View metadata, citation and similar papers at core.ac.uk
brought to you by CORE
provided by Brunel University Research Archive

DEVELOPMENT OF A TIME-DEPENDENT STRUCTURAL RELIABILITY MODEL FOR CIVIL ENGINEERING STRUCTURES

J. M. NICHOLS [(1)], A. K. TOMOR [(2)] and A. BENEDETTI [(3)]

[(1)] *Associate Professor, Department of Construction Science, Texas A&M University, College Station, Brazos County, 77843 and USA, jm-nichols@tamu.edu*

[(2)] *Senior Lecturer, Geography and Environmental Management, University of the West of England (Bristol) Coldharbour Lane, Bristol, UK, Adrienn.Tomor@uwe.ac*

[(3)] *Professor, DICAM, University of Bologna Bologna, Italy, Andrea.Benedetti@unibo.it*

ABSTRACT

The purpose of this paper is to outline the start of the development of a time-dependent Dynamic Structural Reliability Model for civil engineering structures.

The issue of the existing extent of information for the different elements of the model is addressed together with the research required to assure a uniform reliability level for structures. A uniform reliability level provides an ethically sound method of assessment of the relative safety of the structure.

Structural reliability analysis is a long-term research problem that has been aided by the development of sensitive accelerometers, capable of recording thermal vibration of structures. Changes in the natural vibration frequencies over time can be used to identify changes in the structural reliability over time based on real-time behaviour.

KEYWORDS: structural, reliability, dynamic, accelerometer, time dependent

NOTATIONS

β	Reliability index
p_f	Probability of failure
$P()$	Probability function
R	Resistance to load and failure
S	Applied stress regime
H	Horizontal load
V	Vertical load
M_i	Moment capacity at the point i on a frame
L	Length
$G(X)$	Resistance
μ	Mean
σ	Standard deviation
$G(X,t)$	Time dependent resistance
a,s	Constants in equation (6)
AC	Axial stiffness coefficient, for symmetric prismatic beams = 1
S	Bending stiffness coefficient, for a symmetric prismatic beam = 4
CS	Carryover stiffness factor, for a symmetric prismatic beam = 2
E	Young's Modulus
A	Area
I	Second moment of area
L	Length
e	*strain*
ϕ'	*relative change in quantity, i.e. M*
$\phi' M$	Dummy variable for equation 10 to hold the calculation of the impact of end moments induced by variable axial loads in a structural member for stability functions
ω	Dummy variable equal to $0.5\mu L$

1. INTRODUCTION

1.1 Background

Observational data from structures damaged in extreme loads, such as earthquakes, point to the existence of a difference in the probability of failure of different structures in different loading events. Any reasonable ethical view of this observation points to the need for a minimum acceptable level of structure reliability. The accepted definition for an acceptable level of structural reliability is the probability of failure for a given loading event or events. The alternative statement is the return period for failure, such as once in 100 years.

The probability of failure is usually expressed in terms, such as 0.1%, but an alternative number system is often used as a surrogate to describe the probability level, which is the β value. The use of the reliability index β in the Complementary Standard Normal Equation yields the probability of failure. A common usage has been adopted of writing the β as surrogate for the probability of failure, such that a value of β of 5.00 represents a probability of failure of 0.2859E-06. A one to one relationship exists between the reliability index β and the probability of failure p_f, so the use of β as a measure for failure is mathematically acceptable. The range of β is typically 0 to 10. A structure is designed with a target probability of failure or corresponding β value.

The target probability is usually provided by default in modern codes of practice. These codes of practice were derived from a limited number of theoretical and experimental sources. One of the definitive sources is the development of the steel portal frame model in the 1930s [1,2].

1.2 Structural Reliability Methods

Extensive research derived in large part from the early work on the steel portal frame led to the development of the field of structural reliability analysis. A series of definitive classic text books outline the methods used for Structural Reliability Analysis [3-9]. The paper uses the standard definitions provided in these standard textbooks.

Structural reliability is a limit-based approach that has the objective of ensuring that all failure modes of the structure are considered. Melchers [3-6] and others [7-9] clearly outline the methods used for structural reliability analysis and for space reasons only the basic relevant outline is summarized in this section. The standard equation used for Structural Reliability is equation (1):

$$p_f = P(R - S < 0) \tag{1}$$

Although in reality a structure with $R = S$ cannot be considered safe and so equation (1) is expressed as a strict inequality, rather than the $\leq$. There are four components to this equation. The first component is the estimate of the structural capacity of the buildings, R. The second component is the set of applied loads that can be applied to the structure over the lifetime of the structure, S. The third component is the difference between the capacity and the applied loads, which is used to estimate the safety of the building or the probability of failure p_f. The fourth component is the relationship shown in equation (1) that ties the independent elements together as the only real relationship between R and S is the equation.

The standard structural reliability problem assumes that the problem is time invariant or stationary in mathematical terms. Any reading of the extensive literature on building damage shows this to be in error, the standard structural reliability problem is time dependent or non-stationary in mathematical terms. Figure 1 shows a definition sketch for the non-stationary problem with the proability distribution for the capacity and load effects over time and their inevitable overlap.

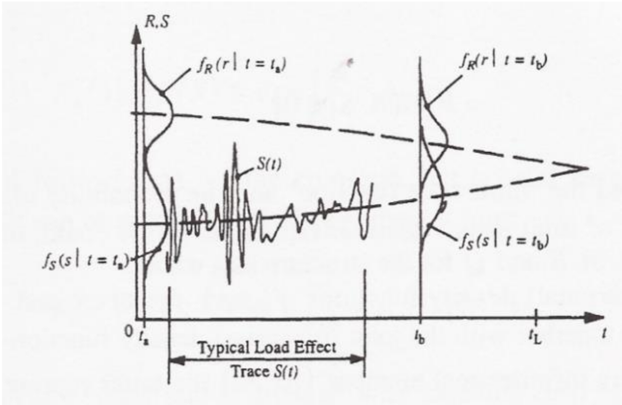

Figure 1. Schematic time-dependent reliability problem (from Melchers [3, 4])

The clear point of Figure 1 is that at some stage the building fails in some fashion. In earthquakes, this failure can be catastrophic for the entire community as shown in Figure 2. This example is taken from central Italy. Here, the clear and present danger to the population for building collapse and death is earthquakes.

Figure 2. 1915 Avezzano earthquake – damage in Via Napoli [10]

A study of the fatality rates in earthquakes in the Abruzzo Region in the 1915 Avezzano earthquake showed that 100% fatality rates occurred in some villages in this region [11], demonstrating the issues relevant to structural reliability methods.

One of the challenges in the last fifty years, since the formal definition of structural reliability measures, is the development of the time dependent structural reliability model. Buildings are subjected to multiple loads over their lifetime, each load doing some damage to the building, but these loads do not necessarily lead to immediate collapse as suggested in Figure 1. However, progressive damage followed by a large event may lead to unexpected catastrophic failure. There are of course many different loads that can damage structures and kill humans, but earthquakes represent a critical issue and provide a critical outset for developing Structural Reliability theory.

The paper addresses the four key components of a proposed time dependent (non-stationary) structural reliability model. The ability to develop non-stationary models relies on the collection of a different type of (high quality) data through monitoring and analysis. Some of this data is analysed near real time and some requires longer periods of data analysis followed by extensive statistical analysis using frequentist [12], Bayesian [13] and Monte Carlo analysis [14].

1.3 Methodology for the proposed method

The paper outlines the development of a time-dependent structural reliability model that is based on measured acceleration data through monitoring. An outline of the four stages of the development of the model is presented below.

1. Time-dependent structural model: Development of a time-dependent non-linear structural model that provides a better representation of the measured data than linear elastic models.
2. Probability distribution of applied loads: Real loads are not constant. Their variability can be characterised through probability distribution (e.g. Gaussian or non-Gaussian). Thermal loading is Gaussian and provides a well-defined basis for long-term monitoring and modelling.
3. Measured frequency data: High quality measurement data for the response of the structure to the applied loads.
4. Bayesian statistics and Monte Carlo analysis: Bayesian statistics is used to determine the existing load probability functions. Monte Carlo analysis is used to determine the probability of failure for a structure based on measured data. The Monte Carlo method requires a suitable structural model, in this case a non-linear model that allows changes in the properties of the structural elements to be modelled as the loads vary.

2. TIME-DEPENDENT STRUCTURAL MODEL

2.1 Introduction

Structural analysis is required for the construction of any modern structure, unlike for historical construction, for example for the buildings shown in Figure 2. The analysis can be a simple set of code rules, such as for light timber framing [15] or detailed finite element analysis for major structures.

Modern structural analysis using codes of practice assume that the main variables are constant with zero variance. The codes of practice build in the variance to the models in the factors used in the code equations [16]. The reason for this method is simplicity of analysis for the practicing engineer, who wants to complete a reasonably accurate design in a minimum time for economic reasons. The development of high sensitivity accelerometers however offers a new possibility to monitor time dependent behaviour of structures in real time. Dynamic properties change with temperature and with changes in the structural condition.

There are therefore two modelling scenarios for the design and monitoring of structures. For simple structures, like houses without dynamic monitoring traditional, essentially static analysis is suitable. For complex structures that are monitored episodically or continuously, time-dependent reliability analysis (based on experimental data) would provide a much more accurate model of the changes in condition than static analysis.

2.2 Standard Model

The proposed structural reliability model is illustrated through a standard model of a steel portal frame that has been used since the 1930s, as shown in Figure 3. V represents the vertical load and H the horizontal load, each with given mean and standard deviation. The height L and the length 2L are standard features of this problem. Points B, C, and D represent the points of plastic failure or sway. The structure is based on the well-studied standard problem, developed by Baker in the 1930s [1] and used by Melchers in this classic text [4]. Melchers and Beck suggested in 2018 [5] that the resistance is time invariant. This may be applicable for a simplified model but is clearly conservative considering the system shown in Figure 1.

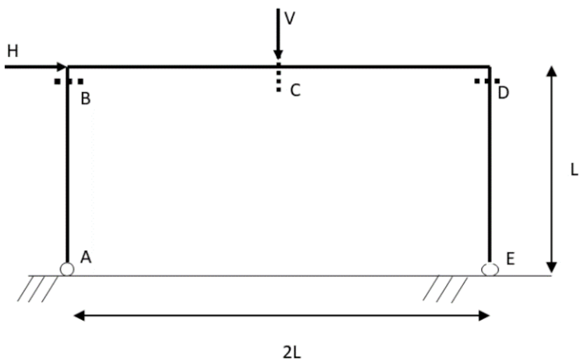

Figure 3. Standard portal frame problem for structural reliability

The first step in the structural capacity analysis is to determine the modes of failure. Figure 4 shows the representation of the failure modes using the simple 1930's nodal graph for the named failure modes (beam, sway and combined) and binary heap representation that have been used for hand calculations for structural analysis programming from the late 1960s. The clear element of Figure 4 is that there is a limited number of failure modes and each can be quantified.

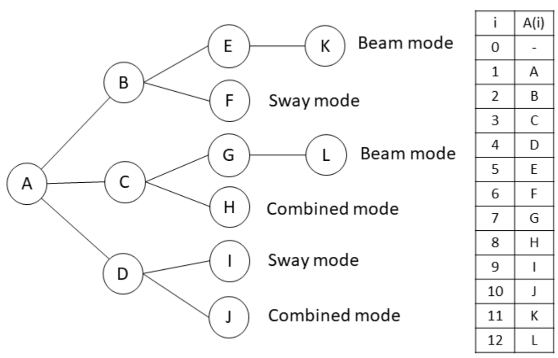

Figure 4. Tree graph and binary heap representations of the failure modes for the portal frame

2.3 Sample Standard Solution

There are a range of analysis programs available to model the structure shown in Figure 3. The limit state function for the modes can be expressed using simple plastic equations as shown in Equation (2) for the beam in the portal frame:

$$M_B + 2M_C + M_D - VL = 0 \quad (2)$$

The traditional assumption for this standard portal frame is that all moments have a capacity of 1, the applied vertical load, V is 1 and the length, L, is 1. The standard deviations are 0.15 for the moments and 0.5 for the vertical load. From that, the resistance of the beam is shown in Equation (3).

$$G(X) = M_B + 2M_C + M_D - V \quad (3)$$

Therefore, the resistance of the beam is $G(X) = 3$. The mean of the resistance is $\mu_G = 3$ and standard deviation σ_G^2 is calculated using in Equation (4).

$$\sigma_G^2 = (0.15)^2 + (2*0.15)^2 + (0.15)^2 + (0.5)^2 \quad (4)$$

From that, the probability distribution β_G can be found as shown in Equation (5) as derived by Melchers.

$$\beta_G = \frac{\mu_G}{\sigma} \quad (5)$$

Hence, β_G is 4.83. Melchers has already calculated the probability of failure p_f for a range of β_G values in Appendix D of [4], that give a probability of $0.7x10^{-6}$ for the current example. This is an estimate for one of the possible failure modes for the simple portal frame model. The static model has an acceptable probability of failure when it is new and at time 0 the structure is considered to fall within acceptable statistical limits. This observation does not mean that the structure cannot collapse, but it will probably require a larger load than the estimated applied load to collapse, as often occurs with earthquake loads.

2.4 Time-dependent model

Condition of structures varies with time. One of the ways to identify the condition of structures is through dyamic characterisation. There has been tremendous development in accelerometer technology during the last two decades to gather high-quality measurement data for the dynamic behaviour of structures. Three grades of accelerometers are currently available. Grade 1, the highest resolution with about 20 - 50 µg resolution, grade 2 with resolution in milli-g terms, commonly used in the automotive industry and grade 3 used in cell phone modems. The sensitivity of grade 1 accelerometers allows not just live loads, but also thermal background vibration to be measured accurately, that forms the basis for the proposed method. The data for in the paper was collected using a SENSR CX1 Grade 1 accelerometer, measuring 2000 times per second in the X, Y and Z directions.

Monitoring can be episodic, a few minutes at a time or continuous. Figure 5 shows an example of acceleration recording over time for a large structure. The majority of the vibration is caused by thermal loading (shown inside the red box) and some additional vibration (outside the red box) by live loading. The relevant frequencies for the recording (caused overwhelmingly by constant thermal loading) can be identified through Fast Fourier transform (FFT) analysis [16, 17]. With available data measured through well defined thermal loading, structural analysis can now be extended to align with the full range of measured natural frequencies of the structure.

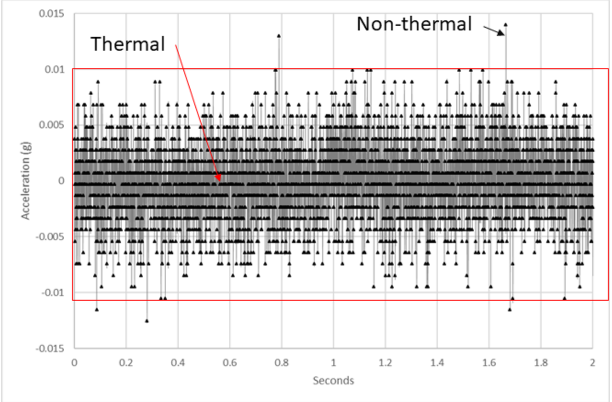

Figure 5. Thermal vibration recording

The extension of the static to dynamic analysis requires the change in the dominant frequencies over time to be incorporated into the model. To demonstrate the process through the portal frame in Figure 3, the resistance of the beam given in Equation (3) can be modified to include the estimate of natural frequencies as shown in Equation (6).

$$G(X,t) = M_B + 2M_C + M_D - aV\sin(st) \quad (6)$$

2.5 Computer Analysis Development

The simple analysis presented above can be completed using a plastic analysis computer program. These programs were developed in the 1960s and to a large extent are unchanged in mathematics, for example ULARC [18]. In terms of model development, there are clear advantages to these early programs. The code is usually short, accurate and coded in Fortran. It is relatively easy to turn these programs into Monte Carlo analysis programs used as the basis for Structural Reliability Analysis. ULARC does not provide natural frequency estimates. Natural frequency estimates can be provided by a range of commercially available programs. Extensive analysis completed in the last decade however points to the need for dynamic analysis using the complete 3D stability functions rather than elastic or plastic analysis, as derived by Ekhande et. al., in 1989 [19]. A full implementation of these equations has been developed using the Harrison base analysis programs [20], which is a more complete solution than the Petersen equations used in some commercial packages [21]. One of the major challenges for coding Structural Reliability Analysis is real time processing of large amount of data that may involve 3D stability function matrices.

3. PROBABILITY DISTRUBUTION OF APPLIED LOADS

The dead load of a structures is essentially fixed. Live loads vary depending on the use of the structure, but are generally relatively constant with time, unless the structure is repurposed. Live loads include traffic, earthquake, wind, flood and other loads. Out of these, earthquake loads are likely to be the most damaging for structures and provide a simple demonstration of the proposed model.

3.1 Load Statistics

Thermal loads are constant at constant temperature. They have a mean value, standard deviation and a probability distribution function. An example of a probability distribution of a thermal load in shown in Figure 6. Thermal loads are of Gaussian (normal) distribution, shape of a bell-curve, but the traffic loads are non-Gaussian. The different types of distributions will be important factors in the next stages of the analysis.

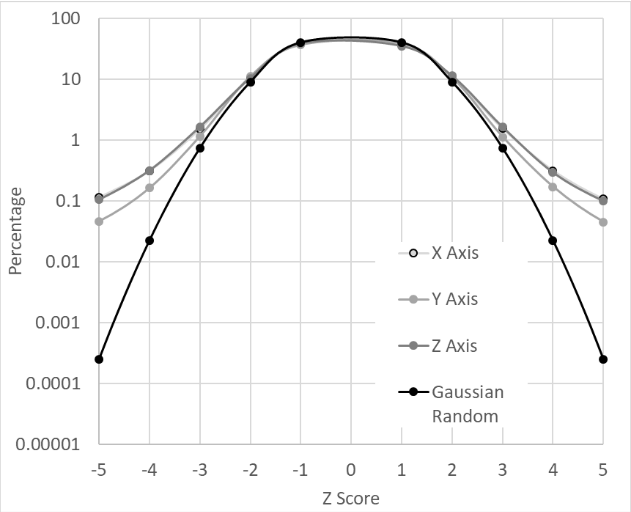

Figure 6. X, Y and Z Direction Acceleration Distribution from Nichols and Tomor [22]

3.2 Underestimation of Earthquake Loads

Earthquake loads form a critical load for large areas of human habitation. The real statistical issue with earthquake loads is the relatively short period of data collection. There are two types of earthquake records: earthquake time trace and observational data. The earthquake time trace goes back to about 1890, although the major development of this system occurred after the start of the Cold War. Observational data is published by NOAA [23] as the world earthquake fatality catalogue and goes back to about 200 BCE. Statistical analysis in the last twenty years shows that the time between fatal earthquakes has been dropping since the 1840's and it continues to drop. The average period between fatal events is less than 20 days now. This statistical analysis shows that the probability distribution for earthquake deaths is Poissonian [24] and as the world population increases the potential maximum probable fatality count in an earthquake increases.

Estimating earthquake loads is extremely difficult, but it is critical for determining the applied stress regime, S curve on Figure 1. An underestimation of the load will statistically reduce the likely lifetime of the structure.

Newmark and Hall [25] used the tripart chart to demonstrate the frequency spectra of earthquakes analysed using Fast Fourier transform methods in an easy to understand format as shown in Figure 7 (see [26] for full details of these events). The figure indicates proposed limiting values for earthquake design, recommended by Newmark and Hall in 1976, the 2010 New Madrid Seismic Zone Maximum Credible estimate (NMSZ-MCE) [27] and by the 1958 Richter estimate [28]. The design event increases in amplitude for all frequencies with time. The interesting observation is that large events often have elements that lie above all three estimates from 1956 onwards, for example the 1989 Nahanni earthquake [29] that occurred in a low seismic risk classified region. In essence, estimating the peak earthquake design loads for any region is problematic because of the short record period. Earthquake loads therefore represent a significant risk in inaccurate statistics and ultra-high variability in their probability distribution.

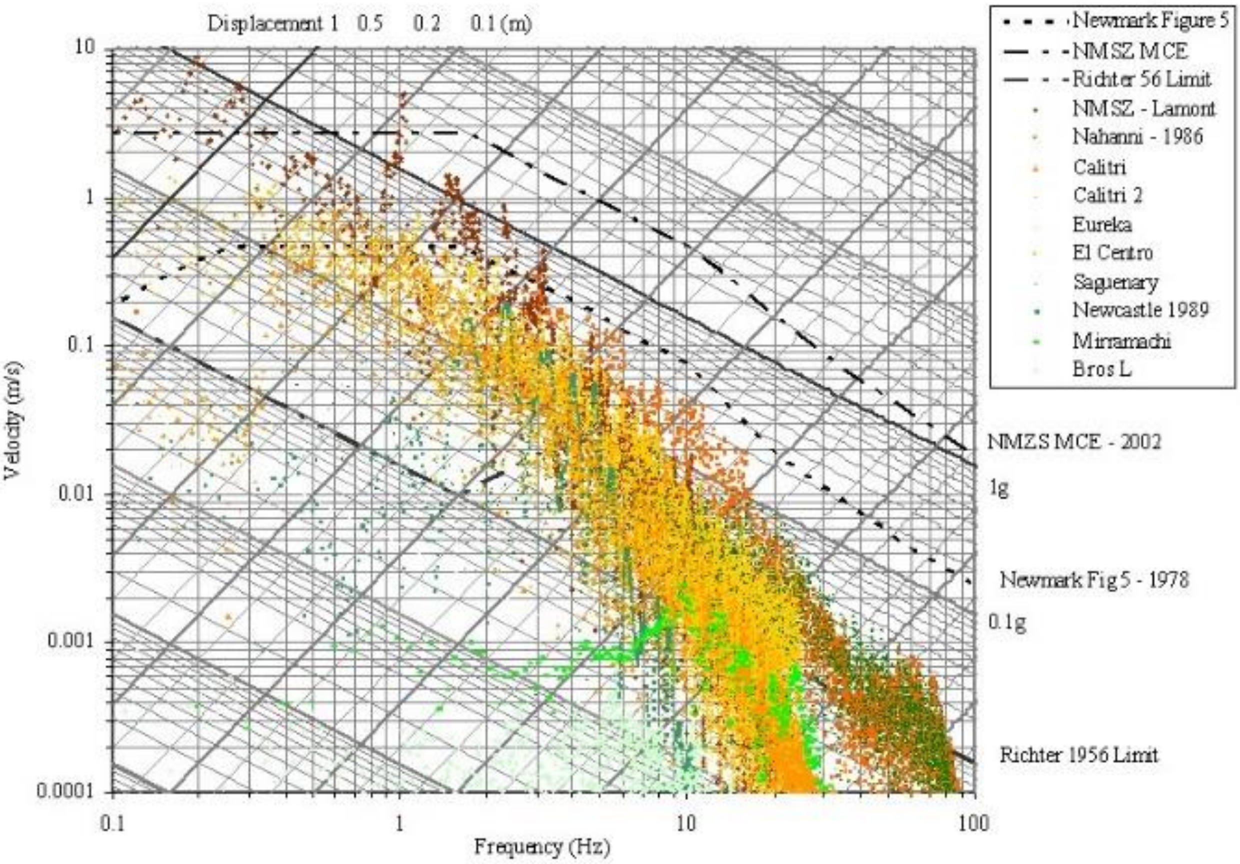

Figure 7. Tripart chart presentation of range of earthquakes

4. MEASURED FREQUENCY DATA

The next step is to identify the natural frequencies of the structure through high quality monitoring. To demonstrate the process, recordings with a Grade 1 SENSR CX1 accelerometer are used with frequencies up to 1000 Hz and 0.122 Hz frequency steps. The monitored data are stored in a MySQL database and analysis programs were developed for real-time processing of the data. During continuous monitoring approximately 10,500 data points are collected each day on the vertical, longitudinal and transverse axes. The time driven data is converted to frequency-driven data through FFT analysis and peak frequencies are plotted every 8 seconds as shown in Figure 8 (approximately one hour of data, 450 plot points, vertical direction). The graph indicates four distinctive frequencies at about 8.5 Hz, 10 Hz, 11.75 Hz and 17.5 Hz. Similar plots are recorded for the horizontal and transverse directions. Frequencies can have one, two or three axes involvement. For the data set shown in Figure 8, the first natural frequency is only observed in the vertical direction, the second, third, and fourth are observed in the vertical as well as in the longitudinal directions. The first frequency observed in the transverse direction (not shown) is about 80 Hz and the peak observed frequency in all directions is about 320 Hz.

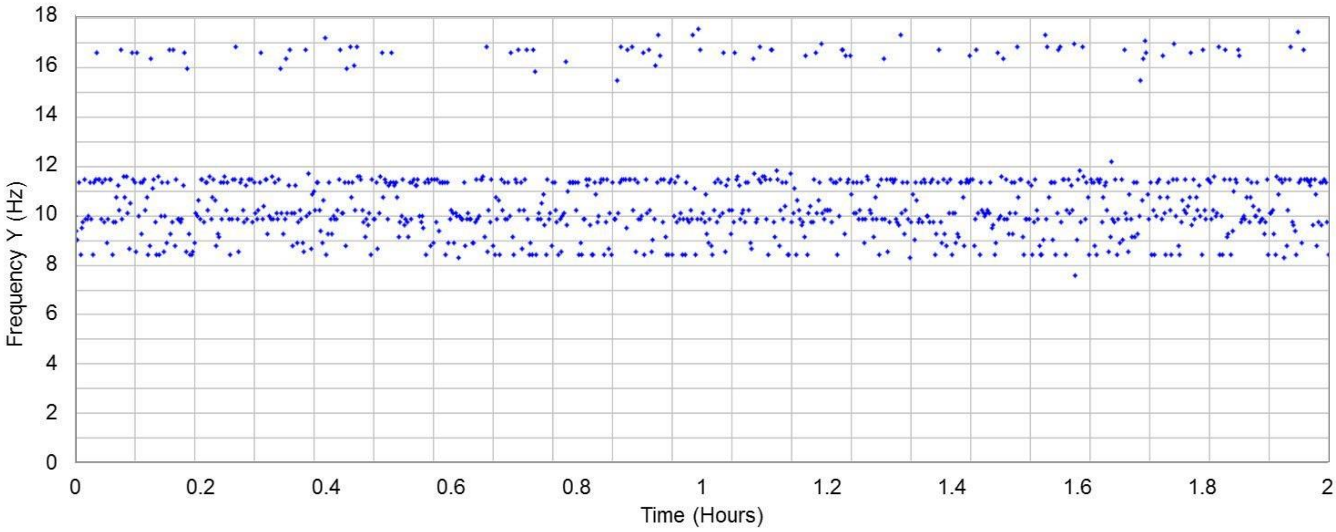

Figure 8. Example of peak frequencies every 8 seconds (ca. 1 hour recording)

The first measured frequency can be easily matched to a linear elastic structural model for specific Young's Modulus and Poisson's' Ratio values. Matching the second and higher natural frequencies is however much more problematic. A linear elastic model generally fails to match the higher frequencies, because minor changes in major variables, such as the Young's Modulus can have significant effects on the higher frequencies. If the frequencies identified by the structural model cannot be matched using linear elasticity, plasticity or partial stability functions [21], the development of a new structural analysis model may be necessary.

For matching higher frequencies for a simple elastic beam with buckling (or any other deformation) as shown in Figure 9 [19], the first step is the development of a beam equation that includes bending capacity and shear deformation capacity coefficients. The model in Figure 9 includes a Py moment [20], that was proposed by Berry (for aircraft spars) [30] and for which the 3D solution was formulated for by Ekhande et al., [19]. The proposed model for the non-linear elastic beam and is given in Equation (7).

$$\begin{bmatrix} P \\ M_{AB} \\ M_{BA} \end{bmatrix} = \begin{vmatrix} AC\frac{EA}{L} & 0 & 0 \\ 0 & S\frac{EI}{L} & CS\frac{EI}{L} \\ 0 & SC\frac{EI}{L} & S\frac{EI}{L} \end{vmatrix} \cdot \begin{bmatrix} e \\ \phi_{AB} \\ \phi_{BA} \end{bmatrix} \qquad (7)$$

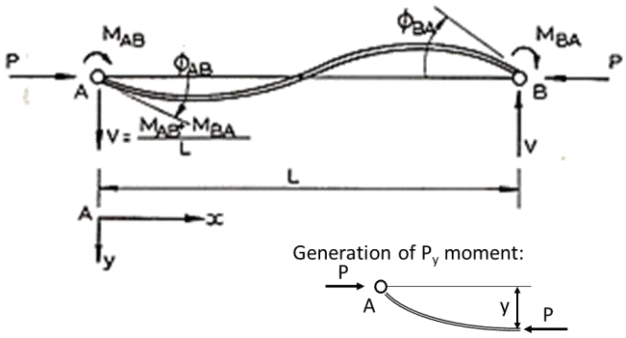

Figure 9. Non-linear elastic beam definition from Harrison [20]

Where S is the bending stiffness coefficient, CS the carryover stiffness factor and AC the axial stiffness coefficient. Using standard theory, the terms AC,S,SC are constant for elastic prismatic beams. For non-linear stiffness theory they are however not constant and can be calculated using Equations (8)-(10). Equation (10) is a complex equation and can be a simplification using an alternative variable termed $\phi' M$, defined in Equation (11). The simplifying terms are the mean defined as $\mu^2 = \frac{P}{EI}$ and ω defined as $\omega = \frac{\mu L}{2}$.

$$S = \frac{\omega(\omega - \coth\omega + \omega\coth^2\omega}{\omega\coth\omega - 1} \tag{8}$$

$$CS = \frac{\omega(\omega + \coth\omega - \omega\coth^2\omega)}{\omega\coth\omega - 1} \tag{9}$$

$$AC = \frac{1}{1 - \dfrac{EA}{4P^3L^2}}(\phi' M) \tag{10}$$

$$\phi' M = \mu L(M_{AB}^2 + M_{BA}^2)(\coth(\mu L) + \mu L\cos ech^2(\mu L)) - 2(M_{AB} + M_{BA})^2 + 2\mu L\cos ech(\mu L) M_{AB} M_{BA}(1 + \mu L\coth(\mu L)) \tag{11}$$

5. BAYESIAN STATISTICS AND MONTE CARLO ANALYSIS

5.1 Bayesian statistics

The next step is to identify the probability distribution of the monitored frequency data for the thermal vibration and other loads. Bayesian statistical analysis provides a relatively straightforward method for estimating the parameters of a probability distribution [31]. Figure 10 shows the probability distribution of the monitored thermal vibration data shown in Figure 8 for temperature against frequency, processed by Bayesian analysis.

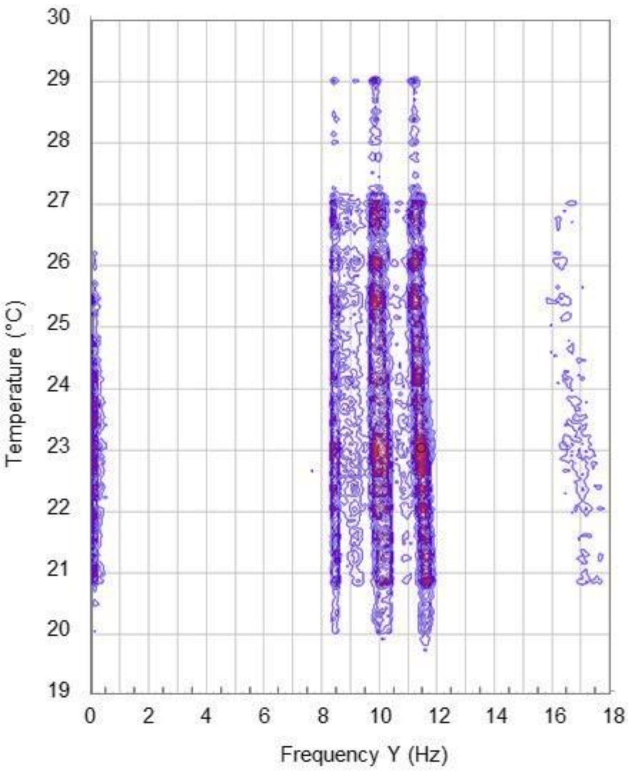

Figure 10. Fast Fourier transform results expressed as a Bayesian result for temperature against frequency

Some of the interesting observations in Figure 10 are:

1. Higher frequencies are increasingly temperature dependent
2. Some of the frequencies are not recorded at higher temperatures
3. Very low frequencies only register below 27ºC, as the structure cools.

Fitting of measured data to structural models is quite a challenge and demonstrates some of the problems still to be overcome, including introducing temperature dependence into the structural model.

5.2 Monte Carlo Analysis

The final step is to determine the probability distribution of loads and failure mechanisms over time, based on measured data and improved structural models and the highest probability of failure for the overall structure.

Using suitable structural models, the Monte Carlo analysis can be used to identify changes in the properties of the structural elements and their probability of failure for the range of applied loads through. the main steps are as follows:

1. Determine the set of variables
2. Determine the probability distributions, means and standard deviations for each variable
3. Develop a set of random numbers for each variable, based on the probability distributions. Identify the probable range of applied loads and resistances
4. Use structural analysis programs to determine the relative safety for each set of random numbers
1. Determine the highest probability of failure for the loads and resistances.

For the original example of a steel portal frame shown in Figure 3, a Monte Carlo analysis has been completed using the proposed method and ULARC program. The estimated probability of failure was found to be within 0.5% of the static estimate based on 1 million iterations.

Further development of the model presents a number of challenges and is likely to utilise structural analysis packages with full stability functions for 3D analysis, modern solvers and eigenvalue solvers.

6. CONCLUSIONS

Buildings are subjected to a combination of loads over their lifetime (including environmental, thermal and earthquake loads), leading to gradual deterioration over time. The statistical characteristics of loads can vary significantly. For example, traffic or earthquake loads are generally non-repeatable with non-Gaussian statistical properties, while thermal loads are repeatable with Gaussian (bell-curve) statistical properties.

Simple structural reliability analysis using constant loads and standard structural models have the danger that certain loads, such as earthquake loads in some configurations can provide high probabilities of failure and little room for error for the estimation of mean and standard deviations for the loads, leading to unexpected structural failure.

A time-dependent Structural Reliability model is proposed for civil engineering structures, based on non-linear structural models, measured thermal loading patterns, probability distribution of applied loads and probability of failure for the structure based on measured data. The development of the program is on the way, but a number of further challenges are yet to be overcome before it can be applied widely.

REFERENCES

1. Baker, J., M.R. Horne, and J. Heyman, *The Steel Skeleton - Plastic Behaviour and Design*. Vol. II. 1965, Cambridge, UK: CUP.
2. Steel Structures Research Committee, *Recommendations for a Code of Practice for the use of Structural Steel in Building (First Report)*. 1931, London, UK: H.M.S.O.
3. Melchers, R.E. *Developing realistic deterioration models*. in *Life-Cycle of Engineering Systems: Emphasis on Sustainable Civil Infrastructure - 5th International Symposium on Life-Cycle Engineering, IALCCE 2016*. 2017.
4. Melchers, R.E., *Structural Reliability Analysis and Prediction*. 1999, Chichester: Wiley.
5. Melchers, R.E. and A.T. Beck, *Structural Reliability Analysis and Prediction*. Vol. Third edition. 2018, Hoboken, NJ: Wiley.
6. Melchers, R.E. and R. Emslie, *Investigations for structural safety assessment of corroded cast iron bridge piers.* Australian Journal of Structural Engineering, 2016. **17**(1): p. 55-66.
7. Ellingwood, B. and A. Tallin, *Structural serviceability: floor vibrations.* Journal of structural engineering, 1984. **110**(2): p. 401-418.
8. Stewart, M., *Structural Reliability*, in *Encyclopedia of Quantitative Risk Analysis and Assessment*. 2008, Wiley: New York, NY.
9. Stewart, M. and S. Lawrence, *Structural Relaibility of Masonry Walls in Flexure.* Masonry International, 2002. **15**(2): p. 48-52.
10. E. Navone & Co, *Photograph of the ruins of Via Napoli, Avezzano (Abruzzo, Italy) after the earthquake of 1915. Italiano: Fotografia delle rovine di via Napoli, Avezzano (Abruzzo, Italia) dopo il terremoto del 1915.* , in *Camera location 42° 01′ 38.89″ N, 13° 25′ 26.47″ E.*, Date 1915 Source Scribner's Magazine (p.423 v.57 - 1915), Editor. 1915, Scribner's Magazine: New York, NY.
11. Nichols, J.M., F. Lopes de Oliveira, and Y.Z. Totoev, *The development of a synthetic fatality function for use in the economic analysis of the rehabilitation and repair of structures*, in *StrucDam Conference 2000*, . 2000, University of San Paulo: Brazil, South America.
12. Johnson, R.A., *Miller and Freund's Probability and Statistics for Engineers*. 2000, Upper Saddle River: Prentice Hall. xii+622.
13. Bretthorst, G.L., *Bayesian Spectrum Analysis and Parameter Estimation*. Lecture Notes in Statistics. 1988, New York, NY: Springer- Verlag.
14. Zapoměl, J., I. Stachiv, and P. Ferfecki, *A novel method combining Monte Carlo–FEM simulations and experiments for simultaneous evaluation of the ultrathin film mass density and Young′s modulus.* Mechanical Systems and Signal Processing, 2016. **66–67**: p. 223-231.
15. Rosowsky, D.V., *Evolution of probabilistic analysis of timber structures from second-moment reliability methods to fragility analysis.* Structural Safety, 2013. **41**: p. 57-63.
16. Brigham, E.O., *The Fast Fourier transform and its applications*. Brigham 1987 ed. 1988, Englewood Cliffs: Prentice.
17. Tomor, A.K. and J.M. Nichols, *Monitoring changes in natural frequency and deterioration for masonry arch bridges with Fast Fourier Transform*, in *Proceedings of the 12th North American Masonry Conference*. 2015: Denver, CO. p. Paper 0082.
18. Sudhakar, A., G.H. Powell, G. Orr, and R. Wheaton, *Computer Program: ULARC: Sample Elasto-plastic Analysis of Plane Frames*. Sudhakar 1972 ed. 1972, Berkeley: UCB.
19. Ekhande, S.G., M. Selvappalam., and M.K.S. Madugula, *Stability Functions for Three Dimensional Beam-Colums.* Journal of Structural Engineering, 1989. **115**(2): p. 467 - 479.
20. Harrison, H.B., *Computer Methods in Structural Analysis*. 1973, Englewood Cliffs, NJ: Prentice.
21. Petersen, C., *Statik und Stabilität der Baukonstruktionen: elasto- und plasto-statische Berechnungsverfahren druckbeanspruchter Tragwerke, Nachweisformen gegen Knicken, Kippen, Beulen*. 1980, Heidelberg: Vieweg.
22. Nichols, J.M. and A.K. Tomor, *Transverse acceleration causing damage in masonry arch bridges*, in *39th IABSE*. 2017: Vancouver, BC.
23. NOAA. *World Earthquake Fatality Catalogue*. 2010 [Accessed 4 May 2016]; Available from: https://www.ngdc.noaa.gov/metaview/page?xml=NOAA/NESDIS/NGDC/MGG/Hazards/iso/xml/G012153.xml&view=getDataView&header=none
24. Nichols, J.M. and J.E. Beavers, *World Earthquake Fatalities from the Past - Implications for the Present and Future.* Natural Hazards Review, 2008. **9**(4): p. 179-189.
25. Newmark, N.M. and W.J. Hall, *Development of Criteria for Seismic Review of Selected Nuclear Power Plants*. 1978, US Nuclear Regulatory Commission: Urbana. p. 49.
26. Nichols, J.M. *Suggested changes to the non-conservative low frequency criteria for the seismic design spectrum used in intraplate regions*. in *ERES 2005*. 2005. Greece.
27. Johnston, A.C. and E.S. Schweig, *The enigma of the New Madrid earthquakes of 1811-1812.* Annual Review of Earth and Planetary Science, 1996. **24**: p. 339-84.
28. Richter, C.F., *Elementary Seismology*. 1958, San Francisco: Freeman. viii+767.
29. Boore, D.M. and G.M. Atkinson, *Spectral scaling of the 1985 to 1988 Nahanni, Northwest Territories, earthquakes.* Bulletin of the Seismological Society of America, 1989. **79**(6): p. 1736-61.
30. Berry, A., *Calculation of Stresses in Aeroplane Wing Spars.* Transactions of the Royal Aeronautical Society, 1919. **1**: p. 3-33.
31. Farrow, M., *Bayesian Statistics* 2013, Newcastle, UK: Newcastle University.