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An Enhanced Algorithm for Complex Permittivity Extraction at Microwave Frequencies

Nawfal Jebbor[1,*], Rachid Chaynane[2], Seddik Bri[3], A. M. Sànchez[4]

[1]Electronics, Instrumentation ans Intelligent Systems team,
ER2TI Laboratory, Department of Physics, Faculty of Sciences and Technics,
Moulay Ismail University of Meknes, Errachidia 52000, Morocco
jebbor.nawfal@gmail.com

[2]Faculty of Applied Sciences, Ibno Zohr University, Ait Melloul 80000, Agadir, Morocco
r.chaynane@uiz.ac.ma

[3]MIN, Electrical Engineering Department, ESTM
Moulay Ismail University of Meknes, Meknes 50000, Morocco
briseddik@gmail.com

[4]Department of Communication Engineering
University of Cantabria Avd. Los Castros, Plaza de la Ciencia, s/n 39005 Santander, Spain
angel.mediavilla@unican.es

***Abstract*— An efficient technique of complex permittivity extraction is employed to characterize low-loss conventional dielectric materials at microwave Ku-band. The computational approach eliminates mathematically the systematic errors of the experimental setup. This method needs two uncalibrated S-parameter measurements. The first is performed with a sample under test and the second is done with an empty rectangular waveguide. Three low-loss dielectric materials (Celotex, Plexiglas and Teflon) are characterized to validate experimentally the extraction method over the Ku-band frequencies [12-18] GHz. The average relative errors between the calibrated and uncalibrated results are then calculated and compared. The proposed method has been improved using the mobile average to the experimental results obtained from the uncalibrated measurements, therefore, the stability is then enhanced.**

***Index Terms-* Low-loss Dielectric Materials, Complex Permittivity, Microwave characterization.**

I. INTRODUCTION

The low-loss dielectric materials find their applications in various fields. Telecommunications and microwave industry applications require precise knowledge of the complex permittivity of the used dielectric materials. However, the rectangular waveguides are widely used for wideband microwave characterization [1-4]. In general, microwave techniques can be categorized into three groups: i) free-space techniques, ii) resonant or cavity perturbation techniques, iii) non-resonant techniques based on a coaxial or rectangular waveguide. The free space methods [5] are employed when the material is available in a large sheet. The free-space techniques are non-destructive and contactless. They are ideally suited for variable incidence of the electromagnetic wave and high-temperature measurements. But unwanted reflections surrounding objects and diffractions from the edges of the sample make the free-space measurements less accurate. The resonant methods [6] are more accurate, but they require elaborate sample preparation and can be applied at narrow frequency band. Microwave non-resonant techniques [7] are widely used over a broadband frequency, even though these techniques are less accurate than the resonant methods. With the help of the Vector Network Analyzer VNA, the reflection and/or transmission coefficients are measured in one- and/or two-port. Using the inverse retrieval techniques, the constituent parameters of the sample under test are extracted. The most commonly used noniterative technique is Nicolson-Ross-Weir (NRW) method [8]. It is well-known that NRW suffers the largely fluctuated values of extracted complex permittivity for low-loss dielectric materials. At frequencies corresponding to a sample length equal to integer multiples of one-half wavelength, the scattering parameter $|S_{11}|$ gets very small and the phase uncertainty is large. The solutions of the NRW technique are proportional to $1/S_{11}$ [9]. Then, the NRW solutions are algebraically unstable as $S_{11} \rightarrow 0$. Therefore, we present here an iterative method based on the S-parameters (Sij) calculation of the rectangular waveguide. This technique is used to overcome the weak points of the NRW method. The Sij parameters are measured in transmission/reflection (T/R) with and without calibration of the VNA. This experimental technique involves placing the dielectric material under test MUT into a WR62 rectangular waveguide and measuring the Sij parameters. A second uncalibrated measurement under the same experimental conditions of an empty waveguide is required to find iteratively the complex permittivity of the MUT. The mathematical approach is rigorous without any approximation, taking into account all reflections of the electromagnetic wave on both sides of the MUT. However, the proposed method is suitable for uncalibrated measurements by the VNA. It has been applied to the determination of the complex relative permittivity of three low-loss dielectric materials at Ku-band for experimental validation. The experimental data are compared with those obtained by calibrating the VNA using the same proposed method.

This technique is a contribution to the existing literature by a practical characterization method of dielectric materials, thereby reducing the drawbacks of other methods. Eliminating the standard calibration manipulations of the Vector Network Analyzer, requiring a single specimen and two measurements without calibration. In addition, the proposed method does not need the exact location of the sample inside the waveguide. The method is iterative, based on the Newton-Raphson root finding algorithm, and converges very quickly. The theoretical study considers all reflections of the electromagnetic wave at the air / sample / air interfaces, unlike other existing methods that consider only the first reflection in their theoretical formalism. The measurements were carried out on three dielectric samples (Celotex, Plexiglas and Teflon). The results of the method were improved by applying mobile average to further reduce the influence of the residual systematic errors of the experimental setup.

The organization of the rest of this paper is as follows. First, we present a theoretical analysis of the measurement cell filled by the sample under test, this theory is based on S-parameters data, and we give an iterative extraction procedure of the complex permittivity in Section II. We next validated our algorithm by simulation and numerical retrieval in Section III. Then, in Section IV, we present the measurement setup, the calibration procedure, and measurements results of characterized low-loss dielectric samples and we discuss the retrieved complex permittivity from calibrated and uncalibrated S-parameters in association with the relative errors. Finally, we recapitulate the main findings of this paper in Section V.

II. EQUATIONS AND RETRIEVAL PROCEDURE

The transmission/reflection (T/R) method is used to extract the complex permittivity of conventional dielectric material [1-3,10-12] based on the S-parameters measurements. The MUT is machined to the same section dimensions of the WR62 waveguide.

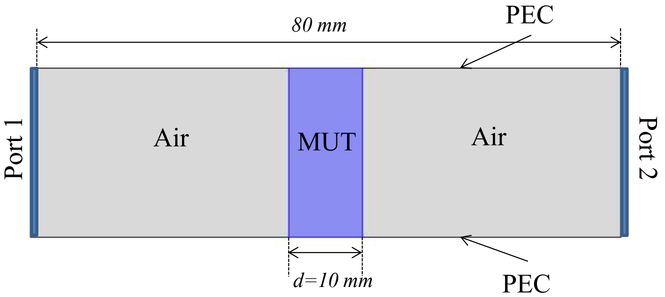

Fig. 1. The rectangular WR62 waveguide is partially filled by the MUT. The top and bottom are perfect electrically conducting (PEC) for simulation case.

The MUT is supposed non-magnetic (μ_r=1) and only the dominant mode TE_{10} scatters into the structure. Figure 1 shows the simulation and the measurement cell partly charged by the MUT. We determine the transmission matrices M_i according to:

$$M_i = \frac{1}{S_{21_i}}\begin{pmatrix} S_{12_i}S_{21_i} - S_{11_i}S_{22_i} & S_{11_i} \\ -S_{22_i} & 1 \end{pmatrix} \quad (1)$$

$$i = 1 \; or \; 2$$

M_1: corresponds to a measurement of the empty waveguide.

M_2: corresponds to a measurement that the waveguide is filled by the MUT sample.

These two transmission matrices can also be formulated as a product of five matrices:

$$\begin{cases} M_1 = x.T_{ref1}.T_1.T_{ref1}^{-1}.y \\ M_2 = x.T_{ref2}.T_2.T_{ref2}^{-1}.y \end{cases} \quad (2)$$

The impedance jumps Air/MUT/Air cause reflections of the incident wave whose transmission matrix is T_{refi}.

$$T_{ref\,i} = \begin{pmatrix} \frac{1}{1-\Gamma_i} & \frac{\Gamma_i}{1-\Gamma_i} \\ \frac{\Gamma_i}{1-\Gamma_i} & \frac{1}{1-\Gamma_i} \end{pmatrix} \quad (3)$$

$$\Gamma_i = \frac{\gamma_0-\gamma_i}{\gamma_0+\gamma_i} \quad (\mu_r^* = 1) \quad (4)$$

$$T_i = \begin{pmatrix} e^{-\gamma_i d} & 0 \\ 0 & e^{\gamma_i d} \end{pmatrix} \quad \text{i= 1 or 2} \quad (5)$$

The errors matrices x and y are pretended maintained during the experiments. They express the regular and/or the systematic errors of the experimental setup like source and load match errors, effects of interconnection wires, hardware imperfections, etc. [1,10-13]. T_i is the transmission matrix of an ideal line with length d and γ the propagation constant.

$$\gamma_0 = j\frac{2\pi}{\lambda_0}\sqrt{1-\left(\frac{\lambda_0}{\lambda_c}\right)^2} \quad (6)$$

$$\gamma_i = j\frac{2\pi}{\lambda_0}\sqrt{\varepsilon_{ri}^*\mu_r^* - \left(\frac{\lambda_0}{\lambda_c}\right)^2} \quad (7)$$

γ_0 and γ_i : The propagation constants in vacuum and the dielectric ε_{ri}^* respectively.

λ_c and λ_0: wavelength cut-off of the waveguide and wavelength in free space respectively.

ε^*_{r1} and ε^*_{r2} are the complex permittivity of the air and MUT sample respectively. To exclude the impact of the two error-ports, a simple procedure based on the product and the inverse matrix is used. Matrix product of M_1 and M_2^{-1} is formulated by equation (8).

$$M_1.{M_2}^{-1} = x.T_{ref1}.T_1.T_{ref1}^{-1}.T_{ref2}.T_2^{-1}.T_{ref2}^{-1}.x^{-1} \quad (8)$$

It is evident that the matrix y is omitted. The equation (8) shows that the matrices $M_1.{M_2}^{-1}$ and $T_{ref1}.T_1.T_{ref1}^{-1}.T_{ref2}.T_2^{-1}.T_{ref2}^{-1}$ are similar, this designates that they have the identical trace [10-13], represented by the sum of the diagonal elements of the square matrix $M_1\,M_2^{-1}$.

$$Tr(M_1.{M_2}^{-1}) = Tr(T_{ref1}.T_1.T_{ref1}^{-1}.T_{ref2}.T_2^{-1}.T_{ref2}^{-1}) \quad (9)$$

Tr(•) is the trace of the square matrix.

Let's

$$f(\varepsilon^*_{r\,2}) = Tr\left(T_{ref1}.T_1.T^{-1}_{ref1}.T_{ref2}.T_2^{-1}.T^{-1}_{ref2}\right) \quad (10)$$

f is a function of $\varepsilon^*_{r\,1}$, λ_0, λ_c and d where $\varepsilon^*_{r\,2}$ is the unique unknown. Multiple complex values of $\varepsilon^*_{r\,2}$ can satisfy the function f. If a good initial guess of $\varepsilon^*_{r\,2}$ is available, solving function f iteratively points directly to the true value of $\varepsilon^*_{r\,2}$.

In this section, we have proposed a meticulous mathematical approach based on the wave cascading matrix without any approximation. This approach considers all reflections of the electromagnetic wave on both sides of the sample through the waveguide. The method is more flexible, and it can be applied to the calibrated or uncalibrated S-parameter measurements. The technique eliminates mathematically the out-port errors of the measurement cell. Two measurements in T/R are enough to evaluate the complex permittivity of the dielectric sample; the first is that the sample holder filled with the reference dielectric (ε^*_{r1}=1-*j*0.00; Air), and the second with the MUT. A precise location of the sample in the waveguide is not needed. The nonlinear function f is solved by using a two-dimensional root-finding algorithm.

III. NUMERICAL VALIDATION

The Finite Element Method (FEM) is well established as a versatile numerical tool for solving eigenvalues and scattering parameters of a great variety of electromagnetic structures. Several structures can be reduced to 2D electromagnetic analysis. This yields a further reduction in the computational requirements. In this paper, we simulate the cells presented in figure 1 by COMSOL MultiPhysics software based on the FEM. COMSOL is employed to solve the field problem of the studied structure for 2D full-wave analysis. The dielectric characterization technique is applied to the calculated S-parameters in order to validate numerically the proposed technique at Ku-band frequencies. The MUT of length d=10 *mm* is filled into the waveguide. The S-parameters (figure 2) are calculated at ports 1 and 2 and the microwave source is implemented only in port 1. The parametric simulation is performed at [12-18] GHz band with 201 frequency points.

With the help of the proposed technique, we track down the real and the imaginary parts of the relative complex permittivity of Teflon implemented as a material property of MUT in the software ε^*_{r1}=2.05-*j*0.01 (figure 3).

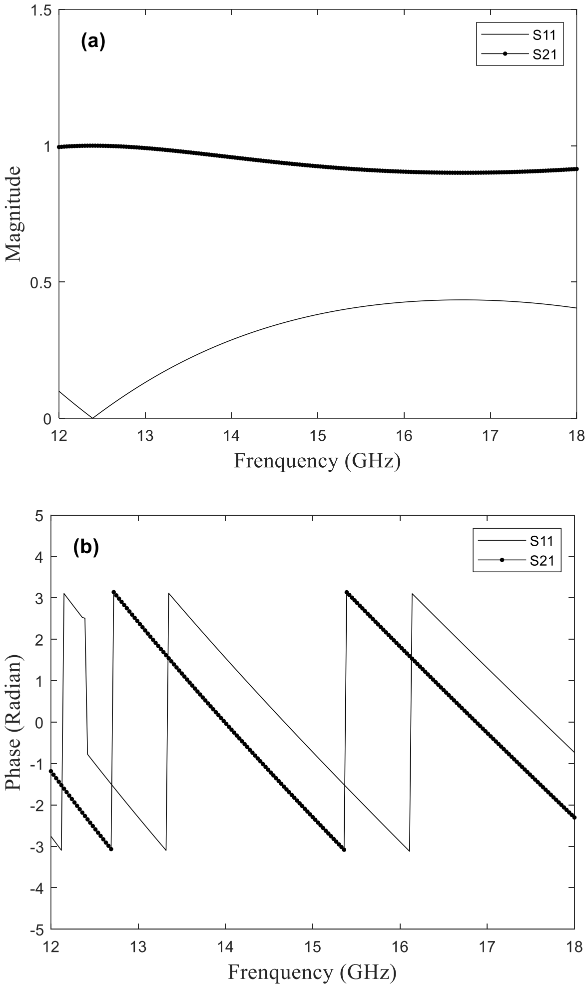

Fig. 2. Magnitude (a) and phase (b) of the calculated S-parameters for the Teflon sample.

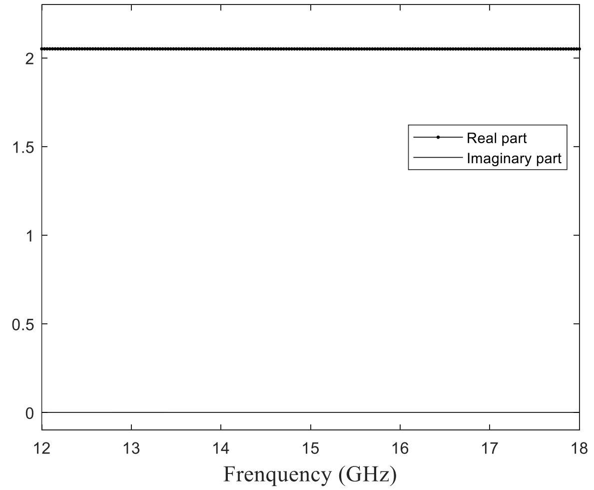

Fig. 3. Numerical retrieval of the relative complex permittivity of Teflon sample.

IV. EXPERIMENTAL SETUP AND RESULTS

We consider the measurement setup shown in figure 4. The MUT with thickness *d*=10 *mm* is located into WR62 rectangular waveguide holder of section (15.8×7.9) mm^2. The E8634A VNA is connected to two coaxial-to-waveguide adapters. We suppose that only the dominant mode TE_{10} propagates in the structure. The dielectric samples are machined to the same waveguide sections.

The uncalibrated S-parameters of empty waveguide and MUT are measured. Then, the computer program can determine the complex permittivity of the MUT. In the subsequent section, the average values of uncalibrated and calibrated results were compared between them to approve the suggested method. The Thru-Reflect-Line TRL calibration technique [14,15] is utilized for calibrating the experimental setup. All measurements are performed at [12-18] GHz band with 201 frequency points. We implement the 4-points mobile average algorithm on the whole frequency domain values of obtained permittivity data in the case of uncalibrated measurements. The proposed method was used to extract the complex permittivity of three low-loss dielectric materials. Given its stability, the relative complex permittivity of air ($\varepsilon^*_{r1}=1-j0.00$) is taken as a reference dielectric and the S-parameters of the empty waveguide are measured only once.

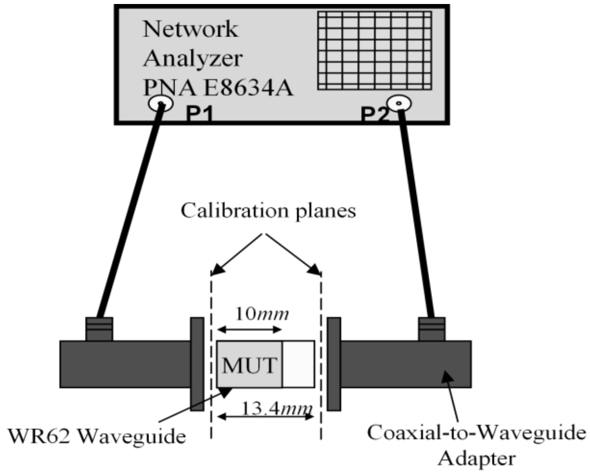

Fig. 4. The experimental setup.

A single specimen of each material is required to derive its dielectric constant. In this paper, we propose to determine the complex permittivity of Celotex, Plexiglas and Teflon sample materials at Ku-band. To validate our proposed method for complex permittivity extraction of conventional low-loss dielectric materials, we first performed S-parameters measurements of our three samples with and without calibration of the VNA. Figures 5.a. and 5.b. show the amplitude and the phase information (S_{11} and S_{21}) of the measured S-parameters for the Celotex sample (S-parameters of the other samples are not shown here for contraction). All the uncalibrated measurements ($S_{11\_uncalib}$ and $S_{21\_uncalib}$) are unstable because they include the systematic errors of the experimental setup. Therefore, there are two ways to cancel or to eliminate these errors: mathematically and/or experimentally. Our performed method incorporates a mathematic error elimination and then a direct extraction of the complex permittivity. Besides, the $S_{11\ calib}$ and $S_{21\ calib}$ data (figures 5.a. and 5.b.) can be, eventually, incorporated in the program for more stable results.

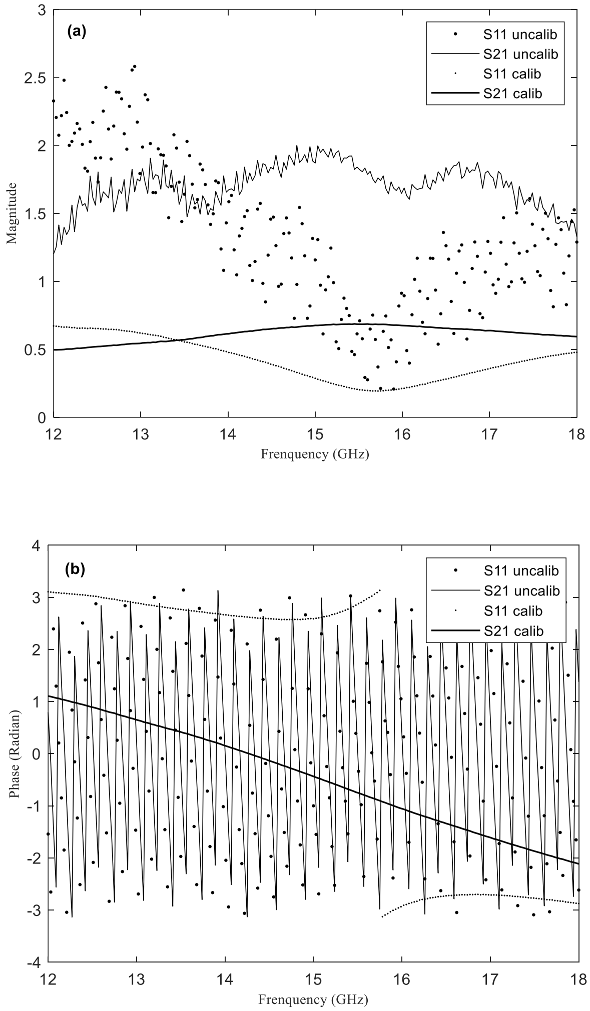

Fig. 5. Magnitude (a) and phase (b) of the measured S-parameters for the Celotex sample.

Table 1 presents the average values of the relative complex permittivity of the MUT $<\varepsilon^*_{calib}>$ and $<\varepsilon^*_{uncalib}>$, and the average relative error percentage on the real and imaginary parts defined as:

$$<\%\text{Error}\,\varepsilon'>=\left|\frac{<\varepsilon'_{uncalib}>-<\varepsilon'_{calib}>}{<\varepsilon'_{calib}>}\right|\times 100$$
$$<\%\text{Error}\,\varepsilon''>=\left|\frac{<\varepsilon''_{uncalib}>-<\varepsilon''_{calib}>}{<\varepsilon''_{calib}>}\right|\times 100 \qquad (10)$$

$\varepsilon^*_{calib}=\varepsilon'_{calib}-j\varepsilon''_{calib}$ and $\varepsilon^*_{uncalib}=\varepsilon'_{uncalib}-j\varepsilon''_{uncalib}$ are the complex relative permittivity of the MUT determined from calibrated and uncalibrated S-parameter measurements respectively. The average values are evaluated using the standard average such as the number of measurements points is equal to 201.

$$<\varepsilon^*_{calib,\,uncalib}>=\frac{\sum_{i=1}^{201}\varepsilon^*_{calib,\,uncalib}}{201} \qquad (11)$$

Table 1: The average complex permittivities and the average relative errors on the real and imaginary parts over the Ku-band frequencies.

MATERIALS	**$<\varepsilon^*_{uncalib}>$**	**$<\varepsilon^*_{calib}>$**	**<% Error ε'>**	**<% Error ε">**
Celotex	4.0112-*j*0.3485	4.0071-*j*0.3454	0.102	0.89
Teflon	2.0544-*j*0.0152	2.0563-*j*0.0165	0.009	7.87
Plexiglas	2.6648-*j*0.0249	2.6648-*j*0.0248	0.000	0.40

Calculations based on measured information have indicated that errors on the real parts are very small (close to zero), but errors on the imaginary parts can be large within 1% except for Teflon (7.87%) for the used materials. The relative errors are caused by the length uncertainty and the experimental conditions are not effectively the same. The Celotex material has significantly high dielectric losses compared to the other studied materials; it appears that the method is well adapted for medium and low-loss materials. The figures 6 and 7 represent the variation of the real and imaginary parts of the complex permittivity over the Ku-band. It is seen that the permittivity values are very stable over the frequency range and near to the average values cited in table 1; except the Celotex that the real part decreases from 4.06 to 3.95. An overall view in both figures, the imaginary parts of all samples are not stable like the real parts, which explain the high average error values in the imaginary parts. Like any other S-parameter based methods, this is a common phenomenon. We note that the proposed method eliminates all operations of assembly/disassembly necessary to calibrate the VNA.

We observed that our proposed method is robust in producing accurate complex permittivity even in the presence of experimental noise effect and the systematic errors cited above in Section II.

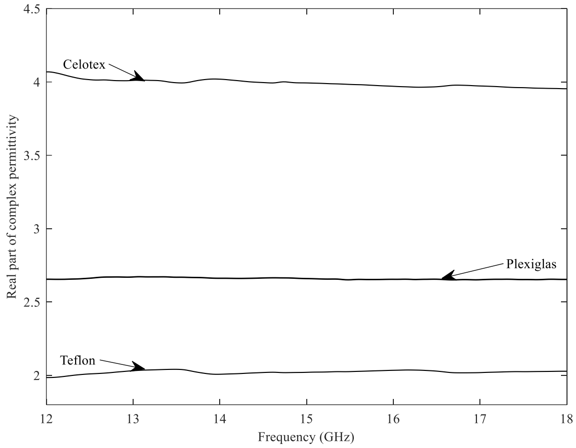

Fig. 6. The measured real part of the relative complex permittivity over the Ku-band using uncalibrated S-parameter measurements.

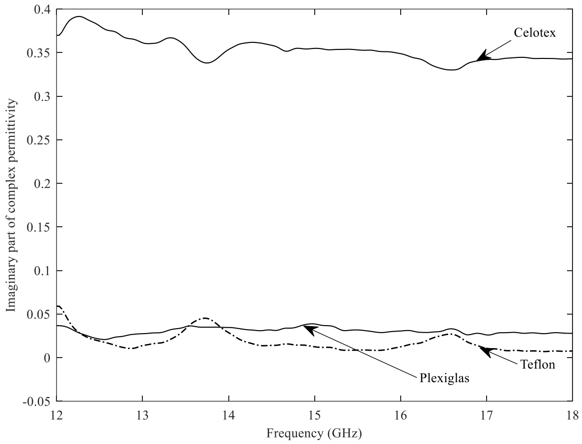

Fig. 7. The measured imaginary part of the relative complex permittivity over the Ku-band using uncalibrated S-parameter measurements.

Contrariwise, the numerical simulations do not consider any noise effect or errors when calculating the Sij-parameters. But, for the case of Teflon sample, for example, the two results are in good agreement (same remark for the other materials studied in this paper).

V. CONCLUSION

An improved method to extract the complex permittivity of low-loss solid dielectric materials is proposed. The method is iterative based on the measurements of S-parameters by the Vector Network Analyzer. The method can eliminate systematic errors by using one specimen of each material. The first measurement is done with the material under test and the second with an empty waveguide. The samples are pre-machined and characterized in similar experimental conditions. The program procedure based on the iterative resolution of nonlinear function is included in the principal program to find iteratively the complex permittivity value of the low-loss dielectric material. The proposed technique is improved by applying a mobile average of four frequency points on the final uncalibrated results. The experimental part presents the application of the proposed method to three dielectric samples (Celotex, Plexiglas and Teflon). A rectangular waveguide in Ku band is used for a wide frequency characterization. The results show the validity of the proposed method for characterizing solid dielectric materials. The relative errors on the real parts are very small, but these of the imaginary parts can be large within 1% except Teflon (7.87%) for the used low-loss dielectric materials. Like any other S-parameters based technique, this is a common phenomenon. Considering the simplicity, accuracy and efficiency over a broad frequency band of the proposed approach, this transmission/reflection method proves that is useful for complex permittivity measurement of low-loss dielectric materials with or without calibration of the Vector Network Analyzer.

REFERENCES

[1] J. Baker-Jarvis, E. J. Vanzura and W. A. Kissick, “Improved technique for determining complex permittivity with the transmission/reflection method”, *IEEE Trans. Microw. Theory Tech.*, Vol. MTT-13, no. 8, pp. 1096-1103, August 1990.

[2] K. J. Bois, L. F. Handjojo and A. D. Benally, “Dielectric plug-loaded two-port transmission line measurement technique for dielectric property characterization of granular and liquid materials”, *IEEE Trans. Instr. Meas.*, Vol. 48, no. 6, pp. 1141-1148, December 1999.

[3] U. C. Hasar, O. Simsek, M. K. Zaterroglu and A. E. Ekinci, “A microwave method for unique and non-ambiguous permittivity determination of liquid materials from measured uncalibrated scattering parameters”, *Progress In Electromagnetics Research, PIER*, Vol. 95, pp. 73-85, 2009.

[4] M. D. Deshpande, C. J. Reddy, P. I. Tiemsin and R. Cravey, "A new approach to estimate complex permittivity of dielectric materials at microwave frequencies using waveguide measurements", *IEEE Trans. Microw. Theory Tech.,* Vol. MTT-45, no. 3, pp. 359-366, April 1997.

[5] F. J. F. Gonçalves, A. G. M. Pinto, R. C. Mesquita, E. J. Silva and A. Brancaccio, "Free-Space Materials Characterization by Reflection and Transmission Measurements using Frequency-by-Frequency and Multi-Frequency Algorithms", *Electronics*, Vol. 7, 260, 18 October 2018.

[6] M. F. Eldessouki, S. W. Kim and S. Li Wei, "Measurement of Complex Permittivity of Materials Using a Double-Ridge Waveguide Resonator", *Key Engineering Materials, Proceedings of International Conference on Material Science and Engineering 2016*, Vol. 723, pp. 154-159, December 2016.

[7] C. Yang, K. Ma, J.-G. Ma, "A Noniterative and Efficient Technique to Extract Complex Permittivity of Low-Loss Dielectric Materials at Terahertz Frequencies", *IEEE Antennas and Wireless Propagation Letters*, Vol. 18, no. 10, pp. 1971-1975, October 2019

[8] W. B. Weir, "Automatic Measurement of Complex Dielectric Constant and Permeability at Microwave Frequencies", *Proc. IEEE*, Vol. 62, no. 1, pp. 33-36, January 1974.

[9] O. Luukkonen, S. I. Maslovski, and S. A. Tretyakov, "A Stepwise Nicolson-Ross-Weir Based Material Parameter Extraction Method", *IEEE Antennas and Wireless Propagation Letters*, Vol. 10, 1295-1298, November 2011.

[10] N. Jebbor, S. Bri et al., "Complex permittivity determination with the transmission / reflection method", *International Journal of Emerging Sciences*, Vol. 1(4), pp.682-695, December 2011.

[11] N. Jebbor, S. Bri et al., "Experimental Complex Permittivity Determination of Low-loss Dielectric Materials at Microwave Frequency Band", *International Journal of Advanced Scientific and Technical Research*, Vol. 2(5), pp.1-10, October 2012.

[12] U. C. Hasar, "A new calibration-independent method for complex permittivity extraction of solid dielectric materials", *IEEE Microwave and Wireless Components Letters,* Vol. 18(12), pp.788-790, December 2008.

[13] U. C. Hasar and O. Simsek, "On the application of microwave calibration-independent measurements for noninvasive thickness evaluation of medium- or low-loss solid materials", *Progress In Electromagnetics Research, PIER,* Vol. 91, pp. 377-392, 2009.

[14] G. F. Engen, and C. A. Hoer, "Thru-Reflect-Line: an improved technique for calibrating the dual six port automatic network analyzer", *IEEE Microw. Theory Tech.*, Vol.27 (2), pp.987-993, December 1979.

[15] "TRL/LRM Calibration", *Agilent Technologies,* No. 16000-95026, January 2004.