

Article



# **Optimization of a Multiple Injection System in a Marine Diesel Engine through a Multiple-Criteria Decision-Making Approach**

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Abstract: In this work, a numerical model was developed to analyze the performance and emissions of a marine diesel engine, the Wärtsilä 6L 46. This model was validated using experimental measurements and was employed to analyze several pre-injection parameters such as pre-injection rate, duration, and starting instant. The modification of these parameters may lead to opposite effects on consumption and/or emissions of nitrogen oxides (NO<sub>x</sub>), carbon monoxide (CO), and hydrocarbons (HC). According to this, the main goal of the present work is to employ a multiple-criteria decision-making (MCDM) approach to characterize the most appropriate injection pattern. Since determining the criteria weights significantly influences the overall result of a MCDM problem, a subjective weighting method was compared with four objective weighting methods: entropy, CRITIC (CRiteria Importance Through Intercriteria Correlation), variance, and standard deviation. The results showed the importance of subjectivism over objectivism in MCDM analyses. The CRITIC, variance, and standard deviation methods assigned more importance to consumption and provided similar results. Nevertheless, the entropy method assigned more importance to consumption and provided a different injection pattern.

Keywords: MCDM; marine engine; injection; emissions; consumption

## 1. Introduction

Pollution levels in recent years have been reaching dangerous limits. Important contributors to global pollution are diesel engines, which are efficient machines but emit important levels of particulate matter (PM), NO<sub>x</sub>, CO<sub>2</sub>, CO, HC, SO<sub>x</sub>, etc. [1–5]. Between these, NO<sub>x</sub> and SO<sub>x</sub> are characteristic of marine diesel engines [6–10]. According to the International Maritime Organization (IMO), NO<sub>x</sub> and SO<sub>x</sub> from ships represent 5% and 13% of global NO<sub>x</sub> and SO<sub>x</sub> emissions, respectively [11]. IMO regulates  $NO_x$  and  $SO_x$  in the shipping sector. Regarding  $SO_x$ , since the sulfur content of the fuel is the reason for SO<sub>x</sub> emission, IMO limits the sulfur content of fuels or requires the use of exhaust gas cleaning systems to reduce sulfur emissions [12]. Regarding NO<sub>x</sub>, IMO imposes even increasing limitations. According to this, several NO<sub>x</sub> reduction procedures have been developed in recent years. Some of them, called primary measurements, operate on the engine performance, such as EGR, water injection, modification of the injection parameters, etc. On the other hand, other  $NO_x$  reduction procedures, called secondary measurements, remove this pollutant from exhaust gases by downstream cleaning techniques, such as selective catalytic reduction (SCR). The present work focuses on pre-injection systems. It is well known that pilot injections reduce NO<sub>x</sub> noticeably [13–17], but sometimes pilot injections can increase consumption and other pollutants such as smoke or hydrocarbons (HC) [18–22], mainly depending on parameters such as injection time, duration, number of pre-injections, dwelling time, etc. Since these

parameters provide conflicting results, a formal tool to establish the most appropriate injection pattern is necessary. According to this, multiple-criteria decision-making (MCDM) approaches constitute a formal tool for handling complex decision-making problems. MCDM methods are complex decision-making tools for choosing the optimal option in cases where there are conflicting criteria. Since the start of the MCDM methods in the 1960s, they were employed in many fields such as sustainability, supply chain management, materials, quality management, GIS, construction and project management, safety and risk management, manufacturing systems, technology, information management, soft computing, tourism management, etc. [23]. One of the handicaps of MCDM methods is the determination of the criteria weights, i.e., the degree of importance for each criterion. It is important to focus on the criteria weights due to their influence on the overall result. According to this, several approaches to define the criteria weights can be found in the literature. Briefly, these approaches can be divided into subjective, i.e., based on the estimations of experts, and objective, i.e., calculated through mathematical expressions. In practice, subjective weights are most commonly used [24]. Contrary to subjective methods, the objective weights are based on mathematical methods and decision-makers have no role in determining the relative importance of criteria. Common objective methods are entropy [25], CRITIC [26], standard deviation [27], variance, mean weight, etc.

The present paper proposes a MCDM approach to select the most appropriate injection pattern using a pilot injection in the marine diesel engine Wärtsilä 6L 46. The pre-injection rate, duration, and starting instants were analyzed and the criteria were specific fuel consumption (SFC) and NO<sub>x</sub>, CO, and HC emissions. These emissions and consumption were characterized through CFD (Computational Fluid Dynamics) analyses. Due to the importance of the criteria weights on the overall result, a comparison of several weighting methods was realized. A subjective criteria weighting method was compared to four objective criteria weighting methods: entropy, CRITIC, variance, and standard deviation.

# 2. Materials and Methods

The Wärtsilä 6L 46 is a four-stroke marine diesel engine, turbocharged and intercooled with direct fuel injection. It has 6 in-line cylinders and each cylinder has 2 inlet and 2 exhaust valves. The standard engine employed in the present work does not implement any NO<sub>x</sub> reduction system such as water injection, EGR, SCR, etc. It incorporates a fast and efficient turbocharging system called single pipe exhaust (SPEX). The cooling system is split into high-temperature and low-temperature stages. Other specifications are provided in Table 1.

Parameter	Value
Output	5430 kW
Speed	500 rpm
Piston displacement	96.4 L/cyl
Bore	460 mm
Stroke	580 mm
Speed	500 rpm
Mean effective pressure	22.5 bar
Mean piston speed	9.7 m/s

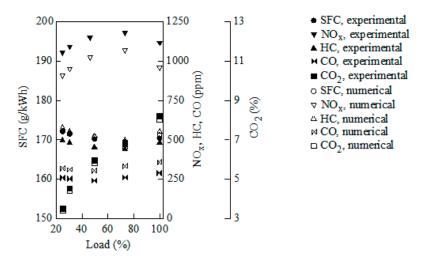
Table 1. Characteristics of the engine at 100% load.

A CFD analysis previously validated with experimental results [28–34] was carried out using the open software OpenFOAM. The simulation was based on the equations of conservation of mass, momentum, and energy and the numerical details are listed in Table 2.

Parameter	Model
Turbulence model	k-e
Combustion kinetic scheme	Ra and Reitz [35] (131 reactions and 41 species)
NO <sub>x</sub> formation kinetic scheme	Yang et al. [36] (43 reactions and 20 species)
NO <sub>x</sub> reduction kinetic scheme	Miller and Glarborg [37] (131 reactions and 24 species)
Fuel heat-up and evaporation model	Dukowicz [38]
Fuel droplet breakup model	Kelvin-Helmholtz and Rayleigh-Taylor [39]

Table 2. Numerical details.

A comparison between the numerical and experimental results is illustrated in Figure 1 and Figure 2. Figure 1 shows the emissions and SFC obtained numerically and experimentally at several loads, and Figure 2 shows the in-cylinder pressure and heat release rate obtained numerically and experimentally at 100% load. As can be seen, both figures show a reasonable correspondence between numerical and experimental results.



**Figure 1.** Specific fuel consumption (SFC) and emissions numerically and experimentally obtained at different loads.

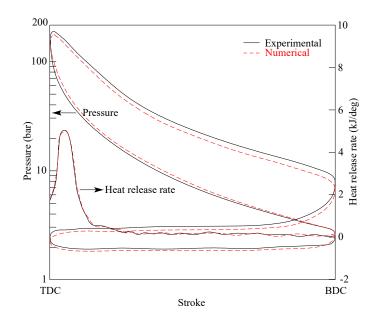


Figure 2. In-cylinder pressure numerically and experimentally obtained at 100% load.

#### 3. Results and Discussion

Once validated, this CFD model was used to provide the data for the MCDM approach. The simulation calculation was carried out simply as a process and as sample results for applying the optimal selection method of multiple injection conditions. The 125 cases illustrated in Figure 3 were analyzed. As can be seen, five pre-injection rates (R): 5%, 10%, 15%, 20%, and 25%; five pre-injection durations (D): 1°, 2°, 3°, 4°, and 5° crank angle (CA); and five pre-injection starting instants (S):  $-22^{\circ}$ ,  $-21^{\circ}$ ,  $-20^{\circ}$ ,  $-19^{\circ}$ , and  $-18^{\circ}$  crank angle after top dead center (CA ATDC), were employed.

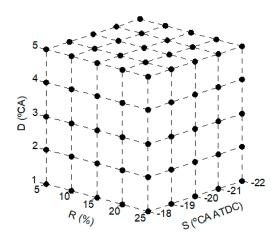


Figure 3. Illustration of the 125 cases analyzed.

Four criteria were analyzed: SFC,  $NO_x$ , CO, and HC. It is worth mentioning that PM emissions should be included in this model. They were not included because current numerical methods do not provide enough accuracy regarding PM [40]. Nevertheless, continuous efforts are being made to develop more models which will provide proper accuracy in the near future.

Table 3 outlines the pre-injection rate, duration, and starting instant, as well as the values of SFC, NO<sub>x</sub>, CO, and HC provided by the CFD model for the 125 cases. According to this, the data matrix is composed of 125 rows and 4 columns. In the remaining of the present work, each datum of the decision matrix will be represented as  $X_{ij}$ , where *i* is the case and *j* is the criteria considered.

	C	р	р		Criter	ion ( <i>j</i> )	
Case (i)	S (°CA ATDC)	R (%)	D- (°CA)	j = 1 SFC (g/kWh)	j = 2 NO <sub>x</sub> (g/kWh)	j = 3 CO (g/kWh)	j = 4 HC (g/kWh)
1	-22	5	1	190.9	7.38	4.65	5.72
2	-22	5	2	189.0	7.83	4.67	5.73
3	-22	5	3	187.5	8.19	4.70	5.76
4	-22	5	4	186.6	8.43	4.74	5.80
5	-22	5	5	186.1	8.58	4.78	5.84
6	-22	10	1	196.4	6.01	4.70	5.78
7	-22	10	2	193.9	6.57	4.73	5.81
8	-22	10	3	192.1	7.00	4.77	5.84
9	-22	10	4	190.8	7.31	4.81	5.89
10	-22	10	5	190.2	7.49	4.86	5.94
11	-22	15	1	200.4	5.06	4.74	5.83
12	-22	15	2	197.5	5.70	4.77	5.86
13	-22	15	3	195.3	6.19	4.81	5.90
14	-22	15	4	193.9	6.53	4.86	5.95
15	-22	15	5	193.1	6.73	4.92	6.02
16	-22	20	1	203.6	4.32	4.77	5.87
17	-22	20	2	200.3	5.01	4.80	5.90

Table 3. Data for the multiple-criteria decision-making (MCDM) problem.

Table 3.	Cont
Table 5.	Cont.

	_			Criterion (j)					
Case (i)	S (°CA ATDC)	R (%)	D (°CA)	j = 1 SFC (g/kWh)	j = 2 NO <sub>x</sub> (g/kWh)	j = 3 CO (g/kWh)	j = 4 HC (g/kWh)		
		20	2	-			-		
18	-22	20	3	197.9	5.55	4.85	5.94		
19	-22	20	4	196.3	5.92	4.91	6.00		
20	-22	20	5	195.5	6.14	4.97	6.07		
21	-22	25	1	206.3	3.70	4.79	5.90		
22	-22	25	2	202.8	4.44	4.83	5.93		
23	-22	25	3	200.2	5.01	4.88	5.98		
24	-22	25	4	198.4	5.41	4.94	6.05		
25	-22	25	5	197.5	5.65	5.02	6.13		
26	-21	5	1	185.2	8.65	4.62	5.68		
27	-21	5	2	183.3	9.11	4.65	5.70		
28	-21	5	3	181.9	9.46	4.67	5.72		
29	-21	5	4	180.9	9.71	4.71	5.76		
30	-21	5	5	180.4	9.85	4.75	5.80		
31	-21	10	1	189.1	7.57	4.67	5.73		
32	-21	10	2	186.6	8.14	4.69	5.76		
33	-21	10	3	184.7	8.57	4.73	5.79		
34	-21	10	4	183.5	8.88	4.77	5.84		
35	-21	10	5	182.9	9.05	4.83	5.89		
36	-21	15	1	191.9	6.83	4.70	5.77		
37	-21	15	2	189.0	7.47	4.73	5.80		
38	-21	15	3	186.8	7.96	4.77	5.84		
39	-21	15	4	185.4	8.31	4.82	5.89		
40	-21	15	5	184.6	8.51	4.88	5.96		
41	-21	20	1	194.1	6.25	4.72	5.81		
42	-21	20	2	190.9	6.94	4.76	5.84		
43	-21	20	3	188.5	7.48	4.80	5.88		
44	-21	20	4	186.9	7.86	4.86	5.94		
45	-21	20	5	186.1	8.07	4.93	6.01		
46	-21	25	1	196.0	5.76	4.74	5.83		
47	-21	25	2	192.5	6.50	4.78	5.87		
48	-21	25	3	189.9	7.08	4.83	5.91		
49	-21	25	4	188.1	7.48	4.89	5.98		
50	-21	25	5	187.3	7.71	4.97	6.06		
51	-20	5	1	181.2	9.61	4.60	5.65		
52	-20	5	2	179.3	10.07	4.62	5.67		
53	-20	5	3	177.9	10.42	4.65	5.69		
54	-20	5	4	176.9	10.67	4.68	5.73		
55	-20	5	5	176.4	10.81	4.73	5.77		
56	-20	10	1	183.9	8.76	4.64	5.69		
57	-20	10	2	181.4	9.32	4.66	5.72		
58	-20	10	3	179.6	9.75	4.70	5.75		
59	-20	10	4	179.3	10.06	4.74	5.80		
60	-20	10	5	178.5	10.00	4.80	5.85		
61	-20	15	1	185.9	8.17	4.66	5.73		
62	-20 -20	15	2	183.0	8.81	4.69	5.75		
63	-20 -20	15 15	2	180.8	9.30	4.69	5.79		
					9.64	4.74			
64 65	-20	15 15	4	179.3 178.6			5.85 5.01		
65	-20	15 20	5	178.6	9.84 7.71	4.85	5.91 5.75		
66 67	-20	20	1	187.4	7.71	4.68	5.75 5.78		
67	-20	20	2	184.2	8.40	4.72	5.78		
68	-20	20	3	181.8	8.93	4.76	5.83		
69	-20	20	4	180.2	9.31	4.82	5.89		
70	-20	20	5	179.4	9.53	4.89	5.96		
71	-20	25	1	188.8	7.32	4.70	5.78		
72	-20	25	2	185.3	8.06	4.74	5.81		
73	-20	25	3	182.6	8.63	4.79	5.86		
74	-20	25	4	180.9	9.04	4.85	5.92		

				lable 3. Co	mt.		
	C	р	D				
Case (i)	S (°CA ATDC)	R (%)	D (°CA)	j = 1 SFC (g/kWh)	j = 2 NO <sub>x</sub> (g/kWh)	j = 3 CO (g/kWh)	j = 4 HC (g/kWh)
75	-20	25	5	180.0	9.27	4.93	6.00
76	-19	5	1	178.9	10.26	4.59	5.62
77	-19	5	2	177.0	10.71	4.61	5.64
78	-19	5	3	175.5	11.07	4.63	5.67
79	-19	5	4	174.6	11.32	4.67	5.70
80	-19	5	5	174.1	11.46	4.71	5.75
81	-19	10	1	180.9	9.55	4.62	5.66
82	-19	10	2	178.4	10.12	4.64	5.68
83	-19	10	3	176.5	10.55	4.68	5.72
84	-19	10	4	175.3	10.86	4.72	5.76
85	-19	10	5	174.7	11.03	4.78	5.82
86	-19	15	1	182.4	9.07	4.64	5.69
87	-19	15	2	179.5	9.71	4.67	5.72
88	-19	15	3	177.3	10.20	4.71	5.75
89	-19	15	4	175.8	10.54	4.76	5.81
90	-19	15	5	175.1	10.74	4.82	5.87
91	-19	20	1	183.5	8.69	4.66	5.71
92	-19	20	2	180.3	9.38	4.69	5.74
93	-19	20	3	177.9	9.92	4.74	5.78
94	-19	20	4	176.3	10.29	4.79	5.84
95	-19	20	5	175.5	10.51	4.86	5.92
96	-19	25	1	184.5	8.37	4.67	5.73
97	-19	25	2	181.0	9.11	4.71	5.76
98	-19	25	3	178.4	9.68	4.76	5.81
99	-19	25	4	176.6	10.08	4.82	5.87
100	-19	25	5	175.8	10.32	4.90	5.95
101	-18	5	1	178.2	10.59	4.57	5.60
102	-18	5	2	176.3	11.05	4.60	5.62
103	-18	5	3	174.8	11.40	4.62	5.65
104	-18	5	4	173.9	11.65	4.66	5.68
105	-18	5	5	173.4	11.79	4.70	5.73
106	-18	10	1	180.0	9.97	4.60	5.63
107	-18	10	2	177.5	10.53	4.63	5.66
108	-18	10	3	175.7	10.96	4.67	5.69
109	-18	10	4	174.4	11.27	4.71	5.74
110	-18	10	5	173.8	11.45	4.76	5.79
111	-18	15	1	181.3	9.53	4.62	5.66
112	-18	15	2	178.4	10.17	4.65	5.68
113	-18	15	3	176.2	10.66	4.70	5.72
114	-18	15	4	174.8	11.01	4.75	5.78
115	-18	15	5	174.1	11.21	4.81	5.84
116	-18	20	1	182.4	9.20	4.64	5.68
117	-18	20	2	179.1	9.89	4.67	5.71
118	-18	20	3	176.7	10.42	4.72	5.75
119 120	-18	20	4	175.1	10.80	4.78	5.81
120	-18	20	5	174.3	11.02	4.85	5.88 5.0
121	-18	25	1	183.3	8.91	4.65	5.69 5.72
122	-18	25	2	179.8	9.65	4.69	5.73
123	-18	25	3	177.1	10.22	4.74	5.77
124	-18	25	4	175.4	10.63	4.80	5.84
125	-18	25	5	174.5	10.86	4.88	5.92

Table 3. Cont.

Each indicator was transformed into its variation in per unit basis through Equation (1).

$$V_{ij} = \frac{X_{ij} - X_{jref}}{X_{jref}} \tag{1}$$

where  $X_{jref}$  is the value corresponding to criterion *j* in the case without pre-injection.

As indicated previously, an important step in a MCDM approach is to determine the criteria weights, and this issue will be treated in Sections 3.1–3.5. Once determined, the adequacy index for each *i*-th case,  $AI_i$ , was calculated through the SAW (simple additive weighting) method, Equation (2). Obviously, the most appropriate solution corresponds to the minimum value of AI.

$$AI_i = \sum_{j=1}^n w_j V_{ij} \tag{2}$$

where  $w_j$  is the weight of the *j*-th criterion and *n* the number of criteria, i.e., four (SFC, NO<sub>x</sub>, CO, and HC).

# 3.1. Subjective Weighting Method

In this method, the levels of importance were set by experts in the field. According to these experts, the same importance was given to consumption (50%) and emissions (50%). Regarding emissions, the importance was also distributed equally between NO<sub>x</sub> ( $33.3^\circ$ %), CO ( $33.3^\circ$ %), and HC ( $33.3^\circ$ %). According to this, the weights of SFC, NO<sub>x</sub>, CO, and HC result in w<sub>1</sub> = 0.5, w<sub>2</sub> = 0.16, w<sub>3</sub> = 0.16, and w<sub>4</sub> = 0.16, respectively.

## 3.2. Entropy Weighting Method

This method measures the uncertainty in the information, and the criteria weights are given by Equation (3).

$$w_{j} = \frac{1 - E_{j}}{\sum_{j=1}^{n} (1 - E_{j})}$$
(3)

In the equation above, 1—Ej represents the degree of diversity of the information related to the *j*-th criterion and  $E_j$  is the entropy value of the *j*-th criterion, given by Equation (4). In this equation,  $p_{ij}$  are the normalized data, Equation (5).

$$E_{j} = -\frac{\sum_{i=1}^{m} p_{ij} \ln(p_{ij})}{\ln(m)}$$
(4)

$$p_{ij} = \frac{V_{ij}}{\sum\limits_{i=1}^{m} V_{ij}}$$
(5)

According to the equations above, the range of the entropy value is 0–1. A low entropy value indicates that the degree of disorder corresponding to criterion *j* is low and thus leads to a high weight.

## 3.3. CRITIC Weighting Method

In this method, the criteria weights are obtained by

$$w_j = \frac{C_j}{\sum\limits_{j=1}^n C_j} \tag{6}$$

where  $C_j$ , Equation (7), represents a measure of the conflict created by criterion *j* with respect to the decision situation defined by the rest of the criteria. As the scores of the alternatives in criteria *i* and *j* become more discordant, the value of  $l_{ij}$  is lowered. The higher the value  $C_j$ , the larger the amount of information transmitted by the corresponding criterion and the higher the relative importance for

the decision-making process. The objective weights are derived by normalizing these values to unity, as indicated above through Equation (6).

$$C_j = \sigma_j \sum_{i=1}^m \left( 1 - l_{ij} \right) \tag{7}$$

where  $\sigma_j$  is the standard deviation of the *j*-th criterion and  $l_{ij}$  is the correlation coefficient, Equation (8). These correlation coefficients represent linear correlation coefficients between the criteria values in the matrix.

$$l_{ij} = \frac{\sum_{k=1}^{m} (V_{ki} - \overline{V}_i)(V_{kj} - \overline{V}_j)}{\sqrt{\sum_{k=1}^{m} (V_{ki} - \overline{V}_i)^2} \sqrt{\sum_{k=1}^{m} (V_{kj} - \overline{V}_j)^2}}$$
(8)

#### 3.4. Variance Weighting Method

The variance procedure method determines the criteria weights in terms of their statistical variances,  $\sigma_i^2$ , through the following equation:

$$w_j = \frac{\sigma_j^2}{\sum\limits_{j=1}^n \sigma_j^2} \tag{9}$$

## 3.5. Standard Deviation Weighting Method

The standard deviation method determines the criteria weights in terms of their standard deviations through the following equation:

$$w_j = \frac{\sigma_j}{\sum\limits_{j=1}^n \sigma_j} \tag{10}$$

To summarize, Table 4 summarizes the criteria weights obtained using these weighting methods. As can be seen, the entropy method assigns an important weight to SFC, while the CRITIC, variance, and standard deviation methods assign  $NO_x$  as the most relevant criterion. Both variance and standard deviation procedures measure the spread, i.e., the degree to which each sample is different from the mean. As can be seen in Table 3 shown above, CO and HC emissions remain practically constant and thus lead to low values of both variance and standard deviation. On the other hand,  $NO_x$  and, to a lesser extent, SFC present more spread and thus higher values of variance and standard deviation. For this reason, the variance and standard deviation methods provide a significant weight to NO<sub>x</sub> and low weights to CO and HC. Since the standard deviation is the square root of the variance, the variance method assigns a higher weight to  $NO_x$  than the standard deviation method. The CRITIC method assigns high values of weights to those criteria with high standard deviation and low correlation with other responses. According to this, the results obtained through the CRITIC method are very similar to those obtained through the standard deviation method, but with less differences between the criteria weights. The entropy method also takes into account the uncertainty in the information and thus assigns low weights to CO and HC. Besides uncertainty, the entropy is based on the degree of disorder and thus assigns an important weight to SFC.

Weighting Mathod	<b>Criteria Weights</b> , $w_j$								
Weighting Method	SFC ( <i>j</i> = 1)	$NO_x (j = 2)$	CO ( <i>j</i> = 3)	HC ( <i>j</i> = 4)					
Subjective	0.50	0.16	0.16	0.16					
Entropy	0.50	0.19	0.16	0.14					
CRITIC	0.27	0.41	0.16	0.16					
Variance	0.09	0.87	0.02	0.02					
Standard deviation	0.20	0.62	0.09	0.09					

**Table 4.** Criteria weights according to the subjective, entropy, CRITIC, variance, and standard deviation weighting methods.

Table 5 outlines the results of the 125 cases analyzed using these procedures. As can be seen, the subjective weighting method provides case 91, with an adequacy index of  $AI_{91} = -0.122$ , as the most appropriate injection pattern. This case corresponds to the  $-19^{\circ}$  CA ATDC pre-injection starting instant, 20% pre-injection rate, and 1° CA pre-injection duration. Since the subjective method assigns an important weight to  $NO_x$ , this 91st solution provides a significant  $NO_x$  reduction with a low increment of SFC, CO, and HC. This solution provides an important pre-injection rate, 20%, due to its importance on NO<sub>x</sub> reduction. Retarding the pre-injection instant also reduces NO<sub>x</sub> noticeably but at expenses of important increments on consumption. This reason leads to the CRITIC, variance, and standard deviation weighting methods to provide case 105, corresponding to the  $-18^{\circ}$  CA ATDC pre-injection starting instant, 5% pre-injection rate, and 5° CA pre-injection duration, as the most appropriate injection pattern, mainly due to the important weight of  $NO_x$  over the other criteria and lower weight of SFC in comparison with the subjective weighting method. A value of  $-18^{\circ}$  CA ATDC leads to important NO<sub>x</sub> reduction with a noticeable SFC penalty. Basically, the NO<sub>x</sub> reduction achieved with a high pre-injection rate or by a late pre-injection rate is reached through a reduction in the combustion temperature, since the high combustion temperatures reached in the combustion chamber are responsible for most  $NO_x$  emitted to the atmosphere [41,42]. On the other hand, the entropy method provides case 25 as the most appropriate injection pattern, with a  $-22^{\circ}$  CA ATDC pre-injection starting instant, 25% pre-injection rate, and 5° CA pre-injection duration. Since the entropy method assigns more weight to SFC and, to a lesser extent, to  $NO_x$ , it provides an earlier pre-injection starting instant, which leads to a reduction in SFC and a higher pre-injection rate, which leads to a reduction in NO<sub>x</sub>.

C	S	R	D			A	I <sub>i</sub>	
Case (i)	(°CA ATDC)	(%)	(°CA)	Subjective	Entropy	CRITIC	Variance	Standard Deviation
1	-22	5	1	-0.007	-0.020	-0.140	-0.376	-0.247
2	-22	5	2	-0.006	-0.017	-0.128	-0.347	-0.227
3	-22	5	3	-0.004	-0.015	-0.118	-0.325	-0.211
4	-22	5	4	-0.001	-0.012	-0.110	-0.309	-0.200
5	-22	5	5	0.002	-0.008	-0.103	-0.299	-0.192
6	-22	10	1	-0.004	-0.020	-0.170	-0.463	-0.302
7	-22	10	2	-0.003	-0.017	-0.155	-0.427	-0.278
8	-22	10	3	-0.001	-0.014	-0.143	-0.399	-0.259
9	-22	10	4	0.003	-0.011	-0.132	-0.380	-0.244
10	-22	10	5	0.007	-0.006	-0.124	-0.368	-0.235
11	-22	15	1	-0.002	-0.019	-0.190	-0.522	-0.340
12	-22	15	2	-0.001	-0.017	-0.173	-0.482	-0.313
13	-22	15	3	0.002	-0.014	-0.159	-0.451	-0.291
14	-22	15	4	0.006	-0.009	-0.148	-0.428	-0.274
15	-22	15	5	0.010	-0.005	-0.138	-0.415	-0.264
16	-22	20	1	0.000	-0.019	-0.206	-0.569	-0.369

**Table 5.** Adequacy index according to subjective, entropy, CRITIC, variance, and standard deviation weighting methods.

<b>Table 5.</b> Co
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<b>c</b> (1)	S	R	D			A	I <sub>i</sub>	
Case (i)	(°CA ATDC)	(%)	(°CA)	Subjective	Entropy	CRITIC	Variance	Standard Deviation
17	-22	20	2	0.002	-0.016	-0.188	-0.525	-0.340
18	-22	20	3	0.004	-0.013	-0.172	-0.491	-0.316
19	-22	20	4	0.008	-0.008	-0.159	-0.467	-0.298
20	-22	20	5	0.014	-0.003	-0.149	-0.452	-0.286
21	-22	25	1	0.002	-0.018	-0.219	-0.608	-0.394
22	-22	25	2	0.004	-0.016	-0.199	-0.561	-0.362
23	-22	25	3	0.006	-0.012	-0.183	-0.525	-0.337
23	-22	25	4	0.000	-0.007	-0.169	-0.499	-0.318
25	-22	25	5	0.011	-0.001	-0.158	-0.483	-0.305
26	-21	5	1	-0.010	-0.020	-0.112	-0.296	-0.195
27	-21	5	2	-0.008	-0.017	-0.100	-0.267	-0.176
28	-21	5	3	-0.006	-0.015	-0.090	-0.245	-0.160
29	-21	5	4	-0.004	-0.012	-0.082	-0.229	-0.148
30	-21	5	5	0.000	-0.008	-0.075	-0.219	-0.141
31	-21	10	1	-0.009	-0.021	-0.136	-0.364	-0.239
32	-21	10	2	-0.007	-0.018	-0.121	-0.329	-0.215
33	-21	10	3	-0.005	-0.015	-0.109	-0.301	-0.196
34	-21	10	4	-0.002	-0.012	-0.098	-0.281	-0.181
35	-21	10	5	0.002	-0.008	-0.090	-0.270	-0.172
36	-21	15	1	-0.008	-0.021	-0.152	-0.411	-0.269
37	-21	15	2	-0.006	-0.019	-0.136	-0.371	-0.242
38	-21	15	3	-0.004	-0.016	-0.121	-0.339	-0.220
39	-21	15	4	0.000	-0.011	-0.110	-0.317	-0.204
40	-21	15	5	0.005	-0.007	-0.101	-0.304	-0.193
41	-21	20	1	-0.007	-0.022	-0.165	-0.448	-0.293
42	-21	20	2	-0.005	-0.019	-0.147	-0.404	-0.263
43	-21	20	3	-0.002	-0.015	-0.131	-0.370	-0.239
44	-21	20	4	0.001	-0.011	-0.118	-0.346	-0.221
45	-21	20	5	0.007	-0.006	-0.109	-0.331	-0.210
46	-21	25	1	-0.006	-0.022	-0.175	-0.479	-0.312
47	-21	25	2	-0.004	-0.019	-0.156	-0.432	-0.280
48	-21	25	3	-0.001	-0.015	-0.139	-0.395	-0.255
49	-21	25	4	0.003	-0.011	-0.126	-0.369	-0.236
50	-21	25	5	0.008	-0.005	-0.115	-0.354	-0.224
51	-20	5	1	-0.011	-0.019	-0.091	-0.236	-0.156
52	-20	5	2	-0.010	-0.017	-0.079	-0.207	-0.137
53	-20	5	3	-0.008	-0.014	-0.068	-0.184	-0.121
54	-20	5	4	-0.005	-0.011	-0.060	-0.168	-0.109
55	-20	5	5	-0.002	-0.008	-0.054	-0.159	-0.102
56	-20	10	1	-0.011	-0.021	-0.110	-0.290	-0.191
57	-20	10	2	-0.010	-0.018	-0.095	-0.254	-0.167
58	-20	10	3	-0.007	-0.015	-0.083	-0.226	-0.148
59	-20	10	4	-0.004	-0.012	-0.072	-0.207	-0.134
60	-20	10	5	0.000	-0.008	-0.064	-0.195	-0.124
61	-20	15	1	-0.011	-0.022	-0.123	-0.327	-0.215
62	-20	15	2	-0.010	-0.019	-0.107	-0.287	-0.188
63	-20	15	3	-0.007	-0.016	-0.092	-0.256	-0.166
64	-20	15	4	-0.003	-0.012	-0.081	-0.233	-0.150
65	-20	15	5	0.001	-0.007	-0.072	-0.220	-0.139
66	-20	20	1	-0.011	-0.023	-0.134	-0.356	-0.234
67 68	-20	20	2	-0.009	-0.020	-0.115	-0.312	-0.205
68 69	-20	20	3	-0.007	-0.016	-0.100	-0.278	-0.181
69 70	-20 -20	20	4	-0.003 0.003	-0.012 -0.007	-0.087 -0.077	-0.254	-0.163 -0.151
70 71	$-20 \\ -20$	20 25	5 1	-0.003	-0.007 -0.023	-0.077 -0.142	-0.240 -0.381	-0.151 -0.250
71 72	$-20 \\ -20$	25 25	1	-0.010 -0.009	-0.023 -0.021	-0.142 -0.123	-0.381 -0.334	-0.250 -0.218
72 73	$-20 \\ -20$	25 25		-0.009 -0.006	-0.021 -0.017	-0.123 -0.106		-0.218
73 74	$-20 \\ -20$	25 25	3	-0.006 -0.002	-0.017 -0.012	-0.106 -0.093	-0.298 -0.272	-0.193 -0.174
74 75	$-20 \\ -20$	25 25	4	-0.002 0.004	-0.012 -0.006	-0.093 -0.082	-0.272 -0.256	-0.174 -0.161
75 76	$-20 \\ -19$	25 5	5 1	-0.004	-0.006 -0.018	-0.082 -0.076		-0.161 -0.130
76 77	-19 -19	5 5	1	-0.011 -0.010	-0.018 -0.015	-0.076 -0.064	-0.195 -0.166	-0.130

				Table	<b>5.</b> Com.			
Casa (i)	S	R	D			A	l <sub>i</sub>	
Case (i)	(°CA ATDC)	(%)	(°CA)	Subjective	Entropy	CRITIC	Variance	Standard Deviation
78	-19	5	3	-0.008	-0.013	-0.054	-0.143	-0.094
79	-19	5	4	-0.005	-0.010	-0.045	-0.127	-0.083
80	-19	5	5	-0.002	-0.007	-0.039	-0.118	-0.075
81	-19	10	1	-0.012	-0.020	-0.092	-0.239	-0.159
82	-19	10	2	-0.010	-0.017	-0.077	-0.204	-0.135
83	-19	10	3	-0.008	-0.014	-0.065	-0.176	-0.115
84	-19	10	4	-0.005	-0.011	-0.054	-0.156	-0.101
85	-19	10	5	-0.001	-0.006	-0.046	-0.145	-0.091
86	-19	15	1	-0.012	-0.021	-0.103	-0.270	-0.179
87	-19	15	2	-0.011	-0.019	-0.086	-0.230	-0.152
88	-19	15	3	-0.008	-0.015	-0.072	-0.199	-0.130
89	-19	15	4	-0.004	-0.011	-0.061	-0.176	-0.114
90	-19	15	5	0.000	-0.006	-0.052	-0.163	-0.103
91	-19	20	1	-0.012	-0.022	-0.112	-0.294	-0.194
92	-19	20	2	-0.011	-0.019	-0.094	-0.251	-0.165
93	-19	20	3	-0.008	-0.016	-0.078	-0.216	-0.141
94	-19	20	4	-0.004	-0.011	-0.065	-0.192	-0.123
95	-19	20	5	0.001	-0.006	-0.056	-0.178	-0.112
96	-19	25	1	-0.012	-0.023	-0.119	-0.315	-0.207
97	-19	25	2	-0.011	-0.020	-0.100	-0.268	-0.176
98	-19	25	3	-0.008	-0.016	-0.083	-0.231	-0.151
99	-19	25	4	-0.004	-0.012	-0.069	-0.205	-0.132
100	-19	25	5	0.002	-0.006	-0.059	-0.190	-0.119
100	-18	5	1	-0.010	-0.016	-0.068	-0.173	-0.116
101	-18	5	2	-0.009	-0.010	-0.056	-0.144	-0.096
102	-18	5	3	-0.007	-0.014	-0.045	-0.122	-0.080
103	-18	5	4	-0.004	-0.008	-0.037	-0.122	-0.068
104	-18	5	5	-0.004	-0.005	-0.031	-0.096	-0.061
105	-18	10	1	-0.011	-0.018	-0.082	-0.213	-0.141
100	-18	10	2	-0.009	-0.015	-0.067	-0.177	-0.117
107	-18	10	3	-0.007	-0.012	-0.055	-0.150	-0.098
100	-18	10	4	-0.004	-0.009	-0.044	-0.130	-0.084
110	-18	10	5	0.004	-0.004	-0.036	-0.118	-0.074
110	-18	15	1	-0.011	-0.019	-0.092	-0.241	-0.159
111	-18	15	2	-0.009	-0.015	-0.072	-0.200	-0.132
112	-18	15	3	-0.009	-0.010 -0.013	-0.073	-0.200 -0.169	-0.132
113	-18	15	4	-0.003	-0.009	-0.049	-0.147	-0.094
114	-18	15	5	0.002	-0.004	-0.049	-0.134	-0.083
115	-18	20	1	-0.011	-0.020	-0.100	-0.262	-0.173
117	-18	20	2	-0.009	-0.017	-0.081	-0.218	-0.143
117	-18	20	3	-0.009	-0.017 -0.013	-0.061	-0.213 -0.184	-0.143
118	-18	20 20	4	-0.007 -0.003	-0.013 -0.009	-0.000	-0.164 -0.160	-0.102
119	-18 -18	20 20	5	0.003	-0.009 -0.004	-0.033 -0.043	-0.100 -0.146	-0.102
120	-18 -18	20 25	1	-0.011	-0.004 -0.020	-0.043 -0.106	-0.146 -0.280	-0.185
121	-18	25 25	2	-0.011 -0.009	-0.020 -0.017	-0.100 -0.087	-0.230 -0.233	-0.153
122	-18	25 25	2	-0.009 -0.007	-0.017 -0.014	-0.087 -0.070	-0.233 -0.197	-0.133
123	-18 -18	25 25	3 4	-0.007 -0.002	-0.014 -0.009	-0.070 -0.057	-0.197 -0.171	-0.128
124 125	-18 -18	25 25	4 5	0.002	-0.009 -0.003	-0.057 -0.046	-0.171 -0.156	-0.109 -0.096
123	-10	23	5	0.003	-0.003	-0.040	-0.150	-0.090

Table 5. Cont.

## 4. Conclusions

The main goal of the present paper is to characterize the most appropriate pre-injection pattern in a marine diesel engine, the Wärtsilä 6L 46, in order to optimize the pilot injection process. A CFD model previously validated with experimental results was employed to obtain data corresponding to a set of 125 injection patterns using pilot injection. The pre-injection rate, duration, and starting instant were varied in the ranges of 5% to 25%, 1° to 5° CA, and -22° to -18° CA ATDC, respectively. Since the manipulation of these parameters has conflicting results on consumption and emissions of NO<sub>x</sub>, CO, and HC, a MCDM approach was employed to select the most appropriate injection pattern. Due to the importance of criteria weights on the overall result, several criteria weighting methods were compared. In particular, a subjective weighting method was compared with four objective weighting methods: entropy, CRITIC, variance, and standard deviation. The CRITIC, variance, and standard deviation methods led to the same injection pattern:  $-19^{\circ}$  CA pre-injection starting angle, 20% pre-injection rate, and 5° CA pre-injection duration. Nevertheless, the entropy method provided a  $-22^{\circ}$  CA pre-injection starting angle, 25% pre-injection rate, and 5° CA pre-injection duration as the most appropriate injection pattern, and the subjective method determined this as a  $-19^{\circ}$  CA pre-injection starting angle, 20% pre-injection rate, and 1° CA pre-injection duration. The main contribution of the present work consists in emphasizing the differences between the results obtained using various methods for the determination of the criteria weights, showing the advantage of subjectivism over objectivism. Based on the overall results, the subjective method is recommended since the criteria weights are defined by experts in the field. In fact, in practical applications, subjective methods are more frequently employed than objective ones. Objective methods are only recommended when the objectivity of the research is too important or when there is no agreement between the weights proposed by the experts.

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## Nomenclature

- AI Adequacy index
- *C* Measure of the conflict
- E Entropy value
- *i* Case
- j Criterion
- *m* Number of cases analyzed
- *n* Number of criteria
- *p* Normalized data
- $\sigma$  Standard deviation
- $\sigma^2$  Statistical variance
- *V* Value in per unit basis
- *w* Criterion weight
- X Value

## Abbreviations

- ATDC After top dead center
- CRITIC Criteria importance through intercriteria correlation
- CA Crank angle
- CFD Computational fluid dynamics
- CO Carbon monoxide
- CO<sub>2</sub> Carbon dioxide
- D Pre-injection duration
- HC Hydrocarbons
- IMO International Maritime Organization

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