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Group decision making in linguistic contexts: an information granulation approach

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Abstract

Group decision making situations are part of today's organizations. It is a type of decision making involving many decision makers which act collectively to choose the best alternative (or alternatives) from a set of feasible alternatives. Usually, numerical values have been used by the decision makers to express their opinions on the possible alternatives. However, as the standard representation of the concepts that humans use for communication is the natural language, words or linguistic terms instead of numerical values should be used by the decision makers to provide their preferences. In such a situation, the linguistic information has to be made operational in order to be fully utilized. In this contribution, assuming that decision makers express their opinions by using linguistic terms, we present an information granulation of such a type of information, which is formulated as an optimization problem in which consistency is maximized by a suitable mapping of the linguistic terms on information granules.

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1. Introduction

One of the most crucial human activities is decision making, whose essence is to find the best opinion, alternative, and so on, from a set of feasible ones. In particular, most of decision making situations in real world usually involve multiple decision makers to make the decision [1]. In such a case, it is called a multiperson decision making situation, being group decision making (GDM) an important class among multiperson decision making settings [2].

GDM is defined as a situation in which there is a set of alternatives and a set of decision makers who provide their preferences concerning the alternatives. The problem here is to find a solution (an alternative or set of alternatives) which is best acceptable by the group of decision makers as a whole. The ideal situation would be one where all the decision makers could convey their preferences on the alternatives in a precise way by means of numerical values.

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Unfortunately, in many cases, decision makers deal with imprecise information or have to verbalize their preferences on qualitative aspects which cannot be evaluated by means of quantitative values. In addition, as decision making is an inherent human ability that is not necessarily rationally guided, it can be based on tacit or explicit assumptions and it does not need complete and precise measurements about the alternatives [3]. This fact has led to researchers to apply the fuzzy sets theory, introduced by Zadeh in 1965 [4], to model the vagueness and uncertainty in GDM situations [5–7].

In recent years, linguistic information have been used to represent the preferences expressed by the decision makers about the alternatives [3,8–11]. The main purpose of using words or sentences in natural language, i.e. linguistic values, instead of numerical ones is that linguistic values are, in general, less specific than numbers, but much more closer to the way that humans verbalize and use their knowledge [3,12]. For instance, if we say "the man is tall" is less specific than "the man measures 2 m". Here, "tall" can be seen as a linguistic value which is less precise and informative than the numerical value "2". Despite its less informative nature, the value "tall" allows humans to naturally convey and deal with information that may be uncertain or incomplete (the speaker may not know the exact man height). As these situations where information is not precise are very common in real world, linguistic variables are a powerful tool to model human knowledge [3].

In GDM situations in which linguistic values are used to represent the opinions given by the decision makers, a mechanism to made operational the linguistic information is required. To do so, linguistic computational models have been presented by researchers [3]:

- The linguistic computational model based on membership functions [13].
- The linguistic computational model based on type-2 fuzzy sets [14].
- The linguistic symbolic computational models based on ordinal scales [15–17].
- The 2-tuple linguistic computational model [18], which is a symbolic computational model that extends the use of indexes.
- The linguistic computational model based on discrete fuzzy numbers [19].

In this contribution, we present an information granulation of the linguistic information in order to made it operational. To do so, granular computing representing and processing information in form of information granules is used [20]. Information granules are complex information entities arising in the process, which is called information granulation, of abstraction of data and derivation of knowledge from information [21]. Here, due to the process of information granulation and the nature of information granules, the definition of a formalism well-suited to represent the problem at hand is required. The resulting information granules are then effectively processed within the computing setting pertinent to the assumed framework of information granulation. In the literature, we can find several formal frameworks in which information granules can be defined, as for example:

- Sets (interval mathematics) [22].
- Fuzzy sets [4,23–25].
- Rough sets [26].
- Shadowed sets [27].
- Probabilities (probability density functions) [28].

Two important questions about information granulation in order to make operational the linguistic information are the following:

- How the linguistic values have to be translated into the entities?
- What optimization criterion can be envisioned when arriving at the formalization of the linguistic values through information granules?

Here, we formulate the information granulation as an optimization problem in which a consistency index is optimized by a suitable mapping of the linguistic values on information granules. To do so, the particle swarm optimization (PSO) [29] is used as the optimization framework, which supports the formation of the information granules. In addition, the granulation formalism considered here concerns intervals (sets), although any other formal scheme of information granulation could be equally utilized.

The rest of this contribution is organized as follows: Section 2 describes both the classical GDM situation and the method used to obtain the consistency level achieved by a decision maker. The information granulation of the linguistic information is presented in Section 3. An experimental example is shown in Section 4 to illustrate it. Finally, some conclusions are pointed out in Section 5.

2. Preliminaries

In this section, on the one hand, we introduced the classical GDM situation, and, on the other hand, we show the concept of consistency and describe how it can be obtained.

2.1. GDM framework

In a classical GDM situation [5,30], there is a problem to solve, a solution set of possible alternatives, $X = \{x_1, x_2, \ldots, x_n\}$ $(n \geq 2)$, and a group of two or more decision makers, $E = \{e_1, e_2, \ldots, e_m\}$ $(m \geq 2)$, characterized by their background and knowledge, who verbalize their preferences about the possible alternatives to achieve a common solution. In a fuzzy context, the objective is to classify the alternatives from best to worst, associating with them some degrees of preference assessed in the $[0, 1]$ interval.

Decision makers can use several preference representation structures to convey their preferences or opinions about the alternatives in a GDM situation. The most common ones that have been widely used in the literature are the following:

- *Preference orderings*. Using this preference representation structure, the opinions of a decision maker $e_l \in E$ about a set of feasible alternatives X are described as a preference ordering $O^l = \{o_1^l, \ldots, o_n^l\}$, where $o^l(\cdot)$ is a permutation function over the index set $\{1, \ldots, n\}$ [31]. Hence, a decision maker gives an ordered vector of alternatives from best to worst.
- *Utility values*. Using this preference representation structure, a decision maker $e_l \in E$ expresses his/her opinions about a set of feasible alternatives X by means of a set of n utility values $U^l = \{u_1^l, \ldots, u_n^l\}$, $u_i^l \in [0, 1]$. Here, the higher the value for an alternative, the better it satisfies decision maker's objective [32].
- *Preference relations*. In this case, the preferences given by the decision maker on X are described by a function $\mu_{P^l} : X \times X \to D$ where $\mu_{P^l}(x_i, x_k) = p_{ik}^l$ can be interpreted as the preference degree or intensity of the alternative x_i over x_k expressed in the information representation domain D. Different types of preference relations can be used according to the domain used to evaluate the intensity of the preference [2,16,17,33]. If D is a linguistic domain, then linguistic values as "High", "Medium", "Low", could be used.

Among the above representation formats, preference relations are the most used for solving GDM problems due to effectiveness in modeling decision processes. In particular, the effort to complete pairwise evaluations is far more manageable in comparison to any experimental overhead we need when assigning membership grades to all alternatives of the universe in a single step, which implies that the decision maker must be able to assess each alternative against all the others as a whole, which can be a difficult task.

Once the decision makers have provided their preferences about the alternatives, a solution set of alternatives has to be chosen. To do so, a selection process is carried out [34,35], which involves two different steps:

- Aggregation of individual preferences into a group collective one in such a way that it summarizes the properties contained in all the individual preferences.
- Exploitation of the collective preference to identify the solution set of alternatives. To do so, we must apply some mechanism to obtain a partial order of the alternatives and, in such a way, select the best alternative(s).

2.2. Consistency

When information is provided by decision makers, an important issue to bear in mind is that of consistency [36–38]. The pairwise comparison helps the decision makers focus only on two elements once at a time. It reduces uncertainty and hesitation while leading to the higher of consistency. However, due to the complexity of most GDM situations, decision makers preferences can be inconsistent. In addition, the definition of a preference relation does not imply any kind of consistency property, although the study of consistency is crucial for avoiding misleading solutions in GDM. It is obvious that consistent information, which does not imply any kind of contradiction, is more relevant or important than the information containing some contradictions. Fortunately, the lack of consistency can be quantified and monitored [39,40], and it will be used as the optimization criterion.

In the following, we describe how characterize the consistency of the fuzzy preference relations because the granulation formalism considered in this contribution to represent the linguistic information concerns intervals in $[0, 1]$.

Definition 1. A fuzzy preference relation P on a set of alternatives X is a fuzzy set on the product set $X \times X$, which is characterized by a membership function $\mu_P : X \times X \to [0, 1]$.

Every value p_{ik} in the matrix P represents the preference degree or intensity of preference of the alternative x_i over x_k: $p_{ik} = 0.5$ indicates indifference between x_i and x_k ($x_i \sim x_k$), $p_{ik} = 1$ indicates that x_i is absolutely preferred to x_k, and $p_{ik} > 0.5$ indicates that x_i is preferred to x_k ($x_i \succ x_k$). Based on this interpretation we have that $p_{ii} = 0.5$ $\forall i \in \{1, \ldots, n\}$ ($x_i \sim x_i$). Since p_{ii}'s do not matter, we will write them as '–' instead of 0.5 [2,38]. Moreover, it is assumed that the matrix is reciprocal, that is $p_{ik} + p_{ki} = 1$ $\forall i, k \in \{1, \ldots, n\}$.

Different properties to be satisfied by the fuzzy preference relations have been proposed in the literature to make a rational choice [40]. Here, in this contribution, we use the additive transitivity property facilitating the verification of consistency when fuzzy preference relations are used to represent the preferences given by the decision makers. In [40], it was shown that additive transitivity for fuzzy preference relations can be seen as the parallel concept of Saaty's consistency property for multiplicative preference relations [41]. The mathematical formulation of the additive transitivity was given by Tanino in [31]:

$$(p_{ij} - 0.5) + (p_{jk} - 0.5) = (p_{ik} - 0.5), \forall i, j, k \in \{1, \ldots, n\} \tag{1}$$

Because the additive transitivity implies additive reciprocity ($p_{ij} + p_{ji} = 1,\ \forall i, j$), it can be rewritten as:

$$p_{ik} = p_{ij} + p_{jk} - 0.5, \forall \in i, j, k\{1, \ldots, n\} \tag{2}$$

A fuzzy preference relation is considered to be "additive consistent" when for every three alternatives encountered in the problem, say $x_i, x_j, x_k \in X$ their associated preference degrees p_{ij}, p_{jk}, p_{ik} fulfill Eq. (2).

Given a reciprocal fuzzy preference relation, Eq. (2) can be used to calculate an estimated value of a preference degree using other preference degrees. Indeed, using an intermediate alternative x_j, the following estimated value of p_{ik} ($i \neq k$) is obtained [37,38,40]:

$$ep_{ik}^{j} = p_{ij} + p_{jk} - 0.5 \tag{3}$$

The overall estimated value ep_{ik} of p_{ik} is obtained as the average of all possible values ep_{ik}^{j}:

$$ep_{ik} = \sum_{j=1; j \neq i,k}^{n} \frac{ep_{ik}^{j}}{n-2} \tag{4}$$

The value $|ep_{ik} - p_{ik}|$ can be used as a measure of the error between a preference value and its estimated one [38].

When information provided is completely consistent then $ep_{ik}^{j} = p_{ik}\ \forall j$. However, because decision makers are not always fully consistent, the evaluation made by a decision maker may not verify Eq. (2) and some of the estimated preference degree values ep_{ik}^{j} may not belong to the unit interval $[0, 1]$. From Eq. (3), it is noted that the maximum value of any of the preference degrees ep_{ik}^{j} is 1.5 while the minimum one is -0.5. In order to normalize the expression domains in the decision model, the final estimated value of p_{ik} ($i \neq k$), cp_{ik}, is defined as the median of the values 0, 1 and ep_{ik}:

$$cp_{ik} = \text{median}\{0, 1, ep_{ik}\} \tag{5}$$

The error assuming values in [0, 1] between a preference value, p_{ik}, and its final estimated one, cp_{ik}, is:

$$\varepsilon p_{ik} = |cp_{ik} - p_{ik}| \tag{6}$$

Reciprocity of $P = (p_{ik})$ implies reciprocity of $CP = (cp_{ik})$, therefore $\varepsilon p_{ik} = \varepsilon p_{ki}$. $\varepsilon p_{ik} = 0$ is interpreted as a situation of total consistency between p_{ik} (p_{ki}) and the rest of entries of P. Obviously, the higher the value of εp_{ik} the more inconsistent is p_{ik} (p_{ki}) with respect to the remaining entries of P.

The above interpretation allows us to assess the consistency degree associated to a reciprocal fuzzy preference relation P as follows [37]:

$$cd = \frac{\sum_{i,k=1;i\neq k}^{n} (1 - \varepsilon p_{ik})}{n^2 - n} \tag{7}$$

When $cd = 1$, the reciprocal fuzzy preference relation P is fully consistent, otherwise, the lower cd the more inconsistent P is.

3. An information granulation of the linguistic information

In this section, we present the granulation process of the linguistic values, which leads to the operational realization of further processing forming a ranking of alternatives considering the opinions expressed by the decision makers. In addition, the optimization framework of this granulation process is described.

3.1. Granulation process

We are interested in GDM situations defined in linguistic contexts, that is, it is assumed that decision makers use linguistic values in the pairwise comparison of alternatives in the preference relation. For instance, linguistic values as “Low” or “High” could be used. It should be pointed out that the linguistic values may be organized in a linear fashion, as there is an apparent linear order among them. Any case, a quantification of the linguistic values is required in order to operate with them.

As the granulation formalism considered in this contribution concerns intervals, the problem of a granular description of linguistic values is concerned with the formation of a family of intervals over the unit interval. Therefore, information granules come in the form of intervals $[a_k, a_{k+1}]$, that is to say, information granules $L_1, L_2, \ldots, L_c$ where $L_1 = [0, a_1)$, $L_2 = [a_1, a_2), \ldots,$ $L_i = [a_{i-1}, a_i), \ldots,$ $L_c = [a_{c-1}, 1]$. These intervals form a partition of the unit interval where $0 < a_1 < \ldots < a_{c-1} < 1$. In such a way, the interval format of granulation of the unit interval is fully characterized by the vector of cutoff points of the granular transformation in the unit interval, $\mathbf{a} = [a_1\ a_2\ \ldots\ a_{c-1}]$.

The two main characteristics of this granulation process are the following:

- The mapping is by no means linear, i.e., a localization of the associated information granules on the scale is not uniform.
- The semantics of the linguistic values allocated in the process of granulation is retained.

Finally, we should point out that a joint treatment of the linguistic values, coming from the decision makers involved in the GDM problem, is considered here. On the one hand, this allows us to deal with the linguistic values in a unified fashion. On the other hand, it allows us to reconcile the semantics of the linguistic values in such a way that the individual consistencies are made comparable and, therefore, could be aggregate to arrive at the joint view at the optimization criterion.

3.2. Optimization framework

The way in which we arrive at the operational version of the information granules specified as intervals is formulated as an optimization problem, which has to be specified in such a way that all details are addressed.

3.2.1. The optimization criterion

First, the optimization criterion needs to be defined. Here, we use the consistency of the preference relations given by the decision makers to obtain the quality of the solution achieved in a GDM situation. In such a way, the solution obtained will be better if the consistency level of each preference relation is high. For this reason, the quality of a given vector of cutoff points is obtained by means of a performance index calculated as the aggregation of the consistency levels measured for all the preference relations given by all the decision makers $\{P^1, \ldots, P^m\}$. The objective is to increase this performance index, which is utilized as optimization criterion.

To optimize the performance index, different alternatives as, for instance, PSO or genetic algorithms could be taken into account. Here, we use PSO as it is very attractive given its less significant computing overhead in comparison with genetic algorithms [42]. Furthermore, PSO offers a significant level of diversity of possible objective functions, playing a role of fitness functions.

3.2.2. PSO environment

The construction of the information granules formalized as intervals is carried out by means of the PSO, which is a viable optimization alternative for this problem. PSO algorithm is a population-based stochastic optimization technique developed by Kennedy and Eberhart [29], which is inspired by social behavior of bird flocking or fish schooling. A particle swarm is a population of particles, which are possible solutions to an optimization problem located in the multidimensional search space. The PSO is well documented in the existing literature with numerous modifications and augmentations [43–46].

On the one hand, what is essential in this setting is finding a suitable mapping between the particle's representation and the problem solution. In this contribution, each particle represents a vector of cutoff points in the unit interval. They are used to represent the intervals into which the linguistic values are translated.

Let us consider a set of five linguistic values (Very Low (VL), Low (L), Medium (M), High (H), and Very High (VH)) with their respective cutoff points (a_1, a_2, a_3, a_4). Then, the following mapping is formed: VL: $[0, a_1]$, L: $[a_1, a_2]$, M: $[a_2, a_3]$, H: $[a_3, a_4]$, and VH: $[a_4, 1]$. If we consider m linguistic values to be used by the decision makers to convey their preferences, this results in $m - 1$ cutoff points, which constitute a particle in the swarm of the PSO. Therefore, in this example, a particle is represented as $[a_1\ a_2\ a_3\ a_4]$.

On the other hand, the performance of each particle during its movement is assessed by means of some fitness function. Here, the aim of the PSO is the maximization of the values of the performance index by adjusting the positions of the cutoff points in the unit interval. When it comes to the formation of the fitness function, its determination has to consider the fact that interval-valued entries of the reciprocal preference relations have to return numeric values of the fitness function. This is carried out as follows: because of information granules are encountered in the form of intervals, series of their realizations being the entries of the preference relations are formed by randomly generating entries coming from the above intervals. To do so, the reciprocal linguistic preference relations $\{P^1, \ldots, P^m\}$ given by the decision makers are sampled to obtain the preference relations $\{R^1, \ldots, R^m\}$ where each entry of R^l, $l = 1, \ldots, m$, is represented by a numerical value drawn from the uniform distribution defined over the corresponding sub-interval of the unit interval, according to the linguistic value of that entry in the reciprocal linguistic preference relation P^l. Therefore, the performance index Q is expressed as follows:

$$Q = \sum_{l=1}^{m} cd_l \tag{8}$$

where cd_l is the consistency degree associated with the reciprocal preference relation R^l. To obtain the consistency degree cd_l, the method described in Section 2.2 is utilized.

Here, the components are intervals, but we need a numeric value of the fitness functions. Therefore, the reciprocal linguistic preference relations $\{P^1, \ldots, P^m\}$ are sampled 500 times. The average of the values of the performance index Q computed over each collection of 500 samples is the fitness function, f, associated with the particle formed by the cutoff points:

$$f = \frac{1}{500} \sum_{i=1}^{500} Q_i \tag{9}$$

A way of the formation of the fitness function is in line with the standard practices encountered in Monte Carlo simulations [47].

Finally, it is important to note that, in this contribution, the generic form of the PSO algorithm is used. Here, the updates of the velocity of a particle are realized in the form $\mathbf{v}(t+1) = w \times \mathbf{v}(t) + c_1\mathbf{a} \cdot (\mathbf{z}_p - \mathbf{z}) + c_2\mathbf{b} \cdot (\mathbf{z}_g - \mathbf{z})$ where "t" is an index of the generation and $\cdot$ denotes a vector multiplication realized coordinatewise. $\mathbf{z}_p$ denotes the best position reported so far for the particle under discussion while $\mathbf{z}_g$ is the best position overall and developed so far across the entire population. The current velocity $\mathbf{v}(t)$ is scaled by the inertia weight (w) which emphasizes some effect of resistance to change the current velocity. The value of the inertia weight is kept constant through the entire optimization process and equal to 0.2 (this value is commonly encountered in the existing literature [42]). By using the inertia component, we form the memory effect of the particle. The two other parameters of the PSO, that is **a** and **b**, are vectors of random numbers drawn from the uniform distribution over the $[0, 1]$ interval. These two update components help form a proper mix of the components of the velocity. The second expression governing the change in the velocity of the particle is particularly interesting as it nicely captures the relationships between the particle and its history as well as the history of overall population in terms of their performance reported so far. The next position (in iteration step "t+1") of the particle is computed in a straightforward manner: $\mathbf{z}(t+1) = \mathbf{z}(t) + \mathbf{v}(t+1)$.

When it comes to the representation of solutions, the particle **z** consists of "$m - 1$" entries positioned in the unit interval corresponding to the search space. One should note that while PSO optimizes the fitness function, there is no guarantee that the result is optimal, rather than that we can refer to the solution as the best one being formed by the PSO.

4. Experimental example

In this section, an experimental example is shown in order to illustrate the approach described in Section 3 and highlight its main characteristics.

Let us suppose a set of four alternatives, $X = \{x_1, x_2, x_3, x_4\}$, and a group of four decision makers, $E = \{e_1, e_2, e_3, e_4\}$. Using the set of five linguistic values $S = \{\mathrm{VL} = \text{Very Low}, \mathrm{L} = \text{Low}, \mathrm{M} = \text{Medium}, \mathrm{H} = \text{High}, \mathrm{VH} = \text{Very High}\}$, the following reciprocal linguistic preference relations are given by the four decision makers:

$$P^1 = \begin{pmatrix} - & \mathrm{VL} & \mathrm{H} & \mathrm{H} \\ Neg(\mathrm{VL}) & - & \mathrm{VL} & \mathrm{H} \\ Neg(\mathrm{H}) & Neg(\mathrm{VL}) & - & \mathrm{VL} \\ Neg(\mathrm{H}) & Neg(\mathrm{H}) & Neg(\mathrm{VL}) & - \end{pmatrix} \quad P^2 = \begin{pmatrix} - & \mathrm{H} & \mathrm{L} & \mathrm{M} \\ Neg(\mathrm{H}) & - & \mathrm{VH} & \mathrm{VL} \\ Neg(\mathrm{L}) & Neg(\mathrm{VH}) & - & \mathrm{VH} \\ Neg(\mathrm{M}) & Neg(\mathrm{VL}) & Neg(\mathrm{VH}) & - \end{pmatrix}$$

$$P^3 = \begin{pmatrix} - & \mathrm{M} & \mathrm{H} & \mathrm{M} \\ Neg(\mathrm{M}) & - & \mathrm{M} & \mathrm{L} \\ Neg(\mathrm{H}) & Neg(\mathrm{M}) & - & \mathrm{VH} \\ Neg(\mathrm{M}) & Neg(\mathrm{L}) & Neg(\mathrm{VH}) & - \end{pmatrix} \quad P^4 = \begin{pmatrix} - & \mathrm{VL} & \mathrm{H} & \mathrm{M} \\ Neg(\mathrm{VL}) & - & \mathrm{VL} & \mathrm{L} \\ Neg(\mathrm{H}) & Neg(\mathrm{VL}) & - & \mathrm{VH} \\ Neg(\mathrm{M}) & Neg(\mathrm{L}) & Neg(\mathrm{VH}) & - \end{pmatrix}$$

Here, it should be noted that because of the linguistic values as represented as intervals, $Neg(s_i)$ denotes the complementary qualitative value of s_i, whose semantics is determined in our approach using the equivalence index characterizing s_i^1. For instance, suppose that $s_i = \mathrm{H}$, and H is represented by the interval $[0.6, 0.8]$. If it is sampled as, for instance, with the number 0.65, $Neg(\mathrm{H})$ will be equal to 0.35.

Once the decision makers have expressed their preferences, our approach is applied. Proceeding with the details of the optimization environment, a generic version of the PSO is used in this contribution. The parameters in the update equation for the velocity of the particle were set as $c_1 = c_2 = 2$, as these values are usually encountered in the existing literature. The size of the swarm consists of 100 particles, and the algorithm was run for 300 generations (or iterations). These values were selected as a result of intensive experimentation.

On the one hand, the progression of the optimization quantified in terms of the fitness function is depicted in Fig. 1. Here, the optimal cutoff points returned by the PSO are: 0.31, 0.37, 0.43, and 0.48, respectively. Therefore, the intervals corresponding to the linguistic values of the set S are: VL: $[0, 0.31]$, L: $[0.31, 0.37]$, M: $[0.37, 0.43]$, H: $[0.43, 0.48]$, and VH: $[0.48, 1]$, respectively. Furthermore, the average value of the performance index Q is equal to 0.751, with a standard deviation of 0.022.

The performance obtained when considering a uniform distribution of the cutoff points over the scale is reported in order to put the obtained optimization results in a certain context. A uniform distribution of the cutoff points is obtained when the points are equal to 0.20, 0.40, 0.60, and 0.80. In such a situation, the average performance index Q assumes the value 0.683 with a standard

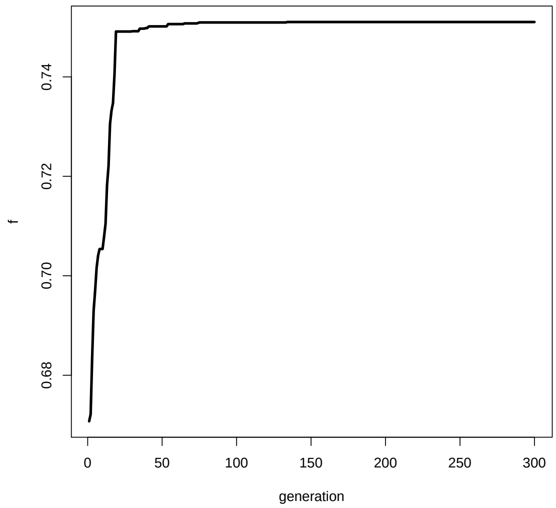

Fig. 1: Fitness function f in successive PSO generations.

deviation of 0.014. Comparing with the values achieved by the optimized cutoff points, the performance index Q takes on now lower values.

Finally, to obtain the ranking of alternatives from best to worst, a selection process (aggregation and exploitation) should be carried out [34,35].

5. Conclusions and future work

In this contribution, an approach to make operational the linguistic information used by the decision makers in GDM situations has been proposed. To do so, an information granulation of the linguistic information and its optimization framework have been described. Also, using this approach, the consistency degree associated to the linguistic preference relations given by the decision makers is increased, as it is utilized as an optimization criterion.

In the future, we propose to continue this research by applying the proposed approach to other formalism as fuzzy sets, shadowed sets, probabilities, rough sets, and so on. For instance, if we deal with probabilistically granulated linguistic values, it could be a source of illumination on possible linkages between fuzzy and probabilistic models of decision making along with some possible hybrid fuzzy-probabilistic schemes.

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