Dynamic modelling and simulation of a prototype race car in MATLAB/Simulink applying different types of electric motors

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ABSTRACT

Nowadays, vehicles with alternative drives are playing an increasingly important role in road transport. Among the various types of alternative-drives, the most widespread ones are hybrid and electric ones, thanks for the rapid development of modern batteries, and hybrid and electrical systems. The above fact establishes the importance of research on various types of electric motors. The Faculty of Engineering of the University of Debrecen has more than a decade of experience in developing prototype race cars with pneumatic and electric drive. For the more effective and conscious development and racing a simulation program has been developed in MATLAB/Simulink environment for the computation of the dynamic functions of a vehicle moving in a linear path. The program is characterized by a modular structure, so the different structural units (vehicle body, front and rear wheels, power train, motor) are modelled and simulated separately. In this study we present models and simulation programs for different electric motor types (series-wound DC and brushless DC motors) in MATLAB/Simulink and apply them in our vehicle dynamics simulation program. From the simulation results the performance of a vehicle – driven by an electric motor – can be predicted in a race situation, consequently the most suitable motor with the optimal characteristics can be selected to it.

1. INTRODUCTION

Nowadays, vehicles with alternative drives are playing an increasingly important role in road transport. The reasons for this are, firstly, regulations and laws on climate change, and secondly, the rapid development of technology. Among the various types of alternative-drives, the most widespread ones are hybrid and electric drives, thanks to the rapid development of modern batteries, and hybrid and electrical systems. The range of electric vehicles that are commercially available today or will be available in the near future can be more than 500 km in many cases, while charging time can be reduced to 10–15 min by using a suitable charger. Thus, the above vehicles are becoming more usable, which is continuously increasing sales while reducing the costs of production. As a result, one of the different electric motor types will soon be found in the drive system of almost every vehicle, which establishes the importance of research on various electric motor types. As a response to the above development trends the Ministry for Innovation and Technology in Hungary has started the Automotive Industry Thematic Program at the University of Debrecen in 2019. In the framework of this program our research group has started to develop simulation programs for electric drives applied in modern road vehicles.

On the other hand, the Faculty of Engineering of the University of Debrecen has more than a decade of experience in developing prototype race cars with electric and pneumatic drives. With these cars, student teams of our Faculty take part in different domestic and

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foreign competitions (MVM Energy Race, Shell ECO Marathon, Pneumobile and Formula Student competitions) announced for university students. At the different competitions the students face several different tasks regulated by strict rules. The race cars have to fulfil the requirements, and additionally, have to be optimally designed for the given competition task. Thus, the designers have to apply different technical solutions and choose the different structural units of the cars (e.g. the electric motor) and their technical characteristics properly. For the more effective and conscious development and racing a simulation program has been realized in MATLAB/Simulink [1] previously for the calculation of the dynamic functions of a vehicle moving on a linear track. The program is characterized by a modular structure thus the different structural units (vehicle body, motor, power train, front and rear wheels) are modelled and simulated separately. Applying the program and supplementing it with an optimization procedure, the optimal values of technical data (e.g. optimal gear ratio in the drive train) can be computed for a given vehicle dynamics aim (racing task). This aim can be e.g. to complete the race in the shortest possible time or with the lowest energy consumption driving above a given minimum speed.

In the recent study we present models and simulation programs for different types of electric motors (series-wound DC (SWDC) and brushless DC (BLDC) motors) and apply them in our vehicle dynamics simulation program. From the simulation results the performance of a car – driven by different types of electric motors – can be predicted in a race situation. Complementing our simulation program with an optimization procedure the most suitable motor with the most advantageous parameters and characteristics can be selected for a given racing task.

2. THE VEHICLE DYNAMICS MODEL AND SIMULATION PROGRAM

Applying the developed dynamics model, and the simulation program based on it [1], the dynamic functions of a vehicle,

moving on a linear path, can be calculated from its technical characteristics and data. Complementing the program with an optimization procedure, the optimal values of technical data can be computed for a given driving dynamics aim [2].

The program takes into consideration almost all the factors that influence the motion of the vehicle. Including the electromagnetic and dynamic characteristics of the motor, air and rolling resistance, moment of inertia of the rotating machine parts, position of the centre of gravity of the vehicle, and the coefficient of friction between the road and the tyres as a function of tyre slip [1].

Beyond the usual driving dynamics functions (acceleration, velocity and position-time functions), the program is capable of calculating the forces on the front and rear wheels and loads on the front and rear axles versus time in tangential and normal direction. Other calculated functions are: rolling and bearing resistance torques, air resistance force, tyre slip, angular speed and torque of the motor versus time.

For modular development and greater clarity, the main vehicle components are organized into separate blocks, which are:

- vehicle body
- motor
- powertrain
- front wheels
- rear wheels

The block diagram of the simulation program is shown in Fig. 1.

The detailed description of the simulation program, together with the input technical characteristics and data that are necessary for running it, can be found in reference [1].

3. MODELING AND SIMULATION OF THE ELECTRIC MOTOR

A main part of the vehicle dynamics simulation program is the "motor simulation block" which is responsible for the
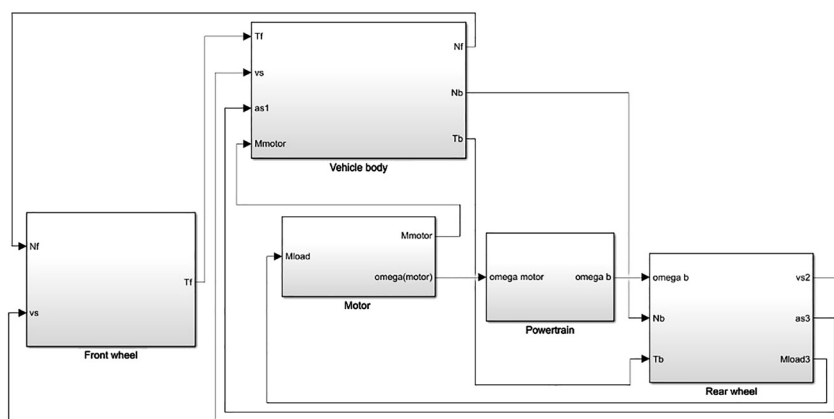

Fig. 1. Block diagram of the vehicle dynamic simulation program [1]

simulation of the electric motor. There are different types of motors (Brushed DC motor (BDC), Brushless DC motor (BLDC), Induction (Asynchronous) motor (IM), Synchronous motor (SM), Switched Reluctance motor (SRM)) that are commercially available for driving electric vehicles. Regarding the student competitions, for the wide range of competition tasks, different types of electric motors with different technical characteristics and parameters can be optimal. Thus, for an optimal choice, the vehicle dynamics simulation program must be run with all the possible different motor types. As a result of this, a decision was made by the authors to model and simulate two of the frequently used electric motor types first, and then the remaining ones in the near future.

3.1. Modeling and simulation of series-wound DC motors

In Fig. 2 the applied model for series-wound DC motors is presented.

The model [3] is based on the following differential equations [4, 5]:

$$U_{batt} - (R_s + R_r + R_{batt} + R_{wire}) \cdot I - (L_s(I) + L_r(I)) \cdot \frac{dI}{dt} - L_{sr}(I) \cdot \omega_r \cdot I = 0. \quad (1)$$

$$M_e(I) - M_{res}(\omega_r) - M_{load}(\omega_r) = J_r \cdot \frac{d\omega_r}{dt} \quad (2)$$

where the electromagnetic torque of the motor can be calculated as:

$$M_e(I) = L_{sr}(I) \cdot I^2 \quad (3)$$

In the equations U_{batt} [V] is the electromotive force of the battery, R_s, R_r [Ω] are the electric resistances of the stator and the rotor windings of the motor, while R_{batt} and R_{wire} [Ω] are the internal resistance of the accumulator and the resultant resistance of the wires connecting the motor to the battery. Characteristics $L_r(I)$, $L_s(I)$ and $L_{sr}(I)$ [H] are the self-dynamic inductances of the rotor and stator windings, and the mutual dynamic inductance, respectively. The above characteristics depend on the intensity of the current flowing through the motor. Characteristic $M_{res}(\omega_r)$ [Nm] is the bearing resistance and brush friction torques together, while $M_{load}(\omega_r)$ [Nm] is the loading torque on the rotor of the motor and J_r [kgm^2] is the rotor's moment of inertia. Characteristics $L_r(I)$, $L_s(I)$, $L_{sr}(I)$ and $M_{res}(\omega_r)$ have to be measured [3], while the value of J_r sometimes can be found in the catalogue of the motor. $M_{res}(\omega_r)$ is also frequently approximated as:

$$M_{res}(\omega_r) = B \cdot \omega_r \quad (4)$$

where ω_r [rad/s] is the angular velocity of the rotor and B [Nms] is a constant. The detailed description of the simulation program based on this model can be found in reference [3], together with the detailed description of the measurement set-up and procedures for the determination of the technical characteristics of the motor, that are necessary as input data for the simulation. In reference [3]
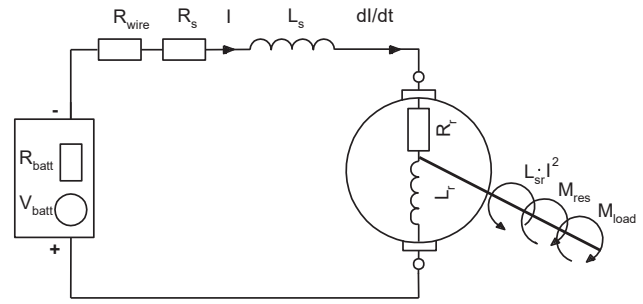

Fig. 2. The applied model for the simulation of series-wound DC motors [3]

all the necessary input data are given for a 4 kW SWDC motor. We use these data when simulating the SWDC motor in the prototype race car in Section 4.

3.2. Modeling and simulation of brushless DC motors

In Fig. 3 the applied model for brushless DC (BLDC) motor with star-connected strator windings is presented.

The model is based on the following differential equations [6, 7]:

$$U_a = R_a \cdot I_a \cdot L_a \cdot \frac{dI_a}{dt} + L_{ab} \cdot \frac{dI_b}{dt} \cdot L_{ac} \frac{dI_c}{dt} + e_a \quad (5)$$

$$U_b = R_b \cdot I_b + L_{ba} \cdot \frac{dI_a}{dt} + L_b \cdot \frac{dI_b}{dt} + L_{bc} \cdot \frac{dI_c}{dt} + e_b \quad (6)$$

$$U_c = R_c \cdot I_c + L_{ca} \cdot \frac{dI_a}{dt} + L_{cb} \cdot \frac{dI_b}{dt} + L_c \cdot \frac{dI_c}{dt} + e_c \quad (7)$$

where U_a, U_b and U_c [V] are output voltages from the inverter, R_a, R_b, R_c, [Ω]; L_a, L_b and L_c [H] are the electrical resistances and the self-inductances of the stator windings, L_{ab}, L_{ac}, L_{ba}, L_{bc}, L_{ca} and L_{cb} [H] are the mutual inductances between the stator winding pairs, e_a, e_b and e_c [V] are the
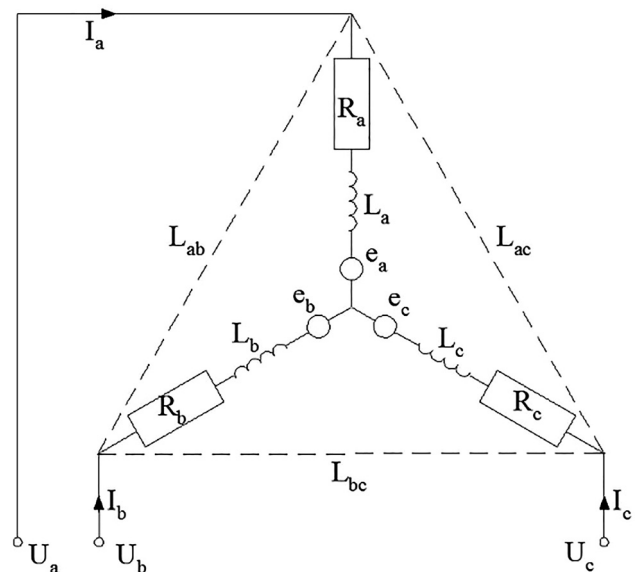

Fig. 3. The applied model for the simulation of BLDC motors

Back EMF values generated in the stator windings by the rotating permanent magnet rotor. The output voltages from the inverter are given as:

$$U_a = U(\theta_e) = \begin{cases} -1, & \text{if } \dfrac{4\pi}{3} \le \theta_e < 2\pi \\ 0, & \text{if } 0 \le \theta_e < \dfrac{\pi}{3} \text{ or } \pi \le \theta_e < \dfrac{4\pi}{3} \\ 1, & \text{if } \dfrac{\pi}{3} \le \theta_e < \pi \end{cases} \quad (8)$$

$$U_b = U\left(\theta_e - \frac{2\pi}{3}\right) \quad (9)$$

$$U_c = U\left(\theta_e - \frac{4\pi}{3}\right) \quad (10)$$

In the above equations the electrical angle can be calculated as $\theta_e = \frac{p}{2} \cdot \theta_r$ [rad], where θ_r [rad] is the angular position of the rotor, and $\frac{p}{2}$ is the number of permanent magnet pole-pairs in the rotor.

The arrangement of windings is symmetrical and the self and mutual inductances can be assumed as constants, thus:

$$R = R_a = R_b = R_c \quad (11)$$

$$L = L_a = L_b = L_c \quad (12)$$

$$M = L_{ab} = L_{ba} = L_{ac} = L_{ca} = L_{bc} = L_{cb} \quad (13)$$

Consequently:

$$U_a = R_a \cdot I_a + L \cdot \frac{dI_a}{dt} + M \cdot \frac{dI_b}{dt} + M \cdot \frac{dI_c}{dt} + e_a. \quad (14)$$

$$U_b = R_b \cdot I_b + M \cdot \frac{dI_a}{dt} + L \cdot \frac{dI_b}{dt} + M \cdot \frac{dI_c}{dt} + e_b. \quad (15)$$

$$U_c = R_c \cdot I_c + M \cdot \frac{dI_a}{dt} + M \cdot \frac{dI_b}{dt} + L \cdot \frac{dI_c}{dt} + e_c. \quad (16)$$

For the Y connected stator windings applying Kirchhoff's nodal rule:

$$I_a + I_b + I_c = 0 \Rightarrow \frac{dI_a}{dt} + \frac{dI_b}{dt} + \frac{dI_c}{dt} = 0 \Rightarrow \text{e.g.} \\ : \quad \frac{dI_b}{dt} + \frac{dI_c}{dt} = -\frac{dI_a}{dt} \quad (17)$$

Thus:

$$U_a = R \cdot I_a + (L - M) \cdot \frac{dI_a}{dt} + e_a \quad (18)$$

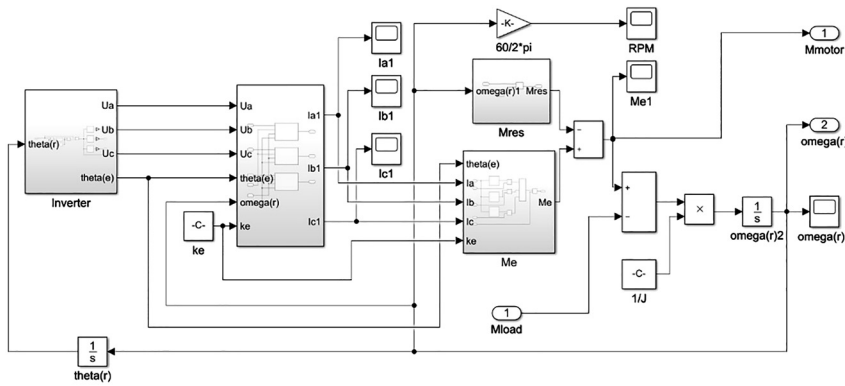$$U_b = R \cdot I_b + (L - M) \cdot \frac{dI_b}{dt} + e_b \quad (19)$$

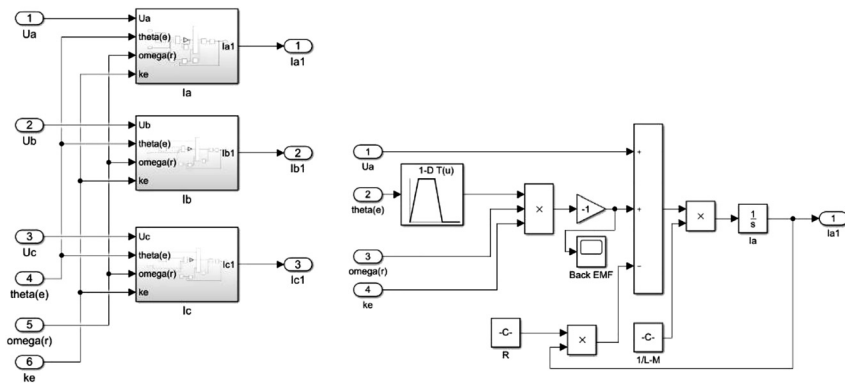*Fig. 4.* The block diagram of the simulation program for BLDC motors

Fig. 5. The internal structure of the program block for the calculation of the phase current intensities
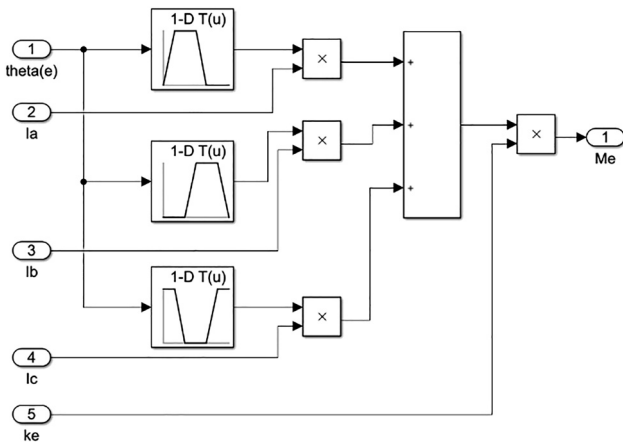
Fig. 6. The internal structure of the program block for the calculation of the electromagnetic torque of the motor
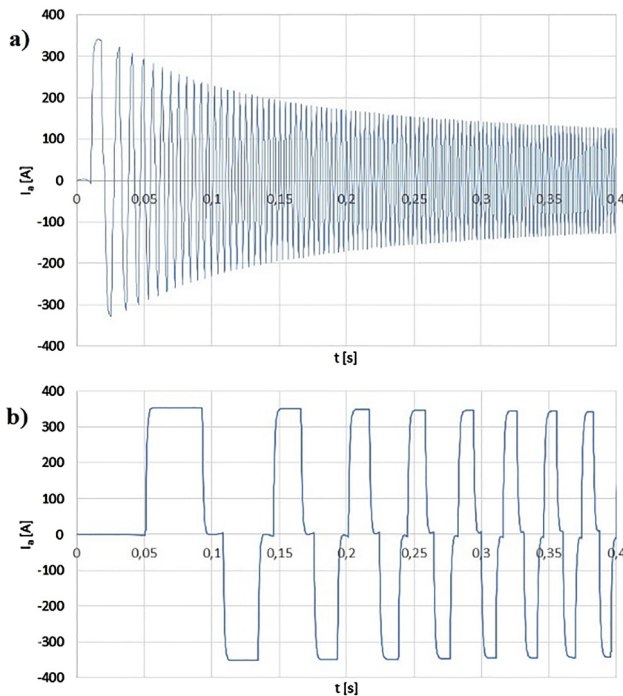

Fig. 7. The intensity of current vs. time in phase *a* without (a) and with (b) loading moment of inertia on the rotor
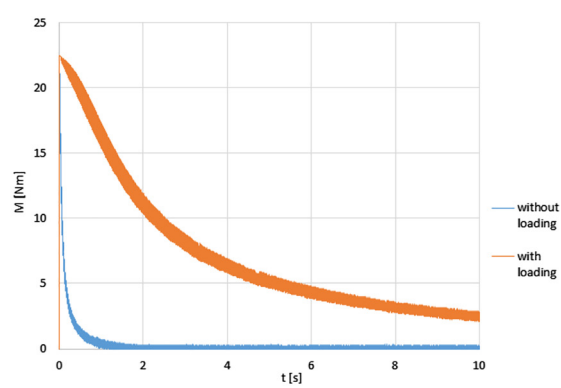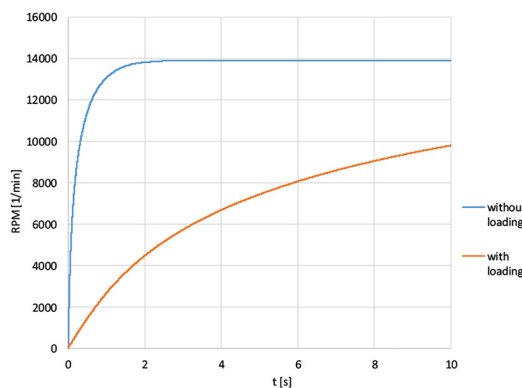
$$U_c = R \cdot I_c + (L - M) \cdot \frac{dI_c}{dt} + e_c \qquad (20)$$

The Back EMF values can be calculated as:

$$e_a = -k_e \cdot \omega_r \cdot F(\theta_e) \qquad (21)$$

$$e_b = -k_e \cdot \omega_r \cdot F\left(\theta_e - \frac{2\pi}{3}\right) \qquad (22)$$

$$e_c = -k_e \cdot \omega_r \cdot F\left(\theta_e - \frac{4\pi}{3}\right) \qquad (23)$$

In the above equations, ω_r [rad/s] is the angular velocity of the rotor, k_e [V/(rad/s)] is the Back EMF constant and $F(\theta_e)$ describes the trapezoidal waveform of Back EMF:

$$F(\theta_e) = \begin{cases} \frac{6}{\pi} \cdot \theta_e - 1, & \text{if } 0 \le \theta_e < \frac{\pi}{3} \\ 1, & \text{if } \frac{\pi}{3} \le \theta_e < \pi \\ -\frac{6}{\pi} \cdot \theta_e + 7, & \text{if } \pi \le \theta_e < \frac{4\pi}{3} \\ -1, & \text{if } \frac{4\pi}{3} \le \theta_e < 2\pi \end{cases} \qquad (24)$$

The equation of motion for the rotor is the same as in Section 3.1 (Eq. 2).

The electromagnetic torque here can be calculated as:

$$M_e = -k_e \cdot \left(F(\theta_e) \cdot I_a + F\left(\theta_e - \frac{2\pi}{3}\right) \cdot I_b + F\left(\theta_e - \frac{4\pi}{3}\right) \cdot I_c \right) \qquad (25)$$

The block diagrams of the simulation program, based on Eqs. (5–25), are shown in Figs. 4–6.

The specifications of the simulated BLDC motor can be found in reference [8]. We also use these data when simulating the BLDC motor in the prototype race car in Section 4. The used input data for the simulation are: rated power: 1.4 kW; rated DC voltage: 100 V, phase resistance: 0.28 Ω, phase inductance: 0.24 mH, number of pole-pairs: 2; line-to-line back EMF constant: 0.0033 V/(r/min); moment of inertia of the rotor: 0.002139 kgm^2, constant B: 0.001 Nms, loading moment of inertia: 0.0688 kgm^2 (The loading moment of inertia can be modelled as a homogenous circular

Fig. 8. RPM and torque of the motor vs. time (M=M$_e$-M$_{res}$).
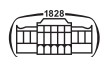
disk applied on the rotor, the value of M_{load} is selected to be zero.) The results of the simulation are shown in Figs. 7–9.

4. SIMULATION OF THE PROTOTYPE RACE CAR

The motion of a prototype race car, accelerating from rest on a linear track, was simulated. The car was designed and constructed at the Faculty of Engineering of the University of Debrecen. The detailed technical description of the car
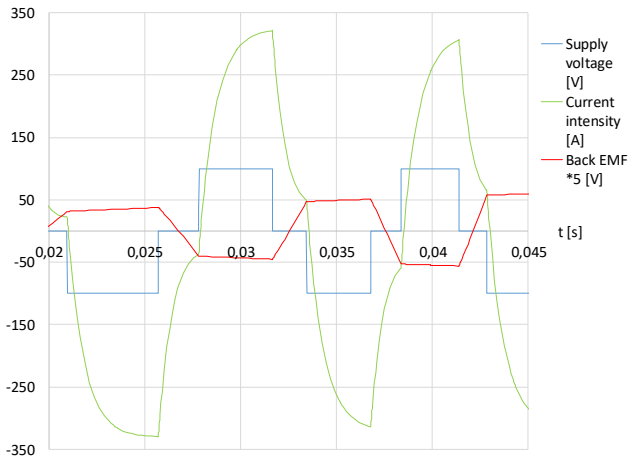

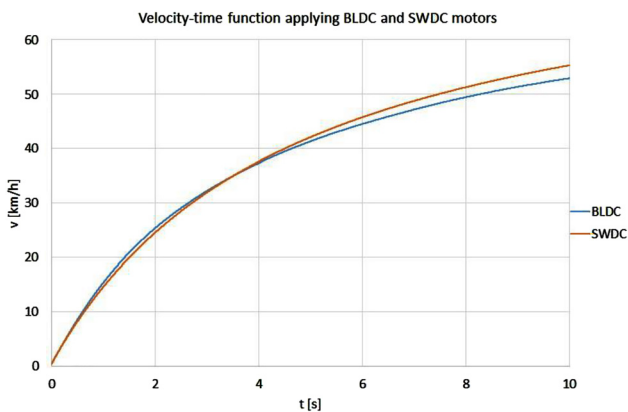*Fig. 9.* Supply voltage, current intensity and Back EMF for phase *a* vs. time

Fig. 10. Velocity-time functions of the prototype race car applying two different types of electric motor
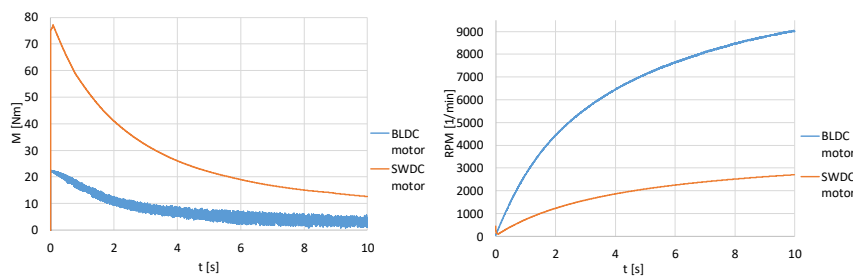
can be found in Reference [1]. The vehicle dynamics simulation program [1] was applied for the simulation both with the SWDC and BLDC motors driving the car. All the vehicle's technical data were the same in both cases (see the list of data in Reference [1]) except for the gear ratio in the chain drive, which was 4 when applying the SWDC and 14 when the BLDC motor. At the above gear ratios, the time the car needed to travel 100 m starting from rest, has a minimum. Fig. 10 shows the obtained velocity-time functions of the vehicle applying the two different motors.

Although the nominal power of the SWDC motor is 4 kW, while the one of the BLDC motor is 1.4 kW, the time the car needed for travelling 100 m, starting from rest, is only slightly different in the two cases (9.64 and 9.76 s, respectively). Nevertheless, it must be emphasized that the BLDC motor is significantly overloaded during the acceleration. Fig. 11 shows the torque and RPM of the two motors vs. time.

5. CONCLUSION

The motion of a prototype race car, accelerating from rest on a linear track, was simulated applying our simulation program that was previously developed in MATLAB/Simulink environment. The program is capable of calculating the dynamics functions of a vehicle, moving on a linear track, from the vehicle's technical data. Complementing the program with an optimization procedure, the optimal values of technical data can be computed for a given driving dynamics aim. Simulations were performed both with SWDC and BLDC motors driving the car. The model and simulation program for the SWDC motor were developed previously, the detailed modelling and simulation procedure of the BLDC motor is presented here. The obtained results show that the vehicle dynamics simulation program, and also the simulation programs for the electric motors work properly. The accuracy of the simulation program for the SWDC motor was checked by test measurements previously. The measured and simulated results are in good agreement. In the near future we also intend to perform test measurements on the BLDC motor and compare the results with the simulated ones. To be able to do that, first of all, we have to measure its electromagnetic and dynamic characteristics. In the long term we also intend to perform telemetry measurements on the prototype race car to check the accuracy of our vehicle dynamics simulation program.

Fig. 11. Torque and RPM of the SWDC and BLDC motors vs. time while the car is accelerating from rest ($M = M_e - M_{res}$)

ACKNOWLEDGEMENT

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