

Biased Randomised Heuristics for Location Routing Problem



Abdullah Ibraheem Almouhanna

A thesis submitted for the degree of
Doctor of Philosophy

Faculty of Technology
Department of Mathematics
Logistic and Operational Research and Analytics Group

January 2019

Declaration

Whilst registered as a candidate for the above degree, I have not been registered for any other degree award.

The results and conclusions embodied in this thesis are the work of the named candidate and have not been submitted for any other academic award.

Abdullah Ibraheem Almouhanna
January 2019

Acknowledgements

My PhD study at the University of Portsmouth has been a wonderful learning experience that I will never forget. It would not have been possible to successfully complete it without personal and professional support, that I have received from many people throughout my PhD journey. Now, it is the time, and it is my chance to extend my gratitude to all of them.

Firstly, I am sincerely grateful to my supervisors Dr. Banafsheh Khosravi and Prof. Djamila Ouelhadj for their invaluable guidance, valuable support, motivation, encouragement, insightful comments, enthusiasm and continuous guidance during the development of this thesis. This study has benefited greatly from the direction and support they have provided over the years. They were always open to discussing and clarifying different concepts.

I would, also, like to thank Prof. Angel A. Juan and Dr. Carlos Quintero-Araujo for their generous, help, and support in the development of different parts of this work.

Also, I would like to thank all of my friends at the Department of Mathematics for making me feel at home while studying my PhD. I am very fortunate to have such true friends who were ready to support me through the most difficult times.

Finally, I would express my heartfelt gratefulness to my mother, my wife, and my children. Without their support and encouragement, I would not have had the ability to finish my study.

At the end, I cannot find the words to express my gratitude to all of you, one by one, to make this dream a reality.

Abstract

In this research work, we consider various classes of the Location Routing Problem namely: (1) Location Routing Problem with Single Depot (LRPSD), (2) Multi-Depot Vehicle Routing Problem (MDVRP), (3) Location Routing Problem with Multi-Depots (LRPMD), and (4) Green Location Routing Problem with Multi-Depot (G-LRPMD). These problems are NP-hard in terms of computational complexities. The interdependence between facility location and vehicle routing has been recognised by practitioners and academics. The LRP is to integrate these two decisions and solve them simultaneously. As both the facility location and the vehicle routing are NP-hard, LRP is also NP-hard. Thus, exact methods are limited to solve the LRP of a practical size. Alternatively, heuristics and metaheuristics have been applied to solve more realistic problems.

Biased Randomised technique has been combined with heuristics to successfully solve the Facility Location Problem (FLP) and the Vehicle Routing Problem (VRP) due to its simplicity, efficiency, and it is a parameter-free heuristic. However, to the best of our knowledge, it has not been combined with a heuristic to solve the LRP.

In this thesis, we have proposed four Biased Randomised heuristics to solve LRPSD, MDVRP, LRPMD and G-LRPMD. Moreover, we have developed a Biased Randomised Variable Neighbourhood Search (BR-VNS) metaheuristic in collaboration with our collaborators at the Internet Interdisciplinary Institute (IN3) in the Universitat Oberta de Catalunya in Spain and Universidad de La Sabana in Colombia.

The first solution method is to solve the LRPSD by a heuristic with four variations. Each variation of the heuristic solves the location by one of the following solution methods namely: clustering, p-median, clustering and p-median, and iterative method. However, in all variations of the heuristic, the routing decisions are made by combining Biased Randomised technique with Clark and Wright heuristic (CWH).

The second solution method is developed to solve the MDVRP by combining Biased Randomised technique with Extended Clark and Wright heuristic (ECWH), which we called Biased Randomised Extended Clark and Wright heuristic (BR-ECWH). The LRPMD is solved by extending the BR-ECWH to consider location decision. Finally, the G-LRPMD is solved by adapting the LRPMD solution method to include constrained distance.

The effectiveness of the suggested solution methods are tested by conducting extensive computational experiments using data sets from the literature. These data sets have many different sizes (such as number of customers, and number of depots) and characteristics (such as distribution of customers, and capacity of depots and vehicles) and are then compared to the best-known solutions in the literature.

The computational analysis indicates the efficiency of the proposed solution methods. The suggested solution methods are also shown to be flexible to solve other classes of the LRP.

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Glossary of symbols and abbreviations

ACO	Ant Colony Optimisation
ACO	Ant Colony Optimization
ALNS	Adaptive Large Neighbourhood Search
BR-CWH	Biased Randomised Clarke and Wright Heuristic
CAOA	Cellular Ant Optimisation Algorithm
COPs	Combinatorial Optimization Problems
CWH	Clark and Wright heuristic
EA	Evolutionary Algorithm
ECWH	Extend Clark and Wright Heuristics
ERUs	Emergency Response Units
EVS	Evolutionary Local Search
EVs	Electric Vehicles
FFA	Ford and Fulkerson Algorithm
FLP	Facility Location Problem
GA	Genetic Algorithms
GCA	Genetic Clustering Algorithm
GHG	Greenhouse-Gas
GIS	Geographical Information System
GRASP	Greedy Randomised Adaptive Search Procedure
GTS	Granular Tabu Search
GVTNS	Granular Variable Tabu Neighbourhood Search
HBMOA	Honey Bees Mating Optimisation Algorithm
HCSA	Hybrid Clonal Selection Algorithm
HESA	Hybrid Electromagnetism Simulated Annealing
ICEVs	Internal-Combustion-Engine Vehicles
ILS	Iterated Local Search
ISOE	Institute for Social-Ecological Research
LRGTS	Lagrangian relaxation with Granular Tabu Search
LRP	Location Routing Problem
LRPDC	Location Routing Problem with Distance Constraint
LRPMD	Location Routing Problem with Multi-Depot
LRPSD	Location Routing Problem with Single Depot
MA	Memetic Algorithm
MACS	Multiple Ant Colony System
MAPM	Memetic Algorithm with Population Management
MCS	Monte Carlo Simulation
MD-TSP	Multi-Depot Traveling Salesman Problem
MDVRP	Multi-Depot Vehicle Routing Problem
MDVRP-IDR	Multi-Depot Vehicle Routing Problem with Inter-Depot Routes

MPNS	Multiple-Phase Neighbourhood Search
PA	Path Relinking
PHLRP	Partitioning-Hub Location Routing Problem
PLRP	Periodic Location Routing Problem
PMDVRP	Periodic Multi-Depot Vehicle Routing Problem
PSO	Particle Swarm Optimisation
PVRP	Periodic Vehicle Routing Problem
SA	Simulated Annealing
S-DVRP	Site-Dependent Vehicle Routing Problem
STSP	Scatter Tabu Search Procedure
TS	Tabu Search
TSBRH	Two-Stage Biased Randomised Heuristic
TSP	Traveling Salesman Problem
VLNS	Very Large Neighbourhood Searches
VNS	Variable Neighbourhood Search
VRP	Vehicle Routing Problem

Chapter 1 Introduction

1.1 Background

Supply chain constitutes one of the main activities that influence the growth of the national economy and society, as it is a vital link between suppliers and customers. Essentially, the supply chain is concerned with flow of materials, services, and information from sources to customers. According to Pishvaei, et al. (2009), in the USA and the UK, 10.5% and 10.6% of the Gross National Product was accounted for by distribution systems, respectively. In European economy, the total annual expenditure on logistics services was 930 billion EUR (Rantasila & Ojala, 2012).

However, supply chain activities also generate huge economic costs and can have a negative impact with regard to the environment. For the economic costs, with the increase in numbers of products being manufactured by companies, supply chain costs will also rise. According to Falsini et al (2009), in average, transportation weights almost 50% of the total supply chain costs. Warehouse costs are about 23.5% of supply chain costs. In terms of the environmental aspects, transportation in supply chain plays an important role in the generation of CO₂ and greenhouse-gas emissions and related externalities, such as air pollution, noise, and traffic congestions (Juan et al, 2016) . Moreover, road transportation alone is responsible for about 18% of total GHG emissions in the EU (Hill et al., 2012).

In the practical world, decisions of location of depots and distribution of goods from these depots are challenging aspects of supply chain. That is because there are several situations which require the optimal location of several depots from which delivery routes originate, servicing a set of customers. This kind of problem is known as the Location Routing Problem (LRP).

The LRP is a popular combinatorial optimisation problem. It is related to both the Facility Location Problem (FLP) and the Vehicle Routing Problem (VRP). Both problems can be viewed as special cases of the LRP. If we require all customers to be directly linked to a depot, the LRP becomes an FLP. If, on the other hand, we fix the depot locations, the LRP reduces to a VRP.

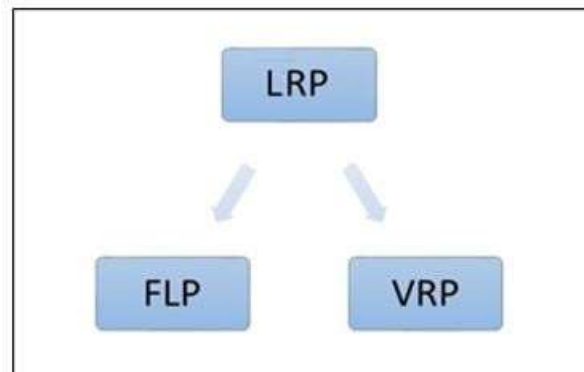


Figure 1.1 LRP and its relation to FLP and VRP

The LRP involves the simultaneous location of depots, assignment of customers to depots, and the determination of their related routes based upon a set of costs, distances, and capacity criteria. Depot location and vehicle routing decisions, if taken independently of one another, may lead to highly sub-optimal planning results (Salhi and Rand, 1989).

Figure 1.1 shows the LRP and its relation to FLP and VRP only. The decision of location is considered as strategic decision, whereas the decision of routing is considered as operational decision.

In several supply chains, it is evident that the location of depots will influence the choice of customers to be included on specific routes, as well as the number and lengths of the routes. Therefore, ignoring routing in depot location decisions may lead to sub-optimal solutions (Salhi and Rand, 1989). Several authors, including Webb (1968), Sussams (1971), Wren and Holliday (1972) and Rand (1976) have noticed the interdependence between these two elements, the FLP and the VRP.

The LRP has evolved over the years with different variations. These different variants arise as a result of using different optimisation criteria such as: type of input data (deterministic or stochastic),

objective function (single objective function or multi-objective function), number of depots (single depot or multi-depots), number of vehicles (limited or unlimited), types of vehicles (homogenous or heterogenous), and constraints such as depot capacity, vehicle capacity, and tour limit (Constrained Distance). We consider four variants of the LRP in this thesis. They are: LRP with Single Depot, the Multi-Depot Vehicle Routing Problem (MDVRP), the LRP with Multi-Depot, and the LRP with Constrained Distance. These four problems are considered because they are very important in real life.

Since the late 60's, there was the perception that the optimal solution of facility location in supply chain is sub-optimal because it does not take routing into account. Several authors including Webb (1968), Sussams (1971), Wren and Holliday (1972), and Rand (1976) had noticed that there was a drawback in using classical location models for distribution planning problems. For instance, Wren and Holliday (1972) noticed that many algorithms that were developed in order to minimise distance between customers and depots, were not appropriate to the case where many customers could be visited in a single journey by the same vehicle. While the close relationship between depot location and optimal routing was underlined by Sussams (1971) as he has mentioned that "Efficient routing is inextricably bound up with depot sitting", and Rand (1976) agrees by the following statement that "Many practitioners are aware of the danger of sub-optimising by separating depot location from vehicle routing". Rand (1976) appreciates the difficulty of solving location routing problems when he stated "Unfortunately, it will rarely be practicable to determine depot locations using vehicle scheduling packages because of the tremendous increase in computational time".

Since the late 70's, facility location and routing have been jointly optimised. Some researchers have formulated and applied LRP models in real life problems. Or and Pierskalla (1979) developed an LRP model for health care by considering location of blood banks with VRP. Jacobsen and Madsen (1980) formulated the newspaper distribution problem in Denmark as an LRP. Nambiar et al. (1981) improved the efficiency of the natural rubber industry in Malaysian by using the LRP. Since then, the LRP plays a major role in both the academic and application field.

The benchmark data sets of the LRP did not appear until 2004 when Barreto (2004) generated the first data set of the LRP and its name is Barreto's data set. Then, Prins et al. (2006a) presented the second data set and it is called Prodhon's data set. While Akca et al. (2009) presented the third data set which is called Akca's data set. Barreto's data set contains a total of 17 instances with 2 to 15 possible depots and the number of customers ranging from 12 to 150, while Prodhon's data set involves a total of 30

instances ranging from 5-10 potential depots and 20-200 customers. Akca's data set includes 12 instances with 5 potential depots and 30-40 customers.

When the LRP emerged in the literature, the solution methods were most dominantly using heuristics. Later, researchers applied exact methods to solve small size problems that did not exceed 20 customers. Subsequently, metaheuristics have been used widely as they can solve more realistic problems in reasonable computational time. However, one of the disadvantages of metaheuristics is parameter fine-tuning.

1.2 Gaps in the literature

As we formerly mentioned, the interdependence between location and routing has been recognised by many researchers and decision makers. Also, we mentioned that LRP consists of the well-known two problems, FLP and VRP. These two problems are shown to be NP-hard. Therefore, the LRP belongs to the class of NP-hard problems.

In early LRP studies, a few heuristics have been developed to solve real life problems with realistic sizes (Jacobsen and Madsen (1980), and Perl (1983)). However, these heuristics do not investigate the solution space efficiently. This is due to the fact that heuristics, generally, get stuck in local optima. On the other hand, in general, heuristic methods have some advantages compared to the other solution methods, such as lower computational time and flexibility and simplicity to implement.

Later, metaheuristics such as Tabu Search (TS), Simulated Annealing (SA), Genetic Algorithms (GA), and Ant Colony Optimisation (ACO) and others have been employed to solve the LRP with high quality solutions (Tuzun and Burke (1999), and Prins et al., (2006)). The main advantage of using these metaheuristics is to search the solution space more effectively. However, the process of searching the solution space may need higher computational time and more parameter fine-tuning.

These methods focus practically, on solution quality and computational time. Although, these two measures are undoubtedly important, there is a lack in the literature of introducing mathematical models and solution methods that concentrate on other important qualities such as simplicity of implementation, flexibility, and handling numerous side constraints that arise in practice. Therefore, these methods do not satisfy requirements of decisions makers in private and public sectors. As such,

there is a need to provide a fast, flexible, efficient method to tackle the LRP. Therefore, the focus of this research is to propose an efficient and fast solution method for the LRP.

Recently, the Biased Randomised technique has been used successfully to solve the VRP (Juan, et al. 2010) and FLP (Cabrera, et al. 2014). However, to the best of our knowledge, previous studies have not implemented it to solve the LRP.

Combining the Biased Randomised technique with classic heuristics improves the performance of heuristics by avoiding possible local optima. Moreover, it does not need any parameter fine-tuning. In addition, at the same time, it can produce alternative high-quality solutions with different properties in a reasonable computational time which gives a chance for the decision maker to choose the suitable solution among them. These methods are practical, efficient, and parameter-free.

1.3 Motivation

1.3.1 Motivation for Location Routing Problem

Improving the supply chain system provides both significant cost savings as well as improved productivity. A company can improve its productivity from 15% to 20% by improving the supply chain (Srivastava and Benton, 1990). This percentage can vary significantly from company to company, and industry to industry. Thus, there is a need for identifying, among other aspects of logistics, ways to lower the supply chain cost.

The two main elements in supply chain which cost more than 60% of total logistics costs, are location of depots (Facility Location Problem, FLP) and the distribution of goods (Vehicle Routing Problem, VRP) (Srivastava, 1993). This means better placed depot locations and better service routes are needed.

The FLP has been identified by several researchers, and several analytical models have been developed to solve it. In addition, the VRP has also been well researched in the literature, and several algorithms have been developed to solve it. Relatively few studies have been made for LRP that brings together the Facility Location Problem and the Vehicle Routing Problem.

These observations have motivated us to study four variants of the LRP that have an important application in the real life. These four problems are: the LRP with Single Depot, the MDVRP, the LRP with Multi-Depots, and the LRP with Constrained Distance. Table 1.1 shows some of applications of the LRP. It contains three columns: paper, optimisation problem, and country. The paper column includes names of authors and year of publishing, while the optimisation problem column includes the application sector in real life, and finally where the model applied is highlighted in the country column.

From a complexity point of view, these four problems are considered as NP-hard problems. Consequently, the size of the LRP restricts use of an exact method which means that heuristics are the best to solve them. On the other hand, heuristics can get stuck at local optima and we need to use a technique such as the Biased Randomised technique to improve their performance.

Paper	Optimisation Problem	Country
Watson-Gandy and Dohrn (1973)	Food and drink distribution	United Kingdom
Bednar and Strohmeier (1979)	Consumer goods distribution	Austria
Or and Pierskalla (1979)	Blood bank location	United States
Jacobsen and Madsen (1980)	Newspaper distribution	Denmark
Nambiar et al. (1981)	Rubber plant location	Malaysia
Perl and Daskin (1984)	Goods distribution	United States
Labbe and Laporte (1986)	Post-box location	Belgium
Semet and Taillard (1993)	Grocery distribution	Switzerland
Kulcar (1996)	Waste collection	Belgium
Murty and Djang (1999)	Military equipment location	United States
Bruns et al. (2000)	Parcel delivery	Switzerland
Chan et al. (2001)	Medical evacuation	United States
Lin et al. (2002)	Bill delivery	Hong Kong
Lee et al. (2003)	Optical network design	Korea
Billionnet et al. (2005)	Telecom network design	France
Gunnarsson et al. (2006)	Shipping industry	Europe
Ukkusuri and Yushimito (2008)	Humanitarian pre-position of supplies for natural disasters	United States
Ambrosino et al. (2009)	Location and delivery for wood distribution	Italy
Govindan et al. (2014)	Distribution of perishable food	Denmark
Park et al (2015)	Location of emergency units on freeways	United States

Table 1.1 Some applications of LRP in real life

1.3.2 Motivation for Multi-Start Biased Randomised technique

Biased Randomisation is a technique which can be integrated in a heuristic to provide an efficient mechanism to solve combinatorial optimisation problems. With this mechanism, a new feasible and potentially good solution is generated every time the procedure is executed.

This framework (integrated Multi-Start Biased Randomised technique with classical heuristics) has demonstrated to be very efficient for solving complex computational optimisation problems. The algorithms produced usually have a single configuration parameter or are even without parameter. This makes the time to deploy the algorithm in a real environment faster, as it avoids the long and complex fine-tuning phase which is usually required by other metaheuristics. Moreover, the results obtained from using this integrated framework are promising.

Recently, Multi-Start Biased Randomised technique has had several applications in combinatorial optimisation problems. Table 1.2 illustrates some of the applications of Multi-Start Biased Randomised technique in different combinatorial problems. It has two columns: paper, and optimization problem. Paper column includes names of authors and year of publishing, while optimisation problem column includes the optimisation problem's name.

Paper	Optimisation Problem
Belloso et al. (2017)	Vehicle Routing Problem with Clustered and Mixed Backhauls
De Armas et al. (2017)	Uncapacitated Facility Location
Belloso et al. (2017)	Fleet Size and Mix Vehicle Routing Problem with Backhauls
Mazza et al. (2016)	Mobile Cloud Computing in Smart Cities
De Armas et al. (2016)	Crew rostering problems in airlines
Dominguez et al. (2016)	Two-Dimensional Vehicle Routing Problem with Backhauls
Dominguez et al. (2016)	Two-dimensional loading vehicle routing problem with heterogeneous fleet
Ferrer et al. (2016)	Non-smooth flow-shop problems
Dominguez et al. (2015)	Two-dimensional Loading HFVRP with Sequential Loading and Items Rotation
Juan et al. (2015)	Multi-Depot Vehicle Routing Problem
Dominguez et al. (2014)	Two-dimensional vehicle routing problem with and without items rotation
Juan et al. (2014)	Flow-Shop Problem
Juan et al. (2013)	Non-smooth routing problems
Gonzalez et al. (2012)	Arc Routing Problem
Juan et al. (2010)	Capacitated vehicle routing problem
Agustin et al. (2016)	Airline crew scheduling
Herrero et al. (2014)	Vehicle Routing Problems with Asymmetric Costs and Heterogeneous Fleets
Carmona et al. (2014)	Optimisation of Aircraft Boarding Processes
Cabrera et al. (2014)	Facility Location Problem in Distributed Computer Systems
Gonzalez-Martin et al. (2014)	Non-smooth Arc Routing Problems

Table 1.2. Some applications of Multi-Start Biased Randomised Technique

This technique has been used with a Clark and Wright heuristic (CWH) which is one of the most known heuristics for the VRP. Also, it has been used with Iterated Local Search to solve FLP. However, to the best of our knowledge, there is no study in the literature which has applied Biased Randomised technique to a classical heuristic to solve the LRP. Therefore, to address the gap in the literature, this thesis will combine the Multi-Start Biased Randomised technique with Extended Clark and Wright Heuristic as a new method to solve the LRP.

1.4 Aims and objectives

This research is to study the LRP problem and how to offer a simultaneous solution method for the LRP using Biased Randomised technique in a nested framework. The main objective of the classic LRP is to minimise the sum of the opening cost of depots, the fixed cost of using vehicles, and the variables cost of vehicles' routing.

The advantage of using the Biased Randomised technique is to improve the performance of a classic heuristic. The new heuristic developed in this study is used to obtain not only one solution, but also many solutions through several iterations. These solutions have the same quality but different characteristics, which helps decision makers to choose the appropriate one. These characteristics are different open depots, different assigning customers to depots, and different assigning customers to routes.

To evaluate the performance of the new heuristic, the solution obtained is compared using the benchmarks in the literature.

In summary, the objectives to be achieved in this research are:

- (1) Build on existing optimisation models of the Location Routing Problem with Single Depot (LRPSD), the Multi-Depot Vehicle Routing Problem (MDVRP), the LRP with Multi-Depot (LRPMD), and the Green LRP with Multi-Depot (G-LRPMD).
- (2) Develop novel methods by combining the Biased Randomised technique with a classic heuristic to handle location and routing decisions simultaneously in the LRP, instead of solving the location problem and routing problem as separate problems. These methods provide effective and promising solutions, and also have a reasonable computational time for larger real-world problems.

(3) Evaluate the performance of the developed Biased Randomised heuristics. The solutions obtained for the Biased Randomised heuristic are evaluated against the benchmarks of the LRP with Single Depot, the LRP with Multi-Depot, and the MDVRP in the literature. For the LRP with Constraint Distance, there are no benchmarks in the literature, therefore, new benchmarks have been generated by modifying the benchmarks of the LRP with Multi-Depot.

1.5 Contribution

To achieve the objectives described in the aims and objectives section, a series of original contributions to the existing research are made. The most relevant ones are summarised as below and are explained in detail in the study.

1.5.1 The Location Routing Problem with Single Depot (LRPSD):

The LRPSD is the simplest variant of the LRP and there are several real-life applications of it such as system computer servers, and collection of money. We propose four Biased Randomised heuristics, namely, Biased Randomised Two-Stage Clustering (BR-TSCH), Biased Randomised Two-Stage p-median (BR-TSPH), Biased Randomised Two-Stage Clustering and p-median (BR-TSCPH), and Biased Randomised Iterated heuristic (BR-IH). The Biased Randomised technique was embedded in these four heuristics.

The experimental results showed that the Biased Randomised heuristics obtained competitive solutions in terms of quality and computational time.

1.5.2 The Multi-Depot Vehicle Routing Problem (MDVRP):

The LRP is a general case of Multi-Depot Vehicle Routing Problem (MDVRP). When the location decision has been made in a problem, LRP reduces to MDVRP which is helpful when we solve the LRP. For this reason, we combined Biased Randomised technique with a classical heuristic, which was proposed by Tillman in 1969 for MDVRP, and we call it Biased Randomised Extended Clark and Wright Heuristics (BR-ECWH). Then, we used the BR-ECWH in a Two-Level heuristic, called Two-

Level Biased Randomised heuristic (TLBRH) to solve the MDVRP. To the best of our knowledge, this combination has not been employed in the literature to solve MDVRP and this solution method for MDVRP is novel. TLBRH performance has been examined in comparison to the Best Known solution in the literature.

To validate the proposed algorithm, computational experiments are conducted on a benchmark set from the literature. The new method has been shown to be very successful in terms of computational time and solution quality. From the perspective of practicality, this algorithm is easy to implement as it has only one parameter.

1.5.3 The Location Routing Problem with Multi-Depot (LRPMD):

In this problem, we move a step ahead to use the Biased Randomised technique to solve the LRPMD. In order to do this, the location decision has been added to the TLBRH which was used to solve the MDVRP. Applying the new algorithm provides good results with excellent computational time. The advantage of this method is that it can be used as an alternative method when decision makers prefer solutions with acceptable quality in a reasonable computational time.

Moreover, we have developed a Biased Randomised Variable Neighbourhood Search (BR-VNS) metaheuristic in collaboration with our collaborators at the Internet Interdisciplinary Institute (IN3) in the Universitat Oberta de Catalunya in Spain and Universidad de La Sabana in Colombia. They are Prof. Angel A. Juan, and Dr. Javier Panadero from Universitat Oberta de Catalunya and Dr. Carlos Quintero-Araujo from Universidad de La Sabana in Colombia.

The experimental results show that both the Biased Randomised heuristic and the Biased Randomised metaheuristic obtained competitive solutions in terms of quality and computational time.

1.5.4 The Green Location Routing Problem with Multi-Depot (G-LRPMD):

Nowadays, supply chain has shifted to use electric vehicles to have greener solutions. Therefore, we propose a new model for LRP where electric vehicles are used, and it can be counted as a Green Location Routing Problem with Multi-Depot (G-LRPMD). However, electric vehicles have a distance limitation and we focus on investigating the LRPMD with constrained distance. The aim of proposing this model is firstly to promote the knowledge transfer to a real-life problem, and secondly to show the efficiency of our proposed model, the Biased Randomised heuristic, and Biased Randomised metaheuristic for this problem. We executed our model and algorithms on modified benchmark data from the literature review, as to the best of our knowledge there are no data sets for G-LRPMD in the literature. In terms of computational experiments, the results reveal promising improvements in terms of computational time and solution quality. This chapter is developed through collaboration work with our collaborators at the Internet Interdisciplinary Institute (IN3) in the Universitat Oberta de Catalunya in Spain and Universidad de La Sabana.

1.6 Thesis structure

This thesis studies the LRP model and related problems such as the MDVRP and the G-LRPMD and develops algorithms to solve them based on Biased Randomised technique combined with classical heuristics. To this end, the thesis is structured including the following chapters:

- Chapter 2 presents an overview of the LRP and MDVRP. This includes exact methods, heuristic approaches, and metaheuristics which have been used in the literature to solve different benchmark problems.
- Chapter 3 introduces four proposed Biased Randomised heuristics to solve the LRPSD.
- Chapter 4 proposes a novel Biased Randomised heuristic implemented on MDVRP. This heuristic is developed using Biased Randomised Extended Clark and Wright Heuristic to solve the MDVRP.
- Chapter 5 proposes a novel Biased Randomised heuristic and Biased Randomised metaheuristic to solve the LRP with Multi-Depot (LRPMD). The heuristic consists of a Two-

Stage Biased Randomised heuristic for LRPMD. The metaheuristic consists of a Biased Randomised heuristic to generate the initial solution and VNS to improve the initial solution.

- Chapter 6 presents a new variant of LRP with Constrained Distance, which is called Green Location Routing Problem with Multi-Depot (G-LRPMD). An attempt was made to examine the performance of our new algorithms on newly generated benchmark data sets.
- Chapter 7 summarises the main achievements of the study. This chapter presents the summary of results, general conclusions and limitations of the study. We also propose possible areas for further research.

In all chapters, the adaptation of some heuristics to solve the LRP and MDVRP and their constraints are considered. This study examines how the solutions to the LRP and MDVRP could be improved by integrating the Biased Randomised technique with a classical heuristic. These methodologies are tested with well-known benchmark problems available in the literature and the results are compared to those from other studies.

1.7 Conclusion

In this chapter, a comprehensive description of the problem addressed in this thesis has been provided. First of all, it presents an introduction, a general overview, and provides a background to the problem given. The gap in the literature has been presented after that to show the importance of this research to cover it. Then, the main two motivations of this study have been enumerated: motivation for the LRP, and motivation for Multi-Start Biased Randomised technique. Furthermore, three main objectives have been listed: building optimisation models of problems addressed in this thesis, proposing novel methods by combining the Biased Randomised technique with classic heuristics, and evaluating the performance of our proposed method by comparing our solutions against the benchmarks. Finally, our contributions, and a brief explanation of this thesis structure have been provided to give a clear picture about the whole thesis.

To understand the academic context of the optimisation problem, which is presented in this thesis, we will present a literature review in the next chapter. The following chapters describe the various aspects of this study. They contain definitions, problem description and models, the description of competitive algorithms to solve the LRP with Single Depot, the MDVRP, the LRP with Multi-Depot, and the LRP

with Constrained Distance. They also provide computational results on well-known benchmarks and finally conclusions.

Chapter 2 Literature review

Over the past few decades, the concept of integrated supply chain has emerged as a new management philosophy which aims to increase distribution efficiency. Such a concept recognises the interdependence among location of depots, allocation of customers to depots, and vehicle route structure. As such, it coordinates a broader spectrum of location and routing options available to logistics managers and consequently avoids the sub-optimisation of distribution solutions. Reflecting the increasing importance of integrated supply chain, an extensive body of combined location routing literature has developed.

In chapter one, it has been illustrated that the LRP is related to the MDVRP because if we fix depot locations, the LRP reduces to the MDVRP. Consequently, the aim of this chapter is to help to summarise and map a comprehensive survey of LRP and of closely related problem, MDVRP literature. Thus, we provide an extensive literature review of LRP in section 2.1 and MDVRP in section 2.2. But before present the survey, the problem description of the LRP and MDVRP is given firstly in section 2.1.1 and 2.2.1, respectively. Then, we give a brief description of the optimisation model for both of them in sections 2.1.2 and 2.2.2, respectively.

2.1 The Location Routing Problem

We mentioned that at the emergence of LRP, the solution methods were heuristics which were applied in a sequential framework. Then, exact methods were applied in small problems. Subsequently, metaheuristics have been used widely as they can solve more realistic problems in reasonable computational time. In this section, we will survey the solution methods that used to solve the LRP.

2.1.1 Problem description

There are various LRP formulations in the literature that could lead to developing different solution methods. We look into an LRP formulation with capacitated depot and capacitated vehicle which is called a general location routing problem as classified by Prins et al. (2007). The problem is to determine the number and locations of depots, assignment of customers to depots, and the corresponding delivery routes, so that the total costs consisting of depot opening cost, transportation cost, and dispatching cost for vehicles are minimised. Each vehicle takes exactly one route starting from the depot, visiting a subset of the customers and returning to the same depot. In addition, customer's demand cannot be split among different routes and the sum of demands in each route must not exceed the vehicle capacity. Furthermore, the total demand of customers assigned to one depot must not exceed its capacity.

2.1.2 Solution methods

There are many attempts to solve the LRP from exact methods to heuristics and metaheuristics. In this section, LRP literature is going to be reviewed, and the characteristics of the solution methodology which has been used is going to be explained.

There are other methods which have been used to solve the stochastic variant of the LRP. For example, simulation methods, and sim-optimisation methods. These kinds of methods will ne be covered in this thesis because the stochastic LRP is not covered. However, it will be mentioned in the future work.

2.1.2.1 Exact methods

Since the LRP combines two NP-hard problems (FLP and VRP), exact methods have been used in only a few studies. The first approach to solve the LRP optimally was by relaxing some constraints of the main problem to generate an initial solution, then improve it by using different methods. Laporte and Nobert (1981) apply this approach to solve the LRP with only a single depot. The initial solution is generated by relaxing sub-tour constraints, then branch-and-bound is used to enforce integrality; finally, sub-tour constraints are added iteratively. The data sets with 20 to 50 customers were solved. Laporte et al. (1983) use the same method to solve a variant of LRP when only one vehicle was assigned to only one depot. Firstly, the problem is solved by relaxing some constraints. Then other constraints are gradually introduced into the problems as they are found to be violated. The algorithm is applied to problems ranging from 20 to 50 customers. This relaxing method is also used to solve the stochastic LRP by Laporte et al. (1986). The initial solution was obtained by using a heuristic approach, then relaxation is used to improve it. Sub-tours and chains between depots will be checked to avoid in third step. Finally, solutions will be checked if they are an integer or not, if not, a branch-and-bound algorithm will be used to achieve an integer solution. Problems with a size of up to 20 customers were solved optimally.

After the huge improvement of computers, branch-and-bound has been used to solve the LRP. Laporte et al. (1988) apply it to solve MDVRP and LRP after transforming the problem into equivalent constrained assignment problems. And instance involves up to 80 customers was solved optimally. Laporte and Dejax (1989) use it for the dynamic LRP and present two solution approaches. The first one is presenting the problem by a suitable network, then using the branch-and-bound algorithm to solve the integer linear program associated with the network. In the second approach, some of the system costs are approximated, and a global solution is then obtained by determining a shortest path on a directed graph.

The branch-and-price algorithm is one of the exact methods that are used for the LRP. Berger et al. (2007) develop a branch-and-price algorithm to provide an optimal solution for the LRP after presenting a set-partitioning-based formulation. Moreover, a set of constraints is identified to reduce the number of constraints. The algorithm is able to provide optimal solutions for problems involving 10 depots and 100 customers. Akca et al. (2009) describe a branch-and-price algorithm based on the set-partitioning formulation to solve LRP with distance constraints. This algorithm is capable of solving an

instance with 40 customers and 5 depots optimally. Cappanera et al. (2003) address LRP to collect obnoxious materials. A Lagrangean relaxation is proposed to decompose the problem into a location sub-problem and a routing sub-problem. A branch-and-bound algorithm is then presented to solve the sub-problems.

The third exact method is branch-and-cut, and branch-and-cut-and-price algorithms. It is applied by Karaoglan et al. (2011) to the LRP with simultaneous pick-up and delivery with the same vehicle. The algorithm implements a local search based on SA to obtain upper bounds. Instances with up to 88 customers and 8 depots can be solved in a reasonable computational time. Instances with 5 depots and 40 customers are solved via branch-and-cut by Belenguer et al. (2011), whereas Contardo et al. (2013) solve instances up to 100 customers by branch-and-cut. Rodríguez-Martín et al. (2014) propose a branch-and-cut algorithm for a variant of LRP. They study the hub location routing problem with one vehicle for each hub. The algorithm succeeds in solving instances of up to 50 nodes. Contardo et al. (2011) employed a branch-and-cut-and-price algorithm to solve instances from 12 to 199 customers and for 2 to 14 depots.

The fourth method is the column-and-cut generation which is presented by Contardo et al. (2014). This approach is capable of solving up to 199 customers and 14 depots. While Ceselli et al. (2014) introduce dynamic column-and-cut generation and branch-and-bound to solve LRP in emergency healthcare systems. This method can solve instances with 10-50 customers and 2-5 depots.

Dynamic programming and radiality constraints are among other strategies that are used to solve the LRP optimally. Baldacci et al. (2011) describe dynamic programming and dual ascent methods for solving the LRP. The instances consist of 20-100 customers and 5-10 depots. In addition, Ocampo et al. (2017) replace sub-tours elimination constraints in VRP and LRP models by radiality constraints, which are used in formulations of electrical power distributions network. These radiality constraints are used to eliminate loops in electrical power network, and ensure every feasible solution consists only of Hamiltonian paths.

The aforementioned exact methods are applied for the LRP with only One-Echelon which consists of some depots and customers; whereas, the Two-Echelon LRP consists of some distribution centers, depots, and customers and it is a variant of the LRP which is also solved optimally. Boccia et al. (2011) design a Two-Echelon freight distribution system for an urban area. They propose an intermediate level of facilities as transit points between platforms and customers. These facilities perform no storage

activities and are devoted to transferring freights coming from platforms on trucks, into smaller vehicles more suitable for distribution in city. Three mixed integer programming models are proposed, aimed at defining location and number of two kinds of capacitated facilities, size of two different vehicle fleets and related routes. XPRESS-MP solver is used to solve three data sets optimally. Crainic et al. (2011) proposed three models for Two-Echelon LRP. These models were solved optimally by XPRESS-MS program. Finally, Contardo et al. (2012) solve instances with 50 customers of Two-Echelon LRP by using a branch-and-cut.

2.1.2.2 Heuristic methods

The LRP is a very difficult problem to solve by using exact algorithms, especially if the number of customers is very large. Therefore, many heuristics are proposed which could be classified into three kinds namely; decomposing procedure, sequential procedure, and nested procedure.

The decomposing procedure consists of two phases and three phases. For the two-phases, the whole problem is divided into two sub-problems – FLP and VRP. The solution obtained in the first phase is used as an input to the second phase such as Aykin (1995), Guerra et al. (2007), Lashine et al. (2006), and Chan and Baker (2005). For the three-phases, the whole problem is divided into three sub-problems; MDVRP, FLP, and MDVRP improvement, such as Perl and Daskin (1985), and Hansen et al. (1994).

Aykin (1995) considers the hub location routing problem and decomposes it into the hub locations problem and the routes problem. The location is solved optimally first, then routing is solved optimally second. Guerra et al. (2007) use the same manner to divide the problem in two phases: location allocation phase and vehicle routing phase. In the first phase, the solution is a set of depots that are selected to be opened based on assigning customers to the nearest depot. Then, in the second phase, a Traveling Salesman Problem (TSP) is resolved with no capacity constraint for the vehicles, then the same TSP is modified to consider vehicle capacity.

Lashine et al. (2006) relax the whole problem by Lagrange relaxation. Then they decompose the problem into two sub-problems: the location allocation problem which solves optimally by Lindo, and the routing problem which solves using a heuristic approach. Chan and Baker (2005) address a new version of LRP which consists of delivery and pick-up service. Moreover, they increase the complexity

of the problem by including limitation on tour length, and asymmetrical distance. The location problem is solved by minimum spanning forest in the first phase, whereas the routing problem is solved by using a modified Clarke and Wright heuristic.

However, Perl and Daskin (1985) decompose the whole problem into three sub-problems and solve it in three phases. The first sub-problem is the MDVRP which is solved using a heuristic approach, while the FLP is considered at the second phase and solved optimally. Finally, the solution is improved at the third phase using a heuristic approach. The heuristic that is used in the first and third phase consists of an initial step and an improvement step. The initial solution is generated in the initial step by assigning customers to depots then using a modified CWH to solve routing. The improvement step involves 2-opt and exchange search. Hansen et al. (1994) improved Perl's heuristic by implementing the heuristic on a PC.

The sequential procedure to solve LRP, in general, consists of two steps. The first step is to generate an initial solution, and the second step is to improve the initial solution. Chien (1993), and Srivastava (1993) have applied this procedure. For Chien (1993), the initial solution is generated by two schemes: random generation, and modified closest-depot rule. The improvement step consists of four local search methods: change-of-vehicle, insertion, swapping, and change-of-facility. Srivastava (1993) develop three heuristics where each one involved the sequential procedure. The initial solution at the first heuristic is generated by opening all depots and using a modified CWH for MDVRP. In the second step, the initial solution is improved by closing depots one each time before closing a depot, an approximate routing costs is used to determine which depot is going to be closed. The procedure of the second heuristic is similar to the first one except the initial solution is solved by only one depot, then improved at the second step, by opening one depot each time. In the third heuristic, the initial solution is generated by clustering customers in groups based on minimal spanning tree. Then, the nearest depots to the cluster centroid are opened. The routing problem is solved using a heuristic approach. Albareda-Sambola et al. (2007) proposed a stochastic model by using recourse for LRP under uncertainty in the number of customers with one vehicle at each depot. The first stage consists of three steps of determining the set of opened depots by using the knapsack problem in the first step, allocating customers to depots in the second step, and designing an initial route by using greedy heuristic in the third step. The second stage involves improvement of the initial route by local search.

The third type of solution method that has been used to tackle the LRP is the nested heuristic. In this procedure, the FLP is considered as a master problem, while the VRP is considered as a secondary

problem. In simple words, nested heuristic generates many solutions based on different combination of depots in an iterative manner. It chooses a configuration of depots to be opened in the first phase, then it solves the routing problem in the second phase, this procedure is iterated, based on a fixed time or another criteria. Nagy and Salhi (1996a) and Nagy and Salhi (1996b) were the first researchers who proposed the nested heuristic. In the location phase, three structures of neighbourhood search are used such as add, drop, and shift structures. Routing cost is estimated on location phase before moving to routing phase which uses a multi-levels heuristic to compute the actual cost.

The difference between these two articles is the approach used for estimation of the route length. Nagy and Salhi (1996a) proposed a formula to estimate the route length based on customer demands, capacity of vehicle, and maximum distance. While for Nagy and Salhi (1996b), the sum of direct distances between depots and customers is used to estimate the route length. Salhi and Fraser (1996) and Nagy and Salhi (1998) use the same method which was proposed on Nagy and Salhi (1996a) to tackle LRP with fleet mix and the many-to-many LRP, respectively.

The clustering procedure was among the methods that were applied to deal with the LRP. Min (1996) developed a three-phase sequential heuristic for LRP. The initial phase aggregates customers into clusters based on the minimum variance method. The second phase allocates clusters to depots by solving the allocation model optimally. The final phase constructs vehicle tours by branch-and-bound algorithm for each tour. Barreto et al. (2007) proposed a sequential heuristic that integrated with a clustering technique. The heuristic consists of four steps including clustering, routing, improvement, and location. In the first step, four clustering techniques are employed, each one involves one or more of the following measures of proximity among groups: single linkage (nearest neighbour), complete linkage (farthest neighbour), group average (average of distance), centroid (gravity centre), ward (the minimum variance method), and saving (modified saving criterion). In the second step, each route is determined optimally by exact method. Improvement to routes is carried out in the third step by 3-opt local search. Finally, each route collapses into one customer to assign routes to depots by solving the location problem optimally.

Lam and Mittenthal (2013) develop a hierarchical clustering consisting of three stages. Firstly, customers are clustered based on geographic location. Secondly, FLP is solved optimally to determine the number, size, and location of depots. In the final stage, a descent heuristic is used to improve the customer-route allocations and the customer-depots allocations. However, stopping criteria for the clustering heuristic may affect the solution quality. Lam et al. (2009) propose two stopping criteria for

the clustering heuristic which are minimum number of clusters and change within cluster variation. The analyses indicate that significant savings can be achieved by considering multiple stopping rules.

Some researchers have studied different versions of LRP such as Two-Echelon of LRP and LRP in a continuous search space. In the first problem, LRP consists of three layers of primary depots, secondary depots, and customers. While, in the second problem, “also known as the infinite set approaches, the depots can be established in a continuous space” Salhi and Nagy (2009). Jacobsen and Madsen (1980) and Nikbakhsh and Zegordi (2010) consider the Two-Echelon LRP. Jacobsen and Madsen (1980) compare three different heuristics. The first heuristics is a tour construction method where the problem is viewed as a spanning tree. The second heuristic is a two-stage heuristic composed of the Alternate Location-Allocation method and the CWH. In the third heuristic, tours are formed by CWH, then facilities are located, and tours are formed again using the new depot locations.

Nikbakhsh and Zegordi (2010) present a two-phase heuristic for the Two-Echelon LRP with a time windows which is based on location-first, allocation routing second. For the initial solution at the first phase, depots to be opened are found sequentially based on the ratio of their fixed cost to their capacity. An unopened depot with the minimum ratio is selected. Then, customers are added to the last opened depot and inserted into routes based on the minimum weighted sum of the routing time, amount of time windows violation and customer priority. Finally, Or-opt heuristic improves the initial route. In the second phase, the initial solution is improved by six neighbourhood search schemes. For a continuous search space, Salhi and Nagy (2009) have presented an iterative heuristic. It considers the end-points of the routes to improve the current location for each depot by solving Weber problems for each depot on the set of the end-points of the routes.

Before choosing and using the aforementioned heuristics, the external environmental characteristics should be considered. Srivastava and Benton (1990) examine the impact of some of these characteristics such as ratio of location to routing cost, number of potential depots, and customer distribution, on the performance of three location routing heuristics (Savings-drop heuristic, Savings-add heuristic, and cluster-routing heuristic). The evaluation results indicate that performance of any heuristic can be affected by the mentioned characteristics.

2.1.2.3 Metaheuristic methods

Several metaheuristic algorithms have been proposed to solve the LRP such as Tabu Search (TS), Simulated Annealing (SA), Greedy Randomised Adaptive Search Procedure (GRASP), Genetic Algorithm (GA), Variable Neighbourhood Search (VNS), Ant Colony Optimization (ACO), and Particle Swarm Optimisation (PSO). An analytical presentation of these algorithms is given next.

a) Tabu Search

Tabu Search is applied to deal with LRP. In the literature of TS approaches for LRP, we can find two main streams which are the two-phase approaches and the three-phase approaches. A two-phases heuristic is presented by Albareda-Sambola et al. (2005). In the first phase, the set of open depots is determined and a priori routes are considered, while in the second phase, the routes are optimised. TS is used at the first and second phase. Tuzun and Burke (1999) introduce a two-phase TS, one seeking a good facility configuration, the other finding a good routing that corresponds to depots configuration. Escobar et al. (2013) propose a two-phase hybrid heuristic for LRP. In the construction phase, clusters and splitting procedure are applied to build an initial solution. In the Improvement phase, a modified Granular Tabu Search (GTS) is applied with a random perturbation procedure to escape from local optimum. Prins et al. (2007) have presented a cooperative metaheuristic to alternate between location phase and routing phase. The location problem is solved by Lagrangean relaxation, while routing phase is handled by GTS with an exchange of information. Lin and Kwok (2006) address the multiple use of vehicles in LRP with two objectives consisting of total cost and workload balance. In this variant of LRP, it is allowed to assign several routes to a vehicle within the vehicle's working time. They employ the TS and SA approaches both simultaneously and sequentially in order to assign routes to vehicles. The FLP is solved using a heuristic approach, while VRP is solved by TS and SA.

Özyurt and Aksen (2007) propose a nested Lagrangean relaxation-based method for LRP. The problem is deconstructed into two sub-problems. The first sub-problem is the FLP which is solved to optimality with CPLEX. In the second one, a TS is developed to solve the MDVRP. Albareda-sambola et al. (2001) formulate the LRP in terms of a network. The linear program solution to the model is taken as a starting point to generate an initial solution. Then, at the first phase, a randomised rounding procedure

is used to obtain integer solutions followed by a local search. In the second phase, a TS algorithm is applied to improve the solution.

A three-phase TS is presented by Huang (2015) to deal with multi-item LRP with random demand and pick-up and delivery. The three phases are location phase to determine the number of opened depots based on the nearest depot, allocation phase to assign customers to opened depots, and routing phase to determine vehicle routing. A modified TS is used in the second and third phase.

The Two-Echelon LRP has also been solved by TS. Boccia et al. (2010) and Crainic et al. (2011) consider the Two-Echelon LRP. Boccia et al. (2010) divide the problem into two sub-problems: location problem and routing problem. The TS is applied in each sub-problem. However, Crainic et al. (2011) deconstruct the problem into two location routing sub-problems, one for each echelon. Then each sub-problem is deconstructed into FLP and MDVRP. An iterative-nested approach involving TS is proposed to combine the solutions of the four sub-problems.

In some papers, the TS is hybridised with another metaheuristic. Hamidi et al. (2012) and Hamidi et al. (2014) present a hybrid GRASP with TS and GRASP with probabilistic TS, respectively for Four-Layer LRP. The method deconstructs the problem into two sub-problems, a location-allocation problem and a routing problem.

b) Simulated Annealing

The Simulated Annealing has been employed to deal with LRP. It has been used, as well as TS, in two frameworks; two phases and three phases. In the two-phase framework, the first phase is to generate an initial solution, and it is subsequently improved at the second phase. Yu et al. (2010), and Jokar and Sahraeian (2012) solve the LRP in two phases. The initial solution is constructed by a greedy heuristic. In phase two, Yu et al. (2010) improve the initial solution by SA with a random neighbourhood structure. Whereas, Jokar and Sahraeian (2012) improve the initial solution by SA using add-drop for locations, and 2-3 opt for routing.

Hassan-Pour et al. (2009) consider stochastic LRP in which availability of depots and routes is limited. This case results from several conditions such as maintenance, capacity limit and breakdown. The problem is considered in two phases. Phase one is to determine the minimum number of depots which

solves optimally by LINGO. Phase two is to determine vehicle routes which solves by SA hybridised by two genetic operators: mutation and crossover. Chen and Imai (2005) study LRP with Two-Echelon. The initial solution is generated randomly, then SA, including swap and 2-opt process is employed to improve the initial solution. Bernal-Moyano et al. (2017) consider LRP with a heterogeneous fleet. They propose a comparison of three metaheuristics: SA, VNS, and probabilistic TS. The computational results show that SA is able to obtain high quality solutions within short computational times.

However, in three-phase framework, location problem is solved in the first phase, routing problem is solved in the second phase, and the global solution is improved in the third phase. Chen and Ting (2007) developed a three-phase hybrid heuristic approach in a sequential manner, combining Lagrangean heuristic and SA. The location problem and customers allocation are solved by Lagrangian heuristic in the first phase, whereas routing problem is solved by SA. In the third phase a global search is performed to improve the solution by SA. Lin and Kwok (2006) address the multiple use of vehicles in LRP with two objectives: total cost and workload balance. Multiple use of a vehicle means that it is allowed to assign several routes to a vehicle within the vehicle's working time constraint. Both TS and SA algorithms are applied in a three-phase framework under two versions: simultaneous and sequential routes assignment to vehicles.

Many variants of LRP have been solved by SA such as LRP with mix fleet, LRP with pick-up and delivery, LRP with auxiliary vehicle, and Open LRP. Wu et al. (2002) address an LRP with mix fleet types. The problem is divided into two sub-problems, location-allocation problem, and vehicle routing problem. Each sub-problem is solved in a sequential and iterative manner by SA embedded in the general framework of the heuristic. Yu and Lin (2015) introduce the open LRP which is motivated by the rise in contracting with third-party logistic companies. The open LRP is different from LRP in that vehicles do not return to the distribution centre after servicing all customers. They propose a SA which uses three local neighbourhood search mechanisms: swap move, insertion move, and 2-opt move.

The LRP with simultaneous pick-up and delivery is addressed by Yu and Lin (2014) and Yu and Lin (2016). In the type of LRP the pick-up and delivery take place at the same time for each customer. Yu and Lin (2014) proposes a multi-start SA which incorporates multi-start hill climbing strategy. While, Yu and Lin (2016) propose a SA which employs three types of local search mechanisms: insertion move, swap move and reverse move.

Mousavi and Tavakkoli-Moghaddam (2013) investigate Two-Echelon LRP with pick-up and delivery. The problem is solved in two stages: location is solved in the first stage by SA and routing is solved in the second stage by hybrid SA and TS. The LRP with auxiliary vehicles is presented by Bashiri et al. (2014) where the length of a route is not restricted by vehicle capacity. The auxiliary vehicle is added to the transportation system as an alternative strategy to cover the limitation of capacity and they are used to deliver goods from depots to vehicles and cannot serve the customers. The problem is solved by SA with two types of local search mechanism: insertion move, and swap move.

c) Greedy Randomised Adaptive Search Procedure

The GRASP is an iterative two-phase search method that has gained considerable popularity in combinatorial optimisation. Each iteration consists of two phases, a construction phase and a local search procedure. In the construction phase, a randomised greedy function is used to build up an initial solution. This randomised technique provides a feasible solution within each iteration. This solution is then exposed for improvement attempts in the local search phase. The final result is simply the best solution found over all iterations. In general, GRASP has not been implemented as a stand-alone approach to solve the LRP. However, it is used with other metaheuristics such as Path Relinking (PA), Evolutionary Local Search (EVS), learning process, VNS, Evolutionary Algorithm (EA), and Honey Bees Mating Optimisation Algorithm (HBMOA). The PA method is combined with GRASP by Prins et al. (2006) and Nguyen et al. (2012). Prins et al. (2006) employ GRASP and PA in a two-phase framework. The first phase executes GRASP based on an extended and randomised version of CWH to generate initial solutions. In the second phase, PA is used to improve the solutions. While Nguyen et al. (2012) complete GRASP by PA for Two-Echelon LRP. The GRASP involves three greedy randomised heuristics to generate initial solutions and two Variable Neighbourhood Descent (VND) procedures to improve them. The optional PA adds a memory mechanism by combining intensification strategy and post-optimisation.

ELS with GRASP is proposed by Duhamel et al. (2010). The initial solutions in GRASP are generated by Randomised Extended CWH. Then, they are improved by local search before applying ELS. The best initial solution is chosen to generate solutions by mutation mechanism. Then, local search is applied to improve them. The local search consists of three neighbourhoods: move, swap, and 2-opt. Nguyen et al. (2010a) and Nguyen et al. (2010b) use GRASP with learning process and GRASP with

ELS and Iterated Local Search (ILS) to solve the Two-Echelon LRP, respectively. Finally, Stenger et al. (2011) combine the GRASP to determine the depot locations and a VNS to optimise the vehicle routes. Prodhon (2011) combine the GRASP with EA for Periodic LRP (PLRP).

Marinakis et al. (2008) propose HBMOA and makes a combined use of a number of different techniques to increase its efficiency. GRASP is used for the initial population of bees and the initial queen in order to have a more competitive queen. Local search is used to decrease the computational time. Finally, adaptive memory procedure is applied in the crossover phase in order to have the fittest broods. Prodhon (2008) study the PLRP and propose an iterative metaheuristic based on Randomised Extended CWH. It consists of three phases: location, combination allocation, and routing. In the first phase, it chooses the depots that will be opened all over horizon using a heuristic approach. A feasible solution is constructed by assigning the customers to a visit combination at the second phase. In the last phase, GRASP is run to improve the result from the previous phase on each period.

d) Variable Neighbourhood Search, Iterated Local Search, and Adaptive Large Neighbourhood Search

Variable Neighbourhood Search (VNS), Iterated Local Search (ILS), and Adaptive Large Neighbourhood Search (ALNS) have been successfully applied to a wide range of practical and complex combinatorial optimisation problems. The general framework for applying these metaheuristics is similar. An initial solution is generated randomly or using a heuristic approach, then the metaheuristic improves the initial solution. The VNS has been applied to solve the standard LRP, PLRP, LRP with probabilistic travel times, LRP where vehicles perform several routes, LRP with non-linear cost functions for each depot, LRP with stochastic demand, and Two-Echelon LRP.

Derbel et al. (2011) present the VNS for solving the LRP without description of the initial solution. Jarboui et al. (2013) integrate VND as the local search in the VNS to solve the LRP. The initial solution is generated by using CWH after opening all depots. Jabal-Ameli et al. (2011) use the VND to solve the LRP. The initial solution is generated by using CWH after opening all depots.

PLRP is solved by Pirkwieser and Raidl (2010) via the VNS. The solution procedure consists of three steps: generate the initial solution, apply VNS, then apply the Very Large Neighbourhood Searches

(VLNS). The initial solution is generated by using CWH after opening the lower bound depots randomly.

Ghaffari-Nasab et al. (2013) study the LRP with probabilistic travel times and present a bi-objective model using two approaches: expected value and chance-constrained programming. The first objective is to minimise the total cost, whereas the second objective is to minimise the maximum good delivery time to customers. The solution method consists of two phases. The VNS is applied to improve the initial solution which is constructed by using CWH after opening all depots.

The LRP when vehicles perform several routes in the same planning period is considered by Macedo et al. (2013) and Macedo et al. (2015). Macedo et al. (2013) apply the VNS, while Macedo et al. (2015) apply the skewed general VNS. The initial solution is generated by a greedy heuristic in both researches.

Melechovský et al. (2005) deal with the LRP with non-linear cost functions for each depot which grows with the total demand handled at this depot. Two methods to find an initial feasible solution, and a metaheuristic to improve the solution, are proposed. The former method for the initial solution opens depots, assigns customers, and builds routes randomly. The latter method is the p-median method. The suggested metaheuristic is a hybrid approach of the VNS and TS.

Marinakis et al. (2016) presented a formulation of LRP with stochastic demands. They treated the problem as a two phase problem. In the first phase, they determined which depots will be opened and which customers will be assigned to them. In the second phase, the VRP with stochastic demands is solved for each of the open depots. A Hybrid Clonal Selection Algorithm (HCSA) with two phases is applied. In the first phase a VNS is used. While in the second phase an ILS algorithm is utilised. Schwengerer et al. (2012) consider Two-Echelon LRP and solve it by VNS. The initial solution is generated by using CWH after opening the lower bound depots randomly. Escobar et al. (2014) propose a Granular Variable Tabu Neighbourhood Search (GVTNS) for the LRP. This heuristic includes a GTS within a VNS.

Derbel et al. (2010) apply ILS to solve the LRP without any explanation about the initial solution. While Rahmani et al. (2015) use it to deal with the multi products Two-Echelon LRP with pick-up and delivery. Two types of local search are proposed: location local search and routing local search. There are no details about the initial solution. Nguyen et al. (2012b) propose multi-start ILS for Two-Echelon

LRP. Tabu list is used to prevent the algorithm from revisiting a known solution. Moreover, a PR procedure is applied to reinforce the ILS.

Finally, ALNS is used by Hemmelmayr et al. (2012) and Hemmelmayr (2015). Hemmelmayr et al. (2012) apply the ALNS for Two-Echelon LRP. The initial solution is constructed by opening the depot that yields the lowest cost, assigning customers to depots randomly, and building routes by CWH. While Hemmelmayr (2015) proposes two sequential and two parallel variants of the VLNS to solve the PLRP. The initial solution is generated by assigning customers randomly to a random combination of opened depot and buliding routes by CWH.

e) Genetic Algorithms, Memetic Algorithm, and Evolutionary Algorithm

The Genetic Algorithm (GA), Memetic Algorithm (MA), and Evolutionary Algorithm (EA) are population-based metaheuristics which have been proved to solve many optimisation problems efficiently. These algorithms are based on the natural mechanism applied to a population of individuals. Those individuals are following some rules to produce new offspring. Those that cannot survive vanish and disappear.

The first use of the GA to tackle the LRP has been proposed by Su (1998) where the initial solution is genereated randomly. Wan and Zhang (2008) consider the Three-Echelon LRP and present a heuristic approach on the basis of GA. Derbel et al. (2012) study a new variant of the LRP with capacitated depots and a single uncapacitated vehicle for each depot. A GA combined with an ILS is applied. Liu et al. (2013) focus on the stochastic LRP with uncertainty in costs and travel time. The problem is formulated by using the chance-constrained goal programming framework. A Simulation-Based GA is developed to solve the problem. The GA handles the optimal solution, while the stochastic simulation addresses uncertain functions.

Chang et al. (2017) consider the multi-objective nonlinear LRP with time windows. In this problem, the customer can be visited more than once. The GA is applied to solve the problem. Finally, Dalfard et al. (2013) present a hybrid GA and SA for Two-Echelon LRP with route length constraints. Martinez-Salazar et al. (2014) consider Two-Echelon LRP with two objectives: reduction of distribution cost and balance of workloads for drivers in the routing stage. They proposed two metaheuristic algorithms based on Scatter Tabu Search Procedure (STSP) and GA.

The MA is used by Prins et al. (2006), Duhamel et al. (2008), Prodhon and Prins (2008) and Karaoglan and Altiparmak (2015). Prins et al. (2006) present the MA with population management to solve the LRP. This approach consists of the MA in which the diversity of a small population of solutions is controlled by accepting a new solution if its distance to the population exceeds a given threshold. Duhamel et al. (2008) design an MA with a modified CWH to generate the initial solutions. In 2008, Prodhon and Prins extend their MA with population management to deal with the PLRP. Finally, Karaoglan and Altiparmak (2015) consider LRP with pick-up and delivery. A MA is proposed to solve this problem.

The last population-based metaheuristic is EA which is proposed by Prodhon (2009) and Koç et al. (2015). Prodhon (2009) hybridise the EA with PR, while Koç et al. (2015) hybridise the EA with VLNS to tackle the LRP with heterogeneous fleet and time windows.

f) Ant colony optimisation and Particle Swarm Optimisation

In this section, two metaheuristics, namely, ACO and PSO are proposed. The ACO is proposed by Dorigo et al. (1996). Since then, many variants of ACO have been developed and applied extensively in the fields of the combinatorial optimisation problems. While the PSO was originally proposed by Kennedy and Eberhart (1997). And since its introduction, PSO has gained rapid popularity and has proved to be a competitive and effective optimisation algorithm in comparison with other metaheuristics. LRP is one of the combinatorial optimisation problems that has been solved by ACO and PSO.

The ACO is employed in three different approaches. The first approach is to apply ACO for VRP while FLP is solved using a heuristic approach or randomly. The second approach is to apply ACO to solve FLP and VRP. The third approach is to apply ACO with another metaheuristic for the other sub-problem.

The first way is used by Nadizadeh et al. (2011), Gao et al. (2016), and Herazo-Padilla et al. (2015). Whereas the second way is used by Ting and Chen (2013) and Bouhafs et al. (2006). Finally, the third way is used by Wang and Sun (2005), and . Bouhafs et al. (2006).

Nadizadeh et al. (2011) cluster customers based on a greedy method. Then, the proper depots are chosen based on the minimum sum of distances with gravity centres to depots. After that, clusters of

customers are allocated to depots based on distance and capacity. Finally, the ACO is used for routing among depots and customers.

Gao et al. (2016) and consider the dynamic LRP and the LRP with stochastic transportation cost and vehicle travel speeds, respectively. Gao et al. (2016) divide the LRP into location-allocation and VRP. To solve location-allocation problem, a k-means clustering algorithm is developed to choose depots to be opened and allocate customers to them. Then the ACO is utilised to handle the VRP in dynamic environments consisting of random and cyclic traffic factors. Herazo-Padilla et al. (2015) use an iterated random selection for depots configuration, then ACO is applied to solve the routing problem. Finally, a simulation model evaluates vehicle routes in terms of their impact on the expected total costs.

The second way of using the ACO is proposed by Ting and Chen (2013) to solve the two sub-problem FLP and VRP. The first ACO is applied to determine the depot set to be opened, and to assign customers to each depot. A VRP for each opened depot is solved by the second ACO. These ACOs are applied iteratively until the stopping criterion is met.

Wang and Sun (2005) deconstruct LRP into location-allocation and VRP. The TS is implemented in location phase to determine a good configuration of depots to be opened, while the ACO is run to solve the routing problem. Bouhafs et al. (2006) propose a metaheuristic approach to solve the LRP based on SA and ACO.

For the PSO, Marinakis (2015) present a PSO for the deterministic and stochastic LRP. The proposed algorithm is a two-phase algorithm that solves the FLP in the first phase and VRP in the second phase. PR and VNS is used to enhance the PSO. The positions of the particles are calculated by the PR, while the VNS is applied in each particle to improve the solutions produced. Marinakis and Marinaki (2008) introduce a hybrid algorithm based on PSO with a Multiple-Phase Neighbourhood Search (MPNS) and a PR.

2.1.3 Real application of Location Routing Problem

The LRP model has been used in different variants of real-life problems. In this section, we show some applications of LRP in healthcare, natural disaster, military, food distribution, communication sector, fuel sector, waste collection and recycling, distribution sector, financial sector, and space science.

2.1.3.1 Healthcare

Or and Pierskalla (1979) consider location of blood banks with VRP. A two-stage heuristic is proposed to solve the problem. In the first stage, a number of VRPs are solved. Based on these results, a saving is calculated and the assignment of hospitals to blood banks is changed. Then a new group of VRP is solved again. When all reasonable exchanges are completed without any improvement, the procedure is terminated. Hua-Li et al. (2012) and Park et al (2015) consider an urban emergency system and emergency response units on freeways.. Hua-Li et al. (2012) propose a bi-objective LRP to maximise the total time satisfaction served and to minimise the total cost. The GA is used to solve the problem. While Park et al (2015) apply the stochastic programming paradigm to solve LRP of Emergency Response Units (ERUs) on freeways. At the first stage, FLP has to be solved before the realisation of uncertainties. The recourse decisions, in the second stage, include assigning vehicles to incidents to minimise the overall expected delay. Ceselli et al. (2014) present a model for the optimisation of logistics operations in emergency healthcare systems. The problem is slightly different of classical LRP. It considers multiple distribution channels when a facility is established. In particular, there are two options for reaching citizens: either by delivering drugs to their homes with a heterogeneous fleet of vehicles, or by establishing distribution centres where the citizens go by their own means to receive treatment or drugs. An exact algorithm is presented, which is based on dynamic column-and-cut generation and branch-and-bound.

2.1.3.2 Natural disasters

Natural disaster has gained attention of researchers, therefore, there are a body of papers in this area. Ukkusuri and Yushimito (2008) develop the LRP model to formulate the humanitarian pre-positioning of supplies for natural disasters. The approach uses a combination of the most reliable path and an integer programming model to find the optimal location of supplies and the most reliable route. Ahmadi-Javid and Seddighi (2013) consider an LRP with a single commodity under a variety of possible disruptions. A heuristic based on deconstructing the main problem into two stages, constructive stage and improvement stage, is proposed. In the constructive stage, an initial solution is randomly built. The improvement stage consists of two phases: location phase and routing phase. In each phase of the second stage, SA algorithm is used to improve the initial solution.

Hassan-Pour et al (2009) and Zhang et al. (2015) study a version of the LRP in which facilities and routes are subject to probabilistic disruption risks under crisis conditions such as maintenance, capacity limit, breakdown, or shut down for unknown causes. Facilities and routing may be partially or completely destroyed. In this condition, serviceability of facilities and routing are not possible. Hassan-Pour et al (2009) apply a two-step approach. Step one is to determine the minimum number of facilities by using a stochastic set-covering problem approach, which is solved by LINGO after formulating it in an integer linear program. Step two is to determine vehicle routes which are solved by SA. The objective functions of step two is to minimise the cost and maximise the probability of delivery to customers by using a multi-objectives function.

Zhang et al. (2015) propose a two-phase approach. In the first phase, an initial solution is generated by using a heuristic approach, while in the second phase, the initial solution is improved. The GTS is used in the second phase. Coutinho-Rodrigues et al. (2012) develop a multi-objective LRP model to design evacuation plans for Coimbra city in Portugal. Six objectives were identified including minimisation of travel distance to shelter, minimisation of risk faced by the population, minimisation of travel distance associated to backup paths, minimisation of risk at the shelters, minimisation of the time required to transfer people from shelter to a hospital when necessary, and minimisation of the number of shelters. The solution is determined by minimising each objective individually in an optimal solution.

Wang et al. (2014) construct a nonlinear integer open location routing model for relief distribution problem considering travel time, the total cost, and reliability with split delivery. The Genetic Algorithm is applied to solve the proposed model. Rath and Gutjahr (2014) consider the LRP that faces international aid organisations after the occurrence of a natural disaster. A three-objective optimisation model is proposed considering – minimising the fixed costs for depots and vehicles, minimising the budget of the operative cost, and maximising the covered demand. An exact solution is used to solve the single-objective problem, whereas the multi-objective problem is solved by using VNS.

Bozorgi-Amiri and Khorsi (2016) model the problem of the humanitarian relief logistics for pre-and post-disaster as LRP. They propose a multi-objective dynamic stochastic programming model. The aim of the model is to minimise the maximum amount of shortages among the affected areas in all periods, the total travel time, and the sum of pre-and post-disaster costs. The first objective pursues fairness, whereas the two other objectives pursue the efficiency goal. The proposed model is solved as a single-objective mixed-integer programming model applying the ϵ -constraint method by using GAMS/CPLEX.

Caunhye et al. (2016) propose a two-stage location routing model, with recourse for integrated preparedness and response planning under uncertainty in risk management for disaster situations. In the first stage, the model sets up warehouses and determines their emergency supply inventory levels. In the second stage, the model plans transshipment and delivery quantities, and vehicle routes for every scenario. The objective of the first stage is to minimise the weighted sum of the total preparedness cost and the worst-case second-stage objective among all scenarios, whereas the second-stage objective is to minimise the total response time. The two-stage model is converted into a single-stage mixed integer model and then implemented in an illustrative example which is solved by using CPLEX.

2.1.3.3 Military

In the military, there are some articles that apply the LRP model to solve military problems. Murty and Djang (1999) address location routing of training National Guard units of the U.S. National Guard. They consider 21 combat vehicle training simulators called mobile trainers and each National Guard unit must train at a station that is not farther than a specified maximum travel distance from its armoury. This problem is studied to find the optimum locations for the home bases for the mobile trainers, the locations of secondary training sites to which the mobile trainers will travel to provide training, and the actual routes that the mobile trainers will take to cover all these secondary training sites. The aim is to allocate each National Guard unit to a training site within the maximum travel distance from its armoury, while simultaneously minimising the mobile trainer fleet mileage and the total distance traveled by all units. Heuristic hierarchical decomposition strategy is used to break the overall problem into three sub-problems. The p-median model is concerned with finding optimum locations for exactly p facilities, to provide a service to a set of customers that involves travel to a nearby facility so as to minimise the total travel. The set covering model is used to allocate an armoury to a training centre. Routing is solved for each facility individually by finding a cycle that covers all customers with minimum distance.

Toyoglu et al. (2012) study replenishment system of ammunition from depots to combat units via transfer points (fixed and mobile). Ammunition is moved by trains from depots to fixed transfer points, then by commercial trucks from fixed transfer points to mobile transfer points. Finally, ammunition is issued from the mobile transfer point to combat units by special ammunition trucks. The flow from depots to fixed transfer points is not included in the model because it is assumed that in the case of war,

there will be enough ammunition at depots and the current rail network structure and equipment are sufficient to carry the demand on time. The locations of the main depots and combat units are known in advance, whereas the location decisions of fixed and mobile transfer points, and the route decisions of commercial and ammunition trucks must be made. The problem is designed as LRP since it contains both location and routing problems. A “route first-location second” heuristic consisting of three phases is proposed. At the first phase, combat units are partitioned into clusters. Each combat unit belonging to a distinct brigade forms a cluster. At the second phase, routes of ammunition trucks from mobile transfer sites to combat units in each cluster are found optimally by CPLEX. Finally, the LRP that decides which locations of transfer points are to be opened, and the routes of commercial trucks from fixed to mobile transfer points, is solved optimally by CPLEX.

Finally, Saricicek and Akkus (2015) consider a hub-location and routing problem for border security in Turkey. The problem consists of selecting hubs among the airports, assigning demand points to hubs, and determining optimal routes for each hub. A p-median model is used to determine the locations of hubs, whereas optimal routes are determined for each hub by solving the mathematical model.

2.1.3.4 Food distribution

Food distribution has seen attention of researchers who use the LRP model. Watson-Gandy and Dohrn (1973) study a depot location with van salesmen problem for a company operating in the food and drink industries in a part of England and Wales. An algorithm is proposed to solve the problem in two phases. The location decisions is made in the first phase by using the Christofides–Eilon approximation algorithm. The routing decisions is solved in the second phase by using the CWH.

Johnson et al. (2002) present a model for delivering hot meals to the homebound, infirm and elderly. They propose a GIS-based heuristic to solve location routing problem by location first-routing second. Ambrosino et al. (2009) study a real-life application related to an Italian company. The company holds 200 food market stores along the national highway network in the north of Italy. A two-phase heuristic is proposed. The first phase determines an initial feasible solution, whereas the second phase improves it by using local search. Boudahri et al. (2013) apply a clustering-based location routing approach to redesign a real agri-food supply chain for poultry products in Algeria.

Menezes et al. (2016) redesign two supply networks in France and Canada. The first one is a supermarket chain while the second one is a recycling network. The LRP model is used to model these

two problems. They divide the LRP into two sub-problems: location and routing. Location is solved optimally by using the p-median model, while routing is solved by the CWH. Bozkaya et al. (2010) investigate LRP when demographics and economic conditions are shifted. The solution approach is tested on an existing of a supermarket chain in Turkey under competition. The objective of the model is to maximise profit, defined as gross profit margin minus logistics costs. They propose a hybrid heuristic consisting of a GA for solving location and TS for solving routing.

Two-Echelon LRP with time windows is introduced by Govindan et al. (2014) for sustainable supply chain network design, and for optimising economic and environmental objectives in a perishable food supply chain network. A hybrid metaheuristic algorithm that combines multi-objective PSO and adapted multi-objective VNS was proposed. Jouzdani and Fathian (2014) study the dairy supply chain in Iran where transportation costs are uncertain. A multi-depot multi-travelling salesman problem formulation of robust location routing problem is proposed.

2.1.3.5 Communication sector

Lin et al. (2002), Lee et al. (2003), and Catanzaro et al. (2011) formulate problems of communication as LRP. Lin et al. (2002) improve the delivery of telecommunication bills to a company's customer in Hong Kong, by using the LRP model. The total cost of delivery bills was minimised significantly by relocating some existing office, setting up new depots, and changing vehicle routing and loading decisions. An approach combining a heuristic with SA is proposed to solve the problem. The method consists of two parts: the initial solution and improvement routing. The initial solution is constructed using a heuristic approach by choosing the minimum number of depots in order to satisfy total demand of customers, allocating each node to the nearest depot, and constructing the initial route by CWH. The SA is applied for improving the routing problem.

Lee et al. (2003) has designed optical Internet access, with wavelength division multiplexing systems, to deliver a high-speed access service. To minimise the total cost of the network while carrying the offered traffic, it is required to find an optimal location of the gateway and an optimal routing of traffic demands in the optical access network. A TS procedure is developed to tackle this complex problem. Catanzaro et al. (2011) consider the Partitioning-Hub LRP (PHLRP) which is a hub location problem with graph partitioning and routing features. The problem arises from the deployment of an internet

routing protocol, and it also finds applications in the strategic planning of freight distribution systems. They introduce a mixed integer programming formulation and provide families of strengthening valid inequalities. The model can solve instances of PHLRP containing up to 20 vertices by using XPRESS solver.

2.1.3.6 Fuel sector

In the Fuel stations sector, Yang and Sun (2015) present an electric vehicles battery swap stations location routing problem, which aims to determine the location of battery swap stations and the routing of electric vehicles simultaneously under driving range limitation. A four-phased heuristic and a two-phased TS based on a modified CWH are proposed to solve the problem. Yildiz et al. (2016) study the refueling station location problem with routing, to locate a given number of refueling stations for alternative fuel vehicles in a road network, to maximise the total flow covered. A branch-and-price algorithm is used to solve the problem.

Xie et al. (2012), Alumur and Kara (2007), and Samanlioglu (2013) propose a model for hazardous waste in the USA and Turkey, respectively. The first two articles consider multi-objective location routing models with two objectives of minimising the total cost and the transportation risk. Whereas, the last one presents a multi-objective location routing model with three objectives including minimising total cost, transportation risk, and total risk for the population around treatment centres. Xie et al. (2012), Alumur and Kara (2007), and Samanlioglu (2013) formulate the problem by a mixed integer linear program and solve it by CPLEX.

Caballero et al. (2007) study the incineration plants for the disposal of solid animal waste in Andalusia (Spain). The problem is to locate two incineration plants and design the routes to serve different slaughterhouses in the same region. A multi-objective location routing problem is presented and solved by a metaheuristic algorithm based on TS. Asefi et al. (2017) propose an LRP model for a municipal solid waste network covering multiple types of wastes. The SA is proposed to deal with this problem.

Rahim and Sepil (2014) and Tunalioğlu et al. (2016) address glass recycling and treat the brown coloured olive oil mill wastewater in Turkey, respectively. Rahim and Sepil (2014) propose a two-phase method. The first phase is to construct an initial solution randomly, while in the second phase, the VNS and exact method is used to improve location and routing, respectively. However, Tunalioğlu

et al. (2016) introduce a multi-period LRP model to solve the problem of the brown coloured olive oil mill wastewater. In this problem, the wastewater is collected from oil mills and delivered to be treated at ultrafiltration facilities using a fleet of vehicles. An ALNS is proposed to solve the problem.

2.1.3.7 Distribution sector

In the distribution sector, there is a main body of research that applies LRP model. Jacobsen and Madsen (1980) consider a two-level location routing model for a distributing newspapers problem in Denmark. The problem is solved by using the spanning tree model and the CWH. Nambiar et al. (1981) improve the efficiency of the natural rubber industry in Malaysia by using location routing model and introducing two heuristics to solve it approximately. The first heuristic considers the location of a single central factory by presupposing that every potential central factory location can serve all collection stations in a region, both in terms of capacity and time constraints. Then, TSP are solved in order to determine the least cost tour from every potential central factory location, to every collection station. A potential central factory with minimum cost will be chosen, then vehicle routing is improved by applying the CWH. The second heuristic assumes that the minimum number of central factories to serve a given area has been determined on the basis of total supply and time constraints. Then, TSP are solved to determine the least cost tour from every potential factory location, to every group. Simple plant location is solved iteratively while the variable costs are the TSP to locate factories and assign collection stations to them. The vehicle routing decision for each factory and its corresponding collection stations is improved by applying the CWH.

Gunnarsson et al. (2006) consider a combined terminal location and ship routing problem at Pulp Company in Scandinavia. Some customers are supplied from the terminals, others are supplied directly from the pulp mills. Two heuristics were proposed. One is developed by relaxation of some constraints, then adding constraints one by one. The second heuristic is also designed based on constraint relaxation, but it is followed by another step to reduce route costs. Marinakis and Marinaki (2008) propose a GA to find an approximate solution for location routing problem of one of the largest companies in Greece, which distributes wood products. Aksen and Altinkemer (2008) model conversion of traditional retailer to e-retailer based on optimisation of location routing problem. The location of depot and delivery vehicles serve two customer types, namely walk-in and online customers. A Lagrangian relaxation based solution method is described to deconstruct the overall

problem into two independent sub-problems namely: FLP and MDVRP. The former problem is solved by the optimisation package GAMS, and the latter one by an augmented LR method.

Çetiner et al. (2010) consider the combined hubbing and routing problem in postal delivery systems. They develop an iterative two-stage solution procedure for the problem. In the first stage, hub locations are determined, and postal offices are multiply allocated to the hubs. The second stage gives the routes in hub, regions that alter the distances between points used in the hub-location problem. The procedure then iterates between two stages by updating the distances used in hubbing, in order to produce a route-compatible hub configuration. De Camargo et al. (2013) study the parcel delivery network design, where several facilities are responsible for assembling flows from several origins, then rerouting them to other facilities where the flows are disassembled, and the packages are delivered to their final destinations. In order to provide this service, local tours are established for the vehicles assigned to each of the processing facilities, which are then responsible for the pick-up and delivery tasks. This application gives rise to the LRP. A formulation for this problem is proposed and solved by CPLEX.

Wang and Mu (2015) study the parcel delivery service by collect-on-delivery problem in China. They model this distribution network as a Two-Echelon LRP and propose an optimisation algorithm by combining SA and PA. Labbé and Laporte (1986) deal with the optimal location of post boxes in an urban or in a rural environment. The problem consists of selecting sites for post boxes which will maximise an appropriate linear combination of user convenience and postal service efficiency. The location problem is solved by using p-median model whereas routing problem is solved by using the general TSP.

Wasner and Zäpfel (2004) consider the transportation networks for parcel service in Austria. They develop a hub location routing model which encompasses the determination of the number, locations of hubs and depots (and their assigned service areas), as well as the routes between demand points and consolidation points. An iterative hierarchical heuristic embedded in a local search with a series of feedback is proposed. The heuristic divides the main problem into three sub-problems; location, allocation, and routing. The solution method is based on solving a sub-problem and using its results to serve as a constraint to solve the other sub-problem. The location problem is solved by local search starting by one depot and increasing the number of depots one by one, until the best number of depots is found. Then, the resulted problem is solved optimally by CPLEX. The postal zones are assigned to depots based on time and distance between a postal zone and the depot locations. The routing is done

by generating a giant tour as a TSP, then improving it by local search after dividing it to several routes based on the capacity of vehicles.

Schittekat and Sörensen (2009) develop a software tool as a decision support tool for Toyota to select the third-party logistics partners in a spare parts network. A TS is used to solve a large LRP. It generates a set of solutions, rather than one solution. This is to increase the negotiating power and the ability to analyse the current network against possible alternatives. It also allows to quickly switch between different transport networks if unexpected events occur.

Stenger et al. (2012) consider a problem that includes relocation as well as subcontracting aspects adapted from a large French small package shipper. They describe the characteristics of two different depot types: self-owned and subcontracted. They develop a location routing model that integrates the choice between the two depot types. The model is solved by means of a hybrid heuristic approach integrating SA and VNS. Lin and Lei (2009) design a distribution system for a Taiwan label-stock manufacturer. They have developed a mathematical model for Three-Echelon LRP. The solution procedure consists of two elements. The first element is a GA for locating DC's and are the big clients included in the first level routing. The second element is a cluster-base heuristic that consists of saving/insertion algorithms and tour improvement/exchange algorithms for finding the first and second level routes.

Singh and Shah (2004) model the collection of tendu patta leaf, which is used to produce tobacco, in India as a facility location problem and a VRP to raise leaf output substantially under existing budget limitations. The problem was solved in sequential and integrated manner. Muñoz Villamizar et al. (2014) consider an urban distribution system in Saint Étienne city, France. Two-Echelon LRP model is used to formulate this problem and to find an exact solution. GAMS is used to program the model and CPLEX solver to solve it.

Ponboon et al. (2016) investigate the impact of three main parameters such as depot location, depot size, vehicle size, and time windows on a distribution network in Osaka, Japan. Nine scenarios from a logistics firm in Osaka were tested with different depot location, depot size, and vehicle size. The branch-and-price algorithm was implemented to ensure the quality of solution. It was found that having large-size depot, serving by large-size vehicle, results in the lowest overall cost.

Laporte et al. (1989) formulate a problem in financial sector as a stochastic LRP. The problem is a collection money from bank branches by armoured vehicles while the daily supply of each branch is a random variable. The problem is solved optimally by branch-and-bound algorithm.

2.1.3.8 Space science

Finally, in space science, Ahn et al. (2012) address LRP with profits which arises in exploration of planetary bodies with different technologies. The problem simultaneously determines base locations, strategies to use at the bases, sites to visit, and routes to carry out the visits, to maximise the sum of profits that can be obtained by visiting sites under resource consumption constraints. Two solution methodologies to solve the problem are proposed which are branch-and-price, and GRASP combined with column generation.

2.2 The Multi-Depot Vehicle Routing Problem

The Multi-Depot Vehicle Routing Problem (MDVRP) is a generalisation of the VRP by serving customers from more than one depot. The MDVRP is a key problem in logistics and supply chain management. Its importance comes from assigning customers to depots and producing detailed routes under a set of constraints simultaneously. The aim of this section is to help summarise and map a comprehensive survey of MDVRP. It is due to the fact that one of the main parts of our proposed methodology focuses on the MDVRP. This is clear as LRP reduces to MDVRP if the depot locations are fixed and the proposed solution method for LRP can benefit from one developed for MDVRP.

In general, there are three main frameworks in the literature which are used to solve the LRP; sequential framework, iterative framework, and nested framework, which we will describe in more detail below. These three frameworks illustrate the role of the MDVRP inside each one of them.

The first framework is called the sequential framework. In this method, a configuration of potential depots is selected initially. Then, the routing problem is solved through two main methods; by treating the whole problem as a MDVRP, or by dividing the whole problem into many VRPs based on the

number of depots. In this framework, there is no iteration for solving location and routing stages, and each stage is solved only once.

The second framework is called the iterative framework. In this method, a configuration of potential depots is selected initially. Then the routing problem is solved by the same two methods. However, the procedure iterates between the location phase and the routing phase to improve both of them. Although the solution of this method is better than the solution of the sequential method, then it treats the location problem and routing problem as if they are on the same footing (Gabor Nagy and Salhi, 1996).

The third framework is called the nested framework. This method embeds the routing stage into the location phase, because the LRP is essentially a location problem, with the routing factor taken into consideration (Nagy and Salhi, 1996). Therefore, the FLP is treated as the main problem, while routing problem is treated as the subordinate problem. In this way, the drawback of the iterated framework can be avoided. In this method, a configuration of potential depots is selected, then, the routing problem is solved by the two main methods which can be MDVRP or VRPs. This procedure is repeated many times by choosing different configurations of potential depots to find out the solution with the total minimum cost of depots and routing. These three frameworks are illustrated in Figure 4.1.

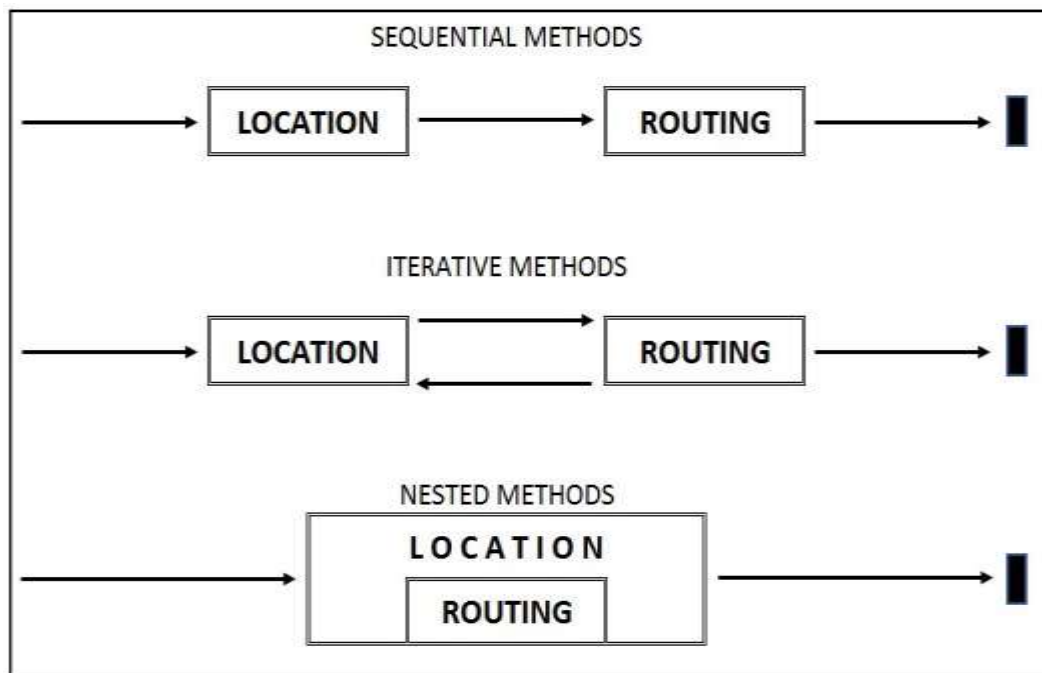


Figure 2.1. Three frameworks to solve the LRP, from (Nagy and Salhi, 1996)

From this, we have seen that the MDVRP is an essential component of the LRP. Therefore, we can develop a new solution method by combining Biased Randomised technique with a classic heuristic in order to achieve a good solution for MDVRP.

2.2.1 Problem description

The MDVRP is a challenging problem because it integrates two combinatorial problems which are an assignment problem and a routing problem. In the assignment problem, each customer is assigned to only one depot, and in the routing problem, each customer must be served by only one vehicle. Therefore, these two combinatorial problems are often interrelated. In this problem, a homogeneous fleet of vehicles with fixed capacity serve a set of customers with known demand, from more than one depot. The capacity of the vehicles cannot be exceeded, and demand of customers must be satisfied; each customer must be served by exactly one vehicle, and each vehicle must depart from and return to the same depot. The aim of the MDVRP is to determine a sequence of customers in a route for each vehicle, where all of them are served so that the total distance traveled by all vehicles is minimised.

2.2.2 Solution methods

There are numerous attempts in the literature to solve the MDVRP with different solution methods from exact methods to heuristics and metaheuristics. In this section, MDVRP literature is reviewed, and the characteristics of various solution methods proposed to solve it are discussed. The proposed solution methods for MDVRP in the literature can be classified to exact methods, heuristics, and metaheuristics.

2.2.2.1 Exact methods

The first approach to solve the MDVRP optimally was by relaxing some constraints of the main problem to generate an initial solution, then improving it by using different methods. Laporte et al. (1981) formulate a linear integer model for the multi TSP and solve it by using a constraint relaxation algorithm. Then, a Gomory cut is introduced to obtain an integer solution. Finally, the algorithm

removes any illegal sub-tour. The algorithm iterates until the solution contains no fractional variable and no illegal sub-tour. Laporte et al. (1984) developed a branch-and-bound algorithm to solve the MDVRP exactly. Their procedure solves a sub-problem that relaxes some constraints of the original problem, adds upper bounds on variables, and branches on the non-integer variables. Laporte et al. (1988) present an exact solution by using the branch-and-bound algorithm for MDVRP and LRP. The optimal solutions are found after transforming the problem into equivalent constrained assignment problems. An instance with up to 80 customers was solved optimally.

Mingozi and Valletta (2003) describe an integer programming formulation and an exact method for solving both the Periodic Vehicle Routing Problem (PVRP) and the MDVRP. The exact method involves the computation of a valid lower bound by means of an additive procedure. This combines different relaxations of the integer formulation to derive an effective feasible solution for the dual problem of the LP relaxation of the integer program. The dual solution is used to generate a reduced integer program which can be solved to optimality by an integer programming solver. Mingozi (2005) describes an exact method for solving the Periodic MDVRP (PMDVRP) using variable pricing in order to reduce the set of variables. The pricing method is based on a bounding procedure for finding near-optimal solutions of the dual problem of the LP relaxation.

Contardo and Martinelli (2014) propose an exact method for the MDVRP based on the solution of vehicle-flow and set partitioning formulations. The first model is solved by the cutting planes method and the second by column-and-cut generation after relaxation of the main model. Ramos et al. (2011) propose a two-step algorithm for MDVRP with multi products. The first step relaxes the original problem which results in a VRP model with a single product. The next step is to solve the routing problem optimally by considering each depot individually. Ramos et al. (2011) study a real case of a recyclable waste collection and model the problem as MDVRP with multi products. A three-step solution method is proposed to tackle the problem. The aim of the first step is to assign customers to depots by relaxation of the MDVRP model with multi products to an MDVRP model with single product. In the second step, assigning the remaining customers is implemented via a greedy heuristic based on the nearest depot. Routing decision is made optimally by the VRP model at the third step.

Montoya-Torres et al. (2016) investigate a collaborative scenario for three firms in Bogota, Colombia. The aim of this scenario is to reduce transport costs, congestion, and environmental impact. Both collaborative and non-collaborative scenarios are compared. The non-collaboration problem is modelled as VRP, whereas the collaboration problem is modelled as MDVRP. The VRP problem is

solved optimally. While the MDVRP is solved in two phases, allocation and routing sequentially. In the first phase, customers are allocated to depots by solving the model optimally, so that each depot has the same number of customers. Then, routing is solved optimally for each VRP resulted for each depot. The computational experiments show that collaborative logistics operations have more advantage in terms of both transportation costs and environmental impacts.

The second approach is using a branch-and-bound algorithm. Carpaneto et al. (1989) use it without transformation to solve a problem with 70 customers. The third approach is the branch-and-cut-and-price, which is applied by Bettinelli et al. (2011) to solve the MDVRP with time windows and a heterogeneous fleet. This method solves instances with 50 customers. Tummel et al. (2013) apply the MDVRP with a heterogeneous fleet and with time windows and assignment restrictions to assign a set of shipments to a set of freight routes so that unused cargo volume is minimised. In this problem, the assignment of each shipment is restricted to a subset of routes. They propose an integer linear program model and solve it by CPLEX and GUROBI. Ramos et al. (2014) address the waste collection systems while accounting for economic and environmental concerns. The problem is modelled as a MDVRP with multi product and two objective functions of distance and CO₂ emission should be minimised. The mathematical problem is solved by the CPLEX.

The branch-and-cut algorithm is used by Benavent and Martinez (2013) for the MDVRP. The proposed approach is capable of solving an instance with 255 customers and 25 potential depots. Bektas et al. (2017) propose a new constraint for MDVRP which eliminates paths between pairs of depots. These inequalities are used in a branch-and-cut algorithm to solve instances with up to 300 clients and 60 depots.

The final approach is to tighten the formulation by adding new constraints. Dondo et al. (2003) add some constraints based on the precedence notion. In the proposed approach, a node precedes another one in a route if it is visited earlier by the same vehicle, but not necessarily immediately before. This approach is implemented for the MDVRP with time windows. The optimal solution is found by choosing the best set of preceding nodes for each pick-up point. Lalla-Ruiz et al. (2016) propose a mixed integer programming formulation for the open MDVRP. New sub-tour elimination constraints are proposed based on ensuring route continuity in terms of demand and distance. The method is capable of solving a problem with 288 customers and 6 depots. Burger and De Schutter (2017) study the Multi-Depot TSP (MD-TSP) and introduce a two-index formulation based on node currents for the

fixed-destination. This formulation reduces the number of binary and continuous variables. The proposed formulation is able to solve problems of up to 170 nodes using general MILP solvers.

2.2.2.2 Heuristic methods

There are many heuristics that have been used to solve the MDVRP. The MDVRP consists of two problems namely, assigning customers to depots, and solving the routing problem. Therefore, heuristics that have been proposed to solve the MDVRP can be categorised into three groups based on the structure of MDVRP itself. These groups are heuristics with one phase, heuristic with two phases, and heuristics with three phases or more.

The first group includes heuristics that solve the two mentioned sub-problems simultaneously. Tillman (1969) extends the CWH to tackle the MDVRP, by modifying the way of computing saving distance to reflect the true savings relative to each depot. The steps of Tillman's heuristic look like the steps of the CWH. In the beginning, each customer is assigned to the nearest depot. Then, routes are constructed based on the modified saving list. When a customer is joined at a depot, it will not be considered at the other depots. To avoid this barrier, Tillman and Hering (1971) improve criteria of assigning customers to depots which is suggested by Tillman (1969). Hence, the best possible choice of saving list is made initially. Another improvement was made by Tillman and Cain (1972) as explained in the following. In their heuristic, distance savings are determined from joining customers on routes, then possible assignments are made as a function of the maximum savings for joining customers on routes. If all possibilities are investigated, this approach will lead to an optimal solution using savings as the criterion to be optimised. However, this is very time consuming. Golden et al. (1972) use an efficient data structures to reduce both the computational time and storage requirement of Tillman and Cain's approach, which permits problems involving hundreds of customers to be solved in a matter of seconds.

For the two-phase framework, Wren and Holliday (1972) and Cassidy and Bennett (1972) develop a two-phase heuristic to generate an initial solution in the first phase and then refine it at the second phase. Wren and Holliday (1972) construct the initial solution by means of a greedy heuristic. This solution is refined in the second phase, by using seven procedures as a local search. While Cassidy and Bennett (1972) generate the initial solution randomly at the first phase, it is then improved by

exchanging nodes one at time between routes, until no further improvement is possible. Perl (1987) generates an initial solution in the first phase by assigning customers to the nearest depot, then solve routing by using a modified CWH. In the second phase, the initial solution is improved by three procedures: 2-opt move, inserting a customer in a route from another one, and exchanging two customers from different routes.

Salhi and Nagy (1999) extend the insertion-based heuristic for the VRP with backhauling to the MDVRP with backhauling. It is based on the idea of inserting more than one backhaul at a time; the improvement solution was in the second phase. Yang and Chu (2000) address the PMDVRP and propose a two-phase heuristic. An initial solution is constructed based on the minimum cost in the first phase. Then, in the second phase, improvement procedures are applied on the initial solution by means of the saving concept. Carlsson et al. (2009) study the min-max MDVRP and present two heuristics which include a linear programming with global improvement and the region partition heuristic. Kazaz and Altinkemer (2003) formulate printed circuit board manufacturing as MDVRP. The printed circuit board consists of two sub-problems, assigning chips to feeder locations in a computerised numerically controlled machine, and sequencing the placement of these components. The assignment problem is solved optimally, while the sequencing is solved by a heuristic using saving method.

The MDVRP with time windows is considered by Giosa et al. (2002) and Chiu et al. (2006). Giosa et al. (2002) present an approximation method consisting of two phases. In the first phase, customers are assigned to depots, while in the second phase, several VRPs are solved separately by CWH. Customers are assigned based on six procedures: parallel assignment, simplified assignment, sweep assignment, cyclic assignment, assignment by clusters, and coefficient propagation. However, Chiu et al. (2006) generate several initial solutions using three heuristics which include saving, insertion, and sweep. Then, the best solution is chosen. The second phase is to improve the best solution by applying two procedures: inter-route and intra-route.

Ramos et al. (2009) treat the recyclable waste collection with three types of recyclable materials and more than one depot, as MDVRP with multi products. The collection sites are assigned to depots based on borderline concepts which divide customers into two groups based on their distance to depots. Then, routing is obtained by the CWH.

Finally, heuristics that consist of three phases and more have been proposed by many researchers. Raft (1982) propose a heuristic to deal with the MDVRP which consists of four phases. In the beginning, the

number of routes and centres of all routes that are required to satisfy all customers are computed. In the first phase, customers are assigned to the nearest route-centre. In the second phase, route-centres and their associated customers are assigned to the nearest depots. In the third phase, several VRPs are solved separately using a heuristic approach. Finally, in the fourth phase, routes are improved by 3-opt. Chao et al. (1993) also propose a three-phase heuristic that assigns customers to depots, builds routes at each depot, and then relies heavily on an improvement procedure to clean up the routes.

Salhi and Sari (1997) and Nagy and Salhi (2005) address the MDVRP and MDVRP with pick-up and delivery, respectively. A three-phase heuristic is proposed for the two problems. The initial solution is generated in the first phase. Then, the initial solution for each depot is improved separately. Finally, the whole solution for all depots is improved simultaneously. Hu et al. (2007) propose a three-phase heuristic for the MDVRP with pick-up and delivery. Firstly, customers are assigned to a depot by using borderline customers. Secondly, an initial solution is generated by clustering customers. In the third stage the solution is improved by an insertion algorithm.

Rahimi-Vahed et al. (2015) propose an iterative modular heuristic for three problems: MDVRP, PVRP, and PMDVRP. The heuristic consists of three sequential phases. At the first phase, customers are clustered. These clusters are assigned to depots at the second phase. Finally, routes are designed for each depot by giant and spilt tour heuristic. Wang et al. (2015) consider the min-max MDVRP to minimise the length of the longest route. They develop a heuristic which consists of three stages – assignment of customer to depots and solve each of them as VRP, then improve the maximal route, and finally, improve all routes by exchanging customers between routes. Gulczynski et al. (2011) combine the MDVRP and split delivery VRP. Customers are classified into borderline and non-borderline. All non-borderline customers are assigned to the nearest depot. Borderline customers are assigned to depots based on a cheapest insertion criterion. Then, the routing problem is solved for each depot separately by using a three-stage heuristic. The initial solution is generated by using a modified CWH. In the second stage, an endpoint mixed integer program with minimum delivery amounts is formulated and solved. The improvement procedure is applied in the third stage by using the enhanced record-to-record travel algorithm, to reduce the total distance travelled by the fleet.

Min et al. (1992) solve the MDVRP with backhauling, by decomposing it into three phases, where the output of one phase becomes the input to the next phase. The first phase aggregates customers into a capacitated cluster. The second phase designs routes and assigns customers to depots. The final phase designs the individual routes. Alemany et al. (2016) study a real-life case of distribution of fuel in the

North of Spain. In this problem, each customer may order different types of fuel, and vehicles use compartments to avoid mixing products of different types. The fleet of vehicles is heterogeneous. Moreover, there are external facilities which might be used to replenish some vehicles. The problem is modelled as MDVRP with a heterogeneous fleet. They propose a three-stage approach to tackle this problem. In the first stage, an assignment map of customers to depots is generated based on their distance to the depots. Then, routing for each depot with its customers is determined independently as a VRP by applying CWH. Finally, a 2-opt local search is used to improve each route.

Hadjiconstantinou and Baldacci (1998) consider a utility company which offers a preventive maintenance for a network of customers. There are a fleet of 17 depot-based mobile gangs dispatched from nine depots to call on 162 customers, with a frequency that can vary from once per day to once every four weeks. Each gang, consisting of two workers, visits in average four customers per day. The problem is addressed as a PMDVRP. The solution is composed the problem into four levels. Firstly, boundaries for each depot service area is defined. Secondly, a PVRP for each depot is solved. At the third level, a VRP for each depot and for each day of the given period is solved. Finally, each tour is solved as a travelling salesman problem.

Afshar-Nadjafi B and Afshar-Nadjafi A (2017) consider the MDVRP with time windows and time-dependent, which means the travel time between locations depends on the departure time. A constructive heuristic consisting of five steps is developed for the problem. In the first step, the sequence of customers is constructed by a greedy heuristic based on their time windows. In the second step, a dynamic probability is used to assign vehicles to customers at the second step. In the third step, routes are constructed in a greedy manner. In the fourth step, start and end depots are determined based on minimum routing cost. Finally, in the fifth step, a local search is applied to improve the solution.

2.2.2.3 Metaheuristic methods

Several metaheuristic algorithms have been proposed to solve the MDVRP: TS, SA, GA, VNS, ACO, and PSO. An analytical presentation of these algorithms which are applied on MDVRP is given next.

a) Tabu Search

TS has been applied to solve the MDVRP in three different frameworks which can be categorized to one-phase, two-phase, and three-phase approaches. Jin et al. (2004) study the MDVRP and present two methods to solve it. They are one-stage and two-stage approaches. The one-stage approach integrates the assignment with the routing in the same level. Assignment customers cost to depots is estimated then each customer is assigned to the depot based on the minimum cost. Then a branch-and-bound algorithm and a TS heuristic is applied for routing. In contrast, the two-stage approach decomposes the problem into two independent sub-problems, assignment and routing, and solves them separately. In the first stage, three assignment methods namely, parallel, simplified and cyclic are used, then, in the second stage, the same branch-and-bound and TS heuristic is applied for routing.

For the two-phases frame, Renaud et al. (1996) and Cordeau et al (1997) propose a TS for MDVRP. Renaud et al. (1996) generate the initial solution by using a heuristic approach, while Cordeau et al (1997) constructs the initial solution by an insertion algorithm. Then, the Tabu Search improves the initial solution. A GTS is proposed by Escobar et al. (2014). The initial solution is constructed by using a heuristic approach. Maischberger and Cordeau (2011) introduce a parallel iterated TS for solving eight different variants of the VRP. They are the VRP, the PVRP, the MDVRP, and the Site-Dependent VRP (S-DVRP), all with or without time windows constraints. In their study, the initial solution is generated by applying a heuristic approach without describing the details.

Aras et al. (2011) study the MDVRP with pricing. In this problem, a firm aims to collect used products to save production cost by re-manufacturing of the parts and components obtained from the collected products. The vehicles are dispatched to a dealer if the acquisition price announced by the firm exceeds the dealer's reservation price. The solution method consists of two stages. The first stage concerns producing an initial solution by sorting all dealers based on their indices and putting all of them on one route only. This route is an infeasible solution therefore there is a penalty cost for overcapacity. The second stage is for improving the solution by TS. Lim and Wang (2005) propose two-stage methodologies by decomposing the problem into two independent sub-problems, assigning and routing, and solves them separately. The assigning stage is performed by applying two criteria – urgency assignment, and group assignment. In the second stage, routes are obtained by TS. Soto et al. (2017) develop a hybrid method including a TS and a multiple neighbourhood search to address the open MDVRP. The initial solution is constructed by an insertion greedy algorithm.

Crevier et al. (2007) address the MDVRP with Inter-Depot Routes (MDVRP-IDR) where vehicles are replenished at intermediate depots along their routes. A three-phase algorithm is proposed. The first phase is an adaptive TS for generating initial routes. The second phase is to determine the feasible routes with the least cost by means of a set partitioning algorithm. Finally, the TS is applied to improve the feasible routes. Zhen and Zhang (2009) consider the MDVRP with inter depot routes in which vehicles may be replenished at intermediate depots along their routes. A three-phase methodology is proposed based on adaptive memory and TS. The initial solution is formed in phase one by using a heuristic approach. Because the initial solution is infeasible for the MDVRP with intermediate depots, the set partitioning algorithm is used to fix the infeasibility at the second phase. Finally, in the third phase, the TS is applied to improve the solution. Shankar et al. (2014) address MDVRP with time windows and use TS within the Geographical Information System (GIS) to obtain an approximate solution. The initial solution is generated by using a heuristic approach.

b) Simulated Annealing

The SA is one of the metaheuristics that has been used widely in the combinatorial optimisation. When it is used to solve MDVRP, it has been applied in a two-phase framework, where another metaheuristic is also involved. Chen et al. (2005) introduce a two-phase heuristic to solve the MDVRP. In the first phase, a random initial solution is generated. The second phase consists of a heuristic and SA approach to improve the initial solution. The improvement heuristic exchanges customers between routes by using 2-opt. Lim and Zhu (2006) consider the MDVRP with fixed distribution. Based on the fact that all sub-routes of an optimal route must be optimal, a randomised best insertion algorithm is proposed to generate an initial solution. Then, an n-opt neighbourhood operator and a SA approach are applied to improve the initial solution.

Ting and Chen (2008) propose a Multiple Ant Colony System (MACS) and SA approach to solve MDVRP with time windows. The algorithm is designed to assign customers to depots firstly by MACS, then solve the routing problem by MACS and finally improve it by SA. Mirabi et al. (2010) address MDVRP in order to minimise the delivery time of the vehicle objective. A two-phase heuristic is proposed to solve the problem. In the first phase, an initial solution is generated by assigning customers to the nearest depot, then routes are built by the means of CWH. In the second phase, local

search and SA are applied to improve the initial solution. Mirabi (2014) propose a Hybrid Electromagnetism Simulated Annealing (HESA) to solve the PMDVRP.

c) Variable Neighbourhood Search, Iterated Local Search, and Adaptive Large Neighbourhood Search

In this section, we will cover three metaheuristics namely; VNS, ILS, and ALNS. These metaheuristics are easy to implement, and flexible to adapt to different problems. Polacek et al. (2004) and Polacek et al. (2008) propose VNS methods to solve MDVRP with time windows. To construct an initial solution, each customer is assigned to the nearest depot. Then, all customers within a depot are ordered by the centre of their time windows. Routes are constructed by insertion based on their time windows. Polacek et al. (2004) improves the initial solution by a VNS, while Polacek et al. (2008) improves the initial solution by introducing two parallel VNS. Xu et al. (2012) study the MDVRP with heterogeneous vehicle and time windows. The problem is solved using a VNS. The initial solution is formed by inserting customers on routes based on their distance to the nearest depot. Kuo and Wang (2012) use the VNS to solve the MDVRP with loading cost. Firstly, the initial solution is generated by two methods. The first method is a random method, while the second method is developed based on the CWH. The VNS is applied afterwards to find the solution. Xu and Jiang (2014) improve VNS for MDVRP with heterogeneous fleet and time windows. The initial solution is obtained by using a clustering algorithm to allocate customers to depots. Then a hybrid operator of insert and exchange are used. After that, VNS is applied to obtain the solution. Imran (2013) and Salhi et al. (2014) apply the policy of borderline customers to assign them to depots for solving MDVRP and MDVRP with heterogeneous vehicles, respectively. The initial solution is obtained for each depot using a greedy heuristic and improved by 2-opt moves. Then, the VNS is applied for improvement. Schmid et al. (2010) model scheduling distribution of ready-mixed concrete that is produced at several plants to construction sites, using heterogeneous fleet as MDVRP. A hybrid procedure is proposed based on a combination of a VNS and an exact method. The VNS approach is used at first to generate feasible solutions and it tries to further improve them. Then, a VLNS determines which variables are supposed to be fixed. Finally, the MILP solver finds the exact solution for the problem.

An ILS is proposed by Li et al. (2015) for MDVRP with simultaneous delivery and pick-up. The proposed algorithm integrates an adaptive neighbourhood selection mechanism into ILS, employs

different structural neighbourhoods in the improvement and perturbation steps, and uses a probability rule to accept a worse solution based on the number of its repetition. The initial solution is generated by assigning each customer to the nearest depot then routing is determined by CWH for each depot individually. Dondo and Cerdá (2009) present a local search algorithm theta that explores a large neighbourhood of the initial solution for the MDVRP with time windows. The initial solution is generated by a clustering heuristic. Pisinger and Ropke (2007) propose an ALNS to solve five different variants of VRP where one of them is the MDVRP. The initial solution used in their proposed local search is found using a heuristic approach. Mancini (2016) study the PMDVRP with heterogeneous fleet. An ALNS is proposed. A greedy heuristic is used to obtain the initial solution.

Tlili et al. (2016) tackle the MDVRP by using an ILS. A constructive heuristic is developed to generate the initial solution by inserting customers one by one into a vehicle route, and when the vehicle capacity is reached, a new empty route is started. Juan et al. (2015) propose a hybrid approach for the MDVRP which combines Biased Randomisation with an ILS. Two Biased Randomised processes are employed to assign customers to depots and to improve routing solutions. Then, routing is constructed by BR-CWH. Calvet et al. (2015) and Calvet et al. (2016) use the approach of Juan et al. (2015) to tackle the MDVRP with Stochastic Demands, and the MDVRP that includes market segmentation issues, in order to maximise benefits, respectively.

Calvet et al. (2015) employ the Monte Carlo Simulation (MCS) techniques to deal with stochastic demand. To reduce the route-failure risk, safety-stocks are included in the algorithm and risk analysis is used. Calvet et al. (2016) consider customers' needs when they are assigned to depots to increase the expected expenditure. Tlili and Krichen (2015) consider a real case of MDVRP at Ezzahra city in Tunisia. An ILS is combined with GIS to design a decision support system to solve the problem and visualise the results. Vidal et al. (2014) introduce ILS and a hybrid GA to tackle the MDVRP with mix fleet.

d) Genetic Algorithms, Memetic Algorithm, and Evolutionary Algorithm

GA, MA, and EA will be covered in this section. These metaheuristics belong to population-based algorithm. In general, they are used to solve optimization problems with high quality solutions.

A GA is proposed by Filipec et al. (1997) to solve non-fixed destination MDVRP. The non-fixed destination problem is an extension of MDVRP with routing, originating and terminating at different

depots. Customers are clustered by mean of Ford and Fulkerson Algorithm (FFA). Then, GA is used to generate radial routes and used again to connect them into complete links. Ho et al. (2008) develop GA to improve two initial solutions. The first one is generated randomly, while CWH and the nearest neighbour heuristic are incorporated to generate the second one. In this paper, the benchmark instances have not been used to compare its results with previous heuristics.

Ombuki-Berman and Hanshar (2009) propose a GA algorithm for MDVRP with two objectives which are minimising the total distance and minimising the number of vehicles used. Lau et al. (2010) deal with MDVRP with multiple products. A GA is proposed using a stochastic search technique to solve the problem. The role of the stochastic search technique is to dynamically adjust the crossover rate and mutation rate after ten consecutive generations. Vidal et al. (2012) addresses three problems which include the MDVRP, the PVRP, and the PMDVRP. They propose a hybrid GA that includes a number of advanced features in terms of solution evaluation, offspring generation and improvement, and population management. Li and Liu (2011) develop a GA for the MDVRP with three objectives which are minimising the computational time, the total cost, and the number of used vehicles. The proposed algorithm is compared with an ILS.

De Oliveira et al. (2016) decompose the MDVRP in sub-problems of VRP and each problem is solved by a cooperative co-EA separately. Bae and Moon (2016) use the MDVRP with time windows to model the delivery and installation of electronics. They develop a heuristic and a GA to identify a near-optimal solution.

Salhi et al (1998) and Thangiah and Salhi (2001) develop a three-stage framework to tackle the MDVRP. In the first stage, the GA algorithm is applied to cluster customers, while routes are obtained in the second stage by using an insertion heuristic. Finally, Salhi et al (1998) improve the solution by local search, while Thangiah and Salhi (2001) improve the solution by post-optimisation routine.

The MDVRP with time windows is addressed and solved using the GA by Yang et al. (2006), Yuan-feng (2008), Liu (2013), Lightner-Laws et al. (2016), Li et al. (2016), and Bi et al. (2017). Yang et al. (2006) propose a GA after obtaining the initial solution randomly. Yuan-feng (2008) uses the heterogeneous fleet and improves a GA after obtaining the initial solution randomly. Liu (2013) adapt a GA based on a HBMOA. Lightner-Laws et al. (2016) use the heterogeneous fleet for a pick-up and delivery service and apply a nested GA. Li et al. (2016) do not require that vehicles return to the same depot and propose a hybrid GA with adaptive local search. Bi et al. (2017) employ a bi-objective

function consisting of minimising traveling distance, and maximising delivery duration, and propose a hybrid multi-objective EA enhanced with a two-phase local search.

A geometric shape based on Genetic Clustering Algorithm (GCA) is proposed by Yücenur and Demirel (2011) for the solution process of the MDVRP. While Bae et al. (2007) develop graphical user interface programming to solve MDVRP by assigning customers firstly to the nearest depot, then using GA to solve the routing problem. Finally, a Two-Echelon MDVRP is introduced by Zhou et al. (2017) which will rise at distribution of e-commerce. A hybrid multi-population GA is proposed.

e) Ant colony optimisation and Particle Swarm Optimisation

ACO and PSO are probabilistic algorithms for solving optimisation problems. These metaheuristics try to improve candidate solutions iteratively.

An ACO is employed by Yu et al. (2011), Yücenur and Demirel (2011), Wang (2013), and Stodola and Mazal (2016) to solve the generic MDVRP. Yu et al. (2011) add a virtual central depot firstly. Then, the ACO is applied. Stodola and Mazal (2016) adapt the ACO to solve the MDVRP. Yücenur and Demirel (2011) propose ACO and GA. The GA is used in the first phase to cluster customers, while in the second phase the ACO is applied for routing. Wang (2013) proposes a Cellular Ant Optimisation Algorithm (CAOA) which combines cellular automaton and ACO to present a high quality solution. Yao et al. (2017) model the fresh seafood delivery problem as MDVRP with an energy cost for keeping fresh seafood in cold conditions as a main feature. The problem is decomposed into VRP, then ACO is used to solve it. Belov and Slastnikov (2017) model the petroleum products delivery problem as MDVRP and apply the ACO with local search to solve it. Islam and Rahman (2012) consider a real-life case of waste collection and formulate it as MDVRP with time windows. The problem is solved by using ACO.

The MDVRP with time windows is addressed by Liu and Yu (2013). An ACO with GA is proposed. The purpose of the GA is to optimise the parameters of the ACO, whereas the ACO is to solve the problem. Ma and Yuan (2010) aim to minimise the waiting time of customers instead of minimising the travelling distance. This aim, minimising the waiting time, is more important than minimising distance in some cases such as emergency, and fast-food delivery. An ACO is proposed to solve this problem. Yang et al. (2011) present multi objectives MDVRP with time windows, and heterogeneous fleet. The

objective function includes three parts: transport cost, deadheading cost, and time cost. A self-adaptive and polymorphic ACO has been introduced to solve this problem. Narasimha et al. (2013) study the min-max MDVRP and solve it by ACO. The main problem is decomposed into many min-max VRP, then each sub-problem is solved individually.

The PSO has been applied by Sombuntham and Kachitvichyanukul (2010), Kachitvichyanukul et al. (2015), and Zhu et al. (2015) to solve the MDVRP with pick-up and delivery and time windows, the MDVRP with multiple pick-up and delivery, and the MDVRP where customer demand consists of two-dimensional weighted items, respectively. To enhance the algorithm and produce high quality solutions, Sombuntham and Kachitvichyanukul (2010) employ multiple social learning structures; Kachitvichyanukul et al. (2015) employ multiple social learning terms, and Zhu et al. (2015) use the local search. Geetha et al. (2012) apply the MDVRP for two real-life problems: the home delivery pharmacy program and waste-collection. The problem is solved by a hybrid metaheuristic consisting of a heuristic, GA, and PSO. The heuristic is for generating the initial solution, while the GA, and PSO are for improving the solution.

2.2.3 Real application of MDVRP

The MDVRP has many applications in our real life. It has been used to solve problems in utilities sectors (Hadjiconstantinou & Baldacci, 1998), reverse logistics (Ramos et al., 2009), in emergency management (Ma & Yuan, 2010), ready-mixed concrete delivery (Schmid et al., 2010), petrol station replenishment (Cornillier et al., 2012), recycl sector (Ramos et al., 2011, Islam & Rahman, 2012, and Ramos et al., 2014), printed circuit board (Kazaz & Altinkemer, 2003), distribution of perishable food (Mancini, 2016), distributing goods (Alemany et al., 2016), distribution of petroleum products (Belov & Slastnikov, 2017), Fresh seafood delivery (Yao et al., 2017). These applications are only an example to show the importance of the MDVRP in our real life.

2.3 Conclusion

The literature review of the LRP and its variants (MDVRP) indicates that these problems are considered in supply chain management as one of the essentials. And these problems are widely studied due to the practical relevance of their applications. Therefore, studies and researches in this area is growing faster in the last decade.

According to Grasas et al. (2017) combining the Biased Randomised technique with heuristics can improve its performance without losing its good properties which increases the chance of obtaining better and diversified solutions.

Moreover, this strategy enables procedures of the deterministic heuristic to be transformed to procedures of the probabilistic algorithm. This means that the new solution method can be run several times to obtain different promising solutions. Combining the Biased Randomised technique with heuristics is relatively simple in terms of implementation, and fast in terms of computational time (Grasas et al., 2017).

On the other hand, the supply chain management is keen to consider environmental issues such as generation of CO₂, and greenhouse-gas emissions to comply with environmental regulations.

From the findings in the literature review, we noticed that heuristics, exact methods, and metaheuristics have been applied to solve the LRP and its variants. Some researchers have attempted to improve the solution quality, while other have attempted to reduce the computational time.

However, the simplicity and flexibility of the solution methods have not been taken into account. On the other hand, environmental issues such as generation of CO₂, and greenhouse-gas emissions have not yet been considered when solving the LRP.

Therefore, this indicates that we should first focus on improving solution methods based on combining Biased Randomised technique with classic heuristics. In addition to this, environmental issues are also considered when solving the LRP to reduce generation of CO₂, and greenhouse-gas emissions.

Chapter 3 Location Routing Problem with Single Depot (LRPSD)

3.1 Introduction

In this Chapter, we study the LRP with Single Depot (LRPSD), which is the simplest variant of the LRP. The LRPSD has many applications in real-life such as system of computer servers, and collection of money. Moreover, we adapted the mathematical model from Laporte and Nobert (1981) for the LRPSD to examine the performance of our four heuristic methods when Biased Randomised technique is applied.

We propose and implement four solution methods based on the combination of a location heuristic and Biased Randomised Clarke and Wright Heuristic (BR-CWH) to obtain location and routing decisions, respectively. The BR-CWH basically integrates the Biased Randomisation technique (Juan et al., 2010) with the CWH (Clarke and Wright, 1964).

We describe in detail the BR-CWH and our four proposed approaches namely: (i) Biased Randomisation Two-Stage Clustering heuristic (BR-TSCH); (ii) Biased Randomisation Two-Stage p-median heuristic (BR-TSPH); (iii) Biased Randomisation Two-Stage Clustering and p-median heuristic (BR-TSCPH); and (iv) Biased Randomisation Iterated heuristic (BR-IH).

The remainder of this chapter is structured as follows: In Section 3.2 we highlight the key contributions of this chapter. In section 3.3 we give a detailed description of the problem and the optimisation model. Section 3.4 presents the details of the solution methods, and in section 3.5 we present the experimental settings and computational results. Section 3.6 provides a summary discussion of the chapter.

3.2 Contribution

The main contribution of this chapter is to propose a novel solution method to solve the LRPSD. The suggested heuristic framework consists of two stages of location and routing to solve the LRPSD by combining Biased Randomised technique with the CWH.

The four mentioned variations are Biased Randomisation Two-Stage Clustering heuristic (BR-TSCH), Biased Randomisation Two-Stage p-median heuristic (BR-TSPH), Biased Randomisation Two-Stage Clustering and p-median heuristic (BR-TSCPH), and Biased Randomisation Iterated heuristic (BR-IH). The similarity between these four heuristics is in the second stage (routing stage) which is solved by the BR-CWH. Whereas, the difference between them is in the first stage (location stage) which is solved by four different methods namely: clustering, p-median, clustering and p-median, and Iterated heuristic.

Another major contribution of the proposed approaches is the incorporation of the location problem into the BR-CWH. The BR-CWH is proposed mainly to deal with the VRP. And the LRP optimisation problem consists of two parts, location decision and routing decision. Therefore, the incorporation of the location problem into the BR-CWH can help to solve the LRP.

The experimental results showed that combining the Biased Randomised technique with the classic heuristic (CWH) is able to obtain alternative and competitive solutions in terms of the quality and computational time.

3.3 Optimisation model

The LRP has many variants of models in the literature based on its application or its characteristics. For example, in an open LRP, vehicles do not return to its depot after delivering goods. Also, there are uncapacitated LRP where depots have unlimited capacity and so on. In this chapter, we considered the LRP with only single depot which is considered as a special case of the general LRP.

The problem is to determine the location of a single depot among potential depots, then determine the corresponding delivery routes to serve all customers from the open depot. The objective function is to minimise the total costs which consist of depot opening cost, variable and fixed costs for vehicles.

Each vehicle takes exactly one route starting from the depot, visiting a subset of the customers and returning to the same depot. In addition, a customer's demand cannot be split among different routes and the sum of demands in each route must not exceed the vehicle capacity.

The LRP model in this research is defined on a complete, weighted, and undirected network $G = (V, E, C)$, where $V = \{1, \dots, n\}$ is a set of nodes representing the depots and customers, and E is a set of undirected edges (i, j) , and $C = (c_{ij})$ is the matrix of traveling cost associated with edge (i, j) in E . In this chapter, developed heuristics only consider a single depot while multi depots will be addressed in Chapter 5. It is assumed that $I \subseteq V$ is a set of potential depots and $J \subseteq V$ is a set of customers. An opening cost f_i are associated with each depot site $i \in I$. A set K of identical vehicles of capacity D is available. When used, each vehicle incurs a fixed cost F and performs a single route. Each customer $j \in J$ has a demand d_j where $d_j \leq D$. Since $d_j \leq D$, there will never be a need for a node (customer) to be visited by more than one vehicle to satisfy its demand.

Figures 3.1 illustrates an example of LRP with single depot. Firstly, in Figure 3.1 (a), there are three potential depots and 18 customers. In Figure 3.1 (b) a single depot is selected to be opened and two are closed. Finally, vehicle routes are computed in Figures 3.1 (c).

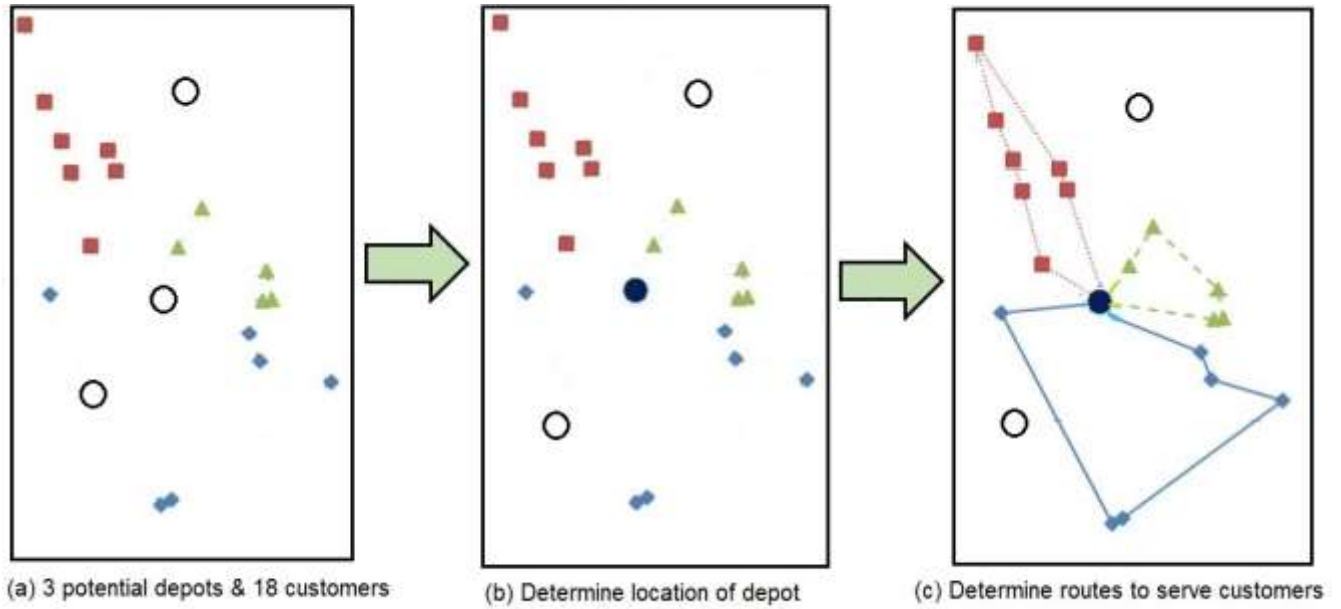


Figure 3.1. An illustrative example of LRPSD

The optimisation model is formulated as a mixed integer linear programming problem. In order to formulate the model, the following notation is introduced.

Sets are defined as follows:

V : Set of nodes, $V = I \cup J$

I : Set of potential depot nodes

J : Set of customers to be served

K : Number of available vehicles (fleet size)

Parameters are defined as follows:

f_i : The fixed cost of opening a depot at i

d_j : Demand of customer j

D : Capacity of each vehicle

F : Fixed cost per vehicle used

c_{ij} : Traveling cost for edge (i, j)

Decision variables are defined as follows:

$$x_{ijk} : \begin{cases} 1, & \text{if vehicle } k \text{ is used on route from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_i : \begin{cases} 1, & \text{if a depot is located at site } i \\ 0, & \text{otherwise} \end{cases}$$

The formulation of the Location Routing with single depot which we adapted from Laporte and Nobert (1981) is as follows:

$$\text{Min } \sum_{i \in I} f_i y_i + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F x_{ijk} \quad (3.1)$$

Subject to

$$\sum_{i \in V} \sum_{k \in K} x_{ijk} = 1 \quad \forall j \in J \quad (3.2)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1 \quad \forall k \in K \quad (3.3)$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \quad \forall k \in K, \quad \forall i \in V \quad (3.4)$$

$$\sum_{i \in V} \sum_{j \in J} d_j x_{ijk} \leq D \quad \forall k \in K \quad (3.5)$$

$$\sum_{i \in I} y_i = 1 \quad (3.6)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K \quad (3.7)$$

$$y_i \in \{0, 1\} \quad \forall i \in I \quad (3.8)$$

$$c_{ij} = \infty \quad \text{when } i = j$$

The objective function (3.1) seeks to minimise the total cost, which includes the fixed cost of the selected facilities and the fixed and variable cost of the vehicles. Constraints (3.2) are the routing constraints that are imposed where each customer has to be visited exactly once by a single vehicle, whereas constraints (3.3) ensure that all routes have to start and end at a depot. Constraints (3.4) are the connectivity constraints to ensure that every vehicle leaves the customer after he has been served. Constraints (3.5) impose the capacity of vehicle, while the constraint (3.6) ensures that only one depot is going to be opened. Constraints (3.7) and (3.8) are integer variables.

3.4 Proposed Biased Randomised heuristics for solving LRPSD

In this section, we propose four heuristics to solve the LRPSD. They are Two-Stage clustering, Two-Stage p-median, Two-Stage clustering and p-median, and Iterated heuristic.

These four heuristics are based on the combination of a location heuristic and the BR-CWH which is proposed by (Juan et al., 2010), which will be explained later, to obtain location and routing decisions, respectively.

The Biased Randomised Clarke and Wright Heuristic (BR-CWH) is explained in section 3.4.1. In section 3.4.1.1 and section 3.4.1.2, Biased Randomised technique and CWH will be explained, respectively. Finally, the explanation of the proposed Biased Randomised heuristics, are given in section 3.4.2.

3.4.1 Biased Randomised Clarke and Wright Heuristic

Recently, Juan et al. (2010) have proposed Biased Randomised technique to induce randomness (non-symmetric) in classical heuristics in an iterative framework. In particular, this new method induces randomness (non-symmetric) to perturbate the greedy behavior slightly of the classical heuristic. Therefore, the deterministic heuristic is transformed into a probabilistic algorithm, and that will make the solution space exploration more efficient.

The multi-start process works together with the Biased Randomised technique to avoid getting into a local minimum, and at the same time, the heuristic converges faster to the near optimal solutions.

The BR-CWH is able to provide high quality solutions which can compete with those provided by much more complex, exact and heuristic approaches, which are usually difficult to implement in practice. Moreover, it can generate hundreds of alternative good solutions in a reasonable time period, offering the decision-maker the possibility of applying various non-aprioristic criteria when selecting the solution that best fits their utility function (Juan et al., 2010).

The solution of CWH is constructed by choosing the edge with the highest savings value. However, in the BR-CWH, a probability of selecting each edge is assigned in the savings list instead.

This probability should be coherent with the savings value associated with each edge. That means edges with higher savings will be more likely to be selected from the list than those with lower savings. If we use the Uniform Randomisation, the logic behind the sorted list will be ineffective in the heuristic. Whereas, using a skewed probability distribution such as geometric distribution or triangular distribution will keep the common sense behind using the heuristic.

Therefore, the geometric distribution with parameter α is employed during the solution structure in the CWH to assign a probability of selecting each edge from the saving list. This means, each time a new edge must be selected from the list of available edges, a geometric distribution is randomly selected to assign exponentially diminishing probabilities to each eligible edge, according to its position inside the savings list (which has been previously sorted by its corresponding savings value).

That way, edges with higher savings values are always more likely to be selected from the list, but the exact probabilities assigned are variable and they depend upon the concrete distribution selected at each step. By iterating this methodology, a random but efficient search process is started. Moreover, this selection process is done without introducing many parameters in the methodology.

The parameter α is selected randomly, from a uniform distribution in $[a, b]$, where $0 < a \leq b < 1$, during the process of solution construction. The parameter α is the probability of choosing the edge with the highest values. Algorithm 3.1 illustrates the pseudo code of introducing the biased randomness into the process of selecting edges from the saving list in the CWH.

```

Input: SavingList
1  $\alpha \leftarrow$  generate random number between ( $a = 0.05$ ) & ( $b = 0.25$ )
2  $\beta \leftarrow$  generate random number between ( $c = 0$ ) & ( $d = 1$ )
3  $\gamma$  (Cumulative Probability)  $\leftarrow 0$ 
4  $n \leftarrow 0$ 
5 foreach edge ( $e$ )  $\in$  SavingList do
6    $Pr(e)$  (Edge Probability)  $\leftarrow \alpha * (1 - \alpha)^n$ 
7    $\gamma \leftarrow \gamma + Pr(e)$ 
8   if  $\beta < \gamma$  then
9     | return ( $e$ )
    end
10  else
11    |  $n \leftarrow n + 1$ 
    end
end

```

Algorithm 3.1. Random Edge-Selection (Juan et al., 2010)

After choosing the edge based on the Biased Randomised technique, the following conditions must be satisfied before joining two customers in one route:

- i) The combined demand on the new route should not exceed the vehicle capacity.
- ii) Customers must not already be on the same route.
- iii) If a customer is connected to two other customers, it is never considered for linking.
- iv) The other restrictions on the systems must be satisfied.
- v) If one or more of the conditions are not satisfied this pair of customers is excluded from further consideration at this depot.

If all the above conditions are satisfied, then these two customers are joined in one route. Figure 3.2 illustrates the flowchart of the BR-CWH.

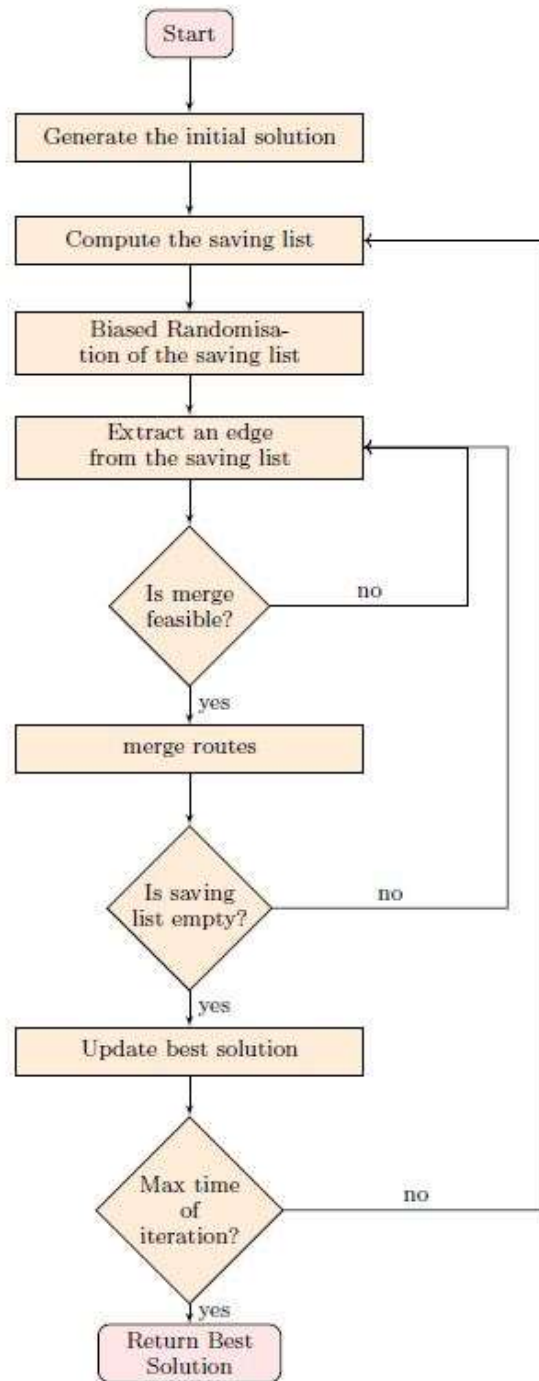


Figure 3.2. Flowchart of the BR-CWH

3.4.1.1 Biased Randomised Technique

A heuristic is defined as a simple procedure which follows a set of common-sense steps in order to solve Combinatorial Optimization Problems (COPs). Therefore, it does not guarantee optimality. However, it is able to provide a good solution in a very short computational time. A heuristic that constructs a good solution by selecting, at each step, the best next option from a list (e.g., list of edges, or list of nodes) which is sorted based on some criteria (e.g., ranking, priority rule, heuristic value) is considered as a deterministic iterative greedy procedure. Therefore, if we run it over and over, we will always get the same result.

If we randomise the order in which the elements (such as edges or nodes) of the list are selected, we will get a different output each time the procedure is executed. This means the randomisation principle will transform a deterministic heuristic into a probabilistic algorithm. Although the uniform randomisation will not be helpful with the logic behind the sorted list in the heuristic, using a skewed probability distribution, such as geometric distribution or triangular distribution, will give more chance for better candidates to be selected (Juan et al., 2010). Figure 3.3 shows the difference between applying the Uniform Randomisation and applying the Biased Randomisation to select an element from the sorted list.

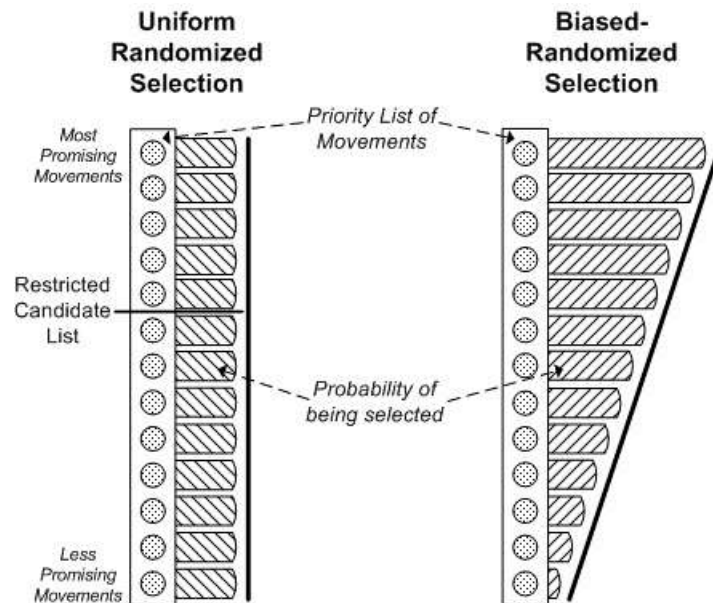


Figure 3.3. Uniform Randomisation vs. Biased Randomisation (Juan et al., 2010)

Recently, the Biased Randomisation technique is integrated in many classic heuristics to provide an efficient mechanism to solve COPs such as the VRP (Juan et al., 2010), and FLP (Cabrera et al., 2014). By applying this mechanism in an iterative framework, a new feasible and potentially good solution is generated every time the procedure is executed.

The developed algorithm usually has a single configuration parameter or is even without any parameter. This makes the time to deploy the algorithm in a real environment faster as it avoids the long and complex fine-tuning phase which is usually required by other metaheuristics. Moreover, using this integrated framework has proven to obtain promising results in low computational times.

3.4.1.2 Clarke and Wright Heuristic

The CWH is proposed by Clarke and Wright (1964) to solve the VRP. It is one of the best-known classical heuristics for solving the VRP.

In the VRP, there are n customers and only one depot in a given area. Additionally, there is a demand $d_j > 0$ of some goods, which have to be delivered and has been assigned to each customer j and this quantity is known in advance. Also, there are a fleet of homogenous vehicles which are stationed at the depot, and each vehicle has a maximum capacity to carry. These vehicles must all start and finish their routes at the depot. The objective of the VRP is to obtain a set of delivery routes from the depot to the various customers to minimise the total distance covered by the entire fleet. It is assumed that the demand of a customer, d_i , is less than the maximum capacity of the vehicles, and the whole demand should be delivered by a single vehicle (i.e. there is no split delivery).

The CWH starts by an initial solution which consists of using n vehicles and assigning one vehicle to each customer. This means, each customer is served by one vehicle in one route, and the total route length of the initial solution is $2 \sum_{i=1}^n d_{0i}$, while d_{0i} is the distance between depot and customer j .

After that, the saving distance is computed for all customers by the equation of the saving distance as below:

$$S_{ij} = d_{i0} + d_{j0} - d_{ij} \quad (3.9)$$

where:

S_{ij} : the saving distance between node i and j

d_{i0} : the distance between node i and the depot

d_{j0} : the distance between node j and the depot

d_{ij} : distance between node i and node j

The saving distance comes from joining two customers i and j in one route and serving them by one vehicle. The total distance traveled, after joining process, is reduced by the amount S_{ij} . The larger S_{ij} is the more desirable to combine i and j in one route. However, customer i and j cannot be combined if in doing so the resulting route violates one or more of the constraints of the VRP, such as the vehicle capacity constraint.

The CWH can be described as follows:

STEP 1: Calculate the savings distance S_{ij} for every two customers (i, j) by using equation (3.9)

STEP 2: Sort the savings distance S_{ij} and list them in descending order to create the savings list.

STEP 3: Process the savings list by beginning with the largest S_{ij} and check the following conditions which must be satisfied before joining two customers in one route:

- i) The combined demand on the new route should not exceed the vehicle capacities.
- ii) Customers i and j must not already be on the same route.
- iii) If a customer is connected to two other customers, it is never considered for joining.
- iv) The other restrictions on the system must be satisfied.
- v) If one or more of the conditions are not satisfied this pair of customers are excluded from further consideration at this depot.

If all the above conditions are satisfied, then these customers are joined in one route.

STEP 4: If the savings list has not been exhausted, return to STEP 3, processing the next entry in the list; otherwise, stop. The solution to the VRP consists of the routes created during STEP 3. Figure 3.4 shows an instance of a VRP (a) and its solution (b).

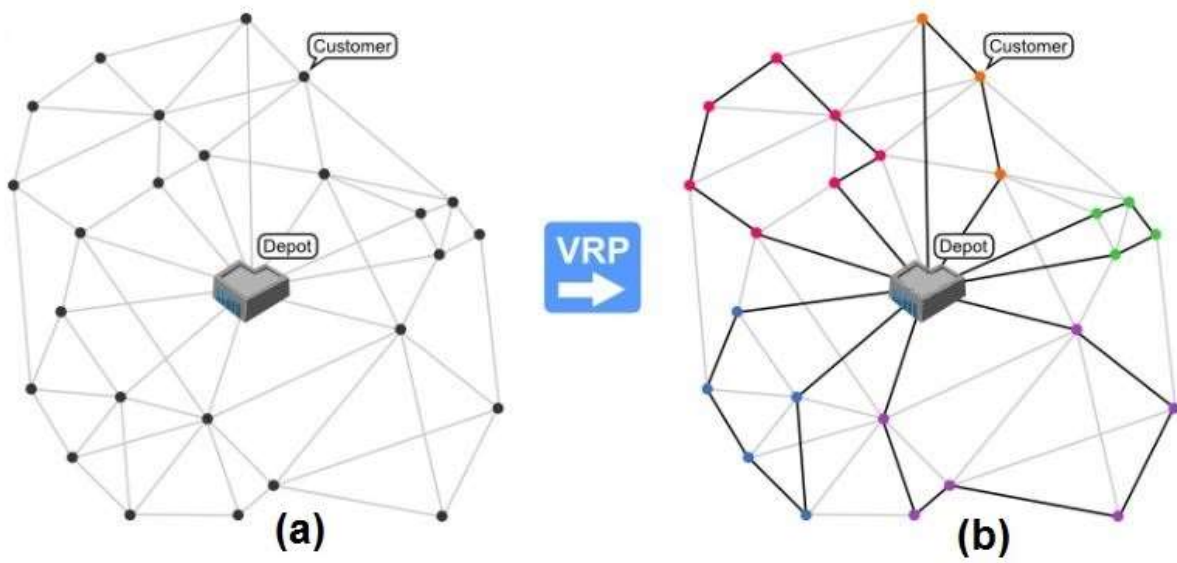


Figure 3.4. An instance of a VRP (a) and its solution (b).

3.4.2 Biased Randomised Two-Stage heuristics

The general framework of heuristics consists of two stages of location and routing which are solved sequentially. Figure 3.5 shows the flowchart of the general framework of the Biased Randomised Two-Stage heuristic which has four variants of the heuristic, called Biased Randomised Two-Stage Clustering heuristic (BR-TSCH), Biased Randomised Two-Stage p-median heuristic (BR-TSPH), Biased Randomised Two-Stage Clustering and p-median heuristics (BR-TSCPH), and Biased Randomised Iterated heuristic (BR-IH).

In the first stage of the first three heuristics, namely BR-TSCH, BR-TSPH, and BR-TSCPH, only one depot is selected to be open among the list of potential candidates, by using clustering technique, p-median model, and clustering and p-median model, respectively. Details of these methods will be explained later in section 3.4.2.1, 3.4.2.2, and 3.4.2.3, respectively. In the second stage of the three heuristics mentioned, routing of customers is determined by applying the BR-CWH.

In the BR-IH, a depot is chosen randomly in the first stage, and routing is solved in the second stage by using the BR-CWH. The heuristic, then, iterates with another randomly chosen depot, and so on. The algorithm checks all the potential depots and keeps the best result in terms of both location and routing costs. In section 3.4.2.4, the BR-IH is explained in detail.

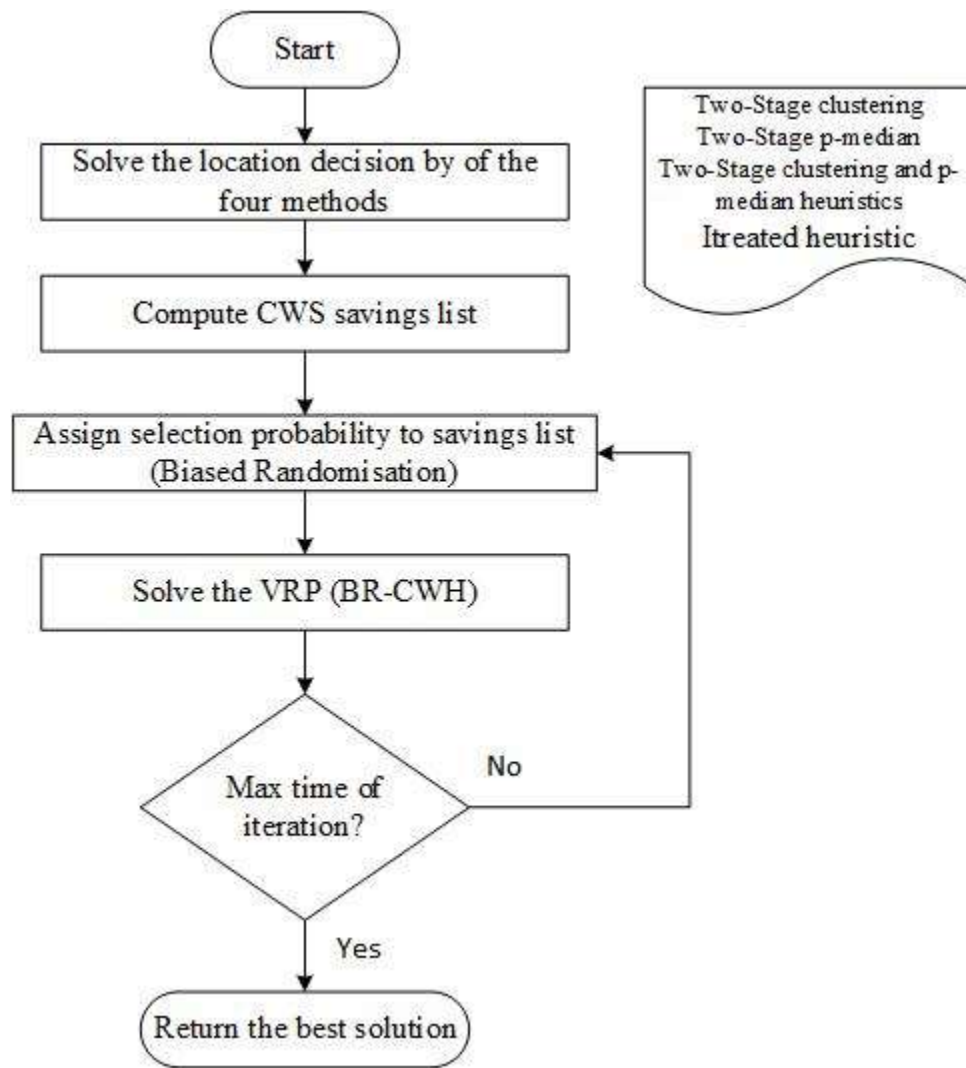


Figure 3.5. Flowchart of the general framework of the Biased Randomised Two-Stage heuristics

3.4.2.1 Biased Randomised Two-Stage Clustering heuristic

In this heuristic, the first stage solves the location problem using clustering and gravity centres, while the second stage solves routing problem using the BR-CWH.

To solve the location problem, we have adapted and modified a heuristic proposed by Salhi and Gamal (2003) to solve the continuous location allocation problem. We have adapted the same method to cover the region of customers with $k_0 \times k_0$ rectangular cells and used the same equation (3.10) to compute the gravity centre of each cell. We have added computing the Euclidean distances between each depot and the gravity centres, and the depot with the minimum sum of distances from the gravity centres is chosen to be the depot of VRP problem.

The benefit of using this heuristic is to deal with the nature of location spread of customers. The locations spread of customers, in some cases, follow the uniform distribution, while in other cases, locations of customers are clustered. Therefore, choosing a depot to be opened without knowing the nature of locations spread of customers may lead to a poor solution (Salhi and Gamal, 2003). Figure 3.6 illustrates two types of the locations spread of customers.

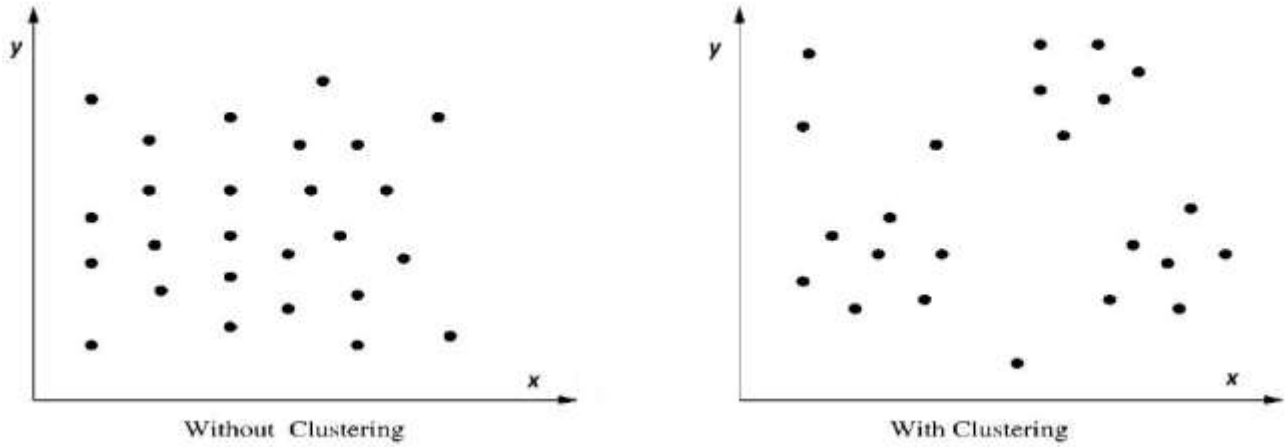


Figure 3.6. Locations spread of customers

The width of the cell in the x -axis is $W_x = (a_{max} - a_{min}) / k_0$ and the length of it in the y -axis is $W_y = (b_{max} - b_{min}) / k_0$. In this method, we consider that the number of cells is constant and equal to k_0 . A cell is defined by its bottom-left corner. The first cell has the bottom-left corner $(A_1, B_1) = (a_{min}, b_{min})$, and subsequent cells, say cell l $(A_l, B_l) = (a_{min} + k_x W_x, b_{min} + k_y W_y)$.

Equation (3.10) is used for each cluster to choose the appropriate depot from the potential options.

$$(A_l, B_l) = \left[\frac{\sum_{j \in J} a_j}{n_l}, \frac{\sum_{j \in J} b_j}{n_l} \right] \quad (3.10)$$

where (A_l, B_l) is the gravity centre of l^{th} cell.

j is the customer j^{th} at cluster l

n_l is the number of customers at cluster l

(a_j, b_j) are the coordinates of the j^{th} customer at cluster l .

By defining the gravity centre of each depot, we can start choosing the best depot among the various candidates. The depot with the minimum sum of distances from the gravity centres is chosen to be the depot of VRP problem. In the second stage, BR-CWH, which is explained in section 3.4.1, uses the potential depot from the first stage to solve the VRP and calculates the routing decision cost and hence

the total cost is determined by BR-TSCH. Algorithm 3.2 illustrates the pseudo code of solving the location problem. Figure 3.7 shows the flowchart of the BR-TSCH.

```

Input:  $N, (x, y)$ 
1 initialize(inputs)
2  $x_{min}$ : minValue  $x$ 
3  $x_{max}$ : maxValue  $x$ 
4  $y_{min}$ : minValue  $y$ 
5  $y_{min}$ : minValue  $y$ 
6  $x_{min}, y_{min} \leftarrow \infty$ 
7  $x_{max}, y_{max} \leftarrow 0$ 
8  $d_{il} \leftarrow \infty$ 
9 foreach  $customer(j) \in N$  do
10   if  $x_j < x_{min}$  then
11      $x_{min} \leftarrow x_j$ 
12   end
13   if  $x_j > x_{max}$  then
14      $x_{max} \leftarrow x_j$ 
15   end
16   if  $y_j < y_{min}$  then
17      $y_{min} \leftarrow y_j$ 
18   end
19   if  $y_j > y_{max}$  then
20      $y_{max} \leftarrow y_j$ 
21   end
22 end
23 foreach  $(k_0 = 2, 3, 4)$  do
24   compute  $W_x = (x_{max} - x_{min})/k_0$  and  $W_y = (y_{max} - y_{min})/k_0$ 
25   To construct  $l$  cells that cover all nodes
26   Where  $l = k_0 \times k_0$ 
27 end
28 foreach cell  $l$  do
29   Compute gravity center as:
30   foreach  $customer(j)$  in cell  $l$  do
31      $a_l$  and  $b_l = 0$ 
32      $a_l = a_l + x_{jl}$  and  $b_l = b_l + y_{jl}$ 
33   end
34    $A_l = a_l/n_l$  and  $B_l = b_l/n_l$ 
35 end
36 foreach depot  $(i)$  do
37   foreach cell  $l$  do
38     compute distance ( $\hat{d}_i$ ) between gravity center ( $A_l, B_l$ ) and depot  $i$ 
39     if  $\hat{d}_i < d_{il}$  then
40        $d_{il} \leftarrow \hat{d}_i$ 
41     end
42   end
43 end
44 return depot  $i$  with the minimum distance  $d_{il}$ 

```

Algorithm 3.2. Pseudocode of the heuristic to solve the location problem

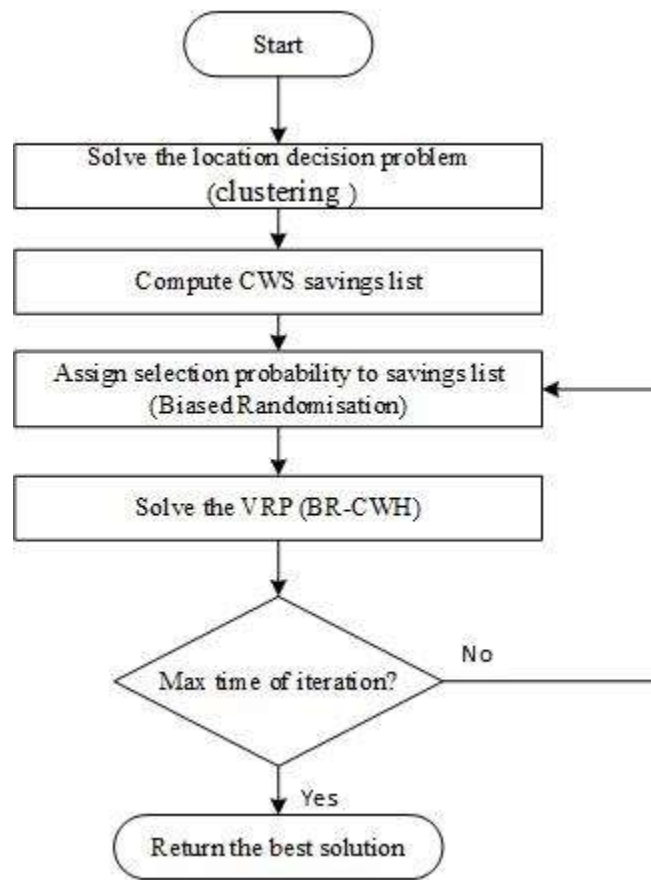


Figure 3.7. Flowchart of the BR-TSPH

3.4.2.2 Biased Randomised Two-Stage p-median heuristic

This heuristic solves the location problem optimally by using the p-median model with $p = 1$, and the routing problem is solved by using the BR-CWH. In the first stage of this heuristic, the p-median model is used to determine the location of the depot which is implemented by CPLEX. The p-median problem is the most practically solved use in discrete location theory because it is very practical in location problems (Daskin and Maass, 2015). On the other hand, the other location problems, such as the p-center problem or covering problem, can be formulated as a p-median problem readily. The p-median problem aims to locate p depots amongst a candidate list of sites and allocate a set of customers to these p depots to serve them (satisfy their demand) with the minimum average distance between customers and its servicing depot. Figure 3.8 illustrates an instance of a p-median (a) and its solution (b) with $p = 2$. In this model, we use only one depot, therefore, $p = 1$.

The problem can be represented by using an undirected network $G = (I, J, E)$, where $I = \{1, \dots, n\}$ is a set of the possible locations of depots, $J = \{1, \dots, m\}$ is a set of customers, and E is a set of undirected edges (i, j) between each possible location in J and each customer in I only. Each customer $j \in J$ has a demand d_j . Furthermore, there is a positive cost ($c_{ij} \geq 0$) associated with the edges in E which represents the traveling cost between i and j .

The model is formulated as a mixed integer linear programming problem. In order to formulate the model, the following notation is introduced.

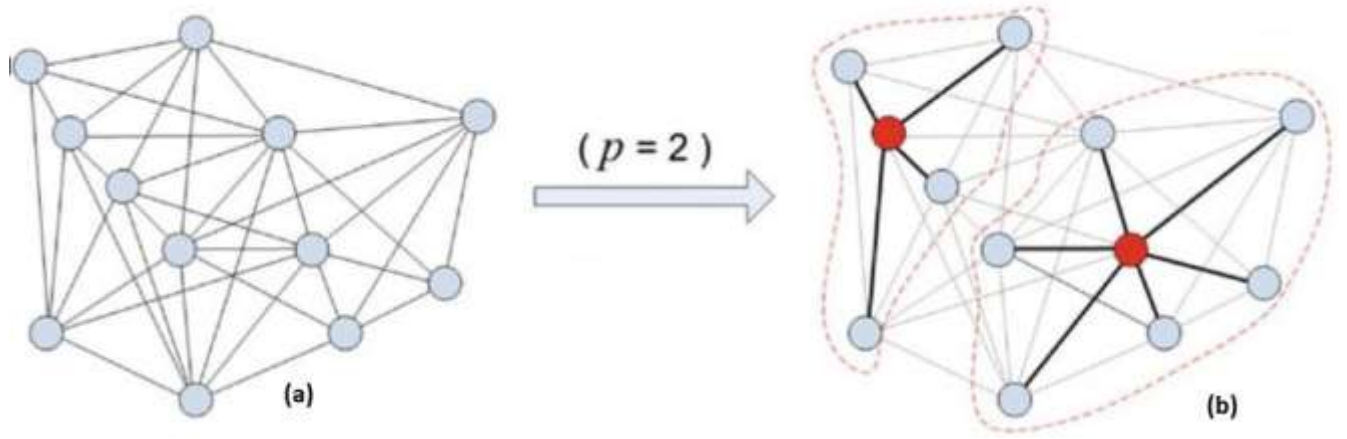


Figure 3.8. An instance of a p -median (a) and its solution with $p = 2$ (b)

p : Number of depots to locate

I : Set of potential depot nodes

J : Set of customers to be served

d_j : Demand of customer j

c_{ij} : Traveling cost for edge (i, j)

Variables are defined as follows:

$y_i : \begin{cases} 1, & \text{if potential depot } i \text{ is opened} \\ 0, & \text{otherwise} \end{cases}$

$z_{ij} : \begin{cases} 1, & \text{if customer } j \text{ is served from depot } i \\ 0, & \text{otherwise} \end{cases}$

The p -median model which is adapted from Rolland et al, (1997) is described as follows:

$$\text{Min } \sum_{i \in I} \sum_{j \in J} d_j c_{ij} z_{ij} \quad (3.11)$$

Subject to

$$\sum_{i \in I} z_{ij} = 1 \quad \forall j \in J \quad (3.12)$$

$$\sum_{i \in I} y_i = p \quad (3.13)$$

$$z_{ij} \leq y_i \quad \forall i \in I, \quad \forall j \in J \quad (3.14)$$

$$y_i \in \{0, 1\} \quad \forall i \in I \quad (3.15)$$

$$z_{ji} \in \{0, 1\} \quad \forall i \in I \quad \forall j \in J \quad (3.16)$$

The objective function (3.11) is to minimise the demand-weighted distance of delivering to customers. Constraints (3.12) ensure that each customer is served by exactly one depot. Constraint (3.13) ensures that p depot is opened, which is equal to one in the developed heuristic. Finally, constraints (3.14) ensure that a customer is not assigned to an unopened depot, and constraints (3.15) and (3.16) indicate integer variables.

In this heuristic p -median is solved optimally by CPLEX, then the potential depot is used in the second stage where VRP is solved by BR-CWH. Figure 3.9 shows the flowchart of the BR-TSPH.

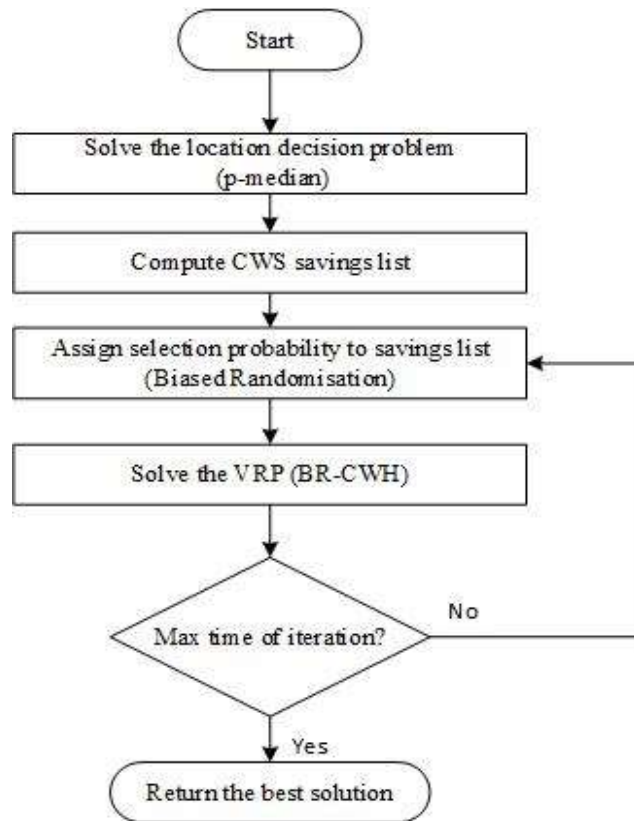


Figure 3.9. Flowchart of the BR-TSPH

3.4.2.3 Biased Randomised Two-Stage Clustering and p-median heuristic

This heuristic combines BR-TSCH and BR-TSPH. Therefore, in the first stage, the location decision is solved by clustering technique and p-median models, and the routing decision is made based on BR-CWH.

The benefit of using this heuristic is its simplicity to deal with the nature of customers locations spread and its ability to find the optimal solution of the p-median model.

In the first stage of this heuristic, the region of customers is covered by $k_0 \times k_0$ rectangular cells which is described in section 3.4.2.1. Then the gravity centre of each cell is computed based on (3.10). Then, the gravity centre is considered as customers in the p-median model, rather than customers themselves. Finally, the p-median is solved optimally by CPLEX. The pseudocode of finding the $k_0 \times k_0$ rectangular cells is similar to the pseudocode in Algorithm 3.2, while the mathematical model of p-median is similar to the mathematical model in section 3.4.2.2.

In the second stage the VRP, which is solved by the BR-CWH algorithm, uses the potential depot from the first stage to calculate the routing decision cost and hence the total cost. Figure 3.10 illustrates the flowchart of the BR-TSCPH.

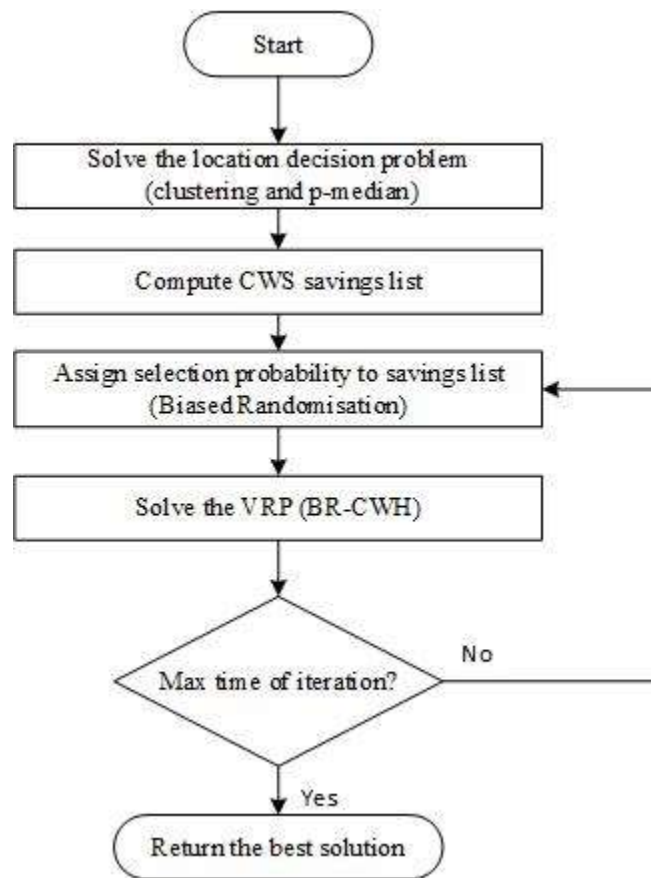


Figure 3.10. Flowchart of the BR-TSCPH

3.4.2.4 Biased Randomised Iterated heuristic

In this heuristic, a depot is randomly chosen in the first stage and the relevant location decision cost is calculated. In the second stage the VRP uses the first randomly chosen depot to calculate the routing decision cost and hence the total cost.

The algorithm continues with the second randomly chosen depot until all potential depots are selected. Therefore, our procedure is considered as complete enumeration. The output is the candidate depot with the minimum cost among all depots.

This heuristic is developed for comparison with the other heuristics so that we can identify if the other heuristics have chosen the optimal depot. Figure 3.11 illustrates the flowchart of the BR-IH.

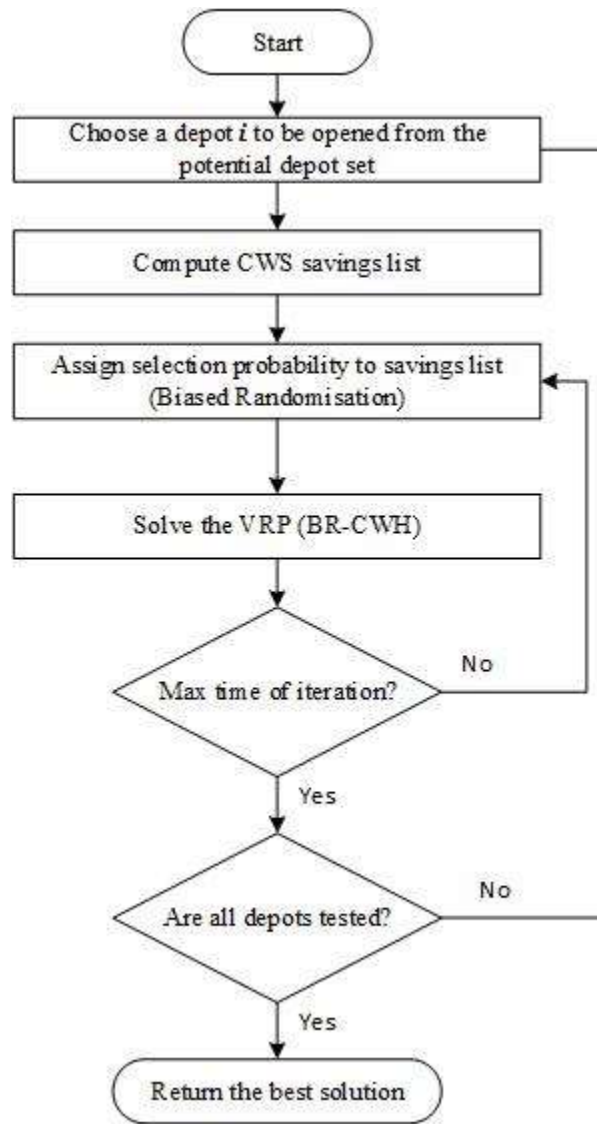


Figure 3.11. Flowchart of the BR-IH

3.5 Computational experiments

The computational experiments are conducted to evaluate the performance of the proposed four heuristics. The four heuristics have been coded using Java applications.

Computational experiments have been performed using a computer with a Core i5, 3.20 GHz processor and 8 GB of RAM. We compare the result of the developed heuristics, BR-TSCH, BR-TSPH, BR-TSCPH, and BR-IH, with the best-known results in the literature.

3.5.1 Data and experimental setting

Since there are no specific benchmarks for the LRPSD in the literature, to the best of our knowledge, we have adapted some instances from the LRP.

There are three benchmark data sets in the literature for the LRP: Barreto's set which is introduced by Barreto (2004), Prodhon's set which is introduced by Prins, et al. (2006a), and Akca's set which is introduced by Akca et al. (2009). The Prodhon's set and Akca's set were not used to test our proposed heuristics because the solution of these two data sets has to contain at least two depots; this means the capacity of only one depot cannot cover the total demand of all customers. Therefore, they are not suitable for the LRPSD. In Barreto's set some instances are suitable for the LRPSD because the capacity of one depot can cover the total demand of all customers, while the other instances are not suitable. Hence, we adapted 10 out of 17 instances, which are suitable based on depot capacity.

Barreto has adapted these 10 instances from the literature related to the LRP and from the literature related to the VRP. These 10 instances consist of 1 instance by Perl (1983), 1 instance by Min et al. (1992), 5 instances by Gaskell (1967), and 5 instances by Christofides and Eilon (1969).

The number of customers vary between 12 and 100, while the number of potential depots, range from 2 to 10 depots. The vehicles capacity varies between 140 and 8000, whereas the capacity of potential depots varies between 280 and 15000. Instance names consist of name of author, number of customers, and number of potential depots, respectively.

Table 3.1 shows the characteristics of the 10 instances which were adapted from Barreto's set. It contains the name of each instance, number of customers in column n , number of potential depots in column m , vehicle capacity in column $V.Q$, and depot capacity in column $D.Q$.

No.	Name	n	m	V.Q	D.Q
1	Perl-12x2	12	2	140	280
2	Gas-22x5	21	5	6000	15000
4	Min-27x5	27	5	2500	9000
5	Gas-29x5	29	5	4500	15000
6	Gas-32x5	32	5	8000	15000
6	Gas-32x5-B	32	5	11000	15000
7	Gas-36x5	36	5	250	15000
8	Chr-50x5	50	5	160	10000
9	Chr-75x10	75	10	140	10000
10	Chr-100x10	100	10	200	10000

Table 3.1. Characteristics of 10 instances from Barreto's set

Figure 3.12 and Figure 3.13 illustrate the distribution of customers and potential depots for two of the instances: Chr-50x5 and Gaskell29x5, respectively. For the instances of Chr-50x5 in Figure 3.12, we can note that the distribution of customers follows a normal distribution.

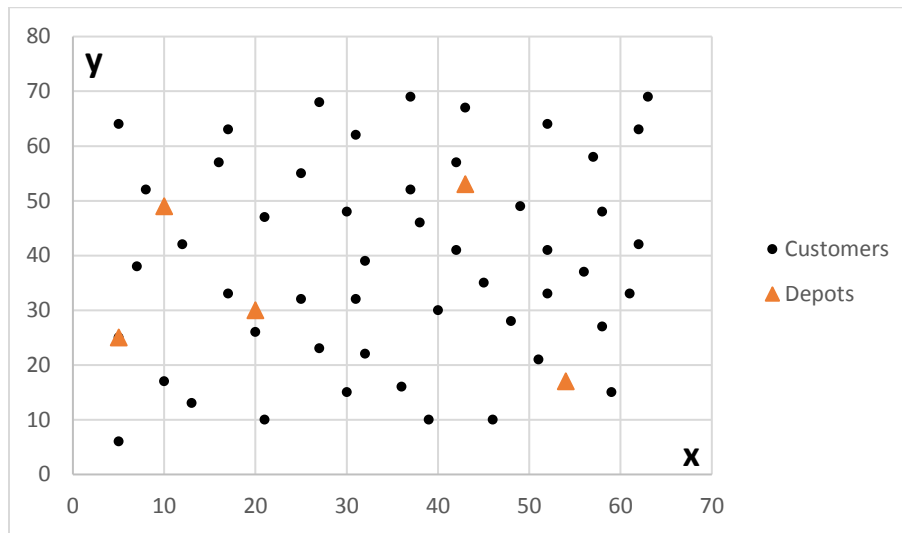


Figure 3.12. Distribution of Customers and depots in Chr-50x5

While the instances of Gaskell29x5 in Figure 3.13, the customer distribution is clustered. However, the distribution of depots location in Chr-50x5 and Gaskell29x5 follows a normal distribution.

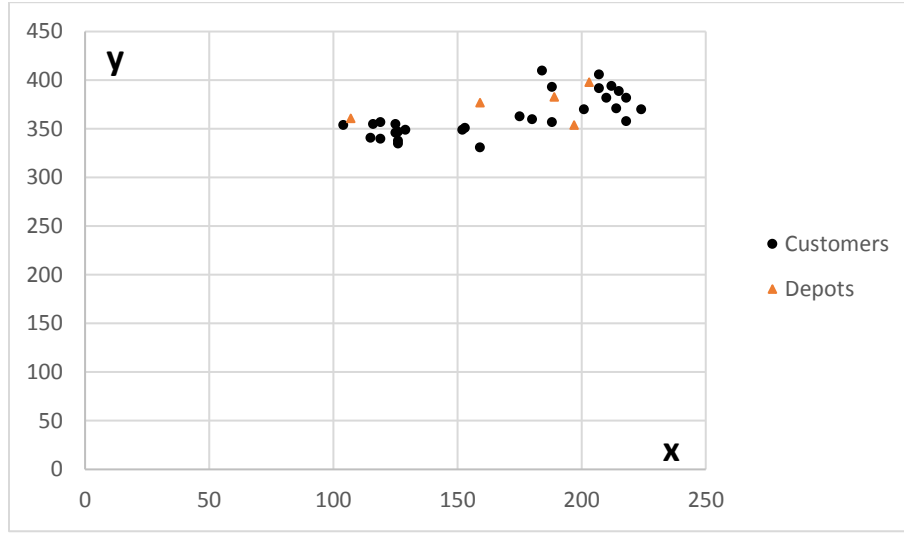


Figure 3.13. Distribution of Customers and depots in Gaskell29x5

This data set is available at http://sweet.ua.pt/sbarreto/_private/SergioBarretoHomePage.htm.

3.5.2 Performance evaluation of the proposed heuristics

In this section, we discuss the results obtained by the proposed four heuristics in order to illustrate their performance. Firstly, we compare the results of BR-TSCH, BR-TSPH, and BR-TSCPH with the result of the BR-IH. Secondly, to evaluate the efficiency of the proposed heuristics, the results have been compared to the best-known solution (BKS) in the literature for the benchmark instances and four other methods namely: Barreto Heuristic which is proposed by Barreto (2004), GRASP which is proposed by Prins, et al. (2006a), Memetic Algorithm with Population Management (MAPM) which is proposed by Prins, et al. (2006b), and Lagrangean relaxation Granular Tabu Search (LRGTS) by Prins, et al. (2007).

Table 3.2 compares the result of three proposed heuristics, BRTSCH, BR-TSPH, BR-TSCPH with the result of the BR-IH. Table 3.3, 3.4, 3.5, and 3.6 present the details of the performance of BR-TSCH, BR-TSPH, BR-TSCPH, and BR-IH, and compare these results with the BKS and the four other methods, Barreto Heuristic, GRASP, MAPM, and LRGTS, respectively.

In Table 3.2, the first column shows the instance names. The second column (Cost) presents the total cost of the Iterated heuristic. The 3rd (cost) and 4th (gap) columns show the total cost of the Two-Stage clustering and its Gap with regard to the Iterated heuristic. The 5th (cost) and 6th (gap) columns show the total cost of the Two-Stage p-median and its Gap with regard to the Iterated heuristic. The 7th (cost)

and 8th (gap) columns show the total cost of the Two-Stage clustering and p-median and its Gap with regard to the Iterated heuristic.

The percentage gap (gap) with respect to the Iterated heuristic is calculated as $\left[\left(\frac{BR_TSCH - BR_IH}{BR_IH}\right) \times 100\right]$. The same formula is used to calculate the percentage gap of the Two-Stage p-median and the Two-Stage clustering and p-median.

The first and second columns in tables 3.3, 3.4, 3.5, and 3.6 show the instance names and BKS values. The 3rd (cost) and 4th (gap/BKS) columns show the total cost of Barreto Heuristic and its Gap with regard to the BKS. The 5th and 6th columns show the total cost of GRASP and its Gap with regard to the BKS. The 7th and 8th columns show the total cost of MAPM and its Gap with regard to the BKS. The 9th and 10th columns show the total cost of LRGTS and its Gap with regard to the BKS. The 11th (cost) column in each table shows the total cost of the Two-Stage clustering, the Two-Stage p-median, the Two-Stage clustering and p-median, and the Iterated heuristic, respectively. The 12th (gap (1)), 13th (gap (2)), 14th (gap (3)), 15th (gap (4)), and 16th (gap (5)) columns compare the total cost of our methods with the BKS, Barreto Heuristic, GRASP, MAPM, and LRGTS, respectively. The percentage gap (gap) with respect to the BKS is calculated as $\left[\left(\frac{\text{our best solution} - BKS}{BKS}\right) \times 100\right]$; the same formula is used to calculate the percentage gap with respect to the Barreto Heuristic, GRASP, MAPM, and LRGTS. The lowest, best solutions which match BKS are indicated in bold.

Name	BKS	BR-IH		BR-TSCH		BR-TSPH		BR-TSCPH	
		Cost	Gap%	Cost	Gap%	Cost	Gap%	Cost	Gap%
Christo69-50x5	565.6	605.56	3.3	606	0.1	609.8	0.7	609.8	0.7
Christo69-75x10	844.4	895.57	3.7	895.57	0	895.6	0	895.6	0
Christo69-100x10	833.4	894.84	6.2	894.84	0	894.8	0	894.8	0
Gaskell67-22x5	585.1	585.1	-0.4	656.47	12.2	629.5	7.6	585.1	0
Gaskell67-29x5	512.1	566.28	10.6	566.28	0	566.3	0	577.7	2
Gaskell67-32x5	562.2	562.27	-3.8	562.27	0	562.3	0	562.3	0
Gaskell67-32x5-B	504.3	504.3	-0.1	504.3	0	504.3	0	504.3	0
Gaskell67-36x5	460.4	460.4	-3.4	460.4	0	460.4	0	460.4	0
Min92-27x5	3062	3266.24	6.6	3296.5	0.9	3296.5	0.9	3296.5	0.9
Perl83-12x2	204	204	--	204	0	204	0	204	0
Average	8133.5	854.46	2.5	864.6	1.3	862.3	0.9	859.1	0.4

Table 3.2. Comparison among the proposed four heuristics

Table 3.2 shows the comparison between results of the four proposed heuristics themselves. The first three heuristics: BR-TSCH, BR-TSPH, and BR-TSCPH are compared with the BR-IH because it

covers all depots. Therefore, we can identify which depot is the optimal one. On the other hand, we have noticed that the Iterated heuristic has the minimum percentage gap with regard to the BKS. It can also be seen that the Iterated heuristic has the minimum cost for all instances.

Figure 3.14 shows the boxplot of the average percentage Gaps with regard to the Iterated heuristic. We can note that the average gap with regard to the BR-IH becomes smaller when we combine clustering technique and p-median model in one method. Figure 3.15 illustrates a chart of the average cost for the four proposed Biased Randomised heuristics. We can notice that the average cost of the BR-IH is the minimum average among the others.

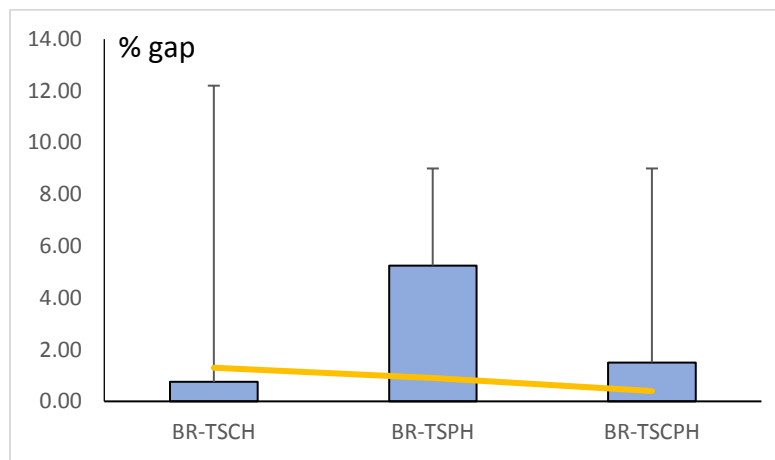


Figure 3.14. Boxplot of the average percentage Gaps with regard to the BR-IH

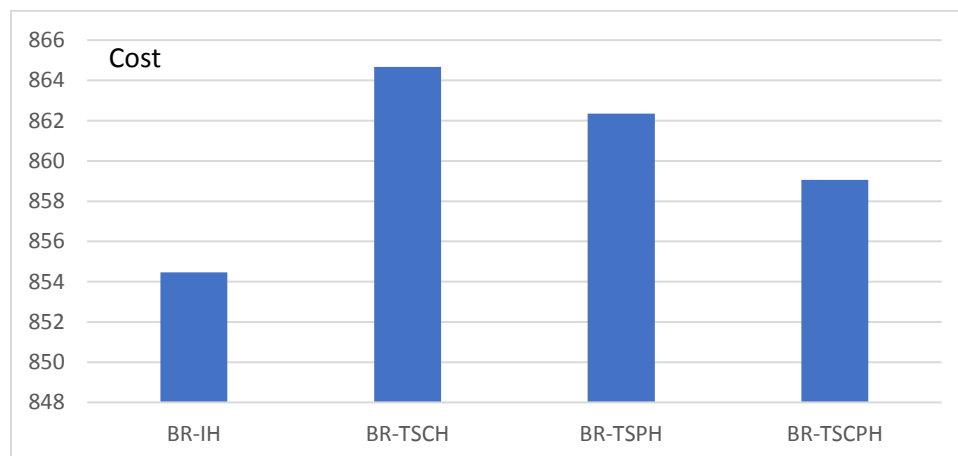


Figure 3.15. Chart of the average cost for the four proposed Biased Randomised heuristics

Name	Barreto Heuristic			GRASP		MAPM		LRGTS		BR-TSCH					
	BKS	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	gap (1) %	gap (2) %	gap (3) %	gap (4) %	gap (5) %
Christo69-50x5	565.6	582.7	3.0	599.1	5.9	565.6	0.0	586.4	3.7	606.00	7.1	4.0	1.2	7.1	3.3
Christo69-75x10	844.4	886.3	5.0	861.6	2.0	866.1	2.6	863.5	2.3	895.57	6.1	1.0	3.9	3.4	3.7
Christo69-100x10	833.4	889.4	6.7	861.6	3.4	850.1	2.0	842.9	1.1	894.84	7.4	0.6	3.9	5.3	6.2
Gaskell67-22x5	585.1	591.5	1.1	585.1	0.0	611.8	4.6	587.4	0.4	656.47	12.2	11.0	12.2	7.3	11.8
Gaskell67-29x5	512.1	512.1	0.0	515.1	0.6	512.1	0.0	512.1	0.0	566.28	10.6	10.6	9.9	10.6	10.6
Gaskell67-32x5	562.2	571.7	1.7	571.9	1.7	571.9	1.7	584.6	4.0	562.27	0.0	-1.6	-1.7	-1.7	-3.8
Gaskell67-32x5-B	504.3	511.4	1.4	504.3	0.0	534.7	6.0	504.8	0.1	504.3	0.0	-1.4	0.0	-5.7	-0.1
Gaskell67-36x5	460.4	470.7	2.2	460.4	0.0	485.4	5.4	476.5	3.5	460.4	0.0	-2.2	0.0	-5.2	-3.4
Min92-27x5	3062	3062	0.0	3062	0.0	3062.0	0.0	3065.2	0.1	3296.5	7.7	7.7	7.7	7.7	7.5
Perl83-12x2	204	--	--	--	--	--	--	--	--	204	0.0	--	--	--	--
Average	8133.5	897.5	2.3	891.2	1.5	895.5	2.5	891.5	1.7	864.7	5.1	3.3	4.1	3.2	4.0

Table 3.3. Results of BR-TSCH

Name	Barreto Heuristic			GRASP		MAPM		LRGTS		Two-Stage p-median					
	BKS	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	gap (1) %	gap (2) %	gap (3) %	gap (4) %	gap (5) %
Christo69-50x5	565.6	582.7	3.0	599.1	5.9	565.6	0.0	586.4	3.7	609.8	7.8	4.7	1.8	7.8	4.0
Christo69-75x10	844.4	886.3	5.0	861.6	2.0	866.1	2.6	863.5	2.3	895.6	6.1	1.0	3.9	3.4	3.7
Christo69-100x10	833.4	889.4	6.7	861.6	3.4	850.1	2.0	842.9	1.1	894.8	7.4	0.6	3.9	5.3	6.2
Gaskell67-22x5	585.1	591.5	1.1	585.1	0.0	611.8	4.6	587.4	0.4	629.5	7.6	6.4	7.6	2.9	7.2
Gaskell67-29x5	512.1	512.1	0.0	515.1	0.6	512.1	0.0	512.1	0.0	566.3	10.6	10.6	9.9	10.6	10.6
Gaskell67-32x5	562.2	571.7	1.7	571.9	1.7	571.9	1.7	584.6	4.0	562.3	0.0	-1.6	-1.7	-1.7	-3.8
Gaskell67-32x5-B	504.3	511.4	1.4	504.3	0.0	534.7	6.0	504.8	0.1	504.3	0.0	-1.4	0.0	-5.7	-0.1
Gaskell67-36x5	460.4	470.7	2.2	460.4	0.0	485.4	5.4	476.5	3.5	460.4	0.0	-2.2	0.0	-5.2	-3.4
Min92-27x5	3062	3062	0.0	3062	0.0	3062.0	0.0	3065.2	0.1	3296.5	7.7	7.7	7.7	7.7	7.5
Perl83-12x2	204	--	--	--	--	--	--	--	--	204	0.0	--	--	--	--
Average	8133.5	897.5	2.3	891.2	1.5	895.5	2.5	891.5	1.7	862.3	4.7	2.9	3.7	2.8	3.5

Table 3.4. Results of BR-TSPH

Barreto Heuristic				GRASP		MAPM		LRGTS		BR-TSCPH					
Name	BKS	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	gap (1) %	gap (2) %	gap (3) %	gap (4) %	gap (5) %
Christo69-50x5	565.6	582.7	3.0	599.1	5.9	565.6	0.0	586.4	3.7	609.8	7.8	4.7%	1.8	7.8	4.0
Christo69-75x10	844.4	886.3	5.0	861.6	2.0	866.1	2.6	863.5	2.3	895.6	6.1	1.0%	3.9	3.4	3.7
Christo69-100x10	833.4	889.4	6.7	861.6	3.4	850.1	2.0	842.9	1.1	894.8	7.4	0.6%	3.9	5.3	6.2
Gaskell67-22x5	585.1	591.5	1.1	585.1	0.0	611.8	4.6	587.4	0.4	585.1	0.0	-1.1%	0.0	-4.4	-0.4
Gaskell67-29x5	512.1	512.1	0.0	515.1	0.6	512.1	0.0	512.1	0.0	577.7	12.8	12.8%	12.1	12.8	12.8
Gaskell67-32x5	562.2	571.7	1.7	571.9	1.7	571.9	1.7	584.6	4.0	562.3	0.0	-1.6%	-1.7	-1.7	-3.8
Gaskell67-32x5-B	504.3	511.4	1.4	504.3	0.0	534.7	6.0	504.8	0.1	504.3	0.0	-1.4%	0.0	-5.7	-0.1
Gaskell67-36x5	460.4	470.7	2.2	460.4	0.0	485.4	5.4	476.5	3.5	460.4	0.0	-2.2%	0.0	-5.2	-3.4
Min92-27x5	3062	3062	0.0	3062	0.0	3062.0	0.0	3065.2	0.1	3296.5	7.7	7.7%	7.7	7.7	7.5
Perl83-12x2	204	--	--	--	--	--	--	--	--	204	0.0	--	--	--	--
Average	8133.5	897.5	2.3	891.2	1.5	895.5	2.5	891.5	1.7	859.1	4.2	2.3%	3.1	2.2	2.9

Table 3.5. Results of BR-TSCPH

Barreto Heuristic				GRASP		MAPM		LRGTS		BR-IH					
Name	BKS	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	(gap/BKS) %	Cost	gap (1) %	gap (2) %	gap (3) %	gap (4) %	gap (5) %
Christo69-50x5	565.6	582.7	3.0	599.1	5.9	565.6	0.0	586.4	3.7	605.6	7.1	3.9%	1.1	7.1	3.3
Christo69-75x10	844.4	886.3	5.0	861.6	2.0	866.1	2.6	863.5	2.3	895.6	6.1	1.0%	3.9	3.4	3.7
Christo69-100x10	833.4	889.4	6.7	861.6	3.4	850.1	2.0	842.9	1.1	894.8	7.4	0.6%	3.9	5.3	6.2
Gaskell67-22x5	585.1	591.5	1.1	585.1	0.0	611.8	4.6	587.4	0.4	585.1	0.0	-1.1%	0.0	-4.4	-0.4
Gaskell67-29x5	512.1	512.1	0.0	515.1	0.6	512.1	0.0	512.1	0.0	566.3	10.6	10.6%	9.9	10.6	10.6
Gaskell67-32x5	562.2	571.7	1.7	571.9	1.7	571.9	1.7	584.6	4.0	562.3	0.0	-1.6%	-1.7	-1.7	-3.8
Gaskell67-32x5-B	504.3	511.4	1.4	504.3	0.0	534.7	6.0	504.8	0.1	504.3	0.0	-1.4%	0.0	-5.7	-0.1
Gaskell67-36x5	460.4	470.7	2.2	460.4	0.0	485.4	5.4	476.5	3.5	460.4	0.0	-2.2%	0.0	-5.2	-3.4
Min92-27x5	3062	3062	0.0	3062	0.0	3062.0	0.0	3065.2	0.1	3266.2	6.7	6.7%	6.7	6.7	6.6
Perl83-12x2	204	--	--	--	--	--	--	--	--	204	0.0	--	--	--	--
Average	8133.5	897.5	2.3	891.2	1.5	895.5	2.5	891.5	1.7	854.5	3.8	1.8%	2.6	1.8	2.5

Table 3.6. Results of BR-IH

Table 3.3 summarises the results of the computational experiments for the BR-TSCH. The proposed heuristic has obtained the same results as the BKS in four instances, while its results are worse in the remaining instances. The main reason for low performance of this heuristic is that the problem is set to have only one depot whereas in the other algorithms, there are more than one depot.

Regarding the other four heuristics, we found that our proposed heuristic has improved three instances with regard to the Barreto heuristic, MAPM, and LRGTS. Whereas, it improved one instance with regard to the GRASP.

The average percentage gaps when comparing the proposed heuristic with BKS, Barreto heuristic, GRASP, MAPM, and LRGTS are 5.1%, 3.3%, 4.1%, 3.2% and 4.0%, respectively. Figure 3.16 shows the boxplot of the average percentage Gaps. We can note from the boxplot that the range of our gap is larger than other methods, which means its performance is less than other methods in terms of solution quality. The average computational time of the BR-TSCH is 2 sec.

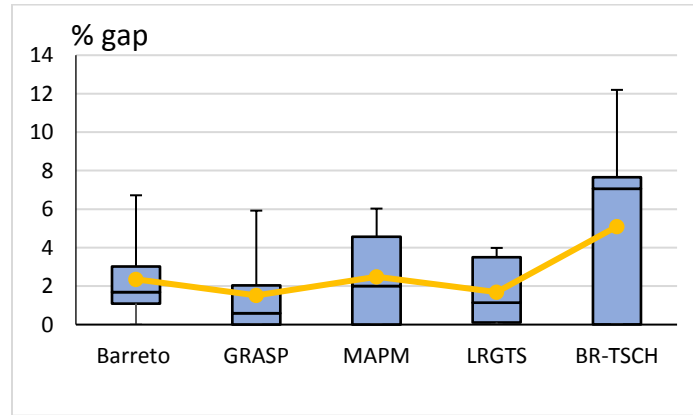


Figure 3.16. Boxplot of the average percentage Gaps for the BR-TSCH

Table 3.4 summarises the results of the computational experiments for the BR-TSPH. The proposed heuristic has obtained the same results of BKS for four instances, while its results are worse in the remaining instances. The main reason for low performance of this heuristic is that the problem is set to have only one depot whereas in the other algorithms, there are more than one depot.

With regard to the other four heuristics, we found that our proposed heuristic has improved three instances when compared to Barreto heuristic, MAPM, and LRGTS, whereas it improved one instance in comparison with the GRASP.

The average percentage gaps when comparing the proposed heuristic with BKS, Barreto heuristic, GRASP, MAPM, and LRGTS are 4.7%, 2.9%, 3.7%, 2.8% and 3.5%, respectively. These percentage gaps are better than the BR-TSCH. Figure 3.17 shows the boxplot of the average percentage Gaps. We can note from the boxplot that the range of our gap is larger than other methods, which means its performance is less than other methods in terms of solution quality. The average computational time of the BR-TSPH is 5.5 sec.

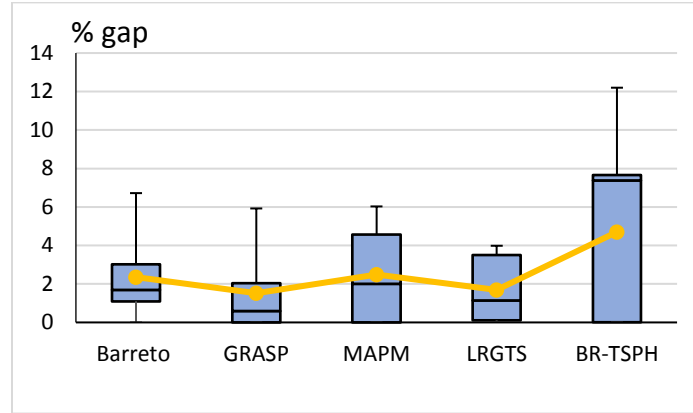


Figure 3.17. Boxplot of the average percentage Gaps for the BR-TSPH

Table 3.5 summarises the results of the computational experiments for the BR-TSCPH. The proposed heuristic has obtained the same results of BKS for five instances, while its results are worse in the remaining instances. This means that combining the BR-TSCH and the BR-TSPH gives a better performance together, than each one alone.

With regard to the other four heuristics, we found that our proposed heuristic has improved four instances with regard to Barreto heuristic, MAPM, and LRGTS. Whereas, it improved one instance regarding to the GRASP.

The average percentage gaps when comparing the proposed heuristic with BKS, Barreto heuristic, GRASP, MAPM, and LRGTS are 4.2%, 2.3%, 3.1%, 2.2% and 2.9%, respectively. These percentage gaps are better than the BR-TSCH and the BR-TSPH. Figure 3.18 shows the boxplot of the average percentage Gaps. We can note from the boxplot that the range of our gap is larger than other methods, which means its performance is less than other methods in term of solution quality. The average computational time of the BR-TSCPH is 3.6 sec.

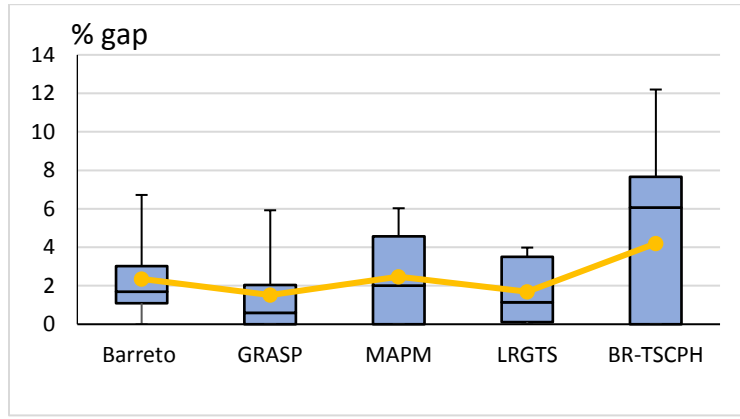


Figure 3.18. Boxplot of the average percentage Gaps for the BR-TSCPH

Table 3.6 summarises the results of the computational experiments for the BR-IH. The proposed heuristic has obtained the same results of BKS for five instances, while its results are worse in the remaining instances.

With regard to the other four heuristics, we found that our proposed heuristic has improved four instances with regard to Barreto heuristic, MAPM, and LRGTS. Whereas, it improved one instance with regard to the GRASP.

The average percentage gaps when comparing the proposed heuristic with BKS, Barreto heuristic, GRASP, MAPM, and LRGTS are 3.8%, 1.8%, 2.6%, 1.8% and 2.5%, respectively. These percentage gaps are better than all three heuristics. Figure 3.19 shows the boxplot of the average percentage Gaps. We can note from the boxplot that the range of our gap is larger than other methods, which means its performance is less than other methods in term of solution quality. The average computational time of the Two-Stage clustering is 5.7 sec.

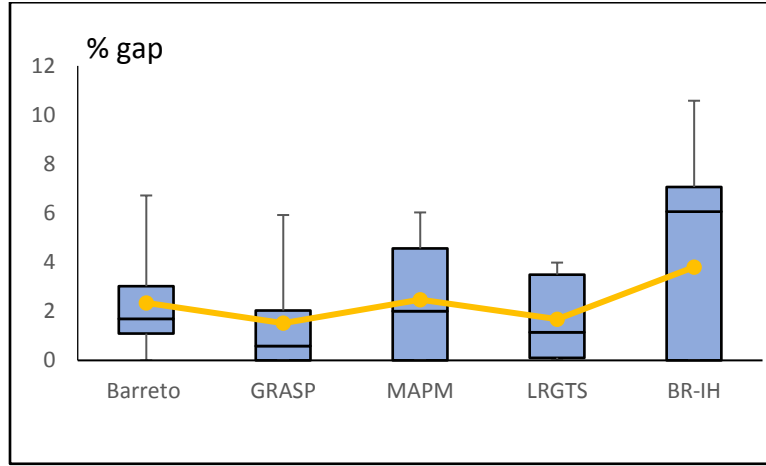


Figure 3.19. Boxplot of the average percentage Gaps for the BR-IH

From all four methods we can note that our methods perform better with small instances and when customer distribution follows a normal distribution. Moreover, total cost of some instances will be less when we open more than one depot.

3.6 Conclusion

In this chapter, a general framework of the heuristic which consists of location heuristic and routing heuristic to solve the LRP with single depot is given. Four variants of the heuristic are developed called Two-Stage clustering, Two-Stage p-median, Two-Stage clustering and p-median, and Iterated heuristic. The first stage is solved by clustering technique, p-median model, clustering technique and p-median model together, and iterated framework, while the second stage of the four heuristics was solved using BR-CHW. This technique has been used to help heuristics to escape from local minima and explore different regions of the search space.

To evaluate the performance of algorithms, computational experiments are carried out for different problem sizes ranging from 12 to 100 customers. Results obtained so far indicate that these four proposed heuristics are suitable to solve the LRP with single depot. This variant of the LRP (LRP with single depot), has some important applications such as server systems, and money collection.

Future study should address two directions for improvement. The first direction is to improve the quality of results by adding a local search to the routing stage. The second direction is to extend the

LRPSD and propose novel mathematical models for the LRP with multi depots, which is covered in chapter 5, and LRP with stochastic demand.

Further research is also required to investigate other heuristics to other extensions of the problem such as an LRP with more than one depot, and Green LRP through the use of electric vehicles. Furthermore, proposing heuristic and metaheuristic approaches to make location and routing decisions in integrated or sequential order, can be very fruitful in terms of quality of the solution.

Chapter 4 Multi-Depot Vehicle Routing Problem (MDVRP)

4.1 Introduction

In this chapter, we address MDVRP in the light of its relation to LRP with Multi-Depot. As we mentioned before, when depot locations are fixed in LRP, the problem reduces to the MDVRP. For that reason, the MDVRP is considered as a special case of the LRP. Therefore, a solution method for the MDVRP can be used to solve the LRP by adding a location decision to it, although this may not necessarily result in a good solution all the time.

The main contribution of this chapter is covered in section 4.2. The MDVRP definition and the optimisation model are presented in section 4.3. In section 4.4, a description of the Tillman's heuristic is given. Section 4.5 outlines the basis of our solution approach. Section 4.6 presents the computational experiments carried out on our method and the analysis of the results. Finally, section 4.7 draws some conclusions and discusses opportunities for future research.

4.2 Contribution

We propose a heuristic to solve the MDVRP which is inspired by a classic heuristic suggested by Tillman (1969). He modified CWH to solve MDVRP and called it the Extended Clarke and Wright Heuristic (ECWH). An advantage of the developed heuristic is that it assigns customers to depots and generates routes for the vehicles at each depot simultaneously. Moreover, it has the same properties of the CWH such as simplicity of implementation, and capability to consider more constraints such as a distance constraint. To the best of our knowledge, this is the first time that Biased Randomised technique has been combined with ECWH to solve MDVRP. We call this new method Biased Randomised Extended Clarke and Wright Heuristic (BR-ECWH).

There are two procedures to solve the MDVRP: (i) decomposing the MDVRP into VRPs by assigning customers to depots, then solving the VRPs separately for each depot and its assigned customers, and (ii) solving the whole MDVRP by assigning customers to depots and building routes for the vehicles at each depot simultaneously. The proposed heuristic brings these two procedures to solve the MDVRP together in a sequential manner; solving the whole MDVRP as one problem by using the BR-ECWH, then improving the solution by using BR-CWH. To the best of our knowledge, this is the first time to bring these two procedures together to solve the MDVRP. Moreover, it is the first time to combine two randomised versions of classical heuristics (BR-ECWH and BR-CWH) to solve the MDVRP.

Furthermore, the iterative framework of this method can present many solutions with the same quality but with different characteristics. For example, the order of customers inside the same route, or it can present different routes with the same cost. These features cannot be obtained from the classic one.

4.3 Optimisation model

In the classical MDVRP, a set of customers are served by a homogeneous fleet of vehicles from multi distribution depots. Each customer has a demand which is known in advance and must be fully satisfied. Each vehicle in the homogeneous fleet has a fixed capacity that must be respected. This means that the total demand of customers in one route will be less than or equal to the vehicle capacity.

Also, each depot has a fixed capacity which also must be respected. Therefore, the total demand of customers that are served from a depot, should not exceed the capacity of that depot. On the other hand, each customer must be served by exactly one vehicle and each vehicle should depart and return to the same depot. There may be a limit on the distance traveled by each vehicle.

The aim of the MDVRP is to assign customers to depots and design a set of routes for the homogeneous fleet of vehicles to serve all customers. In this problem, each vehicle should depart and return to the same depot, and the total distance traveled by the fleet is minimised.

The MDVRP could be defined on a complete, weighted, and undirected network $G = (V, E, C)$, where $V = \{1, \dots, n\}$ is a set of nodes (representing the depots, customers), and E is a set of undirected edges (i, j) , and $C = (c_{ij})$ is the matrix of the traveling cost associated with the edges E . It is assumed that $I \subseteq V$ be a set of depots with a capacity Q_i and $J \subseteq V$ be a set of customers. A set K of identical vehicles of capacity D is available. When used, each vehicle incurs a fixed cost F and performs a single route. Each customer $j \in J$ has a demand d_j where $d_j \leq D$. Since $d_j \leq D$, there will never be a need for a node (customer) to be visited by more than one vehicle to satisfy its demand.

The following figures illustrate a small example of the MDVRP with 2 depots and 21 customers. Figure 4.1 shows how customers are assigned to depots. While Figure 4.2 shows 4 vehicle routes, covering the customer demands through two routes for each depot.

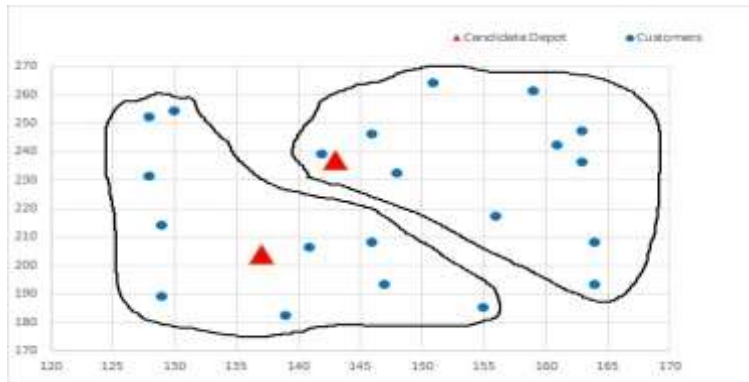


Figure 4.1. Assignment of customers to depots

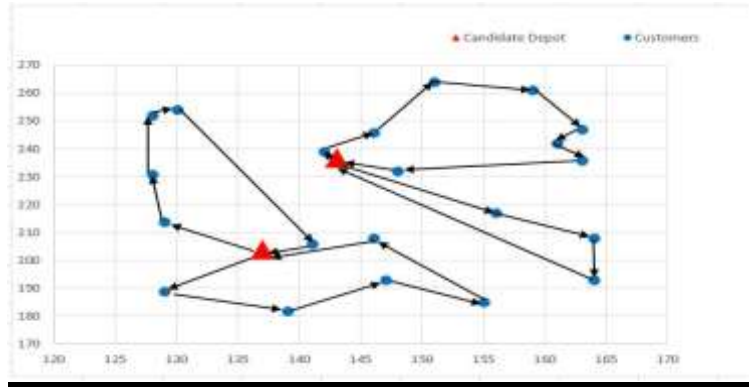


Figure 4.2. Design vehicle routes

The optimisation model of the MDVRP is formulated as a mixed integer linear programming problem. The MDVRP formulation is adapted from (Lim and Wang, 2005). In order to formulate the model, the following notation is used.

Sets are defined as follows:

V : Set of nodes, $V = I \cup J$

I : Set of depot nodes

J : Set of customers to be serviced

K : Number of available vehicles (fleet size)

Parameters are defined as follows:

Q_i : Capacity of depot i

d_j : Demand of customer j

D : Capacity of each vehicle

F : Fixed cost per vehicle used

c_{ij} : Traveling cost for edge (i, j)

Decision variables are defined as follows:

$x_{ijk} : \begin{cases} 1, & \text{if vehicle } k \text{ is used on route from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$

$z_{ij} : \begin{cases} 1, & \text{if customer } j \text{ is served from depot } i \\ 0, & \text{otherwise} \end{cases}$

The MDVRP mathematical model is given as follows:

$$\text{Min } \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F x_{ijk} \quad (4.1)$$

Subject to

$$\sum_{i \in V} \sum_{k \in K} x_{ijk} = 1 \quad \forall j \in J \quad (4.2)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1 \quad \forall k \in K \quad (4.3)$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \quad \forall k \in K, \quad \forall i \in V \quad (4.4)$$

$$\sum_{i \in V} \sum_{j \in J} d_j x_{ijk} \leq D \quad \forall k \in K \quad (4.5)$$

$$\sum_{j \in J} d_j z_{ij} \leq Q_i \quad \forall i \in I \quad (4.6)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K \quad (4.7)$$

$$z_{ij} \in \{0, 1\} \quad \forall i \in I, \quad \forall j \in V, \quad (4.8)$$

$$c_{ij} = \infty \quad \text{when } i = j$$

The objective function (4.1) seeks to minimise the total cost, which includes the fixed and variable cost of the vehicles. Constraints (4.2) are the routing constraints that are imposed whereby each customer has to be visited exactly once by a single vehicle, whereas constraints (4.3) ensure that all routes have to start and end at a depot. Constraints (4.4) are the connectivity constraints to ensure that every vehicle leaves the customer after he has been served. Constraints (4.5) and (4.6) impose the capacity of each vehicle and the capacity of each depot, respectively. Constraints (4.7) and (4.8) determine integer variables.

4.4 Tillman Heuristic for solving MDVRP

Before explaining the proposed approach, it is useful to explain the ECWH which was published by Tillman (1969). The steps of the ECWH are similar to the steps of CWH other than calculation of the saving distance. The algorithm begins with an initial solution in which each customer is assigned to the nearest depot and served by one vehicle. Then, the solution is improved by joining customers on a route that minimises the distance travelled. The customers that are joined on one route are assigned to the depot associated with this improvement.

The savings distance of Clark and Wright is

$$S_{ij} = d_{io} + d_{jo} - d_{ij} \quad (4.9)$$

While:

S_{ij} : savings distance between node i and j

d_{io} : distance between node i and the depot

d_{jo} : distance between node j and the depot

d_{ij} : distance between node i and node j

In the case of the MDVRP, the savings must be calculated to reflect the true savings relative to each depot. The problem occurs in calculating the savings for two customers that are close to one depot and a much greater distance from a second depot. If customers selected to be joined are assigned to a depot based on equation (4.9), customers would be joined and assigned to the more distant depot, which is incorrect. Therefore, to compensate for this, the distance from each depot to each customer is modified by the following equation:

$$\tilde{d}_i^k = \min_s d_i^s - (d_i^k - \min_s d_i^s) \quad (4.10)$$

\tilde{d}_i^k : modified distance between node i and depot k

$\min_s d_i^s$: distance between node i and the nearest depot

d_i^k : distance between node i and depot k

Thus, the savings from joining two customers are then calculated as follows:

$$S_{ij}^k = \tilde{d}_i^k + \tilde{d}_j^k - d_{ij} \quad (4.11)$$

S_{ij}^k : modified savings distance between node i and j to depot k

\tilde{d}_i^k : modified distance between node i and depot k

\tilde{d}_j^k : modified distance between node j and depot k

d_{ij} : distance between node i and node j

The customers selected to be joined are those with the maximum savings where the following conditions must be satisfied:

- i) The combined demand on the new route should not exceed the vehicle capacities.
- ii) Customers i and j must not be on the same route.
- iii) If a customer is connected to two other customers, it is never considered for linking.
- iv) If one or more of the conditions are not satisfied this pair of customers are excluded from further consideration at this depot.

If all the above conditions are satisfied, then these customers are joined at this depot and are eliminated from consideration at the other depots.

Then, the value of \tilde{d}_i^k for the customer that is linked to another customer at depot k is set equal to d_i^k , that is

$$\tilde{d}_i^k = d_i^k \quad (4.12)$$

Then, the savings matrix at this terminal is updated, as required by this change. This completes one iteration and the process is repeated until no more customers can be joined.

4.5 Two-Level Biased Randomised heuristic for solving MDVRP

In this section, a Two-Level Biased Randomised heuristic (TLBRH) is proposed to solve the MDVRP. The proposed approach consists of two levels which are solved sequentially; the Global Level generates a good solution for MDVRP, and the Local Level improves the generated solution by Global Level.

In the Global Level, MDVRP is solved by assigning customers to depots and building routes simultaneously. In the Local Level, MDVRP is decomposed to m VRP and each VRP solution is improved. Figure 4.3 shows the flowchart of the TLBRH.

In the Global Level, the BR-ECWH (explained in the following text), is applied to assign customers to depots and generate a routing solution simultaneously. While in the Local Level, the BR-CWH (explained in Chapter 1), is employed for each depot to improve the routing solution which is allocated to that depot, as proposed by Juan et al. (2010). The main idea behind Biased Randomisation is the introduction of randomness in the greedy constructive heuristic.

The BR-ECWH is chosen to solve the Global Level because of its ability to solve the assigning problem and routing problem simultaneously. Therefore, routing cost of takes into account allocation cost when customers are assigned to the depots. At Local Level, the BR-CWH is applied because it is a fast method and it is able to provide high-quality solutions, which can compete with the ones provided by much more complex exact and metaheuristics approaches, which are usually difficult to implement in practice. Moreover, Biased Randomisation technique enhances our method's performance by introducing randomness to the procedure.

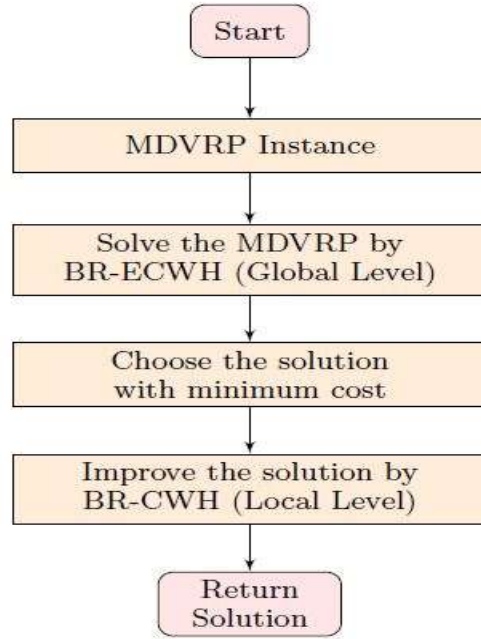


Figure 4.3. TLBRH for the MDVRP

The solution procedure starts by using BR-ECWH to assign customers to depots and find a good routing solution, simultaneously. This procedure is executed in the Global Level of the proposed heuristic. After that, the generated solution by Global Level with the minimum cost is chosen to be improved by BR-CWH without any change in customers' assignment to depots. This procedure is executed in the Local Level of the proposed heuristic.

To the best of our knowledge, this is the first time these two heuristics are combined together in one solution method to solve the MDVRP; the randomised version of the ECWH (BR-ECWH) and randomised version of the CWH (BR-CWH).

The proposed method does not need a lot of parameters therefore there is no need to tune its parameters. Moreover, our method can generate many solutions with the same quality but with different characteristics. Thus, the decision makers can choose amongst these solutions based on their needs.

- **The Global Level:**

The Global Level, as shown in Figure 4.4, starts by generating the dummy solution. The dummy solution consists of assigning each customer to the nearest depot and constructing a route to serve only one customer from its nearest depot. In this solution, we assume that each depot has unlimited capacity and unlimited vehicles to serve all customers that are assigned to it.

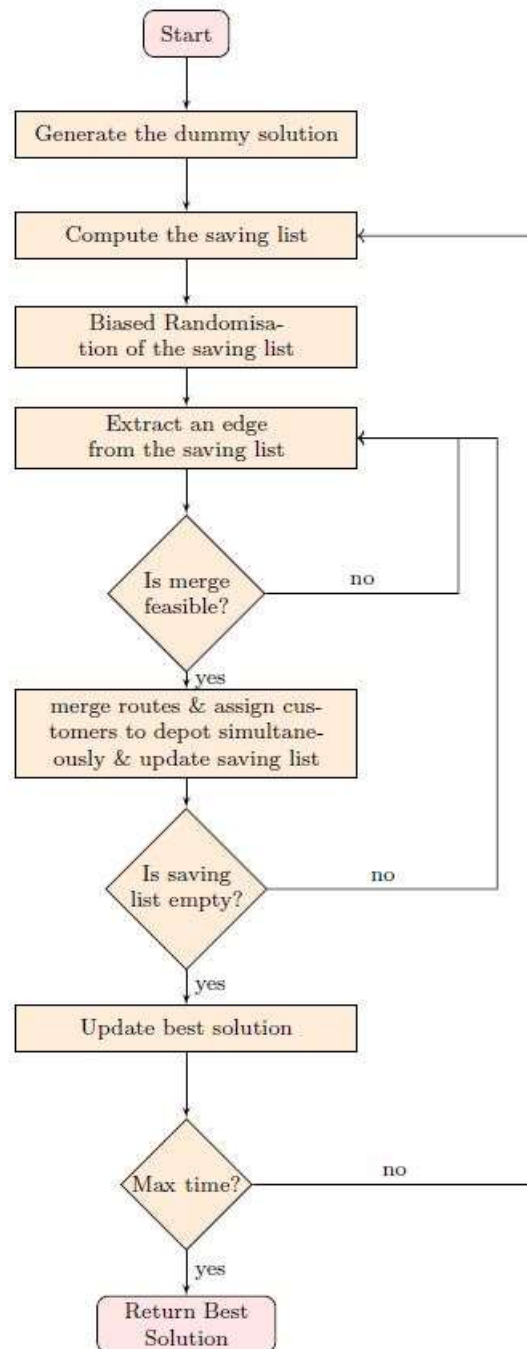


Figure 4.4. Flowchart of the Global Level

After that, the savings list for all customers with consideration of all depots, is generated based on equation (4.11). The savings list allows us to explore the potential savings from merging two customers from the dummy solution in one route. Then, the savings list is sorted in descending order, which means the edge with the highest saving will be at the beginning of the saving list. After that, the Biased Randomised technique is combined with the heuristic after computing the savings list. The way of randomising the saving list was explained in Chapter 3.

The next step is to choose an edge from the saving list to join two customers in one route. In the ECWH, selecting edges is based on their order in the saving list. Therefore, the edge with the highest savings value is chosen first. While in the BR-ECWH, we assign a selection probability, from the geometric distributions, to each edge in the savings list. By doing so, edges at the top of the list with a higher savings value, have a greater chance to be chosen – more than edges with a lower savings value.

After choosing the edge based on the Biased Randomised technique, we check the feasibility of merge. The following conditions must be satisfied before joining two customers in one route:

- i) The combined demand on the new route should not exceed the vehicle capacities.
- ii) Customers must not already be on the same route.
- iii) If a customer is connected to two other customers, it is never considered for linking.
- iv) If one or more of the conditions are not satisfied this pair of customers are excluded from further consideration at this depot.

If all the above conditions are satisfied, then these customers are joined at one route and assigned at this depot simultaneously. And they are eliminated from consideration at the other depots. Then, the value of \tilde{d}_i^k for the customer that is linked to another customer at depot k is set equal to d_i^k .

The savings list is then updated as required by this change, which means one iteration is completed. If the savings list is not empty, the process is repeated, otherwise, the best solution is updated and more customers can be joined. If the maximum time of executing the Global level is reached, the best solution is presented, otherwise the whole heuristic is repeated.

- **Local Level:**

After finishing the Global level, we are going to deal with each depot and its customers as VRP. The aim of this level is to improve the order of customers inside each route. To do that, we adapted the BR-CWH which was proposed by Juan et al. (2010). The complete details of the BR-CWH were given in Chapter 3.

4.6 Computational Experiments

In this section, experimental results are presented. These results include minimising the total cost of the optimisation model given in section 4.3. Preliminary experiments have been conducted to evaluate the performance of the MDVRP solution methods. The TLBRH was coded by using Java applications. Computational experiments have been performed using a computer with a Core i5, 3.20 GHz processor and 8 GB of RAM.

4.6.1 Data sets and experimental setting

There are two benchmark data sets which are available in the literature and they have been used to test the performance of the proposed method.

The first data set consists of 20 MDVRP instances. Some of the instances in this set have been obtained from the literature (Gillett and Johnson, 1976; Chao et al. 1993) while other instances have been adapted from literature related to the VRP (Christofides and Eilon, 1969).

Instances (1 – 7) were generated, originally, for the VRP by Christofides and Eilon (1969). Then, they were modified and adapted by Gillett and Johnson (1976) for the MDVRP. Instances (8 – 11) and (12 – 20) were generated for the MDVRP by Gillett and Johnson (1976) and Chao et al. (1993), respectively.

The number of customers in the first data set varies between 50 and 240 while, the number of depots range from 2 to 6. The vehicle capacity varies between 60 and 500, whereas the number of vehicles available at each depot ranges between 2 and 14.

Table 4.1 shows characteristics of the first data set. It contains names of each instance, number of customers in column n , number of depots in column m , number of vehicles available at each depot k , vehicle capacity in column $V.Q$, and maximum route length allowed L .

No.	Name	n	m	k	$V.Q$	L
1	p01	50	4	4	80	n/a
2	p02	50	4	2	160	n/a
3	p03	75	5	3	140	n/a
4	p04	100	2	8	100	n/a
5	p05	100	2	5	200	n/a
6	p06	100	3	6	100	n/a
7	p07	100	4	4	100	n/a
8	p08	249	2	14	500	310
9	p09	249	3	12	500	310
10	p10	249	4	8	500	310
11	p11	249	5	6	500	310
12	p12	80	2	5	60	n/a
13	p13	80	2	5	60	200
14	p14	80	2	5	60	180
15	p15	160	4	5	60	n/a
16	p16	160	4	5	60	200
17	p17	160	4	5	60	180
18	p18	240	6	5	60	n/a
19	p19	240	6	5	60	200
20	p20	240	6	5	60	180

Table 4.1. Characteristics of the first data set

Figure 4.5 and Figure 4.6 illustrate the distribution of customers and depots for two instances from the first data set, p01 and p12, respectively. In Figure 4.5, we can note that customer distribution follows a normal distribution, while in Figure 4.6 we can see customers are located on rectangular diameters, and depots are located on rectangular centres. This data set is available at <http://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-instances>.

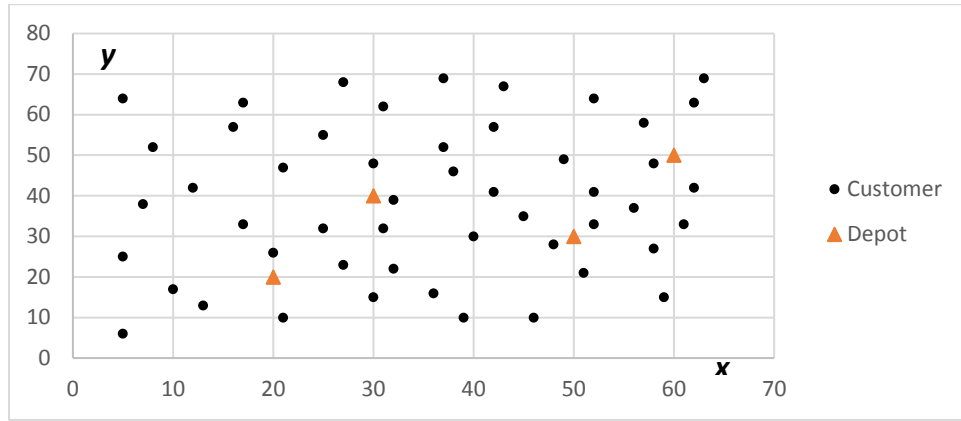


Figure 4.5. Distribution of customers and depots in p01

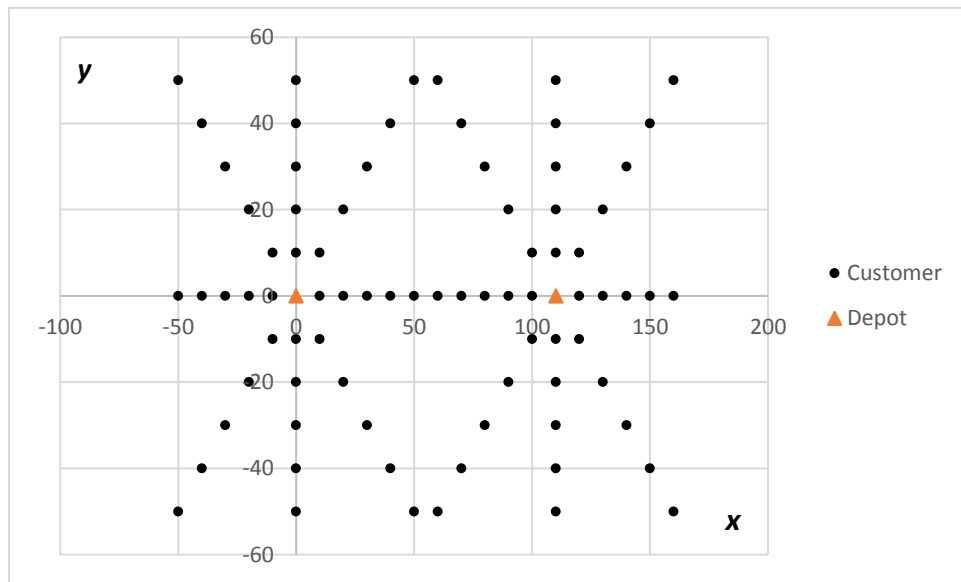


Figure 4.6. Distribution of customers and depots in p12

The second data set consisting of 10 MDVRP instances were introduced by Cordeau et al. (1997). The number of customers in the second data set vary between 48 and 288, while the number of depots range from 4 to 6. The vehicle capacity varies between 170 and 200, whereas the number of vehicles available at each depot ranges between 1 and 6.

Table 4.2 shows characteristics of the second data set. It contains names of each instance, number of customers in column n , number of depots in column m , number of vehicles available at each depot k , vehicle capacity in column $V.Q$, and maximum route length allowed L .

No.	Name	n	m	k	V.Q	L
1	pr01	48	4	1	200	500
2	pr02	96	4	2	195	480
3	pr03	144	4	3	190	460
4	pr04	192	4	4	185	440
5	pr05	240	4	5	180	420
6	pr06	288	4	6	175	400
7	pr07	72	6	1	200	500
8	pr08	144	6	2	190	475
9	pr09	216	6	3	180	450
10	pr10	288	6	4	170	425

Table 4.2. Characteristics of the second data set

Figure 4.7 and Figure 4.8 illustrate the distribution of customers and depots for two instances from the first data set, pr01 and pr07, respectively. In Figure 4.7, we can note that customer distribution follows a normal distribution, while in Figure 4.8 we can see customers are clustered in groups. This data set is available at <http://neo.lcc.uma.es/vrp/vrp-instances/multiple-depot-vrp-instances>.

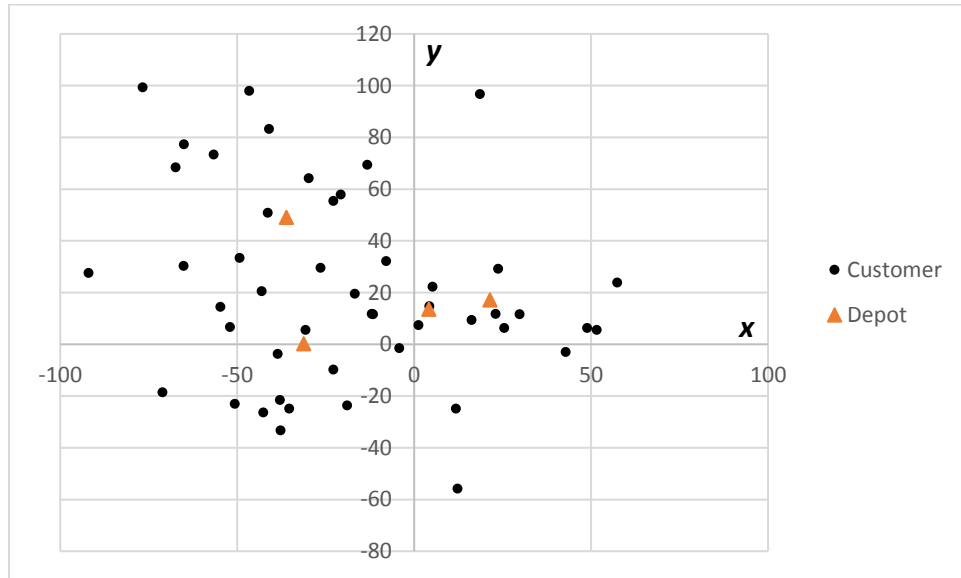


Figure 4.7. Distribution of Customers and depots in pr01

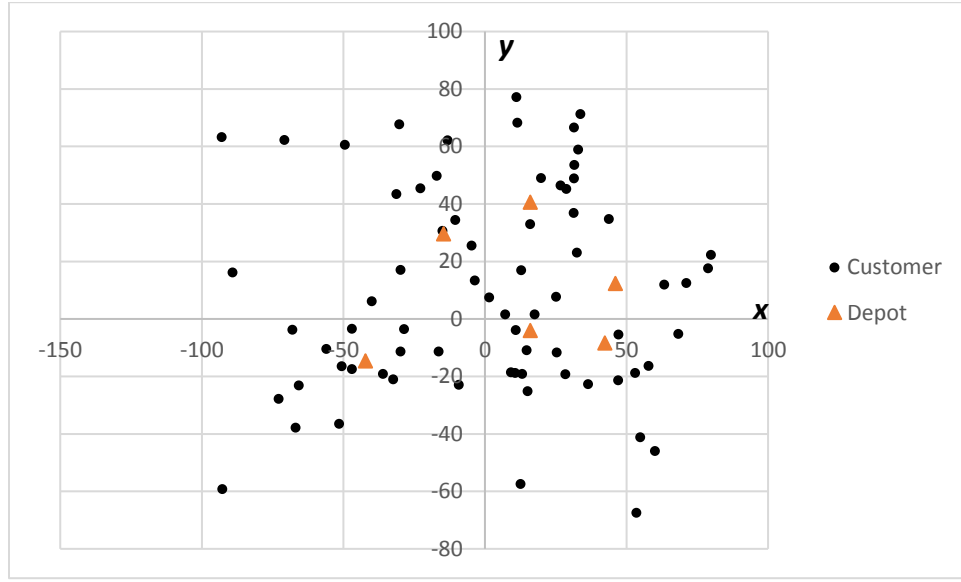


Figure 4.8. Distribution of Customers and depots in pr07

4.6.2 Analysis of the results of the TLBRH

In this section, we discuss the results obtained by the two-level heuristic in order to illustrate the potential of our solution methods. The results have been compared to the best-known solution (BKS) in the literature for the benchmark instances. Table 4.3 and 4.4 present the details of the performance of the proposed method in the first data set and the second data set, respectively.

The first and second column in each table shows the instance name and BKS values. The following columns present the solution obtained by the proposed method, and the percentage gap with respect to the BKS, calculated as $\left[\left(\frac{\text{TLBRH} - \text{BKS}}{\text{BKS}} \right) \times 100 \right]$.

No.	Name	n	BKS	T (min)	TLBRH	Gap %
1	p01	50	576.87	0.53	587.7	1.84
2	p02	50	473.53	0.81	485.06	2.38
3	p03	75	641.19	0.96	660.49	2.92
4	p04	100	1001.59	2.57	1027.15	2.49
5	p05	100	750.03	2.09	768.93	2.46
6	p06	100	876.5	2.06	892.13	1.75
7	p07	100	881.97	2.29	910.9	3.18
8	p08	249	4372.78	19.65	4551.39	3.92
9	p09	249	3858.66	19.3	4007.59	3.72
10	p10	249	3631.11	19.92	3793.09	4.27
11	p11	249	3546.06	19.52	3730.32	4.94
12	p12	80	1318.95	1.09	1320.74	0.14
13	p13	80	1318.95	0.98	1409.67	6.44
14	p14	80	1360.12	0.98	1454.16	6.47
15	p15	160	2505.42	4.18	2608.55	3.95
16	p16	160	2572.23	3.32	2677.27	3.92
17	p17	160	2709.09	3.43	2914.47	7.05
18	p18	240	3702.85	13.98	3884.93	4.69
19	p19	240	3827.06	7.95	4029.77	5.03
20	p20	240	4058.07	9.26	4336.39	6.42
Average				6.74		3.90

Table 4.3. The first data set

From Table 4.3, which summarises the results obtained for the first data set, we can observe that the gap percentage increases with the number of customers in each instance. Figure 4.9 shows the relationship between the gap percentage and the number of customers with correlation coefficient ($R = 0.44$). While the average gap percentage is 3.9%. The average computational time is 8.5 seconds. However, the average computational times for the BKS is 155 seconds.

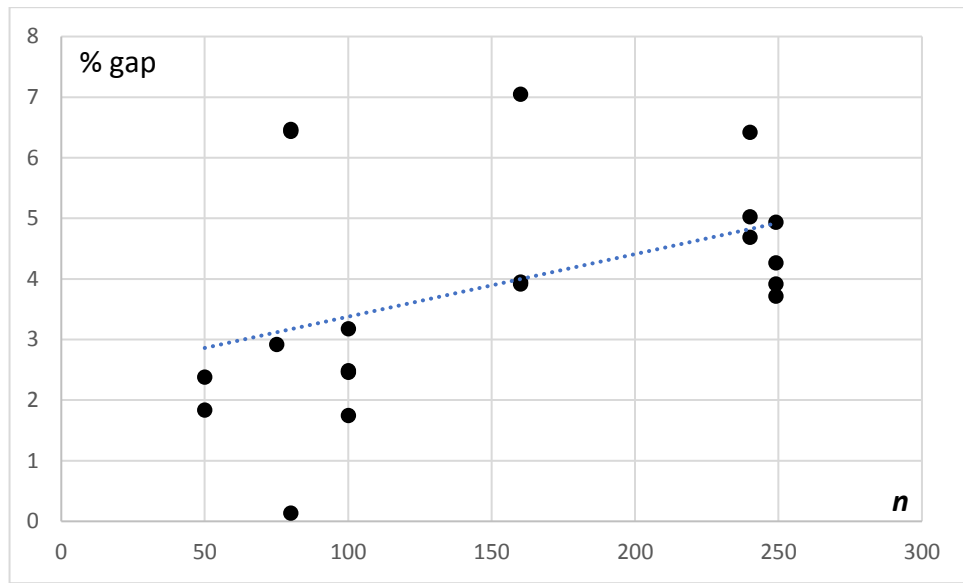


Figure 4.9. The gap based on the number of customers for the first data set

No.	Name	BKS	T (min)	TLBRH	Gap %
21	pr01	861.32	1.02	893.06	3.55
22	pr02	1307.34	3.96	1338.04	2.29
23	pr03	1803.8	6.61	1847.2	2.35
24	pr04	2058.31	11.41	2131.91	3.45
25	pr05	2331.2	20	2411.41	3.33
26	pr06	2676.3	20	2678.443	0.08
27	pr07	1089.56	1.85	1104.44	1.35
28	pr08	1664.85	6.44	1709.77	2.63
29	pr09	2133.2	18.88	2190.62	2.62
30	pr10	2868.26	20	3015.23	4.87
Average			11.02		2.65

Table 4.4. The second data set

From Table 4.4, which summarises the results obtained for the second data set, we can observe the same trend of higher values between the gap percentage and the number of customers in each instance. The gap increases when the number of customers increase, except in instance p06. However, the average gap percentage in this data set is lower than the average gap percentage in the first data set. The reason behind this is believed to be related to the rate of customers for each vehicle. For the first data set, the average rate of customers for each vehicle is 7.3 customer/vehicle, while for the second

data set it is 12 customer/vehicle. The average gap is 2.65%, while the average computational time is 6.3 seconds. However, the average computational times for the BKS is 110 seconds.

When comparing the computational times of the first and second data we find that the computational times of the second data set is longer than the computational times of the first data set. This result is compatible with result of Gillett and Johnson (1976). The average number of depots in the first data set is 4, while the average number of depots in the second data set is 5.

In general, the average gap of our method is 3.9 and 2.65 for the first and the second data set, respectively. These gaps are acceptable when we look at the average computational time, which is reasonably low when compared to other approaches that employ a Biased Randomised technique, such as Juan et al. (2016). They have combined Biased Randomised technique with ILS to solve the the same benchmark of the MDVRP. Their average computational time is 277 seconds.

Moreover, we can observe in Table 4.3 and 4.4 that as the instances get larger, the performance of the two-level deteriorates.

To sum up, the two-level has a considerably good performance with very low computational time, which is highly preferable when a quick solution is required.

4.7 Conclusion

A description of a Biased Randomised heuristic to solve the MDVRP is given. This approach is based on combining a classic constrictive heuristic with the Biased Randomised technique in an iterative framework. Our method consists of two levels; Global Level and Local Level. The Global Level solves the whole MDVRP by using BR-ECHW, while the Local Level improves the solution obtained from the Global Level by applying BR-CWH for each depot with its customers individually. The Biased Randomised technique has been used to help heuristics to escape from local minima and explore different regions of the search space.

To evaluate the performance of algorithms, computational experiments are carried out for different problem sizes ranging from 48 to 288 customers, and number of depots ranging from 2 to 6. The results

obtained so far indicate that this proposed heuristic is suitable to solve the MDVRP as the computational time is short and the average gap is small.

The MDVRP has many real important applications such as food distribution, service sector, drag distribution, and mail delivery.

There are some limitations for the proposed method. Firstly, the two-level method is a heuristic and it seems that it gets stuck in local optima. Secondly, when the rate of customers per vehicles increase, the gap increases; this means its performance deteriorates when the rate increases.

Future study should address two directions for improvement. The first direction is to improve the quality of results by adding a local search to the routing stage, or integrate our method with a metaheuristic such as Tabu Search or VNS. The second direction is to extend the MDVRP and propose novel mathematical models for the split MDVRP.

Finally, the practicality and simplicity of our solution method, with much less parameters, is notable when compared to complex methods in the literature with exhaustive fine-tuning procedures. Therefore, our method can be easily integrated in transport systems for supply chain management.

In a nutshell, the results are promising to extend the suggested solution methods to consider location decision towards the LRP.

Chapter 5 Location Routing Problem with Multi-Depot (LRPMD)

5.1 Introduction

The LRPMD incorporates the three main decisions in supply chain management: the strategic level, the tactical level, and the operational level. These three decisions simultaneously address facility location, customer assignment, and route design. Therefore, the LRPMD is considered as one of the most complex problems in logistics.

It is not surprising that exact methods have been applied to solve only small size LRPMD problems, because the computational time increases exponentially with the problem size (Mousavi and Tavakkoli-Moghaddam, 2013). Alternatively, metaheuristic methods have been used widely to solve the LRPMD, especially after the huge development in computers. While metaheuristics provide solutions with high quality, they consume more computational times compared to heuristics and need more parameters (Juan et al. 2010). Moreover, metaheuristics lack flexibility and need more effort in implementation.

Heuristics have also been used to solve the LRPMD and they are faster than exact methods and metaheuristics (Juan et al., 2010). However, the quality of solutions obtained by heuristics are less than

the quality of solutions obtained by metaheuristics (Juan et al., 2011). In this case, if we can increase the efficiency and effectiveness of heuristics without increasing the computational time, we can, then, increase the quality of the solution.

We have shown the effectiveness of the Biased Randomised technique to improve the performance of a classical heuristic when solving the LRPSD in Chapter 3, and MDVRP in Chapter 4. Therefore, in this chapter, we attempt to examine the ability of this technique on a more complex problem.

Hence, we develop a new and fast heuristic and a metaheuristic to solve the LRPMD. The heuristic combines Biased Randomised technique with ECWH in a nested framework. As we mentioned in Chapter 4, in the nested framework, the routing stage is embedded into the location phase. We have adapted this framework because we consider the LRPMD as a location problem by taking the routing factor into consideration. In this framework, the FLP is the main problem, and the routing problem is the subordinate problem. Therefore, by using this framework, we have chosen not to treat the location and routing problems as if they are on the same footing (Nagy and Salhi, 1996b). The metaheuristic incorporates Biased Randomised technique into a Variable Neighbourhood Search (VNS) framework and therefore called Biased Randomised Variable Neighbourhood Search (BR-VNS).

The general framework of Biased Randomised heuristics consists of two stages, namely the initial stage and the improvement stage. In the initial stage, which is considered as a selection of promising solutions, depots to be opened are determined, customers are allocated to opened depots, and routing to serve customers are designed. The initial solution steps are repeated with different configurations of opened depots, in order to find the most promising solution. In the improvement stage, which is considered as a solution refinement, the best solution obtained from the initial stage, with the minimum cost, is selected to be improved on two levels: Global Level and Local Level. In the global level, we combine the BR-ECWH to improve the customer allocation decision and routing decision. While in the local level we improve routing intensively by applying the BR-CWH for each depot, to improve the routing allocated to that depot, as proposed by Juan et al., (2010) .

On the other hand, the general framework of the Biased Randomised metaheuristic consists of generating an initial solution and improving it by using the VNS. The initial solution is generated by allocating customers to opened depots through Biased Randomised technique. Then the BR-CWH, which is described in Chapter 3, is applied to solve the routing problem.

To show the validity of our proposed method, we carry out computational experiments by using well-known benchmark data sets from the literature. We compare the results obtained by our solution method with the best-known solutions. The computational experiments show that the heuristic generates good quality solutions in a very reasonable computational time.

This chapter consists of the main contributions in section 5.2. The optimisation model of the LRPMD is stated in section 5.3. Section 5.4 outlines the details of the proposed solution methods. Section 5.5 presents the computational experiments carried out and the analysis of the results. Finally, section 5.6 draws some conclusions and discusses opportunities for future research.

5.2 Contribution

We can indicate three main contributions in this chapter. The first two contributions are developing a novel Biased Randomised heuristic and a novel Biased Randomised metaheuristic to solve the LRPMD. The third contribution lies within the first and second contribution and it is about embedding and devising the Biased Randomised technique into both heuristic and metaheuristic, which has been used to solve different combinatorial problems, in a new way.

As for the first contribution, we add a simple, but fast and efficient component, to Tillman's heuristic, to deal with the location decision. As mentioned before, the heuristic by Tillman (1969) was proposed mainly to solve the MDVRP, and to the best of our knowledge it has not been used to solve any other problem. It can be observed clearly that it has not been given the same amount of attention in the literature as the CWH. Therefore, our novel approach improves this classic heuristic to solve the MDVRP, which has resulted after the location decision is made in the LRPMD. As mentioned in Chapter 2, there are three frameworks to solve the LRPMD, the sequential framework, the iterative framework, and the nested framework. In our study, we have shown that the routing problem can be solved in two main steps. Firstly, by treating the whole problem as a MDVRP, and secondly, by dividing the whole problem into many VRPs based on the number of depots. In our solution method, we have used these two methods together. Therefore, we solve the routing stage in the LRPMD as a MDVRP first, then, we improve the solution by dividing MDVRP and re-routing several resulting VRPs.

The second contribution is to develop a Biased Randomised Variable Neighbourhood Search (BR-VNS) metaheuristic to solve the LRPMD. This metaheuristic has been developed in collaboration with our collaborators at the Internet Interdisciplinary Institute (IN3) in the Universitat Oberta de Catalunya in Spain, and Universidad de La Sabana in Colombia.

The last contribution is to combine the Biased Randomised technique with the Tillman's heuristic, which comes after the location component of LRPMD, to improve its performance. The Biased Randomised technique has been applied successfully to improve the performance of classic heuristics, such as CWH (Juan et al., 2010), to solve some combinatorial problems in the literature. In this study, we also make use of its powerful characteristic to randomise search in a heuristic, in order to improve the results.

5.3 Optimisation model

We consider the LRPMD when depots and vehicles have a limited capacity. The problem is to determine the number and locations of depots, assignment of customers to opened depots, and the corresponding delivery routes, so that the total costs, consisting of depot opening costs, and variable and fixed costs for vehicles, are minimised.

Each vehicle takes exactly one route starting from the depot, visits a subset of the customers and returns to the same depot. In addition, customer's demand cannot be split among different routes and the sum of demands in each route must not exceed the vehicle capacity. Furthermore, the total demand of customers assigned to one open depot must not exceed its capacity.

The LRPMD model in this research is defined on a complete, weighted, and undirected network $G = (V, E, C)$, where $V = \{1, \dots, n\}$ is a set of nodes representing the depots and customers, and E is a set of undirected edges (i, j) , and $C = (c_{ij})$ is the matrix of traveling cost associated with edge (i, j) in E . In this study, developed heuristics only consider single depot while multi depots will be addressed in the future. It is assumed that $I \subseteq V$ is a set of potential depots and $J \subseteq V$ is a set of customers. A capacity Q_i and an opening cost f_i are associated with each depot site $i \in I$. A set K of identical vehicles of capacity D is available. When used, each vehicle incurs a fixed cost F and performs a single route.

Each customer $j \in J$ has a demand d_j where $d_j \leq D$. Since $d_j \leq D$, there will never be a need for a node (customer) to be visited by more than one vehicle to satisfy its demand.

Figure 5.1 illustrates an example of LRPMD. Firstly, in Figure 5.1 (a), there are six potential depots and 24 customers. In Figure 5.1 (b), three depots are selected to be opened and three are closed. Figure 5.1 (c) shows how customers are assigned to opened depots. Finally, vehicle routes are calculated in figure 5.1 (d).

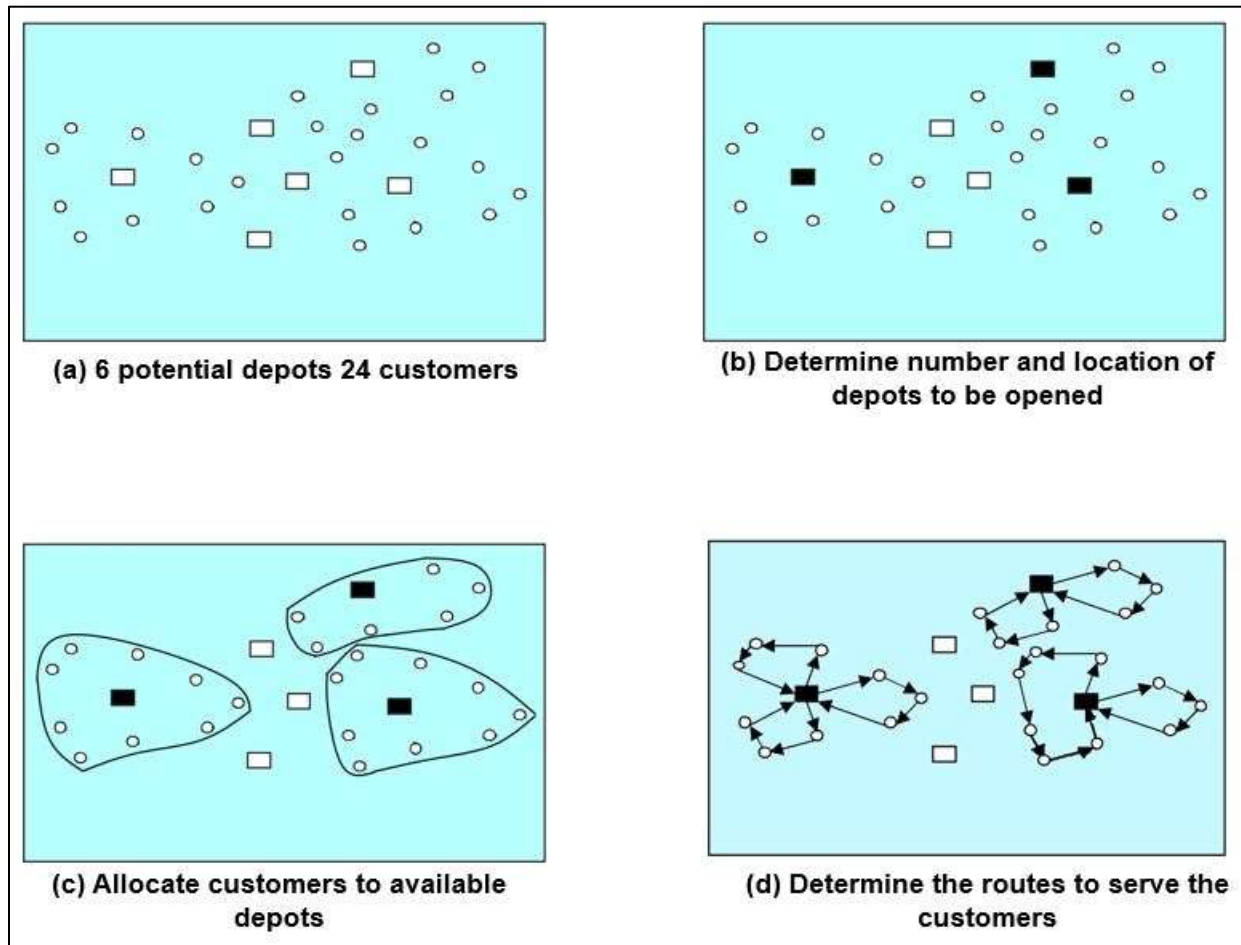


Figure 5.1. An illustrative example of LRPMD

The optimisation model is formulated as a mixed integer linear programming problem and is adapted from Prins et al., (2007). In order to formulate the model, the following notation is introduced.

Sets are defined as follows:

V : Set of nodes, $V = I \cup J$

I : Set of potential depot nodes

J : Set of customers to be serviced

K : Number of available vehicles (fleet size)

Parameters are defined as follows:

f_i : The fixed cost of opening a depot at i

Q_i : Capacity of depot i

d_j : Demand of customer j

D : Capacity of each vehicle

F : Fixed cost per vehicle used

c_{ij} : Travelling cost for edge (i, j)

Decision variables are defined as follows:

$x_{ijk} : \begin{cases} 1, & \text{if vehicle } k \text{ is used on route from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$

$y_i : \begin{cases} 1, & \text{if a depot is located at site } i \\ 0, & \text{otherwise} \end{cases}$

$z_{ij} : \begin{cases} 1, & \text{if customer } j \text{ is served from depot } i \\ 0, & \text{otherwise} \end{cases}$

The LRPMD formulation is as follows:

$$\text{Min } \sum_{i \in I} f_i y_i + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F x_{ijk} \quad (5.1)$$

Subject to

$$\sum_{i \in V} \sum_{k \in K} x_{ijk} = 1 \quad \forall j \in J \quad (5.2)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1 \quad \forall k \in K \quad (5.3)$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \quad \forall k \in K, \quad \forall i \in V \quad (5.4)$$

$$\sum_{u \in J} x_{iuk} + \sum_{u \in V \setminus \{j\}} x_{ujk} \leq 1 + z_{ij} \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K \quad (5.5)$$

$$\sum_{i \in V} \sum_{j \in J} d_j x_{ijk} \leq D \quad \forall k \in K \quad (5.6)$$

$$\sum_{j \in J} d_j z_{ij} \leq Q_i y_i \quad \forall i \in I \quad (5.7)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K \quad (5.8)$$

$$y_i \in \{0, 1\} \quad \forall i \in I \quad (5.9)$$

$$\begin{aligned}
z_{ij} &\in \{0, 1\} & \forall i \in I, \quad \forall j \in V, \\
c_{ij} &= \infty & \text{when } i = j
\end{aligned}
\tag{5.10}$$

The objective function (5.1) seeks to minimise the total cost, which includes the fixed cost of the selected facilities and the fixed and variable cost of the vehicles. Constraints (5.2) are the routing constraints that are imposed: each customer has to be visited exactly once by a single vehicle; whereas constraints (5.3) ensure that all routes have to start and end at a depot. Constraints (5.4) are the connectivity constraints to ensure that every vehicle leaves a customer after he has been served. Constraints (5.5) specify that a customer can be assigned to a depot only if a route linking them is opened. Constraints (5.6) and (5.7) impose both the capacity of vehicle and capacity of the depot. Constraints (5.8), (5.9), and (5.10) define integer variables.

5.4 The proposed Biased Randomised methods

In this section we will describe the proposed methods: the Two-Stage Biased Randomised heuristic (TSBRH), and the Biased-Randomized Variable Neighbourhood Search (BR-VNS).

5.4.1 Two-Stage Biased Randomised heuristic

In this section, a Two-Stage Biased Randomised Heuristic (TSBRH) is proposed in the nested framework to deal with the LRPMD.

In the nested framework, the routing stage is embedded into the location phase. Therefore, the FLP is treated as the main problem, while routing problem is treated as the subordinate problem. This is because the LRP is essentially a location problem, with the routing factor taken into consideration (Nagy and Salhi, 1996). The advantage of this framework is to avoid the drawback of the iterated framework. In this framework, a configuration of potential depots is selected, then, the routing problem is solved. This procedure is repeated many times by choosing different configurations of potential depots to find out the solution with the total minimum cost of depots and routing.

The proposed approach consists of two stages: initial stage, and improvement stage. In the first stage, some depots are selected to be opened among the list of potential candidates. Then, the ECWH

proposed by Tillman (1969), which is explained in Chapter 4, is applied to allocate customers to open depots, and find an initial solution. This stage is repeated for different combinations of depots to investigate the solution space. The best solutions, with best combinations of depots found during the first stage, are then improved throughout the second stage which includes two levels namely, the Global and Local Level. In the Global Level, the BR-ECWH, which is explained in Chapter 4, is applied for the best initial solution resulting from the first stage. In the Local Level, the BR-CWH, which is explained in Chapter 1, is employed for each depot, to improve the routing allocated to that depot, as proposed by Juan et al (2011a). The main idea behind Biased Randomisation is the introduction of randomness in the greedy constructive heuristic. Figure 5.2 shows the flowchart of the proposed method.

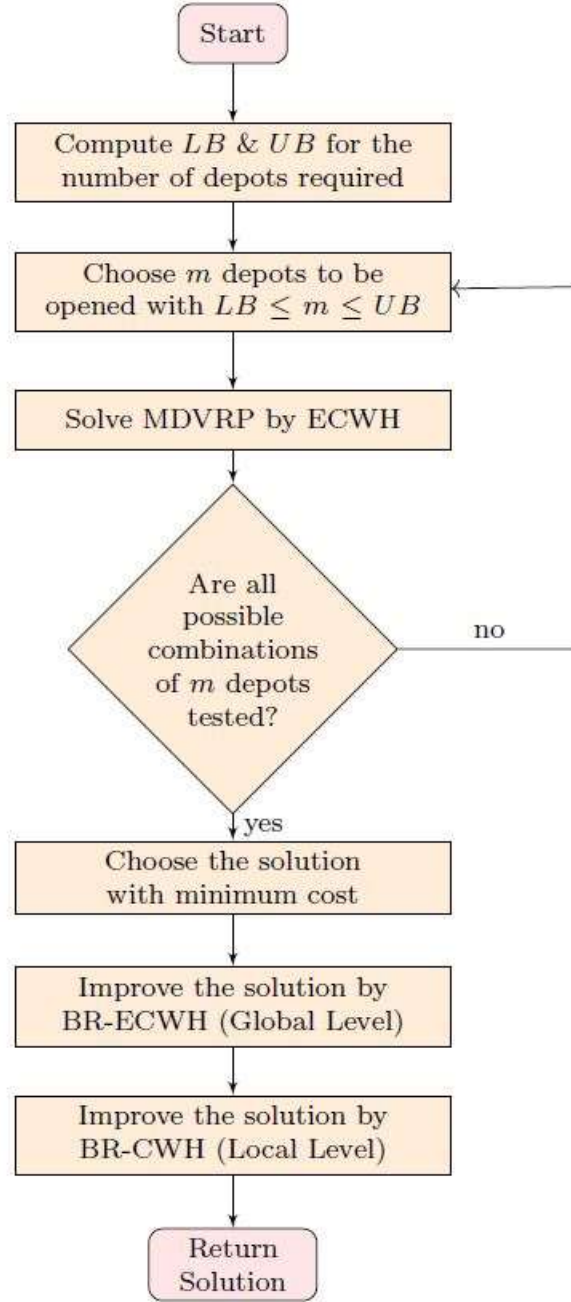


Figure 5.2. Two-Stage Biased Randomised Heuristic for the LRPMD

5.4.1.1 First stage: Selection of promising solutions

The first stage of the proposed approach consists of a fast generation of N feasible and promising solutions for the LRPMD. Each of these solutions is obtained by the following procedure:

- 1) In the first step we determine a lower and an upper bound (LB and UB) for the number of depots to be opened.

- 2) The lower bound is calculated as the quotient between the total demand and the highest depot capacity.
- 3) The upper bound is 60% if the potential depots are 5, 40% if the potential candidates are 8 or 10, and 30% if the potential depots are 15, as suggested in Nagy and Salhi (1996).
- 4) At this point, all combinations of m depots are tested, with $LB \leq m \leq UB$.
- 5) For each of these combinations, the LRPMD is reduced to MDVRP and solved by the ECWH.

The ECWH begins with an initial solution in which each customer is assigned to the nearest depot. Then, the solution is improved by joining customers together on a route, in order to minimise the total travel distance. The customer routes are then assigned to the depot associated with this improvement. The customers selected to be joined are those with the maximum savings where the following conditions must be satisfied. They are:

- 1) The combined demand of the new route should not exceed the vehicle capacity.
- 2) Customers i and j must not be on the same route.
- 3) If a customer is already connected to two other customers, it is never considered for linking.

If one or more of the conditions are not satisfied, this pair of customers is excluded from being served further by the current depot and they are considered in the other depots. If all the conditions are satisfied, then the customers are serviced by the current depot and are eliminated from consideration at the other depot. The whole description of the ECWH is given in Chapter 4.

5.4.1.2 Second stage: Improvement of promising solutions

After choosing depots that will serve customers, the LRPMD is reduced to the MDVRP because the other depots are eliminated from the solution.

As we explained in Chapter 4, the TLBRH consists of two levels, Global Level and Local Level. The TLBRH is employed in the second stage of our current method to solve the LRPMD. It will improve the solution of the LRPMD by reallocating customers and by improving the routing for the best solution found in the first stage. In the following subsections, we will give a reminder about the Global Level and Local Level.

- **Global level**

In this level the customer reallocation and routing are improved for the best solution found in the first stage, by means of the BR-ECWH which is described in Chapter 4.

The BR-ECWH consists of combining Biased Randomisation with ECWH, by randomising the saving list and iterating the heuristic during a given time, which is defined based on the instance size, to get different solutions of similar quality. These are the solutions which are very close to the Best known Solution (BKS).

- **Local level**

In the local level, we apply the BR-CWH which is proposed by Juan et al., (2010) to improve the routing of each depot with its customers individually. The BR-CWH was explained in Chapter 3.

5.4.2 A Biased Randomised Variable Neighbourhood Search (BR-VNS)

In order to generate higher-quality solutions for the G-LRPMD, we developed in collaboration with a Biased-Randomised Variable Neighbourhood Search (BR-VNS) metaheuristic.

Variable Neighbourhood Search (VNS) metaheuristic was firstly introduced in 1977 by P. Hansen and N. Mladenovic. The main reason that it is still widely used nowadays, is that it can escape from local optima exploring successively or at random using different neighbourhoods structures. Therefore, VNS is a very powerful and effective solution method. In addition, VNS is a single-solution metaheuristic and it is a memoryless solution method. This means that there is no information dynamically extracted to be used during the search, as it is the case in nature-inspired solution methods. Due to this fact, it is very adaptable to real-life problems and easy to implement. Also, BR-VNS in this study has very few parameters, which introduces randomness with very few parameters to have more efficiency.

The initial solution in this metaheuristic is generated by allocating customers to depots according to the savings value μ_{ij} associated with serving customer j from depot i . The value μ_{ij} is defined as the saving resulting from the cost difference between serving customer j from depot i and serving customer j from its best alternative depot i^* , i.e.

$$\mu_{ij} = c_{ij} - c_{i^*j} \quad (5.11)$$

Once the savings list has been created for each depot, customers are allocated to depots using a round-robin process. During this process, Biased Randomisation techniques are employed so that different allocations are quickly generated each time the procedure is run. At each turn a depot chooses its next customer according to a geometric distribution with parameter β , as proposed by (Juan et al, 2015). After allocating customers to depots, the BR-CWH is used to solve the routing problem.

During construction of the feasible solutions, the upper and lower bounds (UB/LB) concerning the number of depots to be opened, are computed under the consideration of overall customers' demand and depots' capacities. Subsequently, different random combinations of m depots ($LB \leq m \leq UB$) are generated. The customer allocations and delivery route planning are then optimised during $nInitialIters$ iterations. From the initial solutions, the $nPromising$ most promising ones are stored within a set of *baseSols*. Each potential solution is then further improved through a VNS metaheuristic framework.

The BR-VNS framework is based on construction of different solution neighbourhoods, which are then passed through a given local search operator. For any solution $baseSol \in baseSols$, different shaking procedures are executed to alter the current solution and obtain different neighbourhood structures N_l ($l = 1, 2, \dots, l_{max}$). The shaking procedure consists of randomly exchanging the depot allocation of $\%p$ of all customers. Furthermore, the percentage values applied at this point are increasingly taken from the range $p = 0.05, 0.10, \dots, 0.095$.

After the structure of each *baseSol* has been changed to create a new solution *newSol*, a local search is applied to find the local minimum within the current neighbourhood solution. We have designed three different local search operators namely: customer swap inter-route, inter-depot node exchange, and cross-exchange. In each iteration one of these local search operators is randomly chosen.

In customer swap inter-route, two customers are chosen randomly to swap between different routes of the same depot. In inter-depot node exchange, two customers are chosen randomly to swap between different depots. In cross-exchange three customers (non-consecutive) are chosen randomly to interchange from different depots.

A *newSol* is accepted as the new *baseSol* if the associated cost of the former outperforms that of the latter. Moreover, we also apply a simulated annealing-like acceptance criterion for non-improving solutions, which uses an initial temperature T_0 and a cooling constant *coolingFactor* as described by

Henderson et al (2003). Finally, the current *bestSol* is updated whenever it is outperformed by the *newSol*. This procedure is repeated until a predefined stopping criterion (*maxIter*) is reached. Then, the best found solution is returned to the decision maker. Algorithm 5.1 shows the pseudocode of the BR-VNS.

```

    Input: inputs, parameters
1  initialize(variables)
2  baseSols  $\leftarrow$  createInitialSolutions(inputs, parameters)
3  costs(bestSol)  $\leftarrow$  BigM
4  foreach baseSol  $\in$  initSols do
5       $T = T_0$ 
6      while stopping criteria not reached do
7           $l \leftarrow 1$ 
8          while  $l < l_{max}$  do
9              newSol  $\leftarrow$  shake(baseSol,  $l$ )
10             improving  $\leftarrow$  true
11             while improving do
12                 newSol*  $\leftarrow$  localSearch(newSol)
13                 if costs(newSol*)  $<$  costs(newSol) then
14                     | newSol  $\leftarrow$  newSol*
15                     | end
16                     | else
17                         | improving  $\leftarrow$  false
18                     | end
19                 end
20                 delta  $\leftarrow$  costs(newSol*) - costs(baseSol)
21                 if delta  $< 0$  or (random  $<$  ( $\exp(-\text{delta}/T)$ )) then
22                     | baseSol  $\leftarrow$  newSol*
23                     |  $l \leftarrow 1$ 
24                     | end
25                     | else
26                         |  $l \leftarrow l + 1$ 
27                     | end
28                 end
29                  $T \leftarrow T \times \text{coolingFactor}$ 
30             end
31             end
32             end
33             if costs(baseSol)  $<$  costs(bestSol) then
34                 | bestSol  $\leftarrow$  baseSol
35             end
36         end
37     end
38     return bestSol

```

Algorithm 5.1. Framework of the BR-VNS metaheuristic

5.5 Computational experiments

Computational experiments have been conducted to evaluate the performance of the proposed solution method for the LRPMD. The TSBRH was coded by using Java applications. Computational experiments have been performed using a 2.3 Ghz Quad-Core AMD Opteron(tm) processor with 8 GB of RAM and running under CentOS release 6.6. The average value of results is calculated and it is called Average Total cost.

5.5.1 Data and experimental setting

There are three benchmark data sets which are available in the literature and have been used to test the performance of the proposed method.

The first data set was introduced by Barreto (2004) and its name is Barreto's set because it comes from Barreto's Ph.D. thesis on clustering heuristics for the LRPMD. Some of the instances in this set have been obtained from the literature (Perl, 1983; Min et al., 1992; Daskin, 1995), while other instances have been adapted from literature related to the VRP (Gaskell, 1967; Christofides and Eilon, 1969). The total number of instances in this data set are 17 instances; 3 instances have been obtained from Perl (1983), 2 instances have been obtained from Min et al. (1992), 2 instances have been obtained from Daskin (1995), 6 instances have been adapted from Gaskell (1967), and 4 instances have been adapted from Christofides and Eilon (1969). The number of customers range between 12 and 150, while the number of potential depots, range from 2 to 15 depots. The vehicles capacity varies between 120 and 25000, whereas the capacity of potential depots varies between 280 and 30000000. The instance names include: name of author, number of customers, and number of potential depots.

Table 5.1 shows the characteristics of Barreto's set. It contains names of each instance, number of customers in column n , number of potential depots in column m , vehicle capacity in column $V.Q$, and depot capacity in column $D.Q$. While figure 5.3 and figure 5.4 illustrate the distribution of customers and potential depots for two instances from Barreto's set; Chr-50x5 and Das-150x10, respectively. This data set is available at <http://sweet.ua.pt/sbarreto/private/SergioBarretoHomePage.htm>.

No.	Name	n	m	V.Q	D.Q
1	Perl-12x2	12	2	140	280
2	Perl-55x15	55	15	120	550
3	Perl-85x7	85	7	160	850
4	Min-27x5	27	5	2500	9000
5	Min-134x8	134	8	850	3000
6	Das-88x8	88	8	9000000	25000000
7	Das-150x10	150	10	8000000	30000000
8	Gas-21x5	21	5	6000	15000
9	Gas-22x5	22	5	4500	15000
10	Gas-29x5	29	5	4500	15000
11	Gas-32x5	32	5	8000	15000
12	Gas-32x5b	32	5	11000	15000
13	Gas-36x5	36	5	250	15000
14	Chr-50x5ba	50	5	160	10000
15	Chr-50x5be	50	5	160	10000
16	Chr-75x10ba	75	10	140	10000
17	Chr-100x10	100	10	200	10000

Table 5.1. Barreto's set

Figure 5.3 shows that customer and potential depots distribution follow a normal distribution, while in Figure 5.4, the customers and potential depots are clustered in groups. These two instances were chosen to show the characteristic of instances in Barreto's data set.

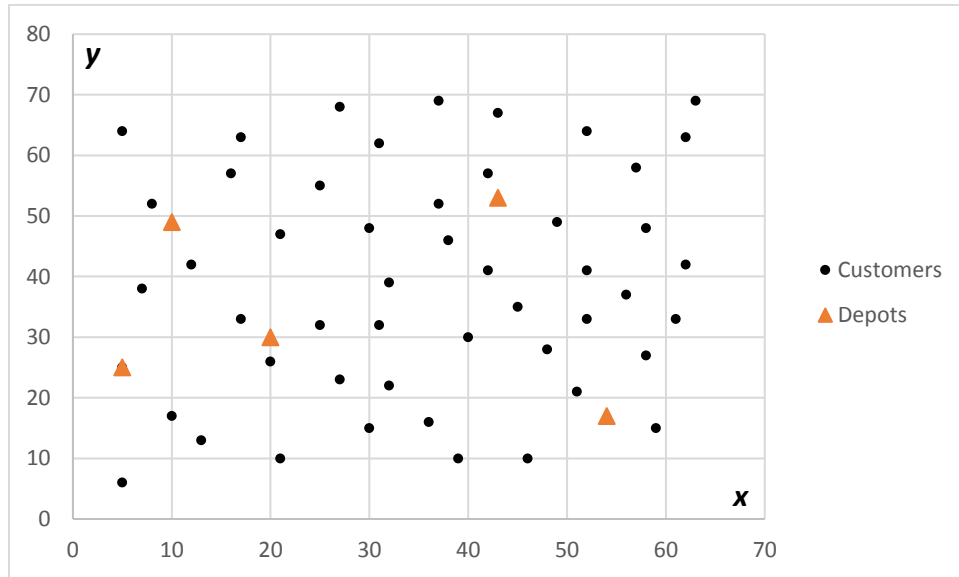


Figure 5.3. Distribution of customers and depots in Chr-50x5

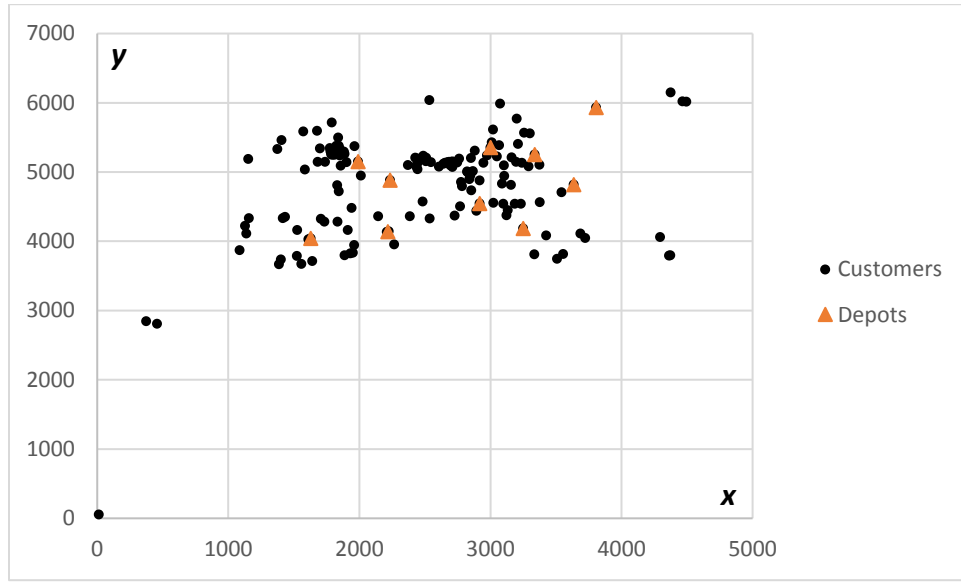


Figure 5.4. Distribution of customers and depots in Das-150x10

The second data set was introduced by Prins, et al. (2006a) and its name is Prodhon's set. This set contains 30 instances with a set of homogenous vehicles and a set of potential capacitated depots. The number of customers is 20, 50, 100, and 200. The vehicles capacity is 70 and 140 while the number of potential depots is 5 and 10. The instance names consist of: number of customers, number of potential depots, number of clusters $\beta \in \{1,2,3\}$, and a letter (a) or (b) based on vehicle capacity; $a = 70$ and $b = 150$. The other data such as depot capacities, customers' demand, and fixed costs were generated as follows:

- 1) Depots' capacity was generated to ensure that there are at least 2 or 3 depots open.
- 2) Customers' demand was generated by using the uniform distribution between 10 and 20.

The main characteristics of Prodhon's set are shown in Table 5.2. The instance name is in the name column, number of customers in column n , number of potential depots in column m , vehicle capacity in column $V.Q$, and depot capacity in column $D.Q$. The depot capacity in some instances is similar, while in the others it is different. In case they are different, we show the minimum and maximum capacity. The data set is available at http://prodhonc.free.fr/Instances/instances_us.htm.

No.	Name	n	m	V.Q	D.Q
1	coord20x5-1a	20	5	70	140
2	coord20x5-1b	20	5	150	300
3	coord20x5-2a	20	5	70	70 - 140
4	coord20x5-2b	20	5	150	150 - 300
5	coord50x5-1a	50	5	70	350 - 420
6	coord50x5-1b	50	5	150	350 - 420
7	coord50x5-2a	50	5	70	350
8	coord50x5-2b	50	5	150	350
9	coord50x5-2BIS	50	5	70	350
10	coord50x5-2bBIS	50	5	150	300
11	coord50x5-3a	50	5	70	350 - 420
12	coord50x5-3b	50	5	150	350 - 420
13	coord100x5-1a	100	5	70	700 - 770
14	coord100x5-1b	100	5	150	700 - 770
15	coord100x5-2a	100	5	70	700 - 840
16	coord100x5-2b	100	5	150	700 - 840
17	coord100x5-3a	100	5	70	770 - 840
18	coord100x5-3b	100	5	150	770 - 840
19	coord100x10-1a	100	10	70	420 - 560
20	coord100x10-1b	100	10	150	420 - 560
21	coord100x10-2a	100	10	70	420 - 560
22	coord100x10-2b	100	10	150	420 - 560
23	coord100x10-3a	100	10	70	420 - 560
24	coord100x10-3b	100	10	150	420 - 560
25	coord200x10-1a	200	10	70	910 - 1190
26	coord200x10-1b	200	10	150	910 - 1190
27	coord200x10-2a	200	10	70	910 - 1260
28	coord200x10-2b	200	10	150	910 - 1260
29	coord200x10-3a	200	10	70	910 - 1190
30	coord200x10-3b	200	10	150	910 - 1190

Table 5.2. Prodhon's set

Figure 5.5, figure 5.6 and figure 5.7 illustrate the distribution of customers and potential depots for three instances from Prodhon's set: coord50x5-1a, coord50x5-2a, and coord50x5-2BIS, respectively.

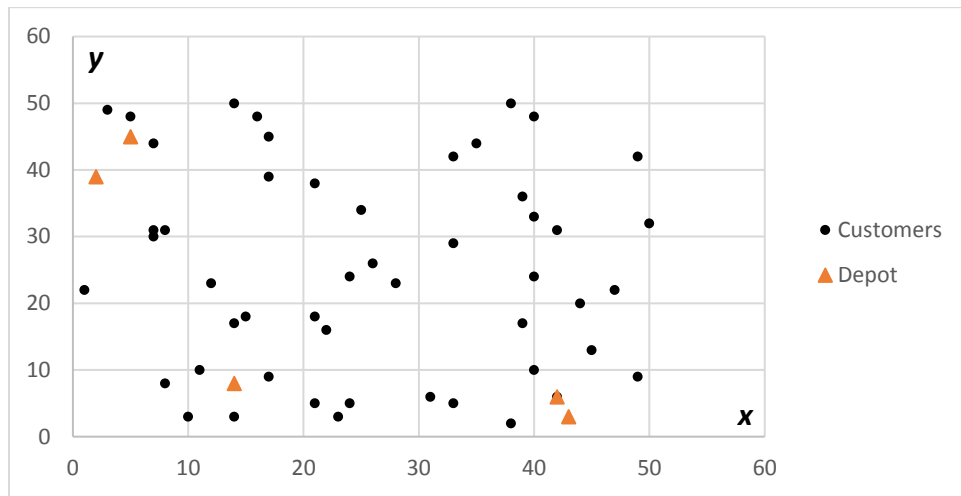


Figure 5.5. Distribution of customers and depots in coord50x5-1a

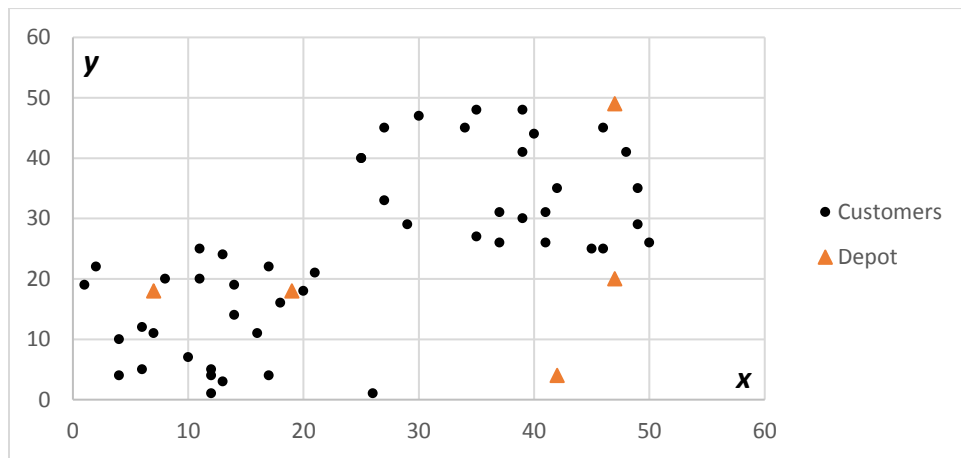


Figure 5.6. Distribution of customers and depots in coord50x5-2a

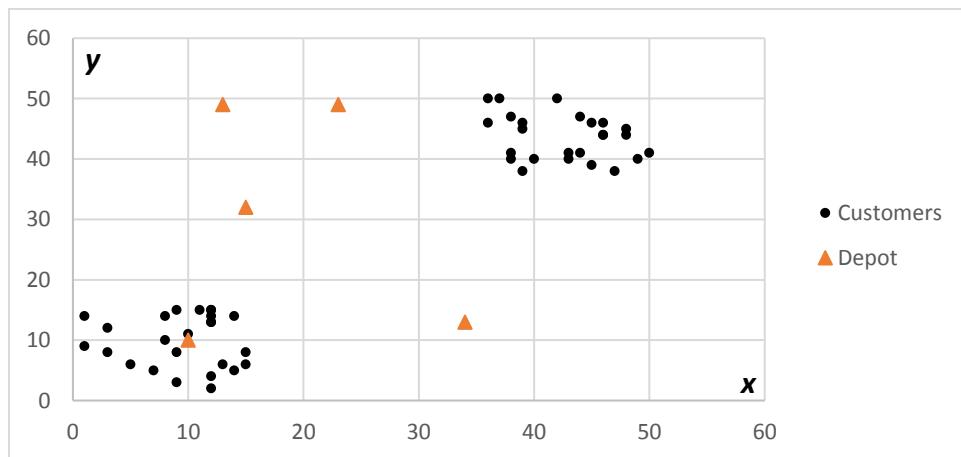


Figure 5.7. Distribution of customers and depots in coord50x5-2BIS

Figure 5.5, shows that customer and potential depot distribution follows a normal distribution. In Figure 5.6, and Figure 5.7 we can see customers and potential depots are clustered by a different factor of clusters in groups. These three instances were picked to show the characteristic of instances in Prodhon's data set.

The third and last data set was introduced in Akca et al. (2009) and its name is Akca's set. This set involves 12 instances: 6 instances of them contain 30 customers, while the other 6 instances contain 40 customers. Each instance has 5 potential depots. Depots' capacity was generated to ensure that at least two depots should be open. The vehicles capacity is from 275 to 390. The number of customer clusters varies from 1 to 3.

The characteristics of each Akca's set is listed in Table 5.3. Column name contains names of instances, n column contains number of customers, column m contains number of potential depots, column $V.Q$ contains vehicle capacity, and column $D.Q$ contains depot capacity. Figure 5.8 and 5.9 illustrate the distribution of customers and potential depots for two instances from Prodhon's set: cr30x5a-1 and cr30x5b-1.

No.	Name	n	m	$V.Q$	$D.Q$
1	cr30x5a-1	30	5	350	1000
2	cr30x5a-2	30	5	350	1000
3	cr30x5a-3	30	5	350	1000
4	cr30x5b-1	30	5	275	1000
5	cr30x5b-2	30	5	275	1000
6	cr30x5b-3	30	5	275	1000
7	cr40x5a-1	40	5	340	1750
8	cr40x5a-2	40	5	390	1750
9	cr40x5a-3	40	5	370	1750
10	cr40x5b-1	40	5	275	1750
11	cr40x5b-2	40	5	275	1750
12	cr40x5b-3	40	5	325	1750

Table 5.3. Akca's set

In Figure 5.8 we can note that customer and potential depots distribution follow a normal distribution. In Figure 5.9 we can see customers and potential depots are clustered in groups. These two instances were chosen to show the characteristic of instances in Akca's data set.

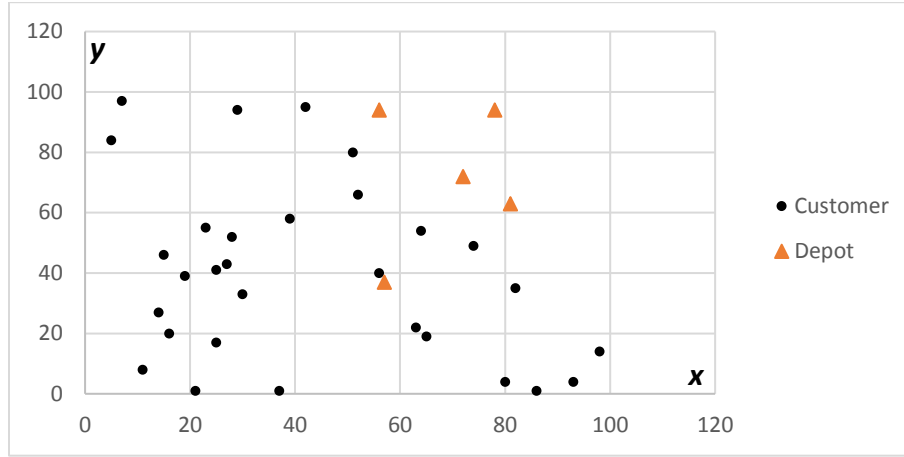


Figure 5.8. Distribution of customers and depots in cr30x5a-1

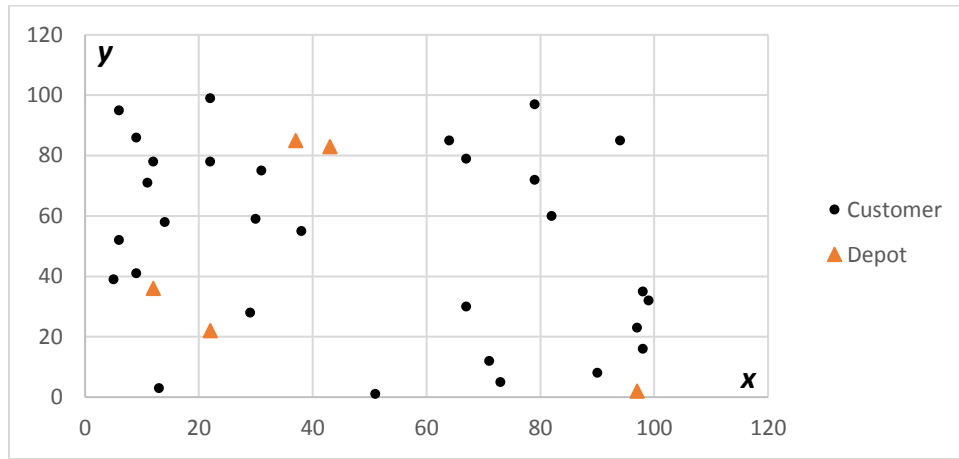


Figure 5.9. Distribution of customers and depots in cr30x5b-1

5.5.2 Analysis of the results

5.5.2.1 Performance evaluation of the TSBRH

In this section, we discuss the results obtained by TSBRH in order to illustrate the potential of our solution methods. To carry out the TSBRH, we used 20 random seeds for each instance. The results have been compared to the best-known solution (BKS) in the literature for the benchmark instances. Moreover, the results have been compared with the top-three performing solution methods (metaheuristics) in terms of percentage gap with respect to the BKS and computational time. These solution methods are: SA for the LRPMD (SALRPMD) which was proposed by Yu et al., (2010),

ALNS which was proposed by Hemmelmayr et al., (2012), and the GRASP followed by an Integer Linear Program (GRASP + ILP) which was proposed by Contardo et al., (2014).

To the best of our knowledge, the Akca's set has been solved by an exact method. The most competitive results based on the computational time are due to Akca et al., (2009), Contardo et al., (2011), and Contardo et al., (2013).

Tables 5.4, 5.5, and 5.6 present the details of the performance of the TSBHRH in Barreto's, Prodhon's, and Akca's set, respectively.

The first and second column in Tables 5.4 and 5.5 show the instance name and BKS values for Barreto's and Prodhon's set, respectively. The 3rd, 4th, and 5th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for SALRPMD. The 6th, 7th, and 8th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for ALNS. The 9th, 10th, and 11th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for GRASP + ILP. The 12th, 13th, and 14th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for our approach (TSBRH).

The first and second column in Table 5.6 shows the instance name and BKS values for the Akca's set. The 3rd, 4th, and 5th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for the Akca et al., (2009). The 6th, 7th, and 8th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for the Contardo et al., (2011). The 9th, 10th, and 11th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for the Contardo et al., (2013). The 12th, 13th, and 14th columns show, respectively the best solution (Z-Best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for our approach (TSBRH).

The percentage gap (GAP) is calculated in all tables with respect to the BKS as $\left[\left(\frac{Z-Best-BKS}{BKS} \right) \times 100 \right]$ and (BS CPU (sec)) for computational times in seconds. The lowest best solutions which match BKS are indicated in bold. At the bottom of Table 5.4 and 5.5 we added the average of the percentage gap, average of the computational times, number of BKS which are obtained by each solution method, and

number of parameters used by each solution method. At the bottom of Table 5.6 we added the average of the percentage gap, average of the computational times, and number of BKS which are obtained by each solution method. The BKS obtained by each method is shown in bold.

As shown in Table 5.4 which summarises the results obtained for Barreto's set, the TSBRH has been capable of matching 7 of the 17 BKSs, while its average gap is 1.59%. Moreover, the average computational time is 3.88 seconds, which is about 2.7% of the average time consumed by SALRPMD, 2.3% of the average time consumed by the ALNS, and 1.8% of the average time consumed by the GRASP + ILP. It should be mentioned that only the TSBRH (our approach) and the GRASP + ILP have been tested with the whole set of instances. In term of solution quality, the best method is SALRPMD with an average gap 0.00%, then ALNS with an average gap 0.17%, and GRASP + ILP with an average gap 0.21%.

There are two general trends for small and large instances that can be observed. In smaller instances with up to 36 customers, our algorithm can match BKS, which is reflected in the GAP column; the only exception is Gaskell-32x5 where the gap is 3.66%. The percentage gap increases when the number of customers and number of potential depots increase.

In general, the average computational time of our algorithm is 3.88 seconds, which indicates the viability of the solution method considering its reasonable gap of 1.59%.

Table 5.5 summarises the results obtained for Prodhon's set. It can be observed that the TSBRH has only achieved 2 of 30 BKSs, with an average gap of 4.44% with respect to the BKS. However, the computational time is quite competitive to other methods. The average computational time is 4.5 seconds, which is about 1.1% of the average time consumed by SALRPMD, 1.0% of the average time consumed by the ALNS, and 0.4% of the average time consumed by the GRASP + ILP. In terms of solution quality, the best method is GRASP + ILP with an average gap 0.12%, then SALRPMD with an average gap 0.42%, and ALNS with an average gap 0.44%.

		SALRPMD				ALNS		GRASP + ILP			TSBRH		
		(Yu et al., 2010)				(Hemmelmayr et al., 2012)		(Contardo et al., 2014b)			Our approach		
		BKS	Z-Best	CPU (sec)	Gap %	Z-Best	CPU (sec)	Gap %	Z-Best	CPU (sec)	Gap %	Z-Best	CPU (sec)
Perl83-12x2	203.98	203.98	6.8	0.00	*	*	*	203.98	0.3	0.00	203.98	2.00	0.00
Gaskell-21x5	424.9	424.9	18.3	0.00	424.9	25	0.00	424.9	1.7	0.00	424.90	2.00	0.00
Gaskell-22x5	585.11	585.11	16.6	0.00	585.11	21	0.00	585.11	2.9	0.00	585.11	2.00	0.00
Min-27x5	3062.02	3062.02	23.3	0.00	3062.02	38	0.00	3062.02	3.5	0.00	3062.02	2.00	0.00
Gaskell-29x5	512.1	512.1	23.9	0.00	512.1	40	0.00	512.1	5.4	0.00	512.10	2.00	0.00
Gaskell-32x5	562.22	562.22	27	0.00	562.22	58	0.00	562.22	6.2	0.00	582.78	2.00	3.66
Gaskell-32x5-2	504.33	504.33	25.1	0.00	504.33	55	0.00	504.33	7.9	0.00	504.33	2.00	0.00
Gaskell-36x5	460.37	460.37	31.7	0.00	460.37	61	0.00	460.37	8.6	0.00	460.37	2.00	0.00
Christ-50x5	565.62	565.62	52.8	0.00	565.6	73	0.00	574.66	17.1	1.60	577.41	2.00	2.08
Christ-50x5-B	565.6	*	*	*	*	*	*	569.49	17.7	0.69	573.45	2.00	1.39
Perl83-55x15	1112.06	1112.06	112.4	0.00	*	*	*	1112.06	47.4	0.00	1129.53	5.10	1.57
Christ-75x10	844.4	844.4	126.8	0.00	853.47	207	1.07	844.58	87.9	0.02	860.98	4.10	1.96
Perl83-85x7	1622.5	1622.5	213.1	0.00	*	*	*	1625.84	81.8	0.21	1634.58	6.20	0.74
Daskin95-88x8	355.78	355.78	226.9	0.00	*	*	*	355.78	209.6	0.00	373.14	2.00	4.88
Christ-100x10	833.43	833.43	330.8	0.00	833.43	403	0.00	840.67	492	0.87	860.98	7.70	3.31
Min92-134x8	5709	5709	522.4	0.00	5712.99	460	0.07	5719.25	750.2	0.18	6012.08	10.00	5.31
Daskin95-150x10	43,919.90	43,919.90	577	0.00	44,309.20	613	0.89	43,952.30	1842.1	0.07	44858.69	10.80	2.14
average	3865.21	3829.86	145.93	0.0	4865.48	171.17	0.17	3641.74	210.72	0.21	3718.61	3.88	1.59
No. of BKS		16		9		10		7					
No. of parameters		7		9		22		1					

Table 5.4. Results of TSBRH for Barreto's set

Instances	BKS	SALRPMD (Yu et al., 2010)			ALNS (Hemmelmayr et al., 2012)			GRASP+ILP (Contardo et al., 2014b)			TSBRH (Our approach)		
		Z-Best	CPU	Gap	Z-Best	CPU	Gap	Z-Best	CPU	Gap	Z-Best	CPU	Gap
coord20x5-1	54,793	54,793	19.8	0.00	54,793	39	0.00	54,793	1.7	0.00	55,089	2.0	0.54%
coord20x5-1b	39,104	39,104	15.0	0.00	39,104	54	0.00	39,104	2.6	0.00	39,104	2.0	0.00%
coord20x5-2	48,908	48,908	19.3	0.00	48,908	38	0.00	48,908	1.5	0.00	50,177	2.0	2.59%
coord20x5-2b	37,542	37,542	15.0	0.00	37,542	67	0.00	37,542	2.8	0.00	37,542	2.0	0.00%
coord50x5-1	90,111	90,111	74.7	0.00	90,111	101	0.00	90,111	15.0	0.00	91,425	2.0	1.46%
coord50x5-1b	63,242	63,242	57.7	0.00	63,242	65	0.00	63,242	18.4	0.00	64,974	2.0	2.74%
coord50x5-2	88,298	88,298	95.0	0.00	88,443	99	0.16	88,298	17.5	0.00	90,007	2.0	1.94%
coord50x5-2b	67,308	67,340	58.6	0.05	67,340	200	0.05	67,373	22.0	0.10	71,321	2.0	5.96%
coord50x5-2BIS	84,055	84,055	74.7	0.00	84,055	107	0.00	84,055	27.3	0.00	85,343	2.0	1.53%
coord50x5-2bBIS	51,822	51,822	66.1	0.00	51,822	98	0.00	51,883	21.0	0.12	55,414	2.0	6.93%
coord50x5-3	86,203	86,456	74.0	0.29	86,203	101	0.00	86,203	16.6	0.00	90,602	2.0	5.10%
coord50x5-3b	61,830	62,700	58.2	1.41	61,830	137	0.00	61,830	22.9	0.00	65,145	2.0	5.36%
coord100x5-1	274,814	277,035	348.6	0.81	275,636	520	0.30	275,457	230.4	0.23	279,264	2.0	1.62%
coord100x5-1b	213,615	216,002	268.9	1.12	214,735	1190	0.52	214,056	230.2	0.21	216,576	2.0	1.39%
coord100x5-2	193,671	194,124	348.6	0.23	193,752	463	0.04	193,708	121.9	0.02	195,980	2.0	1.19%
coord100x5-2b	157,095	157,150	211.5	0.04	157,095	859	0.00	157,178	100.0	0.05	158,862	2.0	1.12%
coord100x5-3	200,079	200,242	250.3	0.08	200,305	454	0.11	200,339	97.3	0.13	202,223	2.0	1.07%
coord100x5-3b	152,441	152,467	196.7	0.02	152,441	684	0.00	152,466	100.1	0.02	154,421	2.0	1.30%
coord100x10-1	287,695	291,043	270.0	1.16	296,877	210	3.19	287,892	2621.8	0.07	329,928	2.0	14.68%
coord100x10-1b	230,989	231,763	202.6	0.34	235,849	188	2.10	234,080	1067.2	1.34	279,514	2.0	21.01%
coord100x10-2	243,590	245,813	260.6	0.91	244,740	136	0.47	243,695	236.1	0.04	261,783	3.2	7.47%
coord100x10-2b	203,988	205,312	199.3	0.65	204,016	261	0.01	203,988	258.5	0.00	220,639	3.0	8.16%
coord100x10-3	250,882	250,882	338.1	0.00	253,801	202	1.16	252,927	723.3	0.82	269,466	3.0	7.41%
coord100x10-3b	204,317	205,009	240.3	0.34	205,609	224	0.63	204,664	584.4	0.17	220,269	3.0	7.81%
coord200x10-1	475,294	481,002	1428.1	1.2	480,883	752	1.18	475,327	3960.4	0.01	501,614	15.0	5.54%
coord200x10-1b	377,043	383,586	1335.8	1.74	378,961	1346	0.51	377,327	4006.0	0.08	394,147	13.9	4.54%
coord200x10-2	449,006	450,848	1795.8	0.41	450,451	1201	0.32	449,291	4943.0	0.06	458,803	8.1	2.18%
coord200x10-2b	374,280	376,674	1245.1	0.64	374,751	1349	0.13	374,575	3486.0	0.08	395,363	7.5	5.63%
coord200x10-3	469,433	473,875	1776.0	0.95	475,373	1251	1.27	469,870	4075.1	0.09	484,669	18.9	3.25%
coord200x10-3b	362,653	363,701	1326.4	0.29	366,902	1137	1.17	363,103	7887.6	0.12	375,890	19.3	3.65%
Average	196470.03	197696.63	422.0	0.42	197852.33	451	0.44	196776.17	1129.0	0.12	206518.46	4.5	4.44%
Number of BKS			10			12			11		2		
Number of parameters			7			9			22		1		

Table 5.5. Results of TSBRH for Prodhon's set

	BKS	Akca et al. (2009)			Contardo et al. (2011)			Contardo et al. (2013)			Our Approach (TSBRH)		
		Z-Best	CPU (sec)	Gap	Z-Best	CPU (sec)	Gap	Z-Best	CPU (sec)	Gap	Z-Best	CPU (sec)	Gap
Cr30 × 5a-1	819.51	819.53	993.30	0.00%	819.51	2.45	0.00%	819.52	3.23	0.00%	837.86	2.00	2.24%
Cr30 × 5a-2	821.50	821.50	10806.50	0.00%	821.50	3.72	0.00%	821.50	8.77	0.00%	881.65	2.00	7.33%
Cr30 × 5a-3	702.30	702.29	917.90	0.00%	702.30	0.50	0.00%	702.30	0.91	0.00%	707.97	2.00	0.81%
Cr30 × 5b-1	880.02	880.02	6420.60	0.00%	880.02	4.57	0.00%	880.02	9.05	0.00%	885.08	2.00	0.57%
Cr30 × 5b-2	825.32	825.32	33.20	0.00%	825.32	1.24	0.00%	825.32	2.55	0.00%	825.32	2.00	0.00%
Cr30 × 5b-3	884.60	884.62	41.70	0.00%	884.60	1.23	0.00%	884.60	3.25	0.00%	884.58	2.00	0.00%
Cr40 × 5a-1	928.10	928.11	10882.80	0.00%	928.10	14.67	0.00%	928.10	140.31	0.00%	933.49	2.00	0.58%
Cr40 × 5a-2	888.42	888.42	11052.90	0.00%	888.42	11.88	0.00%	888.42	86.31	0.00%	899.11	2.00	1.20%
Cr40 × 5a-3	947.30	947.30	10862.00	0.00%	947.30	11.36	0.00%	947.30	76.63	0.00%	963.55	2.00	1.72%
Cr40 × 5b-1	1052.04	1052.07	8084.60	0.00%	1052.04	10.49	0.00%	1052.04	3115.92	0.00%	1059.17	2.00	0.68%
Cr40 × 5b-2	981.54	981.52	862.50	0.00%	981.54	3.77	0.00%	981.54	7.61	0.00%	981.54	2.00	0.00%
Cr40 × 5b-3	964.33	964.32	963.00	0.00%	964.33	2.68	0.00%	964.33	12.33	0.00%	979.80	2.00	1.60%
Average	891.25	891.25	5160.08	0.00%	891.25	5.71	0.00%	891.25	288.91	0.00%	903.26	2.00	1.39%
Number of BKS		12			12			12			3		

Table 5.6. Results of TSBRH for Akca's set

We believe that the size and complexity of Prodhon's data set, is the reason for the performance deterioration of our solution method and not achieving the BKS. We can observe that as the instances get larger, the performance of the TSBRH deteriorates. This behavior can be explained in line with the nature of the solution method. The TSBRH is a heuristic and it seems that it gets stuck in local optima. That explains its high average gap of 4.44%. However, we need to interpret the overall result by also considering the computational time and this is where TSBRH with an average computational time of 4.5 seconds is highly preferable when a quick solution is required.

Although the BKS cannot be reached in the majority of Prodhon's set, which are more challenging in terms of size and complexity, the practicality and simplicity of our solution method with much less parameters are notable compared to other complex methods in the literature with exhaustive fine-tuning procedures.

Similarly, Table 5.6 presents the summary of results for Akca's set. The TSBRH has matched 3 of 12 BKSs with an average gap of 1.39% with respect to the BKS. Moreover, the average computational time is 2.00 seconds, which is about 0.04% of the average time consumed by Akca et al., (2009), 35% of the average time consumed by Contardo et al., (2011), and 0.7% of the average time consumed by Contardo et al., (2013).

These results confirm our previous observation in Barreto's and Prodhon's set about the direct effect of the instance size and customers distribution on the performance of our algorithm. Incidentally it should be noted, that in larger and more complex instances our solution methods struggle and match only a few instances.

In general, TSBRH has a considerably good performance on Barreto's set compared to the other two benchmarks. Moreover, all results confirm our observation about the direct effect of the instance size on the performance of the solution methods. This behaviour can be explained in line with the nature of our solution method, which is a heuristic method that seems to get stuck in local optima.

Also, TSBRH has an average computational time 3.88, 4.5, and 2 seconds, while its average gap is 1.59%, 4.44%, and 1.39% for Barreto's, Prodhon's, and Akca's data set, respectively. Thereby, TSBRH is useful if decision makers prefer to get a solution with reasonable quality within short computational time. This balance between the quality of the solutions and computational time consumed by our approach, makes it an interesting tool to support the design of supply chain management.

Finally, the practicality and simplicity of TSBRH, with much less parameters, is notable when compared to complex methods in the literature with exhaustive fine-tuning procedures.

To sum up, the results are promising to extend the suggested solution methods to consider constrained distance towards a green LRPMD.

5.5.2.2 Performance evaluation of the BR-VNS

In this section, we discuss the results obtained by BR-VNS. Similar to the TSBRH, we used 20 random seeds for each instance. The results have been compared to the best-known solution (BKS) in the literature for the benchmark instances. Moreover, the results have been compared with the same three methods (metaheuristics) in section 5.5.2.1, in terms of percentage gap and computational time with respect to the BKS.

Tables 5.7, 5.8, and 5.9 present the details of the performance of BR-VNS in Barreto's, Prodhon's, and Akca's set, respectively. The table design is similar to tables in section 5.5.2.1 other than the 12th, 13th, and 14th columns which show, respectively the best solution (Z-best), the computational times in seconds (CPU (sec)), and the percentage gap (GAP) for BR-VNS.

As shown in Table 5.7 which summarises the results obtained for Barreto's set, the BR-VNS has been able to match 5 of the 17 BKSs, while its average gap is 0.74%. However, the average computational time is 99.88 seconds, which is about 68.4% of the average time consumed by SALRPMD, 58.4% of the average time consumed by the ALNS, and 47.4% of the average time consumed by the GRASP + ILP.

Table 5.8 summarises the results obtained for Prodhon's set. It can be observed that the BR-VNS has achieved 6 of 30 BKSs, with an average gap of 0.62% with respect to the BKS. The average computational time is 231.8 seconds, which is about 54.9% of the average time consumed by SALRPMD, 51.4% of the average time consumed by the ALNS, and 20.5% of the average time consumed by the GRASP + ILP.

Table 5.9 presents the summary of results for Akca's set. The BR-VNS has matched 8 of 12 BKSs with an average gap of 0.04% with respect to the BKS. The average computational time is 1.00 seconds, which is about 0.02% of the average time consumed by Akca et al., (2009), 17.5% of the average time consumed by Contardo et al., (2011), and 0.4% of the average time consumed by Contardo et al., (2013). In general, our algorithm has a considerably good performance on Akca's

set compared to the other two benchmarks. Finally, the results are promising to extend the suggested solution methods to consider constrained distance towards a green LRPMD.

	SALRPMD				ALNS			GRASP + ILP			BR-VNS		
	(Yu et al., 2010)				(Hemmelmayr et al., 2012)			(Contardo et al., 2014b)			Our approach		
	BKS	Z-Best	CPU (sec)	Gap %	Z-Best	CPU (sec)	Gap %	Z-Best	CPU (sec)	Gap %	Z-Best	CPU (sec)	Gap %
Perl83-12x2	203.98	203.98	6.8	0.00%	*	*	*	203.98	0.3	0.00%	203.98	1.00	0.00%
Gaskell-21x5	424.9	424.9	18.3	0.00%	424.9	25	0.00%	424.9	1.7	0.00%	424.90	2.10	0.00%
Gaskell-22x5	585.11	585.11	16.6	0.00%	585.11	21	0.00%	585.11	2.9	0.00%	586.7	1.10	0.27%
Min-27x5	3062.02	3062.02	23.3	0.00%	3062.02	38	0.00%	3062.02	3.5	0.00%	3062.02	1.10	0.00%
Gaskell-29x5	512.1	512.1	23.9	0.00%	512.1	40	0.00%	512.1	5.4	0.00%	512.10	1.20	0.00%
Gaskell-32x5	562.22	562.22	27	0.00%	562.22	58	0.00%	562.22	6.2	0.00%	562.28	4.30	0.01%
Gaskell-32x5-2	504.33	504.33	25.1	0.00%	504.33	55	0.00%	504.33	7.9	0.00%	504.77	5.20	0.09%
Gaskell-36x5	460.37	460.37	31.7	0.00%	460.37	61	0.00%	460.37	8.6	0.00%	473.66	2.60	2.89%
Christ-50x5	565.62	565.62	52.8	0.00%	565.6	73	0.00%	574.66	17.1	1.60%	565.62	27.90	0.18%
Christ-50x5-B	565.6	*	*	*	*	*	*	569.49	17.7	0.69%	565.6	21.80	0.00%
Perl83-55x15	1112.06	1112.06	112.4	0.00%	*	*	*	1112.06	47.4	0.00%	1119.09	75.90	0.63%
Christ-75x10	844.4	844.4	126.8	0.00%	853.47	207	1.07%	844.58	87.9	0.02%	871.13	88.90	3.17%
Perl83-85x7	1622.5	1622.5	213.1	0.00%	*	*	*	1625.84	81.8	0.21%	1634.78	40.10	0.76%
Daskin95-88x8	355.78	355.78	226.9	0.00%	*	*	*	355.78	209.6	0.00%	356.04	158.70	0.07%
Christ-100x10	833.43	833.43	330.8	0.00%	833.43	403	0.00%	840.67	492	0.87%	849.74	355.30	1.96%
Min92-134x8	5709	5709	522.4	0.00%	5712.99	460	0.07%	5719.25	750.2	0.18%	5839.4	269.40	2.28%
Daskin95-150x10	43,919.90	43,919.90	577	0.00%	44,309.20	613	0.89%	43,952.30	1842.1	0.07%	44005.75	641.30	0.20%
average	3865.21	3829.86	145.93	0.0%	4865.48	171.17	0.17%	3641.74	210.72	0.21%	3655.15	99.88	0.74%
No. of BKS	16				9			10			5		
No. of parameters	7				9			22			8		

Table 5.7. Results of BR-VNS for Baretto's set

Instances	BKS	SALRPMD (Yu et al., 2010)			ALNS (Hemmelmayr et al., 2012)			GRASP+ILP (Contardo et al., 2014b)			BR-VNS (Our approach)		
		Z-Best	CPU	Gap	Z-Best	CPU	Gap	Z-Best	CPU	Gap	Z-Best	CPU	Gap
coord20x5-1	54,793	54,793	19.8	0	54,793	39	0	54,793	1.7	0	54,793	2.1	0.00%
coord20x5-1b	39,104	39,104	15	0	39,104	54	0	39,104	2.6	0	39,104	4.4	0.00%
coord20x5-2	48,908	48,908	19.3	0	48,908	38	0	48,908	1.5	0	48,908	1.0	0.00%
coord20x5-2b	37,542	37,542	15	0	37,542	67	0	37,542	2.8	0	37,542	1.3	0.00%
coord50x5-1	90,111	90,111	74.7	0	90,111	101	0	90,111	15	0	90,402	1.5	0.32%
coord50x5-1b	63,242	63,242	57.7	0	63,242	65	0	63,242	18.4	0	63,242	3.1	0.00%
coord50x5-2	88,298	88,298	95	0	88,443	99	0.16	88,298	17.5	0	88,298	11.6	0.00%
coord50x5-2b	67,308	67,340	58.6	0.05	67,340	200	0.05	67,373	22	0.1	67,853	23.5	0.81%
coord50x5-2bIS	84,055	84,055	74.7	0	84,055	107	0	84,055	27.3	0	84,401	24.6	0.41%
coord50x5-2bBIS	51,822	51,822	66.1	0	51,822	98	0	51,883	21	0.12	51,883	32.7	0.12%
coord50x5-3	86,203	86,456	74	0.29	86,203	101	0	86,203	16.6	0	86,223	10.7	0.02%
coord50x5-3b	61,830	62,700	58.2	1.41	61,830	137	0	61,830	22.9	0	61,844	14.1	0.02%
coord100x5-1	274,814	277,035	348.6	0.81	275,636	520	0.3	275,457	230.4	0.23	277,003	136.9	0.80%
coord100x5-1b	213,615	216,002	268.9	1.12	214,735	1190	0.52	214,056	230.2	0.21	215,702	73.1	0.98%
coord100x5-2	193,671	194,124	348.6	0.23	193,752	463	0.04	193,708	121.9	0.02	194,690	60.2	0.53%
coord100x5-2b	157,095	157,150	211.5	0.04	157,095	859	0	157,178	100	0.05	157,275	71.2	0.11%
coord100x5-3	200,079	200,242	250.3	0.08	200,305	454	0.11	200,339	97.3	0.13	201,299	139.1	0.61%
coord100x5-3b	152,441	152,467	196.7	0.02	152,441	684	0	152,466	100.1	0.02	152,466	106.4	0.02%
coord100x10-1	287,695	291,043	270	1.16	296,877	210	3.19	287,892	2621.8	0.07	294,625	80.2	2.41%
coord100x10-1b	230,989	231,763	202.6	0.34	235,849	188	2.1	234,080	1067.2	1.34	241,396	94.4	4.51%
coord100x10-2	243,590	245,813	260.6	0.91	244,740	136	0.47	243,695	236.1	0.04	244,614	288.0	0.42%
coord100x10-2b	203,988	205,312	199.3	0.65	204,016	261	0.01	203,988	258.5	0	205,019	315.0	0.51%
coord100x10-3	250,882	250,882	338.1	0	253,801	202	1.16	252,927	723.3	0.82	254,667	278.5	1.51%
coord100x10-3b	204,317	205,009	240.3	0.34	205,609	224	0.63	204,664	584.4	0.17	205,746	201.3	0.70%
coord200x10-1	475,294	481,002	1428.1	1.2	480,883	752	1.18	475,327	3960.4	0.01	480,267	999.6	1.05%
coord200x10-1b	377,043	383,586	1335.8	1.74	378,961	1346	0.51	377,327	4006	0.08	379,725	613.4	0.71%
coord200x10-2	449,006	450,848	1795.8	0.41	450,451	1201	0.32	449,291	4943	0.06	450,871	412.9	0.42%
coord200x10-2b	374,280	376,674	1245.1	0.64	374,751	1349	0.13	374,575	3486	0.08	374,720	1094.0	0.12%
coord200x10-3	469,433	473,875	1776	0.95	475,373	1251	1.27	469,870	4075.1	0.09	473,532	586.6	0.87%
coord200x10-3b	362,653	363,701	1326.4	0.29	366,902	1137	1.17	363,103	7887.6	0.12	364,920	1271.1	0.63%
Average	196,470	197,697	422	0.42	197,852	451	0.44	196,776	1129	0.12	198,101	231.8	0.62%
Number of BKS			10			12			11		6		
Number of parameters			7			9			22		8		

Table 5.8. Results of BR-VNS for Prodhon's set

	Akca et al. (2009)				Contardo et al. (2011)			Contardo et al. (2013)			Our Approach (BR-VNS)		
	BKS	Z-Best	CPU (sec)	Gap	Z-Best	CPU (sec)	Gap	Z-Best	CPU (sec)	Gap	Z-Best	CPU (sec)	Gap
Cr30 × 5a-1	819.51	819.53	993.30	0.00%	819.51	2.45	0.00%	819.52	3.23	0.00%	819.51	1.00	0.00%
Cr30 × 5a-2	821.50	821.50	10806.50	0.00%	821.50	3.72	0.00%	821.50	8.77	0.00%	822.01	1.00	0.07%
Cr30 × 5a-3	702.30	702.29	917.90	0.00%	702.30	0.50	0.00%	702.30	0.91	0.00%	702.29	1.00	0.00%
Cr30 × 5b-1	880.02	880.02	6420.60	0.00%	880.02	4.57	0.00%	880.02	9.05	0.00%	880.03	1.00	0.00%
Cr30 × 5b-2	825.32	825.32	33.20	0.00%	825.32	1.24	0.00%	825.32	2.55	0.00%	825.32	1.00	0.00%
Cr30 × 5b-3	884.60	884.62	41.70	0.00%	884.60	1.23	0.00%	884.60	3.25	0.00%	884.58	1.00	0.00%
Cr40 × 5a-1	928.10	928.11	10882.80	0.00%	928.10	14.67	0.00%	928.10	140.31	0.00%	929.58	1.00	0.16%
Cr40 × 5a-2	888.42	888.42	11052.90	0.00%	888.42	11.88	0.00%	888.42	86.31	0.00%	888.78	1.00	0.04%
Cr40 × 5a-3	947.30	947.30	10862.00	0.00%	947.30	11.36	0.00%	947.30	76.63	0.00%	949.47	1.00	0.23%
Cr40 × 5b-1	1052.04	1052.07	8084.60	0.00%	1052.04	10.49	0.00%	1052.04	3115.92	0.00%	1052.04	1.00	0.00%
Cr40 × 5b-2	981.54	981.52	862.50	0.00%	981.54	3.77	0.00%	981.54	7.61	0.00%	981.54	1.00	0.00%
Cr40 × 5b-3	964.33	964.32	963.00	0.00%	964.33	2.68	0.00%	964.33	12.33	0.00%	964.33	1.00	0.00%
Average	891.25	891.25	5160.08	0.00%	891.25	5.71	0.00%	891.25	288.91	0.00%	891.62	1.00	0.04%
Number of BKS			12			12			12		8		

Table 5.9. Results of BR-VNS for Akca's set

5.5.2.3 Performance of TSB RH vs performance of the BR-VNS

In this section, we compare the performance of TSB RH and BR-VNS in terms of best solution, opening cost, distance cost, vehicle cost and average cost. Moreover, we compare their computational time and the percentage gap regarding to the BKS.

Tables 5.10, 5.11, and 5.12, illustrate the comparison between the performance of TSB RH and the performance of BR-VNS for Barreto, Akca, and Prodhon's benchmarks, respectively. The first and second columns in each table show the instance names and the BKS values. The following columns show the average cost obtained using each solution method for different runs (Average Total Cost), the best solution found (BS Total Cost), its associated opening cost (BS Opening Cost), distance-based cost (BS Distance), and (BS Vehicles). This last value corresponds to the number of vehicles in Barreto's and Akca's sets, while it represents the vehicle cost in Prodhon's set. The following columns are: (BS GAP) for the percentage gap of BS values with respect to the BKS, and (BS CPU (sec)) for computational times in seconds. Whenever our best-found solution matches the BKS in the literature, the corresponding value has been indicated in bold.

Considering BS Total Cost columns in Table 5.10, two general trends for small and large instances can be observed. In smaller instances with up to 36 customers, TSB RH tend to perform as well as BR-VNS, and it can even match BKS with the only exception being Gaskell-32x5, where the gap is 3.66%. In larger instances with more than 36 customers, BR-VNS is more effective.

TSB RH has a considerably good performance on Barreto's set compared to the other two benchmarks and can match, employing just a few seconds, 7 out of 17 BKS results in the smallest instances. BR-VNS can also obtain the BKS result in 5 out of 17 instances. In larger instances with more than 36 customers BR-VNS obtains better results compared to TSB RH, although the former also requires larger computational times. However, there are two exceptions: in Christ-75x10 and PERL83-85x7 instances, TSB RH outperforms BRVNS.

In general, the average computational time employed by TSB RH is just of 3.88 seconds, while its average gap is 1.59%. Meanwhile, BR-VNS shows an average computational time of 99.8 seconds, but in that time, it is able to reduce the average gap with respect to the BKS down to 0.74%.

BR-VNS outperforms TSB RH in most of Prodhon's set (Table 5.11). There are two exceptions: coord20x5-1b and coord20x5-2b, where both BR-VNS and TSB RH can match the BKS. In smaller instances with up to 50 customers and 5 depots (e.g., instance coord50x5-2), BR-VNS performs very well and can usually match the BKS result. In general, as expected, the more elaborate VNS

metaheuristic framework tends to outperform the simpler multi-start heuristic, although the latter is able to offer quite competitive solutions in a matter of seconds.

The results in Table 5.12 confirm our previous observation in Barreto's and Prodhon's set about the direct effect of the instance size on performance of both TSBRH and BR-VNS. As expected in larger instances, BR-VNS obtains better solutions in terms of best-found total cost and average total cost. BR-VNS can effectively solve Akca's set and match BKS in 8 out of 12 instances, whereas TSBRH can reach only 3 out of 12 BKS results. Therefore, BR-VNS performs better than TSBRH in terms of both solution quality and computational time.

Figure 5.10 presents a multiple boxplot comparison of the proposed TSBRH and BR-VNS for the LRPMD, for three benchmark sets, in terms of percentage gap. In a nutshell, the results on the LRPMD are promising enough as to extend the suggested solution methods (TSBRH and BR-VNS) to the G-LRPMD.

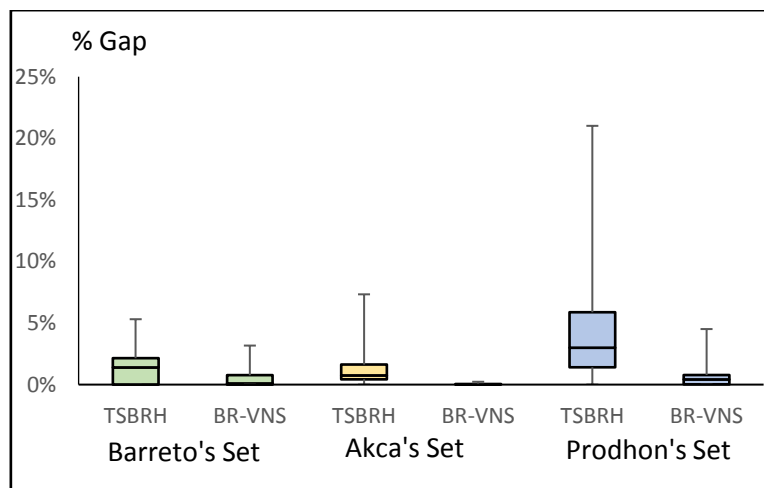


Figure 5.10. The average %gap of the BS Total Cost wrt BKS for LRPMD

Instance Name	BKS	BS		BS Opening Cost		BS Distance		BS Vehicles		Average Costs		BS GAP		BS CPU (sec)	
		TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS
Perl83-12x2	203.98	203.98	203.98	100	100	103.98	103.98	2	2	203.98	203.98	0.00%	0.00%	2.00	1.00
Gaskell-21x5	424.90	424.90	424.90	100	100	324.90	324.90	4	4	424.90	427.73	0.00%	0.00%	2.00	2.10
Gaskell-22x5	585.11	585.11	586.70	50	50	535.11	536.70	3	3	585.11	589.85	0.00%	0.27%	2.00	1.10
Min-27x5	3062.02	3062.02	3062.02	544	544	2518.02	2518.02	4	4	3064.00	3062.34	0.00%	0.00%	2.00	1.10
Gaskell-29x5	512.10	512.10	512.10	100	100	412.10	412.10	4	4	512.10	512.10	0.00%	0.00%	2.00	1.20
Gaskell-32x5	562.22	582.78	562.28	100	50	482.78	512.28	5	4	586.80	564.45	3.66%	0.01%	2.00	4.30
Gaskell-32x5-2	504.33	504.33	504.77	50	50	454.62	454.77	3	3	504.40	508.37	0.00%	0.09%	2.00	5.20
Gaskell-36x5	460.37	460.37	473.66	50	50	410.37	423.66	4	4	463.90	482.86	0.00%	2.89%	2.00	2.60
Christ-50x5	565.62	577.41	565.62	80	80	497.41	485.62	5	5	584.80	577.32	2.08%	0.18%	2.00	27.90
Christ-50x5-B	565.60	573.45	565.60	80	80	493.45	485.60	6	6	576.90	584.66	1.39%	0.00%	2.00	21.80
Perl83-55x15	1112.06	1129.53	1119.09	720	720	409.53	399.09	11	10	1132.00	1125.41	1.57%	0.63%	5.10	75.90
Christ-75x10	844.40	860.98	871.13	120	120	740.98	751.13	12	11	870.50	878.23	1.96%	3.17%	4.10	88.90
Perl83-85x7	1622.50	1634.58	1634.78	1116	1116	518.58	518.78	12	11	1641.00	1643.64	0.74%	0.76%	6.20	40.10
Daskin95-88x8	355.78	373.14	356.04	104	83	268.74	273.04	6	6	376.10	361.45	4.88%	0.07%	2.00	158.70
Christ-100x10	833.43	860.98	849.74	80	80	780.98	769.74	8	8	877.50	857.15	3.31%	1.96%	7.70	355.30
Min92-134x8	5709.00	6012.08	5839.40	1072	804	4940.08	5035.40	10	11	6099.00	5938.31	5.31%	2.28%	10.00	269.40
Daskin95-150x10	43919.90	44858.69	44005.75	15000	15000.0	29858.69	29005.75	11	11	45483.00	44566.97	2.14%	0.20%	10.80	641.30
Average	3637.84	3718.61	3655.15	1145.1	1125.1	2573.55	2530.03	6.47	6.29	3763.88	3699.11	1.59%	0.74%	3.88	99.88

Table 5.10. Results of TSBRH vs BR-VNS for Baretto's set

Instance Name	BKS	OBS		OBS Opening Cost		OBS Distance		OBS Vehicles		Average Costs		OBS GAP		OBS CPU (sec)	
		TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS
coord20x5-1	54793	55089	54793	25549	25549	24540	24244	5000	5000	55114.3	54853.5	0.54%	0.00%	2.0	2.1
coord20x5-1b	39104	39104	39104	15497	15497	20607	20607	3000	3000	39180.1	39118.9	0.00%	0.00%	2.0	4.4
coord20x5-2	48908	50177	48908	24196	24196	20981	19712	5000	5000	51126.5	48908.0	2.59%	0.00%	2.0	1.0
coord20x5-2b	37542	37542	37542	13911	13911	20631	20631	3000	3000	37542.0	37542.0	0.00%	0.00%	2.0	1.3
coord50x5-1	90111	91425	90402	25442	25442	52983	52960	13000	12000	91662.6	90825.2	1.46%	0.32%	2.0	1.5
coord50x5-1b	63242	64974	63242	15385	15385	43589	41857	6000	6000	64996.9	63647.9	2.74%	0.00%	2.0	3.1
coord50x5-2	88298	90007	88298	32714	29319	45293	46979	12000	12000	90148.8	89731.7	1.94%	0.00%	2.0	11.6
coord50x5-2b	67308	71321	67853	29319	29319	35002	32534	7000	6000	71632.7	68336.3	5.96%	0.81%	2.0	23.5
coord50x5-2BIS	84055	85343	84401	19785	19785	53558	52616	12000	12000	86132.9	85131.1	1.53%	0.41%	2.0	24.6
coord50x5-2bBIS	51822	55414	51883	18763	18763	30651	27120	6000	6000	56059.0	52253.6	6.93%	0.12%	2.0	32.7
coord50x5-3	86203	90602	86223	27295	18961	52307	55262	11000	12000	91734.7	86945.1	5.10%	0.02%	2.0	10.7
coord50x5-3b	61830	65145	61844	18590	18961	40555	36883	6000	6000	65301.3	62069.6	5.36%	0.02%	2.0	14.1
coord100x5-1	274814	279264	277003	132890	132890	122374	120113	24000	24000	280528.6	278220.0	1.62%	0.80%	2.0	136.9
coord100x5-1b	213615	216576	215702	132890	132890	71686	71812	12000	11000	217223.2	216868.3	1.39%	0.98%	2.0	73.1
coord100x5-2	193671	195980	194690	102246	102246	69734	68444	24000	24000	196291.6	196373.6	1.19%	0.53%	2.0	60.2
coord100x5-2b	157095	158862	157275	102246	102246	44616	44029	12000	11000	159180.0	157779.1	1.12%	0.11%	2.0	71.2
coord100x5-3	200079	202223	201299	88287	88287	89936	89012	24000	24000	202970.4	201889.1	1.07%	0.61%	2.0	139.1
coord100x5-3b	152441	154421	152466	88287	88287	55134	53179	11000	11000	154988.0	153596.3	1.30%	0.02%	2.0	106.4
coord100x10-1	287695	329928	294625	218910	165068	86018	103557	25000	26000	330413.0	299503.1	14.68%	2.41%	2.0	80.2
coord100x10-1b	230989	279514	241396	208784	158385	58730	71011	12000	12000	280335.9	243836.7	21.01%	4.51%	2.0	94.4
coord100x10-2	243590	261783	244614	150770	145956	86013	75658	25000	23000	263099.1	245505.2	7.47%	0.42%	3.2	288.0
coord100x10-2b	203988	220639	205019	150770	145956	57869	48063	12000	11000	216890.5	205916.8	8.16%	0.51%	3.0	315.0
coord100x10-3	250882	269466	254667	152779	139411	91687	91256	25000	24000	270843.1	256019.7	7.41%	1.51%	3.0	278.5
coord100x10-3b	204317	220269	205746	152779	139411	56490	54335	11000	12000	221102.8	207264.0	7.81%	0.70%	3.0	201.3
coord200x10-1	475294	501614	480267	266151	253840	188463	179427	47000	47000	502331.6	483794.4	5.54%	1.05%	15.0	999.6
coord200x10-1b	377043	394147	379725	253840	253840	116307	103885	24000	22000	396606.8	381698.8	4.54%	0.71%	13.9	613.4
coord200x10-2	449006	458803	450871	280370	280370	130433	123501	48000	47000	460404.5	453413.6	2.18%	0.42%	8.1	412.9
coord200x10-2b	374280	395363	374720	286199	280370	87164	72350	22000	22000	395571.6	375750.5	5.63%	0.12%	7.5	1094.0
coord200x10-3	469433	484669	473532	272528	272528	165141	155004	47000	46000	485020.5	476192.5	3.25%	0.87%	18.9	586.6
coord200x10-3b	362653	375890	364920	234660	234660	119230	108260	22000	22000	376613.4	368464.1	3.65%	0.63%	19.3	1271.1
AVERAGE	196470	206518.47	198101	118061.07	112390.97	71257.4	68810.033	17200	16900	207034.88	199381.623	4.44%	0.62%	4.5	231.8

Table 5.11. Results of TSBRH vs BR-VNS for Prodhon's set

Instance Name	BKS	OBS		OBS Opening Cost		OBS Distance		OBS Vehicles		Average Costs		OBS GAP		OBS CPU (sec)	
		TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS	TSBRH	BR-VNS
cr30x5a-1	819.51	837.86	819.51	200	200	637.86	619.51	5	5	842.34	829.32	2.24%	0.00%	2	1
cr30x5a-2	821.45	881.65	822.01	200	200	681.65	622.01	5	5	883.47	825.78	7.33%	0.07%	2	1
cr30x5a-3	702.29	707.97	702.29	200	200	507.97	502.29	6	6	707.97	706.81	0.81%	0.00%	2	1
cr30x5b-1	880.02	885.08	880.03	200	200	685.08	680.03	5	5	886.84	890.21	0.57%	0.00%	2	1
cr30x5b-2	825.32	825.32	825.32	200	200	625.32	625.32	7	7	825.35	825.32	0.00%	0.00%	2	1
cr30x5b-3	884.58	884.58	884.58	200	200	684.58	684.58	7	7	884.58	884.58	0.00%	0.00%	2	1
cr40x5a-1	928.10	933.49	929.58	200	200	734.61	729.58	7	6	939.26	936.42	0.58%	0.16%	2	1
cr40x5a-2	888.42	899.11	888.78	200	200	699.11	688.78	7	7	901.60	890.80	1.20%	0.04%	2	1
cr40x5a-3	947.26	963.55	949.47	200	200	763.55	749.47	7	6	963.60	953.18	1.72%	0.23%	2	1
cr40x5b-1	1052.04	1059.17	1052.04	200	200	859.39	852.04	9	8	1062.96	1057.87	0.68%	0.00%	2	1
cr40x5b-2	981.54	981.54	981.54	200	200	781.54	781.54	8	8	993.03	989.52	0.00%	0.00%	2	1
cr40x5b-3	964.33	979.80	964.33	200	200	779.04	764.33	8	8	982.47	978.21	1.60%	0.00%	2	1
Average	891.24	903.26	891.62	200.00	200.00	703.31	691.62	6.75	6.50	906.12	897.33	1.39%	0.04%	2	1

Table 5.12. Results of TSBRH vs BR-VNS for Akca's set

5.6 Conclusion

In this chapter we present two solution methods for the LRPMD. The LRPMD has many real-life important applications such as, newspaper distribution, military applications and bill delivery.

A description of the Biased Randomised heuristic and the Biased Randomised metaheuristic was given. The Biased Randomised heuristic (TSBRH) consists of two stages to solve the LRPMD. The first stage is based on combining location decision with a classic constrictive heuristic (ECWH), to find an initial solution. The second stage is based on TLBRH which in Chapter 4, to improve the initial solution which is obtained from the first stage. As we described in Chapter 4, the second stage consists of two levels: Global Level and Local Level. The Global Level reallocates customers to depots by solving the whole MDVRP by using BR-ECHW, while the Local Level improves routing by applying the BR-CWH, which is described in Chapter 3, for each depot with its customers individually. The Biased Randomised technique has been used to help heuristics to escape from local minima and explore different regions of the search space.

The Biased Randomised metaheuristic is developed based on the VNS. The initial solution is generated by employing the Biased Randomised technique to allocate customers to depots; after allocating customers to depots, the BR-CWH is used to solve the routing problem. Then VNS metaheuristic improves the best initial solution.

To evaluate the performance of algorithms, computational experiments are carried out for three data sets with problem sizes ranging from 12 to 200 customers, and number of depots from 2 to 15. The results obtained so far indicate that this proposed heuristic is suitable to solve the LRPMD in terms of the solution quality and computational time.

There are some limitations for the proposed method. Firstly, the TSBRH is a heuristic and it appears to get stuck in local optima. Secondly, when the number of customers increase, the gap become larger, which means its performance deteriorates.

Future study should address two directions for improvement. The first direction is to improve the quality of solution by adding a local search. The second direction is to extend the LRPMD and propose novel mathematical models to consider more realistic problems such as the environmental issues with the Green LRPMD, described in Chapter 6.

Finally, the practicality and simplicity of our solution method, is notable when compared to complex methods in the literature with exhaustive fine-tuning procedures. Therefore, our method can be easily integrated into supply chain management.

Chapter 6 The Green Location Routing Problem with Multi-Depot (G-LRPMD)

6.1 Introduction

Transportation is one of the main activities in supply chain management. That is because it has a huge impact on customer satisfaction, and it also plays an important role in the generation of CO₂ and greenhouse-gas (GHG) emissions, and related externalities such as air pollution, noise, and traffic congestion (Juan et al, 2016). Road transportation alone is responsible for about 18% of total GHG emissions in the EU (Hill et al, 2011). Moreover, higher percentages of CO₂ emissions have been reported in other parts of the world, such as Asia and the Pacific region (United Nations, 2011), and the United States of America (United States Environmental Protection Agency, 2014). Therefore, it becomes necessary to consider more ecological power sources for fueling vehicles in transport.

Internal-Combustion-Engine Vehicles (ICEVs) consume oil and produce a higher percentage of CO₂, greenhouse emissions, and other pollutant effects. It is obvious that a shift from a fossil fuel fleet to an electric-powered fleet is necessary to reduce pollutant emissions in cities. Also, by introducing special taxes, governments are approving policies aimed at decreasing the pollution level generated by transportation. Therefore, from both an environmental and energy stand point,

the use of Electric Vehicles (EVs) should be one of the first priorities for the reduction of primary energy consumption.

In fact, EVs are considered as the next big step in the automobile industry and there is an increasing interest in the use of them. The introduction of the EVs in modern fleets facilitates a shift towards greener road transportation. Furthermore, governments are making noticeable efforts to promote the use of green technologies, such as EVs (Mattila and Antikainen, 2011). However, there are some operational barriers of using the EVs in transportation. Ferreira et al (2011) state that the EVs continue to have a limited autonomy associated with the long charging times, limited charging stations and undeveloped smart grid infrastructures demands. Similar arguments can be found in Achtnicht et al (2012), Wirasingha et al (2008), and Chan et al (2009).

But the main current challenge of using the EVs is the limited driving ranges because of the duration of their batteries. This issue is recognised by different authors as a major challenge (Juan et al, 2016). The ISOE institute, report that the reduced range will remain the main issue concerning electric mobility. According to experts this is not likely to change considerably in the medium term (Institute for Social-Ecological Research, 2017). Electric vehicles with different battery sizes give rise to problems whereby each vehicle will have its own driving range, which needs to be accounted for in route-planning. With EVs becoming more prevalent, an efficient routing of heterogeneous fleets with multiple driving-range vehicles is emerging as a new issue in the transportation industry.

Therefore, we address the LRPMD with constrained distance, which is used to show the effects of the inclusion of EVs during integrated location and routing decisions. In this chapter, we will discuss the Green Location Routing Problem with Multi-Depot (G-LRPMD), which is a natural extension of the LRPMD when EVs are utilised. Also, we develop a Green Two-Stage Biased Randomised Heuristic (G-TSBRH) and Green Biased Randomised VNS, to solve the G-LRPMD. The computational experiments show that the heuristic generates good quality solutions in very reasonable computation time, and the G-BRVNS provides better solutions.

This chapter is organized as follows. The main contribution of the chapter is covered in section 6.2. The G-LRPMD definition and the mathematical model is given in section 6.3. Section 6.4 and 6.5 outline the basis of G-TSBRH and G-BRVNS, respectively. Section 6.6 presents the computational experiments and the analysis of results. In section 6.7, we investigate the effect the distance constraint has on solutions of LRPMD, by comparing results of LRPMD with the results of the G-LRPMD. Finally, section 6.8 concludes the chapter and discusses future research.

6.2 Contribution

Due to the increase in CO₂ and GHG from using ICEVs, there is more interest in the use of green technologies, such as EVs. However, the driving ranges of EVs are limited by the duration of their batteries, which causes new operational challenges. We introduce an ecological fleet of EVs in the LRPMD instead of ICEVs, in order to comply with the shift towards greener road transportation and protecting the environment.

In this chapter, we discuss the Green Location Routing Problem with Multi-Depot (G-LRPMD), which is a natural extension of the LRPMD when EVs are utilised. To tackle the limitation of the driving ranges of EVs, we introduce a new constraint to ensure that EVs will not exceed their driving range. To the best of our knowledge, there is no research reported in the literature that integrates the LRPMD with EVs other than the mentioned collaboration. Therefore, taking environmental issues into account when solving LRPMD, by formulating the mathematical model with a distance constraint, is one of main contributions of this chapter.

The second contribution in this chapter is developing the G-TSBRH to solve this problem. Our proposed method is based on the BR-ECWH for the MDVRP, similar to the solution method in Chapter 5. Also, we proposed the G-BRVNS to solve the G-LRPMD based on the BR-VNS. However, a distance constraint has been added to the model to consider the usage of EVs.

As this problem is quite new in the literature, in order to validate the performance of the proposed approach, three benchmark data sets were generated for G-LRPMD by adapting the classic benchmark instances of the LRPMD. This can be counted as the third contribution of this chapter.

In summary, three main contributions in this chapter are in line with developing a modified mathematical model and novel solution methods, and adapting three classical benchmark data sets for G-LRPMD.

6.3 Optimisation model

G-LRPMD considers the LRPMD with distance constraint. The problem is to determine the number and locations of depots, assignment of customers to open depots, and the corresponding delivery routes, so that the total costs consisting of depot-establishing cost, transportation cost, and

dispatching cost for vehicles are minimised. Each vehicle takes exactly one route starting from the depot, visiting a subset of the customers and returning to the depot. In addition, customer demand cannot be split among different routes and the sum of demands in each route must not exceed the vehicle capacity D . For each vehicle, there is a limited driving range \bar{d} which must be respected. Furthermore, total demand of customers assigned to one depot must not exceed its capacity Q_i .

In general, the G-LRPMD can be defined on a complete, weighted, and undirected network $G = (V, E, C)$, where $V = \{1, \dots, n\}$ is a set of nodes (representing the depots, customers), and E is a set of undirected edges (i, j) , and $C = (c_{ij})$ is the matrix of traveling cost associated with the edges E . The cost is symmetric, i.e. $c_{ij} = c_{ji}$, and it satisfies the triangular inequality $c_{ij} \leq c_{iu} + c_{uj}$.

It is assumed that $I \subseteq V$ be a set of potential depots and $J \subseteq V$ be a set of customers. A capacity Q_i and an opening cost f_i are associated with each depot site $i \in I$. A set K of identical vehicles of capacity D is available. When used, each vehicle incurs a fixed cost F and performs a single route. Each customer $j \in J$ has a demand d_j whereby $d_j \leq D$. Since $d_j \leq D$, there will never be a need for a node (customer) to be visited by more than one vehicle to satisfy its demand. This means that split delivery is not allowed. Figures 6.1 illustrates an example of G-LRPMD compared to the LRPMD. Firstly, in Figure 6.1 (a), there are three depots selected to be opened. Then, in Figure 6.1 (b), customers are assigned to opened depots. Finally, vehicle routes are computed without distance constraints in figure 6.1 (c). Lastly, Figure 6.1 (d) illustrates how the routing plan might be significantly altered due to the introduction of driving range constraints.

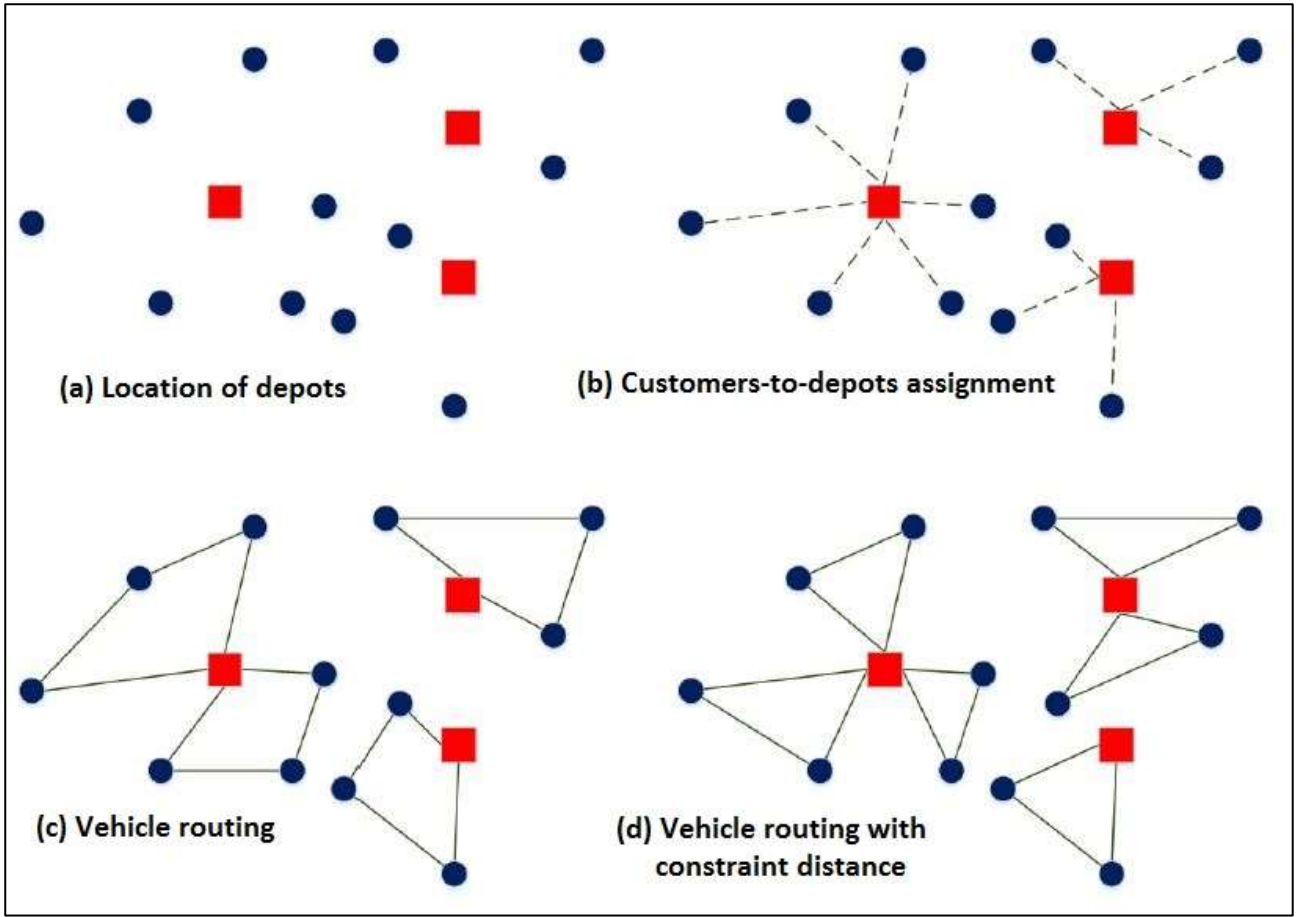


Figure 6.1. An illustrative example of Green LRP

The optimisation model is formulated as a mixed integer linear programming problem and it is inspired by Prins et al., (2007). We have modified the mathematical model in terms of adding the constrained distance to consider the environmental issues when solving the LRPM. In order to formulate the model, the following notation is introduced.

Sets are defined as follows:

V : Set of nodes, $V = I \cup J$

I : Set of potential depot nodes

J : Set of customers to be serviced

K : Number of available vehicles (fleet size)

Parameters are defined as follows:

f_i : The fixed cost of opening a depot at i

Q_i : Capacity of depot i

d_j : Demand of customer j

D : Capacity of each vehicle

F : Fixed cost per vehicle used

c_{ij} : Traveling cost for edge (i, j)

\bar{d} : The maximum distance allowed for each vehicle

Decision variables are defined as follows:

$$x_{ijk} : \begin{cases} 1, & \text{if vehicle } k \text{ is used on route from node } i \text{ to node } j \\ 0, & \text{otherwise} \end{cases}$$

$$y_i : \begin{cases} 1, & \text{if a depot is located at site } i \\ 0, & \text{otherwise} \end{cases}$$

$$z_{ij} : \begin{cases} 1, & \text{if customer } j \text{ is served from depot } i \\ 0, & \text{otherwise} \end{cases}$$

The G-LRPMD formulation is as follows:

$$\text{Min } \sum_{i \in I} f_i y_i + \sum_{i \in V} \sum_{j \in V} \sum_{k \in K} c_{ij} x_{ijk} + \sum_{i \in I} \sum_{j \in J} \sum_{k \in K} F x_{ijk} \quad (6.1)$$

Subject to

$$\sum_{k \in K} \sum_{i \in V} x_{ijk} = 1 \quad \forall j \in J \quad (6.2)$$

$$\sum_{i \in I} \sum_{j \in J} x_{ijk} \leq 1 \quad \forall k \in K \quad (6.3)$$

$$\sum_{j \in V} x_{ijk} - \sum_{j \in V} x_{jik} = 0 \quad \forall k \in K, \quad \forall i \in V \quad (6.4)$$

$$\sum_{u \in J} x_{iuk} + \sum_{u \in V \setminus \{J\}} x_{ujk} \leq 1 + z_{ij} \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K \quad (6.5)$$

$$\sum_{i \in V} \sum_{j \in J} d_j x_{ijk} \leq D \quad \forall k \in K \quad (6.6)$$

$$\sum_{j \in J} d_j z_{ij} \leq Q_i y_i \quad \forall i \in I \quad (6.7)$$

$$\sum_{i \in V} \sum_{j \in V} c_{ij} x_{ijk} \leq \bar{d} \quad \forall k \in K \quad (6.8)$$

$$x_{ijk} \in \{0, 1\} \quad \forall i \in I, \quad \forall j \in J, \quad \forall k \in K \quad (6.9)$$

$$y_i \in \{0, 1\} \quad \forall i \in I \quad (6.10)$$

$$z_{ij} \in \{0, 1\} \quad \forall i \in I, \quad \forall j \in V, \quad (6.11)$$

$$c_{ij} = \infty \quad \text{when } i = j$$

The objective function (6.1) seeks to minimise the total cost, which includes the fixed cost of the selected facilities and the fixed and variable cost of the vehicles. Constraints (6.2) are the routing constraints that impose that each customer has to be visited exactly once by a single vehicle, whereas constraints (6.3) ensure that all routes have to start and end at a depot. Constraints (6.4) are the connectivity constraints to ensure that every vehicle leaves the customer after he has been served. Constraints (6.5) specify that a customer can be assigned to a depot only if a route linking them is opened. Constraints (6.6) and (6.7) impose both the capacity of vehicle and capacity of

depot. Constraints (6.8) guarantee that the route length of each vehicle does not exceed the maximum distance constraint. Constraints (6.9), (6.10), and (6.11) determine integer variables.

6.4 Green Two-Stage Biased Randomised Heuristic for G-LRPMD

Due to the efficiency of the TSB RH proposed and implemented in Chapter 5, we modify it to incorporate the new extension of the LRP with distance constraints – Green Two-Stage Biased Randomised Heuristic (G-TSB RH). Although the general framework of the G-TSB RH is similar to TSB RH, the algorithm needs to be modified to take into account the distance constraints of EVs. In the following, we show the details of modifications and how TSB RH is adapted to G-LRPMD.

In the first stage, some depots are selected to be opened among the list of potential candidates. The main factor in determining the number of opened depots is the total demand of all customers. Therefore, there is no effect of distance constraints on the number of depots. However, the location of opened depots is affected by adding distance constraints, which means the cost of opened depots can increase.

Once the location decision is made by selecting the depots to be opened, the LRPMD is reduced to MDVRP. This means we can apply the ECWH (explained in Chapter 4) proposed by Tillman (1969) to allocate customers to opened depots and find an initial solution. The distance constraints do affect the solution here by increasing the number of vehicles, and total distance. This is due to the fact that the EVs cannot go as far as traditional vehicles due to constrained distances. This stage is repeated for different combinations of depots, looking for the best configuration of depots with the minimum routing cost.

Then, the best solutions found during the first stage are improved throughout the second stage which includes two levels namely, the Global Level and Local Level. In the Global Level, the BR-ECWH, which is proposed and implemented in Chapter 5, is applied for the initial solution resulting from the first stage. Then, in the Local Level, the BR-CWH, which is proposed by Juan et al (2011a) and described in Chapter 3, is employed for each depot, to improve the routing allocated to that depot. In these two levels, again, there will be an effect of distance constraint on the solution by increasing the number of used vehicles, and total distance.

6.5 Green Biased Randomised Variable Neighbourhood Search (BR-VNS)

The BR-VNS, which is proposed and implemented in Chapter 5, shows that it is able to solve the LRPMD efficiently. Therefore, we modify it to incorporate the G-LRPMD, and we called it Green Biased Randomised VNS (G-BRVNS). Although the general framework of the G-BRVNS is similar to BR-VNS, the algorithm needs to be modified to take into account the distance constraints of EVs. In the following, we show the effect of distance constraints on the solution obtained by G-BRVNS.

In the initial solution, some depots are selected to be opened among the list of potential candidates, and customers are allocated to these opened depots. The main factor in determining the number of opened depots is the total demand of all customers. Therefore, there is no effect of distance constraints on the number of depots. However, location of depots is changed after adding the distance constraint.

Once the location decision is made by selecting the depots to be opened, the LRPMD is reduced to MDVRP. Therefore, routing is solved by applying BR-CWH to find the initial solution. The distance constraints do affect the solution by an increased number of vehicles, and total distance. This stage is repeated for different combinations of depots looking for the best configuration of depots with the minimum routing cost.

Then, the best solutions found during the first stage are improved throughout the VNS. Here again, there will be an effect of distance constraints on the solution by an increased number of vehicles, and total distance.

6.6 Computational experiments

Computational experiments have been conducted to evaluate the performance of G-TSBRH and G-BRVNS. The G-TSBRH and BR-VNS were coded by using Java applications. Computational experiments have been performed using a 2.3 Ghz Quad-Core AMD Opteron(tm) processor with 8 GB of RAM and running under CentOS release 6.6.

6.6.1 Data sets and experimental setting

Since there are no specific benchmarks for the G-LRPMD, to the best of our knowledge, we have generated new instances based on the LRPMD ones that have been used in chapter 5. In order to facilitate the comparison, we have adapted all instances from Barret's, Prodhon's, and Akca's sets by means of the following procedure:

- (i) we select a random sample of 10 instances from each benchmark set
- (ii) each of these instances is solved with the BR-VNS metaheuristic as LRP
- (iii) route distances from each solution are sorted according to their distances
- (iv) we select for each set the route length corresponding to the 3rd quartile of them
- (v) this value is rounded to the nearest multiple of 10

It is to note that, at this point we could have selected any other value (instead of the distance of the 3rd quartile) to generate the G-LRPMD instances. As a result, distance is constrained with values of 5500 and 130 for Prodhon's and Akca's sets, respectively. Regarding Barreto's set, we could not get a single value for the constrained distance. Therefore, distance constraint values are 700 for both Min-27x5 and Min92-134x8 instances, 3960 for Daskin95-150x10, and 130 for the remaining instances of this set. It should be mentioned that in the computational experiments, each instance has been run using 10 different random seeds, and the best result is considered as our best solution (BS).

6.6.2 Performance analysis of G-TSBRH and G-BRVNS

In this section, we discuss the results obtained by G-TSBRH and G-BRVNS in order to illustrate the potential of our solution methods. In the computational experiments, each instance has been run using 10 different random seeds. The best result is considered as Best Solution (BS), and the average value of results is considered as Average Total Cost.

Tables 6.1, 6.2, and 6.3 present the results provided by G-TSBRH and G-BRVNS for the G-LRPMD. The following information is gathered in these tables: instance name, BKS, (Average Total Cost) for different runs, best-found solution (BS Total Cost), opening cost (BS Opening Cost), distance-based cost (BS Distance Cost), and (BS Vehicles). This column corresponds to the number of vehicles in Barreto's and Akca's sets, while it reflects the vehicles cost in the Prodhon's set. The last columns correspond to the associated gaps, and computational times in seconds. The

percentage gap between the best solution of G-TSBRH and G-BRVNS is calculated as

$$\left[\left(\frac{BSG-BRVNS - BSG-TSBRH}{BSG-TSBRH} \right) \times 100 \right].$$

We need to emphasise here that our solution methods have considerable benefits and viability in the business context. The G-BRVNS is able to generate a competitive solution in terms of solution quality in a long computational time.

The G-TSBRH is able to generate reasonably good solutions in a matter of seconds, which makes it favorable at operational level when a quick solution is required such as in a communication network. However, the G-BRVNS provides higher-quality solutions by employing more computational time. Moreover, as they can produce many solutions, they can offer the decision maker different scenarios to choose the best solution.

In these tables we notice that computational time increases when the size of instances increases. Size of instances is controlled by two factors, number of potential depots and number of customers.

Also we notice that, in general, the solution quality of the G-BRVNS is better than the solution quality of the G-TSBRH. The average percentage gaps are -4.1%, -5.1%, and -2.88% for Barreto's, Prodhon's and Akca's sets, respectively.

However, the average of the computational times for G-TSBRH is 3.98, 4.6, and 2 sec for the Barreto's set, Prodhon's set, and Akca's set, respectively. The average of the computational times for G-BRVNS is 101.5, 235, and 1 sec for the Barreto's set, Prodhon's set, and Akca's set, respectively. We can claim that G-TSBRH is competitive when a quick solution is required.

Table 6.1 shows results of Barreto' set. We can observe that there are two lines of performance. The first one, the G-BRVNS is faster in smaller instances with up to 29 customers other than one instance, Gaskell-21x5. The second one, the G-TSBRH has considerably lower computational time in larger instances with greater than 29 customers, and outperforms G-BRVNS in 13 out of 17 instances. However, in terms of solution quality, G-BRVNS noticeably outperforms G-TSBRH in terms of solution quality in 14 out of 17 instances. Among the other 3 instances, G-TSBRH can match G-BRVNS in 2 instances, and it outperforms G-BRVNS in 1 instance. This is due to the effectiveness of G-BRVNS in routing decisions. Even when G-TSBRH outperforms G-BRVNS, either higher opening cost or higher vehicle cost make the G-TSBRH overall result worse. Considering the computational times and the average gap of -4.10%, we can claim that G-BRVNS is a viable alternative for strategic decisions, while G-TSBRH is still competitive when a quick solution is required.

For Prodhon's set in table 6.2, the superiority of G-BRVNS in terms of solution quality is apparent and it outperforms G-TSBRH in 24 out of 30 instances. This confirms the type of trend that we have reported on large instances of Barreto's set. Other than 3 relatively smaller instances, G-TSBRH outperforms the G-BRVNS and the computational time of G-BRVNS rapidly increases with the instance sizes and its average computational time goes up rapidly to 235.0, whereas average computational time of the G-TSBRH stays much lower at 4.6. As the average gap is -5.21%, we come to the same decision that G-BRVNS is our choice when a higher quality decision is preferred to a fast but less accurate result provided by G-TSBRH.

Finally, in table 6.3 of Akca's set, results illustrate that G-BRVNS performs considerably better than G-TSBRH in terms of both solution quality and computational time. G-BRVNS outperforms G-TSBRH in 9 out of 12 instances and it matches G-TSBRH result in the other 3 instances. In Akca's set, the number of customers is either 30 or 40 which indicates the instances are not very large. Thus, not surprisingly, similar to smaller instances of Barreto's set, G-BRVNS performs better than G-TSBRH both in terms of solution quality and computational time. G-BRVNS has an average computational time of 1 second and G-TSBRH has a value of 2 seconds. One may argue that the difference of 1 sec between their computational time is negligible. However, with the average gap of -2.88% and little difference in computational time, we can easily recommend G-BRVNS to the decision makers. The Figure 6.2 illustrates the boxplot of comparison of the G-TSBRH and G-BRVNS in term of the average of computational time.

In general, the existing balance between the solution quality and the amount of computational time of the proposed methods, result in their viability in real-life problems. In addition, novelty of the underlying ideas and simplicity of their implementation with regard to re-tuning parameters, when compared to the other approaches in the literature, make the suggested methods even more desirable.

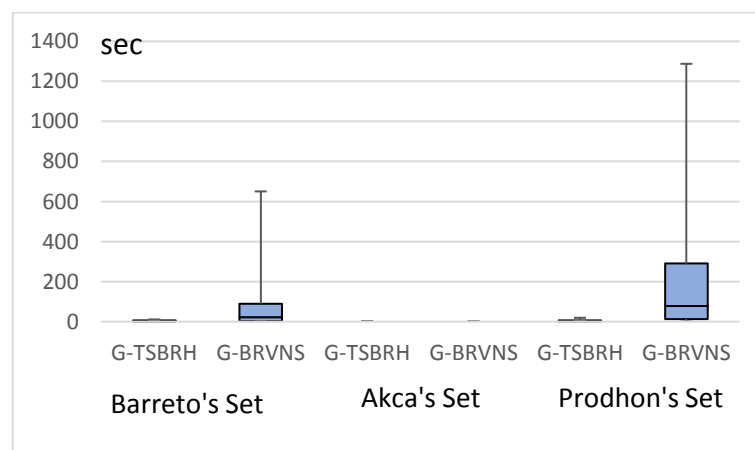


Figure 6.2. The average of the CPU for G-TSBRH and BR-VNS

Instance Name	BS		BS Opening Cost		BS Distance		BS Vehicles		Average Costs		Gap	BS CPU (sec)	
	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS		G-TSBRH	G-BRVNS
Perl-12x2	203.98	203.98	100.0	100.0	103.98	103.98	2	2	203.98	203.98	0.00%	2	1
Gaskell-21x5	465.73	424.90	150.0	100.0	315.73	324.90	5	4	465.73	427.23	-8.77%	2	2.1
Gaskell-22x5	789.74	775.12	150.0	150.0	639.74	625.12	7	7	789.74	789.40	-1.85%	2	1.1
Min-27x5	3770.98	3770.98	816.0	816.0	2954.98	2954.98	6	6	3771.62	3852.04	0.00%	2	1.2
Gaskell-29x5	587.28	529.07	150.0	150.0	437.28	379.07	5	4	587.28	529.23	-9.91%	2	1.3
Gaskell-32x5	681.39	631.43	100.0	100.0	581.39	531.43	6	5	681.39	637.73	-7.33%	2	4.4
Gaskell-32x5-2	680.89	610.74	100.0	100.0	580.89	510.74	6	5	681.29	627.53	-10.30%	2	5.3
Gaskell-36x5	485.42	460.37	100.0	50.0	385.42	410.37	4	4	485.42	463.22	-5.16%	2.1	2.6
Christ-50x5	577.92	565.62	80.0	80.0	497.92	485.62	6	5	581.12	573.36	-2.13%	2.2	33.2
Christ-50x5-B	595.72	570.41	80.0	80.0	515.72	490.41	7	6	598.66	580.84	-4.25%	2.2	22
Perl-55x15	1128.77	1114.32	720.0	720.0	408.77	394.32	11	10	1131.41	1118.73	-1.28%	5.3	76.6
Christ-75x10	860.98	861.88	120.0	120.0	740.98	741.88	12	10	869.75	872.32	0.10%	4.4	89.7
Perl-85x7	1633.93	1628.68	1116.0	1116.0	517.93	512.68	12	12	1643.20	1641.21	-0.32%	6.2	40.4
Daskin-88x8	375.88	355.85	104.4	83.0	271.48	272.85	6	6	378.35	359.71	-5.33%	2	160.1
Christ-100x10	874.35	847.61	80.0	80.0	794.35	767.61	9	8	885.74	856.00	-3.06%	8	360.3
Min-134x8	6283.61	6057.13	1072.0	804.0	5211.61	5253.13	11	11	6307.90	6198.53	-3.60%	10.3	273.2
Daskin-150x10	49506.01	46258.25	15000.0	15000.0	34506.01	31258.25	11	11	50369.91	46950.28	-6.56%	11	650.5
Average	4088.39	3862.73	1178.73	1155.82	2909.66	2706.90	7.41	6.82	4143.09	3922.43	-4.10%	3.98	101.5

Table 6.1. Results obtained using G-TSBRH and G-BRVNS for Baretto's set

Instance Name	BS		BS Opening Cost		BS Distance		BS Vehicles		Average Costs		Gap	BS CPU (sec)	
	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS		G-TSBRH	G-BRVNS
coord20-5-1	55806	55806	25549	25549	24257	24257	6000	6000	55806.0	55806.0	0.00%	2.0	2.2
coord20-5-1b	85036	85036	15497	15497	58539	58539	11000	11000	85036.0	85036.0	0.00%	2.0	4.5
coord20-5-2	51960	49931	24196	22769	21764	22162	6000	5000	51960.0	49931.0	-3.90%	2.0	1.0
coord20-5-2b	54729	54742	21739	21739	26990	27003	6000	6000	54729.0	54742.0	0.02%	2.0	1.3
coord50-5-1	126121	126121	25442	25442	83679	83679	17000	17000	126121.0	126121.0	0.00%	2.0	1.5
coord50-5-1b	121063	120063	25442	25442	79621	79621	16000	15000	121245.0	120562.8	-0.83%	2.0	3.2
coord50-5-2	90094	90132	32714	32714	45380	45418	12000	12000	90149.0	90709.8	0.04%	2.0	11.8
coord50-5-2b	72293	71986	32714	32714	32579	32272	7000	7000	72602.9	72716.9	-0.42%	2.0	23.9
coord50-5-2BIS	192421	191431	19785	19785	145636	144646	27000	27000	192922.7	192069.5	-0.51%	2.0	25.0
coord50-5-2bBIS	113262	71569	19242	19242	80020	43327	14000	9000	113426.6	76185.3	-36.81%	2.0	33.3
coord50-5-3	95981	92883	37954	24492	45027	55391	13000	13000	96231.9	93128.3	-3.23%	2.0	10.9
coord50-5-3b	81009	80081	37954	24492	35055	45589	8000	10000	81812.0	80118.2	-1.15%	2.0	14.4
coord100-5-1	413590	413644	144012	144012	228578	228632	41000	41000	413590.5	414278.5	0.01%	2.0	139.0
coord100-5-1b	396471	394729	144012	144012	216459	215717	36000	35000	397300.4	397300.6	-0.44%	2.3	74.2
coord100-5-2	195892	194604	102246	102246	69464	68358	24000	24000	196161.5	196283.4	-0.66%	2.0	61.1
coord100-5-2b	159192	157847	102246	102246	44946	44601	12000	11000	159645.2	157945.8	-0.84%	2.0	72.3
coord100-5-3	229351	226960	138923	130861	66428	72099	24000	24000	229505.1	228288.5	-1.04%	2.0	141.2
coord100-5-3b	195776	190568	138923	130861	44853	47707	12000	12000	196112.2	191295.0	-2.66%	2.1	108.0
coord100-10-1	334947	318797	226818	202285	82129	90512	26000	26000	335598.9	321346.8	-4.82%	9.9	81.4
coord100-10-1b	295558	277843	226818	202285	55740	61558	13000	14000	296290.0	279505.2	-5.99%	9.0	95.8
coord100-10-2	284612	245110	149586	145956	106026	76154	29000	23000	285053.8	246299.2	-13.88%	3.0	292.4
coord100-10-2b	231583	205412	154095	145956	63488	48456	14000	11000	233555.6	206716.8	-11.30%	3.0	319.8
coord100-10-3	264096	257490	159669	144699	79427	87791	25000	25000	264096.0	258950.4	-2.50%	3.0	282.7
coord100-10-3b	224505	216189	159669	149491	52836	54698	12000	12000	224546.6	216840.1	-3.70%	3.0	204.4
coord200-10-1	584228	544400	311992	266151	216236	222249	56000	56000	584658.2	547674.2	-6.82%	11.4	1012.8
coord200-10-1b	504941	464043	311992	266151	160949	164892	32000	33000	505786.6	469821.1	-8.10%	10.2	621.5
coord200-10-2	502149	450645	280370	280370	167779	123275	54000	47000	503461.8	452903.8	-10.26%	6.5	418.4
coord200-10-2b	432691	374938	280370	280370	122321	72568	30000	22000	433127.9	375985.8	-13.35%	7.7	1108.4
coord200-10-3	531794	472853	317158	272528	167636	154325	47000	46000	533042.4	480580.5	-11.08%	16.5	594.3
coord200-10-3b	439069	385611	317158	272528	98911	91083	23000	22000	445778.8	391509.2	-12.18%	19.1	1287.8
Average	245340.67	229382.13	132809.5	122429.5	90758.4	86219.3	21766.7	20733.3	245978.5	231021.7	-5.21%	4.6	235.0

Table 6.2. Results obtained using G-TSBRH and G-BRVNS for Prodhon's set

Instance Name	BS		BS Opening Cost		BS Distance		BS Vehicles		Average Costs		Gap	BS CPU (sec)	
	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS	G-TSBRH	G-BRVNS		G-TSBRH	G-BRVNS
cr30x5a-1	892.40	892.40	200	200	692.40	692.40	7	7	1103.06	894.27	0.00%	2	1
cr30x5a-2	1006.65	915.54	200	200	806.65	715.54	7	7	1006.67	916.02	-9.05%	2	1
cr30x5a-3	707.97	702.29	200	200	507.97	502.29	6	6	707.97	702.51	-0.80%	2	1
cr30x5b-1	1098.63	952.83	200	300	898.63	652.83	8	6	1098.63	952.83	-13.27%	2	1
cr30x5b-2	971.19	922.65	200	300	771.19	622.65	8	7	971.19	928.53	-5.00%	2	1
cr30x5b-3	984.50	984.50	200	200	784.50	784.50	8	8	985.16	984.79	0.00%	2	1
cr40x5a-1	979.78	979.42	200	200	779.78	779.42	7	7	980.80	981.64	-0.04%	2	1
cr40x5a-2	913.25	899.69	200	200	713.25	699.69	8	7	913.50	903.29	-1.48%	2	1
cr40x5a-3	1009.94	985.36	200	200	809.94	785.36	8	7	1010.02	1000.12	-2.43%	2	1
cr40x5b-1	1137.29	1137.29	200	200	937.29	937.29	10	10	1138.65	1137.29	0.00%	2	1
cr40x5b-2	1163.75	1138.52	200	300	963.75	838.52	9	10	1167.78	1144.27	-2.17%	2	1
cr40x5b-3	993.23	989.55	200	200	793.23	789.55	8	9	999.58	994.23	-0.37%	2	1
Average	988.22	958.34	200.00	225.00	788.22	733.34	7.83	7.58	1006.92	961.65	-2.88%	2	1

Table 6.3. Results obtained using G-TSBRH and G-BRVNS for Akca's set

6.7 Comparison analysis of LRPMD and G-LRPMD

In this section, we will investigate the effect of the distance constraints on solutions of LRPMD and G-LRPMD. For both G-TSBRH and G-BRVNS, we compute the average best solution (BS), the average best solution of opening cost (BS Opening Cost), the average best solution of distance (BS Distance), and the average of the best solution of the vehicle cost (BS Vehicle) for the LRPMD and the G-LRPMD.

6.7.1 TSBRH and G-TSBRH

Table 6.4 summarises the average results obtained from TSBRH for the LRPMD and from G-TSBRH for the G-LRPMD. The bold indicates that the LRPMD is smaller than the G-LRPMD in terms of the BS, BS Opening Cost, BS Distance, and BS vehicles. There is only one exception in Akca's set, the distance constraint does not effect the opening cost of depots.

	BS		BS Opening Cost		BS Distance		BS Vehicles	
	LRPMD	G-LRPMD	LRPMD	G-LRPMD	LRPMD	G-LRPMD	LRPMD	G-LRPMD
Barreto	3718.61	4088.39	1145.1	1178.73	2573.55	2909.66	6.47	7.41
Pordhon	2065.18	2453.41	1180.61	1328.10	7125.74	9075.84	17.20	21.77
Akca	903.26	988.22	200.00	200.00	703.31	788.22	6.75	7.83

Table 6.4. Average solutions of TSBRH and G-TSBRH for LRPMD and G-LRPMD

Figures 6.3, 6.4, 6.5, and 6.6 compare the average between BS, BS Opening Cost, BS Distance, and BS Vehicle obtained by TSBRH for the LRPMD and obtained by G-TSBRH for the G-LRPMD.

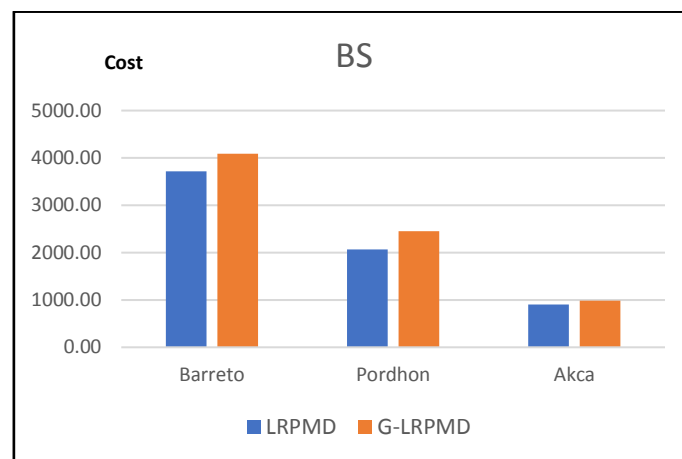


Figure 6.3. Average BS of LRPMD vs G-LRPMD

In Figure 6.3, we can notice that the total cost for the BS of the LRPMD is smaller than the BS of the G-LRPMD, for the three data sets.

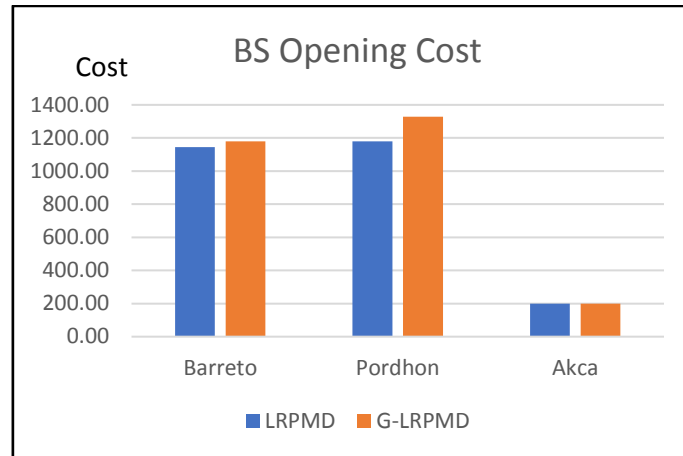


Figure 6.4. Average BS opening cost of LRPMD vs G-LRPMD

Also, in Figure 6.4, we can notice that the cost of opening depots in the LRPMD is smaller than the cost of opening depots in the G-LRPMD in Barreto's and Prodhon's sets. However, Akca's set is not affected by adding the distance constraint.

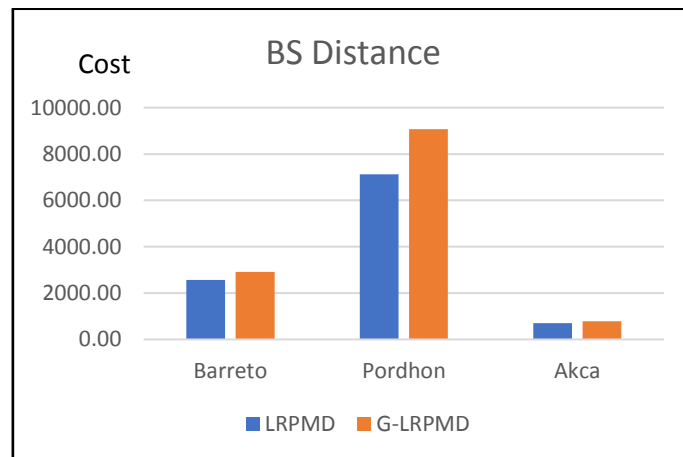


Figure 6.5. Average BS distance of LRPMD vs to G-LRPMD

Finally, in Figure 6.5 and 6.6, it is clear that the BS Distance and the BS vehicles in the LRPMD is smaller than the BS Distance and the BS vehicles in the G-LRPMD for the three data sets.

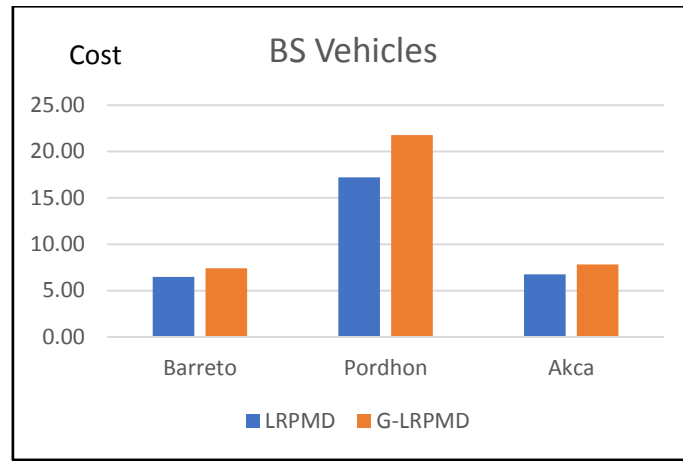


Figure 6.6. Average BS Vehicles of LRPMD vs G-LRPMD

Table 6.8 illustrates the percentage gap of the average of BS, BS Opening Cost, BS Distance Cost, and BS Vehicle between LRPMD and G-LRPMD solution methods for the three data sets. The percentage gap is calculated as $\%gap = \left[\left(\frac{Average\ of\ BS_{LRPMD} - Average\ of\ BS_{G-LRPMD}}{Average\ of\ BS_{LRPMD}} \right) \times 100 \right]$. Similar formula is applied for the BS Opening Cost, BS Distance Cost, and BS Vehicle. The bold indicates that the Prodhon's set has the most increase in the component of cost when compared to Barreto's and Akca's sets.

	BS	BS Opening Cost	BS Distance Cost	BS Vehicles
Barreto	-9.9%	-2.9%	-118.3%	-14.6%
Pordhon	-18.8%	-12.5%	-436.5%	-26.6%
Akca	-9.4%	0.0%	-251.7%	-16.0%

Table 6.5. Percentage gap for LRPMD vs G-LRPMD

As expected, the percentage gap for BS, Opening Cost, BS Distance Cost, and BS Vehicle have increased in all benchmark sets after applying the distance constraint, other than the BS Opening Cost for Akca' set. This fact shows that the characteristics of the benchmark sets play an important role in the performance of the solution matter, in both absence and presence of the distance constraints. Also, the Prodhon's set has the greatest increase in all cost components and thereby the total cost after applying the distance constraint, when compared to the other data set.

We can conclude that the introduction of the distance constraints leads to a noticeable increase in all cost components and hence the total cost, which means employing EVs with limited batteries in order to have greener solutions can be considerably expensive. Therefore, the advancement of EVs technology, which is becoming more rapid recently, is necessary for an overall cheaper and greener solution.

6.7.2 BR-VNS and G-BRVNS

Table 6.6 summarises the average results obtained from BR-VNS for the LRPMD and from G-BRVNS for the G-LRPMD. The bold indicates that the LRPMD is smaller than the G-LRPMD in terms of the BS, BS Opening Cost, BS Distance, and BS vehicles.

	BS		BS Opening Cost		BS Distance		BS Vehicles	
	LRPMD	G-LRPMD	LRPMD	G-LRPMD	LRPMD	G-LRPMD	LRPMD	G-LRPMD
Barreto	3655.15	3862.73	1125.10	1155.82	2530.03	2706.90	6.29	6.82
Pordhon	1981.01	2293.82	1123.91	1224.29	6881.00	8621.93	16.90	20.73
Akca	891.62	958.34	200.00	225.00	691.62	733.34	6.5	7.58

Table 6.6. Average solutions of TSB RH and G-TSB RH for LRPMD and G-LRPMD

Figures 6.7, 6.8, 6.9, and 6.10 compare the average between BS, BS Opening Cost, BS Distance, and BS Vehicle obtained by BR-VNS for the LRPMD and by G-BRVNS for the G-LRPMD.

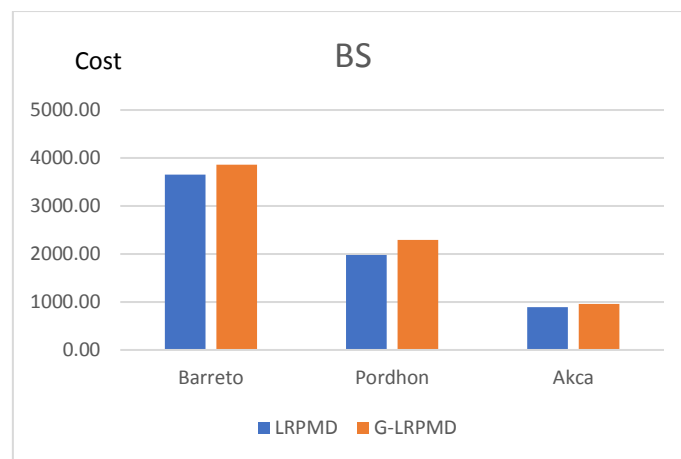


Figure 6.7. BS average of LRPMD wrt to G-LRPMD

In Figure 6.7, we can notice that the total cost for the BS of the LRPMD is smaller than the BS of the G-LRPMD for the three data sets.

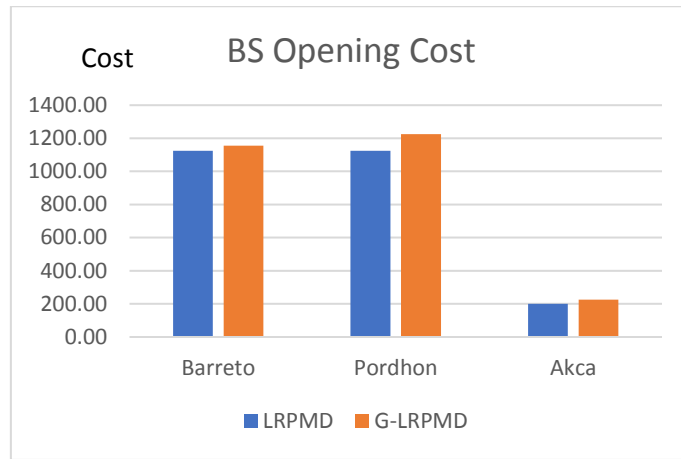


Figure 6.8. BS opening cost average % gap of LRPMD wrt G-LRPMD

In Figure 6.8, we notice that the cost of opening depots in the LRPMD is smaller than the cost of opening depots in the G-LRPMD in Pordhon's sets. However, Barreto's and Akca's sets are affected slightly by adding the distance constraint.

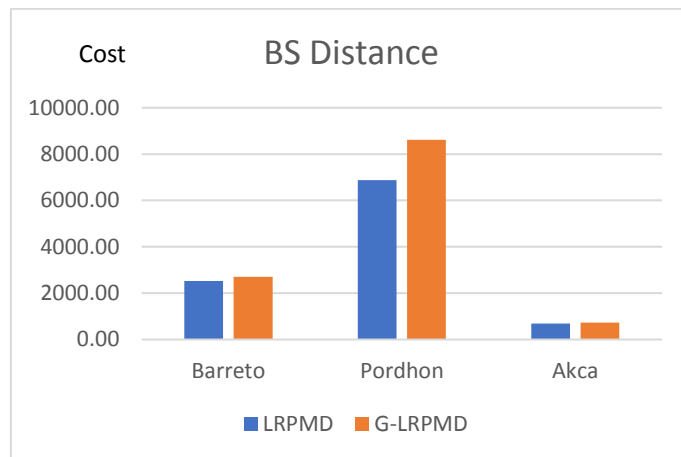


Figure 6.9. BS distance cost average % gap - of LRPMD wrt to G-LRPMD

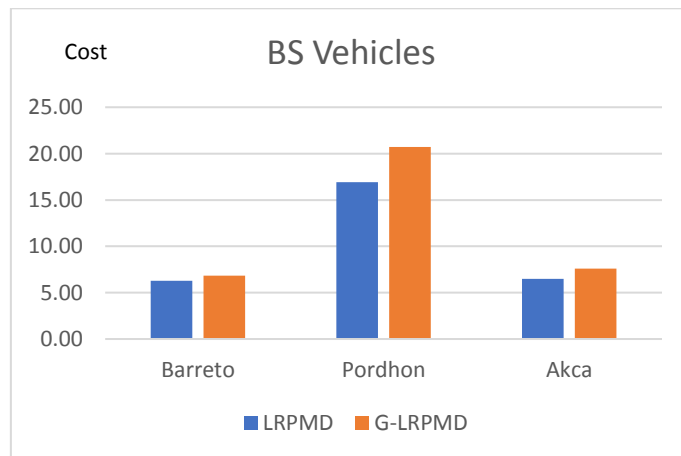


Figure 6.10. Average percentage gap - Vehicles of LRPMD vs Vehicles of G-LRPMD

In Figure 6.9 and 6.10, it clear that the BS Distance and the BS Vehicles in the LRPMD is smaller than the BS Distance and the BS Vehicles in the G-LRPMD, for the three data sets.

Table 6.7 illustrates the percentage gap of the average of BS, BS Opening Cost, BS Distance Cost, and BS Vehicle between LRPMD and G-LRPMD solution methods for the three data sets.

	BS	BS Opening Cost	BS Distance	BS Vehicles
Barreto	-5.7%	-2.7%	-118.9%	-8.4%
Pordhon	-15.8%	-8.9%	-462.0%	-22.7%
Akca	-7.5%	-12.5%	-207.4%	-16.6%

Table 6.7. Percentage gap for LRPMD vs G-LRPMD

As expected, the percentage gap for BS, Opening Cost, BS Distance Cost, and BS Vehicle have increased in all benchmark sets after applying the distance constraint.

We can conclude that the introduction of the distance constraints leads to a noticeable increase in all cost components and hence the total cost, which means employing EVs with limited batteries in order to have greener solutions, can be considerably expensive. Therefore, the advancement of EV's technology, which is becoming more rapid recently, is necessary for an overall cheaper and greener solution.

6.8 Conclusion

In this chapter, we discussed the Green Location Routing Problem with Multi-Depot (G-LRPMD), which considers the use of electrical vehicles. The use of electric vehicles has gained more interest within the delivery fleets and the increasing need for implementation of green transport solutions. Electric vehicles may have different driving ranges and can be limited in the distance they can cover, due to the use of batteries. This imposes additional new operational challenges on the already existing complex issues of the location routing problem.

To the best of our knowledge, this is the first time that green G-LRPMD has been studied. In order to solve the G-LRPMD problem, we present a modified optimisation model considers of EVs. In addition, we modified TSBRH and BR-VNS to consider distance constraint. New benchmark data sets were generated. Computational experiments have been conducted to evaluate the performance of our methods in solving the newly generated instances for the G-LRPMD problem.

The experimental results have shown that G-TSBRH is a fast heuristic that generates good quality solutions in a very short computation time when compared with the G-BRVNS. As the average computational time of G-TSBRH is 3.98, 4.6, and 2 sec, we can claim that it is competitive when a quick solution is required. On the other hand, G-BRVNS is competitive when a high-quality solution is required. To sum up, the proposed methods are more desirable for its novelty of the underlying idea, and for its simplicity in its implementation.

For the future work, we would like to consider different versions of the G-LRPMD by adding stochastic demand, or stochastic travel times, or by using heterogeneous fleets of electric vehicles in terms of its capacity. Also, it would be interesting to examine different configurations of fleets and analyse the trade-off between the associated distance-based cost and determine how "green" each configuration is. It should be noted that when we employ configurations "greener" vehicles, there is an increase in term of number of routes and, therefore, there is an increase in distances and the associated costs.

Chapter 7 Conclusion and Future Research

7.1 Conclusion

This thesis investigates four important topics related to the optimisation of supply chain management; LRPSD, MDVRP, LRPMD, and G-LRPMD. These four problems are very important and are nowadays among the core issues impacting costs and utility of logistics and distribution activities.

Furthermore, these problems possess significant environmental implications and there exist concerns regarding their influence in terms pollution. Henceforth, efficient solution methods geared towards dealing with such complex problems and in support of adequate decision-making processes are developed.

Throughout the course of this thesis both theoretical and experimental contributions are made to previous similar works in the research community.

In the literature, the solution methodology for the LRP and its variants can be divided into three categories namely; heuristic methods, exact methods, and metaheuristics. Heuristics were the first methods used to solve the LRP which applied in a sequential framework. Then, an iterative framework and nested framework were utilised to improve the heuristics performance. Later on,

exact methods were applied to deal with the LRP, but they can only solve small problems that do not exceed 20 customers. Subsequently, metaheuristics have been used widely.

The studied problems are known as NP-hard problems; therefore, approximate algorithms are more suitable to deal with them. Recently, a new technique, Biased Randomisation, was combined with heuristics to improve their performance. The idea of Biased Randomisation is to use a non-uniform random process to enhance the performance of the greedy heuristics. This new method has been applied successfully by Cabrera et al., (2014) and Juan et al., (2010) to solve both FLP and VRP.

Although, the LRP is related to both FLP and VRP, because these problems can be viewed as special cases of the LRP, the combination of the Biased Randomisation technique with heuristics, to the best of our best knowledge, has not been used in the literature to solve it. Therefore, considering the main trends in designing the approximate algorithms, it is significant to propose an efficient solution method based on combining a Biased Randomisation technique with a classic heuristic. This has resulted in a solution method which is able to solve such a complicated problem, even for large size instances.

The main contributions of this thesis include developing several approaches to solve four variants of the LRP. These four problems are, namely, LRPSD, MDVRP, LRPMO, and G-LRPMO. Additionally, we offer two optimisation models, inspired by the literature, for the LRPSD and the G-LRPMO that consider single depot and environmental sustainability impacts due to usage of EVS, respectively.

This thesis consists of six main chapters. Firstly, in Chapter 1, an introduction is provided to illustrate the whole picture of the problems and their importance, and applications in the real life. In Chapter 2, the main theoretical concepts, the development of the main contributions in previous studies, and different solution methods have been reviewed and examined. This effort is carried out in order to know what has been done in this field, to identify the gap in the literature, and to present an original contribution.

Next in Chapter 3, the LRP problem has been analysed when a distribution system with only single depot (LRPSD) is used. A new model for the LRPSD is developed, and at the same time, four heuristics are proposed to solve this problem. The new four heuristics consist of two stages which include location stage and routing stage. In the first stage, clustering technique, p-median model, clustering and p-median, and iterative method, are employed to deal with location problem. In the second stage, all four heuristics have employed the BR-CWH algorithm to deal with the routing problem which is a classic Clark and Wright enhanced by Biased Randomisation. To evaluate the

performance of the proposed solution method, we carried out computational experiments for different problem sizes ranging from 12 to 100 customers. Results obtained so far indicate that these four proposed heuristics are suitable to solve the LRPSD.

Then in Chapter 4, the BR-ECWH combines Biased Randomised technique with ECWH, which is proposed by Tillman (1969) to tackle the MDVRP. The BR-ECWH algorithm is employed to develop a two-level heuristic for solving MDVRP and it consists of two levels; Global level and Local level. In the global level, the BR-ECWH has been applied to solve the assignment problem and routing problem simultaneously. In the local level, the BR-CWH is applied to improve the solution by tackling each depot with its customers individually. We conducted extensive experiments for the for different problem sizes ranging from 48 to 288 customers, and number of depots ranging from 2 to 6. Results of our experiments illustrated that the Two-Level heuristic is suitable to solve the MDVRP as the computational time is short and the average gap is small.

The LRPMD has been studied in Chapter 5 where we present TSB RH and BR-VNS. TSB RH is a Two-Stage heuristic based on the Biased Randomisation technique in a nested framework. The location stage is the first stage in the proposed method which was added to the solution method proposed in Chapter 4. BR-VNS is based on the VNS metaheuristic. The initial solution is generated by using the Biased Randomisation technique, then it is improved by the VNS itself. The competitiveness of our methods has been tested using three well-known sets of the LRPMD benchmarks in the literature. Computational experiments with instances from Barreto's, Prodhon's, and Akca's set reported that the TSB RH is competitive in term of the computational time. However, the BR-VNS is competitive in tern of the solution quality.

The primary contribution of Chapter 6 is to suggest a modified optimisation model to consider an environmental aspect in the LRP, when an electric fleet is used in the distribution system, instead of the fleet with traditional fuel. As EVs have limited travel range, we have a new problem called the Green Location Routing Problem with Multi-Depot (G-LRPMD). Although the framework of the solution method for this new problem, namely G-TSB RH and G-BRVNS, are similar to TSB RH and BR-VNS in Chapter 5, they are different in routing stage due to distance constraints. To the best of our knowledge, this is the first time to study the G-LRPMD and we did not find benchmark instances to test the performance of our method. Therefore, the existing LRPMD benchmarks in the literature are adapted and modified by adding distance constraints. To identify the cost variations, we compare solutions of the G-LRPMD with the LRPMD to examine the effect of adding the distance constraints. We notice that the total cost and number of vehicles, opening cost, and distance cost have increased after adding distance constraint.

7.2 Extensions and future work

This thesis presents very interesting ideas to solve four relevant problems to the LRP and shows promising results for the proposed solution methods. However, there is room for plenty of opportunities to extend the problem and implement alternative algorithms to further improve the solution in future studies. Future alternative extensions to the work can be summarised as follows:

- The model and the solution methods in the current thesis have shown means of successfully applying the four variants of the LRP. One interesting future research is to test different models, including stochastic models, and considering different additional objectives such as: social objectives with regard to the customers and drivers, or quality of service in terms of service time and cost with these approaches.
- Another line of research could be the expansion of the LRPSD and the G-LRPMD models to consider heterogeneous fleets. Furthermore, the use of different types of vehicles such as Hybrid electric vehicles could be considered.
- Another potential study can be based on different variations in the implementation of the Biased Randomisation embedded in the proposed algorithms. For instance, a host of biased (non-symmetric) probabilistic distributions to measure the performance of the developed solution methods and their impact on the results can be considered.
- In order to improve the quality of the solutions achieved by the proposed methods in the current thesis, a potential future work might consist of applying more efficient approaches such as local search algorithms. The methods developed in the current thesis can then be utilised to generate a good quality initial solution, subsequently leading to a good starting point for further search and improving the overall results.
- Another interesting future research is to propose sim-optimisation (sim-heuristic or sim-metaheuristic) to consider the LRP with stochastic travel times, or stochastic demand

7.3 Scientific publications and academic contributions

One of the objectives of this thesis is related to the dissemination of the outcomes in academic conferences and journals. In the following, we include the list of publications, and conference papers.

- Journal publications:
 - 1) The Location Routing Problem using Electric Vehicles with a Constrained Distance
Submitted (Under Review) to Computers and Operations Research (indexed in ISI SCI, 2017 IF = 2.962, Q1; 2017 SJR = 1.916, Q1). ISSN: 0305-0548.

- Conference abstracts
 - 1) Four heuristics for the Location Routing Problem with Single Depot
OR58 Conference, University of Portsmouth. UK, September-2016
 - 2) Two-Stage Biased Randomised heuristic for the Location Routing Problem with Multi-Depot
OR59 Conference, University of Lancaster. UK, September-2018

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FORM UPR16

Research Ethics Review Checklist



Please include this completed form as an appendix to your thesis (see the Research Degrees Operational Handbook for more information)

Postgraduate Research Student (PGRS) Information		Student ID:	UP757166
PGRS Name:	Abdullah Almouhanna		
Department:	Mathematics	First Supervisor:	Banafsheh Khosravi
Start Date: (or progression date for Prof Doc students)	01/02/2015		
Study Mode and Route:	Part-time <input type="checkbox"/> Full-time <input checked="" type="checkbox"/>	MPhil <input type="checkbox"/> PhD <input checked="" type="checkbox"/>	MD <input type="checkbox"/> Professional Doctorate <input type="checkbox"/>

Title of Thesis:	Biased Randomised Heuristics for Location Routing Problem
Thesis Word Count: (excluding ancillary data)	62691

If you are unsure about any of the following, please contact the local representative on your Faculty Ethics Committee for advice. Please note that it is your responsibility to follow the University's Ethics Policy and any relevant University, academic or professional guidelines in the conduct of your study

Although the Ethics Committee may have given your study a favourable opinion, the final responsibility for the ethical conduct of this work lies with the researcher(s).

UKRIO Finished Research Checklist:

(If you would like to know more about the checklist, please see your Faculty or Departmental Ethics Committee rep or see the online version of the full checklist at: <http://www.ukrio.org/what-we-do/code-of-practice-for-research/>)

a) Have all of your research and findings been reported accurately, honestly and within a reasonable time frame?	YES <input type="checkbox"/> NO <input type="checkbox"/>
b) Have all contributions to knowledge been acknowledged?	YES <input type="checkbox"/> NO <input type="checkbox"/>
c) Have you complied with all agreements relating to intellectual property, publication and authorship?	YES <input type="checkbox"/> NO <input type="checkbox"/>
d) Has your research data been retained in a secure and accessible form and will it remain so for the required duration?	YES <input type="checkbox"/> NO <input type="checkbox"/>
e) Does your research comply with all legal, ethical, and contractual requirements?	YES <input type="checkbox"/> NO <input type="checkbox"/>

Candidate Statement:

I have considered the ethical dimensions of the above named research project, and have successfully obtained the necessary ethical approval(s)

Ethical review number(s) from Faculty Ethics Committee (or from NRES/SCREC):	N/A
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If you have *not* submitted your work for ethical review, and/or you have answered 'No' to one or more of questions a) to e), please explain below why this is so:

We did not collect data, we solved mathematical models

Signed (PGRS):		Date: 23/09/2019
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