



Article

Combining a Matheuristic with Simulation for Risk Management of Stochastic Assets and Liabilities

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Abstract: Specially in the case of scenarios under uncertainty, the efficient management of risk when matching assets and liabilities is a relevant issue for most insurance companies. This paper considers such a scenario, where different assets can be aggregated to better match a liability (or the other way around), and the goal is to find the asset-liability assignments that maximises the overall benefit over a time horizon. To solve this stochastic optimisation problem, a simulation-optimisation methodology is proposed. We use integer programming to generate efficient asset-to-liability assignments, and Monte-Carlo simulation is employed to estimate the risk of failing to pay due liabilities. The simulation results allow us to set a safety margin parameter for the integer program, which encourage the generation of solutions satisfying a minimum reliability threshold. A series of computational experiments contribute to illustrate the proposed methodology and its utility in practical risk management.

Keywords: assets and liabilities management; risk management; uncertainty; matheuristics; simulation

1. Introduction

Within the enormous variety of insurance types that we can find, long-term life insurance stands out for its complexity in terms of financial management. The cash flows generated by these insurances extend over several decades and play an important role in the social sphere since they have a close relationship with pensions and retirements and, therefore, with people's vital planning. For this reason, legislation and administrative authorities play a special role in ensuring that insurers faithfully comply with their commitments. The fact that they are extended in the long term, or in the very long term, generates a series of difficulties for their management because the insurer must plan the necessary income with enormous precision to cover its future commitments. Therefore, it is a requirement that the insurer has a range of techniques that allow for matching its assets, as long-term income generators, with its liabilities. Conventionally, we refer to this set of techniques as asset and liability management (ALM) (Ziemba et al. 1998), and it has raised the interest of numerous researchers over the last few years, with a wide variety of approaches being proposed. One of most popular solutions to this asset management problem is cash-flow matching (Iyengar and Ma 2009), whose main objective is to ensure the timely payment of the liabilities. This approach minimises the number of contractual breaches. Due to the volatility of the financial markets, we always have uncertainty regarding income, and this will be linked to the quality of financial assets. Moreover, the credit quality of assets plays a fundamental role, in particular when we deal with bonds, which are widely used in the insurance industry (Gründl et al. 2016). When the default event occurs, the price of the bond is immediately decreased, in such a

33 way that we have lower income. Since [Merton \(1974\)](#), a lot of models have been developed to forecast
34 the price under a default event.

35 Likewise, the obligations cannot be considered as exact or totally predictable. Those liabilities or
36 obligations are the customer's premium that the insurance company receives. In practice, we consider
37 average values for obligations and we can establish certain ranges of dispersion that can be estimated
38 based on the insurer's own experience. Once the premium has been paid, the company invests it in
39 the long term, so that the financial benefit envisaged in the insurance policy is secured. Finally, in the
40 event of the customer's retirement or death, the insurance company needs to have sufficient funds
41 to meet its liability to the customer. Consequently, we are facing a highly complex asset allocation
42 problem, since the amount of assets that an insurer can have is large, and the distribution of liabilities
43 over time does not usually follow any regular pattern, both being stochastic in nature.

44 Heuristic and metaheuristic algorithms have become a new standard when dealing with
45 complex and large-scale portfolio optimisation and risk management problems [Doering et al. \(2019\)](#);
46 [Soler-Dominguez et al. \(2017\)](#). In this paper we explore an asset allocation method by means of
47 heuristic techniques, taking into account the random nature of both assets and liabilities. The goal
48 is to find the most efficient (minimum cost) combination of assets that meets certain requirements:
49 they must generate sufficient income to cover the obligations of the insurer with a high probability. In
50 a recent work, [Bayliss et al. \(2020\)](#) considered a simplified ALM problem, based on the net present
51 value (NPV) concept, in which only one-to-one asset-liability assignment were allowed. Notice that,
52 since we are comparing monetary values of assets that belong to different time periods, it makes sense
53 to consider the NPV associated with each asset in order to make a fairer comparison of assets. Our
54 work goes a step further and allows many-to-many, one-to-many, and many-to-one asset-liability
55 assignments as well. Such an approach increases the efficiency with which liabilities can be covered.
56 This also allows us to address ALM problems regardless of the number of assets and liabilities,
57 as well as their sizes. For addressing large scale instances which could not be solved using exact
58 integer programming techniques, previous approaches were based on the use of greedy heuristics that
59 prioritised larger liabilities over smaller ones. This work, however, proposes an improved approach
60 based on sorting liabilities in ascending due date order, since liabilities with earlier due dates have
61 fewer assets combinations that can be assigned to them. Additionally, assets with earlier maturity dates
62 have higher NPVs, which is what is to be minimised. The main methodological contribution of our
63 approach lies in the introduction of a matheuristic algorithm, which integrates integer programming
64 and Monte-Carlo simulation. In particular, an integer program is solved recursively to generate feasible
65 and efficient asset-liability assignments for a deterministic scenario (where we assume average values
66 for each random variable in the model). After each iteration, the resulting asset-liability assignment
67 mapping (solution) is assessed under a stochastic scenario by using Monte-Carlo simulation, which
68 also provides estimates of the mapping reliability. The simulation outcomes are also employed to
69 update a safety margin parameter of the integer program that controls the minimum ratio between
70 the values of the assets and the liabilities of the generated asset-liability assignments. The proposed
71 approach is then tested in a wide variety of problem instances. The combination of simulation and
72 optimisation methods in NPV-related financial problems under uncertainty has been also explored in
73 [Panadero et al. \(2020\)](#).

74 The rest of the paper is structured as follows: Section 2 provides a brief literature review on
75 ALM. Section 3 introduces a more detailed description of the specific ALM problem considered in this
76 paper. Section 4 proposes a matheuristic algorithm for solving the aforementioned problem. A series
77 of computational experiments are carried out in Section 5, while Section 6 provides an analysis of the
78 obtained results. Finally, Section 7 highlights the most relevant findings of our work and points out
79 future research lines.

80 2. Literature Review on ALM

81 One of the first relevant works on ALM theory is due to [Macaulay \(1938\)](#). This author formulates
82 the theory of duration considering: a fixed liability cash flow, a fixed income as assets, and a constant
83 interest rate. Later, [Hicks \(1975\)](#) introduce the term “corrected duration”, to justify variances in the
84 present value when the interest rate changes. These authors measure duration as a percentage, while
85 the previous one measured duration in terms of time. [Fisher and Weil \(1971\)](#) present the first formalised
86 work about immunisation, defining the condition under which the value of an investment is protected
87 against the variations of the interest rate. A fixed income portfolio, with duration equal to a given
88 investment horizon, is studied in [Fong and Vasicek \(1984\)](#). [Bierwag et al. \(1993\)](#) contribute to the ALM
89 topic with an analysis of the properties of cash flow dispersion in duration hedged portfolios. [Zenios](#)
90 [\(1995\)](#) use a real-world scenario to highlight the mismatch between assets and liabilities in the financial
91 industry, showcasing a case of portfolios containing mortgage-backed securities. [Mulvey et al. \(1997\)](#)
92 discuss an ALM with stochastic and risk factors, where different scenarios are expressed as a tree. This
93 strategy, known as multistage stochastic programming, is also reviewed in more recent publications.
94 Hence, [Kouwenberg and Zenios \(2008\)](#) do not only study the how-to structure of assets along time, but
95 they also consider the associated stochastic nature. [Boender et al. \(2008\)](#) study the role of scenarios in
96 ALM, as a lattice of possibilities for each element in the model, each one with an associated probability.

97 More realistic approaches also exist. For instance, [Kusy and Ziemba \(1986\)](#) study a model with
98 legal, financial, and bank-related policy considerations applied to a Canadian bank. [Oguzsoy et al.](#)
99 [\(1997\)](#) present a multi-period stochastic linear model for ALM in banking. [Mulvey et al. \(2000\)](#) show
100 how Towers Perring applies ALM planning to pension management, modelling stochasticity with
101 scenarios. [Nielsen and Zenios \(1996\)](#) study how to apply a multi-period stochastic program using
102 government bonds, mortgage-backed securities, and derivative products. [Consigli and Dempster](#)
103 [\(1998\)](#) develop a pension fund problem with uncertainty. [Carino et al. \(1998\)](#), [Carino and Ziemba](#)
104 [\(1998\)](#), and [Carino et al. \(1998\)](#) are a series of publications that describe an ALM model adapted
105 to the complexities of the Japanese regulations, such as legal or taxes limitations. In the Iranian
106 regulation framework, [Abdollahi \(2020\)](#) studies a multi-objective ALM programming problem where
107 the constraints are realistic legal conditions of the banking industry. Within a dynamic stochastic
108 control approach, [Sun et al. \(2019\)](#) studies a mean-variance ALM problem where assets and liabilities
109 are both stochastic, and where liabilities transfer part of their risk by means of a reinsurance.

110 [Kouwenberg \(2001\)](#) develops a scenario-generation for the ALM to minimise the expected
111 contribution rates, taking risk into account. [Gondzio and Kouwenberg \(2001\)](#) focus on the computation
112 complexity of the ALM problem, and solve a stochastic model with near 5 million scenarios, more than
113 12 million constraints, and 25 million variables to study a pension fund. [Fleten et al. \(2002\)](#) compares a
114 fixed mix model with a multistage stochastic program. [Dash and Kajiji \(2005\)](#) implement a nonlinear
115 model for the optimisation of property-liability insurers. A Monte-Carlo simulation approach to the
116 analysis of an ALM problem can be found in [Dempster et al. \(2003\)](#). These authors use a model
117 involving global assets and contribution pension plans. [Hibiki \(2006\)](#) compares the results of two
118 different approaches modelling the evolution of assets. These authors check the performance of scenarios
119 trees and hybrid trees with simulation paths. [Zhang and Zhang \(2009\)](#) improve the previous model by
120 introducing new metrics and using a genetic algorithm to solve it. [Consiglio et al. \(2006\)](#) and [Consiglio](#)
121 [et al. \(2008\)](#) study the optimisation problem derived from a liability with complex conditions, leading
122 to a non-linear problem. For a model with just two assets, [Papi and Sbaraglia \(2006\)](#) solve the ALM
123 problem with a recourse algorithm and a method that guarantees convergence. A complete handbook
124 that describes the ALM and the multistage ALM for particular markets can be found in [Zenios and](#)
125 [Ziemba \(2007\)](#). Also, [Kouwenberg and Zenios \(2008\)](#) review different stochastic programming models
126 for ALM, analysing the performance of these models when they are applied to pension funds. [Escudero](#)
127 [et al. \(2009\)](#) introduces a mixed integer-linear model by adding discrete variables into the model to
128 limit the number of transactions and assets in each stage. [Ferstl and Weissensteiner \(2011\)](#) consider a
129 multi-stage ALM under time-varying investment opportunities, using stochastic linear programming.
130 Several other approaches to the ALM problem have been studied recently. Thus, [Zhang and Chen](#)

131 (2016) focus on the mean-variance ALM with constant elasticity of variance. Wei and Wang (2017) focus
132 on random coefficients, while Li et al. (2018) study models with stochastic volatility. Fernández et al.
133 (2018) introduce a stochastic ALM model for a life insurance company using GPUs to run Monte-Carlo
134 simulations. Dutta et al. (2019) uses big data analytics and stochastic linear programming under
135 stochastic scenarios. Li et al. (2019) use a multi-period mean-variance model to analyse the ALM
136 problem with probability constraints. Orlova (2019) develops an algorithm to solve a discrete dynamic
137 process for cash distribution, in which the goal is to minimise the payment of fines for non-timely
138 financing of expenses. This approach solves the problem of financial resources distribution under
139 uncertainty over time. Kopa and Rusý (2020) formulates a complete stochastic program for ALM credit
140 institutions that grant loans to general customers. In this paper, stochastic multi-stage scenarios are
141 considered and the behaviour of the consumer are modelled. This behaviour impacts on the decisions
142 the credit institution has to take and how it has to allocate its assets.

143 3. Problem Description and Formulation

144 When the conditions set out in a contract are met, insurers pay the insured. If they do not have
145 sufficient available funds, they are subject to monetary fines issued by monetary authorities and, most
146 likely, to lost customers. In order to ensure the insurers can meet their liabilities, they perform a process
147 of matching assets to liabilities. Assigning assets to liabilities in an efficient manner is critical to the
148 success of an insurance firm, since assigned (or frozen) assets cannot be used for any other purpose.
149 Assets can only be assigned to liabilities if their maturity date precedes the due date of the liability.
150 The value of the assets assigned to liabilities must equal or exceed the liability values. At the same
151 time, asset maturity values and liability payment values are uncertain, thereby introducing a risk that
152 liabilities cannot be met, even when the expected values imply that they could be met on the average.

153 An asset-liability assignment is the terminology used in this work to refer to a group of assets used
154 to cover a group of liabilities. A feasible solution to the net present value asset-liability management
155 (NPV-ALM) problem consists of a set of asset-liability assignments such that: (i) all liabilities are
156 covered; and (ii) no individual assets or liabilities are part of more than one asset-liability assignment.
157 Furthermore, a solution is also required to be robust under uncertain asset and liability values.
158 Specifically, a solution must meet a minimum reliability level, where reliability is defined as the
159 probability that all liabilities can be paid successfully using their assigned assets. Figure 1 illustrates a
160 single asset-liability assignment consisting of three assets and two liabilities. Notice that, under the
161 expected values for assets and liabilities (dashed lines), the liabilities can be met. However, due to
162 uncertain asset maturity values and liability payment values, there is a risk that the assets fail to cover
163 the liabilities in the assets-liability assignments. If f_i is the probability that asset-liability assignment i
164 fails to cover its liabilities, then the reliability of a set of asset-liability assignment (I) covering all of
165 our liabilities is computed as $r = \prod_{i \in I} (1 - f_i)$. Following Faulin et al. (2008), we employ Monte-Carlo
166 simulation to estimate failure probabilities associated with candidate asset-liability assignments.

167 In this work, we propose a matheuristic algorithm for solving the NPV-ALM problem. A
168 matheuristic integrates mathematical programming techniques with heuristics in order to develop an
169 algorithm that benefits from exact optimisation as well as from fast and efficient heuristic techniques.
170 For the case of the NPV-ALM problem, an integer program (Section 3.2) is used to calculate a set
171 of feasible asset-liability assignment decisions that cover the liabilities. The solution is tested in a
172 simulation to measure its reliability, and the result is employed to tune a safety margin parameter
173 of the integer program. The safety margin parameter controls the minimum ratio between the asset
174 values and the liability values of a generated asset-liability assignment. The process continues until a
175 specified number of iterations have been completed. Section 3.1 formulates the NPV-ALM problem.

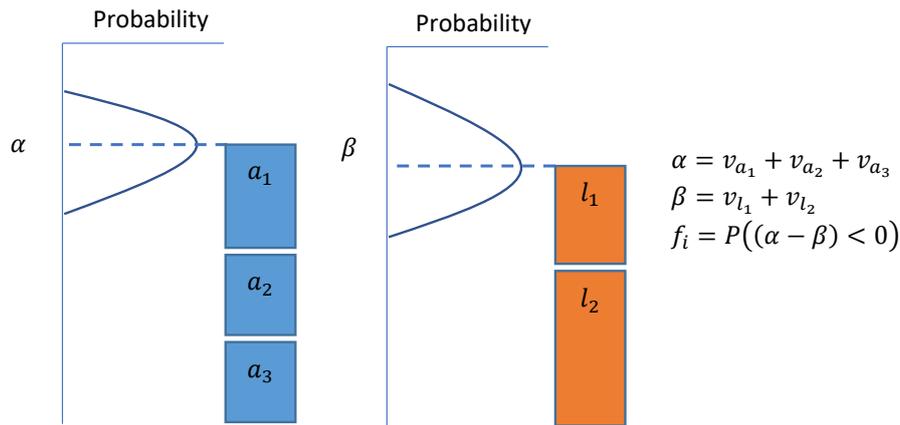


Figure 1. An asset-liability assignment with a failure probability.

176 3.1. A model for the net present value asset and liability management problem

177 Summary of the notation

Sets	
A	: Set of all assets
L	: Set of all liabilities
Stochastic variables	
\bar{v}_a	: The uncertain value of asset a at maturity
\bar{v}_l	: The uncertain value of liability l on its due date
Decision variables	
y_{ga}	: Binary variable indicating whether asset a is selected as part of asset-liability assignment g
z_{gl}	: Binary variable indicating whether liability l is selected as part of asset-liability assignment g
w_a	: Binary variable indicating whether asset a is selected as part of a generated asset-liability assignment
x_l	: Binary variable indicating whether liability l is selected as part of a generated asset-liability assignment
Input parameters	
v_a	: The expected maturity value of asset a
v_l	: The expected value of liability l on its due date
t_a	: The maturity date of asset a
t_l	: The due date of liability l
d	: Discount factor used to calculate the net present value of an asset
r_{min}	: Minimum reliability level
m	: Safety parameter decrease factor
h	: Safety parameter increase factor
Other parameters	
f_g	: Failure probability of asset-liability assignment g
N_g	: Asset-liability assignment g
npv_g	: Net present value associated with Asset-liability assignment g

179 The objective (1) is to minimise the NPV of the assets committed to covering liabilities. In this
 180 context, y_{ga} is a binary decision variable indicating whether asset a is an element of asset-liability
 181 assignment g . Similarly, z_{gl} is a binary decision variable indicating whether liability l is an element of
 182 asset-liability assignment g . Each asset $a \in A$ can only be part of at most one asset-liability assignment,
 183 as specified by Constraint (2). Each liability $l \in L$ can only be part of one selected asset-liability
 184 assignment, as specified by Constraint (3). As a result of Constraints (2) and (3), the maximum number
 185 of asset-liability assignments is $|G| = \min(|A|, |L|)$. A feasible asset-liability assignment requires that
 186 each of the selected assets matures before all of the selected liabilities in the asset-liability assignment.
 187 Constraint (4) introduces a continuous variable ϕ_g representing the latest maturity date of an asset
 188 in asset-liability assignment g . Constraint (5) introduces a continuous variable σ_g representing the
 189 earliest due date of a liability in asset-liability assignment g . Here, H is a large number which ensures
 190 the feasibility of Constraint (5) in asset-liability assignments that the liability l is not part of. Then,
 191 Constraint (6) enforces the time constraints for each asset-liability assignment. Constraint 7 requires
 192 that the sum of the asset values exceeds the value of the covered liabilities by a factor S in each

193 asset-liability assignment g , thus ensuring that our liabilities are covered. Also, S is a multiplicative
 194 safety margin parameter for ensuring that the asset values are able to cover the liabilities under
 195 uncertain asset returns and liability values. Constraints (8) and (9) define the binary decision variables.

$$\min \sum_{g \in G} \sum_{a \in A} y_{ga} \left(\frac{v_a}{(1+d)^{t_a}} \right). \quad (1)$$

$$\sum_{g \in G} y_{ga} \leq 1, \forall a \in A. \quad (2)$$

$$\sum_{g \in G} z_{gl} = 1, \forall l \in L. \quad (3)$$

$$\phi_g \geq y_{ga} t_a, \forall a \in A, \forall g \in G. \quad (4)$$

$$\sigma_g \leq z_{gl} t_l + H(1 - z_{gl}), \forall l \in L, \forall g \in G. \quad (5)$$

$$\phi_g \leq \sigma_g, \forall g \in G. \quad (6)$$

$$\sum_{a \in V} y_{ga} v_a \geq S \sum_{l \in U} z_{gl} v_l, \forall g \in G. \quad (7)$$

$$y_{ga} \in \{0, 1\}, \forall a \in A, \forall g \in G. \quad (8)$$

$$z_{gl} \in \{0, 1\}, \forall m \in L, \forall g \in G. \quad (9)$$

196 3.2. An integer programming model for generating feasible asset-liability assignments

197 Since solution time and memory requirements become an issue when solving the mixed integer
 198 program specified in Section 3.1 for realistic sized problem instances, our heuristic solution approach
 199 is based upon solving an integer program repeatedly to generate a sequence of efficient asset-liability
 200 assignments that cover all of the liabilities. This iterative approach is an alternative to generating
 201 all of the required asset-liabilities assignments in one go. This approach also vastly reduces the
 202 size and complexity of the mathematical programs that need to be solved. This integer program is
 203 denoted as $IP(U, V, k, S)$. Here, U is the set of remaining uncovered liabilities, and V is the set of
 204 available assets currently unassigned to any liabilities. Initially, $U = L$ and $V = A$. Every time a new
 205 asset-liability assignment is generated using the integer program, the selected assets are removed from
 206 V and the selected liabilities are removed from U . The integer program is solved repeatedly until
 207 the set U is empty. The input k is a randomly selected uncovered liability that must be covered by
 208 the next asset-liability assignment generated. This provides a mechanism for randomising the sets
 209 of asset-liability assignments generated. The i^{th} asset-liability assignment generated is denoted as N_i .
 210 It contains the set of selected assets and liabilities. The efficiency of an asset-liability assignment is
 211 measured by the value of the liabilities covered minus the value of the assets used, which encourages
 212 asset-liability assignments to cover as many liabilities as possible with the fewest assets possible.
 213 The net present value of the assigned assets is then subtracted, which captures our overall objective.
 214 Higher values of this efficiency measure correspond to more efficient asset-liability assignments. This
 215 efficiency objective function is expressed by Objective (10). In this expression, x_l is a binary variable
 216 indicating which liabilities, $l \in U$, are part of the generated asset-liability assignment, and w_a is a
 217 binary variable indicating which assets, $a \in V$, are part of the generated asset-liability assignment.

$$\max \sum_{l \in U} x_l v_l - \sum_{a \in V} w_a v_a \left(1 + \frac{1}{(1+d)^{t_a}} \right). \quad (10)$$

218

219 A feasible asset-liability assignment requires that each of the selected assets matures before the selected
 220 liabilities. Constraint (11) expresses this, where t_m is the asset maturity date or liability due date of an
 221 asset or liability $m \in V \cup U$. Also, H is a large number which is used to ensure that Constraint (11)
 222 remains feasible in cases where liabilities are not selected. Optionally, Constraint (11) can be replaced
 223 by a constraint using the same form used in Constraints (4)-(6).

$$w_a t_a \leq x_l t_l + H(1 - x_l), \quad \forall a \in V, \forall l \in U. \quad (11)$$

224

225 Constraint (12) requires that the sum of the asset values exceeds the value of the covered liabilities by
 226 a factor S , where S is a multiplicative safety margin parameter for ensuring that the asset values are
 227 able to cover the liabilities under uncertain asset returns and liability values.

$$\sum_{a \in V} w_a v_a \geq S \sum_{l \in U} x_l v_l. \quad (12)$$

228

229 Constraint (13) states that the randomly selected uncovered liability, k , must be included in the next
 230 asset-liability assignment generated.

$$x_k = 1. \quad (13)$$

231 Constraints (14) and (15) define the binary decision variables.

$$x_l \in \{0, 1\}, \quad \forall l \in U. \quad (14)$$

$$w_a \in \{0, 1\}, \quad \forall a \in V. \quad (15)$$

232 4. Our Matheuristic Approach

233 This section describes our matheuristic algorithm, which combines integer programming and
 234 Monte-Carlo simulation for solving the NPV-ALM problem. This solving approach consists of two
 235 main phases: (i) generation of ‘promising’ solutions; and (ii) simulation and parameter tuning of the
 236 aforementioned solutions. The solution generation phase uses integer programming (specified in
 237 Section 3.2) to generate a set of asset-liability assignments that cover the liabilities. This process is
 238 iterative, i.e., each iteration generates one new asset-liability assignment from the remaining unused
 239 assets and uncovered liabilities. In order to increase the diversity of these solutions, a random factor
 240 is introduced: we randomly select one of the remaining liabilities and add a constraint which forces
 241 this liability to be part of the next asset-liability assignment. The simulation phase is used to measure
 242 the reliability of the generated solution. Monte-Carlo simulation is used estimate the *failure probability*
 243 associated with each asset-liability assignment. This is the probability that the sum of the maturity
 244 values of the assets, in an asset-liability assignment, is less than the corresponding sum of the liabilities.
 245 If the solution is sufficiently reliable, a best solution check is performed to see if the solution has the
 246 lowest associated NPV of any reliable solution found. The reliability result is also used to update the
 247 safety margin parameter of the integer program. The procedure followed is given in Algorithm 1.

248 5. Computational experiments

249 The proposed heuristic has been implemented as a Python application running on a CPU with 3.60
 250 GHz and 16 GB of RAM. Instances from Bayliss et al. (2020) have been used to test the new approach,

Algorithm 1: *AssetLiabilityAssignmentGeneration* ($A, L, r_{min}, \beta, m, h, runs$)

Data: A set of available assets, L set of liabilities, $maxIterations$, r_{min} the minimum reliability level, β geometric distribution parameter, m safety margin decrease factor, h safety margin increase factor, $runs$ the number of Monte-Carlo simulation runs used to estimate asset-liability assignment failure probabilities

- 1 $iteration = 1$, the number of asset-liability assignments generated so far.;
- 2 $bestSolution \leftarrow \emptyset$;
- 3 $bestNPV = \infty$;
- 4 //Initialise the safety margin parameter $S = 1$;
- 5 $S = 1$;
- 6 **while** $iteration \leq maxIterations$ **do**
- 7 //Reset the set of unassigned assets V and uncovered liabilities U ;
- 8 $V \leftarrow A$;
- 9 $U \leftarrow L$;
- 10 $newSolutionNPV \leftarrow 0$;
- 11 $N \leftarrow \emptyset$;
- 12 $i \leftarrow 1$;
- 13 **while** $U \neq \emptyset$ **do**
- 14 //Select an uncovered liability k from an ascending due date sorted list according to a geometric distribution with parameter β ;
- 15 //Solve integer program to obtain the get the next asset asset-liability assignment N_i ;
- 16 $(N_i, npv_i) \leftarrow IP(U, V, k, S)$;
- 17 //Estimate the failure probability f_i of the new asset-liability assignment using Monte-Carlo sampling of asset return and liability values.;
- 18 $f_i \leftarrow simulation(N_i, runs)$;
- 19 $newSolutionNPV \leftarrow newSolutionNPV + npv_i$;
- 20 $U \leftarrow U \setminus N_i$;
- 21 $V \leftarrow V \setminus N_i$;
- 22 $i \leftarrow i + 1$;
- 23 **end**
- 24 //Calculate the reliability r of the new solution;
- 25 $r = \prod_{j=1}^{i-1} (1 - f_j)$;
- 26 //Update the safety margin parameter using the reliability level of the new solution;
- 27 **if** $r \geq r_{min}$ **then**
- 28 //Decrease the safety margin parameter (slowly);
- 29 $S \leftarrow mS$;
- 30 //Check for a new best solution;
- 31 **if** $newSolutionNPV < bestNPV$ **then**
- 32 $bestNPV \leftarrow newSolutionNPV$;
- 33 $bestSolution \leftarrow N$;
- 34 **end**
- 35 **else**
- 36 //Increase the safety margin parameter (relatively quickly);
- 37 $S \leftarrow hS$;
- 38 **end**
- 39 $iteration \leftarrow iteration + 1$
- 40 **end**
- 41 **return** $bestSolution$;

251 plus two new instances that could not be solved with the methodology presented in the former paper.
 252 Table 1 provides the details on the number of assets and liabilities for each instance, discount rate, and
 253 value modifier (if any was employed). Assets and liabilities have been distributed over time using
 254 a random uniform probability distribution from 0 to 100 and from 50 to 150, respectively. Similarly,
 255 values for assets and liabilities have been randomly generated using a uniform probability distribution
 256 from 0 to 1 and from 0 to 0.5, respectively. Asset values from instances 4 and 5 have been modified
 257 to simulate scenarios where its value varies over time, i.e.: given an asset $a \in A$ with a value v_a at
 258 time t_a , a new value v'_a is computed $v'_a = v_a f(t_a, T)$, with $T = \max\{t_a : a \in A\}$ and f the asset value
 259 modifier function. Likewise, instances 6 and 7 consider scenarios with liability values varying over
 260 time: given a liability $l \in L$ with a value v_l at time t_l , a new value v'_l is computed $v'_l = v_l g(t_l, T)$, with
 261 $T = \max\{t_l : l \in L\}$ and g the liability value modifier function. Instance 10 simulates a scenario with
 262 small assets and large liabilities, which encourages the use of multiple assets to cover a liability, while
 263 instance 11 considers a scenario with a few large assets and several small liabilities, to force the use of
 264 a single asset to cover multiple liabilities.

Table 1. Characteristics of the set of instances.

#	Instance	# assets	# liabilities	Discount rate	Asset value modifier	Liability value modifier
1	Control_Instance	1000	200	0.05	-	-
2	Large_x3	3000	600	0.05	-	-
3	Large_x5	5000	1000	0.05	-	-
4	Asset_Value_Increases	1000	200	0.05	t/T	-
5	Asset_Value_Deceases	1000	200	0.05	$1 - (t/T)$	-
6	Liability_Value_Increases	1000	200	0.05	-	t/T
7	Liability_Value_Deceases	1000	200	0.05	-	$1 - (t/T)$
8	Reduced_Discount_Rate	1000	200	0.005	-	-
9	Liabilities_x2	1000	400	0.05	-	-
10	Small_Asset_Large_Liability	1000	200	0.05	0.5	10
11	Large_Asset_Small_Liability	50	1000	0.05	10	0.2

265 Some initial experiments have been performed using instance 1 to set the parameter α associated
 266 with the geometric probability distribution that drives the liability selection and the relative mixed
 267 integer programming optimality gap, $MIPGap$, which is used to terminate the integer programming
 268 algorithm. Experiments to determine α have been carried out in a deterministic scenario, while
 269 experiments to determine $MIPGap$ have been performed with stochastic variables. In this case, a
 270 better performance is attained with $\alpha = 0.75$ and $MIPGap = 0.4$. Figure 2 and Figure 3 present the
 271 results of the numerical tests.

272 Each instance in Table 1 has been solved using the integer programming algorithm presented
 273 in Algorithm 1, with a limit of 100 iterations. A time-limit of 300 seconds has also been imposed to
 274 terminate the algorithm after a solution has been generated if the aforementioned time-limit has been
 275 reached. The minimum reliability r_{min} to consider a solution as feasible in the stochastic scenario is
 276 0.95. The values of the parameters to increase and decrease the safety margin parameter S used are
 277 $m = 0.99$ and $h = 1.1$. In the stochastic scenario, both asset and liability values have been considered
 278 uncertain, with a standard deviation of 5% of its expected maturity value. 500 iterations are executed
 279 for each asset-liability assignment generated in the Monte-Carlo simulation.

280 Table 2 provides the experimental results, compared with the results obtained in Bayliss et al.
 281 (2020). The first column contains the instance number (same as in Table 1). The second column (Cplex)
 282 contains the optimal value for each instance in a one-to-one asset-to-liability mapping. The third
 283 column (BR) contains the results of a previous biased-randomised algorithm, with its associated
 284 reliability values in the next column. Then, column 5 contains the best values obtained in the
 285 deterministic scenario with our matheuristic algorithm. Similarly, the best solutions obtained with a

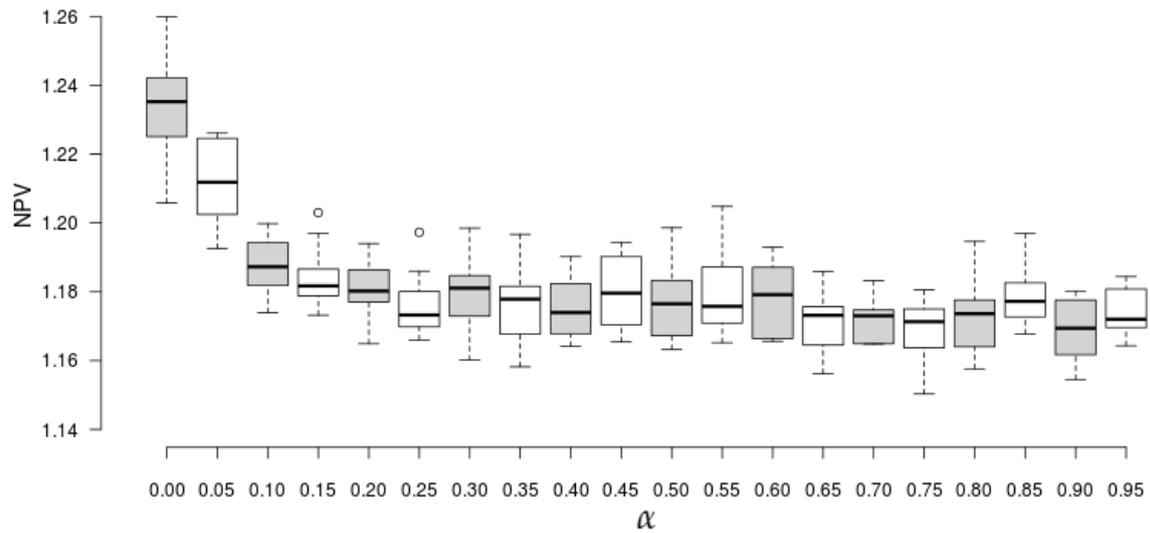


Figure 2. Boxplot comparison of instance 1 results with different α values.

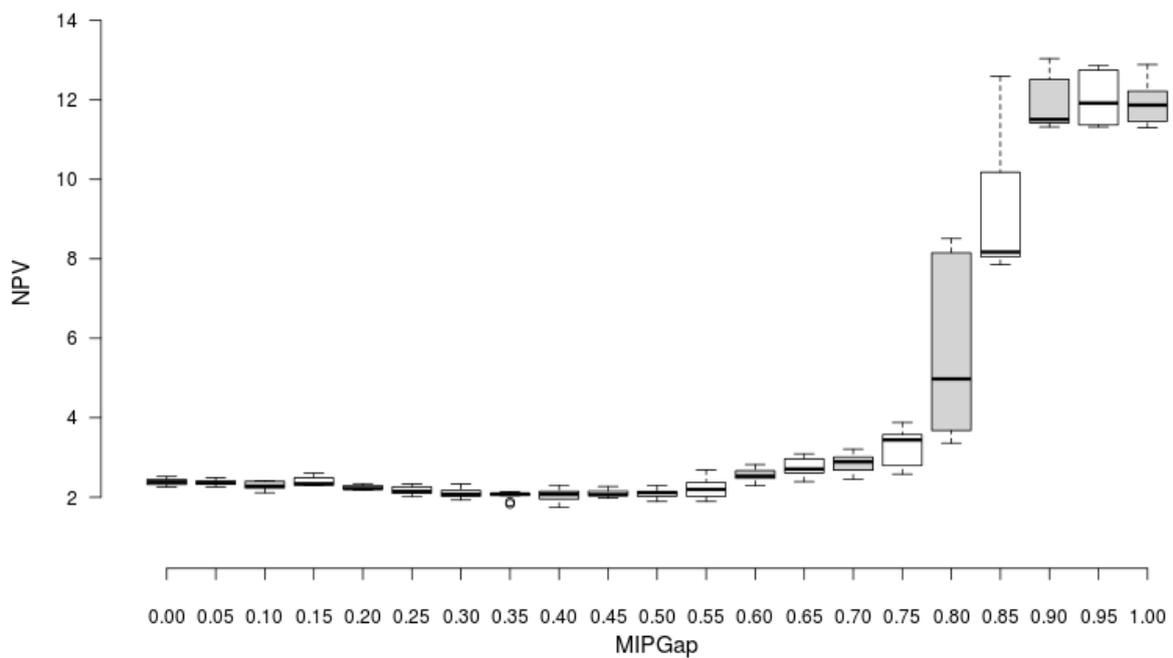


Figure 3. Boxplot comparison of instance 1 results with different MIPGap values.

reliability higher than 0.95 and its reliability are presented in the next two columns. Finally, some gaps between pairs of columns are also provided.

Table 2. Results obtained for each instance.

#	Bayliss et al. (2020)			Our Matheuristic			Gaps			
	Cplex (1)	BR (2)	r (3)	Det. (4)	Stoch. (5)	r (6)	(4) - (1)	(5) - (4)	(5) - (2)	(6) - (3)
1	1.25	1.56	0.95	1.17	1.81	0.95	-6.56%	54.72%	15.84%	0.12%
2	3.73	4.61	0.70	3.51	5.92	1.00	-5.89%	68.65%	28.43%	42.57%
3	OoM	7.7	0.47	5.96	9.43	0.99	-	58.25%	22.44%	111.07%
4	1.22	1.44	0.25	1.18	2.30	0.99	-2.95%	94.06%	59.56%	295.22%
5	3.66	5.85	0.88	1.99	2.97	0.96	-45.73%	49.75%	-49.15%	9.61%
6	5.99	8.53	0.95	3.13	3.72	0.98	-47.69%	18.75%	-56.38%	2.76%
7	10.06	11.65	0.97	9.97	12.28	0.96	-0.88%	23.20%	5.45%	-1.15%
8	33.99	42.81	0.90	34.10	42.64	0.95	0.34%	25.03%	-0.39%	5.69%
9	3.58	4.58	0.84	2.49	5.04	1.00	-30.41%	102.21%	9.99%	18.81%
10	-	-	-	5.25	10.96	0.97	-	108.77%	-	-
11	-	-	-	7.70	11.53	0.96	-	49.79%	-	-

6. Analysis of Results

As it can be seen in Table 2, the stand-alone matheuristic is providing reasonably good solutions when compared with the optimal ones given by Cplex for the deterministic scenario. Actually, Cplex is not able to solve all instances since it gets an “out of memory” (OoM) error for instance 3 (which justifies the need of using matheuristics even for the deterministic case). Also, notice that the cost of the assets-to-liabilities mapping is quite different in the deterministic scenario (*Det.*) and in the stochastic one (*Stoch.*). In other words, the deterministic scenario represents an ‘ideal’ (but not realistic) situation that provides a lower-bound to the real NPV cost under uncertainty conditions. Probably, the most interesting comparison in this table is between columns *BR* and *Stoch.* As one can see, the proposed matheuristic-simulation algorithm is usually able to outperform the previous simulation-optimisation approach proposed in Bayliss et al. (2020). This is mainly due to the fact that the methodology proposed in this paper does not require to assume a one-to-one mapping between assets and liabilities, thus allowing for an increasing number of mapping combinations. The main benefit of using the matheuristic-simulation algorithm is that it treats reliability as a hard constraint, an issue which is very important in the context of meeting liabilities. However, since the matheuristic is a more complex algorithm than *BR*, the 300 second time limit meant that there was not enough time for it to find solutions that met the 95% reliability constraint exactly, allowing it to achieve a low NPV. Notice that the gap between the NPVs of *BR* and the matheuristic are largest when the matheuristic return very reliable solution, while *BR* returns solution with low reliability. Figure 4 highlights the large average reliability gain attained from using the matheuristic, at the expense of a slightly higher NPV on average.

7. Conclusions

This paper proposes a hybrid matheuristic-simulation approach to solve the stochastic version of the asset and liability management problem, where the goal is to minimise the net present value of the assets that are employed to cover the liabilities, while satisfying a reliability constraint. First, a matheuristic is designed by combining integer programming with a heuristic. The heuristic prioritises the selection of liabilities with an earlier maturity date, and it also makes use of a random procedure to increase the diversity of solutions generated. Then, the most promising solutions generated in the previous stage are simulated in a stochastic scenario. For this, a Monte-Carlo simulation is run multiple times in order to obtain estimates of the NPV-cost and the associated reliability of each solution. One of the main novelties of this paper is that approach integrates Monte-Carlo simulation with a matheuristic to provide an algorithm which can guarantee reliable solutions for the asset and liability management

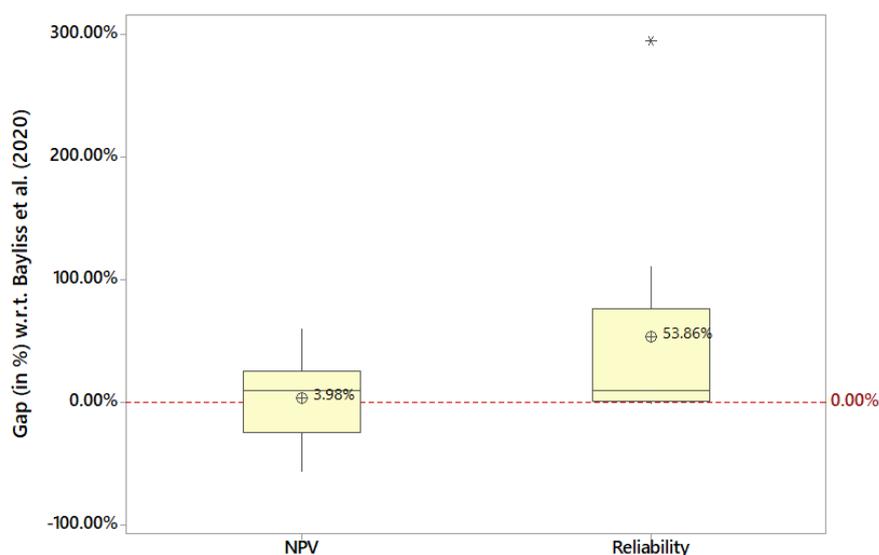


Figure 4. Boxplot comparison of NPV and reliability results w.r.t. a previous work.

320 problem. It also considers the possibility of aggregating different assets, or different liabilities, before
 321 completing the assignment mapping, i.e.: several assets can be aggregated to cover each liability, and
 322 multiple liabilities can be covered by a single asset. To the best of our knowledge, it is the first time
 323 that this many-to-many assignment procedure is considered in the literature on asset and liability
 324 management.

325 The results show that the best deterministic mapping of assets to liabilities is far from being
 326 an optimal solution when uncertainty is present. Hence, simulation-optimisation methods become
 327 necessary to generate high-quality solutions whenever some components of the asset and liability
 328 management problem need to be modelled as random variables instead of deterministic values. In
 329 addition, the numerical experiments show how, by allowing many-to-many assignments between
 330 assets and liabilities, our combined matheuristic-simulation algorithm is able to outperform other
 331 simulation-optimisation approaches. As future work, we plan to: (i) include additional characteristics
 332 in the model so it fully represents the real-life problem that insurance companies and other financial
 333 institutions have to face; and (ii) introduce and test the algorithm in real-life benchmark instances.

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