TWMS J. App. and Eng. Math. V.10, N.3, 2020, pp. 718-726

EDGE IRREGULAR REFLEXIVE LABELING FOR STAR, DOUBLE STAR AND CATERPILLAR GRAPHS

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Abstract. A graph labeling is an algorithm that assignment the labels, traditionally represented by integers, to the edges or vertices, or both, of a network(graph) G. Formally, given a graph $G = (V, E)$, a vertex labeling is a function of V to a set of labels. A graph with such a function defined is called a vertex-labeled graph. Likewise, an edge labeling is a function of E to a set of labels. In this case, the graph is called an edge-labeled graph. We study an edge irregular reflexive $k-$labeling for the star, double star and caterpillar graphs and determine the exact value of the reflexive edge strength for these graphs.

Keywords: edge irregular reflexive labeling, reflexive edge strength, star, double star, caterpillar.

AMS Subject Classification: 05C78, 05C90

1. Introduction

Let $G = (V, E)$ be a simple, finite and undirected graph. Chartrand et al. [1] proposed the following problem. Assign a positive integer label from the set $\{1, 2, ...k\}$ to the edges of a simple connected graph of order at least three in such a way that the graph becomes irregular, i.e the weight (label sum) at each vertex are distinct. What is the minimum value of the largest label k over all such irregular assignments. This parameter of the graph G is well known as the irregularity strength of the graph G. An excellent survey on the irregularity strength is given by Lahel in [2]. For recent results, see the papers by Amar and Togni in [3], Dimitz et al. in [4], Gyarfas in [5] and Nierhoff in [6].

An edge irregular $k-$labeling as a vertex labeling $\phi : V(G) \to \{1, 2, ..., k\}$ was defined, such that for every two different edges xy and $x^{'}y^{'}$ there is $w_\phi(xy) \neq w_\phi(x^{'}y^{'})$, where the weight of an edge $xy \in E(G)$ is $w_\phi(xy) = \phi(x)+\phi(y)$. The minimum k for which the graph G has an edge irregular $k-$labeling is called the edge irregularity strength of the graph

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§ Manuscript received: November 28, 2018; accepted: June 29, 2019.

G, denoted by $es(G)$. In [7] are estimated the bounds of the parameters $es(G)$, and the exact value of the edge irregularity strength for several families of graphs are determined, namely paths, stars, double stars and the Cartesian product of two paths.

Baca *et.al* [8], defined the total labeling $\phi : V(G) \cup E(G) \to \{1, 2, ..., k\}$ to be an edge irregular total $k-$labeling of the graph G if for every two different edge xy and $x^{'}y^{'}$ of G one has $w_\phi(xy) = \phi(x) + \phi(xy) + \phi(y) \neq w_\phi(x^{'}y^{'}) = \phi(x^{'}) + \phi(x^{'}y^{'}) + \phi(y^{'})$. The total edge irregularity strength, $tes(G)$, is defined as the minimum k for which G has an edge irregular total $k-$labeling. Estimated of this parameters are obtained, which provides the precise values of the total edge irregularity strength for paths, cycles, stars, wheels and friendship graphs. Further results on the total irregularity strength can be found in [9, 10, 11, 12, 13, 14].

The comparable issue for the unpredictable naming emerge from a thought of graphs with particular degree. In a simple graph, it isn't conceivable to build a diagram in which each vertex has a one of a kind degree; notwithstanding, this is conceivable in multigraphs (graphs in which we permit various edges between the nearby vertices). The inquiry at that point moved toward becoming: "what is the littlest number of parallel edges between two vertices required to guarantee that the graph show vertex anomaly?" This issue is identical to the marking issue as depicted toward the start of this area.

Ryan et.al, [15], decreed that the vertex labels should represent loops at the vertex. The consequence was two-fold; first, each vertex label was required to be an even integer, since each loop added two to the vertex degree; and second, unlike in total irregular labeling, the label 0 was permitted as representing a loopless vertex. Edges continued to be labelled by integers from one to k.

Thus, they defined labelings $f_e : E(G) \to \{1, 2, ..., k_e\}$ and $f_v : V(G) \to \{0, 2, ..., 2k_v\}$, and then, labeling f is a total $k-$labeling of G defined such that $f(x) = f_v(x)$ if $x \in V(G)$ and $f(x) = f_e(x)$ if $x \in E(G)$, where $k = max\{k_e, 2k_v\}$.

The total $k-$labeling f is called an edge irregular reflexive $k-$labeling of the graph G if for every two different edges xy and $x^{'}y^{'}$ of G, one has $wt(xy) = f_v(x) + f_e(xy) + f_v(y) \neq wt(x^{'}y^{'}) = f_v(x^{'}) + f_e(x^{'}y^{'}) + f_v(y^{'})$. The smallest value of k for which such labeling exists is called the reflexive edge strength of the graph G and is denoted by $res(G)$. For recent results see [16, 17].

The result of this variation was not widely manifest in the labeling strength, but did produce some important outcomes:

$tes(K_5) = 5$ whereas $res(K_5) = 4$

The effect of this change was immediate in the following conjecture where we were able to remove the pesky exception see [18].

Conjecture 1.1. *Any graph G with maximum degree $\Delta(G)$ other than K_5 satisfies:*

$$\text{tes}(G) = \max\left\{\left\lceil\frac{|E(G)|+2}{3}\right\rceil, \left\lceil\frac{\Delta+1}{2}\right\rceil\right\}.$$

In term of reflexive edge irregularity strength, Baca et al. [19] purpose the following conjecture and prove the Theorem 1.

Conjecture 1.2. *Any graph G with maximum degree $\Delta(G)$ satisfies*

$$\operatorname{res}(G) = \max\left\{\left\lceil \frac{|E(G)|}{3} + r \right\rceil, \left\lfloor \frac{\Delta+2}{2} \right\rfloor\right\},$$

where $r = 1$ for $|E(G)| \equiv 2, 3(mod\ 6)$, and zero otherwise.

Theorem 1.1. *For every graph G, $res(C_n) =$* $\begin{cases} \lceil \frac{|E(G)|}{3} \rceil, & \text{if } \ n \not\equiv 2,3(mod\ 6) \\ \lceil \frac{|E(G)|}{3} \rceil + 1, & \text{if } \ n \equiv 2,3(mod\ 6). \end{cases}$

2. Constructing an Edge Irregular Reflexive Labeling

Let us recall the following lemma.

Lemma 2.1. *For every graph G, $res(G) \geq$* $\begin{cases} \lceil \frac{|E(G)|}{3} \rceil, & \text{if } \ n \not\equiv 2,3(mod\ 6) \\ \lceil \frac{|E(G)|}{3} \rceil + 1, & \text{if } \ n \equiv 2,3(mod\ 6). \end{cases}$

The lower bound for $res(G)$ follows from the fact that the minimal edge weight under an edge irregular reflexive labeling in one, and the minimum of the maximal edge weight, that is $|E(G)|$ can be achieved only as the sum of three numbers, at least two of which are even. In this paper, we will determine reflexive edge irregularity strength of star graphs $K_{1,n}$, double star $S_{n,n}$ and caterpillar $S_{n,2,2,m}$.

3. Result for Star graph

In [20] Indriati *et al.* defined the star graph as follows. A star $K_{1,n}$, is a graph constructed from join operation between K_1 and complement of K_n. A vertex with degree n is called a center while vertices with degree 1 are called leaves (pendant) vertices. A star $K_{1,n}$ has $n+1$ vertices and n edges. Let the vertex set of $K_{1,n}$ be $V(K_{1,n}) = \{v, v_i : 1 \leq i \leq n\}$ with v is the center vertex v_i and are the pendant vertices. The edge set of $K_{1,n}$ is $E(K_{1,n}) = \{vv_i : 1 \leq i \leq n\}$. Figure 1 represented the star graphs.

Theorem 3.1. *For star graph $K_{1,n}$ with $n \geq 4$, we have $res(K_{1,n}) = \lfloor \frac{n+2}{2} \rfloor$.*

Proof. As maximum degree of star graph is of central vertex i.e $\triangle = n$, therefore from lemma 1, we have
$res(K_{1,n}) \geq \lfloor \frac{n+2}{2} \rfloor$, let $k = \lfloor \frac{n+2}{2} \rfloor$
In order to prove the upper bound of $res(K_{1,n})$, we described a k−-labeling f from $V \bigcup E :\to \{1, 2, 3, ..., k\}$ as followed:
$f(v) = 0$

$$f(v_i) = \begin{cases} 2\lfloor \frac{i-1}{2} \rfloor, & \text{if } \ 1 \leq i \leq k \\ k, & \text{if } \ k+1 \leq i \leq n, \quad n \equiv 2,3(mod\ 4) \\ k-1, & \text{if } \ k+1 \leq i \leq n, \quad n \equiv 0,1(mod\ 4) \end{cases}$$

$$f(vv_i) = \begin{cases} 1, & \text{if } \ 1 \leq i \leq k, \quad (i \ odd) \\ 2, & \text{if } \ 2 \leq i \leq k, \quad (i \ even) \\ i-k, & \text{if } \ k+1 \leq i \leq n, \quad n \equiv 2,3(mod\ 4) \\ i-k+1, & \text{if } \ k+1 \leq i \leq n, \quad n \equiv 0,1(mod\ 4) \end{cases}$$

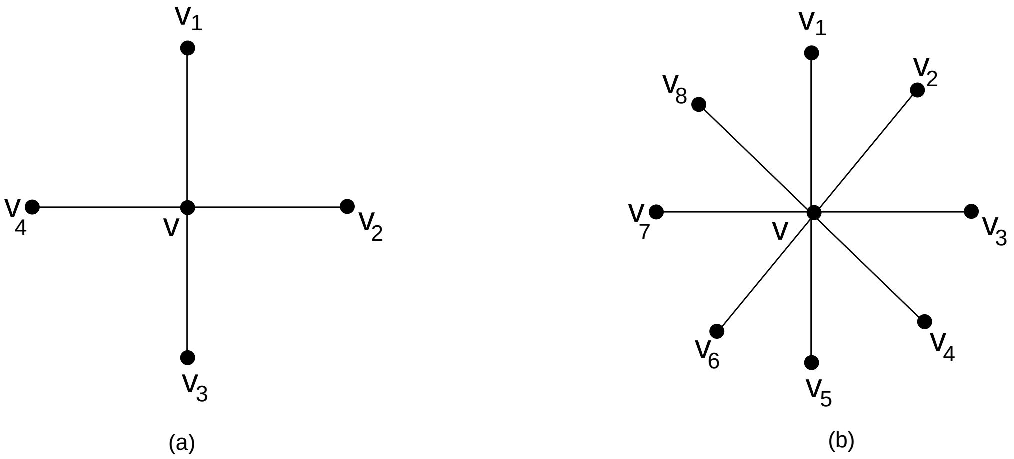

FIGURE 1. (a) Star $K_{1,4}$ (b) Star $K_{1,8}$.

The weights of edges under the labeling f are given as follows:
$w_t(vv_i) = \{i, \quad 1 \leq i \leq n$

It is a matter of routine to check that there are no two edges of the same weight. Thus f is an edge irregular reflexive $k-$ labeling for $K_{1,n}$ and $n \geq 4$. Which completes the proof. □

4. Result for Double Star graph

In [20] Indriati *et al.* defined the double star graphs as follows. A double star denoted by $S_{n,n}$ is obtained by connecting the centers of two isomorphic disjoint star $K_{1,n}$ for $n \geq 3$. One of the pendant vertex of each $k_{1,n}$ is embedded to the center of other $K_{1,n}$. Let the vertex set of $S_{n,n}$ be $V(S_{n,n}) = \{v_i^j : 1 \leq i \leq n-1, \;\; j = 1,2\} \bigcup \{v^j : j = 1,2\}$ and edge set $E(S_{n,n}) = \{v^j v_i^j : 1 \leq i \leq n-1, j = 1,2\}$. Figure 2 represented the Double star graphs.

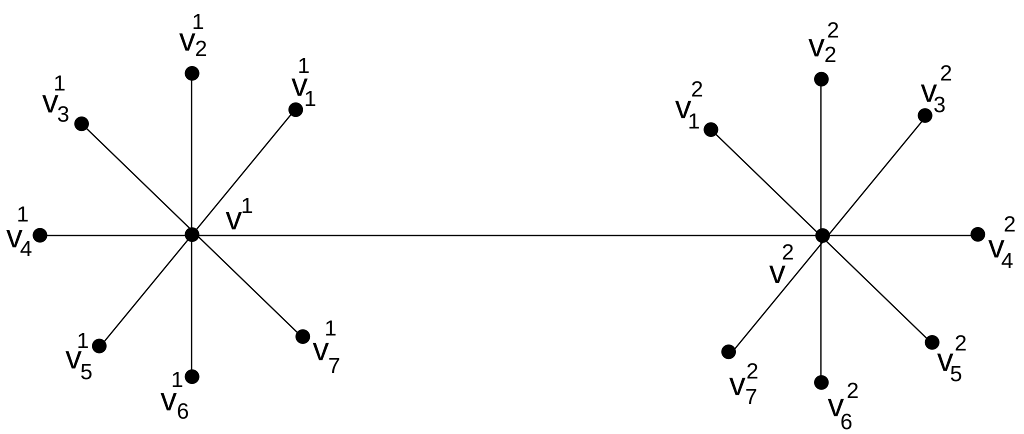

FIGURE 2. Double Star $S_{7,7}$.

Theorem 4.1. *Let $(S_{n,n})$ be the double star graph, then for $n \geq 4$ we have*

$$res(S_{n,n}) = \begin{cases} \lceil \frac{2n-1}{3} \rceil + 1, & \text{if } (2n-1) \equiv 2,3(mod\ 6) \\ \lceil \frac{2n-1}{3} \rceil, & \text{if } (2n-1) \not\equiv 2,3(mod\ 6) \end{cases}$$

Proof. As $|E(S_{n,n})| = 2n - 1$, therefore from lemma 1 we have

$$res(S_n^2) \geq \begin{cases} \lceil \frac{2n-1}{3} \rceil + 1, & \text{if } (2n-1) \equiv 2,3(mod\ 6) \\ \lceil \frac{2n-1}{3} \rceil, & \text{if } (2n-1) \not\equiv 2,3(mod\ 6) \end{cases}$$

Let $k = \begin{cases} \lceil \frac{2n-1}{3} \rceil + 1, & \text{if } (2n-1) \equiv 2,3(mod\ 6) \\ \lceil \frac{2n-1}{3} \rceil, & \text{if } (2n-1) \not\equiv 2,3(mod\ 6) \end{cases}$

In order to prove the upper bound of $res(S_{n,n})$, we described a $k-$ labeling f from $V \bigcup E :\rightarrow \{1, 2, 3, ..., k\}$

$$f(v^1 v_i^1) = \begin{cases} 1, & \text{if } 1 \leq i \leq k, \ (i\ odd) \\ 2, & \text{if } 2 \leq i \leq k, \ (i\ even) \\ i - k, & \text{if } k+1 \leq i \leq n-1, \ n \not\equiv 1(mod\ 3) \\ i - k + 1, & \text{if } k+1 \leq i \leq n-1, \ n \equiv 1(mod\ 3) \end{cases}$$

$$f(v^1 v^2) = \begin{cases} \frac{n}{3} & \text{if } n \equiv 0(mod\ 3) \\ \frac{n-1}{3} + 1 & \text{if } n \equiv 1(mod\ 3) \\ \frac{n-2}{3} & \text{if } n \equiv 2(mod\ 3) \end{cases}$$

$$f(v^2 v_i^2) = \begin{cases} \frac{n}{3} + 1, & \text{if } 1 \leq i \leq k, \ (i\ odd), \ n \equiv 0(mod\ 3) \\ \frac{n}{3} + 2, & \text{if } 2 \leq i \leq k, \ (i\ even), \ n \equiv 0(mod\ 3) \\ \frac{n-1}{3} + 1, & \text{if } 1 \leq i \leq k, \ (i\ odd), \ n \equiv 1(mod\ 3) \\ \frac{n-1}{3} + 2, & \text{if } 2 \leq i \leq k, \ (i\ even), \ n \equiv 1(mod\ 3) \\ \frac{n-2}{3} + 1, & \text{if } 1 \leq i \leq k, \ (i\ odd), \ n \equiv 2(mod\ 3) \\ \frac{n-2}{3} + 2, & \text{if } 2 \leq i \leq k, \ (i\ even), \ n \equiv 2(mod\ 3) \\ i - \frac{n}{3}, & \text{if } k+1 \leq i \leq n-1, \ n \equiv 0(mod\ 3) \\ i - \frac{n-1}{3} + 1, & \text{if } k+1 \leq i \leq n-1, \ n \equiv 1(mod\ 3) \\ i - \frac{n-2}{3} - 2, & \text{if } k+1 \leq i \leq n-1, \ n \equiv 2(mod\ 3) \end{cases}$$

The weights of edges under the labeling f are given as follows:
$w_t(v^1 v_i^1) = i, \qquad 1 \leq i \leq n-1$
$w_t(v^1 v^2) = n$
$w_t(v^2 v_i^2) = n + i, \qquad 1 \leq i \leq n-1$
It is a matter of routine to check that there are no two edges of the same weight. Thus f is an edge irregular reflexive $k-$ labeling for $S_{n,n}$ and $n \geq 4$.
Which completes the proof. □

5. Result for Caterpillar graph

In [20] Indriati *et al.* defined the caterpillar as follows. A caterpillar $S_{n,2,2,m}$ and $n \neq m$ is a class of graph constructed from the double-star $S_{n,m}$ by inserting two vertices on the bridge connecting of the two centers of the two stars. Therefore this caterpillar contain four stars with center of the two end-star have degree n and m respectively, while the center of two star in the middle each has degree two. This graph is a tree with $n+m+1$ edges and $m+n-2$ pendant vertices. We assume that $n > m$, then $\triangle = n$. Figure 3 represented the Caterpillar graphs.

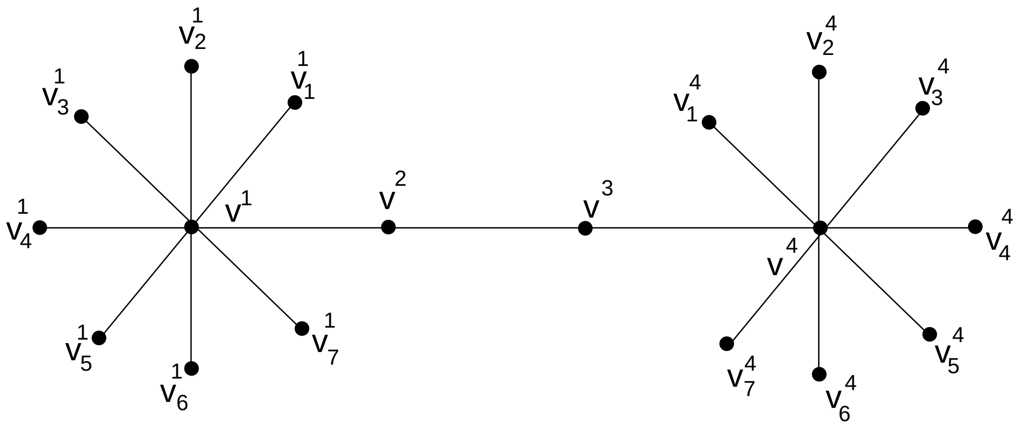

FIGURE 3. Caterpillar $S_{7,2,2,7}$.

Theorem 5.1. *For caterpillar $S_{n,2,2,m}$, $n > m$ and $m \geq 3$ with $n \geq 5$ we have*

$$res(S_{n,2,2,m}) = \begin{cases} \lfloor \frac{n+2}{2} \rfloor, & \text{if } \ n \geq 2m, \ \ (m \ \ odd) \\ \lfloor \frac{n+2}{2} \rfloor, & \text{if } \ n \geq 2m+2, \ \ (m \ \ even) \end{cases}$$

Proof. As $|E(S_{n,2,2,m})| = (m+n+1)$, therefore from lemma 1, we have

$$res(S_{n,2,2,m}) \geq \begin{cases} \lfloor \frac{n+2}{2} \rfloor, & \text{if } \ n \geq 2m, \ \ (m \ \ odd) \\ \lfloor \frac{n+2}{2} \rfloor, & \text{if } \ n \geq 2m+2, \ \ (m \ \ even) \end{cases}$$

Let $k = \begin{cases} \lfloor \frac{n+2}{2} \rfloor, & \text{if } \ n \geq 2m, \ \ (m \ \ odd) \\ \lfloor \frac{n+2}{2} \rfloor, & \text{if } \ n \geq 2m+2, \ \ (m \ \ even) \end{cases}$

In order to prove the upper bound of $res(S_{n,2,2,m})$, we described a $k-$ labeling f from $V \bigcup E :\to \{1, 2, 3, ..., k\}$ as followed:

Let $v^2 = v_n^1, \quad f(v^1) = 0$

$$f(v^j) = \begin{cases} k-1, & \text{if } \ j = 3, 4, \ \ n \equiv 0, 1 (mod \ \ 4) \\ k, & \text{if } \ j = 3, 4, \ \ n \equiv 2, 3 (mod \ \ 4) \end{cases}$$

$$f(v_i^1) = \begin{cases} 2\lfloor \frac{i-1}{2} \rfloor, & \text{if } \ 1 \leq i \leq k \\ k-1, & \text{if } \ k+1 \leq i \leq n, \ \ n \equiv 0, 1 (mod \ \ 4) \\ k, & \text{if } \ k+1 \leq i \leq n, \ \ n \equiv 2, 3 (mod \ \ 4) \end{cases}$$

$$f(v_i^4) = \begin{cases} 2\lceil \frac{i-1}{2} \rceil, & \text{if } \ 1 \leq i \leq m-1, \ \ n \equiv 0, 3 (mod \ \ 4) \\ 2\lfloor \frac{i-1}{2} \rfloor, & \text{if } \ 1 \leq i \leq m-1, \ \ n \equiv 2 (mod \ \ 4) \\ 2\lceil \frac{i}{2} \rceil, & \text{if } \ 1 \leq i \leq m-1, \ \ n \equiv 1 (mod \ \ 4) \end{cases}$$

$$f(v^1v_i^1)=\begin{cases} 1, & \text{if } 1\le i\le k, \ (i \ odd)\\ 2, & \text{if } 2\le i\le k, \ (i \ even)\\ i-k, & \text{if } k+1\le i\le n, \ n\equiv 2,3(mod \ 4)\\ i-k+1, & \text{if } k+1\le i\le n, \ n\equiv 0,1(mod \ 4)\end{cases}$$

$$f(v^4v_i^4)=\begin{cases} \frac{n-2}{2}+1, & \text{if } 1\le i\le m-1, \ (i \ odd), \ n\equiv 2(mod \ 4)\\ \frac{n-2}{2}+2, & \text{if } 2\le i\le m-1, \ (i \ even), \ n\equiv 2(mod \ 4)\\ \frac{n-3}{2}+2, & \text{if } 1\le i\le m-1, \ (i \ odd), \ n\equiv 3(mod \ 4)\\ \frac{n-3}{2}+1, & \text{if } 2\le i\le m-1, \ (i \ even), \ n\equiv 3(mod \ 4)\\ \frac{n}{2}+1, & \text{if } 1\le i\le m-1, \ (i \ odd), \ n\equiv 0(mod \ 4)\\ \frac{n}{2}, & \text{if } 2\le i\le m-1, \ (i \ even), \ n\equiv 0(mod \ 4)\\ \frac{n-1}{2}, & \text{if } 1\le i\le m-1, \ (i \ odd), \ n\equiv 1(mod \ 4)\\ \frac{n-1}{2}+1, & \text{if } 2\le i\le m-1, \ (i \ even), \ n\equiv 1(mod \ 4)\end{cases}$$

$$f(v^jv^{j+1})=\begin{cases} m-1, & \text{if } n\equiv 2(mod \ 4), \ j=2\\ m-2, & \text{if } n\equiv 2(mod \ 4), \ j=3\\ m, & \text{if } n\equiv 3(mod \ 4), \ j=2\\ m-1, & \text{if } n\equiv 3(mod \ 4), \ j=3\\ m+1, & \text{if } n\equiv 0(mod \ 4), \ j=2\\ m, & \text{if } n\equiv 0(mod \ 4), \ j=3\\ m+2, & \text{if } n\equiv 1(mod \ 4), \ j=2\\ m+1, & \text{if } n\equiv 1(mod \ 4), \ j=3\end{cases}$$

The weights of edges under the labeling f are given as follows:
$w_t(v^1v_i^1)=i, \qquad 1\le i\le n$
$w_t(v^4v_i^4)=n+i, \qquad 1\le i\le m$
$w_t(v^2v^2)=m+n+1$
where $v^3=v_m^4$
It is a matter of routine to check that there are no two edges of the same weight. Thus f is an edge irregular reflexive $k-$ labeling for $S_{n,2,2,m}$
Which completes the proof. □

6. Conclusion

In this paper we have determined the edge irregular reflexive labeling for star graphs, $K_{1,n}$ double star $S_{n,n}$ and caterpillar $S_{n,2,2,m}$ for $n>m$. We tried to find the edge irregular reflexive labeling for $S_{n,2,2,m}$ when $m<n<2m(m \ \ odd)$ and $m<n<2m+2(m \ \ even)$ when $m\ge 3$ and $n\ge 5$ but so far without success. So we conclude the paper with the following open problems.

Open Problem 1. *Determined the edge irregular reflexive labeling for caterpillar $S_{n,2,2,m}$ when $m<n<2m(m \ \ odd)$ and $m<n<2m+2(m \ \ even)$ when $m\ge 3$ and $n\ge 5$*

$$res(S_{n,2,2,m})=\begin{cases} \lceil\frac{m+n+1}{3}\rceil, & \text{for } (m+n+1)\not\equiv 2,3(mod \ 6)\\ \lceil\frac{m+n+1}{3}\rceil+1, & \text{for } (m+n+1)\equiv 2,3(mod \ 6)\end{cases}$$

Open Problem 2. *Determine the edge irregular reflexive labeling for caterpillar $S_{n,2,2,m}$ for any choice of m and n.*

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