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Preliminary communication

Determination of the Pile Stiffness Matrix Based on the Pile Load Test Results and the Effect of Pile Interaction

Petar SANTRAČ*, Nenad ŠUŠIĆ, Željko BAJIĆ

Abstract: The paper presents the determination of the elements of the pile stiffness matrix based on pile load test results, made for the needs of modernization of oil refinery for NIS in Pancevo. The tests were carried out on CFA and Franki piles system. However, only the results for CFA piles are shown in this paper. Load tests included axial, tension and horizontal loading. Bearing in mind the geomechanical composition of the terrain and the results of penetration testing (CPT), it has been concluded that the soil resistance can be approximated as a linear function by depth. By approximating the soil by the Winkler model and using the results of the horizontal pile load test, the equivalent gradient of the modulus of soil reaction for the work load level is determined analytically, and then, the pile flexibility coefficients for horizontal load. For vertical load, the coefficient of flexibility is determined directly on the basis of the load-settlement curve. Based on the flexibility coefficient, a matrix of flexibility and a stiffness matrix are determined. The effect of the pile interaction for horizontal and vertical loads was introduced in an approximate way, through the reduction of coefficients of the stiffness matrix. The entire process of determining the stiffness matrix of the pile is illustrated by practical example of a pile group connected by an ideal rigid cap.

Keywords: pile group interaction; pile load test; pile stiffness matrix

1 INTRODUCTION

For the needs of modernization of the NIS oil refinery in Pancevo, depending on the size of the building, shallow and deep foundations were designed. Shallow foundations are made for light objects, on strip and individual foundations, mainly for administrative buildings. Deep foundation has been applied for heavy objects, such as fuel tanks and production facilities. In order to improve the soil bearing capacity and shorten the consolidation time, gravel-piles were used, while for heavy loads, CFA and Franki piles were applied.

This paper includes the load tests of CFP piles in the technical blocks 16 - 22 of the refinery. The pile diameter and the pile length were ø550 mm and 12.8 m. The longitudinal reinforcement bar is 8R ø25, the spiral is R ø8/20 cm and the protective concrete layer is 84 mm.

On the basis of geomechanical site investigations, up to the depth of about 10 m from the terrain's surface, layers of medium compacted, silty and slightly clayey sand (SF-SC) are present. Below them, sand layer (SW) is present up to the depth of about 16m. The penetration resistance of the soil layers increases approximately linearly with depth. The groundwater level during the site investigations (in 2009) was at a depth of 3.9 m below the ground level.

Axial pressure and tension pile load tests as well as horizontal pile load test were carried out. The pile load tests were performed by the IMS Institute at Belgrade. Results and interpretations of load test are given below.

Using the results of the vertical load test, the pile-soil flexibility can be determined directly. With the assumption that the soil can be approximated by the Winkler model, on the basis of the results of horizontal pile load test, pile length and cross section area as well as pile stiffness, it is possible to analytically determine (Hetenyi [1], Barber [2]) the flexibility coefficients for horizontal load.

The obtained coefficients are related to individual pile. In order for the results to be applicable to a pile group, it is necessary to take into account their interaction. This paper presents the analysis of a small pile group with an ideally rigid cap, loaded up to the workload level. The pile group ultimate bearing capacity has not been considered.The interaction between the piles is analysed approximately.

There are a large number of different methods, among which the mostly used for the vertically loaded pile group is the interaction factor method. In this respect, the most commonly used are those proposed by Randolph [3-9], Poulos [10-15], Scott [16], Mandolini [17-18] and others.

The interaction factor depends on the mutual distance of the piles, pile effective radius of action and the share of the pile base in total load capacity.

The group effect for horizontally loaded pile group is estimated by so-called "p-multiplier method", proposed by McVay [19], Mokwa [20], and others. The basis of all the above methods is the reduction of the coefficients of stiffness for each single pile in the group.

For horizontally loaded pile group, the coefficients of reduction depend on the mutual distance of the piles, the position of the pile and the horizontal force direction.

After the pile interaction factor for vertical loading and coefficients of reduction for horizontal loading have been determined, it is possible to estimate the flexibility and stiffness matrix for each pile in the group. The obtained pile matrices contain the stiffness of the soil and the pile at workload level and the pile interaction in the group at horizontal and vertical load. When it comes to the pile interaction and additional shifting from torsional loading, this effect is neglected due to its low significance. The method is illustrated by practical example of the pile group foundation at block 16 - 22 of the refinery.

On the basis of stiffness matrices, using appropriate software, one can easily form a global matrix of stiffness of a pile group connected (fixed or hinged) by an ideally rigid cap. The assumption of an ideally rigid cap is always justified when it comes to a small group. When ignoring the deformation, the cap position can be described with a total of 3 translation and 3 rotation components, which can be determined by kinematic and equilibrium conditions.

2 PILE LOAD TESTS AND PILE STIFFNESS COEFICIENTS

2.1 Vertical Load Test on Pile

The pile compression load test was carried out on the CFA pile No.5 at block 22. The ballast was formed by concrete blocks, overlying on a strong steel beam (I-profile) platform, which were supported on the foundation made by concrete blocks stacked on the ground surface. The total mass of the ballast was 108.0 t. The setup of the pile load test is shown in Fig. 1.The type of test was MLT.

Figure 1 Setup of the pile load test for compression load

The application of the load was done by a hydraulic press, and the settlement was measured with 4 dial gauges, positioned on datum bars resting on immovable supports at a distance of 2.0 m from the pile axis. Along with the dial gauges measurement, precise geodetic one was also done.

The load was applied in increments of 160.0 kN and was maintained till the rate of the pile head displacement is less than 0.05 mm/10 min or for 2 h, whichever occurs first. Firstly, the pile was loaded up to the workload of 650.0 kN and unloaded. After that, it was loaded again, up to 1.5 times the workload around 975.0 kN and unloaded. The measured settlement at the workload is s = 7.89 mm. The results of the load test are shown graphically in Fig. 2.

In the given case, the pile flexibility coefficient of the pile, for vertical loading is $F_{sN} = s/N$ = 7.89/650.0 = 0.0121 m/MN, while the stiffness coefficient, as an inverse value is $K_{Ns} = 1/F_{sN}$ = 82.64 MN/m. The pile ultimate load is determined by the method of hyperbolic extrapolation (Chin-Kondner [21-22], Decourt [23-24]), and it is about N_f = 2.77 MN.

On the CFA pile No.3 at block 16, a tensile load test was performed (Fig. 3). The application of the load was done by a hydraulic press, which rested on the steel beam, from which the force as a tension was transferred to the pile reinforcement bars. The force was incrementally applied (25 - 50 kN) up to the workload Z = 100.0 kN, after which it was unloaded and reloaded again up to 400.0 kN. The displacement was measured with 3 dial gauges, positioned on datum bars resting on immovable supports at a distance of 2.0 m from the pile. Along with the dial gauges measurement, precise geodetic one was also done. The type of test was MLT.

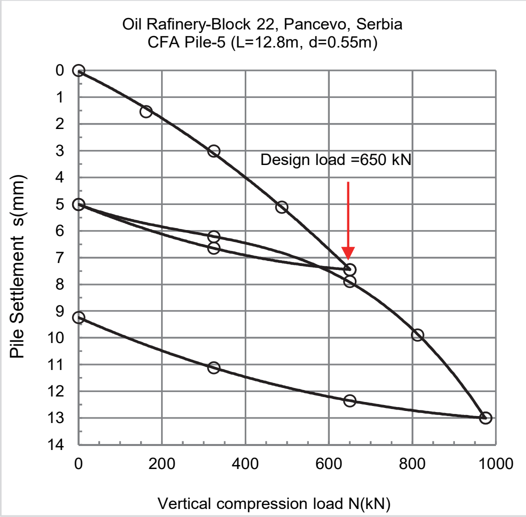

Figure 2 Results of the compression pile load test

Figure 3 Setup of the pile load test for extension load

The results of the tension load test are shown graphically in Fig. 4. For the workload of Z = 100.0 kN, the rise of the pile head was s = 1.44 mm. The coefficient of flexibility is: $F_{sZ} = s/Z$ = 1.44/100.0 = 0.0144 m/MN. The ultimate capacity determined by the method of hyperbolic extrapolation is about Z_f = 63.0 kN.

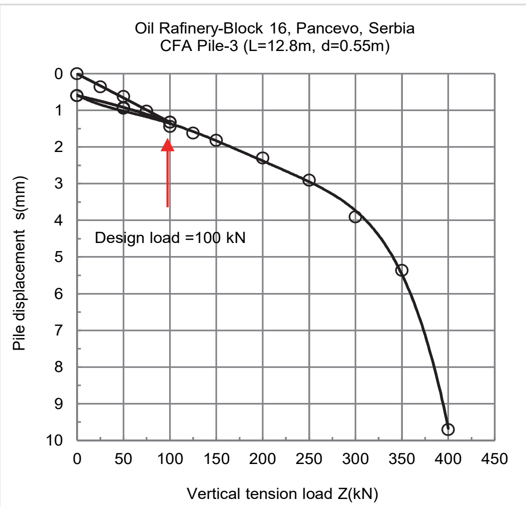

Figure 4 Results of the extension pile load test

2.2 Pile Stiffness Coefficient for Vertical Loading

For the vertically loaded piles, usually an analytical solution (Scott [16], Mylonakis & Gazetas [25]) is used, which connects the pile geometry and axial stiffness, the head settlement and load, with an average soil shear modulus of reaction k_τ along the pile shaft and the modulus of soil reaction k_b at pile base. The axial force N on the pile head can be shown as a product of the pile axial stiffness K_{Ns} and the pile head displacement, according to the expression:

$$N = K_{Ns} \cdot s \ , \ K_{Ns} = \frac{\lambda_\tau L}{f}\left(\frac{E_p A}{L}\right) \ , \ \lambda_\tau = \sqrt{\frac{k_\tau S}{E_p A}} \quad (1)$$

In Eq. (1), f is the compressibility factor, L and S are the pile length and circumference, E_p is the pile modulus of elasticity (29.5 GPa), and A is the pile cross section area.

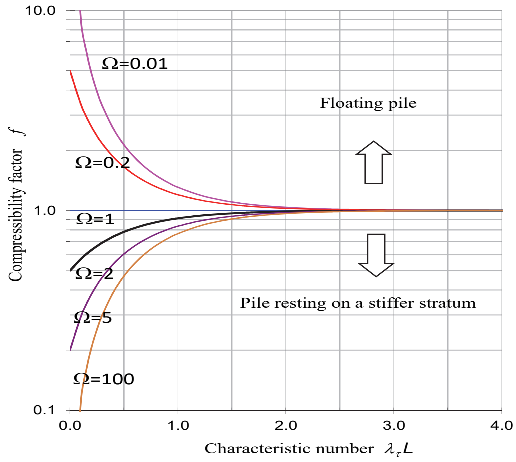

Figure 5 Compressibility factor f

The factor of compressibility can be obtained graphically by the diagram in Fig. 5, or analytically by Eq. (1a).

$$f = \frac{1+\Omega \tanh(\lambda_\tau L)}{\Omega + \tanh(\lambda_\tau L)} \ , \ \Omega = \frac{k_b A_b}{\lambda_\tau E_p A} \quad (1a)$$

In Eq. (1a), $1/\lambda_\tau$ is a characteristic length, A_b is the pile base cross section area and Ω is the transfer function which depends on the ratio of the load carried by the pile base and the pile shaft. The average soil shear modulus of reaction k_τ is [25] approximately:

$$\frac{k_\tau}{G_s} \approx \frac{1.3}{d\pi}\left(\frac{E_p}{E_s}\right)^{\frac{-1}{40}}\left(1+7\left(\frac{L}{d}\right)^{-0.6}\right) \quad (1b)$$

The value of the modulus of soil reaction k_b at pile base is (Scott[16]) approximately:

$$\frac{k_b}{G_b} \approx \frac{2}{1-\nu_b}\frac{1}{d_b} \quad (1c)$$

In Eq. (1b) to Eq. (1c), d and d_b is the pile shaft and pile base diameter, G_s and G_b is the soil shear modulus at pile shaft and pile base and ν_b is the soil's Poisson's ratio. Based on the results of CP-tests shown in Fig. 6, the average cone penetration resistance and sleeve friction of the silty-clayey sand along the pile is around $q_c \approx 4.1$ MPa and $f_S \approx 80.0$ kPa. The cone penetration resistance of the soil at the pile base is around $q_c \approx 12.0$ MPa.

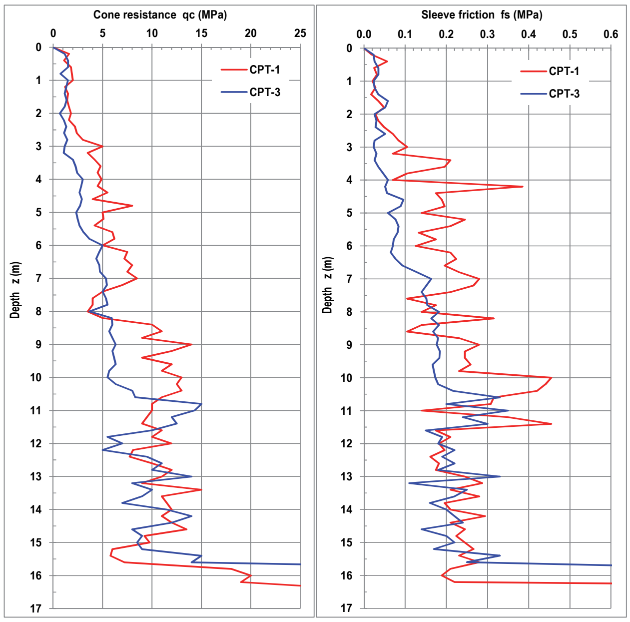

Figure 6 Results of the CPT-s on pile test location

For the cone penetration resistance q_c along the pile, the very conservative value of the modulus of deformation is $E_s \approx 2q_c$, so the soil shear modulus is around $G_s \approx 3.2$ MPa. In the same way, the soil shear modulus at the pile base is around $G_b \approx 9.2$ MPa. The soil shear k_τ and base k_b modulus of reaction and the pile axial stiffness can be obtained by Eq. (1). The results of the calculation are the following:

$$\frac{k_\tau}{G_s} \approx \frac{1.3}{0.55\pi}\left(\frac{29,500}{8.2}\right)^{\frac{-1}{40}}\left(1+7\left(\frac{12.8}{0.55}\right)^{-0.6}\right) = 1.26$$

$$\frac{k_b}{G_b} \approx \frac{2}{1-0.3}\frac{1}{0.55} = 5.19$$

$$k_\tau \approx 4.0 \ \mathrm{MN/m^3} \ , \ k_b \approx 48.0 \ \mathrm{MN/m^3}$$

$$\left.\begin{aligned} \lambda_\tau L &= \sqrt{\frac{4.0 \cdot 1.73}{29,500 \cdot 0.24}}12.8 = 0.313 \\ \Omega &= \frac{48.0 \cdot 0.24}{0.0313 \cdot 29,500 \cdot 0.24} = 0.052 \end{aligned}\right. \rightarrow f(\lambda_\tau, L, \Omega) = 2.36$$

$$K_{Ns} \approx \frac{0.313}{2.36}\left(\frac{29,500 \cdot 0.24}{12.8}\right) = 93.8 \ \mathrm{MN/m}$$

The analytically determined pile axial stiffness coefficient of 93.8 MN/m, approximately corresponds to the value obtained by pile load test: $K_{Ns} = 1/F_{sN} = 82.6$ MN/m.

2.3 Horizontal (Lateral) Load Test on Pile

The lateral load test was carried out on the CFA pile No.1 at block 16. The application of the load on the pile was done by a hydraulic press rested on the steel tube and steel beam supported by 4 piles (Fig. 7). The pile head displacement is measured with 2 dial gauges. At workload of $T = 60$ kN, the measured displacement is $s = 2.3$ mm, so the flexibility coefficient of the pile is: $F_{tT} = t/T = 2.3/60 = 0.0383$ m/MN. The results of the load test load are shown in Fig. 8.

Figure 7 Setup of the pile load test for horizontal load

The compressive strength and modulus of elasticity of the concrete are $f_c = 30.0$ MPa and $E_p = 29.5$ GPa. The ultimate tension strength and modulus of elasticity of the reinforce-ment steel bar are $f_y = 400$ MPa and $E_c = 200.0$ GPa.

During the test, the pile bending capacity was reached at the load of $H_u = 161.0$ kN. The load was increased further up to $H_{max} = 360.0$ kN, where the pile head was displaced to $u_{max} = 22.5$ cm. The moment of inertia of the ideal pile cross section is $I = 0.0045$ m^4, and for the cracked cross section is (phase IIb) $I_{cr} = 0.0015$ m^4. The calculated breakage moment of the pile is $M_u = 0.29$ MNm, which corresponds to the lateral ultimate load of $H_u = 175.0$ kN. This is around the measured ultimate load of 161.0 kN.

Since during the load test the breakage in the pile has occurred before the break in the soil, the pile can be considered long. The soil excavation around the pile after the test has shown horizontal cracks in the pile body at the depth between 1.5 - 2.1 m below the pile head (Fig. 9).

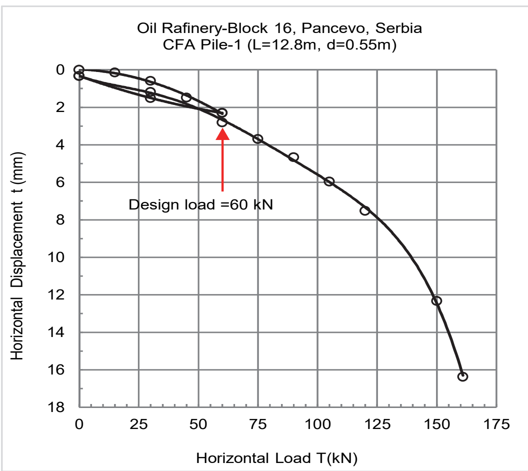

Figure 8 Results of the horizontal pile load test

Figure 9 Cracks on pile after unloading from $T_{max} = 360.0$ kN

2.4 Pile Stiffness Coefficient for Horizontal Loading

Since the penetration test shows that the resistance of the silty-clayey sand layer grows approximately linearly with depth, the interpretation of the load test results can be done by analytical expressions for laterally loaded pile by Winkler soil model [2].

For the linear increase of the horizontal modulus of soil reaction k_h with depth, the horizontal displacement and rotation of the free pile head, in a general case due to the horizontal load and bending moment, are given below:

$$t = F_{tT} \cdot T + F_{tM} \cdot M, \quad \theta = F_{\theta T} \cdot T + F_{\theta M} \cdot M \tag{2}$$

In the above equation, the pile flexibility coefficient loaded with force and bending moment, for a long pile ($\eta L > 4$) in the soil with linearly increasing modulus of soil reaction with depth, can be expressed by the following [2]:

$$F_{tM} = F_{\theta T} = -\frac{1.60}{n_h^{0.4} \cdot (E_p I)^{0.6}}, \quad k_h = n_h \frac{z}{d} \tag{2a}$$

$$F_{tT} = \frac{2.40}{n_h^{0.6} \cdot (E_p I)^{0.4}}, \quad F_{\theta M} = \frac{1.74}{n_h^{0.2} \cdot (E_p I)^{0.8}} \tag{2b}$$

Since the load is applied horizontally without moment, on the basis of the obtained value of the flexibility coefficient F_{tT} and the bending stiffness of the ideal concrete cross-section E_pI, the value of n_h and the characteristic pile length $1/\eta L$, can be calculated according to the Eq. (2b):

$$n_h = \sqrt[0.6]{\frac{2.40}{0.0383 \cdot 132.75^{0.4}}} = 38.0 \ \frac{\text{MN}}{\text{m}^3}$$

$$\eta L = \sqrt[5]{\frac{n_h}{EI}} \cdot L = 10.0 > 4$$

Based on Eq. (2a) to Eq. (2b) and the previously calculated modulus $n_h = 38.0$ MN/m^3, all other flexibility coefficients can be obtained. Otherwise, the $F_{\theta T}$ can be determined if the pile head rotation is measured at horizontal load test, while for the $F_{\theta M}$ a torque load is required.

The flexibility coefficients are: $F_{tM} = -0.0199$ m/MNm, $F_{\theta T} = -0.0199$ rad/MN and $F_{\theta M} = 0.0168$ rad/MNm.

2.5 Pile Stiffness Coefficient for Torsional Load

In order to complete the stiffness matrix, it is necessary to somehow determine the stiffness coefficient of the pile undergoing torsional load. A torsion load test is a non-standard one that is only done if there is a special need.

The pile flexibility coefficient $F_{\theta M}$ for torsion load can be obtained approximately, by the solution of the pile in the elastic continuum [11]. The soil elastic modulus can be constant or linearly increasing with depth. For slender piles ($L/d > 20$) of common stiffness ($10^{-1} < G_p I_0 / G_s d^4 < 10^3$ or $10^{-1} < G_p I_0 / n_G d^5 < 10^4$), by the interpolation method, simple analytical equations can be obtained for the pile stiffness coefficient under torsion load:

$$G_s = \text{const}, \; K_{\mathfrak{M}\theta} = 1/F_{\theta\mathfrak{M}} \approx 2d\sqrt{G_p G_s I_o} \tag{3a}$$

$$G_s = n_G z, \; K_{\mathfrak{M}\theta} = 1/F_{\theta\mathfrak{M}} \approx \sqrt[3]{n_G \left(G_p I_o d\right)^2} \tag{3b}$$

In the Eq. (3), $G_p = 12.1$ GPa is the pile shear modulus, $I_0 = 0.009$ m^4 is the polar moment of inertia, d is the pile diameter and G_S and n_G are the soil shear modulus and the gradient of the shear modulus. Based on the results of CPT (Fig. 6), the q_c - gradient is $n_{qc} \approx 1.1$ MPa/m, so the gradients of soil elastic and shear modulus are: $n_E \approx 2.2$ and $n_G \approx 0.85$ MPa/m. The coefficient of flexibility under torsion load is:

$$F_{\mathfrak{M}\theta} \approx {}^{-3}\!\!\sqrt{0.85(108.9 \cdot 0.55)^2} = 0.0690 \text{ rad/MNm}$$

When flexibility matrix [$\boldsymbol{F}_p$] is determined, the stiffness matrix [$\boldsymbol{K}_p$] can be obtained by inversion of the [$\boldsymbol{F}_p$].

$$\left[\boldsymbol{K}_p\right] = \left[\boldsymbol{F}_p\right]^{-1} = \begin{bmatrix} F_{sN} & 0 & 0 & 0 \\ 0 & F_{tT} & F_{\theta T} & 0 \\ 0 & F_{tM} & F_{\theta M} & 0 \\ 0 & 0 & 0 & F_{\theta\mathfrak{M}} \end{bmatrix}^{-1} \tag{4}$$

After substituting the calculated values of the pile flexibility and stiffness coefficients in Eq. (4), the pile flexibility and stiffness matrix, without pile interaction included, are the following:

$$\left[\boldsymbol{F_p}\right] = \begin{bmatrix} 0.0121 & 0 & 0 & 0 \\ 0 & 0.0383 & -0.0199 & 0 \\ 0 & -0.0199 & 0.0168 & 0 \\ 0 & 0 & 0 & 0.0690 \end{bmatrix}$$

$$\left[\boldsymbol{K_p}\right] = \left[\boldsymbol{F_p}\right]^{-1} = \begin{bmatrix} 82.64 & 0 & 0 & 0 \\ 0 & 67.48 & 79.69 & 0 \\ 0 & 79.69 & 153.52 & 0 \\ 0 & 0 & 0 & 14.49 \end{bmatrix}$$

3 STIFFNESS MATRIX OF THE PILE IN THE GROUP

The stiffness matrix determined in the manner described above cannot be directly applied to the calculation of the displacement of the pile group, since their mutual interaction is not taken into account. Interactions can be ignored only at large mutual distances between piles ($e/d >$ min 6 - 8) which is an extremely rare case in practice.

When, as in this paper, the Winkler soil model is used, the effect of interaction between the piles can be taken into account by reducing the modulus of soil reaction or the pile stiffness coefficient, in the manner described below.

3.1 Interaction Between Vertically Loaded Pilegroup

In the vertically loaded pile group, the pile interaction is introduced by reducing the pile stiffness coefficients. The reduction depends on the pile's position in group, pile length and distances between piles, soil stiffness and on the ratio of the load carried by the pile base and the pile shaft. The redistribution of the pile forces and the increase in the group settlement due to pile interaction is approximated by the method of interaction factors [17], followed by the equation:

$$\alpha_{ij} = A\left(\frac{e_{ij}}{d}\right)^{-B}, \quad \alpha_{ij} = C + D \cdot \ln\left(\frac{e_{ij}}{d}\right), \quad \alpha_{ii} = 1 \tag{5}$$

In Eq. (5), α_{ij} is the factor of interaction between the i^{th} and j^{th} pile and e_{ij} are the mutual distances between piles. The coefficients A, B, C and D are around $A = 0.6$ - 0.9, $-B = 0.6$ - 1.2, $C = 0.95$ - 1.0 and $-D = 0.25$ - 0.3, and can be reliably determined on the basis of the pile load test.

In addition, other, more complex equations can be used, like the method proposed by Randolph &Wroth [5]:

$$\alpha_{ij} = \xi \frac{\ln\left(r_m / e_{ij}\right)}{\ln\left(2r_m / d\right)}, \quad r_m = 2.5L\rho\left(1 - \nu_s\right) \tag{6}$$

In Eq. (6), ξ is the factor of diffraction [25], r_m is the pile radius of influence, ν_s is the soil Poisson's ratio and ρ is the coefficient of inhomogeneity as the ratio of the average soil stiffness along the pile and soil stiffness at the pile base. The diffraction factor comprises the pile axial stiffness, the pile relative length (L/d) and the soil inhomogeneity. It can be determined analytically by Eq. (10) or graphically (Fig. 10).

$$\xi = \frac{\eta + \sin(\eta) + \Omega^2\left(\sin(\eta) - \eta\right)}{2\sin(\eta) + \left(1 + \Omega^2\right) + 4\Omega\cos(\eta)} + \\ + \frac{+2\Omega\left(\cos(\eta) - 1\right)}{2\sin(\eta) + \left(1 + \Omega^2\right) + 4\Omega\cos(\eta)}, \quad \eta = 2L\lambda_\tau \tag{7}$$

For long piles, the diffraction factor tends to 0.5, for piles on stiffer stratum it is between 0 - 0.5, and for floating piles it is between 0.5 - 1.0. After the interaction factors are calculated, the stiffness coefficients of the pile in the group and the settlement factor of the pile group can be calculated by the following equations:

$$K_{\mathrm{Ns},i} = K_{\mathrm{Ns}} \sum_{j}^{n} \alpha_{ij}^{-1},\ R_{\mathrm{S}} = \frac{n}{\sum_{i} \sum_{j} \alpha_{ij}^{-1}} \tag{8}$$

In Eq. (8), $[\boldsymbol{\alpha}]$ is the matrix of the interaction factors, and K_{Ns} and $K_{\mathrm{Ns},1}$ are the axial stiffness of the i^{th} pile without and with pile interaction influence. The term R_s is the pile group settlement factor, which represents the ratio between the settlement of the individual pile and the pile group under the same average load.

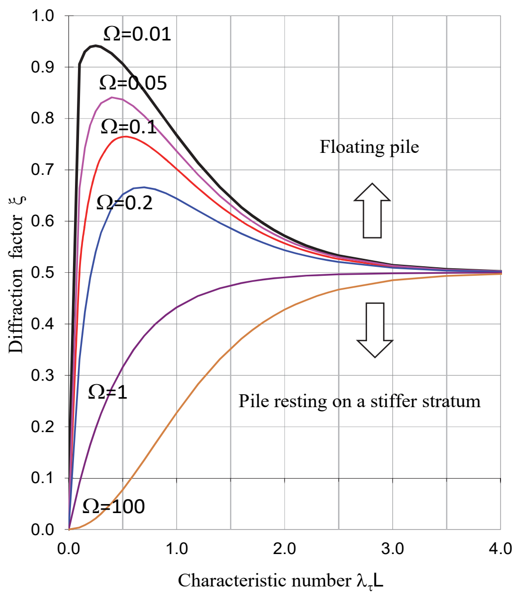

Figure 10 Diffraction (or attenuation) factor ξ

In order to determine the reduced stiffness coefficients according to the Eq. (8), based on the position of the pile in the group, it is first necessary to determine the radius r_m and the diffraction factor ξ. Since the soil average shear modulus along the pile is around $G_s \approx 3.2$ MPa, and the shear modulus at the pile base is around $G_b \approx 9.2$ MPa, the coeffi-cient of inhomogeneity and the radius r_m are:

$$\rho = G_s / G_b = 3.2/9.2 = 0.35$$

$$r_m = 2.5 \cdot 12.8 \cdot 0.35 (1 - 0.3) = 7.84 \text{ m}$$

The pile diffraction factor and the interaction factor are:

$$\left. \begin{aligned} \lambda_\tau L &= \sqrt{\frac{4.0 \cdot 1.73}{29{,}500 \cdot 0.24}} 12.8 = 0.313 \\ \Omega &= \frac{48.0 \cdot 0.24}{0.0313 \cdot 29{,}500 \cdot 0.24} = 0.052 \end{aligned} \right. \rightarrow \ \xi(\lambda_\tau, L, \Omega) = 0.84$$

$$\alpha_{ij} = 0.84 \frac{\ln(7.84/e_{ij})}{\ln(2 \cdot 7.84/0.55)} = 0.67 - 0.25 \ln\left(\frac{e_{ij}}{d}\right)$$

For the analysis of the pile interaction effect, a 4 × 4 pile group, in two-axially symmetric arrangement, connected by an ideally rigid cap is adopted (Fig. 11).

By Eq. (8), the pile group settlement factor is $R_s = 4.0$. The axial stiffness of the piles is: $K_{\mathrm{Ns},1} = 8.43$, $K_{\mathrm{Ns},2} = K_{\mathrm{Ns},5} = 20.32$ and $K_{\mathrm{Ns},6} = 3.23$ MN/m. Due to interaction, the axial stiffness of piles in the central part is reduced mostly, less for the edge and the least at the corner piles. It should be kept in mind that the stiffnesses are rather a mathematical than a physical magnitude, because they contain total settlement, which is the sum of the settlements from the pile force and the forces in the adjacent piles. The physical stiffness is greater because it is the ratio of the force in the pile and the settlement caused only by that force.

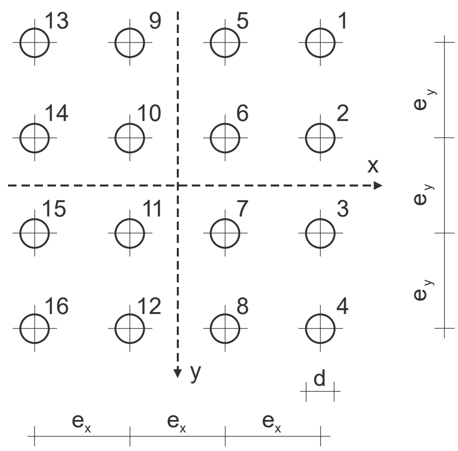

Figure 11 Disposition of the piles under the rigid cap ($e_x = e_y = 3d$)

3.2 Interaction Between Horizontally Loaded Pile Group

For the pile group loaded with horizontal force, the mutual influence is also introduced by reducing the pile stiffness coefficient. The reduction depends on the direction of force, the distance between piles and their position in the group. A method for reducing the piles stiffness coefficient in order to calculate their lateral displacement is known in the literature as the "method of *p*-multiplier". A similar procedure is used in the standard DIN1054 [26].

For a long pile in the soil with a constant Eq. (9a) or linearly variable modulus of soil reaction with depth Eq. (9b), the reduced values for the *i*-th pile are:

$$\lambda_h L > 4 \quad \rightarrow \quad k_{\mathrm{h},\,i} = k_\mathrm{h} \left(\alpha_x \alpha_y\right)_i^{4/3} \tag{9a}$$

$$\eta_h L > 4 \quad \rightarrow \quad n_{\mathrm{h},i} = n_\mathrm{h} \left(\alpha_x \alpha_y\right)_i^{5/3} \tag{9b}$$

The reduction coefficients α depend on the position of the pile in relation to the force direction and piles mutual distance.There are two types of reduction coefficients, that is α_x which depends on the distance of the piles in the force direction e_x and $\alpha_{y\mathrm{A}}$ and $\alpha_{y\mathrm{Z}}$ which depend on the distance of the piles e_y in the direction normal to the force direction.

For the first column, the coefficient $\alpha_x = 1$, while for all other columns is the same and $\alpha_x < 1$. Coefficients $\alpha_{y\mathrm{A}}$ are the same for the final lateral rows and the coefficients $\alpha_{y\mathrm{Z}}$ are the same for all inner rows. The rows and columns are parallel and normal to the direction of force (Fig. 12 and Fig. 13).

If the distance of the piles in the direction normal to the force is $e_y/d \geq 3$, the coefficients $\alpha_{y\mathrm{A}} = \alpha_{y\mathrm{Z}} = 1$. If the distance of the piles in the direction of the force is $e_x/d \geq 6$, the coefficient $\alpha_x = 1$. The reduction is done in the direction of the coordinate axis, separately for the horizontal forces H_x and H_y. According to Fig. 13, the coefficients are:

$$e/d = 1.65/0.55 = 3.0$$

$$\alpha_x = 0.625, \quad \alpha_{y\mathrm{A}} = 1.00, \quad \alpha_{y\mathrm{Z}} = 1.00$$

Starting from the layout of the piles and Eq. (9b), the reduced gradients of the modulus of soil reaction n_h for the effect of the component H_x in the axis $+x$ direction are:

$$i = 1, 2, 3, 4: \quad n_{\mathrm{hx},i} = 38.0(1.0 \cdot 1.0)^{5/3} = 38.0 \ \mathrm{MN/m^3}$$

$$i = 5 - 16: \quad n_{\mathrm{hx},i} = 38.0(0.63 \cdot 1.0)^{5/3} = 17.6 \ \mathrm{MN/m^3}$$

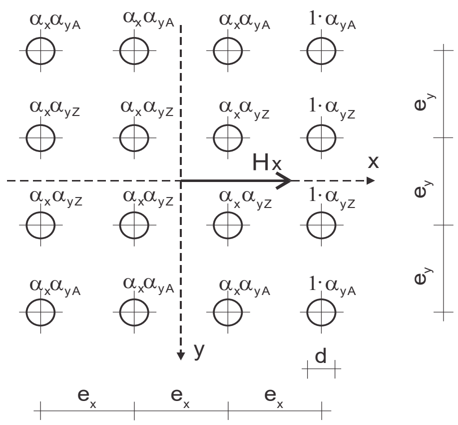

Figure 12 Reduction factors α in dependence of the pile position in the group

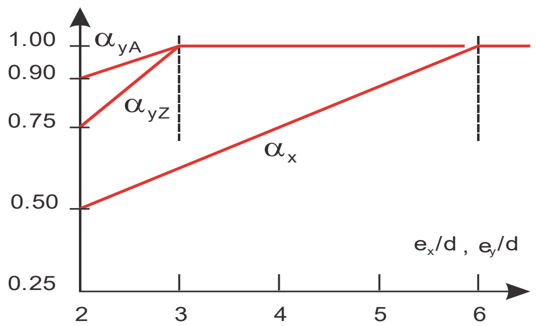

Figure 13 Reduction factors α in dependence of the piles mutual spacing

Based on the reduced values of the modulus of soil reaction, a matrix of flexibility and stiffness can be determined for each pile. For example, for horizontal force in the direction of $+x$ axis, the stiffness matrices for the most characteristic piles 1, 2, 5 and 6 are:

$$[\boldsymbol{K}_p]_{1x} = [\boldsymbol{F}_p]_{1x}^{-1} = \begin{bmatrix} 38.43 & 0 & 0 & 0 \\ 0 & 67.48 & 79.69 & 0 \\ 0 & 79.69 & 153.52 & 0 \\ 0 & 0 & 0 & 14.49 \end{bmatrix}$$

$$[\boldsymbol{K}_p]_{2x} = [\boldsymbol{F}_p]_{2x}^{-1} = \begin{bmatrix} 20.32 & 0 & 0 & 0 \\ 0 & 67.48 & 79.69 & 0 \\ 0 & 79.69 & 153.52 & 0 \\ 0 & 0 & 0 & 14.49 \end{bmatrix}$$

$$[\boldsymbol{K}_p]_{5x} = [\boldsymbol{F}_p]_{5x}^{-1} = \begin{bmatrix} 20.32 & 0 & 0 & 0 \\ 0 & 42.52 & 58.57 & 0 \\ 0 & 58.57 & 131.61 & 0 \\ 0 & 0 & 0 & 14.49 \end{bmatrix}$$

$$[\boldsymbol{K}_p]_{6x} = [\boldsymbol{F}_p]_{6x}^{-1} = \begin{bmatrix} 3.23 & 0 & 0 & 0 \\ 0 & 42.52 & 58.57 & 0 \\ 0 & 58.57 & 131.61 & 0 \\ 0 & 0 & 0 & 14.49 \end{bmatrix}$$

It is obvious that the stiffness coefficients are reduced due to the pile mutual interaction. Keeping in mind the pile symmetry, the matrices for all the other piles in the group can be determined. Based on them, a global stiffness matrix of a pile group with an ideally rigid cap can be formed.

For the direction of the force in the $+y$ axis direction, the reduced gradients of the modulus of soil reaction are:

$$i = 4, 8, 12, 16$$
$$n_{\mathrm{hy},i} = 38.0(1.00 \cdot 1.00)^{5/3} = 38 \ \mathrm{MN/m^3}$$

$$i = 1, 2, 3, 5, 6, 7, 9, 10, 11, 13, 14, 15$$
$$n_{\mathrm{hy},i} = 38.0(0.63 \cdot 1.00)^{5/3} = 17.6 \ \mathrm{MN/m^3}$$

In a similar way, the stiffness matrices can be determined for the horizontal force in the direction of $+y$ axis.

However, the horizontal force is generally not parallel with either axis, but it closes an arbitrary angle φ with the x-axis. Since two different values of n_hx and n_hy cannot be entered at the same time, the equivalent $n_{\mathrm{h}\varphi}$ which depends on the horizontal force direction (φ) must be used.

If n_hx and n_hy are understood as the main radius of the ellipse, then the size $n_{\mathrm{h}\varphi}$ is the radius of the ellipse in the direction of the horizontal force and can be calculated as:

$$n_{\mathrm{h}\varphi,i} = n_{\mathrm{hx},i} n_{\mathrm{hy},i} \left(\frac{1 + \tan^2(\varphi)}{n_{\mathrm{hy},i}^2 + n_{\mathrm{hx},i}^2 \tan^2(\varphi)} \right)^{0.5}$$
$$\tan(\varphi) = \frac{H_y}{H_x} \tag{10}$$

According to Eq. (10), the $n_{\mathrm{h}\varphi}$ value for each individual pile in the group, becomes the function of position in the group and the direction of the horizontal force. It should be kept in mind that the angle φ is between $0 \le \varphi \le \pi/2$. If the horizontal force acts in the direction of -x or -y axis, then it is necessary to adjust the equation to obtain logical results.

4 CONCLUSSION

The paper presents the procedure for determining the stiff-ness matrix of the in the group, based on the results of pile load test, taking into account the pile mutual interaction. The location of the pile load test is in the NIS oil refinery in Pancevo. The tests were performed for vertical compression and extension load and for horizontal load.

Based on the results of geomechanical investigations, it is concluded that the soil strength and deformability grow nearly linearly with depth, so that for the calculation of the pile flexibility coefficient, the soil can be approximated as a Winkler model with a linearly increasing modulus.

By applying this model, all the coefficients of the pile flexibility matrix can be determined, for which otherwise it would be necessary to perform the moment load test. The pile group interaction was determined approximately, using some of the usual practical methods.

According to the design at the location, shallow and deep foundations on piles are provided. To illustrate the methodology, one of the pile groups was adopted, on which the piles were in two-axis symmetry. Unfortunately, there are no results of geodetic surveys, based on which the presented concept in this paper could be fully verified.

It should be emphasized that the entire calculation is linear, since the secant stiffness coefficients of the piles for the workload level are used as constant values. During the calculation, due to the interaction, the stiffness coefficients are changing, but mainly as a mathematical value. This is due to the fact that the force on the pile is divided by total displacement, which is the sum of displacements due to the force on the pile and due to the forces on all the adjacent piles. Physical stiffness, as the ratio of force and the displacement due to this force, varies much less, so the linearity assumption around the workload level is justified.

In any case, the procedure shown here can be extended to nonlinear analysis, by introducing the hyperbolic law to describe the load-displacement curve from pile load test, but then the analysis becomes incomparably more complex.

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Contact information:

Petar SANTRAČ, Prof Dipl.Ing. MSc PhD CEng
(Corresponding author)
Faculty of Civil Engineering Subotica, University of Novi Sad,
Kozaračka 2a, 24000 Subotica, Serbia
E-mail: santrac@geoexpert.rs, santrac@gf.uns.ac.rs

Nenad ŠUŠIĆ, Dipl.Ing. MSc PhD CEng
Institut IMS Beograd,
Bulevar Vojvode Mišića 43, 11000 Beograd, Serbia
E-mail: nenad.susic@institutims.rs

Željko BAJIĆ, Dipl.Ing. MSc CEng
Geo Expertd.o.o. Subotica,
Adolfa Singera 11, 24000 Subotica, Serbia
E-mail: bajic@geoexpert.rs