# A Bargaining-Based Model of Security Design*

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**Abstract**

This paper studies how security design affects project outcomes. Consider a firm that raises capital for multiple projects by offering investors a share of the revenues. The revenue of each project is determined ex-post through bargaining with a buyer of the output. Thus the choice of security affects the feasible payoffs of the bargaining game. We characterize the securities that achieve the firm's maximal equilibrium payoff in bilateral and multilateral negotiations. In a large class of securities, the optimal contract is remarkably simple. The firm finances each project separately with defaultable debt. Welfare and empirical implications are discussed.

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Imagine a firm that raises capital for multiple projects by offering investors a share of the revenues. The revenue of each project is determined ex post through bargaining with a buyer of the output. The firm faces a financing problem because the share which it promises the investors affects the incremental gain from each trade it makes with the buyers and thereby the bargaining outcomes. Suppose that the firm can decide how to bundle the projects and issue securities backed by the pools, what kind of financial contract will it choose?

This paper studies a model of security design when project outcomes depend on bargaining. The interaction between financing and bargaining is a feature of markets in which transactions require specific investments and the terms of trade are determined ex post. To give a few examples, in vertical relationships, input suppliers making relationship-specific investments are locked in negotiations with downstream producers. Likewise, in public procurement projects and natural resource extraction projects, the terms of trade are frequently negotiated (and renegotiated) after the investment stage. A similar situation occurs in patent markets, leveraged buyouts, and labor markets.

Since the work of Brander and Lewis (1986) and Perotti and Spier (1993), the commitment effect of debt-financing on later strategic interaction is understood to affect both the financing decisions and the outcomes in input and output markets. The present paper considers a rich set of financial contracts and endogenously derives the optimal contracts.

In our model, a firm seeks to raise capital for multiple projects through a competitive market of investors. A financial contract, in the form of securities, specifies a sharing rule of the revenue of each project. The firm decides how to bundle the projects and designs securities backed by the pools. At date 0, the firm and the investors agree on the contract. At date 1, the firm negotiates prices with buyers of the projects' outputs. Thus the ex-ante choice of securities affects the feasible payoffs of the bargaining game. We characterize the securities that achieve the firm's maximal equilibrium payoff under complete information.

To illustrate the design problem, Figure 1 depicts the feasible payoffs when the firm negotiates with a single buyer: following an agreement on price $x$, the buyer will get her value

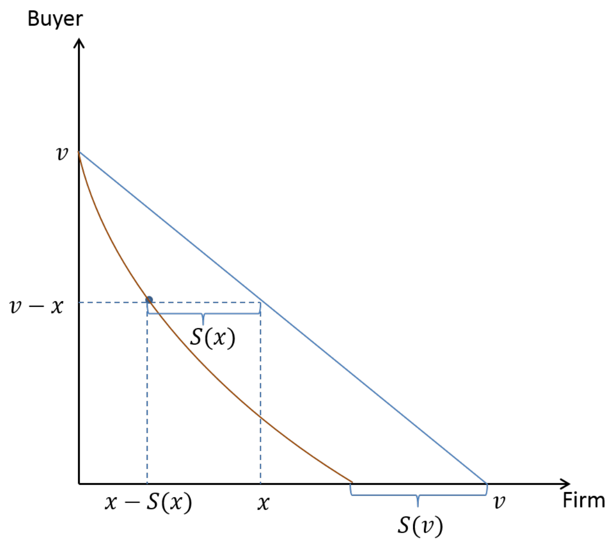


Figure 1: Feasible Bargaining Payoffs

minus the price, $v - x$. The security $S$ specifies a sharing rule of the revenue, and so the firm will get $x - S(x)$. Notice that the shape of the security determines the slope of the payoff frontier. The only restriction we impose is that the payment to investors is positive and increasing in the revenue. Both assumptions are common in the security design literature and satisfied by most securities used in practice.

In our analysis, we first consider the case where the firm negotiates with a single buyer. It turns out that in bilateral bargaining the optimal security is a defaultable debt contract. This contract resolves two issues in the security design problem: 1) the hold-up problem, and 2) the agency incentive problem. The first problem is that since the cost of investment is sunk by the time the buyer arrives, to make the firm-buyer pair internalize the investment cost, the payment to investors should be contingent on the trade price. The second problem is that the contract should create incentives to extract surplus from the buyer, and so the firm's marginal revenue should increase with the trade price.

Debt provides an advantage over equity and other securities in bilateral bargaining because the firm's marginal revenue is inversely related to the securities' steepness. However, our second result shows that when the firm negotiates with multiple buyers and the debt is repaid from the proceeds of the sales, the advantage of debt over equity (and other securities) is attenuated: the more buyers there are, the smaller the firm's share of the social surplus. The reason is that in the bargaining game with debt, an agreement with one buyer over a relatively high price changes the residual debt level and thereby tends to decrease the prices agreed-upon with the other buyers. The firm negotiates with each buyer independently, but the externalities are internalized in equilibrium, working in favor of the buyers and against the firm. In fact, we will see that as the number of buyers increases, the advantage of debt is continually attenuated until debt and equity are equivalent.

The main result of this paper is that in a large class of securities the optimal contract is remarkably simple. The firm takes out multiple loans, each one tied to a distinct transaction. This contract resolves the third issue in the security design problem which is whether to pool or separate the projects. The idea behind the result is that when the firm negotiates with multiple buyers and the security depends on a pool of the proceeds, extracting high surplus from one buyer negatively affects the firm's incremental gain from each trade with the other buyers. A deal-by-deal contract keeps the transactions independent of each other. Importantly, the results do not depend on the details of the bargaining model.

The results have significant empirical and welfare implications. Let us mention a few markets where project financing is common. Infrastructure and natural resource extraction projects are often financed with defaultable debt separately through subsidiaries (see, e.g., Esty 2004). Likewise, private equity firms typically finance leveraged buyouts deal-by-deal with independent loans (see, e.g., Kaplan and Strömberg 2009). The model offers a novel explanation for why a specific type of project financing is prevalent in some markets, but not others. Hold-ups arise when transactions require specific investments, which generate more value in a relationship than outside, and the threat of opportunistic expropriation curbs the incentive to invest. We show that when firms face hold-ups, financing each project with debt separately, rather than jointly, achieves the Pareto efficient allocation. The general idea is that a deal-by-deal contract aligns the firm's incentives with each buyer. We therefore expect to observe project financing when hold-ups arise.

# I Related Literature

This paper contributes to the literature on security design. It is the first paper on security design when firms face hold-ups. Our work is most closely related to the work of DeMarzo et al. (2005) on security design and auctions. In their model, buyers bid for the right to invest in a project by offering the seller a share of the future proceeds. The shape of the securities that buyers can bid with determines the equilibrium of the auction. In the present model, the choice of security determines the equilibrium of the bargaining game.

Previous work has analyzed the problem of designing securities backed by the proceeds of risky assets or projects in the context of costly state verification, hidden action, private information, and liquidity needs.[1] The present model shows that when the proceeds depend on bargaining, security design has several consequences for raising capital, even under complete information and no risk or uncertainty. For instance, the firm prefers debt over equity (or other securities) when there is a single buyer. But as the number of buyers increases, the advantage of debt is continually attenuated until debt and equity are equivalent.

Our work is also related to several other papers. Stole and Zwiebel (1996a,b) consider a related problem where a firm chooses its inputs and organization design to influence subsequent wage negotiations with each of its workers. They show that bargaining externalities impact the equilibrium wages, but the firm's decisions and the resulting bargaining game are substantially different from those in our model.

The commitment effect of debt-financing on later strategic interactions in product markets is well-studied in the literature (see, e.g., Brander and Lewis 1986, Bolton and Scharfstein 1990, Glazer 1994, and Faure-Grimaud 2000). The present paper focuses on the security design problem and the analysis pertains to markets where negotiations, rather than com-

[1] See, e.g., Townsend (1979), Innes (1990), Demarzo and Duffie (1999), DeMarzo (2005), Biais and Mariotti (2005), Axelson (2007), and Antic (2014).

petition, determine prices. We show that the effect of debt-financing crucially depends on the number of buyers: debt is optimal when the firm faces a single buyer, but not multiple buyers. Moreover, when there are sufficiently many buyers, debt and equity are equivalent.

The interaction between financing and ex-post bargaining has been studied in models of liquidation and refinancing decisions, but the negotiations there are between borrowers and creditors[2] (see, e.g., Rajan 1992; Hart and Moore 1998). Several papers examine the effect of debt on later negotiations between firms and unions (see, e.g., Bronars and Deere 1991; Matsa 2010; Hennessy and Livdan 2009; Klasa et al. 2009). Their point is that the firm can issue debt, which provides the lenders with a senior claim to its future cash flow, and thereby shrink the pie that a union or other parties can expropriate in later negotiations. The present model considers a different setting and mechanism. In our model, the firm negotiates with buyers and the investors receive some of the proceeds from the sales. Therefore, the financial contract affects the bargaining outcomes by providing commitment rather than shrinking the pie. This difference has consequences. In our setting, unlike in the above-referenced papers, the effect of debt crucially depends on the number of buyers. We will further elaborate on these differences in Section 4.

# II The Model

We consider a single firm that seeks to finance $N$ costly projects. The cost of project $j$ is $c_j$. The firm will sell the output of each project to a unique buyer. Buyer $j$ values only the output of project $j$ by $v_j$. Projects are profitable $0 < c_j < v_j$ and the total surplus and total costs are denoted by $V = \sum_{j=1}^{N} v_j$ and $C = \sum_{j=1}^{N} c_j$, respectively. At date 0, the firm and investors sign a financial contract. At date 1, the firm negotiates prices with the buyers. The two ingredients, financing and bargaining, are described below.

**Financial Contracts.** The firm can finance projects through a competitive market of

[2]See also Inderst and Vladimirov (ming).

investors who will receive some of the proceeds from the sales in return. A *security* $S : \mathbb{R} \to \mathbb{R}^+$ indicates a payment to investors as a function of the aggregate proceeds of one or several projects. We focus on two types of contracts:

1. We say that projects are financed *jointly* if the contract consists of a single security $S$ that is a function of the aggregate revenues.

2. We say that projects are financed *separately* if the contract consists of $N$ securities $S_1, \ldots, S_N$ and for all $j$, security $S_j$ is a function of only the proceeds from project $j$.

Of course, the firm should be able to finance a subset of the projects jointly and the other projects separately. We consider the two extreme partitions without loss of generality (see Remark 3).

The payment to investors is positive and weakly increasing with the proceeds. These two assumptions are common in the literature on security design[3] and most securities used in practice satisfy them. The assumption that $S \geq 0$ implies that investors cannot commit to paying the firm. Such transfers are rarely observed because they incentivize opportunistic behavior and expose investors to various risks. For instance, firms will try to finance unprofitable projects to receive transfers, which creates screening and monitoring costs for investors. The assumption that $S$ is weakly increasing aligns the interests of the investors with those of the firm. In the absence of monotonicity, the firm can artificially inflate the proceeds and pay the investors less.

There is complete information: every player observes the costs, values, and financial contracts. This assumption simplifies the analysis.

**Bargaining.** An agreement refers to the price paid by a specific buyer and utilities are linear in money. The financial contract divides the proceeds from the sales. We let $G(S)$ and $G(S_1, \ldots, S_N)$ denote the feasible payoff sets of the bargaining games under a joint and

[3]See, e.g., Hart and Moore (1995); Nachman and Noe (1994); Demarzo and Duffie (1999); DeMarzo et al. (2005); Biais and Mariotti (2005).

a separate contract, respectively. Following agreements over prices $x_1, \ldots, x_K$, buyer $j$ will get $v_j - x_j$ if she reached an agreement and nothing otherwise. If no agreement is reached with a buyer, there are no proceeds from that project. Under a joint contract, the firm will get $X_K - S(X_K)$, where $X_K = \sum_{i=1}^{K} x_i$. Under a separate contract, the firm will get[4] $\sum_{i=1}^{K} x_i - S_i(x_i)$.

**Definition.** *A bargaining solution* $\mathcal{F}$ selects a set of agreements, denoted by $\mathcal{F}(G(S))$ and $\mathcal{F}(G(S_1, \ldots, S_N))$, from a feasible payoff set.

To illustrate, Figure 1 depicts the feasible payoff set of a bilateral bargaining game. The bargaining solution may select the agreements that maximize the product of the players' utilities over the feasible payoff set (the Nash solution), or the subgame perfect equilibrium of a sequential bargaining game. One caveat of bargaining theory is that the outcomes may be sensitive to the details of the bargaining model. We consider several bargaining models and show that the optimal contracts do not rely on the details of the bargaining model.

**The Firm's Problem.** The firm designs the financial contract while taking into account the effect on subsequent negotiations and that investors at least break even. Given a bargaining solution, we say that a contract is optimal if the bargaining game has an outcome such that 1) the investors at least break even and 2) no other contract implements a bargaining outcome that makes the firm better off and the investors at least break even.

Thus, the optimal security under a joint contract solves:

$$\max_S X - S(X), \text{ where } X = \sum_{i=1}^{N} x_i, \text{ s.t.}$$

$$1)\ x_1, \ldots, x_N \in \mathcal{F}(G(S))$$

$$2)\ S(X) \geq C$$

The first constraint is that the outcome is implementable, the second is non-negative profits

[4] We will see that it is optimal to set $S_j(0) = 0$, thus $\sum_{i=1}^{N} S_i(x_i) = \sum_{i=1}^{K} S_i(x_i)$.

for investors. Similarly, the optimal securities under a separate contract solve:[5]

$$\max_{S_1,\ldots S_N} \sum_{i=1}^{N} x_i - S_i\left(x_i\right), \text{ s.t.}$$

$$1)\; x_1,\ldots,x_N \in \mathcal{F}\left(G\left(S_1,\ldots,S_N\right)\right)$$

$$2)\; S_i(x_i) \geq c_i, \text{ for all } i$$

In words, the security specifies a sharing rule which determines the firm's incremental gain from each trade. We are looking for the sharing rule that maximizes the firm's payoff, under the restrictions that the payment to investors is sufficiently large.

**Remark 1.** To focus the analysis, we assume complete information, no risk or uncertainty, and no agency or bankruptcy costs. We do not assume that the firm's liability is limited. Section 4 elaborates on the consequences of relaxing the assumptions.

# III Optimal Contracts

We first consider a case where the firm negotiates with a single buyer and then with multiple buyers. We use a simple bargaining model to provide clarity and intuition. In Appendix A, we characterize the equilibrium of a sequential bargaining game.

[5]We can allow for cross-subsidization of projects and require $\sum_{i=1}^{N} S_i(x_i) \geq C$ instead of $S_i(x_i) \geq c_i$, $\forall i$; the results do not change.

## Bilateral Bargaining

A firm with a single project negotiates with a single buyer. We consider the bargaining solution of Nash (1950) which selects the agreements that maximize the product of utilities,[6]

$$\mathcal{F}_N(G(S)) = \{x : x \in \arg\max\{(x - S(x))(v - x)\}\}$$

**Proposition 1.** *Financing a single project with a defaultable debt contract is optimal.*

Let us first compare between debt and equity:

- In the bargaining game with debt, $S(x) = \min\{x, D\}$, the firm receives the remaining proceeds after the debt is paid out. The unique agreement price is $x^* = \frac{v+D}{2}$. Since the buyer will not trade if the price is above her value and the firm will get nothing if the price is below the debt, it is intuitive that the agreement falls halfway between $D$ and $v$ when there is equal bargaining power. If $D = c$, investors break even, and the firm and the buyer split the social surplus, each getting $\frac{1}{2}(v - c)$.

- In the bargaining game with equity, $S(x) = \alpha x$ and $\alpha < 1$, the firm's marginal revenue is constant. The unique agreement price is $x^* = \frac{v}{2}$. If $\alpha\frac{v}{2} \geq c$, investors break even, and the firm will get no more than $\frac{v}{2} - c$.

In the bargaining game with debt, the firm extracts more surplus from the buyer. The proof of Proposition 1 establishes an upper bound on the firm's equilibrium payoffs for all securities. Lemma 1 does most of the work.

**Lemma 1.** *In any outcome in $\mathcal{F}_N(G(S))$ the firm's payoff is no more than the buyer's payoff.*

The proof of Lemma 1 is straightforward. The key observation is that since the security is increasing, the slope of the utility frontier is at most $-1$: if the price $x$ increases by $\Delta$, the

[6] See Herrero (1989) and Zhou (1997) for an extension of Nash's solution to non-convex bargaining problems.

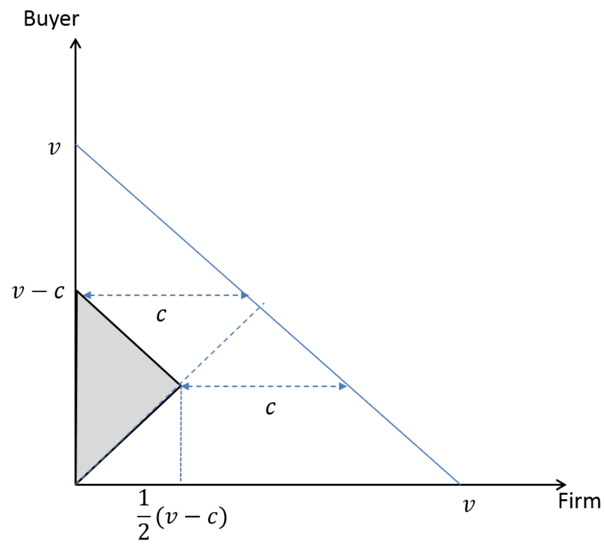


Figure 2: Equilibrium outcomes in which investors break even

buyer's payoff decreases by $\Delta$ and the firm's payoff increases by $\Delta - (S(x+\Delta) - S(x)) \leq \Delta$. Therefore, the agreements that maximize the product of utilities lie above the 45° line, as depicted in Figure 2.

The rest of the proof immediately follows from Lemma 1. Since investors get at least $c$, no more than $v - c$ is left, and the firm's maximal payoff is $\frac{1}{2}(v - c)$. Figure 2 depicts the equilibrium payoffs that the firm can achieve: Lemma 1 implies that the equilibrium payoffs lie above the 45° line (for all securities), and the investors will not break even unless the payoffs are in the shaded region. Finally, in the bargaining game with debt, $S(x) = \min\{x, D\}$ and $D = c$, investors break even, and the firm achieves the maximal payoff.

A defaultable debt contract resolves two issues in the security design problem: 1) the hold-up problem, and 2) the agency incentive problem. The first problem is that since the investment cost is sunk by the time the buyer arrives, to make the firm-buyer pair internalize the investment cost, the payment to investors must be contingent on the trade price. For instance, if the firm's liability were not limited, the debt is paid out regardless of the agreement with the buyer and would not affect the bargaining outcomes.

The second problem is that the contract should create incentives to extract surplus, and so

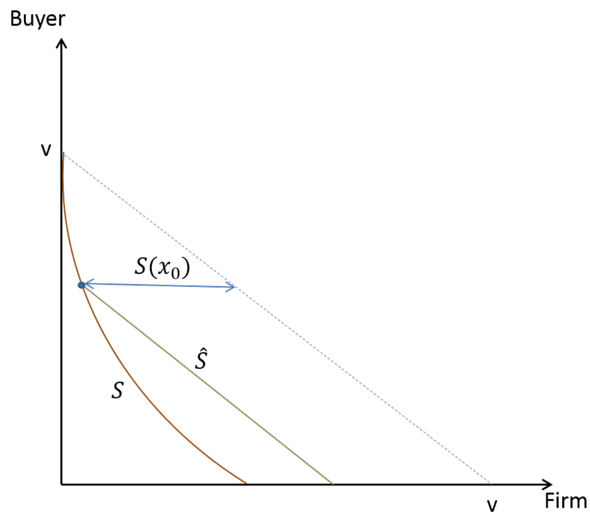


Figure 3: Securities $S$ and $\hat{S}$

the firm's marginal revenue should increase. To provide intuition, let us consider a bargaining solution $\mathcal{F}$ and a security $S$ such that $x_0 \in \mathcal{F}(G(S))$. We can construct another security,

$$\hat{S}(x) = \begin{cases} S(x) & \text{if } x \leq x_0 \\ S(x_0) & \text{if } x > x_0 \end{cases}$$

as depicted in Figure 3. Security $\hat{S}$ increases the firm's marginal revenue at $x_0$, provided that $S'(x_0) \geq 0$. For a large class of bargaining solutions, security $\hat{S}$ will implement an outcome $x_1 \in \mathcal{F}(G(\hat{S}))$ such that[7] $x_1 \geq x_0$. The investors are no worse off under security $\hat{S}$ because $\hat{S}(x_1) = S(x_0)$ and the firm is (weakly) better off. Security $\hat{S}$ is isomorphic to debt in the sense that the payment to investors is constant when proceeds are sufficiently high.

**Remark 2.** We consider the Nash bargaining solution for simplicity. The proof of Proposition 1 can easily be extended to other axiomatic and strategic bargaining models in the literature, including nonsymmetric Nash solutions (see, e.g., Kalai 1977), the bargaining model of Kalai and Smorodinsky (1975), the strategic bargaining model of Rubinstein (1982) (see

[7]This is true for (non)symmetric Nash bargaining solutions and for the bargaining solution of Kalai and Smorodinsky (1975) (because of their monotonicity axiom).

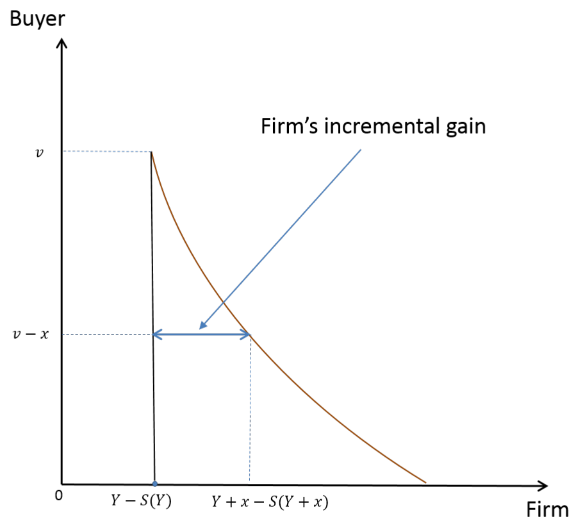


Figure 4: The feasible payoff set $G_j(S, Y)$

Appendix A), and take-it-or-leave-it offers.

## Multilateral Bargaining

A firm with several projects sells output to several buyers. If the security is a function of the pooled proceeds, an agreement with one buyer affects the firm's incremental gain from each trade it makes with the other buyers. We begin with a few definitions. Let $G_j(S, Y)$ denote the feasible payoff set of the bilateral bargaining game between the firm and buyer $j$ when the security is $S$ and the aggregate proceeds in the other transactions are $Y$. Figure 4 depicts the feasible payoffs: if the buyer pays $x$, the firm gets $Y + x - S(Y + x)$; if the firm and the buyer do not reach an agreement, the firm gets $Y - S(Y)$. Therefore, the firm's *incremental gain from trade at price* $x$ is the additional revenue minus the additional payment to investors, $x - (S(Y + x) - S(Y))$.

**Definition.** We say that prices $x_1, \ldots, x_N$ are *bilaterally stable* if $x_j \in \mathcal{F}_{\mathcal{N}}\left(G_j(S, X_{-j})\right)$ where $X_{-j} \equiv \sum_{i \neq j}^{N} x_i$, $\forall j$.

In other words, each price is the outcome of bilateral negotiations, given the other prices.

We borrow the term from Lensberg (1988); the property is due to Harsanyi (1963). Bilateral stability is fundamental property of axiomatic bargaining models and it is supported by various strategic bargaining models in the literature.[8] A bilaterally stable outcome generally exists, but it need not be unique (see Lemma 6 in the Online Appendix).

**Proposition 2.** *If $N$ symmetric projects are financed jointly with debt, the firm will get at most*

$$\pi = \begin{cases} \frac{V}{2} - C & \text{if } (N-1)\frac{v}{2} \geq C \\ \frac{1}{N+1}(V-C) & \text{if } (N-1)\frac{v}{2} < C \end{cases}$$

*where $v_1 = \ldots = v_N = v$.*

*Proof.* Suppose that projects are financed jointly with debt $S(X) = \min\{X, D\}$. The bilateral bargaining game with payoffs $G_j(S,Y)$ is equivalent to a bilateral bargaining game with a debt of $D' = \max\{D-Y, 0\}$ and status-quo payoffs at the origin.[9] The prices $x_1, \ldots, x_N$ are bilaterally stable and the firm's profit is positive if and only if

$$x_i = \frac{1}{2}(v_i + D_{-i}) \text{ where } D_{-i} = \max\left\{D - \sum_{j \neq i}^{N} x_j, 0\right\}, \forall i \tag{1}$$

As long as $v_i = v$ and $D < V$, Equation (1) has a unique solution:

- If $(N-1)\frac{v}{2} \geq D$, the prices $(\frac{v}{2}, \ldots, \frac{v}{2})$ are bilaterally stable because the revenue in any $N-1$ transactions covers the debt.
- If $(N-1)\frac{v}{2} < D$, the prices $\left(\frac{v+D}{N+1}, \ldots, \frac{v+D}{N+1}\right)$ are bilaterally stable.

[8] For axiomatic bargaining models, see, e.g., Lensberg (1988) and Horn and Wolinsky (1988); for strategic bargaining models, see, e.g., Krishna and Serrano (1996); Hart and Mas-Colell (1996); Collard-Wexler et al. (2014).

[9] In the bilateral bargaining game with payoffs $G_j(S,Y)$: if $Y < D$, the residual debt is $D' = D - Y$, and the firm's incremental gain from trade at price $x$ is $x - (S(Y+x) - S(Y)) = x - \min\{x, D'\}$; if $Y \geq D$, the debt is paid out, and the firm's incremental gain is the price $x$.

Since investors break even, $D \geq C$, which establishes the upper bound. $\square$

To illustrate, suppose that the firm has a debt of $D = 100$ and it sells two goods to two buyers with values $v_1 = v_2 = 100$. The (unique) bilaterally stable outcome is $x_1 = x_2 = 66\frac{2}{3}$. If one buyer pays $x_1 \leq 100$, then the residual debt level is $D' = 100 - x_1$, and the other buyer will pay $x_2 = \frac{D'+v_2}{2} = 100 - \frac{x_1}{2}$. In other words, the more one buyer pays, the lower the residual debt level is, and hence the less the other buyer pays.

Proposition 2 is intuitive because the agreement with a buyer depends not just on the debt level, but also on how the debt is expected to change. If the proceeds in the other transactions are sufficiently high, the debt is paid out regardless of the agreement with this buyer and should not affect it. In the first case, $(N-1)\frac{v}{2} \geq C$, debt affects none of the transactions. In the second case, $(N-1)\frac{v}{2} < C$, the effect of debt is preserved, but attenuated: the more buyers there are, the smaller the firm's share of the social surplus.

**Corollary 1.** *If $N$ symmetric projects are financed jointly and $(N-1)\frac{v}{2} \geq C$, debt and equity are equivalent.*

Debt and equity achieve the same project outcomes when there are sufficiently many buyers. However, if each project is financed with debt separately, the transactions are independent of each other, and the firm will get half the social surplus, $\frac{1}{2}(V - C)$. The externalities are the driving force here. The next question is whether the firm can benefit from the externalities with another security. Our main result shows for all securities that are between debt and equity the answer is no.

**Proposition 3** (**Main Result**)**.** *Financing each project with debt separately is optimal within the class of concave securities.*

Lemma 2 does most of the work. It shows that the firm will get no more than the sum of the buyers' payoffs.

**Lemma 2.** *If $S$ is concave and the prices $x_1, ..., x_N$ are bilaterally stable, then $X - S(X) \leq V - X$, where $X = \sum_{i=1}^{N} x_i$.*

*Proof.* First, if the prices $x_1, \ldots, x_N$ are bilaterally stable, the firm's incremental gain from each trade does not exceed the buyer's gain:

$$x_i - S(X_{-i} + x_i) + S(X_{-i}) \leq v_i - x_i, \forall i \tag{2}$$

To see this, note that $x_i \in \mathcal{F}_N\left(G_i(S, X_{-i})\right)$ and we can transform the bilateral bargaining game with payoffs $G_i(S, X_{-i})$ by subtracting the amount $X_{-i} - S(X_{-i})$ from the firm's payoff. The resulting bargaining game is a bilateral game with security $\hat{S}_i(y) \equiv S(X_{-i} + y) - S(X_{-i})$ and a status quo payoffs at the origin. Since the bargaining solution is invariant to linear transformations, $\mathcal{F}_N\left(G_i(S, X_{-i})\right) = \mathcal{F}_N\left(G_i(\hat{S}_i, 0)\right)$, and Lemma 1 implies $x_i - \hat{S}_i(x_i) \leq v_i - x_i$.

Second, concavity implies that the firm's profit function is supermodular,

$$X - S(X) \leq \sum_{i=1}^{N} x_i - (S(X) - S(X_{-i})) \tag{3}$$

To see it, observe that $S(X) - S(X_{-i}) \leq \frac{x_i}{X}\left[S(X) - S(0)\right]$, $\forall i$, and, in sum, $\sum_{i=1}^{N} S(X) - S(X_{-i}) \leq S(X) - S(0)$. Therefore, from (2) and (3) we have that $X - S(X) \leq V - X$. $\square$

Recall in bilateral bargaining the firm will not get more than the buyer for any security (Lemma 1). The same is true in multilateral bargaining when securities are concave functions. The rest of the proof immediately follows. As long as the investors get at least $C$, no more than $V - C$ is left over; and since the firm's payoff does not exceed the total payoff of the buyers, $\frac{1}{2}(V - C)$ is an upper bound.

The general idea behind Proposition 3 is that the incremental gains from trade determine the bargaining outcome. The proof is based on two observations. First, since the security is concave, the firm's profit function is supermodular: the profit is no more than the sum of the incremental gains from trade. Second, since the security is monotonic, the firm's incremental gain from each trade is no more than the buyer's gain. These two observations imply a tight upper bound on the equilibrium payoffs.

**Remark 3.** In principle, the firm can bundle the projects in other ways. For example, it can finance a subset of projects jointly and the others separately. Notice that we can apply Lemma 2 to any security backed by a pool of projects, and hence the firm (weakly) prefers financing each project separately than as a pool. Thus Proposition 3 applies to any partition of the projects.

The proof of Proposition 3 may appear deceptively simple, but in fact, it is a powerful result. The next example demonstrates this.

**Example 1.** There are two identical projects, $c_1 = c_2 = 20$ and $v_1 = v_2 = 100$.

- Each project is financed with debt separately. The firm will get half the social surplus $\frac{1}{2}(V - C) = 80$.
- Projects are financed jointly with debt, $S(X) = \min\{X, D\}$. The investors break even when $D = 40$, and the prices $x_1 = x_2 = 50$ are bilaterally stable. The firm will get 60.
- Projects are financed jointly with equity, $S(X) = \alpha X$. The prices $x_1 = x_2 = 50$ are bilaterally stable. The investors break even when $\alpha = \frac{2}{5}$, and the firm will get 60.
- Projects are financed jointly with security $S_\lambda(X) = \min\{\lambda X, D\}$. This security allocates only the fraction $\lambda$ towards covering the debt, and the rest goes to the firm. For instance, if the fraction $\lambda = \frac{1}{2}$ and the debt level $D = 40$, the prices $x_1 = x_2 = 56$ are bilaterally stable: if one buyer pays 56, only half goes towards the debt and the residual debt is $D' = 40 - \frac{56}{2} = 12$, and hence the price in the other transaction is also 56. The firm's payoff is 72 and the investors break even.

The firm's payoff with security $S_\lambda$ and $\lambda = \frac{1}{2}$ is 72, which is more than what the firm would get with debt or equity, 60, but less than half the social surplus, 80. The intuition is that each buyer would like to wait and negotiate when the residual debt is low. Security $S_\lambda$ allocates only a fraction of the proceeds towards the debt making it less attractive for each buyer to wait. The firm's profit is decreasing in the fraction $\lambda$, but the fraction cannot be too low because the investors will not break even. Securities of this form are concave functions, and thus the firm cannot get more than 80. The firm can get more than half the social surplus with a security that is not concave.

**Example 2 (Non-concave Security).** As in the previous example, there are two identical projects, $c_1 = c_2 = 20$ and $v_1 = v_2 = 100$.

- Projects are financed jointly with the security,

$$S_2(X) = \begin{cases} 0 & \text{if } X < 70 \\ \min\{X - 70, 40\} & \text{if } X \geq 70 \end{cases}$$

  This security backloads the payments to investors: the proceeds between 0 and 70 will go to the firm, the proceeds between 70 and 110 will go to the investors, and the firm will receive the proceeds beyond 110. The prices $x_1 = x_2 = 70$ are now bilaterally stable: if Buyer 1 pays 70, then a debt of $D' = 40$ remains, and Buyer 2 also pays 70.

Intuitively, by backloading the payments to the investors, the firm can use the joint costs as bargaining leverage over each buyer. However, in the bargaining game with security $S_2$ there are many other bilaterally stable outcomes with lower prices (e.g., the prices $50, 20$). In which case, the firm will get less than half the social surplus.

**Proposition 4 (Optimal Non-concave Security).** *Consider $N \geq 2$ symmetric projects where $v_1 = v_2 = \ldots = v_N = v < C$. For any $\epsilon \in [0, \frac{V-C}{2}]$, the firm can extract the maximal* *surplus from the buyers, up to $\epsilon$, with security*

$$S_\epsilon(X) = \begin{cases} \min\{X, D_1\} & \textit{if } X \le a \\ \min\{X - l, D_2\} & \textit{if } X > a \end{cases}$$

*where* $D_1 = C - v + \frac{2}{N}\epsilon$, $D_2 = C$, $l = V - C - \frac{N+1}{N}\epsilon$, *and* $a = (N-1)v - \frac{N-1}{N}\epsilon$.

Notice that if $\epsilon \le \frac{1}{2}(V - C)$, then $S_\epsilon(a) = D_1 = a - l$ and $S_\epsilon$ is increasing and continuous.

*Proof.* The firm's maximal equilibrium payoff is $V - C$, because the investors and the buyers cannot receive negative payoffs. The bargaining game with security $S_\epsilon$ has a bilaterally stable outcome with prices $x_1 = x_2 = \ldots = x_N = v - \frac{\epsilon}{N}$, the firm receives $V - C - \epsilon$, and the investors break even. To see it, consider a subgame where only buyer $i$ remains and the proceeds in the other transactions are $Y = (N-1)v - \frac{N-1}{N}\epsilon = a$. Following an agreement with the buyer on price $x$, the additional payment to investors is $S_\epsilon(Y + x) - S_\epsilon(Y) = \min\{x, v - \frac{2\epsilon}{N}\}$. Thus, the price $x_i = v - \frac{\epsilon}{N}$ is an equilibrium agreement of this subgame.[10] □

This security backloads some but not all of the debt. The advantage of backloading the debt is that the firm can use the cost of several projects as bargaining leverage over each buyer separately. There is also a subtle issue that the security should not backload the entire debt because this could create perverse incentives for the buyers and the firm to agree on relatively low prices and avoid repaying the investors.

To sum up, our main result is that financing each project with defaultable debt separately is optimal within the class of concave securities. The class of concave securities is of importance both from a theoretical and applied perspective. The two most basic securities, debt and equity, are concave, and all convex combinations of debt and equity are also concave securities. To implement non-concave payment schedules, firms must use more sophisticated

[10] Furthermore, if $\epsilon > 0$, then $x_i = v - \frac{\epsilon}{N}$ is the unique outcome which maximizes the Nash product in this bilateral subgame. If $\epsilon = 0$, then $x_i = v$ is a Nash outcome of the subgame, but there are others.

instruments (such as options) which are not always available. We also showed that if such securities are available, the firm can extract the maximal surplus from the buyers by pooling the projects and designing a non-concave security, which backloads some but not all of the debt.

This deal-by-deal contract has several advantages over contracts that use non-concave securities on a pool of projects. First, by financing each project with debt separately, the firm keeps the transactions independent of each other, and so the bargaining game has a simple structure and a unique equilibrium. Furthermore, the outcome does not depend on the details of the bargaining model. If the security depends on the pooled proceeds, in contrast, the bargaining game has a complex structure of interdependent agreements, and usually multiple equilibria.

Second, the optimal financial contracts across various settings in the literature share a fundamental principle that the firm's marginal revenue is increasing in the outcome of the project or asset. For instance, to minimize the losses associated with agency incentive problems or private information on the side of the firm, the optimal contracts should reward the firm when the project's or asset's revenue is relatively high. Convex securities are suboptimal in such settings because they implement the opposite payment schedule, the firm's marginal payoff is decreasing in the revenue.

Third, a deal-by-deal contract based on defaultable debt is robust in the sense that it implements a bargaining game with a unique equilibrium. The non-concave securities in Example 2 and Proposition 4, in contrast, generally lead to bargaining games with multiple equilibria, some of which make the firm worse off than a deal-by-deal contract. For instance, consider the security in Proposition 4 and suppose that the number of buyers $N \geq 4$ and each buyer's value is $v < \frac{N-1}{N}C$, then the prices $(\frac{v}{2}, \dots, \frac{v}{2})$ are an equilibrium (bilaterally stable) whenever $\frac{v}{2} > \frac{C}{N}$, and otherwise, $\frac{v}{2} \leq \frac{C}{N}$, the price $(0, \dots, 0)$ are an equilibrium (this bargaining game may have many other equilibria). The general idea here is that a security that depends on a pool of projects creates interdependencies between the transactions, which tend to generate multiple equilibria in bargaining games.[11]

# IV Discussion

The model considers a simple setting to focus on the security design problem. This section discusses several implications and extensions of the model.

**Bargaining Models.** We considered a simple bargaining solution for clarity and to provide intuition. The main results of this paper do not rely on the details of the bargaining model. The reason is that our characterization of the optimal securities uses weak properties of the bargaining model and the same arguments apply in a large class of models. In Appendix A, we characterize the equilibrium of a sequential bargaining game and show that the optimal securities are the same.

Identifying the class of bargaining models for which the characterization of the optimal securities goes through is an interesting problem. A formal answer is beyond the scope of the present analysis, but we can say that it is a large class. In bilateral bargaining, the result naturally extends to most of the bargaining models in the literature (see Remark 2). In multilateral bargaining, our characterization relies on bilateral stability, which is a fundamental property of axiomatic models and supported by various strategic bargaining models in the literature.

**Hold-ups.** We can naturally extend the model to a setting where the firm also chooses how much to invest in each project, and the investment increases the buyers' surplus. Since the firm makes investment decisions before the buyers arrive, the firm faces a hold-up problem, which curbs the incentive to invest. However, the firm can finance the investment with defaultable debt and make the buyers internalize some of the investment costs. If firms face hold-ups, can raising capital through a financial market create "correct" incentives to invest?

[11]Notice that if $\frac{v}{2} > \frac{C}{N}$, then $D_1 \leq (N-1)\frac{v}{2}$ and also $N\frac{v}{2} < a$ (because $v < \frac{N-1}{N}C$), and thus $(\frac{v}{2}, \dots, \frac{v}{2})$ is bilaterally stable. If $\frac{v}{2} \leq \frac{C}{N}$, then $v \leq D_1$ (because $N \geq 4$) which implies that $(0, \dots, 0)$ is bilaterally stable.

The answer remarkably is yes, provided that the financial contract is optimal. In Appendix B, we show that financing the investment deal-by-deal with defaultable debt overcomes the fundamental hold-up problem. The basic idea is intuitive, a deal-by-deal contract aligns the investment incentives of the firm with each buyer, achieving a socially efficient allocation. If the firm issues debt on a pool of the proceeds, in contrast, the investment decision depends on the number of buyers. When there are relatively few buyers, the allocation is efficient. When there are relatively many buyers, hold-ups lead to inefficient investment decisions.

**Empirical Implications.** As previously mentioned, we observe deal-by-deal contracts based on debt in various markets. For example, infrastructure and natural resource extraction projects are often financed with debt separately through subsidiaries (see, e.g., Esty 2004). Likewise, private equity firms typically finance leveraged buyouts deal-by-deal with independent loans (see, e.g., Kaplan and Strömberg 2009). An open question in the literature is why we observe project financing in some markets, but not others.

The present paper shows that when firms face hold ups, financing each project with debt separately, rather than jointly, achieves the Pareto efficient allocation. Therefore, we expect to observe project financing with defaultable debt when hold-ups arise. There are various other theories in the literature explaining why firms finance investments separately rather than as a pool: a firm may wish to avoid debt overhang or to balance between risk-contamination and co-insurance (see, e.g., Banal-Estañol et al. 2013). However, to explain the difference in financing decisions across markets, most theories in the literature require systematic variation in cross-market risk. The present paper can explain differences in financing decisions without cross-market variation in risk. While the present theory and risk-based theories are not mutually exclusive, the predictions differ across some markets. Further empirical work is needed to test the theories and to understand the causes of cross-market differences in financing decisions.

**Relationship to the Literature on Leverage in Collective Bargaining.** Several papers study the effect of debt on the negotiations between a firm and unions, or other input providers (see, e.g., Bronars and Deere 1991; Matsa 2010; Hennessy and Livdan 2009; Klasa et al. 2009). In these papers, the firm chooses the financial structure before the negotiations. The firm can issue debt, which provides the lenders with a senior claim to the revenues or cash flows. The firm and the union will bargain over the remaining cash flow after the lenders receive their share. In other words, debt shrinks the pie that the union or other parties can expropriate in later negotiations. For instance, if the cash flow is $V$ and the face value of the debt is $D$, then the firm and union bargain over the residual cash flow, $V - D$. The present paper considers a different setting and mechanism. In our model, the firm bargains with buyers, and the "pie" is the buyers' surplus, $V$. For instance, in the bargaining game with debt, following agreements on the prices $x_1, \ldots, x_N$: buyer $i$ gets $v_i - x_i$, the investors receive $S(X) = \min\{X, D\}$, and the firm receives the residual, $\max\{0, X - D\}$. Therefore, debt affects the bargaining game with the buyers by providing commitment rather than shrinking the pie.

Although the two models highlight different mechanisms, the effect of debt on bilateral bargaining games is the same: in both models, the firm receives $\frac{1}{2}(v - D)$. The two models deliver different outcomes in multilateral bargaining games. In our model, when there are sufficiently many buyers, debt does not affect bargaining outcomes, the firm receives $V/2 - D$ (see Proposition 2). In models of collective bargaining, debt always shrinks the pie in later negotiations, irrespective of the number of parties the firm faces. For instance, if the firm negotiates with $N$ unions or other input suppliers and the bargaining power is symmetric, each party should receive $\frac{1}{N+1}(V - D)$.

**Liability, Debt Covenants, and Commitment.** We showed that financing a single project with defaultable debt is optimal. Limiting the firm's liability is a result, not an assumption. The model also has implications concerning the use of certain debt covenants, which provide control rights over specific projects to the investors in some contingencies.

For instance, if the negotiations between the firm and a buyer break down, the investors could step in and negotiate directly with the buyer. The model implies that such covenants are suboptimal, both for the firm and the investors. The reason is that buyers would rather negotiate directly with the investors than with the firm, and increasing the buyers' payoffs in the event of a break down also increases their equilibrium payoffs, thus reducing the surplus that the firm can extract. The same is true for other covenants that affect the negotiations indirectly (e.g., forcing the firm to sell by a certain date).

In our model, at date 0, the firm and the investors can sign a contract that specifies a sharing rule of the projects' revenues. However, our main result does not require strong commitment assumptions on the side of the investors. The reason is that financing a single project with defaultable debt enables the firm to extract the maximal surplus from the buyer. The payout to investors can then be adjusted to resolve any commitment issues. For example, suppose the investors could renegotiate the financial contract ex post, after the firm bargains with the buyer. The investors would renegotiate only if the project's revenue $x$ and the security $S$ satisfy $S(x) < x/2$. It remains optimal to finance the project with defaultable debt, but to avoid the threat of expropriation, the firm might need to promise investors a higher payout. The optimal contract is $S(x) = \min\{x, D\}$ and $D = \max\{c, v/3\}$. Likewise, even if the investors could take over the project and negotiate directly with the buyer, they would not want to do so as long as their payout is greater than $v/2$. The optimal contract remains defaultable debt, $S(x) = \min[x, D]$, and $D = \max[c, v/2]$.

**Competitive Markets.** In various competitive markets, there is a decentralized exchange, where buyers and sellers pairwise bargain over the transaction terms. A natural question is how competition affects the financial contracts, especially when the buyers are on the short side of the market. We contend that in some cases, in equilibrium firms use equity rather than debt, but in other cases, firms use debt. It is difficult to give a definitive answer without taking a stand on the details of the market and the bargaining process. We present two examples that illustrate the key issues.

There are N identical sellers and M identical buyers, the buyer/seller ratio is $r = \frac{M}{N} \leq 1$. Every seller has one project, and every buyer would like to purchase one project. At date 0, every seller chooses how to finance the project. We focus on the choice between debt or equity. At date 1, the sellers and buyers enter the market.

The first example follows models of directed search (see, e.g., Guerrieri et al. 2010), we assume that sellers choose a financial contract to attract buyers.

- **Case 1:** The market operates in the following way. There is a single buyer who observes the financial contracts and chooses which seller to approach. The buyer and the seller alternate offer in the usual way, and the agents discount time. After each round, the buyer can pay a switching cost and start bargaining with another seller. In equilibrium, if multiple sellers are active in the market, the buyer must be indifferent between which seller to approach. If the cost of each project $c \leq v/2$, then each seller uses equity. The reason is intuitive, every seller wants to attract the buyer, and so in equilibrium, they choose the security that is most favorable to the buyer, equity.

Thus, if sellers can use the financial to attract the buyer, in equilibrium firms use equity rather than debt. However, if buyers cannot direct their search, it becomes hard to support an equilibrium with equity.

- **Case 2:** The market operates in the following way. Buyers and sellers meet at random. There are fewer buyers than sellers, and so in each period, every buyer meets a seller with probability 1, and every seller meets a buyer with probability $r \leq 1$. When two agents meet, a randomly chosen proposer makes a take-it-or-leave-it offer. If the offer is accepted, both agents leave the market, and if the offer is rejected, the agents stay. As in the seminal work of Rubinstein and Wolinsky (1985), we consider a stationary environment, and so agents that leave are replaced by clones. The agents discount time at a rate of $\beta < 1$.

  A stationary equilibrium with symmetric strategies is characterized by the buyers' and sellers' continuation values, $v_b$ and $v_s$, and the price offers, $x_b$ and $x_s$. Working backward, if all firms use equity and they receive the same share, $\alpha$, the equilibrium prices converge to[12] $x^* = \frac{r}{1+r}v$, as $\beta \to 1$. If all firms use debt and the face value is the same, $D$, the equilibrium prices converge to[13] $x^* = \frac{1}{1+r}(D + rv)$, as $\beta \to 1$. To support an equilibrium with equity, it must be that the buyer/seller ratio is sufficiently large, $\frac{r}{1+r}v \geq c$. Also, buyers should not be able to observe the financial contract, because if they could, a firm might have profitable deviation to debt.[14] An equilibrium with debt does not suffer from either of these issues. In either case, the equilibrium outcome depends both on the financial contracts and the thickness of the market. The example raises several interesting questions, which we leave for future work.

**Other Securities.** We assumed that securities are monotonic and positive. Relaxing these assumptions alters the results. First, monotonicity of $S$ is important because the firm could extract more than half the social surplus by using a "live or die" security as in Innes (1990). Namely, the firm finances a single project with the following security

$$S(x) = \begin{cases} x & \text{for } x < D \\ c & \text{for } D \leq x \end{cases}$$

That is, the firm will get nothing if the price is low and a large payout if the price is high. For $\epsilon > 0$ and $v - \epsilon < D \leq v$, the corresponding bargaining game has an outcome between $D$ and $v$. Second, the assumption that $S \geq 0$ is important because transfers from the investors to the firm render the sharing rule specified by the security inconsequential. To see this, if

[12]In a stationary equilibrium, sellers propose $v - x_s = \beta v_b$, buyers propose $\alpha x_b = \beta v_s$, and the values satisfy $v_b = v - \frac{1}{2}(x_b + x_s)$ and $v_s = r\alpha\frac{1}{2}(x_b + x_s) + (1-r)\beta v_s$. These equations imply $x^* = \frac{r}{1+r}v$.

[13]In a stationary equilibrium, sellers propose $v - x_s = \beta v_b$, buyers propose $x_b - D = \beta v_s$, and the values satisfy $v_b = v - \frac{1}{2}(x_b + x_s)$ and $v_s = r(\frac{x_b+x_s}{2} - D) + (1-r)\beta v_s$. These equations imply $x^* = \frac{1}{1+r}(D + rv)$.

[14]If the market is large, a deviation by a single firm from equity to debt does not affect the values of other agents, $v_b$ and $v_s$. Therefore, this firm still offers buyers the equilibrium price $x_s$, but the firm could receive a better price offer from buyers.

there is a single project, then investors can pay the firm $t \geq 0$, invest $c$, and sign a debt contract where $D = v - \epsilon$. The two parties can extract the entire surplus from the buyers modulo $\epsilon$. If $t = v - c - \epsilon$, the investors will break even and the firm will get almost the entire social surplus; if $t = 0$, the investors will get it.

**Joint and Separate Contracts.** The two types of contracts, joint and separate, correspond to common financing arrangements: a project can be financed within the firm or separately through a subsidiary. Of course, the firm should be able to finance a subset of the projects jointly and the other projects separately. We considered the two extreme partition without loss of generality (see Remark 3). More general contracts can be written if securities depend on the vector of prices instead of the total revenue, i.e., $S(x_1, \ldots, x_N)$. The analysis and results are easily extended to this case in the Online Appendix.

**Unobservable Contracts.** To simplify, we analyzed the bargaining games under complete information. If the financial contract were unobservable to the buyers, incomplete information may generate additional equilibria in the bargaining game.[15] We conjecture that the firm will not benefit from this uncertainty and that the contracts that achieve the firm's maximal equilibrium payoff are the same as in the complete information case, but a formal proof is beyond the scope of the present paper.

**Risk and Debt Overhang.** The model abstracts from risk and uncertainty. A natural extension of the model would be to a setting where the buyer's surplus is uncertain at the contracting stage. Notice that when there is uncertainty at the contracting stage, the firm and the investors could restructure the financial contract as new information arrives, and if uncertainty is resolved before the bargaining stage, the optimal contract is what we characterized. If the firm and the investors cannot restructure the contract, then additional tensions may arise.

[15] It is well known in the strategic delegation literature that multiple equilibria may arise if the delegation contract is unobservable (see, e.g., Fershtman and Kalai 1997; Koçkesen and Ok 2004).

To illustrate, suppose that the buyer's value is either $v_H$ or $v_L$ and the firm cannot restructure the financial contract. If the firm finances a single project with debt, it may choose not to produce the good once it learns that the buyer's value is low. This debt overhang problem arises whenever there is a production cost of $\epsilon > 0$ and the debt level $D > v_L - 2\epsilon$. Financing the project with equity resolves the debt-overhang problem, but the firm would still face a hold-up problem. The optimal security lies between debt and equity. Also, the firm could eliminate some of the states in which it defaults by financing multiple projects as a pool. The hold-up problem and the debt overhang problem are significant issues for security design that deserve serious attention beyond the scope of this work.

# V Concluding Remarks

This paper presented a model of security design when project outcomes depend on bargaining. The message of our model is that in a large class of securities the optimal contract is remarkably simple. The firm finances the projects separately with defaultable debt. Importantly, the results do not depend on the details of the bargaining model.

The model has significant welfare and empirical implications. We showed that financing an investment deal-by-deal with defaultable debt overcomes the fundamental hold-up problem, leading to efficient investment decisions. The model also provides a novel explanation for why a specific type of project financing is prevalent in some markets, but not others. We think that future empirical work could test this theory and further investigate the causes of the cross-market difference in financing decisions.

Our analysis raises several other questions for future work. In various settings, including housing markets, labor markets, and over-the-counter asset markets, the trading process is decentralized, in that buyers and sellers bargain over the transaction prices. Which kind of financial contract parties choose in a competitive equilibrium could be an interesting question for research.

# VI Appendix A: Sequential Bargaining Game

In this section, we characterize the equilibrium of a sequential bargaining game and show that our main results do not depend on the details of the bargaining model.

## Bilateral Bargaining

A firm with a single project negotiates with a single buyer. We consider the sequential bargaining game of Rubinstein (1982). The firm and the buyer alternate offers in the usual way and following a rejected offer, the game continues with probability $p < 1$ and a breakdown occurs with probability $1 - p$. The game ends when either an offer is accepted or a breakdown occurs. The payoffs are given by $G(S)$, and the bargaining outcomes $\mathcal{F}(G(S))$ are the subgame perfect equilibrium agreements when the probability of a breakdown vanishes.[16]

**Lemma 3.** *In any outcome in $\mathcal{F}(G(S))$ the firm's payoff is no more than the buyer's payoff.*

*Proof.* Assume without loss of generality that $x - S(x)$ is weakly increasing. Let $\overline{x}_F$ and $\overline{x}_B$ be the maximal $SPE$ prices offered by the firm and the buyer, respectively. An equilibrium price $x$ offered by the buyer satisfies $x - S(x) \leq p(\overline{x}_F - S(\overline{x}_F)) - (1-p)S(0)$ and a price accepted by the buyer satisfies $p(v - \overline{x}_B) \leq v - x$. Since $S$ is positive, weakly increasing, and $\overline{x}_B \leq \overline{x}_F$, we have that $\overline{x}_B \leq p\overline{x}_F + (1-p)S(\overline{x}_B)$ and $\overline{x}_F \leq (1-p)v + p\overline{x}_B$. These imply that $p(\overline{x}_F - S(\overline{x}_F)) \leq v - \overline{x}_F$ and[17] $\overline{x}_B - S(\overline{x}_B) \leq p(v - \overline{x}_B)$. If player $i = B, F$ proposes, the firm's equilibrium payoff will be at most $\overline{x}_i - S(\overline{x}_i)$, and the buyer's at least $v - \overline{x}_i$. $\square$

It immediately follows from Lemma 3 that a defaultable debt contract is optimal. If investors get at least $c$, no more than $v - c$ is left, and the firm's maximal payoff is therefore $\frac{1}{2}(v - c)$. Finally, the bargaining game with debt $S(x) = \min\{x, D\}$ and $D = c$, has a

[16] We let $p \to 1$ in order to eliminate a first-move advantage; the analysis trivially extends to any $p > 0$.

[17] To see it, we inject the latter inequality, $\overline{x}_F \leq (1-p)v + p\overline{x}_B$, into the former, $\overline{x}_B \leq p\overline{x}_F + (1-p)S(\overline{x}_B)$, and so $\overline{x}_B \leq p((1-p)v + p\overline{x}_B) + (1-p)S(\overline{x}_B) \Rightarrow \overline{x}_B - S(\overline{x}_B) \leq p(v - \overline{x}_B)$. Likewise, we can inject the former inequality, $\overline{x}_B \leq p\overline{x}_F + (1-p)S(\overline{x}_B) \leq p\overline{x}_F + (1-p)S(\overline{x}_F)$, into the latter $\overline{x}_F \leq (1-p)v + p\overline{x}_B$, and so $\overline{x}_F \leq (1-p)v + p(p\overline{x}_F + (1-p)S(\overline{x}_F)) \Rightarrow p(\overline{x}_F - S(\overline{x}_F)) \leq v - \overline{x}_F$.

subgame perfect equilibrium that achieves the firm's maximal payoff, and investors break even.[18]

**Remark 4.** The sequential bargaining game of Rubinstein (1982) generally has a subgame perfect equilibrium that is close to (and when offers are frequent converges to) the Nash solution (see, e.g., Herrero 1989). But when the payoff set is not convex, the bargaining game may also have outcomes that do not converge to the Nash solution. For example, the security $S(x) = \min\{\lambda x, D\}$, with $\lambda < 1$, is natural to consider: it specifies that only a fraction $\lambda$ of the proceeds go towards repaying the debt and the rest goes to the firm. If $\frac{v}{2} < D < v$, then for $\lambda < 1$ sufficiently large both $x_1 = \frac{v}{2}$ and $x_2 = \frac{v+D}{2}$ can be supported as an $SPE$ of the corresponding bargaining game, but the former does not maximize the Nash product.[19]

**Remark 5.** The sequential bargaining game with debt $S(x) = \min\{x, D\}$ has a class of degenerate equilibria with low prices $x < D$. These equilibria arise because the firm gets nothing from agreements that fall short of the debt, and is therefore indifferent over the range of low prices. However, these outcomes will not survive simple refinements (such as trembling hand). If degenerate outcomes are ruled out, whether by refinements or by an indifference assumption,[20] then the strategic bargaining game with debt has a unique equilibrium that converges to the price $x = \frac{v+D}{2}$ which is the unique Nash solution.

[18] The strategies are stationary: the buyer and the firm always offer prices $x_B = D + \frac{p}{1+p}(v - D)$ and $x_F = D + \frac{1}{1+p}(v - D)$, and on the assumption that an offer will be accepted in the next period, each player accepts offers only if the payoff is no less than the continuation payoff. As $p \to 1$, the prices converge to $x = \frac{v+D}{2}$, which is the unique Nash solution. When $D = c$, investors break even and the firm and the buyer split the social surplus, each getting $\frac{1}{2}(v - c)$.

[19] See Herrero (1989) for more details on the relationship between the axiomatic and the strategic models in a bargaining problem with a non-convex payoff set. Also, a technical method for convexifying the payoff set is to allow for randomization. But this is highly unrealistic in our setting and completely changes the interpretation and trade-offs of the model. For instance, under randomization, a bargaining game with debt is equivalent to a bargaining game with equity.

[20] It is reasonable to assume that the firm would rather wait than reach an agreement from which it gains nothing.

## Multilateral Bargaining

We now consider a multilateral bargaining game. Let us first mention well-known results in bargaining theory. Several papers provide a strategic foundation for bilateral stability (or the stronger property of consistency) in different settings (e.g., Krishna and Serrano (1996), Hart and Mas-Colell (1996), Collard-Wexler et al. (2014)). That is, each paper specifies a sequential bargaining game and shows that the equilibrium outcomes converge, as offers become frequent, to the bilaterally stable (or consistent) outcomes. The latter two papers use equilibrium refinements.

However, applying these results to our model is not straightforward because the bargaining games are different. The feasible payoff sets in our model are not necessarily convex, and hence bilateral bargaining games may have multiple equilibria. The same is true for multilateral bargaining games. The proofs of the above-mentioned results are inductive, relying heavily on convexity to guarantee a unique equilibrium in bilateral bargaining games. Thus the proof techniques do not apply to our model. In what follows we consider a sequential bargaining game and, using refinements, characterize the equilibrium for all concave securities.

**The Bargaining Procedure.** The buyers arrive at the same time and the procedure goes as follows. The firm makes offers simultaneously and privately to each buyer. Each buyer then simultaneously and independently either accept or reject her offer. Any buyer who accepts pays and leaves. The game continues with the remaining buyers. These buyers simultaneously and independently make offers to the firm which chooses which offers, if any, to accept. There is a probability of a breakdown between rounds: following any rejected offer, the game continues with probability $p$ and a breakdown occurs with probability $1 - p$. The game ends when all the goods are sold or a breakdown occurs. When the game ends, the proceeds from the sales are divided with the investors. The payoffs are given by $G(S)$ and $G(S_1, \ldots, S_N)$. Past transactions are observable but offers are private. The solution concept is Perfect Bayesian Equilibrium (Fudenberg and Tirole (1991)).

**Equilibrium Refinements.** For tractability, we will use equilibrium refinements: 1) strategies are *stationary* in that actions depend only on the payoff-relevant state variables: the revenues from previous agreements, and the buyers who have not reached agreements; and 2) the off-equilibrium beliefs are restricted. Since offers are private, each buyer who receives an off-equilibrium offer makes conjectures about the offers made to the other players and the off-equilibrium beliefs can have a large effect on the continuation game generating multiplicity. A buyer has *passive beliefs* if she believes that the other buyers received their equilibrium price. The need for such refinements is common when agreements among pairs of players affect the payoffs of the other players (for instance in vertical contracting when there are several downstream firms; see e.g., McAfee and Schwartz (1994)).

Let the bargaining outcomes $\hat{\mathcal{F}}(G(S))$ be the agreements that are a limit, as $p \to 1$, of $PBE$ agreements with passive beliefs and stationary strategies.

**Proposition 5.** *If the bargaining solution is $\hat{\mathcal{F}}$, financing each project with debt separately is optimal within the class of concave securities.*

The idea of the proof is to bound the equilibrium payoffs of the large game using only the outcomes of bilateral subgames. We can then apply Lemmata 2 and 3 to bound the equilibrium payoffs.

Let $S$ be a continuous and concave security. To characterize the equilibrium payoffs of the multilateral bargaining game, we assume without loss of generality that $Y - S(Y)$ is weakly increasing.[21] Let $x_j^F(Y,p)$ and $x_j^B(Y,p)$ be the maximal $SPE$ prices in a subgame game where only buyer $j$ remains, the firm or buyer makes the first offer, the previous proceeds are $Y$, and the continuation probability is $p < 1$. The functions are well defined (see Lemma 5 below).

[21] For any security that does not satisfy this restriction, there exists a security that does satisfy it and implements the same equilibrium outcomes.

**Lemma 4.** *Let* $\epsilon > 0$*. If the agreements* $y_1, \ldots, y_N$ *are achieved in an equilibrium with passive beliefs and stationary strategies of the multilateral bargaining game, then* $x_j^F(Y_{-j}, p) > y_j - \epsilon$*, where* $Y_{-j} = \sum_{i \neq j}^N y_i$*,* $\forall j$*, provided that* $p < 1$ *is sufficiently large .*

*Proof.* Let $Y = \sum_{i=1}^N y_i$ and $Y_{-j} = \sum_{i \neq j}^N y_i$. To ease notation, we write $x_j^B(Y_{-j})$ and $x_j^F(Y_{-j})$ instead of $x_j^B(Y_{-j}, p)$ and $x_j^F(Y_{-j}, p)$; this should not cause confusion. We focus on equilibria with positive profits,[22] $Y - S(Y) > 0$.

In an equilibrium with passive beliefs, the firm's offers are accepted without delay. Therefore, if the firm offers $y_1, \ldots, y_N$, since buyer $j$ can reject the offer and guarantee a payoff of at least $p\left(v_j - x_j^B(Y_{-j})\right)$, it must be that

$$v_j - y_j \geq p\left(v_j - x_j^B(Y_{-j})\right) = v_j - x_j^F(Y_{-j}) \implies x_j^F(Y_{-j}) \geq y_j$$

The equality follows from standard arguments: the maximal prices are achieved without delay; $p(v - x_j^B(Y)) \geq v_j - x_j^F(Y)$ because the buyer will not accept a higher price; $v_j - x_j^F(Y) \geq p(v - x_j^B(Y))$ because otherwise $x_F(Y)$ is not maximal.

When the buyers propose, the argument is more intricate. Suppose that buyer $j$'s offer is accepted.[23] Following a deviation by buyer $j$ to a lower price, the firm will reject this and possibly other offers. Let us assume that offers $y_1, \ldots, y_K$ are accepted and the others are rejected, where $j > K$. From the previous step, the game will end in the next period with the prices $y'_{K+1}, \ldots, y'_N$. The firm's payoff is $(1-p)\left(Y_K - S\left(Y_K\right)\right) + p\left(Y' - S(Y')\right)$, where $Y_K = \sum_{i=1}^K y_i$ and $Y' = Y_K + \sum_{i=K+1}^N y'_i$, and the equilibrium conditions imply

$$v_j - y_j \geq p(v_j - y'_j) \quad (4)$$

$$Y' \geq Y - \epsilon, \quad (5)$$

[22]The proof can be extended to the case where $Y - S(Y) = 0$, but these equilibria (if they exist) are irrelevant because we are looking for an upper bound on the firm's payoffs.

[23]If buyer $j$'s offer is rejected, then by the previous step $x_j^F(Y_{-j}) \geq y_j$ and we are done.

where (5) holds for $p$ sufficiently large.[24] If either condition (4) or (5) does not hold, there is a profitable deviation for buyer $j$.

We let $Y'_{-j} = Y' - y'_j$ and there are two cases to consider: the more difficult one is when $Y_{-j} > Y'_{-j}$. The key step is given in Lemma 5, which proves that all bilateral subgames satisfy a basic monotonicity property: the firm's payoff $Y + x^F_j(Y)$ is increasing. Therefore, in this case,

$$Y_{-j} + x^F_j(Y_{-j}) > Y'_{-j} + x^F_j(Y'_{-j}) \geq Y' > Y - \epsilon \implies x^F_j(Y_{-j}) > y_j - \epsilon$$

where the weak inequality is true because $x^F_j(Y'_{-j}) \geq y'_j$ (previous step) and the third inequality is (5).

For the second case, suppose that $Y'_{-j} \geq Y_{-j}$. From the previous step $x^F_j(Y'_{-j}) \geq y'_j$ and (4), we get

$$px^F_j(Y'_{-j}) + (1-p)v_j \geq y_j \tag{6}$$

The function $x^F_j(Y)$ is weakly decreasing for many securities.[25] Observe that $x^F_j(Y_{-j}) \geq x^F_j(Y'_{-j})$ and (6) implies $px^F_j(Y_{-j}) + (1-p)v_j \geq y_j$, and we are done. But for some securities, the price $x_j(Y)$ may increase with $Y$. For the case where $x^F_j(Y'_{-j}) > x^F_j(Y_{-j})$, we will show that the difference $x^F_j(Y'_{-j}) - x^F_j(Y_{-j})$ is arbitrarily small for $p < 1$ sufficiently large. The argument relies on stationarity. Given $\delta > 0$, it must be that $Y'_{-j} - Y_{-j} < \delta$ for $p < 1$ sufficiently large. Otherwise, (4) implies $Y' - \delta + (1-p)v_j > Y$ and since strategies are stationary, the firm can deviate and reject $K$ offers and get $(1-p)(Y_K - S(Y_K)) + p(Y' - S(Y'))$, which is profitable when $p$ is sufficiently large. The function $x^F_j(Y)$ is continuous (see Lemma 5) and therefore $x^F_j(Y'_{-j}) - x^F_j(Y_{-j}) < \frac{\epsilon}{2}$ for $p < 1$ sufficiently large. Finally, condition (6) implies $px^F_j(Y_{-j}) + p\frac{\epsilon}{2} + (1-p)v_j \geq y_j$. □

**Lemma 5.** *For $Y \in [0, V - v_j]$, the functions $x^F_j(Y, p), x^B_j(Y, p)$ are well defined, continuous*

[24] If the buyers' offers are accepted immediately, then $Y' > Y$, for all $p < 1$, because $(1-p)(Y_K - S(Y_K)) + p(Y' - S(Y')) \geq Y - S(Y)$.

[25] For example, with debt $x^F_j(Y) = \max\left\{\frac{v_j}{1+p}, \frac{v_j + p(D-Y)}{1+p}\right\}$ is weakly decreasing with $Y$.

*in $Y$, and $Y + x_j^B(Y,p)$ and $Y + x_j^F(Y,p)$ are (strictly) increasing in $Y$.*

*Proof.* Let $p < 1$ and to ease the notation, we write $x_j^B(Y_{-j})$ and $x_j^F(Y_{-j})$ instead of $x_j^B(Y_{-j}, p)$ and $x_j^F(Y_{-j}, p)$.

Let $x_0(Y) = \inf\{x : x - S(Y+x) + S(Y) > 0\}$. Since $S$ is concave, $x_0(Y)$ is decreasing, and the firm's payoff, $Y + x - S(Y+x)$, is strictly increasing in $x$ when $x > x_0(Y)$. Consider the subgame where only buyer $j$ remains and the previous proceeds are $Y$. First, for $Y$ sufficiently large, $v_j > x_0(Y)$, and hence there are gains from trade in this subgame because there exists an agreement such that $v_j - x > 0$ and $Y + x - S(Y+x) > Y - S(Y)$. We now show, by construction, that there exists an $SPE$ in this subgame. We can find a pair $x_B, x_F$, where $v_j > x_F > x_B > x_0(Y)$, that solve

$$v_j - x_F = p(v_j - x_B) \tag{7}$$

$$Y + x_B - S(Y + x_B) = (1-p)\,(Y - S(Y)) + p\,(Y + x_F - S(Y + x_F)) \tag{8}$$

To see this, let

$$g(x,Y) = x - S(Y+x) + S(Y) - p\,[px + (1-p)v_j - S(Y + px + (1-p)v) + S(Y)]$$

Since $g(v_j, Y) > 0 > g(x_0(Y), Y)$, there exists a price $x_B \in (x_0(Y), v_j)$ such that $g(x_B, Y) = 0$ and the prices $x_B$ and $x_F = (1-p)v + px_B$ satisfy (7) and (8). It is straightforward to construct an $SPE$ that supports these prices.[26] The set of equilibrium outcomes is non-empty, closed, and bounded and the maximal prices $x_j^B(Y)$ and $x_j^F(Y)$ exist.

Moreover, the maximal $SPE$ prices $x_j^B(Y)$ and $x_j^F(Y)$ satisfy conditions (7) and (8), the argument is standard: the maximal prices are achieved without delay; $v_j - x_j^F(Y) \geq p(v - x_j^B(Y))$ because otherwise $x_F(Y)$ is not maximal; and $p(v - x_j^B(Y)) \geq v_j - x_j^F(Y)$ because the buyer will not accept a higher price. A similar argument establishes (8) (note that the

[26]The strategies are stationary: the buyer and the firm always offer prices $x_B$ and $x_F$, and on the assumption that an offer will be accepted in the next period, each player accepts offers only if the payoff is no less than the continuation payoff.

firm's payoff $Y + x - S(Y + x)$ is (strictly) increasing in $x$ for $x > x_0(Y)$). Therefore, $v_j - x_j^F(Y) = p(v_j - x_j^B(Y))$ and $x_j^B(Y) = \max\{x : \ g(x, Y) = 0 \text{ and } x \leq v_j\}$.

The function $x_j^B(Y)$ is continuous by the maximum theorem. To show that $Y + x_j^B(Y)$ is (strictly) increasing, we will assume that $S$ is a smooth function. There is no loss of generality here. In the case that the security $S$ has kinks, we can find a smooth security $\hat{S}$ so that the subgame perfect equilibria agreements in the bargaining games with the securities $S$ and $\hat{S}$ are sufficiently close. Observe that $g\left(x_j^B(Y), Y\right) = 0$ and in an $\epsilon$-neighborhood of $(x_j^B(Y), Y)$,

$$\frac{\partial g}{\partial x} > \frac{\partial g}{\partial Y} \tag{9}$$

To see this, $\frac{\partial g}{\partial x} > \frac{\partial g}{\partial Y} \iff 1 + p > S'(Y) + pS'(Y + px + (1-p)v)$, which is true for $\epsilon$ sufficiently small because $1 > S'(Y + x_j^B(Y))$. Additionally,

$$\frac{\partial g}{\partial x}|_{x = x_j^B(Y)} \geq 0 \tag{10}$$

Otherwise, $x_j^B(Y)$ is not the maximal solution to $g(x, Y) = 0$ because $g(v_j, Y) > 0$.

Suppose first that (10) holds with equality, i.e., $\frac{\partial g}{\partial x}|_{x = x_j^B(Y)} = 0$, then (9) implies that $\frac{\partial g}{\partial Y} < 0$ in the neighborhood around $(x_j^B(Y), Y)$. Therefore, for $\Delta > 0$ sufficiently small, $g(x_j^B(Y), Y + \Delta) < 0$, and since $g(v_j, Y + \Delta) > 0$, it must be that $x_j(Y + \Delta) > x_j(Y)$. Thus, $Y + x_j^B(Y)$ is (strictly) increasing.

Otherwise, the inequality in (10) is strict, i.e., $\frac{\partial g}{\partial x}|_{x = x_j^B(Y)} > 0$, and we can use the implicit function theorem,

$$\frac{dx_j^B}{dY} = -\frac{\frac{\partial g}{\partial Y}}{\frac{\partial g}{\partial x}}$$

Since $\frac{\partial g}{\partial x} > \frac{\partial g}{\partial Y}$ and $\frac{\partial g}{\partial x}|_{x = x_j^B(Y)} > 0$, we have that $\frac{dx_j^B}{dY} > -1$, and $Y + x_j^B(Y)$ is (strictly) increasing.

Finally, consider the case where $x_0(Y) \geq v_j$. There are no gains from trade in the bilateral subgame because $x-S(Y+x)+S(Y)\leq 0$ for all $x\leq v_j$. Therefore, $x-S(Y+x)-S(Y)=0$ for all $x<x_0(Y)$ (because $S'_+\leq 1$, ). Hence, any price can be supported as an $SPE$ and therefore $x_j^B(Y)=v_j$. To show continuity, the only non-trivial case is on the boundary: suppose that $Y\to Y_0$ and $x_0(Y_0)=v_j$. By definition $x_j^B(Y_0)=v_j$, and since $v_j\geq x_j^B(Y)\geq \min\{x_0(Y),v_j\}$ and $x_0(Y)\to x_0(Y_0)=v_j$, it follows that $x_j^B(Y)\to x_j^B(Y_0)$ as $Y\to Y_0$. Finally, $Y+x_j^B(Y)$ is (strictly) increasing because $x_j^B(Y)$ is constant.

Note the $x_j^F(Y)$ is also continuous and $Y+x_j^F(Y)$ is (strictly) increasing because $x_j^F(Y)=(1-p)v_j+px_j^B(Y)$.

□

Proposition 5 follows from Lemmata 2, 3 and 4.

*Proof.* If $(y_1^*,\ldots,y_N^*)\in\hat{\mathcal{F}}G(S)$, there exists a sequence of price vectors $\{(y_1^p,\ldots,y_N^p)\}_p$ such that: 1) $y_j^p\to y_j^*$ as $p\to 1$, $\forall j$; and 2) for all $p$, the agreements $y_1^p,\ldots,y_N^p$ are an equilibrium with passive beliefs and stationary strategies of the bargaining game with a continuation probability $p$. Let $Y^p=\sum_{i=1}^N y_i^p$ and $Y_{-j}^p=\sum_{i\neq j}^N y_i^p$. By definition, $x_j^B(Y_{-j}^p,p)$ is an equilibrium outcome of a bilateral bargaining game, and by Lemmata 2 and 3,

$$v_j-x_j^B(Y_{-j}^p,p)\geq x_j^B(Y_{-j}^p,p)-S\left(Y_{-j}^p+x_j^B(Y_{-j}^p,p)\right)+S(Y_{-j}^p),\forall j$$

(i.e., the firm will gain no more than the buyer). Lemma 4 implies that $x_j^B(Y_{-j}^p,p)\geq y_j^p-\epsilon_j^p$, where $\epsilon_j^p\to 0$ as $p\to 1,\forall j$. Thus,

$$v_j-y_j^p\geq y_j^p-S(Y^p)+S(Y_{-j}^p)-2\epsilon_j^p,\forall j$$

(because the firm's payoff is weakly increasing with the price).

Taking $p\to 1$, $v_j-y_j^*\geq y_j^*-S(Y^*)+S(Y_{-j}^*)$, and the rest of the proof follows from Lemma 2 : by concavity, $\frac{y_j^*}{Y^*}\left(S(Y^*)-S(0)\right)\geq S(Y^*)-S(Y_{-j}^*)$, and so $v_j-y_j^*\geq y_j^*-$

$\left(\frac{y_j^*}{Y^*}\left(S(Y^*)-S(0)\right)\right)$, and in sum, $V-Y^* \geq Y^*-S(Y^*)$. Finally, since $S(Y^*) \geq I$, the firm will not get more than $\frac{1}{2}(V-I)$. □

**Remark 6.** To our knowledge, this is the first paper to characterize the equilibrium of multilateral bargaining games with non-convex payoff sets. The technical derivations may also prove useful in other environments where bargaining agreements are between pairs of players and payoffs across pairs are interdependent. For example, consider the negotiations between a single input supplier and downstream producers who subsequently compete in the downstream market, as in Horn and Wolinsky (1988).

# VII Appendix B: The Hold-Up Problem

In this section, we extend the model to a setting in which the firm also chooses how much to invest in each project and the buyers' surplus is increasing with investment. Since the investment decision is made before the buyers arrive, the firm incurs the entire investment cost and only a fraction of the gain. Thus a hold-up problem curbs the incentive to invest. However, the firm can finance the investment with defaultable debt and make the buyers internalize some of the investment costs. We will show that financing the investment deal-by-deal aligns the investment incentives of the firm with each buyer, achieving the Pareto efficient allocation. If the firm issues debt on a pool of the proceeds, the investment decision depends on the number of buyers. When there are relatively few buyers, the allocation is Pareto efficient. When there are relatively many buyers, hold-ups lead to inefficient allocations.

More formally, let us consider a firm that produces $N$ units, one for each buyer. The firm can make an investment which increases the quality of the units. The cost of investment is $I$ and the buyers' values are $v_i = v_j = v(I)$. The function $v$ is increasing, strictly concave, and

$v(0) = 0$. The total surplus is $V(I) = Nv(I)$ and the efficient investment level maximizes the social surplus,

$$I^* = \arg\max V(I) - I$$

We assume that $0 < I^* < \infty$. At date 0, the firm makes investment decisions. At date 1, the buyers arrive and negotiate prices with the firm. The bargaining outcomes are bilaterally stable.

As a benchmark, suppose that the firm finances the investment on its own. Since the cost of investment is sunk, each buyer will pay $\frac{v}{2}$, and the firm will invest

$$I_0 = \arg\max \frac{V(I)}{2} - I$$

Since $v$ is strictly concave, $I_0 < I^*$. Although outcomes with higher investments and higher prices are Pareto-improving, the inability of the firm and the buyers to commit makes them infeasible.

Suppose that the firm can finance the investment with a defaultable debt contract. We first consider the case where the firm issues debt on a pool of the proceeds from the sales. The firm chooses the investment $I$ and the debt level $D$, under the restriction that investors at least break even, $D \geq I$.

**Proposition 6.** *When the firm issues debt on the pool, there exists a finite number $N_0 \geq 2$ such that if the number of buyers $N \leq N_0$, the firm invests $I^*$. Otherwise, the firm invests $I_0$.*

**Proof.** To see this, recall that for a given debt level $D$ and values $v$, the maximal bilaterally stable outcome is for each buyer to pay

$$x = \begin{cases} \frac{v}{2} & \text{if } (N-1)\frac{v}{2} \geq D \\ \frac{v+D}{N+1} & \text{if } (N-1)\frac{v}{2} < D \end{cases}$$

(see Example 1),[27] and the firm's profit $\pi = \max\left\{\frac{V}{2} - D, \frac{1}{N+1}(V - D)\right\}$. Since the firm's profit is decreasing with $D$ and $D \geq I$ (because investors at least break even), the firm optimally sets $D = I$ and chooses an investment level to maximize $\pi(I) = \max\left\{\frac{V(I)}{2} - I, \frac{1}{N+1}(V(I) - I)\right\}$. For a sufficiently large $N$, $I_0 > 0$, and hence there exists a finite number $N_0 \geq 2$ such that

$$\frac{1}{N_0+1}(V(I^*) - I^*) \geq \frac{V(I_0)}{2} - I_0 > \frac{1}{N_0+2}(V(I^*) - I^*)$$

We have that $\arg\max \pi(I) = I_0$ whenever $N > N_0$; and $\arg\max \pi(I) = I^*$ whenever[28] $N \leq N_0$.

To provide intuition, recall that for a given debt level $D$ and surplus $V$, equation (1) in the text showed that the firm's profit is

$$\pi = \begin{cases} \frac{V}{2} - D & \text{if } (N-1)\frac{v}{2} \geq D \\ \frac{1}{N+1}(V - D) & \text{if } (N-1)\frac{v}{2} < D \end{cases}$$

Since the profit is decreasing in the debt level and investors break even, the firm sets $D = I$. When there are relatively few buyers, each buyer internalizes some of the investment costs, the firm receives a fraction of the social surplus and therefore invests to maximize social surplus. However, each buyer fails to internalize the investment cost when there are sufficiently many other buyers, and hold-ups lead to under-investment.

We now consider the case where the firm finances the investment deal-by-deal taking out multiple loans, each one tied to distinct transactions.

**Corollary 2.** *When the investment is financed deal-by-deal, the firm invests $I^*$.*

The argument is identical to the previous one, we assume without loss of generality that

[27]We restrict attention to $V(I) > D$ without loss of generality. Also, since each price is decreasing with the number of units, it should be verified that the firm indeed prefers to sell all the units. It is not hard to check that if the firm sells $m < N$ units, then the gains from selling another unit outweigh the losses from previous units.

[28]When $N = N_0$, the firm may be indifferent between $I^*$ and $I_0$.

each loan is defaultable (otherwise, the loan does not affect the bargaining) and that each loan specifies the repayment[29] $d_i < v_i$. The break-even condition implies that the sum of the payments to investors is no less than the initial investment, $D \equiv \sum_{i=1}^{N} d_i \geq I$. The firm's profit is $\frac{1}{2}(V(I) - D)$, and so it chooses $D = I$ and invests to maximize social surplus.

# References


Antic, N. (2014). Contracting with unknown technologies. *unpublished manuscript, Princeton University*.

Axelson, U. (2007). Security design with investor private information. *Journal of Finance 62*(6), 2587–2632.

Banal-Estañol, A., M. Ottaviani, and A. Winton (2013). The flip side of financial synergies: Coinsurance versus risk contamination. *Review of Financial Studies 26*(12), 3142–3181.

Biais, B. and T. Mariotti (2005). Strategic liquidity supply and security design. *The Review of Economic Studies 72*(3), 615–649.

Bolton, P. and D. S. Scharfstein (1990). A theory of predation based on agency problems in financial contracting. *American Economic Review 80*(1), 93–106.

Brander, J. A. and T. R. Lewis (1986). Oligopoly and financial structure: The limited liability effect. *American Economic Review 76*(5), 956–970.

Bronars, S. G. and D. R. Deere (1991). The threat of unionization, the use of debt, and the preservation of shareholder wealth. *Quarterly Journal of Economics 106*(1), 231–254.

Collard-Wexler, A., G. Gowrisankaran, and R. S. Lee (2014). "nash-in-nash" bargaining: A microfoundation for applied work. NBER working paper 20641.


[29]Otherwise, $d_i \geq v_i$, and any price between 0 and $v_i$ is an equilibrium of the bargaining game.

DeMarzo, P. M. (2005). The pooling and tranching of securities: A model of informed intermediation. *Review of Financial Studies 18*(1), 1–35.

Demarzo, P. M. and D. Duffie (1999). A liquidity-based model of security design. *Econometrica 67*(1), 65–99.

DeMarzo, P. M., I. Kremer, and A. Skrzypacz (2005). Bidding with securities: Auctions and security design. *American Economic Review 95*(4), 936–959.

Esty, B. C. (2004). Why study large projects? an introduction to research on project finance. *European Financial Management 10*(2), 213–224.

Faure-Grimaud, A. (2000). Product market competition and optimal debt contracts: The limited liability effect revisited. *European Economic Review 44*(10), 1823 – 1840.

Fershtman, C. and E. Kalai (1997). Unobserved delegation. *International Economic Review 38*(4), 763–774.

Fudenberg, D. and J. Tirole (1991). Perfect bayesian equilibrium and sequential equilibrium. *Journal of Economic Theory 53*(2), 236–260.

Glazer, J. (1994). The strategic effects of long-term debt in imperfect competition. *Journal of Economic Theory 62*(2), 428 – 443.

Guerrieri, V., R. Shimer, and R. Wright (2010). Adverse selection in competitive search equilibrium. *Econometrica 78*(6), 1823–1862.

Harsanyi, J. C. (1963). A simplified bargaining model for the n-person cooperative game. *International Economic Review 4*(2), 194–220.

Hart, O. and J. Moore (1995). Debt and seniority: An analysis of the role of hard claims in constraining management. *The American Economic Review 85*(3), 567–585.

Hart, O. and J. Moore (1998). Default and renegotiation: A dynamic model of debt. *Quarterly Journal of Economics 113*(1), 1–41.

Hart, S. and A. Mas-Colell (1996). Bargaining and value. *Econometrica 64*(2), 357–380.

Hennessy, C. A. and D. Livdan (2009). Debt, bargaining, and credibility in firm-supplier relationships. *Journal of Financial Economics 93*(3), 382 – 399.

Herrero, M. J. (1989). The nash program: Non-convex bargaining problems. *Journal of Economic Theory 49*(2), 266–277.

Horn, H. and A. Wolinsky (1988). Bilateral monopolies and incentives for merger. *RAND Journal of Economics 19*(3), 408–419.

Inderst, R. and V. Vladimirov (forthcoming). Growth firms and relationship finance: A capital structure perspective. *Management Science*.

Innes, R. D. (1990). Limited liability and incentive contracting with ex-ante action choices. *Journal of Economic Theory 52*(1), 45–67.

Kalai, E. (1977). Nonsymmetric nash solutions and replications of 2-person bargaining. *International Journal of Game Theory 6*(3), 129–133.

Kalai, E. and M. Smorodinsky (1975). Other solutions to nash's bargaining problem. *Econometrica 43*(3), 513–518.

Kaplan, S. N. and P. Strömberg (2009). Leveraged buyouts and private equity. *Journal of Economic Perspectives 23*(1), 121–146.

Klasa, S., W. F. Maxwell, and H. Ortiz-Molina (2009). The strategic use of corporate cash holdings in collective bargaining with labor unions. *Journal of Financial Economics 92*(3), 421–442.

Koçkesen, L. and E. A. Ok (2004). Strategic delegation by unobservable incentive contracts. *The Review of Economic Studies 71*(2), 397–424.

Krishna, V. and R. Serrano (1996). Multilateral bargaining. *Review of Economic Studies 63*(1), 61–80.

Lensberg, T. (1988). Stability and the nash solution. *Journal of Economic Theory 45*(2), 330–341.

Matsa, D. A. (2010). Capital structure as a strategic variable: Evidence from collective bargaining. *Journal of Finance 65*(3), 1197–1232.

McAfee, R. P. and M. Schwartz (1994). Opportunism in multilateral vertical contracting: Nondiscrimination, exclusivity, and uniformity. *American Economic Review 84*(1), 210–230.

Nachman, D. C. and T. H. Noe (1994). Optimal design of securities under asymmetric information. *The Review of Financial Studies 7*(1), 1–44.

Nash, J. F. (1950). The bargaining problem. *Econometrica 18*(2), 155–162.

Perotti, E. C. and K. E. Spier (1993). Capital structure as a bargaining tool: The role of leverage in contract renegotiation. *American Economic Review 83*(5), 1131–41.

Rajan, R. G. (1992). Insiders and outsiders: The choice between informed and arm's-length debt. *Journal of Finance 47*(4), 1367–1400.

Rubinstein, A. (1982). Perfect equilibrium in a bargaining model. *Econometrica 50*(1), 97–109.

Rubinstein, A. and A. Wolinsky (1985). Equilibrium in a market with sequential bargaining. *Econometrica 53*(5), 1133–50.

Stole, L. A. and J. Zwiebel (1996a). Intra-firm bargaining under non-binding contracts. *Review of Economic Studies 63*(3), 375–410.

Stole, L. A. and J. Zwiebel (1996b). Organizational design and technology choice under intrafirm bargaining. *American Economic Review 86*(1), 195–222.

Townsend, R. M. (1979). Optimal contracts and competitive markets with costly state verification. *Journal of Economic Theory 21*(2), 265–293.

Zhou, L. (1997). The nash bargaining theory with non-convex problems. *Econometrica 65*(3), 681–685.