



# HHS Public Access

Author manuscript

*Chronobiol Int.* Author manuscript; available in PMC 2020 December 14.

Published in final edited form as:

*Chronobiol Int.* 2020 April ; 37(4): 564–572. doi:10.1080/07420528.2020.1746798.

## Generalizability of a biomathematical model of fatigue's sleep predictions

Samantha M. Riedy<sup>a,b</sup>, Desta Fekedulegn<sup>c</sup>, Michael Andrew<sup>c</sup>, Bryan Vila<sup>a,d</sup>, Drew Dawson<sup>e,\*</sup>, John Violanti<sup>f,\*</sup>

<sup>a</sup>Sleep and Performance Research Center, Washington State University, Spokane, WA, USA;

<sup>b</sup>Elson S. Floyd College of Medicine, Washington State University, Spokane, WA, USA;

<sup>c</sup>Bioanalytics Branch, Health Effects Laboratory Division, National Institute for Occupational Safety and Health Centers for Disease Control and Prevention, Morgantown, WV, USA;

<sup>d</sup>Department of Criminal Justice and Criminology, Washington State University, Spokane, WA, USA;

<sup>e</sup>Appleton Institute, Central Queensland University, Wayville, SA, Australia;

<sup>f</sup>Department of Epidemiology and Environmental Health, School of Public Health and Health Professions, University at Buffalo, State University of New York, Buffalo, NY, USA

### Abstract

**Introduction:** Biomathematical models of fatigue (BMMF) predict fatigue during a work-rest schedule on the basis of sleep-wake histories. In the absence of actual sleep-wake histories, sleep-wake histories are predicted directly from work-rest schedules. The predicted sleep-wake histories are then used to predict fatigue. It remains to be determined whether workers organize their sleep similarly across operations and thus whether sleep predictions generalize.

**Methods:** Officers (n = 173) enrolled in the Buffalo Cardio-Metabolic Occupational Police Stress study were studied. Officers' sleep-wake behaviors were measured using wrist-actigraphy and predicted using a BMMF (FAID Quantum) parameterized in aviation and rail. Sleepiness (i.e. Karolinska Sleepiness Scale (KSS) ratings) was predicted using actual and predicted sleep-wake data. Data were analyzed using sensitivity analyses.

**Results:** During officers' 16.0 ± 1.9 days of study participation, they worked 8.6 ± 3.1 shifts and primarily worked day shifts and afternoon shifts. Across shifts, 7.0 h ± 1.9 h of actual sleep were obtained in the prior 24 h and associated peak KSS ratings were 5.7 ± 1.3. Across shifts, 7.2 h ± 1.1 h of sleep were predicted in the prior 24 h and associated peak KSS ratings were 5.5 ± 1.2. The

---

**CONTACT** Samantha M. Riedy, samantha.riedy@penmedicine.upenn.edu, Unit for Experimental Psychiatry, Division of Sleep and Chronobiology, University of Pennsylvania Perelman School of Medicine, 1020 Blockley Hall, 423 Guardian Drive, Philadelphia, PA 19104-6021, USA.

\*Shared Senior Authorship

Chronobiology International special issue on Shiftwork and Working Time, 2019

Disclosure Statement

Drew Dawson, PhD derives income from royalties associated with the use of FAID Quantum. The remaining authors do not have any disclosures to report. The findings and conclusions in this report are those of the authors and do not necessarily represent the official position of the National Institute for Occupational Safety and Health, Centers for Disease Control and Prevention.

minute-by-minute predicted and actual sleep-wake data demonstrated high sensitivity (80.4%). However, sleep was observed at all hours-of-the-day, but sleep was rarely predicted during the daytime hours.

**Discussion:** The sleep-wake behaviors predicted by a BMMF parameterized in aviation and rail demonstrated high sensitivity with police officers' actual sleep-wake behaviors. Additional night shift data are needed to conclude whether BMMF sleep predictions generalize across operations.

## Keywords

Biomathematical model; shift work; sleep; sleepiness; policing

---

## Introduction

### Managing fatigue in 24/7 operations

Managing fatigue in 24/7 operations is critical for sustaining performance. Fatigue has historically been managed in operational settings with hours-of-service regulations specifying duty time limitations and minimum rest break requirements (Gander et al. 2016). However, the efficacy of hours-of-service regulations alone to improve safety have been challenged for decades (Gander 2015; Jones et al. 2005; McDonald 1981). These regulations also lack the operational flexibility needed to schedule workers in many operations.

Fatigue risk management systems have been developed in operations such as aviation and rail as a more comprehensive, data-driven regulatory approach to managing fatigue. While hours-of-service regulations provide a single layer of defense against fatigue, fatigue risk management systems include multiple, overlapping layers of defense (Dawson and McCulloch 2005; Gander 2015). Examples of defenses include constructing work schedules with adequate sleep opportunities, evaluating the likelihood of on-duty fatigue, and managing fatigue-related errors before they propagate to fatigue-related incidents or accidents (Dawson and McCulloch 2005). One tool often used in fatigue risk management systems to construct and evaluate work-rest schedules is biomathematical models of fatigue (Dawson et al. 2017).

Biomathematical models of fatigue also serve other purposes in operational settings, including to determine whether fatigue mitigation strategies may be needed (Dean et al. 2007); to develop safety cases in aviation for flights that do not fit within the regulations (Lamp et al. 2019); and, to assess the likelihood of driver or pilot fatigue in post-accident investigations (Pruchnicki et al. 2011). In operational research, biomathematical models of fatigue are useful tools for studying workers' sleep-wake behaviors and the relationships between their work-rest schedules, sleep-wake behaviors, on-duty fatigue, and actual job performance (e.g. Åkerstedt et al. 2008; Hursh et al. 2008; Riedy et al. 2019).

### Biomathematical models of fatigue

Biomathematical models of fatigue quantify the effects of the homeostatic and circadian processes on the temporal profiles of sleep, sleepiness, and fatigue (Hursh et al. 2016; Mallis et al. 2004). There are two types of biomathematical models of fatigue used in operational

settings and operational research: one-step and two-step biomathematical models of fatigue (Kandelaars et al. 2005). One-step biomathematical models predict fatigue at the group-level using actual sleep-wake data (“one-step models”). Since fatigue is predicted at the group-level, all workers with the same sleep-wake behaviors have the same fatigue predictions.

In the absence of actual sleep-wake data, two-step biomathematical models of fatigue are used. Two-step biomathematical models predict workers’ sleep-wake behaviors at the group-level directly from their work-rest schedules. The models then predict fatigue at the group-level using the predicted sleep-wake data (“two-step models”). Since sleep and fatigue are predicted at the group-level, all workers with the same work-rest schedules have the same predicted sleep-wake behaviors and fatigue predictions. To date, one-step biomathematical models of fatigue have been more rigorously tested and validated than two-step biomathematical models of fatigue (Dawson et al. 2011).

### **Two-step biomathematical models in operational settings and research**

Actual sleep-wake data are often unavailable in operational settings and consequently two-step biomathematical models of fatigue are often used in operational settings (Dawson et al. 2017). Obtaining inaccurate sleep and fatigue estimates can have serious implications in both operational settings and operational research. As previously noted, these models are often used in operational settings to assist with work scheduling (e.g. Civil Aviation Safety Authority, 2014). Inaccurate sleep estimates could propagate to inaccurate fatigue estimates and this could ultimately lead to inappropriately classifying a work-rest schedule as likely “safe” or “unsafe” to work. Similarly, two-step biomathematical models of fatigue are used in the development of safety cases in aviation for flights that do not fit within regulations (Lamp et al. 2019). Here, obtaining accurate sleep and fatigue estimates is critical since non-compliant flights deemed “safe” to fly can receive regulatory exemptions for data collection purposes.

In operational research, inaccurate sleep and fatigue estimates may lead to researchers drawing the wrong conclusions on how work-rest schedules, sleep-wake behaviors, and on-duty fatigue are related to workers’ actual job performance. For example, Riedy et al. (2019) found that predicted sleep loss and on-duty fatigue increased the likelihood of a police officer receiving a citizen complaint suggesting that sleep loss and police fatigue affect police-community relationships. These results and conclusions may hinge on whether the two-step biomathematical model of fatigue accurately predicted the police officers’ sleep-wake behaviors.

### **Generalizability of sleep predictions**

There is currently an assumption that sleep predictions generalize across operations. It is not clear, however, whether workers organize their sleep similarly across operations and thus whether the sleep predictions generalize across operations. To date, there is published literature on the parameterization and validation of sleep estimators in aviation (Darwent et al. 2010; Hursh and Waggoner 2017; Ingre et al. 2014; Kandelaars et al. 2005) and rail (Darwent et al. 2012; Gertler et al. 2012). However, these sleep estimators are also used in a number of other operations such as maritime (Hobbs et al.), healthcare (Sagherian et al.

2018), policing (Riedy et al. 2019), and public transit (James et al. 2017). There have likely been validation studies conducted in other operations as well that have not been published due to containing potentially sensitive or proprietary information (James et al. 2018).

Policing is one operation where two-step biomathematical models of fatigue have not been validated. Police officers are often required to work overtime hours and off-duty court hours; and, may choose to have secondary employment (Reaves 2012). It is possible that the work requirements will result in police officers organizing their sleep differently than workers in other operations. There may be more anticipatory behavior where the police officers prepare for possible overtime, more fragmented sleep with on-call hours and/or the stresses associated with policing, and/or greater recovery sleep following long and erratic work hours. Several studies have also found that there is a high prevalence of poor sleep quality among police officers, and this has been associated with various stressors (Gabarino et al. 2002, 2019).

The generalizability of sleep estimators depends on whether workers organize their sleep similarly across operations. If the sleep predictions are generalizable, other operations can start using the two-step bio-mathematical models of fatigue parameterized and validated in aviation and rail. If the sleep predictions are not generalizable, two-step biomathematical models of fatigue still have potential as tools for mitigating fatigue across operations. However, before this happens, the sleep estimator either needs to be re-parameterized or the sleep predictions need to be adjusted to better predict workers' sleep-wake behaviors in the operation of interest. This will be an important step if the sleep predictions do not generalize because inaccurate estimates of workers' sleep-wake behaviors could propagate to inaccurate estimates of on-duty fatigue.

### **Fatigue Audit InterDyne Quantum**

Fatigue Audit InterDyne (FAID) Quantum is analytical software that includes a sleep estimator and one implementation of the Three-Process Model of Alertness – a publicly available biomathematical model of fatigue (Åkerstedt et al. 2004; FAID Quantum User Guide v1.0, 2017). The Three-Process Model of Alertness predicts on-duty sleepiness using sleep-wake data collected in the operation (i.e. one-step approach) or sleep-wake data predicted by FAID Quantum (i.e. two-step approach).

FAID Quantum predicts workers' sleep-wake behaviors directly from their work-rest schedules using two steps (Darwent et al. 2012). First, it predicts the total hours of sleep obtained during non-work hours. Second, it predicts the timing and duration of discrete sleep periods during non-work hours. To date, FAID Quantum's sleep estimator has been parameterized and validated in aviation and rail (Darwent et al. 2010, 2012). It remains to be determined whether its sleep predictions accurately describe the sleep-wake behaviors of workers in other operations and generalize to other operations such as policing.

### **Research objectives**

The objectives of this research include: (1) To compare police officers' actual sleep-wake behaviors to their predicted sleep-wake behaviors; (2) to determine whether misestimations of sleep propagate to misestimations of sleepiness; and, (3) to determine whether a simple

scaling factor can be used to correct for misestimations of sleep (if any). The results from this research are used to assess the generalizability of sleep estimators, and to determine if and how the model should be adjusted in the future to better predict police officers' sleep-wake behaviors.

## Methods

The Buffalo Cardio-metabolic Occupational Police Stress study protocol was approved by the Internal Review Board of the State University of New York at Buffalo, and the National Institute for Occupational Safety and Health (NIOSH) Human Subjects Review Board (IRB), and done in accordance with the Declaration of Helsinki for experiments involving human subjects.

### Buffalo Cardio-metabolic Occupational Police Stress study

A total of 173 police officers employed by the Buffalo, New York Police Department were studied. The officers were enrolled in the Buffalo Cardio-metabolic Occupational Police Stress (BCOPS) study for approximately 15 days between 2011 and 2015. During officers' study participation, they wore an actigraph on their non-dominant wrist to record their movement and infer their sleep-wake behaviors (AMA-32CL; Ambulatory Monitoring Inc., Ardsley, New York, USA). Officers' work data were obtained from the Buffalo, New York Payroll Department. Officers also wrote their work start times and end times in a diary. The payroll work data were cross-checked using the diary entries.

### Fatigue Audit InterDyne Quantum

FAID Quantum predicted police officers' sleep-wake histories directly from their work-rest schedules. The Three-Process Model of Alertness predicted on-duty sleepiness using the actual sleep-wake data and predicted sleep-wake data. Sleepiness predictions were expressed on the 9-point Karolinska Sleepiness Scale (KSS), where higher KSS predictions represented greater likelihood of on-duty sleepiness. KSS ratings across a shift were summarized as peak KSS ratings, which represented the greatest sleepiness predicted during each shift. The default threshold for a high likelihood of on-duty sleepiness is a peak KSS rating of 7 (i.e. a KSS tolerance level [KTL] of 7) (FAID Quantum User Guide v1.0, 2017). Previous research has demonstrated that behavioral and physiological symptoms of sleepiness are evident starting at KSS ratings of 7+ (Åkerstedt et al. 2014).

### Data analyses

All minutes between sleep onset and wake onset were scored as sleep, as the model does not predict wake within sleep periods. Sleep in the 24 h prior to a shift and sleep in the 48 h prior to a shift were calculated using the actual and predicted sleep-wake data. Using the actual sleep-wake data and predicted sleep-wake data, it was determined whether officers had less than the 5 h sleep in the 24 h prior to a shift and/or less than 12 h sleep in the 48 h prior to a shift. For each shift, it was determined whether the peak KSS ratings calculated using the actual and predicted sleep-wake data were greater than a KTL of 7 and greater than a KTL of 8. All data were analyzed using sensitivity and specificity analyses (see Table 1). Thus, the conditions being tested were whether officers' actual sleep and/or predicted sleep

were restricted prior to the shift, and whether on-duty sleepiness predictions were high when using officers' actual sleep-wake data and/or the predicted sleep-wake data.

The predicted sleep periods were scaled to determine if a simple scaling factor could correct for any misestimations of sleep. The scaling factor was the regression coefficient from a mixed-effects model where hours of actual sleep in the 48 h prior to a shift was the dependent variable and hours of predicted sleep in the 48 h prior to a shift was the independent variable. The scaling factor thus represented the rate of change of actual sleep as a function of changes in predicted sleep. The Three-Process Model of Alertness predicted on-duty sleepiness using the scaled sleep periods.

## Results

### Buffalo cardio-metabolic occupational police stress study

The officers in the study sample ( $n = 173$ ) includes 89 police officers, 37 detectives, 36 lieutenants, and 11 police officials with other job titles; 71% of the sample were male officers. Hereupon, the term "officer" is used to describe all 173 police officials. The 173 officers contributed a total of 2,764 days of data and worked a total of 1,494 shifts. On average, each officer participated in the study for  $16.0 \pm 1.9$  days (range: 9–30 days) and worked  $8.6 \pm 3.1$  shifts (range: 1–20 shifts). Shifts were  $9.3 \pm 2.6$  hours in duration (range: 0.3–19.8 hours). They were between the ages of 28–61 (mean  $\pm$  SD:  $46.1 \pm 6.5$  years) and had been with the Buffalo, New York Police Department for 4–39 years (mean  $\pm$  SD:  $19.2 \pm 6.9$  years).

A total of 234 shifts were not preceded by a full 24 hours of sleep-wake data and 336 shifts were not preceded by a full 48 hours of sleep-wake data. The 234 shifts were excluded from analyses examining sleep in the prior 24 h. The 335 shifts were excluded from analyses examining sleep in the prior 48 h and on-duty sleepiness.

### Overall agreement, sensitivity, and specificity

The predicted sleep-wake data demonstrated high overall agreement, sensitivity, and specificity with the actual sleep-wake data at the group-level (see Table 2). Officers primarily worked day shifts and afternoon shifts and, as such, primarily slept during the nighttime hours. To be expected, the model appropriately predicted that the day shift and afternoon shift officers primarily slept during the nighttime hours (see Figure 1). Sleep was observed at all hours-of-the-day, but sleep was rarely predicted during the daytime hours. For example, 11.4% of actual sleep periods and 0.3% of predicted sleep periods included any sleep between 13:00 and 18:00.

Across shifts,  $7.0 \text{ h} \pm 1.9 \text{ h}$  of sleep were obtained and  $7.2 \text{ h} \pm 1.1 \text{ h}$  of sleep were predicted in the 24 h prior to a shift. Similarly,  $14.2 \text{ h} \pm 3.1 \text{ h}$  of sleep were obtained and  $14.5 \text{ h} \pm 1.7 \text{ h}$  of sleep were predicted in the 48 h prior to a shift. Using both the actual and predicted sleep-wake data, most shifts were preceded by at least 5 h of sleep in the prior 24 h and at least 12 h of sleep in the prior 48 h. Shifts were more frequently preceded by less than 5 h of actual sleep (11.4%) than 5 h of predicted sleep (2.3%) in the prior 24 h, and less than 12 h of actual sleep (21.4%) than 12 h of predicted sleep (7.6%) in the prior 48 h. The predicted

sleep-wake data demonstrated low sensitivity and high specificity with the actual sleep-wake data (see Table 2).

Across shifts, the average peak KSS ratings were  $5.7 \pm 1.3$  using the actual sleep-wake data and  $5.5 \pm 1.2$  using the predicted sleep wake data. Using both the actual and predicted sleep-wake data, most shifts had peak KSS ratings less than the KTL of 7 and the KTL of 8. Shifts more frequently had peak KSS ratings greater than 7 using the actual sleep-wake data (17.5%) than the predicted sleep-wake data (11.5%) Similarly, shifts more frequently had peak KSS ratings greater than 8 using the actual sleep-wake data (6.4%) than the predicted sleep-wake data (5.9%). The peak KSS ratings calculated using the predicted sleep-wake data demonstrated low sensitivity and high specificity with the peak KSS ratings calculated using the actual sleep-wake data (see Table 2).

As noted above, sleep in the prior 24 h and sleep in the prior 48 h were slightly overestimated by  $0.2 \text{ h} \pm 1.8 \text{ h}$  and  $0.3 \text{ h} \pm 2.9 \text{ h}$ , respectively, and sleepiness was slightly underestimated by  $0.2 \pm 0.7$  units. However, there was considerable variability around the mean with sleep and sleepiness being both underestimated and overestimated (see Figure 2A).

### Scaling the predicted sleep-wake data

The scaling factor associating how change in predicted sleep is associated with change in actual sleep was 0.7. For every 1 hour increase in predicted sleep, there was a 0.7 hour increase in actual sleep. After adjusting the predicted sleep-wake data,  $4.6 \text{ h} \pm 0.8 \text{ h}$  of sleep were predicted in the 24 h prior to a shift,  $10.1 \text{ h} \pm 1.2 \text{ h}$  of sleep were predicted in the 48 h prior to a shift, and the peak KSS ratings were  $6.9 \pm 1.0$ . The scaling factor did not successfully minimize residuals since there was considerable variability in the misestimation of sleep. The scaling factor simply shifted the sleep and sleepiness distributions (see Figure 2B). On average, sleep was now underestimated, and sleepiness was now overestimated (see Table 2).

## Discussion

This research assessed the generalizability of a biomathematical model of fatigue's sleep predictions. Work data and sleep data were collected from 173 police officers enrolled in the Buffalo Cardio-metabolic Occupational Police Stress study. Sleep was predicted using FAID Quantum's sleep estimator, which, to date, has been parameterized and validated using data collected in the aviation and rail industries. On-duty sleepiness was predicted by the Three-Process Model of Alertness using police officers' actual and predicted sleep-wake data.

During their study participation, the police officers primarily worked day shifts and/or afternoon shifts. The officers had been with the Buffalo Police Department for  $19.2 \pm 6.9$  years and were at a point in their career where they had largely rotated off of the night shift. To be expected given their work schedules, the officers primarily slept during the nighttime hours and the model predicted that the officers would primarily sleep during the nighttime hours. Daytime sleep was the primary discrepancy between the predicted sleep-wake data

and actual sleep-wake data. Sleep was observed at all hours-of-the day, but sleep was rarely predicted during the day.

To appropriately identify fatiguing schedules and mitigate fatigue in the operation, it is important that predicted sleep-wake data demonstrates high sensitivity and specificity with actual sleep-wake data. In the research presented here, the minute-by-minute predicted sleep-wake data demonstrated high overall agreement, high sensitivity, and particularly high specificity with the police officers' minute-by-minute actual sleep-wake data. It could be argued that the sensitivity was relatively low. This argument assumes that one hundred percent is the upper limit for sensitivity. However, even when workers' past sleep-wake behaviors are used to predict their own future sleep-wake behaviors during the same work-rest schedule, the sensitivity still does not approach one hundred percent (Dorrian et al. 2012).

Using the actual and predicted sleep-wake data, most shifts were preceded by at least 5 hours sleep in the prior 24 hours and at least 12 hours sleep in the prior 48 hours, and most shifts had peak KSS ratings less than 7. As a result, the specificities for sleep in the prior 24 h, sleep in the prior 48 h, and on-duty sleepiness were consistently high. Shifts were more frequently preceded by less than 5 hours of actual sleep than predicted sleep in the prior 24 hours, were more frequently preceded by less than 12 hours of actual sleep than predicted sleep in the prior 48 hours, and more frequently had peak KSS ratings greater than 7 using the actual sleep-wake data rather than the predicted sleep-wake data. As a result, the sensitivities for sleep in the prior 24 hours, sleep in the prior 48 hours, and on-duty sleepiness were low.

On average, sleep was slightly overestimated, and sleepiness was slightly underestimated. The average differences were negligible. However, there was considerable variability around the mean with sleep and sleepiness being both underestimated and overestimated. Since there was considerable variability around the mean, the scaling factor did not successfully minimize the discrepancies between the actual sleep-wake data and predicted sleep-wake data. Rather, the scaling factor shifted the sleep and sleepiness distributions. After scaling the data, there was an underestimation of sleep and an overestimation of sleepiness.

Additional data are currently needed to inform whether or not sleep-wake predictions provide a good characterization of police officers' actual sleep-wake behaviors between night shifts. This will be important before concluding that sleep predictions generalize to policing because it is notoriously more difficult to predict sleep between night shifts than day shifts or afternoon shifts. Based on the current set of analyses, changes to the sleep estimator <sup>are</sup> not recommended at this point. While more night shift data are needed to inform changes (if any) to the sleep estimator, it was evident that misestimations of sleep in an operation cannot be corrected for with a simple scaling factor unless sleep is systematically and consistently underestimated or overestimated.

If the sleep predictions generalize to policing, existing two-step biomathematical models of fatigue may provide a useful tool for improving work-rest schedules in policing as well as a useful tool for future operational research with police officers. If the sleep predictions



generalize, this also increases confidence that the sleep predictions may generalize to other operations where the sleep estimators have not yet been validated. If the sleep predictions do not generalize to policing, however, this raises additional questions, such as whether sleep estimators need to be parameterized and validated in each operation; whether sleep predictions generalize across departments or companies within an operation; and whether conclusions previously drawn using two-step biomathematical models need to be corroborated. It is important to note that even if the sleep predictions generalize, it will be at the group-level rather than the individual-level since not all workers with the same work-rest schedules exhibit the same sleep-wake behaviors.

In the future, changes to the sleep estimator (if any) would likely be in the form of re-estimating the model parameters. FAID Quantum predicts the total hours of sleep obtained during non-work hours using a regression model with predictors for sleep propensity, prior shift length, and post shift length. It then predicts the timing and duration of the discrete sleep periods by comparing the sleep propensity rhythm and sleep threshold rhythm. The regression estimates for sleep propensity, prior shift length, and post-shift length would need to be re-estimated if sleep duration is misestimated in an operation. In the current study, re-estimating the regression estimates could minimize the sleep and sleepiness residuals. On the other hand, the sleep propensity rhythm parameters would need re-estimated if sleep timing is misestimated in an operation. In the current study, re-estimating the sleep propensity rhythm parameters could correct for the daytime sleep discrepancies.

### Limitations

Officers primarily worked day shifts and afternoon shifts during their study participation between 2011 and 2015. Sleep between night shifts is notoriously more difficult to predict than sleep between day shifts and afternoon shifts. Additional night shift data are needed to inform if and how the model should be adjusted in the future. Another limitation is that wake within sleep periods were not accounted for. Each minute between sleep onset and wake onset was scored as sleep since models do not predict wake within sleep periods. Thus, sleep is likely even more overestimated than what is shown in the current set of analyses. And, if officers' sleep is fragmented, the on-duty sleepiness predictions are also likely underestimated.

### Funding

This work was supported by the CDC/NIOSH under Grant number 1R01OH009640-01A1 and the NIJ under Grant number 2005-FSBX-0004.

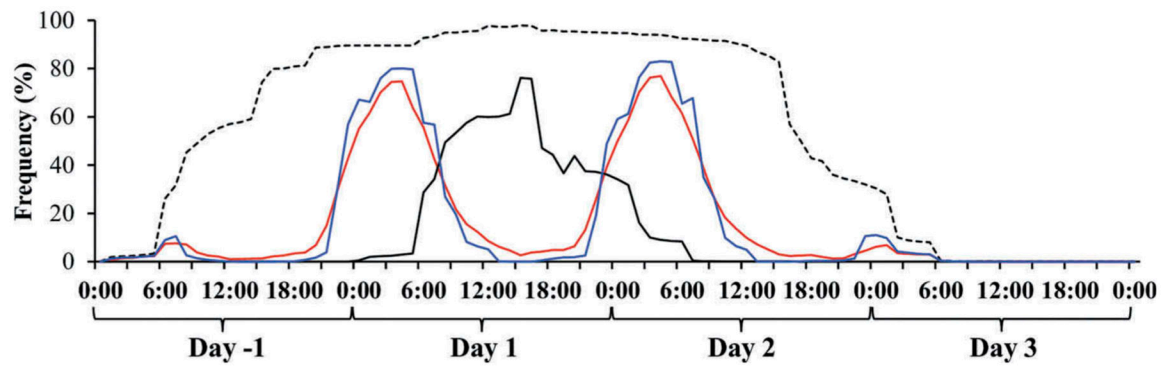
### References

- Åkerstedt T, Anund A, Axelsson J, Kecklund G. 2014 Subjective sleepiness is a sensitive indicator of insufficient sleep and impaired waking function. *J Sleep Res.* 23 (3):240–252. [PubMed: 24750198]
- Åkerstedt T, Connor J, Gray A, Kecklund G. 2008 Predicting road crashes from a mathematical model of alertness regulation—The sleep/wake predictor. *Accid Anal Prev.* 40 (4):1480–1485. [PubMed: 18606281]
- Åkerstedt T, Folkard S, Portin C. 2004 Predictions from the three-process model of alertness. *Aviat Space Environ Med.* 75(3, Suppl):A75–A83. [PubMed: 15018267]

- Civil Aviation Safety Authority. 2014 Biomathematical Fatigue Models Guidance Document. Albert Park, Australia: Dédale Asia Pacific Retrieved 28 Sep 2019 from [https://www.icao.int/safety/fatiguemanagement/ArticlesPublications/biomathematical\\_fatigue\\_models.pdf](https://www.icao.int/safety/fatiguemanagement/ArticlesPublications/biomathematical_fatigue_models.pdf).
- Darwent D, Dawson D, Roach GD. 2010 Prediction of probabilistic sleep distributions following travel across multiple time zones. *Sleep*. 33(2):185–195. [PubMed: 20175402]
- Darwent D, Dawson D, Roach GD. 2012 A model of shift worker sleep/wake behaviour. *Accid Anal Prev*. 45S:6–10.
- Dawson D, Darwent D, Roach GD. 2017 How should a bio-mathematical model be used within a fatigue risk management system to determine whether or not a working time arrangement is safe? *Accid Anal Prev* 99 (Pt B):469–473. [PubMed: 27040118]
- Dawson D, McCulloch K. 2005 Managing fatigue: it's about sleep. *Sleep Med Rev*. 9(5):365–380. [PubMed: 16099184]
- Dawson D, Noy YI, Härrma M, Åkerstedt T, Belenky G. 2011 Modelling fatigue and the use of fatigue models in work settings. *Accid Anal Prev*. 43:549–564. [PubMed: 21130216]
- Dean DA, Fletcher A, SR H, EB K. 2007 Developing mathematical models of neurobehavioral performance for the “real world”. *J Biol Rhythms*. 22(3):246–258. [PubMed: 17517914]
- Dorrian J, Darwent D, Dawson D, Roach GD. 2012 Predicting pilot's sleep during layovers using their own behaviour or data from colleagues: Implications for bio-mathematical models. *Accident Analysis & Prevention*. 45S 45:17–21.
- Gabarino S, de Carli F, Nobili L, Mascialino B, Squarcia S, Penco MA, Beelke M, Ferrillo F. 2002 Sleepiness and sleep disorders in shift workers: A study on a group of Italian police officers. *Sleep*. 25(6):642–647.
- Gabarino S, Guglielmi O, Puntoni M, Bragazzi NL, Magnavita N. 2019 Sleep quality among police officers: implications and insights from a systematic review and meta-analysis of the literature. *Int J Environ Res Public Health*. 16(5):885.
- Gander PH. 2015 Evolving regulatory approaches for managing fatigue risk in transport operations. *Rev Hum Ergon*. 10(1):253–271.
- Gander PH, Wu LJ, van den Berg M, Lamp A, Hoeg L, Belenky G. 2016 Fatigue risk management systems In: Kryger MH, Roth T, Dement WC, editors. *Principles and practice of sleep medicine*. 6th ed. Philadelphia (PA): Elsevier; p. 697–707.
- Gertler J, Hursh S, Fanzone RT 2012 Validation of FAST model sleep estimates with actigraph measured sleep in locomotive engineers; Baltimore, MD: Federal Railroad Administration. U.S. Department of Transportation Report No. DOT/FRA/ORD-12/15.
- Hobbs AN, Gregory KB, Parke BK, Pradhan SK, Caddick Z, Bathurst NG, Flynn-Evans EE San Francisco bar pilot fatigue study. Moffett Field (CA): NASA Ames Research Center Report No. NASA/TM-2018-219934, ARCE-DAA-TN58326.
- Hursh SR, Balkin TJ, Van Dongen HPA. 2016 Sleep and performance prediction modeling In: Kryger MH, Roth T, Dement WC, editors. *Principles and practice of sleep medicine*. 6th ed. Philadelphia (PA): Elsevier; p. 689–696.
- Hursh SR, Raslear TG, Kaye AS, Fanzone JF 2008 Validation and calibration of a fatigue assessment tool for railroad work schedules, final report. Report No. DOT/FRA/ORD-08/04 Washington (D.C.): U.S. Department of Transportation.
- Hursh SR, Waggoner LB 2017 Refining sleep predictions using actigraphy in operational environments: studies with pilots. 10th International Conference on Managing Fatigue Conference website Accessed September 28, 2019 San Diego, CA, USA [https://fatigueconference2017.com/materials/wednesday-am/modeling/Abstract\\_Hursh.pdf](https://fatigueconference2017.com/materials/wednesday-am/modeling/Abstract_Hursh.pdf).
- Ingre M, Van Leeuwen W, Klemets T, Ullvetter C, Hough S, Kecklund G, Karlsson ÅT. 2014 Validating and extending the three process model of alertness in airline operations. *PLoS One*. 9(10):e108679. [PubMed: 25329575]
- James FO, Mapp KJ, Greenwood P, Waggoner LB 2017 Applied fatigue risk management for transit: Biomathematical modeling in the analysis of recorded events. 10th International Conference on Managing Fatigue Conference website Accessed February 9, 2020.
- James FO, Waggoner LB, Weiss PM, Patterson DP, Higgins JS, Lang ES, Van Dongen HPA. 2018 Does implementation of biomathematical models mitigate fatigue and fatigue-related risks in

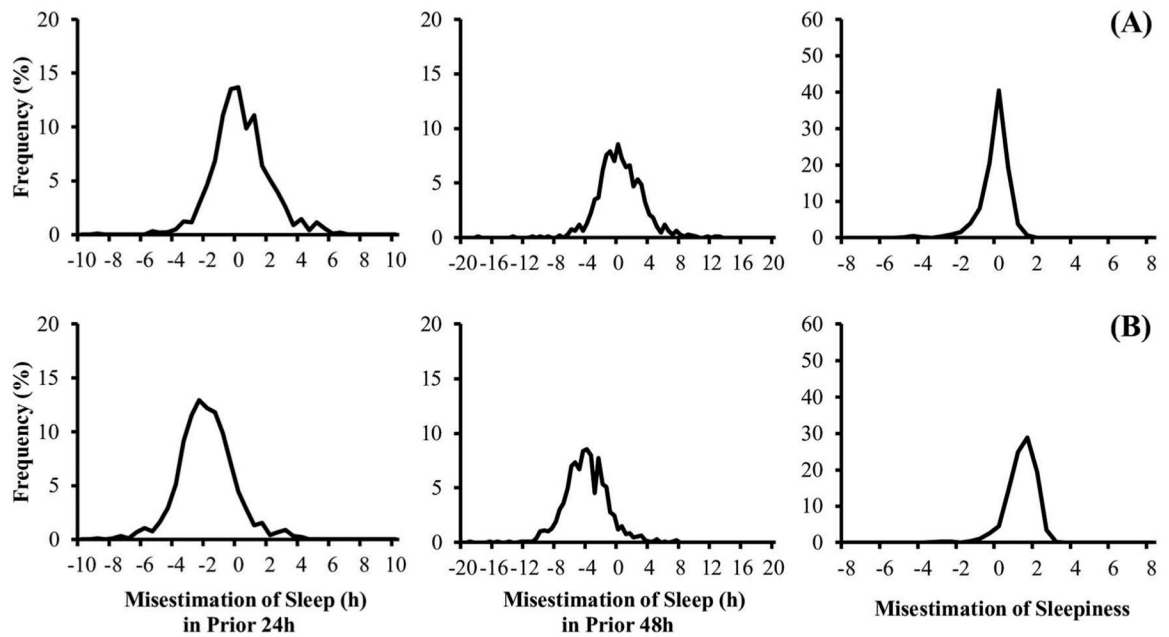
emergency medical services operations? A systematic review. 22(sup1):69–80.  
doi:10.1080/10903127.2017.1384875

- Jones CB, Dorrian J, Rajaratnam SMW, Dawson. 2005 Working Hours Regulations and Fatigue in Transportation: a Comparative Analysis. *Safety Science*. 43(4):225–252.
- Kandelaars KJ, Dorrian J, Fletcher A, Roach GD, Dawson D 2005 A review of bio-mathematical fatigue models: where to from here? Paper presented at the 6th International Conference on Fatigue Management in Transportation, Seattle, WA.
- Lamp A, Chen Jane MC, McCullough DBelenky G. 2019 Equal to or better than: the application of statistical non-inferiority to fatigue risk management. *Accident Analysis & Prevention*. 126:184–190. [PubMed: 29428150]
- Mallis MM, Mejdal S, Nguyen TT, Dinges DF. 2004 Summary of the key features of seven biomathematical models of human fatigue and performance. *Aviat Space Environ Med*. 75(3, Suppl):A4–A14. [PubMed: 15018262]
- McDonald N 1981 Safety and regulations restricting the hours of driving of goods vehicle drivers. *Ergonomics*. 24 (6):475–485.
- Pruchnicki SA, Wu LJ, Belenky G. 2011 An exploration of the utility of mathematical modeling predicting fatigue from sleep/wake history and circadian phase applied in accident analysis and prevention: the crash of comair flight 5191. *Accid Anal Prev*. 43(3):1056–1061. Doi:10.1016/j.aap.2010.12.010 [PubMed: 21376901]
- Quantum Version FAID 1.0. User Guide. 2017 InterDynamics.
- Reaves BA. 2012 Hiring and retention of state and local law enforcement officers, 2008- statistical tables. 10 2012. Washington (D.C.): Bureau of Justice Statistics Report No.: NCJ238251 Accessed 2019 Nov 1 <https://www.bjs.gov/content/pub/pdf/hrsllleo08st.pdf>
- Riedy SM, Dawson D, Vila B. 2019 U.S. police rosters: fatigue and public complaints. *Sleep*. 42(3):1–10. Doi:10.1093/sleep/zsy231
- Sagherian K, Zhu S, Storr C, Hinds PS, Derickson D, Geiger-Brown J. 2018 Bio-mathematical fatigue models predict sickness absence in hospital nurses: an 18 months retrospective cohort study. *Applied Ergonomics*. 73:42–47. [PubMed: 30098641]



**Figure 1.**

Officers' work distributions (**black**), actual sleep distributions (**red**), and predicted sleep distributions (**blue**). Shifts start on day 1 and continue into day 2 if the shift continues past midnight. The pre-duty and post-duty sleep are plotted for sleep in the 24 hours prior to and subsequent to the shift. The percentage of shifts contributing to the figure at each hour is plotted (black dotted line).



**Figure 2.**

Misestimations of sleep in the prior 24 h (left), sleep in the prior 48 h (middle), and on-duty sleepiness (right) prior to scaling the predicted sleep-wake data (top; panel A) and after scaling the predicted sleep-wake data (bottom; panel B). Negative values indicate an underestimation of sleep or sleepiness. Positive values indicate an overestimation of sleep or sleepiness.

**Table 1.**Overall agreement, sensitivity, and specificity definitions<sup>a</sup>.

Measure	Definition
<i>Minute-by-Minute Data</i>	
Overall Agreement	% of sleep and wake minutes correctly predicted by BMMF
Sensitivity	% of minutes of sleep correctly predicted by BMMF
Specificity	% of minutes of wake correctly predicted by BMMF
<i>Sleep in the Prior 24 h</i>	
Sensitivity	% shifts preceded by <5 h correctly predicted by BMMF
Specificity	% shifts preceded by 5 h correctly predicted by BMMF
<i>Sleep in the Prior 48 h</i>	
Sensitivity	% shifts preceded by <12 h correctly predicted by BMMF
Specificity	% shifts preceded by 12 h correctly predicted by BMMF
<i>Sleepiness (KTL of 7)</i>	
Sensitivity	% shifts with peak KSS ratings ≥ 7 correctly predicted by BMMF
Specificity	% shifts with peak KSS ratings < 7 correctly predicted by BMMF
<i>Sleepiness (KTL of 8)</i>	
Sensitivity	% shifts with peak KSS ratings ≥ 8 correctly predicted by BMMF
Specificity	% shifts with peak KSS ratings < 8 correctly predicted by BMMF

<sup>a</sup> Abbreviations: Sleepiness Threshold Level (KTL), Hours (h), Biomathematical Model of Fatigue (BMMF)

**Table 2.**Overall agreement, sensitivity, and specificity results<sup>a</sup>.

Measure	Actual v. Predictions <sup>b</sup>	Actual v. Scaled Predictions <sup>bc</sup>
<i>Minute-by-Minute Data</i>		
Overall Agreement	90.1%	88.3%
Sensitivity	80.4%	61.7%
Specificity	93.0%	96.7%
<i>Sleep in the Prior 24 h</i>		
Sensitivity	7.9%	72.9%
Specificity	98.4%	48.1%
<i>Sleep in the Prior 48 h</i>		
Sensitivity	20.4%	97.5%
Specificity	96.1%	5.4%
<i>Sleepiness (KTL of 7)</i>		
Sensitivity	55.0%	90.0%
Specificity	97.7%	69.1%
<i>Sleepiness (KTL of 8)</i>		
Sensitivity	53.5%	69.0%
Specificity	97.3%	91.9%

<sup>a</sup> Abbreviations: Sleepiness Threshold Level (KTL) and Hours (h)<sup>b</sup> Sensitivity and specificity analyses, which compare (1) the actual sleep-wake data and predicted sleep-wake data, and (2) the sleepiness predictions calculated using the actual sleep-wake data and the sleepiness predictions calculated using the predicted sleep-wake data.<sup>c</sup> Sensitivity and specificity analyses, which compare (1) the actual sleep-wake data and the scaled predicted sleep-wake data, and (2) the sleepiness predictions calculated using the actual sleep-wake data and the sleepiness predictions calculated using the scaled predicted sleep-waked data