

# Determining Damaging Fatigue Cycles under Influence of Random Loadings using the Root-Mean-Square Level

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## ABSTRACT

*The aim of this paper is to investigate the damaging fatigue cycles criterion using the root-mean-square level under the influence of random loads for coil spring. Fatigue life cycle analysis especially in signal processing involves high computational effort because it deals with large quantity of data from vibratory loads obtained from the coil spring. The captured data of frequent low amplitude cycles generally consist of noise or vibrations which are meaningless and not significant for analysis. Therefore, a criterion using the root-mean-square level is proposed in assessing fatigue life of the captured strain signal from the coil spring. Four strain signals were analysed statistically using global statistics and distribution fitting. Fatigue damage was determined using the Morrow model and control charts were used in the classification of predefined damaging cycles. For evaluating the contribution of these cycles to fatigue damage, cycle elimination process was performed. The results showed a significant reduction of 48%–62% in damage values with damage probability ranging from 0.9362 to 0.9999. Hence, the criterion is useful and has potential to be extended in determining damaging cycles in fatigue analysis in indicating the damaging effects for coil spring.*

**Keywords:** damage; fatigue; random; root-mean-square; suspension

## **Introduction**

Fatigue life analysis involves dealing with large quantity of random data which requires a great deal of computational efforts in terms of signal processing technique. Random data especially the frequent low amplitude cycles consist of vibrations that can be meaningless and considered not significant to be included in the analysis. Therefore, it is optimal to start the analysis from the most damaging components in order to optimise the computational efforts [1]. In addition, fatigue load histories collected often contain a lot of noise and makes durability testing very difficult [2]. This problem has motivated the investigation to determine specific criteria to be adopted in fatigue data editing method so that a good compromise between the amount of data removed and the retained damage for analysis could be achieved.

Considerable efforts have been exerted in the fatigue data editing research [3, 4] to increase efficiency in fatigue life assessment. Generally, there are two editing methods used in fatigue life prediction i.e. the amplitude-based and frequency-based [5]. Kadhim et al. [6] proposed computational approach using the fatigue data editing technique by combining with finite element analysis. This proposed method for fatigue life prediction provides the ability to summarise long load histories, hence reducing computational or experimental time. In fatigue life prediction for spot weld, Duraffourg et al. [7] proposed two new criteria based on mechanical damage and fracture mechanics using fatigue data editing technique that is capable to produce damage very close to full load signal damage. More recently, Putra et al. [8] proposed a generation of simulated strain signals to accelerate fatigue tests using wavelet-based data editing. Through this method, the strain signals were shortened up to 36.7% besides reduced testing time up to 33.9%.

This paper investigates and proposes a criterion for determining the damaging fatigue cycles based on the root-mean-square (r.m.s.) level. The definition of damaging cycles was based on suggestions in previous works and the predefined value is tested on four loading histories data collected from the vehicle coil spring component. The significance of these damaging cycles in contributing to fatigue damage is evaluated using cycles elimination technique and the probability of damage values.

## **Theoretical Background**

Fatigue life determination was performed using the Morrow's mean stress model. This model is suitable for cases where the majority of strain signals

are compressive [9] such as the suspension system data. The equation is given by:

$$\varepsilon_a = \frac{\sigma'_f}{E} \left(1 - \frac{\sigma_m}{\sigma'_f}\right) (2N_f)^b + \varepsilon'_f \left(1 - \frac{\sigma_m}{\sigma'_f}\right)^{c/b} (2N_f)^c \quad (1)$$

where the  $\sigma'_f$  as the fatigue strength coefficient,  $b$  is fatigue strength exponent,  $\varepsilon'_f$  is fatigue ductility coefficient,  $\sigma_m$  is the mean stress, and  $c$  is fatigue ductility exponent. To calculate the number of cycles to failure,  $N_f$  for a given value of  $\varepsilon_a$ , numerical or graphical method needs to be adopted.

The Palmgren–Miner rule was used to determine the total damage. This linear damage rule is the most widely used accumulation model and is written as,

$$D_{TOT} = \sum_i \frac{n_i}{N_{fi}} = 1 \quad (2)$$

where  $D_{TOT}$  is the total damage,  $N_{fi}$  is the fatigue life of some materials according to the respective stress or strain level, and  $n_i$  is the number of load cycles in the fatigue test. The conventional process flow of fatigue life and damage determination is given in Figure 1.



The cumulative distribution function of a 2-parameter Weibull can be expressed as follows [10]:

$$F(N_f) = 1 - \exp\left(-\left(\frac{N_f}{\theta}\right)^\beta\right) \quad (3)$$

where  $F(N_f)$  is the fraction failed in time or cycles  $N_f$ ,  $N_{f0}$  is the minimum time or cycles to failure,  $\theta$  is the characteristic life, and  $\beta$  is the Weibull slope or shape parameter.

## Methodology

The strain signals used in this study were the data obtained from real experiment measured using the strain gauge positioned on the car coil spring component that was driven on various road surfaces [8, 11]. The specific type of locations where the signals were recorded is listed in Table 1. The last data was the typical strain history for suspension system developed by the Society of Automotive Engineers (SAE) [2]. The complete data series for each respective loading block is given in Figure 2. It can be observed that the trend of the signals varies according to the different locations. The S2 data recorded on the highway appears to have more stable oscillatory signals compared to the other three signals.

Table 1: The location of each signal obtained

Signal	Type of location/data
S1	Campus
S2	Highway
S3	Rural
SAE	SAE standard data for suspension system

The probability of damage (PoD) concept is defined here for the evaluation of damaging cycles based on the r.m.s. level. The assumption used is that the PoD value derived based on the strain range data which is exhaustive and inclusive. This means that the signals are assumed to be complete signal. From the cumulative distribution function (CDF) of strain range data, the value of PoD can be obtained based on the 2 r.m.s. value of strain range. The schematic diagram plot for PoD determination is demonstrated in Figure 3.

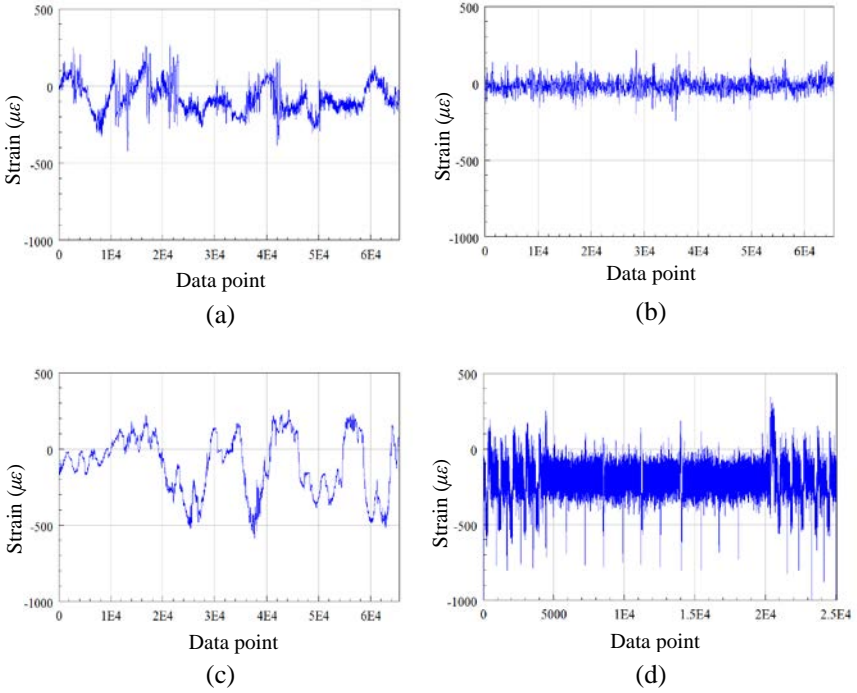


Figure 2: Data series of strain signals for: (a) S1, (b) S2, (c) S3, and (d) SAE

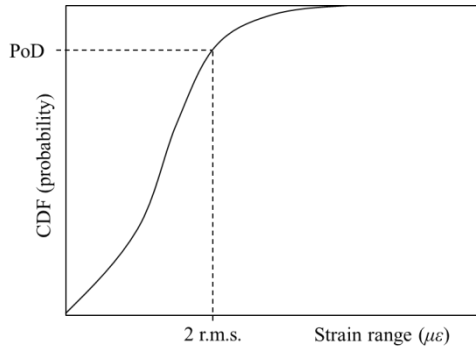


Figure 3: Schematic of CDF plot for PoD determination

## Results and Discussion

The statistical descriptions of strain signals data were obtained by determining the global statistics values. Table 2 depicts the statistics for each signal with bold figures indicating the maximum magnitude. Based on the results, all signals produce negative mean values, with a maximum of  $-15.14$  provided by S2 signal. A negative value of mean is a sign that majority of the strain data in the series are in compression state which is consistent with the behaviour of suspension system data [8]. The r.m.s. of SAE signal has the maximum value of  $246.62 \mu\epsilon$ . This outcome exhibits that SAE signal has the highest oscillatory energy content compared to the other three signals [12]. The high energy content could be explained by the background of the SAE data that has more high amplitude signals [13].

In the context of data distribution, S1 and S2 have positively skewed data while S3 and SAE have the opposite case. All signals do not follow a symmetric distribution due to the non-zero skewness values. Meanwhile, the kurtosis values ranges from 2.04 to 4.45. For a normal distribution, the kurtosis is approximately 3 which is not the case for any of the signals. This finding further supports the non-normality of the data distribution.

Table 2: Global statistics values for each strain signal

Signal	Mean ( $\mu\epsilon$ )	r.m.s. ( $\mu\epsilon$ )	Skewness	Kurtoss
S1	-78.85	126.43	0.36	2.82
S2	<b>-15.14</b>	44.60	<b>0.42</b>	<b>4.45</b>
S3	-100.84	218.79	-0.33	2.04
SAE	-206.63	<b>246.62</b>	-0.37	4.32

Note: The bold figures indicate maximum values for each statistics

The distribution of strain range was investigated based on the suggestions in previous research [1, 14]. The researchers suggested that a multimodal distribution comprises of linear combination of 2 parameter-Weibull would best fit the strain range data. The multimodal distribution is also known as the mixed Weibull distribution. The mixed Weibull probability plot for each signal is demonstrated in Figure 4 has fitting of data at the lower level of strain range appears to be inaccurate. However, in the case of damaging cycles, the distribution fitting of higher strain range is emphasized more. Therefore, the concern is more on the higher level of strain ranges where the damaging cycles are most likely to be located.

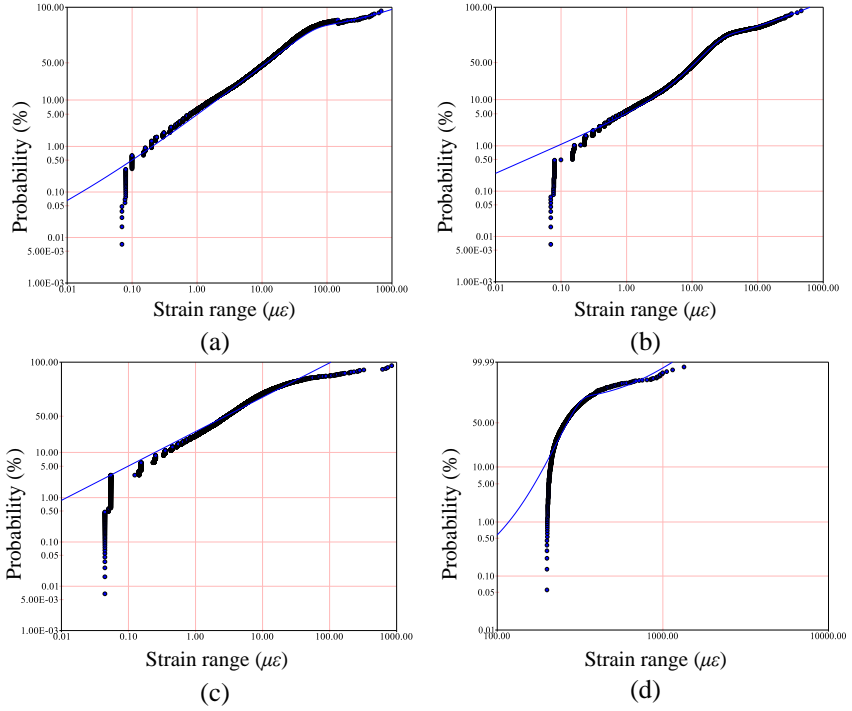


Figure 4: Weibull probability plot of strain range data distribution for: (a) S1, (b) S2, (c) S3, (d) SAE

Fatigue life prediction was carried out by determining fatigue cycles using the Rainflow cycle counting technique. The number of cycles and damage values for each signal are tabulated in Table 3. It is seen that S2 has the highest number of counted cycles (10,300) and SAE has the lowest (1,253). Evaluation on fatigue damage per block shows that the SAE signal has produced the highest damage of  $1.70 \times 10^{-3}$  even though it has the least number of fatigue cycles. Therefore, it is noted that more cycles do not necessarily mean more damage. Instead, fatigue damage values depend on the magnitudes of the strain range, which is the size of cycles [15].

Table 3 : The number of cycles and damage per block for each signal

Signal	Type of location	N (cycles/block)	Fatigue damage (damage/block)
S1	Campus	9,768	$2.85 \times 10^{-4}$
S2	Highway	10,300	$4.78 \times 10^{-5}$
S3	Rural	10,260	$2.12 \times 10^{-4}$
SAE	SAE standard data for suspension system	1,253	$1.70 \times 10^{-3}$

Previous studies by Lambert [16] and Liou et al. [17] found that most of fatigue damage were caused by the stress values greater than 2 to 4 times the r.m.s. for a complete signals. Therefore, the following results are presented to investigate this claim for the strain signals case. To visualize the classification of damaging cycles based on the definition of Lambert [16] and Liou, et al. [17], control charts were used. The boundary or upper control limit (UCL) in the control chart was based on the 2 times r.m.s. values for each signal. The lower control limit (LCL) was set to be zero. The limits are detailed out in Table 4.

Table 4 The upper and lower limit of the control chart

Signal	UCL	$\bar{X}$ (r.m.s.)	LCL
S1	253.0	126.0	0
S2	89.3	44.6	0
S3	437.5	218.8	0
SAE	493.0	247.0	0

The control charts obtained are presented in Figure 5. Overall observation on these control charts shows that the quantity of cycles greater than the UCL varies. For example, S2 appears to have the most number of damaging cycles in contrast to S3 with just several data points higher than the UCL. The exact numbers of damaging cycles with percentage from the total number of cycles in the block are given in Table 5. It can be observed that S2 has 197 damaging cycles compared to S3 with just 4 cycles. It is highlighted here that the percentages of damaging cycles are extremely low ranging from 0.04% to 3.67%.

To evaluate the significance of these damaging cycles to damage value, cycle elimination process was performed. Table 6 shows the reduction in damage values after the elimination. The difference percentage of the original damage and the damage after elimination is given in the last column. The difference values ranging from 48% to 62% showing that the eliminated cycles are indeed contributed significantly to the damage value.



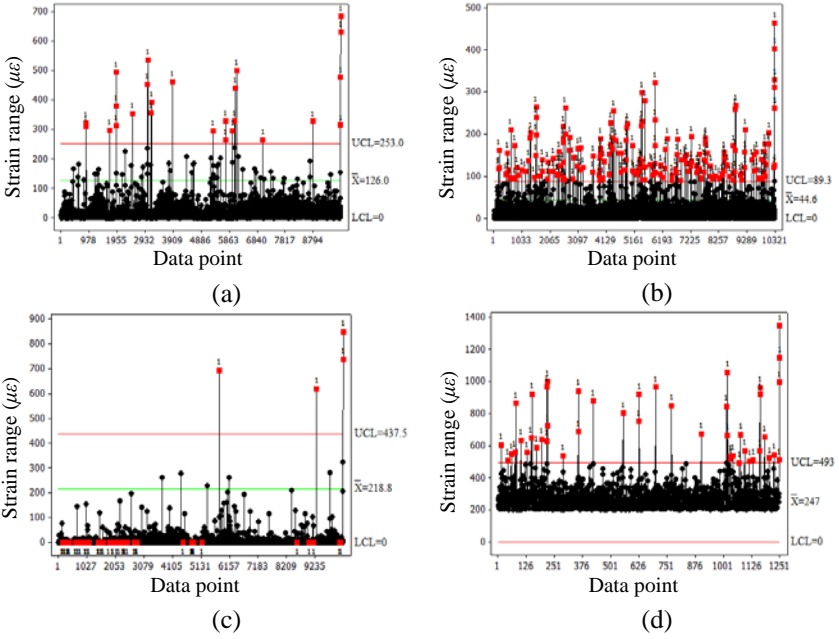


Figure 5: Control chart for the classification of damaging cycles according to the r.m.s. for signals: (a) S1, (b) S2, (c) S3, (d) SAE

Table 5: Total number of damaging cycles and the corresponding estimated damage determined using the Morrow's model

Signal	Cycle > 2 r.m.s		Total no. of cycle
	No.	Percentage (%)	
S1	25	0.26	9,768
S2	197	1.91	10,300
S3	4	0.04	10,260
SAE	46	3.67	1,253

Table 6: Damage reduction after the elimination of damaging cycle

Signal	Original (damage/block)	After (damage/block)	Difference (%)
S1	$2.85 \times 10^{-4}$	$1.45 \times 10^{-4}$	49
S2	$4.78 \times 10^{-5}$	$1.82 \times 10^{-5}$	62
S3	$2.12 \times 10^{-4}$	$1.10 \times 10^{-4}$	48
SAE	$1.70 \times 10^{-3}$	$8.16 \times 10^{-4}$	52

Based on the cumulative distribution function of previously resulting mixed Weibull distribution, the probability of damage values can be determined as summarized in Table 7. The PoD is related to the level of damaging state of a cycle according to the strain range value. The higher the PoD value, the more damaging the cycle is. Therefore, if the cycle is damaging, then the PoD value is also high accordingly. It is evident in Table 7 that the PoD values for the level of strain range greater than 2 r.m.s. are extremely high ranging from 0.9362 to 0.9999. The bar chart for the PoD values is given in Figure 6. Hence, it is clear that these damaging cycles based on 2 times r.m.s value are contributing significantly to the overall fatigue damage.

Table 7: Probability of damage values according to the level of r.m.s.

Signal	2 r.m.s ( $\mu\epsilon$ )	Probability of damage (PoD)
S1	252.86	0.9977
S2	89.20	0.9848
S3	437.58	0.9999
SAE	493.24	0.9362

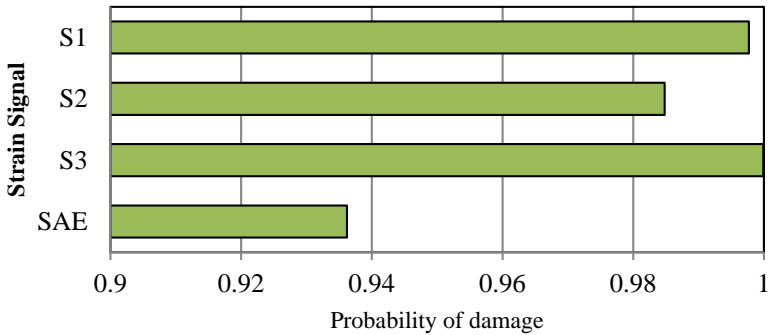


Figure 6: Bar chart of the probability of damage values for all signals

## **Conclusion**

In this study, an investigation of damaging fatigue cycles based on the root-mean-square (r.m.s.) value is presented. The damaging cycles were set according to suggestions from previous research and defined by 2 times the r.m.s. value. Four strain signals data were analysed using global statistics parameters and data distribution fitting to see their behaviour and scatter. Meanwhile, fatigue life analyses were performed where individual cycle was determined using the Rainflow cycle counting technique. Fatigue damage values for all signals were obtained by adopting the Morrow mean stress model. To evaluate the significance of the predefined damaging cycles, control charts were used in the classification of cycles. Reduction in damage values after eliminations of the damaging cycles resulting in significant difference percentages ranging between 48% and 62%. Furthermore, the high probability of damage (0.9362 to 0.9999) obtained in the analysis supports the results that fatigue damage is caused mostly by cycles with strain range greater than 2 times the r.m.s. value. Therefore, this criterion is able to identify damaging cycles and has potential to be extended and further adopted in fatigue data editing for damaging cycles determination.

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