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Return of the capital coefficients matrix

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ABSTRACT

A core ingredient of post-disaster input–output recovery models is the reconstruction of lost production capacity. Therefore, one would expect a set of models endowed with capital coefficients matrices to be available for analysis. However, this is not the case, possibly due to earlier negative experiences with such models. Nevertheless, in this paper, we aim to show that there is a class of problems that can be addressed successfully with a dynamic input–output model with a fully functioning capital coefficients matrix. We put forward that if reconstruction is tightly planned, investment and therewith gross output essentially become pre-determined. This also means that traditional final demand becomes an endogenous residual, with the model being transformed into a distribution and allocation model. We begin with a reordering of variables and equations as proposed in Leontief's dynamic inverse, and then move on directly to the newly proposed model. Suggestions for further work are given.

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1. Introduction

Disasters have adverse consequences for many aspects of human life – mortality, health, infrastructure, livelihood, food and other supplies, among others. They result in disruptions of economic activities and have substantial impact on overall human wellbeing. Many of them, such as floods and hurricanes, lead to substantial capital and infrastructure losses (the so-called direct effects) as well as production shortfalls (the indirect effects). Moreover, these losses have consequences for the allocation of resources which may make forms of centralized coordination inevitable (Trebilcock and Daniels, 2006; Dari-Mattiacci and Faure, 2015).

The omnipresence of disaster occurrences explains the rise of a large number of input–output (IO) based initiatives to model specific aspects thereof (Okuyama, 2007). A most promising one concerns the role of adaptivity concepts via so-called ARIO (adaptive regional IO) models which are equipped with rationing mechanisms, see Hallegatte (2008). Another new point of attention concerns the development of resilience and adaptation strategies, going back to Rose and Liao (2005) and others. Steenge and Bočkarjova (2007) put forward an IO-based model that targets the quickest possible recovery and restoration

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paths to the pre-disaster production level. In a related research line, Li et al. (2013) considered adjustment mechanisms to specifically focus on allocation and distribution problems, while recent contributions have added non-linear programming approaches (Oosterhaven and Többen, 2017).[1]

However, despite these developments, one particular aspect of modern disaster analysis has not been given sufficient attention. The post-disaster recovery stage to a large extent is about reconstruction and rebuilding. Therefore, one would expect the presence of a modern line of research connecting to the IO-based dynamic growth models of, especially, the 1950s, 1960s and 1970s. These models have a very specific characteristic in that they include, next to the standard matrix of direct input coefficients, a so-called *capital coefficients* matrix, also known simply as 'capital' matrix or 'investment' matrix. The columns of this matrix stand for the capital inputs required per unit of additional sectoral output (i.e. the extra capital required to expand production). Leontief (1953a; 1953b) was the first to propose IO models along this line and to provide numerical exercises. Initial expectations were high because this new type of model incorporated much more detail on capital and capital construction than the earlier ones. Unfortunately, the new models were plagued by a range of persistent problems. Despite a number of variants being proposed, problems remained and modellers lost interest. This also meant that interest in the capital coefficients matrix itself waned. This is a pity because in this way access to a wealth of detail about capital construction was lost.

In this paper, we hope to revive the interest in dynamic models featuring a capital matrix, because one particular aspect of the problems associated with these models has not been taken into account so far. Post-disaster reconstruction usually implies, given the amount of production capacity that is left, a strict planning and coordination order reflecting restoration priorities and time preferences. Furthermore, depending on the nature of the disaster, planning may go well into the future, guided by the desire to return as soon as possible to the pre-disaster state of affairs. This timing aspect can be captured by returning for a moment to a specific reordering of the model's variables and equations proposed by Leontief (1970) when introducing another dynamic model, the dynamic inverse (see Section 2.1). That approach received a certain amount of interest, but also suffered from serious problems and interest disappeared.

As we shall see, the reordering referred to above has potential in that it enables us to follow the entire trajectory of investment and gross output in post-disaster recovery activities in a clear perspective over time. Focusing on this recovery path is important for a specific reason: the reconstruction of fixed capital implies the presence of a well-structured perspective on where stakeholders want the economy in question to be in a number of periods ahead, and what needs to be produced in terms of fixed capital, all typically motivated by the desire to return to the pre-disaster situation as soon as possible. Viewed in this way, gross output becomes an exogenous variable.

As we shall see, exploring this path guarantees full use of the capital coefficients matrix without encountering the problems that plagued the earlier dynamic models. There is a second point involved: our proposed alternative also implies a shift in perspective on the recovery activities. Basically, the model's focus shifts towards becoming an allocation and distribution model, possibly with a significant role for external assistance.

[1] For a comprehensive overview of modern research lines, see Koks and Thissen (2016).

The paper is organized as follows. In Section 2, we start with the standard dynamic IO model, the so-called forward-looking model, and the stability problems it is known for. Hereafter we shift briefly to the dynamic inverse and Leontief's reordering of the equations (Section 2.1). Section 3 presents the post-disaster reconstruction interpretation of the reordered set of equations and criteria for the allocation of goods to consumption or investment destinations. Section 4 gives a numerical illustration. Section 5 suggests a new area of application for the capital matrix and provides concluding remarks.

2. Dynamic IO modelling

We start from the well-known dynamic forward-looking model (Leontief, 1953a, 1953b),

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_t + \mathbf{B}(\mathbf{x}_{t+1} - \mathbf{x}_t) + \mathbf{f}_t \tag{1}$$

where $\mathbf{A}$ stands for the matrix of direct input coefficients, $\mathbf{B}$ for the matrix of capital coefficients (or capital matrix, for short), and $\mathbf{x}_t$ and $\mathbf{f}_t$ for, respectively, gross output and final demand (excluding investment) in period t. The final demand for all periods is assumed to be exogenous. Together with a starting value for $\mathbf{x}_t$ (also exogenously given), the model can be solved for $\mathbf{x}_{t+1}$. We obtain

$$\mathbf{x}_{(t+1)} = [\mathbf{I} + \mathbf{B}^{-1}(\mathbf{I} - \mathbf{A})]\mathbf{x}_t + \mathbf{B}^{-1}\mathbf{f}_t \tag{2}$$

from which $\mathbf{x}_{t+1}$ can be directly obtained. Given $\mathbf{f}_{t+1}$ and the endogenously determined value for $\mathbf{x}_{t+1}$, $\mathbf{x}_{t+2}$ can be determined, etc.

Unfortunately, many studies have found that the outcomes obtained for this forward-looking model invariably lead to unrealistic and widely fluctuating outcomes that lack economic interpretation, see e.g. Leontief (1953a; 1953b), Tsukui (1961), Brody (1970) or Meyer and Schumann (1977). Explanations and alternatives have been sought in various directions. The appearance of the inverse of the capital matrix in Equation 2 was a major point of concern. The problem here is that economic theory hardly offers any insight in what this matrix may look like and, e.g. whether its elements will be positive, negative or zero. Another point regarding the capital matrix concerns the question whether or not it has full rank. If not, as we know, it cannot be inverted and special methods are asked for to proceed.[2] All in all, there exists no generally accepted way of addressing the many problems of the forward-looking model and its variants. Consequently, interest in the model and in the capital coefficients matrix has gradually declined.

2.1. *The dynamic inverse*

In this section, we shall take a look at a particular reordering of variables and equations proposed by Leontief (1970). This reordering was in the context of proposing the dynamic

[2] There is an extensive literature on alternatives for Equations 1 and 2. Many articles can be found that discuss specific aspects. Well-known contributions are Jorgenson (1961) and Solow (1959) on the so-called dual instability problem, Tsukui (1961, 1968) on non-linear programming variants, Leontief (1970), Brody (1995) and Miller and Blair (2009, section 13.4.2) on the dynamic inverse as an alternative (see also the next section), Meyer and Schumann (1977) on instabilities in a model for the German Federal Republic, Duchin and Szyld (1985) on aspects of non-negative outcomes, and Fleissner (1990) or Steenge (1990) on specific eigenvalue configurations.

inverse, an alternative to Equation 1, based on the concept of 'working backwards' in solving (1). Leontief reformulated as

$$(\mathbf{I} - \mathbf{A} + \mathbf{B})\mathbf{x}_t - \mathbf{B}\mathbf{x}_{t+1} = \mathbf{f}_t \tag{3}$$

With

$$\mathbf{G} \equiv \mathbf{I} - \mathbf{A} + \mathbf{B} \tag{4}$$

this directly results in (current period $t = 0$)

$$\begin{aligned} \mathbf{G}\mathbf{x}_0 - \mathbf{B}\mathbf{x}_1 &= \mathbf{f}_0 \\ \mathbf{G}\mathbf{x}_1 - \mathbf{B}\mathbf{x}_2 &= \mathbf{f}_1 \\ \mathbf{G}\mathbf{x}_2 - \mathbf{B}\mathbf{x}_3 &= \mathbf{f}_2 \\ \mathbf{G}\mathbf{x}_3 - \mathbf{B}\mathbf{x}_4 &= \mathbf{f}_3 \end{aligned} \tag{5}$$

etc. Next, following IO tradition, he assumed that the final demand vectors for each period are given exogenously.

Clearly, the above equations can be assembled in a single large matrix equation. Letting the time index moves from 0 to 4, we obtain, following Leontief (1970) and Miller and Blair (2009, section 13.4.2), the following 'rectangular' system

$$\begin{bmatrix} \mathbf{G} & -\mathbf{B} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} & -\mathbf{B} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G} & -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G} & -\mathbf{B} \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \end{bmatrix} \tag{6}$$

which is a system of four matrix equations in five unknown vectors, $\mathbf{x}_0$ through $\mathbf{x}_4$.[3]

To solve the above system, Leontief proposed to give the last unknown ($\mathbf{x}_4$ here) a particular value, which can be zero. If we opt for that value, the matrix on the left-hand side of Equation 6 becomes square (because the last column of the left-hand side matrix can be dropped), and we obtain

$$\begin{bmatrix} \mathbf{G} & -\mathbf{B} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G} & -\mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{G} & -\mathbf{B} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{G} \end{bmatrix} \begin{bmatrix} \mathbf{x}_0 \\ \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{f}_0 \\ \mathbf{f}_1 \\ \mathbf{f}_2 \\ \mathbf{f}_3 \end{bmatrix} \tag{7}$$

This is a system of four matrix equations in four unknown vectors, $\mathbf{x}_0$ through $\mathbf{x}_3$, which, given certain conditions regarding invertibility of the square matrix on the left, can be solved for the output vectors in question.

We straightforwardly observe that the conditions for the existence of an economically interpretable growth path are not clear-cut. Clearly, if the matrix on the left is non-singular, we directly can solve for $\mathbf{x}_0$ through $\mathbf{x}_3$ in terms of the final demand vectors $\mathbf{f}_0$ through $\mathbf{f}_3$. Alternatively, we can start from the last row and calculate $\mathbf{x}_3$ given exogenous $\mathbf{f}_3$, thereby

[3] Note that attributing an exogenously given 'starting value' to $\mathbf{x}_0$ would bring us back to the forward-looking model (1), though arranged differently.

assuming that also $\mathbf{G}$ is non-singular. Analogously, we can start from some other vector $\mathbf{x}_4 \neq 0$ and calculate the corresponding vectors $\mathbf{x}_0$ through $\mathbf{x}_3$.

Nonetheless, there are no generally accepted qualitative results guaranteeing an interpretable outcome. For example, we have no theory-based insight in the behaviour of the inverted matrices $\mathbf{G}^{-1}$ or $(\mathbf{I} - \mathbf{A} + \mathbf{B})^{-1}$. Therefore, we also cannot be sure that $\mathbf{x}_3$ and the other $\mathbf{x}$-vectors will have acceptable values. There are other problems. The model requires a truncation, here at period 4. Unfortunately, finding a generally acceptable basis for the truncation involves a number of fundamental questions, some of which require much additional attention, see Leontief (1970) and Brody (1995). This is one reason why the model, although it has received a certain amount of interest, has not established itself as the leading dynamic variant.

3. A shift in perspective

Suppose now, considering Equation 6, that the gross output vectors $\mathbf{x}_0$ to $\mathbf{x}_4$ are determined *exogenously* instead of *endogenously*. In that case, this would provide a way for calculating the final demand vectors $\mathbf{f}_0$ to $\mathbf{f}_3$ as the *residual* or *remaining output* after satisfying intermediate and capital demand. And, evidently, there would be one big advantage: the problems plaguing other model formulations, such as those caused by the requirement to invert matrices $\mathbf{B}$ or $\mathbf{I} - \mathbf{A} + \mathbf{B}$, simply are not there.

Despite this convenient property, such a model is, economically speaking, hardly interesting. In fact, we may wonder what would drive the economy in question under normal circumstances. However, the situation would be completely different if we are dealing with areas hit by a massive disaster, natural or manmade. In fact, the urgency to stabilize and rebuild the economy will dominate the post-disaster situation; in particular decisions regarding what to (re)build and when will dominate the social and political agenda. The reason is straightforward. Reconstruction of fixed capital is an all-important part of the post-disaster activities. This reconstruction, however, normally implies a well-structured perspective on where the economy in question should be in a number of periods ahead, characteristically pushed forward by the desire to return to the pre-disaster situation as soon as possible. Given insight in what is left of capacity a path of (re)investments will be clear and with that the path of gross output. This has an important consequence. In this particular context, the path of gross output – during the recovery period – will become *exogenous*. Furthermore, if the losses and damages are substantial this also will have consequences for the allocation of resources, making forms of centralized coordination inevitable (Trebilcock and Daniels, 2006; Dari-Mattiacci and Faure, 2015). In the next section we shall consider situations like this more closely below.

3.1. Post-disaster recovery

Our point of departure will be Equations 5 and 6. With the disaster causing damage to the productive sectors of the economy and disrupting economic activities, let the extent of the production capacity losses be represented by the diagonal damage fraction matrix $\mathbf{\Gamma}$ comprising proportions of industry damages γ_i $(0 \leq \gamma_i \leq 1)$ in the n sectors, as in

Steenge and Bočkarjova (2007), and determined exogenously by the force of the disaster with

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \gamma_n \end{bmatrix}. \tag{8}$$

Output $\mathbf{x}_t$ is reduced by the damage of the disaster at period t when the catastrophe takes place. For convenience, let it occur at period $t = 0$.[4] We then express the post-disaster surviving output vector $\tilde{\mathbf{x}}_0$, corresponding to the degraded output capacity relative to the initial pre-disaster output $\mathbf{x}_0$, as

$$\tilde{\mathbf{x}}_0 \equiv (\mathbf{I} - \mathbf{\Gamma})\mathbf{x}_0. \tag{9}$$

In effect, relating the interruption caused by the catastrophe to what would have been the economy's growth path opens up the potential of employing the dynamic sequence in Section 2.1 for planning the, say, m periods of recovery, where m is exogenously determined. It is during this phase that market mechanisms may cease to function causing the distribution of goods to become an issue. It is also during this period that policy makers will play a critical role in steering the activities in the economy. Exogenous treatment of the output vectors now becomes a reasonable assumption as policy makers and other stakeholders determine the output recovery path. The pre-determined output vectors may be viewed as the target production over the reconstruction period to restore lost capacities. In formula form, we would obtain

$$\begin{bmatrix} \mathbf{G} & -\mathbf{B} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{G} & -\mathbf{B} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{G} & -\mathbf{B} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{x}}_0 \\ \bar{\mathbf{x}}_1 \\ \vdots \\ \bar{\mathbf{x}}_{m-1} \\ \bar{\mathbf{x}}_m \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{f}}_0 \\ \tilde{\mathbf{f}}_1 \\ \vdots \\ \tilde{\mathbf{f}}_{m-1} \end{bmatrix}. \tag{10}$$

Equation 10 is the disaster recovery version of the dynamic sequence where the post-disaster $\tilde{\mathbf{x}}_0$ and $\tilde{\mathbf{f}}_0$ replace the pre-disaster $\mathbf{x}_0$ and $\mathbf{f}_0$ of the uninterrupted dynamic model while a bar distinguishes the exogenously determined post-disaster output vectors, and a tilde the residual final demand vectors during the disaster recovery.

Analytically, a subject for future work might be the role of the $\mathbf{\Gamma}$ matrix over time. We have seen that reconstruction can be planned immediately after the external shock via a specification of the entire recovery path ahead. Alternatively, we can, at the beginning of each period, determine the value of the corresponding $\mathbf{\Gamma}$ matrix and on that basis determine the corresponding output values. If we choose this option, the $\mathbf{\Gamma}$ diagonal elements, measuring the still lost or seriously damaged stock, can be expected to converge to zero, the rate of convergence depending on the distribution mechanisms in place.

Furthermore, the fixed technology assumption definitively can be relaxed by introducing time indices to the coefficients matrices $\mathbf{A}$ and $\mathbf{B}$ to reflect technological improvement

[4] The time index 0 here applies to the period in which the disaster occurs. The first recovery period after the disaster is denoted by $t = 1$, etc.

over time. This may be quite realistic since the post-disaster recovery phase may be taken as an opportunity for retooling production methods. Although strictly based on IO principles, the proposed model is not a 'rigid' one and can be used as a basis for future flexibility in modelling extensions.

3.2. A shift in focus

Equations 8 and 9 inform us about the shortages in the immediate aftermath of the disaster, while Equation 10 reflects the choices to be made regarding the distribution issues in the subsequent periods. That is the issues in relation to the distribution and allocation of the capacity that is left over. An immediate response on how to allocate the leftover production capacity between destinations, such as sectors, consumers, exports and others, is critical for disaster preparedness. The endogenous surplus for period t of recovery being

$$\tilde{\mathbf{s}}_t = (\mathbf{I} - \mathbf{A})\bar{\mathbf{x}}_t \tag{11}$$

(except for $t = 0$), an allocation scheme is to be proposed where a portion of net output is redistributed to reconstruction efforts to build and restore lost infrastructure and capacity. To that end, the surplus $\tilde{\mathbf{s}}_t$ is visualized to consist of two components, investment on capital for reconstruction $\mathbf{B}\Delta\tilde{\mathbf{x}}_t$ and final consumption $\tilde{\mathbf{f}}_t$,

$$\tilde{\mathbf{s}}_t = \mathrm{B}\Delta\bar{\mathbf{x}}_t + \tilde{\mathbf{f}}_t \tag{12}$$

with $\Delta\bar{\mathbf{x}}_t = \bar{\mathbf{x}}_{t+1} - \tilde{\mathbf{x}}_t$ for $t = 0$ and $\Delta\bar{\mathbf{x}}_t = \bar{\mathbf{x}}_{t+1} - \bar{\mathbf{x}}_t$ for $t = 1, \ldots, m$. Here B again is the matrix of capital coefficients and $\tilde{\mathbf{f}}_t$ the allocation for final consumption of households and government in the face of disaster-induced shortages. The exogenously determined target capacity for the next period drives the amount of investment here. This in turn indicates how much gets allocated for final consumption. Decision-makers in this context can be afforded flexibility on choosing the respective magnitudes such that when the calculated residual final demand is deemed too small, adjustments to the target capacity increase may be made with the extent of the adjustment dependent on the capital matrix.

Continuing, we should note that the above-proposed model should not be seen as an optimization model. For each period, decision-makers will have to determine which external conditions prevail and which room for policy is available. Thus, a flexible menu of choices should be left to the decision-makers, one major reason for its presence being that there is no 'one-size-fits-all' solution to the problems of possibly very distinctive cases and objectives in disaster-stricken economies. In this line, the planning nature of a disaster response may require further attention. Evidently, such a response can be in contrast to the leanings towards market-driven approaches, but may be more realistic in many cases in the aftermath of disasters. After economic disruptions, market adjustment mechanisms may not immediately work. Therefore, having a model like the one we proposed as a 'compass for navigation' is not farfetched.

Regarding the structure of the model, we have been discussing in this section, it may be useful to refer here to Miller and Blair (2009, section 13.4.2). They point out that '[an] issue that arises in many dynamic models, including the input-output system, is which values to specify as fixed in the dynamic process'. As an illustration, they put forward a number of possibilities in varying the choice of initial and terminal conditions or values. Above, we have proposed another variation and put forward a variant in which, when compared to the dynamic inverse, the choices involved in categorizing variables into endogenous or exogenous have been reversed.

4. Numerical illustration

We illustrate the general functioning of the model using the following hypothetical pre-disaster values for a three-sector economy assuming no technological progress and with population growth of 1% so that the economy operates to maintain per capita income:

$$\mathbf{A} = \begin{bmatrix} 0.16 & 0.21 & 0.30 \\ 0.19 & 0.05 & 0.15 \\ 0.20 & 0.20 & 0.10 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0.50 & 0.15 & 0.10 \\ 0.10 & 0.40 & 0.05 \\ 0 & 0.20 & 0.80 \end{bmatrix}, \quad \mathbf{x}_0 = \begin{bmatrix} 400 \\ 300 \\ 500 \end{bmatrix}.$$

Gross output, $\mathbf{x}$, and final consumption, $\mathbf{f}$, thus grow each period by 1%, so that $\mathbf{x}_{t+1} = 1.01 \times \mathbf{x}_t$ and $\mathbf{f}_{t+1} = 1.01 \times \mathbf{f}_t$. The pre-disaster dynamic equation $(\mathbf{I} - \mathbf{A} + \mathbf{B})\mathbf{x}_0 - \mathbf{B}\mathbf{x}_1 = \mathbf{f}_0$ is therefore expressed as

$$\left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.16 & 0.21 & 0.30 \\ 0.19 & 0.05 & 0.15 \\ 0.20 & 0.20 & 0.10 \end{bmatrix} + \begin{bmatrix} 0.50 & 0.15 & 0.10 \\ 0.10 & 0.40 & 0.05 \\ 0 & 0.20 & 0.80 \end{bmatrix} \right) \begin{bmatrix} 400 \\ 300 \\ 500 \end{bmatrix} - \begin{bmatrix} 0.50 & 0.15 & 0.10 \\ 0.10 & 0.40 & 0.05 \\ 0 & 0.20 & 0.80 \end{bmatrix} \begin{bmatrix} 404 \\ 303 \\ 505 \end{bmatrix} = \begin{bmatrix} 120.05 \\ 132.15 \\ 305.4 \end{bmatrix}$$

where the surplus $\mathbf{s}_0 = \begin{bmatrix} 123 \\ 134 \\ 310 \end{bmatrix}$ is composed of investment $\mathbf{B}\Delta\mathbf{x}_0 = \begin{bmatrix} 2.95 \\ 1.85 \\ 4.6 \end{bmatrix}$ and final consumption $\mathbf{f}_0 = \begin{bmatrix} 120.05 \\ 132.15 \\ 305.4 \end{bmatrix}$.

Let the exogenous force of the disaster cause sectoral damages encompassed in the damage fraction matrix $\mathbf{\Gamma}$ as

$$\mathbf{\Gamma} = \begin{bmatrix} 0.42 & 0 & 0 \\ 0 & 0.38 & 0 \\ 0 & 0 & 0.27 \end{bmatrix}.$$

Post-disaster remaining output capacity is then reduced to

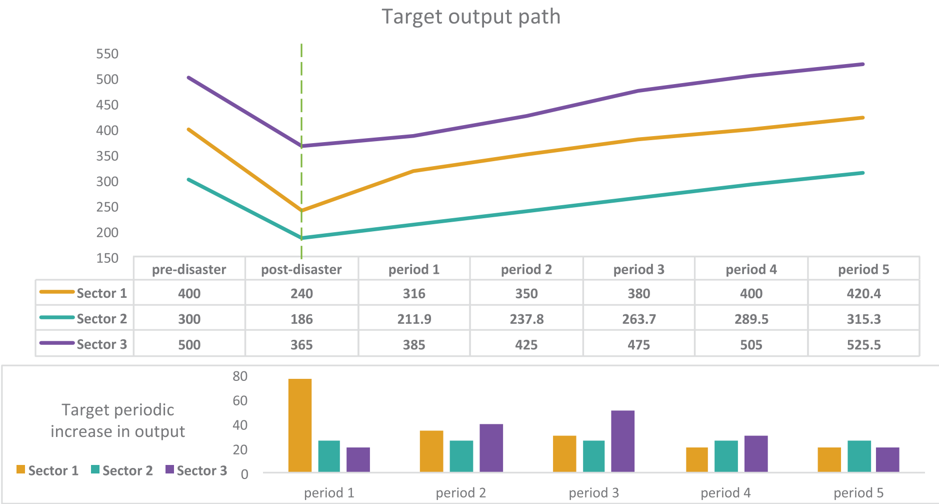
$$\tilde{\mathbf{x}}_0 = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.42 & 0 & 0 \\ 0 & 0.38 & 0 \\ 0 & 0 & 0.27 \end{bmatrix} \right) \begin{bmatrix} 400 \\ 300 \\ 500 \end{bmatrix} = \begin{bmatrix} 240 \\ 186 \\ 365 \end{bmatrix},$$

a stark drop from $\mathbf{x}_0 = \begin{bmatrix} 400 \\ 300 \\ 500 \end{bmatrix}$, while the resulting post-disaster surplus available for allocation to investment and final consumption is now reduced to

$$\tilde{\mathbf{s}}_0 = \left(\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.16 & 0.21 & 0.30 \\ 0.19 & 0.05 & 0.15 \\ 0.20 & 0.20 & 0.10 \end{bmatrix} \right) \begin{bmatrix} 240 \\ 186 \\ 365 \end{bmatrix} = \begin{bmatrix} 53.04 \\ 76.35 \\ 243.3 \end{bmatrix}.$$

Policy makers exogenously plan the recovery path, i.e. where they want the economy to be and when to be there. An illustration of such a plan is shown in Figure 1.

Figure 1. Production capacity recovery plan in terms of the target output path and the target periodic increases in output over the recovery period.



	pre-disaster	post-disaster	period 1	period 2	period 3	period 4	period 5
Sector 1	400	240	316	350	380	400	420.4
Sector 2	300	186	211.9	237.8	263.7	289.5	315.3
Sector 3	500	365	385	425	475	505	525.5

Figure 1 gives the production capacity recovery plan in terms of the target output path and the target periodic increases in output over the recovery period. Both the *pre-disaster* and *post-disaster* headings refer to period 0 where the pre-disaster output corresponds to $\mathbf{x}_0$ while the immediate post-disaster degraded output corresponds to $\tilde{\mathbf{x}}_0$. The periods 1–5 outputs, reflecting the desired recovery path, correspond to the $\bar{\mathbf{x}}_1$ to $\bar{\mathbf{x}}_5$ vectors. The periods 1–5 target periodic increases correspond to $\Delta\bar{\mathbf{x}}_0$ to $\Delta\bar{\mathbf{x}}_4$.

To achieve the target output for the subsequent period, the required investment is

$$\mathbf{B}\Delta\bar{\mathbf{x}}_0 = \begin{bmatrix} 0.50 & 0.15 & 0.10 \\ 0.10 & 0.40 & 0.05 \\ 0 & 0.20 & 0.80 \end{bmatrix} \left(\begin{bmatrix} 316 \\ 211.9 \\ 385 \end{bmatrix} - \begin{bmatrix} 240 \\ 186 \\ 365 \end{bmatrix} \right) = \begin{bmatrix} 43.89 \\ 18.96 \\ 21.18 \end{bmatrix}.$$

The residual final demand which can be domestically sourced is the leftover of the surplus after the investment allocation, $\tilde{\mathbf{f}}_0 = \begin{bmatrix} 9.16 \\ 57.39 \\ 222.12 \end{bmatrix}$. Now, let the exogenously determined minimum final consumption bundle be given by $\mathbf{f}_0^{\min} = \begin{bmatrix} 90 \\ 100 \\ 250 \end{bmatrix}$. The model then provides information about the final demand shortfall which has to be satisfied from external sources. This is the excess of $\mathbf{f}_0^{\min}$ over $\tilde{\mathbf{f}}_0$, equalling $\begin{bmatrix} 80.85 \\ 42.61 \\ 27.88 \end{bmatrix}$.

5. Final remarks

IO has a long and varied history, with contributions ranging across many fields such as international trade, spatial and environmental economics and, more recently, climate change. Still, there is a curious gap in the areas where we find IO-based work: contributions are largely lacking in projects and developments that involve considerable amounts of fixed capital and infrastructure. For example, projects involving housing, factories, roads and railways, the construction of new harbours and airports and so on. In short, IO is absent to a large extent in projects requiring a substantial amount of capital investments.

Above, we briefly recalled that this absence is due to the fact that addressing capital issues requires the presence of a so-called capital coefficients matrix, i.e. a *second* matrix of input coefficients (in addition to the standard direct input coefficients matrix) to capture the input proportions of large-scale investments. Matrices of this second type, known as 'capital coefficients', 'investment' or simply 'capital' matrices, were constructed in the decades following the Second World War, but models endowed with such matrices did not perform well and were gradually sidelined. This is a great pity, because it meant that detailed knowledge of the structure of big investments was also lost.

In this paper, we have returned to a particular IO model involving a capital matrix because, despite the earlier negative experiences, this model is able to perform quite well in a specific situation, i.e. post-disaster reconstruction. The reason that the model is effective is because in many cases that type of activity has a well-defined and well-planned structure of investments and therefore of gross outputs. As a consequence, in terms of IO modelling, causality changes and investments c.q. gross outputs become – to a large part – exogenously determined and traditional final demand therefore becomes endogenous.

Although we focused on post-disaster recovery and reconstruction, our findings may be relevant to others, but comparable, cases as well. Many countries are involved in – often very large-scale projects to counter overcrowding, congestion, insufficient water supply, the danger of flooding and hurricanes, etc. Many of these plans and projects are the result of years of planning and calculation and, therefore, have an internal timing and logic that makes them comparable, despite differing spatial and time scales, to the post-disaster reconstruction efforts we focused on in presenting our views on the role of the capital matrix. Seen in this light, IO modelling may therefore become a much-used instrument for analysing also these large-scale infrastructural developments.

Examples are not difficult to find. One particular case is provided, in many countries, by the transition to a circular economy. The Netherlands, e.g. has decided that its economy will be a circular one by the year 2050 (Potting and Hanemaaijer (eds. et al., 2018). Carrying out this project will require monitoring and fine-tuning of decision-making and activities on a very large scale. Precisely because of its focus on sectoral interconnections at any realistic scale, IO approaches – given some relatively minor modifications – can be expected to be extremely helpful here in many areas and sub-areas if endowed with capital coefficients matrices.

Another example is provided by Indonesian policy. Recently President Widodo formally announced that the country's capital will be relocated to the province of East Kalimantan on the island of Borneo (Afra Sapiie, 2019). The relocation is necessary to solve the multitude of problems experienced by over-populated and over-crowded Java and Jakarta. First estimates indicate that the new location would include new government offices and housing for some 1.5 million civil servants. Relocation costs, in a set of first estimates by Bappenas (the National Development Planning Agency), would amount to almost $33bn, primarily to be funded by the state and public–private partnerships, work beginning not before 2024. Also here, IO models endowed with a capital matrix can be most helpful in providing further analysis and support.

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