

Combined CP violation measurements by the BABAR and Belle Collaborations

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received 6 September 2018

Summary. — We present results of a recent joint analysis campaign by the BABAR and Belle experiments. The approach combines in single physics analyses the 1.1 inverse attobarn collected at the $\Upsilon(4S)$ resonance by the BABAR and Belle experiments at the asymmetric-energy B factories PEP-II at SLAC and KEKB at KEK, respectively. A measurement of the CP violation parameters $\sin(2\beta)$ and $\cos(2\beta)$ by a time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)}h^0$ decays with $D \rightarrow K_S^0\pi^+\pi^-$ decays is reported. The result is $\sin(2\beta) = 0.80 \pm 0.14 \pm 0.06 \pm 0.03$ and $\cos(2\beta) = 0.91 \pm 0.22 \pm 0.09 \pm 0.07$, where the first error is statistical, the second is the experimental systematic uncertainty, and the third is due to the uncertainty of the Dalitz plot amplitude model. First evidence for $\cos(2\beta) > 0$ at the level of 3.7 standard deviations is obtained. The angle β of the CKM Unitarity Triangle is measured to be $\beta = (22.5 \pm 4.4 \pm 1.2 \pm 0.6)^\circ$. The hypothesis of $\beta = 0^\circ$ is ruled out at the level of 5.1 standard deviations, and CP violation is observed in $B^0 \rightarrow D^{(*)}h^0$ decays. The trigonometric multifold solution of $\frac{\pi}{2} - \beta = (68.1 \pm 0.7)^\circ$ is excluded at the level of 7.3 standard deviations, and an ambiguity in the determination of the apex of the CKM Unitarity Triangle is resolved.

1. – Introduction

In the standard model (SM) of electroweak interactions, the only source of CP violation is the single irreducible complex phase in the three-family Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [1, 2]. The first-generation asymmetric-energy B factory experiments BABAR at SLAC (USA) and Belle at KEK (Japan) discovered CP violation in the neutral and charged B meson systems [3-6], and experimentally confirmed the predictions of the Kobayashi-Maskawa theory [2] in many independent measurements. The BABAR and Belle experiments precisely measured the sine of the weak CP -violating phase 2β , $\sin 2\beta$, by time-dependent CP violation analyses of $\bar{b} \rightarrow \bar{c}c\bar{s}$ transitions [7, 8],

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where β is an angle of the Unitarity Triangle defined by the CKM matrix elements V_{ij} as $\arg[-V_{cd}V_{cb}^*/V_{td}V_{tb}^*]$. The current world average is $\sin 2\beta = 0.691 \pm 0.017$ [9], and the corresponding uncertainty of the angle β is 0.7° . The determination of 2β from the measurements of $\sin 2\beta$ is associated with a twofold ambiguity, 2β and $\pi - 2\beta$. This ambiguity can be resolved by measuring the cosine of the weak CP -violating phase 2β . However, the experimentally uncertainties on $\cos 2\beta$ are sizable. The most precise single measurement of $\cos 2\beta$ has an uncertainty of approximately ± 0.36 [10]. No previous single measurement could establish the sign of $\cos 2\beta$ which would resolve the trigonometric ambiguity without any further assumptions.

An elegant approach to measure $\cos 2\beta$ is provided by $B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays, where $h^0 \in \{\pi^0, \eta, \omega\}$ denotes a light unflavored and neutral hadron. The $B^0 \rightarrow D^{(*)}h^0$ decay is mediated only by tree-level amplitudes, predominantly by CKM-favored and color-suppressed $\bar{b} \rightarrow \bar{c}ud$ amplitudes. The $D \rightarrow K_S^0\pi^+\pi^-$ decay proceeds via various resonant and non-resonant intermediate states contributing to the three-body final state. Experimental knowledge of the variations of the relative strong phases as a function of the three-body Dalitz plot phase space of the $D^0 \rightarrow K_S^0\pi^+\pi^-$ decay enables to extract $\cos(2\beta)$ in addition to $\sin(2\beta)$ from the time evolution of $B^0 \rightarrow D^{(*)}h^0$ decays [11].

In an $e^+e^- \rightarrow \Upsilon(4S) \rightarrow B^0\bar{B}^0$ event, the time-dependent decay rate of $B^0 \rightarrow D^{(*)}h^0$ decays depends on the D^0 and \bar{D}^0 decay amplitudes $\mathcal{A}_{D^0} \equiv \mathcal{A}(M_{K_S^0\pi^-}^2, M_{K_S^0\pi^+}^2)$ and $\mathcal{A}_{\bar{D}^0} \equiv \mathcal{A}(M_{K_S^0\pi^+}^2, M_{K_S^0\pi^-}^2)$ as a function of the position within the $D^0 \rightarrow K_S^0\pi^+\pi^-$ Dalitz plot phase space defined by the Lorentz-invariant variables $M_{K_S^0\pi^-}^2 \equiv (p_{K_S^0} + p_{\pi^-})^2$ and $M_{K_S^0\pi^+}^2 \equiv (p_{K_S^0} + p_{\pi^+})^2$, and on the CP -violating weak phase 2β . The decay rate is proportional to

$$(1) \quad \frac{e^{-\frac{|\Delta t|}{\tau_{B^0}}}}{2} [|\mathcal{A}_{\bar{D}^0}|^2 + |\mathcal{A}_{D^0}|^2 - q(|\mathcal{A}_{\bar{D}^0}|^2 - |\mathcal{A}_{D^0}|^2) \cos(\Delta m_d \Delta t)] \\ + 2q\eta_{h^0}(-1)^L [\text{Im}(\mathcal{A}_{D^0}\mathcal{A}_{\bar{D}^0}^*) \cos(2\beta) - \text{Re}(\mathcal{A}_{D^0}\mathcal{A}_{\bar{D}^0}^*) \sin(2\beta)] \sin(\Delta m_d \Delta t).$$

The proper-time interval between the decays of the two B mesons is denoted by Δt . The quantities τ_{B^0} and Δm_d are the neutral B meson lifetime and the mass difference between the physical eigenstates of neutral B mesons, respectively. The variable $q = +1(-1)$ represents the b -flavor content when the second B meson originating from the $\Upsilon(4S)$ decay is tagged as a B^0 (\bar{B}^0). The parameter η_{h^0} is the CP eigenvalue of the h^0 , and L is the angular orbital momentum of the $D^{(*)}h^0$ system. If the D^0 and \bar{D}^0 decay amplitudes \mathcal{A}_{D^0} and $\mathcal{A}_{\bar{D}^0}$ are known, then eq. (1) enables to measure $\sin 2\beta$ and $\cos 2\beta$ by a time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays.

Previous time-dependent Dalitz plot analyses of $B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays by the BABAR and Belle Collaborations [10, 12, 13] could not establish CP violation, obtained results outside of the physical region [12], and used different Dalitz plot amplitude models [10, 12, 13]. This complicates the combination of the individual results.

At the 32nd *Rencontres de Physique de la Vallée d'Aoste*, we presented new results on $\sin(2\beta)$ and $\cos(2\beta)$ obtained by a time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays. The analysis combines the final data sets of the BABAR and Belle experiments in a single measurement. The combined approach enables a unique sensitivity to $\cos(2\beta)$ by effectively doubling the statistics available for the measurement, and by applying common assumptions and the same $D^0 \rightarrow K_S^0\pi^+\pi^-$ decay amplitude

model in the analysis of the data collected by both experiments. The analysis consists of two parts: first, the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decay amplitude model is derived by a Dalitz plot amplitude analysis using a high-statistics $e^+e^- \rightarrow c\bar{c}$ data sample collected by Belle; second, the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decay amplitude model is applied in a time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)} h^0$ with $D \rightarrow K_s^0 \pi^+ \pi^-$ decays reconstructed from BABAR and Belle data to measure $\sin(2\beta)$ and $\cos(2\beta)$.

2. – Determination of the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decay amplitude model using Belle $e^+e^- \rightarrow c\bar{c}$ data

A data set of 924 fb^{-1} recorded at or near the $\Upsilon(4S)$ and $\Upsilon(5S)$ resonances by Belle is used to perform a Dalitz plot amplitude analysis and to extract the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decay amplitude model. This data set contains a high-statistics sample of $e^+e^- \rightarrow c\bar{c}$ events and provides a $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ yield that is about three orders of magnitude larger than in the combined BABAR+Belle measurement of the B meson decay described in sect. 3.

The $D^{*+} \rightarrow D^0 \pi_s^+$ with $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decays are reconstructed, where the charge of the low momentum (“slow”) pion π_s^+ allows to identify the production flavor of the neutral D meson as D^0 or \bar{D}^0 . The signal and background yields are determined by a two-dimensional unbinned maximum-likelihood (ML) fit of the D^0 candidate mass, M_{D^0} , and the $D^{*+} - D^0$ mass difference, ΔM . A total yield of $1\,217\,300 \pm 2\,000$ signal events is obtained. In the signal-enhanced region used to perform the Dalitz plot amplitude analysis, the signal purity is 94%.

The $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ Dalitz plot amplitude model is constructed by combining the isobar ansatz with the K -matrix formalism [14] for the $\pi\pi$ S -wave contributions and the LASS parameterization [15] for the $K\pi$ S -wave contributions. In this model, the decay amplitude can be written as

$$(2) \quad \mathcal{A}(M_{K_s^0 \pi^-}^2, M_{K_s^0 \pi^+}^2) = \sum_{r \neq (K\pi/\pi\pi)_{L=0}} a_r e^{i\phi_r} \mathcal{A}_r(M_{K_s^0 \pi^-}^2, M_{K_s^0 \pi^+}^2) + F_1(M_{\pi^+ \pi^-}^2) + \mathcal{A}_{K\pi_{L=0}}(M_{K_s^0 \pi^-}^2) + \mathcal{A}_{K\pi_{L=0}}(M_{K_s^0 \pi^+}^2),$$

where the symbol F_1 denotes the amplitude for the $\pi\pi$ S -wave contribution parameterized by the K -matrix approach in the P -vector approximation [16], and the symbol $\mathcal{A}_{K\pi_{L=0}}$ represents the LASS amplitude for the $K\pi$ S -wave contribution. The variables a_r and ϕ_r are the magnitude and phase of the r -th intermediate two-body resonant contribution to the three-body final state parameterized by the isobar ansatz. The following intermediate quasi-two-body resonances are included: the Cabibbo-favored $K^*(892)^- \pi^+$, $K_2^*(1430)^- \pi^+$, $K^*(1680)^- \pi^+$, $K^*(1410)^- \pi^+$ modes; the doubly Cabibbo-suppressed $K^*(892)^+ \pi^-$, $K_2^*(1430)^+ \pi^-$, $K^*(1410)^+ \pi^-$ modes; and the CP eigenstates $K_s^0 \rho(770)^0$, $K_s^0 \omega(782)$, $K_s^0 f_2(1270)$, and $K_s^0 \rho(1450)^0$.

The $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ Dalitz plot fit is performed for events in the signal-enhanced region of the flavor-tagged D^0 sample with the signal probability density function (p.d.f.) constructed from eq. (2) with a correction to account for reconstruction efficiency variations in the Dalitz plot phase space due to experimental acceptance effects, and an additional term to account for wrong flavor tags of D mesons. The background is modeled using distributions taken from the M_{D^0} and ΔM sideband regions in data. Free parameters in the fit are the a_r and ϕ_r relative to the $K_s^0 \rho(770)^0$ mode, which

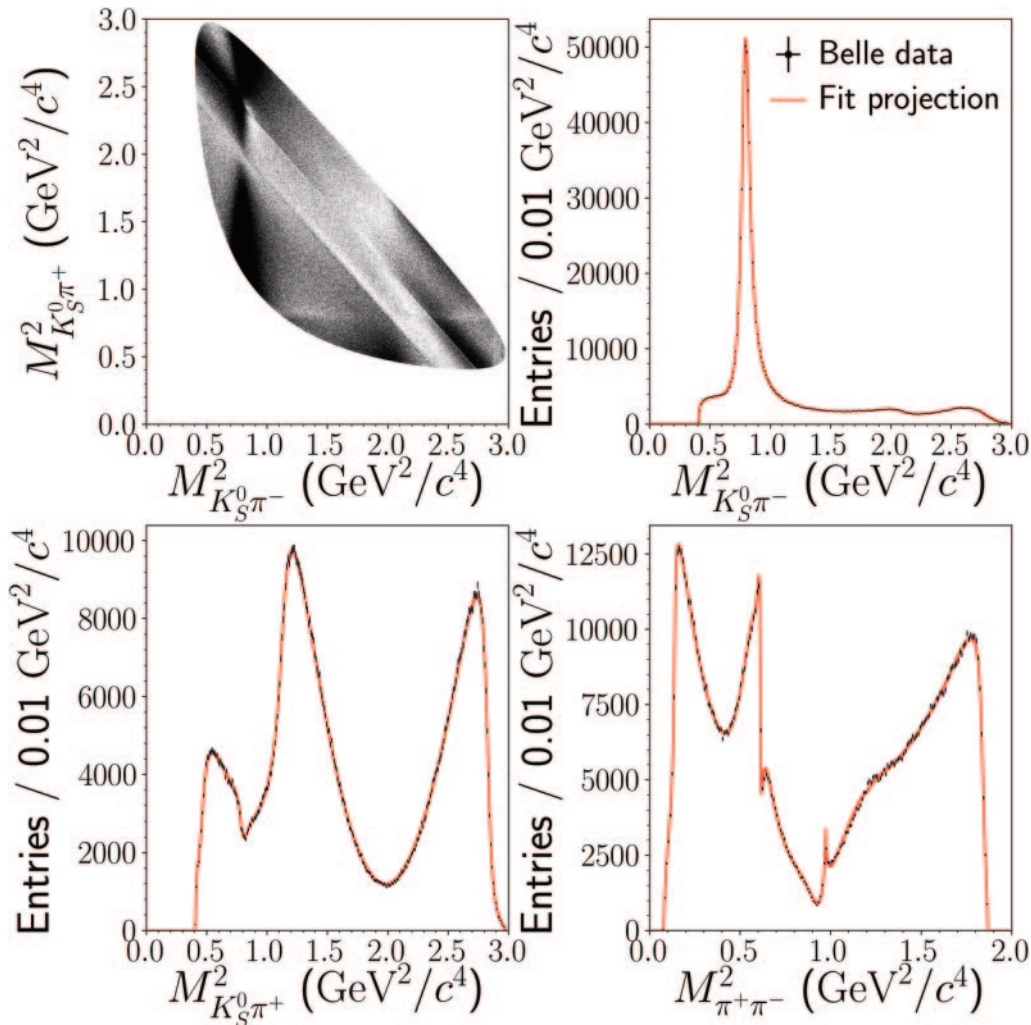


Fig. 1. – The Dalitz plot data distributions reconstructed from Belle $e^+e^- \rightarrow c\bar{c}$ data (points and points with error bars), and projections of the $D^0 \rightarrow K_S^0\pi^+\pi^-$ Dalitz plot fit (red).

is fixed to $a_r = 1$ and $\phi_r = 0$. The masses and widths of the resonances are fixed to the world averages [17] except those of the $K^*(892)^\pm$ which are floated in the fit to improve the fit quality. The LASS parameters, and the complex couplings β_α and production vector f_{1j}^{prod} of the K -matrix are measured in the fit. The remaining K -matrix parameters are fixed to the values of a global analysis of available $\pi\pi$ scattering data [18, 19]. The data distributions and projections of the $D^0 \rightarrow K_S^0\pi^+\pi^-$ Dalitz plot fit are shown in fig. 1. The fit quality is quantified by a two-dimensional χ^2 test. The result is $\chi^2/\text{dof} = 32667/(31321 - 49) = 1.05$ indicating a relatively good quality of the fit compared to previous models of this decay [19-23]. The obtained $D^0 \rightarrow K_S^0\pi^+\pi^-$ decay amplitude model is used as input for the time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays combining BABAR and Belle data described below.

3. – Time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays

The time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)}h^0$ with $D \rightarrow K_S^0\pi^+\pi^-$ decays is performed using data samples collected that contain $(471 \pm 3) \times 10^6 B\bar{B}$ pairs recorded with the BABAR detector and $(772 \pm 11) \times 10^6 B\bar{B}$ pairs recorded with the Belle detector at the $\Upsilon(4S)$ resonance. The light neutral h^0 is reconstructed in the decay modes $\pi^0 \rightarrow \gamma\gamma$, $\eta \rightarrow \gamma\gamma$ and $\pi^+\pi^-\pi^0$, and $\omega \rightarrow \pi^+\pi^-\pi^0$. Neutral D mesons are reconstructed in the decay mode $D \rightarrow K_S^0\pi^+\pi^-$, and neutral D^* mesons are reconstructed in the decay mode $D^* \rightarrow D\pi^0$. Neutral B mesons are reconstructed in the decay modes $B^0 \rightarrow D\pi^0$, $D\eta$, $D\omega$, $D^*\pi^0$, and $D^*\eta$. The $B^0 \rightarrow D^{(*)}h^0$ yields are determined by three-dimensional unbinned ML fits to the distributions of the observables M'_{bc} , ΔE , and NN'_{out} . The beam-energy-constrained mass M'_{bc} is defined as

$$(3) \quad M'_{bc} = \sqrt{E_{\text{beam}}^{*2} - \left(\vec{p}_{D^{(*)}0}^* + \frac{\vec{p}_{h^0}^* \sqrt{(E_{\text{beam}}^* - E_{D^{(*)}0}^*)^2 - M_{h^0}^2}}{|\vec{p}_{h^0}^*|} \right)^2}.$$

The beam-energy-constrained mass provides an observable that is not correlated to the the energy difference $\Delta E = E_B^* - E_{\text{beam}}^*$, where the symbols marked with an asterisk denote observables evaluated in the e^+e^- center-of-mass frame. The observable NN'_{out} is constructed from the output of a neural network multivariate classifier that combines event shape information based on a combination of 16 modified Fox-Wolfram moments [24-26] and that identifies the background that originates from $e^+e^- \rightarrow q\bar{q}$ ($q \in \{u, d, s, c\}$) continuum events. The fit model accounts for contributions from $B^0 \rightarrow D^{(*)}h^0$ signal decays, cross-feed from partially reconstructed $B^0 \rightarrow D^*h^0$ decays, background from partially reconstructed $B^+ \rightarrow D^{(*)0}\rho^+$ decays, combinatorial background from $B\bar{B}$ decays, and background from $e^+e^- \rightarrow q\bar{q}$ ($q \in \{u, d, s, c\}$) continuum events. In total, the $B^0 \rightarrow D^{(*)}h^0$ yields are 1129 ± 48 events for BABAR, and 1567 ± 56 events for Belle. The M'_{bc} , ΔE , and NN'_{out} data distributions and fit projections are shown in fig. 2.

The time-dependent Dalitz plot measurement of the CP violation parameters follows the technique established in the previous combined BABAR+Belle CP violation measurement of $B^0 \rightarrow D_{CP}^{(*)}h^0$ decays [27]. The measurement is performed by maximizing the log-likelihood function:

$$(4) \quad \ln \mathcal{L} = \sum_i \ln \mathcal{P}_i^{\text{BABAR}} + \sum_j \ln \mathcal{P}_j^{\text{Belle}},$$

where the symbol \mathcal{P} represents p.d.f.'s that describe the proper-time interval distributions. The indices i and j denote the events reconstructed from BABAR and Belle data, respectively. The signal p.d.f.'s are constructed from eq. (1) convolved with experiment specific resolution functions to account for the finite vertex resolution [7,28] and including the effect of incorrect flavor assignments [7,29]. The p.d.f.'s for the proper time interval distributions of the combinatorial background from $B\bar{B}$ decays and background from $e^+e^- \rightarrow q\bar{q}$ ($q \in \{u, d, s, c\}$) continuum events account for background from non-prompt and prompt decays convolved with effective resolution functions. The cross-feed from partially reconstructed $B^0 \rightarrow D^*h^0$ decays is modeled by the signal p.d.f. accounting for

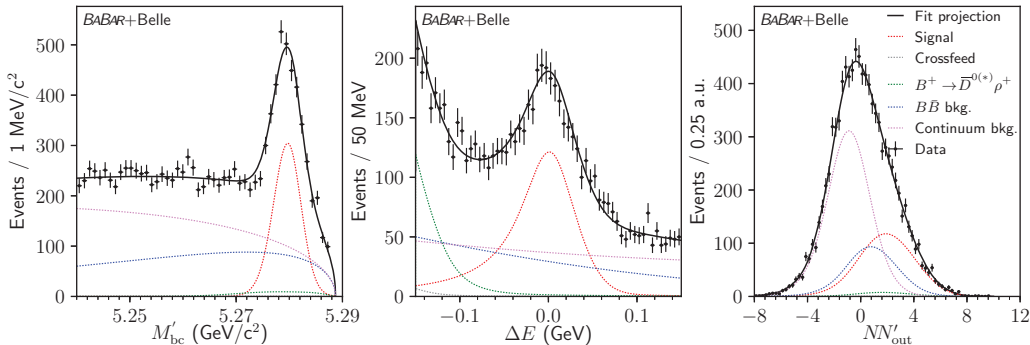


Fig. 2. – The M'_{bc} , ΔE , and NN'_{out} data distributions reconstructed from BABAR and Belle data (points with error bars), and projections of the fits (solid and dotted lines). In plotting the M'_{bc} , ΔE , and NN'_{out} distributions, each of the other two observables are required to satisfy $M'_{bc} > 5.272 \text{ GeV}/c^2$, $|\Delta E| < 100 \text{ MeV}$, or $0 < NN'_{out} < 8$ to select signal-enhanced regions.

the different parameters of the cross-feed contribution, and the background from partially reconstructed $B^+ \rightarrow D^{(*)0} \rho^+$ decays is parameterized by an exponential p.d.f.

The parameters τ_{B^0} , τ_{B^+} and Δm_d are fixed to the world averages [17], and the Dalitz plot amplitude model parameters are fixed to the results of the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ Dalitz plot fit described above. In the fit, the free parameters are $\sin(2\beta)$ and $\cos(2\beta)$. The result of the measurement is

$$(5) \quad \begin{aligned} \sin(2\beta) &= 0.80 \pm 0.14 \text{ (stat.)} \pm 0.06 \text{ (syst.)} \pm 0.03 \text{ (model)}, \\ \cos(2\beta) &= 0.91 \pm 0.22 \text{ (stat.)} \pm 0.09 \text{ (syst.)} \pm 0.07 \text{ (model)}. \end{aligned}$$

The linear correlation between $\sin(2\beta)$ and $\cos(2\beta)$ is $\rho = 5.1\%$. The experimental flavor-tagged proper-time interval data distributions and projections of the fit for different regions of the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ Dalitz plot phase space are shown in fig. 3. The result of the direct measurement of the angle β is

$$(6) \quad \beta = (22.5 \pm 4.4 \text{ (stat.)} \pm 1.2 \text{ (syst.)} \pm 0.6 \text{ (model)})^\circ.$$

The experimental systematic uncertainties on the CP violation parameters are estimated using established methods, described in refs. [7, 8, 27]. The leading experimental systematic uncertainties originate from the applied Δt resolution functions, the decay vertex reconstruction, and a possible fit bias. The uncertainty due to the Dalitz plot amplitude model is estimated by repeating the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ Dalitz plot amplitude analysis with alternative assumptions and variations of the $D^0 \rightarrow K_s^0 \pi^+ \pi^-$ decay amplitude model, and performing the time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)} h^0$ decays using the alternative models as input. The uncertainties due to the Dalitz plot amplitude model are small compared to the statistical uncertainties and the experimental systematic uncertainties.

The significance of the results is evaluated by a likelihood-ratio approach. Including the experimental systematic uncertainties and the Dalitz plot amplitude model uncertainties, the measured value of $\sin(2\beta)$ agrees within 0.7 standard deviations with the world average of $\sin 2\beta = 0.691 \pm 0.017$ [9]. The hypothesis $\cos(2\beta) \leq 0$ is excluded with a

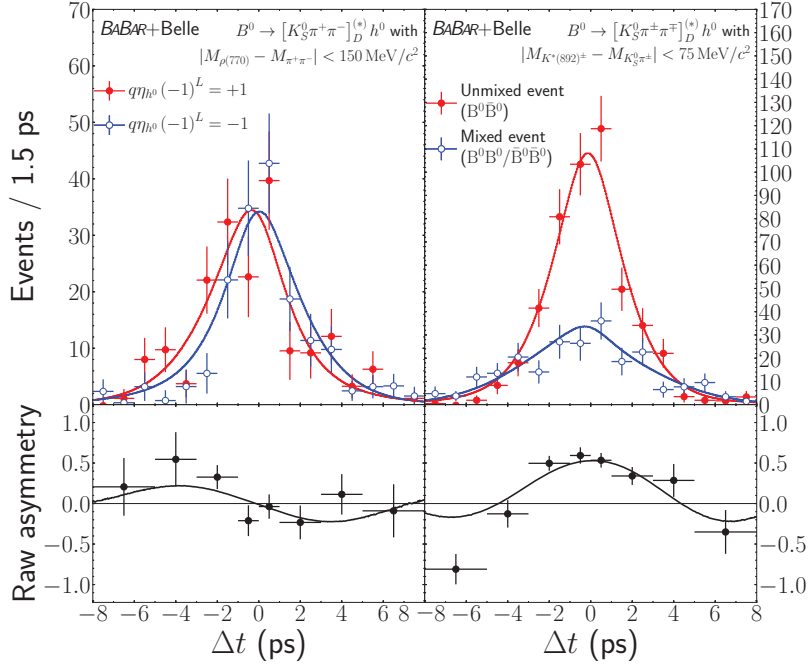


Fig. 3. – The flavor-tagged proper-time interval distributions (data points with error bars) and the corresponding asymmetries for $B^0 \rightarrow D^{(*)} h^0$ candidates associated high-quality flavor tags. The plots in the left column show events in regions of the $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ phase space predominantly populated by CP eigenstates, and the plots in the right column show events in regions predominantly populated by quasi-flavor-specific decays. In the plots, the background has been subtracted using the $sPlot$ technique [30].

significance of 3.7 standard deviations, and the first evidence for $\cos(2\beta) > 0$ is obtained. The measurement excludes the hypothesis $\beta = 0^\circ$ with a significance of 5.1 standard deviations, and an observation of CP violation in $B^0 \rightarrow D^{(*)} h^0$ is reported. The result agrees well with the preferred solution of the Unitarity Triangle, $(21.9 \pm 0.7)^\circ$ [9], and excludes the second solution of $\frac{\pi}{2} - \beta = (68.1 \pm 0.7)^\circ$ with a significance of 7.3 standard deviations. Therefore, the measurement resolves an ambiguity in the determination of the apex of the CKM Unitarity Triangle.

4. – Summary

In summary, we report a time-dependent Dalitz plot analysis of $B^0 \rightarrow D^{(*)} h^0$ decays with $D^0 \rightarrow K_S^0 \pi^+ \pi^-$ decays. The analysis is performed using the final BABAR and Belle data samples, totaling more than 1 ab^{-1} collected at the $\Upsilon(4S)$ resonance. The measurement provides first evidence for $\cos(2\beta) > 0$, and observes CP violation in $B^0 \rightarrow D^{(*)} h^0$ decays. The result directly excludes the trigonometric multifold solution of $\frac{\pi}{2} - \beta$ of the CKM Unitarity Triangle without further assumptions.

REFERENCES

- [1] CABIBBO N., *Phys. Rev. Lett.*, **10** (1963) 531.
- [2] KOBAYASHI M. and MASKAWA T., *Prog. Theor. Phys.*, **49** (1973) 652.
- [3] BABAR COLLABORATION (AUBERT B. *et al.*), *Phys. Rev. Lett.*, **87** (2001) 091801.
- [4] BELLE COLLABORATION (ABE K. *et al.*), *Phys. Rev. Lett.*, **87** (2001) 091802.
- [5] BABAR COLLABORATION (AUBERT B. *et al.*), *Phys. Rev. Lett.*, **93** (2004) 131801.
- [6] BELLE COLLABORATION (CHAO Y. *et al.*), *Phys. Rev. Lett.*, **93** (2004) 191802.
- [7] BABAR COLLABORATION (AUBERT B. *et al.*), *Phys. Rev. D*, **79** (2009) 072009.
- [8] BELLE COLLABORATION (ADACHI I. *et al.*), *Phys. Rev. Lett.*, **108** (2012) 171802.
- [9] HEAVY FLAVOR AVERAGING GROUP (AMHIS Y. *et al.*), *Eur. Phys. J. C*, **77** (2017) 895.
- [10] BELLE COLLABORATION (VOROBYEV V. *et al.*), *Phys. Rev. D*, **94** (2016) 052004.
- [11] BONDAR A., GERSHON T. and KROKOVNY P., *Phys. Lett. B*, **624** (2005) 1.
- [12] BELLE COLLABORATION (KROKOVNY P. *et al.*), *Phys. Rev. Lett.*, **97** (2006) 081801.
- [13] BABAR COLLABORATION (AUBERT B. *et al.*), *Phys. Rev. Lett.*, **99** (2007) 231802.
- [14] CHUNG S. U. *et al.*, *Ann. Phys.*, **507** (1995) 404.
- [15] LASS COLLABORATION (ASTON D. *et al.*), *Nucl. Phys. B*, **296** (1988) 493.
- [16] AITCHISON I. J. R., *Nucl. Phys. A*, **189** (1972) 417.
- [17] PARTICLE DATA GROUP (PATRIGNANI C. *et al.*), *Chin. Phys. C*, **40** (2016) 100001.
- [18] ANISOVICH V. V. and SARANTSEV A. V., *Eur. Phys. J. A*, **16** (2003) 229.
- [19] BABAR COLLABORATION (AUBERT B. *et al.*), *Phys. Rev. D*, **78** (2008) 034023.
- [20] BABAR COLLABORATION (DEL AMO SANCHEZ P. *et al.*), *Phys. Rev. Lett.*, **105** (2010) 081803.
- [21] BELLE COLLABORATION (PENG T. *et al.*), *Phys. Rev. D*, **89** (2014) 091103.
- [22] BELLE COLLABORATION (POLUEKTOV A. *et al.*), *Phys. Rev. D*, **81** (2010) 112002.
- [23] CDF COLLABORATION (AALTONEN T. *et al.*), *Phys. Rev. D*, **86** (2012) 032007.
- [24] FEINDT M. and KERZEL U., *Nucl. Instrum. Methods Phys. Res. Sect. A*, **559** (2006) 190.
- [25] FOX G. C. and WOLFRAM S., *Phys. Rev. Lett.*, **41** (1978) 1581.
- [26] BELLE COLLABORATION (LEE S. H. *et al.*), *Phys. Rev. Lett.*, **91** (2003) 261801.
- [27] BABAR and BELLE COLLABORATIONS (ABDESSELAM A. *et al.*), *Phys. Rev. Lett.*, **115** (2015) 121604.
- [28] TAJIMA H. *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A*, **533** (2004) 370.
- [29] KAKUNO H. *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A*, **533** (2004) 516.
- [30] PIVK M. and LE DIBERDER F. R., *Nucl. Instrum. Methods Phys. Res. Sect. A*, **555** (2005) 356.