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# A Survey on Portfolio Optimisation with Metaheuristics

Prisadarng Skolpadungket, Keshav Dahal

**Abstract**—A portfolio optimisation problem involves allocation of investment to a number of different assets to maximize return and minimize risk in a given investment period. The selected assets in a portfolio not only collectively contribute to its return but also interactively define its risk as usually measured by a portfolio variance. This presents a combinatorial optimisation problem that involves selection of both a number of assets as well as its quantity (weight or proportion or units). The problem is extremely complex due to a large number of selectable assets. Furthermore, the problem is dynamic and stochastic in nature with a number of constraints presenting a complex model which is difficult to solve for exact solution. In the last decade research publications have reported the applications of metaheuristic-based optimisation methods with some success. This paper presents a review of these reported models, optimisation problem formulations and metaheuristic approaches for portfolio optimisation.

**Index Terms**—Metaheuristic, Portfolio optimisation, Investment management, Asset management

## I. INTRODUCTION

One of practical problems in asset management is how to allocate money to invest in different assets in order to achieve the investors risk appetites and return objectives (Markowitz 1992). An investor is assumed to be a rational economic agent who is risk averse. Given a level of return objective, an investor tries to reduce risk as much as possible. To construct a portfolio of assets, a portfolio manager who acts in best interests of the investors, adds a number of assets to form a new asset portfolio that has a different risk-return characteristic than those of individual assets. Choices and quantities of different assets that should be included into the portfolio are the outcomes of portfolio selection process. A portfolio of feasible set of assets that has a minimum risk level and a maximum return level is called an optimal or efficient portfolio.

In the process of investment decision, portfolio managers usually face with abundant choices of investment assets. Also, they may need to make timely decisions in rapidly changing financial market. This represents a tough optimization problem, which continues to present a challenge for efficient optimization solution techniques (Maringer 2005.)

A variety of different techniques have been employed to solve the portfolio optimisation problem. The main drawback of techniques for exact solution is that the number of combinations of states that must be searched increases exponentially with the size of problem and becomes computationally prohibitive (Crama 2003.). Furthermore, these techniques are poor in handling the nonlinear objective and constraint functions and several assumptions are generally required to make the problem solvable using reasonable computational resources (Maringer 2005.) Alternatively, some heuristic-based techniques use algorithms to find approximate solutions for problem instances of NP-hard problems in a reasonable time (Blum 2001.). By using heuristics, the optimisation problems can be tackled in polynomial time with a traded-off of their optimality. In some circumstances of the real world problems, the speed to reach the acceptable approximate solutions is very critical. Feasible near-optimum solutions are acceptable but untimely are not. The simple heuristic solution approaches are based on specialized techniques that work particularly well for a given problem but are only of limited applicability to other problems ((Blum 2003.) Furthermore simple heuristics, based on greedy search algorithms, tend to stop in inferior local optima.

In order to overcome the above limitations, researchers in the last decade have focused much attention on metaheuristic solution techniques (Blum 2003.). Metaheuristics are general intelligent searches that could find the ways out of local optima. Despite their intelligence and generality, performances of metaheuristics depend on problem settings. Over the last decade a number of research publications have been reported on applications of metaheuristic approaches addressing some of portfolio management and optimisation issues.

This paper reviews the portfolio optimisation problem models and the solution methodologies reported in the literature with main focus on the applications of metaheuristic approaches. The next section provides general background to the portfolio models and a brief roadmap on solving methods. Section 3 reviews the portfolio optimisation problem formulations with realistic constraints. Section 4 reviews the applications of metaheuristic approaches to the portfolio problems with mentioning of some traditional approaches for comparison. The final section summarizes the paper and its conclusions.

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## II. GENERAL BACKGROUND AND CONCEPTS

### A. Portfolio Optimisation and Selection

Modern portfolio theory originated in a paper by Harry M. Markowitz in 1952 (Markowitz 1952 cited in Markowitz 1992 and in Elton 1997). The theory stated that an investor should not select assets due to only characteristics that are particular to the assets but she/he need to consider how each asset co-moved with all other assets. Moreover, by taking into account of these co-movements, an investor can construct a portfolio that has less risk given the same expected return than a portfolio constructed by ignoring the interaction between securities (Elton 1997). Since 1952, modern portfolio theory has become a well-developed paradigm and academic field. From simple single period of Markowitz, it has been many strands of developments. Earlier, investor's utility functions had been taken into consideration as well as additional moments rather than variance such as skewness (Tobin 1958, Lee 1977, Kraus 1976, Fama 1965, and Elton 1974 cited in Elton 1997).

An important development that has a high impact on practical implications has been the study of separation theorem that leads to mutual fund theorem. The theory states that if an investor has access to a risk free asset or a comparatively riskless asset, the investor's optimal portfolio is independent of his preferences for expected return and variance (which represent risk). The investor then can construct an optimal portfolio at any expected return level from only a few elements namely a risk free asset and one or few market portfolios or mutual funds. This radically simplified investment decision making. The important outcomes of this strand are the Capital Asset Pricing Model (CAPM) (in the seminal papers, Sharp 1964, Lintner 1965 and Mossin 1966) and the Arbitrage Pricing Theory (APT) (in the seminal papers, Ross 1967 and Roll 1980). However, empirical researches still inconclusive whether the models actually work in reality. Above all, due to practical and cost matters, portfolio managers sometime have still to construct market portfolios or mutual funds from a subset of all available assets. Another strand has been to extend the Markowitz model into multi-period and dynamic models. A comprehensive treatment of this strand of development can be found in Merton 1992. More recently, many researchers have extended the model into stochastic forms (e.g. Gulpinar 2003 and Fernholz 1982, 2002, 2003). However, beside Markowitz mean-variance model, CAPM and APT, other models still have lesser practical applications in the investment management industry.

Markowitz's seminal paper in 1952 was the first work on Modern Finance in general and Modern Portfolio Theory in particular. Markowitz himself has long been regarded as the father of Modern Finance even before he was accolade with Nobel Prize in Economics in 1989. Moreover, the model is the first that defines the concepts of risk and risk management related to financial and investment management. The concept of risk in the model is defined as the volatility of asset prices which is usually represented by the variance or standard deviation (square root of variance) of asset's prices under consideration (even though there are other definitions of risk such as Value-at-Risk or VaR). The risk management as far as

the model concerns is not to eliminate the risk associated with portfolio management but to choose the combination of assets that yield the lowest overall portfolio's variance given an expected portfolio return. The optimum portfolio management by minimising portfolio's variance is equivalent to optimum portfolio by maximising portfolio's return given a portfolio's variance.

The Markowitz model is a well-defined optimisation model and with some modification later by Black (Black 1972) to allow short-selling (allowing negative weights of assets) the model has a closed form solution. By removing some realistic assumptions such as the non-negativity constraints (i.e. no short sell on any assets are allowed); the integer constraints (i.e. shares of assets cannot be divided into lower than their trading units,) etc., the model has a general form that only the assets' expected returns, the variance and covariance of the assets are parameters. On the other hand, if we impose the non-negativity constraint, there exists no general form (closed form) solution for the optimisation problem. Although the model with non-negativity constraint can be solved efficiently by specialised algorithms and other ad hoc methods, imposing other constraints (e.g. the integer constraint or maximum number of asset constraint) will cause large-scale problems became unable to be solved by mixed integer non-linear programming or other exact solution algorithms, within a reasonable time (Busetti 2000).

The portfolio optimisation problems with realistic constraints are NP hard problem especially for those of exact solutions. The methods require complete enumeration where all possible and valid values for the decision variables are tested. The problem will be to select  $k$  out of  $N$  assets and optimize their portfolio weights. Only the complexity of selection of assets alone is  $O(C(N, k))$  e.g. selecting 10 out of 100 assets come with  $C(100, 10)$  or  $1.73 \times 10^{13}$  alternatives. Moreover, for each of the alternatives, the optimal weights must be determined. In this case, the weight must be zero or multiple of 0.1, which have  $10^{10}$  or 100 possible weight structures for each alternative. Obviously, the problem size can quickly get out of our present computation power. There are two ways to cope with the NP hardproblems. First, we can use more realistic models of portfolio optimisation problems and use approximate algorithms, which do not guarantee finding the optimal solution, but search for good enough solutions in a significantly reduced computational time (Blum 2003). Second, we can use approximately simplified unrealistic portfolio optimisation models, which can be solved by standard methods or algorithms.

### B. Solving methods

Heuristics are approximate algorithms. Basic or classical heuristics are greedy algorithms. In combinatorial optimisations, the basic heuristics include local search algorithms and constructive algorithms. Local search algorithms start from initial solutions and repeatedly try to substitute the current solution with a better one in an appropriate vicinity or neighbourhood of the current solutions. Constructive algorithms generate solution from initially empty solution by adding components until a solution is completed. In most of combinatorial optimisations, heuristic methods can

reduce the computation complexity to at most polynomial time (but yield solutions that not sure to be global optima). A drawback of the basic heuristics is that they tends to be trapped with local optima that far inferior than the true or global optima and, for most of times, is not considered good enough. Metaheuristics are to combine basic heuristic methods with some intelligence or guided strategies aimed to avoid the traps.

Essentially, metaheuristics are algorithms for exploring search spaces by using guided strategies that have a dynamic balance between the exploitation of the accumulated search experience (i.e. intensification) and the exploration of search space (i.e. diversification) (Blum 2001). Metaheuristics can be classified into two categories, namely, local search metaheuristics (LSMs) and evolutionary algorithms (EAs). LSMs begin with single solutions and subsequently replaced by another (often but not always the best) solution found in the neighbourhood. They are call exploit-oriented (or intensification) methods because they are often allowed to find a local optima solution. However, they are different form local search algorithms of basic heuristics in such a way that they have some mechanism to strategically guide the search away from trapped local optima. Conversely, EAs make use of a randomly generated population of solutions. The initial population is improved through natural evolution/selection processes. In the processes, the whole or part of population is replaced by newly generated offsprings (often the most suitable ones). As a result, EAs are often called exploration-oriented method. LSMs can be categorised into various models, based on guiding techniques, of which include Simulated Annealing (SA), Tabu Search (TS), Greedy Randomised Adaptive Search Procedure (GRASP) and Variable Neighbourhood Search (VNS). While EAs include Genetic Algorithms (GA), Evolution Strategies (ES), Genetic Programming (GP), Ant Colonies, Estimate of Distribution Algorithms and Scatter Search. Since there are active researches around the world to find new heuristic techniques, the lists are by no mean exhaustive. Moreover, there are also hybrids and other metaheuristics that can fall into neither category (Alba 2005).

Even though, metaheuristics are not problem specific, but very often, in order to reach good solutions, they need to make use of detailed knowledge of the problem domains. As a result, most of efficient metaheuristics are not reusable for different problems or even different instances of the same problem, without redevelopment or in some cases, adjustment of relevant parameters.

### III. PORTFOLIO OPTIMIZATION PROBLEM FORMATION

#### A. The Markowitz Mean-Variance Model

The Markowitz model assumes that investors make their decision in portfolio construction by choosing assets that maximise their portfolio returns at the end of investment period (expected returns). By assuming that investors are risk averse, the simplest model with a number of unrealistic constraints namely, perfect market without taxes, no transaction costs, no short sales, assets are infinitely divisible, the Markowitz portfolio optimisation can be stated mathematically as follows:

$$\text{Min}_{x_i} \quad \sigma_p^2 \quad (1)$$

subject to

$$\sigma_p^2 = \sum_i \sum_j x_i x_j \sigma_{ij} \quad (2)$$

$$r_p = r^* \quad (3)$$

$$r_p = \sum_i x_i r_i \quad (4)$$

$$\sum_i x_i = 1 \quad (5)$$

$$x_i \in \mathfrak{R}_0^+ \quad \forall i \quad (6)$$

Where  $\sigma_{ij}$  is covariance between asset  $i$  and  $j$ , if  $i = j$ , it is variance of asset  $i$ .

$\sigma_p^2$  is variance of the portfolio of assets.

$r_i$  is expected return of asset  $i$

$r_p$  is the expected return of the portfolio

$r^*$  is a predefined level of return

These additional conditions must hold so that the optimisation has a solution:

$$\min_i r_i \leq r_p \leq \max_i r_i$$

$$\sigma_i > 0 \quad \forall i$$

$$\rho_{ij} > -1 \quad \forall (i, j)$$

$$\exists (i \neq j) \text{ such that } r_i \neq r_j$$

Where  $\sigma_i$  is standard deviation (square root of variance) of asset  $i$ .

$\rho_{ij}$  is correlation coefficient of asset  $i$  and asset  $j$

$r_j$  is expected return of asset  $j$

#### B. Some Realistic Constraints

The Markowitz model is a simplified model to focus only a theoretical point of view. In the real world of investment management, portfolio managers face a number of realistic constraints those arise from normal business practices, practical matters and industry regulations. The realistic constraints that are of practical importance include (not exhaustively) integer constraints, cardinality constraints, floor and ceiling constraints, turnover constraints, trading constraints, buy-in threshold and transaction cost inclusions. Integer constraints make the number of any asset include in the portfolio must be integer or indivisible (i.e. cannot be in any fraction of normal trading lot). This may not suffer for metaheuristic optimisations because they are combinatorial but suffers the other optimisation methods that require continuity of the variables. The integer constraints can be expressed as

$$x_i = \frac{n_i}{\sum_{i=1}^N n_i} \quad (11)$$

and

$$n_i \bmod l_i = 0 \quad \forall i \quad (12)$$

Where  $n_i$  is number of unit of asset (share) and  $l_i$  is trading lot of the asset  $i$ .

Cardinality constraints are the maximum number and minimum number of assets that a portfolio manager wishes to include in the portfolio due to monitoring reasons or diversification reasons or transaction cost control reasons (Stein 2005). The constraints can be expressed as follows

$$C_l \leq \sum_{i=1}^N b_i \leq C_u \quad (13)$$

where  $b_i = 1$  if  $x_i > 0$  and  $b_i = 0$ , otherwise

Where  $C_l$  and  $C_u$  are the lowest number of assets and the highest number of assets required to include in a portfolio respectively.

Floor and ceiling constraints define lower and upper limits on the proportion of each asset, which can be hold in a portfolio. These constraints may result from institutional policy in order to diversify portfolio and to rule out negligible holding of assets for the ease of control (Crama 2003). They can be expressed mathematically as follows

$$f_i \leq x_i \leq c_i \quad \forall i \quad (14)$$

Where  $f_i$  and  $c_i$  are the lowest proportion and the highest proportion that asset  $i$  can be held in the portfolio respectively.

Turnover constraints impose upper bound for variations of the asset holding from one period to the next. The constraints are a mean to curb the transaction costs therefore they can be modeled indirectly by incorporating transaction costs and read as follows

$$\max(x_i - x_i^0, 0) \leq B_{iu} \quad \forall i \quad (15)$$

$$\max(x_i^0 - x_i, 0) \leq S_{iu} \quad \forall i \quad (16)$$

Where  $x_i^0$ ,  $B_{iu}$  and  $S_{iu}$  are the holding proportion of asset  $i$  in the initial portfolio, the maximum purchase and the maximum sales of asset  $i$  during the current holding period.

On the other hand, trading constraints impose limits on buying and selling tiny quantities of assets due to practical reasons and can be stated as follows

$$x_i = x_i^0 \quad \text{or} \quad x_i \geq (x_i^0 + B_{il}) \quad \forall i \quad (17)$$

$$\text{and / or} \quad x_i \leq x_i^0 - S_{il} \quad \forall i \quad (18)$$

Where  $B_{il}$  and  $S_{il}$  denote the minimum purchase and sale of asset  $i$  during the current holding period (Crama 2003).

In business of asset trading, stock brokers, bond dealers, etc. are doing their business for money therefore transaction costs associated with purchases and sales of assets are inevitable and should be incorporated in the realistic models. Transaction costs have many forms as follows

$$\text{fixed} : \quad T_i = T_f \quad (19)$$

$$\text{proportion al} : \quad T_i = t_p n_i S_{0i} \quad (20)$$

$$\text{prop. w / floor} : \quad T_i = \max\{t_f, t_p n_i S_{0i}\} \quad (21)$$

$$\text{prop. plus fixed} : \quad T_i = T_f + t_p n_i S_{0i} \quad (22)$$

where  $T_i$ ,  $T_f$ ,  $t_p$ ,  $t_f$ ,  $n_i$  and  $S_{0i}$  are transaction cost of inclusion of asset  $i$  into the portfolio, fixed fee per transaction of purchase asset  $i$ , minimum (floor) fee per-transaction, variable fee per amount (in pound) of purchase and current trading or market price of asset  $i$  (at the time of purchase) respectively.

The transaction costs affect the fund that can be invested in all assets. Let  $V_0$  be the initial endowment the portfolio manager is entrusted to construct a portfolio. And  $n_i$  is the amount of asset  $i$  (assuming the integer constraints hold) then the amount of fund that can be invested in the portfolio will not equal  $V_0$  but will equal (assuming the fund is invested completely on assets)

$$V_0 - \sum_{i=1}^N T_{0i} = \sum_{i=1}^N n_i S_{0i} \quad (23)$$

And if there is no trading during the holding period then the expected portfolio return on the initial endowment for the holding period will be

$$r_p = \frac{\sum_{i=1}^N (n_i S_{0i} [1 + r_i]) - \sum_{i=1}^N (T_{0i} + T_{1i})}{V_0} - 1 \quad (24)$$

Where  $T_{0i}$  and  $T_{1i}$  are (purchase) transaction cost of asset  $i$  at the beginning of the holding period and (sale) transaction cost at the end of the holding period (to convert all assets to cash) respectively.

#### IV. METHODOLOGIES AND ANALYSIS

##### A. Closed Form Solution

By removing the non-negativity constraint in (6) and let  $x_i$  can be any real number (short-selling allowed), the solution of the Markowitz model then has a closed form solution (but far from representing the real world problems). Black (see Black 1972) did this simplification.

##### B. Algorithms for Exact Solutions

The standard Markowitz models with non-negativity constraints are NP-hard, only small problem size i.e. the number of assets (N) are small that can be solved within reasonable time for an exact solution using standard optimisation software (quadratic optimisation tools) (Maringer 2005, Wolfe 1959 cited in Crama 2003). In Jagannathan 2003 (cited in Maringer 2005), the non-negativity constraints are incorporated by modifying the covariance matrix without reducing complexity. Some ad hoc methods are taken advantages of the special structure of the covariance matrix (see Perold 1984 and Bienstock 1996 both cited in Crama 2001). Other researchers investigated some techniques that can be solve only models with only a subset of constraints e.g.

Dembo 1989 using network flow models, Konno 1991 using linear programming models with embedded absolute deviation approach to measure risk, Takehara 1993 (cited in Crama 2003) using an interior point algorithm, Bienstock 1996 (ibid.) and Horniman 2001 (cited in Stein 2005) using branch and cut approaches (as a subset of Mixed Integer Quadratic Programming – MIQP).

### C. Simulated Annealing

Simulated Annealing is the oldest among the metaheuristics. It allows moves toward worse solutions in order to escape from local optima. The probability of such worse moves is diminished like molecules are slowing down when the system's temperature is cooling down (which the name is derived from.) (Blum 2003)

Crama and Schyns (Crama 2003) applied simulated annealing to solve complex portfolio selection problems with floor, ceiling, turnover, trading and cardinality constraints. They encoded a solution of the problem as an  $n$ -dimensional vector  $X$  whose element  $x_i$  represents the holding of asset  $i$  in the portfolio. The quality of a solution is measured by the variance of the portfolio. They use specific approaches to handle specific classes of constraint either by explicitly restricting the solutions to be in feasible region or by penalizing infeasible solutions. They found that the algorithm can be approximate the optimal portfolio frontier for medium size problem (151 assets) within acceptable computing time and can be handle more classes of constraints than those of classical approaches. Also it is quite versatile to apply to different measures of risk other than variance as well as different covariance matrix properties. However, the algorithm still needs to customize and to delicately fine tune parameters to account for different classes of constraints.

### D. Tabu Search

Tabu Search uses the history of the search in the form of tabu list that keeps track of the recently visited solutions and forbids moves toward the list. This metaheuristic uses the tabu list both to avoid local optima and to implement an explorative search strategy (Blum 2003.)

Busetti (Busetti 2000) used Tabu Search/Scatter Search tools in the Opt quest module of Decisioneering Inc.'s Crystal Ball for cardinality constraint case. The results then compared with those of Genetic Algorithms (GA). He found that tabu/scatter search method is unsuitable for optimisation portfolio with cardinality constraints (of the size of 40 assets). Therefore, he concluded that GA is better than tabu/scatter search for this application and problem size. Moreover, the GA applied to portfolio optimisation is effective and robust with respect to quality of solution and speed of convergence. It is also versatility by not relying on restrictive properties of the model, by ease of new constraint addition and by ease of the objective function's modifications. In contrary to other metaheuristic methods, the needs for tailoring, customising and fine-tuning are not an issue for GA, even though these may improve performance of the model in somewhat extent but not necessary.

### E. Ant Colony Search

Ant Colony Search is the imitation of behavior of ants that enable them to find shortest path between food sources and their nest. While moving to/from the nest ants deposit pheromone on the ground. Using the concentration of pheromone, they decide the likelihood of which direction to go (Blum 2003.)

Maringer (Maringer 2005) applied ant colony algorithms to solve optimisation problems in small portfolios (with cardinality constraints). He asserts that the problems can be regard as Knapsack problems with some modifications. Firstly, the "value" of any included asset is dependent on the overall structure of portfolio rather than independent of other (asset) selections as in the knapsack, due to diversification effects. Secondly, the investor has to decide jointly whether to include an asset or not and in what amount (or proportion) of the asset to include in the portfolio. He tested for two cases of choosing three stocks ( $k = 3$ ) and ten ( $k = 10$ ) stocks respectively from S&P 100 (with 161,700 and  $1.73 \times 10^{13}$  alternatives respectively). In the case of standard parameter setting (no evaporation of the pheromone) for  $k = 3$  only 16 % of all run in which the global optimum was found and for  $k = 10$  in just 2 out of the 1,000 independent runs that the global optimal was found. However, when the parameters were set to appropriate values (with some evaporation), for  $k = 3$ , about two third of the run that global optimum was found, and for  $k = 10$ , from half and two third of the run that global optima was found.

### F. Hybrid Local Search

Maringer (Maringer 2003, 2005) also applied hybrid local search algorithm to solve portfolio optimisation problems. The algorithm combine population based heuristic with local search. A crystal-like structure represents a portfolio of assets. Its structure depicts both the assets and their weights. All of the crystals represent the population. The algorithm begins with a random initialization of crystals and their random structure but valid respect to the constraints. Then subsequent iterations consist of three stages; first, modification of crystal (portfolio) structure; second, evaluation and ranking of the modified structure; third, replacement of the poorest crystal in the population. The author has tested the algorithm against other two algorithms, namely, simulated annealing (SA) and simulated annealing with a group of isolate crystals (as a way to introduce evolutionary strategies into SA.) The author conducted such a test in order to determine whether a group of agents outperform individual agents and whether the use of evolutionary strategies improves performance of SA. The results indicate that the hybrid local search (HA) performs the best among the three algorithms in respect to the average deviation from the supposed optimal portfolios (both for DAX data set and FTSE data set). The next is GSA. And the worst is SA. He concluded that introducing evolutionary strategies do improve performances of metaheuristic algorithms in solving portfolio optimisation problems.

### G. Genetic Algorithms

Genetic Algorithms (GA) are population based heuristic algorithms. In GA, solutions are represented as chromosomes

that to be breed by crossover or modified by mutation. Selection processes are used to find optima solutions imitating the natural selection of survival of the fittest (Maringer 2005).

Busetti (Busetti 2000, *ibid.* in section D) compared GA with tabu search and found that GA performs better for portfolio optimisation problems in the problem setting. Streichert et al. (Streichert 2004a) applied the Multi-Objective Evolutionary Algorithm (MOEA) to solve portfolio optimisation problem. However, they are not the first group to applied Evolutionary Algorithms to solve the problem. Tettamanzi et al. (cited in Arnone 1993, and Loraschi 1995a and 1995b) transformed the multi objective optimisation problem into a single-objective problem by using a trade-off function (therefore not a true multi-objective). In their paper, they compared performance of different GA representations of portfolio optimisations with several combinations of real world constraints on the Hang Seng data set with 31 assets. The representations are binary bit-string based genotypes or gray-code encoding and real-valued genotype. They also investigated the size of the bit string from 32 bit ‘continuous’ representation to 7 bit ‘discrete’ representation. They also compare GA with and without Lamarckism (hybrid GA in which the genotype can be modified not only being removed from the population) and Knapsack GA (KGA) with and without Larmarckism. The constraints imposed on the optimisation problems are cardinality and integer (discrete) constraints. The results show that KGA produced better results as well as converged faster than ordinary GA due to more efficient removal of surplus assets. This conclusion is drawn from the fact that, in the problems without constraints, both GA and KGA perform almost the same. They also found that GAs without Larmarckism tend to be premature convergence because the neutrality of the search space cause the GAs to be trapped in the sub-optimal search space but for KGAs, even without Larmarckism, do not have this tendency. For the bit strings, on average the real value coding performed worst in all problem instances. Also, the differences were not much if a constraint was not added. The discrete 7-bit string performed better than the ‘continuous’ 32-bit string because the mutation and crossover operators are more effective. Thus, the hybrid KGA with 7 - bit gray-coding and Larmarckism was the best in the most problem instances with real-world constraints. In another paper (Streichert 2005), the same group of authors introduced an alternative hybrid encoding for evolutionary algorithms, which combines both ‘continuous’ real value and ‘discrete’ binary value together. The algorithm then compared with the different EA representations. When the algorithm and the other EAs without Larmarckism were applied on the problem with only cardinality constraints, the algorithms performed better than those of standard EAs. However, with introducing Larmarckism into the algorithms as well as into all standard EAs, the algorithm’s performance was only comparable to other standard EAs. Same as the results in the previous paper, when added more constraints to the problems, the algorithms with Larmarckism were among the best of all due to its ability to remove some of the neutrality in the search space.

#### H. Hybrid Evolutionary Algorithm

Subbu et al. (Subbu 2005) presented a new hybrid evolutionary multi-objective portfolio optimisation problem algorithm called Pareto Sorting Evolutionary Algorithm (PSEA) that integrates evolutionary computation with linear programming. The problem can be stated as

Max Portfolio Expected Return  
 Min Surplus Variance  
 Min Portfolio Value at Risk  
 Subject to: Duration mismatch  $\leq$  target 1  
 Convexity mismatch  $\leq$  target 2  
 Linear portfolio investment constraints

However, the aim of the paper is only to design algorithm and architecture for portfolio optimisation not to measure or compare performance of the algorithm.

#### V. CONCLUSION

This paper presents an overview of the concepts of portfolio optimisation problem with a bibliographical survey of relevant background, problem settings and the present state of solution techniques. The references provide a representative sample that is relevant to applying metaheuristic optimisation to the problem domain. A clear trend is heading toward hybrid models, which is the blend of population based techniques (e.g. evolutionary algorithm, genetic algorithm) and search based algorithm (e.g. local search, rule-based heuristic). This paper is based on many research articles published from 1952 to date in the field of economics, finance, mathematics, operation research and computer science.

A portfolio optimisation problem involves a huge combination of alternative set of assets. Combining with sophisticate constraints, finding a global optima by an exhaustive search is a NP hard problem. Using pure search based metaheuristics, the search tends to be trapped in local optima. While using pure population based tends to time inefficient. Therefore, the uses of hybrid algorithms can improve the situations if we can blend techniques of both the search based and the population based algorithms that appropriate for solving the problem.

Our future work will be toward improving hybrid metaheuristic implementations to solve the problem with focusing on novel techniques on problem representation, constraint handling, crossover and mutation as well as embedded search heuristics. We also plan to extend the techniques to implement on other portfolio selection models with different definition of risk and return, and also on estimations of the volatility and forecasting of the returns.

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