V_{jt}.

During time interval *t*, stock price fluctuations in asset *j* generate a percentage variance in returns, denoted by σ^2_{jt}. A fraction, $0 \leq \chi^2_{jt} \leq 1$, of asset price variation is attributable to information-based price updating in the absence of trading, while the remaining portion, $\psi^2_{jt} = 1 - \chi^2_{jt}$, occurs in response to order flow imbalances arising during trading. If order flow imbalances arise solely from bets, we can define the resulting "bet volatility" as:

$$\bar{\sigma}_{jt} = \psi_{jt} \cdot \sigma_{jt} \tag{3}$$

where $\bar{\sigma}_{jt}$ is the standard deviation of returns that stem from bet order flow imbalances. Equation (3) indicates that bet volatility, $\bar{\sigma}_{jt}$ can be inferred from the volatility of returns, σ_{jt}, if the volatility multiplier, ψ_{jt}, is known. Finally, if P_{jt} represents the price of asset *j* (stock) price at time *t*,[23] then bet volatility in asset *j* at time *t* is given by:

$$P_{jt} \cdot \bar{\sigma}_{jt} = \psi_{jt} \cdot P_{jt} \cdot \sigma_{jt} \tag{4}$$

To facilitate the empirical implementation of MMI, Kyle and Obizhaeva (2016b) make certain identifying assumptions. Specifically, they assume that the volume multiplier ζ_{jt}, and volatility multipliers, ψ_{jt}, are constant across assets j and through time t, so $\zeta_{jt} = \zeta$ and $\psi_{jt} = \psi$, However, they underline that assuming that volume and volatility multipliers are constants is not important to understand MMI theoretically. Thus, they proceed by making the following restrictions: first, they stipulate a single market maker, so the volume multiplier, $\zeta = 2$; second, they maintain that return volatility stems only from bets, so the volatility multiplier is $\psi = 2$.

[23] This analysis uses UK traded equities, so asset prices, return volatility, bet volume, volatility and mark-to-market values are all denominated in pounds sterling.

Intuitively, this means that expected bet volume equals expected market volume, $(\bar{V}_{jt} = V_{jt})$ and bet volatility becomes identical to return volatility $(\bar{\sigma}_{jt} = \sigma_{jt})$.

The first invariance hypothesis, "invariance of bets" maintains that: *"the distribution of risk (in local currency units) transferred by a bet in units of business time is the same across asset j and time t, in the sense that there exists a random variable $\tilde{I}$ such that for any j and t, the distribution of risk transfers $\tilde{I}_{jt}$ is a market microstructure invariant $\tilde{I}_{jt} \overset{d}{=} \tilde{I}$,"* (Kyle and Obizhaeva, 2016p, p 1352).

The return volatility in one unit of business time $(P_{jt} \cdot \bar{\sigma}_{jt} \cdot \gamma_{jt}^{-1/2})$ multiplied by the distribution of signed bet size $(\tilde{Q}_{jt})$, measures both the direction and size of the risk transferred by a bet per unit of business time (Kyle and Obizhaeva, 2016b). Intuitively, this means that in one unit of business time (γ_{jt}^{-1}), a bet of size $(P_{jt} \cdot \tilde{Q}_{jt})$ generates a standard deviation of mark-to-market gains or losses equal to $P_{jt} \cdot |\tilde{Q}_{jt}| \cdot \bar{\sigma}_{jt} \cdot \gamma_{jt}^{-1/2}$. The amount of risk that a bet transfers per unit of business time, the invariance equation for bets, can be expressed as follows:

$$\tilde{I} \overset{d}{=} \tilde{I}_{jt} := P_{jt} \cdot \tilde{Q}_{jt} \cdot \frac{\bar{\sigma}_{jt}}{\gamma_{jt}^{1/2}} \quad \textbf{(5)}$$

Representing "bet activity" $\bar{W}_{jt}$, as the product of (the value of) expected bet volume, $P_{jt} \cdot \bar{V}_{jt}$ and bet volatility $\bar{\sigma}_{jt}$, then using equation (2) yields:

$$\bar{W}_{jt} := \bar{\sigma}_{jt} \cdot P_{jt} \cdot \bar{V}_{jt} = \bar{\sigma}_{jt} \cdot P_{jt} \cdot \gamma_{jt} \cdot E\left\{\left|\tilde{Q}_{jt}\right|\right\} \quad \textbf{(6)}$$

Combining equation (6) with equation (5), we derive the following empirical implication of the invariance of bets:

$$\gamma_{jt} = \bar{W}_{jt}^{2/3} \cdot \left(E\left\{\left|\tilde{I}\right|\right\}\right)^{-2/3}, E\left\{\left|\tilde{Q}_{jt}\right|\right\} = \bar{W}_{jt}^{1/3} \cdot \frac{1}{P_{jt} \cdot \bar{\sigma}_{jt}} \cdot \left(E\left\{\left|\tilde{I}\right|\right\}\right)^{2/3} \quad \textbf{(7)}$$

Bets are submitted as orders and executed as trades but are not directly observable, as order splitting occurs and a single bet may be implemented over several trading periods. Trade and quote data indicate executed trades which represent a part, but not necessarily the entirety, of a bet. Consequently, Kyle and Obizhaeva (2016b) derive the empirical implications of MMI based on a definition of trading activity, defined as the product of the expected price, P_{jt}, returns volatility, σ_{jt} and observed trading volume, V_{jt} :

$$W_{jt} := P_{jt} \cdot V_{jt} \cdot \sigma_{jt} \quad \textbf{(8)}$$

Intuitively, W_{jt} is the standard deviation of mark-to-market gains or losses on the expected trading volume during one unit of calendar time, and comprises a measure of "total risk transfer" per unit of calendar time. Trading activity relates to bet activity through the following equality[24]:

$$\overline{W}_{jt} = \frac{2\psi}{\zeta} \cdot W_{jt} \quad \textbf{(9)}$$

Kyle and Obizhaeva (2016b) note that equation (7), which describes the implied MMI composition of order flow, incorporates empirical implications of invariance for both the bet arrival rate γ_{jt} and bet size distribution $\tilde{Q}_{jt}$. Intuitively, they imply that if bet activity $\overline{W}_{jt}$ increases by one percent, then the bet arrival rate γ_{jt} increases by 2/3 of one percent and the bet size distribution $\tilde{Q}_{jt}$ moves higher by 1/3 of one percent. The intuition they provide is straightforward. On the basis that calendar time volatility, $\bar{\sigma}_{jt}$, remains unchanged, if the bet arrival rate γ_{jt} accelerates by a factor of 4, then volatility per unit of business time $\bar{\sigma}_{jt} \cdot \gamma_{jt}^{-1/2}$ decreases by a factor of 2. Consequently, for the distribution of $\tilde{I}_{jt}$ to remain invariant, the bet size $\tilde{Q}_{jt}$ must increase by a factor of 2. The implication is that as bet activity increases, the number of bets should increase twice as fast as their size for the distribution of $\tilde{I}_{jt}$ to remain invariant.

In order to test the empirical implications of MMI, Andersen et al. (2018) apply a linear representation of invariance by taking the logarithm of equation (5). They further facilitate the empirical tests of invariance by equating the arrival rate of bets, γ, with the number of trades, N, and bet activity with trading activity. In a sense, they introduce Intraday Trading Invariance (ITI) by assuming that similar relationships that apply for bets hold also for trades occurring over shorter intervals. However, they argue that this assumption may not be valid in practice and that ITI is a purely empirical hypothesis that is motivated by MMI theory and the respective invariance

[24] A proof of equation (9) is given in Appendix AI-1

relationships for bets. We discuss the dataset used by Andersen et al. (2018) and the justification for these simplifications, as well as implications for our own dataset in the Section 3.6.

3.2.2 An Extension to Invariance Model

Following Andersen et al. (2018), in order to provide empirical evidence relating to the MMI, we also use available market data to examine the relationship between the number of trades and trading activity. As previously explained, Kyle and Obizhaeva (2016b) argue that assuming that volume multiplier ζ_{jt} and volatility multiplier ψ_{jt} remain constant is important for empirically examine invariance relationships. They argue that it is a reasonable approximation in their setting of portfolio transitions that they use as a proxy for bets. While this assumption may hold during short time periods and in certain concentrated markets, we argue that stock markets are likely to feel the impact of fragmentation and order shredding compared to other markets. This caution is particularly important when testing invariance relationships for trades. Therefore, this leads us to expect certain differences in our empirical results from those in Andersen et al. (2018), who examine invariance relationships for transactions in E-mini S&P 500 futures contracts market.

Consequently, we propose an extension to the model in Andersen et al. (2018). We argue that the number of bets may relate to the number of trades in a non-linear manner, so that $\gamma \sim N^{\varphi}$, which will affect the slope coefficient directly. Specifically, we assume bets are shredded in an exponential manner, so $\gamma^{1/\varphi}$, implying $N \sim \gamma^{1/\varphi}$. On the basis of this assumption we incorporate this feature onto equation (5), so that

$$\tilde{I}_{jt} := P_{jt} \cdot \tilde{Q}_{jt} \cdot \frac{\bar{\sigma}_{jt}}{N_{jt}^{\varphi/2}} \qquad \textbf{(10)}$$

where φ represents an order shredding factor, such that $\varphi > 0$, N_{jt} is the expected number of trades for asset j at time interval t; all other variables are as previously defined.

Equation (10) is subject to the same set of assumption and restrictions as specified explicitly in Andersen et al. (2018)[25]. As in Andersen et al. (2018), here it is also important to highlight the difference between equation (5) and (10) in that the former refers to bets, whereas the later refers to trades. Equation (10) is a purely empirical hypothesis, distinct from equation (5), though motivated by the invariance of bets. However, it deviates from the ITI framework, in that it is

[25] Briefly it is assumed that the active traders have knowledge reagarding the current volatility and number of trades, and *"consequently they adjust their trades so that to control the risk associated with the change in their asset portfolios. In effect, trade size is random, but "drawn" from a distribution that ensures the invariance relationship holds"* (Andersen et al., 2018, p. 10)

based on the assumption that the number of bets may relate to the number of trades in a non-linear manner. We argue that this assumption is reasonable, as in reality, it is hard to identify bets in order flow and we don't know much about order shredding algorithms used by traders. In a sense, it may be thought of as some approximation to the way trading currently takes place in financial markets. It follows that for $\varphi = 1$, equation (8) becomes similar to ITI invariance relationship as introduced in Andersen et al. (2016).

Given equation (10), the expression for trading activity in (6) yields the following expression for asset j during time t:

$$\bar{W}_{jt} := \bar{\sigma}_{jt} \cdot P_{jt} \cdot \bar{V}_{jt} = \bar{\sigma}_{jt} \cdot P_{jt} \cdot \gamma_{jt} \cdot Q_{jt} \sim \bar{\sigma}_{jt} \cdot P_{jt} \cdot Q_{jt} \cdot N_{jt}^{\varphi} \quad \textbf{(11)}$$

where $Q_{jt} := E\left\{\left|\tilde{Q}_{jt}\right|\right\}$ and N_{jt} is the expected number of trades in asset j at time t.

The equation (11) can be further expressed in terms of trading activity W_{jt} given the equation (9) as follows[26]:

$$\frac{2}{\zeta} \cdot W_{jt} = P_{jt} \cdot N_{jt}^{\varphi} \cdot Q_{jt} \cdot \sigma_{jt} \quad \textbf{(12)}$$

Letting $I := E\left\{\left|\tilde{I}\right|\right\}$, the mean of the invariant distribution, $\tilde{I}$, and taking expectations using equation (10), produces the following equation for any asset j, at time t:[27]

$$E\left\{\left|\tilde{I}\right|\right\} = P_{jt} \cdot E\left\{\left|\tilde{Q}_{jt}\right|\right\} \cdot \frac{\bar{\sigma}_{jt}}{N_{jt}^{\varphi/2}}. \quad \textbf{(13)}$$

Solving (13) for $E\left\{\left|\tilde{Q}_{jt}\right|\right\}$ gives,

$$Q_{jt} = I \cdot P_{jt}^{-1} \cdot \psi^{-1} \cdot \sigma_{jt}^{-1} \cdot N_{jt}^{\varphi/2} \quad \textbf{(14)}$$

Substituting from equation (14) into equation (12) for Q_{jt} generates the expression for the invariance of bets in trading terms arising from our generalized invariance model, namely:

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[26] A proof of equation (12) is given in Appendix AI-2 and equation (15) in Appendix AI-3. Note that here we use the identifying assumption that expected bet volume is given by $\bar{V} := N_{jt}^{\varphi} \cdot E\left\{\left|\tilde{Q}_{jt}\right|\right\}$

[27] Expectations are taken only for the variables $\tilde{I}$ and $\tilde{Q}$ that represent random variables, all other variables represent by definition an expected value.

$$I = \frac{2\psi}{\zeta} \cdot \frac{W_{jt}}{N_{jt}^{3\varphi/2}} \tag{15}$$

As $\tilde{I}$ has an invariant distribution, its constant mean I is independent of trading activity, W_{jt}, the volume multiplier, ζ, the fraction of trading activity arising from order imbalances, ψ, and the expected number of trades (trade arrival rate), N_{jt}. Equation (15) implies that N_{jt} is proportional to $\frac{2\psi}{\zeta} \cdot W_{jt}^{2/3\varphi}$ and constitutes the basis of the empirical tests of invariance we undertake in section 3.

Importantly, note that if the introduced order shredding factor $\varphi = 1$, then equation (15) becomes:

$$I = \frac{W_{jt}}{N_{jt}^{3/2}} \tag{16}$$

which corresponds precisely to the invariance relationship suggested by Kyle and Obizhaeva (2016b) for bets, and by Andersen et al. (2016) in their empirical analysis of trades on the E-mini S&P 500 futures market.

3.3 Methodology

3.3.1 Main Hypothesis

The relationship in equation (15) constitutes the basis of our empirical tests of the invariance hypothesis reformulated to apply to trades. Following Andersen et al. (2016), we apply logs and take expectations of equation (10) to obtain the linear representation:[28]

$$E\{\log \tilde{I}\} = p_{jt} + q_{jt} + \frac{s_{jt}}{2} + \log\psi - \frac{1}{2}\varphi \cdot n_{jt} \tag{17}$$

where p_{jt} is the log of price; q_{jt} is the expected value of $\tilde{q}_{jt}$, the log of signed trade size, $\tilde{Q}_{jt}$; s_{jt} denotes the log value of return volatility σ_{jt}^{2}; and n_{jt} is the log of the number of trades N_{jt}.

[28] As MMI implies an invariant distribution of $\tilde{I}$, using logarithms of means or means of logarithms for the relevant variables of trading activity only makes a marginal difference. For example, we could take logarithms in Equation (14) and continue with the model's derivation. However, following Andersen et al. (2016), we first estimate the logarithms of variables and then their means. A proof of the following equations (18) and (19) is given in Appendix AI-4.

Solving equation (17) for n_{jt} and q_{jt} yields the following two expressions, which hold across time and stocks. Together, they characterize the empirical implications of the invariance of trades.

$$n_{jt} = c_1 + \frac{2}{3\varphi} w_{jt} \tag{18}$$

$$q_{jt} = c_2 + \frac{1}{3} w_{jt} - p_{jt} - \frac{s_{jt}}{2} \tag{19}$$

where $w_{j\tau}$ is the logarithm of trading activity, and the constants c_1 and c_2 in equations (18) and (19) are given by:

$$c_1 = \frac{2}{3\varphi}\left\{\left[\log\psi + \log\left(\frac{2}{\zeta}\right)\right] - E\{\log\tilde{I}\}\right\} \text{ and } c_2 = \frac{2}{3}\left(E\{\log\tilde{I}\} - \log\psi\right) + \frac{1}{3}\log\left(\frac{2}{\zeta}\right)$$

Based on equation (18), an increase in φ implies both a lower slope coefficient and a reduced level of order shredding. We further empirically test this hypothesis in Section 3.5.2.2 and include a discussion of the role of order shredding when interpreting our results in Section 3.6

Moving a step forward, high frequency measures of the trading variables σ_t, N_t and V_t are customarily noisy, as noted by Andersen et al. (2016). To test for an invariance relationship between the number of trades and trading activity requires these trading variables to be directly observable. We divide our sample of $d = 1, \ldots\ldots, D$ trading days into $t = 1, \ldots\ldots, T$ minute non-overlapping intervals (we analyse 5 minute intervals) within each day. This results in a modification of equations (10), (17) and (18) as follows:

$$\tilde{I} \overset{d}{=} \tilde{I}_{jdt} := P_{jdt} \cdot \tilde{Q}_{jdt} \cdot \frac{\bar{\sigma}_{jdt}}{N_{jdt}^{\varphi/2}} \tag{20}$$

$$E\{\log\tilde{I}\} = p_{jdt} + q_{jdt} + \frac{s_{jdt}}{2} + \log\psi - \frac{1}{2}\varphi \cdot n_{jdt} \tag{21}$$

$$n_{jdt} := c_1 + \frac{2}{3\varphi} w_{jdt} \tag{22}$$

where P_{jdt} is the average price for asset j over interval t of day d, Q_{jdt} (number of shares) is the average trade size for asset j during internal t of day d, σ_{jdt} is the volatility of returns for asset j during interval t of day d given information prior to time t, is the number of trades during interval t of day d, and p_{jdt}, q_{jdt}, s_{jdt} and n_{jdt} are the logs of the respective variables

As in Andersen et al. (2016), synergies between the trading strategies of market participants imply that the distribution of $\tilde{I}$ retains MMI characteristics (i.e. i.i.d. across time). Consequently, $\tilde{I}_{jdt}$ and $E\{\log \tilde{I}_{jdt}\}$ in equations (20) and (21) remain constant either during a day, d, or given time interval, t. As trading participants actively acquire real-time information on the state of the market, they form unbiased expectations of the number of transactions, N_{jdt}, trading volume, V_{jdt}, and volatility, σ_{jdt}, over the next time interval (variables are aggregated over 5-minute time intervals). Given these expectations, market participants then select a trade size, Q_{jdt}, reflecting their immediate trading requirements, conditioning on the prevailing market cirnumstances. As realized values may differ from a priori expectations, we require robust estimators for the number of trades, the volatility of returns and the trading volume in order to accurately test the trading invariance hypothesis. We aggregate the logarithms of these 5-minutes observations and average their values across days to mitigate the effect of sampling variation and measurement error. Any remaining intraday fluctuations subsequent to aggregation indicate fluctuations in market expectations.

Let $\tilde{x}_{jdt}$ represent the log of the underlying trading variable, either trade count n, average trade size q, trading volume υ, realized volatility s, or trading activity w, for asset j for interval t of day d that observe in transactions data. The interday average value of this variable observation, for this specific time interval, taken over the entire D days in the sample, is given by:

$$x_{jt} = \frac{1}{D} \cdot \sum_{d=1}^{D} \tilde{x}_{jdt} \ , \ \ t = 1, \ldots\ldots, D \qquad \textbf{(23)}$$

Alternatively, we can calculate the average of the relevant trading variable for all intraday intervals of length t within a given trading day d, as:

$$x_{jd} = \frac{1}{T} \cdot \sum_{t=1}^{T} \tilde{x}_{jdt} \ , \ d = 1, \ldots\ldots, T \qquad \textbf{(24)}$$

where there are T intervals, each of length t, in trading day *d*.

Following Andersen et al. (2001), we calculate realized volatility σ_{jdt} from 10-second returns. We obtain prices in each 10-second time interval by taking the average of the respective log bid and log ask quotes. As tick-by-tick data is not generally reported in continuously-spaced distinct time points, on occasion we obtain the required midpoints by linearly interpolating between the previous and next available midpoint. We compute returns from bid and ask quotes not from trade prices. This avoids the effects of bid–ask bounce and stale prices that may bias the realized volatility estimator (Zhou, 1996, Andersen et al., 2000). We estimate "continuously-compounded returns" using the difference between the 10^{th} and 1^{st} ranked midpoints in each 10-second time interval which accounts for microstructure noise. The realized volatility σ_{jdt} estimator for each 5-minutes interval is then defined as the sum of squared 10-second returns (i.e. 30 squared returns per 5 minutes). We remove from the sample those 5-minutes intervals during which the realized volatility is zero. In contrast to Andersen et al. (2016), there are no 5-minutes intervals without trades that we need to exclude from our sample. The total fraction of omitted observations across all stocks is 5.5%.[29] While the resulting estimator suffers from potential measurement error in relation to actual local volatility, we improve its overall accuracy by summing across different 5-minutes intervals in line with the "error diversification principle" adopted by Andersen et al. (2016).

3.3.2 Empirical Analysis: Market Microstructure Invariance and Alternative Hypotheses

The benchmark empirical hypothesis we investigate, the invariance of trades, proposes a proportionality between the number of trades n_j and trading activity w_j and is given in equation (22). We now formulate and test two variants of this invariance proposition, comparing our findings with an invariance representation of two well-known alternative models in the literature developed by Clark (1973) and Ané and Geman (2000), the MDH-V and MDH-N respectively, which utilize different concepts of business time. These models are briefly discussed in section 1. Here we summarize their distinctive properties and derive appropriate empirical formulations.

The baseline invariance models we estimate are both derived from equation (22). They differ only in their measurement of the trading activity variable, w_{jk}. In its general form, this is given by:

$$n_{jk} := c + \frac{2}{3\varphi} w_{jk} + u_{jk}^{n} \qquad \textbf{(25)}$$

where k represents either different intraday intervals (i.e. $k = t,\ t = 1,\ldots..,T$) or distinct trading days (i.e. $k = d,\ t = 1,\ldots..,D$) for asset j, and u_{jk}^{n} are the regression residuals. Inspection reveals this to be the empirical analogue of the formulation in equation (22).

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29 We also estimate the underlying variables for one-minute intervals, as lower frequency estimators may exhibit upward bias. However, the number of one-minute intervals we exclude from our analysis greatly exceeds the number excluded from the lower frequency, 5-minutes intervals. As omitting more information renders less precise coefficients estimates, we report estimates using the 5-minute frequency.

Model 1: Invariance (Kyle and Obizhaeva, 2016b)

The first invariance model defines trading activity as introduced by Kyle and Obizhaeva (2016b). This is given in equation (8), such that the log of trading activity w_{jk} is equal to $w_{jk} := p_{jk} + \upsilon_{jk} + \frac{1}{2} s_{jk}$, and it restricts $\varphi = 1$. In this case, equation (25) becomes (26):

$$n_{jk} = c + \frac{2}{3} w_{jk} + u_{jk}^{n} \qquad \textbf{(26)}$$

and the trade arrival rate is proportional to trading activity in a specific ratio, namely 2/3.

Model 2: Trading Invariance (Andersen et al., 2016)

The "intraday" trading invariance model of Andersen et al. (2016) investigates whether the invariance of bets can be applied to trades. Tick by tick data on E-mini S&P 500 futures contracts is used to test the specification. The formulation of the general test equation imposes similar restrictions on equation (25) as the Kyle and Obizhaeva (2016b) formulation. It differs from the latter model in that the definition of trading activity omits the price interaction term from equation (10), so that the log of trading activity w_{jk} in this formulation equals $w_{jk} := \upsilon_{jk} + \frac{1}{2} s_{jk}$, the product of expected trading volume and returns volatility. Again note that given the restrictions imposed, the trade arrival rate is proportional to trading activity in the same specific ratio, 2/3, as in Model 1.

Model 3: Mixture Distribution Hypothesis-Volume, MDH-V, (Clark, 1973)

The alternative models we explore also examine the relationship between trading activity and return volatility, but utilize alternative notions of business time to that proposed by MMI. Specifically, Clark (1973) investigates the relationship between trading volume and returns volatility, concluding that expected trading volume is directly proportional to return variation (i.e. ($\sigma_{jdt}^{2} \sim V_{jdt}$) and acts as a proxy for the business time clock. This proportionality can be expressed in log terms as: $s_{jdt} = c + \bar{\upsilon}_{jdt} = c + n_{jdt} + q_{jdt}$ enabling us to specify the following alternative empirical proposition using a similar representation as in equation (25). Following Andersen et al. (2016) and the respective derivation yields the following model:

$$n_{jk} = c + \frac{2}{3}\left[w_{jk} - \frac{3}{2} q_{jk} \right] + u_{jk}^{n} \tag{27}$$

Model 4: Mixture Distribution Hypothesis-Number of Trades, MDH-N, (Ané and Geman, 2000)

In contrast to both MMI and Clark (1973), others maintain that the number of trades (trade count) is a better proxy for business time. Building on earlier work by Jones et al. (1994), Ané and Geman (2000) report a significant relationship between the number of trades and returns variation. The resulting empirical hypothesis indicates that the expected number of trades is proportional to return variation (i.e. $\sigma_{\mathrm{jdt}}^{2} \sim N_{jdt}$), a proportionality which can be written in log terms as: $s_{dt} = c + n_{dt}$. This leads to our second alternative to the main MMI hypothesis which in the manner of equation (25) following Andersen et al. (2016) and the respective derivation we represent as:

$$n_{jk} = c + \frac{2}{3}\left[w_{jk} - q_{jk} \right] + u_{jk}^{n} \tag{28}$$

Finally, the intraday invariance equation (20) reveals that if expected trade size (Q_{jdt}) is constant, there is a proportionality between both expected return volatility and trade count ($\sigma_{jdt}^{2} \sim N_{jdt}$) and expected return volatility and trading volume ($\sigma_{jdt}^{2} \sim V_{jdt}$). In this case, the formulations of Clark (1973), Ané and Geman (2000) and intraday trading invariance become equivalent. In contrast, if variations in trade size and variations in return volatility and number of trades are correlated, the aforementioned proportionality disappears. This outcome is more likely if traders actively control for their risk exposures in business time, which in turn leads to a systematic variation of trade size with both volume and volatility. In this situation, the theories will exhibit contrasting empirical implications.

We proceed to examine the existence of an invariance relationship between the number of trades and the various proposed definitions of trading activity given by equations (26), (27), and (28) outlined above. The empirical implications of this formulation of the equations are that on the basis of the different definitions of the trading activity variable, w_{jk}, the estimated regression coefficient should be $\beta = 2/3$ for every model, thereby facilitating comparisons. Our main empirical objectives are twofold: first, to investigate whether this hypothesized invariance proportionality holds on average, and second to determine which, if any, of the different notions of trading activity can predict it more accurately. We undertake the analysis on variables estimated both as averages across days (interday) and as intraday averages, described in detail section 3.5

3.4 The FTSE 100 Data

We use time-stamped tick data from Thomson Reuters Tick History for the 70 constituent stocks of the FTSE 100 index which trade on the LSE (see list of stocks in Appendix II-Table A1) and remain constituents of the FTSE 100 throughout the sample period. The dataset includes tick-by-tick information on the best available bid and ask quotes, transaction prices, and trading volume (in shares), for the 3 years between 1st January 2007 and 31st December 2009. We focus on the continuous trading period on the LSE from 8 a.m. to 4.30 p.m., Monday to Friday. We exclude 30 days that correspond to holidays or other days with reduced trading activity arising from reduced trading hours. This leads to a total of 754 trading days.

We further divide each trading day into 102 intervals of 5 minute duration. In each interval, we aggregate the observations for trading volume V, number of trades N and average trade size Q so our estimates relate to five-minute values. We compute the realized return volatility, σ, for each 5-minutes interval following the approach explained in subsection 3.1. We apply two aggregation methods which focus on the trading patterns across days (interday) and within a day (intraday). For the 5-minute *averages across days*, we estimate averages of 5 minute intervals using equation (23), aggregate the respective values across 754 trading days and divide by 102 (the number of 5-minute intervals per trading day). For the 5-minute *intraday averages*, we estimate averages of 5 minute intervals using equation (24) across the 102 intervals of 5-minute duration during each trading day and then divide by 754, the number of trading days. The descriptive statistics for the 5-minutes averages across days are reported in Table 1.[30]

[Table 1 in here]

From the descriptive statistics of the overall sample in Panel A, Table 1, we note that the sample stocks trade comparably frequently, with an average of about 48 trades per 5 minute interval, a trading volume of 15 traded shares, and an average trade size of 3 shares. The companies are

[30] The statistics are quantitatively very similar for the 5-minute intraday averages. We report the latter in Appendix II, Table A2.

relatively large with an average market capitalization of 17 billion GBP.[31] To account for any differential impact of market capitalization on invariance, we classify the 70 stocks into 3 groups based on their market capitalizations using monthly values from the LSPD database as provided by WRDS. The list of sample stocks and 3 market cap groups are listed in Appendix II, Table A1. We use two different classification methods: the first classification yields groups of similar size (23 stocks in the high Mcap group, Mkt1, 23 stocks in the medium Mcap group, Mkt2, and 24 stocks in the lowest Mcap group, Mkt3), while the second classification results in groups that each account for an equal proportion (33%) of the total 3-years market capitalization average (PropMkt1, PropMkt2, PropMkt3). In the latter classification, the high Mcap group (PropMkt1) consists only of the top 5 stocks, whose individual market capitalization is very high when compared to the other stocks, and the medium Mcap group (PropMkt2) of the 13 highest capitalized stocks after the top 5. We present the descriptive statistics for the three market cap groups and the group of top 5 stocks with the highest market capitalization in Table 1, Panel B. We can see that the classification by market capitalization results in the high Mcap group (Mkt1) having the highest number of trades, trading volume, and trade size, while the low Mcap group (Mkt3) has the lowest average taken over these measures. However, the groups have a similar average volatility of around 0.3. Analysing the high proportion Mcap group (PropMkt1) separately indicates that this group has quite distinct values, with more than double the trading volume and trade size even when compared to the high Mcap (Mkt1) group.

The descriptive statistics of the model variables Trades (N), Volume (V), Trade Size (Q), Price (P), and Volatility (σ) for each individual stock are presented in Appendix II, Table A3, for 5-minutes averages across days, and in Appendix II, Table A4, for 5-minutes intraday averages. We report the stocks in order of their market capitalization, with BP having the highest and IHG the lowest market capitalization. The mean values in Table A3 indicate that the average number of trades ranges from 19.83 to 126.98, with ABF having the lowest, and BLT the highest number of average trades. Average trading volume ranges from 12.26 (in 1000 shares) to 1915.94 (in 1000 shares) and average trade size ranges from 0.48 (in 1000 shares) to 23.05 (in 1000 shares), with VOD having both the highest average trading volume and trade size. In that respect VOD (which is also included in the PropMkt1 group) seems somewhat distinctive as compared to the other stocks. Finally, average annualized volatility ranges from 0.20 to 0.54, with ABF having the lowest,

[31] Comparing our sample statistics with those from the E-mini futures in Andersen et al. (2016), the average number of trades is approximately 1/15 of the trade count in their sample of E-mini futures. The average volatility of 0.3 is similar to the value of 0.26 in Andersen et al. (2016).

and RBS the highest, annualized volatility. In particular, the top three stocks in term of average annualized volatility belong to the banking sector. As we incorporate the 2008/09 financial crisis in the sample, this may be unsurprising.

Comparing values between Table A3 and Table A4, individual stocks exhibit minor differences in their means for all variables. We observe the largest mean differences for the number of trades and trading volume, and they are especially pronounced for the 23 stocks with the greatest market capitalization. However, the standard deviation of variables differs considerably between the two sets of averages. For the majority of the stocks, the numbers of trades, trading volume and trade size exhibit a higher standard deviation using intraday averages for their estimation. These differences are consistent with the existence of significant intraday trading patterns, which influence the model's variables in a more pronounced fashion than when we compare the same effects during the same 5-minute intervals across trading days.

3.5 Empirical Results

3.5.1 Analysis of Proportionality of Trade Count and Trading Activity

This section discusses the empirical results of panel regressions analysing the proportionality between number of trades and the different definitions of trading activity, introduced by Kyle and Obizhaeva (2016b), Andersen et al. (2016), Clark (1973) and Ané and Geman (2000), respectively. To control both for time and stock effects, we include two-way fixed effects when estimating equations (26), (27), and (28). Next, we investigate model differences in the invariance coefficients when we classify stocks in groups based on market capitalization, trade size, trading volume and number of trades. This will reveal the extent to which these characteristics, if any, affect the invariance proportionality. Ultimately, this approach will verify whether the theoretical 2/3 invariance proportionality for bets applies also to trades on average and reveal which of the four models best captures the market microstructure properties in this specific market.

The two-way fixed effects (stock and time) model we employ has the following form:

$$n_{it} = a + \beta w_{it} + \varepsilon_{it} \;\; (t \in \{1,.......,102\} \text{ or } t \in \{1,.......,754\}; i \in \{1,.......,70\}) \qquad \textbf{(29)}$$

where n_{it} is the logarithm of number of trades for stock *i* at 5-minutes interval *t* and w_{it} is the logarithm of trading activity for stock *i* at 5-minutes interval *t*, defined by equations (26), (27), and (28) respectively; ε_{it} are error terms assumed to be independently distributed across stocks. The number of trades and trading activity are either averages across days (with $t\epsilon\{1,\ldots,754\}$ trading days) or intraday averages (with $t\epsilon\{1,\ldots,102\}$ intraday intervals) as estimated by equations (23) and (24), respectively.

[Table 2 in here]

Table 2 Panel A, presents the coefficient estimates of the OLS regression model given in equation (29) for the four different definitions of trading activity, as specified in Models 1-4, using aggregation across days. Similarly, Panel B depicts the coefficients when we use intraday averages. In both sets of averages (across days and intraday), the null hypothesis for $\beta = 2/3$ is rejected at a 0.1% significance level. Specifically, for the results on averages across days in Panel A, the invariance model (Model 1) and its modification (Model 2) both estimate a lower average proportionality between number of trades and trading activity than invariance theory suggests. In contrast, Model 3 (MDH-V) and Model 4 (MDH-N) yield coefficients that exceed the expected 2/3 proportionality. Comparing the differences between the coefficient estimates of the four models and 2/3, we conclude that Model 4 has the smallest absolute difference of 0.0508, while Model 3 yields the largest difference of 0.0731. In addition, the results for Model 1 and Model 2 suggest that including prices does not influence the average proportionality estimate when measured as an average across days.

As shown in Table 2 Panel B, using intraday instead of across day averages leads to lower coefficient estimates for all models. Model 1 (invariance) and Model 4 (MDH-N) produce almost identical average proportionalities; as do Model 2 (trading invariance) and Model 4 (MDH-V) specifications. We attribute the coefficient differences when variables are averaged across days rather than intraday to the presence of different time and stock fixed effects. In particular, we maintain that time fixed effects are more pronounced using intraday averages due to documented intraday trading patterns. The inference is that the invariance proportionality results may be partly driven by the intraday dynamics of trading activity.[32]

In summary, there is a significant difference in results between the two invariance models and the MDH-N and MDH-V models, according to the definition of trading activity we adopt. Moreover, the estimated relationship between the number of trades and trading activity varies significantly from the models' predicted values. Using across day averages, the MDH-N model (Model 4) yields the closest estimate to the theoretical 2/3 relationship predicted between trading activity

[32] The results are qualitatively similar for the analysis using intraday averages. We report these additional results in the Appendix II, Table A5 and discuss them together in Section 5.3.

and number of trades. However, we note that both invariance models yield smaller standard errors for average invariance coefficient estimates as compared to the other two models, independent of the estimation method we employ for the underlying variables. Thus, the trading activity definitions generated by the two invariance models may provide more robust results on the relationship between trading activity and number of trades than the alternatives. The main difference between the models is the role trade size plays in their respective measure of trading activity. Moreover, the fact that the invariance models exhibit different slope coefficients compared to MDH models gives us reason to infer that trade size appears to be non-constant. In summary, a significant difference in results is apparent between the invariance and both the MDH-N and MDH-V models, which differ according to the definition of trading activity they adopt.

Andersen et al. (2016) argue that trade size, as well as other cross-sectional characteristics, may play a significant role in proportionality estimates. We further analyse the invariance relationships for different subsamples, selected on the basis of market capitalization, trade size and trading activity to offer further insight into the factors affecting the relationship between the number of trades and trading activity.

3.5.2 Cross-Sectional Analysis of a Proportionality between Trade Counts and Trading Activity

3.5.2.1 Influence of Market Capitalization

To provide insight into the different factors influencing the proportionality between trade counts and trading activity, we examine the invariance proportionality for various groups of stock characteristics, namely market capitalization, trade size, trading volume and trade counts, and test for differential effects across these group clusters. We begin by investigating the impact of market capitalization on the relationship between trade counts and trading activity. In section 4, we classify the 70 stocks into 3 groups Mkt1 (high), Mkt2 (medium) and Mkt3 (low) based on their market capitalization. Subsequently, we estimate OLS regressions based on the following modification to the model in equation (32):

$$n_{it} := a + \beta_1 w_{it} + \beta_2 MKT_2 w_{it} + \beta_3 MKT_3 w_{it} + \varepsilon_{it} \quad \textbf{(30)}$$

where n_{it} is the logarithm of number of trades for stock *i* at 5-minutes interval *t*, w_{it} is the logarithm of trading activity for stock *i* at 5-minutes interval *t*, defined by equations (26), (27), and (28), respectively; ε_{it} are the error terms assumed to be independently distributed across stocks; MKT_j (j = 2,3) is a market capitalization dummy variable for "medium" capitalization (j = 2), and "low" capitalization (j = 3) stocks, respectively, taking a value of 1 if the stock belongs to the relevant group and zero otherwise.[33]

Classification according to market capitalization groups yields some revealing results, which we present in Table 3. In the invariance models 1 and 2, we find that the proportionality between trade count and trading activity lies closest to the predicted 2/3 for those stocks in the high market capitalization (Mkt1) group, with the proportionality estimates always being lower in both the medium Mcap (Mkt2) and low Mcap (Mkt3) groups. However, we still reject the hypothesis of $\beta = 2/3$ in all four model specifications. However, the slope coefficient estimates are economically very close to 2/3 which invariance theory suggests.

[Table 3 in here]

We earlier indicated that we compile three further groupings of stocks, each of which contributes 33% of the overall FTSE 100 market capitalization. We conduct a further analysis based on this classification. Table 4 presents the results. We are unable to reject the null hypothesis of $\beta_1 = 2/3$ in the large (PropMkt1) and medium cap (PropMkt2) stock groups in either the invariance model (Model 1) or the trading invariance model (Model 2), thereby providing support for MMI. It appears the inherent stock characteristics of these two groups more closely conform to the requirements of MMI than the remaining stocks in the "small" cap group. Moreover, Panel A of Table 4 indicates the proportionality coefficient estimate for the "small" cap group is statistically significantly below the "large" cap group for all models. No significant differences are found between the invariance model (Model 1) and the trading invariance model (Model 2) with respect to the invariance proportionality estimates for "large" or "medium" cap stocks, while both the MDH-V and MDH-N models indicate that proportionality coefficient for the "medium" cap group is lower on average when compared to the "large" cap group

[Table 4 in here]

In summary, we believe that classifying stocks by market capitalization groups provides interesting insights into the proportionality relationships between trade counts and trading activity for

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[33] We do not estimate independent coefficients on MKT_2 and MKT_3 standalone dummy variables as they are subsumed by stock fixed effects, analyzing only the respective dummy variable interaction terms with the logarithm of trading activity, w_{it}.

different stocks. We further discuss the specific results and their potential interpretation in the following Section 3.5.3.

3.5.2.2 Influence of Trade Size, Trading Volume, and Number of Trades

This section examines the influence of trading volume, number of trades, and trade size on the documented invariance proportionality. Based on the average value of these specific characteristics, we classify the 70 stocks into two groups which we term high and low, employing an indicator dummy variable to distinguish between stocks in high and low characteristic groups.[34] The OLS regressions utilize the following variation of the model in equation (29):

$$n_{it} := a + \beta_1 w_{it} + \beta_2 X w_{it} + \varepsilon_{it} \tag{31}$$

where n_{it} is the logarithm of number of trades for stock i at 5-minutes interval *t*, w_{it} is the logarithm of trading activity for stock i at 5-minute interval *t*, defined by equations (26), (27), and (28) respectively, ε_{it} are the error terms assumed to be independently distributed across stocks, X represents a dummy variable set equal to 1 if a stock is a low value group member of a characteristic group based on trading volume ($TVol$), number of trades ($NTrades$), or trade size ($TSize$), respectively, and zero otherwise.[35]

Tables 5, 6 and 7, respectively, present the coefficient estimates we obtain from the OLS regressions which analyse the three constituent characteristic groups, namely differences in trading volume, number of trades and trade size, for each of the four models.

[Table 5 in here]

Table 5 reports the coefficient estimates for the group classification based on trading volume ($TVol$). The null hypothesis for $\beta_1 = 2/3$ is rejected in all four models at 0.1% significance level. Furthermore, there is no evidence of any significant differences in proportionality coefficients across the different groups of stock classifications. Qualitatively, stocks that belong to the group with low trading volume appear to yield smaller coefficient estimates for the invariance models and higher for MDH models compared to those in the large trading volume group, though those differences are insignificant in most cases. Overall, this finding for the trading volume classification is consistent with the invariant proportionality prediction of MMI. Following the

[34] The high average group incorporates stocks above the 85% percentile measure for these characteristics, whose variation within this top 15% group is comparable to that in the lower 85% for the three chosen characteristics.

[35] We cannot estimate the coefficients on any standalone dummy variables as they are subsumed by stock fixed effects. So we estimate only the respective interaction terms with the logarithm of trading activity.

discussion in Section 3.4, while we can infer a positive relationship between trading volume, trade count, and market capitalization, the proportionality effects the market capitalization groups capture are not manifest across trading volume segmented groups.

[Table 6 in here]

The estimates from the trade count (*NTrades*) classification in Table 6 once again reject the null hypothesis of $\beta_1 = 2/3$ in all four models. With the exception of the MDH-V model, all models report that stocks with lower trade counts evidence lower proportionality coefficients as compared to those with higher average trade counts. The estimates obtained by trade count classification are both qualitatively and quantitatively similar to the estimates we present in table 5 based on market capitalization, although the results differ from those we obtain based on the trading volume classification. As trade size constitutes the link between the trade count and trade volume measures, our results also point towards an important role for trade size in establishing any proportionality relationship between trade counts and trading activity.

[Table 7 in here]

Finally, Table 7 presents the results subsequent to stock classification by trade size *(TSize)*. An inability to reject the null hypothesis of $\beta_1 = 2/3$ in the MDH-N model (Model 4), indicates that large trade size stocks exhibit the trade flow composition implied by MMI on the assumption that the number of trades is proportional to returns variance. The other three models all reject the null hypothesis of $\beta_1 = 2/3$. As invariance theory predicts no statistical difference in the proportionality between large and small trade size groups, the estimates from both the invariance model (Model 1) and the trading invariance model (Model 2) corroborate the theory and are consistent with our expectations. However, there is a positive significant difference for the small trade size stocks in both the MDH-V and MDH-N models. As smaller trade sizes would generally imply a higher degree of intermediation and a smaller amount of risk transfer per trade, the positive difference between large and small trade size groups in these two models contrasts with our prior expectations. What is apparent from the descriptive statistics we present in Table 1 and in Appendix II, Table A3 and A4, is that trade sizes does not correlate with either market capitalization or trading volume, so while the effects it captures play an important role, the nature of this role differs from that incorporated in the latter two variables.

Qualitatively, particularly in the MMI specification, larger trade size stocks universally exhibit smaller regression coefficients as compared to small trade size stocks, thereby violating underlying model assumptions. Following our discussion of order shredding in Section 2.2, we associate smaller slope coefficients with a lower level of order shredding, leading us to conjecture that smaller trade sizes may be a result of a higher level of order shredding in these stocks. Thus, our finding of larger coefficient estimates in smaller trade sizes is consistent with the notion of higher levels of order shredding for small trade size stocks. In a sense, this confirms our initial hypothesis for the extended invariance model in (10).

In summary, we document that the stock characteristics we use in order to classify different groups of stocks affect the estimates of invariance coefficients. While we cannot confirm the precise 2/3 theoretical proportionality suggested by Kyle and Obizhaeva (2016b) for all stocks, in most models our invariance estimates are close to 0.6 and appear generally robust to stock classifications based upon trading volume and trade size. Restricting the analysis to the larger market capitalization stocks, we are able to closely approximate the predicted 2/3 proportionality using both invariance model specifications, and also in the MDH-N model for stocks with large trade size. Overall, there is a significant difference between the invariance models (Model 1 and 2) and the MDH models (Models 3 and 4) in terms of the invariance coefficient estimates. Generally, the invariance models (MDH models) yield consistently lower (higher) coefficient estimates than the 2/3 proportionality predicted in Kyle and Obizhaeva (2016b).

3.6 Discussion

The hypotheses in the preceding empirical analysis all assume that following Kyle and Obizhaeva (2016b), the volume multiplier is $\zeta = 2$ and the volatility multiplier is $\psi = 1$. Andersen et al (2018) use the same set of assumptions to investigate the proportionality between trade counts and trading activity in the E-mini S&P 500 futures market, though they clearly argue that their ITI hypothesis is distinct and purely empirical compared to MMI, while motived by the later. Our dataset is similar in nature to that Andersen et al. (2018) employ, in that we obtain tick-by-tick data for trades and quotes. However, one significant difference in the two datasets concerns the manner in which they report trades. The dataset in Andersen et al. (2018) reports trades as the executable portion of an initiated order. An incoming order can be executed against different passive orders at different price levels, but the data only records the portion executed at the best price level. Although Andersen (2018) argue that due to the high level of liquidity and depth at the market, the unexecuted portion of orders would be negligible, this may decrease the trade size in some instances of market illiquidity. In general, this aggregation would approximate the definition of a bet quite closely and justifies the assumption that the volume multiplier is $\zeta = 2$ and that the same invariance relationships that hold for bets can also potentially hold for trades occurring over shorter intervals. However, order slicing (or order shredding) strategies by both the active and passive traders can generally make it difficult to unravel the initial bet from historical trade data.

In contrast, our dataset has limitations when measuring the magnitude of bets. It records any liquidity demanding order executed against multiple passive limit orders as separate trades, so it captures supply-side intermediation. This contrasts with the dataset used by Andersen et al. (2018) which adopts the perspective of the active trader placing the executable order rather than capturing any intermediation at the trade execution stage. Thus, our dataset may have a tendency to inflate the number of trades and deflate trade size amounts from the levels that theoretical invariance concepts suggest. However, our dataset captures all trades, which can convey different insights. It is important to note that trades in our dataset are not individual bets. Therefore, the assumption that the volume multiplier $\zeta = 2$ and the volatility multiplier $\psi = 1$ would not be expected to hold for all transactions and across stocks, which affects the intercept and possibly also the slope coefficient. In addition, our analysis provides insight into other factors, such as trade size and number of trades, which may influence the relationship between the number of trades and trading activity. As all these factors are affected by order shredding or intermediation activity, they can also underpin a rational explanation for the fact that some of our empirical coefficient estimates can deviate from the stipulated 2/3 invariance proportionality.

In order to incorporate activities such as order shredding or trade intermediation, we contend that our extended invariance model provides one solution to formulating empirical tests of invariance proportionality using trade data. We introduce the order shredding factor φ to the specification of invariance as introduced in Andersen et al. (2018). Following Andersen et al. (2018) and their assumptions on ITI, our extended model is a purely empirical hypothesis that yields a proportionality of $2/3\varphi$, which Consequently, equals 2/3 when we impose a linear relationship between bets and trades as implied by Andersen et al. (2018). We argue that our model generates empirical estimates from trade and quote data, enabling us to draw certain inferences on the relationship between predicted invariance proportionality coefficients and trading activity.

Our empirical estimates of the invariance models for the highest market capitalization group of stocks yield the MMI predicted 2/3 proportionality on average, but also indicate a lower proportionality coefficient for "low" market capitalization stocks, when using averages across days. Both MMI and ITI assume an average of one market maker per bet/trade and that return volatility stems primarily from order flow imbalances caused by bets/trades rather than public information, such as news announcements, respectively. Our results, independent of the criterion for classification and the trading activity notion we employ, imply that these assumptions appear to be more representative of trading in "high" and rather than "low" capitalization stocks and may arise from greater liquidity and depth at the top of the limit-order book. In a sense this may indicate that "large" market cap stocks are more liquid compared to "low" market cap stocks. However, based on our extended invariance model, "low" market cap stocks yield lower coefficient estimates, which is associated with lower level of order shredding. On one hand this implies that these stocks are not illiquid as traders on average are able to execute orders without having to shred them. On the other hand, this result may suggest that bets in large stocks are probably larger than bets in small stocks and so these bets in large stocks are shredded into more pieces than bets in small stocks (e.g. if for simplicity we assume a market in which all bets are simply shredded into 100-share trades).

Coefficient estimates based on volume groups are lower (higher) than invariance theory suggests for invariance (MDH) models. Based on our extended invariance model, qualitatively, stocks that belong to the group with low trading volume are subject to slightly lower level of order shredding on average. Qualitatively similar are the results for transaction counts groups, with those stocks in the lower number of trades group, exhibiting lower level of order shredding on average. We argue that both results are economically reasonable as orders for stocks with thin volumes (either measured by trading volume or number of trades) are expected to be accommodated without the need for order shredding. In addition, small trade size stocks yield higher slope coefficients, possibly due to the presence of higher levels of order shredding serving to decrease trade size for these stocks. This is consistent with the initial hypothesis of our extended invariance model. Higher slope coefficients for lower trade size stocks are present independent of the notion of trading activity used. Nevertheless, large trade size stocks exhibit the trade flow composition implied by MMI on the assumption that the number of trades is proportional to returns variance. In summary, the coefficient estimates when grouping stocks based on major stock characteristics such as trading volume, number of trades and trade size differ from the stipulated 2/3 proportionality partly because of the present of order shredding in the specific market we investigate.

The interpretation we place on the invariance coefficients based on the extended invariance model with order shredding component complements the framework introduced by Kyle and

Obizhaeva (2016b) and that by Andersen et al. (2018). Potentially, another reason for the deviation from the theoretical 2/3 proportionality, assuming that calendar time volatility is constant, is that any change in the number of trades is not followed by changes in trade size of the requisite magnitude. Note that invariance theory suggests that bet size is perfectly negatively correlated with volatility in business time (i.e. calendar time volatility divided by the square root of the bet arrival rate). However, this correlation may not hold for every trade (which are not bets) leading to a possible deviation from the predicted 2/3 proportionality. Examining the correlations between the underlying variables and their impact on the proportionality is an interesting topic for future research.

Moreover, Andersen et al. (2016) suggests any analysis of the type we conduct is also undertaken using intraday averages as well as across day averages. For robustness we undertake this procedure and present the results in Appendix II, Table A5. Employing intraday averages leads to generally lower coefficient estimates than using across day averages, and the null hypothesis of $\beta = 2/3$ is rejected for all models and all classifications. With respect to the different classifications applied, the proportionality is higher for low capitalization, low volume, low trade count and small trade size stocks. We conjecture that the lower proportionality we obtain using intraday averages partly arises from the different time and stock fixed effects present in the data. Daily trading patterns for stock trading activity are extensively documented in the literature. For certain stocks, orders might be shredded/intermediated more often in certain time periods within a daily interval. Note that that our time period includes also both the financial crisis (which significantly affects price, volatility and trading volume in FTSE 100 constituent assets), as well as a significant increase in AT activity (which might lower the trade size), which may have a negative impact on the estimated coefficients. Also, there may be considerable measurement error in the trading activity variables, as well as structural difference between the data samples and the underlying market environments which may negatively impact upon the proportionality coefficient estimates compared to Andersen et al. (2016).

Finally, it is important to underline the difference in our empirical design compared to that employed in Andersen et al. (2018). Their dataset and its inherent characteristics as explained in the previous paragraphs allow also for the empirical investigation of ITI with a model that has different parameter predictions for different notions of trading activity (Andersen et al., 2018, p11, equation (4)). This facilitates the comparison between ITI and other well-known alternatives in the literature, namely MDH-V implied by Clark (1973) and MDH-N implied by Ané and

Geman (2000). In contrast, given our dataset, this design will not have the desirable properties so that enable us to appropriately compare different notions of trading activity and determine which best predicts the hypothesized invariance proportionality between the number of trades and trading activity. Therefore, here we prefer to follow Andersen et al. (2016) and have identical predictions for different regressions that correspond to different notions of trading activity. We argue that this design is more appropriate for the way our dataset reports trades and conveys different insights compared to Andersen et al. (2018).

3.7 Conclusion

This chapter examines the empirical predictions of MMI principles using trading data from FTSE 100 index constituent stocks. We propose an extended invariance model for trades to compare four different notions of trading activity based on the model introduced by Andersen at al. (2018) for trades. Our analysis is motived by MMI proposed in Kyle and Obizhaeva (2016b) using an empirical variation for trades as introduced by Andersen et al. (2018).

Our extension aims to accommodate empirical relationships documented in the stock market, and trading reporting conventions used in the majority of available databases. In the extended model, we introduce a factor to account for the order shredding that is present in stock markets arguing that bets and trades are connected in a non-linear manner. Based on this model , we infer that any lower estimated proportionality, such as we find for "low" market capitalization and trade count stocks, may arise from a higher/lower degree of order shredding/trade intermediation as compared to the original MMI introduced by Kyle and Obizhaeva (2016b) and ITI suggested by Andersen et al. (2018).

Our empirical analysis uses panel specifications and indicates that only the largest stocks in terms of market capitalization exhibit the stipulated 2/3 proportionality between trade counts and trading activity, defining the latter as in Kyle and Obizhaeva (2016b) and Andersen et al. (2016). When we measure trading activity following Ané and Geman (2000), it is large trade size stocks that yield the predicted 2/3 proportionality on average. Independently of testing for the predicted 2/3 proportionality, our results suggest that highly capitalized stocks exhibit higher proportionality coefficients for all definitions of trading activity. We proceed to analyse certain cross-sectional characteristics which might potentially impact upon proportionality estimates in either direction. Classification by trading volume does not seem to influence the estimates of proportionality, while all models except the MDH-V indicate that stocks with larger trade counts exhibit higher measures of proportionality.

Overall, our results provide some empirical support to MMI principles, especially in relation to large capitalization stocks. Furthermore, we provide a framework for further research which may reveal whether the individual characteristics and idiosyncratic risks of different securities and markets influence measures of predicted invariance relationships. It would be interesting to investigate further whether price, tick and lot sizes as stock characteristics have an impact on the invariance proportionality suggested by MMI theory. For example, while the influence of price may prove to be a mechanical relationship (e.g. Andersen et al. (2016) do not include it in their definition of trading activity), the classifications based on tick size may be more economically complex considering the difference in tick size regimes in different markets. Finally, given the existing level of market fragmentation in stock trading, future analysis could focus upon whether the introduction of different trading platforms affects invariance proportionality and if measures differ between more traditional liquidity measures and those liquidity and price impact variables suggested by MMI theory.

Table 1

Descriptive Statistics (5-minutes averages across days)

We estimate 5-minutes averages using equation (23), then aggregate the respective values across 754 trading days (entire sample) and divide by 102 (the number of 5-minutes intervals during which the stocks trade daily) to obtain the mean for each stock.. Then we calculate the variable mean for each group. The number of trades (*N*) is the average trade count, volume (*V*) is the average number of shares (in thousands), trade size (*Q*) is the average number of shares per trade (in thousands) and price (*P*) is the average GBP price, all per 5-minute interval. Volatility (σ) is calculated from 10 second returns and annualized using the following formula, $\sigma_{annual} = \sigma\sqrt{252 \cdot 8.5 \cdot 12}$, where 252 are trading days per year, 8.5 is the number of active trading hours on the LSE, and 12 is the number of hourly 5-minute intervals. Market capitalization (in million GBP) of each group is from monthly stock data. Panel A presents the statistics for the entire sample of 70 stocks. Mkt1, 2, 3 represent the statistics for the largest 23 (Mkt1), the next 23 (Mkt2) and the smallest 24 (Mkt3) stocks ranked by market capitalization, Ex Mkt represents the top 5 stocks with the highest market capitalization.

Panel A: Overall Sample

	Mean	Std Dev	Max	Min
Trades N	48.05	34.13	255.78	10.71
Trading Volume V	15.47	29.19	43.06	7.49
Trade Size Q	2.99	3.41	52.53	0.38
Price P	8.91	7.04	35.89	1.05
Volatility σ	0.3	0.17	2.67	0.14
Market Cap	17,310	21,325	102,918	2,606

Panel B: Market Capitalization Groups

	Mkt1		**Mkt2**		**Mkt3**		**Ex Mkt**	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Trades N	77.9	41.99	37.17	15.87	29.87	12.97	85.95	45.49
Tr.Volume V	289.71	465.38	118.89	96.38	59.61	65.95	627.44	757.47
Trade Size Q	3.54	4.92	3.38	2.37	2.09	2.04	7.45	8.16
Price P	13.74	8.79	4.98	2.66	8.03	5.1	8.93	5.7
Volatility σ	0.32	0.19	0.29	0.15	0.3	0.16	0.25	0.13
Market Cap	39,635	25,003	8,893	3,361	3,981	930	82,060	18,279

Table 2

Average Proportionality between Trade Counts and Trading Activity

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity. Estimates use the model in equation (29). Across day averages in Panel A are from equation (23), and intraday averages in Panel B from equation (24). The trading activity measures are from Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4). All specifications include both stock and time fixed effects. Coefficients are tested against the null hypothesis, $H_0 : \beta = 2/3$. We report two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) in parenthesis for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Estimation based on across days averages				
	Model 1	Model 2	Model 3	Model 4
Constant , a	-0.3293***	1.3702***	6.0623***	3.5300***
	(0.0332)	(0.0234)	(0.0080)	(0.0111)
Invariance Coeff., β	0.5948***	0.5957***	0.7398***	0.7175***
	(0.0164)	(0.0161)	(0.0220)	(0.0200)
R^2	0.9964	0.9965	0.9958	0.9968
Panel B: Estimation based on intraday averages				
	Model 1	Model 2	Model 3	Model 4
Constant , a	0.9594***	2.4396***	4.8291***	3.7293***
	(0.0226)	(0.0258)	(0.0277)	(0.0232)
Invariance Coeff., β	0.4621***	0.3909***	0.3865***	0.4659***
	(0.0160)	(0.0225)	(0.0464)	(0.0409)
R^2	0.9328	0.9046	0.8933	0.9215

Table 3

Proportionality between Trade Counts and Trading Activity by Market Cap Groups

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity, and those relating to the interaction of the two market capitalization dummies, (medium) and (low), with the various trading activity measures, using the model in equation (30). The trading activity measures are from Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4). Estimates for the underlying variables use averages across days from equation (23). All specifications include both stock and time fixed effects. Coefficients are tested against the null hypothesis, $H_0 : \beta_1 = 2/3$. We report two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) in parenthesis for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

	Model 1	Model 2	Model 3	Model 4
Constant , c	-0.0464*	1.14766***	6.5127***	3.9773***
	(0.0225)	(0.0158)	(0.0198)	(0.0057)
Invariance Coeff.,	0.6236**	0.6236**	0.7562***	0.7391***
β	(0.0159)	(0.0158)	(0.0201)	(0.0181)
$MKT_2 \times w_{it}$	-0.0413***	-0.0395***	-0.0317***	-0.0361***
	(0.0091)	(0.0091)	(0.0123)	(0.0106)
$MKT_3 \times w_{it}$	-0.0490***	-0.0484***	-0.0273**	-0.0371***
	(0.0071)	(0.0073)	(0.0066)	(0.0062)
R^2	0.9969	0.9969	0.9960	0.9970

Table 4

Proportionality between Trade Counts and Trading Activity by Market Capitalization Contribution ($PropMKT_i$) Group

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity, and those relating to the interaction of the two market capitalization dummies, (medium) and (low) cap groups, with the various trading activity measures, using the model in equation (30). The trading activity measures are from Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4). Estimates for the underlying variables use averages across days from equation (23). All specifications include both stock and time fixed effects. Coefficients are tested against the null hypothesis, $H_0 : \beta_1 = 2/3$. We report two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) in parenthesis for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

	Model 1	Model 2	Model 3	Model 4
Constant , a	-0.0365	1.1565***	6.5022***	3.9817***
	(0.0225)	(0.0158)	(0.0190)	(0.0055)
Invariance Coeff., β	**0.6339**	**0.6341**	0.7836***	0.7586***
	(0.0169)	(0.0170)	(0.0210)	(0.0191)
$PropMKT_2 \times w_{it}$	-0.0178	-0.0173	-0.0420***	-0.0323**
	(0.0123)	(0.0127)	(0.0104)	(0.0111)
$PropMKT_3 \times w_{it}$	-0.0592***	-0.0583***	-0.0576***	-0.0582***
	(0.0102)	(0.0104)	(0.0095)	(0.0099)
R^2	0.9964	0.9969	0.9960	0.9970

Table 5

Proportionality between Trade Counts and Trading Activity by Trade Volume Group

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity, and those relating to the term interacting the trading volume dummy, , with the various trading activity measures, using the model in equation (31). The trading activity measures are from Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4). Estimates for the underlying variables use averages across days from equation (23). All specifications include both stock and time fixed effects. Coefficients are tested against the null hypothesis, $H_0 : \beta_1 = 2/3$. We report two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) in parenthesis for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

	Model 1	Model 2	Model 3	Model 4
Constant , a	-0.1407***	1.0925***	6.5765***	3.9993***
	(0.0237)	(0.0167)	(0.0191)	(0.0057)
Invariance Coeff., β	0.6011**	0.6052**	0.7304***	0.7137***
	(0.0216)	(0.0213)	(0.0306)	(0.0278)
$TVol \times w_{it}$	-0.0081	-0.0119	0.0128	0.0051
	(0.0140)	(0.0141)	(0.0183)	(0.0168)
R^2	0.9964	0.9965	0.9959	0.9968

Table 6

Proportionality between Trade Counts and Trading Activity by Trade Count Group

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity, and those relating to the term interacting the trade counts dummy, , with the various trading activity measures, using the model in equation (31). The trading activity measures are from Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4). Estimates for the underlying variables use averages across days from equation (23). All specifications include both stock and time fixed effects. Coefficients are tested against the null hypothesis, $H_0 : \beta_1 = 2/3$. We report two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) in parenthesis for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

	Model 1	Model 2	Model 3	Model 4
Constant , α	-0.0694***	1.1368***	6.5293***	3.9898***
	(0.0230)	(0.0161)	(0.0193)	(0.0056)
Invariance Coeff., β	0.6281*	0.6297*	0.7527***	0.7376***
	(0.0165)	(0.0164)	(0.0220)	(0.0198)
$NTrades \times w_{it}$	-0.0470***	-0.0478***	-0.0207	-0.0307**
	(0.0099)	(0.0103)	(0.0111)	(0.0102)
R^2	0.9967	0.9967	0.9959	0.9969

Table 7

Proportionality between Trade Counts and Trading Activity by Trade Size Group

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity, and those relating to the term interacting the trade size dummy, , with the various trading activity measures, using the model in equation (31). The trading activity measures are from Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4). Estimates for the underlying variables use averages across days from equation (23). All specifications include both stock and time fixed effects. Coefficients are tested against the null hypothesis, $H_0 : \beta_1 = 2/3$. We report two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) in parenthesis for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

	Model 1	Model 2	Model 3	Model 4
Constant , a	-0.1870***	1.0654***	6.6187***	4.0087***
	(0.0235)	(0.0166)	(0.0188)	(0.0060)
Invariance Coeff., β	0.5752***	0.5779***	0.7095***	**0.6903**
	(0.0206)	(0.0203)	(0.0292)	(0.0261)
$TSize \times w_{it}$	0.0260	0.0232	0.0435*	0.0382*
	(0.0152)	(0.0153)	(0.0196)	(0.0178)
R^2	0.9965	0.9965	0.9960	0.9969

Appendix I

Proof of Propositions in the text

AI-1. Relationship between expected bet and trading activity

From the specification of bet volume we know that:

$$\bar{V}_{jt} := \frac{2}{\zeta} \cdot V_{jt} \Rightarrow P_{jt} \cdot \bar{\sigma}_{jt} \cdot \bar{V}_{jt} = \frac{2}{\zeta} \cdot V_{jt} \cdot P_{jt} \cdot \bar{\sigma}_{jt} \quad \textbf{(A1)}$$

Trading volatility in currency units is given by:

$$P_{jt} \cdot \bar{\sigma}_{jt} = \psi \cdot P_{jt} \cdot \sigma_{jt} \quad \textbf{(A2)}$$

Combining equations (A1) and (A2), we obtain:

$$P_{jt} \cdot \bar{\sigma}_{jt} \cdot \bar{V}_{jt} = \frac{2}{\zeta} \cdot V_{jt} \cdot P_{jt} \cdot \psi \cdot \sigma_{jt} = \psi \cdot P_{jt} \cdot \frac{2}{\zeta} \cdot V_{jt} \cdot \sigma_{jt} \quad \textbf{(A3)}$$

From the definitions of expected bet (and trading) activity from (3) it can be inferred that:

$$\bar{W}_{jt} = \frac{2\psi}{\zeta} \cdot W_{jt}$$

AI-2. Expressing expected bet activity at any interval t in terms of total volume V_{jt}, price P_{jt}, volatility σ_{jt} and expected trading activity W_{jt}

From the specification of expected bet activity we know that:

$$\bar{W} := P_{jt} \cdot \bar{V}_{jt} \cdot \bar{\sigma}_{jt} \quad \textbf{(A4)}$$

If now we substitute for total volume V_{jt}, price P_{jt} and volatility σ_{jt}, then:

$$\bar{W}_{jt} = P_{jt} \cdot N_{jt}^{\varphi} \cdot E\left\{\left|\tilde{Q}_{jt}\right|\right\} \cdot \psi \cdot \sigma_{jt} \quad \textbf{(A5)}$$

Finally, using the relationship between expected bet and trading activity:

$$\frac{2}{\zeta} \cdot W_{jt} = P_{jt} \cdot N_{jt}^{\varphi} \cdot Q_{jt} \cdot \sigma_{jt}$$

where $Q_{jt} := E\left\{\left|\tilde{Q}_{jt}\right|\right\}$

AI-3. Invariance relationship in terms of trading activity W_{jt} and number of bets N_{jt}

We have shown that:

$$\frac{2}{\zeta} \cdot W_{jt} = P_{jt} \cdot N_{jt}^{\varphi} \cdot Q_{jt} \cdot \sigma_{jt} \quad \textbf{(A6)}$$

We know that,

$$Q_{jt} = I \cdot P_{jt}^{-1} \cdot \psi^{-1} \cdot \sigma_{jt}^{-1} \cdot N_{jt}^{\varphi/2} \quad \textbf{(A7)}$$

Combining the two relationships, and solving for I:

$$I = \frac{2\psi}{\zeta} \cdot \frac{W_{jt}}{N_{jt}^{\frac{3\varphi}{2}}} \quad \textbf{(A8)}$$

AI-4. Invariance relationships in logs (proportionality between trading activity and number of bets and between trading activity and bet size)

Our specification for trading invariance states that:

$$\tilde{I}_{jt} = P_{jt} \cdot \tilde{Q}_{jt} \cdot \frac{\bar{\sigma}_{jt}}{N_{jt}^{\varphi/2}} \qquad \textbf{(A9)}$$

Applying logarithms and expectations then:

$$E\{\log \tilde{I}\} = p_{jt} + q_{jt} + \frac{s_{jt}}{2} + \log\psi - \frac{1}{2}\varphi \cdot n_{jt} \qquad \textbf{(A10)}$$

where $p_{jt} := \log P_{jt}$, $\tilde{q}_{jt} := \log \tilde{Q}_{jt}$, $s_{jt} := \log \sigma_{jt}^2$, $n_{jt} := \log N_{jt}$, $q_{jt} := E\left\{\log \tilde{q}_{jt}\right\}$

Solving for q_{jt} :

$$q_{jt} = E\{\log \tilde{I}\} - p_{jt} - \log\psi - \frac{s_{jt}}{2} + \frac{1}{2}\varphi \cdot n_{jt} \qquad \textbf{(A11)}$$

Taking logarithms of the relationship in (A6),

$$p_{jt} + \varphi \cdot n_{jt} + q_{jt} + \frac{s_{jt}}{2} := \log\left(\frac{2}{\zeta}\right) + w_{jt} \qquad \textbf{(A12)}$$

where $p_{jt} := \log P_{jt}$, $\tilde{q}_{jt} := \log \tilde{Q}_{jt}$, $s_{jt} := \log \sigma_{jt}^2$, $n_{jt} := \log N_{jt}$, $q_{jt} := E\left\{\log \tilde{q}_{jt}\right\}$

Substituting q_{jt} in (A12) from (A11) we obtain the invariance expression for n_{jt} :

$$n_{jt} := c_1 + \frac{2}{3\varphi} w_{jt} \qquad \textbf{(A13)}$$

where $c_1 = \frac{2}{3\varphi}\left\{\left[\log\psi + \log\left(\frac{2}{\zeta}\right)\right] - E\{\log \tilde{I}\}\right\}$

Similarly, in order to obtain the invariance expression for q_{jt} , we for substitute n_{jt} in equation (A11) from the expression in (A13):

$$q_{jt} = E\{\log \tilde{I}\} - p_{jt} - \log\psi - \frac{s_{jt}}{2} + \frac{1}{2}\varphi \cdot \left(c_1 + \frac{2}{2+\varphi} w_{jt}\right) \Rightarrow$$

$$q_{jt} = E\{\log \tilde{I}\} - p_{jt} - \log\psi - \frac{s_{jt}}{2} + \frac{1}{2}\varphi \cdot \left[\frac{2}{3\varphi}\log\psi + \frac{2}{3\varphi}\log\left(\frac{2}{\zeta}\right) - \frac{2}{3\varphi}E\{\log \tilde{I}\}\right] + \frac{1}{2}\varphi\frac{2}{3\varphi} w_{jt} \Rightarrow$$

$$q_{jt} = \left[1 - \frac{1}{3}\right] E\{\log \tilde{I}\} - \left[1 - \frac{1}{3}\right]\log\psi + \frac{1}{3}\log\left(\frac{2}{\zeta}\right) + \frac{1}{3} w_{jt} - p_{jt} - \frac{s_{jt}}{2} \Rightarrow$$

$$q_{jt} = \frac{2}{3}\left(E\{\log \tilde{I}\} - \log\psi\right) + \frac{1}{3}\log\left(\frac{2}{\zeta}\right) + \frac{1}{3} w_{jt} - p_{jt} - \frac{s_{jt}}{2} \Rightarrow$$

$$q_{jt} = c_2 + \frac{1}{3} w_{jt} - p_{jt} - \frac{s_{jt}}{2} \qquad \textbf{(A14)}$$

where $c_2 = \frac{2}{3}\left(E\{\log \tilde{I}\} - \log\psi\right) + \frac{1}{3}\log\left(\frac{2}{\zeta}\right)$

Appendix II

Table A1 - Sample Stocks

LARGE CAPITALIZATION STOCKS

Stock	Abbreviation
BP	**BP**
HSBC HOLDINGS	**HSBA**
VODAFONE	**VOD**
GLAXOSMITHKLINE	**GSK**
ROYAL DUTCH SHELL	**RDSA**
RIO TINTO	**RIO**
ASTRAZENECA	**AZN**
ROYAL BANK OF SCOTLAND GROUP	**RBS**
BRITISH AMERICAN TOBACCO	**BATS**
BG GROUP	**BG**
ANGLO AMERICAN	**AAL**
BHP BILLITON	**BLT**
BARCLAYS	**BARC**
TESCO	**TSCO**
XSTRATA	**XTA**
DIAGEO	**DGE**
LLOYDS TSB GROUP	**LLOY**
STANDARD CHARTERED	**STAN**
UNILEVER	**ULVR**
RECKITT BENCKISER	**RB**
SABMILLER	**SAB**
NATIONAL GRID PLC	**NG**
IMPERIAL TOBACCO GROUP PLC	**IMT**

MEDIUM CAPITALIZATION STOCKS

Stock	Abbreviation
BT GROUP	**BT**
AVIVA PLC	**AV**
PRUDENTIAL PLC	**PRU**
BAE SYSTEMS PLC	**BAE**
CENTRICA PLC	**CNA**
SCOTTISH & SOUTHERN ENERGY	**SSE**
CADBURY SCHWEPPES	**CBRY**
BSB GROUP	**BSY**
MAN GROUP PLC	**EMG**
ROLLS-ROYCE HOLDINGS PLC	**RR**
MORRISON (WM) SUPERMARKETS	**MRW**
MARKS & SPENCER GROUP	**MKS**
SAINSBURY (J)	**SBRY**
WPP PLC	**WPP**
REED ELSEVIER	**REL**
LEGAL & GENERAL GROUP	**LGEN**
COMPASS GROUP	**CPG**
ASSOCIATED BRITISH FOODS	**ABF**
LAND SECURITIES GROUP	**LAND**
OLD MUTUAL PLC	**OML**
ANTOFAGASTA	**ANTO**
PEARSON	**PSON**
SHIRE PLC	**SHP**

LOW CAPITALIZATION STOCKS	
Stock	**Abbreviation**
STANDARD LIFE	**SL**
INTERNATIONAL POWER PLC	**IPR**
KAZAKHMYS	**KAZ**
UNITED UTILITIES	**UU**
SMITH & NEPHEW	**SN**
EXPERIAN GROUP	**EXPN**
BRITISH LAND CO PLC	**BLND**
VEDANTA RESOURCES	**VED**
ROYAL & SUN ALLIANCE INS.	**RSA**
CAPITA GROUP	**CPI**
KINGFISHER	**KGF**
CARNIVAL PLC	**CCL**
CABLE AND WIRELESS	**CW**
SMITHS GROUP	**SMIN**
LIBERTY INTERNATIONAL	**LII**
NEXT	**NXT**
JOHNSON MATTHEY PLC	**JMAT**
BRITISH AIRWAYS	**BAY**
ICAP	**IAP**
SEVERN TRENT PLC	**SVT**
HAMMERSON	**HMSO**
SAGE GROUP PLC	**SGE**
REXAM PLC	**REX**
INTERCONTINENTAL HOTELS GROUP	**IHG**

Source: Thomson Reuters Tick History

Table A2

Descriptive Statistics based on 5-minutes intraday averages

We estimate 5-minutes averages using equation (24), then aggregate the respective values across the102, 5-minutes intervals during which the stocks trade every day, and then divide by 754 (the total number of trading days) to obtain the mean for each stock. Then we calculate the variable mean for each group. The number of trades (*N*) is the average trade count, volume (*V*) is the average number of shares (in thousands), trade size (*Q*) is the average number of shares per trade (in thousands) and price (*P*) is the average GBP price, all per 5-minute interval. Volatility (σ) is calculated from 10 second returns and annualized using the following formula, $\sigma_{annual} = \sigma\sqrt{252 \cdot 8.5 \cdot 12}$, where 252 are trading days per year, 8.5 is the number of active trading hours on LSE, and 12 is the number of hourly 5-minute intervals. Market capitalization (in million GBP) of each group is from monthly stock data. Panel A presents the statistics for the entire sample of 70 stocks. Mkt1, 2, 3 represent the statistics for the largest 23 (Mkt1), the next 23 (Mkt2) and the smallest 24 (Mkt3) stocks ranked by market capitalization, Ex Mkt represents the top 5 stocks with the highest market capitalization.

Panel A: Overall Sample

	Mean	**Std Dev**	**Max**	**Min**
Trades N	48.08	37.67	795.52	5.23
Trading Volume V	154.81	344.6	11300.55	1716.65
Trade Size Q	3	5.4	307.67	0.176
Price P	8.9	7.75	85.4	0.11
Volatility σ	0.3	0.25	22	0.07

Panel B: Market Capitalization Groups

	Mkt1		**Mkt2**		**Mkt3**		**Ex Mkt**	
	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev	Mean	Std Dev
Trades N	78	49.18	37.18	17.94	29.85	14.42	86.08	50.09
Tr.Volume V	290.16	557.08	118.87	123.89	59.54	79.82	628.15	895.84
Trade Size Q	3.55	7.49	3.38	4.09	2.09	3.74	7.45	13.59
Price P	13.74	9.98	4.98	3.07	8.03	5.73	8.93	5.82
Volatility σ	0.31	0.33	0.29	0.19	0.29	0.21	0.25	0.15

Table A3

Descriptive Statistics per Stock using 5-minutes averages across days

This table indicates the average values of number of trades, volume, trade size, price and volatility, separately for each stock and aggregated as 5-minute averages across days. Stocks are ranked based on their market capitalization. For the average across days, the 5-minutes averages are calculated using equation (23), aggregated across 754 trading days (entire sample) and then divided by 102 (number of 5-minutes intervals per trading day). The number of trades (*N*) is the average trade count, volume (*V*) is the average number of shares (in thousands), trade size (*Q*) is the average number of shares per trade (in thousands) and price (*P*) is the average GBP price, all per 5-minute interval.. Volatility (σ) is calculated from 10 second returns and annualized using the following formula, $\sigma_{annual}=\sigma\sqrt{252\cdot 8.5\cdot 12}$, where 252 are trading days per year, 8.5 is the number of active trading hours on the LSE, and 12 is the number of hourly, 5-minute intervals.

	Mean					**Standard Deviation**				
	Trades (N)	**Volume (V)**	**Trade Size (Q)**	**Price (P)**	**Volatility (σ)**	**Trades (N)**	**Volume (V)**	**Trade Size (Q)**	**Price (P)**	**Volatility (σ)**
BP	100.57	563.2	6.35	5.41	0.26	41.54	242.28	0.72	0.01	0.14
HSBA	112.59	466.91	4.62	7.63	0.27	42.4	187.89	0.66	0.03	0.12
VOD	105.16	1915.94	23.05	1.45	0.27	41.35	771.86	10.56	0.003	0.17
GSK	76.68	156.79	2.17	12.22	0.23	30.63	67.34	0.42	0.02	0.11
RDSA	34.76	48.98	1.59	17.95	0.22	14.76	23.09	0.39	0.02	0.11
RIO	120.62	83.18	0.76	35.68	0.42	45.77	37.4	0.18	0.03	0.23
AZN	73.42	59.39	0.86	25.15	0.22	27.86	22.4	0.13	0.01	0.18
RBS	102.04	950.82	9.91	4.58	0.54	29.71	341.36	1.07	0.87	0.3
BATS	57.45	56.59	1.22	17.6	0.23	21.38	22.8	0.4	0.02	0.09
BG	60.29	106.5	2.01	9.85	0.31	21.02	37.46	0.32	0.01	0.13
AAL	101.02	74.65	0.79	24.27	0.4	31.86	23.22	0.12	0.09	0.16
BLT	126.98	191.98	1.86	14.55	0.4	52.72	100.86	1.5	0.03	0.2
BARC	120.32	632.79	5.63	4.26	0.48	39.79	213.29	1.14	0.02	0.22
TSCO	69.47	267.45	4.38	3.98	0.23	25.06	105.58	2.86	0.003	0.13
XTA	103.49	109.35	1.09	22.02	0.46	31.27	37.53	0.16	0.05	0.22
DGE	52.1	88.98	1.94	9.78	0.24	21.59	50.04	0.77	0.09	0.09
LLOY	90.92	608.18	6.66	3.2	0.49	28.17	204.02	1.92	0.01	0.23
STAN	62.34	72.99	1.32	14.16	0.38	19.92	24.62	0.31	0.03	0.19
ULVR	46.86	48.24	1.21	15.66	0.23	18.84	17.92	0.27	0.12	0.11
RB	48.09	21.34	0.48	27.5	0.21	17.47	8.74	0.09	0.02	0.09
SAB	39.59	42.69	1.2	12.35	0.27	13.15	15.02	0.28	0.02	0.11
NG	40.41	85.15	2.23	6.9	0.22	15	34.7	0.5	0.01	0.09
IMT	46.54	32.23	0.86	19.94	0.24	16.56	12.45	0.7	0.06	0.09
BT	47.48	361.02	7.71	2.07	0.27	17.57	140.03	1.81	0.01	0.12
AV	47.13	108.29	2.29	5.42	0.37	16.24	38.07	0.32	0.01	0.14
PRU	52.24	134.9	2.78	5.73	0.38	18.51	44.85	0.73	0.03	0.16
BA	51.19	161.74	3.42	4.1	0.26	17.64	62.76	0.57	0.01	0.12
CNA	40.87	161.86	4.25	3.08	0.24	15.34	62.45	0.97	0.005	0.1
SSE	34.7	32.95	1.03	13.38	0.23	11.78	11.85	0.23	0.01	0.08
CBRY	34.96	90.7	2.9	6.09	0.25	15.61	40.6	0.63	0.01	0.1
BSY	33.54	84.6	3.18	5.35	0.25	13.55	105.22	6.39	0.01	0.1

Table A3 - continued

	Mean					Standard Deviation				
	Trades (N)	Volume (V)	Trade Size (Q)	Price (P)	Volatility (σ)	Trades (N)	Volume (V)	Trade Size (Q)	Price (P)	Volatility (σ)
EMG	40.23	121.93	3.23	4.37	0.36	13.92	50.02	0.67	0.01	0.17
RR	37.61	99.01	2.78	4.26	0.28	13.65	38.66	0.36	0.01	0.18
MRW	33.89	142.39	4.39	2.78	0.25	12.19	52.65	0.57	0.001	0.1
MKS	42.74	134.34	3.12	4.3	0.31	14.39	45.92	0.38	0.01	0.12
SBRY	31.39	117.39	4.36	3.94	0.26	11.75	44.1	2.06	0.01	0.13
WPP	40.27	76.23	2.08	5.65	0.28	16.14	34.4	0.33	0.01	0.12
REL	38.68	68.39	2.1	5.63	0.24	14.38	25.45	0.4	0.004	0.11
LGEN	34.67	292.41	8.92	1.06	0.39	11.9	100.22	2.18	0.003	0.16
CPG	36.32	114.25	3.5	3.37	0.26	13.17	50.14	0.94	0.01	0.1
ABF	19.83	19.47	1.05	8.01	0.2	7.9	7.82	0.21	0.01	0.09
LAND	36.92	34.37	0.95	12.53	0.3	12.69	12.54	0.13	0.03	0.13
OML	33.84	221.76	7.56	1.16	0.39	10.33	71.55	3.01	0.004	0.2
ANTO	36.48	49.34	1.44	6.24	0.43	13.05	18.9	0.2	0.0001	0.19
PSON	32	44.52	1.48	7.19	0.25	12.52	17.33	0.32	0.02	0.14
SHP	30.44	35.83	1.26	10.09	0.25	13.31	15.84	0.4	0.02	0.11
SL	20.32	55.29	3	2.44	0.36	6.86	21.03	0.73	0.004	0.13
IPR	33.68	88.81	2.79	3.51	0.26	11.97	34.25	0.38	0.01	0.12
KAZ	33.43	32.67	1.05	10.49	0.47	33.43	12.87	0.21	0.02	0.24
UU	31.03	42.57	1.44	6.39	0.21	13.54	18	0.25	0.04	0.08
SN	28.58	48.77	1.84	5.62	0.24	11.25	20.76	0.43	0.01	0.23
EXPN	25.17	53.45	2.28	4.74	0.27	9.55	22.59	0.4	0.01	0.1
BLND	40.69	50.88	1.3	8.43	0.32	13.71	18.61	0.2	0.01	0.14
VED	38.29	32.45	0.87	16.11	0.43	12.34	70.04	1.25	0.03	0.18
RSA	28.36	171.07	6.42	1.39	0.3	10.14	62.57	1.42	0.002	0.12
CPI	24.06	26.95	1.31	6.94	0.23	9.31	10.33	0.41	0.004	0.09
KGF	40.49	224.63	6.68	1.79	0.33	15.67	88.01	3.12	0.05	0.14
CCL	32.63	15.7	0.49	19.8	0.28	19.6	9.42	0.11	0.05	0.14
CW	30.96	179.74	6.41	1.6	0.25	13.32	83.86	2.24	0.003	0.11
SMIN	23.71	25.07	1.19	9.53	0.24	9.3	10.1	0.62	0.15	0.09
LII	24.93	22.56	0.94	8.29	0.3	7.36	7.03	0.15	0.03	0.14
NXT	36.86	26.18	0.7	15.72	0.31	11.97	8.78	0.1	0.03	0.12
JMAT	24.5	12.26	0.52	14.98	0.27	10.06	4.76	0.08	0.02	0.14
BAY	40.38	141.47	3.49	2.8	0.37	13.17	51.94	0.45	0.01	0.16
IAP	25.87	39.98	1.73	4.7	0.34	9.25	14.92	0.39	0.02	0.14
SVT	24.16	14.15	0.63	12.8	0.22	9.98	5.68	0.12	0.06	0.09
HMSO	27.22	29.95	1.16	8.79	0.33	9.57	11.41	0.23	0.03	0.16
SGE	22.28	69.98	3.61	2.14	0.26	9	28.66	2.03	0.002	0.1
REX	22.36	41.5	1.94	4.01	0.26	8.52	17.18	0.36	0.02	0.12
IHG	24.5	28.16	1.17	8.47	0.3	9.28	11.6	0.22	0.04	0.19

Table A4

Descriptive Statistics per Stock using 5-minute intraday averages

This table indicates the average values of number of trades, volume, trade size, price and volatility, separately for each stock and aggregated as 5-minute averages intraday. The stocks are ranked based on their market capitalization. For the intraday average, the 5-minutes averages are calculated using equation (24) and aggregated across 102, 5-minutes intervals per trading day, and then divided by 754, the total trading days in the sample. The number of trades (*N*) is the average trade count, volume (*V*) is the average number of shares (in thousands), trade size (*Q*) is the average number of shares per trade (in thousands) and price (*P*) is the average GBP price, all per 5-minute interval.. Volatility (σ) is calculated from 10 second returns and annualized using the following formula, $\sigma_{annual}=\sigma\sqrt{252\cdot 8.5\cdot 12}$, where 252 are trading days per year, 8.5 is the number of active trading hours on the LSE, and 12 is the number of hourly, 5-minute intervals.

	Mean					Standard Deviation				
	Trades (N)	**Volume (V)**	**Trade Size (Q)**	**Price (P)**	**Volatility (σ)**	**Trades (N)**	**Volume (V)**	**Trade Size (Q)**	**Price (P)**	**Volatility (σ)**
BP	100.62	563.41	6.34	5.41	0.26	42.51	304.04	4.16	0.48	0.16
HSBA	112.91	468.47	4.72	7.63	0.26	59.15	257.62	2.84	1.53	0.16
VOD	105.28	1917.35	23.04	1.45	0.27	45.24	1396.99	35.62	0.2	0.18
GSK	76.79	157.04	2.17	12.22	0.23	31.13	104.88	1.43	1.18	0.12
RDSA	34.79	49.08	1.59	17.94	0.22	14.55	27.37	1.21	1.78	0.13
RIO	120.74	83.2	0.76	35.68	0.42	56.05	75.22	0.61	13.77	0.31
AZN	73.49	59.42	0.86	25.15	0.22	29.25	36.41	0.6	2.77	0.21
RBS	102.23	954.16	9.93	4.58	0.55	60.67	781.54	6.55	6.77	0.94
BATS	57.49	56.6	1.22	17.6	0.23	31.2	42.11	1.54	1.41	0.12
BG	60.35	106.6	2	9.85	0.31	23.63	53.42	1.33	1.75	0.16
AAL	101.11	74.7	0.8	24.27	0.4	40.78	34.34	0.47	7.15	0.26
BLT	127.11	192.11	1.88	14.55	0.4	57.36	205.13	4.33	2.87	0.25
BARC	120.55	634.56	5.65	4.26	0.47	58.73	394.21	3.37	2.05	0.4
TSCO	69.5	267.73	4.38	3.98	0.23	29.43	146.74	7.46	0.45	0.15
XTA	103.63	109.44	1.1	22.01	0.46	46.8	71.84	0.55	12.7	0.29
DGE	52.14	89.05	1.96	9.78	0.23	24.15	54.44	1.81	0.89	0.13
LLOY	91	608.85	6.69	3.2	0.48	55.39	635.52	5.02	2.03	0.56
STAN	62.48	73.21	1.32	14.16	0.38	28.8	48.9	1.15	3.16	0.46
ULVR	46.9	48.26	1.22	15.67	0.23	22.79	30.71	1.05	1.54	0.12
RB	48.08	21.34	0.48	27.51	0.21	21.83	10.67	0.3	1.75	0.12
SAB	39.6	42.7	1.2	12.35	0.26	17.78	23.7	0.94	1.93	0.13
NG	40.46	85.18	2.23	6.9	0.22	16.79	58.88	1.76	0.83	0.12
IMT	46.55	32.3	0.87	19.93	0.24	24.36	25.76	2.33	3.19	0.12
BT	47.54	361.45	7.7	2.06	0.27	20.17	223.94	4.69	0.86	0.13
AV	47.16	108.36	2.29	5.42	0.36	18.87	58.36	0.86	1.83	0.25
PRU	52.35	135.14	2.78	5.73	0.37	22.47	80.35	2.25	1.51	0.26
BA	51.2	161.71	3.42	4.1	0.26	19.27	107.84	2.28	0.6	0.14
CNA	40.88	161.84	4.24	3.07	0.24	17.59	95.18	3.21	0.55	0.12
SSE	34.69	32.92	1.03	13.37	0.23	13.64	21.1	0.75	1.81	0.11
CBRY	34.95	90.49	2.9	6.1	0.25	15.92	86.43	2.7	0.79	0.11
BSY	33.47	84.33	3.18	5.36	0.24	16.29	286.66	17.84	0.83	0.13

Table A4 - continued

	Mean					Standard Deviation				
	Trades (N)	Volume (V)	Trade Size (Q)	Price (P)	Volatility (σ)	Trades (N)	Volume (V)	Trade Size (Q)	Price (P)	Volatility (σ)
EMG	40.24	121.94	3.23	4.37	0.36	17.61	89.22	2.43	1.49	0.23
RR	37.63	99.01	2.79	4.26	0.28	14.19	52.99	1.67	0.83	0.22
MRW	33.91	142.24	4.38	2.78	0.25	14.04	94.88	2.75	0.23	0.12
MKS	42.75	134.41	3.12	4.3	0.3	19.58	91.77	1.47	1.72	0.18
SBRY	31.37	117.37	4.48	3.94	0.26	16.86	164.78	7.75	1.02	0.15
WPP	40.26	76.18	2.08	5.65	0.27	17.94	36.35	1.17	1.3	0.14
REL	38.68	68.41	2.11	5.63	0.24	19.58	42.45	1.82	0.68	0.13
LGEN	34.68	292.25	8.9	1.06	0.38	15.21	178.96	5.87	0.38	0.25
CPG	36.32	114.25	3.52	3.38	0.26	14.59	79.56	3.42	0.35	0.14
ABF	19.83	19.42	1.05	8.01	0.2	7.61	12.47	0.77	0.83	0.1
LAND	36.94	34.4	0.96	12.51	0.3	13.99	16.75	0.35	5.55	0.14
OML	33.84	221.69	7.55	1.16	0.39	13.93	112.18	8.07	0.45	0.31
ANTO	36.48	49.37	1.44	6.24	0.43	12.46	26.39	0.94	1.51	0.26
PSON	31.97	44.46	1.48	7.2	0.24	13.15	25.57	0.98	0.9	0.16
SHP	30.4	35.69	1.26	10.1	0.24	13.48	27.29	1.33	1.43	0.11
SL	20.3	55.33	3.01	2.44	0.35	9.5	36.23	2.78	0.49	0.25
IPR	33.66	88.76	2.81	3.51	0.26	13.9	47.3	1.52	0.86	0.14
KAZ	33.4	32.69	1.05	10.49	0.46	13.32	29.48	1.02	4.44	0.3
UU	31.08	42.65	1.44	6.38	0.21	12.39	25.21	0.88	1.11	0.11
SN	28.57	48.73	1.84	5.62	0.24	13.12	28.91	1.32	0.66	0.42
EXPN	25.14	53.24	2.27	4.75	0.27	11.68	39.91	1.74	0.88	0.15
BLND	40.68	50.88	1.3	8.42	0.31	15.72	23.65	0.54	3.97	0.15
VED	38.28	32.46	0.87	16.1	0.43	15.56	182.57	3.44	5.79	0.22
RSA	28.31	170.53	6.41	1.39	0.29	13.19	118.1	4.11	0.14	0.17
CPI	23.98	26.9	1.32	6.94	0.23	12.09	20.13	1.64	0.38	0.13
KGF	40.5	224.6	6.68	1.79	0.33	18.67	159.57	12.34	0.53	0.18
CCL	32.7	15.71	0.5	19.8	0.28	13.61	9.25	0.31	3.79	0.14
CW	30.93	179.48	6.42	1.6	0.25	14.15	130.85	7.4	0.19	0.12
SMIN	23.71	24.97	1.18	9.52	0.24	9.2	21.34	1.9	1.41	0.12
LII	24.9	22.55	0.94	8.28	0.29	9.7	11.91	0.51	3.24	0.15
NXT	36.81	26.13	0.7	15.73	0.31	15.83	15.32	0.33	4.24	0.19
JMAT	24.5	12.25	0.52	14.97	0.27	10.1	7.5	0.29	3.17	0.17
BAY	40.36	141.37	3.49	2.8	0.37	17.28	77.17	1.46	1.34	0.23
IAP	25.83	39.92	1.74	4.69	0.33	13.87	26.92	1.39	1.17	0.25
SVT	24.14	14.14	0.63	12.78	0.22	10.95	8.25	0.44	1.89	0.11
HMSO	27.2	29.94	1.17	8.77	0.33	10.57	16.66	0.8	4.47	0.19
SGE	22.26	69.76	3.6	2.14	0.25	9.17	45.86	5.73	0.31	0.12
REX	22.34	41.43	1.94	4.01	0.26	9.33	28.1	1.25	1.01	0.14
IHG	24.48	28.1	1.16	8.47	0.3	9.71	21.7	1.05	2.56	0.26

Table A5

Proportionality between Trade Count and Trading Activity using 5-minutes intraday averages

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity and different groups of stocks as defined in section 5.2, using intraday averages as defined in equation (24). The results in panels (A) through (E) using intraday averages, have a direct correspondence to those previously reported in Tables (3) through (7), which use across day averages as defined in equation (23). The trading activity measures are from Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4). All specifications include both stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta_1 = 2/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Proportionality between Trade Counts and Trading Activity by Market Cap Groups

	Model 1	Model 2	Model 3	Model 4
Constant , a	0.6989***	1.6860***	4.4929***	3.5299***
	(0.0225)	(0.0258)	(0.0303)	(0.0239)
Invariance Coeff., β	0.4564***	0.3739***	0.36102***	0.4467***
	(0.0213)	(0.0351)	(0.0603)	(0.0588)
$MKT_2 \times w_{it}$	-0.0017	0.0174	0.0472	0.0308
	(0.0213)	(0.0274)	(0.0421)	(0.0450)
$MKT_3 \times w_{it}$	0.0169	0.0341	0.0480	0.0328
	(0.0213)	(0.0275)	(0.0424)	(0.0441)
R^2	0.9329	0.9050	0.8942	0.9218

Panel B: Proportionality between Trade Counts and Trading Activity by Market Cap Contribution (PropMcap) Group

	Model 1	Model 2	Model 3	Model 4
Constant , a	0.7171***	1.6942***	4.5521***	3.5553***
	(0.0217)	(0.0254)	(0.0293)	(0.0236)
Invariance Coeff., β	0.3874***	0.3555***	0.4332***	0.5060***
	(0.0307)	(0.0343)	(0.0283)	(0.0277)
$PropMKT_2 \times w_{it}$	0.0701*	-0.0030	-0.1087*	-0.0956*
	(0.0329)	(0.0428)	(0.0452)	(0.0473)
$PropMKT_3 \times w_{it}$	0.0816**	0.0521	-0.0143	-0.0179
	(0.0292)	(0.0308)	(0.0155)	(0.0178)
R^2	0.9334	0.9057	0.8961	0.9231

Panel C: Proportionality between Trade Counts and Trading Activity by Trade Volume Group				
	Model 1	Model 2	Model 3	Model 4
Constant , a	0.6762***	1.6832***	4.5797***	3.5823***
	(0.0215)	(0.0251)	(0.0290)	(0.0233)
Invariance Coeff., β	0.3953***	0.3052***	0.2995***	0.3752***
	(0.0250)	(0.0343)	(0.0658)	(0.0621)
$TVol \times w_{it}$	0.0857***	0.1149***	0.1250*	0.1249*
	(0.0214)	(0.0282)	(0.0538)	(0.0513)
R^2	0.9345	0.9081	0.8971	0.9249

Panel D: Proportionality between Trade Counts and Trading Activity by Trade Count Group				
	Model 1	Model 2	Model 3	Model 4
Constant , a	0.7206***	1.6957***	4.5583***	3.5570***
	(0.0217)	(0.0252)	(0.0290)	(0.0235)
Invariance Coeff., β	0.4232***	0.3187***	0.3070***	0.3921***
	(0.0272)	(0.0391)	(0.0667)	(0.0686)
$NTrades \times w_{it}$	0.0465	0.0941**	0.1134*	0.0978
	(0.0244)	(0.0331)	(0.0538)	(0.0295)
R^2	0.9332	0.9069	0.8966	0.9235

Panel E: Proportionality between Trade Counts and Trading Activity by Trade Size Group				
	Model 1	Model 2	Model 3	Model 4
Constant , a	0.6467***	1.6757***	4.6751***	3.3603***
	(0.0213)	(0.0249)	(0.0286)	(0.0229)
Invariance Coeff., β	0.3834***	0.2952***	0.2761***	0.3513***
	(0.0216)	(0.0297)	(0.0519)	(0.0482)
$TSize \times w_{it}$	0.1068***	0.1328***	0.1651***	0.1665***
	(0.0186)	(0.0247)	(0.0416)	(0.0390)
R^2	0.9357	0.9095	0.9001	0.9278

CHAPTER 4

Invariance of trades and LSE intraday patterns

Abstract

We investigate market microstructure invariance relationships for each individual stock in FTSE 100 index using four different notions of trading activity. When averages across days are used, the notion of trading activity that implies proportionality between trading volume and returns variance suggests that number of trades are proportional to the respective trading activity in the power of 2/3 for most stocks. The invariance model accurately predicts a different proportionality between the log values of trade counts and trading activity of 1/2 value. Intraday trading patterns in the specific market, the magnitude of trade size and its correlation with the volatility partly explain this value.

4.1 Introduction

In this chapter we further examine empirically invariance principles on trades for FTSE 100 stocks traded on London Stock Exchange using the same dataset as in Chapter 3. Specifically, in this chapter we extend the work in Chapter 3 by investigating whether FTSE 100 stocks individually exhibit any common proportionality between trade counts and trading activity based on the generalised invariance model we develop in chapter 3. Our contribution is threefold. First, we test which of four different notion of trading activity and their respective models specified on invariance terms better describes the intraday patterns of LSE and microstructure properties of our dataset and predicts more accurately common invariance proportionality across FTSE 100 stocks. The definitions of trading activity we use are those introduced by Clark (1973), Ané and Geman (2000), Kyle and Obizhaeva (2016b) and Andersen et al. (2016), which we outline in chapter 3. Second, we split the sample into smaller time periods based on the end of each of the three substantive years in the period under analysis. This enables us to do two things: first, to distinguish proportionality relationships existing at the beginning, during, and at the end of the 2007-08 financial crisis; second, we can now account for how the introduction of alternative trading platforms for LSE traded stocks affects the measured invariance relationships. Given that market microstructure invariance argues that invariance principles hold across time and assets, our goal is to investigate whether invariance predictions continue to hold in different subsamples for all stocks, or if they do not hold, the extent to which the invariance coefficient estimates change for different stocks. In this sense, the choice of the specific subsamples is intuitive, and aims at capturing any potential effects of the financial crisis or changes in the trading patterns for LSE stocks. Finally, given the intraday patterns of LSE, we provide evidence on how the exclusion of minutes with extreme volatility, those manifesting increased trading volume, trade counts and trade size, as well as a positive correlation between trade size and volatility in business time, affect the invariance coefficients estimated by different models, each of which is based on distinct notion of trading activity.

Our principal empirical findings are as follows. The LSE is characterised by extreme realised volatility at the opening. This may indicate a less efficient price discovery process due to less homogeneous beliefs regarding the direction of prices. Also, during the first minutes of active trading, trade size appears to be higher. At the closing of active trading, increases in trading volume, trade counts and realised volatility reveal an attempt by traders to unwind their positions and/or rebalance their portfolios. When averages across days are used and all minutes of active trading are included, the notion of trading activity that implies a proportionality between trading volume and returns variance more accurately predicts the stipulated 2/3 proportionality between the number of trades and trading activity for 70% of the stocks, except those with high on average volatility. This result is consistent even if the minutes that are characterised by extreme volatility and increases in trading volume, trade counts and trade size are excluded. The use of intraday averages leads to lower estimates of proportionality for all notions of trading activity, likely due to measurement errors and sampling variation which bias the coefficients estimates.

However, we find that the invariance models predict a 1/2 proportionality for 86% of the stocks. These differences in the estimated proportionalities across models may be partly driven by the magnitude of trade size which has a different impact on the models we investigate. Based on our extended invariance model, the 1/2 proportionality value implies that bets in the futures markets are larger than bets in stocks and thus are shredded into more pieces assuming that on average bets shredded into same size trades in these two markets. Results also suggest that the specific proportionality is partly driven by the positive correlation between trade size and volatility in business time in the first/last 10 minutes and that orders are shredded more in the period between the first/last 10 minutes of active trading. When we examine this invariance relationship for each year in our sample, we find that invariance model does not indicate a unified order flow pattern for all stocks, and that stock and/or industry specific characteristics are important for traders when they choose the order size or shred their orders. The results for pre-crisis and in-crisis periods are similar, especially for those stocks that belong on the top of FTSE 100 in terms of market capitalisation. Stocks for which the 1/2 proportionality does not hold in any of the two periods are characterised by the highest or lowest GBP trade size during the pre-crisis period. The direction and magnitude of changes in the underlying variables appears to be critical in determining whether the proportionality will differ between pre-crisis and in-crisis periods.

The remainder of the chapter proceeds as follows. Section 2 reviews the intraday patterns of London Stock Exchange. Section 3 explains the methodology and the main and alternative empirical hypotheses. Section 4 concerns with the data and descriptive statistics regarding the underlying variables. Section presents and discusses the main empirical results. Section 7 focuses on the empirical results in relation to smaller sub-periods. Section 7 concludes.

4.2 Intraday patterns of the London Stock Exchange

In order to investigate the invariance relationships between the number of trades and trading activity for each individual stock in high frequencies, it is important to analyse the intraday patterns of the UK equity market and more specifically that of London Stock Exchange. Understanding the way trading takes place on the LSE and the extent to which intraday patterns affect the behaviour of the underlying variables in our model is crucial in explaining the individual invariance coefficients. In the following paragraphs, we discuss the changes that LSE has undergone in terms market organisation, the pre- and post-trading services offered, and LSE rules regarding market participants.

4.2.1 During 1980s[36]

The first steps towards the new era of trading on the LSE began in the early 1980s. During this period, the LSE undergoes a series of reforms. One major change is the introduction of the Unlisted Securities Market (USM)[37] in November 1981. The aim of the USM is to provide small sized UK companies with a market where their securities can trade. Until that point LSE had been unwilling to grant full quotation on the main market to this type of companies, due to their size, associated business risk and absence of an acceptable earnings record. As a result, new and small companies struggled to raise equity capital to finance their business and growth, and mergers or acquisitions by larger companies are often the only viable solution. The USM, despite its inherent risks[38], offers a trading forum to these companies, contributing significantly to the growth in the British economy. Another major change is the design of a new index in cooperation with the Financial Times, which would replace the FT Index. The new index, known today as FTSE 100, started operating in January 1984 (Michie, 1999).

Although these two major changes improve the status of LSE, they are not enough to cope with the impending aftermath of the suppression of fixed commissions and single capacity, and the introduction of statutory financial regulation (SROs), known as “Big Bang”. A new trading system that would ensure functionality of the market after the “Big Bang” was necessary. By 21 October 1986, along with the information network, TOPIC, and the settlement system, TALISMAN, a new screen-based trading system, named SEAQ (Stock

[36] Michie (1999) provides an excellent review of the history of LSE from its opening until the 2000s.
[37] USM belongs to the main market, although defined as separate part thereof.
[38] Due to limited turnover, a few investors could control new issuance and the market could easily be manipulated.

Exchange Automated Quotation), became ready for operation, imitating the one used by NASDAQ. TOPIC was used as the base for this new system, which was qualified to manage 8 or 9 transactions a second, slightly slower than NASDAQ. At the time of the "Big Bang" date, 27th of October 1986, SEAQ operated smoothly. However, TOPIC fails twice due to overloading. The success of SEAQ at this point is such that gradually trading shifts from the traditional trading-floor (Michie, 1999).

Despite its similarities with the NASDAQ trading system, SEAQ has several exclusive features. First, market makers are required to post two-sided quotes during the Mandatory Quote Period (MQP), for the securities that they are registered to trade. They also have to guarantee to deal at these quotes with all members, and fully disclose their transactions (Abhyankar et al., 1997, Demarchi and Foucault, 2000). These prices are binding for a Nominal Market Size (NMS), decided by the LSE[39]. Under the new system market makers qualify for stamp duty relief, short-selling accommodation and access to the Inter-Dealer Brokers' system (IDB). Second, at least until early 1990s, institutional investors dominate trading on the LSE[40], contrasting with the more retail-based market of NASDAQ. Consequently, the predominant characteristic of order-flow on LSE is the fact that large sparse trades occur on only one side of the market[41]. Finally, the trading system in LSE includes two periods outside MQP, one before its opening and one after its closing. In contrast, the trading system in NASDAQ does not allow for such periods (Abhyankar et al., 1997)

4.2.2 During 1990s

In the years following Big Bang, the LSE has to face unanticipated challenges and solve emerging problems regarding its trading that the initial success of SEAQ never foreshadows. In the established new regime, transactions settlement proved to be a major issue for the stock exchange, with the existing paperless settlement system, TALISMAN, being unable to handle the overwhelming volume of business[42]. Also, by the end of 1987, the quality of price information SEAQ is delivering appeared to be below requisite market standards. Given that the electronic market is becoming an integral part of securities trading on London, both SEAQ

[39] Abhyankar et al. (1997) report that in 1991 the NMP depended on customer turnover in each stock over the previous twelve months, expressed in a number of shares, and ranged from 500 to 200,000 shares

[40] Abhyankar et al. (1997) report that around 60% of trades came from institutional investors.

[41] However, given that both trading systems are quote-driven, market makers in both stock exchanges come up against a more fragmented order flow compared to that in a specialist system (e.g. that of NYSE)

[42] As Miclie (1999) states, until then time settlement takes place only after the end of the fortnightly account and not after the transaction actually occurs. As a result, the amount of transactions that are not cleared is predicted to be between £3 to £4 billion in August 1987

and TALISMAN need to be upgraded. In this new financial environment, LSE gradually starts to lose its crucial role in the British financial system, while it has to cope with the escalating competition from other stock exchanges. The attempt to exert further control over the securities market by both the governments and central banks make things even more complicated. In the UK, the statutory bodies responsible for regulating LSE seek to expand their influence to all its operational features. During the 1990s all the provisional rules and member privileges that grant LSE a competitive advantage gradually cease to exist. In addition, one by one, the functions (both regulatory and trading related) that the stock exchange historically performs are being undertaken by other bodies, forcefully modified, or sold. As a result, the responsibility of the LSE is confined to an arguably[43] difficult task, namely supplying an organised market, regulated by rules imposed by external bodies in order to minimise trading risks (Michie, 1999).

The most tangible examples of these changes are those concerned with two main income resources of LSE: information and settlement services. The former are provided, as mentioned, through TOPIC and the latter through TALISMAN. The increasing competition in the information gathering and provision landscape, as well as the considerable investments by other electronic providers, obligates the stock exchange to sell TOPIC to REUTERS in December 1994 and offload its trade confirmation service, SEQUAL. The situation is quite different with respect to settlement services, which LSE is forced to relinquish, not because of competition, but rather because of its failure to deliver the required standards. After the early 1980s, the stock exchange is already attempting to replace TALISMAN with TAURUS, a better settlement system. This undertaking stops in 1984 due to resistance from external bodies that provide the same services. The replacement activity resumes in 1987, but further delays and the failure of the LSE to design a settlement system that allows for the new trading conditions, leads to the project being dropped in 1993. Instead, a 10-days rolling settlement system is initiated in 1994 and then replaced by a 5-days rolling settlement system in 1995; however, there is still the need for a more modern settlement system. To satisfy this need, the Bank of England launches a new system, CREST, in 1996, which can manage up to 170,000 transactions a day. The new system replaces TALISMAN by the third quarter of 1997 (Michie, 1999).

[43] Trading was changing, the speed of transactions became higher and the magnitude of turnover greater

The loss of these services, although reducing the revenues of stock exchange at this date, creates a great opportunity: LSE could now focus on the critical role of providing an organized international securities market. The existing quote-driven trading system, however, is not adequate for this endeavour and needs to be replaced by an order-driven system in which orders could be executed and matched electronically via a central computer without further intermediation. The necessity for this change becomes apparent after the crash of 1987, when the disadvantages of the system in place are uncovered. For example, during Black Monday the less actively traded stocks are not as marketable as before and thus with the declining turnover, market makers cannot post close bid and ask quotes. The widening of bid-ask spreads[44] disheartens investors from trading in these stocks leading to a vicious circle. Similarly, market-makers are more often picked off in highly actively traded stocks. As a result, many major companies either abandon market-making or decide to trade only in a sub-set of stocks that they consider less risky or quote even wider spreads, actions that compound the situation (Michie, 1999).

Except for the obvious need to introduce better rules and regulations[45], as well as an upgraded electronic trading system, LSE has to find the right balance between the needs and requirements of market makers and those of brokers/dealers and other market participants. On the one hand, the idea of a specialist system is out of the question because that would favour only the market makers. On the other hand, an order-driven system, despite certain obvious benefits, will potentially entail the risk of less liquid markets, and thus risk depriving the LSE of an important competitive advantage. However, externally, the increasing domestic competition, due to the new regulatory framework and technological convergence, and internally, the realisation that market-making in many securities simultaneously can be costly, open the route for the switch to an order-driven system. At the beginning of the fourth quarter of 1997, the new order-driven trading system, SETS (Stock Exchange Trading Service), is introduced, replacing SEAQ for all FTSE 100 stocks, as well as roughly 30 more stocks (Michie, 1999, Demarchi and Foucault, 2000).

The change from a dealer's market to an order-driven one is anything but smooth. Initially, only a third of all trades execute through SETS, while the majority still trade with the market makers and/or the Retail Services Providers to avoid the abnormal prices observed at the

[44] Miclie (1999) discloses that the bid-ask spreads before 1987 were around 3%, whereas in 1990 they fluctuate between 1000 to 2000 bps.

[45] Demanded not only by the new market conditions, but also by government bodies (e.g. the Office of Fair Trading) that want to minimise the possibility of anti-competitive practices

opening and closing of the trading day in SETS (Michie, 1999). Demarchi and Foucault (2000) highlight that until the second half of 1998, only orders of medium size are executed via SETS (less than 10 NMS[46]). Indicative of this picture is the fact that SETS accounts for almost 70% of small-sized, and approximately 50% of medium-sized transactions, while transactions greater than 20 NMS generally take place outside SETS (Board and Wells, 1998). That intrinsic market fragmentation in a sense handicaps the liquidity of SETS. To deal with the situation, in late 1990s, LSE increases the NMS to 20, initiated a new way of computing the closing price[47] and modifies the structure of the call auction that takes place before the opening of the trading day. In the new setting, all registered members of LSE have the option to trade outside the central limit order book (Demarchi and Foucault, 2000). Despite these drawbacks, SETS offers market participants the option of splitting their orders swiftly, improved competition for smaller and medium trades and providing lower trading costs and more pre-trade transparency (Gemmill, 1998, Michie, 1999, Demarchi and Foucault, 2000).

Progressively, the stock exchange begins to focuses on its core functions aiming to drop all the remaining services which other bodies can provide more efficiently, are outside its current scope, or in which it has failed to succeed. Nevertheless, its attempt to shut down the USM in 1993[48] generates much opposition, not only from market participants, but also from the UK government. As a result, the stock exchange replaces USM with AIM (Alternative Investment Market) in June 1995. This new market for small companies inherits the risks of the previous one, mainly because LSE cannot apply the same set of rules and regulations that it has in place for large companies. Despite the continuous criticism by the government and media, as well as the competition from similar markets for small companies, such as OFEX and EASDAQ, AIM gradually became a successful trading forum where smaller companies can raise funds cheaply and investors can trade in potential growth stocks (Michie, 1999).

[46] Nominal Market Size (NMS) is measured in shares and varies depending on the liquidity of each stock. Initially it represents at least 2% of the average trading volume in each specific stock. With the introduction of MiFID I the NMS stops being controlled by LSE and is set at an EU level. According to ESMA (2014, p. 51), "under MiFID I, the average daily turnover (ADT) is used to determine when an order should be considered to be large in scale compared to normal market size. The ADT is calculated by dividing the yearly turnover by the number of trading days and this calculation is made for each share on an annual basis. The shares are grouped within five different classes and the result of the annual ADT calculation determines whether the share should be reclassified and moved to another class. The higher the ADT, the higher the minimum threshold for the large in scale waiver is".

[47] "A weighted average of the transaction prices in the last 10 minutes of trading day" (Demarchi and Foucault, 2000, p. 83). Now, LSE defines the closing price as the last price at the time the market closes (based on the results of the closing auction or the midprice of the best bid and offer prices at the time the market closes).

[48] Initially LSE tries to open the market to more small companies, relaxing the rules for a full quotation, but this attempt is not successful.

4.2.3 From 2000s until the present

Entering the millennium, LSE starts to establish its dominance in the UK stock market, improve its current services and expand operations in other financial products such as ETFs. The first decade of the 21st century is characterised by several unique features. First, fixed income securities, currency and commodities gradually become popular among investors. Second, the emergence of private electronic platforms and OTC derivatives trading make it more difficult for the traditional exchanges to generate earnings (Augar, 2016). Finally, the introduction of algorithmic trading underlines the need for new regulations and for a considerable upgrade in the electronic trading systems of the stock exchanges, to ensure market stability and transparency, offer speed to the new types of trader, and protect slower traders from being picked-off. For example, LSE introduces circuit breakers to restrain the illiquidity at fixed intervals and the automatic execution of trades at any price. This allows for a cooling-off period and help market participants to cope with the unexplained volatility that caused uncertainty (Foresight, 2012).

Around the world major stock exchanges began merging with each other in an attempt to offer better services and win a greater market share. In order for London Stock Exchange to remain in a leading position, actions are required, not only in terms of new products, services and customers, but also investment in new technologies. To cover the first need, LSE merged with Borsa Italiana in the fourth quarter of 2007, establishing the LSE Group, with the goal of diversifying the provision of its products and its customer base (Willey, 2007)[49]. The following years found LSE participating in several mergers, acquisitions and joint ventures with other stock exchanges to strengthen its position. Most important was the acquisition of 60% stake in Turquoise in 2009. In addition, the stock exchange constantly improved the services and trading venues provided to market participants. Table 1 summarises the characteristics of each offered service to date and Figure 1 presents the current trading process in SETS, the main limit order book of LSE.

Except the stock exchange's expansion in size and services, the relaxation of restrictions and other changes in the trading landscape after the introduction of MiFID I, along with the growing trend of algorithmic trading and hedge funds, created another need. For LSE to remain competitive, a technologically advanced electronic trading system that would offer speed in trading, facilitate the execution of trades and increase system capacity was is crucial.

[49] LSE had previously rejected takeover by NASDAQ

Accordingly, LSE launches TradElect in 2007, a trading system capable of trading in 10ms on average, as compared to the previous 140ms and handling 3000 trades per second (MacDonald, 2007). For a year the new Microsoft-based system performs efficiently, with a real reduction in latency up to 6ms and a significant increase in the number of trades per day. For these reasons, the LSE plans to allow more stocks to trade via the new system, to increase its capacity in terms of messages per second, and improve connectivity and functionality (LSE, 2008).

However, the unanticipated crash of TredElect for more than 7 hours in September 2008, in a day during which high volumes are expected, casts shadows over the previous proven ability of the trading system. Despite attempts to initiate improvements, LSE recognises that the performance of TredElect, given its operating costs, is not as high as expected. Despite the initial thoughts to design a new in-house built trading system, LSE decides to replace TredElect with Millennium Exchange, a Linux-based system, after the acquisition of the company that designed it in 2009. By 2011, Millennium Exchange fully replaces TredElect, offering members ultra-low latency, superior functionality and flexibility. This is the trading system which LSE currently uses, while it has sold its technology to other stock exchanges around the world. LSE continues to be one of the world's leading stock exchanges, constantly introducing new venues and upgrading its services to provide market participants with unrivalled speed, transparency and flexibility.

[Table 1 in here]

[Figure 1 in here]

4.3 Methodology

This chapter incorporates the invariance model for trades we introduce in Chapter 3 as an extension of intraday trading invariance (ITI) model introduced by Andersen et al. (2018) and motivated by the market microstructure invariance (MMI) theory for bets suggested by Kyle and Obizhaeva (2016b). This is described in equation (10). The empirical methodology is also quite similar to the previous chapter. Specifically, to account for the common noise that the variables measuring expected trading σ_t, N_t and V_t have in high frequencies, we aggregate the logarithms of 5-minutes observations for these variables and average across days based on equation (23) and intraday based on the equation in (24)[50] in a similar fashion to the previous chapter. In order to investigate the empirical proportionality between the number of trades and trading activity for the individual stocks in our sample, we employ the same four different notions of trading activity as in Chapter 3, namely: those suggested by Kyle and Obizhaeva (2016b), Andersen et al. (2016), Clark (1973) and Ané and Geman (2000) with the respective models given in (26), (27) and (28).

Following Andersen et al. (2018) and the respective restrictions for ITI in this chapter we test the following null and alternative hypotheses:

$H_0 : \beta = 2/3$, $H_1 : \beta \neq 2/3$

The models relating to the various notions of trading activity on which our analysis is based take the following form:

Mode1 (Kyle and Obizhaeva (2016b): $n_{jk} := c + \frac{2}{3} w_{jk} + u^n_{jk}$ **(32)**

Model 2 (Andersen et al. (2016): $n_{jk} := c + \frac{2}{3} w_{a,jk} + u^n_{jk}$ **(33)**

Model 3 Clark (1973): $n_{jk} = c + \frac{2}{3}\left(w_{jk} - \frac{3}{2} q_{jk} \right) + u^n_{jk}$ **(34)**

Model 4 Ané and Geman (2000): $n_{jk} = c + \frac{2}{3}\left(w_{jk} - q_{jk} \right) + u^n_{jk}$ **(35)**

In contrast to Chapter 3, where we estimate an average proportionality between the number of trades and trading activity, while controlling for any stock and time fixed effects with panel regressions, in this chapter we focus on the proportionality between number of trades and trading activity for individual stocks. We use OLS regressions for each stock in our sample, based on the four models presented above. First, we investigate whether the stipulated 2/3 proportionality between the number of trades and trading activity holds in the FTSE 100 equity market. Second, we seek to determine which of the four models best captures the market microstructure properties in the specific market context. We do that by employing both

[50] Following previous work on invariance in high frequencies we calculate the realised volatility σ_{jdt} from 10-second returns

averages across days, based on equation (23), and intraday averages based on equation (24), as estimators for the underlying variables.

4.4 Data and Descriptive statistics

In this section, we discuss the intraday innovations in realized volatility, σ, trading volume, V, number of trades, N, and average trade size, Q. These variables, which are pivotal components of the invariance model, are influenced by the unique features of the LSE stock market during the period under analysis. As already explained, understanding these characteristics and their potential impact on the invariance relationships is important in interpreting any proportionality between the number of trades and trading activity.

This chapter uses the same dataset as in Chapter 3. Specifically, time-stamped tick data is obtained from Thomson Reuters Tick History for 70 stocks trading on the London Stock Exchange and also listed in the FTSE 100 (Appendix-Table A1). The dataset includes tick-by-tick information on the best available bid and ask quotes, transaction prices, and trading volume (in shares), for 3 years between 1st January 2007 and 31st December 2009. We select only the stocks that remain constituents of the FTSE 100 throughout the sample period. Our selection procedure ensures that we eliminate any potential survivorship bias that may impact the results. To reveal any potential impact of market capitalisation on invariance, we classify the 70 stocks which remain constituents of the FTSE 100 throughout the sample period, into 3 groups on the basis of their market capitalisations[51]. Each group accounts for approximately 33% of the total 3-years market capitalization average[52].

We consider trades for these stocks during the time span the London Stock Exchange is open for continuous trading, namely from 8am to 4.30pm, Monday to Friday. We exclude from the sample the 30 days that correspond to holidays or other days with reduced trading activity arising from reduced trading hours. This leads to a total of 754 trading days. Each trading day is further divided into 102, 5-minute intervals. In each of these intervals, we aggregate the observations for trading volume V, number of trades N and average trade size Q so that we

[51] We estimate the average market capitalization for each stocks using monthly values from LSPD database as provided by WRDS

[52] In Chapter 3 we use also an extra classification based on market capitalisation, ranking stocks based on their market capitalisation and then creating 3 groups of stocks with similar number of stocks in each group. As their results for average proportionality between the number of trades and trading activity are quite similar between the two different approaches of grouping stocks based on market capitalisation, here we only report one of them.

estimate their five-minute values. The realised return volatility σ for each 5-minutes interval is computed from 10-second returns following Andersen et al. (2016).

4.4.1 Intraday Volatility

Various financial papers document a U-Shaped intraday pattern of returns volatility in stock markets. Wood et al. (1985), Jain and Joh (1988) and McInish and Wood (1990) examine the NYSE market and show that returns volatility of stocks traded follow a U-Shaped pattern. Brock and Kleidon (1992) and Foster and Viswanathan (1993) also find the same pattern in returns volatility of NYSE stocks. Stoll and Whaley (1990) confirm that volatility is higher at the opening of NYSE market in all common stocks traded and argue that it is more pronounced for high-volume stocks. However, they assert that the prices observed during the first minutes of trading tend to reverse later in the day. Madhavan et al. (1997) discover the same U-Shaped pattern in returns volatility of NYSE stocks, although the volatility of ask prices tends to decrease throughout the day.

Lockwood and Linn (1990) provide evidence that the variances of hourly intraday returns in the Dow Jones Industrial Average decline from the opening till early afternoon and increase after that point. Hamao and Hasbrouck (1995) state that returns volatility has a U-shape in Tokyo Stock Exchange traded stocks. However, George and Hwang (1995) explain that returns volatility is only high for a small number of high actively traded stocks, while Chang et al. (1993) do not record high returns volatility in the opening minutes. Choe and Shin (1993) claim that returns volatility is higher in the morning and afternoon period than in the closing minutes of the same market. In addition, Chan et al. (1995) prove that returns volatility in NASDAQ stocks follows a U-Shaped pattern. Werner and Kleidon (1996), Abhyankar et al. (1997) and Cai et al. (2004) also observe U-shaped pattern in the returns volatility of LSE stocks. Yet, Andersen et al. (2000) discover two independent U-shaped patterns in the returns volatility of Nikkei 225 index that can be connected to information asymmetry around the opening and closing of the market in the morning and afternoon.

The fact that no complete consensus exists in the literature is demonstrated by the number of papers that document that returns volatility is heavier at the opening of the stock exchanges than the closing, so that its pattern resembles more an L-shape rather than U-Shape. For example, Fleming and Remolona (1999), Tian and Guo (2007), Eaves and Williams (2007), Pascual and Veredas (2009) and Glezakos et al. (2011) all support the L-shaped pattern in returns volatility of the markets they investigate. Similarly, other studies report that returns volatility follows a reverse J-shape pattern which is similar to the L-shaped one. For instance, Ozenbas (2006) argues that this pattern is present not only in NYSE and NASDAQ, but also in LSE, Deutsche Boerse and Euronext, Hussain (2011) finds the same for DAX between 200nd 2005, whereas Harju and Hussain (2011) shows that this patters in returns volatility exists in FTSE 100, XDAX30, SMI and CAC40.

In this chapter, we estimate realised returns volatility as the sum of squared returns over 10s for every 5-minutes interval over the trading day for all 70 FTSE 100 stocks. We then average the volatility estimate for each 5-minute interval across all 754 trading days, so that we obtain 102 five minutes realised volatility observations for each stock. Returns are calculated by linearly interpolating from the average of two closest log bid and log ask prices, as in Andersen et al. (2001).

Graph 1 presents the estimated intraday realized volatilities for all stocks in the sample. Returns volatility for the majority of the stocks follows an L-shaped/reverse J-shaped pattern and not a U-shape. This confirms with the conclusions of Werner and Kleidon (1996), Abhyankar et al. (1997) and Cai et al. (2004) that all examine the same market. Volatility is higher at the opening and decreases gradually throughout the day. Spikes during US macroeconomic announcements at 13.30 and after the opening of US stock market at 15:00 are apparent for all stocks[53]. A possible explanation is that price discovery is less efficient at the morning opening because during this period there are more heterogeneous opinions regarding the direction of prices after the preceding non-trading hours. In a sense, this reflects the more pronounced difference in the way market participants interpret the aggregated overnight information and subsequently incorporate the information into their quoted prices. As far as the closing minutes are concerned, the spike in returns volatility can be attributed to market participants attempting to unwind their positions/rebalance their portfolios before the closing auction. Finally, the fact that returns volatility is not very high at the closing of the market, so that it exhibits a U-shaped pattern, likely suggests that TredElect, the electronic system (see the previous section), improves price discovery during the trading day, so prices remain less volatile than previous studies suggest.

[Graph 1 in here]

[53] Other spikes for some stocks are obvious at certain time intervals. This can be attributed to stock specific characteristics which are outside the scope of this chapter. Here we investigate the presence of a common pattern in the data that will help interpret any invariance proportionality.

4.4.2 Intraday Trading Volume

Several papers investigating the intraday patterns of trading activity variables contend that trading volume exhibits also a U-Shaped pattern. In papers analysing the NYSE, Jain and Joh (1988), Brock and Kleidon (1992), Gerety and Mulherin (1992) and Foster and Viswanathan (1993) all present evidence that trading volume (number of shares traded) exhibits a U-shape during trading day. According to Gerety and Mulherin (1992) overnight volatility is responsible for this pattern in trading volume of NYSE stocks, whereas as reported by Atkins and Basu (1995) public announcements after trading appear also to contribute to this pattern. Chan et al. (1995) provide evidence of a similar volume pattern on for NASDAQ stocks. In contrast, Werner and Kleidon (1996) and Abhyankar et al. (1997) document a two-humped pattern in trading volume, with peaks at 9:00am and 3:00pm, while the market is open for active trading from 8:30am to 4.30pm London time. Ellul et al. (2002) and Cai et al. (2004) display similar results for the pattern in the volume of stocks traded in SETS, with a spike around mid-morning and a growing volume trend after the opening of US markets until the closing of active trading.

Similar to the process we undertake for realised volatility, trading volume is averaged for each 5-minutes interval across all 754 trading days. Graph 2 depicts the pattern of trading volume for all 70 FTSE 100 socks in the sample. In contrast to Werner and Kleidon (1996), Abhyankar et al. (1997), Ellul et al. (2002) and Cai et al. (2004) that report a two-humped pattern, our analysis actually indicates a four-humped pattern in trading volume. Corroborating Ellul et al. (2002) and Cai et al. (2004), there is a spike around 10:15 that reflects market response to UK macroeconomic announcements such as average earnings, industrial production, producer price index, retail sales, retail price index and others[54]. This spike can also be attributed to the Exchange Delivery Settlement Price auction for FTSE 100 Index Options contracts and FTSE 100 Index Futures contracts that takes place between 10.10am-10.15 am[55]. In addition, trading volume increases steadily after 14:30, when the US stock market opens, until the close of active trading on the LSE. However, two more peaks are present in our dataset: the lowest occurs around 12:00 and the other around 13:30. The former can be connected to the opening of FOREX market trading in New York and the second to US macroeconomic announcements including GDP, retail sales, PPI and CPI, trade balance index of leading indicators and others (Andersen et al., 2003). The discrepancies in the documented

[54] Note that important macroeconomic announcement become available to journalists around 9.30 am
[55] More information about the specific auction is provide in Figure 1

pattern as compared to previous papers might stem from changes in the way trading takes place on the LSE. The period under analysis also differs from that in other papers in a meaningful way. Werner and Kleidon (1996) and Abhyankar et al. (1997) both analyse variables of trading activity in SEAQ, whereas Ellul et al. (2002) and Cai et al. (2004) look at SETS. As explained previously, after 2007 LSE replaced SETS with TradElect, which offered higher speed, increased flexibility and better functionality. In addition, these changes accommodate the emergence of algorithmic traders at the beginning of the 2000s. The accessibility and speed of the new trading system could possibly allow market participants to more quickly react to, and incorporate in their quotes, any new information arriving in the market around the two aforementioned spikes.

[Graph 2 in here]

4.4.3 Intraday Number of trades and trade size

Some literature contends that trade counts, the number of trades, also follows a U-Shaped pattern within the trading day. For example, Jain and Joh (1988), Chan et al. (1995) and Blau et al. (2009) suggest that there exists a U-Shaped pattern in the number of trades on both the NYSE and NASDAQ. In our analysis, we estimate trade counts for each 5-minute interval by averaging the corresponding observations across 754 trading days. It is apparent from Graph 3 that the number of trades for the 70 FTSE 100 stocks does not exhibit this U-shaped pattern, but the resulting pattern rather resembles that for trading volume, at least in the period after 12:00. For the majority of stocks, trade counts first accentuate in the five minutes after the opening of the market. The peak in the trading volume which occurs at around 10:15 is not accompanied by a similar spike in the number of trades. However, following the trading volume pattern, there exist two considerable spikes at 13.30 when US macroeconomic indicators are announced and at 14.30, when the US stock market opens. Graph 4 displays the trade size for 70 FTSE 100 as an average for each 5-minute interval across 754 trading days. Despite some individual spikes or bottoms during the trading day, which can be attributed to stock specific characteristics, trade size is high mainly at the opening and is somewhat less clearly elevated at the closing of the market. Nevertheless, the pattern of trade size, in conjunction with the patterns in the number of trades, volume and volatility yields some interesting observations.

First, it is obvious that for the majority of the stocks at the opening, market participants trade more shares with each order, despite the high returns volatility in the first minutes of trading. In line with Amihud and Mendelson (1987) this possibly suggests that at the opening, traders have a higher possibility of executing their order at price better than their limit price as compared to the rest of trading day, and thus they trade more shares. According to Atkins and Basu (1995) and Barclay and Hendershott (2003) this evidence may reflect the information advantage of some traders stemming from information asymmetry in the pre-open auction and/or accumulated fresh information that becomes available overnight. French and Roll (1986) also highlight that return volatility can be considered as a blunt measure of risk and an implied measure of the level of information. That could also explain why returns volatility is high per trade, trade size and volume mainly around the time the market opens.

Second, the aforementioned trading pattern is less apparent during the period surrounding the closing of the market. At this time, more orders are executed, but evidence a smaller number of shares bought or sold per order as compared to the opening period. This is consistent with Amihud and Mendelson (1987) who find that trading volume at the opening is higher on average. Stoll and Whaley (1990) underline that this pattern is more obvious for low-volume stocks. However, for most stocks trade size increases slightly in the last 5 to 10 minutes. This trading pattern, primarily present at the opening, is identical to "intraday stealth trading" reported by Chakravarty (2001), Barclay and Warner (1993) and Blau et al. (2009). Traders that are informed prefer to buy/sell more shares per trade (i.e. transfer large trade sizes) when trading volume is higher (e.g. at the opening/closing of the market) so that they cloak their information advantage. For the same reason, during periods of low volume, they prefer to split their orders into more trades. Earlier, Kyle (1985) argues that aggressive trading, or order shredding, is a characteristic of informed traders, whereas Admati and Pfleiderer (1988) allege that informed traders prefer to trade during periods of high liquidity and volume. In recent years, algorithmic trading has made the aforementioned trading strategies easier to implement.

Finally, the information related explanations of the trading behaviour at the opening probably also applies to trading behaviour around 10.15, when UK macroeconomic announcements become available. Trade counts decrease, while trade size and volume increase around this time, although returns volatility remains almost unchanged for most stocks. The latter can be explained by the fact that this type of information is publicly announced and is available to everyone, and not simply to a select number of traders, as is more likely at the opening and closing. In contrast, this is not the case when US macroeconomic indicators are announced around 13:30 or when the US stock market opens for trading at 14:30. During these periods, there is a considerable increase in the number of trades for all stocks, a smaller increase in trading volume and returns volatility, and a decrease in trade size. Intuitively, this means that during these periods market participants trade more, but they buy or sell smaller amount of shares per trade. This is similar to the pattern Andersen et al. (2016) report for the E-mini S&P 500 futures contract market.

[Graph 3 in here]

[Graph 4 in here]

4.5 Main Empirical Results

Having established the intraday trading patterns of London Stock Exchange during the period under analysis, in this section we focus on estimation of the invariance coefficients for each stock in our sample. Table A2 in Appendix shows the results of OLS regressions for different notion of trading activity based on the models in equations (32), (33), (34) and (35), respectively. The underlying variables are averages of observations for the same 5-minutes interval[56] across all trading days in the sample, as defined by equation (23). The invariance model (Model 1) and the alterative (Model 2) reject the null hypothesis of $H_0: \beta = 2/3$ proportionality between the transaction counts and trading activity, for all stocks at 0.1% significance level. In contrast, the MDH models indicate that for some stocks the investigated invariance relationship cannot be rejected. Table 2 summarizes some important descriptive statistics for the overall estimated invariance coefficients for the individual stocks which we report in Table 3. Specifically, the MDH-V model (Model 3) predicts the required 2/3 proportionality for 49 stocks (70% of the sample), whereas the MDH-N model (Model 4) exhibits the specific 2/3 relationship for 16 of these stocks. The average and median estimated value for the invariance coefficients is approximately 1/2 for both invariance models with a standard deviation around 0.03 for both.

[56] As explained, intervals with zero realised volatility or zero number of trades are excluded from the analysis. Here we report only 5-minutes intervals results, because the percentage of exclusions in 1-minute intervals is greater and thus the estimation becomes less accurate. In contrast to Andersen et al. (2016) that report similar invariance relationships for 1-minute and 5-minutes intervals, we find that in our sample the relationship between the number of trades and trading activity is not consistent in 1-minute intervals. The reason is most likely that FTSE 100 stocks are less actively traded compared to the E-mini S&P future contracts used in Andersen et al. (2016).

[Table 2 in here]

The average value of invariance coefficients for MDH-V (Model 3) is 0.6316 with a standard deviation of 0.04, whereas for the MDH-N model the average value is 0.5873 with a standard deviation of around 0.04. Given the values of adjusted R-squared in Table A2 in Appendix, it appears that the MDH-V model better fits the data for all stocks as compared to other models, albeit the standard errors of invariance coefficients are slightly smaller. Analysing the results further, we note that MDH-N model predicts 2/3 proportionality only for the lower capitalization stocks. In contrast with the average invariance coefficients based on the panel regressions reported in Chapter 3, the coefficient estimates for individual stocks are smaller in each respective model. This finding highlights the potential presence of fixed effects that lower the proportionality between the number of trades and trading activity, independently of the definition used for the latter. Taking into account the notion of trading activity implied by each model, the stipulated 2/3 invariance proportionality is present in transactions for the specific stocks and period we investigate when we consider the notion of trading activity that implies a proportionality between trading volume and returns variance. The 2/3 invariance proportionality is also present for some stocks when the notion of trading activity that implies a proportionality between trade counts and returns variance is used

Proceeding, we conduct similar analysis for the models in (32), (33), (34) and (35), but now using intraday averages as estimators of the underlying variables, defined by equation in (24). Table A3 in Appendix summarizes the results of the OLS regressions for all four models. Analogous to the results reported in Chapter 3 employing panel specifications for intraday averages, the null hypothesis of $\beta = 2/3$ is rejected for all four models at a 0.1% significance level, while in all models the coefficient estimates for the majority of the stocks are higher than the average coefficients in the respective panels. This is most apparent for the alternative invariance (Model 2) and MDH-V models (Model 3). At this point, when we consider also the differences in coefficient estimates between individual stocks and panels when averages are estimated across days, the above result highlights that stock fixed effects have a negative impact on the proportionality between the numbers of trades and trading activity[57]. The adjusted R-squared is higher for the MDH models as compared to the invariance models. Allegedly, the value of the standard deviation of trade size, when it is estimated as an intraday average, may be a potential reason why the adjusted R-squared is lower in invariance models.

[57] Chapter 3 highlights that stock effects are more pronounced when averaging across days than intraday.

The variation in this variable across stocks can also explain the substantial difference among the resulting coefficients estimates in the invariance models. In similar fashion to the results in Table A2, the coefficient estimates for the same stock vary considerably between the invariance and MDH models[58]. Also, including or excluding price in the definition of trading activity has a more significant impact on the invariance coefficients, when the underlying variables are estimated as intraday averages. Although the impact of price may be mechanical, the fact that the coefficients are lower and do not converge to a value that is constant across all stocks in the sample, when the underlying variables are intraday averages, is intriguing. A possible explanation is that innovations in the underlying variables for some stocks are more apparent when averaging intraday than across days and this may affect the coefficient estimates.

To further examine invariance theory in the context of different definitions of trading activity, we plot the logarithm of trade counts (n_t) against the logarithm of trading activity (w_t) for all four models under investigation. Figures 2 and 3 present the scatterplots between the number of trades and trading activity of all stocks for invariance and MDH models respectively. Note that x-axis in each scatterplot represents a different notion of trading activity, whereas y-axis (logarithm of trade counts) is the same for all scatterplots. The variables are averages of 5-minutes interval across all days based on equation (23). For each respective graph, the solid red line represents a line with 2/3 slope as suggested by invariance theory, whereas the blue dashed-dot line corresponds to a line with 1/2 slope based on the majority of the invariance coefficients for Models 1 and 2 in Table A2. The diamond symbol depicts the stocks with high-, the cross those with medium- and the circle those with low market capitalisations.

Upon inspection of Figures 2 and 3, it is apparent that Model 1 (invariance model) yields pairs of trade counts and trading activity for all stocks that are less dispersed compared to the other three models. Given the differences between the four notion of trading activity we use, price and trade size appear to play an important role in investigating the microstructure properties of our sample in the invariance framework. In all graphs, points that appear to be outliers (right hand side of the graphs) overwhelmingly refer to the first 5 minutes of trading activity in LSE. As we described in the previous section, during these minutes trade counts increase, while trading volume, volatility and trade size are considerably high and gradually decrease quickly in the subsequent 5-minutes intervals. This leads to an extreme value of the trading activity,

[58] The only difference is that all coefficients are lower compared to those produced when averaging across days

independently of the definition we employ. Also, it is apparent that the fitted 2/3 solid red line, better fits the data points in the graphs for Models 3 and 4. Intuitively, changing the intercept of the solid red line would reveal that it coincides with the individual regression lines estimated for the MDH-V model (Model 3) for 49 out of 70 stocks, and for 16 out of 70 stocks in the MDH-N model (Model 4). In contrast, the fitted blue dashed-dot line with a slope of 1/2 better fits the data points in the graphs for both Model 1 and 2, reflecting the fact the invariance coefficient estimates for the majority of stocks are closer to this value when using their respective trading activity definitions (see Table A2).

[Figure 2 in here]

[Figure 3 in here]

In summary, using averages across days as estimators for underlying variables, we are able to confirm a 2/3 invariance proportionality between trade counts and trading activity, when examining invariance relationships for trades at the level of individual stocks in FTSE 100. In contrast to Andersen et al. (2018) and Benzaquen et al. (2016), the specific value is only predicted for 70% of the stocks when we use the notion of trading activity as suggested by Clark (1973) and for 23% of the stocks (i.e. mainly the low market cap stocks) when we use the notion of trading activity as suggested by Ané and Geman (2000). Apparently, in the ITI framework, MDH-V model more precisely predicts the investigated invariance relationship in our sample for the period under investigation at the level of individual stocks. However, it fails to do so for stocks with high average volatility (see Table A3-Appenix II-Chapter 3). This finding is in contrast to the findings in Chapter 3, where we use panel specifications, and potentially indicates that fixed effects play an important role when examining invariance relationships.

In addition, similar to Chapter 3, none of the employed models can accurately predict the 2/3 proportionality when we use intraday averages as estimators for the underlying variables. This is also in contrast to Andersen et al. (2016) who are able to confirm the stipulated invariance relationship in the time series dimension. However, closer inspection reveals that there are groups of stocks that converge to certain proportionality values between the number of trades and trading activity. Certainly, all the coefficients estimates in Table A3 (Appendix) are lower when compared to those we present in Table A2 (Appendix), for all stocks and trading activity definitions. One reason is that averaging observations intraday reveals certain problems in terms of measurement error and sampling variation which may serve to bias the coefficients estimates. Intuitively, there are innovations in market expectations concerning the nature of the underlying variables that are more pronounced during each trading day than across days, and this affects the invariance proportionality. Another reason may be the institutional structure of the specific market and the way trading takes place (how and when traders prefer to shred their orders, when algorithmic traders or informed market participants trade). Finally, intraday averages exhibit more upward bias for days that include more intervals with zero trades and/or zero realized volatility, and thus the estimates of the invariance coefficients are less accurate.

Nevertheless, it is interesting the fact that invariance models (Model 1 & 2) are able to predict a fairly precise proportionality between trade counts and trading activity that is not 2/3 but rather 1/2 for 86% of the stocks in our sample, when we use averages across days. For the remaining 10 stocks, the invariance models do not predict 1/2 proportionality, though the invariance coefficients remain quite close to 1/2 for the majority of these stocks. This may reflect stock specific characteristics, a complete discussion of which is beyond the intended scope of this chapter. From Table 2, it is also obvious that predicted coefficients especially by invariance model (Model 1) have smaller standard deviation (i.e. more stocks compared to other models converge to 1/2 proportionality between trade counts and trading activity). This can be confirmed by the fact that Model 1 yields pairs of trade counts and trading activity for all stocks that are less dispersed compared to the other three models (see Figure 1 & 2). We further investigate and attempt to explain this difference in proportionalities in the following sections.

4.6 A theoretical interpretation of different proportionalities

The interpretation of all the above results follows the ITI framework and the respective assumptions, as motivated by MMI theory for bets. In Chapter 3, we explain how ITI is connected to MMI theory and justify the logic behind this connection. An assumption that is made by both Kyle and Obizhaeva (2016b) and Andersen et al. (2018) in their respective samples is that volume multiplier is $\zeta = 2$ and volatility multiplier is $\psi = 1$. Generally, this assumption of constant multipliers may hold during short time periods and in certain concentrated markets. But the specific market we investigate is likely to feel the impact of fragmentation and order shredding which as we show in Chapter 3 can affect the invariance coefficient estimates on average.

One potential reason for the lower proportionality that the invariance models estimate may be the way our dataset reports trades. Andersen et al. (2018) base their analysis on a similar set of assumptions, though distinct, to MMI theory in their empirical tests of ITI in the E-mini S&P 500 futures contract market, because their dataset facilitates this connection. Specifically, the way their dataset reports trades (aggregation of ticks for each price level) makes it possible to infer equivalent invariance relationships for trades as for bets. Although, this facilitates the empirical testing of invariance principles for trades, in line with Chapter 3, we argue that this approach does not capture how trading takes place in every market. For example, ticks at the same price level might refer to a different bet, or ticks from the same bet might be executed in different price levels. Thomson Reuters Tick History does not report the aggregate ticks for each price level, and thus trade counts also reflect the order flow stemming from intermediation on the supply side. Specifically, our dataset reports trades as they arrive and are executed in the market in the manner described by the trading example for an individual stock in Figure 4. This feature may indeed hamper the empirical tests of invariance as suggested by Kyle and Obizhaeva (2016b) and Andersen et al. (2018), because transaction counts will appear inflated and trade sizes smaller in comparison to the levels that invariance as a concept implies.

[Figure 4 in here]

Trade size is an important component of the invariance theory. While the MDH-N model excludes trade size, the MDH-V includes trade size in the denominator of the trading activity measure, while it is present in the numerator for MMI and ITI, interacting with the number of trades. This leads to the possibility that the difference in coefficient estimates across the models arise mainly from the varying influence of trade size on the trading activity measure. This effect of the trade size may be futher exarcebated by the fact that for stocks trading on the LSE there is a threshold size for large orders, compared to NMS, which is unique to each stock. Specifically, having a threshold for the maximum order size might cause a bet to intermediate more often than the initial assumptions suggest or cause an order to be shredded across different days. Thus, when examining invariance principles from the perspective of the subsequent trades this may have an impact on the invariance proportionality estimates.

Another potential reason why Models 1 and 2 do not exhibit the theoretical 2/3 invariance proportionality is that innovations in trade counts are not followed by innovations in trade size of the requisite magnitude when calendar time volatility is assumed to be constant, as ITI theory by Andersen et al. (2018) implies. Specifically, motivated by MMI theory, ITI suggests that business time volatility (i.e. calendar time volatility divided by the square root of the trade arrival rate) is perfectly negatively correlated with trade size. Our extended invariance model in (10) implies that for any order shredding factor $\varphi \neq 1$, trades do not perfectly reflect bets (i.e. there isn't a linear relationship), and thus this perfectly negative correlation may not be present in every trade, leading to a lower proportionality between trade counts and trading activity (we further examine this in the next section). However, given that the coefficient estimates for invariance models suggest a different proportionality the model in (10) implies a perfectly negative correlation between volatility in business time and trade size with an order shredding factor of $\varphi \approx 1.33$. This is actually greater than $\varphi = 1$ which Andersen et al. (2018) accepts when testing for ITI in trades regarding S&P 500 futures contracts market. If we accept that invariance theory holds empirically for trades, comparing the two markets this intuitively suggests that across the intraday pattern, orders regarding stocks in FTSE 100 are shredded less on average compared to orders in the S&P 500 future contracts market. Given that futures markets are more liquid in principle compared to stock markets, this finding is interesting. However, in the MMI theory framework, this implies that bets in the futures market, investigated by Andersen et al. (2018), are larger than bets in stocks traded in FTSE 100 and thus are shredded into more pieces if we accept that on average bets shredded into same size trades.

In a sense some of these factors as discussed above are capture by the fixed effects when we use panels in Chapter 3. This can potentially explain why there are differences in the coefficient estimates when we investigate invariance relationships for individual stocks and the market as a whole. We proceed by examining the role that the opening and closing minutes play to the invariance relationship between the number of trades and trading activity, given that these minutes are characterized by extreme volatility and increases in trading volume, trade counts and trade size, as well as different correlation than the correlation that invariance theory implies for the underlying variables. Hereafter we focus only on averages across days as estimators for the underlying variables, given that the intraday averages are less accurate independent of the model employed, for reasons we explain before.

4.7 Invariance when first/last 10 minutes of trading are excluded

Until now we have examined the invariance relationship in high frequencies using tick by tick data for 70 FTSE 100 stocks including trades from the opening (8:00am) till the closing (4.30pm) of active trading in London Stock Exchange(LSE). The main purpose here is to examine whether excluding these minutes will affect the proportionality between the number of trades and trading activity that we get when all active trading hours are included. Based on the analysis of the intraday patterns, the microstructure properties of the underlying variable and the analysis on the entire sample it is clear that trading has different characteristics mainly at the opening and closing of the market compared to the rest period. The behaviour of variables of trading activity potentially induces noise to the results of invariance in our sample. This noise stems mainly from high returns volatility and volume in the first minutes, as well as high number of trades and volume in the last minutes of active trading. Consequently, the trade size, which is important variable in the invariance framework, is also affected.

For these reasons, invariance relationships between the number of trades and trading activity is investigated in a sample that does not include the first and last 10 minutes of trading. Results of the OLS regressions regarding all four notion of trading activity are represented by the models in equations (32), (33), (34) and (35) are reported in Table A4 (Appendix). The underlying variables are averages of respective observations for 5-minutes intervals across all days as defined by equation (23). Following the assumptions for ITI in the previous tests, the null ($H_0 : \beta = 2/3$) and alternative ($H_0 : \beta \neq 2/3$) hypotheses are similar to those for the entire sample. Removing the first and last 10 minutes of active trading from our sample leads to an increase of invariance coefficients for all models. This effect is more pronounced for the invariance models (Models 1 and 2), for which coefficients estimates have increased around 14% on average. The increase in the proportionality between trade counts and trading activity is smaller for MDH models (4.4% and 7.5% on average for Model 3 and Model 4, respectively). Also, all models appear to fit the data better (i.e. adjusted R-squared are higher for all stocks) compared to when the entire trading day is included in the analysis. Table 3 reports a summary of the coefficients estimates of Table A4. The null hypothesis for $H_0 : \beta = 2/3$ is still rejected for almost all the stocks by the invariance models, though coefficient estimates approach this value more than when all minutes of trading are included. Both models (Model 1 and 2) predict 2/3 invariance proportionality for 5.71% of the stocks. The average and median coefficients estimates are higher than those we report in Table 2. On the contrary, the exclusion of trading minutes does not have a significant impact on the MDH-V model (Model 3) other than slightly increasing the average and median coefficient values. This is not the case for the MDH-N model (Model 4) which appears now to predict the required proportionality for more stocks (57.14%). These results again suggest that the extreme volatility and increased trade size which is apparent at the opening of active trading have a considerable negative impact on the invariance coefficients estimated by the invariance models and the MDH-N model[59]. The same reasoning, albeit its influence is somewhat diminished, applies to the increased volatility, trading volume and number of trades the market experiences at the close of active trading[60]. Although there are some changes in terms of which stock exhibits or do not exhibit the required invariance coefficient, the MDH-V model is still able to predict 2/3 proportionally for the majority of the stocks. This indicates that the specific model is not noticeably affected by the extreme values of the underlying variables at the opening and closing of the active trading. Overall, if we treat the invariance proportionality as an indicator of liquidity, the results for all models show that liquidity improves during the trading day, while it is lower in the first and last 10 minutes of trading activity.

[Table 3 in here]

Another potential explanation as to why there is a manifest increase in the value of coefficients estimated by the invariance models lies in the correlation between trade size and volatility in business time (volatility divided by the square root of trade counts). As we previously explain, MMI theory implies that there is a perfect negative correlation between the bet size and volatility in business time as long as calendar time volatility remains constant. In a similar fashion ITI theory suggest similar correlation for trade size and volatility in business time. Table A5 in Appendix depicts the correlation between the trade size and volatility in business time for all 70 stocks, in the first and last 10 and 15 minutes, both in the entire sample and excluding the opening and closing 10 or 15 minutes. Using all trading minutes, the correlation is positive for all but one stock (SL) with the highest being 0.8470 (BLT) and the lowest in absolute terms 0.0324 (IMT). There is also a strong positive correlation for the majority of the

[59] Given that the MDH-N model excludes trade size in the notion of trading activity, the results imply mainly the impact of extreme volatility on the estimates of the specific model.

[60] We have contacted similar analysis by excluding only the first 10 minutes of active trading and the results do not vary considerably. This suggests that the behavior of the underlying variables at the opening have a greater impact on invariance proportionality than that at the closing of active trading.

stocks in the first and last 10 minutes of active trading[61]. Excluding these minutes, the positive correlation decreases. Indeed, many more stocks now exhibit a negative correlation between trade size and volatility in business time. The highest value for the correlation is now 0.4161 (HSBA) and the lowest in absolute terms is 0.0034 (EMG).

On the basis of the extended invariance model in (10), assuming that $\varphi = 1$ (i.e. the model is similar to ITI theory), this change in the direction of the correlation, from positive to negative together with the decrease in the values of the positive correlations, may partially explain why the invariance coefficients significantly increase when we exclude the first/last 10 minutes. If we assume $\varphi > 1$, as it is the case for the entire sample, results also indicate that orders in stocks for FTSE 100 are shredded more in minutes following the opening 10 minutes and before the closing 10 minutes. Specifically, as the invariance estimates for Model 1 & 2 increase when we exclude the first/last 10 minutes of trading activity, φ becomes smaller compared to the entire sample (i.e. the estimated proportionality is now 7/12), but still greater than 1. This implies increase level of order shredding in the minutes between the first/last 10 minutes of active trading. This finding also corresponds to the results we depict in the relevant graphs, both for the number of trades (Graph 3) and trade size (Graph 4).

In summary, excluding the first/last 10 minutes of trading activity yield some interesting results. The MDH-V model can still predict a 2/3 proportionally for the majority of the stocks, whereas the MDH-N model can now predict the required proportionality for more stocks. In contrast, the coefficient estimates for the invariance models are increased, converging towards a value of 7/12. In the invariance universe, changes in trading activity are followed by changes in the number of trades and shifts in the distribution of trade size in a specific proportional way (i.e. 2/3 and 1/3, respectively). Results on the entire sample reveal that this proportionality for the majority of the stocks is 1/2 (i.e. according to invariance that implies 1/2 change in the number of trades and 1/2 shift in the distribution of trade size). We argue that on top of all the other reasons stemming from intraday patterns, the way that our dataset reports trades and the fact that trades are not bets, this revealed proportionality is partly driven by the positive correlation between trade size and volatility in business time in the first/last 10 minutes. Excluding those minutes leads to a negative, or at least a less positive correlation between trade

[61] Table A5 reports perfect positive/negative correlations for the first/last 10 minutes, but this is due to the fact we include fewer observations. When we add in another 5 minutes, the correlations are no longer perfect. However, this does not change the strong positive correlation for the majority of the stocks for the specific minutes under consideration. Also, the results are similar when we exclude 10 or 15 minutes. In this chapter we chose to exclude the first/last 10 minutes when estimating the invariance coefficients.

size and volatility in business time for all stocks and thus to a shift in the proportionality more towards the number of trades than trade size with an increase in trading activity. Based on our extended invariance model this also indicates an increase in the level of order shredding after the first 10 minutes and before the last 10 minutes of trading activity. Intuitively, this suggests that in the 10 first/last minutes traders buy/sell more stocks per trade as trading activity increases than they do during the remaining minutes of trading day.

From the analysis until this point it is apparent that MDH-V model best describes the invariance properties in our sample as proposed by ITI theory. However, particularly the invariance model (Model 1) as motivated by Kyle and Obizhaeva (2016b) predicts a different proportionality which we partly explain in the previous sections. To further investigate this finding, i.e. the 1/2 proportionality between the number of trades and trading activity, we proceed to conduct some further tests across subsample periods. The main goal is to examine the extent to which yearly characteristics and/or the financial crisis play a role in the specific estimated invariance proportionality existing between the number of trades and trading activity.

4.8 Invariance in subsample periods

4.8.1 Year by Year Analysis

First we divide our sample into the three separate years for which we have data and examine the relationship between the number of trades and trading activity. The goal here is to check whether the estimated 1/2 proportionality when we use the entire sample also holds for each individual year. Intuitively, this test will reveal whether the implied order flow composition remains the same across shorter time periods. Full results of the OLS regressions based on the model (25), including the constant term, standard errors and adjusted R-squared for the three substantive years are presented in Table A6 in the Appendix. The underlying variables are interday averages of respective observations for 5-minutes intervals across all days in the sample, as defined by equation (23). Given the proportionality that Model 1 predicts in the previous analysis, the null hypothesis is $H_0 : \beta = 1/2$. Table 4 reports only the invariance coefficients of 70 FTSE stocks for the entire period as depicted in Table A2 (Appendix) and for 2007, 2008 and 2009 from Table A6 (Appendix), respectively. Coefficients in bold indicate that we cannot reject the null hypothesis for $H_0 : \beta = 1/2$. Estimates of invariance proportionality drops in value for 34 of the stocks between 2007 and 2008 and for 44 stocks between 2008 and 2009. However, the proportionality between the number of trades and trading activity is not significantly different from 1/2 for 45 stocks during all three years and at least for one year for all stocks. Also, the constant term is significantly different from zero for almost all stocks only in 2009, while in the other two years it is not significant for the majority of the stocks.

[Table 4 in here]

Taking into account the coefficients when the whole sample period is used, it appears that the 1/2 proportionality is close to the average of the acquired proportionalities for the respective years. Interestingly, the invariance coefficient for all stocks in the high capitalisation group deviates from 1/2 at least during one of the substantive years. Based on the assumption of 1/2 invariance proportionality and the implied composition of trade size and number of trades, the findings do not suggest a unified order flow pattern for all stocks, but rather indicate that traders are affected by stock and/or industry specific characteristics when deciding their order size or how to shred their orders, at least for the period under analysis. That would partly explain why the invariance coefficients for some stocks increase/decrease across years. In addition, as we explain changes regarding the class of large trade waiver (i.e. by how much an order is allowed to exceed the NMS) to which a stock belongs due to MiFID I, based on a stock's average daily turnover calculated on an annual basis, may also potentially explain the change in the invariance coefficient estimates. For example, an increase in the size of an order which is allowed for a specific stock may lead to large orders or executed trades and thus higher measures of trading activity. In turn, whether this increase in trading activity is proportional or not to an increase in the number of trades can potentially affect the respective invariance coefficients[62]. However, over the entire sample period, any such differences between the three years do not manifest themselves, and overall the implied order-flow composition is the same (i.e. 1/2 proportionality) for the majority of the stocks.

4.8.2 Pre-Crisis and After Crisis Analysis

To further investigate the estimated 1/2 proportionality between trade counts and trading activity, we divide the sample into two periods: a pre-crisis period between 1st January 2007 and 30th June 2008 and an in-crisis period between 1st of July 2008 and 31st of December 2009.

[62] Investigating the extent at which NMS plays a role in the invariance coefficient estimates and whether it can potentially explain the 1/2 proportionality between trade counts and trading activity is a very interesting topic for future research.

Given the period of the analysis, the main purpose here is to check whether the outbreak and duration of the 2008-09 financial crisis leads to a different proportionality. We choose July 2008 as our break point as the third quarter of 2008 appears to be an appropriate time in terms of the extent to which the 2008 global financial crisis starts to have an impact on the UK economy (real GDP reduction, the emergence of a credit crunch, a decrease in major economic indicators, and the FSTE 100 index's sharpest drop since its creation).We report the full results of the OLS regressions, based on the model (25), including the constant term, standard errors and adjusted R-squared for both pre-crisis and in-crisis periods in Table A7 in the Appendix. To make the analysis easier, we present only the invariance coefficients for 70 FTSE 100 stocks, both during pre-crisis and in-crisis periods, in Table 5. The underlying variables are averages of respective observations for 5-minutes intervals across all days in the sample, as defined by equations (23).

[Table 5 in here]

First, the results reveal that for 46 stocks, the requisite 1/2 proportionality is present in both pre-crisis and in-crisis period, while for 3 stocks the respective coefficient is not 1/2 during either periods. All 49 stocks exhibit the same relationship between trade counts and trading activity as when the entire period is examined. This pattern does not hold for the remaining 21 stocks in our sample, for which there is a change in either accepting or rejecting a null hypothesis for $H_0 : \beta = 1/2$ between the two periods. Specifically, the null hypothesis for $H_0 : \beta = 1/2$ is rejected for pre-crisis period for 15 of these stocks, whereas the exact opposite holds for the remaining 6 stocks. Upon inspection of Table 4, it is apparent that there is a change in the statistical significance of the invariance coefficients between pre-crisis and in-crisis period for all the stocks that belong to the high market cap group. There is also a reduction in the coefficient estimation values for the majority of the stocks that do not exhibit 1/2 proportionality in any of the two periods. The majority of the specific stocks are characterised by the highest/lowest GBP-denominated trade size ($P \cdot Q$) during the pre-crisis period.

The changes in the coefficients between the pre-crisis and in-crisis periods are independent of the significance of the coefficients tested against the null hypothesis of $H_0 : \beta = 1/2$. Specifically, even if the coefficient estimates do not reject the null hypothesis in both periods, they may exhibit a decline or an increase for some stocks between the two periods. Also, these changes alone are not enough to explain the fact that for some stocks, the proportionality between the number of trades and trading activity is not significantly different from 1/2 in both periods, while for others it does not. On the contrary, what appear to be important is the direction, as well as the magnitude of the increase/decrease in the underlying variables or the way that market participants trade in the specific stocks (e.g. how and when they split their orders).

4.9 Conclusion

In this chapter we examine invariance relationships for trades regarding individual stocks that remain FTSE 100 constituents throughout our 2007-2009 sample period. For this purpose we employ four different notion of trading activity based on the generalised invariance model we propose in Chapter 3. However, here we do not focus on the average proportionality between the number of trades and trading activity, but rather investigate the invariance principles separately for each stock.

Analysing the intraday patterns of London Stock Exchange, we find that realised volatility is extremely high during the first 5 minutes of active trading, spikes at the time of UK macroeconomic announcements, the opening of the New York FOREX market, around US macroeconomic announcements and the opening of the US stock market, whereas it also increases slightly at the closing. Price discovery appears to be less efficient at the morning opening, because during this period there are more heterogeneous opinions regarding the direction of prices after non-trading hours. It also appears that market participants attempt to unwind their positions and/or rebalance their portfolios before the closing auction. Trading volume has a four-humped pattern during the period under analysis, whereas market participants trade more shares within each order for the majority of the stocks at the opening. Both trading volume and number of trades spike considerably during the US macroeconomic announcements and the opening of the US stock market.

The results of the OLS regressions show that the MDH-V model more accurately predicts the 2/3 proportionality between the number of trades and trading activity for 70% of the stocks stipulated by ITI invariance theory as motivated by the MMI theory. Intuitively, based on the MDH theory introduced by Clark (1973), this suggests that for these specific stocks, the trading volume is proportional to the returns variance. This result is also consistent when we exclude from the sample the first/last 10 minutes of trading activity which are characterised by extreme volatility and increases in trading volume, trade counts and trade size, as well as different correlation than the correlation that invariance theory implies for the underlying variables. However, the model fails to predict 2/3 proportionality for those stocks that have high on average volatility. In addition, none of the employed models can accurately predict the 2/3 proportionality when we use intraday averages as estimators for the underlying variables mainly due to measurement error and sampling variation and /or the institutional structure of the market and intraday trading patterns.

However, we find that the invariance models are able to predict a fairly precise proportionality between trade counts and trading activity that is not 2/3 but rather 1/2 for 86% of the stocks in our sample, when we use averages across days. This proportionality may be a feature of our dataset or the specific market or driven by the magnitude of trade size which has a different impact on the models we investigate. The latter may be further exacerbated by the fact that for stocks trading on the LSE there is a threshold size for large orders, compared to NMS, which is unique to each stock. Based on our extended invariance model and the stipulated proportionality by MMI and ITI theories, the 1/2 proportionality value implies that bets in the futures market, investigated by Andersen et al. (2018), are larger than bets in stocks traded in FTSE 100 and thus are shredded into more pieces if we accept that on average bets shredded into same size trades in these two markets.

We also show that this proportionality is partly driven by the positive correlation between trade size and volatility in business time in the first/last 10 minutes. Excluding these minutes from the analysis leads to an increase in the coefficient estimates predicted by invariance models that now converge to a value of 7/12. On the basis of the extended invariance model this finding implies increased level of order shredding in the minutes between the first/last 10 minutes of active trading. Further analysis on the specific proportionality value on a year per year basis does not suggest a unified order flow pattern for all stocks, but rather indicates that traders are affected by stock and/or industry specific characteristics when deciding their order size or how to shred their orders, at least for the period under analysis. The change of coefficient estimates for some stocks from one year to another may be driven by changes in the maximum allowed order size relative to NMS that is unique for each stock and is calculated annually. These differences are no longer manifest when we use the entire sample in the estimation. The results also indicate a change in the statistical significance of the invariance coefficient between the pre-crisis and in-crisis periods for all the stocks that belong to the high market cap group. Also, the direction and magnitude of the increase/decrease in the underlying variables between the pre-crisis and in-crisis periods determine any changes in proportionality.

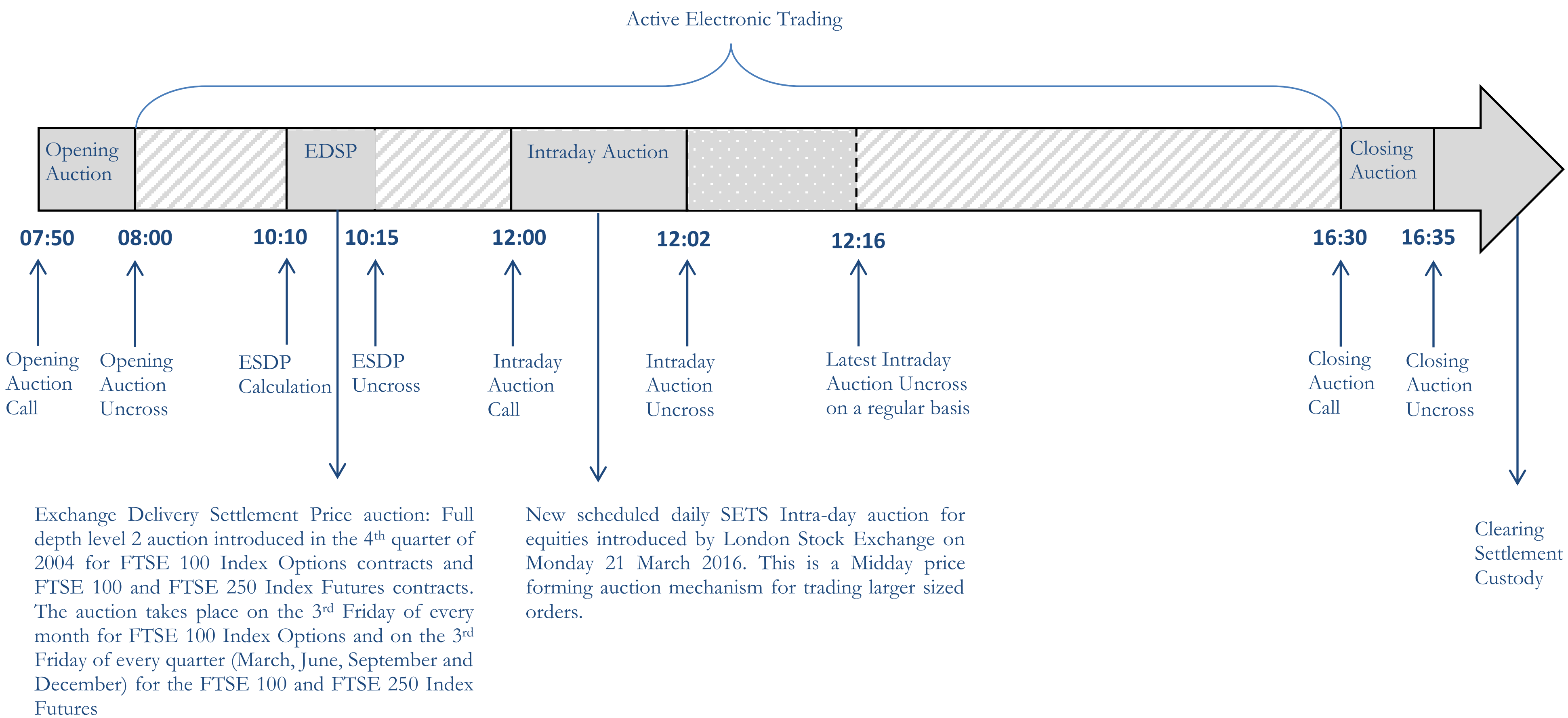
At this point it is important to underline once more that the analysis in this chapter, similar to Chapter 3, concerns itself with trades, which are different from the concept of bets on which the MMI theory by Kyle and Obizhaeva (2016b) is built. Also, significant differences exist in the way that trades are reported in the dataset between the LSE and the E-mini S&P500 futures contract market, which means the results are not directly comparable with those in Andersen et al. (2018). However, given that trades are components of bets, this research is complementary to other papers discussing invariance theory and provides a different interpretation of the invariance proportionality for trades regarding individual stocks. The chapter also attempts to link it to the intraday patterns of specific equity market. Given the increasing level of market fragmentation which is appearing in global stock markets, it should be interesting to examine in future research the extent to which stock and industry characteristics, as well as the introduction of different trading platforms, affect invariance principles, and intuitively liquidity across various platforms.

Table 1- Main domestic trading services of London Stock Exchange

SETS
The main electronic order book of London Stock Exchange. This is an order-driven electronic market that accommodates also the provision of liquidity by market makers and guarantees 2-way prices. Trading in SETS includes FTSE100, FTSE250 and FTSE Small Cap Index constituents, Exchange Traded Funds, Exchange Trading Products and liquid AIM, Irish and London Standard listed securities. The type of orders supported in SETS are passive orders, stop orders, stop limit orders, hidden limit orders, mid-price pegged orders and executable quotes. There are two clearers available pre and post trading and counterparty protection for the market participants. The trading process in SETS is depicted in Figure 1. LSE also operates a version of SETS on a modified trading cycle that supports Securitised Derivatives
SETSqx
Stock Exchange Electronic Trading Service-quotes and crosses is a trading forum for securities that are less liquid than those traded on SETS. It is a non-electronic quote-driven market for market makers, while it includes 5 electronic auctions a day at 08:00, 9:00, 11:00, 2pm and 4:35pm. It supports named or anonymous electronic orders, which are cleared centrally. SETSqx supports two types of order book model contingent on the security having registered market makers who provide non-electronic quotes.
SEAQ
The non-electronically service for executing quotes of London Stock Exchange. This trading venue allows market makers to post quotes in AIM securities (not traded on SETS or SETSqx) and a number of fixed interest securities.

Source: London Stock Exchange

Figure 1-Trading process in SETS

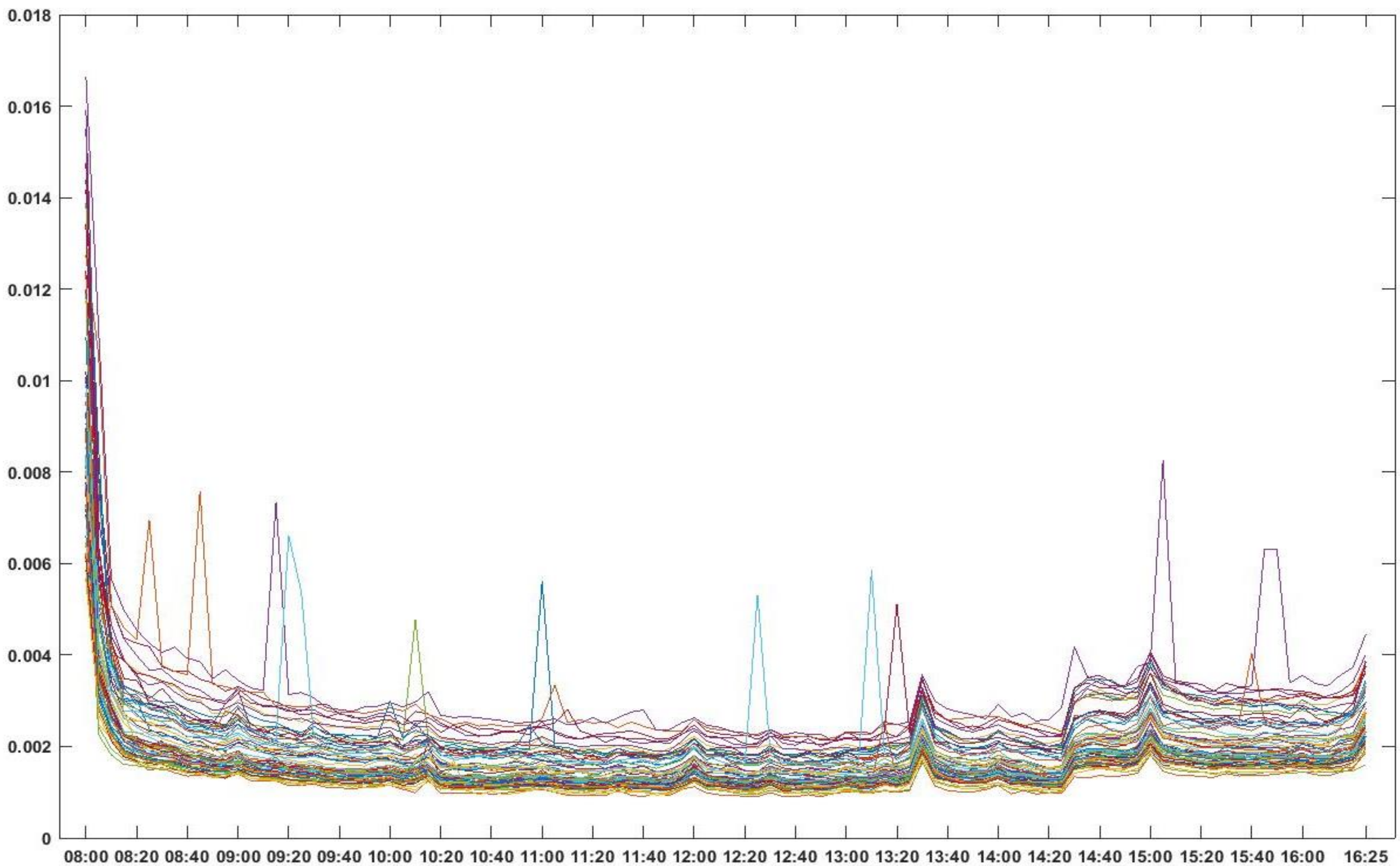


Source: London Stock Exchange

Graph 1

Realised Volatility

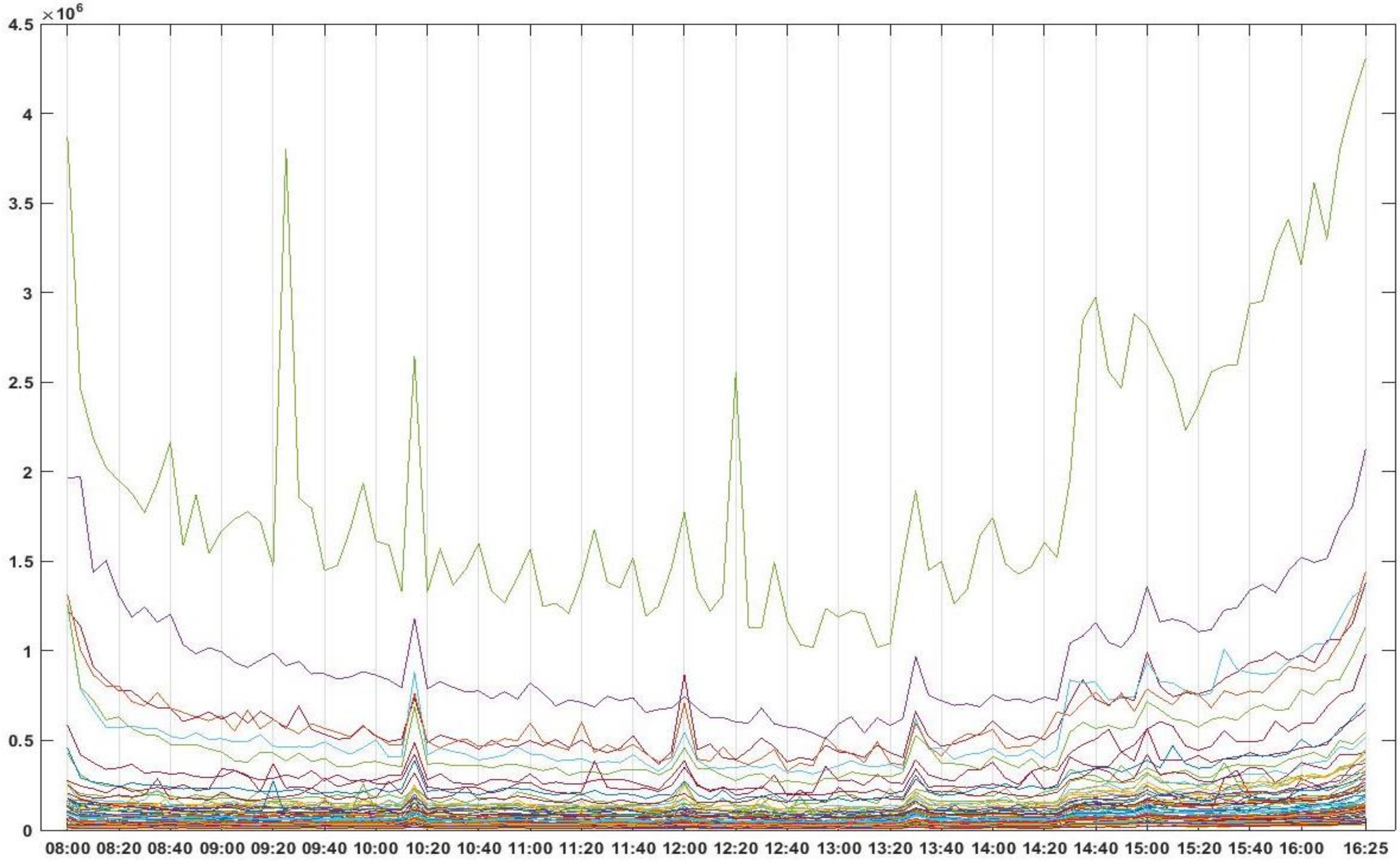
Realised volatility for all 70 stocks in the sample as average across days based on the equation (23). For each substantive 5-minutes interval over the trading day the realised volatility is estimated as the sum of squared returns over 10s during the specific time span.



Graph 2

Trading Volume

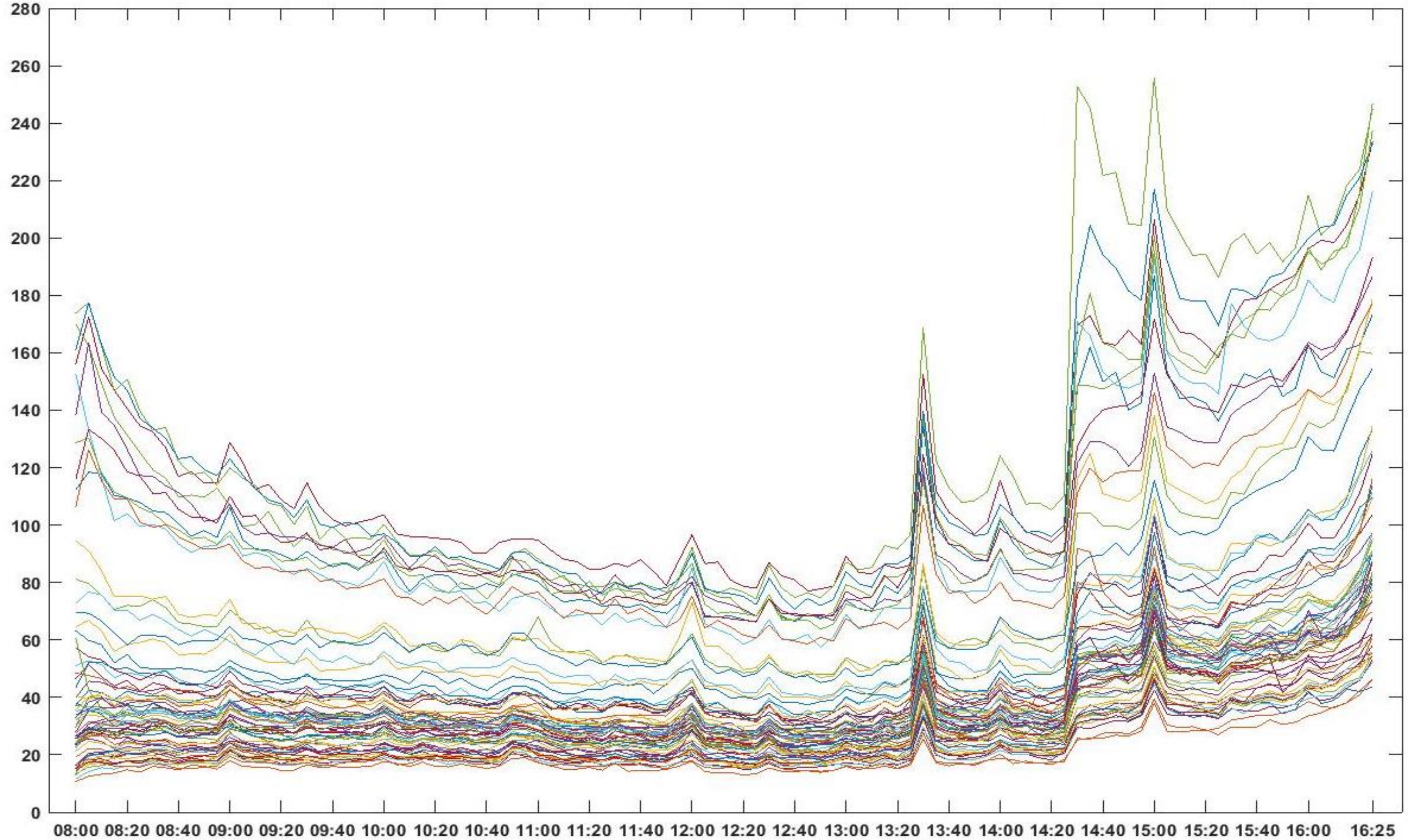
Trading volume for all 70 stocks in the sample as average across days based on the equation (23)



Graph 3

Number of Trades

Number of trades for all 70 stocks in the sample as average across days based on the equation (23)



Graph 4

Trade size

Trade size for all 70 stocks in the sample as average across days based on the equation (23)

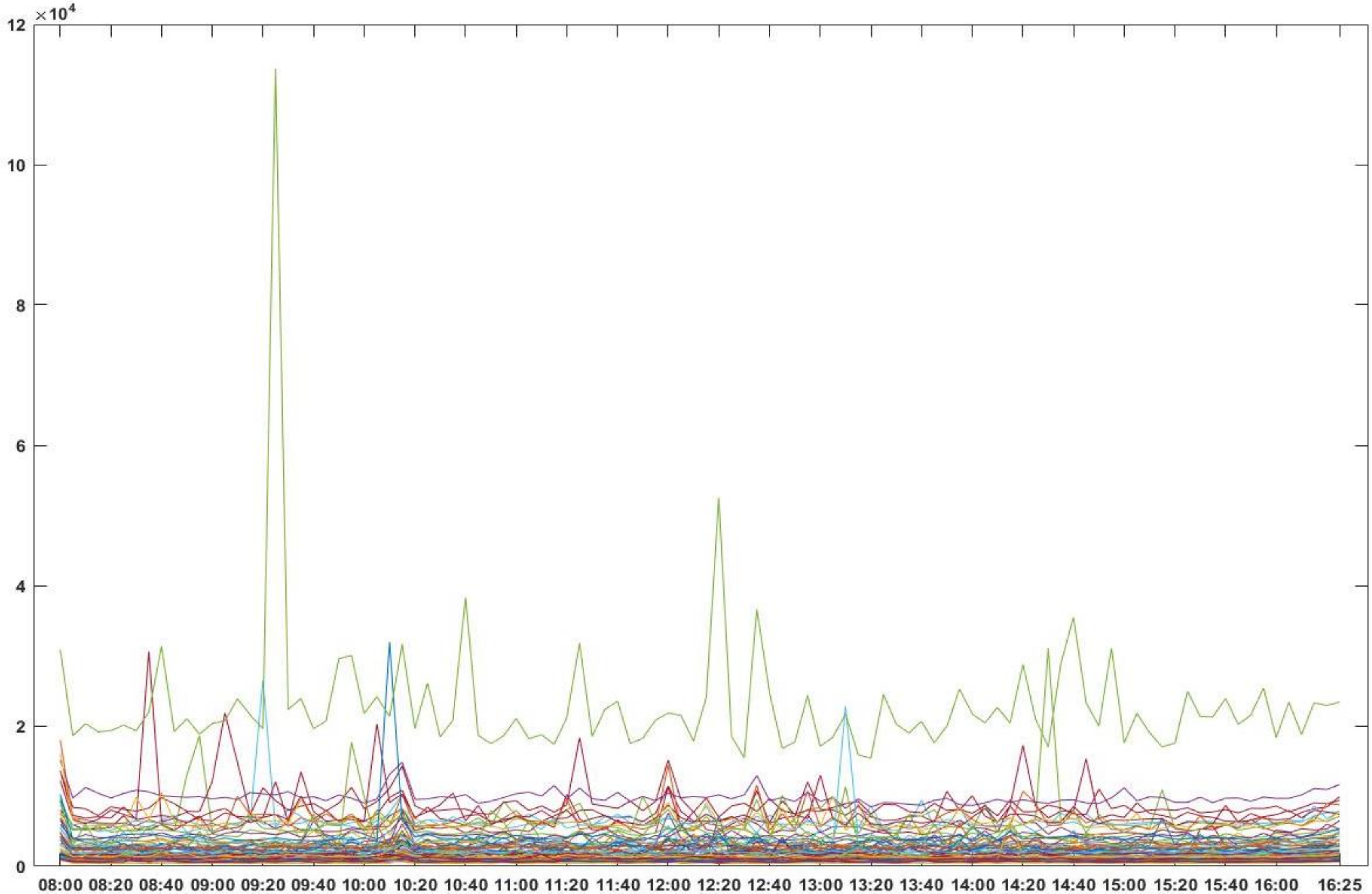


Table 2

Invariance coefficients for estimation using averages across days

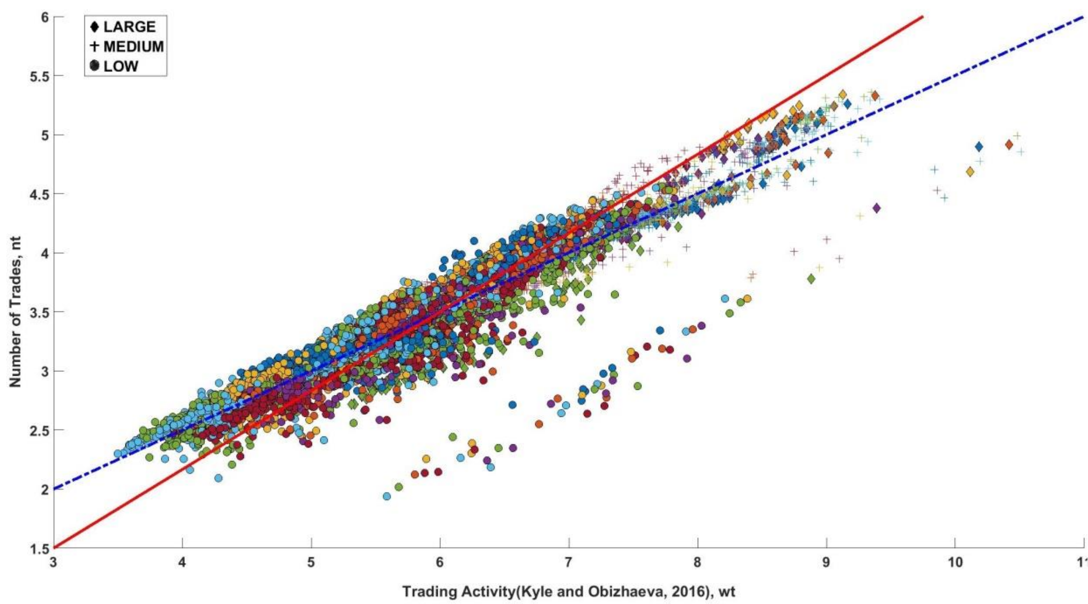
This table reports the number and percentage of stocks for which the 4 models under investigation can predict the 2/3 proportionality between the number of trades and trading activity as suggested by invariance theory. Also, it presents the average, standard deviation and median of the invariance coefficients estimates for four notion of trading activity. The different notions of trading activity are those introduced by Kyle and Obizhaeva (2016) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4).

	Model 1	Model 2	Model 3	Model 4
No of Stocks with 2/3 proportionality	0	0	49	16
% of Stocks with 2/3 proportionality	0%	0%	70%	22.86%
Average	0.5071	0.5096	0.6316	0.5873
Std. Dev.	0.0330	0.0322	0.0418	0.0377
Median	0.5077	0.5107	0.6366	0.5892

Figure 2-Scatterplot for averages across days

Scatterplot of the logarithm of trade counts against the logarithm of trading activity. Model 1 represents the invariance model as introduced by Kyle and Obizhaeva (2016b) and Model 2 the alternative invariance model suggested by Andersen et al. (2016). The variables are averages of 5-minutes interval across all days. For each respective trading activity notion, the solid red line represents a line with 2/3 slope, as implied by invariance theory, and the blue dashed-dot line corresponds to line with 1/2 slope.

Model 1



Model 2

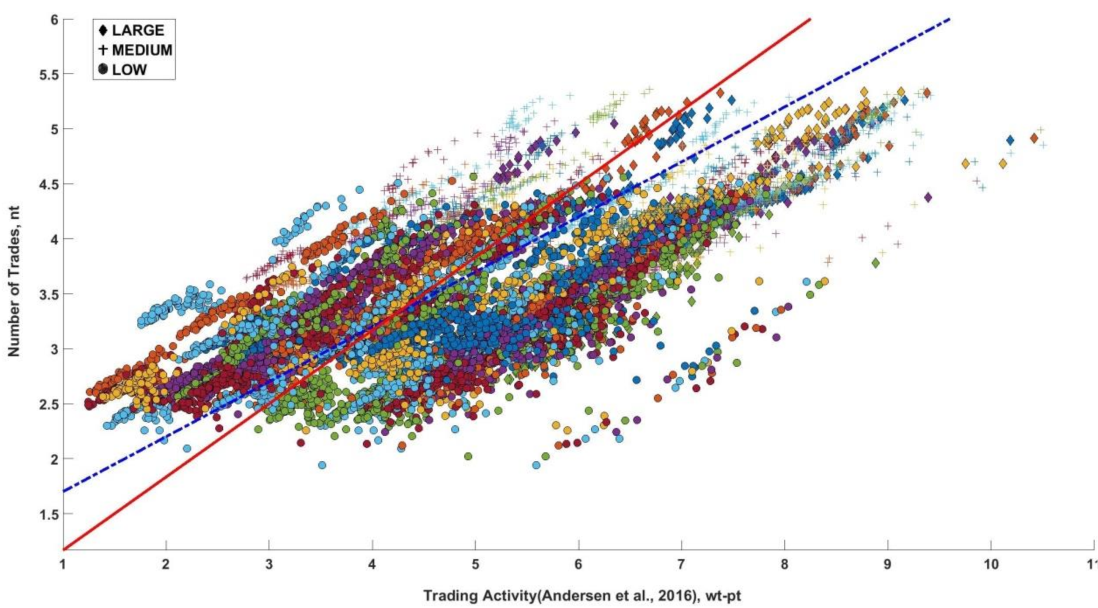
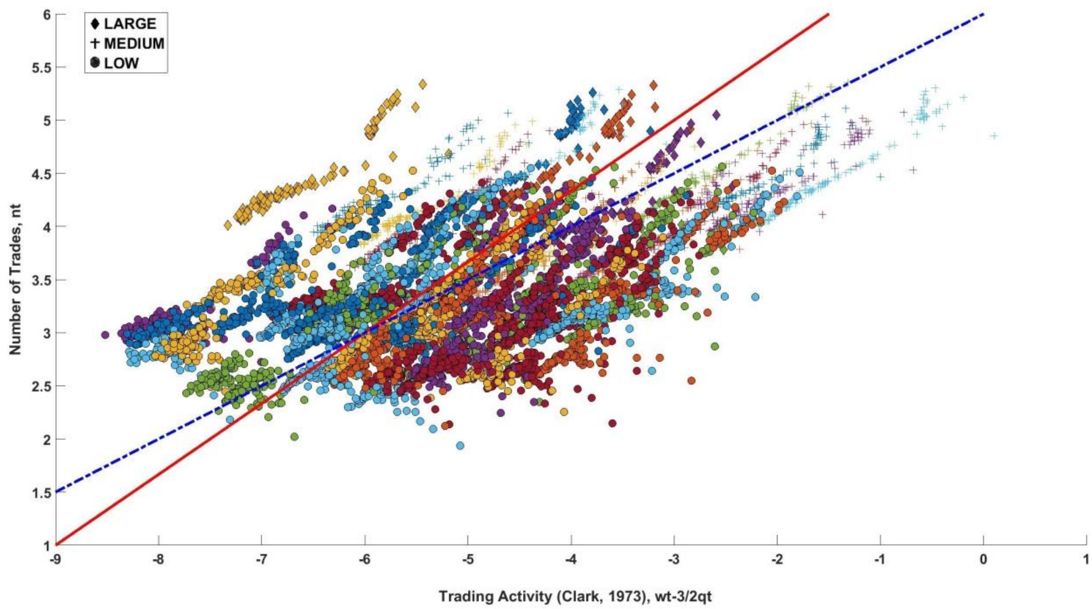


Figure 3-Scatterplot for averages across days

Scatterplot of the logarithm of trade counts against the logarithm of trading activity. Model 3 refers to the notion of trading activity introduced by Clark (1973) and Model 4 the notion suggested by (Ané and Geman (2000)). The variables are averages of 5-minutes interval across all days. For each respective trading activity notion, the solid red line represents a line with 2/3 slope, as implied by invariance theory, and the blue dashed-dot line corresponds to line with 1/2 slope.

Model 3



Model 4

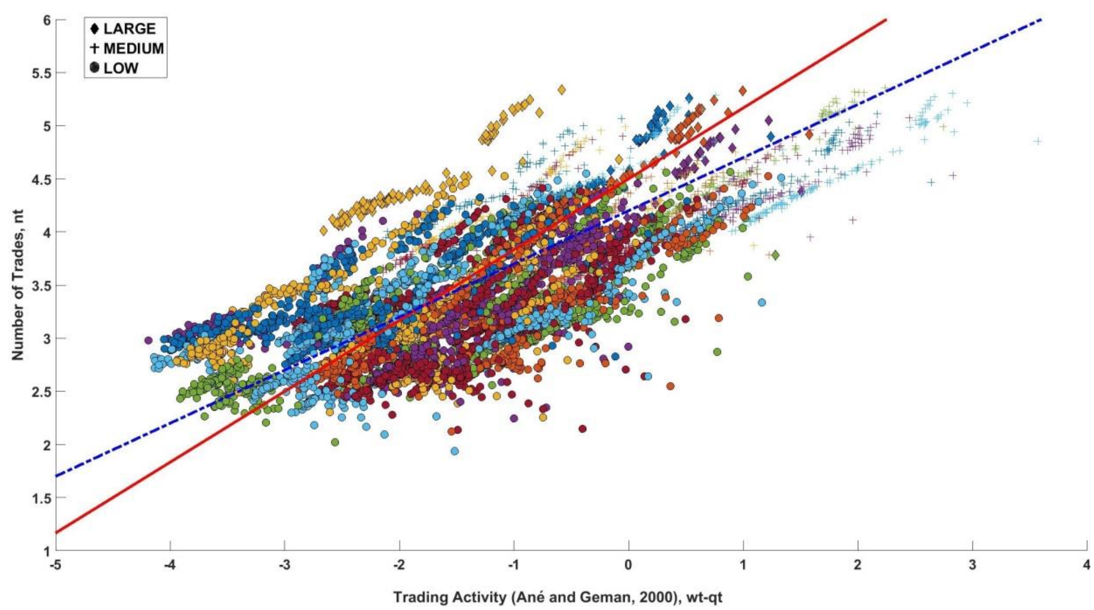


Figure 4-Trading example on FTSE 100 (Individual stock)

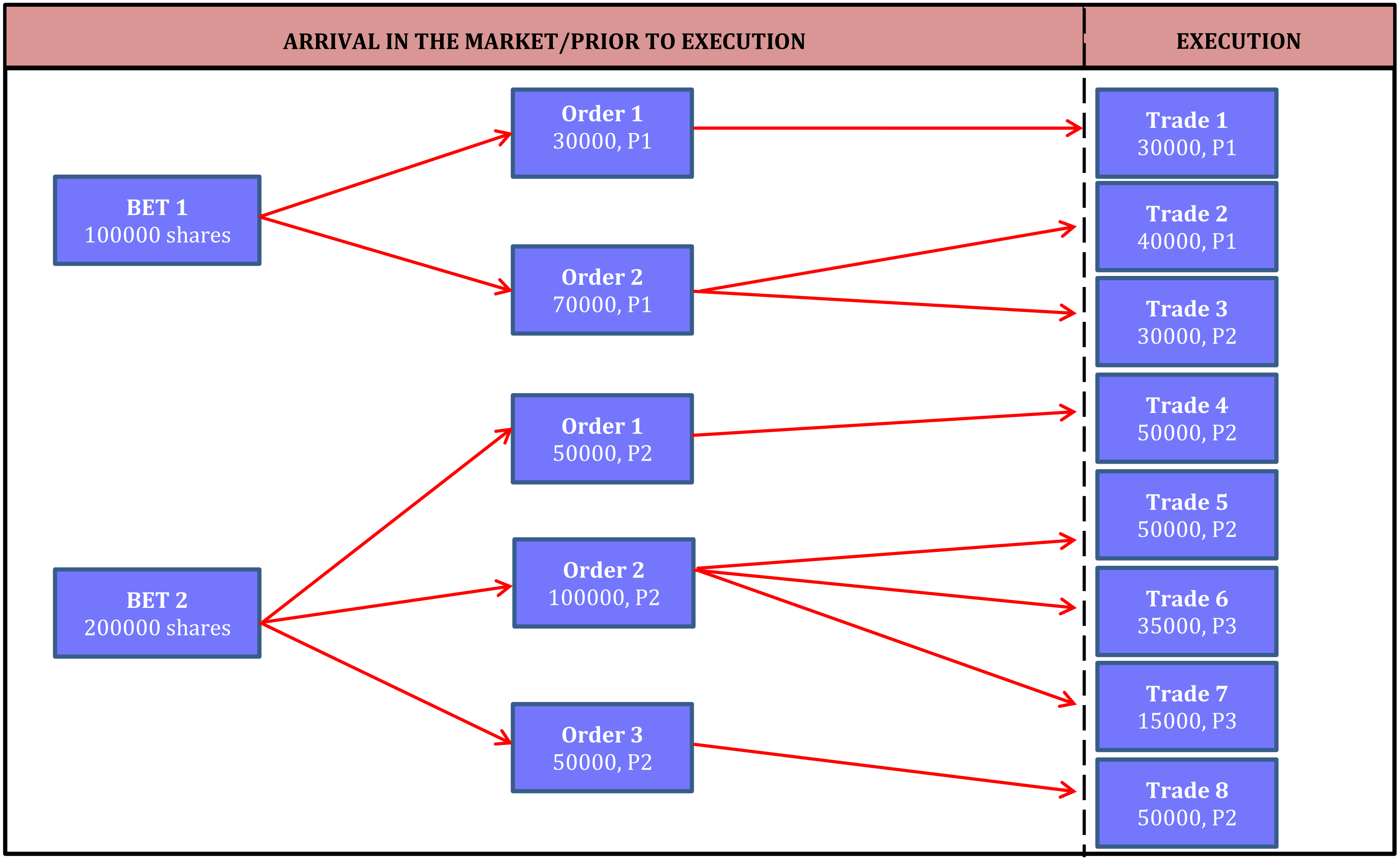


Table 3

Invariance coefficients for estimations across days

This table reports the number and percentage of stocks for which the 4 models under investigation can predict the 2/3 proportionality between the number of trades and trading activity when the first and last 10 minutes of activity trading are excluded. Also, it presents the average, standard deviation and median of the invariance coefficients estimates for four notion of trading activity. The different notions of trading activity are those introduced by Kyle and Obizhaeva (2016) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, Ané and Geman (2000) (Model 4).

	Model 1	Model 2	Model 3	Model 4
No of Stocks with 2/3 proportionality	4	4	48	40
% of Stocks with 2/3 proportionality	5.71%	5.71%	68.57%	57.14%
Average	0.5768	0.5794	0.6593	0.6311
Std. Dev.	0.0348	0.0340	0.0399	0.0370
Median	0.5795	0.5833	0.6661	0.6378

Table 4
Invariance Proportionalities

This table reports the invariance proportionalities estimates of OLS regressions for 70 FTSE 100 stocks based on the model (25), for the entire sample period and the respective 3 years, 2007, 2008 and 2009

	BP	**HSBA**	**VOD**	**GSK**	**RDSA**	**RIO**	**AZN**	**RBS**	**BATS**	**BG**
Full Sample	**0.5332**	**0.5160**	**0.4764**	**0.5404**	0.5415	**0.5117**	**0.5072**	0.4327	**0.5190**	**0.4848**
Year2007	0.5751	0.5584	0.4386	0.5714	**0.5289**	**0.4934**	**0.5085**	**0.4844**	**0.5436**	**0.5032**
Year2008	0.5485	**0.5167**	**0.4734**	**0.5311**	0.5443	**0.5039**	**0.5151**	0.4473	**0.5032**	**0.4679**
Year2009	**0.5257**	**0.5083**	**0.5087**	**0.5281**	0.5532	**0.5288**	**0.4983**	0.3727	**0.5133**	**0.4952**
	AAL	**BLT**	**BARC**	**TSCO**	**XTA**	**DGE**	**LLOY**	**STAN**	**ULVR**	**RB**
Full Sample	**0.4941**	0.5616	**0.4965**	**0.4678**	**0.4757**	**0.5166**	**0.4836**	**0.4742**	**0.5387**	**0.4825**
Year2007	**0.4685**	**0.5306**	**0.5032**	**0.4789**	0.4517	**0.4643**	**0.5213**	**0.4644**	**0.5306**	**0.4845**
Year2008	**0.4736**	0.5664	**0.5053**	**0.4653**	0.4449	**0.5086**	**0.4952**	**0.4593**	**0.5301**	**0.4812**
Year2009	**0.5336**	0.5815	**0.4883**	**0.4683**	**0.5208**	**0.5465**	**0.4546**	**0.4985**	**0.5530**	**0.4819**
	SAB	**NG**	**IMT**	**BT**	**AV**	**PRU**	**BAE**	**CNA**	**SSE**	**CBRY**
Full Sample	**0.4995**	**0.5240**	**0.4984**	0.4395	**0.4858**	**0.5185**	**0.4621**	**0.5081**	**0.5191**	0.5606
Year2007	**0.5083**	**0.4996**	**0.4931**	**0.4950**	**0.4996**	**0.5145**	**0.4472**	**0.4995**	**0.4957**	0.5601
Year2008	**0.5062**	**0.5276**	**0.4912**	**0.5078**	**0.5071**	**0.5276**	**0.4688**	**0.5493**	**0.5056**	0.5595
Year2009	**0.4898**	**0.5240**	**0.5083**	0.3578	**0.4594**	**0.5120**	**0.4747**	**0.4798**	**0.5463**	**0.5438**
	BSY	**EMG**	**RR**	**MRW**	**MKS**	**SBRY**	**WPP**	**REL**	**LGEN**	**CPG**
Full Sample	**0.5379**	**0.4693**	**0.4940**	**0.5160**	**0.5023**	**0.5008**	**0.5225**	**0.5219**	**0.4603**	**0.5028**
Year2007	**0.5499**	**0.4874**	**0.4929**	**0.5010**	**0.5032**	**0.4797**	**0.5230**	**0.5422**	**0.5454**	**0.4959**
Year2008	0.5656	**0.4645**	**0.5246**	**0.5199**	**0.5051**	**0.5259**	**0.5514**	**0.5349**	0.4135	**0.5132**
Year2009	**0.5005**	**0.4624**	**0.4795**	**0.5221**	**0.5058**	**0.4997**	**0.4972**	**0.5069**	0.4410	**0.4984**
	ABF	**LAND**	**OML**	**ANTO**	**PSON**	**SHP**	**SL**	**IPR**	**KAZ**	**UU**
Full Sample	**0.5568**	**0.4951**	0.4066	**0.4953**	**0.5408**	**0.5252**	**0.5073**	**0.5036**	**0.4611**	**0.5546**
Year2007	0.6215	**0.4895**	**0.4531**	**0.4889**	**0.5406**	**0.5248**	**0.5070**	**0.4963**	**0.4578**	**0.5490**
Year2008	**0.5622**	**0.5222**	0.3767	**0.4758**	**0.5533**	**0.5142**	0.5586	**0.5159**	**0.4552**	0.5687
Year2009	**0.4941**	**0.4675**	0.3889	**0.5192**	**0.5271**	**0.5292**	**0.4529**	**0.4979**	**0.4792**	**0.5356**
	SN	**EXPN**	**BLND**	**VED**	**RSA**	**CPI**	**KGF**	**CCL**	**CW**	**SMIN**
Full Sample	**0.5218**	**0.5140**	**0.4839**	**0.4776**	**0.4754**	0.5592	**0.5084**	0.5603	**0.5368**	**0.5171**
Year2007	**0.5415**	**0.4755**	**0.5019**	**0.4629**	**0.5660**	0.5805	**0.5376**	**0.5275**	**0.4963**	**0.5150**
Year2008	**0.5396**	**0.5509**	**0.4778**	**0.4797**	**0.5387**	0.5880	**0.5318**	0.5633	0.5919	**0.5177**
Year2009	**0.4900**	**0.5003**	**0.4764**	**0.4882**	0.3451	**0.5081**	**0.4734**	0.5750	**0.4885**	**0.5168**
	LII	**NXT**	**JMAT**	**BAY**	**IAP**	**SVT**	**HMSO**	**SGE**	**REX**	**IHG**
Full Sample	**0.4777**	**0.5025**	**0.5579**	**0.4631**	**0.5295**	0.5708	**0.5054**	**0.5503**	**0.5158**	**0.5225**
Year2007	**0.5076**	**0.5134**	0.5721	**0.4475**	0.5814	**0.5632**	**0.5579**	0.6206	0.5920	**0.5344**
Year2008	**0.4789**	**0.4967**	0.5828	**0.4816**	**0.5395**	0.5971	**0.5225**	0.5802	**0.5054**	**0.5265**
Year2009	**0.4569**	**0.5018**	**0.5270**	**0.4656**	**0.4809**	**0.5386**	**0.4578**	**0.4876**	**0.4895**	**0.5089**

Table 5

Invariance Coefficients for pre-crisis (Jan 2007- Jun 2008) and in-crisis (Jul 2008-Dec 2009) periods

This table reports the invariance proportionalities estimates of OLS regressions for 70 FTSE 100 stocks based on the model (25), for pre-crisis (Jan 2007- Jun 2008) and in-crisis (Jul 2008-Dec 2009) periods

	BP	**HSBA**	**VOD**	**GSK**	**RDSA**	**RIO**	**AZN**
Pre-Crisis	0.5614	0.5417	0.4514	0.5649	**0.5203**	**0.4885**	**0.5141**
In-Crisis	**0.5389**	**0.5115**	**0.5012**	**0.5286**	0.5616	**0.5299**	**0.5044**
	RBS	**BATS**	**BG**	**AAL**	**BLT**	**BARC**	**TSCO**
Pre-Crisis	**0.4767**	**0.5312**	**0.4914**	**0.4634**	0.5386	**0.5009**	**0.4730**
In-Crisis	0.3948	**0.5102**	**0.4882**	**0.5215**	0.5823	**0.4956**	**0.4705**
	XTA	**DGE**	**LLOY**	**STAN**	**ULVR**	**RB**	**SAB**
Pre-Crisis	0.4474	**0.4732**	**0.5015**	**0.4732**	**0.5280**	**0.4927**	**0.5125**
In-Crisis	**0.4987**	**0.5465**	**0.4754**	**0.4797**	**0.5492**	**0.4743**	**0.4922**
	NG	**IMT**	**BT**	**AV**	**PRU**	**BAE**	**CNA**
Pre-Crisis	**0.5199**	**0.4992**	**0.5102**	**0.5135**	**0.5263**	**0.4528**	**0.5288**
In-Crisis	**0.5269**	**0.4994**	0.4022	0.4674	**0.5143**	**0.4756**	**0.4958**
	SSE	**CBRY**	**BSY**	**EMG**	**RR**	**MRW**	**MKS**
Pre-Crisis	**0.5102**	0.5614	0.5632	**0.4881**	**0.500**	**0.5105**	**0.5051**
In-Crisis	**0.5300**	0.5583	**0.5229**	**0.4562**	**0.4935**	**0.5232**	**0.5081**
	SBRY	**WPP**	**REL**	**LGEN**	**CPG**	**ABF**	**LAND**
Pre-Crisis	**0.5020**	**0.5423**	**0.5493**	**0.5066**	**0.5108**	0.6198	**0.4938**
In-Crisis	**0.5060**	**0.5094**	**0.5114**	0.4308	**0.5023**	**0.5152**	**0.4954**
	OML	**ANTO**	**PSON**	**SHP**	**SL**	**IPR**	**KAZ**
Pre-Crisis	**0.4504**	**0.4922**	**0.5509**	**0.5200**	**0.5560**	**0.5114**	**0.4640**
In-Crisis	0.3807	**0.5040**	**0.5369**	**0.5318**	**0.4653**	**0.4995**	**0.4690**
	UU	**SN**	**EXPN**	**BLND**	**VED**	**RSA**	**CPI**
Pre-Crisis	0.5769	**0.5538**	**0.5146**	**0.501**	**0.4744**	0.5807	0.5973
In-Crisis	**0.5363**	**0.5037**	**0.5135**	**0.4742**	**0.4807**	0.3999	**0.5306**
	KGF	**CCL**	**CW**	**SMIN**	**LII**	**NXT**	**JMAT**
Pre-Crisis	**0.5410**	**0.5419**	**0.5359**	**0.5293**	**0.5007**	**0.5085**	0.5928
In-Crisis	**0.4956**	0.5741	**0.5267**	**0.5112**	**0.4641**	**0.5012**	**0.5348**
	BAY	**IAP**	**SVT**	**HMSO**	**SGE**	**REX**	**IHG**
Pre-Crisis	**0.4528**	0.5798	0.5868	**0.5602**	0.6293	0.5775	**0.5497**
In-Crisis	**0.4755**	**0.4955**	0.5589	**0.4722**	**0.5064**	**0.4906**	**0.5072**

Appendix

Table A1

Table A1- Stocks and their abbreviations

LARGE CAPITALISATION STOCKS	
Stock	**Abbreviation**
BP	**BP**
HSBC HOLDINGS	**HSBA**
VODAFONE	**VOD**
GLAXOSMITHKLINE	**GSK**
ROYAL DUTCH SHELL	**RDSA**
MEDIUM CAPITALISATION STOCKS	
Stock	**Abbreviation**
RIO TINTO	**RIO**
ASTRAZENECA	**AZN**
ROYAL BANK OF SCOTLAND GROUP	**RBS**
BRITISH AMERICAN TOBACCO	**BATS**
BG GROUP	**BG**
ANGLO AMERICAN	**AAL**
BHP BILLITON	**BLT**
BARCLAYS	**BARC**
TESCO	**TSCO**
XSTRATA	**XTA**
DIAGEO	**DGE**
LLOYDS TSB GROUP	**LLOY**
STANDARD CHARTERED	**STAN**
LOW CAPITALISATION STOCKS	
Stock	**Abbreviation**
UNILEVER	**ULVR**
RECKITT BENCKISER	**RB**
SABMILLER	**SAB**
NATIONAL GRID PLC	**NG**
IMPERIAL TOBACCO GROUP PLC	**IMT**
BT GROUP	**BT**
AVIVA PLC	**AV**
PRUDENTIAL PLC	**PRU**
BAE SYSTEMS PLC	**BAE**
CENTRICA PLC	**CNA**
SCOTTISH & SOUTHERN ENERGY	**SSE**
CADBURY SCHWEPPES	**CBRY**
BSB GROUP	**BSY**
MAN GROUP PLC	**EMG**
ROLLS-ROYCE HOLDINGS PLC	**RR**
MORRISON (WM) SUPERMARKETS	**MRW**
MARKS & SPENCER GROUP	**MKS**
SAINSBURY (J)	**SBRY**
WPP PLC	**WPP**
REED ELSEVIER	**REL**
LEGAL & GENERAL GROUP	**LGEN**
COMPASS GROUP	**CPG**
ASSOCIATED BRITISH FOODS	**ABF**
LAND SECURITIES GROUP	**LAND**

OLD MUTUAL PLC	**OML**
ANTOFAGASTA	**ANTO**
PEARSON	**PSON**
SHIRE PLC	**SHP**
STANDARD LIFE	**SL**
INTERNATIONAL POWER PLC	**IPR**
KAZAKHMYS	**KAZ**
UNITED UTILITIES	**UU**
SMITH & NEPHEW	**SN**
EXPERIAN GROUP	**EXPN**
BRITISH LAND CO PLC	**BLND**
VEDANTA RESOURCES	**VED**
ROYAL & SUN ALLIANCE INS.	**RSA**
CAPITA GROUP	**CPI**
KINGFISHER	**KGF**
CARNIVAL PLC	**CCL**
CABLE AND WIRELESS	**CW**
SMITHS GROUP	**SMIN**
LIBERTY INTERNATIONAL	**LII**
NEXT	**NXT**
JOHNSON MATTHEY PLC	**JMAT**
BRITISH AIRWAYS	**BAY**
ICAP	**IAP**
SEVERN TRENT PLC	**SVT**
HAMMERSON	**HMSO**
SAGE GROUP PLC	**SGE**
REXAM PLC	**REX**
INTERCONTINENTAL HOTELS GROUP	**IHG**

Source: Thomson Reuters Tick History

Table A2

OLS regression results (5-minutes averages across days)

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity based on the models in (32), (33), (34) and (35) when underlying variables are estimated as averages across days based on equation (23). The different notions of trading activity are those introduced by Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, (Ané and Geman (2000)) (Model 4). The volume multiplier is assumed to be $\zeta = 2$ and the volatility multiplier $\psi = 1$. Coefficients are tested against the null hypothesis $H_0 : \beta = 2/3$. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

		Model 1			Model 2			Model 3			Model 4		
	Stocks	***c***	β	$\bar{R}^2$	***c***	β	$\bar{R}^2$	***c***	β	$\bar{R}^2$	***c***	β	$\bar{R}^2$
High Market Capitalization	BP	0.2504	0.5332***	0.8896	1.1362***	0.5347***	0.8902	7.5877***	**0.6690**	0.9399	4.0743***	0.5623***	0.8439
		(0.1462)	(0.0187)		(0.1150)	(0.0187)		(0.0804)	(0.0168)		(0.0154)	(0.0240)	
	HSBA	0.3935*	0.5160***	0.8765	1.4039***	0.5196***	0.8766	7.2885***	**0.6594**	0.9578	4.6022***	0.6110***	0.9378
		(0.1520)	(0.0193)		(0.1144)	(0.0194)		(0.0597)	(0.0138)		(0.0096)	(0.0157)	
	VOD	0.8486***	0.4764***	0.8122	1.0129***	0.4774***	0.8126	8.5409***	0.6199*	0.9028	5.5183***	0.5682***	0.8746
		(0.1725)	(0.0228)		(0.1645)	(0.0228)		(0.1342)	(0.0202)		(0.0423)	(0.0214)	
	GSK	0.2551	0.5404***	0.8525	1.5974***	0.5416***	0.8525	6.5978***	**0.6525**	0.9093	4.2107***	0.6144*	0.8939
		(0.1604)	(0.0224)		(0.1052)	(0.0224)		(0.0786)	(0.0205)		(0.0119)	(0.0210)	
	RDSA	-0.1489	0.5415***	0.8932	1.4115***	0.5416***	0.8933	6.0388***	**0.6755**	0.9468	3.6655***	0.6285*	0.9330
		(0.1155)	(0.0186)		(0.0625)	(0.0186)		(0.0679)	(0.0159)		(0.0164)	(0.0168)	
Medium Market Capitalization	RIO	0.3926**	0.5117***	0.8910	2.1812***	0.5121***	0.8915	5.3713***	0.6167***	0.9497	3.4927***	0.5810***	0.9337
		(0.1439)	(0.0178)		(0.0819)	(0.0178)		(0.0214)	(0.0141)		(0.0288)	(0.0154)	
	AZN	0.5922***	0.5072***	0.8036	2.2243***	0.5072***	0.8037	5.8308***	**0.6298**	0.8973	3.8224***	0.5875***	0.8684
		(0.1699)	(0.0249)		(0.0904)	(0.0249)		(0.0614)	(0.0212)		(0.0152)	(0.0227)	
	RBS	1.0986***	0.4327***	0.8549	1.3610***	0.4328***	0.8555	7.2511***	0.5082***	0.8642	4.9764***	0.4816***	0.8632
		(0.1332)	(0.0177)		(0.1222)	(0.0177)		(0.1153)	(0.0200)		(0.0274)	(0.0191)	
	BATS	0.4641**	0.5190***	0.8203	1.9534***	0.5185***	0.8199	5.9586***	**0.6342**	0.8797	3.8753***	0.5944**	0.8629
		(0.1545)	(0.0241)		(0.0859)	(0.0242)		(0.0814)	(0.0233)		(0.0134)	(0.0236)	
	BG	0.5392**	0.4848***	0.7872	1.6493***	0.4831***	0.7872	6.2738***	0.6039**	0.8692	4.0743***	0.5623***	0.8439
		(0.1726)	(0.0251)		(0.1154)	(0.0250)		(0.0936)	(0.0233)		(0.0154)	(0.0240)	
	AAL	0.5623***	0.4941***	0.8507	2.1072***	0.4952***	0.8522	5.5229***	0.5857***	0.9309	3.6712***	0.5562***	0.9085
		(0.1589)	(0.0206)		(0.0943)	(0.0205)		(0.0325)	(0.0159)		(0.0240)	(0.0176)	
	BLT	0.0149	0.5617***	0.8955	1.5071***	0.5618***	0.8963	6.2400***	**0.6488**	0.9645	3.9559***	0.6222**	0.9474
		(0.1547)	(0.0191)		(0.1039)	(0.0190)		(0.0330)	(0.0124)		(0.0167)	(0.0146)	
	BARC	0.5685**	0.4965***	0.8390	1.1534***	0.5050***	0.8423	7.1882***	0.5888***	0.9115	4.7257***	0.5585***	0.8910
		(0.1721)	(0.0216)		(0.1449)	(0.0217)		(0.0836)	(0.0183)		(0.0132)	(0.0194)	
	TSCO	0.9498***	0.4678***	0.7776	1.5828***	0.4694***	0.7781	7.2622***	0.5970***	0.8574	4.8089***	0.5505***	0.8321

		(0.1631)	(0.0249)		(0.1295)	(0.0249)		(0.1326)	(0.0242)		(0.0383)	(0.0246)	
	XTA	0.6532***	0.4757***	0.8488	2.0083***	0.4770***	0.8516	5.7101***	0.5903***	0.9150	3.7701***	0.5499***	0.8953
		(0.1560)	(0.0200)		(0.0983)	(0.0198)		(0.0420)	(0.0179)		(0.0225)	(0.0187)	
	DGE	0.4703**	0.5166***	0.8280	1.6331***	0.5190***	0.8290	6.6387***	**0.6711**	0.8856	4.2144***	0.6159*	0.8710
		(0.1486)	(0.0234)		(0.0960)	(0.0234)		(0.1045)	(0.0240)		(0.0222)	(0.0236)	
	LLOY	0.7095***	0.4836***	0.7973	0.9860***	0.5024***	0.8032	7.3995***	0.5899***	0.8822	4.8709***	0.5545***	0.8576
		(0.1762)	(0.0242)		(0.1594)	(0.0247)		(0.1162)	(0.0214)		(0.0289)	(0.0225)	
	STAN	0.6020***	0.4742***	0.8261	1.8347***	0.4761***	0.8273	5.8269***	0.5927***	0.9198	3.8172***	0.5524***	0.8925
		(0.1482)	(0.0216)		(0.0919)	(0.0216)		(0.0592)	(0.0174)		(0.0106)	(0.0191)	
Low Market Capitalization	ULVR	0.2352	0.5387***	0.7851	1.7144***	0.5388***	0.7849	6.0208***	**0.6480**	0.8893	3.8507***	0.6124*	0.8579
		(0.1725)	(0.0280)		(0.0963)	(0.0280)		(0.0879)	(0.0227)		(0.0185)	(0.0248)	
	RB	0.7844***	0.4825***	0.7506	2.3830***	0.4825***	0.7505	5.4711***	**0.6375**	0.8470	3.6072***	0.5833**	0.8193
		(0.1606)	(0.0276)		(0.0703)	(0.0276)		(0.0813)	(0.0269)		(0.0147)	(0.0272)	
	SAB	0.4217**	0.4995***	0.7811	1.6673***	0.5004***	0.7815	5.9543***	**0.6223**	0.8502	3.8334***	0.5786**	0.8287
		(0.1570)	(0.0263)		(0.0919)	(0.0263)		(0.1076)	(0.0260)		(0.0239)	(0.0262)	
	NG	0.4093*	0.5240***	0.7755	1.4215***	0.5233***	0.7745	6.7241***	**0.6372**	0.8379	4.3425***	0.5987*	0.8203
		(0.1652)	(0.0280)		(0.1117)	(0.0281)		(0.1422)	(0.0279)		(0.0422)	(0.0279)	
	IMT	0.5763**	0.4984***	0.7456	2.0613***	0.4984***	0.7452	5.7184***	**0.6449**	0.8122	3.7002***	0.5929*	0.7936
		(0.1713)	(0.0289)		(0.0860)	(0.0290)		(0.1064)	(0.0308)		(0.0179)	(0.0301)	
	BT	0.9623***	0.4395***	0.7572	1.1823***	0.4477***	0.7608	7.3064***	0.5368***	0.8267	4.9099***	0.5034***	0.8060
		(0.1505)	(0.0247)		(0.1369)	(0.0249)		(0.1683)	(0.0244)		(0.0646)	(0.0246)	
	AV	0.5555**	0.4858***	0.7691	1.3004***	0.4938***	0.7742	6.5680***	0.5955***	0.8646	4.2888***	0.5583***	0.8351
		(0.1659)	(0.0265)		(0.1238)	(0.0265)		(0.1179)	(0.0234)		(0.0337)	(0.0247)	
	PRU	0.2642	0.5185***	0.8019	1.1255***	0.5224***	0.8044	6.6241***	0.6150*	0.8922	4.2611***	0.5835***	0.8652
		(0.1685)	(0.0256)		(0.1252)	(0.0256)		(0.1031)	(0.0213)		(0.0267)	(0.0229)	
	BAE	0.8742***	0.4621***	0.7649	1.5183***	0.4626***	0.7653	7.0026***	0.6002*	0.8351	4.5962***	0.5499***	0.8136
		(0.1555)	(0.0255)		(0.1201)	(0.0255)		(0.1472)	(0.0265)		(0.0456)	(0.0262)	
	CNA	0.5347**	0.5081***	0.7458	1.0941***	0.5084***	0.7467	7.3172***	**0.6186**	0.8199	4.7626***	0.5820**	0.7992
		(0.1699)	(0.0295)		(0.1374)	(0.0294)		(0.1807)	(0.0288)		(0.0671)	(0.0290)	
	SSE	0.3919*	0.5191***	0.7356	1.7397***	0.5176***	0.7344	6.0697***	**0.6610**	0.8158	3.8680***	**0.6135**	0.7946
		(0.1741)	(0.0309)		(0.0947)	(0.0309)		(0.1314)	(0.0312)		(0.0320)	(0.0310)	
	CBRY	0.0565	0.5606***	0.8243	1.0559***	0.5624***	0.8248	7.0706***	**0.7085**	0.8847	4.3613***	**0.6552**	0.8666
		(0.1486)	(0.0257)		(0.1030)	(0.0258)		(0.1369)	(0.0254)		(0.0447)	(0.0256)	
	BSY	0.2809	0.5379***	0.7802	1.1548***	0.5421***	0.7829	7.0770***	**0.6941**	0.8706	4.4225***	**0.6385**	0.8428
		(0.1559)	(0.0284)		(0.1093)	(0.0284)		(0.1485)	(0.0266)		(0.0538)	(0.0274)	
	EMG	0.5831***	0.4693***	0.7549	1.2215***	0.4727***	0.7576	6.6741***	0.5972*	0.8306	4.3119***	0.5513***	0.8068
		(0.1590)	(0.0266)		(0.1223)	(0.0266)		(0.1487)	(0.0268)		(0.0479)	(0.0268)	
	RR	0.5538***	0.4940***	0.7693	1.2439***	0.4968***	0.7712	6.8617***	**0.6171**	0.8277	4.4432***	0.5732***	0.8102
		(0.1523)	(0.0269)		(0.1144)	(0.0269)		(0.1606)	(0.0280)		(0.0554)	(0.0276)	
	MRW	0.4132*	0.5160***	0.7295	0.9361***	0.5166***	0.7302	7.4854***	**0.6529**	0.8174	4.7583***	0.6052*	0.7903

	(0.1728)	(0.0312)		(0.1411)	(0.0312)		(0.1993)	(0.0307)		(0.0785)	(0.0310)	
MKS	0.4631**	0.5023***	0.7570	1.1132***	0.5095***	0.7623	6.9053***	0.6107*	0.8291	4.4764***	0.5736**	0.8074
	(0.1695)	(0.0283)		(0.1312)	(0.0283)		(0.1561)	(0.0276)		(0.0515)	(0.0278)	
SBRY	0.4816**	0.5008***	0.7451	1.1174***	0.5069***	0.7498	7.0326***	**0.6366**	0.8193	4.4979***	0.5885**	0.7968
	(0.1544)	(0.0291)		(0.1163)	(0.0291)		(0.1832)	(0.0297)		(0.0710)	(0.0295)	
WPP	0.4037*	0.5225***	0.7825	1.2671***	0.5275***	0.7852	6.8596***	**0.6630**	0.8359	4.3589***	**0.6119**	0.8198
	(0.1580)	(0.0274)		(0.1123)	(0.0274)		(0.1530)	(0.0292)		(0.0472)	(0.0285)	
REL	0.4601**	0.5219***	0.7720	1.3477***	0.5239***	0.7729	6.7419***	**0.6405**	0.8420	4.3600***	0.5995*	0.8210
	(0.1554)	(0.0282)		(0.1076)	(0.0282)		(0.1480)	(0.0276)		(0.0505)	(0.0278)	
LGEN	0.7370***	0.4603***	0.6784	0.6717***	0.4684***	0.6869	7.3751***	0.5368***	0.7453	4.9393***	0.5114***	0.7247
	(0.1722)	(0.0315)		(0.1732)	(0.0314)		(0.2407)	(0.0312)		(0.1055)	(0.0313)	
CPG	0.5632**	0.5028***	0.7211	1.1717***	0.5029***	0.7207	7.3629***	**0.6703**	0.8168	4.6590***	**0.6106**	0.7880
	(0.1710)	(0.0311)		(0.1338)	(0.0311)		(0.1911)	(0.0316)		(0.0711)	(0.0315)	
ABF	0.3058*	0.5568***	0.7603	1.4583***	0.5574***	0.7611	6.4131***	**0.6789**	0.8062	4.1117***	**0.6378**	0.7953
	(0.1324)	(0.0311)		(0.0692)	(0.0310)		(0.1840)	(0.0331)		(0.0755)	(0.0322)	
LAND	0.4832*	0.4951***	0.6899	1.6713***	0.4960***	0.6911	5.9818***	**0.6404**	0.8028	3.8281***	0.5898*	0.7670
	(0.1887)	(0.0330)		(0.1100)	(0.0329)		(0.1328)	(0.0316)		(0.0330)	(0.0323)	
OML	1.0858***	0.4066***	0.6906	1.0738***	0.4122***	0.6973	6.9604***	0.4961***	0.7672	4.7446***	0.4653***	0.7432
	(0.1422)	(0.0270)		(0.1407)	(0.0270)		(0.2058)	(0.0272)		(0.0910)	(0.0272)	
ANTO	0.4204*	0.4953***	0.7565	1.3129***	0.4953***	0.7567	6.2035***	**0.6284**	0.8343	3.9625***	0.5799*	0.8085
	(0.1631)	(0.0279)		(0.1131)	(0.0279)		(0.1297)	(0.0278)		(0.0360)	(0.0281)	
PSON	0.3100	0.5408***	0.7397	1.3645***	0.5427***	0.7407	6.5589***	**0.6628**	0.8296	4.1944***	**0.6224**	0.8032
	(0.1689)	(0.0319)		(0.1071)	(0.0319)		(0.1540)	(0.0299)		(0.0535)	(0.0306)	
SHP	0.3222*	0.5252***	0.8283	1.5198***	0.5275***	0.8297	6.2963***	**0.6789**	0.8574	3.9554***	**0.6217**	0.8504
	(0.1262)	(0.0238)		(0.0725)	(0.0238)		(0.1311)	(0.0275)		(0.0396)	(0.0259)	
SL	0.3625**	0.5073***	0.8079	0.7818***	0.5119***	0.8122	6.8661***	**0.6169**	0.8072	4.4168***	0.5787**	0.8121
	(0.1122)	(0.0246)		(0.0908)	(0.0245)		(0.2048)	(0.0300)		(0.0852)	(0.0277)	
IPR	0.5509**	0.5036***	0.7175	1.1670***	0.5036***	0.7180	7.0715***	**0.6316**	0.7864	4.5685***	0.5866*	0.7656
	(0.1661)	(0.0314)		(0.1279)	(0.0313)		(0.2012)	(0.0327)		(0.0771)	(0.0323)	
KAZ	0.5216*	0.4611***	0.7217	1.5267***	0.4642***	0.7255	5.5762***	0.5831***	0.7881	3.6204***	0.5385***	0.7662
	(0.1652)	(0.0284)		(0.1028)	(0.0284)		(0.1240)	(0.0301)		(0.0282)	(0.0296)	
UU	0.3645*	0.5546***	0.7861	1.3778***	0.5557***	0.7875	6.7696***	**0.6715**	0.8686	4.3632***	**0.6320**	0.8437
	(0.1455)	(0.0288)		(0.0932)	(0.0287)		(0.1407)	(0.0260)		(0.0542)	(0.0271)	
SN	0.4287**	0.5218***	0.7742	1.3192***	0.5231***	0.7756	6.7350***	**0.6526**	0.7937	4.3154***	0.6055*	0.7905
	(0.1415)	(0.0280)		(0.0939)	(0.0280)		(0.1877)	(0.0331)		(0.0672)	(0.0310)	
EXPN	0.3629**	0.5140***	0.8141	1.1287***	0.5192***	0.8185	6.8161***	**0.6604**	0.8102	4.3012***	0.6072*	0.8173
	(0.1210)	(0.0244)		(0.0839)	(0.0243)		(0.1895)	(0.0318)		(0.0680)	(0.0285)	
BLND	0.6183**	0.4839***	0.6788	1.5671***	0.4897***	0.6841	6.2882***	**0.6322**	0.7785	4.0538***	0.5787*	0.7459
	(0.1907)	(0.0331)		(0.1249)	(0.0330)		(0.1541)	(0.0335)		(0.0418)	(0.0336)	
VED	0.4278*	0.4777***	0.7564	1.7138***	0.4780***	0.7572	5.3351***	0.5994*	0.8355	3.4438***	0.5555***	0.8094
	(0.1640)	(0.0269)		(0.0919)	(0.0269)		(0.0898)	(0.0265)		(0.0155)	(0.0268)	

RSA	0.6872***	0.4754***	0.6129	0.8317***	0.4767***	0.6144	7.6914***	**0.6136**	0.7329	4.9687***	0.5665**	0.6947
	(0.1848)	(0.0375)		(0.1729)	(0.0375)		(0.2811)	(0.0368)		(0.1302)	(0.0373)	
CPI	0.2848*	0.5592***	0.7774	1.3652***	0.5598***	0.7775	6.6179***	**0.6987**	0.8535	4.1982***	**0.6524**	0.8338
	(0.1365)	(0.0297)		(0.0799)	(0.0298)		(0.1567)	(0.0288)		(0.0627)	(0.0290)	
KGF	0.5359**	0.5084***	0.7321	0.7574***	0.5164***	0.7382	7.6890***	**0.6360**	0.8041	4.9470***	0.5910*	0.7819
	(0.1729)	(0.0305)		(0.1572)	(0.0306)		(0.2113)	(0.0312)		(0.0832)	(0.0310)	
CCL	0.1610	0.5603***	0.8653	1.8201***	0.5609***	0.8653	5.5254***	**0.6838**	0.9028	3.4936***	**0.6387**	0.8910
	(0.1149)	(0.0220)		(0.0518)	(0.0220)		(0.0824)	(0.0223)		(0.0230)	(0.0222)	
CW	0.4006**	0.5368***	0.7740	0.6482***	0.5367***	0.7740	8.1603***	**0.6846**	0.8203	5.1510***	**0.6307**	0.8071
	(0.1455)	(0.0288)		(0.1323)	(0.0288)		(0.2367)	(0.0319)		(0.1019)	(0.0307)	
SMIN	0.3998**	0.5171***	0.7493	1.5518***	0.5186***	0.7519	6.2387***	**0.6795**	0.8330	3.9369***	**0.6228**	0.8099
	(0.1414)	(0.0297)		(0.0758)	(0.0296)		(0.1521)	(0.0303)		(0.0554)	(0.0300)	
LII	0.5881***	0.4777***	0.6650	1.5317***	0.4833***	0.6721	5.9270***	**0.6078**	0.7321	3.8587***	0.5615**	0.7116
	(0.1635)	(0.0337)		(0.0962)	(0.0335)		(0.1829)	(0.0365)		(0.0633)	(0.0355)	
NXT	0.4467*	0.5025***	0.7210	1.8032***	0.5049***	0.7219	5.6150***	**0.6279**	0.8062	3.6302***	0.5837**	0.7789
	(0.1760)	(0.0311)		(0.0928)	(0.0311)		(0.1147)	(0.0306)		(0.0243)	(0.0309)	
JMAT	0.2370	0.5579***	0.7668	1.7319***	0.5581***	0.7677	5.5362***	**0.6460**	0.8156	3.5974***	**0.6170**	0.8021
	(0.1438)	(0.0306)		(0.0634)	(0.0305)		(0.1287)	(0.0305)		(0.0415)	(0.0305)	
BAY	0.7084***	0.4631***	0.7361	1.1002***	0.4687***	0.7406	6.9734***	0.5926*	0.7937	4.5332***	0.5441***	0.7742
	(0.1591)	(0.0275)		(0.1344)	(0.0276)		(0.1833)	(0.0300)		(0.0645)	(0.0292)	
IAP	0.3433*	0.5295***	0.7832	1.1350***	0.5315***	0.7851	6.4952***	**0.6366**	0.7801	4.1918***	0.5991*	0.7849
	(0.1314)	(0.0277)		(0.0901)	(0.0276)		(0.1938)	(0.0336)		(0.0725)	(0.0312)	
SVT	0.2384	0.5708***	0.7798	1.6862***	0.5704***	0.7803	5.9525***	**0.6849**	0.8476	3.8170***	**0.6470**	0.8285
	(0.1391)	(0.0301)		(0.0640)	(0.0301)		(0.1316)	(0.0289)		(0.0464)	(0.0293)	
HMSO	0.3876*	0.5054***	0.7185	1.3862***	0.5094***	0.7236	6.1154***	**0.6409**	0.7692	3.8991***	0.5922*	0.7546
	(0.1610)	(0.0314)		(0.0984)	(0.0313)		(0.1726)	(0.0349)		(0.0561)	(0.0336)	
SGE	0.3446*	0.5503***	0.7663	0.7373***	0.5539***	0.7700	7.5711***	**0.6767**	0.8218	4.8339***	**0.6329**	0.8063
	(0.1346)	(0.0302)		(0.1122)	(0.0301)		(0.2226)	(0.0313)		(0.1018)	(0.0308)	
REX	0.4614***	0.5158***	0.7659	1.1337***	0.5205***	0.7713	6.8004***	**0.6448**	0.8004	4.3761***	0.6004*	0.7942
	(0.1292)	(0.0283)		(0.0914)	(0.0282)		(0.1997)	(0.0320)		(0.0819)	(0.0304)	
IHG	0.2621	0.5225***	0.7825	1.3365***	0.5266***	0.7860	6.0947***	**0.6484**	0.7932	3.8649***	0.6022*	0.7917
	(0.1369)	(0.0274)		(0.0805)	(0.0273)		(0.1653)	(0.0329)		(0.0542)	(0.0307)	

Table A3

OLS regression results (5-minutes intraday averages)

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity based on the models in (32), (33), (34) and (35) when underlying variables are estimated as intraday averages based on equation (24). The different notions of trading activity are those introduced by Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, (Ané and Geman (2000)) (Model 4). The volume multiplier is assumed to be $\zeta = 2$ and the volatility multiplier $\psi = 1$. Coefficients are tested against the null hypothesis $H_0 : \beta = 2/3$. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

		Model 1			Model 2			Model 3			Model 4		
	Stocks	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$
High Market Capitalization	BP	3.0513***	0.1721***	0.1487	3.2228***	0.1913***	0.1954	6.5533***	0.4499***	0.8030	4.6915***	0.5052***	0.7476
		(0.1178)	(0.0150)		(0.0876)	(0.0141)		(0.0393)	(0.0081)		(0.0090)	(0.0107)	
	HSBA	1.1397***	0.4195***	0.4783	1.8247***	0.4463***	0.6504	6.5743***	0.4925***	0.8445	4.5888***	0.5588***	0.8515
		(0.1262)	(0.0160)		(0.0706)	(0.0119)		(0.0340)	(0.0077)		(0.0069)	(0.0085)	
	VOD	2.7244***	0.2244***	0.2851	2.6401***	0.2472***	0.3462	7.2375***	0.4228***	0.8004	5.2557***	0.4365***	0.7362
		(0.0990)	(0.0129)		(0.0903)	(0.0124)		(0.0514)	(0.0077)		(0.0193)	(0.0095)	
	GSK	1.9048***	0.3084***	0.5030	2.5938***	0.3259***	0.5601	5.8938***	0.4661***	0.8499	4.1930***	0.5016***	0.8827
		(0.0805)	(0.0112)		(0.0499)	(0.0105)		(0.0277)	(0.0071)		(0.0046)	(0.0067)	
	RDSA	0.6979***	0.4035***	0.6186	1.8806***	0.3975***	0.6258	5.0143***	0.4321***	0.8030	3.5650***	0.4948***	0.8661
		(0.0717)	(0.0115)		(0.0379)	(0.0112)		(0.0337)	(0.0078)		(0.0078)	(0.0071)	
Medium Market Capitalization	RIO	0.7446***	0.4681***	0.6586	2.5806***	0.4245***	0.6792	5.0208***	0.3645***	0.6670	3.7341***	0.4438***	0.7497
		(0.0993)	(0.0123)		(0.0492)	(0.0106)		(0.0157)	(0.0094)		(0.0181)	(0.0093)	
	AZN	1.5253***	0.3695***	0.6233	2.7992***	0.3459***	0.6016	5.2414***	0.4225***	0.8216	3.8617***	0.4804***	0.8931
		(0.07160	(0.0105)		(0.0377)	(0.0103)		(0.0213)	(0.0072)		(0.0050)	(0.0061)	
	RBS	1.9233***	0.3218***	0.7169	2.8519***	0.2148***	0.3503	5.3023***	0.1693***	0.4017	4.6192***	0.2163***	0.5030
		(0.0560)	(0.0074)		(0.0747)	(0.0107)		(0.0457)	(0.0075)		(0.0170)	(0.0078)	
	BATS	0.7738***	0.4692***	0.7355	2.2105***	0.4426***	0.6853	5.3492***	0.4570***	0.8966	3.8604***	0.5094***	0.9327
		(0.0658)	(0.0103)		(0.0393)	(0.0109)		(0.0204)	(0.0057)		(0.0045)	(0.0050)	
	BG	0.8335***	0.4413***	0.7425	2.0145***	0.4023***	0.6152	5.1747***	0.3280***	0.7667	4.0178***	0.4074***	0.8635
		(0.0653)	(0.0095)		(0.0538)	(0.0116)		(0.0273)	(0.0066)		(0.0056)	(0.0059)	
	AAL	0.6326***	0.4849***	0.7209	2.3654***	0.4386***	0.7841	5.3283***	0.4872***	0.8423	3.6984***	0.5344***	0.8808
		(0.0850)	(0.0110)		(0.0388)	(0.0084)		(0.0163)	(0.0077)		(0.0103)	(0.0072)	
	BLT	0.9224***	0.4491***	0.6098	2.5438***	0.3702***	0.4954	5.6137***	0.4075***	0.8429	4.0733***	0.5003***	0.9165
		(0.1062)	(0.0131)		(0.0746)	(0.0136)		(0.0178)	(0.0064)		(0.0069)	(0.0055)	
	BARC	0.8613***	0.4588***	0.6608	1.9560***	0.3840***	0.6923	7.1457***	0.5802***	0.7540	4.7234***	0.5638***	0.7591
		(0.0955)	(0.0120)		(0.0626)	(0.0093)		(0.0557)	(0.0121)		(0.0097)	(0.0116)	
	TSCO	1.6283***	0.3627***	0.6134	2.0751***	0.3727***	0.6835	6.4577***	0.4494***	0.8525	4.6944***	0.4734***	0.8658

		(0.0691)	(0.0105)		(0.0483)	(0.0092)		(0.0377)	(0.0068)		(0.0112)	(0.0068)	
	XTA	0.4033***	0.5077***	0.6483	2.5555***	0.3660***	0.6660	5.1170***	0.3310***	0.4280	3.9136***	0.4161***	0.5359
		(0.1066)	(0.0136)		(0.0475)	(0.0094)		(0.0342)	(0.0139)		(0.0187)	(0.0141)	
	DGE	1.1498***	0.4076***	0.6858	2.0242***	0.4207***	0.7472	5.5926***	0.4294***	0.8299	4.1007***	0.4730***	0.8750
		(0.0639)	(0.0101)		(0.0367)	(0.0089)		(0.0314)	(0.0071)		(0.0071)	(0.0065)	
	LLOY	0.6314***	0.4923***	0.5836	1.7304***	0.3863***	0.7080	5.5663***	0.2538***	0.1973	4.6110***	0.3497***	0.3196
		(0.1103)	(0.0152)		(0.0586)	(0.0090)		(0.1020)	(0.0186)		(0.0272)	(0.0186)	
	STAN	0.4513***	0.4955***	0.7796	1.9215***	0.4544***	0.7650	5.4943***	0.4930***	0.8455	3.8181***	0.5467***	0.9120
		(0.0659)	(0.0096)		(0.0393)	(0.0092)		(0.0268)	(0.0077)		(0.0050)	(0.0062)	
Low Market Capitalization	ULVR	0.6573***	0.4687***	0.6919	2.0168***	0.4472***	0.6721	5.2751***	0.4527***	0.8334	3.8118***	0.5343***	0.9189
		(0.0704)	(0.0114)		(0.0396)	(0.0114)		(0.0292)	(0.0074)		(0.0055)	(0.0058)	
	RB	0.8102***	0.4775***	0.7893	2.4465***	0.4553***	0.7713	4.9285***	0.4551***	0.8566	3.6022***	0.5059***	0.9137
		(0.0525)	(0.0090)		(0.0235)	(0.0090)		(0.0212)	(0.0068)		(0.0047)	(0.0057)	
	SAB	0.5275***	0.4805***	0.7516	1.9332***	0.4213***	0.7122	5.4627***	0.5026***	0.8417	3.8220***	0.5637***	0.9235
		(0.0602)	(0.0101)		(0.0345)	(0.0098)		(0.0335)	(0.0079)		(0.0063)	(0.0059)	
	NG	0.9896***	0.4251***	0.7868	1.7342***	0.4437***	0.8114	5.5476***	0.4051***	0.7513	4.1412***	0.4557***	0.8447
		(0.0475)	(0.0081)		(0.0311)	(0.0078)		(0.0440)	(0.0085)		(0.0116)	(0.0071)	
	IMT	0.5419***	0.5038***	0.7812	2.0730***	0.4935***	0.8144	5.2798***	0.5164***	0.8473	3.6884***	0.5539***	0.8923
		(0.0577)	(0.0097)		(0.0260)	(0.0086)		(0.0283)	(0.0080)		(0.0062)	(0.0070)	
	BT	2.3158***	0.2149***	0.4883	2.1673***	0.2669***	0.5767	5.1095***	0.2173***	0.4227	4.1829***	0.2210***	0.4540
		(0.0496)	(0.0080)		(0.0462)	(0.0083)		(0.0640)	(0.0092)		(0.0243)	(0.0088)	
	AV	1.1058***	0.3972***	0.6649	1.9628***	0.3514***	0.6802	5.9325***	0.4681***	0.6486	4.1841***	0.4744***	0.7031
		(0.0646)	(0.0103)		(0.0412)	(0.0088)		(0.0635)	(0.0126)		(0.0112)	(0.0112)	
	PRU	0.5489***	0.4745***	0.8020	1.5298***	0.4389***	0.8308	6.1249***	0.5110***	0.8084	4.2384***	0.5608***	0.9076
		(0.0566)	(0.0086)		(0.0355)	(0.0072)		(0.0443)	(0.0091)		(0.0082)	(0.0065)	
	BAE	1.3930***	0.3759***	0.6345	1.7609***	0.4097***	0.7069	5.7830***	0.3796***	0.7544	4.3777***	0.4188***	0.7907
		(0.0637)	(0.0104)		(0.0456)	(0.0096)		(0.0443)	(0.0079)		(0.0146)	(0.0079)	
	CAN	1.1248***	0.4041***	0.7527	1.4861***	0.4226***	0.7940	6.0763***	0.4205***	0.7649	4.4268***	0.4347***	0.7972
		(0.0489)	(0.0084)		(0.0368)	(0.0078)		(0.0536)	(0.0085)		(0.0191)	(0.0080)	
	SSE	0.7836***	0.4488***	0.8353	1.8584***	0.4772***	0.8527	5.1582***	0.4433***	0.7947	3.7495***	0.4839***	0.8786
		(0.0408)	(0.0073)		(0.0222)	(0.0072)		(0.0352)	(0.0082)		(0.0078)	(0.0066)	
	CBRY	1.0122***	0.3939***	0.7235	1.7623***	0.3834***	0.7355	5.7650***	0.4637***	0.6988	4.1629***	0.5320***	0.8605
		(0.0513)	(0.0089)		(0.0336)	(0.0084)		(0.0603)	(0.0111)		(0.0142)	(0.0078)	
	BSY	0.8192***	0.4359***	0.6824	1.5301***	0.4400***	0.7588	6.1623***	0.5296***	0.8195	4.3095***	0.5800***	0.9013
		(0.0596)	(0.0108)		(0.0350)	(0.0090)		(0.0513)	(0.0091)		(0.0145)	(0.0070)	
	EMG	1.1180***	0.3785***	0.6949	1.5451***	0.4013***	0.7745	5.8849***	0.4544***	0.7505	4.1547***	0.4601***	0.7883
		(0.0551)	(0.0091)		(0.0367)	(0.0079)		(0.0533)	(0.0096)		(0.0164)	(0.0087)	
	RR	1.2317***	0.3724***	0.6608	1.8629***	0.3488***	0.6797	5.9374***	0.4551***	0.7227	4.2683***	0.4835***	0.7979
		(0.0553)	(0.0097)		(0.0375)	(0.0087)		(0.0593)	(0.0103)		(0.0182)	(0.0089)	
	MRW	1.2074***	0.3703***	0.6908	1.5641***	0.3749***	0.7072	6.1222***	0.4422***	0.7436	4.4238***	0.4710***	0.8211

	(0.0503)	(0.0090)		(0.0402)	(0.0088)		(0.0619)	(0.0095)		(0.0209)	(0.0080)	
MKS	1.1892***	0.3792***	0.6957	1.8138***	0.3569***	0.7144	6.7046***	0.5758***	0.8107	4.3708***	0.5173***	0.8020
	(0.0552)	(0.0092)		(0.0386)	(0.0082)		(0.0576)	(0.0101)		(0.0179)	(0.0094)	
SBRY	1.4151***	0.3197***	0.5770	1.6426***	0.3706***	0.7136	5.9587***	0.4616***	0.7024	4.2210***	0.4731***	0.7648
	(0.0537)	(0.0100)		(0.0348)	(0.0086)		(0.0681)	(0.0110)		(0.0237)	(0.0096)	
WPP	0.7870***	0.4542***	0.6315	1.5310***	0.4615***	0.7893	6.2372***	0.5437***	0.8308	4.2861***	0.5669***	0.8405
	(0.0730)	(0.0126)		(0.0357)	(0.0087)		(0.0472)	(0.0089)		(0.0154)	(0.0090)	
REL	1.0670***	0.4081***	0.5184	1.5819***	0.4579***	0.6299	5.6892***	0.4437***	0.7268	4.1551***	0.4844***	0.7344
	(0.0794)	(0.0143)		(0.0491)	(0.0128)		(0.0540)	(0.0099)		(0.0205)	(0.0106)	
LGEN	1.0485***	0.4001***	0.6163	1.4357***	0.3280***	0.4542	6.5856***	0.4353***	0.5437	4.7245***	0.4499***	0.6057
	(0.0635)	(0.0115)		(0.0727)	(0.0131)		(0.1123)	(0.0145)		(0.0449)	(0.0132)	
CPG	1.0523***	0.4115***	0.6987	1.6876***	0.3794***	0.6627	5.8328***	0.4163***	0.7363	4.3461***	0.4681***	0.8171
	(0.0544)	(0.0099)		(0.0428)	(0.0099)		(0.0557)	(0.0091)		(0.0190)	(0.0081)	
ABF	0.8908***	0.4178***	0.7610	1.7726***	0.4107***	0.7449	5.3716***	0.4902***	0.7046	3.8375***	0.5166***	0.8088
	(0.0365)	(0.0085)		(0.0199)	(0.0088)		(0.0648)	(0.0116)		(0.0220)	(0.0092)	
LAND	0.9391***	0.4148***	0.7577	1.8647***	0.4370***	0.7922	4.7773***	0.3525***	0.6100	3.6493***	0.3895***	0.6867
	(0.0491)	(0.0086)		(0.0275)	(0.0082)		(0.0439)	(0.0103)		(0.0116)	(0.0096)	
OML	1.4002***	0.3434***	0.5520	1.7479***	0.2804***	0.4401	6.1391***	0.3882***	0.5788	4.5076***	0.3962***	0.6063
	(0.0599)	(0.0113)		(0.0607)	(0.0115)		(0.0918)	(0.0121)		(0.0394)	(0.0116)	
ANTO	0.7058***	0.4456***	0.7103	1.9279***	0.3405***	0.5206	5.0519***	0.3794***	0.7170	3.8216***	0.4572***	0.8191
	(0.0606)	(0.0104)		(0.0485)	(0.0119)		(0.0410)	(0.0087)		(0.0108)	(0.0078)	
PSON	0.7991***	0.4477***	0.7705	1.7477***	0.4268***	0.7890	5.7842***	0.5114***	0.8149	4.0512***	0.5352***	0.8758
	(0.0473)	(0.0089)		(0.0273)	(0.0080)		(0.0462)	(0.0089)		(0.0134)	(0.0073)	
SHP	0.9805***	0.3981***	0.7241	1.8604***	0.4105***	0.7737	5.7153***	0.5566***	0.7333	3.8713***	0.5648***	0.8387
	(0.0477)	(0.0090)		(0.0249)	(0.0081)		(0.0586)	(0.0122)		(0.0142)	(0.0090)	
SL	0.7192***	0.4269***	0.7534	1.1130***	0.4212***	0.7809	5.7365***	0.4509***	0.7165	4.0938***	0.4724***	0.7783
	(0.0410)	(0.0089)		(0.0306)	(0.0081)		(0.0714)	(0.0103)		(0.0292)	(0.0092)	
IPR	0.9568***	0.4258***	0.7505	1.5142***	0.4168***	0.7052	5.9841***	0.4542***	0.7613	4.3080***	0.4752***	0.8103
	(0.0475)	(0.0090)		(0.0403)	(0.0098)		(0.0573)	(0.0093)		(0.0206)	(0.0084)	
KAZ	0.1298***	0.5281***	0.7865	1.9836***	0.3351***	0.5444	4.8814***	0.4139***	0.6732	3.5808***	0.4914***	0.7769
	(0.0583)	(0.0100)		(0.0411)	(0.0112)		(0.0441)	(0.0105)		(0.0107)	(0.0096)	
UU	1.3073***	0.3658***	0.6952	1.7706***	0.4313***	0.7827	5.2823***	0.3953***	0.6993	3.9437***	0.4138***	0.7503
	(0.0449)	(0.0088)		(0.0271)	(0.0083)		(0.0516)	(0.0094)		(0.0181)	(0.0087)	
SN	0.9488***	0.4163***	0.7292	1.5722***	0.4445***	0.7943	5.8124***	0.4900***	0.8249	4.0971***	0.5034***	0.8590
	(0.0469)	(0.0092)		(0.0279)	(0.0082)		(0.0472)	(0.0082)		(0.0168)	(0.0074)	
EXPN	0.8978***	0.4033***	0.6810	1.5737***	0.3870***	0.7121	5.9168***	0.5092***	0.7574	4.1868***	0.5589***	0.8690
	(0.0501)	(0.0101)		(0.0313)	(0.0090)		(0.0631)	(0.0105)		(0.0193)	(0.0079)	
BLND	1.1965***	0.3821***	0.6554	1.6694***	0.4623***	0.8273	5.2802***	0.4126***	0.6649	3.8645***	0.4160***	0.6848
	(0.0586)	(0.0101)		(0.0293)	(0.0077)		(0.0496)	(0.0107)		(0.0142)	(0.0103)	
VED	0.1273***	0.5269***	0.8073	1.8463***	0.4375***	0.6471	4.7576***	0.4273***	0.7809	3.4308***	0.4963***	0.8582
	(0.0572)	(0.0094)		(0.0407)	(0.0118)		(0.0288)	(0.0083)		(0.0063)	(0.0074)	

RSA	1.7177***	0.2627***	0.5713	1.7505***	0.2741**	0.5892	4.9855***	0.2587***	0.4577	3.9644***	0.2765***	0.5266
	(0.0418)	(0.0083)		(0.0394)	(0.0083)		(0.0795)	(0.0103)		(0.0349)	(0.0096)	
CPI	0.7089***	0.4650***	0.8155	1.6396***	0.4532***	0.8096	5.4013***	0.4739***	0.8333	3.9081***	0.5132***	0.9017
	(0.0371)	(0.0081)		(0.0218)	(0.0080)		(0.0428)	(0.0077)		(0.0142)	(0.0062)	
KGF	1.2444***	0.3786***	0.5967	1.6478***	0.3394***	0.6485	7.0319***	0.5394***	0.7919	4.8197***	0.5465***	0.8222
	(0.0646)	(0.0113)		(0.0473)	(0.0091)		(0.0687)	(0.0101)		(0.0254)	(0.0093)	
CCL	0.8410***	0.4284***	0.7643	2.1611***	0.4059***	0.7871	4.9363***	0.5208***	0.7801	3.4218***	0.5341***	0.8512
	(0.0452)	(0.0087)		(0.0181)	(0.0077)		(0.0371)	(0.0101)		(0.0079)	(0.0081)	
CW	1.2031***	0.3736***	0.6540	1.3308***	0.3834***	0.6725	6.2921***	0.4325***	0.7383	4.5923***	0.4614***	0.7944
	(0.0504)	(0.0099)		(0.0451)	(0.0098)		(0.0703)	(0.0094)		(0.0290)	(0.0086)	
SMIN	0.9824***	0.3923***	0.7684	1.8049***	0.4152***	0.8074	5.0950***	0.4498***	0.6862	3.7051***	0.4901***	0.8187
	(0.0377)	(0.0079)		(0.0194)	(0.0074)		(0.0562)	(0.0111)		(0.0161)	(0.0084)	
LII	1.0915***	0.3721***	0.6582	1.6342***	0.4471***	0.7853	4.9266***	0.4078***	0.6425	3.5996***	0.4130***	0.6773
	(0.0478)	(0.0098)		(0.0247)	(0.0085)		(0.0560)	(0.0111)		(0.0195)	(0.0104)	
NXT	0.7927***	0.4410***	0.8662	2.1801***	0.3761***	0.8117	5.3527***	0.5573***	0.8019	3.6146***	0.5571***	0.8993
	(0.0360)	(0.0063)		(0.0204)	(0.0066)		(0.0383)	(0.0101)		(0.0066)	(0.0068)	
JMAT	0.6334***	0.4723***	0.8227	1.9306***	0.4564***	0.7892	4.9749***	0.5112***	0.7747	3.5204***	0.5538***	0.8812
	(0.0377)	(0.0080)		(0.0183)	(0.0086)		(0.0428)	(0.0101)		(0.0106)	(0.0074)	
BAY	1.1157***	0.3912***	0.7650	1.4486***	0.3966***	0.7490	6.1334***	0.4551***	0.7130	4.3286***	0.4504***	0.7641
	(0.0460)	(0.0079)		(0.0411)	(0.0084)		(0.0646)	(0.0105)		(0.0209)	(0.0091)	
IAP	0.7126***	0.4507***	0.8612	1.4634***	0.4287***	0.8100	5.3659***	0.4394***	0.7935	3.9016***	0.4695***	0.8645
	(0.0316)	(0.0066)		(0.0253)	(0.0076)		(0.0479)	(0.0082)		(0.0169)	(0.0068)	
SVT	0.7854***	0.4502***	0.8254	1.8566***	0.4848***	0.8397	4.8322***	0.4369***	0.7585	3.5590***	0.4734***	0.8375
	(0.0350)	(0.0075)		(0.0168)	(0.0077)		(0.0416)	(0.0090)		(0.0130)	(0.0076)	
HMSO	1.2508***	0.3348***	0.5765	1.7992***	0.3758***	0.7031	4.3937***	0.2920***	0.4434	3.4636***	0.3201***	0.5079
	(0.0539)	(0.0105)		(0.0284)	(0.0089)		(0.0596)	(0.0119)		(0.0206)	(0.0115)	
SGE	1.2614***	0.3393***	0.5879	1.4367***	0.3607***	0.6666	6.2777***	0.4939***	0.7604	4.3625***	0.4894***	0.7851
	(0.0468)	(0.0104)		(0.0352)	(0.0093)		(0.0722)	(0.0101)		(0.0312)	(0.0093)	
REX	0.9682***	0.4004***	0.6488	1.3954***	0.4368***	0.7658	5.3405***	0.4106***	0.5895	3.9437***	0.4392***	0.6577
	(0.0492)	(0.0107)		(0.0287)	(0.0088)		(0.0784)	(0.0125)		(0.0317)	(0.0115)	
IHG	0.8807***	0.3966***	0.7280	1.6849***	0.4054***	0.7897	5.8666***	0.6028***	0.8163	3.7957***	0.5626***	0.8522
	(0.0445)	(0.0088)		(0.0229)	(0.0076)		(0.0525)	(0.0104)		(0.0154)	(0.0085)	

Table A4

OLS regression results when the first and last 10 minutes of active trading are excluded

This table reports the coefficient estimates from OLS regressions for different definitions of trading activity based on the models in in (32), (33), (34) and (35) when the first and last 10 minutes of active trading are excluded. The underlying variables are estimated as averages across days based on equation (23). The different notions of trading activity are those introduced by Kyle and Obizhaeva (2016b) (Model 1), Andersen et al. (2016) (Model 2), MDH-V, Clark (1973) (Model 3) and MDH-N, (Ané and Geman (2000)) (Model 4). The volume multiplier is assumed to be $\zeta = 2$ and the volatility multiplier $\psi = 1$. Coefficients are tested against the null hypothesis $H_0 : \beta = 2/3$. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

		Model 1			Model 2			Model 3			Model 4		
	Stocks	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$
High Market Capitalization	BP	-0.1806	0.5898***	0.9441	0.7984***	0.5916***	0.9449	7.7186***	0.6956*	0.9701	4.7902***	**0.6596**	0.9652
		(0.1131)	(0.0146)		(0.0884)	(0.0145)		(0.0597)	(0.0124)		(0.0101)	(0.0127)	
	HSBA	-0.1338	0.5846***	0.9433	1.0067***	0.5895***	0.9442	7.3810***	**0.6801**	0.9815	4.6191***	**0.6510**	0.9766
		(0.1138)	(0.0146)		(0.0848)	(0.0146)		(0.0415)	(0.0095)		(0.0060)	(0.0102)	
	VOD	0.2882*	0.5527***	0.9083	0.4784***	0.5538***	0.9088	8.7728***	**0.6538**	0.9470	5.6249***	0.6190**	0.9369
		(0.1337)	(0.0178)		(0.1272)	(0.0178)		(0.1047)	(0.0157)		(0.0328)	(0.0163)	
	GSK	-0.3271	0.6240*	0.9384	1.2216***	0.6258*	0.9388	6.7353***	**0.6867**	0.9500	4.2290***	**0.6664**	0.9487
		(0.1154)	(0.0162)		(0.0750)	(0.0162)		(0.0619)	(0.0160)		(0.0083)	(0.0157)	
	RDSA	-0.5152***	0.6031***	0.9486	1.2228***	0.6031***	0.9486	6.1126***	0.6924*	0.9754	3.6974***	**0.6623**	0.9692
		(0.0873)	(0.0143)		(0.0466)	(0.0143)		(0.0480)	(0.0112)		(0.0119)	(0.0120)	
Medium Market Capitalization	RIO	-0.0562	0.5690***	0.9612	1.9340***	0.5690***	0.9612	5.4041***	0.6375**	0.9819	3.4399***	0.6150***	0.9779
		(0.0930)	(0.0116)		(0.0526)	(0.0116)		(0.0135)	(0.0088)		(0.0170)	(0.0094)	
	AZN	0.0609***	0.5879***	0.9088	1.9526***	0.5880***	0.9088	5.9373***	**0.6641**	0.9510	3.8155***	**0.6393**	0.9398
		(0.1276)	(0.0189)		(0.0671)	(0.0189)		(0.0449)	(0.0153)		(0.0098)	(0.0164)	
	RBS	0.7059***	0.4864***	0.9074	1.0111***	0.4850***	0.9055	7.4958***	0.5499***	0.9039	5.0452***	0.5281***	0.9072
		(0.1174)	(0.0158)		(0.1087)	(0.0159)		(0.1056)	(0.0182)		(0.0251)	(0.0171)	
	BATS	-0.0300	0.5992***	0.9293	1.6894***	0.5987***	0.9289	6.0793***	**0.6665**	0.9308	3.8961***	**0.6436**	0.9319
		(0.1062)	(0.0168)		(0.0585)	(0.0168)		(0.0651)	(0.0185)		(0.0095)	(0.0177)	
	BG	0.0004	0.5655***	0.9109	1.2980***	0.5631***	0.9106	6.4281***	**0.6404**	0.9360	4.1052***	0.6152**	0.9297
		(0.1224)	(0.0180)		(0.0816)	(0.0179)		(0.0690)	(0.0170)		(0.0109)	(0.0172)	
	AAL	0.0347	0.5643***	0.9508	1.8030***	0.5648***	0.9512	5.5801***	0.6120***	0.9772	3.6276***	0.5976***	0.9719
		(0.0999)	(0.0130)		(0.0590)	(0.0130)		(0.0197)	(0.0095)		(0.0136)	(0.0103)	
	BLT	-0.4936***	0.6262**	0.9603	1.1738***	0.6256**	0.9603	6.2965***	**0.6689**	0.9882	3.9295***	**0.6574**	0.9835
		(0.1040)	(0.0129)		(0.0698)	(0.0129)		(0.0200)	(0.0074)		(0.0094)	(0.0086)	
	BARC	-0.0062	0.5706***	0.9229	0.6707***	0.5797***	0.9250	7.3499***	0.6230**	0.9570	4.7536***	0.6073***	0.9494
		(0.1321)	(0.0167)		(0.1107)	(0.0168)		(0.0620)	(0.0134)		(0.0095)	(0.0142)	

	TSCO	0.4396***	0.5485***	0.8995	1.1800***	0.5507***	0.9005	7.4951***	**0.6379**	0.9280	4.9035***	0.6068**	0.9202
		(0.1208)	(0.0186)		(0.0952)	(0.0186)		(0.0995)	(0.0180)		(0.0288)	(0.0181)	
	XTA	0.1352	0.5439***	0.9511	1.6921***	0.5438***	0.9517	5.7895***	0.6222***	0.9722	3.7299***	0.5957***	0.9677
		(0.0970)	(0.0125)		(0.0610)	(0.0124)		(0.0254)	(0.0107)		(0.0126)	(0.0110)	
	DGE	0.0299	0.5888***	0.9267	1.3557***	0.5913***	0.9280	6.8257***	0.7119*	0.9397	4.2694***	**0.6706**	0.9420
		(0.1056)	(0.0168)		(0.0673)	(0.0167)		(0.0804)	(0.0183)		(0.0162)	(0.0169)	
	LLOY	0.1472	0.5631***	0.9002	0.4626***	0.5860***	0.9078	7.5835***	0.6228***	0.9345	4.9392***	0.6044***	0.9267
		(0.1371)	(0.0190)		(0.1211)	(0.0190)		(0.0914)	(0.0167)		(0.0227)	(0.0173)	
	STAN	0.0754	0.5536***	0.9371	1.5155***	0.5556***	0.9379	5.9248***	0.6198***	0.9640	3.8255***	0.5985***	0.9583
		(0.0987)	(0.0146)		(0.0606)	(0.0145)		(0.0418)	(0.0122)		(0.0064)	(0.0127)	
Low Market Capitalization	ULVR	-0.3105*	**0.6305**	0.9092	1.4207***	**0.6307**	0.9090	6.1488***	**0.6792**	0.9462	3.8889***	**0.6644**	0.9364
		(0.1232)	(0.0202)		(0.0681)	(0.0203)		(0.0642)	(0.0164)		(0.0131)	(0.0176)	
	RB	0.2880**	0.5719***	0.9139	2.1827***	0.5718***	0.9136	5.5324***	**0.6550**	0.9090	3.6230***	0.6259*	0.9126
		(0.1024)	(0.0178)		(0.0441)	(0.0178)		(0.0641)	(0.0210)		(0.0100)	(0.0197)	
	SAB	-0.0200	0.5765***	0.9346	1.4175***	0.5777***	0.9351	6.1239***	**0.6608**	0.9366	3.8868***	0.6315*	0.9380
		(0.0916)	(0.0155)		(0.0531)	(0.0155)		(0.0729)	(0.0174)		(0.0153)	(0.0165)	
	NG	-0.1167	0.6168*	0.9144	1.0744***	0.6161*	0.9133	6.9536***	**0.6800**	0.9189	4.4434***	**0.6592**	0.9196
		(0.1118)	(0.0192)		(0.0755)	(0.0193)		(0.1054)	(0.0205)		(0.0305)	(0.0198)	
	IMT	0.0714	0.5874***	0.9068	1.8212***	0.5875***	0.9065	5.8301***	**0.6743**	0.8821	3.7306***	**0.6444**	0.8936
		(0.1119)	(0.0191)		(0.0555)	(0.0192)		(0.0872)	(0.0250)		(0.0131)	(0.0226)	
	BT	0.5744***	0.5061***	0.8675	0.8237***	0.5164***	0.8728	7.4990***	0.5637***	0.8828	5.0250***	0.5442***	0.8791
		(0.1207)	(0.0201)		(0.1083)	(0.0200)		(0.1444)	(0.0208)		(0.0547)	(0.0205)	
	AV	0.1422	0.5546***	0.8970	0.9955***	0.5631***	0.9015	6.7015***	0.6204**	0.9315	4.3512***	0.5986***	0.9220
		(0.1185)	(0.0191)		(0.0871)	(0.0189)		(0.0865)	(0.0171)		(0.0246)	(0.0177)	
	PRU	-0.1302	0.5811***	0.9125	0.8369***	0.5850***	0.9146	6.7450***	**0.6382**	0.9514	4.3106***	0.6205**	0.9416
		(0.1190)	(0.0183)		(0.0876)	(0.0181)		(0.0715)	(0.0146)		(0.0187)	(0.0157)	
	BAE	0.4163***	0.5400***	0.8972	1.1692***	0.5406***	0.8976	7.2414***	**0.6416**	0.9146	4.7015***	0.6059**	0.9113
		(0.1121)	(0.0186)		(0.0862)	(0.0185)		(0.1112)	(0.0199)		(0.0340)	(0.0192)	
	CNA	0.0681	0.5929***	0.8988	0.7224***	0.5929***	0.8996	7.5202***	**0.6491**	0.8951	4.8885***	**0.6311**	0.8992
		(0.1152)	(0.0202)		(0.0926)	(0.0201)		(0.1422)	(0.0226)		(0.0504)	(0.0214)	
	SSE	-0.0520	0.6017***	0.9171	1.5106***	0.5999***	0.9157	6.1325***	**0.6734**	0.8910	3.9154***	**0.6497**	0.9031
		(0.1024)	(0.0184)		(0.0555)	(0.0185)		(0.1013)	(0.0239)		(0.0227)	(0.0216)	
	CBRY	-0.4297***	**0.6487**	0.9334	0.7257***	**0.6511**	0.9343	7.3100***	0.7510***	0.9374	4.4755***	0.7152*	0.9381
		(0.1003)	(0.0176)		(0.0687)	(0.0175)		(0.1069)	(0.0197)		(0.0332)	(0.0186)	
	BSY	-0.1137	0.6138**	0.9269	0.8849***	0.6183**	0.9295	7.2088***	0.7157**	0.9411	4.5156***	**0.6802**	0.9386
		(0.0951)	(0.0175)		(0.0655)	(0.0173)		(0.1021)	(0.0182)		(0.0352)	(0.0177)	
	EMG	0.1847	0.5391***	0.9028	0.9194***	0.5428***	0.9054	6.8527***	0.6276*	0.9180	4.4022***	0.5966***	0.9148
		(0.1063)	(0.0180)		(0.0807)	(0.0178)		(0.0190)	(0.0268)		(0.0336)	(0.0185)	
	RR	0.2169*	0.5569***	0.9089	0.9959***	0.5599***	0.9107	7.0244***	**0.6435**	0.9114	4.5339***	0.6130**	0.9123
		(0.1003)	(0.0179)		(0.0746)	(0.0178)		(0.1176)	(0.0204)		(0.0394)	(0.0193)	

MRW	-0.0351	0.6010**	0.8868	0.5746***	0.6015**	0.8876	7.7075***	**0.6852**	0.9041	4.9023***	**0.6570**	0.9011
	(0.1193)	(0.0218)		(0.0970)	(0.0217)		(0.1479)	(0.0227)		(0.0567)	(0.0221)	
MKS	0.0937	0.5669***	0.8775	0.8290***	0.5747***	0.8832	7.0997***	**0.6433**	0.9146	4.5682***	0.6182*	0.9049
	(0.1276)	(0.0215)		(0.0972)	(0.0212)		(0.1137)	(0.0200)		(0.0382)	(0.0203)	
SBRY	0.0844***	0.5801***	0.8917	0.8247***	0.5862***	0.8963	7.2570***	**0.6709**	0.9119	4.6347***	**0.6400**	0.9079
	(0.1074)	(0.0205)		(0.0795)	(0.0202)		(0.1313)	(0.0212)		(0.0504)	(0.0207)	
WPP	0.0089	0.5946***	0.9141	0.9915***	0.6003***	0.9171	7.0572***	**0.6986**	0.9187	4.4508***	**0.6612**	0.9185
	(0.1056)	(0.0185)		(0.0738)	(0.0183)		(0.1113)	(0.0211)		(0.0337)	(0.0200)	
REL	0.0186	0.6058**	0.9150	1.0484***	0.6082**	0.9162	6.9411***	**0.6758**	0.9238	4.4653***	**0.6524**	0.9231
	(0.1021)	(0.0187)		(0.0700)	(0.0187)		(0.1064)	(0.0197)		(0.0353)	(0.0191)	
LGEN	0.4794***	0.5106***	0.8141	0.4176***	0.5177***	0.8203	7.5219***	0.5544***	0.8274	5.0486***	0.5403***	0.8248
	(0.1341)	(0.0248)		(0.1342)	(0.0246)		(0.1993)	(0.0257)		(0.0860)	(0.0253)	
CPG	0.1572***	0.5806***	0.8942	0.8591***	0.5809***	0.8940	7.4758***	**0.6871**	0.8938	4.7599***	**0.6496**	0.8969
	(0.1104)	(0.0203)		(0.0862)	(0.0203)		(0.1463)	(0.0240)		(0.0512)	(0.0224)	
ABF	0.0662	0.6189**	0.9243	1.3478***	0.6194**	0.9250	6.5193***	**0.6952**	0.8929	4.2041***	**0.6702**	0.9076
	(0.0756)	(0.0180)		(0.0389)	(0.0179)		(0.1366)	(0.0244)		(0.0516)	(0.0217)	
LAND	0.0385	0.5768***	0.8864	1.4238***	0.5775***	0.8873	6.0786***	**0.6604**	0.8994	3.8811***	**0.6316**	0.8970
	(0.1187)	(0.0210)		(0.0685)	(0.0209)		(0.0950)	(0.0224)		(0.0226)	(0.0217)	
OML	0.7655***	0.4711***	0.8581	0.7588***	0.4762***	0.8632	7.1866***	0.5246***	0.8626	4.8933***	0.5064***	0.8627
	(0.1011)	(0.0194)		(0.0993)	(0.0192)		(0.1617)	(0.0212)		(0.0694)	(0.0205)	
ANTO	0.1191	0.5505***	0.9017	1.1123***	0.5501***	0.9013	6.3385***	**0.6546**	0.9228	4.0211***	0.6172**	0.9170
	(0.1068)	(0.0185)		(0.0739)	(0.0185)		(0.0901)	(0.0192)		(0.0246)	(0.0188)	
PSON	-0.1427	**0.6308**	0.9009	1.0875***	**0.6331**	0.9021	6.7139***	**0.6906**	0.9101	4.2909***	**0.6709**	0.9088
	(0.1113)	(0.0212)		(0.0697)	(0.0212)		(0.1143)	(0.0220)		(0.0384)	(0.0216)	
SHP	-0.0240	0.5949***	0.9431	1.3331***	0.5973***	0.9444	6.5043***	0.7199*	0.9217	4.0454***	**0.6744**	0.9319
	(0.0777)	(0.0148)		(0.0438)	(0.0147)		(0.1023)	(0.0213)		(0.0288)	(0.0185)	
SL	0.1349	0.5607***	0.9113	0.6000***	0.5654***	0.9155	7.1342***	**0.6545**	0.8797	4.5617***	0.6222*	0.8947
	(0.0800)	(0.0178)		(0.0636)	(0.0174)		(0.1688)	(0.0246)		(0.0675)	(0.0217)	
IPR	0.1287	0.5879***	0.8974	0.8484***	0.5878***	0.8977	7.3105***	0.6683	0.8897	4.7117***	**0.6407**	0.8947
	(0.1056)	(0.0202)		(0.0809)	(0.0201)		(0.1476)	(0.0239)		(0.0541)	(0.0223)	
KAZ	0.2126	0.5175***	0.8854	1.3448***	0.5198***	0.8872	5.7285***	0.6170*	0.9103	3.6699***	0.5816***	0.9037
	(0.1088)	(0.0189)		(0.0672)	(0.0188)		(0.0817)	(0.0197)		(0.0187)	(0.0193)	
UU	0.0500	0.6214*	0.9141	1.1864***	0.6223*	0.9155	6.8776***	**0.6892**	0.9305	4.4448***	**0.6664**	0.9266
	(0.0967)	(0.0193)		(0.0611)	(0.0192)		(0.1042)	(0.0191)		(0.0387)	(0.0190)	
SN	0.0518	0.6011***	0.9176	1.0782***	0.6023***	0.9189	7.0418***	**0.7042**	0.8889	4.4638***	**0.6681**	0.9022
	(0.0912)	(0.0183)		(0.0598)	(0.0182)		(0.1444)	(0.0253)		(0.0492)	(0.0223)	
EXPN	0.1414	0.5627***	0.9375	0.9815***	0.5679***	0.9412	6.9649***	**0.6832**	0.8908	4.3933***	**0.6403**	0.9121
	(0.0723)	(0.0148)		(0.0489)	(0.0144)		(0.1455)	(0.0243)		(0.0487)	(0.0202)	
BLND	0.1966	0.5610***	0.8610	1.2974***	0.5675***	0.8669	6.4296***	**0.6601**	0.8851	4.1228***	**0.6250**	0.8785
	(0.1307)	(0.0229)		(0.0840)	(0.0226)		(0.1117)	(0.0241)		(0.0299)	(0.0236)	
VED	0.1085	0.5331***	0.9029	1.5448***	0.5331***	0.9030	5.4424***	0.6279*	0.9326	3.4658***	0.5944***	0.9242

	(0.1071)	(0.0177)		(0.0596)	(0.0177)		(0.0587)	(0.0171)		(0.0099)	(0.0173)	
RSA	0.3806**	0.5425***	0.8100	0.5479***	0.5435***	0.8110	7.7307***	**0.6169**	0.8195	5.0715***	0.5914**	0.8181
	(0.1300)	(0.0266)		(0.1214)	(0.0266)		(0.2252)	(0.0294)		(0.0996)	(0.0283)	
CPI	0.0429	0.6169**	0.9330	1.2347***	0.6177**	0.9332	6.6455***	**0.7014**	0.9202	4.2573***	**0.6730**	0.9280
	(0.0762)	(0.0168)		(0.0441)	(0.0168)		(0.1147)	(0.0210)		(0.0417)	(0.0190)	
KGF	0.1594	0.5788***	0.8677	0.4145***	0.5875***	0.8737	7.9056***	**0.6663**	0.8867	5.0807***	**0.6362**	0.8823
	(0.1284)	(0.0229)		(0.1152)	(0.0227)		(0.1645)	(0.0242)		(0.0641)	(0.0236)	
CCL	-0.0054	0.5966***	0.9507	1.7612***	0.5973***	0.9507	5.5993***	0.7002*	0.9554	3.5270***	**0.6625**	0.9546
	(0.0713)	(0.0138)		(0.0316)	(0.0138)		(0.0573)	(0.0154)		(0.0154)	(0.0147)	
CW	0.1965*	0.5820***	0.9090	0.4655***	0.5818***	0.9088	8.2311***	**0.6920**	0.8951	5.2405***	**0.6525**	0.9024
	(0.0933)	(0.0187)		(0.0848)	(0.0187)		(0.1794)	(0.0240)		(0.0731)	(0.0218)	
SMIN	0.1075	0.5835***	0.9143	1.4086***	0.5848***	0.9167	6.2868***	**0.6867**	0.8954	4.0004***	**0.6499**	0.9045
	(0.0852)	(0.0181)		(0.0447)	(0.0179)		(0.1205)	(0.0238)		(0.0402)	(0.0214)	
LII	0.1602	0.5707***	0.8794	1.2900***	0.5765***	0.8868	6.1664***	**0.6527**	0.8556	3.9833***	**0.6240**	0.8659
	(0.1031)	(0.0215)		(0.0588)	(0.0209)		(0.1371)	(0.0272)		(0.0451)	(0.0249)	
NXT	0.1534	0.5578***	0.8824	1.6592***	0.5603***	0.8830	5.6793***	**0.6419**	0.9015	3.6619***	0.6126*	0.8967
	(0.1161)	(0.0207)		(0.0606)	(0.0207)		(0.0813)	(0.0215)		(0.0168)	(0.0211)	
JMAT	-0.0156	0.6169**	0.9163	1.6378***	0.6169**	0.9169	5.6315***	**0.6651**	0.9085	3.6551***	**0.6495**	0.9130
	(0.0879)	(0.0189)		(0.0380)	(0.0188)		(0.0910)	(0.0214)		(0.0282)	(0.0203)	
BAY	0.3934***	0.5208***	0.8718	0.8357***	0.5267***	0.8761	7.2271***	**0.6323**	0.9017	4.6490***	0.5916***	0.8926
	(0.1158)	(0.0203)		(0.0968)	(0.0201)		(0.1302)	(0.0212)		(0.0467)	(0.0208)	
IAP	0.1589**	0.5727***	0.9248	1.0167***	0.5742***	0.9259	6.6797***	**0.6658**	0.9727	4.2869***	**0.6335**	0.9022
	(0.0778)	(0.0166)		(0.0528)	(0.0165)		(0.1416)	(0.0244)		(0.0499)	(0.0212)	
SVT	-0.0442	**0.6373**	0.9256	1.5729***	**0.6365**	0.9258	6.0373***	**0.7007**	0.9222	3.8827***	0.6800	0.9258
	(0.0836)	(0.0183)		(0.0378)	(0.0183)		(0.0948)	(0.0207)		(0.0315)	(0.0195)	
HMSO	0.0742	0.5710***	0.8846	1.2054***	0.5746***	0.8890	6.2576***	**0.6668**	0.8642	3.9802***	**0.6328**	0.8736
	(0.1060)	(0.0209)		(0.0636)	(0.0206)		(0.1336)	(0.0268)		(0.0415)	(0.0244)	
SGE	0.1345	0.6028***	0.9153	0.5669***	0.6061**	0.9184	7.6542***	**0.6864**	0.9068	4.9312***	**0.6575**	0.9120
	(0.0821)	(0.0186)		(0.0674)	(0.0183)		(0.1593)	(0.0223)		(0.0691)	(0.0207)	
REX	0.1741**	0.5840***	0.9243	0.9390***	0.5881***	0.9291	6.9861***	**0.6724**	0.8815	4.5044***	**0.6427**	0.9008
	(0.0763)	(0.0170)		(0.0525)	(0.0165)		(0.1570)	(0.0250)		(0.0591)	(0.0216)	
IHG	0.0278	0.5735***	0.9101	1.2091***	0.5772***	0.9130	6.2785***	**0.6823**	0.8899	3.9497***	**0.6432**	0.7917
	(0.0904)	(0.0183)		(0.0522)	(0.0181)		(0.1232)	(0.0244)		(0.0392)	(0.0218)	

Table A5

Correlations between trade size and volatility in business time (volatility divided by the square root of trade counts)

Minutes	**AAL**	**ABF**	**ANTO**	**AV**	**AZN**	**BAE**	**BARC**	**BATS**	**BAY**	**BG**
All	0.3441	0.2915	0.5142	0.6411	0.3328	0.3363	0.1933	0.0068	0.5370	0.3346
First 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
First 15'	0.9311	0.9981	0.8926	0.9502	0.9999	0.9386	0.9995	0.9651	0.9731	0.9723
Last 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Last 15'	0.9867	0.9540	0.5703	0.8699	0.9674	0.9944	0.9999	0.8025	0.9765	0.3972
First/Last 10' Excluded	0.0491	-0.0571	0.1500	0.1242	0.0999	0.1263	0.0205	-0.1054	0.1251	0.1647
First/Last 15' Excluded	0.0328	0.0027	0.0559	0.1427	0.1285	0.1695	0.0290	-0.0756	0.1069	0.1981
Minutes	**BLND**	**BLT**	**BP**	**BT**	**CBRY**	**CCL**	**CNA**	**CPG**	**CPI**	**CW**
All	0.4405	0.8470	0.3134	0.2672	0.0996	0.6215	0.2653	0.2313	0.2442	0.1228
First 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
First 15'	0.9843	0.9950	0.9809	0.9933	0.9996	0.9838	0.9783	0.9869	0.9839	0.9924
Last 10'	1.0000	1.0000	-1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	-1.0000	1.0000
Last 15'	0.9998	0.9943	0.5029	0.9930	-0.3430	0.9716	0.6013	0.8395	-0.6900	0.2046
First/Last 10' Excluded	0.0880	0.1647	0.0929	-0.0594	-0.1236	-0.1978	-0.0117	-0.1414	-0.0303	-0.0252
First/Last 15' Excluded	0.1597	0.1797	0.1460	-0.0353	-0.0864	-0.2226	0.0235	-0.1273	0.0236	0.0048
Minutes	**DGE**	**EMG**	**EXPN**	**GSK**	**HMSO**	**HSBA**	**IAP**	**IHG**	**IMT**	**IPR**
All	0.0773	0.4072	0.1410	0.1633	0.2992	0.5190	0.1235	0.1633	0.0324	0.4340
First 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	-1.0000	1.0000	1.0000	1.0000
First 15'	0.9812	0.9986	0.5928	1.0000	0.9331	0.9568	0.5649	0.7055	0.9878	0.9163
Last 10'	1.0000	1.0000	-1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	-1.0000
Last 15'	0.9757	0.9724	0.0386	0.8975	0.9086	0.8850	0.4265	0.9922	0.9873	-0.0847
First/Last 10' Excluded	0.0745	0.0034	-0.0159	-0.0260	-0.0670	0.4161	-0.0913	0.1204	-0.0698	-0.0196
First/Last 15' Excluded	0.1020	-0.0151	-0.1131	0.0285	-0.0897	0.4211	-0.0734	0.1075	-0.0651	-0.0929
Minutes	**JMAT**	**KAZ**	**KGF**	**LAND**	**LGEN**	**LII**	**LLOY**	**MKS**	**MRW**	**NG**
All	0.2917	0.1911	0.0452	0.6746	0.1965	0.4448	0.3926	0.3537	0.3585	0.2459
First 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
First 15'	0.9999	0.9631	0.9809	0.9810	0.9790	0.9760	0.8071	0.9597	0.9457	0.7744
Last 10'	-1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	-1.0000	1.0000	-1.0000	-1.0000
Last 15'	0.3370	0.9739	0.7499	0.9810	0.9495	0.9003	0.3804	-0.0623	-0.0833	0.4605
First/Last 10' Excluded	-0.0343	0.0352	-0.0429	0.0705	0.0619	-0.1056	-0.0386	0.0159	0.0056	0.0478
First/Last 15' Excluded	0.0493	0.0276	-0.0136	0.0471	0.0961	-0.0689	-0.0411	0.0437	-0.0115	0.1080

Minutes	NXT	OML	PRU	PSON	RB	RBS	RDSA	REL	REX	RIO
All	0.6639	0.1608	0.6824	0.3206	0.5036	0.4047	0.1648	0.2165	0.3941	0.4461
First 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
First 15'	0.9985	0.9926	0.7482	0.9422	0.9935	0.7659	0.9973	0.9429	0.9970	0.9232
Last 10'	-1.0000	-1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Last 15'	0.2962	-0.9596	0.9869	0.7837	0.7931	0.9524	-0.8489	0.7815	0.9614	0.9735
First/Last 10' Excluded	0.0737	0.1086	0.0440	0.1008	-0.0379	0.0930	-0.0467	0.0261	-0.1286	0.1560
First/Last 15' Excluded	0.0597	0.1585	0.0511	0.0974	0.0113	0.0707	-0.0339	0.0609	-0.0623	0.1639
Minutes	**RR**	**RSA**	**SAB**	**SBRY**	**SGE**	**SHP**	**SKY**	**SL**	**SMIN**	**SN**
All	0.3522	0.6427	0.1928	0.1803	0.0605	0.1153	0.2638	-0.0430	0.1188	0.6242
First 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
First 15'	0.9846	0.9984	0.9974	0.7869	0.9284	0.9670	0.9761	0.9549	0.9908	0.9998
Last 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	-1.0000	1.0000	1.0000	1.0000
Last 15'	0.9460	0.9918	0.9807	0.9930	0.9802	0.9159	-0.9980	0.4452	0.9587	0.8286
First/Last 10' Excluded	-0.2161	0.1146	-0.0873	-0.0715	-0.0338	-0.1141	-0.0115	-0.2154	-0.1572	-0.1792
First/Last 15' Excluded	-0.2119	0.1882	-0.0479	-0.1443	-0.0393	-0.1157	0.0111	-0.2131	-0.1572	-0.1279
Minutes	**SSE**	**STAN**	**SVT**	**TSCO**	**ULVR**	**UU**	**VED**	**VOD**	**WPP**	**XTA**
All	0.3363	0.1151	0.2795	0.2361	0.2247	0.4266	0.3484	0.0696	0.4686	0.4754
First 10'	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
First 15'	0.9675	0.9784	0.9928	0.9964	0.9869	0.9686	0.9883	0.9795	0.9795	0.9824
Last 10'	1.0000	1.0000	1.0000	1.0000	-1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
Last 15'	0.8227	0.9258	0.9948	0.9305	-0.2423	0.8714	0.9449	0.6929	0.9815	0.9933
First/Last 10' Excluded	-0.0218	-0.0561	0.1195	0.1888	0.0539	0.0947	0.0898	0.0239	-0.0498	0.1018
First/Last 15' Excluded	0.0248	-0.0529	0.1673	0.2319	0.0864	0.1003	0.1160	0.0408	0.0015	0.0868

Table A6

OLS regression results for each year

This table reports the coefficient estimates for each substantive year in the sample (2007, 2008 and 2009) when underlying variables are estimated as averages across days based on equation (23). OLS regressions are based on the model in (25) which uses the definition of trading activity introduced by Kyle and Obizhaeva (2016b). Coefficients are tested against the null hypothesis $H_0: \beta = 1/2$. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

		2007			2008			2009		
	Stocks	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$
High Market Capitalization	BP	-0.5319***	0.5751***	0.9516	0.0038	0.5485***	0.8847	0.7867***	**0.5257**	0.8312
	(4/7)	(0.1054)	(0.0129)		(0.1607)	(0.0197)		(0.1703)	(0.0236)	
	HSBA	-0.2714	0.5584***	0.8878	0.2983	**0.5167**	0.8737	0.8403***	**0.5083**	0.8765
	(5/9)	(0.1538)	(0.0197)		(0.1591)	(0.0195)		(0.1471)	(0.0190)	
	VOD	0.7700***	0.4386***	0.8226	0.8518***	**0.4734**	0.8299	0.9612***	**0.5087**	0.7696
		(0.1593)	(0.0202)		(0.1686)	(0.0213)		(0.1935)	(0.0277)	
	GSK	-0.2987*	0.5714***	0.9051	0.2598	**0.5311**	0.8568	0.6883***	**0.5281**	0.7916
	(4/7)	(0.1380)	(0.0184)		(0.1627)	(0.0216)		(0.1761)	(0.0269)	
	RDSA	-0.2699*	**0.5289**	0.9036	-0.2309	0.5443***	0.8693	0.0377	0.5532***	0.8976
	(5/9)	(0.1062)	(0.0172)		(0.1387)	(0.0210)		(0.1081)	(0.0186)	
Medium Market Capitalization	RIO	0.3111*	**0.4934**	0.8754	0.4277**	**0.5039**	0.9028	0.5179**	**0.5288**	0.8756
	(1/2)	(0.1477)	(0.0185)		(0.1383)	(0.0164)		(0.1560)	(0.0198)	
	AZN	0.3509*	**0.5085**	0.8233	0.4854*	**0.5151**	0.7974	0.9253***	**0.4983**	0.7910
		(0.1607)	(0.0234)		(0.1864)	(0.0258)		(0.1623)	(0.0255)	
	RBS	0.4859***	**0.4844**	0.9005	0.9687***	0.4473***	0.8231	1.6891***	0.3727***	0.8294
	(1/2)	(0.1263)	(0.0160)		(0.1712)	(0.0206)		(0.1064)	(0.0168)	
	BATS	0.0752	**0.5436**	0.8504	0.5643**	**0.5032**	0.8072	0.7284***	**0.5133**	0.8121
	(5/9)	(0.1395)	(0.0227)		(0.1695)	(0.0244)		(0.1494)	(0.0245)	
	BG	0.2933	**0.5032**	0.7480	0.6001**	**0.4679**	0.7856	0.6563***	**0.4952**	0.8282
		(0.1906)	(0.0290)		(0.1812)	(0.0243)		(0.1481)	(0.0224)	
	AAL	0.5350**	**0.4685**	0.8283	0.7523***	**0.4736**	0.8522	0.4681**	**0.5336**	0.8585
		(0.1621)	(0.0212)		(0.1588)	(0.0196)		(0.1598)	(0.0215)	
	BLT	-0.0155	**0.5306**	0.9174	-0.0418	0.5664***	0.9037	0.1516	0.5815***	0.8617
	(5/9)	(0.1266)	(0.0158)		(0.1569)	(0.0184)		(0.1806)	(0.0232)	
	BARC	0.2443	**0.5032**	0.8708	0.4612*	**0.5053**	0.8294	0.9214***	**0.4883**	0.8231
		(0.1524)	(0.0193)		(0.1882)	(0.0228)		(0.1735)	(0.0225)	
	TSCO	0.6242***	**0.4789**	0.8435	0.9332***	**0.4653**	0.7512	1.2111***	**0.4683**	0.7492
		(0.1352)	(0.0205)		(0.1896)	(0.0266)		(0.1609)	(0.0269)	

	XTA	0.5654***	0.4517***	0.8218	0.9376***	0.4449***	0.8414	0.5467***	**0.5208**	0.8624
		(0.1599)	(0.0209)		(0.1565)	(0.0192)		(0.1581)	(0.0207)	
	DGE	0.5976***	**0.4643**	0.8289	0.4745***	**0.5086**	0.8354	0.5357**	**0.5465**	0.7841
		(0.1319)	(0.0210)		(0.1548)	(0.0224)		(0.1663)	(0.0285)	
	LLOY	0.1485	**0.5213**	0.8758	0.5447**	**0.4952**	0.7865	1.2506***	**0.4546**	0.7733
		(0.1371)	(0.0195)		(0.1964)	(0.0256)		(0.1739)	(0.0245)	
	STAN	0.4694**	**0.4644**	0.7529	0.7492***	**0.4593**	0.7852	0.5876***	**0.4985**	0.8985
		(0.1785)	(0.0264)		(0.1706)	(0.0239)		(0.1109)	(0.0167)	
Low Market Capitalization	ULVR	0.0260	**0.5306**	0.8221	0.3090	**0.5301**	0.7613	0.3810*	**0.5530**	0.7796
	(5/9)	(0.1504)	(0.0245)		(0.1953)	(0.0295)		(0.1671)	(0.0292)	
	RB	0.6107***	**0.4845**	0.7657	0.7951***	**0.4812**	0.7440	0.9439***	**0.4819**	0.7447
		(0.1488)	(0.0266)		(0.1773)	(0.0280)		(0.1548)	(0.0280)	
	SAB	0.1755	**0.5083**	0.7711	0.4031*	**0.5062**	0.7820	0.6357***	**0.4898**	0.7965
		(0.1613)	(0.0275)		(0.1716)	(0.0266)		(0.1375)	(0.0246)	
	NG	0.4444**	**0.4996**	0.7632	0.3733*	**0.5276**	0.7910	0.5384**	**0.5240**	0.7535
		(0.1641)	(0.0277)		(0.1689)	(0.0270)		(0.1620)	(0.0298)	
	IMT	0.3810*	**0.4931**	0.7248	0.7034***	**0.4912**	0.7394	0.6649***	**0.5083**	0.7708
		(0.1743)	(0.0302)		(0.1866)	(0.0290)		(0.1524)	(0.0275)	
	BT	0.3795*	**0.4950**	0.8205	0.4048*	**0.5078**	0.7799	1.6658***	0.3578***	0.6833
		(0.1503)	(0.0230)		(0.1804)	(0.0268)		(0.1231)	(0.0242)	
	AV	0.4038*	**0.4996**	0.7901	0.2646	**0.5071**	0.7622	0.9244***	**0.4594**	0.7628
		(0.1602)	(0.0256)		(0.1824)	(0.0281)		(0.1548)	(0.0255)	
	PRU	0.1359	**0.5145**	0.8145	0.1806	**0.5276**	0.7712	0.4746**	**0.5120**	0.8165
		(0.1618)	(0.0244)		(0.1957)	(0.0286)		(0.1509)	(0.0241)	
	BAE	0.7361***	**0.4472**	0.7251	0.8340***	**0.4688**	0.7564	1.0215***	**0.4747**	0.7991
		(0.1703)	(0.0274)		(0.1704)	(0.0264)		(0.1340)	(0.0237)	
	CNA	0.4857**	**0.4995**	0.7165	0.2238	**0.5493**	0.7487	0.8347***	**0.4798**	0.7773
		(0.1811)	(0.0312)		(0.1968)	(0.0316)		(0.1345)	(0.0255)	
	SSE	0.4325*	**0.4957**	0.6712	0.4624**	**0.5056**	0.7485	0.3670*	**0.5463**	0.7740
		(0.1985)	(0.0344)		(0.1752)	(0.0291)		(0.1481)	(0.0293)	
	CBRY	-0.1822	0.5601***	0.8137	0.0704	0.5595***	0.8131	0.3879**	**0.5438**	0.8438
		(0.1646)	(0.0266)		(0.1598)	(0.0267)		(0.1188)	(0.0233)	
	BSY	-0.0372	**0.5499**	0.8093	0.0873	0.5656***	0.7882	0.7247***	**0.5005**	0.7545
	(5/9)	(0.1485)	(0.0265)		(0.1730)	(0.0291)		(0.1400)	(0.0284)	
	EMG	0.2976	**0.4874**	0.7817	0.5260**	**0.4645**	0.7613	0.8705***	**0.4624**	0.7263
		(0.1581)	(0.0256)		(0.1658)	(0.0258)		(0.1519)	(0.0282)	
	RR	0.3834*	**0.4929**	0.7549	0.3178	**0.5246**	0.7581	0.8388***	**0.4795**	0.8033
		(0.1594)	(0.0279)		(0.1796)	(0.0294)		(0.1225)	(0.0236)	
	MRW	0.3082	**0.5010**	0.7066	0.3528	**0.5199**	0.7307	0.5989***	**0.5221**	0.7505
		(0.1850)	(0.0321)		(0.1880)	(0.0314)		(0.1450)	(0.0299)	

MKS	0.2373	**0.5032**	0.7464	0.4352*	**0.5051**	0.7003	0.6533***	**0.5058**	0.8126
	(0.1802)	(0.0291)		(0.2143)	(0.0328)		(0.1278)	(0.0241)	
SBRY	0.2756	**0.4797**	0.7565	0.3255	**0.5259**	0.7200	0.7752***	**0.4997**	0.7709
	(0.1564)	(0.0270)		(0.1844)	(0.0326)		(0.1234)	(0.0271)	
WPP	0.1737	**0.5230**	0.8040	0.2166	**0.5514**	0.7545	0.7669***	**0.4972**	0.7930
	(0.1487)	(0.0257)		(0.1912)	(0.0313)		(0.1363)	(0.0252)	
REL	0.0487	**0.5422**	0.7885	0.3088	**0.5349**	0.7638	0.8750***	**0.5069**	0.7829
(5/9)	(0.1532)	(0.0279)		(0.1746)	(0.0296)		(0.1366)	(0.0265)	
LGEN	0.2114	**0.5454**	0.7252	1.0468***	0.4135***	0.5847	0.8274***	0.4410***	0.7451
(5/9)	(0.1871)	(0.0334)		(0.1918)	(0.0346)		(0.1358)	(0.0256)	
CPG	0.4387*	**0.4959**	0.6863	0.4708*	**0.5132**	0.7070	0.7567***	**0.4984**	0.7681
	(0.1803)	(0.0333)		(0.1973)	(0.0328)		(0.1384)	(0.0272)	
ABF	-0.0707	0.6215	0.7890	0.1982	**0.5622**	0.7546	0.7143***	**0.4941**	0.7628
(3/5)	(0.1400)	(0.0319)		(0.1499)	(0.0319)		(0.1007)	(0.0274)	
LAND	0.3971*	**0.4895**	0.7006	0.2541	**0.5222**	0.7102	0.8111***	**0.4675**	0.6686
	(0.1877)	(0.0318)		(0.2010)	(0.0331)		(0.1698)	(0.0327)	
OML	0.7472***	**0.4531**	0.6637	1.2927***	0.3767***	0.6311	1.2092***	0.3889***	0.7582
	(0.1713)	(0.0320)		(0.1594)	(0.0286)		(0.1062)	(0.0218)	
ANTO	0.3642	**0.4889**	0.6805	0.4606*	**0.4758**	0.7085	0.4564***	**0.5192**	0.8404
	(0.1907)	(0.0333)		(0.1863)	(0.0303)		(0.1269)	(0.0225)	
PSON	0.1541	**0.5406**	0.7120	0.2028	**0.5533**	0.7542	0.5710***	**0.5271**	0.7540
(5/9)	(0.1815)	(0.0341)		(0.1756)	(0.0314)		(0.1489)	(0.0299)	
SHP	0.0983	**0.5248**	0.8367	0.4032*	**0.5141**	0.7799	0.4981***	**0.5292**	0.8557
	(0.1296)	(0.0231)		(0.1557)	(0.0271)		(0.0984)	(0.0216)	
SL	0.2911**	**0.5070**	0.8361	-0.0067	0.5586***	0.8122	0.7538***	**0.4529**	0.7491
	(0.0996)	(0.0223)		(0.1357)	(0.0267)		(0.1083)	(0.0260)	
IPR	0.4842*	**0.4963**	0.6535	0.4767**	**0.5159**	0.7362	0.6855***	**0.4979**	0.7573
	(0.1924)	(0.0359)		(0.1762)	(0.0307)		(0.1334)	(0.0280)	
KAZ	0.4064*	**0.4578**	0.6423	0.5455**	**0.4552**	0.6720	0.5542***	**0.4792**	0.8201
	(0.1881)	(0.0339)		(0.1898)	(0.0316)		(0.1308)	(0.0223)	
UU	0.2326	**0.5490**	0.7910	0.1843	0.5687***	0.7882	0.7054***	**0.5356**	0.7822
	(0.1467)	(0.0280)		(0.1643)	(0.0293)		(0.1221)	(0.028)	
SN	0.1534	**0.5415**	0.7676	0.2612	**0.5396**	0.7705	0.8139***	**0.4900**	0.7981
	(0.1476)	(0.0296)		(0.1662)	(0.0293)		(0.1091)	(0.0245)	
EXPN	0.3761**	**0.4755**	0.7914	0.1259	**0.5509**	0.8003	0.6494***	**0.5003**	0.8255
	(0.1266)	(0.0243)		(0.1425)	(0.0273)		(0.1019)	(0.0229)	
BLND	0.2942	**0.5019**	0.6922	0.6174**	**0.4778**	0.6862	0.8978***	**0.4764**	0.6723
	(0.2024)	(0.0332)		(0.1945)	(0.0321)		(0.1712)	(0.0330)	
VED	0.3489*	**0.4629**	0.7231	0.4621*	**0.4797**	0.7040	0.4797***	**0.4882**	0.8194
	(0.1672)	(0.0285)		(0.2005)	(0.0309)		(0.1344)	(0.0228)	
RSA	0.0709	**0.5660**	0.7067	0.2360	**0.5387**	0.6620	1.4448***	0.3451***	0.5167

	(0.1917)	(0.0362)		(0.2088)	(0.0382)		(0.1342)	(0.0331)	
CPI	0.0458	0.5805***	0.7633	0.0974	0.5880***	0.7997	0.6737***	**0.5081**	0.7718
	(0.1465)	(0.0321)		(0.1485)	(0.0293)		(0.1123)	(0.0275)	
KGF	0.1064	**0.5376**	0.7449	0.3518	**0.5318**	0.7030	0.9739***	**0.4734**	0.7673
	(0.1787)	(0.0313)		(0.2125)	(0.0343)		(0.1319)	(0.0259)	
CCL	0.2297	**0.5275**	0.8470	0.0595	0.5633***	0.8523	0.2793**	0.5750***	0.8771
	(0.1172)	(0.0223)		(0.1311)	(0.0233)		(0.1028)	(0.0214)	
CW	0.4285***	**0.4963**	0.8033	0.0800	0.5919***	0.7850	0.8049***	**0.4885**	0.7045
	(0.1259)	(0.0244)		(0.1686)	(0.0308)		(0.1414)	(0.0314)	
SMIN	0.2690	**0.5150**	0.7222	0.3390*	**0.5177**	0.7677	0.5866***	**0.5168**	0.7623
	(0.1635)	(0.0317)		(0.1424)	(0.0283)		(0.1185)	(0.0287)	
LII	0.2828	**0.5076**	0.6968	0.4834**	**0.4789**	0.6992	0.9169***	**0.4569**	0.6333
	(0.1675)	(0.0332)		(0.1648)	(0.0312)		(0.1473)	(0.0345)	
NXT	0.3251	**0.5134**	0.6646	0.4220*	**0.4967**	0.7252	0.5715***	**0.5018**	0.7744
	(0.2087)	(0.0362)		(0.1876)	(0.0304)		(0.1361)	(0.0269)	
JMAT	0.0505	0.5721***	0.7327	0.1314	0.5828***	0.7489	0.4583***	**0.5270**	0.8154
(4/7)	(0.1667)	(0.0343)		(0.1723)	(0.0335)		(0.1022)	(0.0249)	
BAY	0.6912***	**0.4475**	0.6868	0.4976**	**0.4816**	0.7445	0.8943***	**0.4656**	0.7609
	(0.1804)	(0.0300)		(0.1783)	(0.0280)		(0.1289)	(0.0259)	
IAP	0.0728	0.5814***	0.8086	0.1930	**0.5395**	0.7581	0.6773***	**0.4809**	0.8084
	(0.1260)	(0.0281)		(0.1687)	(0.0303)		(0.0969)	(0.0233)	
SVT	0.1816	**0.5632**	0.7150	0.0579	0.5971***	0.8048	0.5076***	**0.5386**	0.8050
	(0.1676)	(0.0353)		(0.1485)	(0.0292)		(0.1055)	(0.0264)	
HMSO	-0.0729	**0.5579**	0.6996	0.1640	**0.5225**	0.7473	0.9050***	**0.4578**	0.7218
	(0.1964)	(0.0363)		(0.1627)	(0.0302)		(0.1298)	(0.0282)	
SGE	-0.2579	0.6206***	0.8318	0.0871	0.5802***	0.7560	0.9090***	**0.4876**	0.7510
	(0.1327)	(0.0277)		(0.1609)	(0.0328)		(0.1049)	(0.0279)	
REX	-0.0393	0.5920***	0.7615	0.4145**	**0.5054**	0.7204	0.7833***	**0.4895**	0.8305
	(0.1552)	(0.0329)		(0.1500)	(0.0313)		(0.0927)	(0.0220)	
IHG	0.0445	**0.5344**	0.7279	0.1821	**0.5265**	0.7958	0.5288***	**0.5089**	0.8103
	(0.1724)	(0.0325)		(0.1397)	(0.0265)		(0.1086)	(0.0245)	

Table A7

OLS regression results for pre-crisis (Jan 207-Jun 2018) and in-crisis period (Jul 2018- Dec 2019)

This table reports the coefficient estimates for pre-crisis and in-crisis periods when underlying variables are estimated as averages across days based on equation (23). OLS regressions are based on the model in (25) which uses the definition of trading activity introduced by Kyle and Obizhaeva (2016b). Coefficients are tested against the null hypothesis $H_0 : \beta = 1/2$. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

		Pre-Crisis			In-Crisis		
	Stocks	c	β	$\bar{R}^2$	c	β	$\bar{R}^2$
High Market Capitalization	BP	-0.3426**	0.5614***	0.9307	0.5218**	**0.5389**	0.8563
		(0.1230)	(0.0152)		(0.1673)	(0.0220)	
	HSBA	-0.0762	0.5417*	0.8763	0.6754***	**0.5115**	0.8822
		(0.1592)	(0.0202)		(0.1473)	(0.0186)	
	VOD	0.7762***	0.4514*	0.8299	0.9040***	**0.5012**	0.7918
		(0.1586)	(0.0203)		(0.1883)	(0.0255)	
	GSK	-0.1974	0.5649**	0.8890	0.5819***	**0.5286**	0.8190
		(0.1479)	(0.0199)		(0.1713)	(0.0247)	
	RDSA	-0.1618	**0.5203**	0.8715	-0.1333	0.5616***	0.9062
		(0.1232)	(0.0199)		(0.1115)	(0.0180)	
Medium Market Capitalization	RIO	0.3983**	**0.4885**	0.8795	0.4273**	**0.5299**	0.8944
		(0.1452)	(0.0180)		(0.1466)	(0.0181)	
	AZN	0.3574*	**0.5141**	0.8010	0.7948***	**0.5044**	0.8091
		(0.1756)	(0.0255)		(0.1643)	(0.0244)	
	RBS	0.6060***	**0.4767**	0.8841	1.4946***	0.3948***	0.8292
		(0.1395)	(0.0172)		(0.1231)	(0.0178)	
	BATS	0.2309	**0.5312**	0.8302	0.6689***	**0.5102**	0.8172
		(0.1503)	(0.0239)		(0.1559)	(0.0240)	
	BG	0.3768*	**0.4914**	0.7364	0.6280***	**0.4882**	0.8308
		(0.1988)	(0.0292)		(0.1524)	(0.0219)	
	AAL	0.6380***	**0.4634**	0.8306	0.5138**	**0.5215**	0.8639
		(0.1614)	(0.0208)		(0.1581)	(0.0206)	
	BLT	0.0053	0.5386*	0.9099	0.0414	0.5823***	0.8819
		(0.1366)	(0.0169)		(0.1721)	(0.0212)	
	BARC	0.3701*	**0.5009**	0.8523	0.7318***	**0.4956**	0.8265
		(0.1671)	(0.0207)		(0.1776)	(0.0226)	

	TSCO	0.7238***	**0.4730**	0.8001	1.1113***	**0.4705**	0.7615
		(0.1574)	(0.0235)		(0.1683)	(0.0262)	
	XTA	0.6841***	0.4474*	0.8109	0.6618***	**0.4987**	0.8732
		(0.1671)	(0.0215)		(0.1483)	(0.0189)	
	DGE	0.5864***	**0.4732**	0.8259	0.4347**	**0.5465**	0.8121
		(0.1393)	(0.0216)		(0.1637)	(0.0261)	
	LLOY	0.3960*	**0.5015**	0.8258	0.9382***	**0.4754**	0.7847
		(0.1660)	(0.0229)		(0.1805)	(0.0247)	
	STAN	0.4899**	**0.4732**	0.7631	0.6789***	**0.4797**	0.8689
		(0.1798)	(0.0262)		(0.1268)	(0.0185)	
Low Market Capitalization	ULVR	0.1317***	**0.5280**	0.7937	0.3343	**0.5492**	0.7774
		(0.1663)	(0.0268)		(0.1783)	(0.0292)	
	RB	0.6007***	**0.4927**	0.7569	0.9556***	**0.4743**	0.7455
		(0.1596)	(0.0277)		(0.1617)	(0.0275)	
	SAB	0.2197	**0.5125**	0.7663	0.5846***	**0.4922**	0.7981
		(0.1694)	(0.0281)		(0.1459)	(0.0246)	
	NG	0.3665*	**0.5199**	0.7687	0.4605**	**0.5269**	0.7764
		(0.1689)	(0.0283)		(0.1639)	(0.0281)	
	IMT	0.4474*	**0.4992**	0.7200	0.6947***	**0.4994**	0.7651
		(0.1820)	(0.0309)		(0.1639)	(0.0275)	
	BT	0.2839	**0.5102**	0.7967	1.3607***	0.4022***	0.7297
		(0.1690)	(0.0256)		(0.1372)	(0.0243)	
	AV	0.2992	**0.5135**	0.7763	0.7451***	0.4674***	0.7694
		(0.1731)	(0.0274)		(0.1584)	(0.0254)	
	PRU	0.1029	**0.5263**	0.7890	0.3975*	**0.5143**	0.8124
		(0.1814)	(0.0271)		(0.1586)	(0.0246)	
	BAE	0.7825***	**0.4528**	0.7116	0.9389***	**0.4756**	0.8030
		(0.1794)	(0.0286)		(0.1396)	(0.0234)	
	CNA	0.3328	**0.5288**	0.7208	0.6825***	**0.4958**	0.7715
		(0.1913)	(0.0327)		(0.1525)	(0.0268)	
	SSE	0.3859*	**0.5102**	0.7005	0.3920*	**0.5300**	0.7630
		(0.1918)	(0.0331)		(0.1603)	(0.0294)	
	CBRY	-0.1012	0.5614*	0.7991	0.2250	0.5583*	0.8456
		(0.1713)	(0.0280)		(0.1286)	(0.0237)	
	BSY	-0.0536	0.5632*	0.8042	0.5352***	**0.5229**	0.7646
		(0.1553)	(0.0276)		(0.1551)	(0.0288)	
	EMG	0.3077	**0.4881**	0.7839	0.8103***	**0.4562**	0.7337

	(0.1570)	(0.0255)		(0.1589)	(0.0273)	
RR	0.3903*	**0.5000**	0.7397	0.6762***	**0.4935**	0.7917
	(0.1721)	(0.0295)		(0.1386)	(0.0252)	
MRW	0.3121	**0.5105**	0.6945	0.5000**	**0.5232**	0.7567
	(0.1953)	(0.0336)		(0.1558)	(0.0295)	
MKS	0.3342	**0.5051**	0.6826	0.5345***	**0.5081**	0.8082
	(0.2153)	(0.0342)		(0.1408)	(0.0246)	
SBRY	0.2712	**0.5020**	0.7248	0.6358***	**0.5060**	0.7640
	(0.1733)	(0.0307)		(0.1401)	(0.0279)	
WPP	0.1152	**0.5423**	0.7878	0.6438***	**0.5094**	0.7814
	(0.1627)	(0.0280)		(0.1535)	(0.0268)	
REL	0.0716	**0.5493**	0.7717	0.7324***	**0.5114**	0.7858
	(0.1658)	(0.0297)		(0.1447)	(0.0265)	
LGEN	0.5206*	**0.5066**	0.6447	0.8530***	0.4308*	0.7215
	(0.2111)	(0.0373)		(0.1414)	(0.0266)	
CPG	0.4319*	**0.5108**	0.6637	0.6442***	**0.5023**	0.7712
	(0.2010)	(0.0361)		(0.1483)	(0.0272)	
ABF	-0.0387	0.6198***	0.7696	0.5297***	**0.5152**	0.7749
	(0.1527)	(0.0337)		(0.1104)	(0.0276)	
LAND	0.3929*	**0.4938**	0.6826	0.5790**	**0.4954**	0.6973
	(0.1981)	(0.0334)		(0.1792)	(0.0324)	
OML	0.8213***	**0.4504**	0.6542	1.2415***	0.3807***	0.7421
	(0.1770)	(0.0325)		(0.1137)	(0.0223)	
ANTO	0.3820	**0.4922**	0.6682	0.4257**	**0.5040**	0.8228
	(0.2020)	(0.0344)		(0.1353)	(0.0233)	
PSON	0.1447	**0.5509**	0.7016	0.4396**	**0.5369**	0.7762
	(0.1898)	(0.0357)		(0.1513)	(0.0286)	
SHP	0.2341	**0.5200**	0.7987	0.4036**	**0.5318**	0.8465
	(0.1491)	(0.0259)		(0.1102)	(0.0225)	
SL	0.0727	**0.5560**	0.8580	0.6034***	**0.4653**	0.7516
	(0.1052)	(0.0225)		(0.1188)	(0.0266)	
IPR	0.4375*	**0.5114**	0.6778	0.6418***	**0.4995**	0.7525
	(0.1905)	(0.0350)		(0.1466)	(0.0285)	
KAZ	0.4248*	**0.4640**	0.6235	0.5531***	**0.4690**	0.8003
	(0.2048)	(0.0358)		(0.1373)	(0.0233)	
UU	0.0910	0.5769**	0.7972	0.6024***	**0.5363**	0.7813
	(0.1532)	(0.0289)		(0.1364)	(0.0282)	

SN	0.1024	**0.5538**	0.7572	0.6725***	**0.5037**	0.7988
	(0.1625)	(0.0312)		(0.1231)	(0.0251)	
EXPN	0.2134	**0.5146**	0.8059	0.5045***	**0.5135**	0.8152
	(0.1291)	(0.0251)		(0.1163)	(0.0243)	
BLND	0.3475	**0.5010**	0.6813	0.8313***	**0.4742**	0.6839
	(0.2068)	(0.0340)		(0.1755)	(0.0320)	
VED	0.3288	**0.4744**	0.7175	0.5261***	**0.4807**	0.7870
	(0.1802)	(0.0296)		(0.1512)	(0.0249)	
RSA	0.0172	0.5807*	0.6984	1.1362***	0.3999**	0.5639
	(0.2008)	(0.0379)		(0.1602)	(0.0349)	
CPI	-0.0003	0.5973**	0.7752	0.5180***	**0.5306**	0.7839
	(0.1485)	(0.0320)		(0.1256)	(0.0277)	
KGF	0.1698	**0.5410**	0.7008	0.7620***	**0.4956**	0.7666
	(0.2051)	(0.0351)		(0.1496)	(0.0272)	
CCL	0.1674	**0.5419**	0.8477	0.1801	0.5741***	0.8731
	(0.1212)	(0.0228)		(0.1121)	(0.0218)	
CW	0.2924*	**0.5359**	0.8008	0.5566***	**0.5267**	0.7367
	(0.1383)	(0.0266)		(0.1532)	(0.0313)	
SMIN	0.2401	**0.5293**	0.7233	0.5175***	**0.5112**	0.7759
	(0.1636)	(0.0325)		(0.1231)	(0.0273)	
LII	0.3346	**0.5007**	0.6797	0.7773***	**0.4641**	0.6632
	(0.1750)	(0.0341)		(0.1513)	(0.0328)	
NXT	0.3529	**0.5085**	0.6535	0.5128***	**0.5012**	0.7793
	(0.2142)	(0.0368)		(0.1462)	(0.0265)	
JMAT	-0.0094	0.5928**	0.7556	0.4139**	**0.5348**	0.7834
	(0.1652)	(0.0335)		(0.1252)	(0.0279)	
BAY	0.6692***	**0.4528**	0.6757	0.7382***	**0.4755**	0.7788
	(0.1905)	(0.0312)		(0.1372)	(0.0252)	
IAP	0.0413	0.5798**	0.7908	0.5572***	**0.4955**	0.7932
	(0.1434)	(0.0296)		(0.1174)	(0.0252)	
SVT	0.0865	0.5868*	0.7421	0.3645**	0.5589*	0.8113
	(0.1642)	(0.0344)		(0.1196)	(0.0268)	
HMSO	-0.0802	**0.5602**	0.7168	0.7177***	**0.4722**	0.7321
	(0.1867)	(0.0350)		(0.1397)	(0.0284)	
SGE	-0.2738	0.6293***	0.8087	0.7316***	**0.5064**	0.7552
	(0.1471)	(0.0304)		(0.1185)	(0.0286)	
REX	0.0766	0.5775*	0.7118	0.6578***	**0.4906**	0.8279

	(0.1706)	(0.0365)		(0.0995)	(0.0222)	
IHG	0.0095	**0.5497**	0.7363	0.4397***	**0.5072**	0.8208
	(0.1719)	(0.0327)		(0.1121)	(0.0236)	

CHAPTER 5

Invariance of transaction costs in the FTSE 100

Abstract

We examine market microstructure invariance (MMI) principle for transaction costs in FTSE 100 constituent stocks. We empirically test the MMI predictions concerning the invariance of transaction costs using three common proxies for transaction costs, namely quoted, effective, realized spread on trade data. We find that the predicted -1/3 proportionality is present in average daily patterns in our sample for all three proxies of transaction costs, with larger trades having a negative impact on this proportionality when the underlying variables are estimated as intraday averages. Market fragmentation does not impact the estimated invariance coefficients, though it may lead to a further reduction of percentage transaction costs on LSE per unit of volatility. Finally, the invariance prediction holds for a consolidated market, but the lower reduction in the realised spreads suggests a greater impact of large trades in the alternative platforms, and is also consistent with the view that only few market makers benefit from an increase in trading activity.

5.1 Introduction

Measuring market liquidity, which I define as the facility with which an order arriving in the market can be executed in a timely fashion at a price close to the fundamental value of the relevant security, can be an elusive undertaking. From this definition, it follows that the more the actual trade execution price deviates from the securities fundamental price, the more illiquid the underlying market. Market liquidity can vary across securities and through time, and understanding the determinants of such time variation is of vital importance for market participants and policy makers. When market liquidity decreases, investors are unable to purchase and sell an asset at similar prices, especially when they are considering transacting in larger trade sizes, and thus they are paying larger trading costs in order to undertake round-trip transactions. Such a market is characterised by wider bid-ask spreads. Practitioners may incur losses if market liquidity suddenly shrinks due to the resulting impact on their portfolio returns. Professionals that provide security trading services are constantly seeking the market/trading venue with the highest liquidity or attempting to time their trades to minimise their trading costs. Therefore, developing measures which efficiently capture market liquidity is a key for the fluent operation of financial markets and designing appropriate trading regulations.

Measuring market liquidity is not easily undertaking, as it involves several dimensions, inter alia trading costs, depth, and execution speed among others. However, most commentators agree that direct trading costs are an important component which enters into an accurate assessment of market illiquidity, so quantifying such costs, and ascertaining how they change in response to variation in important trading characteristics exhibited by the underlying assets, becomes an important undertaking for market participants. There exists an extensive microstructure literature focusing on transaction costs, their measures, as well as their role in the trading process and their relation to market liquidity, price discovery and generally market quality. I selectively survey this extensive literature in chapter 2 of this thesis. This explains that bid-ask spread, one of the most commonly used direct measures of transaction costs, mainly exist due to liquidity provision costs such as order processing costs, adverse selection costs and inventory risk (Stoll (1978), O'Hara, 1995, Foucault et al., 1997, Madhavan, 2000). Elements of the spread can also be seen as providing compensation for immediacy (Demsetz, 1968).

Based on one important determinant of the spread, namely adverse selection costs, Glosten and Milgrom (1985) develop a theoretical model for a pure dealership market, on the basis of which they show that adverse selection costs alone can explain the existence of a spread. Kyle (1985) develops a theoretical model that is based on information asymmetry, and discusses how trade prices react to order size (determinants of market depth) and in turn how this affects transaction costs. Other papers combine the idea of information asymmetry with order-processing costs or inventory risks and/or relax assumptions relating to competitive conditions, market orders, risk neutrality, and market structure (Easley and O'Hara, 1987, Admati and Pfleiderer, 1988, Hasbrouck, 1988, Klemperer and Meyer, 1989, Easley and O'Hara, 1992, Madhavan, 1992, Biais et al., 1998, Bloomfield et al., 2005, Calcagno and Lovo, 2006). De Jong et al. (1996) find that on the Paris Bourse, the adverse selection component of the bid-ask spread increases slightly with order size, but the order-processing component decreases. Using NYSE data, Huang and Stoll (1997) report that the order-processing component accounts for the majority of the bid-ask spread, and that the adverse selection component does not increase with the trade size in contrast with the prediction of Kyle (1985). Madhavan et al. (1997) note that on the NYSE, the adverse selection component exhibits a downward trend through the trading day (high at the beginning of active trading and then decreasing) in contrast to the operating cost component. In the same spirit, Bouchaud et al. (2009) argue that under normal trading conditions, the main determinant of the spread in liquid, competitive and electronic market (i.e. markets where both order processing and inventory costs should be low) is the impact induced by adverse selection.

The aforementioned papers constitute a small but representative fraction of microstructure papers that analyse transaction costs, the trading process and liquidity determination. We maintain that given the tendency to increased trading activity on electronic markets, adverse selection considerations and the consequent impact it has on securities markets, appears to be the most logical route along which to develop models analysing the determinants of transaction costs. Based on the MMI approach and the idea of bets (i.e. risk transfers) and their consequent market impact due to adverse selection introduced by Kyle and Obizhaeva (2016b), Kyle and Obizhaeva (2016a) propose a novel way to construct transaction costs models to measure such market impact costs using scaling laws. They demonstrate that these scaling laws, at least approximately, match empirical patterns in the trading behaviour of Russian stocks.

In this chapter, based on the invariance concept and the proposed scaling laws for transaction costs, we investigate the empirical predictions of MMI in a different equity market context than Kyle and Obizhaeva (2016a), specifically for FTSE 100 constituent stocks trading on the London Stock Exchange (LSE). The chapter contributes to the literature as follows. First, we investigate the proportionality between transaction costs per unit of volatility and trading activity as suggested by MMI theory for trades on the subset of 70 equities which remain constituents of the FTSE 100 stocks trading on the LSE during our sample period. As far as we know this is the first study in the literature to investigate this specific empirical invariance prediction in equity markets[63]. For the purposes of this chapter, following Kyle and Obizhaeva (2016a), we conjecture that there exists a proportionality between the number of trades and bets, provided that tick and minimum trade size or other microstructure elements adjust across stocks so that they have an identical impact on trading.

Second, we examine the stipulated invariance proportionality using different proxies for transaction costs. Specifically, apart from the quoted bid-ask spread which Kyle and Obizhaeva (2016a) also use, we employ both effective and realised spreads in order to capture different aspects of transaction costs and test whether the theoretical proportionality between the transaction costs per unit of volatility and trading activity in the power of -1/3 holds. Third, we investigate whether the MMI empirical prediction changes when we account for trading activity and volume traded on alternative trading platforms for the same stocks, and we account for their impact of market fragmentation on transaction costs in the invariance framework. We further explore this idea in a hypothetical consolidated market where a market participant has simultaneous access to different trading platforms.

Our principal empirical findings are as follows. The -1/3 proportionality between percentage transaction costs per unit of volatility and trading activity, as indicated by the transaction cost microstructure invariance prediction, is present in daily patterns on average in our sample for all three proxies of transaction costs. Spreads become narrower with an increase in trading activity during specific 5-minute time intervals than within specific days provided that returns volatility is constant. Larger trades have a negative impact on the proportionality when underlying variables are estimated as intraday averages, whereas they are more likely to occur during specific days rather than specific 5-minutes intervals across trading days. Market fragmentation does not change the statistical significance of estimated invariance coefficients, though it may lead to a further reduction of the percentage transaction costs on LSE per unit of volatility. Trading activity and volume traded on Chi-X have the greatest impact on LSE

[63] We confront the same data issues that we face in the previous chapters. Please refer to the introduction in Chapter 3 for more information

percentage transaction costs. Moreover, invariance proportionality also holds for the consolidated market. However, the lower reduction in the realised spreads we observe for a given increase in trading activity may suggest a greater impact of large trades in the alternative platforms, or alternatively that only a few market makers are able to benefit from the increase in trading activity.

Finally, this chapter contributes to the existing literature relating to market microstructure invariance and the related scaling laws, as well as to the discussion concerning the extent to which its empirical predictions hold across different market settings. Related papers include Kyle and Obizhaeva (2016a), Kyle and Obizhaeva (2016b), Kyle et al. (2016), Bae et al. (2016), Kyle et al. (2014) and Andersen et al. (2016). Our results complement those of microstructure research which discusses the components, measures and models of transaction costs, and also studies which investigate transaction costs in the presence of market fragmentation. Relevant studies include the papers by Battalio (1997), Battalio et al. (1997), Bessembinder and Kaufman (1997), Battalio et al. (1998), Battalio and Holden (2001), Boehmer and Boehmer (2003), Degryse et al. (2015) and Gresse (2017) who analyses market fragmentation and its impact on transaction costs).

The remainder of the chapter proceeds as follows. Section 2 briefly summarizes the theoretical background for the development of a transaction cost model and the invariance of transaction costs, as suggested in Kyle and Obizhaeva (2016a). Section 3 explains the methodology and the main empirical hypothesis. Section 4 concerns with the data and descriptive statistics regarding the underlying variables and presents the empirical results of the analysis. Section 5 concludes.

5.2 Theoretical background

5.2.1 On the development of a transaction cost model

In practice trading stocks or other securities can be expensive. Kyle and Obizhaeva (2016a) assume that G_{jt} denotes the price impact cost of executing a bet of size Q_{jt} in asset j at time t. They make the identifying assumption that G_{jt} is a function of (i.e. depends on) bet size Q_{jt} (units of shares), the share price P_{jt} (units of local currency per share), the trading volume V_{jt} (units of shares/day), the returns variance σ_{jt}^2 (units per day) and the average cost in local currency units of executing the bet $\bar{C}_B$ (units of local currency). It follows that:

$$G_{jt} := g(Q_{jt}, P_{jt}, V_{jt}, \sigma^2_{jt}, \bar{C}_B) \quad \textbf{(36)}$$

In practice, the quantity G_{jt} is dimensionless (i.e. a number), measured in bps, with $G_{jt} \geq 0$. Moreover, G_{jt} is a function only of its parameters[64] and not any other characteristics of asset j at time t. So, any proposed model of transaction costs should take into account the above restrictions. Dimensional analysis highlights the importance of the units used being consistent. In the above equation all underlying arguments are measured in units of currency, shares or time. According to the Buckingham π theorem any physically valid function with N variables, even if its form is unknown, it can be rewritten as a combination of $N-K$ dimensionless parameters formulated from the original variables, where K is the number of physical dimensions involved. Given that the function (36) involves 3 dimension and 5 physical variables then it follows that:

$$G_{jt} = g(\pi_{1_{jt}}, \pi_{2_{jt}}) \quad \textbf{(37)}$$

where $\pi_{1_{jt}}$ and $\pi_{2_{jt}}$ are two dimensionless parameters ($N-K=5-3=2$)

Kyle and Obizhaeva (2016a) propose that these two dimensionless variables, $\pi_{1_{jt}} = L_{jt}$ and $\pi_{2_{jt}} = Z_{jt}$, are defined as:

$$L_{jt} := \left(\frac{m^2 \cdot P_{jt} \cdot V_{jt}}{\sigma^2_{jt} \cdot \bar{C}_B} \right)^a, \quad Z_{jt} := \frac{P_{jt} \cdot Q_{jt}}{L_{jt} \cdot \bar{C}_B}, \quad G_{jt} = g(L_{jt}, Z_{jt}) \quad \textbf{(38)}$$

where m^2 is a dimensionless constant and a is an exponent. Initially a does not play any significant role given that L_{jt} is a dimensionless parameter.

Kyle and Obizhaeva (2016a) refine further their proposed trading cost model with the introduction of a conservation law in the form of leverage neutrality. The idea behind leverage neutrality is quite simple and intuitive, drawing inferences from the capital structure theorem of Modigliani and Miller (1958). Specifically, adding cash or riskless debt in the "package" of risky asset traded, should not affect the percentage cost G_{jt} of trading the risky asset[65]. More formally, if a stock is levered up by a factor of A, following a cash dividend payment of

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[64] That is why the function in (37) is defined as g and not g_{jt}

[65] Leverage changes (i.e. changes in the debt/equity ratio of a firm) should not have an impact on the economics of the risk transfer as described by a bet of Q_{jt} shares.

$(1-A^{-1})P_{jt}$ (units of local currency), which is financed with cash or riskless debt, then the variables that potentially can affect G_{jt} will change as presented in Table 1:

[Table 1 in here]

Substituting the variables in Table 1 into the expression for L_{jt} and setting $a = 1/3$ reveal that if the stock is levered by a factor of A, the dimensionless parameter L_{jt} becomes L_{jt}/A, which in turn means the dimensionless parameter Z_{jt} remains unchanged. It follows that the percentage cost G_{jt} of executing a bet with size Q_{jt} will also change by a factor of A[66].

Therefore, imposing the restriction of leverage neutrality suggests that for any A, the function f should be homogeneous, so that:

$$g(A^{-1}L_{jt}, Z_{jt}) = A \cdot f(L_{jt}, Z_{jt}) \overset{A=L_{jt}}{\Rightarrow} L_{jt}^{-1} \cdot f(1, Z_{jt}) = g(L_{jt}, Z_{jt}) = G_{jt} \tag{39}$$

Then for a univariate function f which is defined as $f(Z_{jt}) := g(1, Z_{jt})$, the percentage cost function G_{jt} can be written as:

$$G_{jt} = \frac{1}{L_{jt}} f(Z_{jt}), \quad L_{jt} := \left(\frac{m^2 \cdot P_{jt} \cdot V_{jt}}{\sigma_{jt}^2 \cdot \bar{C}_B} \right)^{1/3}, \quad Z_{jt} := \frac{P_{jt} \cdot Q_{jt}}{L_{jt} \cdot \bar{C}_B} \tag{40}$$

Given the initial identifying assumption of the parameters that may affect G_{jt} in (36), the equations for L_{jt} and Z_{jt} in (38) and the scaling and constraints implied by dimensional analysis and leverage neutrality, the transaction cost function in (40) takes a more general specification:

$$G_{jt} = g(Q_{jt}, P_{jt}, V_{jt}, \sigma_{jt}^2, \bar{C}_B) = \left(\frac{\sigma_{jt}^2 \cdot \bar{C}_B}{m^2 \cdot P_{jt} \cdot V_{jt}} \right)^{1/3} f\left[\left(\frac{\sigma_{jt}^2 \cdot \bar{C}_B}{m^2 \cdot P_{jt} \cdot V_{jt}} \right)^{1/3} \cdot \frac{P_{jt} Q_{jt}}{\bar{C}_B} \right] \tag{41}$$

If bet size $\tilde{Q}_{jt}$ is assumed to be a random variable, which takes positive values for purchases and negative for sells, with $E\{\tilde{Q}_{jt}\} = 0$, and the dimensionless constant m^2 is chosen in a way

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[66] This is because the cost (in currency units) of executing this bet does not change while the value (in currency units) of the bet changes inversely proportionally with P_{jt}, from $P_{jt} \cdot |Q_{jt}|$ to $(P_{jt}/A) \cdot |Q_{jt}|$

that $E\left\{\left|Z_{jt}\right|\right\}=1$, then Z_{jt} can be viewed as *"scaled bet size"*[67] . Also taking expectations of the equation for Z_{jt} in (41) indicates that the parameter $1/L_{jt}$ is actually a ratio of the average cost in local currency units $\bar{C}_B$ and to average value of the bet in local currency units $E\left\{P_{jt}\left|\tilde{Q}_{jt}\right|\right\}$ as shown in equation (42). Kyle and Obizhaeva (2016a) argue that $1/L_{jt}$ estimates the "value-weighted expected market impact cost" of a bet as a ratio of value traded in local currency units. In this sense it can be treated as an *"illiquidity index"*, with its inverse serving as a *"liquidity indicator"*[68]:

$$E\left\{\left|Z_{jt}\right|\right\}=1 \Rightarrow \frac{1}{L_{jt}}=\frac{\bar{C}_B}{E\left\{P_{jt}\left|Q_{jt}\right|\right\}}=\left(\frac{\sigma_{jt}^2\cdot\bar{C}_B}{m^2\cdot P_{jt}\cdot V_{jt}}\right)^{1/3} \Rightarrow E\left\{P_{jt}\left|Q_{jt}\right|\right\}=L_{jt}\cdot\bar{C}_B \qquad \textbf{(42)}$$

The expression (42) suggests that more liquid markets are associated with more bets of larger sizes. In addition, from the definition of the number of bets $\gamma_{jt}=V_{jt}/E\left\{\left|\tilde{Q}_{jt}\right|\right\}$ as introduced in in Kyle and Obizhaeva (2016b) and the definitions of both L_{jt} and Z_{jt} it can be inferred that as liquidity increases the number of bets increases twice as fast as compared to their size:

$$\gamma_{jt}=\frac{\sigma_{jt}^2\cdot L_{jt}^2}{m^2} \qquad \textbf{(43)}$$

The equation (43) indicates that there exists a proportionality between the illiquidity measure $1/L_{jt}$ and the returns volatility in one unit of business time $\sigma_{jt}/\gamma_{jt}^{1/2}$, the latter describing the price movement caused by the arrival of each bet in the market.

Utilising the definition of trading activity from Kyle and Obizhaeva (2016b), namely the product of trading volume and returns volatility[69] denoted in local currency units, $W_{jt}:=P_{jt}\cdot V_{jt}\cdot\sigma_{jt}$, the definitions of $1/L_{jt}$ in (42) and γ_{jt} in (43) can be rewritten as follows:

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[67] This is because it expresses the bet size Q_{jt} as a multiple of the average unsigned bet size $E\left\{\left|\tilde{Q}_{jt}\right|\right\}$

[68] The definition of $1/L_{jt}$ implied in equation (42) is similar to that suggested in Kyle and Obizhaeva (2016b). In a sense, the scaling constant m^2 accounts for the volume multiplier ζ, the volatility multiplier ψ , the average amount of risk in local currency units a bet transfers per unit of business time, $E\left\{\left|\tilde{I}\right|\right\}$, so that we can directly use the trading volume V_{jt} and returns variance σ_{jt}^2 when we refer to bets.

[69] Trading activity can be considered as a measure of the total risk transferred per day. It is leverage neutral and is measured in local currency units per time to the power of 3/2.

$$\frac{1}{L_{jt}} = \sigma_{jt}\left(\frac{m^2 W_{jt}}{C}\right)^{-1/3}, \quad \gamma_{jt} = \left(\frac{W_{jt}}{mC}\right)^{2/3} \tag{44}$$

5.2.2 Market microstructure invariance contribution

The idea of developing models with the use of dimensional analysis and conservation laws, such as leverage neutrality, is an interesting approach that opens limitless possibilities for market microstructure literature. However, any model will require measures for its underlying variables in order to yield valid empirical predictions. For example in the case of the theoretical transaction cost model (41), variables such as the price of the security P_{jt}, the trading volume V_{jt} and returns variance σ_{jt}^2 constitute asset characteristics that can be directly acquired or estimated with data that is publicly available. In contrast, the bet size Q_{jt} is only known to the respective trader that initiates the transaction and is therefore, private information. As for the transaction cost estimates, some can easily be obtained or estimated, such as the bid-ask spreads, while others require access to confidential information. What is also difficult to capture is how the dimensionless parameters m^2 and $\bar{C}_B$ may or may not remain constant across different assets.

Kyle and Obizhaeva (2016b) introduce market microstructure invariance theory as the empirical hypotheses that when bet sizes are estimated as risks that bets transfer in business time, both the risks and the transaction costs in local currency units that are associated with these bets remain constant. Based on these invariance principles, Kyle and Obizhaeva (2016a) argue that the value in currency units of the cost $\bar{C}_B$ and the scaling variable m^2 remain constant across both time and securities. The former principle is based on the insight that the allocation of resources across time and assets by market participants is such that $\bar{C}_B$ is balanced, and the latter principle is based on the conjecture that information signals on which the sizes of a bet are decided, do not vary across time and securities. Intuitively, the aforementioned invariant variables can potentially assist in connecting the microscopic and macroscopic properties of trading in an asset. For example if these invariance principles hold in a specific market then the liquidity index in (40) becomes a measure that is directly proportional to volume (in local current units) per unit of returns variance to the power of 1/3 and can be readily estimated from publicly available data. This aligns with the general belief of traders that transaction costs are higher in markets with low volume and high volatility. MMI also provides an alternative to other well-known liquidity measures such as those developed in Amihud (2002) and Pástor and Stambaugh (2003), and is somewhat similar to the definition of "market temperature"[70] in Derman (2002), where "temperature" is actually the product of calendar-time volatility and square root of trading frequency (number of intrinsic-time ticks that occur for a stock). In analogous fashion, the illiquidity measure in (44) is proportional to trading activity in the power of 1/3 per unit or returns volatility.

However, market microstructure invariance does not suggest a specific functional form for the function f in equation (41). Assuming as in Kyle and Obizhaeva (2016a) that f is a power function of the form $f(Z_{jt}) = \alpha \left| Z_{jt} \right|^{\omega}$ with the constant $\alpha \neq 0$, then for different values of ω, we can obtain the following special cases from equation (40) that are all consistent with invariance:

1. Proportional transaction cost ($\omega = 0$):

$$G_{jt} = \alpha \cdot \frac{1}{L_{jt}}, \qquad \alpha = consant \tag{45}$$

2. Linear market impact cost ($\omega = 1$):

$$G_{jt} = \alpha \cdot \frac{P_{jt} \cdot \left| Q_{jt} \right|}{\bar{C}_B \cdot L_{jt}^2}, \qquad \alpha = consant \tag{46}$$

3. Square-root market impact cost ($\omega = 1/2$):

$$G_{jt} = \alpha \cdot \sigma_{jt} \left(\frac{\left| Q_{jt} \right|}{V_{jt}} \right)^{1/2}, \qquad \alpha = consant \tag{47}$$

The liner market impact model aligns closely with the ideas formulated in price-impact models that are contingent on adverse section (e.g. Kyle (1985)), whereas the square root specification is in accordance with the empirical findings presented in certain econophysics papers (e.g. (e.g. Gabaix et al. (2006a)). However, the functional form of the cost function may correspond to a higher power law as reported by Almgren et al. (2005).

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[70] The idea of market temperature is that stocks that trade more frequently (i.e. stocks that are hotter) in the short run will lead to a short-term expectation for greater returns.

5.3 Methodology

The above analysis provides a way to investigate the invariance of transaction costs by testing its empirical implications as suggested by equations for the transaction costs model, the illiquidity measure and the number of bets. As in Chapter 3, the focus of this chapter, when examining the specific invariance principle, is on trades rather than bets. MMI theory is based on the concept of bets, and such bets are difficult to observe in practice, as they are executed through sequences of smaller orders to reduce transaction costs. To test the invariance of transaction costs in higher frequencies (i.e. for transactions occurring in short intervals) certain operational assumptions are required. For the purposes of this chapter, in similar fashion to Kyle and Obizhaeva (2016a), we conjecture that there exists a proportionality between the number of trades and bets, provided that tick and minimum trade size or other microstructure elements adjust across stocks so that they have an identical impact on trading. As in Andersen et al. (2018) we argue that this condition that allows for testing invariance relationships for trades is strict and may not be valid in practice. However, it is a purely empirical hypothesis that is directly motivated by the invariance of transaction costs for bets and the MMI theory, as introduced by Kyle and Obizhaeva (2016b).

Therefore, to fix things let TC_{jt} denote the transaction cost in local currency units for executing a trade in asset j at time t. Provided that the underlying assumptions for the connection between bets and trades holds, an empirical implication/hypothesis of invariance for trades is given by substituting for $1/L$ in (45) from definition in (44) and then applying logs:

$$\log\left(\frac{TC_{jt}}{P_{jt}\cdot\sigma_{jt}}\right)=c-\frac{1}{3}\log W_{jt} \quad \textbf{(48)}$$

Note that in accordance with dimensional analysis in the previous section, the transaction cost G_{jt} in equation (45) should be dimensionless. Therefore, TC_{jt} is divided by P_{jt} (in local currency units) to acquire the desired outcome as σ_{jt} is also in local currency units. Given that invariance suggests that the parameters $\bar{C}_B$ and m^2 remain constant across both time and assets, they have been moved from the definition of $1/L$ into the constant term c for the empirical estimation. The above equation (48) implies a proportionality between the transaction costs per unit of volatility and the trading activity in the power of $-1/3$.

For the empirical analysis, we estimate the underlying variables in 5-minute intervals as in Chapter 3 of the thesis. Intervals with zero number of trades or zero volatility are excluded from the estimation[71]. Realised volatility is estimated as the sum of 10 second squared returns[72] over the specific time span. Furthermore, we introduce the following three commonly used proxies for the transaction cost, TC_{jt}: 1) the quoted spread (in local currency units) which is defined as $QS_{jt} = Ask_{jt} - Bid_{jt}$ following Boehmer and Boehmer (2003), 2) the effective spread (in local currency units) which is defined as $ES_{jt} = I_{\text{trade}}\left(TP_{jt} - MP_{jt}\right)\times 2$ following Lee (1993) and Huang and Stoll (1996) and 3) the realised spread (in local currency units) which is defined as $2\times I_{\text{trade}}\left(TP_t - MP_{j,t+5}\right)$ following Huang and Stoll (1996). Following the classification rule as introduced by Lee and Ready (1991), if a trade registers above (below) the prevailing midpoint at that moment it is classified as buyer-initiated (seller-initiated). To match each trade with respective quotes in both effective and realised spreads we employ similar approach as in Holden and Jacobsen (2014). We use the interpolated timing rule where each trade is matched with the quotes that prevail in the prior millisecond[73], while we account for withdrawn quotes. For asset j and time t, Ask_{jt} is the best ask price (i.e. the lowest price at which a trader is willing to sell), Bid_{jt} is the best bid price (i.e. the highest price at which a trader is willing to buy), MP_{jt} is the midpoint of the best bid and ask prices, TP_t is the trade price, $MP_{j,t+5}$ is the midpoint of the best bid and ask prices 5 minutes after the execution of the trade and I_{trade} is a trade indicator that takes +1 if the trade is a buy and -1 is the trade is a sell. All transaction costs proxies are estimated on a tick frequency and then averaged for each respective 5-minute interval.

To mitigate the effects of sampling variation and any potential measurement errors we also aggregate the logarithms of the 5-minute observations regarding underlying variables and average their value intraday based on the following equation:

[71] The 5-minute frequency is preferred compared to 1-minute as it yields less intervals with zero number of trades or zero volatility. In this way more information regarding the underlying variables is included rendering the coefficients estimates more accurate.

[72] Changes in the midpoints between the best bid and ask prices

[73] In contrast to Lee and Ready (1991) that uses 5-seconds frequency for matching, in order to avoid erroneous sequencing of quotes when classifying trades, we prefer to use the interpolated timing rule (millisecond matching), due to the prevalence of high-frequency trading in recent years.

$$v_{jd} = \frac{1}{T} \cdot \sum_{t=1}^{T} \tilde{v}_{jdt} \ , \ d = 1, \ldots\ldots, T \qquad \textbf{(49)}$$

where there are T intervals of length t in a trading day d and v_{jd} represents the intraday average in logs of the relevant underlying variables.

Alternatively, we can calculate the average of the relevant trading variables across days for each distinct 5-minute interval as:

$$v_{jt} = \frac{1}{D} \cdot \sum_{d=1}^{D} \tilde{v}_{jdt} \ , \ d = 1, \ldots\ldots, D \qquad \textbf{(50)}$$

where there are D intervals of length t over the entire sample of D days and v_{jt} represents the average across days in logs of the relevant underlying variables.

Based on the approaches we outline above, the baseline model for the estimation of the invariance hypothesis in equations (48) takes the following form:

$$y_{jl} = c + \beta_{jl} w_{jl} + u_{jl}^{n} \qquad \textbf{(51)}$$

where l represents either distinct trading days (i.e. $l = d,\ t = 1, \ldots.., D$) or different intraday intervals ($l = t,\ t = 1, \ldots.., T$) for asset j and during this l time span, y_{jl} is average of logarithm of the percentage transaction cost per unit of volatility, w_{jl} is the average of logarithm of trading activity and u_{jl}^{n} are the regression residuals.

Based on the model in equation (51), we formally test the null hypothesis $H_0 : \beta_{jl} = -1/3$ in this chapter using the aforementioned proxies for transaction costs.

5.4 Empirical Results

5.4.1 Data and descriptive statistics

To examine the invariance relationship in equation (51) we use time-stamped tick data we obtain from Thomson Reuters Tick History for the 70 constituent stocks of the FTSE 100 index which trade on the LSE and remain constituents of the FTSE 100 throughout the sample period (see a complete list of stocks in Appendix I-Table A1). The dataset includes tick-by-tick information on the best available bid and ask quotes, transaction prices, and trading volume (in shares), for the 3 years between 1st January 2007 and 31st December 2009.

We focus on the continuous trading period on the LSE from 8am to 4.30pm, Monday to Friday. We exclude 30 days that correspond to holidays or other days with reduced trading activity arising from reduced trading hours. This leads to a total of 754 trading days. We further divide each trading day into 102 intervals of 5 minute duration. In each interval, we aggregate the observations for trading volume, number of trades and average trade size, price and transaction costs so our estimates relate to five-minute values.

Table 2 reports the summary statistics for the sample of 70 FTSE 100 stocks. The market capitalisation $MCap_{ij}$ is the 3-year average of monthly values from the LSPD database as provided by WRDS for each stock. The number of trades N_{jt}, trading volume V_{jt} and GBP volume $P_{jt}V_{jt}$ are daily averages across 754 days for each respective stock. The transaction costs and percentage transaction costs are averages of the transaction costs and percentage transaction costs respectively at the end of each five minute interval. Returns volatility σ_{jt} is the daily average of the square-root of the sum of ten-second returns across 754 days for each respective stock. Market capitalisation average of the 70 FTSE 100 is 17.31 billion, with a range of 100.31 billion. The difference between the max and min market capitalisation stocks is an institutional characteristic of FTSE 100. Such difference is also apparent for the average number of trades, trading volume and GBP volume per day. The maximum average daily volatility is 0.055 and the minimum is 0.016 among the stocks of our sample. The average quoted spread ranges from 0.0011 to 0.0500 GBP and the average effective spreads vary from 0.0016 to 0.0286, whereas their mean values are very similar across stocks. The average realised spread ranges from 0.0011 to 0.0127 across stocks and its mean is 0.0041, which are lower than the means of quoted and effective spreads as expected. The means of the average percentage quoted and effective spreads are 12.29 and 11.76 bps, respectively and higher than the mean of the average percentage realised spreads (5.54 bps) across stocks. The range of the average percentage quoted spread is approximately 15bps, that of average percentage effective spread 32bps and that of average percentage realised spread 19bps. Results indicate that for the period under analysis the trades are executed at the best bid and ask spreads most of the time, whereas there is an evident temporary price impact. The fact that effective spreads are a bit lower on average when compared to quoted spreads indicates that the market provides price improvement, namely actual transaction prices are better than quoted prices (a lower execution price than the ask for purchases and a higher execution price than the bid for sales).

[Table 2 in here]

5.4.2 The London Stock Exchange Market

As explained in the previous section to obtain robust estimates of the underlying variables for the analysis, we calculate an average of 5-minute observations across the 102 intervals during each of 754 trading days using equation (49) or an average of 5-minute observations across the 754 days for each of 102 intervals using equation (50). These intraday averages serve as the observations for the investigation of the invariance hypothesis. To control for both time and stock effects, we employ the following two-way fixed effects model when estimating equation (54)[74].

$$y_{it} = a + \beta w_{it} + \varepsilon_{it} \;\; (t \in \{1, \ldots\ldots, 754\} \; or \; t \in \{1, \ldots\ldots, 102\}; i \in \{1, \ldots\ldots, 70\}) \qquad \textbf{(52)}$$

where for stock i during a trading day t (or time interval t), y_{it} is the intraday (or across days) average logarithm of the percentage transaction cost per unit of volatility. This is $(QS / P\sigma)_{it}$ for percentage quoted spreads, $(ES / P\sigma)_{it}$ for percentage effective spreads and $(RS / P\sigma)_{it}$ for percentage realised spreads, w_{it} is the intraday (or across days) average of the logarithm of trading activity and ε_{it} are the error terms which are assumed to be independently distributed across stocks.

Table 3, Panel A, presents the coefficient estimates of the OLS regression model specified in equation (52) for different definitions of trading costs when intraday averages are used (Model 1 refers to quoted spreads, Model 2 refers to effective spreads and Model 3 refers to realised spreads). The invariance hypothesis that the proportionality coefficient β is equal to $-1/3$ is accepted by all three models. Using quoted spreads as a proxy for transaction costs yields estimations with the lowest standard errors. Results indicate that on average the percentage transaction costs per unit of volatility are proportional to trading activity to the power of $-1/3$ per day. This is consistent with the findings reported in Kyle and Obizhaeva (2016b) for portfolio transitions regarding US stocks. Assuming that returns volatility remains constant, the coefficients estimates suggest that if trading activity increases by one unit the quoted,

[74] Poolability test shows the presence of time and stock fixed effects. In contrast with Kyle and Obizhaeva (2016a) and Andersen et al. (2016) that employ a pooled model here we use a two-way fixed effects model.

effective and realised spreads should drop by 1/3 percent for the invariance of transaction costs to hold.

Table 3, Panel B, presents the coefficient estimates when we use averages across days. All coefficients estimates are higher than the ones reported in Panel A, with the largest differences observed for the realised spreads. The null hypothesis for $\beta = -1/3$ is accepted when we employ the percentage effective costs as our proxy for transaction costs, although it is rejected at the 5% (1%) significance level when employing the percentage quoted (realised) spreads as a proxy for transaction costs in Model 1 (Model (3)), respectively. In the case of the percentage quoted spreads estimates, while the null hypothesis is rejected statistically, we believe that it still remains economically close to the predicted value of $-1/3$, taking into account the standard error (0.0233) and the absolute difference between the coefficient estimate and that predicted by invariance theory is, $\left|-1/3)-(-0.3881)\right| \approx 0.0548$. As for the percentage realised spreads the difference between the coefficient estimate and that predicted by invariance theory is $\left|-1/3-(-0.4009)\right| \approx 0.0676$[75]. Assuming that returns volatility remains constant, the above results suggest that on average in a 5-minutes interval in our sample, if trading activity increases by one unit then the percentage transaction costs decrease, although it is only for percentage effective spread measure that proportionality is statistically significantly equal to the value predicted by invariance theory.

[Table 3 in here]

Interestingly, whether we employ intraday or across day averages, the findings suggest that invariance theory always correctly implies an estimated inverse proportionality between the trading activity and percentage transaction costs per unit of volatility for the three different proxies of transaction costs. The fact that an increase in the average trading activity is followed by a more pronounced decrease in average percentage transaction costs in the same 5-minutes intervals across days rather than looking at the same time interval within days may also reveal certain features of market dynamics. Specifically, ceteris paribus market makers appear to revise their quotes with an increase in the trading activity more during certain specific 5-minute intervals of the trading day. If we adopt the view that the main components of the spread are inventory risk, order processing cost and adverse selection, the empirical results suggest that the impact of these risk parameters exhibits a more pronounced variation during the same

[75] In a broad sense even this coefficient estimate can be seen as economically significant to what invariance theory predicts, given that the difference is between 0.0479 and 0.0873, if the standard error is included.

specific 5-minute intervals across days than within a specific trading day, and that an increase in average market liquidity will be followed by a greater reduction in their magnitude than that estimated when we average observations intraday. These results may be linked also to intraday trading dynamics, as some types of traders are known to be more active at certain times of the trading day. For example, if informed institutional investors/leveraged funds are more active, early morning or late afternoon, then the adverse selection and the risk exposure of markets and consequently the spreads will be larger at these times as well. Also, it is important to note that theoretical invariance relationships refer to bets and not trades. The assumption that there is a proportionality between bets and trades may not hold precisely as expected if a bet arriving in the market is split to different orders and trades re-allocated across different 5-minute intervals during a trading day. Also, the effect of an increase in the trading activity appears to be lower in the case of the effective spread indicating that during some days (or 5-minute intervals) there are large trades or hidden orders that will induce an increase in the midquote on average that will counter the decrease of the effective spread. Finally, if the increase in average trading activity exerts such an impact on the market, it may be that market makers are willing to reduce their profits, as measured by realised spread, on average both intraday or across days, in exchange of faster unwinding of their positions (more risk transfers).

We proceed to further explore this conjecture by examining the role of large trades and their impact on the relationship between percentage transaction costs per unit of volatility and trading activity on LSE. We do this by including a dummy variable capturing large trade size in the model (52), where we define a large trade as one that lies above the 90th percentile of all executed trades in the sample period, measured in the number of shares traded. Our aim is to examine whether average (daily or 5-minutes) trade sizes for each stock that lie above 90th percentile[76] of the trade size distribution of the sample have an impact on the specific invariance proportionality and towards which direction. For intraday averages the 90th percentile corresponds to a trade size of approximately 4540 shares, whereas for averages across days is slightly lower at approximately 4320 shares. To control for both time and stock effects, we use the following variation of the model in (52), and we investigate the null hypothesis of $H_0 : \beta_1 = -1/3$:

$$y_{it} = a + \beta_1 w_{it} + \beta_2 TSize_{it} w_{it} + \varepsilon_{it} \quad (t \in \{1,.......,754\} \ or \ t \in \{1,.......,102\}; i \in \{1,.......,70\}) \quad \textbf{(53)}$$

[76] We believe that trade sizes that lie above the 90th percentile are extreme enough to capture any impact on the invariance proportionality.

where for stock i during a trading day t (or time interval t), y_{it} is the intraday (or across days) average logarithm of the percentage transaction cost per unit of volatility, namely: $(QS/P\sigma)_{it}$ for percentage quoted spreads, $(ES/P\sigma)_{it}$ for percentage effective spreads and $(RS/P\sigma)_{it}$ for percentage realised spreads, w_{it} is the intraday (or across days) average of the logarithm of trading activity, $TSize_{it}$ is dummy that takes 1 for average trade sizes (intraday or across days) that are above the 90th percentile and 0 otherwise[77] and ε_{it} are the error terms assumed to be independently distributed across stocks.

Results in Table 4 show that accounting for higher average trade sizes (i.e. large trades) have no significant impact upon the coefficient estimates either for intraday (Panel A) or across days averages (Panel B). Consistent with Table 3, the null hypothesis is accepted by all three models in Panel A and only for effective spreads in Panel B. Results indicate that an increase in the trading activity that involves a large trade size is followed by a slightly lower reduction in percentage transaction costs across days on average, compared to those arising from smaller or medium trade sizes, provided that returns volatility remains constant. This result is expected as traders need to pay higher cost for executing larger trades. The measured difference in this effect between effective and realised spreads indicates the market impact of these large trades. It can be noted that it does not have a statistically significantly differential impact on the percentage transaction costs across 5-minutes intervals, although the impact of large trade sizes is positive on average for effective percentage trade costs. This may indicate that large trades occur more during specific days rather than during specific 5-minutes intervals each. This might be driven by the fact that order splitting may occur in only during specific days rather than the same 5-minutes intervals within each day based on the needs, information and strategy of the respective traders. In a similar fashion to Table 3 results, an increase in the trading activity involving any trade size is followed by a reduction of percentage transaction costs per unit of volatility of a greater magnitude in the same 5-minutes intervals across days rather than looking at the same time interval within days.

[Table 4 in here]

[77] We do not estimate the independent coefficient of a standalone dummy variable as it is subsumed by stock fixed effects.

5.4.3 Market Fragmentation

The findings in the previous section provide our benchmark estimates for the validity of market microstructure invariance for transaction costs on the London Stock Exchange. We now proceed to examine whether the coefficient estimates are also influenced by the introduction of alternative trading platforms for the FTSE 100 stocks which occurs during the period under analysis. The implementation of the first directive on markets in financial instruments (MiFID I) in November 2007 allows for alternative trading platforms to co-exist along with the traditional regulated markets such as the London Stock Exchange. The ensuing market fragmentation and its potential impact on transaction costs, as well as the consequent decision of traders to divert their trades away from LSE towards these new platforms are potentially crucial factors we should consider when examining invariance relationships in the sample of FTSE stocks[78].

Prior to the introduction of MiFID I, trading in secondary market is concentrated on primary exchanges. MiFID I revokes the concentration rule for EU member states, thus allowing market participants to execute their transactions via different trading platforms together submitting orders to the traditional exchanges (Schacht et al., 2009; Gentile and Fioravanti, 2011). The main aim of the specific directive is to enhance both market quality and integration. It attempts to do this by increasing the competition between various order-execution venues, by improving their transparency to the greatest degree possible and by creating a unified framework that guarantees the protection of investors (Degryse et al., 2015). In this context, MiFID 1 defines three categories of trading venues available to market participants: 1) Regulated Markets (RMs), 2) Multilateral Trading Facilities (MTFs) and 3) Systematic Internalizers (SIs).Trades that are executed outside these venues are considered OTC. Under MiFID I, RMs and MTFs are both multilateral trading systems with identical functionalities. Although both allow for primary listings, only RMs are given the legal authority to list regulated financial instruments. Consequently, this means MTFs mainly focus on other trading services, a function equivalent to trading networks (ECNs) in the U.S. The SIs are investment firms that can execute orders outside RMs and MTFs on their own accounts or against other clients' orders. Under MiFID I, SIs are treated as mini-exchanges subject to certain pre- and post-trade transparency requirements (Gresse, 2017).

[78] Due to lack of data we only report trading volume and number of trades after May 2008. However, we believe this is sufficient to provide a picture of how trading is split for FSTE 100 stocks across different trading categories and platforms.

MiFID I provides for best execution prices, with order execution policies being required to be published to the clients, and involving prompt client order-handling, and transparency obligations (Schacht et al., 2009). The introduction of this specific directive has certain implications for European financial markets. Indeed, MiFID I is the regulatory precursor which heralds:: 1) a steady fashion towards electronic trading across all asset classes, in particular equities; 2) the existing electronic order books and trading systems gradually becoming more efficient and attracting more investors 3) diversification in the reporting of transactions based on their size, help to reduce market distortions; 4) data consolidation, providing a full picture of a given security's liquidity; 5) increasing competition in the financial markets, not only in terms of the trading choices available to the investors, but also in terms of post-trade services.

At the time of the study the main MTFs for FTSE 100 constituent stocks are Chi-X, BATS (Europe) and Turquoise. BATS Europe is launched on 31st October 2008 as subsidiary of the U.S. exchange BATS. Chi-X is initially a platform owned by broker Instinet and which merges with BATS in 2011, and from July 2007 it starts trading FTSE 100 constituent stocks. Finally, Turquoise opens on 22nd of September 2008 underwritten by a group of investment banks and is later acquired by the LSE at the end of 2009 (Gresse, 2017). Figure 1 shows the percentages of trading volume and number of trades across different trading categories available to market participants between May 2008 and December 2009. Our analysis focuses only on the lit order book where most volume is traded and where the highest number of trades occurs. However, it is important to note that a salient feature of the transactions during the specific period is that those occurring OTC involve a greater number of shares per trade than those occurring in other trading categories.

Moving a step forward, Figure 2 presents the percentages of the lit order book trading volume and number of trades occurring across different trading platforms, both regulated markets and multilateral trading facilities, for FTSE 100 shares between May 2008 and December 2009. The London Stock Exchange attracts the majority of the volume traded (75.57%) and still has the highest number of trades (60.29%), followed by Chi-X, Turquoise and BATS, which are the three most active MTFs during the specific period.

[Figure 1 in here]

[Figure 2 in here]

Based on the above information we investigate the impact of trading market fragmentation on invariance principles. We decide to only use data from trading on Chi-X, Turquoise and BATS, which along with LSE, account for 99.45% of total trading volume and 98.76% of total number of trades for FTSE 100 during the period under review. Analogous to the LSE dataset, the data for the aforementioned platforms includes tick-by-tick information on the best available bid and ask quotes, transaction prices, and trading volume (in shares). The data for Chi-X spans a period between 7th April 2008 and 31st December 2009, for Turquoise 1st of September 2008 and 31st of December 2009 and for BATS between 7th November 2008 and 31st December 2009[79]. We focus on the continuous trading period on the LSE from 8am to 4.30pm, Monday to Friday. We exclude days that correspond to holidays or other days with less trading activity arising from a reduction in trading hours. This leads to a total of 437 trading days for Chi-X, 336 days for Turquoise and 286 days for BATS.

First, on the basis of equation (51), we investigate whether the simultaneous presence of alternative platforms affects the estimated invariance proportionality and the percentage transaction costs per unit of volatility on LSE. In this respect, we introduce two different proxies for market fragmentation. The first, $Waltratio$, is consistent with the invariance framework and is defined as the percentage of total trading activity in the alternative platforms to the total trading activity for a specific stock during a specified time interval. The second, $Concratio$ is similar to the Herfindahl-Hirschman Index for market concentration/fragmentation (see for example Weston (2000), Bennett and Wei (2006) and Gresse (2017)) and is defined as the percentage of total volume traded in the alternative platforms to total trading volume for a specific stock during a specified time interval. Coefficients estimates are based on the two-way fixed effects model in (54). We only include data on LSE after 7th April 2008, the first date we have information on trading activity on Chi-X. We use both intraday and across days averages of 5-minutes observations for the underlying variables based on equations (49) and (50) and compare the resulting coefficient estimates with those obtained when only the trading activity on LSE is taken into account for the same time span:

$$y_{it} = a + \beta_1 w_{it} + \beta_2 \Omega_{it} + \varepsilon_{it} \;\; (t \in \{1, \ldots\ldots, 437\} \; ; i \in \{1, \ldots\ldots, 70\}) \qquad \textbf{(54)}$$

[79] Although Chi-X started trading on FTSE 100 stocks in July 2007, we have data on this platform only from April 2008. Thus, we treat April 2008 as the date when Chi-X trades in FTSE 100 stocks can potentially have an impact on the LSE.

where for stock i during a trading day t, y_{it} is the intraday (or across days) average of the logarithm of percentage transaction cost per unit of volatility, namely: $(QS / P\sigma)_{it}$ for percentage quoted spreads, $(ES / P\sigma)_{it}$ for percentage effective spreads and $(RS / P\sigma)_{it}$ for percentage realised spreads, w_{it} is the intraday (or across days) average of the logarithm of trading activity, Ω_{it} is the either the intraday (or across days) average of $Waltratio$ or the intraday (or across days) of $Concratio$ and ε_{it} are the error terms assumed to be independently distributed across stocks

In Table 5, Panel A we present the results based on the model in (52), when only LSE trading activity is included. The coefficients estimates, either for intraday averages or averages across days for all three models are higher in value than those reported in Table 3, when the entire sample is used. However, the rejection or acceptance of null hypothesis for $\beta_1 = -1/3$ remains the same in every individual case. Coefficient invariance estimates, in Panel B, when $Waltratio$ is included in the model, are also similar, both for intraday and across days averages, although perhaps slightly more inflated, especially in the latter set. Economically, in terms of the values of the coefficients, this may be attributable to the ensuing competition between market makers in LSE and other platforms following market fragmentation. Statistically, our findings indicate that market fragmentation in relation to the specific FTSE 100 stocks, during the period under analysis, does not significantly impact, the invariance proportionality between the percentage transaction costs per unit of volatility and trading activity on LSE[80]. However, some specific features of our results are noteworthy. In particular, the results in Panel B reveal an interesting correlation between $Waltratio$ and percentage transaction costs per unit of volatility on LSE. Specifically, in all the models and independently of the estimation method (i.e. intraday or across days averages), $Waltratio$ is negatively correlated with the transaction costs per unit of volatility on LSE. Intuitively, provided that returns volatility remains constant, $Waltratio$ moves in the opposite direction than the direction of percentage transaction costs on LSE, on average during a 5-minutes interval or during a trading day.

[Table 5 in here]

[80] Similar appears to be the effect of financial crisis that is present in the specific time span

This finding is consistent with papers that investigate the relationship between market fragmentation and transaction costs (see for example (Gresse (2017)). Also, this negative correlation is more obvious when averages across days are used, indicating that this pattern is stronger on average during specific 5-minutes intervals as compared to specific trading days. As with the values of the estimated invariance coefficients, this correlation may be induced by the competition across platforms. Intuitively, as investors and their trades move to other platforms, LSE market makers will need to offer better spreads to attract traders or if the spreads offered in LSE are not better than other platforms trades will most likely move away from LSE.

Our results remain very similar when we include $Concratio$ instead of $Waltratio$, in the model in (54), as depicted in Table A2 in the Appendix. When intraday averages are used, all three models again accept the null hypothesis for a $-1/3$ proportionality between percentage transaction costs per unit of volatility and trading activity on LSE, whereas only Model 2 (effective spreads) yields the predicted proportionality, when averages across days are employed. The values of the estimated invariance coefficients are very close to those reported in Table 5, Panel B. Again, market fragmentation does not appear to statistically affect the invariance proportionality independently of the estimation method. This suggests the underlying rationale is indeed robust to fragmentation considerations. We note that the negative correlation between $Concratio$ and the LSE percentage transaction costs per unit of volatility is greater than that between $Waltratio$ and the LSE percentage transaction costs per unit of volatility in all models and set of averages. Specifically, this negative correlation is greater by 0.30 to 0.45 compared to $Waltratio$ and is stronger during specific 5-minutes intervals than within trading days for all percentage transaction costs proxies.

To further examine the impact of fragmentation on the invariance principle and LSE percentage transaction costs, we repeat the same analysis, but now we include $Waltratio$ for each respective platform instead of the total across all markets. We estimate the relationship arising from the model in (55). The data starts on 7th of November 2008, the date from which we have continuous data on trading for all platforms. We use again intraday or across days averages of 5-minutes observations for the underlying variables and compare coefficient estimates with those we generate when only the trading activity on LSE is taken into account. We obtain the coefficient estimates from the following two-way fixed effects model:

$$y_{it} = a + \beta_1 w_{it} + \beta_2 \Omega chix_{it} + \beta_3 \Omega bats_{it} + \beta_4 \Omega turq_{it} + \varepsilon_{it}$$
$$(t \in \{1,......., 286\} ; i \in \{1,......., 70\}) \quad (55)$$

where for stock i during a trading day t, y_{it} is the intraday (or across days) average of the logarithm of percentage transaction cost per unit of volatility, namely: $(QS / P\sigma)_{it}$ for percentage quoted spreads, $(ES / P\sigma)_{it}$ for percentage effective spreads and $(RS / P\sigma)_{it}$ for percentage realised spreads), w_{it} is the intraday (or across days) average of the logarithm of trading activity, $\Omega chix_{it}$ is the intraday (or across days) average of $Waltratio$ on Chi-X, $\Omega bats_{it}$ is the intraday (or across days) average of $Waltratio$, $\Omega turq_{it}$ is the (or across days) intraday average of $Waltratio$ on Turquoise and ε_{it} are the error terms assumed to be independently distributed across stocks.

The results based on the model in (55) using intraday averages are given in Table 6. Upon inspection of Panel A, which only considers LSE trading activity, the coefficient estimates remain close in terms of their value to those we report for the entire sample in Table 3, Panel A and those in Table 5, Panel A. Independent of the percentage transaction costs proxy we employ, the invariance coefficients remains statistically significantly equal to $-1/3$ for all three models. The coefficients are also qualitatively similar in magnitude to the previous estimates for averages across days (Table A3, Panel A in the Appendix). When we include $Waltratio$ (Panel B) in the model, invariance coefficients are somewhat higher than those in Panel A, but the null hypothesis of $\beta = -1/3$ is still accepted by all three models. These findings are consistent with the results reported for $Waltratio$ for total trading activity and volume across all alternative platforms, respectively. The null hypothesis is rejected for all models when averages across days are used (Table A3, Panel B, in the Appendix).

[Table 6 in here]

This implies that the average reduction in percentage transaction costs is stronger during specific 5-minutes intervals than trading days, when accounting for individual platform $Waltratio$[81]. However, economically the result is the same. In line with the results for

[81] We understand that part of this increase may be caused by a bias induced in the model due to the inclusion of the extra variables. However, we do not think this is sufficient to explain the magnitude of the specific increase.

Waltratio we report in Table 5, market fragmentation does not appear to statistically impact the invariance proportionality, independently of the estimation method used.

Nevertheless, both sets of findings, either those for intraday averages (Table 6, Panel B) or those analysing averages across days (Table A3, Panel B) suggest that *Waltratio* in Chi-X is more negatively correlated with the percentage transaction costs per unit of volatility on LSE compared to other platforms, except from the percentage realised spreads per unit of volatility on LSE, when we use averages across days. The latter appear to have slightly higher negative correlation with the *Waltratio* on Turquoise. On the contrary, the negative correlation between *Waltratio* of the remaining two platforms and percentage transaction costs on LSE varies with the estimation method for the underlying variables and the proxy for percentage transaction costs. Specifically, the percentage spreads per unit of volatility on LSE are negatively correlated on average with the trading activity on Turquoise, when we employ within days estimation; however, this relationship is significant only for percentage realised spreads when we use averages across days. Interestingly, the trading activity on BATS is not correlated with the percentage transaction costs per unit of volatility on LSE, except from the case when percentage realized spreads are employed as a proxy and averages across days is used as an estimation method. Summarizing the above results, trading on Chi-X appears to be more correlated with the percentage transaction costs on LSE due to the fact that the majority of the volume and number of trades occurring outside LSE go through the specific platform. Also, results suggest that trading activity on Turquoise is negatively correlated with LSE percentage transaction costs on average during specific days, whereas trading activity on BATS is only correlated with the LSE percentage realised spreads during specific 5-minutes intervals.

5.4.4 Consolidated Market

To this point, we test invariance in the context of one market, LSE, and examine how the trading activity and volume on alternative platforms affect the consequent empirical implications of microstructure invariance, as well as the percentage transaction costs on the LSE. This analysis, although somewhat intuitive, does not take into account the fact that market participants during the period under investigation have simultaneous access to all platforms on which to post orders (limit or market) for a specific stock. For example, a bet (and the consequent orders and trades) for a stock can arrive in LSE and another platform at the same time or orders linked to the same originating bet for a certain stock can simultaneously be placed in different platforms because of immediacy requirements.

Therefore, it may be interesting to examine invariance from the perspective of a hypothetical consolidated market and a "global" market investor who is connected to several trading venues. In this respect, we combine trades, trading volume and transaction costs of LSE, ChiX, BATS and Turquoise so that for each of the 70 stocks in our sample we create a dataset that includes all trading information at a millisecond frequency. The sample period is between January 2007 and December 2009. Specifically, for trades that occur at the same millisecond we use the average price and the total volume across platforms. The quoted spreads are then calculated as the differences between the best ask and best bid present during each millisecond across all competing markets. The effective spreads are estimated as twice the absolute difference between the average global trade price and the midpoint of the best ask and best bid quotes across all platforms. The realised spreads are computed as twice the difference between the global average trade price and the midpoint of the best ask and best bid across all platforms, 5 minutes after the trade takes place. They are positive for buy initiated and negative for sell initiated trades.

To examine the two invariance principles we follow the same methodology for obtaining robust estimators for underlying variables. We first employ intraday averages (average of 5-minute observations across the 102 intervals during each of 754 trading days) based on equation (49) and alternatively averages across days (average of 5-minute observations across 754 days for each of 102 intervals) based on equation (50). The two-way fixed effects model for the invariance hypothesis in equation (51) has the following form:

$$gy_{it} = a + \beta gw_{it} + \varepsilon_{it} \;\; (\, t \in \{1, ..., 754\} \text{ or } t \in \{1, ..., 102\}; i \in \{1, ..., 70\}) \qquad \textbf{(56)}$$

where for stock i during a trading day t (or time interval t), gy_{it} is the intraday (or across days) average logarithm of the percentage consolidated transaction cost per unit of volatility, namely: $(QS / P\sigma)_{it}$ for consolidated percentage quoted spreads, $(ES / P\sigma)_{it}$ for consolidated percentage effective spreads and $(RS / P\sigma)_{it}$ for consolidated percentage realised spreads, gw_{it} is the intraday (or across days) average of the logarithm of consolidated trading activity and ε_{it} are the error terms.

We report invariance estimates for this consolidated market in Table 7. When we use intraday averages (Panel A), we accept the null hypothesis for $\beta = -1/3$ for the effective spreads (Model 2) and realised spreads (Model 3) on average. Using quoted spreads as a proxy for transaction costs (Model 1) leads us to reject the null hypothesis at 5% significance level, although the value remains close in economic terms to that predicted by invariance, $|0.3333-(-0.3620)| \approx 0.0286$. All coefficients are qualitatively similar in terms of values to those we report in Table 3, Panel A when only the LSE market is considered, except from that those for the quoted spreads which is apparently higher in Table 7. This result may be attributable to differences between the quoted spreads for LSE and the consolidated market's quoted spreads, as well as competition between market makers and the option a trader possesses to divert their trades to alternative platforms when they find better prices. Statistically, the effect of an increase in trading activity is the same for LSE and the consolidated market regarding percentage effective and realised spreads. However, economically, we see that the reduction in effective spreads on average is greater in the consolidated market. In turn, this implies that larger orders are also executed in the alternative platforms and the consolidated market depth is greater than LSE market alone.

[Table 7 in here]

Overall, the reduction in realised spreads in slightly lower on average in the consolidated market, which may indicate that market makers in the alternative platforms do not reduce their realised spreads as much as the LSE competitors, or in other words the market impact of trades may be greater in the alternative platforms. In contrast, when we use averages across days, coefficient estimates increase in comparison to those we report in Table 4, Panel B, and the null hypothesis is rejected for all models. This is consistent with findings regarding the LSE market, although the coefficients estimates for the consolidated market are inflated as compared to those we report in Table 3, Panel B. We observe the greatest difference in the quoted spreads and effective spreads. This shows that the average impact of a change in consolidated trading activity on the percentage transaction costs per unit of volatility is greater during specific 5-minutes intervals than during specific days. Finally, in both panels, the standard errors are lower than those we report for the LSE venue in Table 4.

Finally, analogous to our analysis of the LSE market, we investigate whether large trades have a different impact on invariance proportionality in the consolidated market. For intraday averages the 90^{th} percentile for the consolidated market corresponds to a trade size of approximately 4540 shares, very similar to the LSE market, whereas for averages across days it is lower, at approximately 4000 shares. Estimates are based on the following two-way fixed effects model:

$$gy_{it} = a + \beta_1 gw_{it} + \beta_2 TSize_{it} gw_{it} + \varepsilon_{it} \quad \textbf{(57)}$$

$$(t \in \{1,......., 754\} \ or \ t \in \{1,......., 102\}; i \in \{1,......., 70\})$$

where for stock i during trading day t (or time interval t), gy_{it} is the intraday (or across days) average logarithm of the percentage consolidated transaction cost per unit of volatility, namely: $(QS / P\sigma)_{it}$ for consolidated percentage quoted spreads, $(ES / P\sigma)_{it}$ for consolidated percentage effective spreads and $(RS / P\sigma)_{it}$ for consolidated percentage realised spreads, gw_{it} is the intraday (or across days) average of the logarithm of consolidated trading activity, $TSize_{it}$ is a dummy that takes 1 for average trade sizes (the intraday or across days) that are above the 90th percentile and 0 otherwise and ε_{it} are the error terms assumed to be independently distributed across stocks.

The results for the consolidated market we present in Table 8 confirm that accounting for large trades does not significantly change the coefficient estimates. We accept the null hypothesis for the same model specifications as in Table 7 (all three models in Panel A and for the effective spread specification in Panel B). In similar vein to the findings for the LSE market, an increase in trading activity that involves larger trade sizes causes a slightly lower reduction in average percentage transaction costs across days as compared to that resulting from small or medium trade sizes, providing that returns volatility remains constant. In contrast, it does not have a statistically significantly different impact on the percentage transaction costs interday across the same a 5-minutes interval. However, this effect is positive only for effective spreads. As we explain earlier in relation to the LSE, this might imply that large trades happen more during specific days rather than being confined to the same specific 5-minute time intervals every day. The results for the consolidated market indicate that this is a characteristic trading pattern for FTSE 100 stocks during the period under analysis.

[Table 8 in here]

5.5 Conclusion

In this chapter we empirically investigate scaling laws concerning transaction costs based on microstructure invariance theory as proposed by Kyle and Obizhaeva (2016a) and provide tests of its underlying implications introduced by Kyle and Obizhaeva (2016b). Overall, our results for share trading on the London Stock Exchange are supportive of the existence of the predicted proportionality between percentage transaction costs and trading activity proposed by invariance theory. The suggested scaling laws appear to match empirical patterns in the data for the majority of the trading activity during the period under analysis in our sample of FTSE 100 stocks. Our results in general corroborate the empirical findings Kyle and Obizhaeva (2016a) report for their sample of Russian stocks.

Specifically, using three different proxies for transaction costs, we find that invariance theory correctly implies an inverse proportionality between the trading activity and percentage transaction costs per unit of volatility for different proxies of transaction costs. The predicted invariance proportionality of -1/3 is present throughout the average daily patterns in our sample. Moreover, market makers on average offer better prices as trading activity increases during specific 5-minute intervals across days than within specific trading days, providing returns volatility is constant. Drawing inferences from the microstructure literature, our results suggest that the parameters of transaction costs such as inventory risk, order processing cost and adverse selection costs are reduced more during certain 5-minute trading intervals than other 5-minute intervals. An increase in the average daily trading activity involving larger trades leads to a smaller reduction in the average daily transaction costs per unit of volatility of the respective stocks. These larger trades tend to occur on average within particular days rather than the same specific 5-minutes interval across days. Market makers are willing to reduce their profits with an increase in trading activity on average, both across days or 5-minute intervals, in exchange for a faster unwinding of their positions (more risk transfers), thereby reducing their inventory risk.

The statistical significance of the invariance proportionality results remains unaltered when we amend the model to include ratios of trading activity and volume as indicators of market fragmentation. However, results suggest that the trading activity (or volume) on the alternative platforms relative to the total trading activity (or volume) is negatively correlated with the percentage transaction costs on LSE, both during a 5-minutes interval or during a trading day, providing that return volatility remains constant. This finding is consistent with the view that competition between market venues affects transaction costs and the way trading takes place. This is also consistent with several microstructure papers that examine the effect of fragmentation on transaction costs, including Battalio (1997), Battalio et al. (1997), Battalio et al. (1998), Battalio and Holden (2001), Gresse (2017) and others. The highest negative correlation appears to be between the trading activity on Chi-X and LSE percentage transaction costs, which is consistent with the observation that the majority of the volume and number of trades in the relevant shares occurring outside LSE goes through this specific platform. Turquoise trading is negatively correlated with LSE percentage transaction costs on average on particular days, whereas trading on BATS is only negatively correlated with LSE percentage realised spreads on average during the same specific 5-minutes interval every day.

Results from analysis of the consolidated hypothetical market show that invariance proportionality still generally holds for different proxies of transaction costs, with estimates resembling those of the LSE. Although, invariance relationships hold in the consolidated data, the performance of MMI is relatively worse. This fact may potentially indicate that market fragmentation is an important factor to consider in the invariance framework for equity markets. Finally, the lower reduction in realised spreads, on average, in the consolidated market suggests that either the market impact of trades is greater in the alternative platforms or that not all market makers take advantage of increases in liquidity via the increase in trading activity (see for example Degryse et al. (2015)).

Overall, our results provide evidence that invariance relationships and scaling laws regarding transaction costs and trading activity hold approximately in the sample under consideration. Further investigation as to whether the empirical predictions invariance proposes are valid in different periods or samples is warranted. Also examining other proxies for transaction costs and their empirical relationship to trading activity, or assuming a different price impact function may constitute a worthwhile task for future research. Finally, future papers may focus on investigating whether invariance predictions change when transaction costs are classified based on the trade size or other stock characteristics or when platform characteristics such as latency or high frequency trading are taken into account.

Table 1

Changes in the underlying variables of transaction cost model for risky asset as a result of dividend pay-out

Variables	Description
P_{jt}/A	As the price of the share including dividend is conserved it follows that the price of the share without the dividend should be P_{jt}/A
Q_{jt}	The bet size will not change as the risk transferred by a bet remains the same
V_{jt}	Trading volume will not change
$\bar{C}_B$	The cost in local currency units of executing the bet remains the same as the cash dividend has zero risk
$A^2\sigma_{jt}^2$	The return variance increase as each share has the same risk in local currency units $P_{jt}\sigma_{jt}$

Table 2

Summary statistics for the sample of 70 FTSE 100 stocks (Average and Percentiles)

The table summarises descriptive statistics of variables for the sample of 70 FTSE 100 stocks: GBP market capitalisation $MCap_{ij}$ (in billions), average number of trades N_{jt} per day, average trading volume V_{jt} (in millions) per day, average GBP trading volume $P_{jt}V_{jt}$ (in million pounds) per day, average daily returns volatility σ_{jt}, average quoted, effective and realized spreads (in pounds), as well as their percentage versions (in hundredths of a percent).

Variables	Average	Min	p5	p50	p95	Max
$MCap_{ij}(pounds)$	17.31	2.61	2.9	7.05	70.83	102.92
$N_{jt}(no\ of\ trades)$	4,869	1,961	2,225	3,846	11,814	12,921
$V_{jt}(no\ of\ shares)$	15.72	1.23	1.77	7.54	63.00	195.06
σ_{jt}	0.025	0.016	0.017	0.022	0.041	0.055
$P_{jt}V_{jt}(pounds)$	78.02	13.84	16.16	43.46	283.24	357.61
$QS_{jt}(pounds)$	0.0096	0.0011	0.0019	0.0070	0.0230	0.0500
$ES_{jt}(pounds)$	0.0091	0.0016	0.0021	0.0070	0.0221	0.0286
$RS_{jt}(pounds)$	0.0041	0.0011	0.0012	0.0037	0.0089	0.0127
$QS_{jt}/P_{jt}(bps)$	12.29	6.05	7.35	11.72	18.73	21.05
$ES_{jt}/P_{jt}(bps)$	11.76	5.58	7.07	11.06	17.81	37.59
$RS_{jt}/P_{jt}(bps)$	5.54	2.23	2.91	5.00	9.00	20.61

Table 3

Average Proportionality between Percentage Transaction Costs per unit of volatility and Trading Activity

This table reports the coefficient estimates from OLS regressions for different definitions of trading costs based on the model in (52) using intraday averages based on equation (49) in panel A and averages across days based on equation (50) in panel B. Model 1 uses quoted spreads as a proxy for transaction costs, Model 2 uses effective spreads as proxies for transaction costs and Model 3 uses realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Within-day estimates

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	1.5792*** (0.0255)	1.1161*** (0.0289)	1.0727*** (0.0369)
Invariance Coef., β	**-0.3309** (0.0265)	**-0.2926** (0.0287)	**-0.3163** (0.0361)
R^2	0.8386	0.8023	0.6535

Panel B: Estimation based on 5-minutes intervals across days

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.3629*** (0.0356)	1.9446*** (0.0411)	2.4079*** (0.0626)
Invariance Coef., β	-0.3881* (0.0233)	**-0.3504** (0.0230)	-0.4009*** (0.0197)
R^2	0.9880	0.9793	0.9360

Table 4

Average Proportionality between Percentage Transaction Costs per unit of volatility and Trading Activity when controlling for trade size

This table reports the coefficient estimates from OLS regressions for different definitions of trading costs based on the model in equation (52) using intraday averages based on equation (49) in panel A and averages across days based on equation (50) in panel B. β_2 is the coefficient of the interaction term of trading activity and a dummy that takes 1 for average trade sizes (the intraday or across days) that are above the 90th percentile and 0 otherwise. Model 1 uses quoted spreads as a proxy for transaction costs, Model 2 uses effective spreads as proxies for transaction costs and Model 3 uses realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Within-day estimates

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	1.6614*** (0.0252)	1.2055*** (0.0286)	1.1298*** (0.0368)
Invariance Coef., β	**-0.3441** (0.0272)	**-0.3070** (0.0296)	**-0.3256** (0.0368)
$TSize_{it}$	0.0244*** (0.0052)	0.0263*** (0.0050)	0.0177*** (0.0034)
R^2	0.8434	0.8077	0.6560

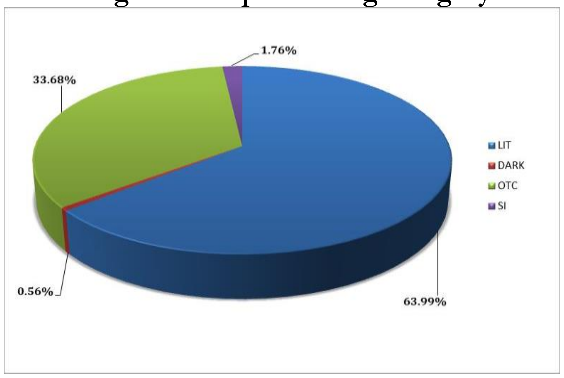
Panel B: Estimation based on 5-minutes intervals across days

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.3512*** (0.0359)	1.9479*** (0.0415)	2.3702*** (0.0633)
Invariance Coef., β_1	-0.38677* (0.0235)	**-0.3508** (0.0235)	-0.3964*** (0.0191)
β_2	-0.0014 (0.0026)	0.0004 (0.0003)	-0.0044 (0.0026)
R^2	0.9880	0.9793	0.9361

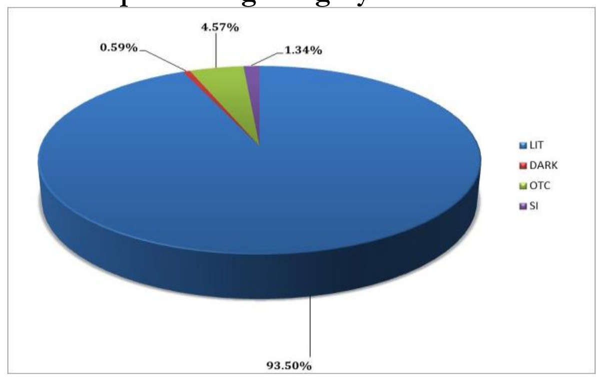
Figure 1-Breakdown of trading volume and trades across market categories

The graphs show the %trading volume and %trades across different trading categories available to market participants regarding the FTSE 100 between May 2008 and December 2009. *LIT* refers to lit order book, *DARK* to dark pools, *OTC* to over-the-counter and *SI* to systematic internalisers transactions.

%Trading Volume per trading category



%Trades per trading category

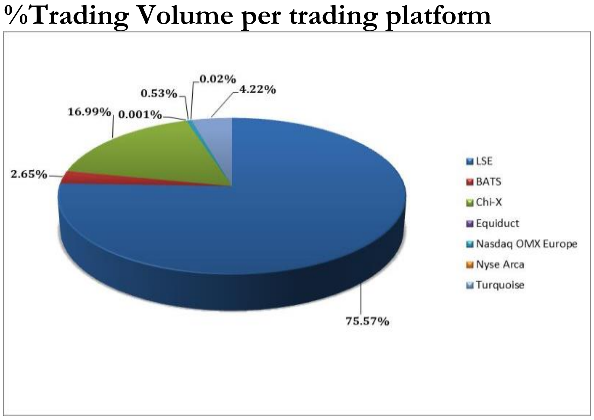


Source: Fidessa

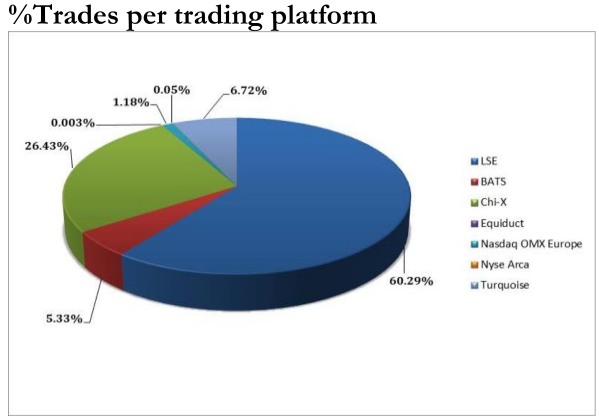
Figure 2- Breakdown of lit trading volume and trades across platforms

The graphs show the %trading volume and %trades of lit order book trading across different trading platforms regarding the FTSE 100 between May 2008 and December 2009.

%Trading Volume per trading platform



%Trades per trading platform



Source: Fidessa

Table 5

Invariance Proportionality while controlling for Waltratio

Panel A shows the coefficients estimated when only the trading activity on LSE is considered between April 2008 and December 2009 based on the model in (52). Panel B shows the coefficients, when percentage of total trading activity occurring on the alternative platforms relative to total trading activity is included, based on the model in (54). Model 1 is based on quoted spreads, Model 2 on effective spreads and Model 3 on realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta_1 = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Estimation without the aggregate trading activity on alternative platforms			
Within-day estimates			
	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	1.8642*** (0.0292)	1.4297*** (0.0331)	1.3786*** (0.0416)
Invariance Coef., β	**-0.3812** (0.0376)	**-0.3524** (0.0405)	**-0.3782** (0.0489)
R^2	0.8346	0.7824	0.6497
Estimation based on 5-minutes intervals across days			
	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.3839*** (0.0400)	1.8768*** (0.0470)	2.3705*** (0.0723)
Invariance Coef., β	-0.4086** (0.0271)	**-0.3689** (0.0286)	-0.4186** (0.0287)
R^2	0.9851	0.9753	0.9109
Panel B: Estimation when aggregate trading activity on alternative platforms is included			
Within-day estimates			
	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.1087*** (0.0308)	1.7647*** (0.0348)	1.5973*** (0.0440)
Invariance Coef., β_1	**-0.3942** (0.0381)	**-0.3702** (0.0408)	**-0.3899** (0.0496)
β_2	-0.4641*** (0.0846)	-0.6405*** (0.0959)	-0.4203*** (0.0720)
R^2	0.8374	0.7879	0.6522
Estimation based on 5-minutes intervals across days			
	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.7354*** (0.0482)	2.2762*** (0.0569)	2.8267*** (0.0895)
Invariance Coef., β_1	-0.4299** (0.0295)	**-0.3930** (0.0311)	-0.4477*** (0.0313)
β_2	-0.6992*** (0.1808)	-0.7968 (0.2075)	-0.8649*** (0.1483)
R^2	0.9855	0.9758	0.9118

Table 6

Invariance Proportionality while controlling for individual Waltratio (Within-day estimates)

Results are based on intraday averages estimation for underlying variables based on equation in (49). Panel A shows the coefficients estimated when only the trading activity on LSE is considered between November 2008 and December 2009 based on the model in (52). Panel B shows the coefficients, when percentage of trading activity occurring on each respective alternative platform relative to total trading activity is included, based on the model in (55). Model 1 is based on quoted spreads, Model 2 on effective spreads and Model 3 on realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta_1 = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Estimation without the aggregate trading activity on alternative platforms

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	1.9280*** (0.0330)	1.5229*** (0.0376)	1.4714*** (0.0471)
Invariance Coef., β	**-0.3921** (0.0501)	**-0.3680** (0.0376)	**-0.3990** (0.0630)
R^2	0.8320	0.7807	0.6473

Panel B: Estimation when individual Waltratio is included

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.3375*** (0.0377)	2.0837*** (0.0426)	1.9325*** (0.0538)
Invariance Coef., β_1	**-0.4187** (0.0526)	**-0.4045** (0.0560)	**-0.4292** (0.0669)
β_2	-1.0264*** (0.2258)	-1.3871*** (0.2409)	-1.0423*** (0.2248)
β_3	0.1471 (0.2943)	0.1907 (0.3330)	-0.2402 (0.2682)
β_4	-0.5146** (0.1819)	-0.7456*** (0.2095)	-0.4722* (0.1919)
R^2	0.8377	0.7916	0.6533

Table 7

Average Proportionality between Percentage Transaction Costs per unit of volatility and Trading Activity for the consolidated market

This table reports the coefficient estimates from OLS regressions for different definitions of trading costs for the consolidated market based on the model in equation (56) using intraday averages based on equation (49) in panel A and averages across days based on equation (50) in panel B. Model 1 uses quoted spreads as a proxy for transaction costs, Model 2 uses effective spreads as proxies for transaction costs and Model 3 uses realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Within-day estimates

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.1144*** (0.0289)	1.6244*** (0.0280)	1.5314*** (0.0331)
Invariance Coef., β	-0.3620* (0.0131)	**-0.3099** (0.0137)	**-0.3087** (0.0167)
R^2	0.8124	0.8015	0.7009

Panel B: Estimation base on across days averages

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	3.0158*** (0.0327)	2.5917*** (0.0342)	2.7691*** (0.0513)
Invariance Coef., β	-0.4333*** (0.0200)	-0.4030*** (0.0187)	-0.4235*** (0.0118)
R^2	0.9910	0.9874	0.9603

Table 8

Average Proportionality between Percentage Transaction Costs per unit of volatility and Trading Activity for the consolidated market when controlling for trade size

This table reports the coefficient estimates from OLS regressions for different definitions of trading costs for the consolidated market based on the model in equation (57) using intraday averages based on equation (49) in panel A and averages across days based on equation (50) in panel B. β_2 is the coefficient of the interaction term of trading activity and a dummy that takes 1 for average trade sizes (the intraday or across days) that are above the 90th percentile and 0 otherwise. Model 1 uses quoted spreads as a proxy for transaction costs, Model 2 uses effective spreads as proxies for transaction costs and Model 3 uses realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Within-day estimates

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.1996***	1.7026***	1.5821***
	(0.0284)	(0.0276)	(0.0331)
Invariance Coef., β_1	-0.3745**	**-0.3213**	**-0.3162**
	(0.0131)	(0.0136)	(0.0165)
β_2	0.0297***	0.0273***	0.0179***
	(0.0057)	(0.0062)	(0.0039)
R^2	0.8191	0.8078	0.7038

Panel B: Estimation based on across days averages

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	3.0110***	2.5932***	2.7628***
	(0.0328)	(0.0343)	(0.0515)
Invariance Coef., β_1	-0.4327***	-0.4032***	-0.4227***
	(0.0201)	(0.0188)	(0.0186)
β_2	-0.0013	0.0004	-0.0018
	(0.0012)	(0.0002)	(0.0023)
R^2	0.9910	0.9874	0.9603

Appendix

Table A1: Stocks and their abbreviations

Stock	Abbreviation	Stock	Abbreviation
BP	**BP**	BT GROUP	**BT**
HSBC HOLDINGS	**HSBA**	AVIVA PLC	**AV**
VODAFONE	**VOD**	PRUDENTIAL PLC	**PRU**
GLAXOSMITHKLINE	**GSK**	BAE SYSTEMS PLC	**BAE**
ROYAL DUTCH SHELL	**RDSA**	CENTRICA PLC	**CNA**
RIO TINTO	**RIO**	SCOTTISH & SOUTHERN ENERGY	**SSE**
ASTRAZENECA	**AZN**	CADBURY SCHWEPPES	**CBRY**
ROYAL BANK OF SCOTLAND GROUP	**RBS**	BSB GROUP	**BSY**
BRITISH AMERICAN TOBACCO	**BATS**	MAN GROUP PLC	**EMG**
BG GROUP	**BG**	ROLLS-ROYCE HOLDINGS PLC	**RR**
ANGLO AMERICAN	**AAL**	MORRISON (WM) SUPERMARKETS	**MRW**
BHP BILLITON	**BLT**	MARKS & SPENCER GROUP	**MKS**
BARCLAYS	**BARC**	SAINSBURY (J)	**SBRY**
TESCO	**TSCO**	WPP PLC	**WPP**
XSTRATA	**XTA**	REED ELSEVIER	**REL**
DIAGEO	**DGE**	LEGAL & GENERAL GROUP	**LGEN**
LLOYDS TSB GROUP	**LLOY**	COMPASS GROUP	**CPG**
STANDARD CHARTERED	**STAN**	ASSOCIATED BRITISH FOODS	**ABF**
UNILEVER	**ULVR**	LAND SECURITIES GROUP	**LAND**
RECKITT BENCKISER	**RB**	OLD MUTUAL PLC	**OML**
SABMILLER	**SAB**	ANTOFAGASTA	**ANTO**
NATIONAL GRID PLC	**NG**	PEARSON	**PSON**
IMPERIAL TOBACCO GROUP PLC	**IMT**	SHIRE PLC	**SHP**
STANDARD LIFE	**SL**	BRITISH LAND CO PLC	**BLND**
INTERNATIONAL POWER PLC	**IPR**	VEDANTA RESOURCES	**VED**
KAZAKHMYS	**KAZ**	ROYAL & SUN ALLIANCE INS.	**RSA**
UNITED UTILITIES	**UU**	CAPITA GROUP	**CPI**
SMITH & NEPHEW	**SN**	KINGFISHER	**KGF**
EXPERIAN GROUP	**EXPN**	CARNIVAL PLC	**CCL**
CABLE AND WIRELESS	**CW**	JOHNSON MATTHEY PLC	**JMAT**
SMITHS GROUP	**SMIN**	BRITISH AIRWAYS	**BAY**
LIBERTY INTERNATIONAL	**LII**	ICAP	**IAP**
NEXT	**NXT**	SEVERN TRENT PLC	**SVT**
HAMMERSON	**HMSO**	REXAM PLC	**REX**
SAGE GROUP PLC	**SGE**	INTERCONTINENTAL HOTELS GROUP	**IHG**

Source: Thomson Reuters Tick History

Table A2

Invariance Proportionality while controlling for Concratio

The table shows the coefficients, when the percentage of total volume traded on the alternative platforms relative to the total trading volume is included, based on the model in (53). Model 1 is based on quoted spreads, Model 2 on effective spreads and Model 3 on realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta_1 = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

	Intraday Averages		
	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.1434***	1.7697***	1.7292***
	(0.0306)	(0.0346)	(0.0434)
Invariance Coef., β_1	**-0.3947**	**-0.3688**	**-0.3939**
	(0.0376)	(0.0403)	(0.0491)
β_2	-0.5834***	-0.7130***	-0.7719***
	(0.1029)	(0.1101)	(0.1013)
R^2	0.8385	0.7885	0.6571
	Averages across days		
	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.7409***	2.2989***	2.7985***
	(0.0445)	(0.0524)	(0.0830)
Invariance Coef., β_1	-0.4271**	**-0.3907**	-0.4407***
	(0.0294)	(0.0311)	(0.0312)
β_2	-1.0475***	-1.2398	-1.1300***
	(0.2301)	(0.2568)	(0.1936)
R^2	0.9857	0.9763	0.9122

Table A3

Invariance Proportionality while controlling for individual Waltratio (Across days estimates)

Results are based on averages across days estimation for underlying variables based on equation in (50). Panel A shows the coefficients estimated when only the trading activity on LSE is considered between November 2008 and December 2009 based on the model in (52). Panel B shows the coefficients, when percentage of trading activity occurring on each respective alternative platform relative to total trading activity is included, based on the model in (55). Model 1 is based on quoted spreads, Model 2 on effective spreads and Model 3 on realised spreads as proxies for transaction costs. All specifications include stock and time fixed effects. Coefficients are tested against the null hypothesis $H_0 : \beta_1 = -1/3$. Two-way clustered robust standard errors (with the use of heteroscedasticity-corrected covariance matrices) are reported in parenthesis only for the coefficient estimates. *, **, and *** denote significance at the 5%, 1%, and 0.1% level, respectively.

Panel A: Estimation without the aggregate trading activity on alternative platforms

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.4288*** (0.0425)	2.1402*** (0.0502)	2.5175*** (0.0783)
Invariance Coef., β	-0.4200*** (0.0309)	**-0.3891** (0.0328)	-0.4460** (0.0357)
R^2	0.9809	0.9690	0.8822

Panel B: Estimation when individual Waltratio is included

	Model 1: Quoted Spreads	Model 2: Effective Spreads	Model 3: Realised Spreads
Constant , a	2.8842*** (0.0539)	2.5708*** (0.0637)	3.2090*** (0.1020)
Invariance Coef., β_1	-0.4497*** (0.0353)	-0.4256* (0.0372)	-0.4969*** (0.0403)
β_2	-0.9992*** (0.2001)	-1.2212*** (0.2234)	-1.0113*** (0.1805)
β_3	-0.5875 (0.3496)	-0.5190 (0.3872)	-1.2511*** (0.2990)
β_4	-0.2926 (0.3944)	-0.4349 (0.4682)	-0.6869** (0.2527)
R^2	0.9816	0.9700	0.8842

CHAPTER 6

Conclusion

6.1 Remarks

In this thesis I empirically examine certain key propositions derived from market microstructure invariance (MMI) theory, introduced by Kyle and Obizhaeva (2016b). I utilize data on trading in FTSE 100 index constituent stocks for the three year period between 2007 and 2009 inclusive to conduct the analysis. Specifically, I empirically investigate the theoretical prediction of the existence of a proportionality between trade counts and trading activity, using both panel model specifications (Chapter 1) and then proceeding to examine the contribution of individual stock characteristics (Chapter 2). Subsequently, I turn my attention to a second implication of MMI, namely the existence of a designated proportionality between percentage transaction costs per unit of volatility and trading activity, again using panel model specifications (Chapter 3). In general, the empirical findings provide a significant degree of support for the empirical existence of both expected proportionality relationships.

The first substantive chapter (chapter 3) introduces an extended formulation of the ITI model as introduced by Andersen et al. (2018) as motivated by the invariance of bets proposed in Kyle and Obizhaeva (2016b). Given the assumption that bets and trades are connected in a nonlinear manner, in contrast to Andersen et al. (2018) the model introduces an order shredding factor as a single component. Based on this extended model formulation, I argue that any deviations from the stipulated proportionality between trade counts and trading activity in the power of 2/3 can potentially be explained by higher (lower) degree of order shredding and potentially higher (lower) intermediation. Using four different notion of trading activity taken from the literature, namely those implied by Kyle and Obizhaeva (2016b), Andersen et al. (2016), Clark (1973) and Ané and Geman (2000), panel estimation specifications, I find that the theoretical 2/3 proportionality is: (i) linked more to intraday patterns rather than patterns evident across days, (ii) holds only for certain subsamples and (iii) depends on the definition of trading activity I employ. Specifically, classification of stocks in terms of their market capitalization reveals that only "large" and "medium" market cap stocks exhibit the stipulated 2/3 proportionality between trade counts and trading activity, when the latter is defined as in Kyle and Obizhaeva (2016b) and Andersen et al. (2016). Large trade size stocks also yield the predicted 2/3 proportionality, but only when trading activity is based on the notion that the number of trades in proportional to returns variance, as suggested by Ané and Geman (2000). Results based on all trading activity definitions, except that implied by Clark (1973), suggest that stocks with larger trade counts exhibit higher measures of proportionality, while the magnitude of trading volume does not seem to impact upon estimates of proportionality coefficients.

The second empirical chapter (chapter 4), examines the invariance proportionality between the number of trades and trading activity separately for each stock. The model based on the definition of trading activity implied by Clark (1973) (i.e. based on the idea that trading volume is proportional to the returns variance) generates estimates of the 2/3 power proportionality predicted by MMI theory for 70% of the stocks when averages across days are employed, except from those stocks with high on average volatility. This finding is consistent even when the first/last 10 minutes of trading activity are excluded from the analysis. However, the models of trading activity suggested by Kyle and Obizhaeva (2016b) and Andersen et al. (2016), respectively, predict a 1/2 invariance proportionality for 86% of the stocks. This proportionality is partly driven by the intraday trading patterns in LSE, the magnitude of trade size and its correlation with the volatility in business time. Based on the extended invariance model this value also implies that bets in S&P 500 E-mini future contracts market are larger than bets in stocks traded in FTSE 100 and thus are shredded into more pieces if we accept that on average bets shredded into same size trades in these two markets.

Excluding the first/last 10 minutes of trading activity from the analysis leads to an increase in the coefficient estimates predicted by invariance models that now converge to a value of 7/12. On the basis of the extended invariance model this finding implies increased level of order shredding in the minutes between the first/last 10 minutes of active trading. Year by year analysis, based on the trading activity definition of MMI theory and the null hypothesis of 1/2, suggests that there is no unified order flow pattern for all stocks, but rather traders are affected by stock and/or industry specific characteristics when deciding upon the trade size. The impact of financial crisis on the proportionality values we investigate is more obvious for stocks that belong to the high market cap group. I believe that the deviations from the 1/2 proportionality value found for the enter sample are partially driven by the GBP trade size.

The third chapter investigates the invariance of transaction costs as proposed by MMI, and the consequent empirical prediction of a proportionality between percentage transaction costs per unit of volatility and trading activity. Complementing the previous work of Kyle and Obizhaeva (2016a) Russian stocks, I use three common proxies of transaction costs, inter alia: quoted spreads, effective spreads and realized spreads. I find that the suggested scaling laws for transaction costs match patterns in the data regarding trades for FTSE 100 stocks on the LSE. In general, the results are in line with the empirical findings for Russian stocks in Kyle and Obizhaeva (2016a). The expected proportionality between percentage transaction costs per unit of volatility and trading activity to the power of -1/3, is present in daily patterns in the sample on average, whereas the percentage transaction costs fall more as trading activity increases during comparable 5-minute intervals across trading days rather than within particular days, providing returns volatility is constant. I discuss whether an increase in trading activity that involves trades with larger sizes leads to a smaller decrease in average daily transaction costs per unit of volatility and whether these trades are likely to happen on average within specific days rather than specific 5-minutes intervals across days. The reduction in percentage transaction costs per unit of volatility arising from increases in trading activity indicates that market makers may prefer to exchange profits for faster unwinding of their positions.

The proportionality relationships linked to transaction costs remain robust to incorporating variables capturing ratios of trading activity and volume on alternative platforms which I use as indicators of market fragmentation. However, I find a negative correlation between percentage transaction costs on the LSE and trading activity (volume) on alternative platforms relative to total trading activity (total volume), assuming that returns volatility remains constant. This is consistent with the interpretation that market makers on LSE reduce their spreads when confronted with competition from alternative platforms. This finding is consistent with Battalio (1997), Battalio et al. (1997), Battalio et al. (1998), Battalio and Holden (2001), Gresse (2017) and other microstructure papers that examine the effect of fragmentation on transaction costs. Among the alternative platforms, I find the highest negative correlation to exist between the trading activity on Chi-X and LSE percentage transaction costs. The trading activity on Turquoise is negatively correlated with the LSE percentage transactions costs mostly within particular days, whereas that on BATS is negatively correlated with LSE percentage realised spreads during specific 5-minutes intervals of each trading day. Finally, I show that even if the stock market is treated as a consolidated market, invariance proportionality still approximately holds for different proxies of transaction costs, finding broadly similar results to those I report for the LSE alone. However, the fact that the performance of MMI is relatively worse may potentially indicate that market fragmentation is an important factor to consider in the invariance framework for equity markets. Finally, I contend that the fact that there is a lower reduction in realized spreads on average in the consolidated market either reveals a greater impact of trades in the alternative platforms, or the fact that not all market makers take advantage of increases in liquidity as trading activity increases to reduce their quoted spreads (see for example Degryse et al. (2015)).

6.2 Implications, Limitations and Future Research

Market microstructure invariance (MMI) theory is one of the most novel ideas in market microstructure in recent years. The theory aims to provide a map of understanding the way financial markets operate and how order flow imbalances move prices, as well as facilitating the development of accurate measures of liquidity.

This thesis contributes to the empirical investigation of MMI theory and the calibration of invariance predictions for equity markets, and provides explanations of deviations from the stipulated empirical predictions of invariance principles. First, the thesis shows that order shredding is a factor to consider when empirically investigating invariance and when explaining deviations from the empirical prediction of proportionality between trade counts and trading activity in the power of 2/3. Second, it reveals that this aforementioned proportionality holds on average when stock and time fixed effects are considered for certain group of stocks, which are formulated based on specific stock characteristics, and that the definition of trading activity is important. Third, it argues that intraday patterns of the market affect this proportionality for each individual stock and that trade size and its correlation with volatility in business time is an important determinant of proportionality estimates. Fourth, the thesis finds that the empirical prediction of an inherent proportionality relationship between percentage transaction costs per unit of volatility and trading activity is indeed accurate, and appears to be robust across different proxy measures of transaction costs. It also continues to hold when market fragmentation considerations are taken into account. Refining the invariance predictions for trades in equity market based on these findings is an important task for future research. Finally, this thesis and the underlying analysis contribute to the respective time deformation literature and market microstructure literature that explores the relationship between order flow imbalances and price innovations.

The invariance predictions implied by MMI principles are not expected to hold everywhere and anytime, but rather serve as an important benchmark. At this point it is important to underline that the thesis examines empirically MMI theory based on trades, which are different from the underlying concept of bets, on the basis of which the theory is formulated. Therefore, any deviations of the proportionalities I investigate from the values predicted by invariance values are only to be expected. Electronic order handling, regulatory changes, the introduction of different trading platforms all likely encourage the splitting of intended orders (bets) into several actual trades with more intermediation over time. This makes the empirical investigation of invariance principles for trades (intending to capture the theoretical invariance of bets) difficult at best. This problem is exacerbated by the way in which trades are recorded in the majority of available databases. Although this thesis provides a potential way to overcome this problem, further research exploring how bets manifest in trades is needed.

The results of this thesis provide overall empirical evidence that the proposed microstructure invariance relationships especially those relating to the invariance of transaction costs, together with their scaling laws hold approximately for trades. Further investigation of whether they continue to hold in different sample periods or for different types of securities and markets is paramount. In relation to the invariance of bets, using alternative econometric approaches instead of two way fixed effects models, while accounting for market frictions is a worthy task for future research. Finally, in relation to the invariance of transaction costs future research can focus on examining other proxies for transaction costs and their relationship to trading activity. This can be undertaken by assuming a different impact function, classifying transaction costs based on the trade size or other stock characteristics, or examining whether empirical predictions change when platform characteristics such as latency or high frequency trading are taken into account.

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