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A cosmological signature of the Higgs instability: Primordial black holes

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Summary. — A remarkable property of the Standard Model Higgs potential is that it develops an instability at a scale of the order of 10^{11} GeV. The cosmological implications of this feature give us the possibility to get an insight of physics at scales which are inaccessible at colliders. We show in the paper by ESPINOSA J. R. *et al.*, *Phys. Rev. Lett.*, **120** (2018) 121301, that a possible cosmological signature of the instability could be the generation of dark matter in the form of primordial black holes, seeded by the fluctuations of the Higgs field generated during inflation.

1. – The instability of the Standard Model Higgs vacuum

It has been known for a long time that the potential of the Standard Model (SM) Higgs boson is unstable at high energies [1]. The running quartic coupling λ of the Higgs potential (neglecting the mass term which is subdominant at our scales of interest)

$$V(h) = \frac{1}{4}\lambda(h)\, h^4 \qquad (1)$$

becomes negative at a scale $h \sim 10^{11}$ GeV for the central measured values of the top quark and Higgs boson masses (see fig. 1).

The first question we could raise is whether today we should worry for a quantum tunnelling process which brings the background value of the Higgs field from the electroweak scale to the scale of this instability, beyond the barrier. The answer is that the probability for this tunnelling is absolutely negligible, and the lifetime of the electroweak vacuum is enormously larger than the age of the Universe. Still, it is remarkable that in the parameter space $(m_{\text{Higgs}}, m_{\text{top}})$ we happen to live in a very narrow region corresponding to a metastable vacuum, dangerously lying at the boundary between an absolutely stable region (where $\lambda(h)$ never turns negative) and an unstable one (whose lifetime would be shorter than the age of the Universe).

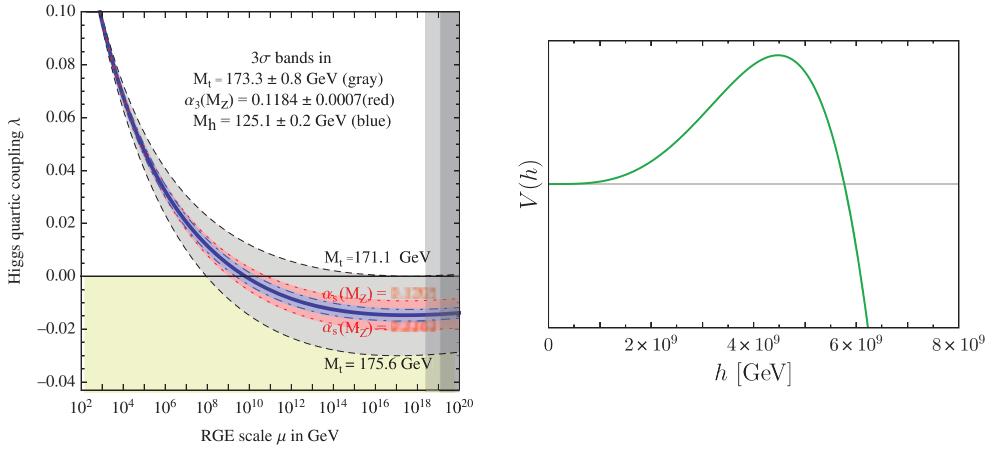


Fig. 1. – Left: running of the Higgs quartic coupling λ as a function of the renormalisation group scale, to be associated with h (from the last reference in [1]). Right: qualitative sketch of the Higgs potential.

Other two crucial epochs of the early Universe concerning the metastability of the Higgs vacuum are the phase of primordial inflation, when the Universe is believed to have inflated exponentially, and the subsequent reheating phase.

During the inflationary epoch, in which the geometry of the Universe was close to a de Sitter space, any (effectively massless) scalar field is subject to quantum fluctuations of the order of $\pm H/(2\pi)$, where H is the Hubble rate. Thus, the background value $h_\mathrm{c}(t)$ of the Higgs field randomly fluctuates on a time scale H^{-1} with quantum jumps [2]

$$\Delta_\mathrm{q} h_\mathrm{c} \sim \pm \frac{H}{2\pi}. \tag{2}$$

Depending on the value of H, these fluctuations could lead the Higgs field beyond the potential barrier, and make it roll towards its true vacuum at larger field values. This vacuum has large negative energy, which can eventually overcome the positive energy density of the inflaton and determine the corresponding region to be an anti-de Sitter (AdS) bubble. Once inflation is over, this bubble would then expand at the speed of light. Given that our observable Universe lies in the electroweak vacuum, we know that there never was such an AdS region in our past lightcone.

After the end of inflation, the Universe undergoes a thermal phase dubbed reheating, during which all the SM particles are created. During this phase, the interactions of the Higgs field with the thermal bath have the overall effect of stabilising the potential through a thermal contribution $V_T(h)$ [3]

$$V_0(h) + V_T(h) = \frac{1}{4}\lambda(h) h^4 + \frac{1}{2} m_T^2 h^2, \qquad m_T^2 \simeq 0.12\, T^2 \exp\left(-\frac{h}{2\pi T}\right). \tag{3}$$

If the reheating temperature T_RH is high enough and h_c is not too far from the origin, thermal corrections can rescue the Higgs, bringing it back around the electroweak vacuum.

The phenomenology of the Higgs instability in the early Universe is very suggestive and discloses the possibility to probe the Physics of the Higgs potential at extremely high energy scales through cosmology. Apart from deriving constraints on inflationary models, could we have some positive probe of the metastability of the Higgs vacuum? Apart from the catastrophic possibility of tunnelling today beyond the potential barrier, we argue that a signature of the Higgs instability could have arisen if the Higgs field probed the unstable region at the end of inflation, and was rescued back in time by thermal corrections at reheating.

Section **2** describes the mechanism that we consider, and sect. **3** discusses the generation of Primordial Black Holes (PBH) as an outcome. Another signature of this same process is the excitation of a stochastic background of gravitational waves (GW), as exposed in ref. [4].

2. – A cosmological signature: excitation of the Higgs fluctuations on small scales

We assume that the Higgs potential turns negative at a scale around 10^{11} GeV. Figure 2 (left) shows the running of $\lambda(h)$ we consider in [5], which corresponds to $m_{\rm Higgs} = 125.09\,{\rm GeV}$, $m_{\rm top} = 172\,{\rm GeV}$, $\alpha_S = 0.1184$.

Therefore, during the epoch of primordial inflation, the Higgs field background is subject to quantum fluctuations (eq. (2)), where we take $H = 10^{12}\,{\rm GeV}$. Figure 2 (right) shows a qualitative sketch of the dynamics of the Higgs field background that we consider. Initially, $h_{\rm c}(t)$ randomly fluctuates on patches of size H^{-1}. Occasionally, the Higgs field could go beyond the potential barrier, but the quantum diffusion will prevail as long as it is not counterbalanced by the drift predicted by the classical evolution of the field, $\Delta h_{\rm c} \sim \dot{h}_{\rm c}\,\Delta t$ with $\Delta t \sim H^{-1}$ and $h_{\rm c}$ following the equations of motion

$$\ddot{h}_{\rm c} + 3H\dot{h}_{\rm c} + V'(h_{\rm c}) = 0. \tag{4}$$

When the classical drift overcomes the quantum diffusion, $h_{\rm c}$ begins to slowly roll down the negative potential: this happens when

$$\overbrace{\frac{V'(h_{\rm c})}{3H^2}}^{\text{classical}} \gtrsim \overbrace{\frac{H}{2\pi}}^{\text{quantum}} . \tag{5}$$

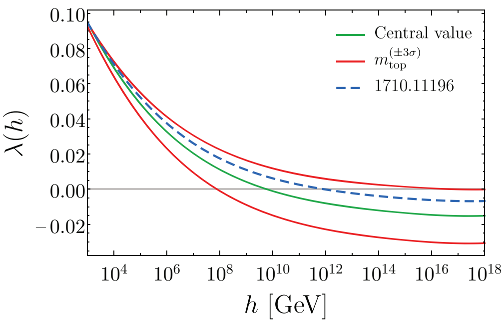
From this starting point t_* we follow the classical evolution of the field $h_{\rm c}$.

In the meantime, the fluctuations of the Higgs field are excited. We define them through $h(t,\vec{x}) = h_{\rm c}(t) + \delta h(t,\vec{x})$. The equations of motion for the Fourier transform δh_k (at the linear order in δh_k) read(1)

$$\delta\ddot{h}_k + 3H\delta\dot{h}_k + \frac{k^2}{a^2}\,\delta h_k + V''(h_{\rm c})\,\delta h_k = 0. \tag{6}$$

At early times, when $k \gg aH$, the third term in eq. (6) prevails and makes δh_k oscillate around zero. When the corresponding mode leaves the Hubble radius at the time t_k,

(1) We have checked that the term describing the feedback of the metric is negligible.



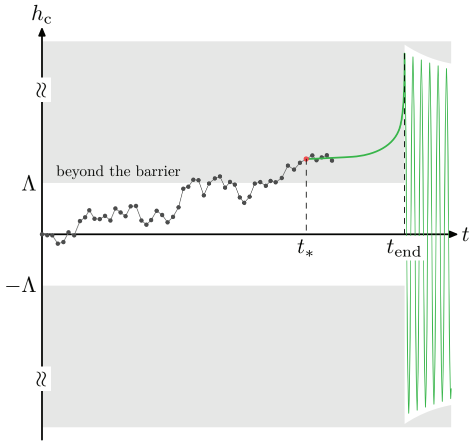


Fig. 2. – Left: running of $\lambda(h)$ we consider in [5], together with the values corresponding to the central and extremal values of $m_{\rm top}$ (last reference in [2]). Right: qualitative sketch of the dynamics of the Higgs background for the mechanism we describe.

determined by $k = a(t_k)H$, the fourth term in eq. (6) is the leading one. For modes $k > k_*$, where k_* is the mode leaving the Hubble radius at t_*, at t_k the background is already slowly rolling down the negative potential, and the fourth term has a negative sign. Therefore, it acts as a driving term for the equation, and sources a tachyonic instability for the fluctuations δh_k, which grow importantly after they leave the Hubble radius.

The proper way to quantify them in a gauge-invariant way is through the comoving curvature perturbation ζ, which is conserved on super-horizon scales. We can split ζ in two components: the first one, which is dominant for scales $k < k_*$, is the standard one coming from the inflaton, which determines the large scale structures we observe today. The last contribution, relevant for small scales $k > k_*$, comes from the Higgs and gives the largest contribution to ζ, in the last e-folds of inflation. In the flat gauge, ζ can be written then as

$$(7) \qquad \zeta = H\frac{\delta\rho_{\rm tot}}{\dot\rho_{\rm tot}} = \underbrace{\frac{\dot\rho_{\rm inflaton}}{\dot\rho_{\rm tot}}\zeta_{\rm standard}}_{\text{standard inflaton contr.}} + \underbrace{H\frac{\delta\rho_h}{\dot\rho_{\rm tot}}}_{\text{contr. from } h \text{ at } k>k_*} .$$

We have described so far what happens while the Higgs background rolls down the negative potential. At some point, before $h_{\rm c}$ goes too far, inflation must end and reheating has to take place, in order for the thermal corrections to "rescue" the Higgs as described in sect. **1**. For simplicity, we assume an instantaneous reheating: during inflation the Hubble rate is exactly constant, and at the time $t_{\rm e}$ all the inflaton energy density is instantaneously converted into radiation at a temperature $T_{\rm RH}$. The positive thermal contribution to $V'(h)$ from eq. (3) stops the tachyonic growth in eqs. (4) and (6) if $T_{\rm RH} \gtrsim h_{\rm c}(t_{\rm e})$, and makes $h_{\rm c}$ and δh_k oscillate around zero. Figure 3 shows the evolution of $h_{\rm c}$ and δh_k throughout the phases we described. Then, shortly after $t_{\rm e}$, the Higgs field decays into radiation: the damping rate of the Higgs, evaluated at 2 loops, is about $\sim 10^{-3}T$, so that within a few hundreds of oscillations all the Higgs energy density is

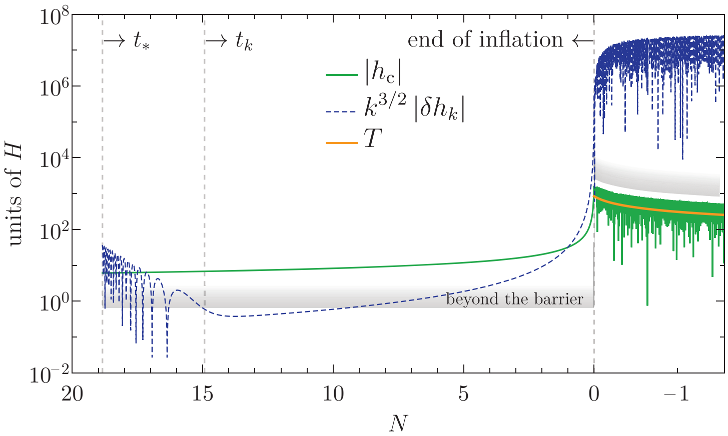


Fig. 3. – Evolution of the Higgs background and its fluctuation during the phases we described.

converted into radiation energy density.

The final outcome of the mechanism is then the generation of large adiabatic perturbations on small scales $k > k_*$. Figure 4 (left) shows the power spectrum of the curvature perturbations on small scales (in units of k_*), defined as usual by

$$\mathcal{P}_\zeta = \frac{k^3}{2\pi^2}\,|\zeta_k|^2\ . \tag{8}$$

If these fluctuations are large enough, when the corresponding modes re-enter the Hubble radius during the radiation dominated epoch they can seed the formation of PBH.

3. – Generation of primordial black holes

In this section we briefly introduce the PBHs, first proposed many decades ago [6] and discussed in depth in the recent reviews [7] and in the references therein. We conclude by presenting the PBH mass spectrum obtained through the mechanism we describe.

PBHs are black holes which could have formed in the early Universe, during the radiation dominated era, and not as the final stage evolution of massive stars. They would behave as collisionless cold dark matter, and their phenomenology would be similar to the one of Massive Astrophysical Compact Halo Objects (MACHOs), with some distinguishing features. They could lie in any possible mass range, *a priori*: this vast domain is then delimited by two key constraints. If they were lighter than $\lesssim 10^{-18} M_\odot$, they would have evaporated by now, and if they were heavier than $\gtrsim 10^{3-4} M_\odot$, they would have affected the Cosmic Microwave Background (CMB) through spectral distortion.

They form from the collapse of large overfluctuations in the radiation energy density, when the corresponding scale re-enters the Hubble radius, as sketched in fig. 4 (right).

If the density overfluctuation is large enough, the mass contained within a sphere of radius H^{-1} collapses into the PBH (up to an efficiency factor $\gamma \approx 0.2$):

$$M_{\rm PBH} = \gamma \frac{4\pi}{3}\,\rho\, H^{-3} = 13.3\gamma\left(\frac{10^{13}\ {\rm GeV}}{H}\right) e^{2N} \approx M_\odot\, e^{2(N-36)}, \tag{9}$$

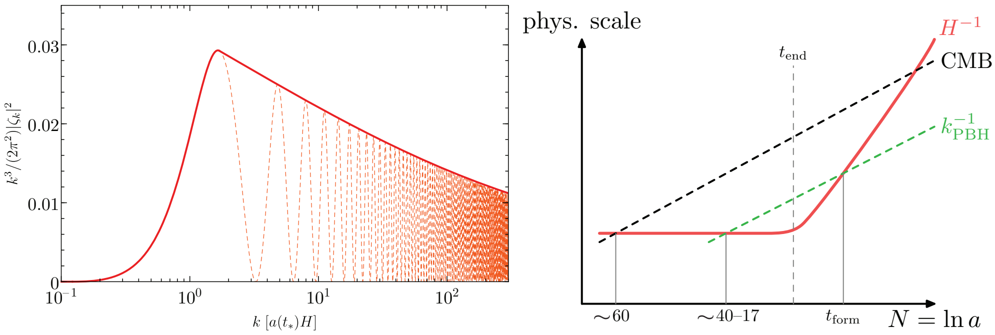


Fig. 4. – Left: power spectrum of the curvature perturbations at small scales, with k expressed in units of k_*. Right: sketch showing the formation time for PBHs.

where in the last step we plugged a Hubble rate during inflation $H = 10^{12}$ GeV and we introduced the number of e-folds N from t_k until the end of inflation. As a condition for the collapse, we take the density contrast

$$\Delta(t,\vec{x}) = \frac{4}{9}\left(\frac{1}{aH}\right)^2 \nabla^2\zeta(\vec{x}), \tag{10}$$

to overcome a threshold density of $\Delta_c \approx 0.45$ (for a more refined discussion on the threshold, see [8]). The Δ defined in eq. (10) physically corresponds to the density contrast and to the spatial curvature of the metric: when the spacetime curvature becomes large enough, even radiation has to fall into the gravitational well and collapses against its own pressure.

The recipe we apply for computing the PBH abundance is then the following: given the power spectrum, one derives the variance of the smoothed density contrast $\sigma_\Delta(M)$, which allows to compute the probability of exceeding the threshold Δ_c by assuming a Gaussian probability distribution(2) for Δ:

(11)

$$\sigma^2_\Delta\Big(M(k)\Big) = \int_0^\infty \mathrm{d}\ln q \underbrace{W^2(q,k)}_{\text{Gaussian window}} \underbrace{\frac{16}{81}\left(\frac{q}{k}\right)^4 \mathcal{P}_\zeta(k)}_{\mathcal{P}_\Delta(k)}, \quad \beta(M) = \int_{\Delta_c}^\infty \frac{\mathrm{d}\Delta}{\sqrt{2\pi}\,\sigma_\Delta} e^{-\frac{\Delta^2}{2\sigma^2_\Delta}}.$$

After formation, the density of PBHs goes as $\rho_{\rm PBH} \sim a^{-3}$ until matter-radiation equality, from when they scale as the rest of matter. By accounting for this, the final abundance today of PBHs is given by

$$f(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm CDM}} = \frac{\beta(M)}{1.6\cdot 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g_*(T_f)}{106.75}\right)^{-1/4}\left(\frac{M}{10^{-15}M_\odot}\right)^{-1/2}. \tag{12}$$

(2) Non-Gaussianities play an important role, given that we look at the tail of the distribution [9], so that eq. (11) can only be an approximation.

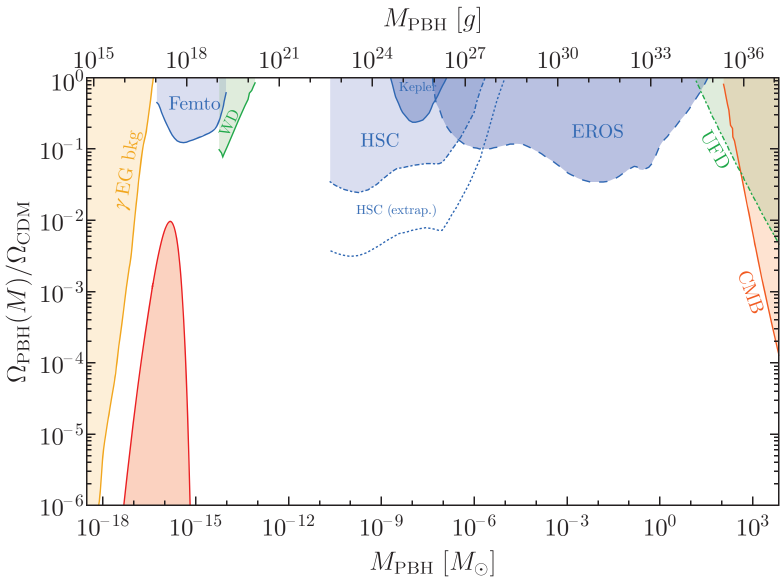


Fig. 5. – Example of a mass function of PBHs obtained with our mechanism (red line), superimposed with observational constraints (see [5] for details).

The result of this procedure for the running of $\lambda(h)$ that we consider is shown in fig. 5, together with the observational constraints.

Two important effects on the evolution of the PBH mass function must be cited: the accretion of matter throughout their history, and their merging, which could be relevant given that they are expected to be generated strongly clustered on large scales. Their effect is to shift the distribution of PBHs to higher masses and higher energy densities. These effects being beyond our scope, we just account for an increase of 10^2 in the PBH abundance due to accretion, and the mass function shown in fig. 5 yields an energy density $\Omega_{\rm PBH} = 0.01\,\Omega_{\rm CDM}$.

Let us comment now on the effect of the choice of parameters for our mechanism (which are $\lambda(h)$, t_*, $h_{\rm c}(t_*)$) on the PBH mass function. The position of the peak depends on t_*, the total number of e-folds of evolution. For the central running of $\lambda(h)$, one obtains too light PBHs. Thus, we need a slower evolution of $h_{\rm c}$, *i.e.*, a "less negative" λ.

The height of the peak depends instead on the amplitude of $\mathcal{P}_\zeta$, so on how much the fluctuations grew before the end of inflation. This is controlled by the precise values of t_* and $h_{\rm c}(t_*)$, together with $\dot{h}_{\rm c}(t_*)$ (which we assume to be zero). Given the exponential dependence of $\beta(M)$ on σ_Δ (eq. (11)), one needs an important fine-tuning to achieve a PBH abundance today which can account for the totality of dark matter. Such an unpleasant feature is present though in any model for the formation of PBHs.

More importantly, in the scenario we describe there is no candidate for DM apart from PBHs: in the regions where they were not formed, there would be no dark matter and thus large scale structures would not have been formed. We can invoke then anthropic explanations for this fine-tuning: without PBHs there would be no large scale structures today.

It must be added that the evolution we describe occurs with the same initial conditions on a patch of the size of the Hubble rate at t_k, which is much smaller than the observable

Universe today [10]. Neighbour patches would typically display slightly different initial conditions, and in particular $h_c(t_e)$ could be beyond the value which can be rescued by thermal corrections, leaving a catastrophic AdS expanding region. See the second reference in [10] for a solution to this potential issue.

4. – Conclusions

The metastability of the Higgs potential is an important byproduct of the Standard Model. Understanding its implications throughout the cosmological history could give us precious information about particle physics at extremely high scales. We propose in ref. [5] a possible observational signature of this feature, which would arise if the Higgs probed the unstable region at the end of inflation and was rescued back by reheating. Large density fluctuations at small scales would be created, with the formation of PBHs which could potentially constitute the totality of dark matter today. Another potential and independent signature would be a background of stochastic gravitational waves [4]. We are living today in a very exciting era for the experimental study of PBHs and GWs, and we can hope to learn important pieces of physics beyond the Standard Model with these probes.

* * *

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