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Heating in collisionless plasmas: Do collisions play an effective role?

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Summary. — High temperature and low density plasmas are ubiquitous in the Universe. These systems often exhibit a turbulent dynamics, characterized by the cross-scale coupling of fluid and kinetic scales and by the inhomogeneous development of coherent spatial and temporal structures. At smaller scales the energy of fluctuations is dissipated and plasma is eventually heated. Despite collisions are usually ruled out from the description of these systems, it has been recently shown that plasma collisionality may be locally enhanced, owing to the presence of fine structures in velocity space. In this perspective, further insights are here given by comparing the characteristic times of collisional dissipation with the time scales of other collisionless processes, such as nonlinear coupling and linear instability onset. When taking into account fine velocity structures, these characteristic times could be comparable. A novel scenario for the description of heating in weakly-collisional plasmas, even including inter-particle collisions, is finally proposed.

1. – Introduction

Understanding the mechanisms of energy dissipation in hot and dilute plasmas is a complex problem, concerning the description of near-Earth environments, such as the solar wind and planetary magnetospheres, and astrophysical systems, such as supernovae remnants [1]. The study of near-Earth plasmas, where *in-situ* spacecraft missions provide accurate measurements of both particle distribution functions and electromagnetic fields, can give useful insights for comprehending the dynamical processes also occurring in far objects.

Since several nonlinearities are important in these systems, a strongly turbulent behavior is often observed [2]. The fluctuations energy, injected at large scales, is transfered along the inertial range of the spectrum, producing smaller scales fluctuations. When fluctuations with characteristic spatial and temporal scales of the order of particles motions (e.g. proton gyro-motion) are excited, a kinetic approach is mandatory to analyze the plasma dynamics. *In-situ* observations and kinetic numerical simulations indicate that the dynamics changes at kinetic scales: spectra become usually steeper [2] and strong perturbations of the particle velocity distribution function (VDF) (e.g. temperature anisotropy and non-gyrotropy, field-aligned beams and ring-like structures) are routinely recovered [3-7]. The presence of such velocity space structures indicates that the plasma is free to explore states which are not at the thermal equilibrium. Plasma modeling at kinetic scales is usually carried out by means of kinetic collisionless models, which couple Vlasov equations for particles and Maxwell equations for electromagnetic fields [8-16].

At small scales, energy is dissipated and plasma is eventually heated. Several dissipative mechanisms are introduced also within the collisionless framework, however, in absence of inter-particle collisions, the system is still reversible. To properly introduce concepts such as thermodynamic heating, intrinsically related to the irreversible information degradation, collisions need hence to be included in the system description.

It is worth to remark that the collisionless hypothesis, usually adopted to model the solar wind, is based on the quasi-Maxwellian assumption for estimating the characteristic times for collisionless and collisional processes. Within this approximation, collisional effects are extremely slow compared to other physical processes, therefore collisions are ruled out by the system description [17]. However, the particle VDF is strongly perturbed by plasma turbulence at kinetic scales and presents fine velocity space structures. In ref(s). [18, 19], it has been shown, for a homogeneous and force-free plasma and by modeling collisions with the nonlinear Landau operator [20, 21], that the dissipation of fine velocity structures occurs on characteristic times much smaller than the Spitzer-Harm time, which is the characteristic times associated with collisional processes within the quasi-Maxwellian assumption. Hence, collisions may compete with other processes for dissipating energy.

The collisional enhancement obtained by modeling collisions with the nonlinear Landau collisional operator is related to the presence of velocity gradients in the operator structure. The choice of the proper collisional operator is still matter of debate. The Landau operator can be derived from the Liouville equation but is not the most general collisional operator that can be adopted for describing collisions in plasmas. The most general operator is indeed the Lenard-Balescu operator [22,23], which can also be derived from the Liouville equation ad is a nonlinear Fokker-Planck-like operators, involving velocity space integrals and derivatives. It holds a H-theorem for the Gibbs-Bolzmann entropy growth and solves the usual divergence, occurring at large parameters of impact, by introducing the plasma dispersion function. The numerical implementation of the Lenard-Balescu operator is nowadays prohibitive, since it requires the evaluation of a multi-dimensional integral in the complex plane, due to the presence of the dispersion function. The Landau operator preserves most of the characteristics of the Lenard-Balescu operator. The only difference concerns the solution of the large impact parameter divergence: in the Landau operator case an *ad-hoc* cut-off, that eliminates the role of the dispersion function, is introduced.

In this direction, it has also been pointed out that taking into account nonlinearities of the collisional operator is crucial to properly quantify the effect of collisions with respect to other processes [19]. This aspect should be remembered when collisions are modeled through simplified operators, such as the Dougherty one [24-28], or linearized operators, such as the Lenard-Bernstein one [29, 30]. The properties of the Dougherty operator have been recently investigated in detail by means of numerical simulations [31, 32] and analytical calculations [33].

Here we extend the analysis give in ref(s). [18, 19], by directly comparing the characteristic times of collisional effects with other collisionless processes, such as instability growth rate and nonlinear coupling. Collisions are modeled with the nonlinear Landau operator. It is shown that, when fine velocity structures are retained, collisional characteristic scales can be be compared to the ones relative to other processes. This gives the possibility of speculating on a potential scenario to describe heating in plasmas. Plasma turbulence at kinetic scales strongly perturbs the particle VDF, generating intense gradients in velocity space. The presence of such gradients naturally enhances the effect of collisions, which fast smooth out these velocity structures. The collisional dissipation of fine velocity space structures ultimately heats the plasma, since the ordered energy, contained in these structures, is irreversibly degraded. Note that, i) obtaining fast collisional characteristic times is crucially connected to the high-resolution description of the full phase-space, i.e. to the possibility of performing accurate *in-situ* measurements and numerical simulations; and ii) collisional characteristic times are smaller when velocity space structures become finer [18, 19]. We argue that, in a natural situation, which - we remark - is only partially accessible due to the resolution limitations of *in-situ* instruments and numerical tools, the production of far from equilibrium structures may be effectively balanced by the enhancement of collisions, that are the physical ingredient able to irreversibly heat the system.

The paper is organized as follows. In sect. **2** we review the usual methods adopted to model collisions in weakly collisional plasmas. Based on results obtained in ref(s). [18,19], the comparison of characteristic times of collisionless mechanisms and collisional dissipation is then given in sect. **3**. Finally, we conclude and summarize in sect. **4**.

2. – Modeling collisions in weakly collisional plasmas

The Landau operator has the following form:

$$\frac{\partial f(\mathbf{v})}{\partial t} = \pi \left(\frac{3}{2}\right)^{\frac{3}{2}} \frac{\partial}{\partial v_i} \int d^3v' \, U_{ij}(\mathbf{u}) \left[f(\mathbf{v}') \frac{\partial f(\mathbf{v})}{\partial v_j} - f(\mathbf{v}) \frac{\partial f(\mathbf{v}')}{\partial v'_j} \right] , \tag{1}$$

being f normalized such that $\int d^3v f(\mathbf{v}) = n = 1$ and $U_{ij}(\mathbf{u})$

$$U_{ij}(\mathbf{u}) = \frac{\delta_{ij} u^2 - u_i u_j}{u^3} , \tag{2}$$

where $\mathbf{u} = \mathbf{v} - \mathbf{v}'$, $u = |\mathbf{u}|$ and the Einstein notation is introduced. In eq. (1), and from now on, time is scaled to the inverse Spitzer-Harm frequency ν_{SH}^{-1} [17] and velocity to the particle thermal speed v_{th}.

The Landau operator should be introduced at the right hand side of the Vlasov equation, coupled to the Maxwell system for the electromagnetic fields. However, solving numerically that system of equations is computationally demanding. Hence, as in ref(s). [18, 19], we reduce to the case of force-free, homogeneous plasma.

3. – Collisional vs collisionless characteristic times

In ref(s). [18,19], it has been shown that collisions can be enhanced by the presence of fine structures in velocity space. Results indicate that, when the VDF shows small scale velocity structures, the approach towards the equilibrium (described in terms of entropy growth, due to the dissipation of non-equilibrium velocity space features) occurs on several characteristic times. The entropy is the proper variable to describe the approach towards the equilibrium, since it is intimately related to the irreversible information degradation.

It has been also proved that finer are velocity space structures, smaller are the collisional characteristic times (i.e. faster is the effect of collisions). To be quantitative, we remark that the typical characteristic times for proton-proton collisions in the solar wind, within the quasi-Maxwellian assumption, is:

$$\tau_{pp} \sim \nu_{pp}^{-1} \tag{3}$$

where ν_{pp} is the characteristic collisional frequency, whose order is about the Spitzer-Harm frequency $\nu_{SH} \simeq 8 \times (0.714\pi n e^4 \ln\Lambda)/m^{0.5}(3k_BT)^{3/2}$, where n, e, m and T are the proton density, charge, mass, temperature and $\ln\Lambda$ and k_B are the Coulombian logarithm and the Boltzmann constant. By introducing the typical values for the solar wind [34], it is possible to get:

$$\tau_{pp} \sim \eta[10^4, 10^5]\Omega_{cp}^{-1} \tag{4}$$

where we introduced $\eta \leq 1$ to mimic the effect of fine velocity space structures. In the quasi-Maxwellian approach $\eta = 1$, while, by retaining fine structures, $\eta \ll 1$. Note that, if fine velocity space structures are not retained ($\eta = 1$) and by inserting the typical parameters for the solar wind, one gets the usual estimation $\tau_{pp} \sim T_{tr} \sim 10^5 s$, where $T_{tr} = 1au/V_{SW}$ is the travel time of a solar wind parcel from the Sun to the Earth and V_{SW} is the typical solar wind speed [35].

The collisional characteristic time of eq. (4) should be compared with the characteristic times related to other physical processes. Here we refer to two different physical phenomena typically invoked in the description of solar wind. The first process is the presence of a nonlinear coupling of turbulent fluctuations, whose typical characteristic time is $\tau_{nl}(k)$ [36]. By choosing the wavenumber k in the dissipative range ($k \gtrsim 1/d_p$, being d_p the proton skin depth), one gets:

$$\tau_{nl} \sim [10, 10^2]\Omega_{cp}^{-1} \tag{5}$$

The second process regards the onset of linear instabilities, whose nature can be both fluid or kinetic [37-40]. Here we are not focused on the nature of the instability but we aim to extrapolate a characteristic time associated to this phenomenon. The typical instability growth rate, needed to explain some of the solar wind observed features (e.g. the presence of temperature anisotropies) is $\gamma_I^{-1} \sim [10^{-2}, 10^{-3}]\Omega_{cp}^{-1}$, therefore the characteristic time is:

$$\tau_I \sim \gamma_I^{-1} \sim [10^2, 10^3]\Omega_{cp}^{-1} \tag{6}$$

Note that comparing eq(s). (5–6) is generally useful for understanding if turbulence dominates over the onset of linear phenomena. In the linear instability calculation, it is usually assumed a scale separation between the unstable fluctuations and the equilibrium background [41], which could be invalidated by the presence of turbulence. If $\tau_{nl} \lesssim \tau_I$, turbulence dominates and instability does not onset.

By comparing eq. (4) with eq(s). (5–6), one easily figures out that collisions are much slower than the other processes within the quasi-Maxwellian approach ($\eta = 1$): $\tau_{pp} \gg \tau_I \gtrsim \tau_{nl}$. Within this framework, collisions do not hence play any effective role in the plasma dynamics. By retaining fine structures as suggested by ref(s). [18, 19], τ_{pp} tends to be smaller. The specific value of η depends on the particular fine structures recovered in the proton VDF. In ref. [18], the cases of mild and intense fine structures have been investigated and the analysis provides $\eta = 10^{-1}$ and $\eta = 10^{-3}$, respectively. Although the former value is still too high to make collisions efficients, the latter produces characteristic collisional times τ_{pp} of the same order of τ_I or τ_{nl}.

One can imagine that, in a realistic turbulent environment, the production of fine structures in velocity space continues until that η is such that τ_{pp} is comparable with τ_I or τ_{nl}. Then, by dissipating fine velocity structures, collisions contribute to heat the plasma. A stationary condition, where the turbulent energy which perturbs the particle VDF is balanced by the collisional dissipation could in principle also be achieved. From a time scales perspective, this last scenario is possible only if fine structures are considered.

4. – Conclusions

In this paper, results described in ref(s). [18, 19] have been extended by reporting a comparison of characteristic times of collisional and collisionless processes. It has been shown that, if the effect of collision is modeled by taking into account the presence of fine structures in velocity space - naturally produced in turbulent weakly collisional plasmas, collisional effects may be compared with other typical processes occurring in the system. The production of such out-of-equilibrium features has been discussed by proposing a potential scenario where (a) turbulence at kinetic scales creates these distortions in the particle VDF, (b) these velocity space structures enhance the effect of collisions until the level when (c) collisions efficiently contribute to heat the plasma. Future works will aim to extend the analysis to the self-consistent case.

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REFERENCES

[1] Vaivads A., Retinó A., Soucek J., Khotyaintsev Yu.V., Escoubet C.P, Alexandrova O, ... and Wimmer-Schweingruber, R.F., *J. Plasma Phys.*, **82(5)** (2016) 905820501.

[2] Bruno R. and Carbone V., *Living Rev. Sol. Phys.*, **2** (2005) 4.

[3] Marsch E., *Living Rev. Sol. Phys.*, **3(1)** (2006) 1.

[4] Servidio S., Valentini F., Perrone D., Greco A., Califano F., Matthaeus W.H. and Veltri P., *J. Plasma Phys.*, **81(1)** (2015) 325810107.

[5] Valentini F., Perrone D., Stabile S., Pezzi O., Servidio S., De Marco R., ... and Veltri P., *New J. Phys.*, **18** (2016) 125001.

[6] Pezzi O., Parashar T.N., Servidio S., Valentini F., Vásconez C.V., Yang Y., Malara F., Matthaeus W.H. and Veltri P., *Astrophys. J.*, **834** (2017) 166.

[7] Pezzi O., Parashar T.N., Servidio S., Valentini F., Vásconez C.V., Yang Y., Malara F., Matthaeus W.H. and Veltri P., *J. Plasma Phys.*, **83** (2017) 905830105.

[8] Valentini F., Trávníček P., Califano F., Hellinger P. and Mangeney A., *J. Comput. Phys.*, **225(1)** (2007) 753.

[9] Perrone D., Valentini F. and Veltri P., *Astrophys. J.*, **741** (2011) 43.
[10] Perrone D., Valentini F., Servidio S., Dalena S. and Veltri P., *Astrophys. J.*, **762** (2013) 99.
[11] Perrone D., Bourouaine S., Valentini F., Marsch E. and Veltri P., *J. Geophys. Res.: Space Phys.*, **119** (2014) 2400.
[12] Cerri S.S., Servidio S. and Califano F., *Astrophys. J. Letters*, **846** (2017) L18
[13] Groselj D., Cerri S.S., Navarro A.B., Willmott C., Told D., Loureiro N.L., Califano F. and Jenko F., *Astrophys. J.*, **847** (2017) 28.
[14] Pezzi O., Servidio S., Perrone D., Valentini F., Sorriso-Valvo L., Greco A., Matthaeus W.H. and Veltri P., *Phys. Plasmas*, **25** (2018) 060704.
[15] Perrone D., Passot T., Laveder D., Valentini F., Sulem P.L., Zouganelis I., Veltri P. and Servidio S., *Phys. Plasmas*, **25** (2018) 052302.
[16] Franci L., Landi S., Verdini A., Matteini L. and Hellinger P., *Astrophys. J.*, **853** (2018) 26
[17] Spitzer L. Jr, *Physics of Fully Ionized Gases*, (Interscience Publishers, New York) 1956.
[18] Pezzi O., Valentini F. and Veltri P., *Phys. Rev. Lett.*, **116** (2016) 145001.
[19] Pezzi O., *J. Plasma Phys.*, **83** (2017) 555830301.
[20] Landau L.D., *Phys. Z. Sowjet.*, **154** (1936) ; translated in *The Transport Equation in the Case of the Coulomb Interaction*, in *Collected papers of L. D. Landau*, (Pergamon Press, Oxford), 1981.
[21] Rosenbluth M.N., MacDonald W.M.and Judd D.L., *Phys. Review*, **107(1)** (1957) 1.
[22] Lenard A., *Annals Physics*, **10.3** (1960) 390.
[23] Balescu R., *Phys. Fluids*, **3** (1960) 52.
[24] Dougherty J.P., *Phys. Fluids*, **7** (1964) 1788.
[25] Dougherty J.P. and Watson S.R., *J. Plasma Phys.*, **1** (1967) 317.
[26] Dougherty J.P., Watson S.R. and Hellberg M.A., *J. Plasma Phys.*, **1** (1967) 327.
[27] Pezzi O., Valentini F., Perrone D. and Veltri P, *Phys. Plasmas*, **20** (2013) 092111; **21** (2014) 019901.
[28] Pezzi O., Valentini F. and Veltri P., *The European Physical Journal*, **68** (2014) 128.
[29] Lenard A. and Bernstein I.B., *Phys. Rev.*, **112** (1958) 1456.
[30] Pezzi O., Camporeale E. and Valentini F., *Phys. Plasmas*, **23** (2016) 022103.
[31] Pezzi O., Valentini F. and Veltri P., *J. Plasma Phys*, **81(1)** (2015) 305810107.
[32] Pezzi O., Valentini F. and Veltri P., *Phys. Plasmas*, **22(4)** (2015) 042112.
[33] Pezzi O., Valentini F., Servidio S., Camporeale E. and Veltri P., *Fourier-Hermite decomposition of the collisional Vlasov-Maxwell system: implications for the velocity space cascade*, submitted to *Plasma Phys. Control. Fus.*
[34] Huba J.D., *NRL: Plasma formulary*, (Naval Research Lab, WashingtonDC Beam Physics Branch), 2004.
[35] Kasper J.C., Lazarus A.J. and Gary S.P., *Geophys. Res. Lett.*, **29** (2002) 20.
[36] Matthaeus W.H., Oughton S., Osman K.T., Servidio S., Wan M., Gary S.P., Shay M.A., Valentini F., Roytershteyn V. and Karimabadi H., *Astrophys. J.*, **790** (2014) 1557.
[37] Hellinger P., Trávníček P., Kasper J.C. and Lazarus A.J., *Geophys. Res. Lett.*, **33** (2006) L09101.
[38] Bale S.D., Kasper J.C., Howes G.G., Quataert E., Salem. C and Sundkvist D., *Phys. Rev. Lett.*, **103** (2009) 211101.
[39] Maruca B.A., Kasper J.C. and Bale S.D., *Phys. Rev. Lett.*, **107** (2011) 201101.
[40] Klein K.G., Alterman B.L., Stevens M.L., Vech D. and Kasper J.C., *Phys. Rev. Lett.*, **120** (2018) 205102.
[41] Gary S.P., *Theory of space plasma microinstabilities* (Cambridge university press, Cambridge) 2005.