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A New Nonlinear Control Design Strategy for Fixed Wing Aircrafts Piloting

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This paper proposes a novel nonlinear feedback control strategy for velocity and attitude control of fixed wing aircrafts. The key feature of the control design strategy is the introduction of a virtual control input in order to deal with the underactuation property of such vehicles and to indirectly control the orientaion of the aircraft. As such, the proposed strategy consists of three control loops each realising a specific task. Simulations are carried out by using the jetstream-3102 aircraft in a real-time virtual Simulation Platform for the development of Aircraft Control Systems (SP-ACS). The proposed approach of control is based model for that we have introduces an identification part before test and validation. We use the Total Least Squares Estimation technique (TLSE) to identify the aerodynamic parameters, which are unknown, variable, classified. Each aerodynamic coefficient is defined as the mean of its numerical values. All other variations are considered as modeling uncertainties that will be compensated by the robustness of the piloting law. Simulation results on Jetstream-3102 aircraft show very good performance in terms of convergence towards the desired reference trajectories and in terms of robustness with respect to modeling uncertainties.

Keywords: Attitude and speed controller; nonlinear control; aerodynamic flight; autopilot; nonlinear systems; underactuation system; TLSE.

1. Introduction

Defining proper control strategies for aircraft control is essential to avoid fatal accidents. The most common fatal accidents are: loss of control in flight, controlled flight into terrain and runway excursion during approach and landing. For this reason, it is imperative to adopt a proper control strategy of attitude and speed of aircrafts.[1] In general, when designing a controller, the control system designer, usually based its design on the mathematical model of the aircraft and used appropriate mathematical tools to demonstrate the convergence or robustness properties of the controller. In this regards, many control design strategies has been proposed for attitude and speed in the presence of both modelled and unmodelled uncertainties and by using different methods such as linear quadratic control,[1] feedback linearization technique,[23] and,[4] eigenstructure assignment,[5],[6] H_∞ robust control,[7],[8] dynamic inversion,[9],[10] backstepping[11]and sliding mode control, to mention a few. Traditional flight control systems use PID control with scheduled gains. It is well-known that this approach is very important and convenient for conventional aircrafts of the second and third generation. However, gain scheduling suffers from the inherent deficiency of relying on time-invariant linear models based on small perturbations of the full nonlinear aircraft model at a particular point in the flight envelope. In addition, the dynamic properties deteriorate when the scheduling parameters, such as speed and pitch angle, change rapidly over the small time intervals (see eg.,[912] and,[1314]).

However, it is not always wise to choose one control design method based on the mathematical description of the system rather than another because it may not be possible Practical meaning. In effect, in[1] it is shown that it is important to consider the aircraft as an energy system, whereby the energy gain or loss is distributed into the kinetic and the potential energy of the aircraft. This translates into the fact that it is not possible to control the thrust independently to the rudder, elevator and the aileron deflections. Also, it is important to realise that aircraft systems are underactuated systems. This means that there are less actuators than the degrees of freedom, meaning that it is not always possible to control some variables of the aircraft directly using the available actuators.

Indeed, the aircraft is a twelve (12) order system with four (04) actuators. The twelve 12 state variables are:

- $X = (x, y, z)^T \in \mathbb{R}^3$ which is the aircraft position expressed in the earth fixed reference frame R_E;
- $W = (u, v, w)^T \in \mathbb{R}^3$ is the inertial speed vector expressed in the body reference frame R_B;
- $\Phi = (\phi, \theta, \psi)^T \in \mathbb{R}^3$ are the Euler angles describing the orientation of the aircraft relative to R_E;
- $\Omega = (p, q, r)^T \in \mathbb{R}^3$ the angular velocity of the aircraft expressed in the body fixed reference frame

The four (04) control variables are:

- $U = (\delta_a, \delta_e, \delta_r)^T \in \mathbb{R}^3$ where $\delta_a, \delta_e, \delta_r$, are the aileron, elevator and rudder deflections respectively and
- F_T which is the thrust force due to the propulsion system.

The dynamics of the angular velocity Ω and the inertial speed vector W are directly affected by the control U and F_T, while the position X and the orientation Φ are not. Instead, X and Φ are coupled with W and Ω respectively. On the other hand, since four (04) inputs variables can only control four (04) states variables, one must find judicious ways to affect and indirectly control the rest of the state variables.

In this paper, we propose a novel strategy for velocity and attitude control of an aircraft based on the above considerations. For this, we first reduce the model of the aircraft by considering the norm of the inertial speed $V = \|W\|^2 = W^T W$ rather than its individual components. In that case, the model is reduced to a systems of ten (10) variables. Next, the pilot has to ensure that the aircraft does not stall. For this, we have to ensure that the derivative of the position X does not escape to infinity. Therefore, the non-stalling condition reduces to ensuring that $V \leq M$. As the result, the aircraft model can be reduced to an 8th order. The input variable F_T is used to track a reference speed trajectory V_{ref}. Next a virtual control Ω_v is introduced to indirectly control the orientation Φ towards a desired reference trajectory Φ_{ref}. This is a crucial point in the design strategy since Φ is not directly affected any real actuators as mentioned above. The input U is then used to steer Ω towards a desired reference trajectory Ω_{ref}. Another key point in the design strategy is that the reference angular velocity Ω_{ref} is chosen in such a way that it permit the virtual control input to track the desired orientation. The derived controller U is dependent on the control F_T ensuring indirectly a natural distribution between the kinetic and potential energy of the aircraft. As such, three (03)-control loops are designed each realising a specific control objective. Furthermore, the three control loop are design so as to make sure that the aircraft does not stall. This is measured by ensuring that $\|\dot{X}\|$ remain bounded. The proposed control strategy is model-based, where the aerodynamic coefficients define the behavior of the aircraft in the flight range and throw out the dynamic model, which are unknown, variable and classified. For this reason, knowledge of different aerodynamic coefficients turns out to be of great importance for the development of control laws.

In general, these coefficients determined through wind tunnel testing, analytical methods or system identification.[15] Latter provides a method for developing dynamic system models and identifying their parameters. System identification is the determination of a model describing the relationship between its inputs and outputs; this model is able to predict a future response given the input.[16] In the field of aircraft control the following definition[17] has been widely adopted to describe system identification: “Given the system responses, what is the model? ”. Within the field of Aeronautics our equivalent system takes the form of a dynamic model consisting of stability and control derivatives which characterise the aircraft under investigation. In this work to deal with the identification of aircraft aerodynamic coefficients, based on our previous work[18,20,21] we adopt the Total Least Squares Estimation technique (TLSE). TLSE use the Singular Value Decomposition (SVD), it has the interesting propriety of giving the best approximation of the augmented measurement matrix, by another matrix with the same dimension, but with a lesser range, in the sense of the least squares. In addition to the dimension reducing propriety, the SVD has the advantage of being able to estimate the invert of any matrix, whether it is square or rectangular, and most of all, whether it is singular or not. We applied TLSE technique to the aerodynamic coefficients identification problem and each aerodynamic coefficient derivative is defined as the mean of its numerical values. All other variations are considered as modeling uncertainties that will be compensated by the robustness of the piloting law. The performance of the proposed control design is evaluated using the jetstream-3102 aircraft in a real-time virtual Simulation Platform for the development of Aircraft Control Systems (SP-ACS), latest was developed in our previous work,[18],[20].[21] The obtained results prove the effectiveness of the proposed control strategy. It is shown that the autopilot delivers a very good performance in terms of convergence towards the desired reference trajectories and in terms of robustness with respect to modeling uncertainties.

The main contributions of this work can be summarised in five (05) points given as follows:
(i) In the developed nonlinear feedback control strategy for velocity and attitude control of fixed wing aircrafts, first, we reduced the model of the aircraft by considering the norm of the inertial speed $V = \|W\|^2 = W^T W$ rather than its individual components. Next, the non-stalling condition reduces to ensuring that $V \leq M$, as the result, the aircraft

model can be reduced to an 8th order.
(ii) The key feature of the control design strategy is the introduction of a virtual control input in order to deal with the underactuation property of such vehicles and to indirectly control the orientation of the aircraft.
(iii) Also, we consider the aircraft as an energy system, the derived controller: rudder, elevator and the aileron deflections is dependent on the thrust control ensuring indirectly a natural distribution between the kinetic and potential energy of the aircraft.
(iv) We deal with the aerodynamic coefficients identification by means of the Total Least Squares Estimation technique (TLSE).
(v) Finally, the simulation and the validation of the developed autopilot is carried out through a Simulation Platform of Aircraft Control Systems (SP-ACS), which is an environment framework based on Software In the Loop (SIL) methodology and we use Microsoft Flight Simulator (FS2004) as the environment for plane simulation.
This paper is organized as follows: in the next section, the dynamic model of a fixed wing aircraft is introduced. In Section three (03), the autopilot design methodology is illustrated. The identification procedure are introduced in Section four (4). In section five (05) the SP-ACS and simulation results of the identification are given and the performance of the proposed design methodology is shown via simulation using a Jetstream-3102 aircraft flight in the SP-ACS. Finally, some conclusions are drawn in section (06).

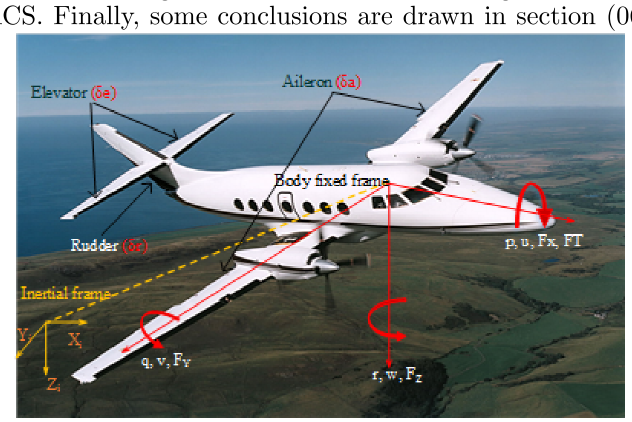

Fig. 1. Jetstream-3102 aircraft withe the referential frames configuration.

Notations: Throughout this paper, the following notations are employed:
- $C_\theta = \cos(\theta)$, $S_\theta = \sin(\theta)$; $T_\theta = \tan(\theta)$;
- All matrices are denoted with bold uppercase letter;
- The supescript $'^T'$ denotes the transpose of a matrix.

2. Aircraft aerodynamical model

In what follows, a brief description of the main features of the aircraft is introduced as well as its dynamic model. The system structure is presented and a model reduction is performed in order to facilitate the development of a speed and attitude control design. The considered aircraft is a British Aerospace (Jetstream-3102), which is a fixed wing, twin turboprop aircraft as illustrated in Fig.1. This type of aircraft has as control inputs the throttle setting command (δ_{th}), and the deflection angles of the three control surfaces: elevator (δ_e), ailerons (δ_a) , and rudder (δ_r) (see FIG 1).[18] The wing surface area s, the wingspan b, the mean aerodynamic chord $\bar{c}$, and the mass m of the aircraft are considered to be constant and their values are given in the Table 1.

Table 1. Parameters of Jetstream-3102

Parameter	Symbol	Value	Unit
Mass	m	6890	lbs
Length	L	28.88	ft
Wingspan	b	46	ft
Wing surface area	S	280	ft^2
Wing root chord	$\bar{c}$	6.5	ft

There are several works, in the literature, that deal with aircraft modelling (see e.g.,[2425]). The main difference between the modelling of a fixed wing aircraft compared to a rotating wing one lies in their distinct aerodynamics coefficients and the type of the propulsion employed. Hereafter, we recall the major modelling aspects of a fixed wing aircraft. As shown in Fig.1, the roll–pitch–yaw convention are adopted using the Euler angles $\Phi = (\phi, \theta, \psi)_T$.[18] The forces F and moments M_G acting on the aircraft at the center of gravity are issued from three major sources: gravity (F_G), engine thrust (F_E) and aerodynamic forces (F_A);[18] that is

$$F = F_G + F_E + F_A \tag{1}$$

$$M_G = M_E + M_A \tag{2}$$

The gravitational force F_G is directed along the normal of the earth plane and is considered constant over the attitude envelope. More precisely,

$$F_G = mg\zeta \tag{3}$$

where

$$\zeta = (\,-S_\theta \; S_\phi C_\theta \; C_\phi C_\theta\,)^T \tag{4}$$

and g is the acceleration due to gravity. The trust force F_E is written in the body frame reference as

$$F_E = F_{prop} \begin{pmatrix} \cos\beta_m \cos\alpha_m \\ \sin\beta_m \\ \cos\beta_m \sin\alpha_m \end{pmatrix} \delta_{th} \tag{5}$$

where F_{prop} is the max engine propulsion force, δ_{th} is throttle setting command, α_m and β_m are the pitch and yaw setting respectively. For small angles, we have $F_E = [F_{prop}(\cos\beta_m \cos\alpha_m)\delta_{th}, 0, 0]^T$ or equivalently $F_E = [F_T, 0, 0]^T$, with $F_T = l\delta_{th}$ and $l = F_{prop}(\cos\beta_m \cos\alpha_m)$. Throughout this work, we assume that $\alpha_m = 0.0349rad$

and $\beta_m = 0.02rad$ so that l is constant. Using Newton-Euler convention, in the body-fixed reference frame the force and aerodynamic moments is given as:

$$F = m\frac{dW}{dt} + \Omega \times W \tag{6}$$

$$M_G = \frac{d(I_G\Omega)}{dt} + \Omega \times I_G\Omega \tag{7}$$

where I_G is the moment of inertia, $\Omega = (p, q, r)^T$ is the angular velocity of the aircraft and $W = (u, v, w)^T$ is the inertial speed vector of the aircraft. The aerodynamic force

$$F_A = (F_x, F_y, F_z)^T = p_a s(C_x, C_y, C_z)^T \tag{8}$$

where $p_a = \frac{1}{2}\rho V$ is the aerodynamic pressure with ρ being the ambient air density and $V = W^T W$ aircraft velocity and C_x, C_y, C_z are the aerodynamic coefficients given as,[22],[23]:[18]

$$\begin{cases} C_x = C_{x,0} + C_{x,1}\alpha + C_{x,2}\alpha^2 + C_{x,3}q\frac{\bar{c}}{\sqrt{V}} \\ \quad +C_{x,4}\delta_a + C_{x,5}\delta_e + C_{x,6}\delta_r + C_{x,7}F_T \\ C_y = C_{y,0} + C_{y,1}\beta + C_{y,2}\beta^2 + C_{y,3}p\frac{b}{2\sqrt{V}} \\ \quad +C_{y,4}\delta_a + C_{y,5}\delta_e + C_{y,6}\delta_r + C_{y,7}F_T \\ C_z = C_{z,0} + C_{z,1}\alpha + C_{z,2}\alpha^2 + C_{z,3}q\frac{\bar{c}}{\sqrt{V}} \\ \quad +C_{z,4}\delta_a + C_{z,5}\delta_e + C_{z,6}\delta_r + C_{z,7}F_T \end{cases} \tag{9}$$

Note that the above coefficients are given up to a second order Taylor approximation in the side-slip angles and up to a first order Taylor approximation in the control inputs. Now as the influence of the of δ_a and δ_r are minimal in the x-direction, we assume that $C_{x,4} = C_{x,6} = 0$. By a similar reasoning and taking into account physical structural consideration, we have $C_{y,2} = C_{y,5} = C_{z,2} = C_{z,4} = C_{z,6} = 0$.[18] Hence, the above expression simplifies into:

$$\begin{cases} C_x = C_{x,0} + C_{x,1}\alpha + C_{x,2}\alpha^2 + C_{x,3}q\frac{\bar{c}}{\sqrt{V}} \\ \quad +C_{x,5}\delta_e + C_{x,7}F_T \\ C_y = C_{y,0} + C_{y,1}\beta + C_{y,3}p\frac{b}{2\sqrt{V}} + C_{y,4}\delta_a \\ \quad +C_{y,6}\delta_r + C_{y,7}F_T \\ C_z = C_{z,0} + C_{z,1}\alpha + C_{z,3}q\frac{\bar{c}}{2\sqrt{V}} + C_{z,5}\delta_e + C_{z,7}F_T \end{cases} \tag{10}$$

From equations (1), (3) and (6), the dynamics of the inertial speed, W, is given by

$$\dot{W} = \mathbf{R}_1(\Omega)W + g\zeta + \frac{1}{m}F_A + \frac{1}{m}B_0F_T \tag{11}$$

with $U = (\delta_a, \delta_e, \delta_r)^T$

$$\mathbf{R}_1(\Omega) = \begin{pmatrix} 0 & r & -q \\ -r & 0 & p \\ q & -p & 0 \end{pmatrix}, B_0 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

By developing equation (11) become

$$\dot{W} = \mathbf{R}_1(\Omega)W + \Psi + \mathbf{B}_2U + \frac{1}{m}B_3F_T \tag{12}$$

where $\Psi = g\zeta + \tilde{\Psi}$ with

$$\tilde{\Psi} = \frac{\rho s V}{2m}\begin{bmatrix} C_{x,0} + C_{x,1}\alpha + C_{x,2}\alpha^2 + C_{x,3}q\frac{\bar{c}}{\sqrt{V}} \\ C_{y,0} + C_{y,1}\beta + C_{y,3}\frac{pb}{2\sqrt{V}} \\ C_{z,0} + C_{z,1}\alpha + C_{z,2}q\frac{\bar{c}}{\sqrt{V}} \end{bmatrix}$$

$$\mathbf{B}_2 = \frac{1}{2}\rho s V\begin{pmatrix} 0 & C_{x,4} & 0 \\ C_{y,3} & 0 & C_{y,4} \\ 0 & C_{z,3} & 0 \end{pmatrix}, B_3 = \frac{1}{2}\rho s V\begin{pmatrix} C_{x,5}+1 \\ C_{y,5} \\ C_{z,4} \end{pmatrix}$$

In other words,

$$F_A = m\tilde{\Psi} + m\mathbf{B}_2U + m(B_3 - B_0)F_T \tag{13}$$

The propulsive forces can also create moments if the thrust does not act through the aircraft center of gravity. We assume the engine is mounted in such a way that the thrust point lies in the body axes xz-plane and offsetted from the center of gravity by Z_{TP} in the body-axes z-direction so that $M_E = (0, F_TZ_{TP}, 0)^T$. The moments caused by aerodynamic forces M_A and aerodynamic moments coefficients are given by:[18]

$$M_A = \frac{1}{2}\rho s V(bC_l, \bar{c}C_m, bC_n)^T \tag{14}$$

where C_l, C_m and C_n are given by a first order Taylor approximation in the various variables involved the aerodynamic moments coefficients as well as taking in account the physical constraints:[18]

$$\begin{cases} C_l = C_{l,1}\beta + \frac{b}{2\sqrt{V}}(C_{l,2}p + C_{l,3}r) + C_{l,4}\delta_a + C_{l,5}\delta_r \\ C_m = C_{m,0} + C_{m1}\alpha + \frac{\bar{c}}{2\sqrt{V}}(C_{m,2}\dot{\alpha} + qC_{m,3}) + C_{m,4}\delta_e \\ C_n = C_{n,1}\beta + \frac{b}{2\sqrt{V}}(rC_{n,2} + pC_{n,3}) + C_{n,4}\delta_a + C_{n,5}\delta_r \end{cases} \tag{15}$$

It is important to note that the above two Taylor approximations in the aerodynamic moment and coefficients leads to modeling errors and uncertainties on the systems parameters. Consequently, we get:

$$\dot{\Omega} = \gamma(\Omega) + \frac{1}{2}\rho s V\mathbf{P}_1\Pi(\Omega, W) + \frac{1}{2}\rho s V\mathbf{P}_1\mathbf{B}_1U + P_2F_T \tag{16}$$

where:

$$\gamma = \begin{pmatrix} q(a_1p + a_2r) \\ a_5pr - a_6(p^2 - r^2) \\ q(a_8p - a_1r) \end{pmatrix}, \mathbf{P}_1 = \begin{pmatrix} a_3 & a_4 & 0 \\ 0 & 0 & a_7 \\ a_4 & a_9 & 0 \end{pmatrix}$$

$$\Pi = \begin{bmatrix} b\left(C_{l,1}\beta + \frac{b}{2\sqrt{V}}(C_{l,2}p + C_{l,3}r)\right) \\ \bar{c}\left(C_{n,1}\beta + \frac{b}{2\sqrt{V}}(rC_{n,2} + pC_{n,3})\right) \\ b\left(C_{m,0} + C_{m,1}\alpha + \frac{\bar{c}}{2\sqrt{V}}(C_{m,2}\dot{\alpha} + qC_{m,3})\right) \end{bmatrix}$$

$$P_2 = \begin{pmatrix} 0 \\ a_7Z_{TP} \\ 0 \end{pmatrix}, \mathbf{B}_1 = \begin{pmatrix} bC_{l,4} & 0 & bC_{l,5} \\ \bar{c}C_{n,4} & 0 & \bar{c}C_{n,5} \\ 0 & bC_{m,4} & 0 \end{pmatrix}$$

The dynamics of the aircraft position $X = (x, y, z)^T$, is given by:[18]

$$\dot{X} = \mathbf{R}_0(\Phi)W \tag{17}$$

with

$$\mathbf{R}_0(\Phi) = \begin{pmatrix} C_\theta C_\psi & S_\phi S_\theta C_\psi - C_\phi S_\psi & C_\phi S_\theta C_\psi + S_\phi S_\psi \\ C_\theta S_\psi & S_\phi S_\theta S_\psi + C_\phi C_\psi & C_\phi S_\theta S_\psi - S_\phi C_\psi \\ -S_\theta & S_\phi C_\theta & C_\theta \end{pmatrix}$$

In the earth fixed reference frame, the rotational velocity is described by the variables $\dot{\phi}$, $\dot{\theta}$ and $\dot{\psi}$. However, in the body-fixed frame, the rotational velocity is described by roll p, pitch q and yaw rates, r. The relation between those two sets of variables are given by:

$$\dot{\Phi} = \mathbf{\Gamma}(\Phi)\,\Omega \tag{18}$$

and

$$\mathbf{\Gamma}(\Phi) = \begin{pmatrix} 1 & T_\theta S_\phi & T_\theta C_\phi \\ 0 & C_\phi & -S_\phi \\ 0 & \frac{S_\phi}{C_\theta} & \frac{C_\phi}{C_\theta} \end{pmatrix}$$

In summary, the dynamical behavior of the aircraft model using Newton–Euler convention, is given by:[18]

$$\begin{cases} \dot{X} = \mathbf{R}_0 W \\ \dot{W} = \mathbf{R}_1 W + \mathbf{\Psi} + B_2 U + \frac{1}{m} B_3 F_T \\ \dot{\Omega} = \gamma + \frac{1}{2}\rho s V \mathbf{P}_1 \Pi + \frac{1}{2}\rho s V \mathbf{P}_1 \mathbf{B}_1 U + P_2 F_T \\ \dot{\Phi} = \mathbf{\Gamma}\Omega \end{cases} \tag{19}$$

where we have dropped the arguments for simplicity of notations.

3. Autopilot design methodology

The main aim of the present work is to design an autopilot in order to track a desired attitude and velocity in spite of modeling errors and/or uncertainties on parameters that can affect the aircraft model. For this, one has first to make some observation about the aircraft systems' structure. From the above equations (19), one can see that the system possesses the structure as illustrated in Fig 2, where for simplicity, we have denoted:

$$\begin{cases} g(.) = \mathbf{R}_1 W + \mathbf{\Psi} + B_2 U + \frac{1}{m} B_3 F_T \\ f(.) = \gamma + \frac{1}{2}\rho s V \mathbf{P}_1 \Pi + \frac{1}{2}\rho s V \mathbf{P}_1 \mathbf{B}_1 U + P_2 F_T \end{cases} \tag{20}$$

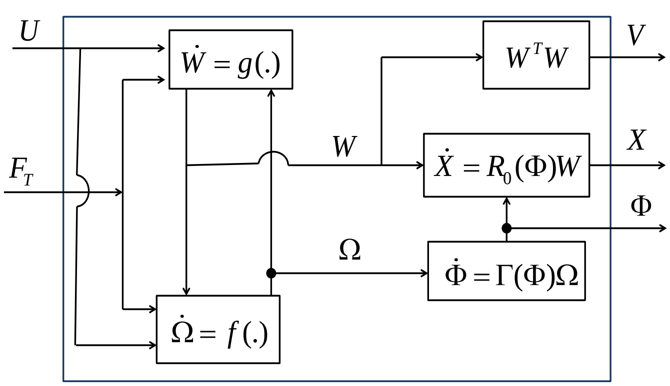

Fig. 2. System structure

As mentioned in the introduction, it can be observed that the dynamics of angular velocity Ω and inertial speed W are directly affected by the control inputs U and thrust force F_T while the other two variables X and Φ are not. The dynamics of the Φ and X are indirectly affected by the actuators U and F_T through their tight coupling with Ω and W. Now, it is well-known that the four (04) inputs variables U and F_T can only control four (04) states variables. Therefore,we need to find judicious ways to indirectly control the rest of the state variables. For this we start by reducing the model of the aircraft given by (19) by taking into account the practical consideration of piloting. In effect, the pilot does not control the individual components of the velocity but rather its magnitude or norm:

$$V = W^T W = \|W\|^2 \tag{21}$$

The dynamics of V is given by:

$$\begin{aligned} \dot{V} &= 2W^T \dot{W} \\ &= 2W^T \left(\mathbf{R}_1 W + \Psi + \mathbf{B}_2 U + \frac{1}{m} B_3 F_T \right) \end{aligned}$$

Since $\mathbf{R}_1$ is skew-symmetric, we have $W^T \mathbf{R}_1 W = 0$, so that

$$\dot{V} = 2W^T \Psi + 2W^T \mathbf{B}_2 U + \frac{2}{m} W^T B_3 F_T \tag{22}$$

Next, the pilot has to ensure that the aircraft does not stall. For this, we have to ensure that the derivative of the position X does not escape to infinity. As a result, we impose the following condition:

$$\|\dot{X}\|^2 \leq M \tag{23}$$

where $M > 0$. Note that since $\mathbf{R}_0$ is an orthogonal matrix (i.e. $\mathbf{R}_0^T = \mathbf{R}_0^{-1}$), we have

$$\|\dot{X}\|^2 = \dot{X}^T \dot{X} = W^T W = V \tag{24}$$

Therefore, the non-stalling condition reduces to ensuiring that $V \leq M$. As the result, the above aircraft model can be reduced to an 8th order system described by:

$$\begin{cases} \|\dot{X}\|^2 = V \\ \dot{V} = 2W^T \Psi + 2W^T \mathbf{B}_2 U + \frac{2}{m} W^T B_3 F_T \\ \dot{\Omega} = \gamma + \frac{1}{2}\rho s V \mathbf{P}_1 \Pi + \frac{1}{2}\rho s V \mathbf{P}_1 \mathbf{B}_1 U + P_2 F_T \\ \dot{\Phi} = \mathbf{\Gamma}\Omega \end{cases} \tag{25}$$

Based on the above observations, we adopt the following design strategy, which is also illustrated in Fig 3:

- First, we introduce a virtual control Ω_v to control the orientation Φ towards the desired reference trajectory Φ_{ref}. We refer this controller as *Controller 1* as depicted in Fig 3.
- Next, we use the input variable F_T to design a controller to track a given speed trajectory V_{ref}. This is *Controller 2* in Fig 3.
- Finally, we employ the input U is to steer Ω towards a given reference trajectory Ω_{ref}; which is chosen in such a way that it permit the virtual control input to track the desired orientation Φ_{ref}. This is referred to as *Controller 3* Fig 3.

In what follows, we detail the development of each controllers.

3.1. *Design of Controller 1*

Let $\Phi_{ref} = (\phi_{ref}, \theta_{ref}, \psi_{ref})^T$ be the desired orientation. We consider Ω as a virtual control input that we rename as Ω_v. Our objective is to steer $\Phi = (\phi, \theta, \psi)^T$ to $\Phi_{ref} = (\phi_{ref}, \theta_{ref}, \psi_{ref})^T$ using the virtual control input Ω_v since Φ is not directly affected by the real actuators. We have equation (18):

$$\dot{\Phi} = \mathbf{\Gamma}\Omega_v$$

we seek for a controller of the form:

$$\Omega_v = -\mathbf{\Gamma}^{-1}\mathbf{K}_0\Phi + \mathbf{\Gamma}^{-1}u_1 \tag{26}$$

where $\mathbf{K}_0 = diag(k_{0,1}, k_{0,2}, k_{0,3})$ is a gain matrix with $k_{0,i} > 0, i = 1, 2, 3$ and

$$u_1 = \dot{\Phi}_{ref} + \mathbf{K}_0\Phi_{ref} \tag{27}$$

is an additional control that is chosen such that $\Phi \to \Phi_{ref}$ as $t \to +\infty$. In other words,

$$\Omega_v = -\mathbf{\Gamma}^{-1}\mathbf{K}_0(\Phi - \Phi_{ref}) + \mathbf{\Gamma}^{-1}\dot{\Phi}_{ref} \tag{28}$$

Under the above virtual control, the closed-loop system simplifies to:

$$\dot{\Phi} - \dot{\Phi}_{ref} = -\mathbf{K}_0(\Phi - \Phi_{ref}) \tag{29}$$

By setting $e_\Phi = \Phi - \Phi_{ref}$, one can see that $\dot{e}_\Phi = -\mathbf{K}_0 e_\Phi$ which shows that $e_\Phi(t) \to 0$ when $t \to +\infty$. We have therefore to make sure that $\Omega(t) \to \Omega_v$ when $t \to +\infty$. This will be realised by Controller 3 subsequently.

3.2. *Design of Controller 2*

Consider again the dynamics of V; that is:

$$\dot{V} = 2W^T(\Psi + \mathbf{B}_2U) + \frac{2}{m}W^TB_3F_T \tag{30}$$

Equivalently, we can write:

$$\dot{V} - \dot{V}_{ref} = -\dot{V}_{ref} + 2W^T(\Psi + \mathbf{B}_2U) + \frac{2}{m}W^TB_3F_T \tag{31}$$

where $V_{ref} = W_{ref}^T W_{ref} = \|W_{ref}\|^2$ is a desired time-varying speed. We choose the aerodynamic and thrust forces such that:

$$-\dot{V}_{ref} + \frac{2}{m}W^TB_3F_T = -l_1(V - V_{ref}) - V_{ref} \tag{32}$$

where $l_1 > 1$. That is,

$$F_T = \frac{1}{\frac{2}{m}W^TB_3}\left(\dot{V}_{ref} - l_1(V - V_{ref}) - V_{ref}\right) \tag{33}$$

Note that

$$\begin{aligned} W^TB_3 &= \frac{1}{2}\rho V s\,(\,u\ v\ w\,)\begin{pmatrix} C_{x,5} + k \\ C_{y,5} \\ C_{z,4} \end{pmatrix} \\ &= \frac{1}{2}\rho V s\,[u\,(k + C_{x,5}) + vC_{y,5} + wC_{z,4}] \neq 0. \end{aligned}$$

Then in closed loop we have:

$$\dot{V} - \dot{V}_{ref} = -l_1(V - V_{ref}) - V_{ref} + 2W^T(\Psi + \mathbf{B}_2U) \tag{34}$$

Setting $e_V = V - V_{ref}$, we have

$$\begin{aligned} \dot{e}_V &= -l_1e_V - V_{ref} + 2W^T(\Psi + \mathbf{B}_2U) \\ &\leq -l_1e_V - V_{ref} + \left|2W^T(\Psi + \mathbf{B}_2U)\right| \end{aligned}$$

By using the Cauchy-Schwarz inequality, we get

$$\begin{aligned} \dot{e}_V &\leq -l_1e_V - V_{ref} + \|W\|^2 + \|\Psi + \mathbf{B}_2U\|^2 \\ &\leq -l_1e_V - V_{ref} + V + \|(\Psi + \mathbf{B}_2U)\|^2 \end{aligned}$$

since $V = \|W\|^2$. Consequently,

$$\dot{e}_V \leq -(l_1 - 1)e_V + \|\Psi + \mathbf{B}_2U\|^2 \tag{35}$$

It is therefore clear that if $\|\Psi + \mathbf{B}_2U\|^2$ is bounded, one can choose l_1 large enough so that $e_V(t) \to 0$ asymptotically when $t \to +\infty$. The boundedness of $\|\Psi + \mathbf{B}_2U\|^2$ is ensured by Controller-3 hereafter.

3.3. *Step 3: Design of Controller 3*

The purpose of Controller 3 is to make sure that Ω tracks Ω_v as $t \to +\infty$. This will be done using the control input U. Consider again the angular velocity equation from system (25):

$$\dot{\Omega} = \gamma + \frac{1}{2}\rho V s\mathbf{P}_1\Pi + \frac{1}{2}\rho V s\mathbf{P}_1\mathbf{B}_1U + P_2F_T \tag{36}$$

which can be equivalently written as

$$\dot{\Omega} - \dot{\Omega}_v = -\dot{\Omega}_v + \gamma + \frac{1}{2}\rho sV\mathbf{P}_1\Pi + \frac{1}{2}\rho sV\mathbf{P}_1\mathbf{B}_1U + P_2F_T \tag{37}$$

Proceeding as before, we impose

$$\begin{aligned} \dot{\Omega} - \dot{\Omega}_v &= -\dot{\Omega}_v + \gamma + \frac{1}{2}\rho sV\mathbf{P}_1\Pi + \frac{1}{2}\rho sV\mathbf{P}_1\mathbf{B}_1U + P_2F_T \\ &= -\eta(\Omega - \Omega_v) \end{aligned}$$

with $\eta > 0$. Then,

$$\begin{aligned} U = \frac{1}{\frac{1}{2}\rho sV}(\mathbf{P}_1\mathbf{B}_1)^{-1}\Big[&\dot{\Omega}_v - \eta(\Omega - \Omega_v) \\ &-P_2F_T - \frac{1}{2}\rho sV\mathbf{P}_1\Pi - \gamma\Big] \end{aligned} \tag{38}$$

In closed-loop, we have

$$\dot{\Omega} - \dot{\Omega}_v = -\eta(\Omega - \Omega_v) \tag{39}$$

Set $e_\Omega = \Omega - \Omega_v$, then

$$\dot{e}_\Omega(t) = -\eta e_\Omega(t) \tag{40}$$

so that

$$e_\Omega(t) = e^{-\eta t} e_\Omega(0) \tag{41}$$

From this we can see that $e_\Omega(t) \to 0$ when $t \to +\infty$. In other words, $\Omega \to \Omega_v$ when $t \to +\infty$.

Remark 3.1. Note that to further improve the convergence of the controller, one can add an integral term in the controller so that

$$\begin{aligned} U = & \frac{1}{\frac{1}{2}\rho s V} \left(\mathbf{P}_1 \mathbf{B}_1\right)^{-1} \Big[\dot{\Omega}_v - \eta\left(\Omega - \Omega_v\right) \\ & + \eta \int \left(\Omega - \Omega_v\right) dt - P_2 F_T - \frac{1}{2}\rho s V \mathbf{P}_1 \Pi - \gamma\Big] \end{aligned} \tag{42}$$

so that in closed-loop we have

$$\dot{e}_\Omega(t) = -\eta e_\Omega(t) - \mu \int e_\Omega(t) dt \tag{43}$$

Summary of result: To summarise, we can state that under the following control laws:

$$\Omega_v = -\mathbf{\Gamma}^{-1}\mathbf{K}_0(\Phi - \mathbf{K}_0^{-1}\Omega) \tag{44}$$

$$\begin{aligned} U = & \frac{1}{\frac{1}{2}\rho s V} \left(\mathbf{P}_1 \mathbf{B}_1\right)^{-1} \Big[\dot{\Omega}_v - \eta\left(\Omega - \Omega_v\right) \\ & - P_2 F_T - \frac{1}{2}\rho s V \mathbf{P}_1 \Pi - \gamma\Big] \end{aligned} \tag{45}$$

$$F_T = \frac{1}{\frac{2}{m} W^T B_3} \left(\dot{V}_{ref} - l_1\left(V - V_{ref}\right) - V_{ref}\right) \tag{46}$$

$$\dot{\Phi}_{ref} = -\mathbf{K}_0 \Phi_{ref} + \Omega_v \tag{47}$$

where

- $l_1 > 1$, $\eta > 0$ and $\mathbf{K}_0 = diag(k_{0,1}, k_{0,2}, k_{0,3})$ is a gain matrix with $k_{0,i} > 0, i = 1, 2, 3$
- Ω_v and V_{ref} are the desired orientation and speed respectively,

the aircraft overall closed-loop system (48) converges towards the desired trajectories Φ_{ref} and V_{ref} while avoiding stalling condition:

$$\begin{cases} \|\dot{X}\|^2 = V \\ \dot{\Omega} = \dot{\Omega}_v - \eta\left(\Omega - \Omega_v\right) \\ \dot{\Phi} = \dot{\Phi}_{ref} - \mathbf{K}_0\left(\Phi - \Phi_{ref}\right) + \left(\Omega - \Omega_v\right) \\ \dot{V} = \dot{V}_{ref} - l_1\left(V - V_{ref}\right) - V_{ref} + 2W^T\left(\Psi + \mathbf{B}_2 U\right) \end{cases} \tag{48}$$

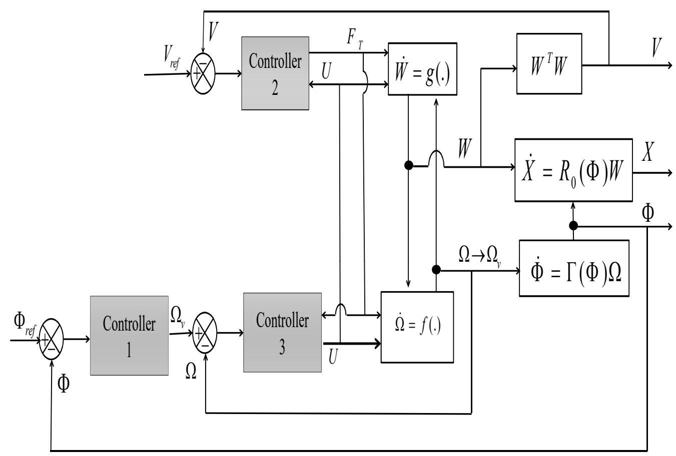

Fig. 3. Control design architecture

4. Aerodynamic coefficients identification

The aerodynamic coefficients given by equations (15, 9) define the aircraft's behaviour over flight range throw the dynamic model given by equation(19), which are unknown, variable and classified. For that the knowledge of the different aerodynamic coefficients turns out to be of great importance for the development of control law. In this paper to deal with the identification of aircraft aerodynamic coefficients, based in our previous work,[20,21] we adopt the Total Least Squares Estimation technique (TLSE). TLSE is based on the use of the SVD decomposition, it has the interesting propriety of being able to estimate the invert of any matrix, whether it is square or rectangular, and most of all, whether it is singular or not.[26] The expression of aerodynamic coefficients equations 15 and 9 can be represented using vector and matrix notation as flow,

$$Y = A\Theta \tag{49}$$

where $Y_{6\times 1}$ is the the dimensional vector of the variable to be explained, $A_{6\times 32}$ is the dimensional matrix of explanatory variables, and $\Theta_{32\times 1}$ dimensional vector of system parameters where:

$$\begin{aligned} Y &= [C_x, C_y, C_z, C_l, C_m, C_n]^T_{6\times 1} \\ A &= [A_1, A_2, A_3, A_4, A_5]^T_{6\times 32} \\ \Theta &= [\Theta_1\ \Theta_2\ \Theta_3\ \Theta_4\ \Theta_5]^T_{1\times 32} \end{aligned}$$

with:

$$\begin{aligned} \Theta_1 &= \left[C_{x,1}\ C_{y,1}\ C_{z,1}\ C_{l,1}\ C_{m,1}\ C_{n,1}\right]^T \\ \Theta_2 &= \left[C_{x,2}\ C_{m,2}\right]^T \\ \Theta_3 &= \left[C_{x,3}\ C_{y,3}\ C_{z,3}\ C_{l,2}\ C_{l,3}\ C_{m,3}\ C_{n,2}\ C_{n,3}\right]^T \\ \Theta_4 &= \left[C_{x,5}\ C_{y,4}\ C_{y,6}\ C_{z,5}\ C_{l,4}\ C_{l,5}\ C_{m,4}\ C_{n,4}\ C_{n,5}\right]^T \\ \Theta_5 &= \left[C_{x0}\ C_{y,0}\ C_{z,0}\ C_{m,0}\ C_{x,7}\ C_{y,7}\ C_{z,7}\right]^T \end{aligned}$$

$$A_1 = \begin{pmatrix} \alpha & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta & 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta \end{pmatrix}, A_5 = \begin{pmatrix} 1 & 0 & 0 & 0 & F_T & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & F_T & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & F_T \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} \alpha^2 & 0 \\ 0 & \dot{\alpha}\frac{\bar{c}}{2V} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}, A_4 = \begin{pmatrix} \delta_e & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_a & \delta_r & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_e & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_a & \delta_r & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_e & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \delta_a & \delta_r \end{pmatrix}$$

$$A_3 = \begin{pmatrix} q\frac{\bar{c}}{2V} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & p\frac{b}{2V} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & q\frac{\bar{c}}{2V} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p\frac{b}{2V} & r\frac{b}{2V} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & q\frac{\bar{c}}{2V} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r\frac{b}{2V} & p\frac{b}{2V} \end{pmatrix}$$

Rewriting the linear model of equation (49) as:

$$[\,A\,|Y\,]\begin{bmatrix} \Theta \\ -1 \end{bmatrix} = \mathbf{0} \quad (50)$$

The singular value decomposition of $[\,A\,|Y\,]$ is:

$$[\,A\,|Y\,] = U\Sigma V^T \quad (51)$$

for $n = 32$, $m = 6$ and $d = 1$: $\Sigma_{m\times(n+d)} = diag(\sigma_1, ..., \sigma_{n+d})$, $\sigma_1 \geqslant ... \geqslant \sigma_{n+d}$ be the singular values of $[\,A\,|Y\,]$, and define the partitioning:[26]

$$V: = \begin{bmatrix} V_{11} & V_{12} \\ V_{21} & V_{22} \end{bmatrix} \begin{matrix} n \\ d \end{matrix}, \Sigma: = \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{matrix} n \\ d \end{matrix}$$

The total least squares solution is given by:[26]

$$\hat{\Theta}_{tlse} = -V_{12}V_{22}^{-1} \quad (52)$$

this solution exists if and only if V_{22} is non-singular. In addition, it is unique if and only if $\sigma_n \neq \sigma_{n+1}$. In what flow we give the basic total least squares algorithm.[26]

Algorithm 1 Basic total least squares algorithm

Input: $A \in \mathbb{R}^{m\times n}$ and $Y \in \mathbb{R}^{m\times d}$
Compute the singular value decomposition :
$[\,A\,|Y\,] = U\Sigma V^T$
if V_{22} *is nonsingular* **then**
set $\hat{\Theta}_{tlse} = -V_{12}V_{22}^{-1}$
else
Output a message that the problem as no solution and stop.
Output: $\hat{\Theta}_{tlse}$ a total least squares solution of $Y \approx A\Theta$

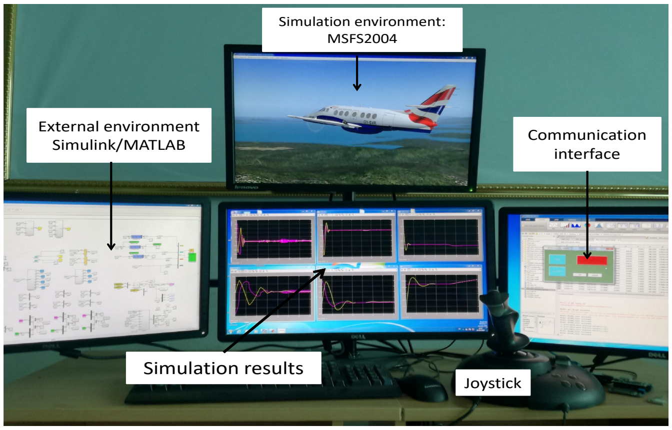

Fig. 4. Simulation Platform of Aircraft Control Systems (SP-ACS).

5. Simulation results

In this section we introduce, firstly, the employed Simulation Platform of Aircraft Control Systems (SP-ACS), then we present the aerodynamic coefficients identification results. Finely, the simulation and the validation of the developed autopilot is carried out firstly using a MATLAB/Simulink model for Jetstream-3102 aircraft given in Figure 7, this model use the aerodynamic coefficients estimated by TLSE for autopilot parameters tuning. Secondly the validation was carried out in a the SP-ACS for trajectory tracking.

5.1. *Simulation Platform of Aircraft Control Systems (SP-ACS)*

Since it is very complicated and difficult to access real test and evaluation systems, we resort to simulation tools in order to validate the performance of the designed controller.[19] For that reason, we have realized in our previous work[20,21] a system-level Simulation Platform for the development of Aircraft Control Systems(SP-ACS). The SP-ACS, as illustrated in Figure 4, is composed of three parts:
(i) **Simulation environment**: we use a commercial Flight Simulator(MSFS2004) developed by Microsoft. This simulator includes several simulated aircrafts in its library which are piloted by test pilots and they demonstrate that the flight simulation are very close on their usual flights with the real aircrafts.
(ii) **External environment**: This environment is built using *Real Time Windows Target* module of Simulink/MATLAB. In this environment the different aircraft control laws are implemented.
(iii) **Communication interface**: This is a real time interface that was implemented between the simulation environment (MSFS2004) and the external environment in order to read and write the sensors, actuators data and parameters. By exploiting IPC (Inter-Process Communication) using a buffer of $64ko$, the dynamic link li-

brary called FSUIPC.dll (Flight Simulator Universal Inter-Process Communication)[27] allows external applications to read and write in and from MSFS2004. To read and write a variable, one needs to know its address in the FSUIPC table, its format, and the necessary communications. For example, the indicated air speed is read as a signed long($S32$) at the address ($0x02BC$). The elevator deflection δ_e is read and write as a signed long ($S16$) at the address ($0x0BB2$).

5.2. *Identification results*

In this section we present the aerodynamic coefficients identification results based on TLSE technique. Using the Simulation Platform of Aircraft Control Systems (SP-ACS), the investigation aircraft is the jetstream-3102, the piloting controls are sent by using the PPjoy (virtual joystick), several flight tests were conducted by changing the simulation parameters (speed, weather, time of day, ...). Figures Fig.5 and Fig.6 present some results of aerodynamic coefficients derivatives. They are function of the time and their values are around the intrinsic values. The obtained estimation of aerodynamic coefficients derivatives by TLSE, are defined as the mean values of those coefficients given in Table 2.

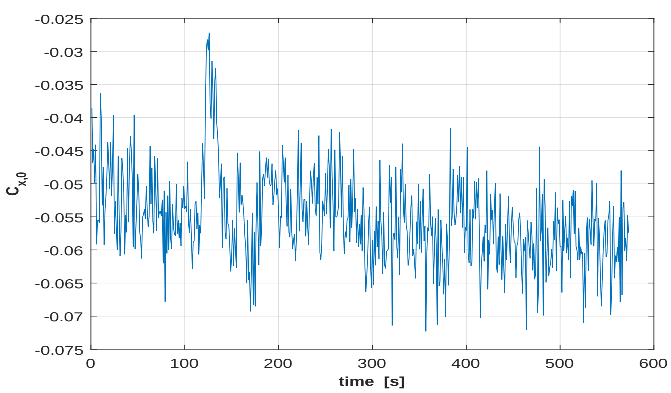

Fig. 5. Aerodynamic coefficients derivatives $C_{x,0}$.

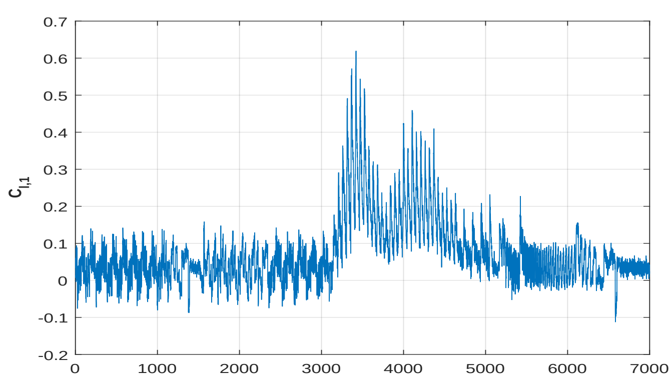

Fig. 6. Aerodynamic coefficients derivatives $C_{l,1}$.

5.3. *Autopilot parameters tuning*

In this section we investigate the autopilot parameters (η, K_0, l_1) tuning for a fixed references (attitude and speed), for that we start by using a MATLAB/Simulink model for the investigated aircraft (jetstream 3102) before working with the Simulation Platform of Aircraft Control Systems (SP-ACS). Using the identification parts results this model was developed, as illustrated in Fig.7, to reproduce the aerodynamic behavior of the aircraft. Simulation results are carried out corresponding to roll, pitch and yaw angles and the reference speed respectively as: $\Phi_{ref} = (\phi_{ref}, \theta_{ref}, \psi_{ref})^T = (1.5, 0.2, 1)^T$ and $V_{ref} = 80[m/s]$.

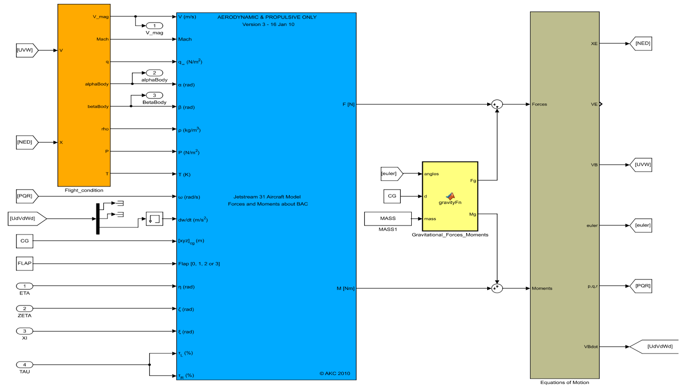

Fig. 7. The developed Jetstream-3102 aircraft model.

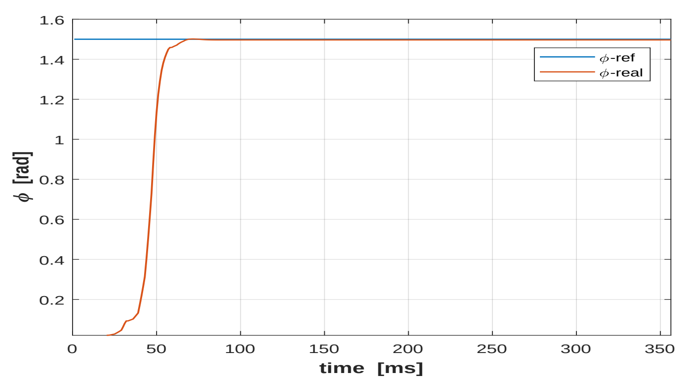

Fig. 8. Tracking of roll angle ϕ.

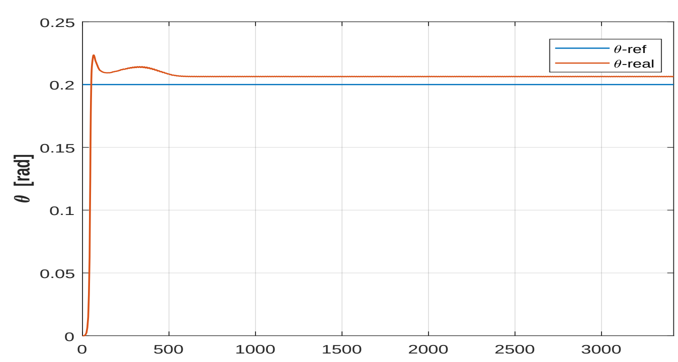

Fig. 9. Tracking of pitch angle θ.

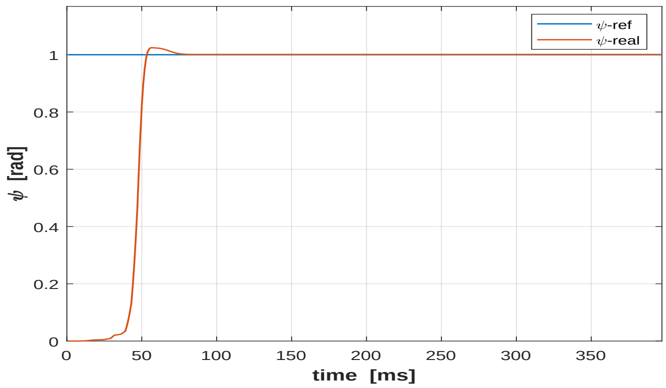

Fig. 10. Tracking of yaw angle ψ.

Figures: Fig.8, Fig.9, Fig.10 and Fig.11 shows the tracking results for respectively roll (ϕ), pitch(θ), yaw (ψ) angles and speed, for the choosing gains: $\eta = 1500$, $l_1 = 250$ and $\mathbf{K}_0 = diag(150, 20, 200)$.

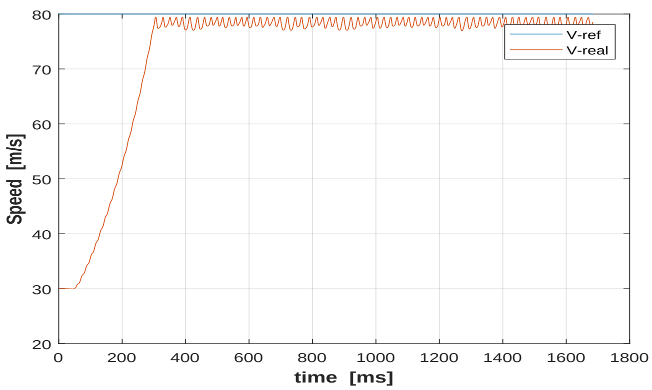

Fig. 11. Speed V tracking.

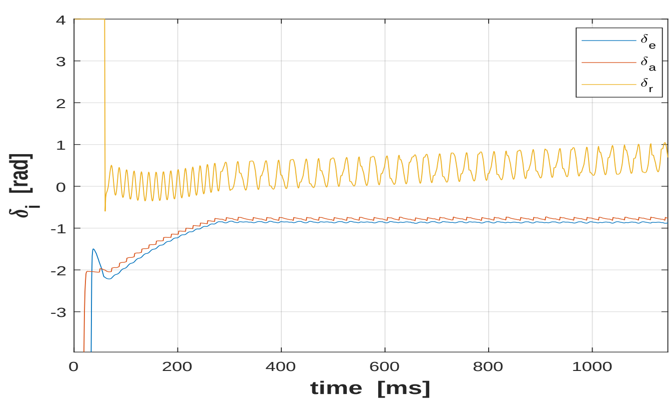

Fig. 12. Control surfaces deflection.

According to the obtained results after several simulation we make the following observation:

- The parameter η affects the rise time of the system and at the same time the amplitude of the overflow. A large value of η reduces the rise time and increases the overflow. Therefore, the choice of these gains is based on a trade-off between reduction of the rise time and the overflow.
- The parameter $\mathbf{K}_0$ affect the tracking precision of attitude. Therefore, one have to make a compromise between precision, rising time and overflow.

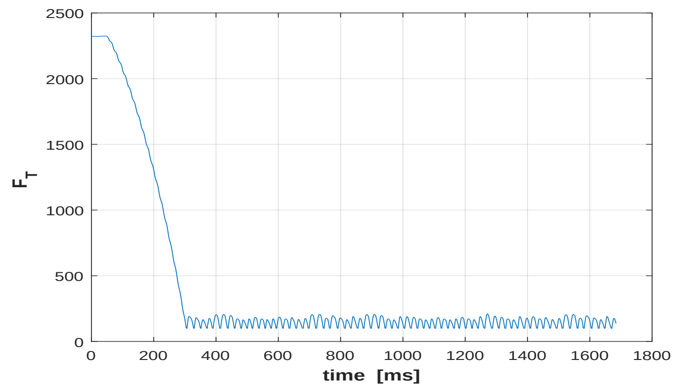

Fig. 13. Thrust force (F_T).

5.4. *Autopilot evaluation in Simulation Platform of Aircraft Control Systems (SP-ACS)*

To carry out the simulations in the Simulation Platform of Aircraft Control Systems (SP-ACS), we first run the Flight Simulator FS2004 and the interface with the Real Time Windows Target module of Simulink/Matlab. The aircraft taking off was carried out using the Joystick. Then, we run our software to transmit the control outputs ($\delta_a, \delta_e, \delta_r$ and F_T) calculated by the designed autopilot in order to track the desired velocity and attitude references.

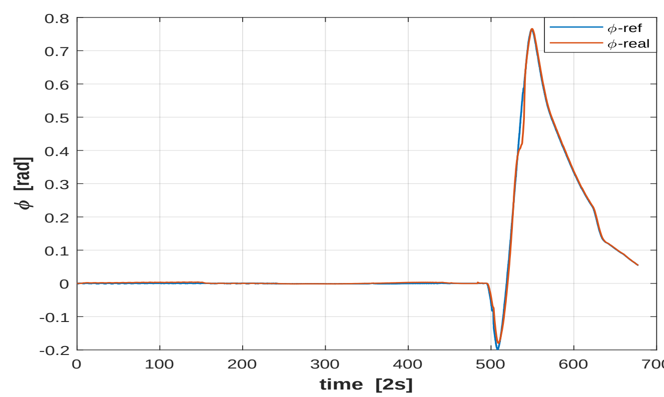

Fig. 14. Tracking of bank angle (ϕ).

The references are the speed V expressed in [m/s], the attitude (Φ) expressed in [rad] collected from a simulated flight in the (SP-ACS). To satisfy the actuators bounds, the input signals to the upper and lower saturation values of the control laws are used and a scaled functions are added to respect the virtual Joystick (PPjoy) bounds: upper limit is 62767, lower limit is 1.

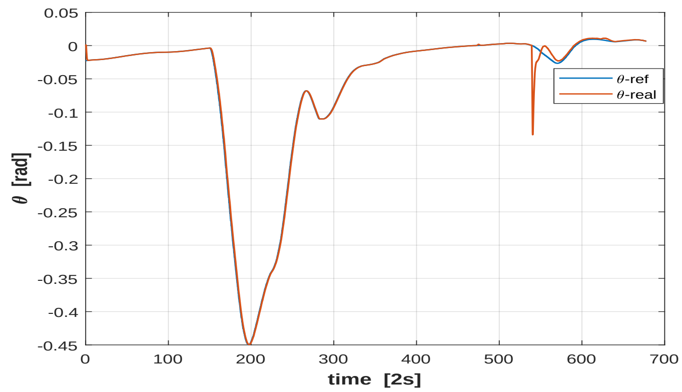

Fig. 15. Tracking of pitch angle (θ).

For the model, the different aerodynamic coefficients used in equations (10) and (15) are listed in Table 2. Notice that the aircraft aerodynamic model contain a singularity when $\theta = \pi/2$ where the terms $tg(\theta)$ and $sec(\theta)$ are infinite. Such conditions occur in aerobatic maneuvers where the aircraft loops or climbs at a near vertical angle.

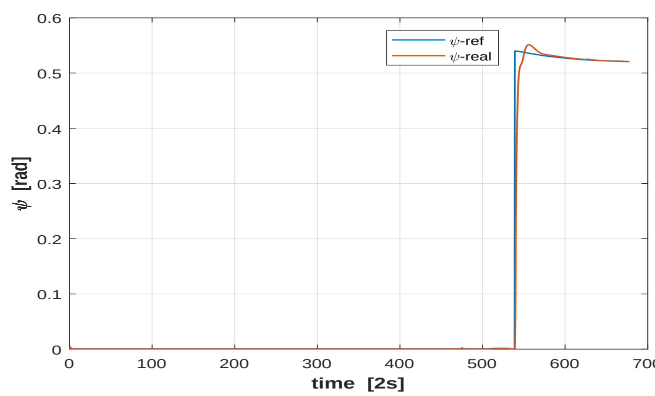

Fig. 16. Tracking of heading angle (ψ) tracking.

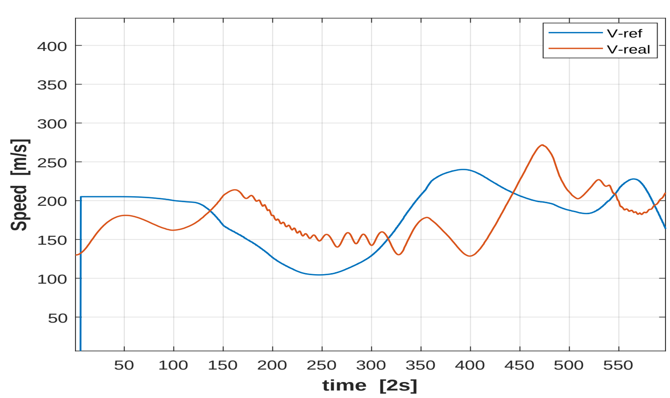

Fig. 17. Tracking of speed (V).

To overcome these problems, the pitch angle can be constrained so that the computation results in a valid floating-point number; for example, $tan(89.5) = 114.6$ and this value can be used in computations when the pitch attitude is between 89.50 and 90.50. The numerical error introduced by this approximation only occurs at this extreme flight attitude where its effects on the aircraft behavior may not be apparent. The developed autopilot was tested for trajectory tracking purpose. For that we use a reference attitude and speed signal from a recorded flight. The latter is subdivided into three different flight phases: the first phase is an accelerated level flight, the second one is an accelerated climb and the last one is a turn.

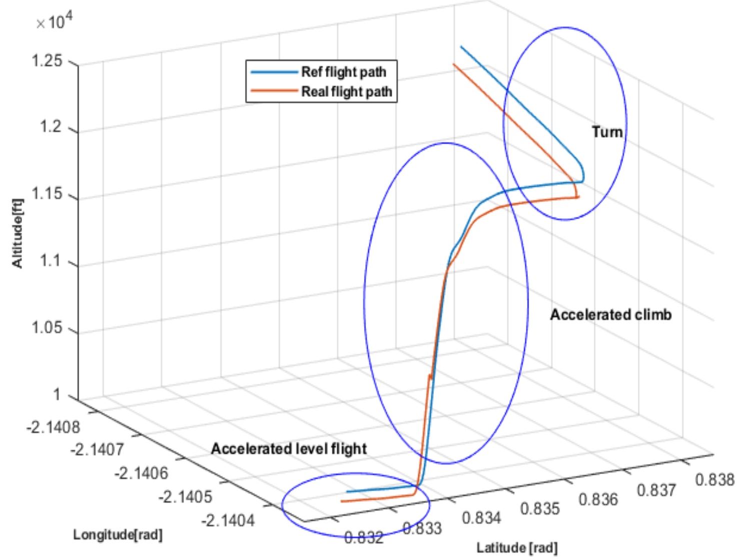

Fig. 18. Simulation and real trajectory.

(i) **Accelerated level flight**: lasts for a duration of [0, 290]s, in this flight phase bank and heading are zero with the pitch variations being small and the speed is increasing. The developed autopilot generate an increasing thrust control signal allows the motors to generate the necessary force F_T (Fig.22) which allows speed tracking, at the same time the aileron deflection is small and rudder deflection are zero because this control signals are not used in this flight step only elevator deflection (Fig.20) is used to track the pitch angle.

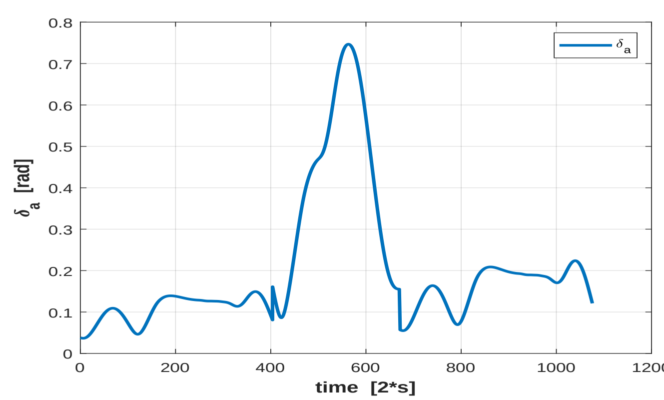

Fig. 19. Ailerons control surface deflection (δ_a).

(ii) **Accelerated climb flight**: lasts for a duration of [290, 640]s, in this flight phase, the controller reduce the thrust signal and increase the elevator deflection to increase pitch angle and at the same time reduce speed that allow the aircraft to make a climb. After some time the controller reduce elevator deflection to reduce pitch angle that correct the altitude and increase the thrust to track the increasing longitudinal speed. (iii) **Turn**: Finally, to make a turn lasts for a duration of [780, 1200]s, the controller reduce the thrust control signal to compensate the altitude lost and generate an aileron deflection signal that create a bank angle that allow the aircraft to turn and change the heading. Fig.14, Fig.15, Fig.16 and Fig.17 illustrate the desired and realised bank, pitch, heading angles and longitudinal speed respectively. One can see clearly that the developed controller allow a good tracking with some errors. The controller output aileron (δ_a), elevator (δ_e) and thrust(F_T) signals are depicted in Fig.19, Fig.20 and Fig.22 respectively. The rudder deflection (δ_r) Fig.21 signal is zero because it is used only for landing and taking off. We can see that the controller outputs are realisable and respect the actuators limits constraints. The developed autopilot allow the aircraft to track a desired trajectory by means of attitude and speed tracking as illustrated in Fig.18. In the last phase of the flight we notice a loss of altitude mainly caused by the error in the speed tracking.

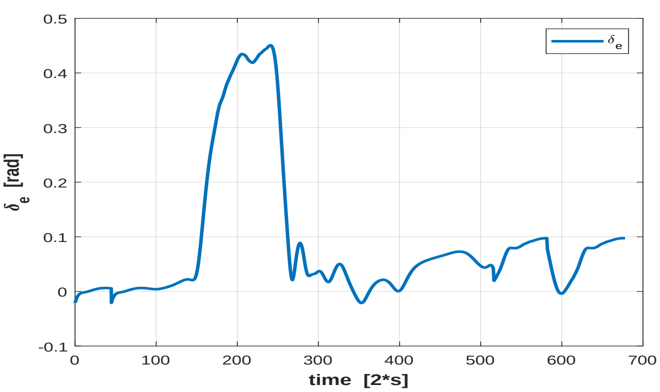

Fig. 20. Elevator control surface deflection (δ_e).

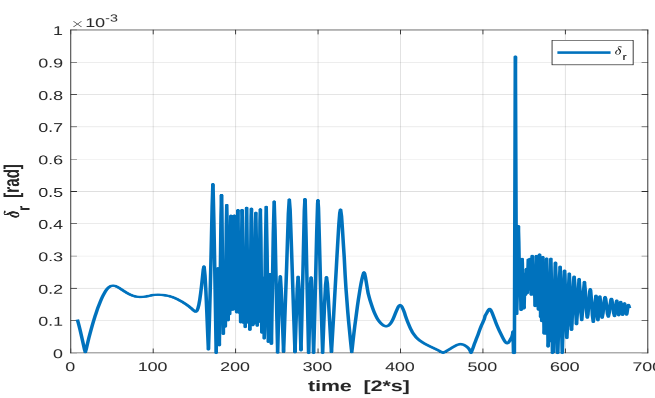

Fig. 21. Rudder control surface deflection (δ_r).

According to the obtained results we make the following observation: This strategy of control give a good tracking of attitude references but to the detriment of the speed control as shown on Fig.17. This is normal since the convergence is asymptotic rather than exponential. The asymptotic convergence of the speed is not critical in practice. In fact, it is more important to have a precise and exponential convergence of the orientation rather than the speed.

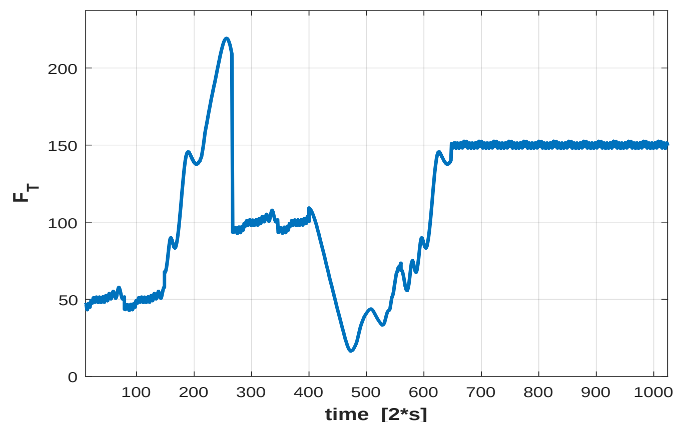

Fig. 22. Thrust force calculated by autopilot (F_T).

6. Conclusion

In this paper we have proposed a new nonlinear feedback control design methodology for velocity and attitude control. For this we first reduce the aircraft model so that its is suitable for the specific control design objective. The proposed strategy consists of three control loops each realising a specific task. The key feature of the control strategy is the introduction of a virtual control input in order to cater for the underactuation property of such vehicles. After the reduction of aircraft model, the input variable F_T is used to track a reference speed trajectory V_{ref}. Since Φ is not directly affected any real actuators, a virtual control Ω_v is introduced to indirectly control the orientation Φ towards a desired reference trajectory Φ_{ref}, this is a first crucial point in the design strategy. Another key point is that the reference angular velocity Ω_{ref} is chosen in such a way that it permit the virtual control input to track the desired orientation. The derived controller U is dependent on the control F_T ensuring indirectly a natural distribution between the kinetic and potential energy of the aircraft. As such, the three (03) control loop are design so as to make sure that the aircraft does not stall. This is measured by ensuring that $\|\dot{X}\|$ remain bounded.

The effectiveness of the developed autopilot for attitude and longitudinal velocity tracking was demonstrated on a Jetstream-3102 aircraft flying in a real-time virtual Simulation Platform for the development of Aircraft Control Systems (SP-ACS). We have introduces an identification part based on the Total Least Squares Estimation technique (TLSE)to identify the aerodynamic parameters, which are unknown, variable, classified and used in the expression of the piloting law. Each aerodynamic coefficient is defined as the mean of its numerical values. All other variations are considered as modeling uncertainties that was compensated by the robustness of the piloting law. Simulation results show very good performance in terms of convergence towards the desired reference trajectories and in terms of robustness with respect to modeling uncertainties. Finally the methodology developed here can be easily extended to other underactuated mechanical systems.

Table 2. Jetstream-3102 Aerodynamic coefficients derivatives.

Coef	**Description**	**Value**
$C_{x,0}$	F_x force at zero angle of attack and sideslip	-0.055
$C_{x,1}$	F_x force due to the angle of attack	-0.48
$C_{x,2}$	F_x force due to the angle of attack variation	0.85
$C_{x,3}$	F_x force due to the angular velocity q	-0.76
$C_{x,5}$	F_x force due to elevator deflection	-0.78
$C_{x,7}$	F_x force due to the Thrust force	0.89
$C_{y,0}$	F_y force at zero angle of attack	0.05
$C_{y,1}$	F_y force due to sideslip angle	0.03
$C_{y,3}$	F_y force due to the angular velocity$p$	0.21
$C_{y,4}$	F_y force due to aileron deflection	-0.15
$C_{y,6}$	F_y force due to rudder deflection	0.054
$C_{y,7}$	F_y force due to the Thrust	0.42
$C_{z,0}$	F_z force at zero angle of attack	1.74
$C_{z,1}$	F_z force due to the angle of attack	0.17
$C_{z,3}$	F_z force due to the angular velocity q	-0.305
$C_{z,5}$	F_z force due to elevator deflection	0.01
$C_{z,7}$	Lift due to Thrust	0.1
$C_{l,1}$	Roll moment due to the sideslip angle	0.073
$C_{l,2}$	Damping from angular velocity$p$	0.1
$C_{l,3}$	Roll moment due to angular velocity r	-0.097
$C_{l,4}$	Roll moment due to aileron deflection	-0.22
$C_{l,5}$	Roll moment due rudder deflection	0.024
$C_{m,0}$	Pitch moment	0.1
$C_{m,1}$	Pitch moment due to the angle of attack	0.1
$C_{m,2}$	Pitch moment du to angle of attack variation	0.1
$C_{m,3}$	Damping from angular velocity q	0.1
$C_{m,4}$	Pitch moment due to the elevator deflection	0.1
$C_{n,1}$	Yaw moment due to the side slip angle	-0.39
$C_{n,2}$	Yaw moment due to angular velocity$r$	0.048
$C_{n,3}$	Yaw moment due to angular velocity p	0.042
$C_{n,4}$	Yaw moment due to aileron deflection	-0.1
$C_{n,5}$	Yaw moment due to the rudder deflection	-0.2

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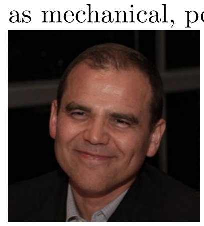

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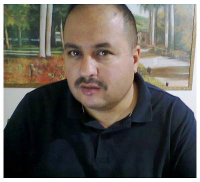

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