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Biomorphodynamics of river banks in vegetated channels with self-formed width

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Abstract

Laboratory and field studies investigating the mutual interaction between riparian vegetation dynamics and river morphodynamics have revealed that riparian vegetation may play an important role in the evolution of channel beds and river banks. In order to disentangle this still debated question, field and modeling techniques have helped to explore and better understand the time and spatial scales of such processes. Simple morphodynamic models for river evolution have typically used a constant discharge to describe in-channel processes and basic relationships for river bank dynamics. In order to overcome these limits we propose a longitudinally integrated dynamical model that describes the bank pull - bar push mechanisms in channels with symmetric cross section. Different hydrographs (constant, periodic and stochastic discharge) are applied to investigate channel width and vegetation biomass evolution trajectories and equilibrium values. Results show the interplay of riparian vegetation and water flow in controlling channel width evolution and the trajectories of channel adjustment to flow perturbations. These results also highlight the limit of adopting a constant discharge when describing mutual flow and vegetation processes affecting channel evolution.

In addition, under stochastic forcing, the model shows the existence of a range of flood frequencies for which the cooperation between the hydrologic time scales and that characterizing vegetation colonization induces a regular pattern in channel width time variations (coherence resonance). Finally, model application to real case studies confirm the possibility to use the model to interpret long-term river evolutionary trajectories in realistic applications. *Keywords:* bio-morphodynamic model, dynamical system, bank deposition, vegetation colonization, channel width temporal adjustments, stochastic water discharge

1. Introduction

The mutual interaction between riparian vegetation and river processes has been found to control the morphological evolution of lateral infinitely erodible channels (Gurnell, 2014). Plants growing on the river banks modify sediment properties driving the width toward which the river tends to adjust (Micheli et al., 2004; Allmendinger et al., 2005). Channel width is the result of the balance between erosional and depositional processes controlling the migration rate of the river banks.

Whether a channel undergoes narrowing, widening, or maintaining a balance between the advancing/retreating rate of the banks, thus retaining an almost constant channel width, depends on how water flow interacts with the sediment budget, river morphology, and the properties of the floodplain, i.e. vegetation coverage and sediments type. The rate at which river banks retreat is associated with fluvial erosion processes and bank failure mechanisms (Darby et al., 2007; Rinaldi et al., 2008).

16 In vegetated floodplains, plants exert a positive feedback through their
17 roots on the stability of river bank reducing bank failure frequency (Micheli
18 and Kirchner, 2002; Gasser et al., 2019). The strengthening effect of plant
19 roots and the presence of fine sediments provide cohesion to the sediment
20 of the bank, generating a cohesive layer on the top of a non-cohesive one
21 (Pizzuto, 1984), which may affect bank erosion (Thorne, 1990).

22 Laboratory experiments exploring the interaction between flow, sediments,
23 and vegetation have confirmed field observations that vegetation can control
24 bank erosion, channel width, and river planform evolution (Tal and Paola,
25 2010; Van Dijk et al., 2013), towards single thread channels (Braudrick et al.,
26 2009). Whilst many field and laboratory studies indicate a positive effect of
27 vegetation on bank stability, other field observations have revealed that in
28 forested margin, trees-toppling associated to fluvial erosion further increases
29 the bank retreat rate (Pizzuto et al., 2010).

30 The above ground biomass also plays a fundamental role in the dynamics
31 of river banks. During floods, riparian plants interact with overbank flow
32 inducing deposition of fine sediments and aggrading the bar surface, which
33 generates benches that contribute to the accretion of the bank (Erskine et al.,
34 2009). This favours the progressive shifting of the bank margin and induces
35 cross section narrowing (Friedman et al., 1996; Thorne, 1998; Erskine et al.,
36 2012) with the channel possibly adjusting toward a new morphological equi-
37 librium (Eekhout et al., 2014).

38 Field observations on active branches of meandering rivers confirmed this
39 be a fundamental mechanism in the evolution of natural meander bends
40 (Nanson, 1981; Gurnell and Petts, 2006; Zen et al., 2017). Vegetation en-

41 croachment growing on bar deposits or dead wood stranded on the point bar,
42 retain sediments, moisture, and nutrients, allowing the creation of a suitable
43 environment for seeds to sprout and grow (Gurnell et al., 2001). Once es-
44 tablished, plants consolidate bar sediment through their root systems, thus
45 increasing the resistance of the bar to erosion by flow and generating addi-
46 tional protection for the bank. The reduced channel width then increases the
47 erosion at the cutting bank during the subsequent flood pushing it away (i.e.
48 bar push). The newly-generated wider cross section will allow new sediments
49 to be exposed, and colonized, leading the inner river bank to advance again
50 (i.e. bank pull).

51 Modeling research has explored the above biomorphodynamic processes to
52 provide insights into their physical controls. Because bank erosion represents
53 a relevant problem for river management this has long received considerable
54 research attention. Numerical models of river bank flow-related dynamics
55 have been proposed to understand how vegetation biomass reinforces river
56 banks (Pollen and Simon, 2005; Langendoen et al., 2009) or how its presence
57 can promote bank failure (Wiel and Darby, 2007). Simple models have in-
58 cluded the overall influence of vegetation on bank stability as an increased
59 friction angle or bank strength to explore river planform morphology (Millar,
60 2000; Eaton et al., 2010). Other modeling effort have included the presence
61 of vegetation on physically-based morphodynamic models for river evolution
62 to investigate how its presence influence river planform (Murray and Paola,
63 2003; Crosato and Saleh, 2011). Finally, more complex bio-morphodynamic
64 models have coupled riparian vegetation dynamics with river morphodynam-
65 ics to explore the evolution of river bars (Bertoldi et al., 2014; Caponi and

66 Siviglia, 2018) and meandering bends (Perucca et al., 2007; Oorschot et al.,
67 2016; Zen et al., 2016).

68 Although riparian vegetation seems to play a crucial role in controlling
69 river morphodynamics in vegetated river corridors (Camporeale et al., 2013;
70 Gurnell, 2014) there is also evidence that this is not the *sine qua non* condi-
71 tion to establish a stable single thread morphology. Laboratory observations
72 on the erosion of channel banks have shown that fine sediments mixed with
73 coarse non-cohesive sediments provide sediment cohesion, which prevents
74 runaway widening and maintains single thread channel evolution (Parker,
75 1998; van Dijk et al., 2013). Such results have been further supported by
76 studies conducted on meandering rivers wandering in non-vegetated flood-
77 plains such as in modern desert basins (Santos et al., 2019), and on the
78 surface of Mars (Matsubara et al., 2015). These studies have revealed how
79 rivers preserving single thread channels are seen to be devoid of riparian vege-
80 tation (Ielpi, 2018). Other experiments focusing on the dynamics of bar push
81 and bar pull provided evidence of the fact that fine sediments deposited on
82 the coarse sediments of a point bar generates morphological structures sim-
83 ilar to scroll bars (van de Lageweg et al., 2014). The application of models
84 on the long-term planform evolution of meandering rivers has also proven a
85 long-term control on the erosion rate of meanders banks. As a consequence
86 of their spanning back and forth during their evolution river meanders self-
87 build their floodplain, locally modifying its erodibility properties (Bogoni
88 et al., 2017).

89 Morphodynamic models for river planform evolution have traditionally
90 adopted the hypothesis of i) a constant channel width, and ii) constant dis-

91 charge. In particular, such models take advantage of the fact that the relevant
92 erosion processes occur at a different temporal scale of the in-channel mor-
93 phodynamics (Ikeda et al., 1981; Seminara, 2006). Modeling efforts within
94 the past years have introduced a local imbalance between the advance and
95 retreat rates of the opposite banks to explore channel width temporal varia-
96 tions (Parker et al., 2011; Eke et al., 2014; Lopez Dubon and Lanzoni, 2018;
97 Monegaglia et al., 2019). In this type of models the two opposites banks
98 can either erode or deposit proportionally to the excess of shear stress (lon-
99 gitudinal velocity) experienced at the bank toe through a erosion (E_e) and
100 deposition (E_d) coefficient. The proposed simplified closures are of the type
101 $\xi_E = E_e (\tau_s - \tau_c)$ when $\tau_s > \tau_c$ and $\xi_D = E_d (\tau_s - \tau_c)$ when $\tau_s \leq \tau_c$; where τ_s
102 and τ_c is the shear stress and its critical value for sediment movement respec-
103 tively. Although, the authors have referred to E_d as the coefficient account-
104 ing for vegetation dynamics and their interaction with alluvial sediments, the
105 aforementioned relationship for bank evolution can also be applied for other
106 physical controls responsible for the advancing of the floodplain margin (e.g.
107 sediment mixture).

108 The local expansion and contraction of the channel width in time can not
109 be explored by models based on a constant, formative, discharge (Pizzuto,
110 1994). Flow variability in time is fundamental to the erosional and deposi-
111 tional processes in both bare and vegetated channels. In the latter, periods
112 of low discharge allow vegetation to colonise and grow. Weather during a
113 flood vegetation decays or survives, stabilizing the alluvial sediments of the
114 bank, depends on how the hydrological and biological processes interact at
115 relevant/different time scales (Perona et al., 2012).

116 Although the influence of flow stochasticity and river morphology on the
117 vegetation dynamics has largely been investigated (e.g. Camporeale and Ri-
118 dolfi, 2006; Doulatyari et al., 2014; Vesipa et al., 2017; Bertagni et al., 2018),
119 its feedback on the river morphology, and on the interaction between vegeta-
120 tion and river processes, in particular, is poorly understood. Perucca et al.
121 (2007) used the model proposed by Camporeale and Ridolfi (2006) to link the
122 timescales of the vegetation growing on the floodplain with the lateral mi-
123 gration of evolving meandering channels. To account for the frequency with
124 which the channel is found to be morphodynamically active, Eke et al. (2014)
125 introduced a flood intermittent factor while keeping the water discharge con-
126 stant. Zen et al. (2016) proposed a hybrid biomorphodynamic lumped model
127 for bank accretion to quantify the influence of river flow variability on the in-
128 channel bio-morphodynamics controlling the bar push - bank pull mechanism
129 in evolving meander bends.

130 Recently, Davidson and Eaton (2018) have tried to overcome the constant-
131 discharge approach by including randomness in investigating river channel
132 morphology evolution. In their lumped model a series of random-yearly
133 floods erode the channel banks, while a random coefficient, accounting for
134 plant colonisation controls channel narrowing. The model allowed the au-
135 thors to obtain channel cross sections whereby the generated geometry de-
136 pended both on the water discharge and the history contingency.

137 In this work we propose a simple lumped model for river biomorphody-
138 namics where the two banks can either erode or advance toward the center
139 of the channel as a result of the mutual interaction (push-pull mechanism)
140 between near-bank fluvial processes and vegetation dynamics growing on the

141 banks. The model is used to investigate i) the role of flow discharge (exter-
142 nal driver of the system) in selecting the channel width to which the channel
143 tends to adjust, and ii) how the interaction between vegetation dynamics,
144 sediment erosion and water discharge temporal scale influences channel width
145 time and spatial scales. To these aims different types of hydrographs includ-
146 ing deterministic and stochastic external forces are adopted.

147 The work is structured as follows. In Section 2 the model is presented
148 along with the assumptions adopted and used to investigate the dynamical
149 properties of the system. In Section 3, results obtained for the different water
150 discharge that includes constant, periodic and stochastic flow are presented
151 and discussed. Finally, Section 4 presents the application of the proposed
152 model to three real cases and Section 5 is the conclusion section of the paper.

153 **2. Methods**

154 We propose a model that mimics the key mechanisms for which plants
155 growing onto the river bar stabilise the sediments and contribute to chan-
156 nel narrowing until geomorphic relevant floods erode the bank and uproot
157 riparian vegetation. Temporal variation in channel width are thus related to
158 vegetation dynamics, which in turn is modulated by the channel flow rate.
159 Although trees toppling can destabilise the bank increasing the rate of bank
160 retreat, in the model only the positive feedback of vegetation that increases
161 channel bank stability is accounted for. Hereinafter, an asterisk will indicate
162 dimensional quantities. Figure 1 presents a schematic of the lumped model
163 proposed.

164 *2.1. Bio-morphodynamic Model and related Assumptions*

165 We model an erodible straight channel characterised by homogeneous
166 non-cohesive sediments, with a representative size diameter of d_s^* , and a bed
167 slope, S . To keep the model simple, a series of hypotheses have been intro-
168 duced. We consider a wide channel, such that the hydraulic radius can be
169 approximated with the water depth; the channel cross section is rectangu-
170 lar and symmetrical, thus the banks either advance or retreat of the same
171 quantity. Moreover, the model is lumped in the sense that it is integrated
172 along the streamwise direction following the work of Cantelli et al. (2007),
173 and Tealdi et al. (2011).

174 Other considerations on the temporal scales of the processes are worth
175 some further explanation/clarification. In-channel morphodynamic processes
176 occur at a time scales faster than that characterising river banks migration,
177 and colonisation and stabilisation by vegetation dynamics. Because of this,
178 the sediment flux between the channel and floodplain region can be consid-
179 ered negligible, and bank advance and retreat can be modeled as continu-
180 ous processes (Howard, 1992; Lanzoni and Seminara, 2006). Under these
181 hypotheses, bank retreat is linearly related to the excess of near-bank shear
182 stress. We assume that the material eroded at the bank is immediately trans-
183 ported out of the section without modifying the channel bed. In the model,
184 we account for the shear stress value through the dimensionless Shields num-
185 ber and the relationship for river bank erosion (Partheniades, 1965) reads

$$\frac{dw^*}{dt^*} = E^* (\tau_s - \tau_{form})^\alpha, \quad (1)$$

186 where w^* is the channel width, t^* is the time, τ_s is the longitudinal Shields
187 number, E^* is a erodibility coefficient accounting for bank material properties

188 and α is a positive constant assumed to take the value 1 (Darby and Rinaldi,
 189 2007). The value τ_{form} represents the critical value of the Shields number for
 190 bank erosion, such that if the Shields number exceeds this value the channel
 191 will widen.

192 Based on the assumption that rivers adjust themselves such to maintain
 193 a bankfull Shields number (Parker et al., 1998), Eke et al. (2014) showed
 194 that the equilibrium channel characteristics can be estimated once channel
 195 slope, grain size, friction coefficient and bankfull discharge are known. Thus,
 196 by following Eke et al. (2014), the formative Shields number in the erosion
 197 relationship (1) for the channel bank τ_{form} has been set equals to the equi-
 198 librium value of near-bank Shields number associated to the bankfull flow
 199 discharge, Q_0^* and width w_0^* , in a non vegetated channel. This implies that,
 200 if there are no changes in water flow or channel width and vegetation does
 201 not grow, then channel banks are neither eroded nor aggraded. Thus, in the
 202 model, the bankfull condition represents the reference state of the river.

203 Previous works modeled river banks advancements by adopting a closure
 204 relationship similar to equation (1) for bank erosion. In this work, bank
 205 advancement is directly related to vegetation dynamics, which has direct
 206 influence on sediment processes. By growing during the low flow, vegetation
 207 consolidates the exposed sediments of the fluvial bar, and the channel narrows
 208 according to the relationship

$$\frac{dw^*}{dt^*} = -r_1^* \rho^*(t^*) w^*(t^*), \quad (2)$$

209 where r_1^* is a coefficient accounting for the rate of colonization of the vege-
 210 tation, ρ^* is the vegetation biomass. The negative sign is introduced since
 211 we consider channel widening as positive. The narrowing term is linearly

212 related to the channel width to account for the fact that when the channel
213 is wide the reduced water depth promotes vegetation growth, thus channel
214 narrowing.

215 In order for vegetation to populate the bank during low flow an aggrading
216 surface connecting the river bed with the floodplain is required, e.g the typical
217 transverse profile of fluvial bars. Lumped models studying channel bank
218 erosion have adopted a trapezoidal section and expressed the shear stress
219 experienced at the inclined sidewalls as a fraction of the channel bed shear
220 stress (Cantelli et al., 2007; Tealdi et al., 2011). To keep the problem at
221 the minimum level of complexity, we consider the bank vertical whereby the
222 vegetation is able to withstand bank erosion and thus a rectangular cross-
223 section (Figure 1). In addition, the overbank flow is neglected assuming that
224 all the hydrograph peaks can be contained within the main channel.

225 When the channel is eroding the banks, and thus widening the section,
226 we assume a rectangular cross-section without bed forms, which is consistent
227 with the analysis of Zen et al. (2014). Building up from the work of Tubino
228 (1991), (Zen et al., 2014) showed that during high flow the generation of bars
229 is hampered but that the widening process promotes channel bed instability
230 leading to the deposition of bars. Channel widening promotes both chan-
231 nel bed instability and vegetation colonisation by lowering the water depth.
232 Therefore, we assume that when the discharge is not high enough for bank
233 erosion, a bar generated during a previous formative event is present and its
234 exposed sediments can be colonised by vegetation.

235 By taking advantage of the different time scales between river hydrody-
236 namics and bank dynamics, we can also interpret changes in water flow rate

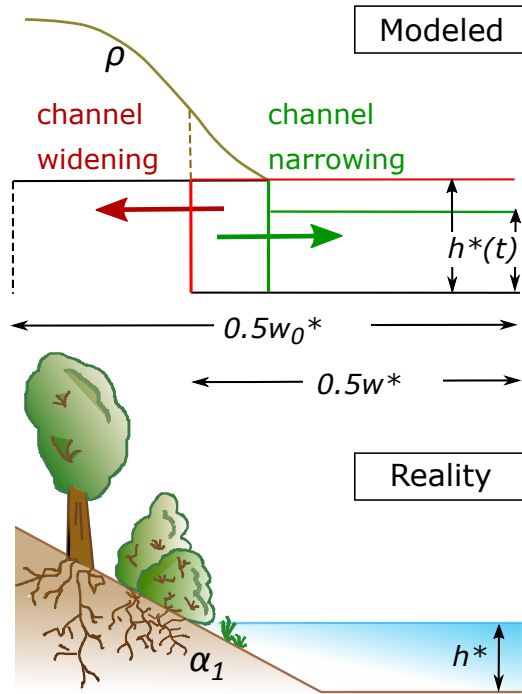


Figure 1: Sketch of the modeling framework. w_0^* is the initial channel width, w^* is the time dependent channel width, h^* is the water depth associated to the water discharge Q^* flowing into the channel of width w^* , slope S and sediment size d_s^* , ρ is the continuous distribution of vegetation biomass, and α_1 accounts for the sediment cohesion induced by plant roots (equation (4)). The red and green arrow indicate channel widening due to erosion and channel narrowing due to plant colonisation respectively.

237 as a sequence of instantaneous uniform flows and assume the river slope to
 238 be constant in time. At the reach scale, the river channel slope changes on
 239 larger temporal scales than those associated with channel width adjustments.
 240 Therefore, we assume the channel responds to changes in water discharge
 241 by modifying its width rather than aggrading-degrading the bed (Howard,
 242 1980). Under the hypothesis of normal flow and shallow flow approxima-
 243 tion, the shear stress is equal to the product of the fluid density, gravity
 244 acceleration, bed slope and water depth, and the Shields number reads

$$\tau_s(t^*) = \frac{h^*(t^*)S}{\Delta d_s^*} = \frac{1}{\Delta d_s^*} \left(\frac{SQ^*}{C^*w^*} \right)^{(2/3)}, \quad (3)$$

245 where Δ is the relative submerged weight of the sediment (1.65 for quartz
 246 material), h^* is the water depth, Q^* is the related water flow discharge, and
 247 C^* denotes the Chezy friction coefficient.

248 In order to account for the increased resistance to sediment erosion due
 249 to the presence of plant roots (Yang et al., 2018), the critical value for the
 250 Shields number is defined as follows (Zen et al., 2016):

$$\tau_{form} = \tau_{form,0}(1 + \sigma_1\rho^*), \quad (4)$$

251 where $\sigma_1 \leq 1$ is a positive coefficient linking below-ground biomass (plant
 252 roots) with the above-ground biomass ρ , and $\tau_{form,0}$ is the threshold value for
 253 the near-bank Shields number in absence of vegetation. Vegetation growth
 254 dynamics follows a logistic curve, whereas vegetation decays because of chan-
 255 nel bank erosion when the longitudinal shear stress falls above its formative

256 threshold. The biological dynamics is described by the following equations

$$\frac{d\rho^*(t^*)}{dt^*} = \frac{r^*}{t_v^*} \rho^*(t^*) [\beta^* - \rho^*(t^*)] \quad (5)$$

$$\frac{d\rho^*(t^*)}{dt^*} = -\rho^*(t^*) \tau_\Delta(t^*) E^* \frac{1}{w^*(t)} \quad \tau_\Delta > 0, \quad (6)$$

257 where β^* represents the vegetation carrying capacity (maximum biomass
 258 reachable under optimal conditions), r^* is the curve growth rate that deter-
 259 mines the time scale t_v^* representing the time vegetation takes to grow from
 260 the 5% to 95% of the carrying capacity, and $\tau_\Delta = (\tau_s(t^*) - \tau_{form}(t^*))$. In
 261 equation (6) vegetation decay is inversely related to channel width to ac-
 262 count for the increase in water depth associated with channel narrowing that
 263 promotes vegetation removal, and the subsequent channel widening. The
 264 temporal evolution of the two states variables of the system, namely river
 265 channel width w^* , and vegetation biomass ρ^* , is thus described by the fol-
 266 lowing system of non-linear, coupled, ordinary differential equations

$$\frac{dw^*(t^*)}{dt^*} = E^* \tau_\Delta(t) \Theta(\tau_\Delta) - r_1^* \rho^*(t^*) w^*(t^*) \quad (7)$$

$$\frac{d\rho^*(t^*)}{dt^*} = \frac{r^*}{t_v^*} \rho^*(t^*) [\beta^* - \rho^*(t^*)] - \rho^*(t^*) \tau_\Delta(t) \Theta(\tau_\Delta) E^* \frac{1}{w^*(t)}; \quad (8)$$

267 where the parameter $\Theta(\tau_\Delta)$ is the Heaviside step function which sets to null
 268 the positive term of equation (7) when the Shields number falls below its
 269 formative value. Hence, channels narrow because of colonising vegetation
 270 on the bank. While growing, vegetation increases sediments resistance to
 271 erosion thus allowing the bank to advance (second term of the right-hand-
 272 side of equation (7)). This reduces the channel section, which in turn induces
 273 vegetation removal, bank erosion and channel widening during subsequent
 274 floods (Figure 1).

275 It is worth clarifying that there is a high degree of uncertainty in the choice
 276 of the values of the constants accounting for the vegetation colonisation rate,
 277 r_1 , and the increased resistance to sediment erosion provided by plant roots,
 278 σ_1 . This is mainly due to the absence in the literature of physically based
 279 relationships that would allow such parameters to be estimated on the base of
 280 measurable characteristics of the river channel. Because of this uncertainty,
 281 in the following analysis we will explore a range of values to quantify their
 282 influence on the overall channel width and vegetation biomass.

283 *Dimensionless equations*

284 We first rewrite the model in dimensionless form. To this aim, let us
 285 introduce the following dimensionless quantities:

$$\begin{aligned}
 t^* = tt_0^* = tt_v^*, \quad w^* = w_0^*w, \quad Q^* = Q_0^*Q, \quad E^* = E\frac{w_0^*}{t_0^*}, \quad (9) \\
 \rho^* = \rho\beta^*, \quad r^* = \frac{r}{\beta^*}, \quad r_1^* = r_1r_2^* = r_1\frac{r}{t_0^*\beta^*},
 \end{aligned}$$

286 having denoted with t_0^* a typical temporal scale of the process, i.e. the time
 287 t_v^* , and $r_2^* = r^*/t_0^*$. The Shields number can be therefore expressed as:

$$\tau_s = \frac{1}{\Delta d_s^*} \left(\frac{SQ_0^*}{C^*w_0^*} \right)^{(2/3)} \left[\frac{Q(t)}{w(t)} \right]^{(2/3)} = \tau_{s,0} \left[\frac{Q(t)}{w(t)} \right]^{(2/3)}; \quad (10)$$

288 where $\tau_{s,0}$ is the Shields number associated to the initial, bankfull, channel
 289 configuration. The model (7, 8) can now be rewritten as:

$$\frac{dw(t)}{dt} = E\tau_\Delta(t)\Theta(\tau_\Delta) - \gamma\rho(t)w(t) \quad (11)$$

$$\frac{d\rho(t)}{dt} = r\rho(t)\left[1 - \rho(t)\right] - \rho(t)\tau_\Delta(t)E\frac{1}{w}. \quad (12)$$

290 The parameter $\gamma = r_1r$ accounts for the rate of colonisation of vegetation
 291 having set r constant and equal to 5.88, which allows the vegetation to grow

292 from 5% to 95% of its maximum in a time equals to t_v^* ($t = 1$ in dimensionless
293 quantities).

294 2.2. River hydrology

295 To define the influence of hydrologic stochasticity on the control of veg-
296 etated channel width we will explore the response of the system to either
297 deterministic or stochastic hydrologic forcing. The use of a characteristic
298 discharge is usually adopted in analytical models for river morphodynamics.
299 The value is normally chosen so as to generate the same equilibrium geome-
300 try produced by the long-term hydrograph. In this analysis, the equilibrium
301 bankfull geometry in the absence of vegetation corresponds to the bankfull
302 discharge, $Q_0 = 1$.

303 We will also explore the response of the system forced with a periodic
304 flow oscillation of the type

$$Q(t) = Q_0 + 0.5\sin(f2\pi t); \quad (13)$$

305 where f is the signal frequency that can be interpreted as mean hydrograph
306 fluctuations. It is worth mentioning that a periodic pattern in the hydrolog-
307 ical signal can be observed in natural catchments characterised by a nivo-
308 pluvial regime.

309 For the stochastic analysis, we will use a synthetic hydrologic signal char-
310 acterized by a series of instantaneous flood events occurring randomly with
311 random magnitude, hereinafter denoted to as ‘jumps’, followed by an expo-
312 nential deterministic decay. Under this assumption the stochastic hydrolog-
313 ical signal is described by the Langevin equation

$$\frac{dQ}{dt} = -\frac{Q}{\theta} + \sum_{i=1}^{\infty} p_i(t)\delta(t - t_i); \quad (14)$$

314 where θ indicates the rate of discharge decay after the positive jump p_i oc-
 315 curred, and $\delta(\cdot)$ is the Dirac delta distribution. We also assume both jumps
 316 intensity, p_i , and their interarrival time to be distributed according to expo-
 317 nential functions whose parameters are, respectively, the jumps average, α ,
 318 and their average daily frequency, λ . In this case and under the assumption
 319 that jumps occur as a independent and identically distributed uncorrelated
 320 process, equation (14) describes the dynamics of the so-called Compound
 321 Poisson Process - CPP (Doulatyari et al., 2014). The probability distribu-
 322 tion of the discharge values, p_Q , generated from these hypothesis is a gamma
 323 function with mean μ_Q and variance σ_Q^2 can be estimated analytically (Botter
 324 et al., 2007; Ridolfi et al., 2011). For the sake of completeness let us intro-
 325 duce the coefficient of variation C_v , that indicates the magnitude of variation
 326 around the series mean value, as $C_v = \sigma_Q^2/\mu_Q$.

327 In lowland alluvial rivers the rising limb of the hydrograph is characterized
 328 by a mild slope. Yet, in such catchments the falling limb last longer compared
 329 to that typical of mountains streams, which increases the correlation, θ of the
 330 hydrological signal. Thus, the stochastic process generates correlated rising
 331 limbs as a sequence of jumps and exponential decays.

332 **3. Results and Discussion**

333 *3.1. Linear stability analysis*

334 We study the isoclines and the nullclines (i.e. $dw/dt = 0$, $dp/dt = 0$)
 335 of the system to determine the nature of the equilibrium point and how it
 336 is influenced by the system parameters. When the system is forced with
 337 deterministic hydrological action the equilibrium values for the two state

338 variables, w and ρ , can easily be assessed analytically by equating their tem-
 339 poral derivatives to zero. The first equation, (11), provides the equilibrium
 340 value for the vegetation biomass

$$\rho_{eq} = \frac{1}{1 + r_1} \quad (15)$$

341 which can be substituted into the second equation, (12), to derive the equi-
 342 librium value for channel width. However, the influence of channel width on
 343 the Shields number introduces a strong non linearity in the equation that
 344 does not allow to obtain an explicit relationship for the equilibrium channel
 345 width $w_{eq}(Q, E, \sigma_1, \tau_{s,0}, \gamma)$. Yet, such a value can be computed once assigned
 346 the characteristic for channel geometry, hydrology and vegetation type, by
 347 solving the following equation

$$\xi_2 w_{eq}^{2.5} - \xi_1 w_{eq} = \xi_0 Q, \quad (16)$$

348 where the coefficients ξ_i , $i = \{0, 1, 2\}$ read:

$$\xi_2 = 1, \quad \xi_1 = \left(E \tau_{s,0} \frac{1 + r_1 + \sigma_1}{\gamma} \right)^{3/2}, \quad \xi_0 = \left(E \tau_{s,0} \frac{(1 + r_1)}{\gamma} \right)^{3/2}. \quad (17)$$

349 Computing the equilibrium values is further complicated by the dependency
 350 of the Shields formative value on vegetation. The influence of riparian veg-
 351 etation on sediment erosion through equation (1) and (4), does not allow us
 352 to impose $\tau_\Delta > 0$ to obtain an analytical solution for the equilibrium of the
 353 system, preventing a stability analysis to be performed. To overcome this
 354 issue we introduce a new continuous function, $\widetilde{\tau}_\Delta$, which is able to provide an
 355 analytical approximation for the Heaviside step function originally adopted.
 356 The excess of Shields number, τ_Δ , is therefore interpreted as a transcritical

357 bifurcation (Strogatz, 2018) of a function switching its stable condition be-
 358 tween zero, when the Shields number falls below its critical value, and the
 359 excess of Shields number itself otherwise. The equation describing the tran-
 360 scritical bifurcation is solved coupled with the equations for channel width
 361 and vegetation biomass dynamics, and the new complete system now reads:

$$\begin{aligned}
 \frac{dw(t)}{dt} &= E\widetilde{\tau}_\Delta(t) - \gamma\rho(t)w(t) \\
 \frac{d\rho(t)}{dt} &= r\rho(t)\left[1 - \rho(t)\right] - \rho(t)\widetilde{\tau}_\Delta(t)E\frac{1}{w} \\
 \frac{d\widetilde{\tau}_\Delta}{dt} &= (\tau_\Delta\widetilde{\tau}_\Delta(t) - \widetilde{\tau}_\Delta(t)^2 + \xi)\frac{1}{t_\Delta},
 \end{aligned} \tag{18}$$

362 where t_Δ is a temporal scale indicating the rapidity with which the approxi-
 363 mation function $\widetilde{\tau}_\Delta$ tends to the solution obtained by adopting the Heaviside
 364 function and ξ is a small value (assumed equals to 0.01) that is added to
 365 avoid the approximation function to stick to zero. In the following section
 366 we will explore the influence of the channel bank erodibility, E , river hydrology
 367 Q , and vegetation characteristics r_1 and σ_1 on the two state variables of
 368 the system, namely w and ρ .

369 The new system of ODE equations (18) allows us now to compute the
 370 equilibrium condition for which the temporal derivative are set equal to zero
 371 and study its stability. To do this we linearise the system around the equi-
 372 librium point $\{w_{eq}, \rho_{eq}, \widetilde{\tau}_{\Delta,eq}\}$ and investigate its eigenvalues. Because one
 373 of the three eigenvalues associated to the system refers to the approxima-
 374 tion function, only the two eigenvalues λ_1, λ_2 associated to the physical
 375 state variables of the system w and ρ will be considered. Figure 2a presents
 376 the dependency of the equilibrium values for the channel width from water
 377 discharge, Q , and the coefficient σ_1 accounts for sediment strengthening by

378 plants root. The value presented by the vegetation biomass at the equilibrium
379 is dictated only by the colonisation coefficient r_1 , as also noted in equation
380 (15) (Figure 2b). For the sake of brevity the linearised system is reported
381 in Appendix A. The system shows two equilibrium points, one where bank
382 erosion processes balance that of bank colonisation and another one where
383 the unvegetated channel would reach its equilibrium width when the Shields
384 number equals its threshold value for bank erosion ($\tau_{\Delta} = 0$).

385 Figure 2d shows that when the value of the vegetation colonisation rate,
386 r_1 , is close to zero, the eigenvalues are different negative real numbers, while
387 for $r_1 \geq 0.2$ the two eigenvalues are complex conjugates with the real part
388 invariably negative. Since the real component of the eigenvalues is invariably
389 negative the equilibrium point is an attractive point. This means that bank
390 advancing, induced by vegetation dynamics, and erosion adjust their rate in
391 time until an equilibrium channel width is reached. If the sediment supply
392 is assumed constant, the way the two bank processes cooperate is directly
393 controlled by the vegetation type and the hydrology of the channel. There-
394 fore, in the following, we will explore different river hydrology and different
395 type of vegetation, i.e distinct value of the colonisation parameter r_1 and
396 the constant σ_1 accounting for the increased resistance to sediment erosion
397 associated with plant roots.

398 In absence of vegetation colonisation ($r_1 \leq 0.2$), or presence of sparse
399 vegetation coverage, the equilibrium point of the system (w_{eq}, ρ_{eq}) behaves
400 as a stable node directly attracting all the trajectories on the phases plane
401 (Kaplan and Glass, 1995). Therefore, when vegetation type presents a low
402 colonisation rate, vegetation biomass increases, reducing the channel width

403 monotonically, or vice versa, until the equilibrium width is reached. When
404 the eigenvalues are complex conjugates, i.e. $r_1 > 0.2$, the equilibrium point
405 of the system is a stable focus (Kaplan and Glass, 1995) and the fluvial
406 system behaves in a different way. The system state variables present an
407 exponentially decaying oscillation at the rate $e^{\lambda_i t}$ (since the eigenvalues λ_i
408 are negative) which organise all the trajectories lying on the phase plane in a
409 spiral path around the equilibrium point. Therefore, the higher the absolute
410 value of the real component of the eigenvalues, the faster the system reaches
411 the equilibrium condition. Figure 2d shows how the stable focus of the system
412 become rapidly attractive when $r_1 > 0.2$.

413 When the channel is fed with a variable discharge, the increase in water
414 flow induces channel widening via bank erosion. The widened cross section
415 allows the vegetation to colonise the sediments and grow, advancing the
416 bank and narrowing the channel. This promotes bank erosion that widens
417 the channel removing vegetation biomass, and the cycle restart. However,
418 with every cycle, the survived vegetation will reduce the amount the bank
419 retreats, diminishing also the space available for vegetation to grow, thus the
420 amount the bank advances. When reported in the phase plan the values of
421 the state variable of the system organise on a spiral path. The phase plane
422 presents a more simple path in the case the channel is fed with a constant
423 discharge. Here, vegetation encroachment at the bank narrows the channel
424 section increasing the shear stress, thus promoting bank erosion and biomass
425 removal. As a consequence the channel widens until the equilibrium width
426 value is reached.

427 Because channel bank accretion and erosion are mutually related, when

428 the colonisation rate r_1 increases these two bank processes keep pace with
429 each other, maintaining the equilibrium channel width. Yet, a rapid vegetation-
430 related channel narrowing induces strong erosion at the bank leading to an
431 overall reduced vegetation biomass at the equilibrium (Figure 2b). Con-
432 versely, the parameter σ_1 , accounting for the increased resistance to sediment
433 erosion, modifies the equilibrium channel width without affecting the equi-
434 librium value associated to the vegetation biomass. In this case, the absolute
435 value of the complex and real component of the eigenvalues, respectively in-
436 creases and decreases linearly with the constant σ_1 . Therefore, increasing
437 in plant root strength will augment the attractive force of the equilibrium
438 point while extending the time required to reach the equilibrium by intro-
439 ducing bigger oscillations of the parameters. By hampering bank erosion,
440 plant roots bring an unbalance between the bank processes that promotes
441 the development of narrow channels. Such influence on channel geometry be-
442 comes stronger for higher water flow (Figure 2a). Despite the discharge value
443 influencing the equilibrium channel width, it marginally affects the dynami-
444 cal property of the system with the equilibrium point that remains a stable
445 focus and it is reached at almost the same rate for discharge values bigger
446 than the reference value $Q = 1$, i.e. the real component of the eigenvalues
447 shows very small variations in Figure 2c.

448 *3.2. Deterministic behavior: Constant discharge*

449 We first explore the case of three non vegetated channels fed with a con-
450 stant discharge equal to the bankfull discharge, $Q = 1$, and different initial
451 channel width (Figure 3a dashed lines). Since the threshold Shields number
452 for bank erosion has been set as equal to the Shields number associated to

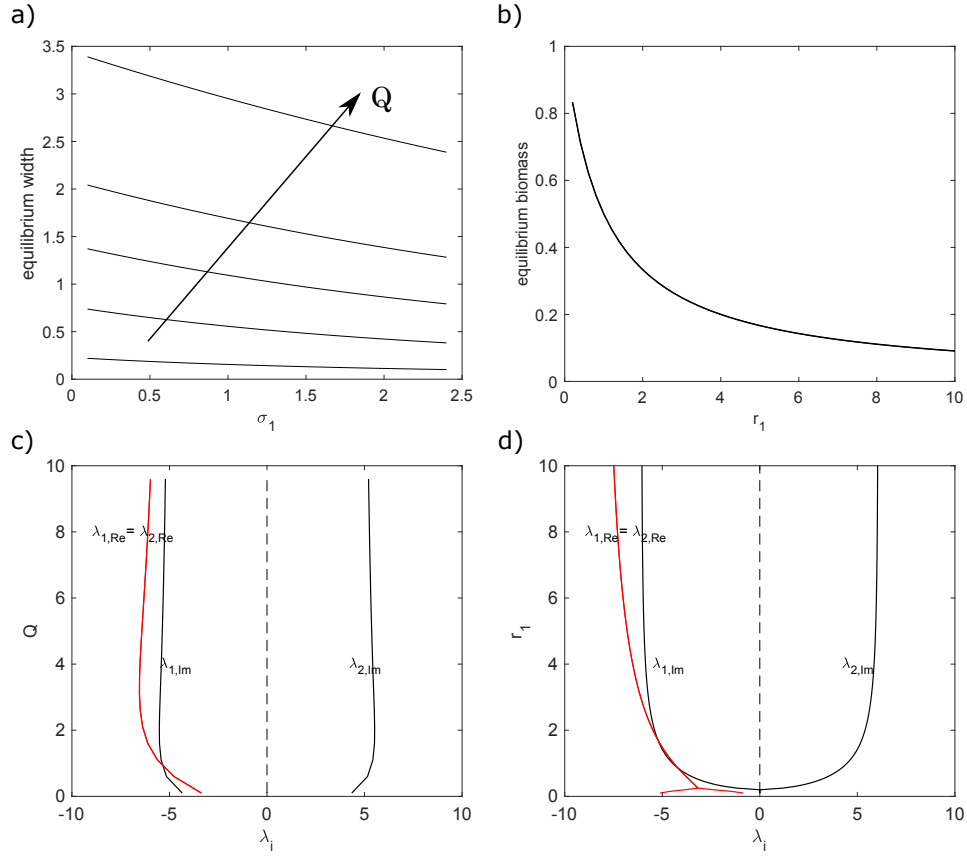


Figure 2: The upper panels show the equilibrium state for the a) channel width against the coefficient σ_1 accounting for the role played by plants root and water discharge, Q ; and b) the vegetation biomass for different rate of colonisation, r_1 . The lower panels show the eigenvalues for the system (18) associated with the variables w and ρ for different values of c) water discharge Q and d) colonisation rate r_1 . When not specified $r_1 = 2$ and $Q = 1 - E = 100$, $\sigma_1 = 0.2$.

453 bankfull conditions, i.e. $w(t = 0) = 1$ and $Q(t = 0) = 1$, a channel present-
454 ing unitary width maintains its geometry with time. A wider channel, e.g.
455 $w(t = 0) = 2$, because of the water depth and near bank shear stress will
456 favour deposition at the bank and thus section contraction. However, as the
457 Shields number falls below its threshold value for bank erosion, the channel
458 will neither erode nor deposit thus maintaining the initial channel width. On
459 the other hand, a narrower channel, $w(t = 0) = 0.5$, due to the higher flow
460 velocity becomes erosional, hence widening its section. This progressively
461 diminishes bank erosion until the system reaches its actual bankfull width
462 and the Shields number equals its threshold value. Conversely, a vegetated
463 channel adjusts to a new equilibrium condition by either eroding or deposit-
464 ing, regardless of its initial geometry (Figure 3a). Indeed, the presence of
465 vegetation activates the narrowing term $dw/dt < 0$ in equation (11) which is
466 proportional to the rate of colonization r_1 . Figure 3a (green continuous line)
467 shows that for both the bankfull-reference and wider channel, $w(t = 0) = 1$
468 and $w(t = 0) = 2$, respectively, vegetation growth promotes channel nar-
469 rowing. On the other hand, the narrower channel, $w(t = 0) = 0.5$, initially
470 increases its width until the channel is wide enough to allow bank advances
471 to reduce the channel width and adjust it to a stable value.

472 Figure 3b and c show, respectively, how the colonisation rate r_1 and
473 the constant σ_1 influence these processes by modifying their temporal scales
474 and the overall equilibrium condition of the channel. In particular, when
475 the colonisation process is rapid vegetation growth it is not able to keep pace
476 with the rate of bank advance that rapidly increases the shear stress inducing
477 bank erosion, thus vegetation removal. The faster the channel narrows the

478 lower is the amount of vegetation biomass on the bank, as well as the time
479 needed to reach the equilibrium. Because no changes are introduced on the
480 property of the bank and the shear stress is inversely related to the channel
481 width, therefore the amount that the channel narrows is proportional to
482 that which it widens. Therefore, for $r_1 \geq 1$ the channel tends to recover the
483 same equilibrium width, but it presents lower values of vegetation biomass.
484 On the other hand, changes in the root influence on sediment erosion, i.e.
485 σ_1 , determine different equilibrium width without affecting the equilibrium
486 vegetation biomass Figure 3c. In this case the constant accounting for an
487 increased resistance to sediment erosion exerts two effects: i) promotes bank
488 advance and vegetation growth by protecting the bank, ii) reduce channel
489 widening. This does not modify the overall vegetation biomass, but it reduces
490 channel width and increases the rate at which the banks shift. Therefore, the
491 stronger the influence on bank erosion exerted by plant roots the narrower the
492 channel. Such a result is in agreement with field observations on vegetated
493 evolving channels (Allmendinger et al., 2005). Overall, the colonisation rate
494 and the roots strength control the equilibrium vegetation biomass and the
495 equilibrium channel width respectively, and the related time scales. In Figure
496 3b and c the black line indicates the trajectory associated with $r_1 = 2$ and
497 $\sigma_1 = 0.2$.

498 From the mathematical point of view, the behaviour of the system in
499 Figure 3b is explained with the conversion of the equilibrium from a stable
500 node to a stable focus and with the increased attraction of this latter, i.e.
501 smaller temporal scales for high r_1 values already discussed in the analysis
502 of the eigenvalues. Moreover, the overshooting that characterises the curves

503 of Figure 3b is associated with the complex component of the eigenvalues
 504 of the system and therefore it is observed only for value of r_1 greater than
 505 0.2 (see Figure 2d). A similar behaviour is observed when the protection to
 506 erosion exerts by the plant root increases as reported in Figure 3c. However,
 507 in this case, the real component of the eigenvalues decreases, whilst the
 508 complex component increases. The complex component is still responsible
 509 for the amplitude of the overshooting that progressively increases while the
 510 reduced variation of the real component leaves the time required to reach the
 511 equilibrium almost unchanged.

512 Figure 4a and c present the trajectories of the system for different initial
 513 values of channel width and vegetation biomass. The blue and green isolines
 514 refers to the value of $\frac{dw}{dt}$ and $\frac{d\rho}{dt}$, respectively. The point where the two
 515 zero-isolines meet each other is the equilibrium point (black dot). Wide and
 516 equilibrium channels always undergo a narrowing process, $\frac{dw}{dt} < 0$, associated
 517 with an increase of vegetation biomass, $\frac{d\rho}{dt} > 0$, until a maximum value after
 518 which channel width changes slightly while the biomass decreases abruptly.
 519 The different trajectories reported on the phase plan of Figure 4a and c show
 520 how the initial conditions control the attractiveness of the equilibrium point,
 521 with narrower channels, i.e. $w(t = 0) < 1$, reaching the equilibrium condition
 522 faster than the wider ones, i.e. $w(t = 0) \geq 1$.

523 We now explore the influence of the erodibility coefficient, E , on the dy-
 524 namics (Figure 4a and c). By increasing bank erodibility the equilibrium
 525 channel width, also increases without influencing vegetation biomass. Sim-
 526 ulation runs conducted for different water discharge ranging from 0.5 to 2
 527 with $r_1 = 2$ reveal a similar behaviour with the river system adjusting to

528 larger sections for large discharges without modifying the vegetation biomass
529 (red circle in Figure 4b); such that the higher the discharge the wider the
530 equilibrium channel. The equilibrium value for vegetation biomass is indeed
531 controlled by the parameter r_1 that accounts for the ratio at which channel
532 banks advance reducing the channel width. Figure 4b and d show that the
533 influence of the parameter ratio r_1 on the equilibrium value of the channel
534 width is low for $Q \leq 1$ and it becomes even lower when bank erodibility
535 increases (almost vertical lines in Figure 4d). The temporal trajectories re-
536 ported in the phases plan also provide a visualization of the system flow and
537 how it is attracted by the equilibrium point that behaves as a stable focus.

538 Overall, in channels fed with a constant discharge the presence of vege-
539 tation leads to an equilibrium cross section narrower than that estimated by
540 adopting the bankfull discharge. Indeed to recover the bankfull geometry (i.e
541 dimensionless channel width = 1) the vegetated channel should be fed with a
542 constant discharge higher than its bankfull value, with the increased amount
543 set proportional to the ratio r_1 . In other words, if the sediment cohesion
544 induced by plant roots is neglected the model should be fed with a water
545 discharge lower than the bankfull value to obtain realistic estimates of the
546 channel width. This result agrees with the findings of Bolla Pittaluga et al.
547 (2014) and Lanzoni et al. (2015) who applied a one-dimensional morphody-
548 namic model to the Magra River (Italy) and Po River (Italy), respectively,
549 to explore the role of the formative discharge.

550 3.3. *Deterministic behavior: Periodic discharge*

551 The response of the system undergoing periodical fluctuations of the wa-
552 ter discharge around its bankfull value $Q = 1$ shows a behaviour consistent

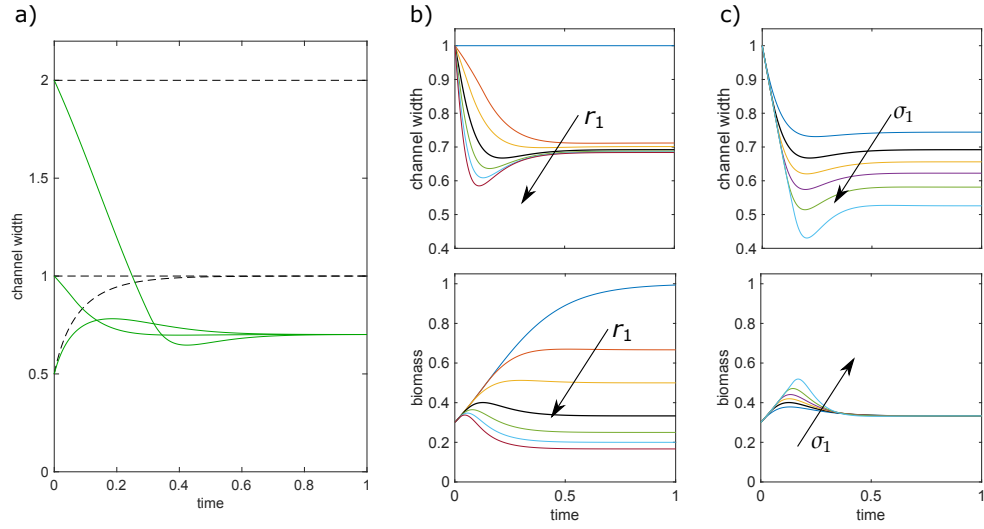


Figure 3: Evolutionary trajectories. a) Comparison between evolutionary trajectories for a non vegetated channel (dashed black line) and a vegetated channel (continuous green line) for different initial values of dimensionless channel width, $w = 0.5, 1.0, 2.0$, $w = 1$ indicates the bankfull channel width. b) and c) show how the parameter r_1 and σ_1 influence the same trajectory reported in a) for a vegetated channel initially showing a bankfull cross-section ($w = 1$) and the related biomass $\rho - r_1 = 0:5$, $\sigma_1 = 0:1$. Other relevant parameters are $E = 100$, $\sigma_1 = 0.2$, $r_1 = 2$ in a), $Q = 1$.

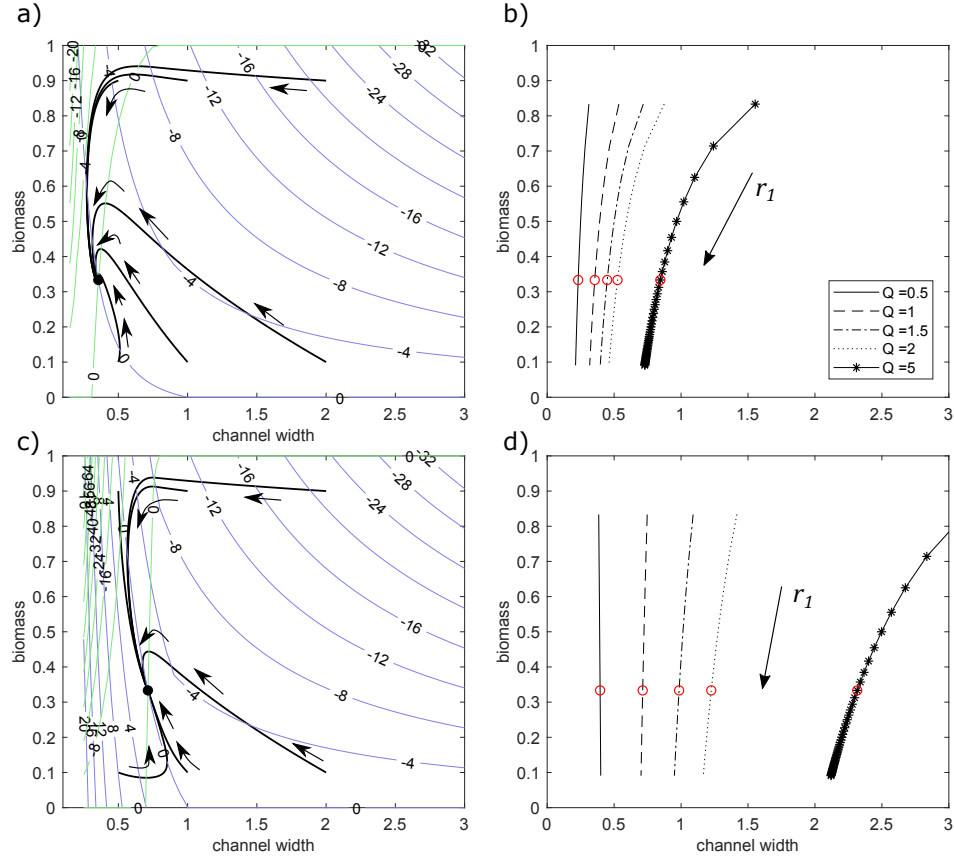


Figure 4: Panels a) and c) phases plane for the system (18) for different initial conditions of vegetation biomass 0.1, 0.9 and channel width 0.5, 1.0, 2.0, and values of erodibility coefficient a) $E = 10$, c) $E = 100$. The blue and green lines represent the 'isolines', which are the locus of points where the time derivative $dw/dt = 0$ and $d\rho/dt = 0$, respectively. Panels b) and d) report the equilibrium point of the system for different values of the dimensionless water discharge $Q = 0.5:5$, respectively for b) $E = 10$, and d) $E = 100$. When not specified the relevant parameters for the simulation are $r_1 = 2$ and $Q = 1$. The red circle on panels b) and d) indicates the equilibrium point of the system associated to $r_1 = 2$.

553 with that observed for the case of constant discharge. The comparison of
554 two deterministic trajectories, respectively associated with a constant and
555 periodical discharge, shows that after a transitory phase the system reaches
556 a steady phase in which it oscillates around an equilibrium state at the same
557 frequency of the hydrologic forcing (Figure 5). The system subject to a peri-
558 odic discharge undergoes narrowing, then the increased shear stress promotes
559 bank erosion widening the channel. As a consequence the lower depth allows
560 vegetation to colonise the sediment of the bank and narrow down the sec-
561 tion. As a result, the two signals of channel width and vegetated biomass
562 oscillate with the same frequency but with opposite phases. This behaviour
563 is controlled by the inertia of the system and the interaction between the
564 bank erosion and colonisation temporal scales.

565 According to equation (13), the frequency of the hydrologic signal indi-
566 cates the number of complete cycles of flow increases and decreases within
567 a characteristic vegetation time t_v . Because the hydrologic time scales are
568 directly associated with the vegetation timescales, when the signal frequency
569 is low the limited channel widening is associated with an initial increase in
570 water flow rate promoting vegetation growth. As a consequence, the channel
571 undergoes narrowing allowing vegetation to grow further and resulting in a
572 net increase in vegetation biomass. At this point, because of the new over-
573 narrow section, any increase in water flow will remove vegetation biomass via
574 bank erosion. Figure 5 shows that the amplitude of such temporal variation
575 for both channel width and vegetation biomass is inversely related to the sig-
576 nal frequency. There is however an initial phase, which last almost half of the
577 typical vegetation time t_v , where the state variables of the system present a

578 trend regardless the frequency characterising the hydrologic forcing. At this
579 stage, the channel width limits the erosional power of the flow, generating
580 a suitable environment for the vegetation to grow. This unbalance between
581 bank advance and retreat results in a negative trend for the channel width
582 and a net increase for vegetation biomass. The channel keeps narrowing until
583 a width of around 0.8 when the erosional processes keep pace with vegetation
584 dynamics and the system oscillates around its equilibrium configuration.

585 It is worth noting that a higher frequency of the hydrologic signal narrows
586 the evolutionary trajectory to that experienced by the channel when fed
587 with a constant bankfull discharge (dashed line in Figure 5). However, the
588 reduced period of time for which the discharge is higher than the bankfull
589 value promotes channel widening. As a result, the system oscillates around a
590 channel width larger than that to which the bankfull discharge would have set
591 the channel. This is particularly evident for the simulations with frequency f
592 = 5 (Figure 5). The amount of time for which the water flow is above or below
593 the average is the same regardless of the number of time the average value
594 is crossed and the fast oscillation does not allow changes in the system that
595 slightly oscillate around the equilibrium condition. This result highlights the
596 limit of using deterministic hydrograph as input for river morphodynamic
597 models.

598 *3.4. Stochastic behavior*

599 While interpreting the response of the system subject to a deterministic
600 forcing is quite straightforward, this may not be the case for stochastic forc-
601 ing. We start with studying the evolution of the system to a sequence of
602 CCP flood events. Figure 6 shows that in channels where bank colonisation

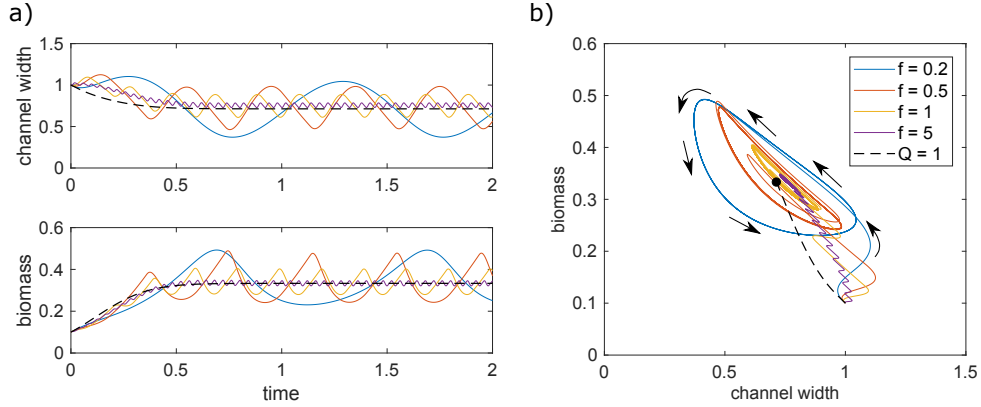


Figure 5: Response of the system to a periodic discharge for different frequencies with which the discharge value oscillates around the formative dimensionless discharge Q_o - $Q(t) = Q_o + 0.5\sin(f2\pi t)$ with frequencies f ranging from 0.2 to 5. Panels a) shows the temporal trajectories for both channel width and vegetation biomass; b) the same trajectories presented in a) are reported on the phase plane channel width - biomass.

603 is hampered by environmental conditions, $r_1 = 0$, the channel keeps its width
604 constant until a flood event able to erode channel banks occurs. In this case
605 vegetation growing on the channel banks is removed. In vegetated chan-
606 nels characterised by vegetation encroachments on the banks, $r_1 > 0$, during
607 low/non formative flow vegetation colonises the banks and grows, reducing
608 the river channel width with a rate that is proportional to r_1 (increasing
609 trajectory slope in Figure 6b). During the subsequent formative flood the
610 reduced channel cross-section increases the near-bank shear stress promoting
611 channel bank erosion and thus vegetation removal. Therefore, flood effects
612 on river channel morphology are not related only to floods magnitude but
613 also to the geometry presented by the channel when a flood occurs. As a
614 consequence initially non-formative floods can generate high bank erosion in

615 channels that undergoes narrowing because of the mutual interaction between
616 riparian vegetation and river morphodynamics. (Figure 6b).

617 Model runs performed by forcing the system with a Poissonian stochastic
618 discharge of the type described in equation (14) reveal that the system rapidly
619 loses the configuration set by the initial condition and starts oscillating,
620 randomly, around a steady state configuration. Monte Carlo simulations
621 (only fifty of them are reported in Figure 7 for clarity) allow for extracting
622 the average channel width and vegetation biomass (black solid line in Figure
623 7) and to obtain the average evolutionary trajectories of the system as well
624 as its equilibrium point. Figure 7 also shows the histogram of the channel
625 width and vegetation biomass values. Interestingly the average evolutionary
626 trajectory, differ from those obtained by forcing the system with constant
627 discharges: equal to the average-CPP discharge (dashed black line in Figure
628 7) and the bankfull formative discharge, $Q = 1$ (dashed white line in Figure
629 7), respectively. However, the trajectory associated to $Q = 1$ is quite close
630 to the average trajectory, especially in its final part, showing an equilibrium
631 width slightly larger than the average one. The equilibrium values for the
632 trajectory associated with $Q = 1$ and the average trajectory are, respectively,
633 $(0.69, 0.33)$ and $(0.61, 0.35)$. Nevertheless, this even small difference is due to
634 a change in the system dynamics. By adopting a constant bankfull discharge,
635 the water depth is kept, on average, at higher values than those present in the
636 channel during the low flow periods generated by the stochastic hydrology.
637 This hampers the colonisation process that advances the channel banks and
638 promotes bank erosion, generating a wider channel.

639 It is instructive to compute the probability distribution of the equilib-

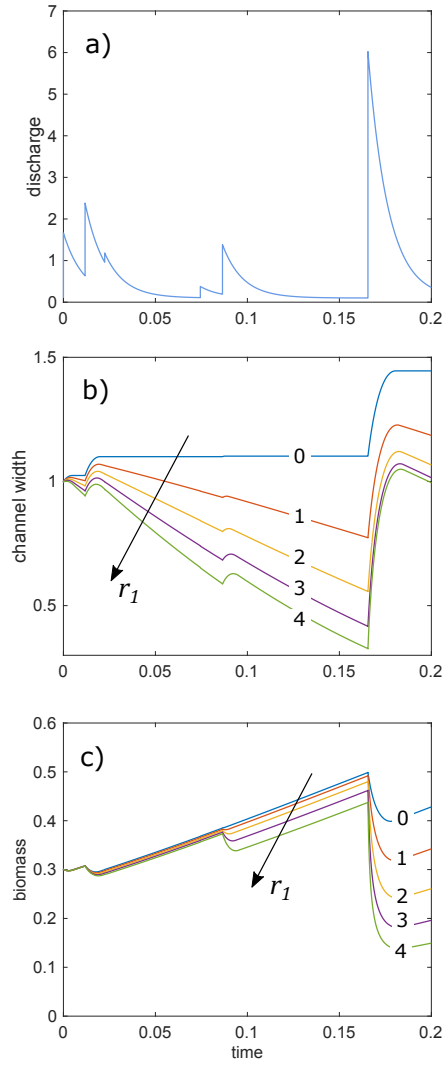


Figure 6: Response of the system to the stochastic hydrograph reported in panel a) for different values of the colonization parameter r_1 ranging from 0 to 4; b) channel width and c) vegetation biomass. Other relevant parameters are: $\mu_Q = 0.6 \text{ m}^3/\text{s}$, $\tau = 2 \text{ day}$, $\lambda = 0.02 \text{ day}^{-1}$, $E = 200$, and Initial Conditions $w = 0.1$, $\rho = 0.3$.

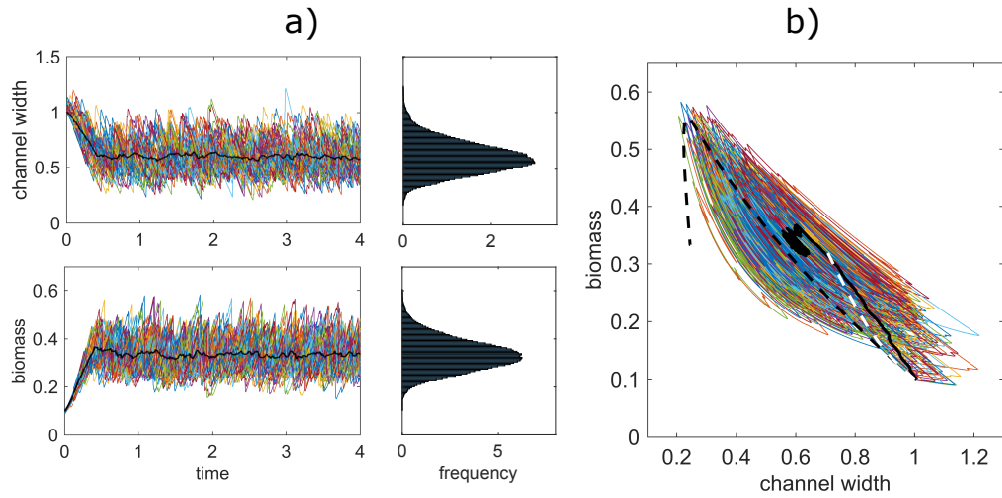


Figure 7: Response of the system to a stochastic hydrologic forcing described by a poissonian process. a) series of 50 distinct stochastic trajectories both for the channel width and the vegetation biomass, with relative histogram. The black continuous line represents the average value for all the 50 realizations at each time. b) the phases plan associated to the trajectories in a). The solid black line represents the average values obtained from the stochastic-runs, while the black and white dashed lines represent the deterministic behaviour of the system fed with a constant discharge equal to μ_Q and 1, respectively. Other relevant parameters: $E = 100$; $\tau = 10$ day; $\lambda = 0.05$ day^{-1} ; $\mu_Q = 0.3$ m^3/s ; $r_1 = 2$; $\sigma_1 = 0.2$.

640 rium channel width, p_w . The latter is readily obtained from the probability
641 distribution of the water discharge following a classic derived distribution
642 procedure, once the discharge is expressed as a function of the channel width
643 by rearranging the equation (16). The analytical expression for the probab-
644 ity density function of the equilibrium channel width (expressed in equation
645 16) associated with the distribution of the water discharge reads

$$p_w = \frac{e^{-\frac{\phi}{\alpha}} \alpha^{-\lambda\tau} (-\xi_1 - 2.5w^{1.5}\xi_2) \phi^{\lambda\tau-1}}{\Gamma[\lambda\tau]\xi_0} \quad (19)$$

646 with $\phi = -(w\xi_1 + w^{2.5}\xi_2)\xi_0^{-1}$ and $\Gamma[\cdot]$ the complete Gamma function (Abramowitz
647 and Stegun, 1965). Notice that this pdf is only attained in the limit of a pro-
648 cess always at equilibrium, which is not the case where all dynamics have
649 comparable time scales. Equation (19) does not allow the statistics of the
650 distribution to be computed analytically. Therefore, both the average and
651 standard deviations of the distribution have been estimated numerically.

652 Figure 8 shows the dependency of the pdf of channel width at the steady
653 state from different parameters of the system, including the average fre-
654 quency with which floods occur, λ , the colonisation rate, $r1$, and the constant
655 accounting for plant roots σ_1 . By increasing the average frequency, λ , the
656 coefficient of variation, C_v , decreases since the discharge mean, μ_Q , linearly
657 increases faster than the variance of the signal, σ_Q^2 - Figure 8a. As a con-
658 sequence, high floods frequencies induce, on average, wider channels with a
659 distribution presenting larger variance, σ_w^2 , compared to that associated to a
660 river characterised by sporadic floods, i.e. low λ values (Figure 8b).

661 As already observed in the deterministic part of the analysis, the colonisa-
662 tion rate does not affect channel width which is controlled by the strength of
663 plant roots. Different rates of colonisation do not change the average channel

664 width and have little influence on the overall probability distribution (Fig-
665 ure 8c). Conversely, the channel adjusts to lower average channel width and
666 presents a more peaked and narrow distribution when plant roots are efficient
667 in trapping and retain the bank sediments, i.e. high value of the constant σ_1 ,
668 in accordance with field observations (Allmendinger et al., 2005) (Figure 8d).
669 We finally explore the role of the channel bank erodibility, E that shows, as
670 expected, an opposite behaviour compared to that of plant roots. Low chan-
671 nel bank erodibility determines more peaked distribution, i.e. lower variance,
672 and lower mean values, when compared with the case of river with highly
673 erodible channel banks. While the variance increases as the erodibility of the
674 channel banks increases, increasing E above 200 slightly modifies the average
675 channel width. As for the periodical case presented above, changes in the
676 average frequency λ with which floods occur (for a given value of τ) control
677 the amplitude of the oscillations of the signal for channel width and biomass.
678 However, a more interesting and less trivial behaviour emerges when the sys-
679 tem is forced with a stochastic signal. With this purpose we performed a
680 Fourier analysis of channel width temporal variations away from the tran-
681 sitions due to the influence of initial conditions, paying attention to have a
682 signal that extends at least around 5 times the characteristic time scale of
683 the biomorphodynamic process investigated (t_v). Variations in channel width
684 were analysed on the frequency domain by computing the Power Spectrum.
685 The structure of the signal emerging from the frequency analysis was visual-
686 ized through a fitting curve computed as the Fourier series of the first eight
687 frequencies given by $w_{Fourier} = a_0 + \sum_{i=1}^8 [a_i \sin(\omega_i t) + b_i \cos(\omega_i t)]$, where a_0
688 is the average value around which the channel width oscillates at the steady

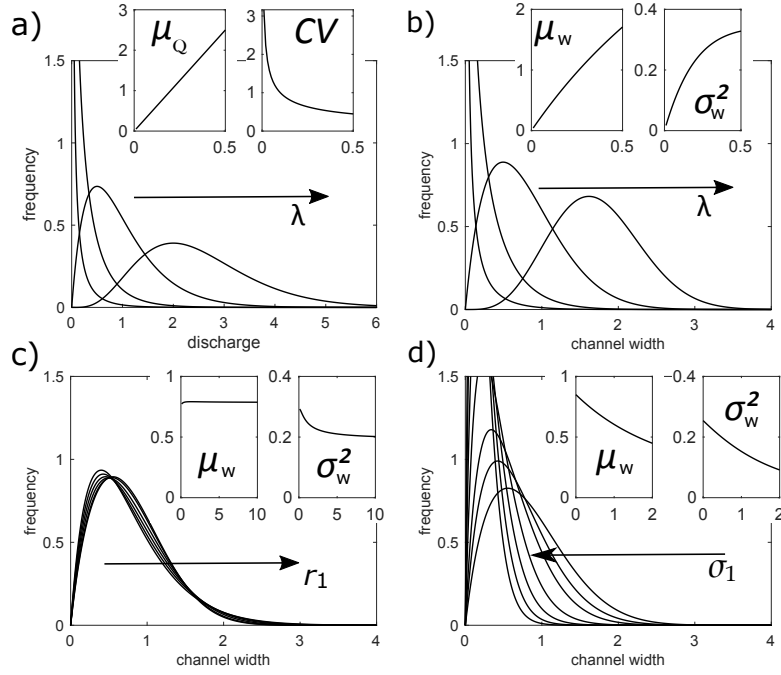


Figure 8: Behaviour of the probability density distribution for the equilibrium channel width, p_w , of a vegetated channel with erodible banks and forced with a CCP characterised by a distribution p_Q . The two upper panels show the dependency of both a) p_Q and b) p_w from the average frequency λ with which the floods occur. Panel c) and d) show, respectively, the dependency of p_w on the colonisation rate of the vegetation, r_1 and the increased resistance to sediment erosion due to plant roots, σ_1 . The close ups present the average value, μ_x and variance, σ_x^2 associated with the distribution p_x . Other relevant parameters: $E = 100$; $\tau = 10$ day; $\mu_Q = 0.5$ m^3/s ; and when not modified $\lambda = 0.2$ day^{-1} ; $r_1 = 2$; $\sigma_1 = 0.2$.

689 state. Finally, we estimate the autocorrelation function of the signal to inter-
690 pret any changes in correlation, i.e. memory of the process, associated with
691 the mutual interaction between vegetation dynamics and river processes.

692 Results show that when the average frequency λ is big (low value of vari-
693 ation coefficient C_v) the forcing signal is weak and channel width oscillates
694 without a clear pattern (Figure 9a), whereas for low frequency values (high
695 value of variation coefficient C_v) the hydrograph presents sporadic almost
696 uncorrelated peaks that generate a narrower channel nevertheless width os-
697 cillations do not present a regular organization (Figure 9c). However, there
698 exists a range of intermediate frequencies for which the system seems to re-
699 cover a more regular (i.e. coherent) variation of the channel width (Figure
700 9b). This appears more clearly if we consider the signal in the frequency
701 domain and analyse its power spectrum (right column Figure 9). In this new
702 domain the intermediate frequency (Figure 9b) shows a spectrum peaked
703 around $\omega = 7$ while high frequencies present a quite flat spectrum (Figure
704 9a) and the very low frequencies present a spectrum that tends to become flat
705 again since several frequencies show high power. Therefore, only for inter-
706 mediate λ the system selects a dominant frequency and the response signal
707 to a random noise organizes according to a more regular oscillatory pattern.
708 The Fourier expansion of the signal, for the first eight modes, (red curve in
709 the central panels of Figure 9) visualizes the main oscillating structure for
710 the channel width temporal trajectory.

711 This process for which noise induces a coherent response in time is known
712 as coherence resonance and it is generated from the interaction of the noise
713 with an intrinsic time scale of the deterministic component of the dynamics

714 system (Ridolfi et al., 2011). In this case the temporal scale of bank erosion
715 (or vegetation removing), t_b , associated with the Poisson process needs to
716 be compared with that of the deterministic vegetation growth, t_0 . When
717 floods occur with low frequency (i.e. $t_b \gg t_0$) the channel principally
718 narrows with sporadic, random, channel bank erosion that increases channel
719 width. Conversely, when the interval time between two consecutive floods is
720 low (i.e. $t_b \ll t_0$) the vegetation keeps being removed via bank erosion and
721 the channel undergoes predominantly widening with only random narrowing
722 processes that reduce the section. This sporadic occurrence of channel widen-
723 ing or narrowing with random intensities prevent the formation of a regular
724 pattern. However, when floods occur with an intermediate frequency, under
725 the condition $t_b \ll t_0$, the vegetation-related channel narrowing counter-
726 acts bank erosion and the response of the system shows a quasi-oscillatory
727 behavior.

728 Simulation runs conducted for different r_1 for a frequency below (0.01)
729 and above (0.03) the intermediate frequency $\lambda = 0.02$ reveal that the coloni-
730 sation rate itself is not able to reorganize the response of the system in a
731 regular structure (no significant difference in the power spectrum of the sig-
732 nal). However, the colonisation rate does influence the autocorrelation of the
733 signal with an abrupt decrease of the temporal integral scale, i.e. represen-
734 tative time for which the process loose its memory (autocorrelation becomes
735 null), as soon as the colonisation parameter r_1 becomes different from zero.
736 The time integral scale value decreases until a value of r_1 around 4 after
737 which higher values of the colonisation rate do not affect the autocorrela-
738 tion of the signal. Overall, the control on the channel width exerted by the

739 colonisation ratio is relevant only for low values of the parameter r_1 (say <
740 2). This is also supported by the analysis of the eigenvalues that showed how
741 the complex component of the eigenvalues is kept almost constant for value
742 of r_1 larger than 2.

743 **4. Model applications**

744 We applied the model to three study cases: the vegetated flume from the
745 work of Tal and Paola (2010), the restoration project of the Lunterse Beek
746 stream (Eekhout et al., 2014), and that of the River Thur, Switzerland. The
747 three cases were chosen because they represent three single thread reaches
748 whereby changes in channel width can be related to vegetation dynamics.
749 Since the initial configuration of the channel in the laboratory experiments
750 of Tal and Paola (2010) was a braided network, the model has been applied
751 only to the second phase of Run A where a stable single thread channel was
752 formed. Data from the field and laboratory were used to define the initial
753 channel characteristics while the parameter of the model were adjusted to fit
754 the measured valued for channel width in time. The values are reported in
755 Table 1.

756 Results from model applications to the cases of the run A from Tal and
757 Paola (2010) and the Lunterse Beek stream are shown in Figure 10. For
758 the application to the flume run, the model shows good agreement with
759 the observed evolutionary trend of the single thread reach formed in the
760 flume. The channel progressively narrows due to a net increase in vegetation
761 biomass, until it oscillates around an equilibrium channel width. In the
762 first 4 floods the modeled channel width quantitatively agreed with the wet

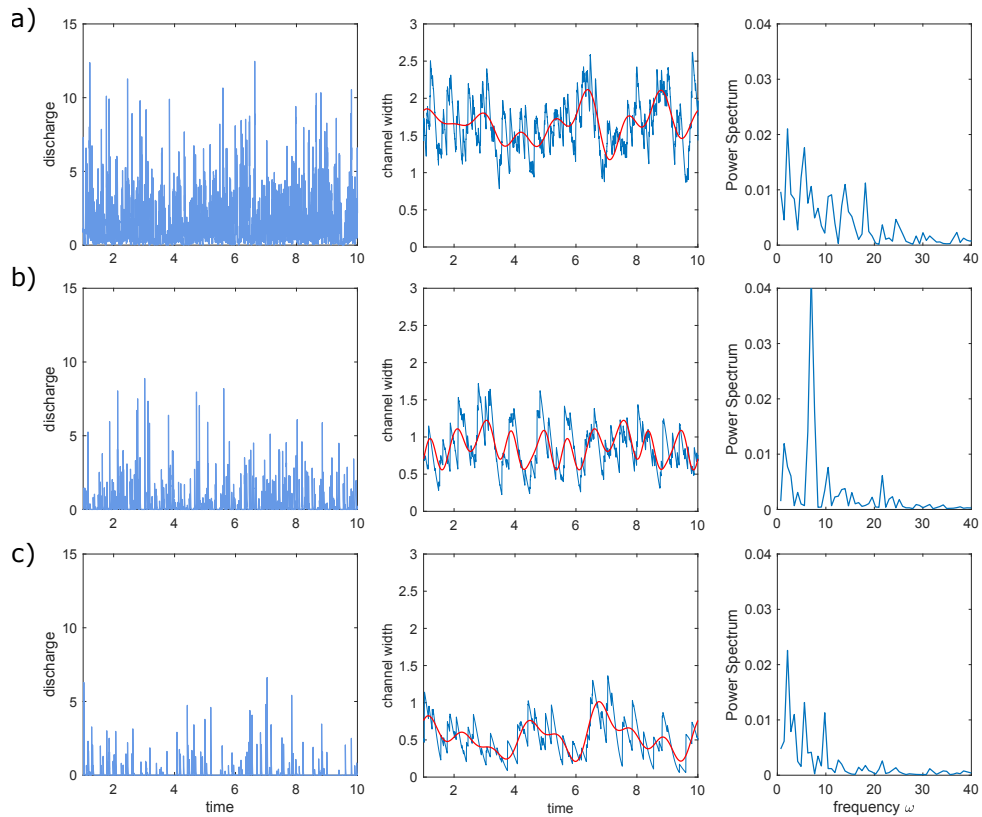


Figure 9: Behaviour of the system subject to a stochastic hydrograph with $\tau = 10$ day, $\mu_Q = 1.5$ m³/s and $\lambda =$ a) 0.08, b) 0.02, and c) 0.008 day⁻¹. On the left column is reported the stochastic hydrograph, on the central and right columns are presented, respectively, the channel width variations and width signal spectrum associated to the hydrograph on the left. The red continuous line superimposed to the channel width signal on the central panels is the Fourier expansion of the signal computed using the first eight frequencies ω reported on the spectrum panel. Other relevant parameters: $E = 200$; $r_1 = 2$; $\sigma_1 = 0.2$.

763 width measured during the experiment, while for the second part of the
764 experiment run the model generates larger channel width variation than those
765 measured in the flume (Figure 10a). For the case of the Lunterse Beek
766 stream, the model results are compared with both the channel condition
767 documented by photographs from different dates (Eekhout et al., 2014) and
768 the measurements of channel width (Vargas-Luna et al., 2016). After an
769 initial increasing in channel width associated to occurrence of floods around
770 100 days from the start of the observation period, the channel underwent
771 narrowing. The subsequent period of low flow allowed the vegetation to grow,
772 restricting the channel cross section until other floods occurred around day
773 400 widening the channel. However, the increase in water flow discharge was
774 not enough to entirely remove the vegetation that after 500 days could grow
775 narrowing the channel again. Both channel width and vegetation dynamics
776 are well described by the model (Figure 10b). Differently from the case of the
777 flume experiment, in this case the whole predicted evolutionary trajectory
778 shows a good quantitative agreement with the field measurements and field
779 observations, i.e. historical images.

780 Finally we study a 1.5 km section of the Thur River, Switzerland, that
781 was restored in 2002 to promote the formation of fluvial bars and increase the
782 riparian environment biodiversity. To this aim, river managers and engineers,
783 removed bank protection allowing the river to adjust its channel width that
784 was previously set to 55 m. The active channel width has been estimated from
785 the sequence of aerial images covering a period from 2002 to 2018 (Figure 11
786 by taking an average of the position of the banks (red lines in the pictures
787 in Figure 11) and neglecting the local increase in channel width due to the

788 meandering of the channel. The channel width was accounted as the portion
789 of bare channel bed that could have been reworked during a flood.

790 Model results show that the first floods occurring after the restoration of
791 the channel at the end of year 2002 widen the channel by setting its average
792 channel width to a value almost twice the initial one, i.e. 100 m. In the
793 following three years, the channel narrows to a width of around 90 m, with
794 a flood occurring at the end of the summer 2005 that sets back the channel
795 banks recovering a width of 100 m. This event slightly influences vegetation
796 dynamics and vegetation biomass keeps growing during the following years
797 narrowing the channel. Low magnitude floods generate marginal variations
798 in channel width and vegetation biomass until the end of the summer 2013.
799 Here the channel experiences the biggest flood and bank erosion increases
800 the channel width to 112 m, by reducing the vegetation biomass slightly
801 above 0.6. In the following years, the absence of significant floods allows the
802 vegetation to grow and the channel width recovers to a value of around 80
803 m. The comparison of model results with the images collected from different
804 dates shows a quantitative agreement for the channel width. In addition,
805 the model catches the temporal dynamics of vegetation biomass. From the
806 images it is possible to observe a low presence of vegetation at the end of
807 2005, and vegetation encroachment on the exposed bar in September 2008
808 that grew extending the vegetated portion of bar as observed in the picture
809 from 2009. Starting from 2013 vegetation progressively grows reaching a
810 coverage in 2016 that remain almost unchanged in 2018 (see images in Figure
811 11).

812 Vegetation encroachment onto river bars initiates pioneer morphological

813 features that can evolve in mature morphological structures, e.g. islands,
814 stable banks or floodplain. Whether vegetation succeeds in colonising the
815 exposed sediment and grow or it is uprooted before increasing sediment co-
816 hesion depends on both river flow and morphology. Therefore, depending on
817 the development stage of vegetation growing on river banks or bars, a flood
818 may be able to remove vegetation and rework the channel bed (e.g. channel
819 widening), or depositing fine sediments promoting vertical accretion of bar
820 or bank advance (i.e channel narrowing). According to this concept the evo-
821 lutionary trajectory of a river is seen as a sequence of stable and unstable
822 phases resulting from the mutual influence between water flow, vegetation
823 and sediments. The switch between one phase to another is controlled by the
824 water flow and its effect is moderated by the vegetation which is able to en-
825 gineer the river channel. Therefore, a river system remains in a stable phase
826 until a flood competent for reworking the channel bed occurs, leading to an
827 unstable condition and, at the same time, starting a new cycle (i.e colonised
828 exposed sediments) that will generate a new stable phase (Corenblit et al.,
829 2014).

830 The simple model proposed provides satisfactory results when applied to
831 the three real cases, with more accurate results for the real channels rather
832 than the laboratory one. In particular, when compared with the labora-
833 tory observations the model overestimated the variation in channel width
834 but was capable to correctly predict the overall evolutionary trajectory ob-
835 served in the flume, with a Pearson correlation coefficient between observed
836 and modelled data equals to 0.69. Results from the application to real scale
837 reaches showed better agreement between the values predicted by the model

838 and those extracted from the field with a Pearson correlation coefficient esti-
839 mated of around 0.53 and 0.84 for the case of the LunterseBeek stream and
840 the Thur River respectively. Despite the low correlation value shows by the
841 LunterseBeek case, Figure 10b shows the capability of the model to correctly
842 interpret the evolution and the magnitude of the changes, both for channel
843 width and vegetation biomass, as it was observed in the months following
844 the restoration project of the stream. The comparison between modelled and
845 observed channel width for the three study cases is reported in Figure 12.

846 The analysis conducted for the case of the river system subject to a
847 stochastic hydrological forcing helps us to interpret how changes in vegetation
848 characteristics, or river hydrology, may affect the overall channel width for
849 the three study cases presented above. In particular, results reported in Fig-
850 ure 8 show that the colonisation process does not affect river channel width,
851 while the increased resistance to sediment erosion provided by plant roots, i.e.
852 high σ_1 , significantly narrows the channel cross-section. On the other hand,
853 the colonisation rate, r_1 , was found to control the vegetation biomass, with
854 higher values of r_1 determining lower values of vegetation biomass for the
855 effect of increased erosional power due to a narrower channel cross-section.
856 Finally, an increase in flood frequency, λ , promotes the formation of larger
857 channels by augmenting the intensity of the near-bank erosion processes.

858 It is therefore evident that application of the model requires a preliminary
859 calibration of the parameters against observed data. This is due to a lack of
860 physical relationships describing the interaction between sediment processes
861 and vegetation biomass in the literature. The development of physical rela-
862 tionships linking vegetation biomass to sediment dynamics would allow the

863 parameter of the model to be estimated on the base of measurable property
864 of the river system, and the model to be universally applied. This prob-
865 lem affects also spatially distributed models for river morphodynamics. On
866 one hand, such models show the advantage of being able to provide a more
867 detailed description of the flow field and sediment dynamics, including bar
868 deposits, thanks to a stronger physical basis for sediment processes. On
869 the other hand, by adopting relationships for vegetation dynamics similar to
870 those we adopted in our model, they still require a preliminary calibration of
871 the parameters when investigating bio-morphological processes (e.g. Bertoldi
872 et al. 2014; Oorschot et al., 2016; Zen et al., 2017; Caponi and Siviglia, 2018).
873 In addition, this type of model requires considerable computational and time
874 efforts to obtain the final result.

875 Model results have provided evidence that the model developed is able
876 to capture the essential behaviour of the system and could be used, once
877 calibrated against real observations, to predict long term river evolution with
878 extremely low computational effort. The lump model could be used to inform
879 a spatially distributed model for river morphodynamics, such that results
880 from the former would help choosing the input parameters for the latter by
881 pre-selecting river future evolutionary trajectories.

882 **5. Conclusions**

883 We proposed a simple bio-morphodynamic model to investigate the tem-
884 poral scales of channel width variation and how these relate with that of the
885 hydrologic forcing.

886 The model is in the form of a dynamical system of two non-linear ordinary

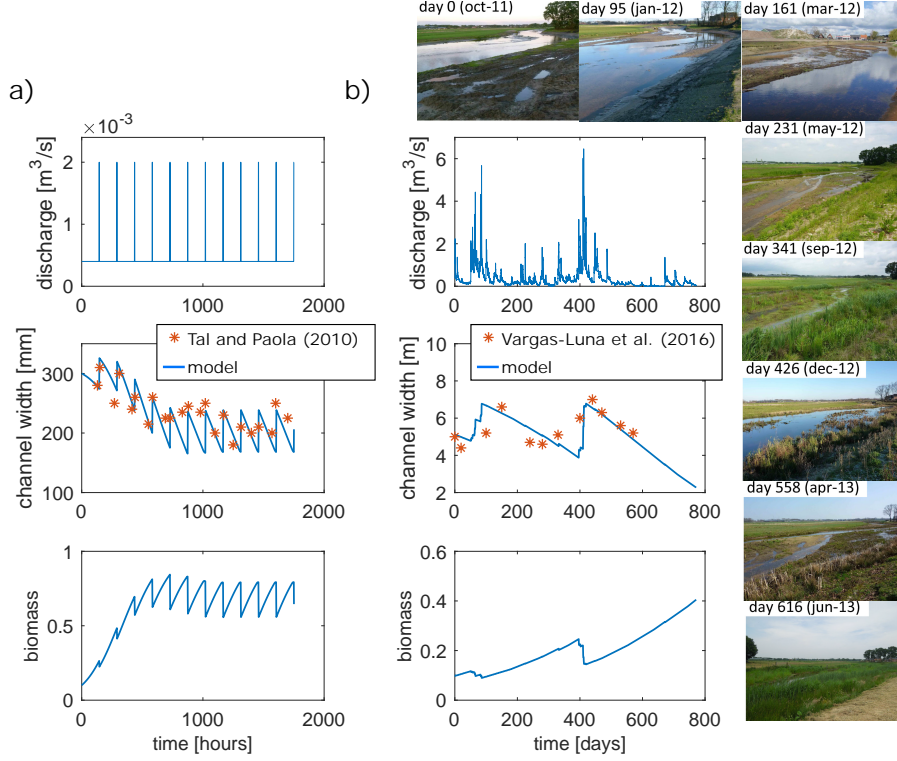


Figure 10: Results from model application to the study case of a) the laboratory run A of Tal and Paola (2010) and b) the Lunterse Beek (Eekhout et al., 2014). The channel width measurements associated to this latter case refer to Vargas-Luna et al. (2016) while the hydrograph can be found at <https://www.joriseekhout.com/publications/>.

	Q_0^*	w_0^*	S	d_s^*	t_v^*	E^*	r_1	σ_1
	$[m^3/s]$	$[m]$	$[m/m]$	$[m]$	$[years]$	$[m/s]$	$[-]$	$[-]$
Lab flume	$4 \cdot 10^{-4}$	0.3	0.015	$0.5 \cdot 10^{-3}$	2	$9.5 \cdot 10^{-7}$	0.45	0.5
Lunterse Beek	1.4	5	$0.96 \cdot 10^{-3}$	$2.58 \cdot 10^{-4}$	4	$1.58 \cdot 10^{-5}$	3	0.3
River Thur	200	55	10^{-3}	0.02	5	$7.3 \cdot 10^{-3}$	1.5	0.04

Table 1: Model input and coefficients for the application to the flume from the experimental run of Tal and Paola (2010), the Lunterse Beek (Eekhout et al., 2014), and the River Thur (Pasquale and Perona, 2014; Schirmer et al., 2014).

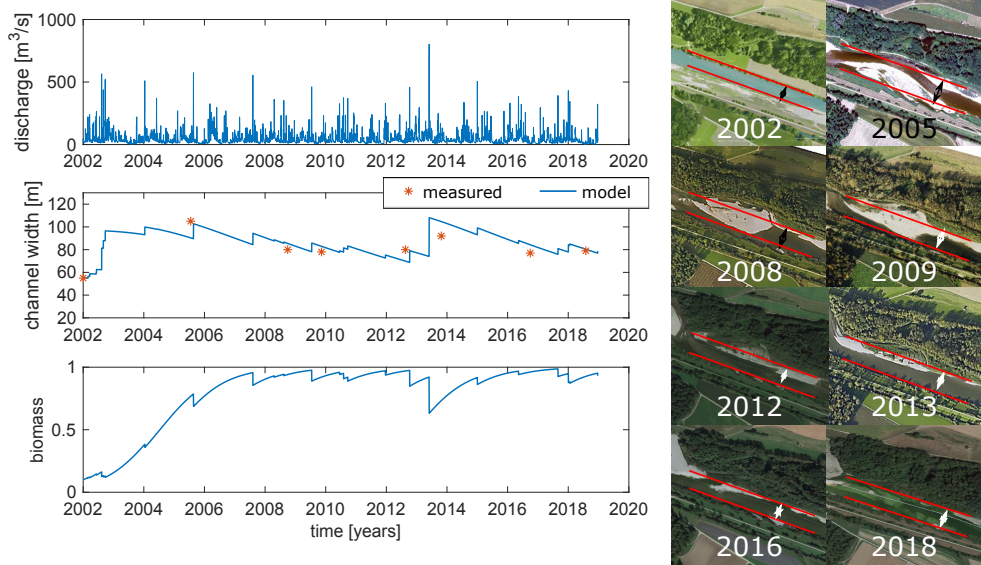


Figure 11: Results from model application to the study case of the Thur River. The exact dates the pictures were taken are: 30/09/2008, 10/11/2009, 19/08/2012, 24/10/2013, 30/09/2016/31/07/2018. The date associated to the pictures from 2002 and 2005 is not known. The estimated channel width, in chronological order, is: 55 m, 105 m, 80 m, 78 m, 80 m, 92 m, 77 m, 80 m. Sources Google Earth.

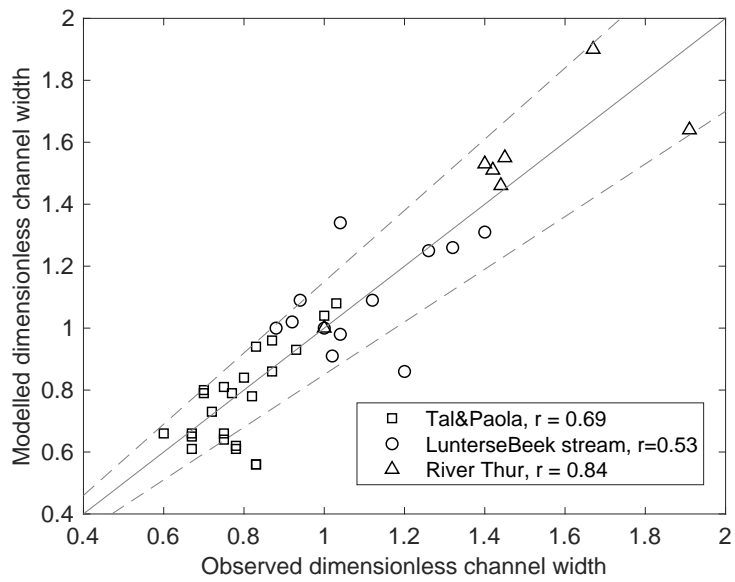


Figure 12: Comparison between observed and modelled channel width for the three study cases: the experimental run of Tal and Paola (2010), the Lunterse Beek (Eekhout et al., 2014), and the River Thur (Pasquale and Perona, 2014; Schirmer et al., 2014). The values are reported in dimensionless form, r is the Pearson correlation coefficient, the solid line is the line of equality, and the dashed lines represent the borders of the 15% confidence interval.

887 differential equations to mimic the interaction between riparian vegetation
888 dynamics and river flow in controlling river channel width temporal adjust-
889 ments. Two distinct mechanisms are considered for channel narrowing and
890 widening occurring at different time scales: the channel narrows because of
891 the vegetation encroachment onto the river bar and widens when the shear
892 stress allows the river bank to be eroded. Different from previous model
893 adopting simplify closure to describe the stabilizing action of the vegetation
894 on the bank (Eke et al., 2014; Lopez Dubon and Lanzoni, 2018; Monegaglia
895 et al., 2019) in this work channel narrowing is directly associated to vegeta-
896 tion dynamics. Furthermore, to fully couple bank and vegetation dynamics
897 vegetation decays during the erosion of the bank and a linear relationship
898 links increases in the critical Shields number for sediment movement with
899 the vegetation biomass.

900 The obtained results have highlighted the limit of adopting simplified
901 discharges in morphodynamic models or regime models to interpret realistic
902 response of the channel. We argued that the use of a constant discharge
903 in morphodynamic models that neglect the root-induced sediment cohesion
904 may overestimate the channel width when compared with real channel cross
905 sections. In addition, the system forced with a high-frequency periodic dis-
906 charge reduced both channel width and vegetation biomass, while increase
907 in flood frequency should generate wider cross sections.

908 The variable-discharge simulations verify that a vegetated channel does
909 not reach a final equilibrium but, because of the cyclical repeat of channel
910 widening and narrowing, keeps oscillating around an asymptotic value as
911 forced by the water flow. Furthermore, these simulations revealed the exis-

912 tence of an intermediate frequency of floods occurrence for which stochastic
913 bank erosion and deterministic vegetation growth interact leading to a co-
914 herent response of the system in time.

915 The satisfactory results obtained from model application have revealed
916 the potential of the model to be used to interpret the evolutionary trajectories
917 of a channel, once the model parameters have been opportunely calibrated.
918 Because of the high uncertainty affecting the choice of the parameters as-
919 sociated to vegetation dynamics, we have explored how model results are
920 affected by changes in the parameters r_1 and σ_1 accounting, respectively, for
921 the colonisation process and roots action on sediment erosion. We argued
922 how such a problem also affects spatially distributed models for river mor-
923 phodynamics, which able to provide a more detailed description of in-channel
924 flow field, when investigating bio-morphological interactions.

925 The present work takes advantage of the minimalist approach and the di-
926 mensionless form of the system to explore the interaction between vegetation,
927 water flow and river morphology time scales, by using a low computational
928 effort. The analysis is a first step to include a stochastic dynamic paradigm
929 in a bio-morphodynamic model for river evolution whereby bank properties
930 are directly related to vegetation dynamics which in turn are controlled by
931 both channel morphology and water flow. At the state of the art, model
932 results could inform physically based bio-morphodynamic models for river
933 evolution to optimize the modelling resources. However, in order to obtain
934 quantitative tools that can also support river managers decisions, research
935 effort is required to quantify the increased resistance to sediment erosion in-
936 duced by the plant root system. This will also allow model parameters to be

937 estimated on based on measurable properties of the system rather than to
938 be calibrated.

939 The model can be included in morphodynamic models for meandering
940 rivers evolution to overcome the limit posed by the simplified closure de-
941 scribing bank deposition and further extend the results obtained by Zen et al.
942 (2016) and Davidson and Eaton (2018). The updated model could be used to
943 investigate the temporal scales of lateral migrating meander bends and relate
944 them to the hydrologic forcing and spatial scales of scroll bars formation in
945 meandering rivers floodplain (Zen et al., 2017; Strick et al., 2018).

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948 that contributed to improve the quality of the paper.

949 **Appendix A: the linear system**

950 The linearized version of the system (18) obtained after expanding in
951 series the three unknown of the problem, $w, \rho, \tilde{\tau}_\Delta$, around their equilibrium
952 values $w_{eq}, \rho_{eq}, \tau_{\Delta,eq}$ reads:

$$\begin{aligned} a_{11}w_1(t) + a_{12}\rho_1(t) + a_{13}\tilde{\tau}_{\Delta,1}(t) + \frac{dw(t)}{dt} &= 0 \\ a_{21}w_1(t) + a_{22}\rho_1(t) + a_{23}\tilde{\tau}_{\Delta,1}(t) + \frac{d\rho(t)}{dt} &= 0 \\ a_{31}w_1(t) + a_{32}\rho_1(t) + a_{33}\tilde{\tau}_{\Delta,1}(t) + \frac{d\tilde{\tau}_\Delta(t)}{dt} &= 0, \end{aligned} \tag{A.1}$$

953 where the coefficients a_{ij} with $i,j = 1,2,3$ are:

$$\begin{aligned}
a_{11} &= \gamma\rho_0; & a_{12} &= w_0\gamma; & a_{13} &= -E; \\
a_{21} &= -\frac{E\rho_0\tilde{\tau}_{\Delta,eq}}{w_0^2}; & a_{22} &= -1 + 2\rho_0 + \frac{E\tilde{\tau}_{\Delta,0}}{w_0}; & a_{23} &= \frac{E\rho_0}{w_0}; \\
a_{31} &= \frac{2t_{\Delta}\left(\frac{Q}{w_0}\right)^{2/3}\tau_{s,0}\tilde{\tau}_{\Delta,0}}{3w_0}; & a_{32} &= t_{\Delta}\tau_{c,0}\tilde{\tau}_{\Delta,0}; & & \\
a_{33} &= t_{\Delta}\left[\tau_{c,0}(1 + \rho_0) - \left(\frac{Q}{w_0}\right)^{(2/3)}\tau_{s,0} + 2\tilde{\tau}_{\Delta,0}\right]; & & & &
\end{aligned} \tag{A.2}$$

954 having denoted with $\{w_0, \rho_0, \tau_{\Delta,0}\}$ a stable condition of the system, e.g. the
955 initial bankfull condition.

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