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A stochastic optimization method with in-pit waste and tailings disposal for open pit life-of-mine production planning

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ABSTRACT

Environmental responsibility and the sustainable development of mineral resources are a topic of critical importance to the mining industry and at the same time relate to operational and rehabilitation costs to be considered in technical studies. Open pit mining operations impact their local environment in terms of their modification of the landscape and local ecosystems. Many of these impacts are the result of the transportation of large volumes of materials mined and shifted from and to different locations. External stockpiles and waste dumps occupy considerable space as well as involve substantial transportation costs to move materials from open pits to stockpiles and then move them back to the pit for rehabilitation after the end of exploitation. Depending on the shape of the deposit and the intended design of the pit, a desirable option may be to place it directly in the free spaces within the pit, instead of storing all waste and tailings materials in stockpiles and/or waste/tailings dumps. This paper presents a new mathematical formulation integrating to life-of-mine planning and the maximization of net present value, with the related waste and tailings disposal kept within the mined-out parts of a pit, using a stochastic integer program that manages geological uncertainty including metal grades, material types and related chemical compositions. In addition to the traditional variables related to the materials being extracted from the ground in the form of mining blocks, strips of ground following the dip of a pit are considered within the pit as decision variables, and the optimization process aims to optimally define both the sequence of extraction of mining blocks and the reservation of strips needed to store waste materials. An application at an iron ore mine demonstrates the feasibility, applied aspects and advantages of the proposed method.

1. Introduction

Life-of-mine (LOM) planning is a core element of production forecasting, financial valuation and environmentally responsible development of open pit mining projects and operations. The optimization of the ore and waste extraction sequence and generation of related performance forecasting are undertaken based on operational research methods with the objective of maximizing discounted cash flows over the LOM while accounting for operational constraints and considerations (Whittle, 1999; Hustrulid and Kuchta, 2006; Ramazan and Dimitrakopoulos, 2004; Newman et al., 2010; Dimitrakopoulos, 2018). Among the core LOM planning considerations, waste management is a particularly important concern. Waste dumps and stockpiles represent significant volumes of material that substantially impact the local environment, while the available space for waste storage is often limited. As a result, material is first moved to a stockpile and then moved back to the open pit during the rehabilitation phase, leading to considerable costs and efforts. An alternative approach is to store waste and tailings

directly into the mined-out areas of the open pit during its operation, which reduces the usage and size of external stockpiles as well as waste transportation and related costs. However, disposing material inside the pit during mining operations can have severe consequences on production, as an in-pit areas of storage may automatically sterilize potential underlying ore. As a result, it is critical to simultaneously optimize the extraction sequence of materials represented by mining blocks and the in-pit waste disposal to optimally define the mining production policy. The literature on this topic is limited, and some related work is found in Zuckerberg et al. (2007), who present an extended version of BHP's mine planning software Blazor, named Blazor-InPitDumping (BlazorIPD). Their concept is the following: a period of extraction is assigned to aggregates of mining blocks (or AGGs), and different processing decisions are assigned to subdivisions. Then, the percentage of an AGG going to waste can either go to an external stockpile or to a zone inside the pit. If it goes to the pit, it is associated with a “refill AGG”, which is an aggregate larger than an AGG, but that also respects precedence constraints. BlazorIPD produces a schedule that reduces the

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external stockpile by taking advantage of the free space in the open pit. However, the sparse disposition of waste material within the pit that can result from the optimization may be problematic from an operational point of view. In addition, aggregating mining blocks can misrepresent mining selectivity and provide misleading forecast results. Furthermore, this approach, like traditional mine planning methods, relies on a deterministic optimization in which all parameters used in the related mathematical model are considered known with certainty. In mine planning, geological information describing the pertinent properties of materials being mined is critical and at the same time largely uncertain, introducing substantial risks in mine plans, production forecasts and assessments. The adverse effects of uncertainty associated with the geological attributes of interest of mineral deposits (meal grades, material types, geometallurgical and other rock properties, deleterious materials and so on) have been repeatedly documented in the past (Ravenscroft, 1992; Dowd, 1994; Vall  e, 2000; Dimitrakopoulos et al., 2002; Godoy, 2002; Dimitrakopoulos, 2011; others). The problem originates from the sparse drill hole data on which the mined orebody model is based. Deterministic optimization methods, such as the conventional ones frequently employed in LOM planning, use as input an orebody model generated from the available drilling data using estimation techniques and base the optimization process on this average-type (estimated) single representation of the mineral deposit at hand. Estimated orebody models of a deposit are smooth representations of reality and underrepresent both global proportions of materials and their local variability (eg. Dimitrakopoulos et al., 2002; Godoy, 2002), leading to poorly informed mine plans and production schedules. In terms of waste management, a smoothed representation of the grade distribution of a deposit tends to minimize the waste tonnage forecasts and results in unexpected additional material sent to the waste dump. To avoid this and address inevitable uncertainty, a set of stochastic simulated realizations of the mineral deposit may be used. These simulated models of the mineral deposit are equally probable representations of the actual orebody, given the available data, and reproduce the variability of the deposit's grade (Goovaerts, 1997; Journel and Kyriakidis, 2004). Instead of basing the optimization on a single representation of the ore body, a stochastic optimizer uses this set of simulated scenarios of the orebody to obtain a life-of-mine production schedule that maximizes net present value while minimizing risk in meeting production forecasts. Ramazan and Dimitrakopoulos (2005, 2013) propose a two-stage Stochastic Integer Programming (SIP) model with fixed recourses, which successfully optimizes the open pit mine scheduling under geological uncertainty. Several applications of such models with diverse solution approaches can be found in the literature. Most of these studies focus on heuristic methods to solve this complex problem: the aggregation of blocks technics (Menabde et al., 2007; Ramazan, 2007; Boland et al., 2009; Del Castillo and Dimitrakopoulos, 2016), the sliding time window method (Dimitrakopoulos and Ramazan, 2008; Cullenbine et al., 2011) and the topological sorting algorithm (Chicoisne et al., 2012). Metaheuristic methods have also been developed to tackle larger scale problems and full mining complexes, which account for the whole mineral value chain with multiple mines and processing streams. For example, Montiel and Dimitrakopoulos (2015), Montiel et al. (2016) and Goodfellow and Dimitrakopoulos (2016, 2017) use variations of simulated annealing and manage to optimize the mineral value chain. The notion of mining complexes is promising in terms of waste and tailings management, since the opportunities given by the diverse processing streams and the modelling of stockpiles can allow a better representation of the material exchanges between the different components of a mine.

The topic of in-pit waste disposal was also addressed in Zuckerberg et al. (2007); however, the present study considers a stochastic framework and additional operational considerations. The proposed method is tested in an iron ore mining project that aims to limit the space required for the external stockpiles for the waste and tailings

material by refilling the pit during exploitation. The motivation for applying in-pit waste disposal originates from different factors, including limiting environmental impacts, accounting for constraints in available space surrounding the pit, and reducing the cost of waste transport and rehabilitation. In the deposit illustrated in this study, the shape of the deposit (low dip layers and long strike length) is used as an advantage, allowing for the material to be stored in bands or strips oriented toward the dip; the corresponding storage zone remains continuous and grows from one period to another. This orientation assures that the storage location remains contained without occupying space around the open pit, with the condition that the ore below an individual zone must also have been extracted prior to using it for storage. In general, all of the required constraints must be jointly addressed, starting with those associated with the extraction sequence of mining blocks, such as block accessibility, production capacity and blending constraints. The novelty of the proposed approach explicitly considers in-pit storage considerations and simultaneously accounts for geological uncertainty within the stochastic optimization framework, a topic particularly relevant in terms of waste production forecasts and management.

In the following sections, the description of the proposed SIP model, referred to as the open pit mine planning stochastic integer program with in-pit tailings disposal (OMPSIP-ITD), is presented. Subsequently, a case study at an iron ore deposit located in Labrador, Canada, details the applied aspects and contributions of the proposed model. The results are presented in terms of the material disposal inside the pit and the quality of production forecasts, including discounted cash flows and production targets along with their quantified risk in terms of meeting forecasts due to geological uncertainty. Finally, conclusions and insights for future research are presented.

2. The OMPSIP-ITD mathematical model

In this section, the proposed stochastic mathematical programming formulation for open pit mine production planning including the integration of in-pit waste disposal (OMPSIP-ITD), is detailed. The mathematical model is a two-stage stochastic integer program with fixed recourse (Birge and Louveaux, 2011) that simultaneously optimizes the extraction sequence and destination policy (Ramazan and Dimitrakopoulos, 2005, 2013; Spleit and Dimitrakopoulos, 2017; Rim  l   et al., 2017) and the in-pit storage, which introduces several new notations, variables and constraints. In particular, at each period, a top and a bottom strip are considered to delimitate the storage zone in the pit. All the strips in-between are reserved for storage. Several hypotheses are made and described in the model.

2.1. Notation

The diverse sets, indices, parameters and variables used in the following OMPSIP-ITD formulation are described below.

2.1.1. Sets

- $\mathcal{B} = \{i = 1, \dots, N\}$ Set of blocks in the ore body;
- $\mathcal{P} = \{p = 1, \dots, P\}$ Set of considered periods for the schedule;
- $\mathcal{D} = \{0, 1\}$ Set of destinations available for the blocks = 0(waste dump) $d = 1(\text{mill})$;
- $\mathcal{S} = \{s = 1, \dots, S\}$ Set of scenarios (equiprobable ore body stochastic simulations);
- $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$ Set of blocks' characteristics, $\mathcal{C}_1 = \{c_1 = 1, \dots, C_1\}$ linear characteristics (tonnages, trucks hours...), $\mathcal{C}_2 = \{c_2 = 1, \dots, C_2\}$ non-linear characteristics (grades);

$G(\mathcal{B}, \mathcal{A})$ Oriented graph representing the precedence relationships between blocks. On Fig. 1, $(b, e) \in \mathcal{A}$, which means that block $b \in \mathcal{B}$ is a predecessor of block $e \in \mathcal{B}$;

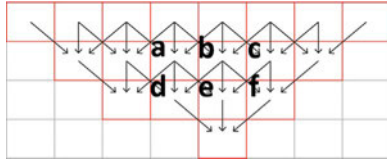


Fig. 1. Precedence relationships between blocks.

$\Gamma_i^+ = \{b \in \mathcal{B}(i, b) \in A\}$ Set of direct successors of block i . On Fig. 1, $\Gamma_b^+ = \{d, e, f\}$;
 $\Gamma_i^- = \{a \in \mathcal{B}(a, i) \in A\}$ Set of direct predecessors of block i . On Fig. 1, $\Gamma_e^- = \{a, b, c\}$;
 Γ_i^{-Tot} Set of the all cone of predecessors of block i . On Fig. 1, $\Gamma_e^{-Tot} = \{a, b, c\} \cup \Gamma_a^{-Tot} \cup \Gamma_b^{-Tot} \cup \Gamma_c^{-Tot}$;
 $\mathcal{K} = \{k = 1, \dots, K\}$ Set of strips considered for storage, $k = 0$ being the southeast strip, $k = K$ the northeast one;
 \mathcal{B}_k Set of blocks which belong to strip k ;

2.1.2. Parameters

$v_{i,d,s}$ Economic value of block i in scenario s if it is sent to destination d ;

This economic value depends on several parameters:

$$v_{i,d,s} = \begin{cases} -E_{waste}^{cost} \cdot t_{i,s} - TH_{i,d}^{cost} \cdot TH_{i,d} & \text{if } d = 0 \\ R_{i,s} - P_{conc}^{cost} \cdot conc_{i,s} - E_{ore}^{cost} \cdot t_{i,s} - TH_{i,d}^{cost} \cdot TH_{i,d} & \text{if } d = 1 \end{cases}$$

With:

$R_{i,s}$ Revenue from selling the metal content of block i ;
 $conc_{i,s}$ Concentrate tonnes of block i in scenario s , $conc_{i,s} \in \mathcal{C}_i$;

Where:

$$conc_{i,s} = t_{i,s} \cdot Rec_{i,s}$$

$Rec_{i,s}$ Weight recovery of block i in scenario s , obtained from the simulation of the Davis Tube Weight Recovery (used in the case study);

P_{conc}^{cost} Processing cost of concentrate material per tonne;

E_{ore}^{cost} Extraction cost of ore material per tonne;

E_{waste}^{cost} Extraction cost of waste material per tonne;

$TH_{i,d}$ Truck hours needed to send block i to destination d ;

TH^{cost} Cost per truck hour;

$t_{i,s}$ Tonnes of block i in scenario s ;

$q_{c_1,i,s}$ Quantity of characteristic c_1 of block i in scenario s ;

$g_{c_2,i,s}$ Grade c_2 in scenario s of block i ;

$target_{c,p}^{\pm}$ Minimum (-) and maximum (+) targets of quantity or grade c in period p ;

$pen_{c,p}^{dev\pm}$ Penalty cost of deviation from the targets of quantity or grade c in period p (excess +, shortage -);

r Discount rate taking into account the time value of money;

$d_p = \frac{1}{(1+r)^{p-1}}$ Discount factor;

π Maximum number of blocks to be stored outside of the pit;

γ Percentage of ore in a strip which needs to be extracted before storage within this strip;

N_k Number of blocks within strip k ;

2.1.3. Variables

2.1.3.1. Extraction variables (first stage).

$$x_{i,d,p} = \begin{cases} 1 & \text{if block } i \in \mathcal{B} \text{ is sent to destination } d \in \mathcal{D} \text{ by period } p \in \mathcal{P}; \\ 0 & \text{Otherwise} \end{cases}$$

To simplify the notation, we set $x_{i,d,p=0} = 0, \forall i \in \mathcal{B}, \forall d \in \mathcal{D}$.

The expression ‘‘by period $p \in \mathcal{P}$ ’’ means that block i was extracted

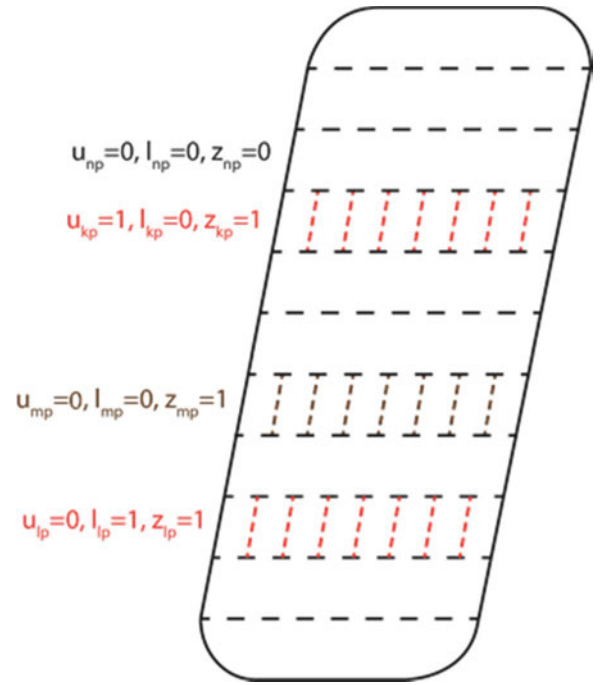


Fig. 2. Top view of the strips to store the tailings.

prior to or at period p , a formulation used to facilitate the branching during the solving process (Caccetta and Hill, 2003).

2.1.3.2. Deviation variables (second stage).

$dev_{c,p,s}^{\pm} \in \mathbb{R}^+$ Deviations from the targets in terms of characteristics $c \in \mathcal{C}$ for scenario $s \in \mathcal{S}$, during period $p \in \mathcal{P}$ (excess +, shortage -);

2.1.3.3. In-pit storage variables (first stage).

$$u_{k,p} = \begin{cases} 1 & \text{if } k \in \mathcal{K} \text{ is the top strip at period } p \in \mathcal{P} \\ 0 & \text{Otherwise} \end{cases}$$

$$l_{k,p} = \begin{cases} 1 & \text{if } k \in \mathcal{K} \text{ is the bottom strip at period } p \in \mathcal{P} \\ 0 & \text{Otherwise} \end{cases}$$

$$z_{k,p} = \begin{cases} 1 & \text{if strip } k \in \mathcal{K} \text{ is available for storage at period } p \in \mathcal{P} \\ 0 & \text{Otherwise} \end{cases}$$

$y_{k,p} \in \mathbb{R}^+$ Amount of tailings stored in strip $k \in \mathcal{K}$, period $p \in \mathcal{P}$

Fig. 2 shows the top view of the strips as defined: the strips go toward the dip in a West-East direction. The top and bottom strips (respectively $u_{k,p} = 1$ and $l_{k,p} = 1$) are identified with red dotted lines. All the strips between them are available for storage ($z_{m,p} = 1$).

2.2. General stochastic formulation of OMPSIP-ITD

This section describes the OMPSIP-ITD formulation used in the rest of the study.

2.2.1. Objective function

$$\max Z = \overbrace{\frac{1}{S} \sum_{i \in \mathcal{B}} \sum_{d \in \mathcal{D}} \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} d_p \cdot v_{i,d,s} \cdot (x_{i,d,p} - x_{i,d,p-1})}^{\text{Part 1}} - \overbrace{\frac{1}{S} \sum_{c \in \mathcal{C}_1 \cup \mathcal{C}_2} \sum_{p \in \mathcal{P}} \sum_{s \in \mathcal{S}} d_p \cdot (pen_{c,p}^{dev+} \cdot dev_{c,p,s}^+ + pen_{c,p}^{dev-} \cdot dev_{c,p,s}^-)}^{\text{Part 2}}$$

2.2.2. Constraints

2.2.2.1. Reserve constraints.

$$x_{i,d,p} - x_{i,d,p-1} \geq 0 \quad \forall i \in \mathcal{B}, \forall d \in \mathcal{D}, \forall p \in \mathcal{P} \quad (1)$$

$$\sum_{d \in \mathcal{D}} x_{i,d,p} \leq 1 \quad \forall i \in \mathcal{B}, \forall p \in \mathcal{P} \quad (2)$$

2.2.2.2. Precedence constraints.

$$\sum_{d \in \mathcal{D}} x_{i,d,p} \leq \sum_{d \in \mathcal{D}} x_{j,d,p} \quad \forall i \in \mathcal{B}, \forall j \in \Gamma_i^-, \forall p \in \mathcal{P} \quad (3)$$

2.2.2.3. Capacities constraints.

$$\begin{aligned} \text{Upper bound} \quad & \sum_{i \in \mathcal{B}} (q_{c_1,i,s} (x_{i,d,p} - x_{i,d,p-1})) - dev_{c_1,p,s}^+ \\ & \leq target_{c_1,p}^+ \quad \forall c_1 \in \mathcal{C}_1, \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \end{aligned} \quad (4.1)$$

$$\begin{aligned} \text{Lower bound} \quad & \sum_{i \in \mathcal{B}} (q_{c_1,i,s} (x_{i,d,p} - x_{i,d,p-1})) + dev_{c_1,p,s}^- \\ & \geq target_{c_1,p}^- \quad \forall c_1 \in \mathcal{C}_1, \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \end{aligned} \quad (4.2)$$

2.2.2.4. Grade quality constraints.

$$\begin{aligned} \text{Upper bound} \quad & \sum_{i \in \mathcal{B}} (g_{c_2,i,s} i_{i,s} (x_{i,d,p} - x_{i,d,p-1})) - dev_{c_2,p,s}^+ \\ & \leq target_{c_2,p}^+ \sum_{i \in \mathcal{B}} (i_{i,s} (x_{i,d,p} - x_{i,d,p-1})) \quad \forall c_2 \in \mathcal{C}_2, \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \end{aligned} \quad (5.1)$$

$$\begin{aligned} \text{Lower bound} \quad & \sum_{i \in \mathcal{B}} (g_{c_2,i,s} i_{i,s} (x_{i,d,p} - x_{i,d,p-1})) + dev_{c_2,p,s}^- \\ & \geq target_{c_2,p}^- \sum_{i \in \mathcal{B}} (i_{i,s} (x_{i,d,p} - x_{i,d,p-1})) \quad \forall c_2 \in \mathcal{C}_2, \forall p \in \mathcal{P}, \forall s \in \mathcal{S} \end{aligned} \quad (5.2)$$

The objective function and the constraints (1) to (5.2) correspond to a typical formulation for a stochastic optimization of an open pit mine planning under geological uncertainty. Brief explanations follow; for more details the reader can refer to Dimitrakopoulos and Ramazan (2008), Ramazan and Dimitrakopoulos (2013) or Rim  l   et al. (2017). The objective function is composed of two parts. Part1 aims to optimize the average discounted cash flow over the set of scenarios, while Part2 penalizes the deviations from the production targets. This second part is essential for the robustness of the schedule to the geological uncertainty.

The set of constraints (1) assures that if a block is extracted at a given period, it remains extracted for the later periods. Constraints (2) allow any block to be sent to only one destination. Constraints (3) define the accessibility of the blocks; that is, for a block to be extracted, its direct predecessors have to be extracted too, respecting the precedence constraints for the stability. The sets of constraints (4.1) and (4.2) enforce the scheduled production to respect the quantities targets by allowing deviations $dev_{c_1,p,s}^\pm$, which are penalized in the objective function. Constraints (5.1) and (5.2) have a similar role for the grade targets. Given the nonlinearity of the average grade, it is actually the element's amount that is controlled. Other constraints of mining the earliest period of extraction are also implemented but will be described later in this paper. Constraints (6) to (15) aim to model the storage inside the mined-out pit. A detailed description of these constraints is given below.

2.2.2.5. Uniqueness of the top and bottom strips.

$$\begin{cases} \sum_{k \in \mathcal{K}} u_{k,p} \leq 1 \\ \sum_{k \in \mathcal{K}} l_{k,p} \leq 1 \end{cases} \quad \forall p \in \mathcal{P} \quad (6)$$

For each period, the storage zone is only defined by a maximum of one top strip $u_{k,p}$ and a bottom one $l_{k,p}$. They correspond to the red-dotted strips on Fig. 2.

2.2.2.6. Order of the top and bottom strips.

$$\sum_{k \in \mathcal{K}} k^*(u_{k,p} - l_{k,p}) \geq 0 \quad \forall p \in \mathcal{P} \quad (7)$$

By definition, the top strip must be further north than the bottom strip. Eq. (7) states that for each period the top strip is further north than the bottom strip.

2.2.2.7. Increasing dimension of the tailings storage zone.

$$\begin{cases} \sum_{k \in \mathcal{K}} k \cdot u_{k,p} \geq \sum_{k \in \mathcal{K}} k \cdot u_{k,p-1} \\ \sum_{k \in \mathcal{K}} k \cdot l_{k,p} \leq \sum_{k \in \mathcal{K}} k \cdot l_{k,p-1} \end{cases} \quad \forall p \in \mathcal{P} \setminus \{1\} \quad (8)$$

From one period to another, the size of the in-pit storage zone can increase or remain steady because, once positioned, the tailings and waste material are not moved again to limit the re-handling costs. Eq. (8) ensures that, between two consecutive periods, the top strip can only be translated to the north and the bottom strip to the south.

2.2.2.8. Strips' availability for storage.

$$z_{k,p} = \sum_{j=k}^{K-1} (u_{j,p} - l_{j+1,p}) + u_{K,p} \quad \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (9)$$

A strip is defined as available for storage ($z_{k,p} = 1$) only if it is located between the top and bottom one. The set $\{k \in \mathcal{K} \mid z_{k,p} = 1\}$ defines the in-pit zone reserved for storage. Fig. 3 gives explanations about these constraints. The axis corresponds to the strips, from South to North, the blue strips are the top and bottom ones and the red strip is the one considered in Eq. (9). For a given strip k and a given period p , constraints (9) check the strips that are further north than strip k (the yellow ones) to look for the bottom and top strips. If only the top strip is met, strip k is in the storage zone (case 1); if both the top and the bottom strips are found, strip k is located further south than the storage zone (case 2); finally, if neither the top nor the bottom strips are met, strip k is further north (case 3). Case 4 justifies the "j + 1" in Eq. (9). Indeed, if strip k is also the bottom strip, it must be available for storage so only the strip above is checked for not being the bottom one.

2.2.2.9. Allowance of storage within a strip.

$$y_{k,p} \leq N_k \cdot z_{k,p} \quad \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (10)$$

A strip can be filled with material only if it has been defined as available for storage.

2.2.2.10. Storage available quantity within a strip.

$$\sum_{p'=1}^p y_{k,p'} \leq \sum_{i \in \mathcal{B}_k} \sum_{d \in \mathcal{D}} x_{i,d,p} \quad \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \quad (11)$$

Per strip, the maximum volume of tailings that can be stored is the volume of ore that has been extracted within this strip. An assumption is made concerning the volume occupied by the tailings: on average, any extracted block results in 80% of its volume as tailings to be stored. This is justified by the swelling of the mining blocks (typically an increase of 20–30% of the volume after extraction) and the average iron grade of 30%, which, for the blocks sent to the mill, will be extracted and will reduce the resulting volume. Both phenomena compensate for each other.

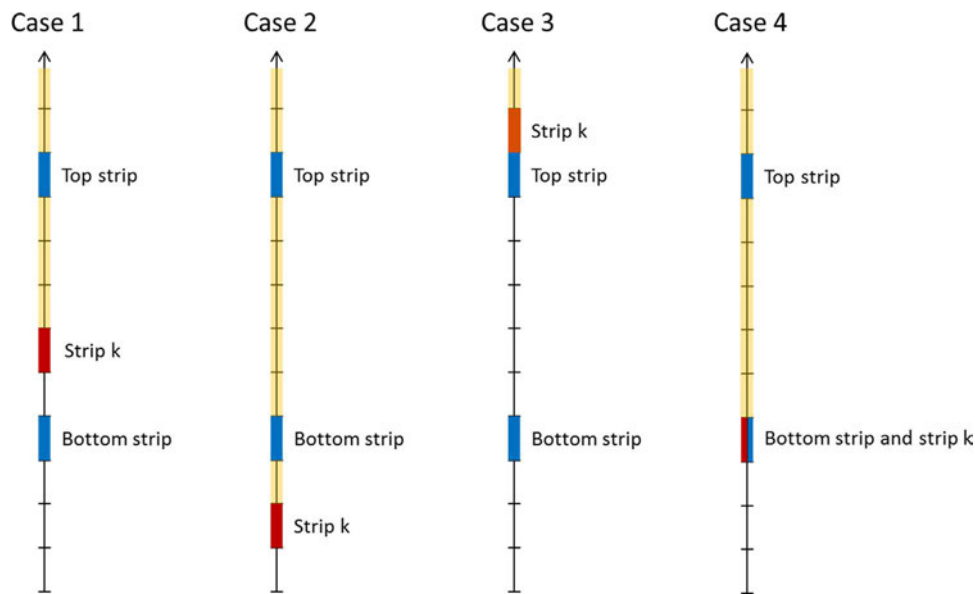


Fig. 3. Strips' availability for storage graphs.

2.2.2.11. Only the extracted material can be stored.

$$\sum_{k \in \mathcal{K}} y_{k,p} \leq \sum_{i \in \mathcal{B}} \sum_{d \in \mathcal{D}} (x_{i,d,p} - x_{i,d,p-1}) \quad \forall p \in \mathcal{P} \tag{12}$$

For each period, the amount of stored material must be less than what has been extracted during this period because tailings or waste are moved just once.

2.2.2.12. Blocks that can still be extracted.

$$\sum_{d \in \mathcal{D}} (x_{i,d,p} - x_{i,d,p-1}) \leq 1 - z_{k,p} \quad \forall k \in \mathcal{K}, \forall i \in \mathcal{B}_k, \forall p \in \mathcal{P} \tag{13}$$

A block positioned in a strip that has been defined as available for storage cannot be extracted. In other words, the remaining blocks are inaccessible as they are covered by tailings.

2.2.2.13. Maximum amount of tailings stored outside of the pit.

$$\sum_{i \in \mathcal{B}} \sum_{d \in \mathcal{D}} x_{i,d,p} - \sum_{k \in \mathcal{K}} \sum_{p \in \mathcal{P}} y_{k,p} \leq \pi \tag{14}$$

A maximum amount of storage at external stockpiles located outside the pit, i.e. π , is defined. Above this amount, the storage of tailings must be done inside the pit.

2.2.2.14. Extracted ore condition for storage.

$$\gamma \cdot N_k \cdot z_{k,p} \leq \sum_{i \in \mathcal{B}_k} \sum_{d \in \mathcal{D}} x_{i,d,p} \quad \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \tag{15}$$

In order to respect governmental rules about accessibility of ore reserves, all the ore has to be extracted or at least a certain proportion γ of it before storing in a strip. These constraints are illustrated in Fig. 4,

which shows a cross-section of a strip considered for storage and the mining blocks of the mineralized layer. The green-dotted lined blocks have been extracted (let V_1 be the corresponding total volume) and the red blocks have not (volume V_2). Constraints (15) guarantee that for each strip, the inequality $V_1 \geq \gamma \cdot (V_1 + V_2)$ is respected before starting to store tailings.

3. Case study

3.1. The deposit and application specifics

In this section, the OMPSIP-ITD model is applied to an iron ore deposit located in Labrador, Canada, that is being considered for development, and a feasibility study is undertaken. The deposit has a 10 km South-North dimension of only 2 km wide, which allows different extraction zones far away from each other from one period to another. The deposit is well adapted to in-pit storage because of its particular large size and flat shape of a low dip (6° East) and a 10 km North-South orientation with an about 2 km width, which allows different extraction zones far away from each other from one period to another once the bottom layer is extracted. In addition, the low dip of the deposit's lithology facilitates both the circulation of trucks and the stable disposal of waste and tailings. The concept applied in this case study is to define a dip oriented toward strips that, once their ore has been extracted, will be reserved for waste. The available volume within a strip is the volume of extracted blocks. It should be noted that, in order to reduce computational time and given that the reserve tonnage is far superior to the production target, only the northeast half of the deposit is considered in this application.

More specifically, the schedule is performed on 3177 mining blocks

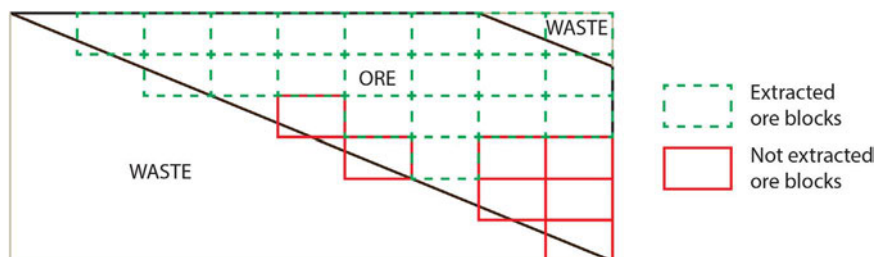


Fig. 4. Cross section of mandatory ore block extraction within a strip.

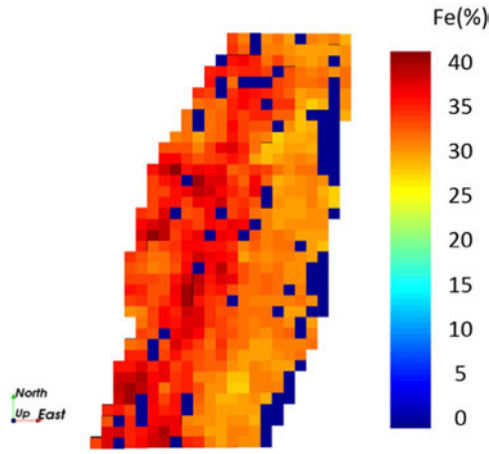


Fig. 5. A FeH grade simulation, top view.

of size 100 m * 100 m* 15 m for 10 periods with 2 destinations and 38 strips of storage. In this case study, the space of storage outside of the pit is limited to 500 blocks, after which all the tailings must be stored inside the pit. Also, the percentage of extracted ore within each strip before storage γ is set to 75%. This value was chosen to prevent the in-pit storage from sterilizing a high proportion of ore blocks while providing enough flexibility to satisfy the production targets (a value of 100% is likely to deteriorate the blending or to send ore blocks to the waste dump). However, depending on the specific requirements, one could adapt this value in a different case study. A set of 10 stochastic simulations is used as an input (Fig. 5 shows one example). The quantity constraints are in terms of concentrate tonnes of iron per year, while the quality constraints concern the average DTWR (Davis Tube Weight Recovery representing the recoverable iron) average grade per year and the average grade of silica, which constitutes the most prominent impurity in the concentrate product. Geological uncertainty was quantified by a set of 10 stochastic simulations generated by a direct block simulation with min/max autocorrelation factors (Boucher and Dimitrakopoulos, 2009), which determine for every mining block of the deposit the iron grade (FeH%), DTWR (%), silica grade (Sio2%) and density. The OMPSIP-ITD model contains 64,680 binary variables, 680 continuous ones and around 413,000 constraints, from which the in-pit storage only (constraints 6–15) represent 1140 binary variables, 380 continuous ones and around 33,000 constraints. This model is too complex to be solved by the solver Cplex; thus, a sliding time window heuristic (STWH), adapted from the method used in Dimitrakopoulos and Ramazan (2008) and Cullenbine et al. (2011), is used here instead.

Fig. 6 illustrates the STWH approach. It is an iterative method that, at each iteration, relaxes binary variables, except for a few consecutive periods (namely the “sliding window”). Once the resulting model is solved, the first period of the window is fixed, the window is moved one period further, and the process is reiterated.

In this case study, a sliding window of one period is considered, and some of the latest periods are merged when relaxed in order to reduce

the size of the problem. For instance, on Fig. 6, iteration #1 considers only the extraction variables of period 1 as binary and relaxes the others, and periods 4 and 5 are merged into one period with doubled capacity of production. In addition, the earliest periods of extraction constraints are modified. Rather than using equations (16) (as given in Rimélé et al., 2017), we define these constraints by the Eq. (16'), which are qualified as updated in the earliest periods of extraction.

$$x_{i,d,p} = 0 \quad \forall i \in \mathcal{B}, \forall d \in \mathcal{D},$$

$$\forall p \in \left\{ \bigcup_{c_1 \in \mathcal{C}_1} \left(t \in \mathcal{P} \forall s \in \mathcal{S}, \sum_{t'=1}^t target_{c_1,t}^+ - \Delta_{c_1,t'} \leq \sum_{j \in \Gamma_1^{Tot} \cup \{i\}} q_{c_1,j,s} \right) \right\} \quad (16)$$

$$x_{i,d,p} = 0 \quad \forall i \in \mathcal{B} \Phi^k, \quad \forall d \in \mathcal{D},$$

$$\forall p \in \left\{ \bigcup_{c_1 \in \mathcal{C}_1} \left(t \in \mathcal{P} \forall s \in \mathcal{S}, \sum_{t'=k}^t target_{c_1,t}^+ - \Delta_{c_1,t'} \leq \sum_{j \in (\Gamma_1^{Tot} \cup \{i\}) / \Phi^k} q_{c_1,j,s} \right) \right\} \quad (16')$$

where $\forall c_1 \in \mathcal{C}_1, \forall t' \in \mathcal{P}, \Delta_{c_1,t'} \in \mathbb{R}^+$ are parameters used to keep the flexibility of the stochastic formulation, which allows deviations from the production targets. Typically, one can take $\Delta_{c_1,t'} \cong \frac{1}{4} * target_{c_1,t'}$.

At each iteration k , the STWH method aims to determine which blocks are extracted in period $p = k$. Φ^k refers to the set of blocks that have been previously assigned to a period anterior to k . These definitions ensure that only the remaining blocks are considered, that is, the blocks that have not already been extracted by the end of period k . For each of these blocks, the total quantities of its remaining predecessors are defined. Then, these quantities are compared with the production targets from period k . If the block is not reachable for a given period, the corresponding variable can be set to 0. These new constraints (16') allow for the fixing of more variables at each iteration. With the previous version, constraints (16) mainly fixe variables associated with the first periods.

3.2. Results

The results are presented in terms of mineability, profitability and risk associated with geological uncertainty. First, Fig. 7 presents an overhead view of the disposal of the tailings and waste material inside of the pit parallel to the schedule. The schedule is represented by colors according to the periods of extraction of the blocks (from 1 to 10). Blocks in grey are those not extracted (-1). It is noteworthy that the sequence of extraction is spatially continuous; that is, the blocks extracted at the same period are located close to each other. For the tailings part, a bar diagram shows which strips are free (blue ones) and which ones are available for storage (red ones) for all the periods (years 1–10). Within the red part of the bars (zone reserved for the tailings), the volume (in terms of the number of blocks) available for waste storage is provided. The storage zone respects the previously defined constraints: a continuous zone between the top and bottom strip (constraints (9)) and a growing storage volume capacity from one

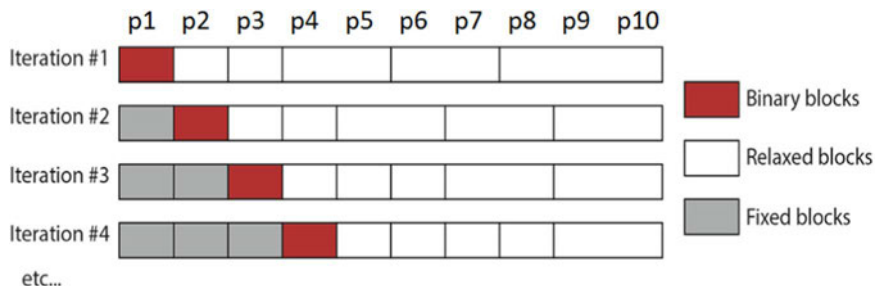


Fig. 6. The sliding time window heuristic (STWH) method.

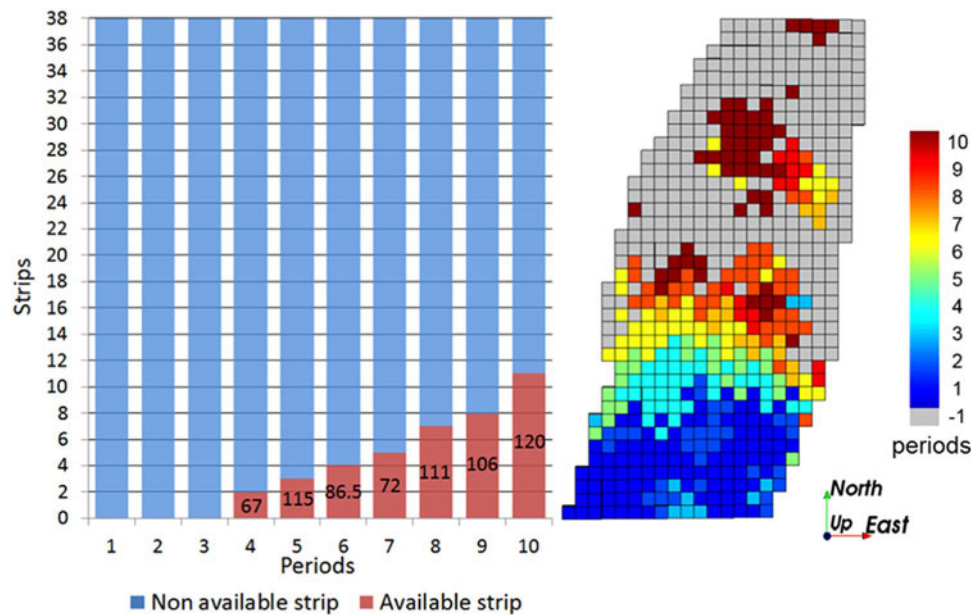


Fig. 7. Top view of the tailings disposal per period and schedule.

Table 1
Tailings storage per strip and period.

#period	4	5	6	7	8	9	10
#strip	2	1	3	1	3	4	5
nb tailings blocks	67	56.7	57.8	4.3	11.2	71	72
					72	39	32
						74	69
							51

period to another (constraints (8)). Additionally, the correspondence with the schedule shows that no blocks are extracted in a strip reserved for storage at a given period (constraints (13)). In total, 1177 blocks have been extracted over the 10 years: 677 blocks of tailings have been stored inside the pit and 500 outside. Table 1 gives more details about the strips used for storage, including during which period and for how many blocks of tailings.

For a more explicit view of storage at an individual strip, two cross sections of strip number 3 are presented in Fig. 8. The first graph shows the periods of extraction of the blocks: all the blocks are extracted between periods 1 and 3. In total, 69 blocks have been extracted in this strip and 23 remained undisturbed (i.e, exactly 75% of the blocks were extracted as requested by constraints (15)). The second graph represents an interpretation about how the storage within this strip would occur. From the optimization, 57.8 blocks of tailings are stored in the third strip during period 5 and 11.2 during period 6. With the assumptions on the average volume generated by an extracted block (80% justified by both the swelling of the blocks and the extracted metal), the tailings represent an equivalent volume of respectively 46 and 9 blocks.

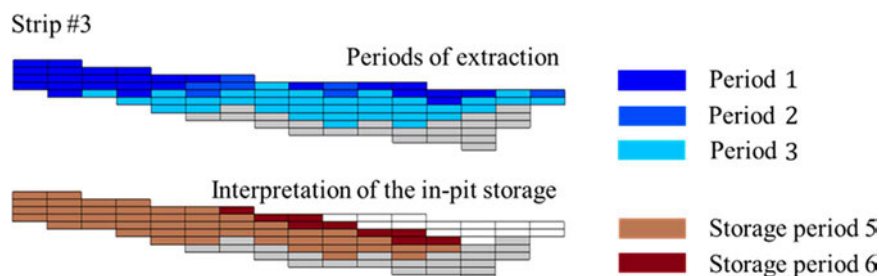


Fig. 8. Cross section of strip #3: periods of extraction and interpretation of the in-pit storage.

In order to keep the deeper blocks on the East side of the deposit accessible, the first tailings are positioned on the west side of the strip.

Fig. 9 shows the destinations of the individual blocks; most of the blocks are considered as ore by the optimizer and sent to the mill. In terms of quality constraints, the risk profiles are all satisfactory. Fig. 10, Fig. 11 and Fig. 12 present (respectively) the concentrate tonnes, the average silica grade and the average Davis Tube Weight Recovery (grade of magnetically recoverable iron) for all the simulations, including the simulation ensemble average and their targets. The average concentrate production is close to the target with a low distribution, and the silica and the DTWR are within the range of tolerance.

The gap in terms of the objective value in CPLEX between the final binary solution and the relaxed model (relaxation of the extraction variables, not the strips) is 1.76%. ($gap^{Cplex} = \frac{Obj^{relaxed} - Obj^{STWH}}{Obj^{relaxed}} = 1.76\%$). This result demonstrates the good performance of the sliding time window heuristic method (with grouped periods) to solve this model close to optimality.

For the discounted cash flow (DCF), which is equal to the objective value of CPLEX but without the artificial penalty costs of deviation, the risk is very well controlled, as can be seen in Fig. 13 and Fig. 14. The results are given relative to the average of the simulations. Without the penalty costs, one can observe that the average of the simulations has a higher DCF than the relaxed model. After 10 years, the difference between the highest and lowest DCF over the set of simulations is equal to 1.17% of the average value ($\frac{DCF_{max} - DCF_{min}}{DCF_{average}} = 1.17\%$).

The final objective value has also been compared to the same model but without considering the in-pit storage (without constraints 6–15); this model could be solved with an exact method (no heuristic

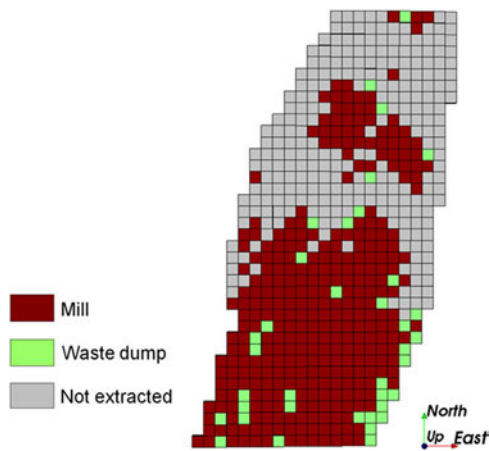


Fig. 9. Top view destinations.

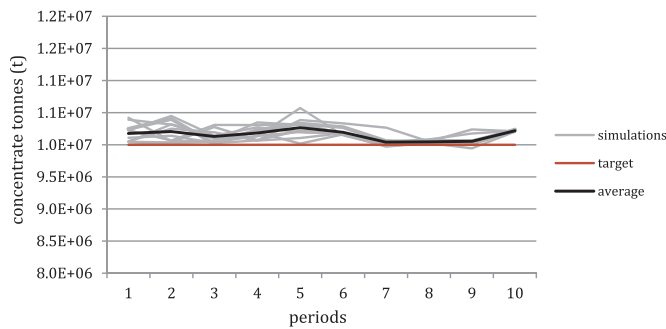


Fig. 10. Concentrate tonnes of iron.

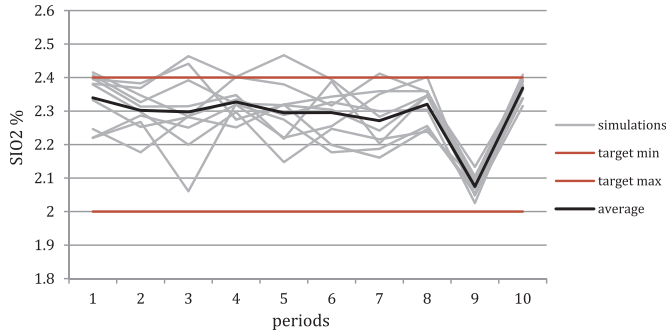


Fig. 11. Average silica grade.

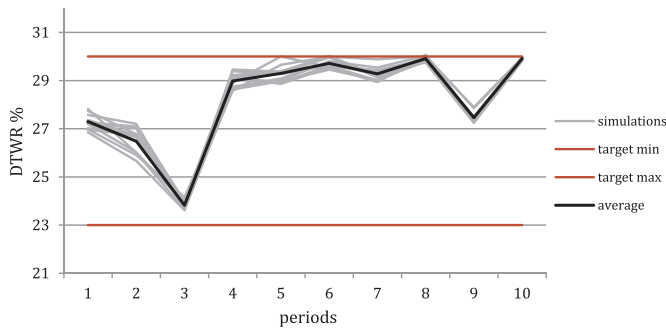


Fig. 12. Average DTWR grade.

involved). The result shows the interest of defining strips for storage because the gap between the solution obtained from the OMPSIP-Without In-pit Tailings Disposal (OMPSIP-WITD) model and the OMPSIP-ITD model $\left(\frac{Obj_{OMPSIP-WITDExact} - Obj_{OMPSIP-ITDSTWH}}{Obj_{OMPSIP-WITDExact}}\right)$ is only of

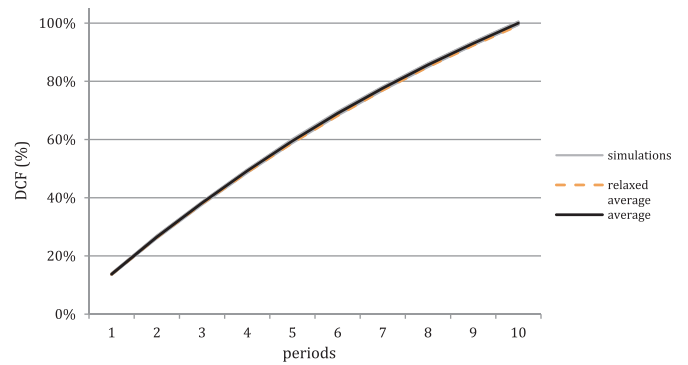


Fig. 13. Cumulative DCF.

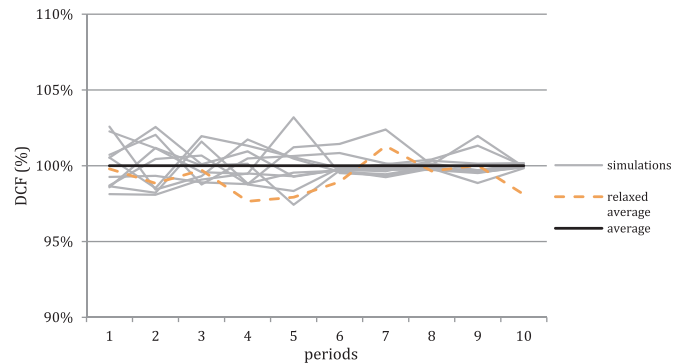


Fig. 14. DCF per period.

1.77%, even though the savings of not re-handling the tailings for the rehabilitation are not considered.

The amount of material stored inside the pit can be converted into an equivalent dump area in order to estimate the volume saved in external stockpiles. Fig. 15 presents the dimensions of the corresponding stockpile; it would represent a volume of 81 million m³, which, assuming a height of 100 m, would require a waste storage area with a diameter of 1224 m on the ground.

4. Conclusions

The environmentally responsible development of mineral resources is critically important to the mining industry and at the same time important in managing operational and rehabilitation costs. Given the importance of the topic, the present paper presents a new stochastic mathematical formulation that allows for incorporating the disposal of mining waste and tailings materials directly inside of the pit into the life-of-mine planning and production optimization. The stochastic nature of the proposed formulation manages uncertainty in terms of ore grades, material types and their related chemical compositions, which together are critical to both ore production and rehabilitation. The practical aspects and contributions of the proposed method were demonstrated with an application at an iron ore deposit located in Labrador, Canada, under development. The flat shape of the deposit facilitates the proposed approach, while waste disposal is considered by strips compliant with the deposit's dip. Operational constraints require a single continuous and growing zone of storage from one period to another, with the ore present in this zone having already been

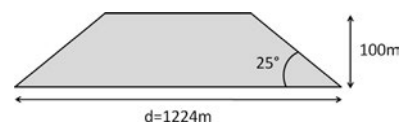


Fig. 15. Equivalent dump area.

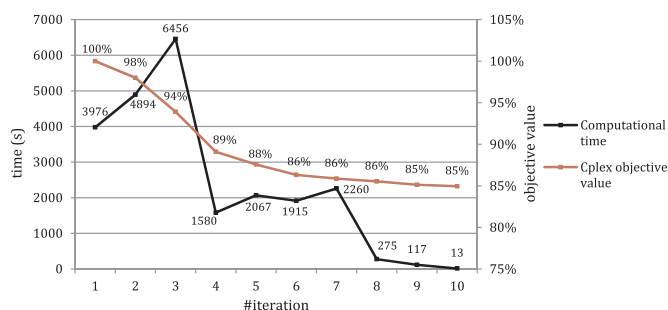


Fig. 16. Iteration computational time and objective value vs it#1 of the sliding time window heuristic approach used in the case study.

extracted. The results of the case study are shown to be very satisfactory, enforcing tailings and waste storage inside the pit. Comparisons show that providing new storage within the pit of the iron ore deposit will save considerable costs in re-handling during the rehabilitation phase of the project, reduce the impact on the local environment and provide a solution to a limited external space for material storage, all at a comparable cost to not directly integrating waste management into the optimization. While the proposed model is suitable for low-dip sedimentary type deposits, a natural extension of this study could aim to adapt the model to more complex shape deposits. Integrating the proposed approach to the simultaneous stochastic optimization of mineral value chains (Montiel et al., 2016; Goodfellow and Dimitrakopoulos,

Appendix. – Some computational details

Fig. 16 illustrates the characteristics of the sliding time window solving method. On the first ordinate axis, the required computational time for each iteration of the method is shown. In terms of processing time, the method required 6 h 32 min using CPLEX v12.4 and a 2.8 GHz i7-2600S processor and 8 GB of RAM. The second ordinate axis shows the decrease of the objective value per iteration (as compared to the first iteration); the first period was defined as binary. The assessment of a decreasing objective value is not surprising, since the problem is more constrained at each iteration (binary constraints) and the periods are ungrouped (especially for the first iterations).

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2017) is a natural extension of the work presented herein.

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