Attainment of Equilibrium via Marshallian Path Adjustment: Queueing and Buyer Determinism

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We examine equilibration in a market where Marshallian path adjustment can be enforced, or not, as a treatment: a posted offer market either with buyer queueing via value order, or random order, respectively. We derive equilibrium predictions, and run experiments crossing queueing rules with either human or deterministically optimizing robot buyers under both locally stationary and non-stationary marginal cost. Results on rate of convergence to competitive equilibrium are obtained, and Marshallian path adjustment is established as conducive to attaining competitive equilibrium.

Marshallian path adjustment, wherein the highest outstanding-value buyer and the lowest outstanding-cost seller transact at each point in time is a dynamic that allows ultimate attainment of the competitive equilibrium price despite prior trades occurring at other prices, and which exhausts all gains from trade. Prior work has reported evidence consistent with Marshallian path adjustment as an emergent phenomenon (Cason and Friedman, 1997; Plott, Roy, and Tong, 2013). Instead of attempting to observe Marshallian path adjustment as an endogenous phenomenon, we vary its

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presence or absence exogenously as a treatment, allowing observation of market equilibration—or perhaps its failure—in the presence or absence of Marshallian path adjustment.

Our test-bed is the posted offer market. Within the posted offer market lies a particular design attribute—queueing protocol—that is well suited as our key treatment variable. By means of different queuing protocols, one can exogenously enforce or preclude Marshallian path adjustment in a posted offer market: if one imposes value order buyer queueing (Beckman, 1965; Levitan and Shubik, 1972; Vives, 1986) and a flat supply curve, then trades are executed in the same order as that prescribed by Marshallian path adjustment.[1] Conversely, one can "switch off" Marshallian path adjustment by replacing value order queueing with exogenously randomized order buyer queueing, which effectively precludes Marshallian path adjustment.[2] We thus present a diagnostic test of a foundational theoretical concept (Marshallian path adjustment) and explore its implications for equilibration and efficiency.

If queueing order is endogenously determined, i.e. by speed of subject response, then ordering varies as a function of anything influencing response speed. One influence on response speed may be excess rents (Smith, 1962): higher available surplus from transacting (itself a function of induced value or cost) might lead to quicker response; in the limit, *de facto* value order

[1]By contrast, call markets impose Marshallian path adjustment as part of their normal operation.

[2]Readers with an industrial organization background may be familiar with work in duopoly settings that varies buyer queueing and with a discussion over which form of queueing is more realistically descriptive of particular industries. Our approach takes a market design perspective. In particular, we take advantage of the remarkable and useful coincidence (isomorphism) between Marshallian path adjustment and value order queueing in a particular market (posted offer) for the purpose of investigating equilibration in a competitive (five seller, five buyer) environment.

queueing might thereby emerge. However, if variation across subjects in (unobservable) calculation speeds, manual dexterity, comprehension of the strategic implications of the market rules, and so on, overwhelms the influence of excess rents on response speed, then queueing order might appear uncorrelated with excess rents, and in this sense appear random. Between the preceding extremes lies a continuum of cases, over which ordering might be "affiliated" with excess rents (induced values) to a varying degree. Our design imposes, exogenously, two extremes of this continuum of queueing possibilities.

Both value order and random order queueing support multiple pure strategy equilibria (for particular marginal cost settings), including competitive equilibria. One does not know ex ante which, if any, of these equilibria might occur, and whether the price and quantity outcomes that do eventuate will be the same or different across the different queueing rules. As it happens, our results demonstrate that previously observed poor performance of the posted offer market can likely be attributed to the absence of enforced Marshallian path adjustment (due to use of queueing rules other than value order), and not one-time, one-sided posting of prices. More generally, given individual posting of offers and human buyers, the presence or absence of an exogenously enforced Marshallian path adjustment appears to be sufficient to span either convergence to the competitive equilibrium or not. One does not need to switch institutions, say as from Chamberlin (1948) to Smith (1962), to observe variation so wide in attainment of the competitive equilibrium allocation.

1. BACKGROUND

Appreciation of the problem of equilibration, and in particular of out-of-equilibrium trades, dates back at least to the 19th century. Different resolutions include: Walras' employment of an institution, *tâtonnement*, which determines prices centrally and precludes out-of-equilibrium trade; Green's (1974) Edgeworth-inspired employment of a coalition formation process which operates on a decentralized basis and which precludes out-of-equilibrium trade; and Marshall's (1961) suggestion as to how a sequence of trades, pairing the buyer with the highest outstanding value with the seller with the lowest outstanding cost, might eventually generate a price at the crossing of supply and demand, even if all trades except the ultimate, marginal trade take place at prices other than the competitive equilibrium price.[3]

Plott et al. (2013) examine whether Marshall's suggested trajectory of trades—here and elsewhere referred to as Marshallian path adjustment—might be an emergent property of the double auction, and report data suggestive of such a pattern of trades.[4] Collins, James, Servátka, and Woods (2017), hereafter CJSW, find that the combination of individually posted prices and Marshallian Path Adjustment is associated with attainment of competitive equilibrium in an advance production (market entry) setting.[5]

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[3]Hahn and Negishi (1962) also allow for out-of-equilibrium trades; rather than directly enforcing an order over buyers' (sellers') actions, they require that individual and aggregate excess demands not be allowed to have persistent opposite signs, i.e. those who would satisfy excess supply (demand) at a uniform price call must do so.

[4]In a follow-up study, Plott and Pogorelskiy (2017) examine price dynamics in a call market with two calls per trading period. While that paper tries to shed light on conjectures about possible Walrasian or Newtonian dynamic influences on bid or ask adjustments on either side of the calls, at any given call the mechanics of trade follow Marshallian path adjustment. The institution implemented by Plott and Pogorelskiy (2017) is related to the uniform-price double auction (McCabe et al., 1984).

[5]The uniform *ex post* market clearing price institution that is embedded in the market entry game can also be argued not to violate Marshallian path adjustment. To see this, consider a comparison with the quantities-only clearing-house (CHQ) introduced by Friedman and Ostroy (1995). CHQ works by accepting quantity messages from agents and imputing limit prices for those agents from their actual cost or valuation parameters. CJSW's uniform price market (the market entry game) treats demand equivalently.

A method by which Marshallian Path Adjustment can be turned on or off turns out to be manipulation of the buyer queueing rule. Edgeworth (1925) pioneered the study of buyer queuing in a setting where each firm has capacity that is both insufficient to supply the entire market and fixed, or at least less easily changed than prices. The work of Beckman (1965) and Levitan and Shubik (1972) further examine the role of buyer queueing as a critical determinant of residual demand. These latter works specify the mechanics of value order buyer queueing in a duopoly.[6]

Vives (1986) finds that given sufficiently large aggregate capacity and number of firms, value order queueing ("surplus maximizing rationing" in his terminology), can lead to attainment of the competitive price. While our parameterization is directly linked to that of the market entry game (Selten and Güth, 1982) rather than Vives' setting, we too provide theoretical results that the competitive equilibrium is a possible outcome under value order queuing. Which of the possible outcomes, *of which the competitive equilibrium is just one*, eventuates is of course an empirical question, addressed in our results section.[7] (See subsection A.1 of the appendix for a

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Further, supply in CJSW's uniform price market can be viewed as equivalent to a (single-unit-per-producer) CHQ if producers (entrants) are held to have submitted a "quantity equals one" message at that juncture; their marginal cost, being sunk, is imputed as equal to zero. Consequently, the value-order queuing used in CJSW's posted offer advance production treatment imposes Marshallian Path Adjustment.

[6] Levitan and Shubik (1972) make special mention of the importance of the choice of buyer queueing rule in obtaining particular equilibria, and also acknowledge that, "The actual shape of contingent demand cannot be specified generally from *a priori* reasoning. It will depend upon priorities in service of customers and … needs specific empirical investigation and model building" (pg. 119).

[7] We are first and foremost interested in value order queueing as a means to exogenously enforce Marshallian path adjustment in order to document its effect on equilibration. However, let us also point out that value order queueing may be an emergent phenomenon in naturally-occurring settings in its own right, for example, in the operation of call markets.

more detailed survey of related literature.)

Marshallian path adjustment might be disrupted if buyers do not engage in deterministic demand revealing behavior. Human buyers might possess notions—e.g. fairness norms and/or expectations of either a relative (split) or absolute (hourly compensation) nature—prompting them to view some offers as unacceptable even if profitable (Cox, Friedman, and Gjerstad, 2007); alternatively, they might make mistakes. Thus, human buyers might refuse profitable offers or accept offers at a loss (neither is possible with robot buyers programmed to accept an offer if and only if profit is greater than or equal to zero). Any of the preceding could as a byproduct disrupt Marshallian path adjustment, even when value order queueing is in use. Despite this, prior studies using value order (see Table A.1 in Appendix A) exclusively use robot buyers, creating a confound in the existing literature. To eliminate this confound, our design provides, *ex ante*, for variation in buyer determinism and then controls, *ex post*, for any effect thereof, from whatever source(s).

Finally, our design draws on the literature on incomplete information market experiments, wherein often each participant knows only their own marginal cost (resale value). Smith (1962), Plott and Smith (1978), and Forsythe, Palfrey, and Plott (1982) examine whether markets arrive at Pareto optimal outcomes despite each agent lacking information about their opponents' costs or valuations (and hence their payoff functions). More recent examples examining duopoly/oligopoly (and attainment, or not, of Nash Equilibrium) include Cox and Walker (1998), Huck, Normann, and Oechssler (1999) (especially their NOIN treatment), and Friedman, Huck, Oprea, and Weidenholzer (2015). These studies suppress information about opponents in order to test conjectures recurrent in economics from Smith (1776) through Hayek (1945) to the present day, as to whether systemic benchmark allocations can be achieved given decentralized, private information (or perhaps missing information). Hence our design keeps marginal costs as private information (while the demand curve, overall, is disclosed to sellers to allow calculation of their respective, conditional payoffs).

2. DESIGN

Our design is based on that used in the POAP (i.e. posted offer with advance production) treatments in CJSW (2017), which were constructed so as to be payoff-equivalent at competitive equilibrium to an empirically-studied parameterization of the market entry game.[8] Thus the design incorporates:

1. five buyers, each capable of buying one unit, comprising a market demand curve with resale values {8,6,4,2,0};
2. five sellers, each capable of producing and selling one unit;[9]
3. an advance production environment, wherein sellers must choose whether or not to produce and pay for their unit before entering the market and all production decisions are known to all, before pricing;[10]
4. the posted offer institution, wherein those sellers who have become producers post a price for their unit; and
5. a buyer queueing method.

In addition, variation of the determinism of buyer behavior is included in this design as a treatment allowing a robustness check on the impact of

[8]Our design is also comparable, though less closely, to that of Mestelman and Welland (1988). See further discussion in CJSW (2017, p. 279, 289).

[9]Note also that restricting buyers and sellers to one unit each is recommended by Plott et al. (2013) when Marshallian path adjustment is the object of study.

[10]There is a payoff of 1 to staying out of the market and a payoff of 1 for entering the market; the latter is added to the profit or loss from producing and, should there be a sale, selling the unit produced as a condition of advance production. This shift does not change any Nash equilibria, or prices and quantities at the margin, but necessitates that profits in equilibrium will be 1 rather than zero. This design choice is implemented to facilitate comparison with other designs in a larger research program which elsewhere nests the market entry game (CJSW, 2017).

TABLE I

MATRIX OF TREATMENTS

	Marshallian Path Adjustment	
	Enforced, per Value Order Queue	Not Enforced, per Random Order Queue
Human Buyers	VOHB	ROHB
Robot Buyers	VORB	RORB

Marshallian path adjustment, and more finely a scale comparison between the effects of queueing method and buyer determinism (see Table I).[11]

In a given round, play unfolds as follows.[12] First, the sellers are informed of their respective costs, MC. Costs are identical across sellers in a given round; i.e. $MC_i = MC$, $\forall i$, but sellers are informed only of their own costs.[13] Buyers' resale values are disclosed at the beginning of each round to all sellers, but buyers know only their own resale value.[14] Resale values

[11]VORB in this paper is the same parameterization as POAP in CJSW (2017). The reference acronym is changed for ease of reference for the reader within the present study.

[12]Complete instructions are reproduced in Appendix C.

[13]Subjects knowing only their own cost (resale value) information is a feature of numerous experimental studies. Two studies which feature private but identical marginal costs across sellers (as ours does) are Smith and Williams (1989) and Cason and Williams (1990).

[14]Disclosing the demand curve to all sellers allows each seller to calculate their own payoff for any possible choice of (non-)production and pricing that they might consider, contingent upon their beliefs about other sellers' choices. Beliefs might eventuate such that outcomes theoretically-derived under complete information emerge nonetheless under conditions of incomplete information. Thus the complete information equilibria from our theory section are still interesting benchmarks, comparable to the role played by the competitive equilibrium prediction in Plott and Smith (1978), the rational expectations prediction in Forsythe et al. (1982), or the Cournot equilibrium prediction in Cox and Walker (1998). Note also that such beliefs could be affected if one were to change aspects of the design, and might as such support different empirical results. For example, making marginal cost of each seller public might be argued to support attempts at greater collusion by sellers or greater withholding by buyers, or possibly both, which might lead

are the same across all rounds.

Next, the sellers, having already been informed in the instructions as to the buyer queueing rule in effect in that session, make their production decisions. Hereafter, we denote sellers who chose to produce in advance as *producers*. All subjects are then informed of the number of producers (equivalently, number of units for sale), m, in that round.

Subsequently producers, each with a unit for sale, make their respective pricing decisions, which must come from the set $\{0,2,4,6,8,10\}$. Buyers are then released, as per the queueing rule in place in that session, to evaluate the offers that round, each buyer either to buy one unit, or zero, as they so choose. Human buyers also earn a commission of 0.1, distinct from surplus from trading, each time they buy a unit, thus eliminating indifference in favor of procurement when a buyer's value equals the lowest available ask.[15] After the end of the round, the number of producers and the prices posted are displayed to all, while each individual's payoff is displayed privately, as feedback.

There are 96 rounds in an experimental session. Each session is divided into 6 blocks of 16 rounds. Within each such block, during the first 4 rounds MC (identical across subjects, but private information) is varied randomly without replacement through $\{2,4,6,8\}$. During the middle 8 rounds, $MC = 4$ throughout. During the final 4 rounds, MC varies randomly but without replacement via a different drawing than the first 4 rounds. (The same MC sequence is used in all sessions.) This alternation of blocks allows for capture of observations under (locally) stationary conditions as well as under non-stationary conditions. A stationary environment is so common

to results similar to Ruffle (2000). We thank an anonymous referee for pointing this out.

[15]The use of commissions conditional on completing a trade was first discussed in published work by Smith (1976), and first used in practice by Plott and Smith (1978) in experiments conducted prior to the former publication. See also Ketcham et al. (1984).

as to be arguably a default setting in market experiments. A non-stationary environment allows interesting additional comparisons—not least another robustness check on conclusions about treatment effects—and is specifically suggested by Plott et al. (2013, pg. 193) as a way to allow the separation of the equilibrating properties of Marshallian path adjustment from possible development of price expectations otherwise. Thus, we implement alternating stationary and non-stationary segments.

All subject groups are disjoint, and no subject participated in more than a single session. Each group consists of five human sellers (and in human buyer sessions, also five human buyers) and is fixed throughout that session, while there are two concurrent, unrelated groups per session. All experiments took less than two and a half hours. Payoffs consisted of one period randomly selected, after the experiment, from each of the six blocks, plus a show-up fee.[16] Experiments were conducted in z-Tree (Fischbacher, 2007) and subjects were recruited using ORSEE (Greiner, 2015). All sessions were run at the New Zealand Experimental Economics Laboratory at the University of Canterbury.

3. THEORY

For a posted offer with advance production, and a value order buyer queue, the pure strategy equilibria fall into three categories: (1) the competitive market equilibrium with marginal cost pricing, (2) (tacitly) collusive

[16]This method is that used by Duffy and Hopkins (2005) and thereafter by CJSW (2017). It is implemented here, as it was in those prior studies, as the most appropriate method (given the informational conditions imposed in the experiment) to avoid incentive and protocol issues that could otherwise arise if earnings are accrued cumulatively throughout the experiment. If earnings had instead accrued cumulatively, then draw-down of earnings or even bankruptcy on the part of subjects would have been possible, due to the advance production (equivalently, pre-committed entry) environment. Subject payments are detailed in subsubsection A.2.1 of the appendix.

pure strategy equilibria involving fewer units produced supporting higher prices, and (3) (tacitly) collusive equilibria involving some units produced not sold, supporting higher prices. There are also mixed strategy equilibria, involving slightly different advance production (equivalently, entry) probabilities than would be predicted under an otherwise similarly parameterized market entry game; this difference stems from a difference in pricing possibilities.[17] All of these results for the value order queueing version of the posted offer with advance production are established in CJSW (2017).

Our point of departure for characterizing equilibria under random order queueing is to point out where they begin to differ from those under value order queueing. As per (1) above, under value order queueing there is a pure strategy Nash equilibrium (PSNE) in which each agent nominates their asking price via $P_i = P(m)$, the market clearing price. That is, each agent, i, posts as their asking price, P_i, a number equal to the price coordinate on the demand curve, $P(m)$, given that m units are known to have been produced in the first stage of the game. (For ease of exposition, we shall refer to $P_i = P(m)\ \forall i$, as "pricing via the demand curve".) For $m = 3$ additional uniform price equilibria, featuring unsold units as per (3) above, arise even with value order queueing. The set of equilibria then gets richer still when queueing is instead done in random order, because here there may now exist an incentive for producers to post higher-than-Bertrand asks in anticipation of encountering high-valuation buyers later in the queue than would be possible under value order. Notably, the $m = 3$ case under random order queueing supports two asymmetric pricing equilibria. For $m > 3$, pricing via the demand curve is not a PSNE under random order queueing

[17] Since the market entry game employs an embedded administered pricing rule, while the posted offer with advance production allows free individual posting of prices, the possible gains or losses associated with entry can vary across the two games, thus altering the probability of entry which establishes indifference with not entering.

(unlike for value order queueing).[18]

By the time of the pricing subgame, (1) marginal cost is sunk, and (2) the number of producers is known and common knowledge; therefore, at that juncture, asking prices consistent with pure strategy Nash equilibria depend only on the number of producers, m. Thus, we can meaningfully divide analysis of equilibria between the pricing subgame and the game composed of the advance production decision and pricing subgame jointly. We provide such a taxonomy of equilibria in Table II. Panel A presents all pure strategy Nash equilibria in the pricing subgame, not all of which are subgame perfect. (Those PSNE that are not subgame perfect may still arise empirically, and thus need to be accounted for.) Panel B presents subgame perfect equilibria; these involve asking prices, number of producers, and marginal costs.

We identify two types of (tacitly) collusive equilibria. In one, at the advance production stage, quantity is restricted to less than the competitive amount of advance production, permitting higher equilibrium prices; we term this collusive production. In the other, at the pricing stage, some or all of the m producers may possibly price above the competitive price, resulting in less than m units taken by buyers; we term this collusive pricing. As Table II shows, given our study's parameterization, random order and value order each support collusive pricing pure strategy equilibria, and the collusive pricing equilibria possible under value order are a subset of the collusive pricing equilibria possible under random order.[19]

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[18]Further alternatives, in the form of strategies exploiting repetition of play, are explored in subsubsection B.3.1 of the appendix. There we demonstrate theoretically a possible collusive repeated game equilibrium (and what is required for its implementation by sellers).

[19]Even more strikingly, random order does not support any pricing via the demand curve—a necessary condition for competitive equilibrium—when there are more than three producers ($m > 3$). It might also be noted that, for uniform pricing equilibria, the

TABLE II

DESCRIPTION OF PURE STRATEGY AND SUBGAME PERFECT NASH EQUILIBRIA

Panel A. Asking Prices Supporting Pure Strategy Nash Equilibria in the Pricing Subgame Given Advance Production

Number of Producers	Posted Asking Prices: Value Order Queuing	Random Order Queuing
$m = 1$	**{8}**	**{8}**
$m = 2$	**{6,6}**	**{6,6}**
$m = 3$	**{4,4,4}**,{6,6,6}	**{4,4,4}**,{4,4,6},{4,6,6},{6,6,6}
$m = 4$	**{2,2,2,2}**,{4,4,4,4}	{4,4,4,4}
$m = 5$	**{0,0,0,0,0}**,{2,2,2,2,2},{4,4,4,4,4}	{2,2,2,2,2},{4,4,4,4,4}

Note: Asking prices supporting PSNE in the pricing stage are listed for value and random order queueing, given demand schedule {8,6,4,2,0}, with marginal cost sunk. A subset of these PSNE in the pricing subgame support the pure SPNE (for advance production and pricing) enumerated in Panel B. Asking prices in **bold** are consistent with pricing via the demand schedule, i.e. each seller i posting a price $P_i(m)$ where m is the number of producers and $P(\cdot)$ is the demand schedule.

Panel B. Advance Production (Quantities) and Asking Prices Supporting Subgame Perfect Nash Equilibria

Marginal Cost of Production	Number of Producers	Posted Asking Prices: Value Order Queueing	Random Order Queueing
$MC = 8$	$m = 0$	∅	∅
	$m = 1$	{8}	{8}
$MC = 6$	$m = 2$	**{6,6}**	**{6,6}**
$MC = 4$	$m = 3$	**{4,4,4}**,{6,6,6}	**{4,4,4}**,{4,4,6},{4,6,6},{6,6,6}
$MC = 2$	$m = 4$	**{2,2,2,2}**	—
	$m = 5$	{4,4,4,4,4}	{4,4,4,4,4}

Note: Asking prices supporting pure SPNE are listed for value and random order queueing, given marginal cost $MC = \{8, 6, 4, 2\}$ and demand schedule {8,6,4,2,0}. The PSNE in the pricing subgame are enumerated in Panel A. Cells for which no PSNE in the pricing stage support a SPNE have dashes. Asking prices in **bold** are consistent with pricing via the demand schedule.

Special attention is warranted in the case of three producers ($m = 3$). In this case, both value order and random order support the competitive equilibrium. However, they each also support collusive pricing equilibria, and there are important differences among these. Under random order queueing, each producer is indifferent, in expectation, between pricing at 4 or 6, while under value order queueing, in equilibrium agents must ask the same price. Under random order queueing, all producers pricing at 6 is robust to trembles (Selten, 1975), but under value order queueing, all producers pricing at 6 is *not* robust to trembles.

4. RESULTS

4.1. *Overview of Results*

The treatments we evaluate are described in Table I. For each of these four treatments, we consider data generated by four groups. This totals 16 groups formed from 80 human sellers, 40 human buyers, and 40 robot buyers. Across 96 periods, this yields 7,680 seller-level observations, 3,840 human-buyer-level observations, and 1,536 group-level observations.

With this data, the questions that we address include: (1) do we observe convergence to any equilibria; (2) do we observe convergence to different equilibria when exogenous enforcement of Marshallian path adjustment is or is not in place due to use of different queuing treatment; (3) does the rate of convergence vary by queuing treatment; (4) does the selection of equilibria or speed of convergence vary by buyer behavior treatment (robot or human); (5) are there other possible effects of human, rather than robotic, buyer

payoffs of the pricing subgame (to producers and buyers) are the same under random order and value order. This occurs because, under both value and random order: (1) equilibrium pricing at or above m steps down the demand curve, and (2) uniform pricing, together guarantee that all buyers with a higher value than the uniform asks will receive an opportunity to purchase (at the same price), with lower-value buyers not able to purchase profitably, regardless of queuing order.

behavior; and (6) how does all of this interact with the advance production decision preceding the pricing subgame?

To first provide a visual overview of results, Figure 1 graphically illustrates production choices, asking prices, quantity sold, and efficiency from the stationary marginal cost periods of the experiment for one session from each treatment. (The stationary periods, for which $MC = 4$ unchanging, are 5 through 12, 21 through 28, and so on.) Open dots represent unfilled asks and closed dots represent filled asks (realized transactions); the vertical coordinate for either denotes the price at which the ask was made. The left-to-right position of a dot within the vertical bar associated with the period denotes which seller made the ask (e.g. left-most position, seller 1; … ; right-most position, seller 5). The number of transactions in a period is the number interior to the bottom of the graph, while the efficiency in that period, measured as the fraction of (expected) total surplus realized, is the number immediately exterior to the bottom of the graph. Aggregate efficiency under VORB, VOHB, RORB, and ROHB is respectively 98.9%, 93.2%, 92.6%, and 85.2%. (We present similar graphs for all sessions and details on efficiency and buyer behavior in subsections A.2.2 and A.6.1 of the appendix, respectively.)

Value order sessions for both human and robot buyers converge to the competitive equilibrium of three units produced, all priced at 4, while random order sessions exhibit a wide range of behaviors anticipated in the theory presented in section 3, including non-uniform pricing equilibria detailed there. As we shall see, econometric analysis supports this characterization.

4.2. *Convergence to Competitive Equilibrium*

Likely the foremost question in market experiments is whether or not competitive equilibrium is attained. There are two distinct, but nested, senses in which competitive equilibrium might be judged as occurring. The less

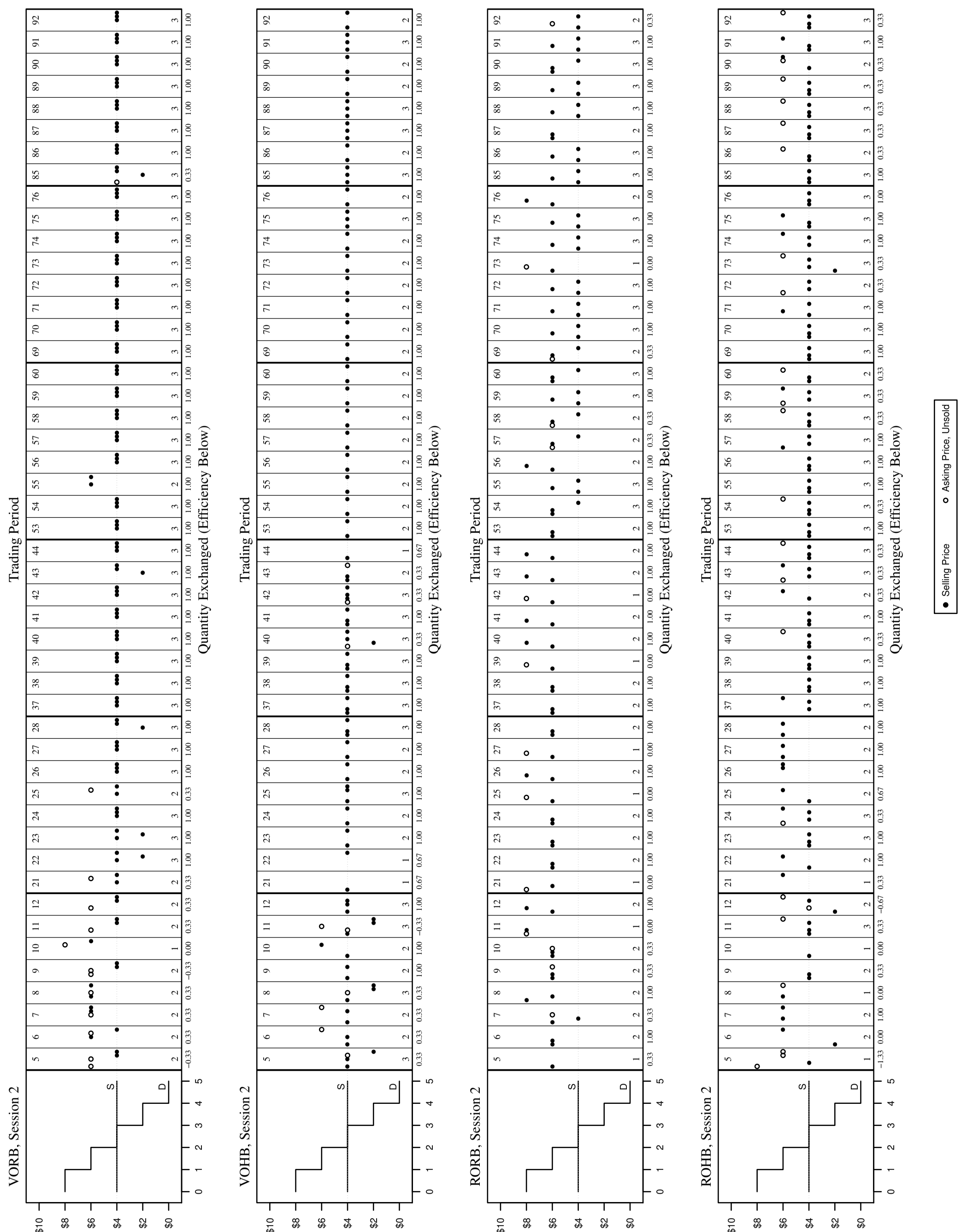

Figure 1: Asks, Prices, and Quantities in Stationary Periods of Session 2 of Each Treatment.

restrictive sense is that familiar from market experiments with production-to-order (e.g. Plott and Smith, 1978): are the competitive price and quantity predictions observed?[20] The more restrictive sense also applies that criterion, but adds another: are the individual producers who deliver the competitive equilibrium quantity the same from one round to the next? This additional criterion isolates instances where asymmetric PSNE play is responsible for the occurrence of competitive equilibrium quantity (as opposed to, say, turn-taking in production, which would be admitted under the less restrictive standard).

We present results addressing both standards of equilibrium attainment, for both stationary and non-stationary rounds. Whether CE price and quantity have occurred is straightforward to document; thus for the looser standard of CE attainment we present a probit where the dependent variable is 1 if both CE price and CE quantity occur, and 0 otherwise. For the tighter standard we further refine the dependent variable, such that it receives the value 1 only if all of the preceding is true and a particular stable pattern (described next) manifests in the identities of the individual producers.

We evaluate the emergence of the *temporally stable, individual* advance production decisions that support competitive equilibrium in pure strategy play as follows. Loosely following Duffy and Hopkins (2005), we identify *ex post* the players who are most likely to be producers, by reference to late round behavior, and having done so then track (from the first round onwards) the adjustment process that leads to the outcome observed at

[20]The minimum criteria we set for observation of CE are the observation of both: (1) a SPNE number of entrants, given marginal cost, and (2) uniform pricing at the price implied by the demand curve, given the number of entrants observed under (1). Note that the preceding conditions imply zero economic profits. This includes the cases where $MC = 6$ and two sellers choose to produce (and price at 6), and where $MC = 8$ and one seller chooses to produce (and price at 8), and where $M = 8$ and no seller produces.

the end of each session.[21] This categorization and tracking process which we employ is suited to uncover assortative behavior, for instance a pecking order in advance production for which frequency of advance production is monotonic on the part of each individual seller. Under such a pecking order, the seller who produces when $MC = 8$ should *a fortiori* produce when $MC = 6$, and so on. The seller who joins in at $MC = 6$ should produce at MC=4, and so on.

Graphically, over a block of four periods, the ordering described above would in stationary ($MC = 4$ only) rounds lead to a configuration such as that in the left-hand panel of Figure 2, while in non-stationary rounds ($MC = \{2, 4, 6, 8\}$) would lead to a configuration such as the two right-hand panels of Figure 2.

As examples of the evolution of actual advance production decisions—possibly towards the assortative benchmarks in Figure 2—among a group of sellers in Session 2 of each treatment see the data presented in Figure 3. In reading the graphs, in left-to-right temporal order of blocks, we would point out the following particular phenomena as being of interest. First, for any group, the final block does not exhibit the same entry pattern as the first block; there is change in advance production decisions over the course of the experiment. Second, there are instances of adjacent blocks exhibiting iden-

[21]Which players are predicted to engage in advance production is determined by the following algorithm adapted from Duffy and Hopkins (2005), implemented over four-period blocks of data: (1) begin in final block $\tau = \{T\}$ with all $I = \{1, 2, 3, 4, 5\}$ sellers, (2) order the I sellers by number of periods of production for block(s) τ and prior block $\min\{\tau\} - 1$, denoting this $\theta_t(I)$ and $\theta_{t-1}(I)$ respectively; (3) breaking any ties in ordering by the number of period of production in $t = 1, \ldots, T$; then (4) if $\min\{\tau\} - 1 = 1$ or if $\theta_{t-1} = \theta_t$, return θ_{t-1} as the predicted ordering, else (5) update $\tau = \{\min\{\tau\} - 1, \ldots, T\}$ and return to step 2. For ease of exposition, we describe advance production decisions exactly matching the assortative pattern predicted by this algorithm as "temporally stable" in identities, as for instance, in Table III.

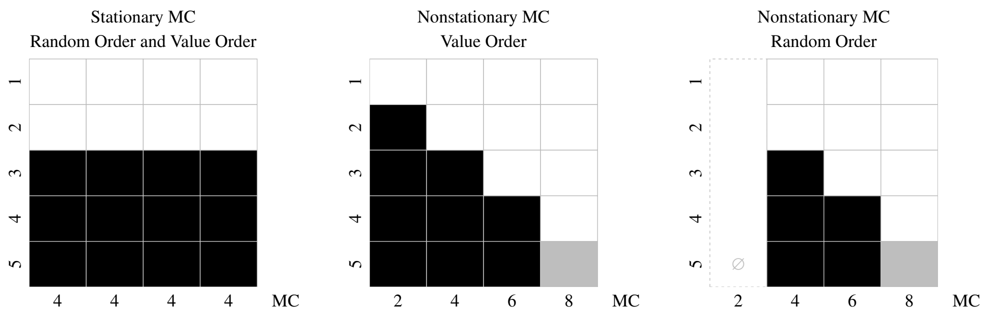

Figure 2: Competitive Equilibrium Arrangement of Advance Production

Note: Each row shows the identity of each seller (vertical axis labeling, 1 . . . ...5). Each column shows whether a given seller would engage in advance production (black) or not (white) within a block of four contiguous rounds given all of the following: (a) marginal cost in that round is as listed at the bottom of that column, on the horizontal axis, (b) there is a "pecking order" in advance production that respects monotonicity at the individual level (and *a fortiori* at the group level), and (c) total number of producers is that which would be observed given both SPNE advance production decisions and zero economic profits (i.e. a SPNE number of producers and uniform pricing at the level then implied by the demand curve). (Note the following special cases. First, that gray is used for the $MC = 8$ cases as either advance production by one seller, or no production at all, each feature SPNE play and zero economic profits. Second, that in the case of random order queueing and $MC = 2$, there are no SPNE featuring the uniform pricing that would be implied by the demand curve; thus the $MC = 2$ column for random order is occupied by the null set.)

tical patterns; stability of advance production—associated with pure strategy play—may be emerging over the course of the experiment. Third, the patterns emerging in actual play may resemble the monotonic-in-advance-production, pure strategy benchmarks seen in Figure 2; the VORB data contain more instances conforming to such benchmark configurations than do, say, the ROHB data. In subsubsection A.2.2 of the appendix we present visualizations of advance production data for all sessions, and there include the $MC = 2$ data from RO sessions in figures otherwise constructed as per Figure 3. In subsection A.4 of the appendix, we present a non-parametric analysis of all advance production data that shows a significant and downward ordinal time trend in instability among the set of producers, given stationary environment, for all treatments except RORB.

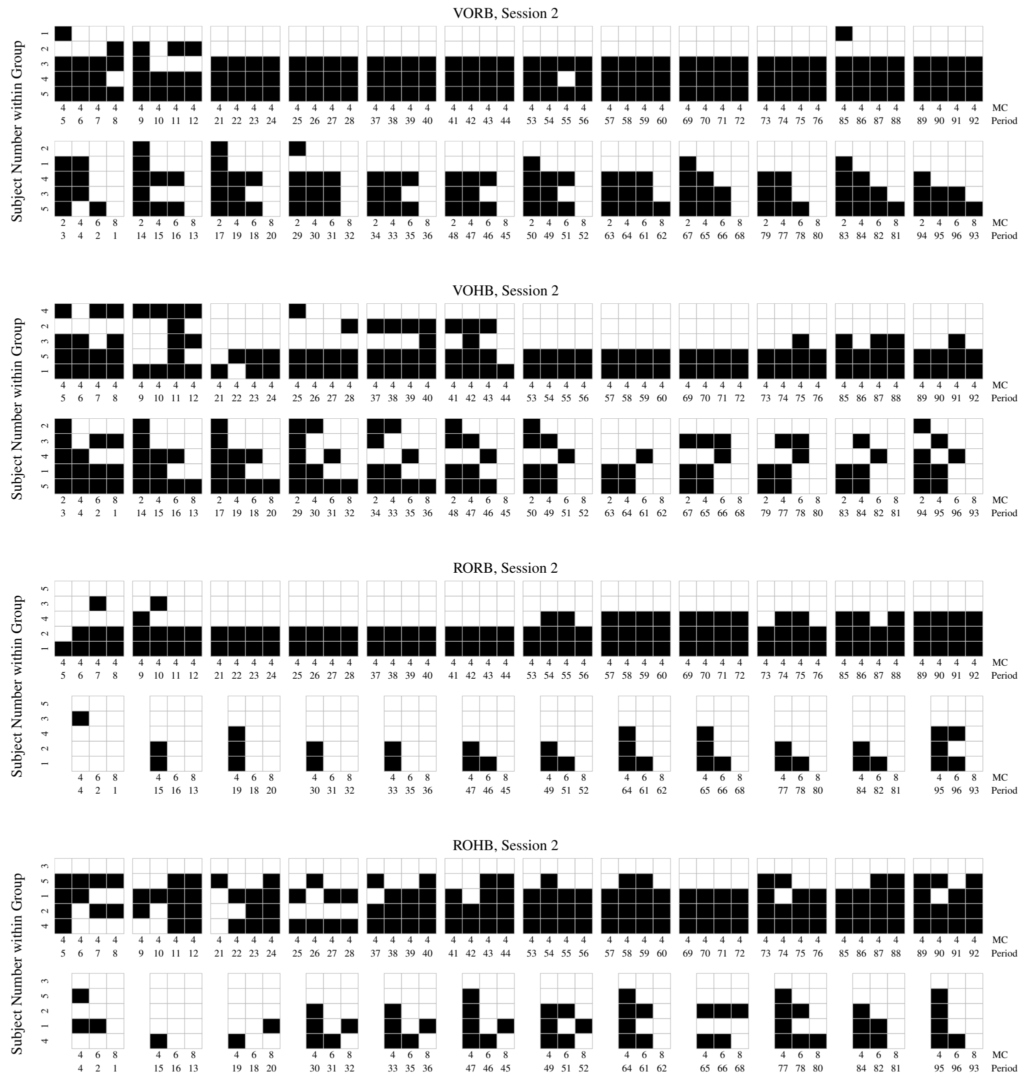

Figure 3: Advance Production in Session 2 of Each Treatment

Note: Session 2 seller decisions to engage in advance production (black) or not (white). Y-axis: subject numbers within group (corresponding also to subjects' respective horizontal locations in Figure 1) arrayed in a monotonic "pecking order" of advance production, per the algorithm described in footnote 21. X-axis: lexicographic sorting; first, by chronology across four-contiguous-period blocks; second by (weakly) ascending marginal cost (MC) within each block; third, for stationary rounds, chronologically within each block. This arrangement of the data is constructed for visual comparison to Figure 2 (i.e. under RO, no competitive equilibrium prediction exists when $MC = 2$, thus above and in Figure 2 the column above $MC = 2$ is null). Advance production data arrayed as in Figure 2, but with $MC = 2$ data also included for RO sessions, is found in Figures A19 through A22 of the appendix, constituting all sessions.

TABLE III

MARGINAL EFFECTS OF PROBIT ON ATTAINMENT OF COMPETITIVE EQUILIBRIUM

	Stationary		Non-Stationary	
	CE $P\&Q$	+ Stable Identities	CE $P\&Q$	+ Stable Identities
	(1)	(2)	(3)	(4)
Robot Buyers	-0.2478^{***}	-0.1483	-0.0584	-0.0190
	(0.0823)	(0.1060)	(0.0724)	(0.0558)
Random Order	-0.0998	-0.1438	-0.0308	-0.0707
	(0.0948)	(0.0879)	(0.0746)	(0.0560)
Period	0.0015^{***}	0.0043^{***}	0.0033^{***}	0.0028^{***}
	(0.0012)	(0.0013)	(0.0007)	(0.0009)
Period × Robot Buyers	0.0015^{***}	0.0018	0.0004	-0.0004
	(0.0012)	(0.0016)	(0.0011)	(0.0010)
Period × Random Order	-0.0020^{***}	-0.0035^{**}	-0.0013	-0.0002
	(0.0014)	(0.0015)	(0.0012)	(0.0010)
Observations (Groups)	768 (16)	768 (16)	672 (16)	672 (16)

Notes: Reported are marginal effects on attainment of asking prices and quantities (Models 1 and 3) and stable seller identities (Models 2 and 4) consistent with competitive equilibrium. Stable seller identities are defined in footnote 21. Corresponding standard deviations are in parentheses. Standard errors are clustered at group level, with 4 groups per each of the 4 treatments. Competitive equilibrium is not attainable in RO under $MC = 2$ (see Figure 2); therefore we omit $MC = 2$ for RO sessions in Models 3 and 4. (We report estimates with $MC = 2$ observations omitted for both VO and RO treatments in Table A3).

*** Significant at the 1 percent level.
** Significant at the 5 percent level.

Table III presents probit marginal effects on attainment of equilibrium. Starting with the stationary rounds, we see that under Model 2 (using the more stringent standard of CE, including stability of the identities of individual entrants) the approach to CE over time is strongly impeded by random order queueing; the probit estimations (reported in subsection A.3 of the appendix) underlying the marginal effects imply that expected time to 95% realization of individually stable CE is 124 rounds under value order, but 342 rounds under random order, given robot buyers in each case. A similar finding emerges in Model 1 under the looser CE standard (price and aggregate quantity, only), with the additional proviso that in this case some credit for the hastening of CE is attributed to the presence of robot buyers; the underlying probit model implies that estimated time to 95% realization of CE price and quantity is 108 rounds with robot buyers but 146 rounds with human buyers, given value order queueing in each case.

The non-stationary rounds present a contrast. The fastest predicted convergence result under Model 3 (Model 4) is 239 rounds (311 rounds) to 95% realization of competitive equilibrium, using the VORB (VOHB) treatment combination. Clearly, convergence to either standard of CE (without or with stable individual seller identities) is impeded by a non-stationary environment.

The contrast in results across stationary and non-stationary environments may illustrate consequences of agents not knowing other agents' marginal costs, impacts of imposing a (non-)stationary experimental environment, and most especially possible interactions there between. That is, in a stationary environment, with repetition, agents with limited, private information have in prior studies been observed to achieve outcomes originally predicted by theorists under the assumption that agents had full information and common knowledge (e.g. Forsythe et al. (1982); see Background section). We observe that here, too. But what we do not observe is similarly speedy attainment of CE when instead a non-stationary environment is imposed. This is consistent with the following conjecture: in a repeated, stationary environment agents may be (better) able to make sufficient inferences, from opponents' past play, as to underlying motivations and/or parameterizations (e.g. opponents' MC) such that attainment of outcomes theoretically derived under richer information conditions is not precluded.[22] We demonstrate the sorting that would be expected under fictitious play in subsubsection B.3.2 of the appendix.

4.3. *Equilibria Beyond Competitive Equilibria*

Having thus far focused on attainment of CE, what happens more generally? For instance, what if SPNE entry decisions are not observed—is some PSNE in the pricing subgame reached nonetheless? Note also that once the entry stage in a round has passed, marginal cost is sunk (and thus information partitions with respect to marginal cost are no longer a consideration under many standard theories of choice).

As the theory section shows, there are many pure strategy Nash Equilibria under each of value order and random order queueing; Table IV reports the proportion of trading periods in which equilibrium asking prices were observed, disaggregated by treatment and number of producers. (A longer discussion of this table follows in subsection A.5 of the appendix.)

We also note that price dispersion differs across queueing methods. [23] Table V reports the mean and standard deviations of transaction prices, disaggregated by treatment and number of producers.[24]

[22]We thank an anonymous referee for pointing this out.

[23]Even in cases with unique NE, possibility of errors or near-indifference may give rise to significant price dispersion. See Baye and Morgan (2004).

[24] Note that in Table V we report descriptive statistics of *transaction* prices and in Table IV we report the proportion of *asking* prices consistent with PSNE. For instance, the upper-left cell of Table IV reports that in the case of $m = 1$ producing seller in

TABLE IV

Proportion of Asking Prices Consistent with Pure Strategy Nash Equilibrium in the Pricing Subgame

Number of Producers	PSNE	Value Order Queueing: Human Buyers (VOHB)	Value Order Queueing: Robot Buyers (VORB)	Random Order Queueing: Human Buyers (ROHB)	Random Order Queueing: Robot Buyers (RORB)
$m = 1$	**{8}**	0.71	0.64	0.62	0.55
$m = 2$	**{6,6}**	0.54	0.62	0.45	0.29
$m = 3$	Any PSNE	0.75	0.73	0.88	0.89
	{4,4,4} (CE)	0.73	0.69	0.26	0.03
	{4,4,6}	—	—	0.18	0.45
	{4,6,6}	—	—	0.40	0.32
	{6,6,6}	0.02	0.04	0.04	0.09
$m = 4$	Any PSNE	0.46	0.44	0.09	0.13
	{2,2,2,2} (CE)	0.05	0.02	—	—
	{4,4,4,4}	0.41	0.42	0.09	0.13
$m = 5$	Any PSNE	0.13	0.32	0.12	0.09
	{0,0,0,0,0} (CE)	0.00	0.00	—	—
	{2,2,2,2,2}	0.00	0.00	0.00	0.00
	{4,4,4,4,4}	0.13	0.32	0.12	0.09

Note: Reported are proportions of asking prices consistent with the PSNE in the pricing subgame described in Panel A of Table II. Cells for which there are no asking prices consistent with PSNE have dashes. Competitive equilibria are denoted CE. Asking prices in **bold** are consistent with pricing via the demand schedule.

Across all cases, the standard deviation of prices under value order queueing is approximately 1.13, while it is 1.37 under random order queueing; prices are more dispersed under random order queueing ($p < 0.0001$, via a Siegel-Tukey test of non-parametric dispersion). Across all cases, the standard deviation of prices with human buyers is 1.27, while it is 1.29 with robot buyers; there is no significant difference in dispersion ($p \approx 0.1457$ via a Siegel-Tukey test).[25] Theoretically, random order queueing supports

VOHB, 22 of 31 (71%) of asking prices were consistent with PSNE (i.e. at 8); 19 of those 22 asks at 8 resulted in a transaction, as did all 9 asks at other prices, for a total of 28 transactions in the upper-left cell of Table V.

[25] We also perform the nonparametric Fligner-Killen test on homogeneity of variances; we find a significant difference between random and value order queueing ($p < 0.0001$),

TABLE V

Description of Prices and Price Dispersion

Number of Producers	Value Order Queueing: Human Buyers (VOHB)	Value Order Queueing: Robot Buyers (VORB)	Random Order Queueing: Human Buyers (ROHB)	Random Order Queueing: Robot Buyers (RORB)
$m = 1$	6.93 (1.76, 28)	7.25 (1.42, 24)	6.90 (1.66, 60)	6.34 (2.23, 41)
$m = 2$	5.08 (1.16, 145)	5.65 (0.83, 139)	5.41 (1.09, 186)	6.42 (1.04, 99)
$m = 3$	4.06 (0.45, 421)	4.13 (0.64, 463)	4.51 (0.97, 262)	4.99 (1.06, 350)
$m = 4$	3.50 (0.91, 131)	3.72 (0.82, 171)	4.04 (0.75, 151)	4.12 (1.09, 195)
$m = 5$	2.99 (1.01, 56)	3.43 (0.91, 56)	3.62 (0.79, 73)	3.71 (0.81, 110)

Note: Reported are mean transaction prices, with standard deviations and number of observations, respectively, in parenthesis below each. Excluding a seller who repeatedly took negative payoffs in the $m = 1$ monopoly case in RORB, the mean transaction price in the upper right cell would become 7.19.

non-competitive strategies—and resultant price dispersion—which will only be used given the possibility of prolonged availability of high-value buyers (Ronayne and Myatt, 2019). Our empirical results are consistent with that theoretical finding, and with random order queueing promoting convergence to some non-uniform pricing equilibria identified in our Theory section. Finally, note that in addition to the differences in price dispersion, there are differences in level of prices across treatments that are significant. We present further discussion and statistical tests in subsection A.6 of the appendix.

but there would only be a significant difference between human and robot buyers if one admitted type I error at 10% level ($p \approx 0.0604$).

5. CONCLUSIONS

In order to appreciate the most wide-reaching implications of our results, one might first consider this surprising finding: the posted offer market's canonical characterization as a poorly equilibrating form of market is not entirely accurate.[26] This characterization rests on a large number of results obtained using queuing protocols where the direct proportionality between excess rents and response speed conjectured by Smith (1962) was at best not enforced, and in the case of exogenously randomized ordering effectively precluded (see appendix Table A1 for examples).[27] This current paper provides the previously missing comparison between random and value order queuing when there are more than two agents on both sides of the market. In the presence of the counterfactual created by varying (within an otherwise identical setting) value order or random order queueing, prior results take on new meaning: poor equilibration appears likely due, at the margin, largely to issues around buyer queueing, rather than the rules of the posted offer market itself.

The ultimate point here is not so much to exonerate the posted offer market as to try to identify the real culprit in past poor equilibration—still at large and possibly active in other forms of market. We thus make the following testable conjecture: any combination of environment and institution which actively precludes Marshallian Path Adjustment will gen-

[26]For instance, of posted-offer markets (Plott, 1982, pg. 1498) writes "adjustment to equilibrium tends to be from above and either converges to equilibrium more slowly [than in double auctions] or does not converge at all".

[27]Note also that if endogenous random order (colloquially known as "first come, first served") is driven by buyer characteristics (e.g. manual dexterity as applied to the use of a computer mouse) which are unobservable to the sellers, then from the point of view of the sellers endogenous order may appear, subjectively, indistinguishable from exogenous random order. As such, sellers facing endogenous ordering might at least temporarily strategize and respond as if they were facing exogenous random ordering.

erate inefficient allocations relative to some combination which does not preclude Marshallian Path Adjustment (but is otherwise similar). Random order buyer queueing appended to the posted offer market actively precludes Marshallian Path Adjustment, while value order queueing actively enforces it. (The presence of human buyers might in principle disrupt Marshallian Path Adjustment, relative to demand revealing robot buyers, but there is only limited empirical support for that in our data.) We conjecture that other forms of allocative process thus distinct would have similar disparities in empirical allocative efficiency.

This conjecture can be integrated with the finding from CJSW (2017) that individual posting of offers, rather than an ex post market clearing price, aids equilibration. Only when both (a) Marshallian path adjustment is imposed and (b) prices are informed by individual posting of offers is the competitive equilibrium allocation reliably attained. Removing either (a or b) leads to supra-competitive pricing or cycling, respectively. Interestingly, the double auction allows individual posting, and may encourage Marshallian path adjustment as an emergent phenomenon; whether its robust equilibration has roots in these traits is an intriguing topic for future research.

We also conjecture that the price discovery encouraged by value order queueing has real effects on entry. That is, by allowing speedier attainment of pricing via the demand curve during the pricing subgame (see Table A6 in subsection A.5 of the appendix), value order queueing then indirectly allows speedier resolution (i.e. sorting) in the entry subgame.

The above conjectures offer testable predictions, and provisional guides to market design. While said conjectures apply more broadly than just to issues of buyer and/or seller queueing, we also note that queueing issues specifically are at the heart of current work on high-frequency trading (Aldrich and Friedman, 2019; Budish et al., 2015) wherein endogenous purchase of trading priority (e.g. by server colocation privileges) is the unregulated or default state of affairs. To the extent that the traders with the highest surplus associated with trading are best able to purchase priority, then a familiar form emerges: value order queueing. A proposed countermeasure to this state of affairs is to replace continuous trading with frequent batch auctions that pool orders over time, erasing time priority stemming from emergent value order queueing; order execution proceeds instead along other familiar lines: exogenous random ordering, imposed by design. Which forms of queuing either emerge, exist as a matter of default, or are imposed by design will likely vary across allocation problems. However, subject to coming into use, each queuing structure will have at the margin a distinct effect on market dynamics. We hope that our results offer a low cost source of insight for wide-ranging market design efforts in future.

REFERENCES

Aldrich, E. M. and D. Friedman (2019): "Order Protection Through Delayed Messaging," *Available at SSRN 2999059*.

Baye, M. R. and J. Morgan (2004): "Price Dispersion in the Lab and on the Internet: Theory and Evidence," *The RAND Journal of Economics*, 35, 449.

Beckman, M. J. (1965): "Bertrand-Edgeworth oligopoly revisited," in *Operations Research Verfahren, III*, ed. by R. Henn, Meisenheim: ... Verfahren, 55–68.

Brown-Kruse, J. L. (1991): "Contestability in the Presence of an Alternate Market: An Experimental Examination," *The RAND Journal of Economics*, 22, 136–147.

Budish, E., P. Cramton, and J. Shim (2015): "The High-Frequency Trading Arms Race: Frequent Batch Auctions as a Market Design Response," *The Quarterly Journal of Economics*, 130.

Camerer, C. and T. Ho (1999): "Experience-weighted attraction learning in normal form games," *Econometrica*, 67, 827–874.

Cason, T. N. and D. Friedman (1997): "Price Formation in Single Call Markets," *Econometrica*, 65, 311–345.

Cason, T. N. and A. W. Williams (1990): "Competitive equilibrium convergence in a posted-offer market with extreme earnings inequities," *Journal of Economic Behavior* *and Organization*, 14, 331–352.

Chamberlin, E. H. (1948): "An Experimental Imperfect Market," *Journal of Political Economy*, 56, 95–108.

Collins, S. M., D. James, M. Servátka, and D. Woods (2017): "Price-Setting and Attainment of Equilibrium: Posted Offers Versus An Administered Price," *Games and Economic Behavior*.

Coursey, D., R. M. Isaac, M. Luke, and V. L. Smith (1984a): "Market Contestability in the Presence of Sunk (Entry) Costs," *The RAND Journal of Economics*, 15, 69–84.

Coursey, D., R. M. Isaac, and V. L. Smith (1984b): "Natural Monopoly and Contested Markets: Some Experimental Results," *The Journal of Law and Economics*, 27, 91–113.

Cox, J. C., D. Friedman, and S. Gjerstad (2007): "A tractable model of reciprocity and fairness," *Games and Economic Behavior*, 59, 17–45.

Cox, J. C. and M. Walker (1998): "Learning to play Cournot duopoly strategies," *Journal of Economic Behavior & Organization*, 36, 141–161.

Davidson, C. and R. Deneckere (1986): "Long-Run Competition in Capacity, Short-Run Competition in Price, and the Cournot Model," *The RAND Journal of Economics*, 17, 404–415.

Davis, D. D. and A. W. Williams (1991): "The Hayek Hypothesis in Experimental Auctions: Institutional Effects and Market Power," *Economic Inquiry*, 29, 261–274.

Davis, D. D. and B. J. Wilson (2008): "Strategic buyers, horizontal mergers and synergies: An experimental investigation," *International Journal of Industrial Organization*, 26, 643–661.

Duffy, J. and E. Hopkins (2005): "Learning, Information, and Sorting in Market Entry Games: Theory and Evidence," *Games and Economic Behavior*, 51, 31–62.

Edgeworth, F. Y. (1925): *Papers relating to political economy*, London: Macmillan.

Erev, I., E. Ert, A. E. Roth, E. Haruvy, S. M. Herzog, R. Hau, R. Hertwig, T. Stewart, R. West, and C. Lebiere (2010): "A choice prediction competition: Choices from experience and from description," *Journal of Behavioral Decision Making*, 23, 15–47.

Fischbacher, U. (2007): "z-Tree: Zurich Toolbox For Ready-Made Economic Experiments," *Experimental Economics*, 10, 171–178.

Forsythe, R., T. R. Palfrey, and C. R. Plott (1982): "Asset valuation in an experimental market," *Econometrica*, 50, 537–567.

Friedman, D., S. Huck, R. Oprea, and S. Weidenholzer (2015): "From imitation to collusion: Long-run learning in a low-information environment," *Journal of Economic Theory*, 155, 185–205.

Friedman, D. and J. Ostroy (1995): "Competitivity in Auction Markets: An Experimental and Theoretical Investigation," *The Economic Journal*, 105, 22.

Green, J. R. (1974): "The Stability of Edgeworth's Recontracting Process," *Econometrica*, 42, 21.

Greiner, B. (2015): "Subject pool recruitment procedures: organizing experiments with ORSEE," *Journal of the Economic Science Association*, 1, 114–125.

Hahn, F. H. and T. Negishi (1962): "A Theorem on Non-Tatonnement Stability," *Econometrica*, 30, 463.

Hayek, F. A. (1945): "The use of knowledge in society," *The American economic review*, 35, 519–530.

Huck, S., J. Leutgeb, and R. Oprea (2017): "Payoff information hampers the evolution of cooperation," *Nature Communications*, 8, 1–5.

Huck, S., H.-T. Normann, and J. Oechssler (1999): "Learning in Cournot oligopoly–An experiment," *The Economic Journal*, 109, 80–95.

Ketcham, J., V. L. Smith, and A. W. Williams (1984): "A Comparison of Posted-Offer and Double-Auction Pricing Institutions," *The Review of Economic Studies*, 51, 595–614.

Kreps, D. M. and J. A. Scheinkman (1983): "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes," *The Bell Journal of Economics*, 14, 326–337.

Kruse, J. B. (1993): "Nash Equilibrium and Buyer Rationing Rules: Experimental Evidence," *Economic Inquiry*, 31, 631–646.

——— (2008): "Simulated and Real Buyers in Posted Offer Markets," in *Handbook of Experimental Economics Results*, ed. by C. R. Plott and V. L. Smith, Amsterdam: Elsevier, 71–76.

Lepore, J. J. and A. P. Shafran (2013): "Consumer Rationing and Cournot Outcomes: Experimental Evidence," *Southern Economic Journal*, 79, 727–746.

Levitan, R. and M. Shubik (1972): "Price Duopoly and Capacity Constraints," *International Economic Review*, 13, 111.

Linde, J., J. Sonnemans, and J. Tuinstra (2014): "Strategies and evolution in the minority game: A multi-round strategy experiment," *Games and economic behavior*, 86, 77–95.

Marshall, A. (1961): *Principles of Economics*, Principles of Economics, New York:

Macmillan, 8th ed.

McCabe, K. A., S. Rassenti, and V. L. Smith (1984): "Designing a Uniform-Price Double Auction: an Experimental Evaluation," in *The Double Auction Market: Institutions, Theories, and Evidence*, ed. by D. Friedman and J. Rust, Reading, MA.

Mestelman, S. and D. Welland (1988): "Advance Production in Experimental Markets," *The Review of Economic Studies*, 55, 641–654.

Pangallo, M., T. Heinrich, and J. D. Farmer (2019): "Best reply structure and equilibrium convergence in generic games," *Science Advances*, 5, eaat1328.

Plott, C., N. Roy, and B. Tong (2013): "Marshall and Walras, disequilibrium trades and the dynamics of equilibration in the continuous double auction market," *Journal of Economic Behavior and Organization*, 94, 190–205.

Plott, C. R. and K. Pogorelskiy (2017): "Call Market Experiments: Efficiency and Price Discovery through Multiple Calls and Emergent Newton Adjustments," *American Economic Journal: Microeconomics*, 9, 1–41.

Plott, C. R. and V. L. Smith (1978): "An Experimental Examination of Two Exchange Institutions," *The Review of Economic Studies*, 45, 133–153.

Plott, C. R. a. (1982): "Industrial Organization Theory and Experimental Economics," *Journal of Economic Literature*, 20, 1485–1527.

Rapoport, A. (1995): "Individual Strategies in a Market Entry Game," *Group Decision and Negotiation*, 4, 117–133.

Romero, J. and Y. Rosokha (2019): "The evolution of cooperation: The role of costly strategy adjustments," *American Economic Journal: Microeconomics*, 11, 299–328.

Ronayne, D. and D. P. Myatt (2019): "A Theory of Stable Price Dispersion," Economics Series Working Papers 873, University of Oxford, Department of Economics.

Ruffle, B. J. (2000): "Some factors affecting demand withholding in posted-offer markets," *Economic Theory*, 16, 529–544.

Selten, R. (1975): "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4, 25–55.

Selten, R. and W. Güth (1982): "Equilibrium Point Selection in a Class of Market Entry Games," in *Games, Economic Dynamics, and Time Series Analysis*, Heidelberg: Physica-Verlag HD, 101–116.

Smith, A. (1776): *An Inquiry into the Nature and Causes of the Wealth of Nations*, McMaster University Archive for the History of Economic Thought.

Smith, V. and A. W. Williams (1989): "The Boundaries of Competitive Price Theory: Convergence, Expectations, and Transactions Costs," in *Advances in Behavioral*

Economics, ed. by L. Green and J. H. Kagel, Norwood, N.J.

Smith, V. L. (1962): "An experimental study of competitive market behavior," *Journal of Political Economy*, 70, 111–137.

——— (1976): "Experimental economics: Induced value theory," *The American Economic Review, Papers and Proceedings*, 66, 274–279.

Sundali, J. A., A. Rapoport, and D. A. Seale (1995): "Coordination in Market Entry Games with Symmetric Players," *Organizational Behavior and Human Processes*, 64, 203–218.

Vives, X. (1986): "Rationing rules and Bertrand-Edgeworth equilibria in large markets," *Economics Letters*, 21, 113–116.

APPENDIX A: ADDITIONAL ANALYSIS (FOR ONLINE PUBLICATION)

A.1. *Additional Background*

A.1.1. *Market Equilibration*

Appreciation of the problem of equilibration, and in particular of out-of-equilibrium trades, dates back at least to the 19th century. Different resolutions include: Walras' employment of an institution, *tâtonnement*, which determines prices centrally and precludes out-of-equilibrium trade; Green's (1974) Edgeworth-inspired employment of a coalition formation process which operates on a decentralized basis and which precludes out-of-equilibrium trade; and Marshall's (1961) suggestion as to how a sequence of trades, pairing the buyer with the highest outstanding value with the seller with the lowest outstanding cost, might eventually generate a price at the crossing of supply and demand, even if all trades except the ultimate, marginal trade take place at prices other than the competitive equilibrium price.

In characterizing behavior in the double auction, Plott et al. (2013) examine whether Marshall's suggested trajectory of trades—here and elsewhere referred to as Marshallian path adjustment—might be an emergent property of the double auction. That is, for reasons not yet fully understood, trades within the double auction might, at any point in time, have a tendency to match the highest value buyer and lowest cost seller. Plott et al. report data suggestive of such a pattern of trades.

In a follow-up study, Plott and Pogorelskiy (2017) examine price dynamics in a call market with two calls per trading period.[28] While that paper tries to shed light on conjectures about possible Walrasian or Newtonian dynamic influences on bid or ask adjustments on either side of the calls, at any given

[28] The institution implemented by Plott and Pogorelskiy (2017) is related to the uniform-price double auction (McCabe et al., 1984).

call the mechanics of trade follow Marshallian path adjustment. That is, at each call bids and asks are paired by the experimenter until no further pairs with bid exceeding ask remain; the midpoint of the closest (and last) such bid-ask pair becomes the uniform clearing price for all transacting pairs (see pp. 7, 8, and 14). Thus Walrasian or other adjustment *across* calls can only take place given Marshallian mechanics have been implemented at *each* call.

The observation of endogenous Marshallian path adjustment within the double auction is a welcome clue about the generally reliable equilibration in that institution. Furthermore, both such emergent behavior and the implicit construction of Marshallian path adjustment that takes place as part of the regular operation of a call market constitute evidence that such adjustment may be a survival-positive trait for evolved institutions used in commercial settings.

Further suggestion of the possible importance of Marshallian path adjustment is found in Collins, James, Servátka, and Woods (2017), hereafter, CJSW. CJSW compare equilibration across two forms of market that share payoff equivalence given competitive pricing: the posted offer market with advance production environment (POAP), and a uniform ex-post market-clearing price, centrally administered. Both markets are implemented in the same advance production environment. Furthermore, the pairing of the uniform *ex post* market clearing price institution with an advance production environment is isomorphic to the market entry game (Selten and Güth, 1982). Implementing value order queueing of buyers and a flat supply curve in POAP; CJSW enforce a sequence of trades that satisfy Marshallian path adjustment, as is further discussed in the next section.[29] CJSW find that

[29]The uniform *ex post* market clearing price institution can also be argued not to violate Marshallian path adjustment. To see this, consider a comparison with the quantities-only clearing-house (CHQ) introduced by Friedman and Ostroy (1995). CHQ works by accepting quantity messages from agents and imputing limit prices for those agents from

while the uniform price institution generates cycling prices that may not converge to the competitive equilibrium, the POAP converges to competitive equilibrium. This happens despite individually posted offers supporting additional equilibria to those possible given the uniform, centrally administered ex-post clearing price (one that is implicitly embedded in the market entry game).

A crucial next step for this literature is to ask: "What happens to equilibration if one switches Marshallian path adjustment off"? Since one can modularly switch out the buyer queueing rule in the posted offer market, the posted offer is a natural venue in which to address this question. Results under a value order queueing, Marshallian path adjustment treatment may then be compared with data resulting when an alternative form of buyer queueing—say, random order—is used instead. Is Marshallian path adjustment critical to attainment of the competitive equilibrium allocation in this case? The theory and results sections of this present paper will address these questions in terms of predictions and empirical results, respectively.

We next review how queuing order, which can be used to enforce Marshallian path adjustment, and degree of determinism in buyer behavior, have been dealt with in the literature.

A.1.2. *Buyer Queuing*

The buyer queueing rule in effect in a given market can radically change the predictions across markets that otherwise are identically parameterized, and which further have in common the use of a particular trading institution (e.g. posted offer). For instance, in the duopoly case, if one or both

their actual cost or valuation parameters. CJSW's uniform price market (the market entry game) treats demand equivalently. Further, supply in CJSW's uniform price market can be viewed as equivalent to a (single-unit-per-producer) CHQ if producers (entrants) are held to have submitted a "quantity equals one" message at that juncture; their marginal cost, being sunk, is imputed as equal to zero.

firms have capacity sufficient to serve only a "small" portion of the demand curve, or if the residual demand curve faced by the higher-pricing firm is not constructed by first removing the highest valuation buyers from the overall market demand curve, then there is scope for pricing above marginal cost even in the presence of Bertrand price competition.

The above described line of inquiry was initiated by Edgeworth (1925), focusing on the role of firms' production capacity, which for each firm was held to be insufficient to supply the entire market and also fixed, or at least less easily changed than prices. The work of Beckman (1965) and Levitan and Shubik (1972) brings focus onto the role of buyer queueing, as a critical determinant of residual demand. These latter works specify the mechanics of value order buyer queueing in a duopoly. Algebraically, for total demand curve $Q = a - p$, and (symmetric) firm capacity k, where $k \leq a$, quantity demanded will be split across firms according to

$$(1) \qquad Q_i(p_i|p_j) = \begin{cases} a - p_i, & \text{if } p_i < p_j, \\ \frac{1}{2}(a - p_i), & \text{if } p_i = p_j, \\ a - k - p_i, & \text{if } p_i > p_j. \end{cases}$$

Operationally, the highest valuation buyer gets to choose first, and buys from the lower priced firm first; the higher priced firm can only receive business once the lower priced firm has sold all its units.[30] We shall refer to the generalized, m-seller version of this method of buyer queueing as *value order queueing*. (This is the means by which the Marshallian adjustment path can be exogenously enforced—at least where buyers are deterministically programmed to maximize their own point-in-time surplus.)

[30] Levitan and Shubik (1972) make special mention of the importance of the choice of buyer queueing rule in obtaining particular equilibria, and also acknowledge that, "The actual shape of contingent demand cannot be specified generally from a priori reasoning. It will depend upon priorities in service of customers and…needs specific empirical investigation and model building" (pg. 119).

Vives (1986) finds that given sufficiently large aggregate capacity and number of firms, value order queueing ("surplus maximizing rationing" in his terminology), can lead to attainment of the competitive price. While our parameterization is directly linked to that of the market entry game rather than Vives' setting, in deriving our theoretical predictions in the next section, we too find that the competitive equilibrium is a possible outcome under value order queuing. Which of the possible outcomes, of which the competitive equilibrium is just one, eventuates is of course an empirical question, addressed in our results section.[31]

Queueing of market participants arises in the experimental literature, too. Chamberlin (1948) provides a very early example of an economics experiment. In setting up a real time market, Chamberlin, like all subsequent experimenters, had to make, by commission, omission and/or default, decisions regarding market and experimental design. Ultimately he allows for unstructured and unregulated markets in which buyers and sellers negotiate prices while freely moving around a room. Chamberlin finds that this "random meeting" of buyers and sellers generates systematic deviations from competitive equilibrium. In the experimental literature on the posted offer, random order buyer queueing has been the default; for an early example see Ketcham, Smith, and Williams (1984, pg. 598), with Davis and Williams (1991) and other papers cited in our reference section being just a small sampling of subsequent experimental posted offer studies using random order queueing.

Much less has been done with value order queueing in experiments. To our

[31]We are first and foremost interested in value order queueing as a means to exogenously enforce Marshallian path adjustment in order to document its effect on equilibration. However, let us also point out that value order queueing may be an emergent phenomenon in naturally-occurring settings in its own right, for example, in the operation of call markets.

knowledge the only experimental studies with posted offers that implement both value order queueing and queues incorporating randomness in ordering do so in duopoly. Kruse (1993) examines variation of buyer queueing rule as a treatment in experimental posted offer duopoly markets, and finds higher prices and greater price dispersion under random order queueing. Lepore and Shafran (2013) use proportional queueing instead of random order and find qualitatively similar results to Kruse.[32] However the parameters in these prior studies—understandably, given the focus on duopoly—do not support the competitive equilibrium as a pure strategy Nash Equilibrium (ours do). Thus the kind of comparison we seek in this paper was precluded by the design of that earlier work.

In Table A1, we summarize the experimental literature employing the posted offer by its queuing rule.

A.1.3. *Robustness to Variation in Buyer Determinism*

The buyers' task in a posted offer market is simple in operation: they face only a binary decision on whether or not to buy (a single unit) from among the offers remaining at the time of their choice opportunity. But this still allows substantial complication in buyer behavior in practice. Compare human buyers to "demand revealing" robot buyers.[33] Trivially, human buyers

[32]Related theoretical studies are as follows. In a duopoly setting, Kreps and Scheinkman (1983) derive Cournot pricing, despite value order queuing, given a particular timeline and structure of firms' capacity choices. Also in a duopoly setting, Davidson and Deneckere (1986) find that replacing value order queueing with alternatives such as proportional queueing rule out the Cournot result obtained by Kreps and Scheinkman (1983), finding instead a wider range of prices, and lower profits resulting from unsold goods.

[33]In this context, demand revealing means that a buyer will attempt to maximize surplus at each point in time with no attempt to influence the future, and will not reject an offer as long as it offers non-negative surplus and buying capacity has not yet been exhausted.

Table A1: Summary of Experimental Posted Offer Literature by Queuing Rule

Exogenous random order	First come first served (Endogenous random order)	Proportional rationing	Value order (Efficient rationing)
Buchheit and Feltovich (2011)	Helland, Moen and Preugschat (2017)	Jacobs and Requate (2016)	Davis (2003)
Isaac and Smith (1985)		Lepore and Shafran (2013)	Davis, Holt and Villamil (1990)
Millner, Pratt and Reilly (1990)			Goodwin and Mestelman (2010)
Ruffle (2000)			Jacobs and Requate (2016)
			Kruse (1993)
			Lepore and Shafran (2013)
Borck, Engelmann, Müller and Normann (2002)			Davis, Holt and Villamil (1990)
Cason and Williams (1990)			Davis and Korenok (2009)
Cason, Friedman and Milam (2003)			Davis and Wilson (2000)
Cason, Gangadharan and Nikiforakis (2011)			Collins, James, Servátka, and Woods (2017)
Coursey and Smith (1983)			Collins, James, Servátka, and Vadovič
Davis and Williams (1986)			
Davis and Williams (1991)			
Davis and Wilson (2008)			
Deck and Wilson (2003)			
Deck and Wilson (2006)			
Holt and Sherman (1990)			
Isaac and Reynolds (1992)			
Ketcham, Smith and Williams (1984)			
Plott and Smith (1978)			
Collins, James, Servátka, and Vadovič			

can make errors. Perhaps more interestingly, human buyers might withhold demand (refusing to buy, even when doing so generates positive surplus for them) in an effort to negotiate with sellers (albeit across periods). That is, by refusing to buy for a "small" profit in a given round, a buyer might influence sellers to lower their prices in subsequent rounds, potentially allowing for a net increase in buyer surplus over the entirety of their interaction.

The possibility of this kind of buyer behavior is addressed in Cason and Williams (1990), Davis and Williams (1991), Ruffle (2000), and Davis and Wilson (2008), who find evidence suggestive of demand withholding by human buyers in the posted offer markets they present. The meta-analysis over Coursey, Isaac, and Smith (1984b), Coursey, Isaac, Luke, and Smith (1984a), and Brown-Kruse (1991) conducted by Kruse (2008) compares deviations in pricing from long-run competitive equilibrium (i.e. minimum of average total cost) between sessions using robot buyers and those using human buyers. Kruse finds that deviations are lower in the presence of human buyers, and that this is not inconsistent with tougher negotiation on the part of human buyers—or at least that the threat thereof is taken into account by sellers.

Since our design crosses "human buyer versus robot buyer" and "random order queueing versus value order queueing", we are able to gauge relative magnitudes of any effect each treatment might have, holding constant all other aspects of the experimental design. Most important, we can check robustness: for instance, are results on the effect of queuing on equilibration robust to variation in buyer determinism?

A.2. *Additional Data Analysis*

A.2.1. *Buyer and Seller Earning Distributions*

Figure A1 illustrates the distribution of raw, unadjusted earnings in experimental currency units (ECUs) for sellers and buyers in the experiment. Subjects were paid at a rate of exchange of 5 NZD per ECU, in addition to a 5 NZD show-up payment and 5 NZD deposit. The 6.25% (5 of 80) of sellers who experienced negative raw earnings of less than one ECU (to the left of the dashed line) forfeited their deposit and received only a 5 NZD show-up payment. The histogram includes the 0.1 ECU commision paid to buyers for each purchase, with final payments rounded up (unannounced) to the nearest whole amount in NZD.

Sellers receive a payoff of 1 ECU per round in all competitive equilibria wherein pricing is as off the demand curve. (See Table C1 for a full enumeration of PSNE payoffs.) In this case, sellers would be paid at a rate of 5 NZD per ECU for six periods, earning a total of 30 NZD in addition to the 5 NZD show-up fee and 5 NZD deposit, for a total of 40 NZD. In realized payoffs, subjects earned an average of just under 37.94 NZD.

A.2.2. *Additional Graphical Illustration and Overview of Results*

Figure 1 graphically illustrates production choices, asking prices, quantity sold, and efficiency from the stationary marginal cost periods of the experiment for the second session of each treatment. Here, in each of sixteen Figures A2 through A17, we extend the analysis described in subsection 4.1 by presenting for each session a figure consisting of four panels. Each panel contains a temporarily ordered sequence of trading periods data specific to a particular value of marginal cost.

We note the following. The value order sessions for both human and robot buyers exhibit tendency to converge during the stationary marginal

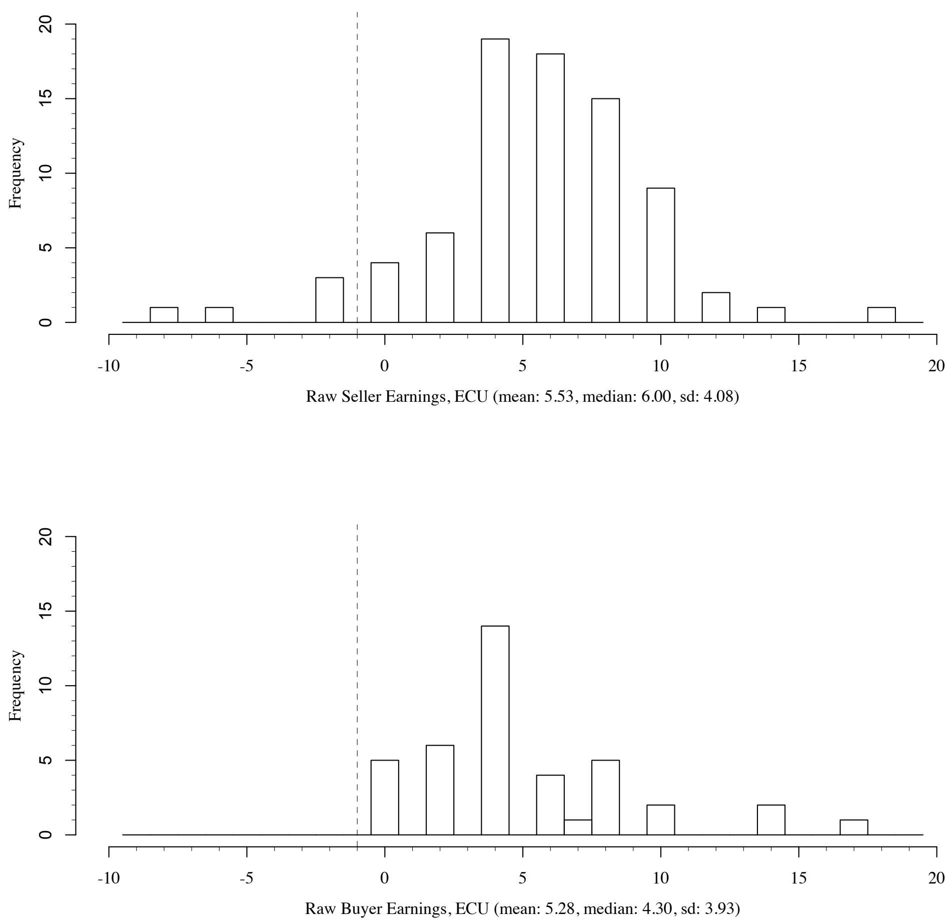

Figure A1: Distribution of Raw Earnings in ECUs for Buyers and Sellers

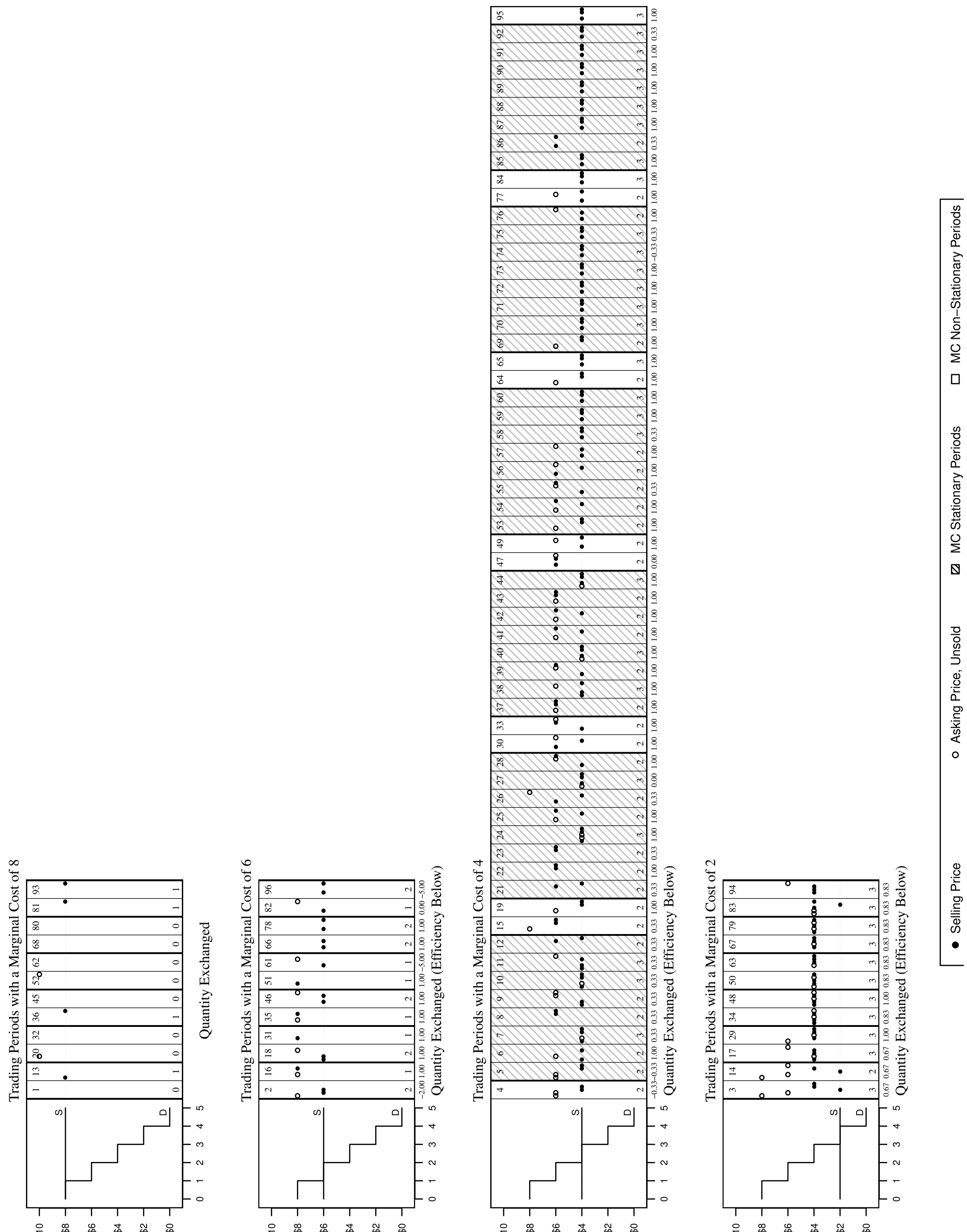

Figure A2: Asks, Prices, and Quantities in Session 1 of Value Order Robot Buyers (VORB) Treatment

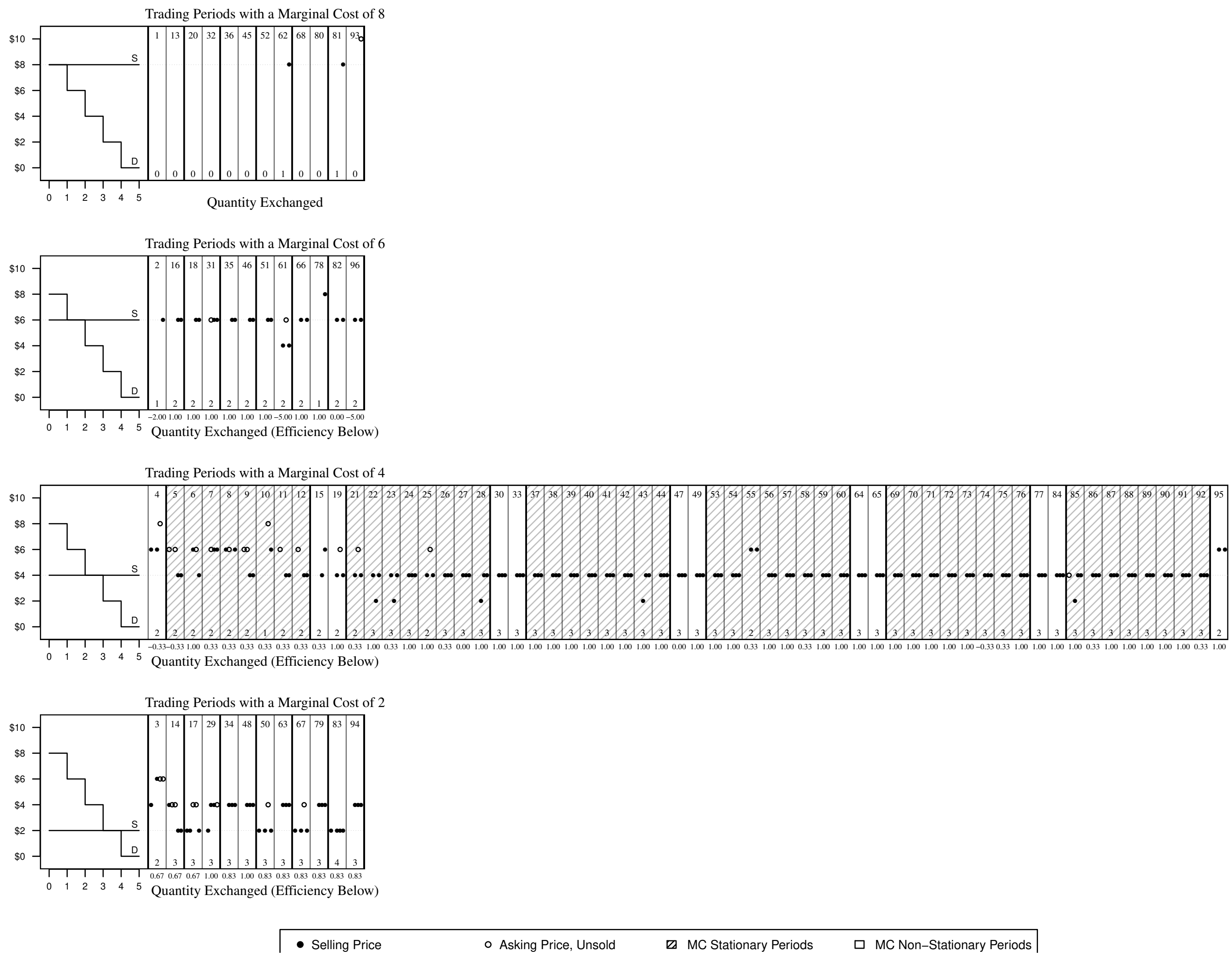

Figure A3: Asks, Prices, and Quantities in Session 2 of Value Order Robot Buyers (VORB) Treatment

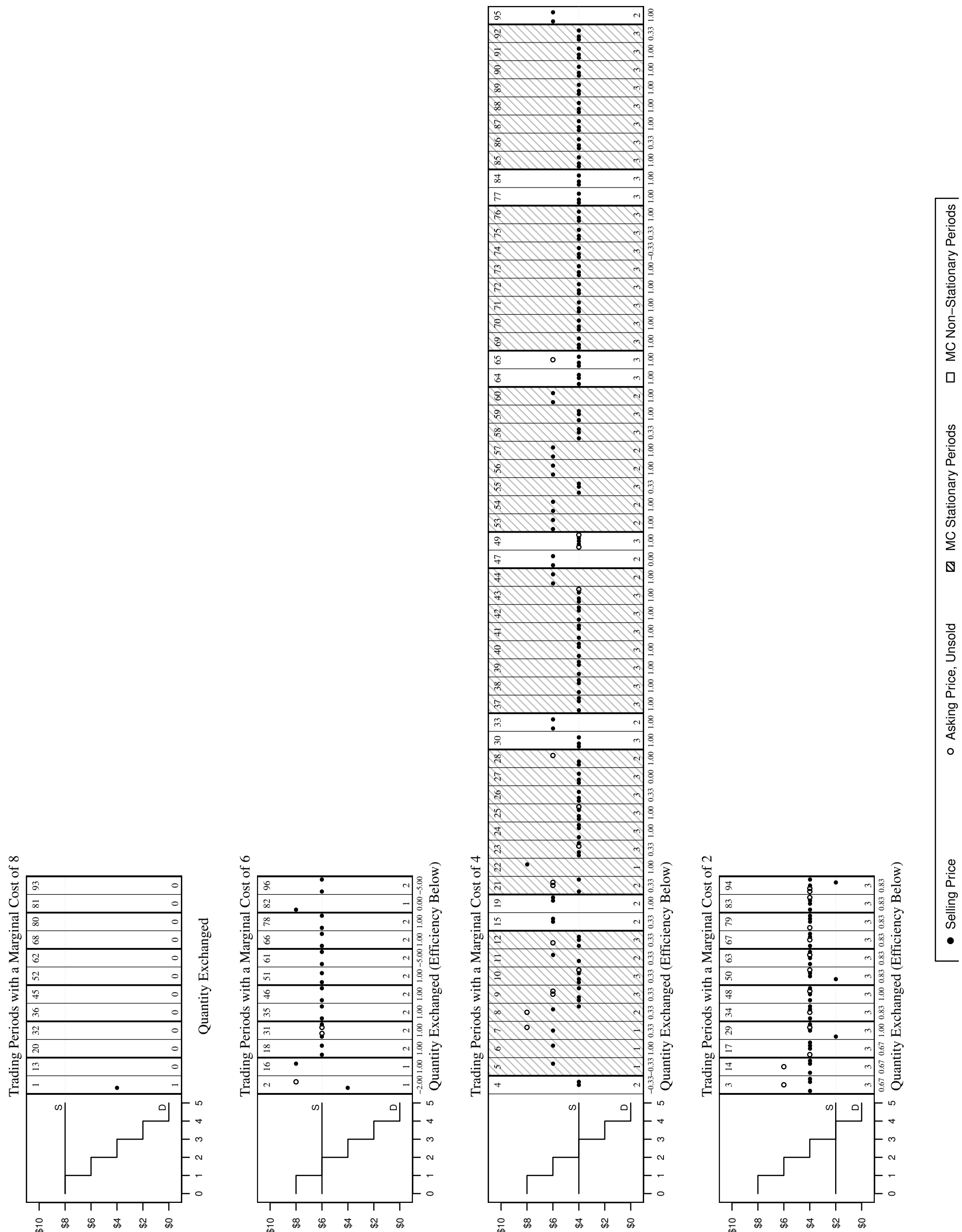

Figure A4: Asks, Prices, and Quantities in Session 3 of Value Order Robot Buyers (VORB) Treatment

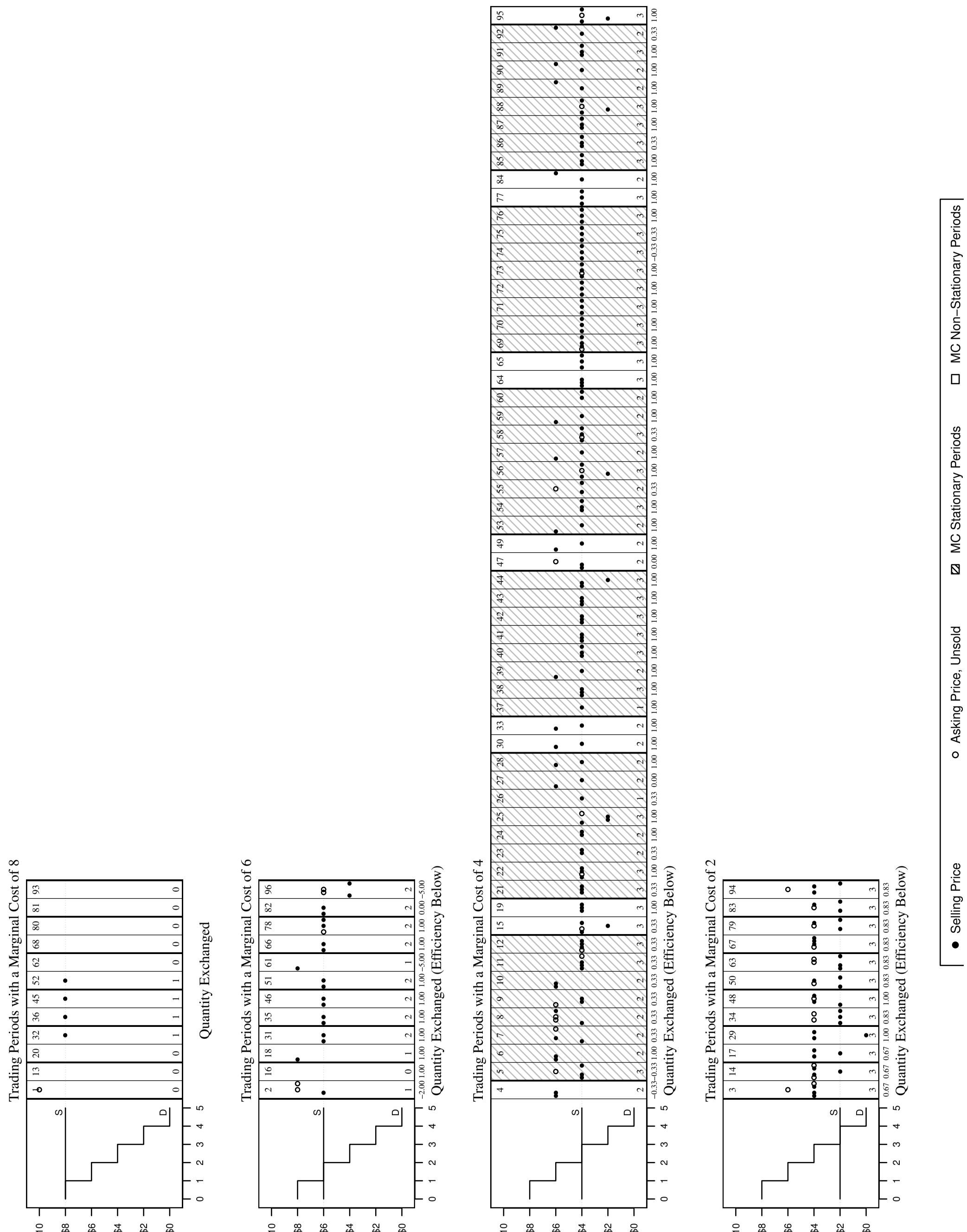

Figure A5: Asks, Prices, and Quantities in Session 4 of Value Order Robot Buyers (VORB) Treatment

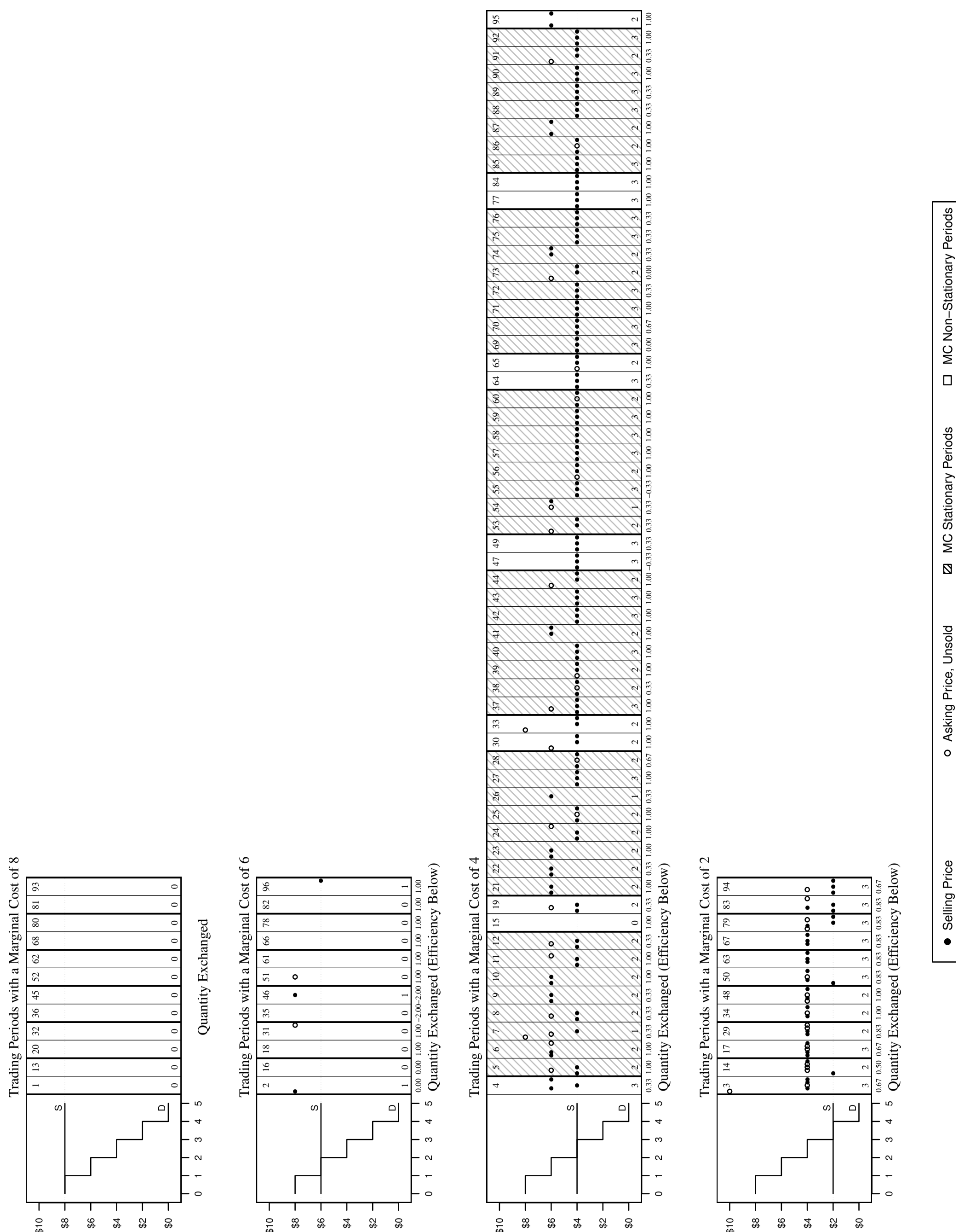

Figure A6: Asks, Prices, and Quantities in Session 1 of Value Order Human Buyers (VOHB) Treatment

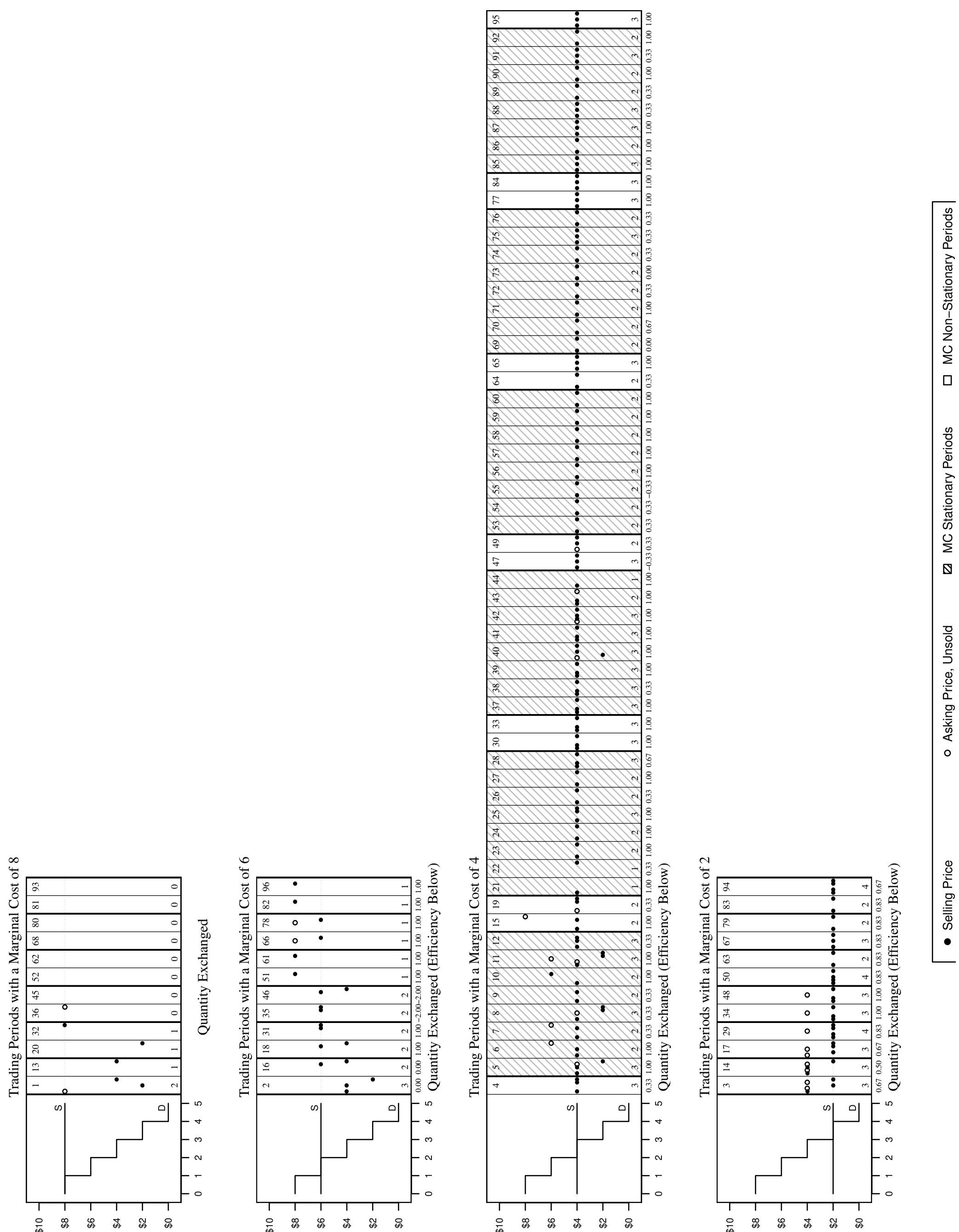

Figure A7: Asks, Prices, and Quantities in Session 2 of Value Order Human Buyers (VOHB) Treatment

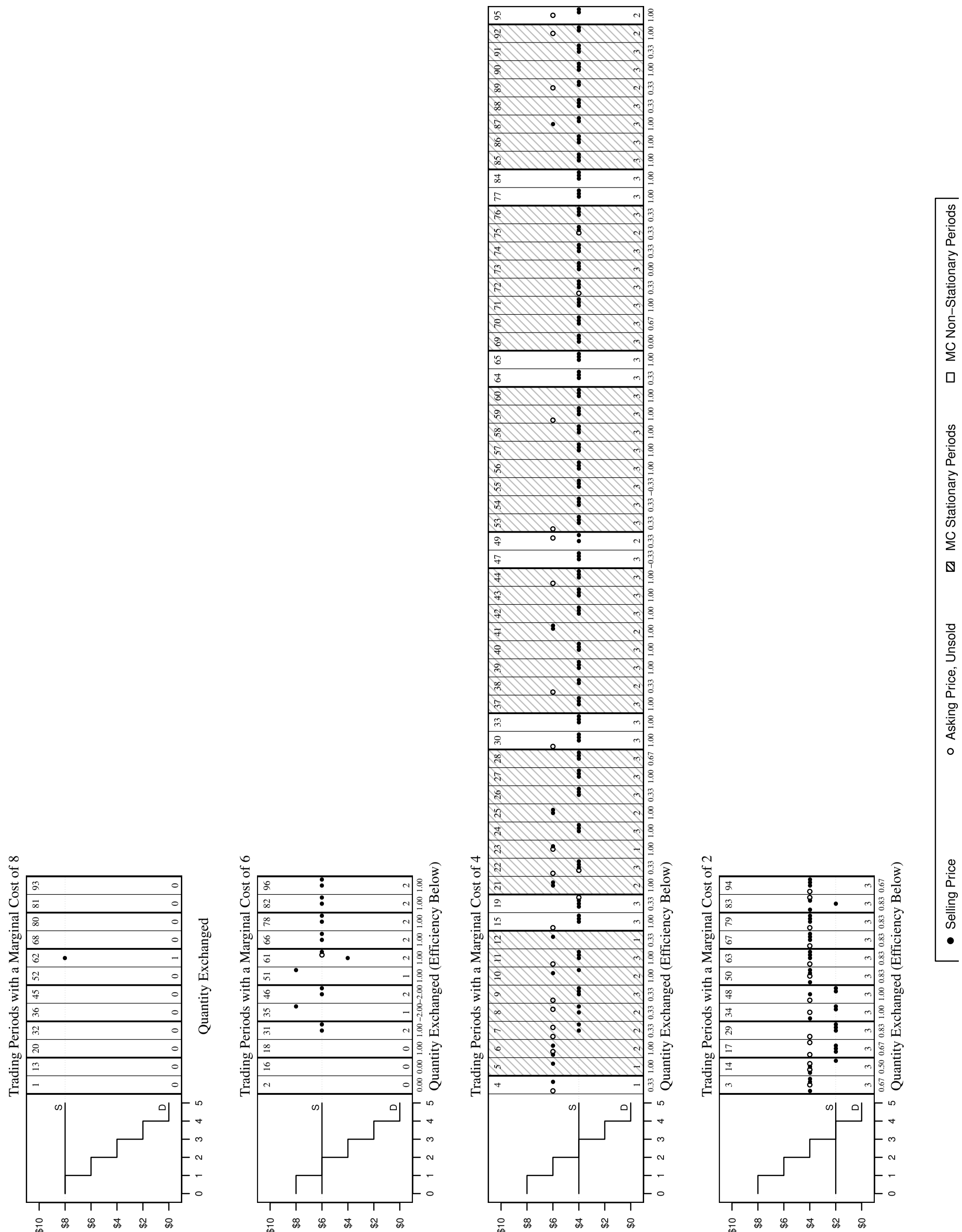

Figure A8: Asks, Prices, and Quantities in Session 3 of Value Order Human Buyers (VOHB) Treatment

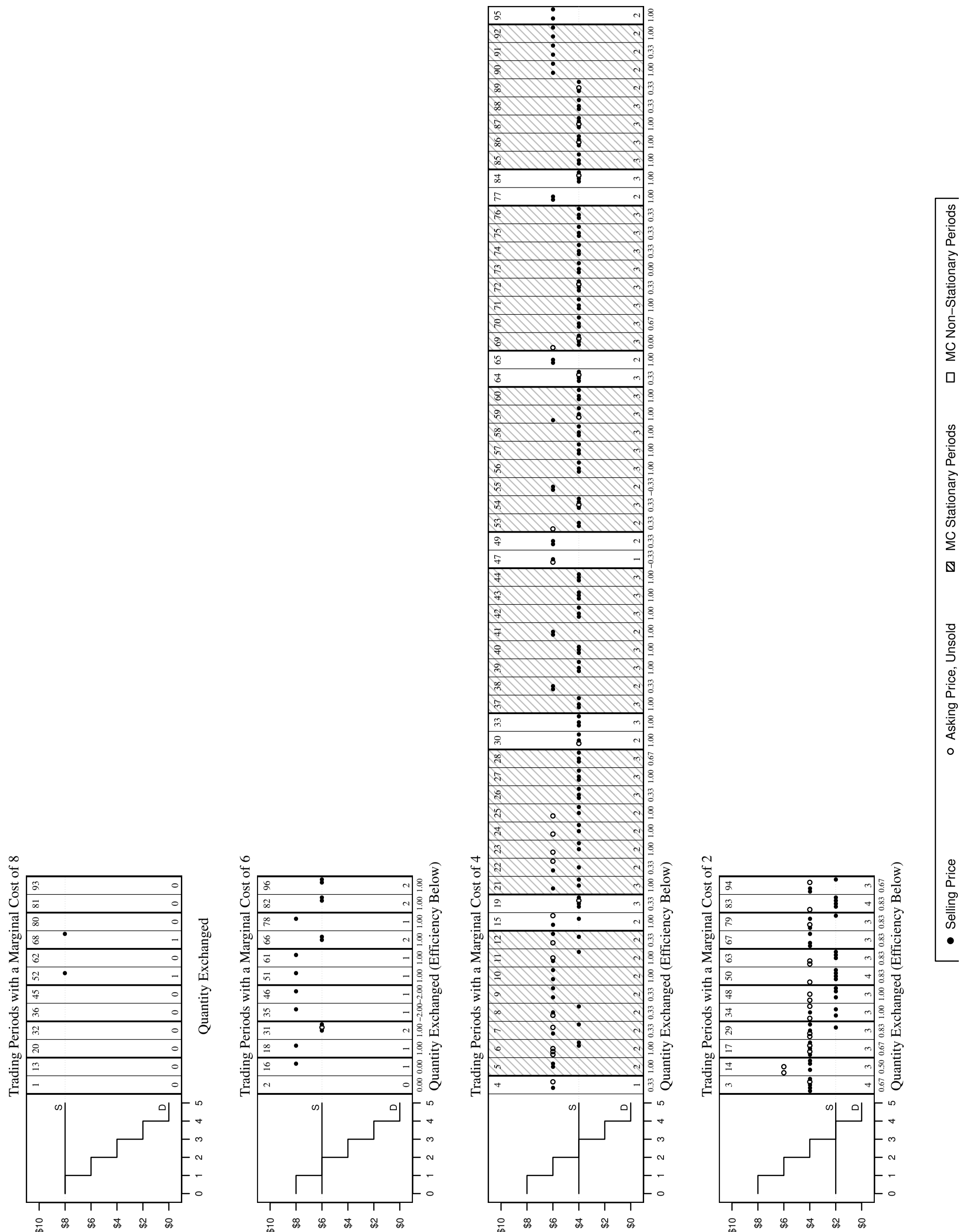

Figure A9: Asks, Prices, and Quantities in Session 4 of Value Order Human Buyers (VOHB) Treatment

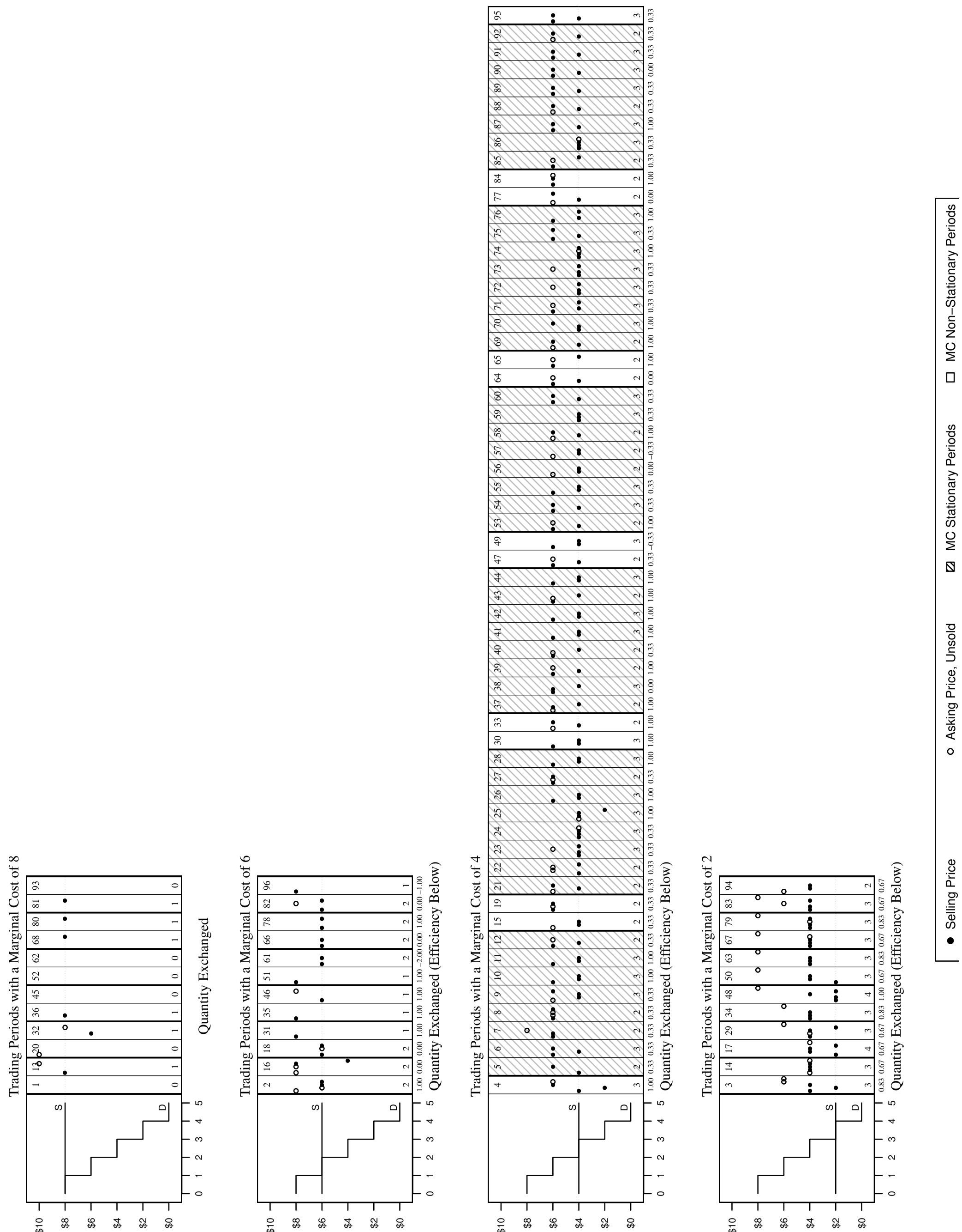

Figure A10: Asks, Prices, and Quantities in Session 1 of Random Order Robot Buyers (RORB) Treatment

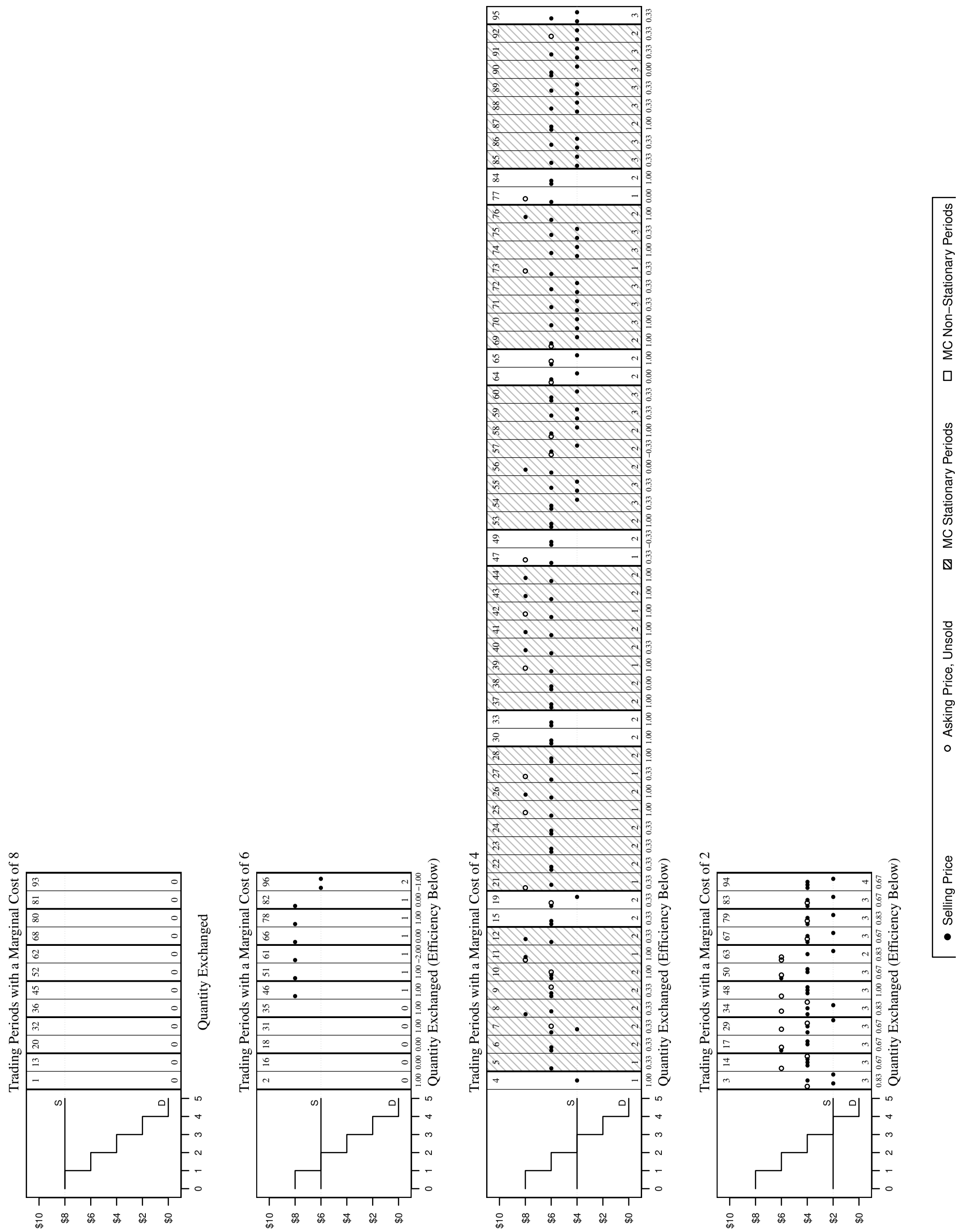

Figure A11: Asks, Prices, and Quantities in Session 2 of Random Order Robot Buyers (RORB) Treatment

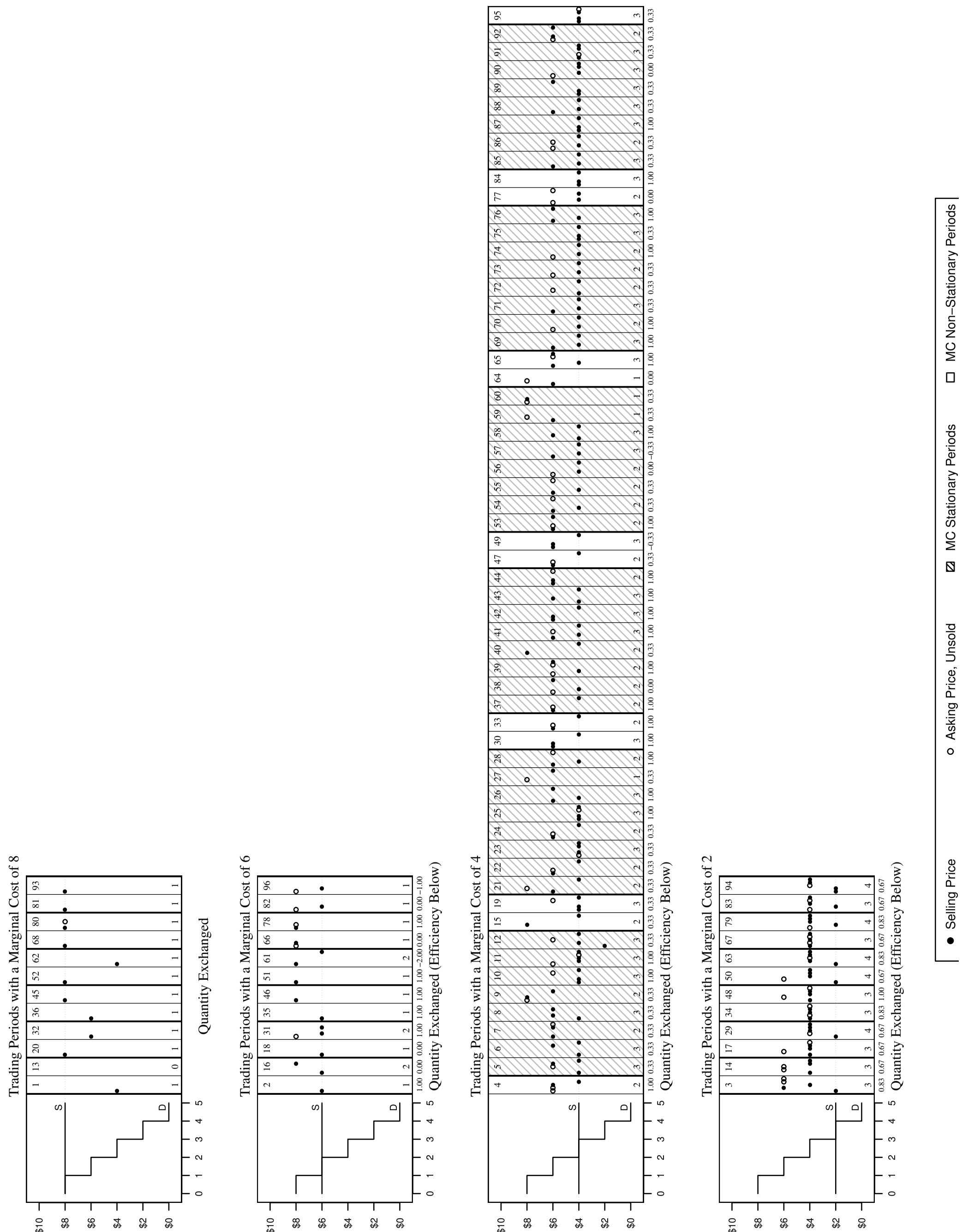

Figure A12: Asks, Prices, and Quantities in Session 3 of Random Order Robot Buyers (RORB) Treatment

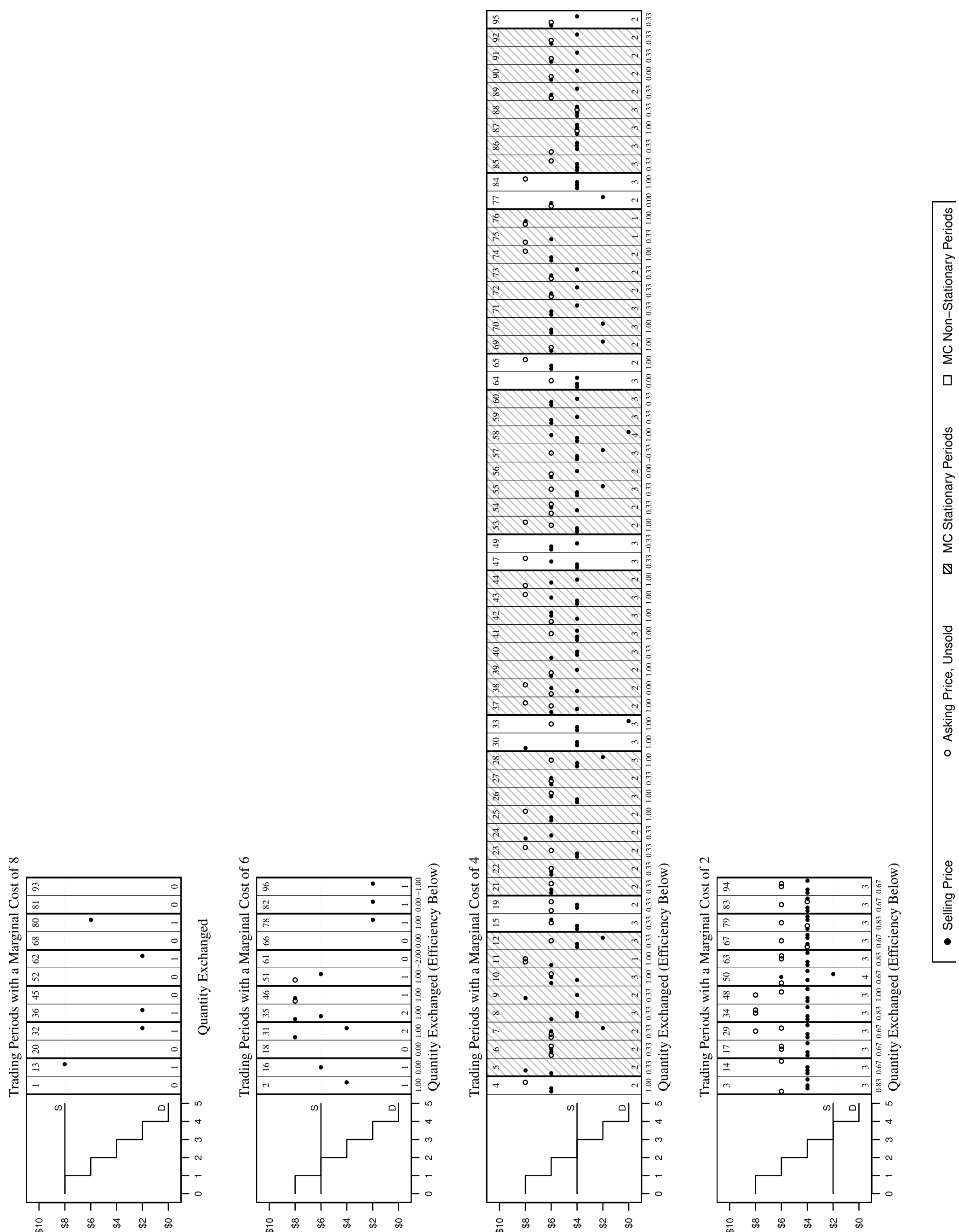

Figure A13: Asks, Prices, and Quantities in Session 4 of Random Order Robot Buyers (RORB) Treatment

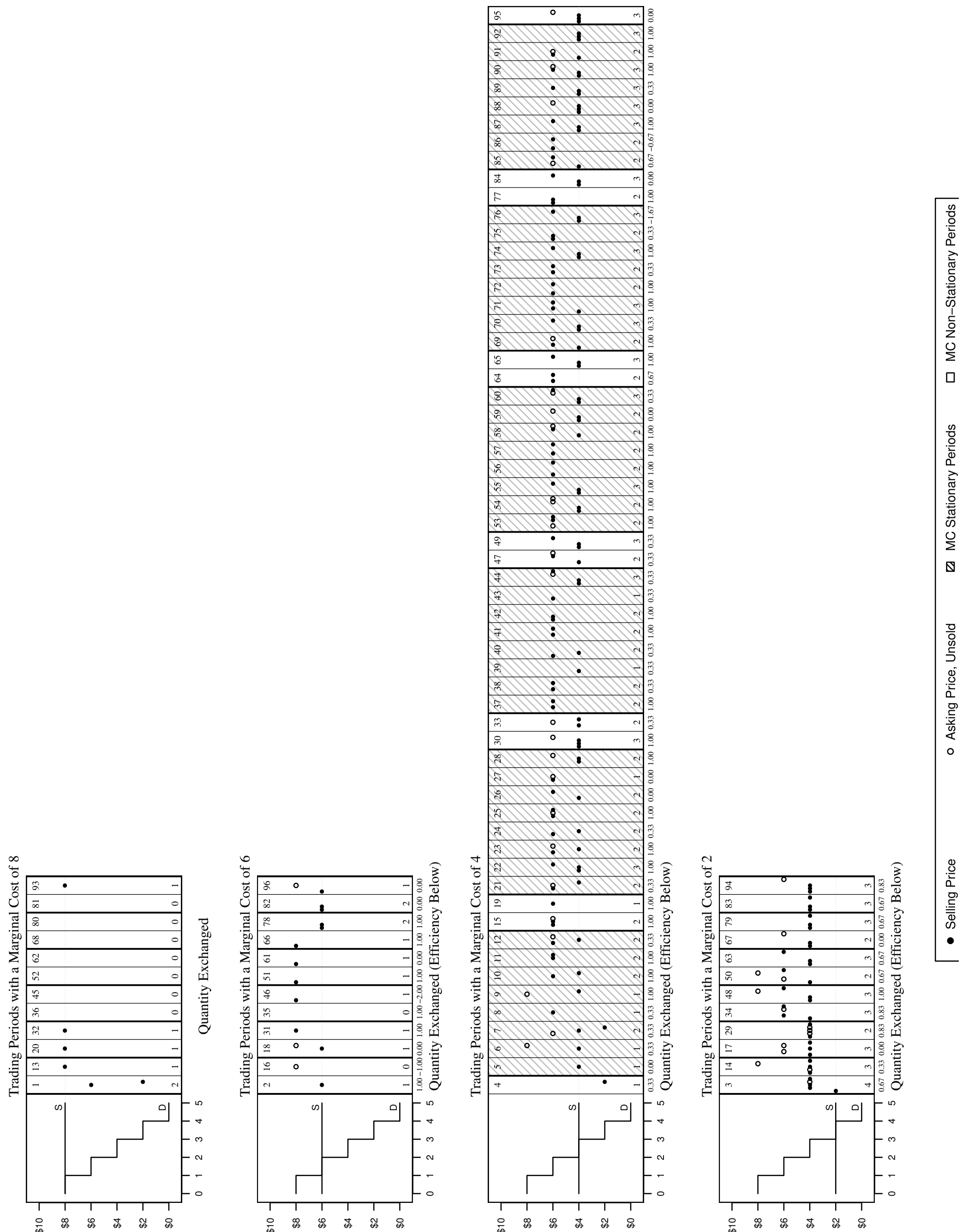

Figure A14: Asks, Prices, and Quantities in Session 1 of Random Order Human Buyers (ROHB) Treatment

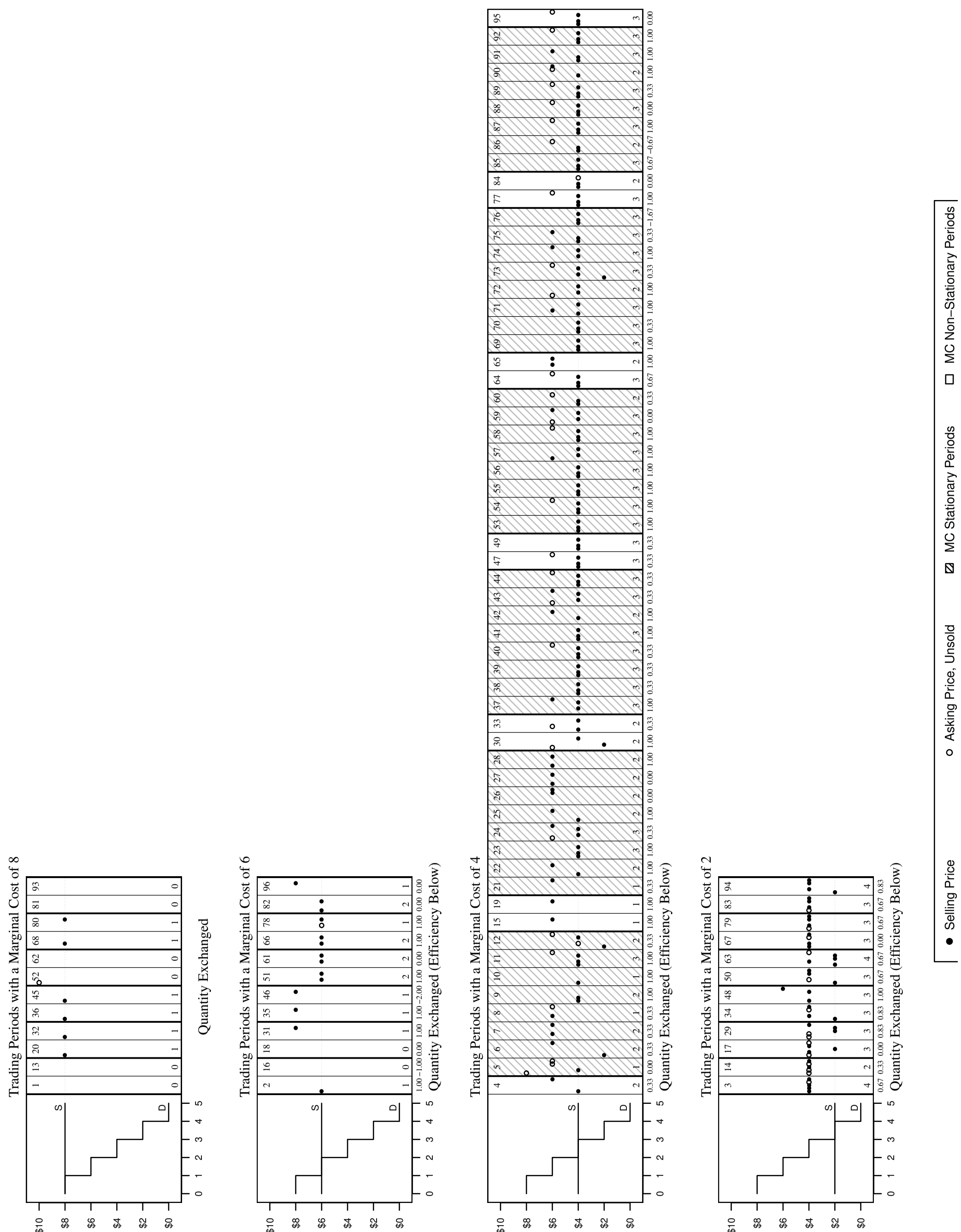

Figure A15: Asks, Prices, and Quantities in Session 2 of Random Order Human Buyers (ROHB) Treatment

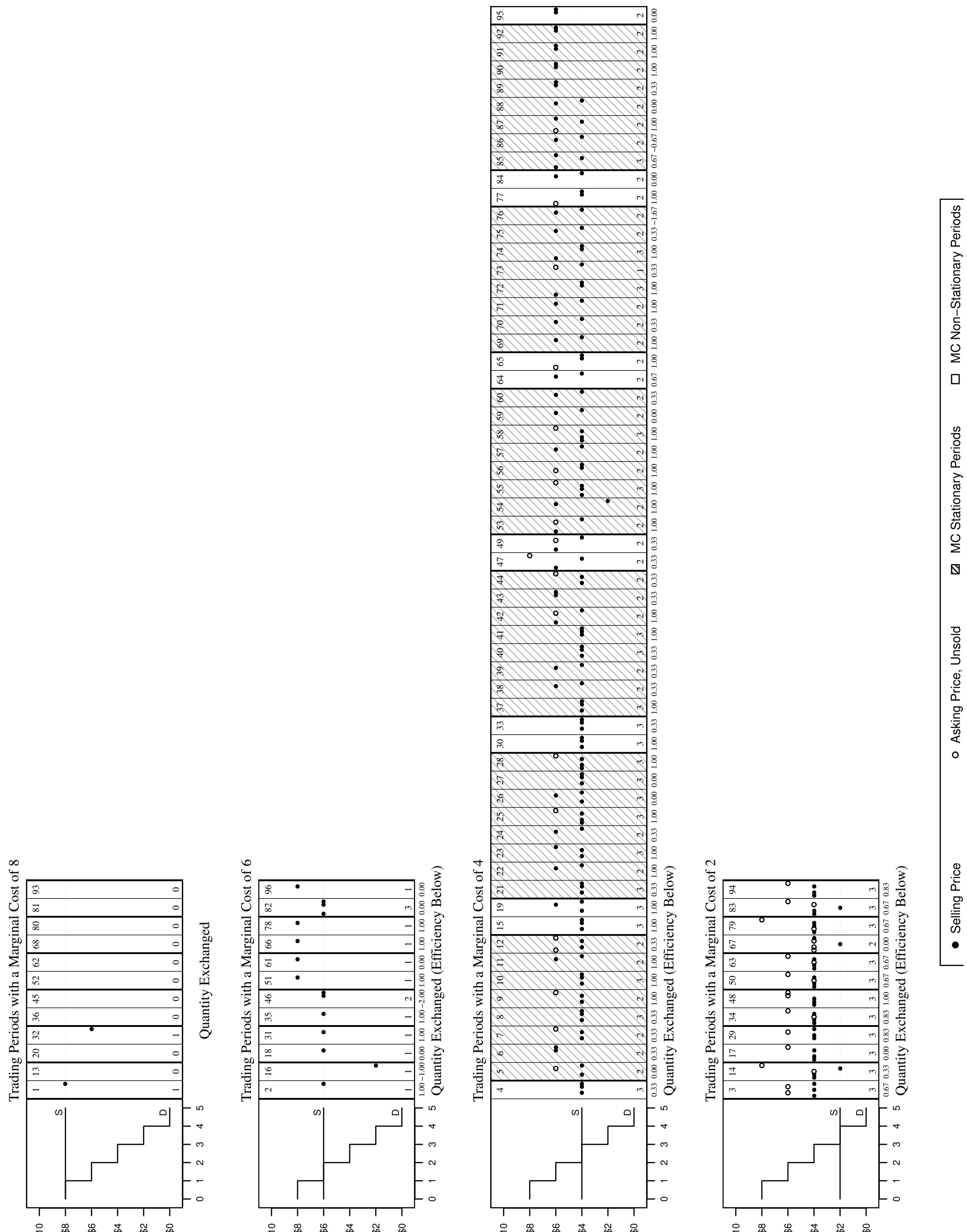

Figure A16: Asks, Prices, and Quantities in Session 3 of Random Order Human Buyers (ROHB) Treatment

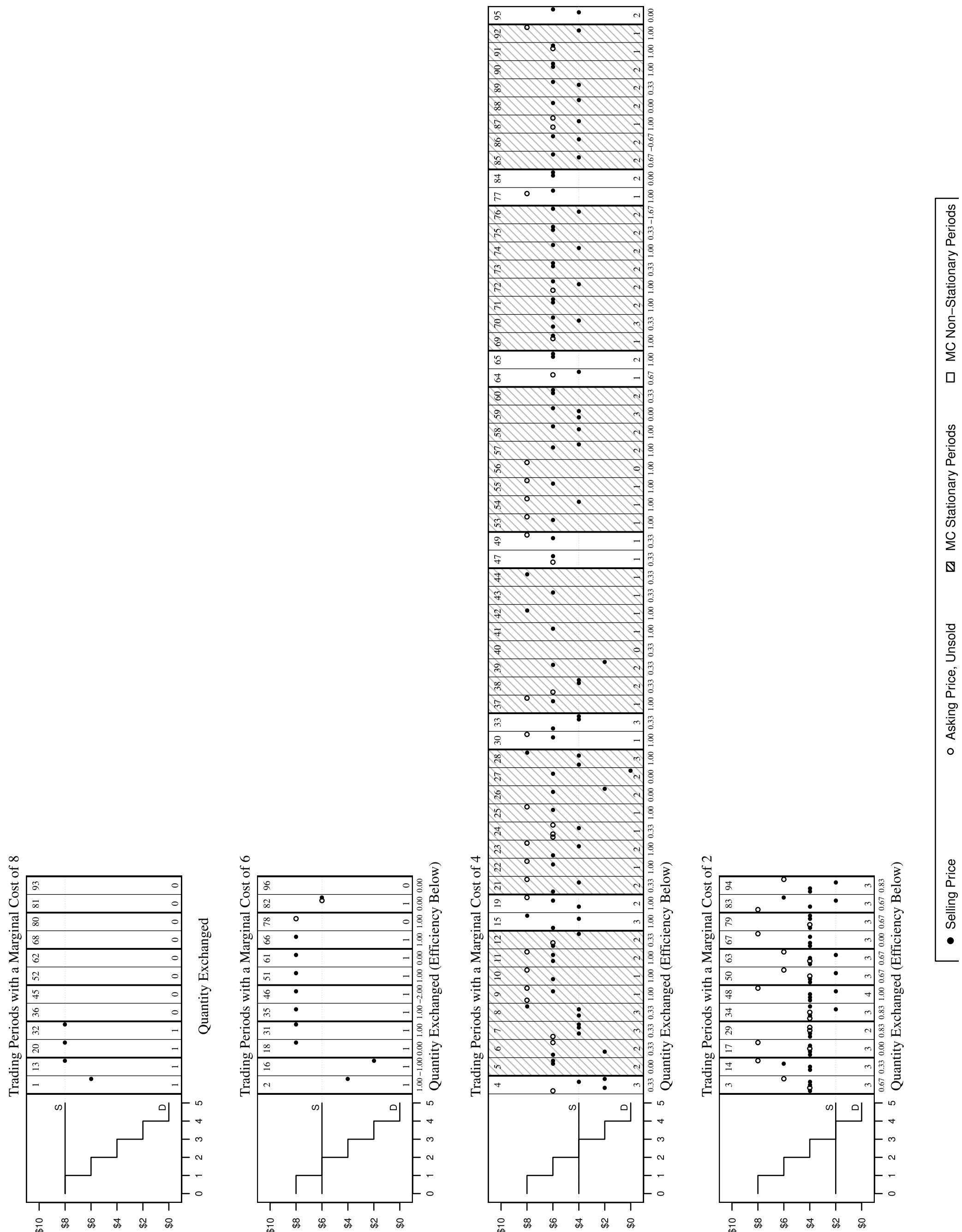

Figure A17: Asks, Prices, and Quantities in Session 4 of Random Order Human Buyers (ROHB) Treatment

cost periods to the competitive equilibrium of three units produced, all three transacting at a uniform price of 4. In contrast, the random order sessions exhibit a wide range of behaviors anticipated in the theory presented in section 3 during the stationary marginal cost periods, including non-uniform pricing equilibria detailed there; and occasionally even the high uniform price, low quantity, equilibrium featuring two units produced, with both traded at a price of 6.

Looking across the cross-section of eight random order groups, at the time series of eight periods in the final stationary segment, we observe 2 out of 64 periods exhibiting three produced units, all transacting at a uniform price of 4. Whereas for value order 43 of the 64 periods (eight final stationary periods across eight value order groups) show three produced units, all transacting at a uniform price of 4. As we shall see, econometric analysis supports the conjecture that the competitive equilibrium is much more readily reached under Marshallian path adjustment (i.e. with value order queueing in use) than otherwise (i.e. with random order queueing instead).

We explain how we evaluate the emergence of the temporally stable, individual advance production decisions that support competitive equilibrium in pure strategy play in subsection 4.2. Here, as a complement to Figure 2, as present an alternative arrangement for advance production decisions that support subgame perfect pure strategy play in Figure A18. Additionally, as examples of the evolution of actual entry decisions—possibly towards the assortative benchmarks in Figure 2 and/or Figure A18—we present all advance production from the experiment in each of four Figures A19 through A22.

A.3. *Additional Analysis of Convergence to Competitive Equilibrium*

Table III reports marginal effects of a probit on attainment of equilibrium, according to each standard (requiring individual seller stability, or

TABLE A2

Probit on Attainment of Competitive Equilibrium

	Stationary		Non-Stationary	
	CE $P\&Q$	+ Stable Identities	CE $P\&Q$	+ Stable Identities
	(1)	(2)	(3)	(4)
Constant	−0.8346***	−1.2384***	−0.5192***	−0.7972***
	(0.1837)	(0.2020)	(0.1657)	(0.1742)
Robot Buyers	−0.9118***	−0.6969**	−0.1513	0.0545
	(0.2595)	(0.2953)	(0.2026)	(0.2126)
Random Order	−0.4600*	−0.6757**	−0.0799	−0.2042
	(0.2683)	(0.3343)	(0.2054)	(0.2170)
Period	0.0171***	0.0205***	0.0086***	0.0079***
	(0.0033)	(0.0035)	(0.0029)	(0.0030)
Period × Robot Buyers	0.0146**	0.0085*	0.0011	−0.0012
	(0.0045)	(0.0049)	(0.0036)	(0.0037)
Period × Random Order	−0.0217***	−0.0165***	−0.0035	−0.0004
	(0.0047)	(0.0054)	(0.0036)	(0.0038)
Observations (Groups)	768 (16)	768 (16)	672 (16)	672 (16)

Notes: Reported are probit estimates on attainment of asking prices and quantities (Models 1 and 3) and stable seller identities (Models 2 and 4) consistent with competitive equilibrium. Corresponding (unclustered and non-robust) standard deviations are in parentheses. Marginal effects with cluster-robust standard errors at the group level are reported in Table III

*** Significant at the 1 percent level.
** Significant at the 5 percent level.
* Significant at the 10 percent level.

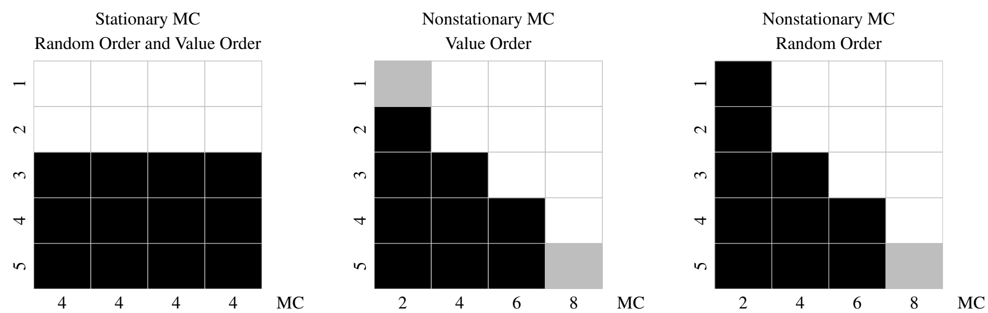

Figure A18: Arrangement of Advance Production Supporting Subgame Perfect Nash Equilibria

Note: Each column shows the identity of each seller (vertical axis labeling, 1 . . . ...5). Each column shows whether a given seller would engage in advance production (black) or not (white) within a block of four contiguous rounds given all of the following: (a) marginal cost in that round is as listed at the bottom of that column, on the horizontal axis, (b) there is a "pecking order" in advance production that respects monotonicity at the individual level (and *a fortiori* at the group level), and (c) total number of producers is that which would be observed given both advance production decisions consistent with an SPNE. (Note the following special cases. First, that gray is used for the $MC = 8$ cases as either one producer, or no production at all, each support SPNE play. Second, gray is also used in the case of value order queueing and $MC = 2$, as both four or five producers can each support SPNE play.)

not), for stationary and for non-stationary rounds. In Table A2 we report the results of the probit regression from which the marginal effects were computed. These estimates inform the time-to-convergence predictions reported in subsection 4.2.

Competitive equilibrium is not attainable in RO under $MC = 2$; therefore we omit $MC = 2$ for RO sessions in Models 3 and 4 of Table III. We report estimates with $MC = 2$ observations omitted for both VO and RO treatments in Table A3.

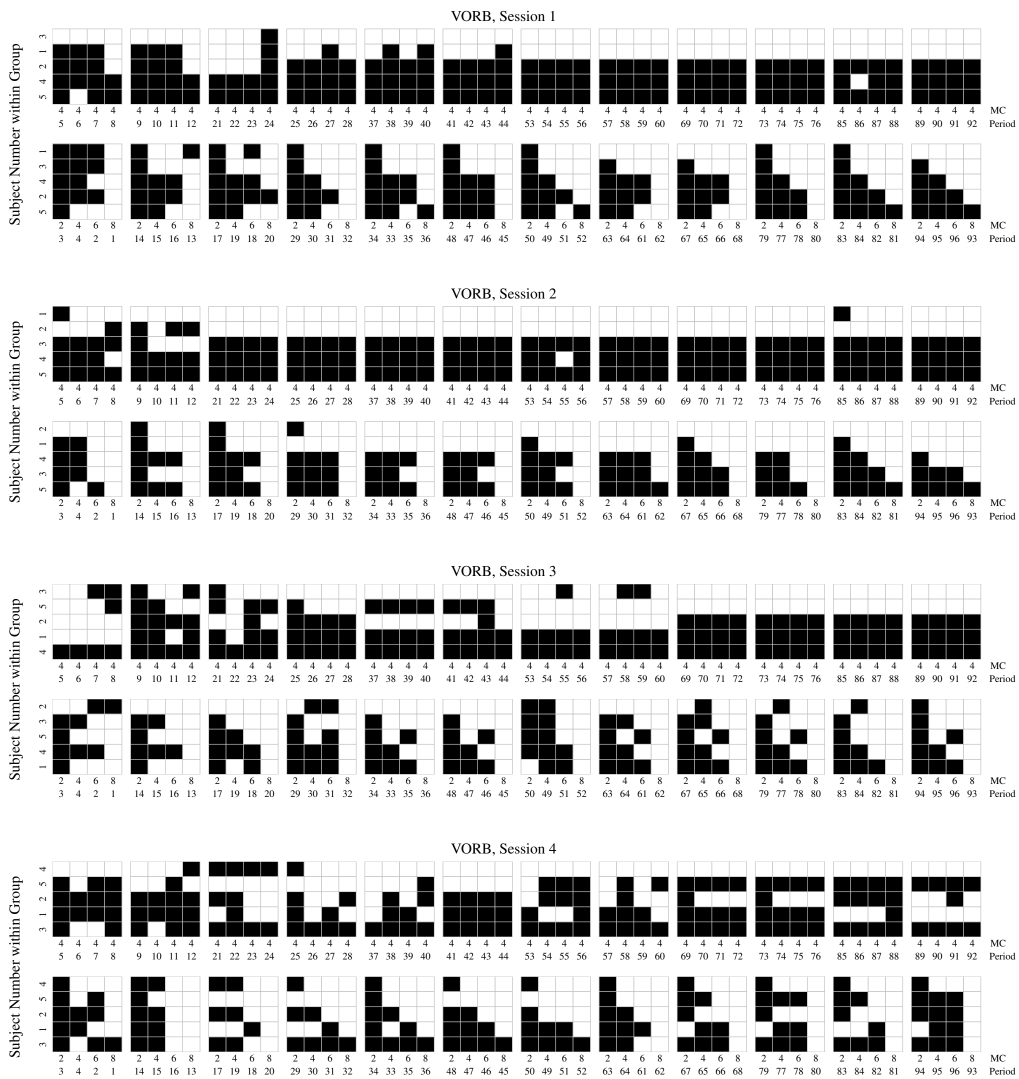

Figure A19: Advance Production in Value Order Robot Buyer (VORB) Treatment

Note: Seller decisions to engage in advance production (black) or not (white). Y-axis: subject numbers within group (corresponding also to subjects' respective horizontal locations in Figures A2 through A17) arrayed in order of advance production, per the algorithm described in footnote 21. X-axis: lexicographic sorting; first, by chronology across four-contiguous-period blocks; second by (weakly) ascending marginal cost (MC) within each block; third, for stationary rounds, chronologically within each block.

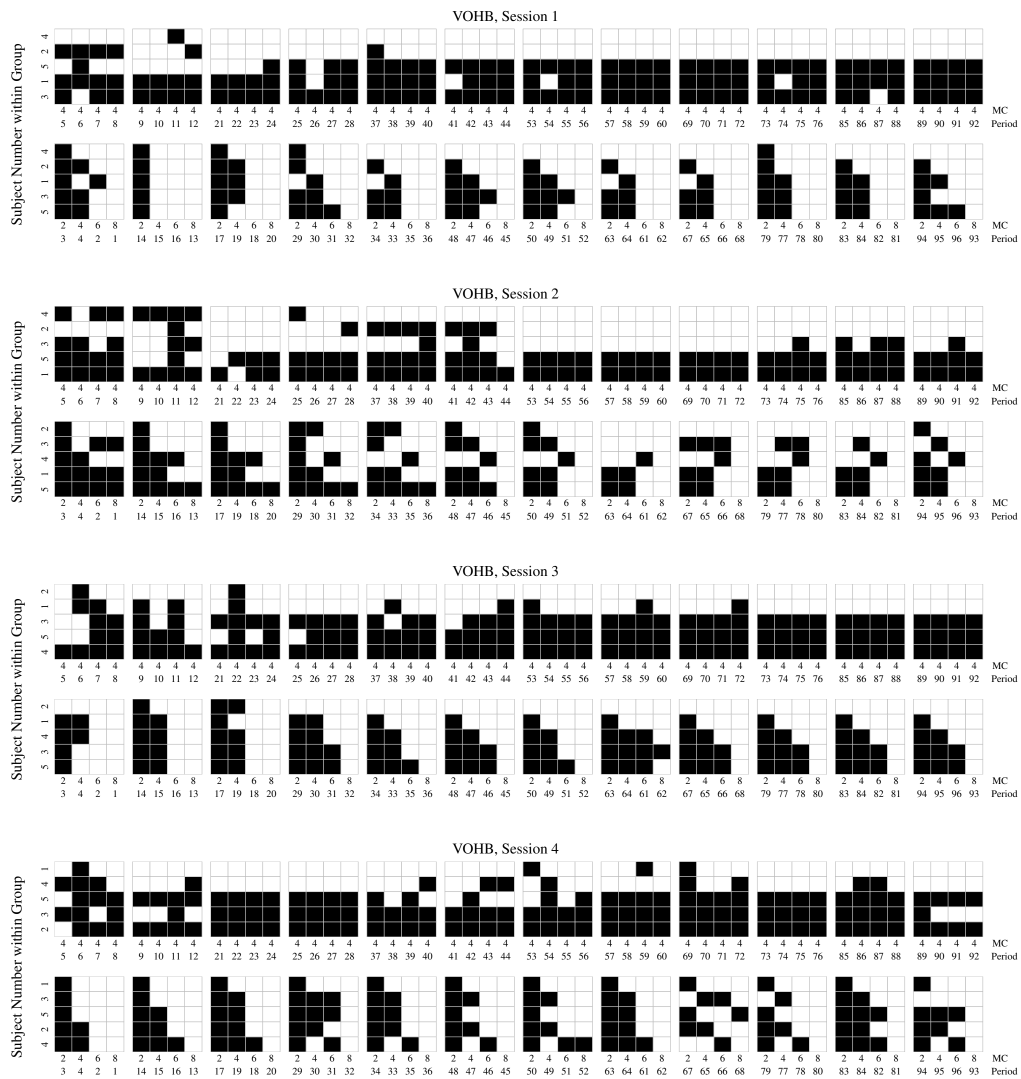

Figure A20: Advance Production in Value Order Human Buyer (VOHB) Treatment

Note: Seller decisions to engage in advance production (black) or not (white). Y-axis: subject numbers within group (corresponding also to subjects' respective horizontal locations in Figures A2 through A17) arrayed in order of advance production, per the algorithm described in footnote 21. X-axis: lexicographic sorting; first, by chronology across four-contiguous-period blocks; second by (weakly) ascending marginal cost (MC) within each block; third, for stationary rounds, chronologically within each block.

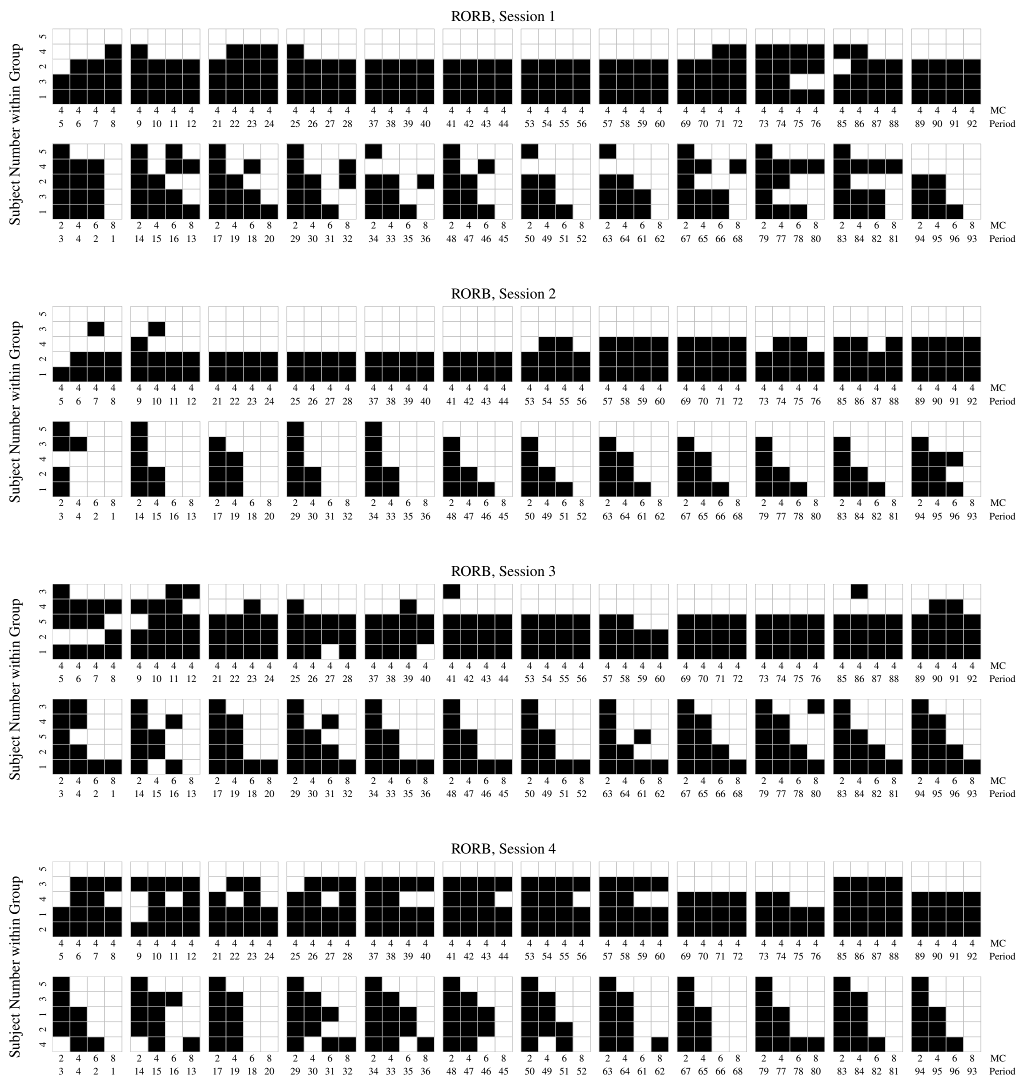

Figure A21: Advance Production in Random Order Human Buyer (RORB) Treatment

Note: Seller decisions to engage in advance production (black) or not (white). Y-axis: subject numbers within group (corresponding also to subjects' respective horizontal locations in Figures A2 through A17) arrayed in order of advance production, per the algorithm described in footnote 21. X-axis: lexicographic sorting; first, by chronology across four-contiguous-period blocks; second by (weakly) ascending marginal cost (MC) within each block; third, for stationary rounds, chronologically within each block.

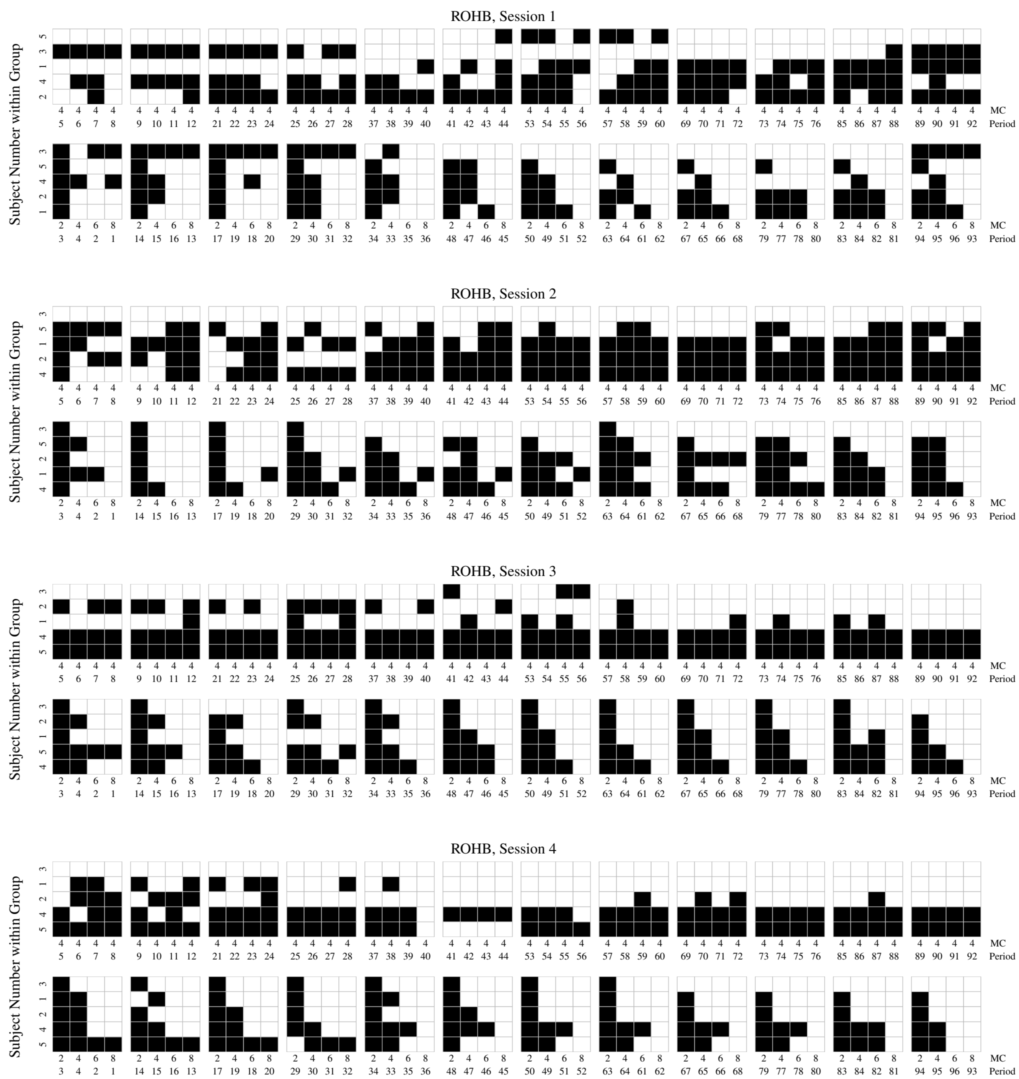

Figure A22: Advance Production in Random Order Human Buyer (ROHB) Treatment

Note: Seller decisions to engage in advance production (black) or not (white). Y-axis: subject numbers within group (corresponding also to subjects' respective horizontal locations in Figures A2 through A17) arrayed in order of advance production, per the algorithm described in footnote 21. X-axis: lexicographic sorting; first, by chronology across four-contiguous-period blocks; second by (weakly) ascending marginal cost (MC) within each block; third, for stationary rounds, chronologically within each block.

TABLE A3

MARGINAL EFFECTS OF PROBIT ON ATTAINMENT OF COMPETITIVE EQUILIBRIUM IN NON-STATIONARY PERIODS

	Non-Stationary	
	CE $P\&Q$	+ Stable Identities
	$(3')$	$(4')$
Robot Buyers	-0.0671	0.0276
	(0.0863)	(0.0663)
Random Order	-0.1319	-0.1595^{**}
	(0.0845)	(0.0649)
Period	0.0042^{***}	0.0035^{***}
	(0.0010)	(0.0012)
Period × Robot Buyers	0.0005	-0.0007
	(0.0014)	(0.0012)
Period × Random Order	-0.0022	-0.0006
	(0.0014)	(0.0012)
Observations (Groups)	576 (16)	576 (16)

Notes: Reported are marginal effects on attainment of asking prices and quantities (Model $3'$) and stable seller identities (Model $4'$) consistent with competitive equilibrium. Corresponding standard deviations are in parentheses. Standard errors are clustered at group level, with 4 groups per each of the 4 treatments. $MC = 2$ observations are omitted for both VO and RO treatments.

*** Significant at the 1 percent level.
** Significant at the 5 percent level.

A.4. *Analysis of Trends in Individual Stability of Advance Production Decisions*

In this section, we evaluate the emergence (or not) of stable advance production decisions in stationary and non-stationary portions of the data without considering attraction to any particular outcome (e.g. competitive equilibrium).

We begin by example, illustrating in Table A4 a series of transition matrices for session 2 of the VORB treatment, following Table 6 of Sundali et al. and Table 4 of CJSW (2017). To do this, we first divide the 48 stationary and non-stationary periods of the experiment each into 12 contiguous four-period blocks. Then, we tabulate a 2×2 matrix that summarizes the overlap (or lack thereof) across the ("Produce" or "Do Not" produce) decisions observed in a given period and those observed in the first immediately prior period ordered (lexicographically) in marginal cost and then chronology.[34]

Equivalently, it can be said in reference to Figures A19 through A22, that a table like Table A4 would summarize the overlap (or lack thereof) in advance production decisions between each cell (row and column) of a block and the same cell (row and column) in the preceding and subsequent blocks.

As in Sundali et al. (1995) and CJSW, the off-diagonal cells of these matrices do not all contain a count of zero, and are therefore inconsistent with instant and complete adoption of stable strategies. To more compactly summarize instability in advance production decisions, we follow Sundali et al. by computing for each transition matrix an index of change (IC) as the sum of off-diagonal entries in each matrix over the sum of entries in the

[34] Thus, decisions in non-stationary periods are compared to the identically parameterized period in the prior block, and decisions in stationary periods are compared to the decision in the period of the prior block for which the same number of periods have elapsed within each block.

TABLE A4

Transition Matrices Between Adjacent Blocks of Stationary $MC = 4$ Periods of Session 2 of VORB

		Block 2	
		Do Not	Produce
Block 1	Do Not	4	3
	Produce	4	9

IC=0.35

		Block 3	
		Do Not	Produce
Block 2	Do Not	5	3
	Produce	3	9

IC=0.00

		Block 4	
		Do Not	Produce
Block 3	Do Not	8	0
	Produce	0	12

IC=0.00

		Block 5	
		Do Not	Produce
Block 4	Do Not	8	0
	Produce	0	12

IC=0.00

		Block 6	
		Do Not	Produce
Block 5	Do Not	8	0
	Produce	0	12

IC=0.00

		Block 7	
		Do Not	Produce
Block 6	Do Not	8	0
	Produce	1	11

IC=0.05

		Block 8	
		Do Not	Produce
Block 7	Do Not	8	1
	Produce	0	11

IC=0.05

		Block 9	
		Do Not	Produce
Block 8	Do Not	8	0
	Produce	0	12

IC=0.00

		Block 10	
		Do Not	Produce
Block 9	Do Not	8	0
	Produce	0	12

IC=0.00

		Block 11	
		Do Not	Produce
Block 10	Do Not	7	1
	Produce	0	12

IC=0.05

		Block 12	
		Do Not	Produce
Block 11	Do Not	7	0
	Produce	1	12

IC=0.05

Note: Transition matrices summarize the overlap (or not) across decisions to produce in advance ("Produce") or not ("Do Not") observed in a given period and those observed in the first immediately prior four-contiguous-period block ordered (lexicographically) in marginal cost and then chronology. IC is the index of change, or the proportion of observations in the off-diagonal cells.

TABLE A5

Tests of Monotonic Trend in Indices of Change by Spearman Rank Correlation

Treatment	Stationary		Non-Stationary	
	Correlation	P-value	Correlation	P-value
VORB	−0.8808	0.0003	−0.5057	0.1125
VOHB	−0.8342	0.0014	−0.3489	0.2929
RORB	−0.2187	0.5183	−0.3250	0.3295
ROHB	−0.7123	0.0139	−0.3180	0.3406

Note: Reported are Spearman rank correlations and p-values of a test between indices of change (ICs) and the chronological ordering of the blocks from which the ICs were calculated.

matrix.

We next compute 32 sets of 11 transition matrices across stationary and non-stationary portions of each session of each treatment, then add the matrices across all four sessions to create 8 sets of 11 summary matrices across stationary and non-stationary portions of each treatment.[35] From this, we then compute 11 ICs for each of the 8 sets of matrices.

Finally, we seek to characterize any monotonic (but not necessarily linear) relationship that may exist between the ICs calculated for each treatment. We proceed by conducting a Spearman rank correlation test between the ICs and the chronological ordering of the blocks from which the ICs were calculated. We report the results of these tests for stationary and non-stationary portions of each treatment in Table A5. We note from the table that the calculated correlations reflect a negative trend in the index of change in all treatments (and therefore less instability in advance production decisions); however, correlation is only significantly different from zero at conventional levels for the stationary portions of the VORB, VOHB, and ROHB treatments.[36]

[35] We do not report all transition matrices, but note that these are calculable from Figures A19 through A22.

[36] The reader may of course test for a *significantly negative* trend by dividing the

A.5. *Additional Analysis of Consistency with Pure Strategy Nash Equilibrium in the Pricing Subgame*

We now addend subsection 4.3 with additional consideration of Table IV. We observe that attainment of pricing consistent with PSNE is overall less frequent for random order queuing than value order queueing (and this difference is significant; $p < 0.0001$ in χ^2 test over proportions). Equally interesting is the single exception to this finding at a more disaggregated level: the $m = 3$ subset of the data. When $m = 3$, random order queueing allows attainment of some PSNE in almost 90% of cases, but more often of the non-uniform pricing equilibria that exist only under random order queueing ({4,6,6} and {4,4,6}), and of the uniform supra-competitive pricing equilibria for three producers ({6,6,6}), than of the competitive equilibrium. In the $m = 3$ case, value order queuing tends to competitive equilibrium in a far greater percentage of rounds than does random order queueing.

A different notable phenomenon is observed within the random order treatments when $m = 3$. Human buyers are associated with observation of competitive equilibrium pricing and allocation 26% of the time, while robot buyers are associated with observation of competitive equilibrium pricing and allocation only 3% of the time ($p < 0.0001$ in χ^2 test over proportions). Conditional on random order queueing being in use, and on there already being three entrants, human buyers may have some context-specific efficacy in bringing about the price component of the competitive equilibrium allocation.

In the $m = 1$, 2, 4, and 5 cases, we observe more instances of pricing consistent with some PSNE under value order queueing than under random order queueing.[37] This suggests that value order queueing may lead to adop-

p-values reported in Table A5 in half.

[37]Given discreteness in pricing, one might wonder how likely it is to observe pricing consistent with PSNE by chance. Assuming a uniform likelihood of each price

tion of pricing consistent with PSNE faster than random order queueing.

To evaluate convergence to PSNE pricing even in those cases where entry decisions were not consistent with SPNE, we run two random effect probit regressions with cluster-robust standard errors at the group level. The dependent variable in each regression is realization, or not, of PSNE pricing in the pricing subgame. We do this for (1) uniform PSNE subgame pricing only (as per competitive equilibrium), and (2) all (uniform and non-uniform) PSNE subgame pricing. We report these results in Table A6.

A.6. *Additional Analysis of Pricing and Price Dispersion*

We further addend subsection 4.3 with additional consideration of Table V. for the cases with the same equilibria under both value and random order queuing ($m = 1$ and $m = 2$ producers), the mean transaction prices are 5.63 with value order queuing and 6.00 with random order queueing (there is no difference in central tendency; $p \approx 0.8747$, via a rank sum test over differences in the mean price across the 16 groups). For cases in which the theoretically predicted equilibria differ under value order and random order, i.e. $m \geq 3$ producers, the mean transaction prices are 3.90 with value order queuing and 4.39 with random order queueing ($p \approx 0.0011$, via a rank sum test).

An overview of the impact of human or robot buyer behavior is also possible. Across all cases, the mean transaction price is 4.40 with human buyers and 4.58 with robot buyers ($p \approx 0.6454$, via a rank sum test over

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{0,2,4,6,8,10} being chosen, pricing consistent with PSNE would emerge randomly in approximately 0.167 of trials when $m = 1$, and in around 0.027 of trials when $m = 2$, under either queueing method. Under value order queuing, pricing consistent with PSNE would emerge randomly in around 0.009 of trials when $m = 3$, in 0.0015 of trials when $m = 4$, and in 0.0004 of trials when $m = 5$. Under random order queuing, pricing consistent with PSNE would emerge randomly in around 0.037 of trials when $m = 3$, in 0.0008 of trials when $m = 4$, and in 0.0001 of trials when $m = 5$.

TABLE A6

MARGINAL EFFECTS OF PROBIT ON PURE STRATEGY NASH EQUILIBRIUM SUBGAME PRICING

	Uniform Pricing PSNE	All PSNE
	(1)	(2)
Robot Buyers	−0.1124	−0.0808
	(0.0628)	(0.0585)
Random Order	−0.1533**	−0.0173
	(0.0619)	(0.0588)
Period	0.0063***	0.0045***
	(0.0012)	(0.0007)
Stationary $MC = 4$	0.0241	0.0544
	(0.0453)	(0.0437)
Period × Robot Buyers	0.0004	0.0016*
	(0.0012)	(0.0012)
Period × Random Order	−0.0043***	−0.0017*
	(0.0013)	(0.0013)
Period × Stationary $MC = 4$	−0.0011	0.0012
	(0.0012)	(0.0009)
Observations (Groups)	1,382 (16)	1,382 (16)

Notes: Reported are marginal effects on PSNE subgame pricing as described in Panel A of Table II, with standard deviations in parentheses below each. Standard errors are cluster-robust at group level, with 4 groups per each of the 4 treatments, and 96 periods per group; 153 periods with no producers are omitted.

*** Significant at the 1 percent level.
** Significant at the 5 percent level.
* Significant at the 10 percent level.

differences in the mean price across the 16 groups).[38]

A.6.1. *Allocative Efficiency, Division of Surplus, and Buyer Behavior in the Pricing Stage*

To maximize immediate surplus, buyers should select the lowest available profitable ask when queued. By construction, robot buyers do this 100% of the time. Human buyers are, however, not constrained in this manner, and do not always do so. Reasons might include making mistakes, a desire to punish producers, and so on.

So, do human buyers leave money on the table in practice? Yes. Take ROHB, for example: there are no less than 5 instances in the periods of non-varying marginal cost ($MC = 4$) where a buyer leaves a full-step-of-the-demand-curve's worth of surplus, \$2.00, untouched by the mouse-click needed to take it. What kind of impact does the possibility of this kind of behavior, not possible with robot buyers, have on efficiency? We report descriptive statistics in Table A7.[39]

[38] In the reported rank sum tests over mean prices, we treat each of the 16 groups as the unit of observation. If we instead treated transactions as independent units of observation, we find a significant and higher central tendency in mean transaction prices under random order than under value order queuing ($p < .0001$), as well as a significant difference in the central tendency of mean transaction prices between human and robot buyers ($p \approx 0.0007$).

[39] Note that realized surplus can be negative for producers, as MC is subtracted when producing in advance. Note also that in random order treatments, the surplus that might be realized depends upon the order in which buyers enter the queue. (Consider an example with robot buyers where seller asks are $\{8,6\}$. Under value order queuing, consumer surplus will always be 2. Under random order queueing, consumer surplus will be 0 if the buyer with a value of 6 is queued before the buyer with value 8, and 2 if the higher-valued buyer comes first; with each ordering equally likely, the expected consumer surplus is 1.) Thus, in the first two rows of Table A7, we report fractions of deterministic surplus for the value order queuing, and fractions of *expected* surplus for random order queuing. In expectation, the fraction of consumer surplus realized by buyers (second row) in RORB

TABLE A7

Mean Efficiency and Distribution of Surplus

Measure	Value Order Queueing Human Buyers VOHB	 Robot Buyers VORB	Random Order Queueing Human Buyers ROHB	 Robot Buyers RORB
Efficiency (Fraction of [Expected] Total Surplus Realized)	0.9315 (0.2222)	0.9891 (0.0732)	0.8517 (0.3657)	0.9256 (0.2574)
Fraction of (Expected) Consumer Surplus Realized by Buyers	0.9788 (0.1348)	1.0000 (0.0000)	1.0026 (0.5318)	0.9878 (0.4096)
Fraction of Realized Total Surplus Accrued to Buyers	0.8447 (0.3472)	0.8533 (0.3364)	0.6261 0.3442)	0.5807 (0.3580)

Note: Reported are proportions described in the left-most column, with standard deviations in parenthesis below each. In the first two rows, we report fractions of deterministic surplus for value order queuing, and of expected surplus for random order queuing, as explained in footnote 39.

Under random order queueing, realized buyer surplus—net of demand withholding by buyers, price concessions by producers worried about demand withholding, mistakes, etc.—is higher when humans are the buyers (the third row of Table A7). However, this effect pales in comparison to the increase in percentage of total surplus (and also raw, dollar-denominated surplus) that occurs with a change in buyer queueing type: a switch to value-order queueing is far more effective at gaining surplus for buyers (and overall) than any uncoordinated efforts by individual buyers.[40]

To take a more granular look at buyer decision-making, we consider the ways it differs from the deterministic demand-revealing benchmark (i.e. "robot" behavior) in the VOHB treatment. In this treatment, 50% (10 of 20) of human buyers committed at least one deviation from the demand-revealing benchmark, with approximately 1.67% (32 of 1920) of human buyer decisions being categorized as deviations, and approximately 0.52%

approaches 1 in the limit, but diverges in this case (1.0026) because the sample is finite and particular orderings can thus be oversampled relative to the population.

[40]Greater impact of buyer withholding might be observed if buyers are large relative to the market, e.g. Ruffle (2000) employs two buyers rather than five.

TABLE A8

Correlations between Frequency of Advance Production and Asking Prices Among Sellers

		Mean Price	Price of 10	8	6	4	2
Pearson Correlation		0.3277	0.2208	0.1595	0.2343	-0.0975	-0.3312
P-value	Independent	0.0030	0.0491	0.1576	0.0365	0.3897	0.0027
	Clustered	0.0046	0.0448	0.3897	0.0821	0.5518	< 0.0001

(10 of 1920) resulting in a loss to the buyer greater than the commission payment. Of these deviations, approximately 84% (27 of 32) were forgone profitable trades, 13% (4 of 32) were taking a price above value, and 3% (1 of 32) were taking a profitable ask greater than the lowest available ask.

A.6.2. *Analysis of Correlation between Advance Production and Pricing*

In Table A8, we report Pearson product moment correlation coefficients. These are between the cross-section of frequency of advance production across sellers and the cross-section of asking prices (for units both transacted and not transacted) offered by sellers. The leftmost column reports the correlation between the frequency of advance production and the mean asking price for each of seller, while the rightmost five columns report the correlation between the frequency of advance production and the frequency of the offer of a specific asking price in $\{10, 8, 6, 4, 2\}$ for each seller.[41] All 80 sellers are included. We test whether each correlation coefficient is different from zero in two ways. First, we calculate a t-test statistic and report the associated p-value in where we assume the samples follow independent normal distributions. Second, we standardize each variable and run a linear regression of advance production on asking price with standard errors clustered by group; the estimated slope coefficient is then necessarily the

[41] Sellers were permitted to ask at 0; because this occured for only 4 of 7680 recorded asks, we exclude this in the table.

correlation coefficient, and the affiliated p-value is reported in the table.

This positive and significant correlation between advance production and mean asking price indicates that sellers who more frequently choose to engage in advance production also price higher, on average. This interpretation is consistent with the positive correlations (some significant) between advance production and frequency of pricing above 4 and negative correlations (some significant) between advance production and frequency of pricing below 4.

A.6.3. *A Unifying Framework for Attributes Impacting Equilibration*

Table A9 presents statistical results over a wide range of experiments nesting and unifying the Market Entry Game and the Posted Offer Market with Advance Production. We do this by merging the data presented in this study with that reported in CJSW (2017).

This statistical analysis employes additional explanatory variables, as there are more possible design settings across this larger pool of experiments. Classic results on the Market Entry Game (Rapoport, 1995) employ a global shifter, "capacity", changes in which can be shown to be equivalent to shifts in the demand curve; such studies also represent payoff information as payoff functions, rather than via demand and supply curves; we vary both of the preceding, as a control. Thus "individual-level shifter" refers to parametrization changes by the experimenter being implemented at the local (e.g. individual marginal cost level) rather than demand/capacity level, while "demand curve representation" refers to information being disclosed to subjects as demand and supply curves, rather than abstract payoff functions. The Market Entry Game has implicit in it an ex post market clearing price rule, while the Posted Offer allows individual sellers to post their own (respective) offers; the "individual-price posting" variable tracks which condition is in effect in a given session. All other variables are as defined in the main text of this paper.

Collectively, these variables allow us to parse exhaustively the influences on attainment of competitive equilibrium across the Market Entry Game and the Posted Offer with Advance Production, with implications for yet other institutions; attainment of competitive equilibrium is hastened by individual posting of offers, robot (demand revealing) buyers, and a stationary demand and supply environment; it is delayed by random order buyer queueing.

APPENDIX B: DERIVATION OF PURE STRATEGY NASH EQUILIBRIA (FOR ONLINE PUBLICATION)

A derivation of the pure strategy Nash equilibria (PSNE) and subgame perfect Nash equilibria (SPNE) under value order queueing is provided in CJSW (2017). Below we consider a more general case including random order queuing, including both analytic and numerical derivation of PSNE and SPNE for the parameters used in the experiment.

B.1. *Enumeration of Subgame Perfect Nash Equilibria*

In Table C1 we provide a description of the (expected) payoffs for pure strategy Nash equilibria. Note that the payoff for opting out of advance production is 1, so a seller earning a payoff of 1 is indifferent between advance production and not, holding pricing constant.

B.2. *Derivation of Pure Strategy Nash Equilibria*

B.2.1. *Overview of the Strategic Environment*

We begin with a short overview of the strategic environment. Consider a market with M sellers and N buyers. Each buyer j has a valuation v_j for a single unit of a good (in our experiment $v_j \in \{0, 2, 4, 6, 8\}$). The sellers independently decide whether to produce a single unit of a good. The

TABLE A9

MARGINAL EFFECTS OF PROBITS ON ADVANCE PRODUCTION (QUANTITY) NECESSARY TO SUPPORT COMPETITIVE EQUILIBRIUM

	Competitive Equilibrium Production	
	(1)	(2)
Individual-level Shifter	−0.0419	−0.0405
	(0.0621)	(0.0556)
Demand Curve Representation	0.0781	0.0762
	(0.0770)	(0.0701)
Individual-price Posting	−0.0646	−0.0609
	(0.0750)	(0.0725)
Robot Buyers	−0.0743	−0.0687
	(0.0746)	(0.0816)
Random Order	0.0043	0.0038
	(0.0757)	(0.0892)
Stationary $MC = 4$ (Capacity)	−0.0050	−0.0049
	(0.0359)	(0.0480)
Period	−0.0027	−0.0025
	(0.0013)	(0.0022)
Period × Individual-level Shifter	0.0006	0.0006
	(0.0008)	(0.0011)
Period × Demand Curve Representation	−0.0011	−0.0010
	(0.0009)	(0.0013)
Period × Individual-price Posting	0.0041***	0.0039**
	(0.0009)	(0.0015)
Period × Robot Buyers	0.0040***	0.0038**
	(0.0009)	(0.0019)
Period × Random Order	−0.0025***	−0.0023
	(0.0010)	(0.0018)
Period × Stationary $MC = 4$ (Capacity)	0.0023***	0.0022**
	(0.0007)	(0.0007)
Observations (Groups)	3360 (36)	3360 (36)

Note: Reported are marginal effects on attainment of levels of production necessary to support competitive equilibrium contingent on exogenously imposed marginal cost or market capacity. Corresponding standard deviations are in parentheses. In Model 1, random effect is at group level; and in Model 2, standard errors are cluster-robust; each with 4 groups per each of the 9 treatments, and 96 periods per group, except where $MC = 2$ omitted for random order treatments (due to nonexistence of CE in RO when $MC = 2$).

*** Significant at the 1 percent level.
** Significant at the 5 percent level.

Table C1: Asking Prices Supporting Pure Strategy Nash Equilibria in the Pricing Subgame, with (Expected) Payoffs Stratified by Advance Production

Marginal Cost of Production	Number of Producers	Posted Asking Prices: Value Order Queueing	Posted Asking Prices: Random Order Queueing	(Excepted) Payoffs: Value Order Queueing	(Excepted) Payoffs: Random Order Queueing
$MC = 8$	$m = 1$	**{8}**	**{8}**	{1}	{1}
	$m = 2$	**{6,6}**	**{6,6}**	{−1,−1}	{−1,−1}
	$m = 3$	**{4,4,4}**,{6,6,6}	**{4,4,4}**,{4,4,6},{4,6,6},{6,6,6}	{−3,−3,−3},{−3,−3,−3}	{−3,−3,−3},{−3,−3,−3},{−3,−3,−3},{−3,−3,−3}
	$m = 4$	**{2,2,2,2}**,{4,4,4,4}	{4,4,4,4}	{−5,−5,−5,−5},{−4,−4,−4,−4}	{−4,−4,−4,−4}
	$m = 5$	**{0,0,0,0,0}**,{2,2,2,2,2},{4,4,4,4,4}	{2,2,2,2,2}{4,4,4,4,4}	{−7,−7,−7,−7,−7},{−5.4,−5.4,−5.4,−5.4,−5.4},{−4.6,−4.6,−4.6,−4.6,−4.6}	{−5.4,−5.4,−5.4,−5.4,−5.4},{−4.6,−4.6,−4.6,−4.6,−4.6}
$MC = 6$	$m = 1$	**{8}**	**{8}**	{3}	{3}
	$m = 2$	**{6,6}**	**{6,6}**	{1,1}	{1,1}
	$m = 3$	**{4,4,4}**,{6,6,6}	**{4,4,4}**,{4,4,6},{4,6,6},{6,6,6}	{−1,−1,−1},{−1,−1,−1}	{−1,−1,−1},{−1,−1,−1},{−1,−1,−1},{−1,−1,−1}
	$m = 4$	**{2,2,2,2}**,{4,4,4,4}	{4,4,4,4}	{−3,−3,−3,−3},{−2,−2,−2,−2}	{−2,−2,−2,−2}
	$m = 5$	**{0,0,0,0,0}**,{2,2,2,2,2},{4,4,4,4,4}	{2,2,2,2,2}{4,4,4,4,4}	{−5,−5,−5,−5,−5},{−3.4,−3.4,−3.4,−3.4,−3.4},{−2.6,−2.6,−2.6,−2.6,−2.6}	{−3.4,−3.4,−3.4,−3.4,−3.4},{−2.6,−2.6,−2.6,−2.6,−2.6}
$MC = 4$	$m = 1$	**{8}**	**{8}**	{5}	{5}
	$m = 2$	**{6,6}**	**{6,6}**	{3,3}	{3,3}
	$m = 3$	**{4,4,4}**,{6,6,6}	**{4,4,4}**,{4,4,6},{4,6,6},{6,6,6}	{1,1,1},{1,1,1}	{1,1,1},{1,1,1},{1,1,1},{1,1,1}
	$m = 4$	**{2,2,2,2}**,{4,4,4,4}	{4,4,4,4}	{−1,−1,−1,−1},{0,0,0,0}	{0,0,0,0}
	$m = 5$	**{0,0,0,0,0}**,{2,2,2,2,2},{4,4,4,4,4}	{2,2,2,2,2}{4,4,4,4,4}	{−3,−3,−3,−3,−3},{−1.4,−1.4,−1.4,−1.4,−1.4},{−0.6,−0.6,−0.6,−0.6,−0.6}	{−1.4,−1.4,−1.4,−1.4,−1.4},{−0.6,−0.6,−0.6,−0.6,−0.6}
$MC = 2$	$m = 1$	**{8}**	**{8}**	{5}	{5}
	$m = 2$	**{6,6}**	**{6,6}**	{5,5}	{5,5}
	$m = 3$	**{4,4,4}**,{6,6,6}	**{4,4,4}**,{4,4,6},{4,6,6},{6,6,6}	{3,3,3},{3,3,3}	{3,3,3},{3,3,3},{3,3,3},{3,3,3}
	$m = 4$	**{2,2,2,2}**,{4,4,4,4}	{4,4,4,4}	{1,1,1,1},{2,2,2,2}	{2,2,2,2}
	$m = 5$	**{0,0,0,0,0}**,{2,2,2,2,2},{4,4,4,4,4}	{2,2,2,2,2}{4,4,4,4,4}	{−1,−1,−1,−1,−1},{0.6,0.6,0.6,0.6,0.6},{1.4,1.4,1.4,1.4,1.4}	{0.6,0.6,0.6,0.6,0.6},{1.4,1.4,1.4,1.4,1.4}

Note: Asking prices and payoffs supporting pure strategy Nash equilibria are listed for value and random order queueing, given marginal cost $MC = \{8, 6, 4, 2\}$ and demand schedule {8,6,4,2,0}, with marginal cost sunk. A subset of these PSNE in the pricing subgame support the SPNE (for advance production and pricing) enumerated in Panel B of Table II; the PSNE in the pricing subgame are enumerated in Panel A of Table II. Cells for which no PSNE in the pricing stage support a SPNE have dashes. Asking prices in **bold** are consistent with pricing via the demand schedule.

marginal cost of production is MC (in our experiment $MC \in \{2, 4, 6, 8\}$). Once the production decisions have been made, the supply (the number of units produced) m is revealed and the firms set prices (in the experiment the prices were restricted to multiples of $r = 2$ in the set $\{0, 2, 4, 6, 8, 10\}$). Then, the market opens and buyers arrive either in the descending order of their valuations (the value order queuing case) or in random order (the random order queuing case). A trade results in the following payoffs for the buyer and the seller respectively:

$$v_j - P_i + \delta \text{ and } P_i - MC,$$

where δ is small and positive utility from trading intended to break indifference when the seller's price equals the valuation $P_i = v_j$. (Without loss of generality, we can in the analysis of the pricing subgame ignore sellers' entry/exit subsidies, which are both equal to 1, and serve only to shift profit from 0 to 1.)

We will characterize SPNE of the game while restricting attention to pure pricing strategies. In the spirit of backward induction, we first take a look at buyers' subgames. In each subgame, one buyer with valuation v_j makes a buy or pass decision with respect to m available objects with prices $\{P_1, ..., P_m\}$. The following lemma lays out buyer's optimal response.

Lemma 1 *The buyer has a dominant (behavior) strategy to buy an object k iff $v_j \geq P_i$, where $P_i = min\{P_1, ..., P_m\}$.*

The argument follows directly from buyer's rationality. The buyer chooses to buy if and only if his valuation exceeds the lowest of the available prices.

Next we move on to pricing subgames. Each subgame is parameterized by the number of sellers m who have produced and have an object to sell. Because the cost is sunk at this point, each seller's objective is to maximize their revenue. We characterize subgame perfect equilibria in pure strategies, which, however, coincide with PSNE. Behavior will depend on the buyer arrival process. We examine two different types of queuing: the random order and value order.

B.2.2. *Pricing Subgame: Value Order Queuing*

By Lemma 1, the cheapest objects are always bought first. Because buyers arrive in the descending order of their valuations, if there is a trade in the (buyer's) subgame j, then there must have been trades in all preceding buyer subgames $\{1, ..., j-1\}$. Similarly, if there is no trade in subgame j, then there will be no trade in all subsequent subgames $\{j+1, ..., m\}$. Given these observations, we first establish that there is no PSNE in which different sellers offer different prices.

Lemma 2 *In any PSNE: $P_k = P_l$, $k, l \in \{1, ..., m\}$.*

Proof. Suppose this is not the case and all sellers do not offer the same price. Order the prices in the ascending order. Now define the marginal seller to be the one with the price P_i at which the the demand meets the supply, i.e., at which the i-th buyer still buys but $i+1$-th buyer does not. Note, if multiple sellers price at P_i, then each one of them is a marginal seller. By definition, the probability that the marginal seller makes a sale is

$$(2) \qquad q(P_i) = max\left[1, \frac{d(P_i) - s(P_i - r)}{s(P_i) - s(P_i - r)}\right].$$

In the second term of the max operator, $d(P_i) - s(P_i - r)$ is the difference between the number of objects demanded at P_i and the number supplied at $P_i - r$ (where r is the smallest price increment); similarly, $s(P_i) - s(P_i - r)$ is the difference between the number of objects supplied at P_i and $P_i - r$. The ratio can be interpreted as the number of buyers who are available and willing to buy (at P_i) per object offered in the market.

Suppose that P_i maximizes the marginal seller's revenue. If this were not the case, then he could profitably deviate by lowering the price. (It is never optimal to price higher than P_i as that would result in no sale and zero revenue.) If P_i is set optimally, then it cannot be profit-maximizing for any other seller to set a lower price, $P_k < P_i$. To see this, note that if the marginal seller prices optimally, then

$$(3) \qquad q(P_i)P_i \geq P_i - r.$$

The right-hand side is the expected revenue from dropping the price by the smallest price increment (r) and trading with certainty. So it follows that if $q(P_i)P_i \geq P_i - r$, then it must be that $q(P_i)P_i \geq P_k$.

However, if any one of the sellers with the lower price, P_k, switched to P_i, then this seller would get $q'(P_i)P_i$, where $q'(P_i) > q(P_i)$. To understand the last inequality, let us express $q(P_i)$ as $(d - s_{-r})/(s - s_{-r})$. Then, $q'(P_i) = (d-(s_{-r}-1))/(s-(s_{-r}-1)) = (d-s_{-r}+1)/(s-s_{-r}+1)$. In words, the one seller who upped his price from P_k to P_i will now compete for objects with all the incumbent P_i-sellers – hence, the +1 in the numerator; and, because this seller set his price at P_i, there is an additional object being offered (at P_i) – hence, the +1 in the denominator.

Since, $q'(P_i)P_i > q(P_i)P_i > P_i - r$, a P_k-seller can improve his revenue by pooling with the marginal seller(s) at P_i. ■

Next, let us define a market clearing price, $P(m)$, to be equal to the m-th highest valuation among the buyers.

Proposition 3 *Under the value order queueing, uniform pricing at the market clearing level, $P_i = P(m)$, is a PSNE.*

Proof. We need to verify that there is no opportunity to profitably deviate. Suppose all sellers set $P_i = P(m)$ and consider a seller k. In the supposed equilibrium, the k-seller sells and gets $P(m)$. Dropping a price below $P(m)$ is clearly not profitable as that would give a revenue $P_k < P_i = P(m)$. Raising the price above $P(m)$ is also not profitable since that would result in no sale and zero revenue. ■

We have established (in Proposition 3) that pricing at the market clearing level is always a PSNE and, by Lemma 2, there is no (asymmetric) equilibrium involving different prices. It is easy to see that uniform pricing below the market clearing level cannot be an equilibrium because any seller could profitably deviate by rising his price to $P(m)$.

It remains to examine whether uniform pricing above the market clearing level ($P_i > P(m)$) could be an equilibrium. Pricing above market clearing is not ruled out by Lemma 2. Suppose all sellers price at $P_i > P(m)$. This would be an equilibrium if a seller could not unilaterally deviate to a lower price $P_i - r$, i.e., if the following deviation condition holds $q(P_i)P_i \geq P_i - r$. The deviation condition is more likely to hold when P_i is close to the market clearing level $P(m)$ and the minimum price increment is large. In fact, when $r \to 0$ the deviation condition ($q(P_i)P_i \geq P_i - r$) is bound to fail.

For our experimental parametrization, all equilibria (in pure strategies) for value order queuing are summarized in Table II in the main text. The market clearing equilibria fall under the proposition 3. The remaining uniform pricing equilibria can be readily verified by checking that unilateral deviation is not profitable, i.e., $q(P_i)P_i > 0$ for upward deviation and $q(P_i)P_i > P_i - r$ for downward deviation.[42] For the sake of completeness, let us list them below:

- **Subgame** $m = 1$**:** (8)
- **Subgame** $m = 2$**:** $(6, 6)$
- **Subgame** $m = 3$**:** $(4, 4, 4), (6, 6, 6)$
- **Subgame** $m = 4$**:** $(2, 2, 2, 2), (4, 4, 4, 4)$
- **Subgame** $m = 5$**:** $(0, 0, 0, 0, 0), (2, 2, 2, 2, 2), (4, 4, 4, 4, 4)$

[42]Equilibria can be identified through a numerical, exhaustive algorithm such as that reported in subsubsection B.3.3.

B.2.3. *Pricing Subgame: Random Order Queuing*

In this section we consider a random queuing of buyers under which buyers' arrival in the market is independent of their valuations. Here we show that random queuing alters the incentives of sellers in the pricing subgame. For instance, it is not true anymore that market clearing pricing is always a PSNE; nor it is true that a uniform pricing PSNE always exists. Let us explore these claims in more detail. We start by developing some intuition.

The following lemma establishes a lower bound on equilibrium pricing.

Lemma 4 *$P(m)$ dominates pricing at any $P_i < P(m)$.*

Proof. At any profile of prices, where the seller i prices below the market clearing level, $P_i < P(m)$, the sale is guaranteed $q(P_i) = 1$. Hence, the seller can do better by increasing the price to $P(m)$ and getting a revenue $P(m)$ $(> P_i)$. ■

Next, let us focus on uniform pricing equilibria. We state the following (non-)existence result.

Proposition 5 *Under the random order queuing, a uniform pricing PSNE need not exist.*

Proof. By Lemma 4, no seller will set a lower price than $P(m)$. So suppose that sellers price uniformly at $P_i \geq P(m)$. Each expects to get $q(P_i)P_i = (d(P_i)/m)P_i$, where the probability of sale is the number of buyers who are willing to buy (with $v_j \geq P_i$) divided by the number of available objects at P_j. Consider what would happen if one of the sellers set a higher price, $P'_k > P_i$. Then, that seller's object would not be considered by buyers until possibly the very last subgame. Because the buyers arrive in random order, the probability that the k-buyers's valuation in the last subgame exceeds the price is $d(P'_k)/m$. A deviation to P'_k will not be profitable when

$$\text{(4)} \qquad \frac{d(P_i)}{m}P_i \geq \frac{d(P'_k)}{m}P'_k \iff \frac{d(P_i)}{d(P'_k)} \geq \frac{P'_k}{P_i},$$

for all $P'_k > P_i$.

Next, consider a deviation to a lower price $P''_k < P_i$. This puts the seller in the position of being the very first one to sell. As long as $P_i < max\{v_j\}$, the probability of making a sale is 1. From this, it follows that the optimal P''_k is $P_i - r$. A deviation to a lower price is not profitable when

$$\text{(5)} \qquad \frac{d(P_i)}{m}P_i \geq P''_k \iff \frac{d(P_i)}{m} \geq \frac{P_i - r}{P_i}.$$

Satisfying both (4) and (5) may not be possible when (i) $r \to 0$ and (ii) the market clearing pricing is in the inelastic region of the demand curve. (i) essentially forces $P_i \to P(m)$ in order for (5) to hold. But (ii) implies

(6)
$$(P(m)+r)d(P(m)+r) > P(m)d(P(m)) \iff \frac{d(P(m))}{d(P(m)+r)} < \frac{P(m)+r}{P(m)},$$

which violates (4). ■

There may be asymmetric equilibria in which different sellers set different prices or equilibria in mixed strategies. However, there is not much more we could say with a high degree of generality. Therefore, we turn to our experimental parametrization and characterize all pure strategy Nash equilibria in pricing subgames.

We start by noting that no seller will ever set a price greater than 8. If he did, he would price himself out of the market and earn zero revenue. For this reason, in what follows, we do not consider deviations to prices that are higher than 8.

- **Subgame $m = 1$:** The seller is a monopolist, and hence, prices at 8.

- **Subgame $m = 2$:** In the duopoly case there are only three candidates for PSNE: $\{(6,6),(6,8),(8,8)\}$. $(8,8)$ is not an equilibrium as there is a profitable deviation to 6. By (5), $1/2 < 6/8$. $(6,6)$ is an equilibrium as both, (4) and (5), are satisfied. The former holds because $2/1 > 8/6$; the latter holds because $2/2 > 4/6$. It follows that $(6,8)$ cannot be an equilibrium.
- **Subgame $m = 3$:** First note that, in equilibrium, no seller will ever price at 8. The most the seller can earn by pricing at 8 is when the other two sellers also price at 8. The expected revenue is $(1/3) \times 8$. However, dropping the price to 4 would guarantee a sale and a revenue equal to $4 > 8/3$. This leaves four equilibrium candidates: $\{(4,4,4)$, $(4,4,6)$, $(4,6,6)$, $(6,6,6)\}$. All four are equilibria.

  Note that at $(6,6,6)$ a deviation to 4 is not (strictly) profitable as $(2/3) \times 6 = 4$. At $(4,4,4)$ both, (4) and (5), hold and (4) holds with equality: $3/2 = 6/4$. Lastly, the revenue from pricing at 4 when prices are $(4,4,6)$ is 4, which is the same as the expected revenue from pricing 6 when prices are $(4,6,6)$, i.e., $(2/3)\times 6$. Together, these observations imply that both $(4,4,6)$ and $(4,6,6)$ are also equilibria.
- **Subgame $m = 4$:** First, no seller wants to price at 8. If all four sellers priced at 8, there is a profitable deviation to 6 – since $(1/4) \times 8 < 6$. If at least one of the sellers prices below 8, then the maximum a seller can earn is less than $(1/4)\times 8$. By setting a price at 2 he can guarantee himself a revenue of 2. Given this, we now argue that no seller will ever price at 6 as well. The highest expected revenue from pricing at 6 is $(2/4) \times 6$, which happens only in the case when all other sellers also price at 6. Otherwise, his revenue is less than 3. Notice, however, that deviation to 4 gives a revenue of 4 as long as at least one of the other sellers prices at 6; and he expects to earn $(3/4) \times 4$ when all other sellers price at 4.

The remaining equilibrium candidates are various permutations of 2 and 4. $(2,2,2,2)$ is not an equilibrium as there is a profitable deviation to 4, i.e., (4) is violated: $4/3 < 4/2$. Because this is the most unfavorable pricing case for a seller who prices at 4, the deviation condition has to hold as well for the next three equilibrium candidates: $\{(2,2,2,4),(2,2,4,4),(2,4,4,4)\}$. It follows that $(4,4,4,4)$ is an equilibrium.

- **Subgame** $m = 5$**:** As in the previous case ($m = 4$), no seller would price at either 8 or 6. Parallel arguments apply and we will not repeat them here.

  No seller will ever price at 0. With zero price he would earn zero while pricing at 2 yields a strictly positive expected revenue, i.e., at least $(4/5) \times 2$. This leaves us with several equilibrium candidate pricing profiles that are various permutations of 2 and 4. Observe first that $(4,4,4,4,4)$ is an equilibrium. A unilateral deviation to 2 would give 2, which is less than the expected revenue from pricing at 4: $(3/5) \times 4$. $(2,2,2,2,2)$ is also an equilibrium. Deviating to 4 would give the seller zero revenue. From this we can also conclude that no pricing profile which involves a unilateral deviation from one of the two equilibria can itself be an equilibrium. Among the remaining pricing profiles none is an equilibrium. To see this, let us restrict attention to pricing profiles where at least two sellers price at 2 and another two sellers price at 4. For all these profiles there is a profitable deviation from 4 to 2. By pricing at 2 the seller can guarantee a revenue of 2, which is more than the most he can get from keeping the price at 4: $(1/3) \times 4$.

In summary, the pure strategy equilibria in the pricing subgame, for each possible m are as follows:

- **Subgame** $m = 1$**:** (8)
- **Subgame** $m = 2$**:** $(6,6)$

- **Subgame** $m = 3$**:** $(4, 4, 4), (4, 4, 6), (4, 6, 6), (6, 6, 6)$
- **Subgame** $m = 4$**:** $(4, 4, 4, 4)$
- **Subgame** $m = 5$**:** $(2, 2, 2, 2, 2), (4, 4, 4, 4, 4)$

B.2.4. *Advance Production*

Lastly, let us take a look at the advance production decisions. Each of the sellers has a binary choice to produce or not. Denote the production strategy as $\sigma \in [0, 1]$. We focus on equilibria in symmetric production strategies. Denote by $E(m) = P_1, ..., P_m$ an equilibrium, in a pricing subgame $m \in \{1, 2, 3, 4, 5\}$. Let $\pi_i(E(m))$ denote the seller i's expected revenue in the pricing subgame when sellers price according to equilibrium $E(m)$. (Seller entry/exit subsidies do affect decisions at this stage, and are included in the analysis thereof.) The seller's expected profit from producing an object is given by

$$\sum_{l=0}^{4} \binom{4}{l} \sigma^l (1-\sigma)^{4-l} \pi_i(E(l+1)) - MC + 1 \tag{7}$$

The profit from not producing is 1. The seller will choose to produce when

$$\sum_{l=0}^{4} \binom{4}{l} \sigma^l (1-\sigma)^{4-l} \pi_i(E(l+1)) \geq MC. \tag{8}$$

In our experiment $MC \in \{2, 4, 6, 8\}$. Let us first focus on equilibria in pure strategies. There are only two candidates for subgame perfect equilibria in symmetric pure production strategies: (i) none of the sellers produces ($\sigma = 0$) and (ii) all sellers produce ($\sigma = 1$). When $MC = 8$, not producing is an equilibrium for either type of queuing. The reason is that the seller can break even only in the case when she is the only one producing. In all other cases, he is better off not producing (because the prices are less than 8). However, in any symmetric equilibrium, $\sigma > 0$, all pricing subgames are reached with positive probability. Hence, the expected revenue falls short of the cost of production. This rules out any symmertic equilibrium with $\sigma > 0$.

The second candidate, $\sigma = 1$, can be also sustained as an equilibrium, for either type of queuing, when either (i) $MC = 2$ and $P_5 = (2, 2, 2, 2, 2)$, or (ii) when $MC = 4$ and $P_5 = (4, 4, 4, 4, 4)$ in the $m = 5$ subgame. The reason is simple: when all other sellers produce, it is optimal for the seller to produce when the expected revenue in the $m = 5$ subgame at least covers his cost of production. That is the case under (i) and (ii).

For all other parametric configurations, a symmetric subgame perfect equilibrium involves mixed production strategies. Depending on which pricing equilibria are played in various subgames $m \in \{1, 2, 3, 4, 5\}$, the equilibrium production probability σ satisfies the following indifference condition that balances the costs and benefits of producing a unit:

$$
\begin{aligned}
MC = &\binom{4}{0}\sigma^0(1-\sigma)^4\pi_i(E(1)) + \quad \binom{4}{1}\sigma^1(1-\sigma)^3\pi_i(E(2)) + \\
&\binom{4}{2}\sigma^2(1-\sigma)^2\pi_i(E(3)) + \quad \binom{4}{3}\sigma^3(1-\sigma)^1\pi_i(E(4)) + \\
&\binom{4}{4}\sigma^4(1-\sigma)^0\pi_i(E(5)).
\end{aligned} \tag{9}
$$

There is a unique mixture σ for each combination of pricing equilibria in various subgames $\{E(1), ..., E(5)\}$, with the exception of the cases where equilibria are in pure strategies.

Deriving the subgame perfect equilibrium predictions for the advance production decisions requires that we specify pricing equilibria in all six pricing subgames $m \in 0, ..., 5$. Because of the multiplicity of such equilibria under both types of queuing, there are too many possibilities to consider. There is also no satisfactory equilibrium selection criterion that we could rely on to pick the most likely candidate for each subgame. Instead of explicitly considering all combinations of possible equilibria, we examine equilibria that give us the upper and the lower bound on subgame perfect productions decisions. In other words, we consider the maximal and the minimal selection form the equilibrium set, derive the subgame perfect production probabilities and highlight the differences between the two types of queuing.

We begin by noting that the maximal selection from the equilibrium set under the random queuing and the value queuing is the same: $\{\{8\},\{6,6\},\{6,6,6\},\{4,4,4,4\},\{4,4,4,4,4\}\}$; the minimal selection, however, is not. Under the value order queuing, the minimal selection is $\{\{8\},\{6,6\},\{4,4,4\},\{2,2,2,2\},\{0,0,0,0,0\}\}$, while under the random order queuing we have $\{\{8\},\{6,6\},\{4,4,4\},\{4,4,4,4\},\{2,2,2,2,2\}\}$. Hence, for $MC > 2$ we would expect strictly more production to take place under the value order queueing than under the random order queuing.

Table II enumerates the PSNE, including the subgame perfect PSNE incorporating the advance production stage.

B.2.5. *Numerical Derivation*

The functions below, coded in R, may be used to calculate the pure strategy Nash equilibria and sub-game perfect Nash equilibria for the parameters used in the study. The calls `spne(order="value")` and `spne(order="random")` enumerate the SPNE under value and random order queueing, respectively.

```
# Function: permutations
# Calculates the permutations of a vector sequence, x; returns a matrix.
permutations <- function(x,prefix=c())
{
    if(length(x)==0) { return(prefix); }
    do.call(rbind,sapply(1:length(x),FUN=function(idx)permutations(x[-idx],c( prefix,x[idx])),simplify = FALSE));
}

# Function: prob.sale
# Takes a vector of asks and for each returns the probability of a sale for each ask.
# The demand argument is a sequence of values in any order, e.g. {8,4,6,2,0}.
# The order argument takes {"random","value"}, for the order of buyer action.
prob.sale <- function(ask,demand=seq(8,0,-2),order="value")
{
    # Order asks accendingly.
    ask.order <- order(ask);
    ask <- ask[ask.order];
```

```
    # Define variables for storage of parameters and state-dependent outcomes.
    if (order=="random")     { value <- permutations(demand); }
    else if (order=="value") { value <- matrix(sort(demand,decreasing=TRUE),nrow=1,ncol=length(demand)); }
    else                     { stop("Argument \"order\" must be {\"random\",\"value\"}."); }
    bought <- matrix(0,dim(value)[1],length(ask)); # Create a matrix to store sales.

    # In each state,...
    for (state in 1:dim(value)[1])
    {
        # For each buyer,...
        for (buyer in 1:dim(value)[2])
        {
            # Look through the vector of asks...
            bought.buyer <- 0;
            for (seller in 1:length(ask))
            {
                # And buy the first unsold surplus-generating unit.
                if (bought.buyer==0&bought[state,seller]==0&value[state,buyer]>=ask[seller])
                {
                    bought[state,seller] <- 1;
                    bought.buyer <- 1;
                }
            }
        }
    }

    # Find the probabiliy of a sale.
    probability <- colMeans(bought);

    # For equivalent asks, merge probabilities (i.e., random tie-breaking).
    for (offer in unique(ask))
    {
        probability[offer==ask] <- mean(probability[offer==ask]);
    }

    # Return probability of sale, reordering if not submitted in value order.
    probability[ask.order] <- probability;
    return(probability)
}

# Function: expected.value
# Takes a vector of asks and for each returns the expected value to each seller.
# The demand argument is a sequence of values in any order, e.g. {8,4,6,2,0}.
# The order argument takes {"random","value"}, for the order of buyer action.
expected.value <- function(ask,demand=seq(8,0,-2),order="value")
{
    return(prob.sale(ask,demand,order)*ask);
}

# Function: psne
# Calculates the pure strategy Nash equilibria.
# The m argument takes the number of sellers (i.e. entrants).
# The demand argument is a sequence of values in any order, e.g. {8,4,6,2,0}.
# The order argument takes {"random","value"}, for the order of buyer action.
psne <- function(m,demand=seq(8,0,-2),order="value")
{
    # If there is only one seller, return the highest value on the demand curve.
    if (m==1) { return(max(demand)); }
```

```
# Get all possible pure strategies for stage 2 and create storage for expected values.
asks.j <- unique(t(combn(sort(rep(demand,m-1)),m-1)));           # Find all combinations of opponent asks.
asks.i <- kronecker(rep(1,dim(asks.j)[1]),sort(demand));         # Create a vector of possible responses.
asks   <- cbind(kronecker(asks.j,rep(1,length(demand))),asks.i); # Combine to create a matrix of asks.
colnames(asks) <- NULL;                                          # Clear column names from cbind.

# Iterate through asks, calculating expected values for each.
ev   <- matrix(NA,dim(asks)[1],dim(asks)[2]);
for (ask in 1:dim(asks)[1])
{
    ev[ask,] <- expected.value(asks[ask,],demand,order);
}

# Get unique combinations of opponent strategies and create space to store best responses.
br    <- matrix(NA,nrow=dim(asks.j)[1],ncol=length(demand))
br.ev <- matrix(NA,nrow=dim(asks.j)[1],ncol=length(demand))

# Search through all unique opponent strategies for best responses.
for (j in 1:dim(asks.j)[1])
{
        asks.index <- apply(as.matrix(asks[,-m]),1,identical,asks.j[j,]); # Find relevant rows.
        ev.response <- ev[asks.index,m];                                  # Find expected value of all responses.
        br.index    <- as.logical(ev.response==max(ev.response));         # Find best responses.
        br[j,1:sum(br.index)]    <- asks[asks.index,m][br.index];         # Store best responses strategies.
        br.ev[j,1:sum(br.index)] <- ev[asks.index,m][br.index];           # Store expected value of best responses.
}

# Create a list of possible equilibria.
x <- matrix(NA,0,m)
for (col in 1:dim(br)[2])
{
    x.add <- cbind(asks.j,br[,col]);
    x <- rbind(x,na.omit(x.add))
}

# Determine if possible equilibria are, in fact, pure startegy Nash equilibria (PSNE).
equilibrium <- rep(TRUE,dim(x)[1])
for (row in 1:dim(x)[1])
{
    # Get unique permutations of the row, with other players strategies sorted as per the asks above.
    x.permutations <- unique(permutations(x[row,]));
    for (r in 1:dim(x.permutations)[1]) { x.permutations[r,-m] <- sort(x.permutations[r,-m]); }
    x.permutations <- unique(x.permutations);

    # Go through all permutations of the row, finding the best reponse.
    for (r in  1:dim(x.permutations)[1])
    {
        # All permutations must be a best response to other players' strategies to support PSNE.
        row.match <- apply(as.matrix(x[,-m]),1,identical,x.permutations[r,-m])
        equilibrium[row] <- as.logical(any(x[row.match,m]==x.permutations[r,m])*equilibrium[row]);
    }
}

# Eliminate reundant equilibria.
x.equilibrium <- x[equilibrium,]
if (is.matrix(x.equilibrium))
{
    for (r in 1:dim(x.equilibrium)[1]) { x.equilibrium[r,] <- sort(x.equilibrium[r,]); }
```

```
        x.equilibrium <- unique(x.equilibrium);
    }

    # Return a list of pure strategy equilibria.
    return(x.equilibrium)
}

# Function: prob.purchase
# Takes a vector of asks and for each returns the probability of a purchase for each buyer value on the demand curve.
# The demand argument is a sequence of values in any order, e.g. {8,4,6,2,0}.
# The order argument takes {"random","value"}, for the order of buyer action.
# The surplus argument takes {"total","buyer","seller"}, for the surplus to be calculated.
# The h argument is the marginal cost of producing one unit, or the cost of entry.
expected.surplus <- function(ask,demand=seq(8,0,-2),order="value",surplus="total",h=NA)
{
    # Order asks accendingly.
    ask.order <- order(ask);
    ask <- ask[ask.order];

    # Define variables for storage of parameters and state-dependent outcomes.
    if      (order=="random") { value <- permutations(demand); }
    else if (order=="value")  { value <- matrix(sort(demand,decreasing=TRUE),nrow=1,ncol=length(demand)); }
    else                      { stop("Argument \"order\" must be {\"random\",\"value\"}."); }
    bought <- matrix(0,dim(value)[1],length(ask));    # Create a matrix to store sales.

    # Define variables for storage of surplus.
    if (surplus=="buyer"|surplus=="total")  { surplus.buyer  <- matrix(0,dim(value)[1],length(demand)); }
    if (surplus=="seller"|surplus=="total") { surplus.seller <- matrix(0,dim(value)[1],length(ask)); }
    if (surplus!="seller"&surplus!="buyer"&surplus!="total")
{
    stop("Argument \"surplus\" must be {\"total\",\"buyer\",\"seller\"}.");
}
    if ((surplus=="seller"|surplus=="total")&is.na(h)) { stop("Argument \"h\" must be specified."); }

    # In each state,...
    for (state in 1:dim(value)[1])
    {
        # For each buyer,...
        for (buyer in 1:dim(value)[2])
        {
            # Look through the vector of asks...
            bought.buyer <- 0;
            for (seller in 1:length(ask))
            {
                # And buy the first unsold surplus-generating unit.
                if (bought.buyer==0&bought[state,seller]==0&value[state,buyer]>=ask[seller])
                {
                    bought[state,seller] <- 1;
                    bought.buyer <- 1;

                    if (surplus=="buyer"|surplus=="total")
{
surplus.buyer[state,buyer] <- value[state,buyer]-ask[seller];
}
                    if (surplus=="seller"|surplus=="total") { surplus.seller[state,seller] <- ask[seller]-h; }
                }
            }
        }
```

```
        # If no seller sells, they still incur their entry cost.
        if (surplus=="seller"|surplus=="total")
        {
            for (seller in 1:length(ask))
            {
                if(bought[state,seller]==0) { surplus.seller[state,seller] <- -h; }
            }
        }
    }

    # For sellers with the same ask, calculate the expected surplus, rather than surplus by order in the buyer queue.
    for (offer in unique(ask))
    {
        surplus.seller[ask==offer] <- mean(surplus.seller[ask==offer]);
    }

    # Find and return expected surplus.
    if      (surplus=="buyer")  { return(colMeans(surplus.buyer)); }
    else if (surplus=="seller") { return(colMeans(surplus.seller)); }
    else if (surplus=="total")  { return(sum(colMeans(surplus.buyer))+sum(colMeans(surplus.seller))); }
}

# Function: spne
# Calculates and enumerates the subgame perfect Nash equilibria.
# The demand argument is a sequence of values in any order, e.g. {8,4,6,2,0}.
# The order argument takes {"random","value"}, for the order of buyer action.
# The h argument is the marginal cost of producing one unit, or the cost of entry.
spne <- function(demand=seq(8,0,-2),order="value",mc=seq(8,2,-2),list.all.psne=FALSE)#h=seq(8,2,-2))
{
    # Declare the entry subsidy. (Does not affect PSNE or SPNE since it is added in all cases.)
    v <- 1;

    # Create space to store the SPNE.
    x <- data.frame(h=numeric(),m=numeric(),asks=character(),surplus=character(),stringsAsFactors=FALSE);

    # Iterate through marginal costs.
    for (h in mc)
    {
        # Iterate through the possible number of entrants.
        for (m in 1:length(demand))
        {
            # Calculate PSNE pricing given m entrants (and turn vectors in row matrices).
            psne.m <- psne(m,demand=demand,order=order);
            if (!is.matrix(psne.m)) { psne.m <- matrix(psne.m,nrow=1); }

            # Iterate through strategies in PSNE and determine if they support entry.
            for (s in 1:dim(psne.m)[1])
            {
                # Equilibrium occurs where there is no positive incentive to enter or exit.
                # Equilibrium can also occur where there is a positive incentive to enter if there are max entrants.
                pi <- as.character(expected.surplus(psne.m[s,],demand=demand,order=order,surplus="seller",h=h)+v);
                if (list.all.psne|(all(surplus==v)|(m==length(demand)&all(surplus>=v))))
                {
                    x <- rbind(x,data.frame(h,m,paste(as.character(psne.m[s,]),collapse=","),paste(pi,collapse=",")));
                }
            }
        }
    }
    # Apply user-friendly column names.
```

```
    colnames(x) <- c("h","m","asks","surplus");

    # Return a list of subgame perfect equilibria.
    return(x);
}
```

B.3. *Dynamic Incentives in the Repeated Game*

B.3.1. *Collusion*

We now explore dynamic incentives in our experiment. Recall that the experimental environment consists of several stationary and nonstationary sequences of rounds. In each stationary sequence $MC = 4$; in the non-stationary sequence, each value of $MC \in \{2, 4, 6, 8\}$ comes up exactly once in a scrambled order. Throughout the experiment subjects encounter each value of MC at least $16 \times 2 = 32$ times. We will refer to this game as a large finitely-repeated game (LFRG).

When it comes to LFRG, if there are multiple subgame perfect Nash equilibria (SPNE), then it is typically the case that there are too many of them to enumerate. This is true in our case as well. We will therefore not attempt to characterize the whole set of equilibria. Instead, we will focus on a specific question of whether players can use dynamically optimized strategies to collude on some output-price pair that is more profitable than playing the best SPNE in every stage. In doing so, we will also assume that there is no discounting and our players are able to coordinate on asymmetric stage-game SPNE.

First, let us define the best collusive outcome as an output-price policy that maximizes the total stage-game profit. The table below gives the optimal collusive outcomes for each value of MC.

MC	2	4	6	8
Price profile	(6,6)	{(6,6),(8)}	(8)	(8)

Note, other collusive arrangements are possible, e.g., $(4, 4, 4)$ in $MC = 2$

case, but they are less profitable.

In the next claim we argue that players are unable to benefit from collusion when the marginal cost is higher than 2.

Proposition 6 *In stage-games where $MC \in \{4, 6, 8\}$, players cannot achieve a greater stage-game payoff by colluding, than by playing one of the stage-game SPNEs.*

Proof. The proof follows directly from the observation that in all of these cases, when $MC > 2$, one of the profit-maximizing outcomes is also a stage-game SPNE. Because these equilibria are asymmetric, only some players (who happen to produce) benefit. All players, however, can achieve the greatest expected profit by taking turns as they go through the rounds of the experiment (please see below where we explicitly illustrate this procedure). ■

This leaves us with the case of $MC = 2$. In rounds where $MC = 2$, colluding on $(6, 6)$ can generate greater total profit than playing the most profitable stage-game SPNE $(4, 4, 4, 4)$. Next we will construct an SPNE of the LFRG in which players collude in almost all $MC = 2$ rounds. We take the standard route of building a carrot-and-stick trigger strategy by exploiting the fact that some stage-game SPNEs yield higher and some yield lower profit.

Let T denote the last round and consider the following strategy profile.

Equilibrium: *On-path: in all rounds $t \leq T - 4$, players produce and price according to the schedule in the table below.*

Round MC	*2*	*4*	*6*	*8*
Price profile	*(6,6)*	*(6,6)*	*(8)*	*(8)*

In the last four rounds players play the lowest price stage-game SPME corresponding to the actual round MC as stated in Table 2, panel B.

Off-path: in any off-path pricing subgame (where m is from what it should be on-path according to the table above), players play the lowest price NE *as stated in Table 2, panel A of the paper; once the history of play is off-path, then in each subsequent round players play the lowest price stage-game SPNE corresponding to the actual round MC as stated in Table 2, panel B.*

To ensure that all players benefit from collusion, players take turns entering the market by cycling in a clock-wise direction. To help visualize this, consider the player circle shown below:

1

5 *2*

4 *3*

In the first round, only the player 1 enters if $MC \in \{6, 8\}$; or players 1 and 2 enter if $MC \in \{2, 4\}$. Then, in every subsequent round, the entering player is the one (if $MC \in \{6, 8\}$) or the two (if $MC \in \{2, 4\}$) who are immediate successors in terms of the player-index to the previous round entrants in the clock-wise direction.

Let us now verify that this constitutes an SPNE of our LFRG. In the last $T - 12$ periods players play a stage-game SPNE on equilibrium path. Hence, there are no incentives to deviate. Similarly, in all earlier rounds where $MC > 2$ the trigger strategy prescribes a stage-game SPNE. Hence, there are no incentives to deviate in those rounds as well. This leaves us with the rounds (except the last one) where $MC = 2$

Deviation in any one of those rounds triggers the "punishment phase". Hence, if we can prevent a deviation in one of the $MC = 2$ rounds, we prevent a deviation in all of them. Consider an $MC = 2$ round. On the equilibrium path, exactly two players produce and price at 6. These two players cannot do any better, and hence, do not have a profitable deviation. The remaining four players do not produce. Consider one of those four players. The only alternative to on-path pay is to produce a unit. Then, all players observe that there are 3 rather than the expected 2 market entrants, i.e., firms know one of them has deviated. The play is off-the-equilibrium path and players price via the demand schedule: $(4,4,4)$. Hence, the deviating player gains $4-2=2$ relative to saying out of the market. However, form that point on all players produce and price via the demand schedule if $MC>2$ in all subsequent rounds. In particular, in all stationary rounds in $t \in \{T-12,...,T-4\}$, where $MC=4$, the players produce-and-price $(4,4,4)$, earning zero net gain relative to staying out of the market. If the play remained on-path, the players would have played $(6,6)$ which yields the producing players a net gain of $3-1=2$ relative to staying out of the market. And because on the equilibrium path the players take turns producing and there are $8>5$ stationary rounds, it is guaranteed that each of the players would have gained at least 2 from staying on-the-equilibrium path. Hence, a deviation is not profitable.

This equilibrium is not the punishment "tightest" trigger strategy equilibrium that can be designed, but it is one of the most transparent ones. Our objective here was to clearly illustrate that collusion is possible in our game, but if it were to happen it is likely going to be restricted to rounds in which $MC=2$. If players were to collude in other rounds they would have to do it at production-price profiles that are less profitable than what they can achieve without any collusion in an individual round SPNE. It is worth highlighting that along the way of constructing this trigger strategy equilibrium we have used some possibly strong assumptions, such as, that players can coordinate on asymmetric strategies and are able to cycle across rounds, that they know everyone's MC and that they know the structure of the rounds (stationary/non-stationary) forthcoming. None of this was, however, a common knowledge in our experiment.

B.3.2. *Fictitious Play and Learning with Incomplete Information*

Let us examine more closely whether subjects who lack information on the payoffs of their opponents are able to lock into one of the equilibria over time. One of learning or evolutionary models in the literature might shed some light on this question. There is a wide variety of such models that have been applied to different economic games. For instance, evolutionary models have been used to sift through a myriad of dynamic strategies in repeated minority game (Linde, Sonnemans, and Tuinstra, 2014) and repeated prisoner's dilemma game (Romero and Rosokha, 2019). Reinforcement learning and fictitious play have been studied and tested in the context of the market entry game in Duffy and Hopkins (2005). More sophisticated models, such as, experience weighted attraction (Camerer and Ho, 1999) and inertia, sampling and weighting (I-SAW) model (see Erev et al. 2010), have been pitted against one another in a market entry prediction competition where the latter outperforming the former. A battery of learning and evolutionary models have been examined within the context of normal form game play in Pangallo et al. (2019). This work uncovers a relationship between the process of best-reply dynamics and convergence of various learning algorithms. Finally, models of imitation along with several other learning heuristics (such as, win-continue, lose-reverse) have been examined in the context of a repeated Cournot game (i.e., Friedman et al. 2015). The results point to a two-tier model where the initial learning via imitation is later replaced with a more cooperative heuristic (also see Huck et al. 2017).

Out of this multitude of possibilities we pick a model that is simple, workable and has been applied in the same economic context that we work with, i.e., the market entry game. In this sense Duffy and Hopkins (2005), hereafter DH, is probably the closest to what we do. They compare the outcomes of reinforcement learning and fictitious play in a market entry game. The latter converges more rapidly and has a better fit with the experimental data. In the long run (over a span of 96 periods) the learning process converges to a noisy equivalent of asymmetric Nash equilibrium in pure strategies. This is indeed what we observe in the experiment. The setup of DH differs from ours in three important ways. First, DH consider a "bare" market entry game. The pricing subgame is hardwired in the payoff function. Second, DH assume that players have a complete information regarding their payoffs, i.e., there is no uncertainty regarding marginal costs. And third, the payoff function does not change between rounds of repetition. Let us address each of these in turn.

In order to reconcile the first of the above differences between DH's market entry game and our advance production posted offer market (which can be thought of as a relaxation of the market entry game), we will in our simulations impose an assumption about pricing. To this end, notice that conditional on producing an item and entering the market, the uncertainty regarding marginal costs of the opponents is a non-issue. At that point marginal costs are sunk, and hence, in the pricing subgame all firms are strategically identical. One could therefore broadly divide the learning problem into two parts: (i) learning production and (ii) learning pricing. To address our case as simply and directly as we can, we will abstract from learning pricing, and focus on learning production (i.e. entry). Thus we will assume that pricing is via the demand schedule following the competitive pricing rule.

The second difference between our experiment and DH has to do with the uncertainty regarding marginal costs. When it comes to stochastic fictitious play (as well as reinforcement learning), however, whether marginal costs are known or unknown should not make any difference. But this might matter for the actual behavior in the experiment. Our simulations confirm the former. Our experiments then demonstrate that behavior is also qualitatively consistent with the simulations.

Lastly, the third difference has to do with variation in the payoff function between rounds. Our experiment involved an alternating pattern of blocks of stationary and non-stationary rounds. Again, our simulations demonstrate that even with this alternating pattern, the production strategy profile tends toward the (noisy equivalent of) asymmetric pure strategy equilibrium.

Our stochastic fictitious play model involves the following structure. The payoffs are:

$$\pi_i(s_{i,t} = 1; MC_t, m) = 8 - 2(m-1) - MC_t$$

and

$$\pi_i(s_{i,t} = 0; MC_t, m) = 0,$$

where $s_{i,t} \in \{0,1\}$ represents the production decision (1 denotes "produced a unit" and 0 that the firm stayed out of the market) and MC_t is a production cost for firm i in a given round t (i.e., $MC_{i,t} = MC_t$). Firms' beliefs are captured by h_t which is a vector with elements $h_{i,t}$ indicating the frequency of production decisions up to round t for each firm i. $h_{i,t}$ is updated according to the following rule

$$h_{i,t+1} = \frac{\delta h_{i,t} + s_{i,t}}{\delta t + 1}.$$

The δ parameter reflects limited memory and discounts the more distant past in an exponential manner.

The decision rule is $\sigma_{i,t}$ which gives the probability of producing a unit. In round t, for a given h_t, each decision maker evaluates his expected payoff from producing a unit $E\pi_{i,t}(s_{i,t} = 1; MC_{i,t})$ and from staying out of the market $E\pi_{i,t}(s_{i,t} = 0; MC_t)$. Let $\Delta_{i,t}$ denote the difference. The firm i's decision of whether to produce or not is governed by a perturbed best-reply rule

$$\sigma_{i,t} = \begin{cases} 1 - \varepsilon^{\theta(1+\Delta_{i,t})} & \text{if} \quad \Delta_{i,t} \geq 0 \\ \varepsilon^{\theta(1-\Delta_{i,t})} & \text{if} \quad \Delta_{i,t} < 0 \end{cases},$$

where ε is an error term and θ is the payoff salience parameter.

We present outcomes of the simulations for three scenarios: 1. stationary rounds only (where $MC_{i,t} = 4$ in all rounds); 2. non-stationary rounds only (where in each block of 4 rounds $MC_{i,t}$ is chosen at random without replacement from $\{2,4,6,8\}$); and 3. the case that corresponds to our experimental implementation, where blocks of 8 stationary and non-stationary rounds alternate in regular intervals. We ran 1000 simulations for each market over the span of 200 rounds. Parameters were set at the following values $\varepsilon = 0.05$, $\theta = 1$, $\delta = 0.9$, and $h_{i,0} \in []0,1]$ was determined randomly. The results are illustrated respectively in Figures C1, C2, and C3.

For each round, we first order the $h_{i,t}$ from the lowest to the highest. Then, in the first panel of each figure we display the lowest $h_{i,t}$ in each of the 1000 simulations; the second panel in a row shows the same but for the second lowest $h_{i,t}$; etc. In each panel we also display the path through cross-sectional sample means as well as the 95% confidence region highlighted in gray color.

We make three observations: 1. when marginal costs are stationary, the learning process seems to converge toward (a noisy equivalent of) asymmetric equilibrium strategy profile with 2 or 3 firms producing; 2. when marginal costs are not stationary, we also get a convergence but to a different strategy profile, namely one where all firms randomize with equal probability; 3. in the setting which mirrors our experimental implementation we get a similar tendency toward the asymmetric strategy profile as in the first case, but this time the non-stationary blocks clearly add some noise and disturbance to the process.

B.3.3. *Numerical Derivation*

The code below, when compiled in Python, may be used to calculate the figures reported in Figures C1, C2, and C3.

```
#Needs numpy, numba and decimal
import numpy as np
import itertools as itrt
from numba import jit
```

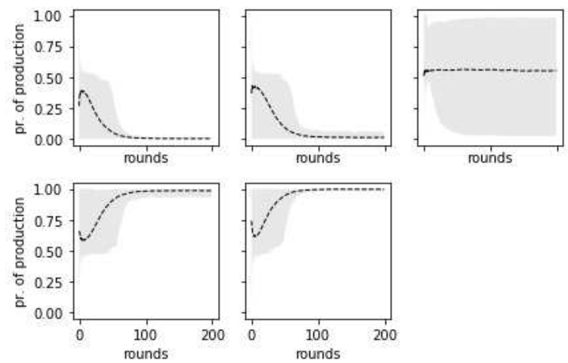

Figure C1: Simulations with Stationary Rounds Only

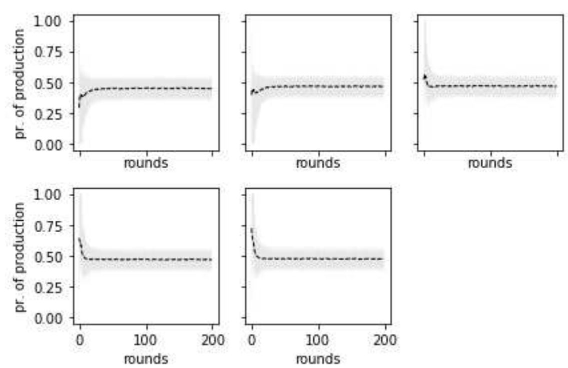

Figure C2: Simulations with Nonstationary Rounds Only

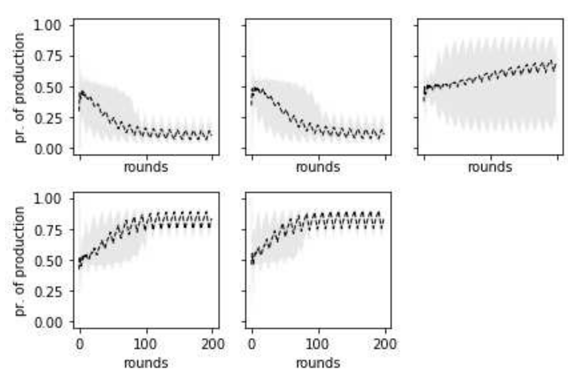

Figure C3: Simulations with Alternating Blocks

```
import decimal

def Error_rule_simple(err, u_one, u_two, theta):
    if u_one >= 0:
        Pr_SM = 1 - np.power(err,theta*(1+u_one))
    else:
        Pr_SM = 0 + np.power(err,theta*(1+np.absolute(u_one)))
    #print(Pr_SM)
    return Pr_SM

def Get_p_mc(step, MC, P, stationary):
    if stationary == 1:
        return [el - 4 for el in P], MC
    else:
        return [el - MC[0] for el in P], MC

def Outcome_distr(sig_noti,P_mc):
    num_noti = len(sig_noti)
    num_mrk_out = len(sig_noti)+1
    sig_noti_rec = np.ones(num_noti) - sig_noti

    #create permuations matrix for all possile outcomes
    I_sig = np.array(list(itrt.product(range(2), repeat=num_noti)))
    I_sig_rec = np.subtract(np.ones(I_sig.shape),I_sig).astype(int)

    #multiply by the probabilities cell-by-cell
    Pr_all_sig = I_sig * sig_noti
    Pr_all_sig_rec = I_sig_rec * sig_noti_rec
    Pr_all = Pr_all_sig + Pr_all_sig_rec

    Pr_all = np.prod(Pr_all, axis = 1)

    #sum instances for same market outcomes
    Out_sum = []
    for i in range(num_mrk_out):
        Out_sum.append([1 if i==sum(el) else 0 for el in I_sig])
    Out_sum = np.array(Out_sum)

    #pr. distr. over market outcomes
    Dstr_out = np.dot(Out_sum, Pr_all)

    #calculate expected payoff
    Exp_pi = np.dot(Dstr_out, P_mc)

    return Exp_pi

def Update_beliefs_actions_FL(act_counter, sigma, P_mc, err, theta, delta):
    Exp_pi_enter = []
    for i in range(len(sigma)):
        #create sigma not i vector
        sig_noti = [sigma[j] for j in range(len(sigma)) if j != i]

        Exp_pi_enter.append(Outcome_distr(sig_noti,P_mc))

    for i in range(len(sigma)):
        #comapre exp. payoffs and bump counter
        exp_pi = Exp_pi_enter[i]
        max_pi = 6
        #print(exp_pi)
```

```
        #deterministic rule
        #if 1 < exp_pi:
        #stochastic rule
        act_counter[i,:] = delta*act_counter[i,:]
        if np.random.rand() < Error_rule_simple(err, exp_pi, 0, theta):
            act_counter[i,0] += 1
        else:
            act_counter[i,1] += 1

        #recalculate sigma
        sigma[i] = act_counter[i,0]/sum(act_counter[i,:])

    return act_counter, sigma

def Run_fict_learning(err, theta, delta, it, players, P, stationary):
    #probabilities of mrk. entry for the 5 players
    sigma = np.random.rand(players)
    #counter of how many times each player chose the given strategy {enter, stay out}
    act_counter = np.zeros((players,2))

    Results[0][it][:] = sigma

#This is the part where to switch between:
    #only stationary (set synch_switch = 1 #*********** to synch_switch = 5 #*********** and comment out synch_switch = 1 #$$$$$$$$$$$$$) vs.
    #nonstationary (set set synch_switch = 1 #*********** and enable synch_switch = 1 #$$$$$$$$$$$$$) vs.
    #alternating (set set synch_switch = 1 #*********** and comment out synch_switch = 1 #$$$$$$$$$$$$$)
    synch_switch = 5 #***********
    MC = []
    for step in range(maxit):
        if len(MC) <= 1:
            if synch_switch == 1 or synch_switch == 4:
                MC = [2,4,6,8]
                np.random.shuffle(MC)
                if synch_switch >= 4:
                    synch_switch = 1
            else:
                MC = [4,4,4,4]
            synch_switch += 1
        else:
            MC = MC[1:]
        #synch_switch = 1   #$$$$$$$$$$$$$
        P_mc, MC = Get_p_mc(step, MC, P, stationary)

        #print('+++++++++++++++ run round +++++++++++++++')
        act_counter, sigma = Update_beliefs_actions_FL(act_counter, sigma, P_mc, err, theta, delta)

        Results[step][it][:] = sorted(sigma)

    return Results

#parameters
maxit = 200
no_iterations = 1000
err = 0.05
theta = 1
delta = 0.9

#if rounds are stationary then 1
```

```
stationary = 0
#players
players = 5
#market payoffs
P = [8,6,4,2,0]

global Results
Results = np.zeros((maxit, no_iterations, players))

for it in range(no_iterations):
    Run_fict_learning(err, theta, delta, it, players, P, stationary)


#Process results and plot
x = []
pl_one = []
pl_two = []
pl_three = []
pl_four = []
pl_five = []

indx = 1
for el in Results:
    for els in el:
        x.append(indx)
        pl_one.append(els[0])
        pl_two.append(els[1])
        pl_three.append(els[2])
        pl_four.append(els[3])
        pl_five.append(els[4])
    indx += 1

x = np.zeros((maxit))
ln_one = np.zeros((maxit,3))
ln_two = np.zeros((maxit,3))
ln_three = np.zeros((maxit,3))
ln_four = np.zeros((maxit,3))
ln_five = np.zeros((maxit,3))

_round = 0
for el in Results:
    x[_round] = _round
    ln_one[_round,0] = np.mean(Results[_round,:,0])
    ln_one[_round,1] = sorted(Results[_round,:,0])[50]
    ln_one[_round,2] = sorted(Results[_round,:,0])[-50]
    ln_two[_round,0] = np.mean(Results[_round,:,1])
    ln_two[_round,1] = sorted(Results[_round,:,1])[50]
    ln_two[_round,2] = sorted(Results[_round,:,1])[-50]
    ln_three[_round,0] = np.mean(Results[_round,:,2])
    ln_three[_round,1] = sorted(Results[_round,:,2])[50]
    ln_three[_round,2] = sorted(Results[_round,:,2])[-50]
    ln_four[_round,0] = np.mean(Results[_round,:,3])
    ln_four[_round,1] = sorted(Results[_round,:,3])[50]
    ln_four[_round,2] = sorted(Results[_round,:,3])[-50]
    ln_five[_round,0] = np.mean(Results[_round,:,4])
    ln_five[_round,1] = sorted(Results[_round,:,4])[50]
    ln_five[_round,2] = sorted(Results[_round,:,4])[-50]
    _round += 1
```

```
#import numpy as np
import pylab as plt

fig, axs = plt.subplots(2, 3, sharex=True, sharey=True)

axs[0,0].plot(x, ln_one[:,0], lw=1, label='mean', color='black', ls='--')
axs[0,0].fill_between(x, ln_one[:,1], ln_one[:,2], facecolor='lightgray', alpha=0.5,
                label='95% range')
axs[0,0].set_xlabel('rounds')
axs[0,0].set_ylabel('pr. of production')


axs[0,1].plot(x, ln_two[:,0], lw=1, label='mean', color='black', ls='--')
axs[0,1].fill_between(x, ln_two[:,1], ln_two[:,2], facecolor='lightgray', alpha=0.5,
                label='95% range')
axs[0,1].set_xlabel('rounds')


axs[0,2].plot(x, ln_three[:,0], lw=1, label='mean', color='black', ls='--')
axs[0,2].fill_between(x, ln_three[:,1], ln_three[:,2], facecolor='lightgray', alpha=0.5,
                label='95% range')
axs[0,2].set_xlabel('rounds')

axs[1,0].plot(x, ln_four[:,0], lw=1, label='mean', color='black', ls='--')
axs[1,0].fill_between(x, ln_four[:,1], ln_four[:,2], facecolor='lightgray', alpha=0.5,
                label='95% range')
axs[1,0].set_xlabel('rounds')
axs[1,0].set_ylabel('pr. of production')


axs[1,1].plot(x, ln_five[:,0], lw=1, label='mean', color='black', ls='--')
axs[1,1].fill_between(x, ln_five[:,1], ln_five[:,2], facecolor='lightgray', alpha=0.5,
                label='95% range')
axs[1,1].set_xlabel('rounds')

axs[1,2].axis('off')

plt.tight_layout()
plt.show()
```

APPENDIX C: INSTRUCTIONS (FOR ONLINE PUBLICATION)

Note: Subject instructions did not have headings indicating the treatment; these are provided for the convenience of the reader only.

C.1. *Instructions Given at the Beginning of Each Session*

C.1.1. *Overview*

You are about to participate in an experiment in the economics of decision-making. If you follow these instructions carefully your decisions might earn you a considerable amount of money which will be paid to you in cash at the end of the experiment. If you have a question at any time, please feel free to ask the experimenter. We ask that you not talk otherwise for the duration of the experiment. Also, please turn off your cell-phone and do not use the computer for any other purpose than your participation in the experiment requires. If you break these rules, we will have to exclude you from the experiment and from all payments. Each of you is seated at a computer workstation. You will use these computer workstations to enter information by means of mouse clicks and/or typing. (If you happen to have entered a number and things do not work, please make sure you also haven't hit the space bar).

C.1.2. *Earnings*

Every participant will get 5 NZD as a show up fee for today's session, and a separate 5 NZD as a deposit. In addition, you will have the opportunity to earn money in the experiment.

The experiment will consist of 6 separate segments. At the beginning of each segment you will receive detailed instructions. You will be paid for one randomly chosen decision from each segment. Note that each of your decisions has an equal chance of being chosen and you do not know in advance which will be chosen, so think about each decision carefully. Your earnings from all 6 segments will be summed with your deposit in order to determine how much you are paid, in total, within the actual experiment. (Your show up fee is separate and you get to keep it regardless.)

Note that it is also possible to lose money in a given segment. If the sum over all 6 segments' earnings is negative, but between zero and -4.99 NZD, this sum will be subtracted from your 5 NZD deposit and you will only get to keep the remainder of the deposit. If such a sum exceeded -5.00 NZD, you would lose your entire deposit, and then your total earnings from the experiment itself would be set to equal zero. If the sum over the 6 segments is greater than or equal to zero, you will get to keep both that non-negative sum of earnings over the 6 segments plus the entire deposit. Again, you will get to keep the show up fee regardless.

The payoffs in today's experiment will be denoted in experimental currency unit (ECU).

$$1 \text{ ECU} = 5 \text{ NZD}$$

Your ECUs will be converted to dollars at this rate, and you will be paid in NZD when you leave the lab. The more ECUs you earn, the more dollars you earn.

C.2. *Instructions for Treatment Value Order Human Buyer (VOHB)*

C.2.1. *Seller Instructions*

This Segment

In the rounds about to begin, and which will continue until further notice, there are 10 human participants: 5 acting as sellers and 5 acting as buyers. You are a seller. In each round, you will have the opportunity to make a decision between one of two possible actions. Once all participants have made their decisions, a second screen will appear which will report to you your payoff resulting from that round's events, and also the determinants of that payoff—namely your decision, and the decisions of others also participating. (More on this below.) There will be multiple rounds. Throughout these rounds you will stay in the same group of 5 sellers, while the group of 5 buyers will also stay the same.

The Sequence of Play in a Round

The first computer screen you see in each round asks you to make a decision between two actions: IN or OUT. You enter your decision by using the mouse to fill in the radio-button next to the action you wish to take. If you want to choose action IN, fill in the circle next to IN by clicking on it with the mouse; If you want to choose action OUT, fill in the circle next to OUT by clicking on it with the mouse. Once all participants have entered their decisions, a second screen will appear. This second screen reminds you of your decision for the round, informs you of your payoff for the round, and informs you of other determinants of your payoff (e.g. the decisions taken by other participants). Your payoff represents an amount in ECU that could be paid to you in cash (if the given round is randomly selected for payoff) as will be explained below.

How Payoffs are Determined

Payoffs are determined as follows:

- If you choose OUT your payoff for the round is equal to 1 (this is true in each round).
- If you choose IN, your payoff will be equal to $1 + \text{Price} - \text{MC}_i$. The components of this payoff are given by the following:
  - Price will be determined by (a) what you nominate as a price (which must be an even number) and (b) whether a buyer chooses to purchase from you at the price you nominate. There are 5 human buyers, each of whom can re-sell a purchased unit to the experimenter, such that:
    One buyer has a resale value of 8.
    One buyer has a resale value of 6.
    One buyer has a resale value of 4.
    One buyer has a resale value of 2.
    One buyer has a resale value of 0.
    (Note also that at the beginning of each round, you will be informed of the number of units at which the demand schedule and

the supply schedule intersect in that round.)

- The buyers will get to choose (among units listed for sale) in descending order of resale value—that is, the buyer with the highest resale value chooses first, the buyer with the second highest resale value chooses second, and so on.
- A buyer who purchases a unit will receive a commission. (This will be credited directly to the buyer, by the computer, such that no funds and no additional actions from any sellers are involved.) The commission is larger than zero, but the exact amount is the buyer's private information.
- If no buyer purchases from you (in a round in which you have chosen IN), price will equal 0 for purposes of determining your payoff in that round.
- You have an individual marginal cost of supplying a unit, MC_i (which may vary by round).

For example, if you choose IN, and $\mathrm{MC}_i = 2$, and you nominate a price equal to 4, and a buyer purchases your unit, then your payoff from choosing IN would be: $1 + 4 - 2$, which equals 3.

As another example, suppose all of the numbers in the first example stayed the same, except MC_i which was instead equal to 6. Then your payoff from choosing IN would be: $1 + 4 - 6$, which equals -1.

As another example, suppose all of the numbers in the first example stayed the same, except the price you nominated was 6. Then your payoff from choosing IN would be: $1 + 6 - 2$, which equals 5.

Are there any questions before we begin?

C.2.2. *Buyer Instructions*

How Payoffs are Determined

In this part of the experiment there are 5 buyers and 5 sellers in a market. You are a buyer.

You have a resale value for a unit of the good in that market; this means that if you purchase a unit of the good, you can re-sell it to the experimenter for the amount of the stated resale value.

- If you buy a unit, your payoff for that round would equal:

$$\text{Payoff} = (\text{Resale value} - \text{price paid}) + 0.10.$$

(The 0.10 is a commission you receive if you buy a unit.)

- If you do not buy a unit, your payoff for that round would be zero.

In any one round you can buy at most 1 unit.

Buyers take turns a evaluating offers to sell; the order of turns is in descending order of buyers' respective resale values; starting with the buyer with the highest resale value, and continuing to the buyer with the second highest resale value, and so on.

C.3. *Instructions for Treatment Value Order Robot Buyer (VORB)*

This Segment

In the rounds about to begin, and which will continue until further notice, there are 5 human participants acting as sellers and 5 robots acting as buyers. In each round, you will have the opportunity to make a decision between one of two possible actions. Once all participants have made their decisions, a second screen will appear which will report to you your payoff resulting from that round's events, and also the determinants of that payoff—namely your decision, and the decisions of others also participating. (More on this below.) There will be multiple rounds. Throughout these rounds you will stay in the same group of 5 human participants as sellers (with 5 robots as buyers).

The Sequence of Play in a Round

The first computer screen you see in each round asks you to make a decision between two actions: IN or OUT. You enter your decision by using the mouse to fill in the radio-button next to the action you wish to take. If you want to choose action IN, fill in the circle next to IN by clicking on it with the mouse; If you want to choose action OUT, fill in the circle next to OUT by clicking on it with the mouse. Once all participants have entered their decisions, a second screen will appear. This second screen reminds you of your decision for the round, informs you of your payoff for the round, and informs you of other determinants of your payoff (e.g. the decisions taken by other participants). Your payoff represents an amount in ECU that could be paid to you in cash (if the given round is randomly selected for payoff) as will be explained below.

How Payoffs are Determined

Payoffs are determined as follows:

- If you choose OUT your payoff for the round is equal to 1 (this is true in each round).
- If you choose IN, your payoff will be equal to $1 + \text{Price} - \text{MC}_i$. The components of this payoff are given by the following:
  - Price will be determined by (a) what you nominate as a price (which must be an even number) and (b) whether a robot buyer chooses to purchase from you at the price you nominate. There are 5 robot buyers, each of whom can re-sell a purchased unit to the experimenter, such that:
    One buyer has a resale value of 8.
    One buyer has a resale value of 6.
    One buyer has a resale value of 4.
    One buyer has a resale value of 2.

One buyer has a resale value of 0.
(Note also that at the beginning of each round, you will be informed of the number of units at which the demand schedule and the supply schedule intersect in that round.)

– The robot buyers are programmed to choose (among units listed for sale) in descending order of resale value—that is, the robot buyer with the highest resale value chooses first, the buyer with the second highest resale value chooses second, and so on. A robot buyer chooses the lowest priced unit available, provided that resale value is greater than or equal to the price (otherwise it will not purchase at all).

– If no robot buyer purchases from you (in a round in which you have chosen IN), price will equal 0 for purposes of determining your payoff in that round.

– If multiple units are listed at a given price, then the robot buyers may purchase all, none, or one or some but not all units. In the last case (in which only one or some but not all units are purchased) a random tie-breaker is employed to determine which of the units are purchased or not.

– You have an individual marginal cost of supplying a unit, MC_i (which may vary by round).

For example, if you choose IN, and $\mathrm{MC}_i = 2$, and you nominate a price equal to 4, and a buyer purchases your unit, then your payoff from choosing IN would be: $1 + 4 - 2$, which equals 3.

As another example, suppose all of the numbers in the first example stayed the same, except MC_i which was instead equal to 6. Then your payoff from choosing IN would be: $1 + 4 - 6$, which equals -1.

As another example, suppose all of the numbers in the first example stayed the same, except the price you nominated was 6. Then your payoff from choosing IN would be: $1 + 6 - 2$, which equals 5.

Are there any questions before we begin?

C.4. *Instructions for Treatment Random Order Human Buyer (ROHB)*

C.4.1. *Seller Instructions*

This Segment

In the rounds about to begin, and which will continue until further notice, there are 10 human participants: 5 acting as sellers and 5 acting as buyers. You are a seller. In each round, you will have the opportunity to make a decision between one of two possible actions. Once all participants have made their decisions, a second screen will appear which will report to you your payoff resulting from that round's events, and also the determinants of that payoff—namely your decision, and the decisions of others also participating. (More on this below.) There will be multiple rounds. Throughout these rounds you will stay in the same group of 5 sellers, while the group of 5 buyers will also stay the same.

The Sequence of Play in a Round

The first computer screen you see in each round asks you to make a decision between two actions: IN or OUT. You enter your decision by using the mouse to fill in the radio-button next to the action you wish to take. If you want to choose action IN, fill in the circle next to IN by clicking on it with the mouse; If you want to choose action OUT, fill in the circle next to OUT by clicking on it with the mouse. Once all participants have entered their decisions, a second screen will appear. This second screen reminds you of your decision for the round, informs you of your payoff for the round, and informs you of other determinants of your payoff (e.g. the decisions taken by other participants). Your payoff represents an amount in ECU that could be paid to you in cash (if the given round is randomly selected for payoff) as will be explained below.

How Payoffs are Determined

Payoffs are determined as follows:

- If you choose OUT your payoff for the round is equal to 1 (this is true in each round).
- If you choose IN, your payoff will be equal to $1 + \text{Price} - \text{MC}_i$. The components of this payoff are given by the following:
  - Price will be determined by (a) what you nominate as a price (which must be an even number) and (b) whether a buyer chooses to purchase from you at the price you nominate. There are 5 human buyers, each of whom can re-sell a purchased unit to the experimenter, such that:
    One buyer has a resale value of 8.
    One buyer has a resale value of 6.
    One buyer has a resale value of 4.
    One buyer has a resale value of 2.
    One buyer has a resale value of 0.
    (Note also that at the beginning of each round, you will be informed of the number of units at which the demand schedule and the supply schedule intersect in that round.)
  - The buyers will get to choose (among units listed for sale) in random order. That is, any buyer is equally likely to be, say, the one who gets the opportunity to choose in a given round (similarly for second, third, fourth, and fifth to choose in a given round). A fresh random ordering will be made each round (i.e. the random order in which buyers get to choose will be redrawn each round).

– If no buyer purchases from you (in a round in which you have chosen IN), price will equal 0 for purposes of determining your payoff in that round.

– You have an individual marginal cost of supplying a unit, MC_i (which may vary by round).

For example, if you choose IN, and $\mathrm{MC}_i = 2$, and you nominate a price equal to 4, and a buyer purchases your unit, then your payoff from choosing IN would be: $1 + 4 - 2$, which equals 3.

As another example, suppose all of the numbers in the first example stayed the same, except MC_i which was instead equal to 6. Then your payoff from choosing IN would be: $1 + 4 - 6$, which equals -1.

As another example, suppose all of the numbers in the first example stayed the same, except the price you nominated was 6. Then your payoff from choosing IN would be: $1 + 6 - 2$, which equals 5.

Are there any questions before we begin?

C.4.2. *Buyer Instructions*

How Payoffs are Determined

In this part of the experiment there are 5 buyers and 5 sellers in a market. You are a buyer.

You have a resale value for a unit of the good in that market; this means that if you purchase a unit of the good, you can re-sell it to the experimenter for the amount of the stated resale value.

- If you buy a unit, your payoff for that round would equal:

$$\text{Payoff} = (\text{Resale value} - \text{price paid}) + 0.10.$$

(The 0.10 is a commission you receive if you buy a unit.)

- If you do not buy a unit, your payoff for that round would be zero.

In any one round you can buy at most 1 unit.

Buyers take turns a evaluating offers to sell; the order of turns is random. That is, any buyer is equally likely to be, say, the one who gets the opportunity to choose in a given round (similarly for second, third, fourth, and fifth to choose in a given round). A fresh random ordering will be made each round (i.e. the random order in which buyers get to choose will be redrawn each round).

C.5. *Instructions for Treatment Random Order Robot Buyer (RORB)*

This Segment

In the rounds about to begin, and which will continue until further notice, there are 5 human participants acting as sellers and 5 robots acting as buyers. In each round, you will have the opportunity to make a decision between one of two possible actions. Once all participants have made their decisions, a second screen will appear which will report to you your payoff resulting from that round's events, and also the determinants of that payoff—namely your decision, and the decisions of others also participating. (More on this below.) There will be multiple rounds. Throughout these rounds you will stay in the same group of 5 human participants as sellers (with 5 robots as buyers).

The Sequence of Play in a Round

The first computer screen you see in each round asks you to make a decision between two actions: IN or OUT. You enter your decision by using the mouse to fill in the radio-button next to the action you wish to take. If you want to choose action IN, fill in the circle next to IN by clicking on it with the mouse; If you want to choose action OUT, fill in the circle next to OUT by clicking on it with the mouse. Once all participants have entered their decisions, a second screen will appear. This second screen reminds you of your decision for the round, informs you of your payoff for the round, and informs you of other determinants of your payoff (e.g. the decisions taken by other participants). Your payoff represents an amount in ECU that could be paid to you in cash (if the given round is randomly selected for payoff) as will be explained below.

How Payoffs are Determined

Payoffs are determined as follows:

- If you choose OUT your payoff for the round is equal to 1 (this is true in each round).
- If you choose IN, your payoff will be equal to $1 + \text{Price} - \text{MC}_i$. The components of this payoff are given by the following:
  - Price will be determined by (a) what you nominate as a price (which must be an even number) and (b) whether a robot buyer chooses to purchase from you at the price you nominate. There are 5 robot buyers, each of whom can re-sell a purchased unit to the experimenter, such that:
    One buyer has a resale value of 8.
    One buyer has a resale value of 6.
    One buyer has a resale value of 4.
    One buyer has a resale value of 2.
    One buyer has a resale value of 0.
    (Note also that at the beginning of each round, you will be informed of the number of units at which the demand schedule and the supply schedule intersect in that round.)
  - The robot buyers are programmed to choose (among units listed for sale) in random order. That is, any robot buyer is equally likely to be, say, the one who gets the opportunity to choose in a given round (similarly for second, third, fourth, and fifth to

choose in a given round). A fresh random ordering will be made each round (i.e. the random order in which buyers get to choose will be redrawn each round). A robot buyer chooses the lowest priced unit available, provided that resale value is greater than or equal to the price (otherwise it will not purchase at all).

– If no robot buyer purchases from you (in a round in which you have chosen IN), price will equal 0 for purposes of determining your payoff in that round.

– If multiple units are listed at a given price, then the robot buyers may purchase all, none, or one or some but not all units. In the last case (in which only one or some but not all units are purchased) a random tie-breaker is employed to determine which of the units are purchased or not.

– You have an individual marginal cost of supplying a unit, MC_i (which may vary by round).

For example, if you choose IN, and $\mathrm{MC}_i = 2$, and you nominate a price equal to 4, and a buyer purchases your unit, then your payoff from choosing IN would be: $1 + 4 - 2$, which equals 3.

As another example, suppose all of the numbers in the first example stayed the same, except MC_i which was instead equal to 6. Then your payoff from choosing IN would be: $1 + 4 - 6$, which equals -1.

As another example, suppose all of the numbers in the first example stayed the same, except the price you nominated was 6. Then your payoff from choosing IN would be: $1 + 6 - 2$, which equals 5.

Are there any questions before we begin?