Icarus

The Precession Constant and its Long-Term Variation

--Manuscript Draft--

Manuscript Number:	ICARUS-D-20-00066R1
Article Type:	Letter
Keywords:	Precession constant; dynamical flattening and Earth rotation; mantle convection; glacial isostatic adjustment; modern climate change
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Abstract:	The dynamical flattening of the Earth, H , related to the precession constant, is a fundamental astro-geodetic parameter that appears in studies of the Earth's rotation and orbital evolution. We present numerical predictions and observations of the variation in H over time scales ranging from tens of millions of years to decades. The geophysical processes controlling this variation include solid-state convection in the rocky mantle of the Earth that drives plate tectonics, isostatic adjustments due to ice age loading, and ice-ocean mass transfer linked to modern global climate change. The time dependence of H is complex and non-linear, and thus, in contrast to previous suggestions, cannot be captured by a constant rate parameter.
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Opposed Reviewers:	W Richard Peltier, PhD Professor, University of Toronto wrp@physics.utoronto.ca Prof. Peltier has a long-standing animosity toward my group and my students. He would be incapable of an objective review. My opposition to him seeing any of my work before publication is absolute. If the journal were to consider sending him the manuscript - which is unlikely given that he is not an expert - I would ask that the paper

	be withdrawn.
Response to Reviewers:	

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March 15, 2020

Dear Editor:

We have submitted a revised version of the manuscript “The Precession Constant and its Long-Term Variation” for your continued consideration as a Letter to *Icarus*. The manuscript is co-authored by Siavash Ghelichkhan, Jocelyn Fuentes, Mark J. Hoggard, Fred D. Richards and Jerry X Mitrovica (corresponding author). We note that the order of the first and second listed authors of the original manuscript has been switched in the revised version. This reflects the effort required to revise the article and the change has been fully agreed upon by all authors.

As you will note from the response letter attached to this submission, we have comprehensively addressed all of the comments raised in the single, positive review that we received. We believe that these changes have strengthened the manuscript and we genuinely enjoyed considering and addressing the interesting issues that the reviewer raised.

Given the combination of astronomical and geological issues discussed in the manuscript, we continue to believe that *Icarus* is the ideal journal in which to present our results.

We look forward to hearing from you in due course,

Sincerely,

Jerry X. Mitrovica
Frank B. Baird, Jr., Professor of Science

Dear Editor:

We have submitted a revised version of the manuscript “The Precession Constant and its Long-Term Variation” (ICARUS-D-20-00066). The revised manuscript received a single, positive review that raised three points of concern. In the material below, we respond comprehensively to each point, and indicate the associated changes to the text. The reviewer’s comments appear in blue and our responses are in black – text quoted from the original or revised manuscript is indented. We note that the revised manuscript switches the order of the first and second authors. All authors have agreed on this change, which reflects the work that was performed in revising the manuscript.

Reviewer #1: The paper presents more realistic computations of the Earth's precession constant variations, which has important implications for precession and obliquity. Given these important implications and the significant differences between the calculations presented in the paper and previous models assuming a single rate term, this paper would be a valuable addition. However, some issues need to be addressed before I can recommend it paper for publication.

We thank the reviewer for the positive assessment of the novelty and importance of our manuscript.

We recognized after reading the review that our original manuscript included sloppy terminology that may have played a role in some of the reviewer’s comments. Our manuscript is concerned with changes in the ellipticity or flattening of the Earth and in the text of the manuscript we used these terms interchangeably with “precession constant”. The latter formally refers to the annual rate of precession of the equinoxes, while the former is a measure of the flattening (or ellipticity) of the Earth. The precession constant is controlled by the dynamical flattening (or ellipticity), but in a formal sense they are not the same thing. It has been common in the literature to equate the two terms, but to do so is to be imprecise – as we were.

A variety of language is also used in the literature to distinguish the terms, for example Burša et al. (2008) write: “The coefficient associated with the precession constant, *H*, which is often called dynamical ellipticity or dynamical flattening …”. The subtle distinction between the two is important and we have revised the text to make sure we do not obscure the distinction.

Major comments:

Eq. 2 is only valid for deformations that do not involve a spherical component of degree-0 (see the paragraph below Eq. 9 in Rochester and Smylie). Therefore, it is only valid for an incompressible interior unless the forcings responsible for the deformations do not involve degree-0, which is difficult to justify for mantle convection, ice age perturbations, and recent melting of glaciers. The incompressible interior assumption needs to be justified and stated clearly.

The reviewer is correct to point out that a degree-0 term will impact the inertia tensor of the Earth – there are many examples in our previous work on rotational dynamics of terrestrial planets where we make this connection explicit (e.g., Matsuyama et al., *JGR*, v. 111, 2006; equation 3). The contribution of the degree-0 term is commonly left out of the expression for the perturbation in the inertia tensor (as in equation 2 of the original manuscript - see, for example, Burša et al., 2008) because it impacts all three principal axes (*A*, *B* and *C*) identically. That is, while a degree-0 volume term would introduce an inequality in equation (2), it would have relatively small impact on our expressions for the dynamical flattening or ellipticity (or the results based upon them).

Before addressing this issue in more quantitative terms, we first respond to the reviewer's argument that "Therefore, it is only valid for an incompressible interior unless the forcings responsible for the deformations do not involve degree-0, which is difficult to justify for mantle convection, ice age perturbations, and recent melting of glaciers". In fact, the forcings associated with ice age perturbations and the recent melting of glaciers have no degree-0 component because these processes conserve mass. That is, the surface mass load driving these signals includes changes in both ice *and* sea level components – and conserving the total surface mass represents a key constraint in the calculations. As we wrote on Line 109 of the manuscript:

> Our prediction of perturbations to the precession constant (Figure 1B) is generated using a theory of ice age dynamics that involves a gravitationally self-consistent treatment of sea level changes (Kendall et al., 2005).

We have revised this sentence to read (line 112):

> Our prediction of perturbations to the precession constant (Figure 1B) is generated using a theory of ice age dynamics that involves a gravitationally self-consistent treatment of sea level changes constrained to conserve the total (ice plus ocean) surface mass (Kendall et al., 2005).

All such calculations ensure that the change in ice-plus-ocean mass is identically zero.

In regard to mantle convection, the situation is more nuanced. The reviewer writes: "(equation 2) is only valid for an incompressible interior". First, the equations governing mantle convection that we solve incorporate compressibility in the form of the anelastic liquid approximation, which ensures a proper treatment of compressibility effects arising from the orders of magnitude increase in pressure with depth in the mantle, as indicated within the original (and revised) Appendix:

Once the temporal evolution of the mantle flow field has been successfully reconstructed, we calculate the time history of dynamic ellipticity, *H*. For this purpose, we solve the governing, coupled system of Stokes and Poisson's equations using an instantaneous flow methodology that includes the effects of self-gravitation and compressibility and assumes a free-slip (no tangential stress) boundary condition (Corrieu et al., 1995).

Furthermore, our formalism for reconstruction of mantle structure comes from an explicit treatment of compressibility effects in the geodynamic adjoint equations (Ghelichkhan & Bunge 2016, cited in the Appendix). So, our governing equations include compressibility. However, this does not mean that the Earth is subject to net volume changes because an increase in density in one mantle location is balanced by a decrease elsewhere. Nevertheless, the main process that will drive a degree-0 change in the shape of the Earth is secular cooling, a process that is not included in our mantle convection simulations and is ignored in our equation (2). How big is this term?

To answer that question, we can augment equation (3) of the original manuscript (corrected for the sign error noted below) to the form that accounts for a degree-0 perturbation to the inertia tensor:

$$\delta H = \left[3/2 - H\,(1 + \delta C_{(0,0)}/\,\delta C_{(2,0)})\right]\,\delta C_{(2,0)}/C$$

This expression differs from the equation in the manuscript through the addition of the term $\delta C_{(0,0)}/\,\delta C_{(2,0)}$.

How much has the Earth's radius changed with time due to cooling and thermal contraction? There is general consensus that the temperature of the mantle during the Archean Period was of the order of 100°C greater than today (Ganne & Feng, *Geochem. Geophys. Geosys.*, v. 18, 2017), or about 1° over 50 Myr. A simple scaling analysis that equates the change in volume (δV) due to a change in temperature (δT) (i.e., $\delta V = V\,\alpha\,\delta T$, where α is the coefficient of thermal expansion, ~3×10^{-5} K^{-1}) with the change in volume due to a small change in radius ($\delta V = 3V\,\delta r/a$, where a is the radius of the Earth), yields the following expression for the change in radius: $\delta r = \alpha\,\delta T\,a/3$. Plugging in the above numbers, yields a contraction of ~65 m since 50 Ma. This agrees well with estimates shown in Tsuchiya et al. (*Geosci. Frontiers.*, v. 4, 2013), ~80 m over the same time period and we will adopt this value for δr due to contraction. Perturbing the expression for the moment of inertia of a sphere yields $\delta C_{(0,0)} = (4/5) M_e a \delta r$ (where δr is a negative number, and M_e is the Earth's mass).

Next, we turn to the term $\delta C_{(2,0)}$. We can simply look up this value from results of our convection simulation, but we can gain some physical insight by considering the change in the flattening, *f*, of the Earth (i.e., the difference in the equatorial and polar radius). From Figure 1A, the peak-to-peak perturbation to $\delta H/H$ reaches 0.0043 over the past 50 Myr. A paper by members of our group (Morrow et al., *Geophys. J. Int.*, v. 191, 2012) derives the following relationship between the flattening and the perturbation $\delta H/H$ (see their equation 3): $f = 1.06 \times 10^4 \ \delta H/H$, and thus the result in our Figure 1A is equivalent to a decrease in the flattening of ~ 45m over this same period. Perturbing the expression for the moment of inertia of an ellipsoid of revolution yields $\delta C_{(2,0)} = (4/15) M_e a f$ (where f is a negative number). Thus, $\delta C_{(0,0)} / \delta C_{(2,0)}$. ~ 5, and $H(1+\delta C_{(0,0)} / \delta C_{(2,0)})$~6H~0.02 (which is << that the leading term in the above equation, 3/2). We conclude that ignoring the impact on $\delta H/H$ of thermal contraction introduces an error that is of order 1%.

To address this issue in the manuscript, we have performed a series of revisions. First, above equation (2), we have changed the original text:

> Perturbing Equation (1), and using the fact that the trace of the inertia tensor is invariant during deformation (e.g., Rochester and Smylie, 1974)

to read

> Perturbing Equation (1), and using the fact that the trace of the inertia tensor is invariant during non-uniform deformation (e.g., Rochester and Smylie, 1974) … (NB. we comment in Section 3 on the impact of uniform deformation on these equations)

Furthermore, we have added the following text as a penultimate paragraph to the Results section:

> The calculations in Figure 1, since they are based on Equation (2), do not include the impact on the inertia tensor of a uniform, degree-0 deformation of the Earth. If we included this spatially uniform signal in the theory, Equation (3) would be revised to
>
> $$\delta H = [3/2 - H\,(1 + \delta C_{(0,0)} / \delta C_{(2,0)})]\ \delta C_{(2,0)}/C, \qquad (8)$$
>
> where the subscript denotes the spherical harmonic degree and order of the structure contributing to the inertia perturbation. In our calculations of perturbations in $\delta H/H$ due to ice mass changes (Figure 1B), we include complementary sea level changes and the total mass of the surface load is conserved (i.e., it has no degree-0 component). The same must be true for the processes responsible for the observations that form the basis of Figure 1C. Thus, in these cases, there is no degree-0 deformation, $\delta C_{(0,0)}=0$, and the above expression collapses to Equation (3). While our calculations of the perturbation to $\delta H/H$ driven by mantle convection adopt a compressible flow model, any changes in the volume of the Earth are negligible. However, these calculations do not include secular cooling and thermal contraction of the Earth. Estimates of this process suggest that the reduction in Earth radius over the past 50 Myr due to thermal contraction has been ~80 m (Tsuchiya et al., 2013). Using this value, and the results in Figure 1A, yields the estimate

$\delta C_{(0,0)}/ \delta C_{(2,0)} \sim 5$, and $H(1+\delta C_{(0,0)}/ \delta C_{(2,0)}) \sim 6H$. This value is of order 1% of the leading term of 3/2 in Equation (8) and neglecting it in adopting Equation (4) remains justified.

L60-61: The conservation of the Earth's rotational angular momentum needs to be justified. What is conserved is the angular momentum of the Earth-Moon system, which has contributions from Earth's rotational angular momentum and the orbit angular momentum. This seems important given that the authors are considering tidal dissipation perturbations to dH/dt.

In fact, in the original manuscript we did not consider tidal dissipation perturbations to the dynamical flattening in our Figure 1 (though it is mentioned in the Introduction). Nevertheless, the reviewer's comment regarding conservation of angular momentum is of course correct, but our text at this point in the manuscript was intended simply as a statement of the impact of the various processes that we are considering (mantle convection, ice age dynamics, modern ice melting) on the rotation rate – it was not intended as a suggestion that these processes are the only ones that can influence the rotation rate. In this regard, since none of these processes changed the rotation rate significantly over the longest time window we consider in the manuscript (50 Myr in Figure 1A), each process can be considered to have an independent effect on the rotation rate. Consider, as an example, the tidal dissipation process the reviewer is referring to. Geological records (e.g., Williams, *Geophys. Res. Lett.*, v. 24, 1997) suggest that the rotation rate at 620 Ma was ~10% faster than today (8×10^{-5} rad/s at 650 Ma versus 7.3×10^{-5} rad/s at present); assuming linearity, this would indicate a small reduction in the rotation rate of < 1% since 50 Ma. (The same upper bound would emerge if one used the present-day tidal dissipation rate to estimate the change in rotation rate at 50 Ma; see Laskar et al., *Astron. Astrophys.*, v. 270, 1993.)

In any case, it is important to emphasize that Equation (5) of the original manuscript that the reviewer is pointing to has no bearing on the focus of our study – that is, predictions of $\delta H/H$ arising from mantle convection, ice age dynamics and modern melting of ice sheets and glaciers – and we have thus deleted it from the manuscript.

L68-71. The connection between the precession constant and the frequency of a perturbations is not clear. This should be explained in more detail, including an equation relating the precession constant and the frequency of the perturbation.

This relationship is a standard proportionality in the case when the precession frequency is out of resonance with any gravitational forcing in the solar system (see, e.g., Williams, *Astron. J.*, v. 108, 1994). That publication, together with Laskar et al. (1993) show the relevant equations and we have added citations to both of them at this point in the text.

Minor comments:

Writing Eq. 1 also explicitly in terms of J2 would help readers make the connection between the precession constant and J2, especially for those who are not familiar with the definition of J2.

To address this comment, we have revised the sentence above the original Equation 7 (now Equation 6):

> Comparing equations (1) and (6) yields the following relationship between perturbations in H and J_2

to read:

> Combining equations (1) and (5) yield the following relationship,
>
> $$H = M_e a^2/C\ J_2. \qquad (6)$$

L45-48: I'm assuming that tidal dissipation results in a perturbation in dH/dt due to the perturbation on the rotation rate? It might be worth discussing the connection between tidal dissipation and dH/dt in more detail.

We agree, but feel that this discussion is best suited to the end of the Results section where we can point out that tidal dissipation, which is not modeled, also impacts the dynamical flattening. In particular, we have added the following as a final paragraph of that section:

> The results in Figure 1 do not include the impact on the dynamical flattening of an additional process mentioned in the introductory section, namely, tidal dissipation. The present level of tidal dissipation is slowing the Earth's rotation at a rate of $d\Omega/dt\ /\Omega = 8.8 \times 10^{-18}\ s^{-1}$ (e.g., Quinn et al., 1991) and the dynamical flattening will be approximately proportional to Ω^2. While the variation of tidal dissipation over time is uncertain, any effort to estimate the total change in dynamical flattening from all geophysical processes must include this contribution.

Eq. 3, the term in the square brackets should be [3/2-H] instead of [3/2+H]

Thank you for catching this typo. We have made the correction.

Eq. 7, the right-hand-side is missing a term associated with the perturbation of C. After some algebra and using H<<1, it does reduce to the expression in Eq. 7 but this should be explained in more detail. In particular, the same approximation used in Eq. 4 is being used here.

Agreed. We should have made the underlying approximation clear in the original manuscript. We have revised the following text that appeared above the original Equation 7

> Comparing equations (1) and (6) yields the following relationship between perturbations in H and J_2

to read:

> Equations (1) and (5) yield the following relationship,
>
> $$H = M_e a^2/C\, J_2. \qquad (6)$$
>
> Taking the first variation of this expression, and once again using the fact that H << 1, yields

Figure 1A. The authors could discuss the physics behind the change in the sign of dH/dt around 20 Ma. Is there a major change in the convection perturbations at this time?

To investigate the physics of changes in dynamic flattening, we examined the so-called “dynamic geoid response functions”, which relate mantle structure to the corresponding gravitational equipotential figure of the Earth. Dynamic response functions encapsulate the gravitational signal of both internal density anomalies and the associated viscous boundary deflections of the CMB and surface. At the longest wavelengths (e.g., degree 2), these functions are positive in the upper mantle, but negative in the lower 1000 km of the mantle.

Current seismic tomography models and inferences of present-day mantle structure based upon model reconstructions of plate tectonics suggest the persistence of degree-two buoyancy distribution in the deep mantle, with two slow velocity anomalies beneath Africa and the Pacific Ocean that have been girdled by a continuous subduction system since the breakup of Pangea. In the transition zone, density anomalies are generally found to be anti-correlated with this deeper mantle structure.

A key implication of these theoretical and observational arguments is that present-day transition zone anomalies and deep mantle anomalies are both expected to constructively contribute to the geoid signal (due to the sensitivity kernel switching sign). Thus, an increase in dynamic flattening would be expected if either transition zone or deep mantle degree-2, order-0 anomalies have increase in amplitude through time, and vice versa. These contributions vary depending on the growth of Rayleigh-Taylor instabilities at the base of the mantle (the location of plume inception), and episodic motion of subducting slabs. Our analysis indicates that the general increase in $\delta H/H$ up to 20 Ma is caused by an increase in the degree-2, order-0 component of density anomalies in the deep mantle, accompanied by a further decrease in the transition zone (i.e. both anomalies grow up to this point). From 20 Ma to present, the transition zone anomalies begin to decrease in amplitude as slab material starts to sink beneath the transition zone, and this causes $\delta H/H$ to begin to decrease towards the present day. We have summarized this rather technical explanation by adding the following text to the manuscript (line 98):

> Our investigation of the evolving mantle heterogeneity in the adjoint model indicates that the increase in $\delta H/H$ from 50 Ma to 20 Ma is driven by an increase in the amplitude of long-wavelength density anomalies at the base of the upper mantle (the so-called transition zone) and the base of the lower mantle (i.e., above the fluid outer core). The subsequent change in trend reflects a progressive weakening of the transition zone signal after 20 Ma.

Fig. 1C. The connection between the negative dH/dt and melting from polar ice sheets could be made clearer by discussing the perturbations to the moments of inertia and J2.

Agreed. After the following text on line 129:

> Finally, we turn our attention to recent variations in the dynamical flattening on decadal time scales. Figure 1C shows the observed change in *H* across the satellite period, relative to 2012, derived from the results of Cheng et al. (2013). As discussed earlier, a change in the trend of the $\delta H/H_0$ time series, or equivalently J_2 (Equation 5), took place around the year 1990.

we have added:

> Prior to that date, the trend is dominated by the above-noted reduction in oblateness (and polar moment of inertia) since ~6 ka driven by the ongoing effects of the ice age. This trend continues after 1990, but the onset of significant melting of ice sheets at that time contributes an increase in oblateness (as ice melts near the poles and mass redistributes toward lower latitudes); the net signal is characterized by a reduced trend (i.e., the magnitude of dH/dt and dJ_2/dt decreases).

Once again, we thank the reviewer for raising the above issues. The associated revisions have improved the manuscript and we genuinely enjoyed the process of considering and addressing the very interesting points raised in the review.

Highlights

The dynamical flattening *H* appears in studies of Earth rotation and orbital evolution

Modern climate, ice age dynamics and mantle convective flow all drive variations in *H*

We quantify this variability using modern observations and new geophysical modeling

The variation in *H* is highly non-linear on time scales ranging from decades to 10^8 yr

Manuscript submitted to Icarus (Letter) on 30th August 2020

The Precession Constant and its Long-Term Variation

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Abstract

The dynamical flattening of the Earth, H, related to the precession constant, is a fundamental astro-geodetic parameter that appears in studies of the Earth's rotation and orbital evolution. We present numerical predictions and observations of the variation in H over time scales ranging from tens of millions of years to decades. The geophysical processes controlling this variation include solid-state convection in the rocky mantle of the Earth that drives plate tectonics, isostatic adjustments due to ice age loading, and ice-ocean mass transfer linked to modern global climate change. The time dependence of H is complex and non-linear, and thus, in contrast to previous suggestions, cannot be captured by a constant rate parameter.

Key words: Precession constant, dynamical flattening, mantle convection, glacial isostatic adjustment, climate change, Earth rotation

1 Introduction

The dynamic flattening of the Earth, H, is a measure of the difference between the polar moment of inertia (C) and the mean of the equatorial moments of inertia (A, B) of the planet:

$$H = \frac{1}{C}\left[C - \frac{1}{2}(A + B)\right]. \tag{1}$$

H is a fundamental parameter in precession and nutation theories of the Earth, as well as a series of other rotational normal modes of widely varying frequency (e.g. Wahr, 1981; Dehant & Capitaine, 1996; Chao, 2017). Dynamic flattening also plays an important role in a range of global geophysical studies – either explicitly or implicitly – through its connection to changes in the planetary spin rate (or, alternatively, "length-of-day" in geodesy) or dynamical form factor, J_2. For example: (1) satellite-based estimates of the secular rate of change of J_2 after ~1990 are thought to be impacted by the onset of significant polar ice sheet melting (Cox & Chao, 2002); (2) variations in J_2 associated with ongoing, residual effects of the last ice age, as well as with tidal dissipation and other factors, combine to explain the slowing of the

Earth's rotation rate over the past three millennia that has been estimated from ancient eclipse observations (Stephenson & Morrison, 1984, 1995; Stephenson, 2003; Mitrovica *et al.*, 2015); (3) perturbations in the dynamical flattening driven by mass changes arising from ice age effects and solid-state convective mantle flow alter Milankovitch (precession, obliquity) band variations in climate proxy records (Laskar *et al.*, 1993; Forte & Mitrovica, 1997; Mitrovica *et al.*, 1997; Pälike & Shackleton, 2000; Lourens *et al.*, 2001; Morrow *et al.*, 2012); and (4) geological measurements of the period of Earth's rotation during the Proterozoic Eon ($\sim$ 620 Ma) that are based on tidal rhythmites reflect long-term tidal breaking and dissipation in the Earth-Moon-Sun system (Williams, 1997), which would also be manifest as a trend in H.

The above discussion raises the question: Is the rate of change of dynamical flattening constant and, if not, what is the temporal structure of its variability? Burša *et al.* (2008) estimated that the long-term variation in $dH/dt = -8.45\times10^{-11}$ yr^{-1} from satellite data over the period 1979–2002 (Cox & Chao, 2002). They argue that this rate should be treated as a fundamental astro-geodetic parameter and suggest that the trend may remain valid for the past 650 Myr; this argument is based on the fact that the current rate of tidal breaking of the Earth's rotation rate would, if applied over this long time period, lead to a rotation period at 650 Ma (henceforth "Ma" denotes "million years ago") relatively close to the geological inference of ~21.8 hours. Putting aside geophysical modeling of variations in H, there are a variety of reasons to be sceptical of this argument. First, as noted above, the trend in the dynamical form factor, or dJ_2/dt, has varied significantly across the satellite period. For example, Roy & Peltier (2011) estimate rates of -3.7×10^{-11} yr^{-1} for the period 1976–1992 and -0.9×10^{-11} yr^{-1} for 1992–2009 (equivalent values for dH/dt are -11.1×10^{-11} yr^{-1} and -2.7×10^{-11} yr^{-1}, respectively). The estimate of $dH/dt = -8.45\times10^{-11}$ yr^{-1} in Burša *et al.* (2008), based on the results of Cox & Chao (2002), is thus a time-weighted average of these two values. Second, ongoing isostatic adjustment in response to ice age loading over the last few million years dominates the pre-1992 variation in H, and thus any trend in the precession constant over this period cannot be constant, but will instead reflect the time scales of ice age cyclicity. Third, the current dynamic flattening of the Earth is known to exceed the form of a rotating planet in hydrostatic equilibrium by $\sim$ 1% (Nakiboglu, 1982; Chambat *et al.*, 2010). This excess flattening is driven by convective flow in the mantle and will thus vary over the timescale associated with that process, which is tens of millions of years. Finally, the present rate of tidal breaking of the Earth's rotation must be anomalously high, because a back-projection of that rate leads to the so-called "time-scale problem" of Lunar origin (i.e., the Moon's orbital radii would place it at the Roche limit only ~2 billion years ago; Kaula & Harris, 1975). The likely resolution of this problem is that ocean tidal dissipation would have been lower during the time of the Pangean supercontinent (~340–170 Ma), and during previous supercontinent periods, with a consequent reduction in rates of change of both H and Earth's rotation period during these times (Hansen, 1982).

In this article, we describe new predictions of the variation in the dynamical ellipticity over time scales ranging from tens of millions of years to centuries, based on geophysical modeling of changes in Earth's shape associated with mantle convective flow over the past 50 million years and ice mass flux across the Plio-Pleistocence glacial cycles (i.e., the past 3 Myr). We also map a recent, satellite-derived time series of J_2 into a variation in H from 1976–2012. The predictions, together with the satellite derived time series, provide a measure of the natural — and human-induced — variability in Earth's dynamical flattening. This variability is complex, and it cannot be captured by a constant rate term.

2 Mathematical Background

Perturbing Equation (1), and using the fact that the trace of the inertia tensor is invariant during non-uniform deformation (e.g. Rochester & Smylie, 1974), i.e.,

$$\delta A + \delta B + \delta C = 0, \tag{2}$$

yields the following expression for the variation in H

$$\delta H = \left[\frac{3}{2} - H\right]\frac{\delta C}{C}. \tag{3}$$

(NB. we comment in Section 3 on the impact of uniform deformation on these equations). Since $H \sim 0.00327$, this expression can be approximated as

$$\delta H = \frac{3}{2}\frac{\delta C}{C}. \tag{4}$$

The dynamical form factor, J_2, is defined as

$$J_2 = \frac{1}{M_e a^2}\left[C - \frac{1}{2}(A + B)\right], \tag{5}$$

where M_e and a are the mass and radius of the Earth, respectively. Combining equations (1) and (5) yields the following relationship

$$H = \frac{M_e a^2}{C} J_2. \tag{6}$$

Taking the first variation of this expression, and once again using the fact that $H << 1$, yields

$$\delta H = \frac{M_e a^2}{C}\delta J_2. \tag{7}$$

Since $C \sim \frac{1}{3}M_e a^2$, the scaling factor on the right-hand side of Equation (7) is ~ 3. This simple relationship was applied to relate expressions for dJ_2/dt and dH/dt that were used in Section 1.

In the results below, we will consider predictions and observations of the relative perturbation in the dynamic flattening, $\delta H/H_0$, where H_0 is the present-day value (0.003274) and the perturbation δ is defined relative to this value (Figure 1). As a guide to interpreting the impact of such signals, if a dynamic ellipticity of value H' is connected to an orbital frequency (for example, of precession or obliquity variations) of f', then, in the absence of resonance effects, the perturbation $\delta H/H'$ would yield a proportional perturbation in the associated frequency of $f'\delta H/H'$ (Laskar *et al.*, 1993; Williams, 1994).

3 Results

Plate tectonics is driven by thermochemical convection within the Earth's mantle, a process that also leads to perturbations in the shape of the solid surface, core-mantle boundary, and gravitational field of the planet on a wide range of spatial scales. In the 1980s, global geophysical research focused on numerical and theoretical modeling of the process using constraints from satellite-derived estimates of Earth's long-wavelength gravity field (e.g., Richards & Hager, 1984; Ricard *et al.*, 1984; Hager *et al.*, 1985). These efforts combined tomographic models of seismic velocity variations in the mantle with experimental constraints from mineral physics on the mapping between these velocities and density (or, equivalently, buoyancy), with the goal of constraining the depth-dependent variation of mantle viscosity. While this approach provides invaluable insights on mantle dynamics, trade-offs between mantle buoyancy and viscosity render results subject to considerable uncertainty (Thoraval & Richards, 1997). Subsequent work therefore extended these studies to consider a wider range of present-day observations, including plate velocities, perturbations to surface topography, and excess ellipticity of the core-mantle boundary as inferred from the period of the Earth's free core nutation (e.g. Forte & Peltier, 1987; Lithgow-Bertelloni & Richards, 1998; Gurnis *et al.*, 2000; Forte & Mitrovica, 2001; Simmons *et al.*, 2006).

A number of studies have extended the present-day snapshot of mantle dynamics, the focus of the above analyses, to model the time history of the system. These analyses were generally based on "backward advection" of the governing field equations under the caveat that thermal diffusion is treated as negligible, since it is not temporally reversible in a unique sense and is not tractable due to numerical instabilities (Steinberger & O'Connell, 1997; Conrad & Gurnis, 2003; Moucha *et al.*, 2008). A major limitation of this approach is that it produces transient behaviour within the thermal boundary layers (regions at the base and top of the convecting mantle, which are dominated by conductive heat transport), resulting in model simulations undergoing an initial jump prior to reaching steady-state; this jump contaminates the most recent period of model evolution. These issues are avoided in more sophisticated adjoint treatments that solve the full field equations in a forward sense and therefore rigorously incorporate thermal diffusion (e.g., Bunge *et al.*, 2003; Ismail-Zadeh *et al.*, 2004; Zhou & Liu, 2017; Li *et al.*, 2017; Price & Davies, 2018; Ghelichkhan & Bunge, 2018).

Here, we adopt the adjoint methodology of Ghelichkhan & Bunge (2016) to track relative changes in the dynamical ellipticity driven by mantle convection over the past 50 Myr (Figure 1A). Details of the calculation are provided in the Appendix. Our simulation yields a perturbation in the magnitude of H of order 0.1% since 50 Ma, with an increase in dynamic flattening until 15 Ma, followed by a decrease of comparable magnitude in the subsequent 15 Myr (Figure 1A). Our investigation of the evolving mantle heterogeneity in the adjoint model indicates that the increase in $\delta H/H$ from 50 Ma to 20 Ma is driven by an increase in the amplitude of long-wavelength density anomalies at the base of the upper mantle (the so-called transition zone) and the base of the lower mantle (i.e., above the fluid outer core). The subsequent change in trend reflects a progressive weakening of the transition zone signal after 20 Ma.

This variation in H is significantly smaller than predicted by a previous backward advection simulation (Forte &

Mitrovica, 1997), and it has significant implications for the stability of Earth's precession and obliquity parameters. In particular, Laskar *et al.* (1993) has shown that if the dynamic flattening were perturbed downward by ∼ 0.2% relative to the present day value, these parameters would experience a non-linear perturbation due to a passage through the $s_6 - g_6 + g_5$ resonance that is associated with perihelion of Jupiter and Saturn and the node of Saturn. We conclude that such a passage is unlikely to have occurred over the past 50 Myr (c.f. Forte & Mitrovica, 1997).

Next, we turn to variability in the dynamical ellipticity associated with ice age dynamics over the past 3 Myr. Over this period, the Earth was subject to glacial cycles of increasing magnitude, and an obliquity-paced periodicity of ∼40 kyr until ∼800 ka, followed by the so-called "Mid-Pleistocene transition" to cycles of period ∼100 kyr (Lisiecki & Raymo, 2005). The last such cycle occurred from ∼120–6 ka, with the Last Glacial Maximum reached at 26 ka, and it involved a mass flux equivalent to ∼ 130 m of global average sea level change (Austermann *et al.*, 2013). Our prediction of perturbations to the dynamical flattening (Figure 1B) is generated using a theory of ice age dynamics that involves a gravitationally self-consistent treatment of sea level changes constrained to conserve the total (ice plus ocean) surface mass (Kendall *et al.*, 2005) and it requires, on input, models for the radial profile of mantle viscosity and the full space-time geometry of ice mass changes. For the former, we adopt the same viscosity model used in our convection simulation to generate Figure 1A, and for the latter, we use the ice history developed by Raymo *et al.* (2011).

Since the Earth is currently in an interglacial period, with high-latitude glaciation near a minimum, the mean perturbation of H relative to present day represents a reduction in the flattening of 0.12%. Over the same period, the convection-induced perturbation to H reaches 0.015% of the present-day value, and thus ice age dynamics dominate the perturbation in dynamical flattening across this 3 Myr time scale. The temporal variability in Figure 1B reflects the history of forcing, with the above-noted transition in the period of cyclicity and a general change in the magnitude of variability at ∼800 ka. Across the current interglacial (i.e., since 6 ka), the polar regions of the Earth are continuing to rebound from subsidence associated with 26–6 ka ice unloading, and this process is reflected in the gradual reduction in flattening that persists to the present day.

Finally, we turn our attention to recent variations in the dynamical flattening on decadal time scales. Figure 1C shows the observed change in H across the satellite period, relative to 2012, derived from the results of Cheng *et al.* (2013). As discussed earlier, a change in the trend of the $\delta H/H_0$ time series, or equivalently J_2 from Equation (5), took place around the year 1990. Prior to that date, the trend is dominated by the above-noted reduction in oblateness (and polar moment of inertia) since ∼6 ka that is driven by ongoing effects of the ice age. This trend continues after 1990, but the onset of significant modern melting of ice sheets at that time contributes to an increase in oblateness (as ice melts near the poles and mass redistributes toward lower latitudes), resulting in a net signal that is characterized by a reduced trend (i.e., the magnitude of dH/dt and dJ_2/dt decreases).

In more quantitative terms, the rate of change in $\delta H/H_0$ prior to 1990 is -3.4×10^{-8} yr^{-1}, and it decreases in magnitude by approximately a factor of four to -0.8×10^{-8} yr^{-1} in the period 1990–2012. The ice age calculation of Figure 1B predicts a contribution to the present-day rate of change of $\delta H/H_0$ of approximately -5.1×10^{-8} yr^{-1}, and correcting the two observed rates for this signal yields residuals of $\sim 1.7 \times 10^{-8}$ yr^{-1} and $\sim 4.3 \times 10^{-8}$ yr^{-1}, respectively. In the earlier period, 1976–1990, the remaining contributor to the signal is associated with melting of glaciers driven by global climate change. Mitrovica *et al.* (2015) estimated the rate of change of J_2 due to this glacier melting as $\sim 2.0 \pm 0.3 \times 10^{-11}$ yr^{-1}; this converts to a rate of change in $\delta H/H_0$ of $\sim 1.8 \times 10^{-8}$ yr^{-1}, a value which is in agreement with the (observed minus ice age-corrected) residual cited above ($\sim 1.7 \times 10^{-8}$ yr^{-1}). In the period after 1990, the larger ice age-corrected signal (4.3×10^{-8} yr^{-1}) reflects the onset of major melting from the polar ice sheets (Cox & Chao, 2002; Cheng *et al.*, 2013). The best fit linear form across the full time series, i.e., 1976–2012, is characterized by a rate of change of $\delta H/H_0$ of $\sim -2.1 \times 10^{-8}$ yr^{-1}.

The calculations in Figure 1, since they are based on Equation (2), do not include the impact on the inertia tensor of a uniform, degree-0 deformation of the Earth. If we included this spatially uniform signal in the theory, Equation (3) would be revised to

$$\delta H = \left[\frac{3}{2} - H\left(1 + \frac{\delta C_{(0,0)}}{\delta C_{(2,0)}}\right)\right]\frac{\delta C_{(2,0)}}{C}, \tag{8}$$

where the subscripts denote the spherical harmonic degree and order of the structure contributing to the inertia perturbation. In our calculations of perturbations in $\delta H/H$ due to ice mass changes (Figure 1B), we include complementary sea level changes and the total mass of the surface load is conserved (i.e., it has no degree-0 component). The same must be true for the processes responsible for the observations that form the basis of Figure 1C. Thus, in these cases, there is no degree-0 deformation, $\delta C_{(0,0)} = 0$, and the above expression collapses to that in Equation (3). While our calculations of the perturbation to $\delta H/H$ driven by mantle convection adopt a compressible flow model, any changes in the volume of the Earth are negligible. However, these calculations do not account for secular cooling and thermal contraction of the Earth. Estimates of this latter process suggest that the reduction in Earth radius over the past 50 Myr due to thermal contraction has been ~80 m (Tsuchiya *et al.*, 2013). Using this value and the results in Figure 1A yields an estimate of $\delta C_{(0,0)}/\delta C_{(2,0)} \sim 5$, and therefore $H(1 + \delta C_{(0,0)}/\delta C_{(2,0)}) \sim 6H$. This value is of order 1% of the leading term of $\frac{3}{2}$ in Equation (8), and neglecting it in adopting Equation (4) remains justified.

The results in Figure 1 do not include the impact on the dynamical flattening of an additional process mentioned in the introductory section, namely, tidal dissipation. The present level of tidal dissipation is slowing the Earth's rotation at a rate of $(d\Omega/dt)/\Omega = 8.8 \times 10^{-18}$ s^{-1} (e.g. Quinn *et al.*, 1991) and the dynamical flattening will be approximately proportional to Ω^2. While the variation of tidal dissipation over time is uncertain, any effort to estimate the total change in dynamical flattening from all geophysical processes must include this contribution.

4 Final Remarks

The dynamical flattening of the Earth, a parameter associated with the precession constant, plays an important role in a wide range of applications in astronomy, geodesy and geophysics, including astronomical observations of nutations, investigations of the stability of the orbital elements (precession, obliquity) controlling Milankovitch forcing of ice age climate, and the evolution of the Earth-Moon-Sun system over billion-year time scales. Burša *et al.* (2008) highlighted the importance of recognizing the time dependence in the precession constant within astronomical analyses. However, they suggested that the variation in H could be captured by a constant rate term computed by fitting a linear form through a satellite time series of J_2 extending from 1979–2002 – they derived a value for dH/dt of -8.45×10^{-11} yr^{-1}, or equivalently a rate of change of $\delta H/H_0$ of $\sim 2.6\times10^{-8}$ yr^{-1} – and advocated that the rate be adopted as a fundamental astro-geodetic parameter. In contrast to this view, we have shown in Figure 1 that time dependence of the dynamic ellipticity is highly non-linear, even when considering only the last 40 years of satellite-based measurements (Figure 1C). The full complexity of the time series of $\delta H/H_0$ in Figure 1 reflects the suite of geophysical processes that perturb the Earth's flattening, including mantle convection, ice age dynamics, and modern global climate change.

5 Acknowledgements

SG is indebted to D.R. Davies for guidance and support. We acknowledge funding and support from Harvard University (JJF, JXM), National Aeronautics and Space Administration grant NNX17AE17G (MJH, JXM), the American Chemical Society Petroleum Research Fund grant 59062-DNI8 (MJH), Imperial College Research Fellowship scheme and the Schmidt Science Fellows program (FDR)

Appendix

Time-evolution in our adjoint treatment (Ghelichkhan & Bunge, 2016) is constrained by assimilating a history of plate motions (Young *et al.*, 2019), and the initial buoyancy field (i.e., at 50 Ma) is iteratively optimized through comparison of the final, present-day buoyancy field predicted by the flow model with the buoyancy field inferred from seismic tomography. This procedure typically converges after 12-15 iterations.

Two other fields need to be prescribed in this procedure, the radial viscosity structure used in the flow calculation and the present-day mantle buoyancy field to which the prediction of the flow model at the present day is compared. We consider each, in turn. All other material parameters and boundary conditions are adopted from Colli *et al.* (2018).

We use a radial viscosity profile derived from a joint inversion of data related to mantle convection and ice age dynamics (Mitrovica & Forte, 2004). The viscosity model, which we also adopt in the ice age calculations described in the main text, is characterized by a three order of magnitude increase in viscosity from the shallow mantle beneath the lithosphere (10^{20} Pa s) to 2000 km depth (10^{23} Pa s), followed by a reduction of comparable magnitude toward the core-mantle-boundary.

To construct the present-day mantle buoyancy field, we use lower mantle shear wave velocities from the recent tomography model LLNL-G3D (Simmons *et al.*, 2012). Upper mantle velocity structure is prescribed from the higher resolution surface wave tomography model SL2013sv (Schaeffer & Lebedev, 2013), smoothly blended into the deeper mantle model over the depth range 250–350 km. To convert seismic velocities into density, we first calculate anharmonic velocities and densities as a function of pressure and temperature for a pyrolitic mantle composition using the thermodynamic database of Stixrude & Lithgow-Bertelloni (2011) and the Perple_X Gibbs free-energy minimisation software (Connolly, 2005). Next, anharmonic velocities are corrected for anelasticity using the Q5 attenuation model of Cammarano *et al.* (2003), adopting the solidus of Hirschmann (2000) in the upper $\sim$ 250 km and Andrault *et al.* (2011) in the deeper mantle. Tomographically inferred velocity variations as a function of depth are then used to query the resulting lookup table and extract corresponding values of density. To prevent the continental lithosphere from actively participating in convection, densities within the lithosphere are set to the radial average using the lithosphere-asthenosphere boundary map of Hoggard *et al.* (2020).

Once the temporal evolution of the mantle flow field has been successfully reconstructed, we calculate the time history of dynamic ellipticity, H. For this purpose, we solve the governing, coupled system of Stokes and Poisson's equations using an instantaneous flow methodology that includes the effects of self-gravitation and compressiblity and assumes a free-slip (no tangential stress) boundary condition (Corrieu *et al.*, 1995).

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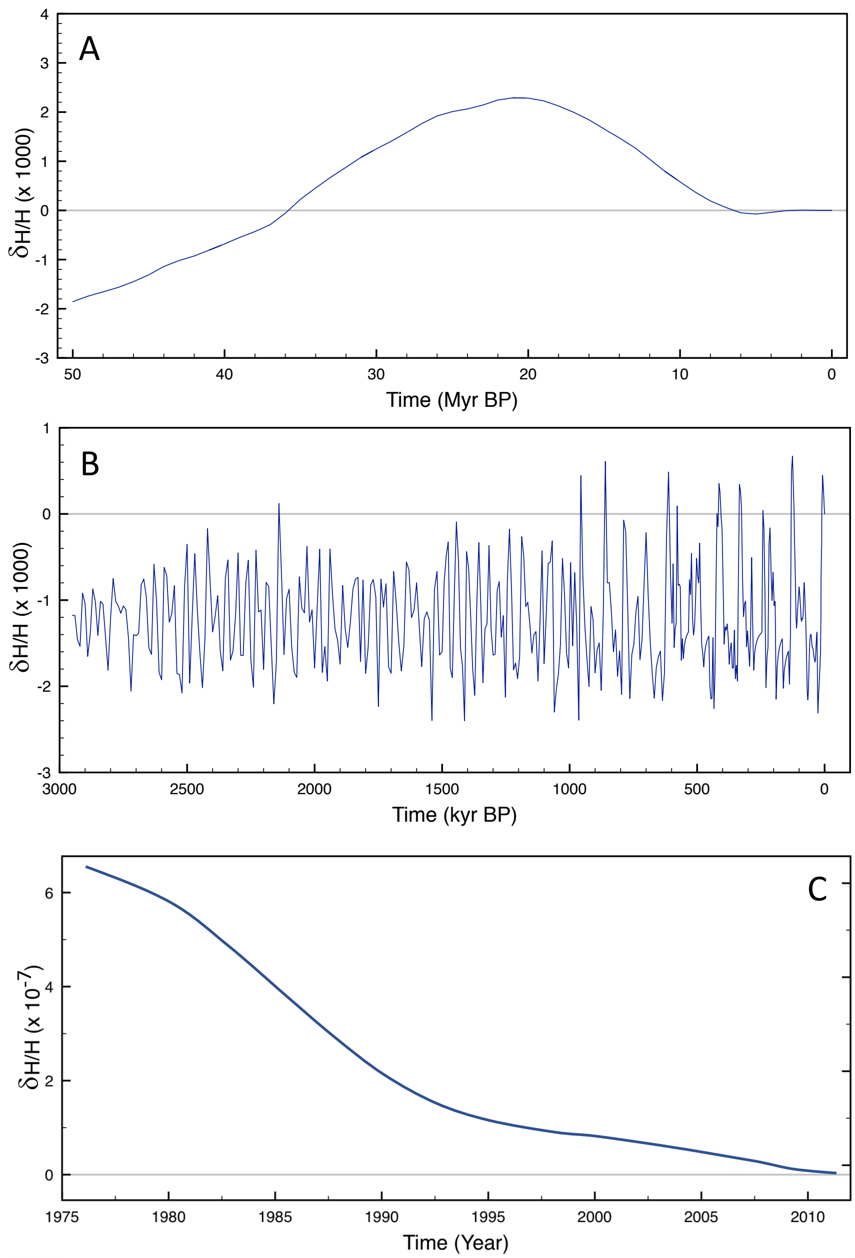

Figure 1: (A) Relative perturbation in the precession constant of the Earth predicted using the adjoint reconstruction of mantle convective flow over the past 50 Myr (see text). (B) As in (A), except for a reconstruction of changes in the precession constant since 3 Ma predicted from a simulation of ice age dynamics alone (see text). The numerical predictions in frames (A) and (B) are both based on a common model of the Earth's radial viscosity profile that was derived by simultaneously inverting a large suite of global geophysical data related to mantle convection and ice age dynamics (Mitrovica & Forte, 2004; see text). (C) Time series of the relative perturbation in the precession constant estimated from satellite altimetry records since 1975. The time series is computed from the results of Cheng *et al.* (2013). All results are plotted relative to the present day value, or in the case of frame (C), 2012.