DYNAMIC PERFORMANCE OF SIMPLY-SUPPORTED RIGID-PLASTIC SQUARE PLATES SUBJECT TO LOCALISED BLAST LOADING

N. Mehreganian,

[a] *Department of Civil & Environmental Engineering, Skempton Building, South Kensington Campus, Imperial College London, London SW7 2AZ, UK*

A. S. Fallah [*],

[b] *Department of Mechanical, Aerospace and Civil Engineering, Brunel University London, Uxbridge UB8 3PH, UK*

L.A. Louca

[a] *Department of Civil & Environmental Engineering, Skempton Building, South Kensington Campus, Imperial College London, London SW7 2AZ, UK*

ABSTRACT

This paper presents the theoretical solution to the response of a square plate undergoing plastic deformation due to a generic localised blast pulse. A representative localised blast load function was considered with spatial distribution of constant pressure over a central zone and exponentially decaying outside that zone. Considering an appropriate moment function and ignoring the membrane, transverse shear and rotary inertia effects, the static plastic collapse was initially found, whereby the analysis was extended to dynamic case by assuming a kinematically admissible, time dependent velocity profile. The analytical model, which was validated against the numerical results obtained through ABAQUS hydrocode, showed close correlation in terms of the permanent transverse deflection profile. In order to consider the effect of temporal pulse shape, the results were formulated for the rectangular as well as exponentially and linearly decaying pulses. For blast loads of high magnitude, the pressure load was replaced by an impulsive velocity. The calculations were simplified utilising the dimensionless form and the results were corroborated with the theoretical and experimental results from the literature. The model thus showed improvements in predicting the final deformation of square plates over the previous models of simplified loading function.

Keywords: Limit analysis, localised blast, impulsive loading, plastic hinge, plastic collapse pressure

---

[*] To whom correspondence should be addressed:

Email: as3@imperial.ac.uk (Arash S. Fallah)

Tel.: +44 (0) 2075945140

Fax: +44 (0) 2075941974

Notations

The following symbols are used in this paper:

Latin Upper Case

$A_i - G_i$	Loading parameters; [various]
$\dot{D}$	Internal Energy dissipation rate; $[ML^2T^{-3}]$
$\dot{E}$	External Work rate; $[ML^2T^{-3}]$
H	Plate characteristic thickness; $[L]$
I	Impulse; $[MLT^{-1}]$
L	Plate characteristic side length; $[L]$
M_0	Maximum moment of the plate per unit length; $[MLT^{-2}]$
T	End of motion time; $[T]$
V_0	Impulsive velocity; $[LT^{-1}]$
$\ddot{W}$	Maximum transverse acceleration; $[LT^{-2}]$

Latin Lower Case

f(z), g(z)	Moment parameter function; $[MLT^{-2}]$
a	Exponent (loading parameter); [1]
b	Exponent (loading parameter); $[L^{-1}]$
p_1	Dynamic plastic collapse pressure; $[ML^{-1}T^{-2}]$
p_c	Static plastic collapse pressure; $[ML^{-1}T^{-2}]$
r_e	Loading constant central zone radius; $[L]$
$\dot{w}$	Transverse velocity; $[LT^{-1}]$
$\ddot{w}$	Transverse acceleration; $[LT^{-2}]$

Greek Lower case

α	Lower bound Static collapse coefficient; [1]
β	Upper bound Static collapse coefficient; [1]
ξ	Plastic hinge Generalised coordinate; [1]
ε_1	Impulse parameter; $[L]$
$\dot{k}_x$	Curvature rate in x direction; $[T^{-1}]$
$\dot{k}_y$	Curvature rate in y direction; $[T^{-1}]$
$\dot{k}_{xy}$	Warping curvature rate; $[T^{-1}]$
λ	Dimensionless kinetic energy; [1]
η	Dynamic overloading factor; [1]
μ	Areal density (ρH); $[ML^{-2}]$
τ	Duration of the pulse; $[T]$
ω_0	(r_e/L); [1]
σ_0	Static yield stress; $[MLT^{-2}]$

Introduction

With increasing risk of threats associated with blast loads in recent years, the dynamic performance of protective plated structures subject to short-term, high intensity pulse loads has been a topic of interest in the realm of civil, aeronautical, mechanical, defence and military engineering. This has become primarily significant for design purposes due to many major incidents which have occurred in the UK, in which explosions caused severe damage to structures (Chen et al., 2015). Attempts to mitigate the structural damage due to blast loads have, consequently, drawn the attention of engineers to plated metallic structures with the intention of protecting equipment, systems and people. One of the common forms into which plated structures cast is the quadrangular plates.

Most structural elements are made of steel or other alloys with high post-yield load carrying capacity, as well as enhanced energy-absorption. Since these elements can undergo large plastic deformations due to extreme dynamic loading, it is essential to investigate the blast response of such elements using theoretical methods suitable for inclusion of such effects. In theoretical calculations, the structure is often idealised to behave either as an elastic-perfectly plastic or a rigid-perfectly plastic medium. The latter model- in which the elastic effects are ignored for simplicity (and without great loss of accuracy)- is appropriate for the assessment of dynamic response of blast-loaded structures, provided the loading is treated as a short duration pulse and the ratio of kinetic energy to elastic strain energy is considerably high (Zheng et al., 2016).

In the previous analytical investigations (Florence, 1966; Jones, 1997; Jones et al., 1970; Li and Huang, 1989; Nwankwo, Soleiman Fallah, Langdon, et al., 2013; Wierzbicki T and Florence AL, 1970), however, certain limitations have been implemented, viz., loading has been characterised as a uniformly distributed pressure with rectangular temporal pulse shape, which renders the results suitable for response to global blast loads. Nevertheless, blast loads due to the proximal charges of short stand-off distance will lead to localised responses with much higher local deformation and strains as well as the possibility of triggering potential tearing mechanisms, a fact which necessitates the investigation of alternative loading functions. In the sequel, there is a summary on different blast loads using associated characteristics for blast loading functions. The equations proposed by researchers (Fallah et al., 2014; Jones, 1997; Karagiozova et al., 2010; Youngdahl, 1971) have been used to lift restrictions on temporal and spatial distributions of pulse loads as well as providing a realistic yet accurate approximation of localised blast load which proves feasible for such types of loading.

Localised Blast loads

The blast loading is a high-pressure load arisen either from deflagration, (e.g. propagation of flame in a gas explosion), or chemical detonation within a high explosive charge (e.g., Trinitrotoluene (TNT) explosion) (Schultz et al., 1999). Due to the rather instantaneous chemical reaction within a charge, the latter is characterised with much higher pressure and flame propagation velocity than the former. Furthermore, the duration of detonation is generally much lower than that of deflagration and the rise time to maximum pressure for detonation is virtually zero in contradistinction to the finite rise time for deflagration processes. Besides, depending on the proximity of the blast source, the loading can be classified as global (e.g. far-field explosions) or localised (e.g. buried land mines). While researchers have proposed a few definitions for the sake of classification of loading depending on certain attributes (Jacob et al., 2007; Micallef et al., 2015), the concern has been regarding the structural response rather than the definitions.

References (Gharababaei and Darvizeh, 2010) and (Jacob et al., 2007) used the ratio of stand-off distance to plate radius to discern the localised load from global blast, while an empirical relationship by Hopkinson and Cranz (DOD., 2008; Neuberger et al., 2007) describing a scaling parameter as ratio of stand-off-distance to the cubic root of TNT equivalent charge weight can be used as an index to gauge the structural response type. References (Chung Kim Yuen et al., 2012; Neuberger et al., 2007, 2009) incorporated the Hopkinson Cranz scaling law to define a scaling model that correlates the response of circular plates to uniform blasts in various load case scenarios. However, Micallef et al (Micallef et al., 2015) found the stand-off to charge diameter ratio a more theoretically sound scaling factor to gauge the loading nature.

Static and Dynamic performance of the plate

The first study on static collapse of plates was conducted by Hopkins and Prager (Hopkins and Prager, 1953) who implemented the limit analysis theorems to determine the load carrying capacity of simply supported rigid-perfectly plastic circular plates. The loading was subsequently considered with rectangular pulse shapes, the dynamic effect of which was investigated by researchers (Xue and Hutchinson, 2004; Yuan and Tan, 2013) establishing that the ratio of the blast duration to the total plate response time is pivotal in idealisation of the blast with zero period, i.e. uniform momentum impulse. Nwankwo et al. examined the temporal and spatial distribution of peel and shear stresses in beams' adhesive lap joints which were subject to a transverse, uniformly distributed blast load. (Nwankwo, Soleiman Fallah and Louca, 2013). (Youngdahl, 1971) discussed the strong dependence of the dynamic plastic deformation of plated structures on the pulse shape. An empirical relationship was proposed to eliminate the pulse shape effect which was incorporated in the design and analysis of two-dimensional structural members. He proposed a relationship which linked the maximum plastic deformation to square of effective impulse multiplied by a function of effective pressure. This relationship was widely used for decades prior to being theoretically established by researchers as the upper bound for the actual displacement while the response time was found to be the lower bound on the actual response time (Li and Jones, 2005).

Rajendran and Lee (Rajendran and Lee, 2009) presented a detailed review of the pressure pulse from air and underwater explosions. The study included a description of blast wave detonation as well as shock wave propagation, various forms of pressure loads, and the plate wave interaction. Methods of calculating response to such shocks were proposed by researchers (Balden and Nurick, 2006; Rajendran and Narasimhan, 2006; Wierzbicki and Nurick, 1996). (Cox and Morland, 1959) obtained theoretical solutions for dynamic plastic response of simply supported square plates as well as the response of n-sided polygonal plates subjected to uniform dynamic load. Jones with co-authors presented an extensive series of theoretical and experimental research on various structural elements subject to spatially uniform pressure loads of rectangular temporal pulse shape (Jones, 1968, 1971, 2013; Jones et al., 1970; Jones and Griffii, 1971; Li and Jones, 2000). In most cases, Jones' analytical models for impulsive loading provided solutions in good agreement with experimental works, when the ratio of kinetic energy to maximum strain energy stored elastically was more than ten (Zheng et al., 2016). (Komarov and Nemirovskii, 1986) extended further the analyses of (Jones, 1997) to the dynamic case with travelling plastic hinges to obtain the incipient plastic deformation in each of the two stages of motion.

For blasts with high magnitude impulses, large deformations are expected and for thin shells the membrane forces dominate the structural behaviour. Yuan et al. proposed an analytical model to calculate the large deformation of elastic-perfectly plastic beam systems. They investigated the influence of the catenary (membrane) action in conjunction with its interaction with bending and shear actions in the plastic limits. Three distinct failure deformations mode of beams subject to dynamic (non-impulsive) uniform loaded were investigated (Yuan et al., 2016). The effect of membrane action was similarly studied by (Jones, 1997, 2012, 2014b) for blast loaded plates and for plates struck by single and repeated masses, the latter being a phenomenon called pseudo-shakedown. Chen and Yu (Chen and Yu, 2014) investigated the membrane effects on beams with inclusion of transient state in the velocity profile. Other researchers (Yu and Chen, 1992) extended analysis of Komarov and Nemirovskii (Komarov and Nemirovskii, 1986) to include the effect of catenary action on circular panels. It was shown, theoretically, that the inclusion of transient phase-in which the velocity profile is time dependent- in analysis gave better agreement with experimental results than analysis of (Jones, 1997). Jones later presented a mathematical procedure for the strain rate sensitive ductile plate under impact and explosive loading. His equations to predict the strain rate behaviour of plates were not only dependent on the material constants D and q (from Cowper Symonds equation), but also on the flow stress and density of the plate material (Jones, 2014a).

Zheng et al. discussed the strain energy of a stiffened plate subject to uniform pressure load in terms of bending strain energy, elastic-plastic membrane strain energy in plastic zones and strain energy of the stiffeners. The plate deformation was represented by a cosine shape function to represent the global deformation of central zone and a linear shape function used for simplified rigid-plastic model in the surroundings parts. The generalised displacements were obtained numerically and used to determine the deflection time history (Zheng et al., 2016).

Nurick and co-authors (Cloete et al., 2006; Nurick et al., 2016; Nurick and Balden, 2010; Nurick and Martin, 1989; Nurick and Radford, 1997) conducted a series of experiments to determine the dynamic plastic deformation of rectangular and circular plates and developed a range of dimensionless parameters, which are of practical significance in the study of dynamic response of plates subject to localised and global blast loads. The numerical and experimental studies on localised blast loading effects were further investigated by (Bonorchis and Nurick, 2007, 2009, 2010) and (Jacob et al., 2007) to account for the effects of boundary conditions and stand-off distance.

The previous studies on rectangular plates have focused on temporally rectangular pulse loads of a spatially uniform pressure loading profile. The purpose of the study in this paper is to derive a theoretical model, which determines the collapse load and dynamic plastic deformation of square plates subject to localised blast loads. Similar work was done on circular plates, thick plates and membranes (Micallef et al., 2014) ,(Micallef et al., 2012, 2016).

For this purpose, this paper is presented in seven sections: following the introduction, the initial assumptions are discussed, the fundamental energy equilibrium equations and the characteristics of the localised blast load function are presented. Then, in Section 3, the solutions for the static collapse pressure load of a square plate are obtained. This is followed by an investigation into the dynamic collapse phenomenon and displacement solutions for each phase of motion. Subsequently, the dimensionless kinetic energy-displacement relation is obtained for impulsive loading. In Section 6, the proposed analytical model is validated against the numerical results obtained through a Finite Element model set up in ABAQUS 6.14-4/Explicit® and the experimental studies extracted from the literature in. Finally, Section 7 presents the concluding remarks of the study.

Statement of the problem

The purpose of the current study is to extend, within the framework of limit analysis, the theoretical studies mentioned in Section 1 to cases including localised blast loads through implementation of a modified loading function(Fallah et al., 2014; Karagiozova et al., 2010), (Micallef et al., 2012). While the theoretical studies from the literature have focused on the general solutions for dynamic plastic deformation of plates subject to impulsive loads of uniform distribution, the objective of this work is to derive the general solutions for square plated structures subject to any form of blast loading, i.e. localised or global.

Since the localised blasts affect only a small area of the structure severely, it is expected that boundary conditions are not significant and full plate action may not be required (Micallef et al., 2012). Certain other assumptions and the fundamental equilibrium equations are discussed hereunder.

Assumptions

For the simplicity of the analyses to be conducted, the following assumptions are made throughout the study:

1- In view of the Kirchhoff-Love plate theory, the quadrangular plate studied herein is assumed to be sufficiently 'thin' such that the effects of transverse shear and rotatory inertia can be neglected but not thin to the extent that in-plane actions can have a considerable effect on the plate response. Consequently, it is assumed that bending action is predominant and its effect transcends those of the membrane, transverse shear or rotatory inertia.

2- Furthermore, the blast pressure load is assumed to be exerted on the structural elements laterally, such that the plate material particles follow straight trajectories normal to the un-deformed plate mid-surface. Consequently, it is assumed that in-plane displacement components vanish from equilibrium equations (Wierzbicki and Nurick, 1996). Moreover, the plate maintains its uniform thickness throughout the motion and through-thickness dilatational waves are not considered.

Material properties and Loading

An initially flat, monolithic, ductile metallic square plate with side length of $2L$ and thickness of H, with simply supported boundary conditions all along the periphery, is subjected to a representative axi-symmetric localised blast load (Micallef et al., 2012; Nurick and Balden, 2010). Due to geometrical symmetry, only one quarter of the plate is considered in the analyses.

In most works of the literature (Micallef et al., 2015), (Bonorchis and Nurick, 2009; Karagiozova et al., 2010; Langdon et al., 2005), the localised blast load, which is a function of temporal and spatial variables, is assumed multiplicative, i.e. $\mathrm{p(x,y,t)} = f(x,y)q(t)$. The temporal part of the piecewise continuous loading function is considered as a rectangular pulse shape given by Eqn. (1) and shown in Fig. 1, whereas the spatial form is assumed to be constant over the central region and decay exponentially outside this zone, as given by Eqn. (2) and depicted in Fig. 2. The influence of alternative pulse shapes will be discussed in Appendix A.

$$p_1(t) = \begin{cases} 1 & for\ 0 \le t \le \tau \\ 0 & for\ t \ge \tau \end{cases} \quad \textbf{(1)}$$

$$p_2(r) = \begin{cases} p_0 & 0 \ \le r \le r_e \\ ap_0 e^{-br} & r_e \ \le r \le \mathrm{L} \end{cases} \quad \textbf{(2)}$$

The constant 'a' depends on the loading central diameter, $a = e^{br_e}$, and 'b' can be found through regression analysis on the pressure time histories obtained numerically or experimentally. The Loading constant central zone parameter r_e is referred to as the loading radius hereinafter.

Governing equations

The rate of plastic energy dissipation for a rectangular plate with infinitesimal displacements is found from Eqn. (3).

$$\dot{D} = \iint M_x \dot{k}_x + M_y \dot{k}_y + 2M_{xy} \dot{k}_{xy} dxdy \quad \textbf{(3)}$$

where $\dot{k}_x = -\partial^2 \dot{w}/\partial x^2$, $\dot{k}_y = -\partial^2 \dot{w}/\partial y^2$, $\dot{k}_{xy} = -\partial^2 \dot{w}/$ vectors in x, y and xy directions which are perpendicular to the corresponding portion of the yield surface. In what follows we shall, as is customary, denote differentiation with respect to time by placing a dot above a letter. The rate of external work on a finite plate area A is:

$$\dot{E} = \int (p(x, y, t) - \mu \ddot{w}) \dot{w} dA \quad \textbf{(4)}$$

For a rectangular plate element, which undergoes infinitesimal displacement normal to the plane mid-surface, when subject to lateral loads, the governing equations of motion are given by Eqn.'s (5)-(7):

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = \mu \ddot{w} - p(x, y, t) \quad \textbf{(5)}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0 \quad \textbf{(6)}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} - Q_y = 0 \quad \textbf{(7)}$$

where M_x, and M_y are bending moments per unit length (generalised stresses) in generalised coordinates and M_{xy} is the twisting moment per unit length. In a similar fashion, Q_x and Q_y are the shear forces per unit length in the Cartesian coordinate system, according to the normality requirements of plasticity (Jones, 1997).

Yield Surface

The principle bending moments according to (Timoshenko, S.;Woinosky-Kreiger, 1959) are given by Eqn.'s (8)-(9). Provided the principle moments are arranged in this fashion, it can be assumed that the Johansen yield criterion in two-dimensional moment space governs the plastic flow. The associated yield surface, together with

Tresca criterion, is shown in the Fig. 3 the condition satisfying it is given as Max $\{|M_1|, |M_2|\} \leq M_0$, where M_0 i.e. the maximum plastic moment per unit length is found by Eqn. (10).

$$\mathrm{M}_1 = (M_x + M_y)\,/2 - \frac{1}{2}\left[(M_x - M_y)^2 + 4M_{xy}^2\right]^{\frac{1}{2}} \quad \textbf{(8)}$$

$$\mathrm{M}_2 = (M_x + M_y)\,/2 + \frac{1}{2}\left[(M_x - M_y)^2 + 4M_{xy}^2\right]^{\frac{1}{2}} \quad \textbf{(9)}$$

$$M_0 = \frac{\sigma_0 H^2}{4} \quad \textbf{(10)}$$

Cox & Morland (Cox and Morland, 1959) investigated a particular theoretical solution to dynamic plastic deformation of square plates subject to uniformly distributed rectangular pressure pulse. It was found convenient to introduce an auxiliary dimensionless coordinate z, which is equated to $z = (x_1 + y_1)/\sqrt{2}L$ along the central axis. However, the Cartesian coordinates can be related to the polar coordinate r whereby the blast load is defined (Eqn. (2)), which lies on the equipotential surface with z for the range $0 < z < 1$, as it is assumed that the maximum loading range is on the inscribing circle to the plate, giving $p(x, y, t) = p(z, t)$. Fig. 4 and Eqn. (11) show the relationship between z and the in-plane Cartesian coordinates.

$$r = zL = \sqrt{x^2 + y^2},\ 0 < r < L \quad \textbf{(11)}$$

From the isotropy of stress-moment at the plate centre, $M_x = M_y = M_0$ and $M_{xy} = 0$, while along the diagonal plastic hinge lines-which constructs the collapse mechanism, $M_y = M_0$ when ($y = 0$ and $0 \leq x \leq \sqrt{2}L$) and $M_x = M_0$ when ($x = 0$ and $0 \leq y \leq \sqrt{2}L$). Rearranging the Eqn.'s (5)-(7) and eliminating the shear force reactions leads to:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} = \mu\ddot{w} - p(x, y, t) \quad \textbf{(12)}$$

For brevity in analyses in the sequel, the solution to this Ordinary Differential Equation (ODE) is obtained by defining the generalised stresses (bending moments) and loads in terms of parameter z, as follows.

Static collapse pressure

Lower bound calculations

For the problem discussed earlier, the lower bound calculations can be conducted by noting that the distribution of bending moment must satisfy the static equilibrium ($\mu\ddot{w} = 0$) of the generalised stresses and must nowhere violate the yield criterion. Using the boundary conditions, equation of motion (Eqn. (12)), and considering the principle moments in Eqn.'s (8) and (9), we can assume that the generalised stresses are attributed to a moment distribution function as represented in Eqn.'s (13)-(15):

$$M_x = M_0 + \mathrm{x}^2\mathrm{f(z)} \quad \textbf{(13)}$$

$$M_y = M_0 + \mathrm{y}^2\mathrm{f(z)} \quad \textbf{(14)}$$

$$M_{xy} = \mathrm{xyf(z)} \quad \textbf{(15)}$$

These equations should satisfy the yield condition of Fig. 3 and Eqns. (8)-(9); viz., for any coordinates $0 \leq x \leq \sqrt{2}L$ and $0 \leq y \leq \sqrt{2}L$:

$$\mathrm{M}_1 = M_0 \quad \textbf{(16a)}$$

$$-M_0 \leq \mathrm{M}_2 \leq M_0 \quad \textbf{(16b)}$$

Eqn. (16a) can be obtained by elementary calculations of (8) with the aid of Eqns. (13)-(15). Thus, the above conditions determine the plastic flow in the regime AD of the yield criterion. The principle moment M_2 can be evaluated in (17), when using Eqns. (11), (13)-(15) in (16b):

$$M_2 = M_0 + z^2 f(z) \quad \textbf{(17)}$$

The admissibility of Eqn. (17) will be verified in section 4.1.3. Combining Eqns. (13) - (15) into Eqn. (12) and incorporating Eqn. (11) yields:

$$6f + 6z\frac{\partial f}{\partial z} + z^2\frac{\partial^2 f}{\partial z^2} = -p_0\ , \qquad 0 \leq z \leq r_e/L \quad \textbf{(18)}$$

$$6f + 6z\frac{\partial f}{\partial z} + z^2\frac{\partial^2 f}{\partial z^2} = -ap_0e^{-bLz} \qquad r_e/\mathrm{L} \le z \le 1 \tag{19}$$

which are valid for all coordinates across the plate provided the moments are arranged as per Eqns. (13)-(15). Therefore, on integration, a piecewise general solution to differential Eqn. (18) is:

$$f(z) = \begin{cases} -\frac{p_0}{6} + \frac{A_1}{z^2} + \frac{B_1}{z^3} & 0 \le z \le r_e/L \quad \textbf{(20a)} \\ \frac{-ap_0e^{-bLz}(bLz+2)}{(bLz)^3} + \frac{C_1}{z^2} + \frac{C_2}{z^3} & r_e/\mathrm{L} \le zL \le \mathrm{L} \quad \textbf{(20b)} \end{cases}$$

The boundary conditions satisfying the bending moment distributions are given by $M_x = M_y = M_0$, $M_{xy} = Q_x = Q_y = 0$ at the plate centre, suggesting that the plastic flow in the centre of the plate to be governed by corner A of Johansen yield criteria. The arbitrary constants $A_1 - C_2$ are found by applying these boundary conditions, together with the continuity of moment and shear at $z = r_e/L$, and the boundary as follows.

$$\begin{cases} A_1 = B_1 = 0 & \textbf{(21a)} \\ C1 = \frac{-p_0((br_e)^2 + 2br_e + 2)}{2(Lb)^2} & \textbf{(21b)} \\ C2 = \frac{p_0((br_e)^3 + 3(br_e)^2 + 6br_e + 6)}{3(Lb)^3} & \textbf{(21c)} \end{cases}$$

The bending moment in an arbitrary section defined by normal n to the circle passing through the section is given by transformation formula (Eqn.(22)) according to (Braestrup, 2007; Timoshenko, S.;Woinosky-Kreiger, 1959):

$$M_n = M_x\sin^2\phi + M_y\cos^2\phi + 2M_{xy}\sin\phi\cos\phi \tag{22}$$

It transpires that, at simply supported plate boundary ($z = 1$), $M_n = 0$ and $\phi = 45$. Evaluating Eqns. (20a-b) and (21a-c) in Eqn. (17) yields the lower bound for static collapse pressure as:

$$p_0 = p_c = \frac{\mathrm{M}_0}{\propto L^2} \tag{23}$$

where

$$\propto = \left(\frac{3Lr_e{}^2 - 2r_e{}^3}{6L^3}\right) + \frac{(ae^{-Lb}(Lb+2) + b^2 r_e(L - r_e) + bL - 2br_e - 2)}{L^3 b^3} \quad \textbf{(24)}$$

Upper Bound Calculations

The upper bound to the static collapse load of the square plate can be calculated by employing the principle of virtual velocities. In this manner, the rate of external work to the rate of internal energy dissipation through plastic work, i.e. $\dot{D} = \dot{E}$, Eqns. (3)-(4) require a kinematically admissible velocity field. It is assumed that the velocity profile is of a conical shape, which is the same as the transverse deflection profile throughout the entire static phase.

$$\dot{w} = \dot{W}(1 - z) \quad \textbf{(25)}$$

While it is physically reasonable to assume the conical shape in Fig. 5 for the velocity profile, it is also mathematically evident that for the case of $b \to 0$ (the case of uniform load) and $b \to \infty$ (i.e., the case of point load) the velocity shape function is of the form described in (25) and thus there is no reason as to why an alternative profile exists in the range of $0 < b < \infty$, i.e., the case studied hereunder. Thus, it is reasonable, as a first attempt, to assume the velocity field as this profile. With this assumption, the external work rate will furnish to the expression in (26):

$$\dot{E} = 2L^2 \dot{W}_0 \left(\int_0^{r_e/L} p_u (1 - z) z dz + \int_{r_e/L}^{1} p_u (1 - z) z a e^{-bLz} dz \right) \quad \textbf{(26)}$$

Evaluating the integrals of (3), (26), the upper bound plastic collapse load is given as:

$$p_u = \frac{M_0}{\beta L^2} \quad \textbf{(27)}$$

and

$$\beta = \frac{ae^{-bL}(Lb+2) + b^2 r_e(L - r_e) + bL - 2br_e - 2}{(Lb)^3} + \frac{3Lr_e{}^2 - 2r_e{}^3}{6L^3} \quad \textbf{(28)}$$

It is evident from Eqns. (28) and (24) that, $\beta = \alpha$ thus the upper bound and lower bound are identical, hence $p_c = p_u = M_0/\beta L^2$ is the exact plastic collapse pressure. For the value of $b = 0$, $r_e = L$ the Eqn. (23) simplifies to $p_c = 6M_0/L^2$ which corresponds to collapse load in the case of uniform pressure load. The variation of load parameter β as a function of centre region radius and parameter b is shown in Fig. 6. It is also evident from Fig. 7 that principle moment distribution (ratio of M_2/M_0) is heavily dependent on the loading exponent b.

Dynamic Analyses

The dynamic analyses in this section are conducted by including the inertia term in equilibrium Eqns. (5)-(7). The kinematic relations in dynamic analyses are distinguished by two distinctive cases, as follows:

i. Case i: where $1 \le \eta \le \eta_{crit}$

ii. Case ii: where $\eta \ge \eta_{crit}$, in which η is the dynamic overloading factor defined as $\eta = p_1/p_c$ and η_{crit} is defined in Eqn. (38).

It is pragmatic to introduce the following parameters

$$\ddot{\bar{w}} = M_0/\mu L^2 \;,\; \bar{\tau} = \eta\tau, \quad \bar{m} = M_2/M_0 \qquad \textbf{(29a-c)}$$

Case i- $1 \le \eta \le \eta_{crit}$

During this case, we have assumed that the velocity profile is the same as the static case discussed in section 30. However, the loading involves the temporal part of pulse shape in Eqn. (1). It is convenient to investigate the structural response in two distinctive phases, i.e., $0 \le t \le \tau$ and $\tau \le t \le T$, where τ is the duration of pulse load and T the end of motion time.

First phase of motion $0 \le t \le \tau$

During the first phase of motion, we maintain the temporal part to be of rectangular form (Eqn. (1)), while the spatial part follows Eqn. (2). Thus, the plastic flow of hinge lines is controlled by regime AB of the Johansen yield criterion. The ODE's in this phase are:

$$6g_1 + 6z\frac{\partial g_1}{\partial z} + z^2\frac{\partial^2 g_1}{\partial z^2} = \mu\ddot{w} - p_1 \qquad \mathbf{0 \le z \le r_e/L} \qquad \textbf{(30a)}$$

$$6g_2 + 6z\frac{\partial g_2}{\partial u} + z^2\frac{\partial^2 g_2}{\partial u^2} = \mu\ddot{w} - ap_1e^{-bLz} \qquad \mathbf{r_e/L \le z \le 1} \qquad \textbf{(30b)}$$

which have the same form as before except the static function $f(z)$ from Eqns. (18) and (19) is replaced with dynamic function $g(z)$. The general solutions to differential Eqns. (30a-b) are:

$$g_1(z) = \begin{cases} \frac{\mu\ddot{W} - p_1}{6} - \frac{\mu\ddot{W}z}{12} + \frac{A_1}{z^2} + \frac{B_1}{z^3} & 0 \le z \le r_e/L \quad \textbf{(31a)} \\ \frac{-ap_1e^{-bLz}(bLz+2)}{(bLz)^3} + \frac{\mu\ddot{W}}{6} - \frac{\mu\ddot{W}z}{12} + \frac{D_1}{z^2} + \frac{E_1}{z^3} & r_e/\mathrm{L} \le z \le 1 \quad \textbf{(31b)} \end{cases}$$

By employing the continuity of generalised stresses and shear forces at $z = 0$ and $z = r_e/L$, the integration constants are determined as follows:

$$\begin{cases} A_1 = B_1 = 0 & \textbf{(32a)} \\ D1 = \frac{-p_1((br_e)^2 + 2br_e + 2)}{2(Lb)^2} & \textbf{(32b)} \\ E1 = \frac{p_1((br_e)^3 + 3(br_e)^2 + 6br_e + 6)}{3L^3b^3} & \textbf{(32c)} \end{cases}$$

An expression for $\ddot{W}_1$ is found as per Eqn. (33) by combining Eqns. (31a)-(32c), considering Eqns. (17), (23) and (29a) and invoking the boundary conditions at plate corners i.e. $\overline{m} = 0$:

$$\ddot{W}_1 = 12\ddot{\overline{w}}(\eta - 1) \quad \textbf{(33)}$$

Two time integrations of Eqn. (33) then yields the displacement field as $W_1(t) = (6\ddot{\overline{w}}(\eta - 1)t^2)$, when appreciating zero integration constants due to initial boundary conditions, i.e.$\dot{W}_1(0) = W_1(0) = 0$

Second phase of motion $\tau \le t \le T$

The second phase of motion is characterised by the fact that loading is absent and all motion is due to intrinsic inertia effects, the Eqn. (33) is applicable, although with $\eta = 0$ it becomes:

$$\ddot{W}_2 = -12\ddot{\overline{w}} \quad \textbf{(34)}$$

Again, the subsequent transverse displacement is achieved by two time integrations of Eqn. (34), while the constants of integration are determined by employing the continuity of the transverse velocity and displacement fields a $t = \tau$, presented in Eqn. (35).

$$W_2 = -6\ddot{w}\,(t^2 - 2\bar{\tau}t + \eta\tau^2) \qquad \textbf{(35)}$$

The second phase ceases at $t = \bar{\tau}$, which is when the transverse velocity $\dot{W}_2$ vanishes. Hence, the permanent displacement field at any point $0 \le z \le 1$ can be written as:

$$w_f = 6\,\ddot{w}\bar{\tau}^2(\eta - 1)(1 - z)/\eta \qquad \textbf{(36)}$$

Static and kinematic admissibility

It is essential to verify whether the mathematical treatment in 0 are statistically admissible, i.e. the Eqn. (16a) and inequality (16b) are not violated. Whilst Eqns. (13)-(15) satisfy $\mathrm{M}_1 = \mathrm{M}_0$, it is necessary to demonstrate that the shear forces at the centre vanish (i.e. $Q_x|_{z=0} = 0$ and $(\partial^2 M_x)/(\partial x^2) > 0$ (or $(\partial^2 M_y)/(\partial x^2) > 0$). While the former condition is clearly satisfied from Eqn. (31a), the latter condition, requires:

$$\frac{\partial^2 M_x}{\partial x^2} = \frac{\left(\left(6\left(2L^4z^3 - L^4z^4 - \frac{5}{2}z^2L^2x^2 + 1/2x^4\right)\right)(\eta - 1)M_0 - p_1L^6z^3\right)}{3L^6z^3} > 0 \qquad \textbf{(37)}$$

Which is obtained with the aid of variational parameters. Elementary calculation gives:

$$\eta \le \left|\frac{12\beta}{12\beta - 1}\right| = \eta_{crit} \qquad \textbf{(38)}$$

A similar procedure to establish the kinematic admissibility of the velocity profile is achieved by satisfying Eqn. (16b), i.e. it is required to show that $-\mathrm{M}_0 \le \mathrm{M}_2 = (\mathrm{M}_x + \mathrm{M}_y)/2 + 1/2[(\mathrm{M}_x - \mathrm{M}_y)^2 + 4\mathrm{M}_{xy}{}^2]^{1/2} \le \mathrm{M}_0$. This inequality simplifies to the following:

$$-2\mathrm{M}_0 \le (\mathrm{x}^2 + \mathrm{y}^2)\mathrm{g}(\mathrm{z}) \le 0 \qquad \textbf{(39)}$$

The right-hand side of the inequality-at the plate centre- requires that $(\mu\ddot{W} - \ p_1)/6 - \mu\ddot{W}z/12 \leq 0$, which, when using Eqns. (33), (23), will lead to an identical expression to Eqn. (38).

In the case of $r_e = L$ and $\beta = \frac{1}{6}$; the right-hand side of the inequality simplifies to the condition for the case of uniformly distributed load, i.e. $\eta \leq 2$.

In the second phase of motion, the Eqn. (16b) (or (37)) must still be satisfied, but with setting $\eta = 0$, which yields:

$$-2M_0 \leq \frac{-2\,M_0(x^2 + y^2)}{L^2} \leq 0 \qquad \textbf{(40)}$$

By considering the fact that $(x^2 + y^2)/L^2$ is universally positive (Eqn. (11)), Eqn. (40) is valid for all values of ω_0 and L at plate centre. A plot of bending moment distribution for various values of η is shown in Fig. 10. In this case, the loading parameter $b = 50$ and $\omega_0 = 0.125$, which are found by regression analyses on numerical results of registered pressure time history for an Armour steel model B4 found in (Mehreganian et al., 2018)

Case ii: $\boldsymbol{\eta \geq \eta_{crit}}$

First phase of motion

For the blast loads with high magnitudes, $\eta > \eta_{crit}$ then $p_1 >> p_c$ so the right-hand side of Eqn. (39) is no longer valid, since a yield violation occurs near the plate centre. This requires the velocity field profile to be modified. It is therefore assumed that the velocity profile is governed by three distinguishable phases, as the incipient plastic hinge forms in the central part of the plate (Fig. 8). It is also assumed that in this phase, the plastic flow in the plate centre is characterised by corner A of the yield condition in Fig. 3, which governs the central part of the plate for $0 \leq z \leq \xi_0$, whereas the remaining part of the plate $\xi_0 \leq z \leq 1$ is governed by the regime AB. Thus, the velocity profile will be of the form:

$$\dot{w} = \dot{W}_2 \qquad 0 \leq z \leq \xi_0 \qquad \textbf{(41)}$$

$$\dot{w} = \dot{W}_2 \frac{(1-z)}{(1-\xi_0)} \qquad \xi_0 \leq z \leq 1 \qquad \textbf{(42)}$$

where ξ_0 is time independent. It is also assumed that $r_e/L \leq \xi_0$. Thus, the moment function $g_1(z)$ is replaced by $g_2(z)$ in the following form:

$$g_2(z) = \begin{cases} \dfrac{\mu\ddot{W} - \ \ p_1}{6} + \dfrac{A_3}{z^2} + \dfrac{B_3}{z^3} & 0\ \le zL \le r_e \quad \textbf{(43a)} \\ \dfrac{-ap_1 e^{-bLz}(bLz+2)}{(bLz)^3} + \dfrac{\mu\ddot{W}}{6} + \dfrac{D_3}{z^2} + \dfrac{E_3}{z^3} & r_e\ \le zL \le\ \xi_0 L \quad \textbf{(43b)} \\ \dfrac{-ap_1 e^{-bLz}(bLz+2)}{(bLz)^3} + \dfrac{\mu\ddot{W}}{6(1-\xi_0)} - \dfrac{\mu\ddot{W}z}{12(1-\xi_0)} + \dfrac{F_3}{z^2} + \dfrac{G_3}{z^3} & \xi_0\mathrm{L}\ \le zL \le\ L \quad \textbf{(43c)} \end{cases}$$

$$\begin{cases} A_3 = B_3 = 0 \\ D_3 = \dfrac{-p_1((br_e)^2 + 2br_e + 2)}{2(Lb)^2} \\ E_3 = \dfrac{p_1((br_e)^3 + 3(br_e)^2 + 6br_e + 6)}{3L^3b^3} \end{cases} \quad \textbf{(44)}$$

The integration constants in Eqns. (43a-b) are obtained by applying the boundary conditions of $Q_x = 0$, $M_x = M_y = M_0$ at the midspan and owing to the continuity of shear force at $z = r_e/L$. It transpires that at the interval $0 \le z \le r_e/L$, $\mu\ddot{W}_2 = p_1$ Thus, using the kinematic conditions of $W_2 = \dot{W}_2 = 0$ at $t = 0$, the maximum displacement will become:

$$W_2 = \frac{\ddot{w}\eta t^2}{2\beta} \quad \textbf{(45)}$$

The expression for arbitrary moment function in the interval $\xi \le z \le 1$, is similar to Eqn. (31b), except the parameter $\ddot{W}$ is now replaced with $\ddot{W}/(1-\xi_0)$ while the integration constants are F_3 and G_3. The succeeding conditions of $Q_x = 0$ and $M = M_0$ at the interval $r_e/L < z < \xi_0$ would furnish these constants as:

$$\begin{cases} F_3 = \dfrac{p_1\left(3a(-1+\xi_0)(Lb\xi_0+1)e^{-Lb\xi_0} + L^2b^2\xi_0^2\left(\xi_0 - \frac{3}{2}\right)\right)}{3L^2b^2(1-\xi_0)} \\ G_3 = \dfrac{p_1\left(4a(-1+\xi_0)[\ (Lb\xi_0)^2 + 2Lb\xi_0 + 2]e^{-Lb\xi_0} + (Lb\xi_0)^3\left(\xi_0 - \frac{4}{3}\right)\right)}{4L^3b^3(-1+\xi_0)} \end{cases} \quad \textbf{(46)}$$

The expression (43c) should satisfy the at boundary condition of bending moment, i.e. $\bar{m} = 0$ at $z = 1$, which yields an expression of the incipient plastic hinge and the dynamic overloading factor in Eqn. (47).

$$\eta = \frac{12L^3b^3\beta}{\gamma} \tag{47}$$

where $\eta = p_1/p_c$, and the parameter γ is:

$$\gamma = 12a[b^2L^2(1-\xi_0)\xi_0 + (1-2\xi_0)bL - 2]e^{-Lb\xi_0} + 12a(Lb+2)e^{-Lb} - 3(1-\xi_0)^2(\xi_0+1/3)L^3b^3 \tag{48}$$

The distribution of bending moment with various values of ξ_0 is presented in Fig. 11. For impulsive loading, $\eta \to \infty$. thus $\gamma \to 0$ which occurs when the plastic hinges form at the supports, i.e. $\xi_0 \to 1$. The expression of ξ_0 in Eqn. (48) is highly nonlinear which can be solved with the aid of numerical methods.

Second phase of motion $\tau \le t \le T_1$

In this phase $p_1 = 0$ however, due to initial velocity at first phase, motion continues during the second phase. Concerning this, ξ_0 from Eqn. (42) is substituted by an active plastic hinge ξ, which moves inwards as demonstrated in Fig. 9. For the region $0 \le z \le \xi$, the equilibrium with $p_1 = 0$ predicts $\mu\ddot{W}_2 = 0$. Therefore, by implementing the continuity of the velocity parameter at $t = \tau$, the deformation and velocity are as in Eqn. (49), (50).

$$\dot{W}_2 = \ddot{\bar{w}}\bar{\tau}/\beta \tag{49}$$

$$W_2 = \frac{\ddot{\bar{w}}\bar{\tau}}{2\beta}(t-\tau) \tag{50}$$

For the second region, the differential Eqn.'s (30a) and (30b) will change to:

$$z^2\frac{\partial^2 g_2}{\partial z^2} + 6z\frac{\partial g_2}{\partial z} + 6g_2 = \mu\ddot{w}_2 = \mu\ddot{W}_2\left(\frac{1-z}{1-\xi}\right) + \mu\dot{W}_2\dot{\xi}\left(\frac{1-z}{(1-\xi)^2}\right) \tag{51}$$

The general solution of which is:

$$g_2 = \frac{\mu\ddot{W}_2(2-z)}{12(1-\xi)} + \frac{\mu\dot{W}_2\dot{\xi}(2-z)}{12(1-\xi)^2} + \frac{D_4}{z^2} + \frac{E_4}{z^3} \tag{52}$$

The integration constants are obtained by conditions of $Q_x = 0$ and $M_x = M_y = M_0$ at $0 \le z \le \xi$, the continuity of Q_x and M_x at $z = \xi$ gives:

$$\begin{cases} D_4 = \dfrac{p_1\tau\dot{\xi}\xi^2(2\xi-3)}{6(\xi-1)^2} \\ E_4 = \dfrac{-p_1\tau\dot{\xi}\xi^3(3\xi-4)}{12(\xi-1)^2} \end{cases} \tag{53}$$

An expression in terms of the travelling hinge ξ is found by employing the simply supported boundary condition (Eqn. (22)) at corners of the plate:

$$(3\xi^2 - 2\xi - 1)\dot{\xi} = \frac{12\beta}{\eta\tau} \tag{54}$$

An expression for the travelling hinge displacement is attained through time integration of $\dot{\xi}$ in (55), by ensuring that the plastic hinge at $t = \tau$ is stationery:

$$\xi^3 - \xi^2 - \xi - 1 = 12\beta t/\eta\tau + \bar{\xi} \tag{55}$$

where

$$\bar{\xi} = \xi_0^3 - \xi_0^2 - \xi_0 - 1 - \frac{12\beta}{\eta} \tag{56}$$

The integration constant $\bar{\xi}$ is determined by evaluating the continuities $\xi = \xi_0$ at $t = \tau$ in Eqn. (55). Subsequently, the second phase completes when $\xi = 0$, which occurs at time $T_1 = -(\bar{\xi} + 1)\eta\tau/12\beta$. The influence of dynamic load factor on the length of active plastic hinge (with $\tau = 50\mu s$) is presented in Fig. 12. Substituting this expression into Eqn. (50) gives the transverse displacement at the end of this phase as:

$$W_2 = -\frac{\ddot{w}\eta\tau^2(\eta\bar{\xi} + 6\beta + \eta)}{12\beta^2} \tag{57}$$

Final phase of motion $T_1 \leq t \leq T$

The final phase of the plate motion will essentially develop since the kinetic energy from the previous phase has to be somehow dissipated. The transverse velocity profile is the same as Fig. 5 as the plastic hinge closes ($\xi = 0$). The incipient deformation is identical to the circumstance of infinitesimal blast loads, viz., the condition of inequality (38). Therefore, the solution to velocity and displacement fields at this phase is achieved by time integration of (34) and equating it with Eqns.(35), (49), (50), (57) at $t = T_1$, The transverse displacement at this phase is equivalent to:

$$W_3 = -\frac{p_1\left[\tau^2\eta^2(\bar{\xi} + 1)^2 + 24\beta\eta\tau\left(t\bar{\xi} + \frac{1}{2}\tau\right) + 144\beta^2t^2\right]}{24\eta\beta\mu} \tag{58}$$

recalling the parameter $\bar{\xi}$ is defined as $\xi_0^3 - \xi_0^2 - \xi_0 - 1 - 12\beta/\eta$. The plate rests when $\dot{W}_3 = 0$, which occurs at time T (Eqn. (59)).

$$T = -\frac{\tau\bar{\xi}\eta}{12\beta} \tag{59}$$

Thus, Eqn. (58) will simplify to Eqn. (60):

$$W_f = -\frac{\ddot{w}\,\eta\tau^2\left[\left(\bar{\xi} + \frac{1}{2}\right)\eta + 6\beta\right]}{12\beta^2} \tag{60}$$

Static and Kinematic Admissibility

It is evident that the continuity requirements of generalised stresses i.e. $Q_x = Q_y = 0$, $M_x = M_y = M_0$ are satisfied at $z = 0$, $z = \xi_0$ and $z = \xi$ for the first, second, and third phases of motion. Furthermore, the Eqn. (16a) is satisfied throughout the entire motion, irrespective of the phase of motion. It can also be observed that the inequality (16b) is satisfied for $0 \le z \le \xi$ during the entire phases of motion.

Impulsive Loading

A blast load of rectangular pulse shape with very short duration ($\tau \to 0$) and very high amplitude ($\eta \to \infty$ or $p_1 \gg p_c$) is known as impulsive loading. In the case of impulsive loading, the total change in momentum equals the total impulse imparted upon the system, hence the conservation of momentum implies that:

$$\int_0^{\frac{re}{L}} 8L^2\tau p_1 z dz + \int_{\frac{re}{L}}^1 8L^2\tau p_1 a e^{-bLz} z dz = \int_0^1 8L^2\mu V_0 z dz \tag{61}$$

The solution to the Eqn. (61) yields:

$$V_0 = \frac{\varepsilon_1 \tau p_1}{L^2\mu} \tag{62}$$

where $\varepsilon_1 = \frac{r_e^2 b^2 - 2e^{-Lb}(bL+1) + 2r_e b + 2}{b^2}$. Consequently, the Eqn. (60) can be recast in the form:

$$\frac{W_f}{H} = -\frac{\lambda}{12\eta}\left[\left(\bar{\xi} + \frac{1}{2}\right)\eta + 6\beta\right] \tag{63}$$

in which $\lambda = \frac{\mu V_0^2 L^2}{M_0 H}\left(\frac{L^4}{\varepsilon_1^2}\right)$ is the non-dimensional kinetic energy. For the case of $r_e = L$ and $\eta \to \infty$, $r_e \to L$, $\beta = \frac{1}{6}$ and $W_f/H \cong \lambda/8$, which conforms to results for the case of uniform pressure load. A plot of normalised deflection vs. the dimensionless kinetic energy is shown in Fig. 15 for the case of $\omega_0 = 0.7$, $b = 50$ for various values of load ratio. It can be observed that with increase of η the plot is only marginally different from impulsive load, i.e. $\eta \to \infty$.

Fully Clamped Square Plate

Although in practical applications the protective plate elements are designated with fully-clamped conditions, the foregoing analysis for the simply supported plates can simply be extended to the case of fully-clamped plate: the edge conditions of the moment, denoted by $\bar{m} = -1$, yields the static plastic collapse as $p_c = 2M_0/\beta L^2$; thus, the foregoing results may be furnished for the fully-clamped plates by merely changing M_0 to $2M_0$ in the parameter $\ddot{\bar{w}}$ and associated expressions accordingly.

However, it should be noted that, in contradiction to the global blasts, the boundary effects are not significant for the localised blast because such a blast impacts a small area of the plated structure (Micallef et al., 2014). Furthermore, the difference of the permanent deformation of uniformly, impulsively loaded clamped and simply supported plates is insignificant in Mode I (large inelastic deformation) (Yuen and Nurick, 2001).

Numerical analyses

Limitations of the study and Material model

The analysis performed in Section 0 was predicated on the assumptions of Kirchhoff –Love plate theory, which ignores the effects of transverse shear and rotatory inertia. Taking this limitation into account, it could be assumed that for the range of $0.01 < H/L < 0.02$ and under infinitesimal loading conditions, the bending action dominates the structural behaviour, such that the build-up of membrane action associated with the plastic collapse can be disregarded.

Whilst considering the limitations of study, in this section, the analytical solutions are validated against numerical simulations. The simulations are conducted in commercial Finite Element (FE) software ABAQUS 6.14-4/Explicit®, which is capable of simulating the dynamic response for blast loading scenarios through analyses of various degrees of complexity. For the sake of numerical analyses, the model parameters are as follows:

$$H = 4mm$$
$$L = 200mm$$
$$\sigma_0 = 330MPa$$
$$\mu = 30.4kg/m^2$$

These properties are typical of the protective plates made of mild steel, provided the strain rate sensitivity is ignored. The loading parameters were selected consistent with constant values of $p_1 = 20MPa$, $b = 50$, $\tau = 30\mu s$ and the rectangular pulse shape considered in the analytical analyses of previous sections.

Finite Element (FE) model

A full 3-Dimensional FE model was set up in ABAQUS for rectangular plate of length $2L$, discretised with S4(R) shell elements of double curvature with hourglass control and pinned along the periphery. These elements incorporate reduced integration formulation to prevent shear locking. Shear locking is a major issue in computational formulations of beams and plates leading to artificial over-stiffening. Due to symmetry, only a quarter of the plate is modelled. The convergence of the displacements was satisfied with a mesh of element length to thickness ratio of 1, giving a total of 2500. For each case of $\omega_0 = r_e/L$, a pressure matrix corresponding to Cartesian coordinates was constructed by utilisation of Eqns. (2) and (11). This pressure matrix was mapped directly onto the panel. For the case of ω_0=0.5, a contour plot of transverse displacement and stress distributions is shown in Fig. 13.

We can see from Fig. 17 that for the range of $H/L = 2\%$ the numerical results for displacement compare favourably with the analytical results. The position of the stationery plastic hinge is obtained numerically for various loading radii and plotted in Fig. 16. It is interesting to note that for most loading constant central zone radii, the length of ξ_0 (which should satisfy $\eta\gamma - 12\beta^3L^3b^3 = 0$ from Eqn. (47)) is predicted at~ 0.89th of the plate length. For close-in blasts with small loading radii/plate length ratio, the length of the stationery plastic hinge decreases accordingly. It turns out that in the case of $r_e/L = 0.1$, for example, a solution of ξ_0 in Eqn. (47). Exists at the plate centre, in addition to the hinges formed near the supports. Numerical observations on the position of first maximum equivalent plastic strain ($\bar{\varepsilon}_p$) corroborate with this statement.

The theoretical solution for the impulsive loading from Eqn. (63) is validated in Fig. 14 against the experimental results on ARMOX steel and mild steel MS4 specimens obtained from (Jacob et al., 2004; Langdon et al., 2015), respectively. The tests from Jacob et al (Jacob et al., 2004) were those of Series I plates. The duration time of $15\mu s$, typical of impulsive load cases, was chosen and the loading parameter $r_e = 25mm$ was taken as the charge radius, whilst the loading decay parameter 'b' - typically in the range of $50 \leq b \leq 100$- was evaluated by curve fitting methods on the registered pressure loads from the numerical results of (Mehreganian et al., 2018). It is also important to note the deviation of data from the predicted curve when the dimensionless kinetic energy increases ($\lambda > 200$), the limit beyond which the membrane action significantly affects the transverse deflections.

It should be noted that, for the case of ARMOX steel, the material is capable of higher energy absorption, while the deformation is not significantly affected by strain rate sensitivity. However, the response can be affected by the material elasticity which can pose a difficulty when implementing limit analysis. Regardless of this limitation, the application of modified load function considerably enhances the prediction of plate response as opposed to the previous models introduced by researchers (Jones, 1997), and (Komarov and Nemirovskii, 1986) valid for uniform pressure.

Concluding remarks

This paper deals with a theoretical model devised to predict the transverse dynamic plastic displacement field of a generic simply supported, monolithic, square plate subjected to localised blast loading. A piecewise continuous load function formerly studied by (Karagiozova et al., 2010) and extended by (Micallef et al., 2015), and (Micallef et al., 2012) was incorporated into the analyses, which is universal and adjustable, through alteration of its parameters, to replicate various loading scenarios from proximal (localised) to distal (global) blast loads.

The plate was assumed thin, to enable making use of Kirchhoff-Love theory as opposed to more sophisticated Mindlin-Reissner plate theory investigated by (Toolabi et al., 2018) using numerical methods. As such, transverse shear and rotatory inertia effects can be ignored without loss of accuracy. The plate was, however, assumed thick enough not to be rendered a membrane. The analyses were performed by means of limit analysis and the incipient velocity profile was governed by the travelling plastic hinge in the three stages of analysis for high amplitude loads (i.e. $p_1 \gg p_c$ or $\eta > \eta_{crit}$).

Close agreement was found when correlating the results of the theoretical model with the corresponding FE model for different load distributions. The transverse deflection-impulsive load relation was validated for different cases of load distributions. It was concluded that the analytical model yields satisfactory results for low impulse where bending effect is dominant, while for larger impulses on the thin plate, the membrane effects become significant. Moreover, the theoretical solutions for impulsive load give better estimate than the previous rigid-plastic model proposed earlier by (Jones, 1997), or (Komarov and Nemirovskii, 1986) when validated against the experimental data for close-in low impulse (kinetic energy) blast loads (i.e., $\lambda < 200$). The significance of the study lies in the fact that its results are applicable to plates made of materials with little or no strain rate sensitivity, such as aluminium and high strength armour steel. Structures made of armour steel undergo less deflection with higher energy absorption capacity when compared to their Mild steel counterparts.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

Declaration of Conflicting interest

The Authors declare that there is no conflict of interest.

References

Balden VH and Nurick GN (2006) Numerical simulation of the post-failure motion of steel plates subjected to blast loading. *International Journal of Impact Engineering* 32(1–4): 14–34.

Bonorchis D and Nurick GN (2007) The effect of welded boundaries on the response of rectangular hot-rolled mild steel plates subjected to localised blast loading. *International Journal of Impact Engineering* 34(11): 1729–1738.

Bonorchis D and Nurick GN (2009) The influence of boundary conditions on the loading of rectangular plates subjected to localised blast loading - Importance in numerical simulations. *International Journal of Impact Engineering*, Elsevier Ltd 36(1): 40–52. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2008.03.003.

Bonorchis D and Nurick GN (2010) The analysis and simulation of welded stiffener plates subjected to localised blast loading. *International Journal of Impact Engineering*, Elsevier Ltd 37(3): 260–273. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2009.08.004.

Braestrup MW (2007) Yield line theory and concrete plasticity. In: *Morley Symposium on Concrete Plasticity and its Application*, pp. 43–48.

Chen A, Louca LA and Elghazouli AY (2015) Blast assessment of steel switch boxes under detonation loading scenarios. *International Journal of Impact Engineering*, Elsevier Ltd 78: 51–63. Available from: http://linkinghub.elsevier.com/retrieve/pii/S0734743X14002875.

Chen FL and Yu TX (2014) Membrane Factor Method for large deflection response of beams and plates to intense dynamic loading. 141: 59–71. Available from: http://library.witpress.com/viewpaper.asp?pcode=SUSI14-006-1.

Chung Kim Yuen S, Langdon GS, Nurick GN, et al. (2012) Response of V-shape plates to localised blast load: Experiments and numerical simulation. *International Journal of Impact Engineering*, Elsevier Ltd 46: 97–109. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2012.02.007.

Cloete TJ, Nurick GN and Palmer RN (2006) The deformation and shear failure of peripherally clamped centrally supported blast loaded circular plates. *International Journal of Impact Engineering* 32(1–4): 92–117.

Cox AD and Morland LW (1959) Dynamic plastic deformations of simply-supported square plates. *Journal of Mechanical Physics and Solids* 7(March 1959): 229–241.

DOD. (2008) *Unified Facilities Criteria (UFC): Structures to Resist the Effects of Accidental Explosions (UFC 3-340-02). Department of Defence*, Washington DC.: Department of Defence.

Fallah AS, Micallef K, Langdon GS, et al. (2014) Dynamic response of Dyneema® HB26 plates to localised blast loading. *International Journal of Impact Engineering*, Elsevier Ltd 73: 91–100. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2014.06.014.

Florence a (1966) Circular plate under a uniformly distributed impulse☆. *International Journal of Solids and Structures*, Pergamon Press Ltd. 2(1): 37–47. Available from: http://linkinghub.elsevier.com/retrieve/pii/0020768366900059.

Gharababaei H and Darvizeh A (2010) Experimental and Analytical Investigation of Large Deformation of Thin Circular Plates Subjected to Localized and Uniform Impulsive Loading#. *Mechanics Based Design of Structures & Machines* 38(2): 171–189. Available from: http://search.ebscohost.com/login.aspx?direct=true&db=aph&AN=50791808&site=eds-live&scope=site.

Hopkins HG and Prager W (1953) The Load carrying capacity of circular plates. *Journal of the Mechanicsa nd Physics of Solids* 2(June): 1–18.

Jacob N, Yuen SCK, Nurick GN, et al. (2004) Scaling aspects of quadrangular plates subjected to localised blast loads - Experiments and predictions. *International Journal of Impact Engineering* 30(8–9): 1179–1208.

Jacob N, Nurick GN and Langdon GS (2007) The effect of stand-off distance on the failure of fully clamped circular mild steel plates subjected to blast loads. *Engineering Structures* 29(10): 2723–2736.

Jones N (1968) Impulsive Loading of a Simply Supported Circular Rigid Plastic Plate. *Journal of Applied Mechanics* 35(1): 59. Available from: http://link.aip.org/link/JAMCAV/v35/i1/p59/s1&Agg=doi.

Jones N (1971) A theoretical study of the dynamic plastic behaviour of beams and plates with finite-deflections. *International Journal of Solids and Structures* 7(33): 1007–1029.

Jones N (1997) *Structural Impact*. 1st ed. Ca (ed.), Cambridge: Cambridge University Press.

Jones N (2012) Impact loading of ductile rectangular plates. *Thin-Walled Structures*, Elsevier 50(1): 68–75. Available from: http://dx.doi.org/10.1016/j.tws.2011.09.006.

Jones N (2013) The credibility of predictions for structural designs subjected to large dynamic loadings causing inelastic behaviour. *International Journal of Impact Engineering*, Elsevier Ltd 53: 106–114. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2011.12.008.

Jones N (2014a) Dynamic inelastic response of strain rate sensitive ductile plates due to large impact, dynamic pressure and explosive loadings. *International Journal of Impact Engineering*, Elsevier Ltd 74: 3–15. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2013.05.003.

Jones N (2014b) Pseudo-shakedown phenomenon for the mass impact loading of plating. *International Journal of Impact Engineering*, Elsevier Ltd 65: 33–39. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2013.10.009.

Jones N and Griffii N (1971) An experimental study into the dynamic plastic bheaviour of wide beams and rectangular plates. *International Jounral of Mechanical Sciences* 13: 721–735.

Jones N, Uran TO and Tekin SA (1970) The dynamic plastic behavior of fully clamped rectangular plates. *International Journal of Solids and Structures* 6(12): 1499–1512.

Karagiozova D, Langdon GS, Nurick GN, et al. (2010) Simulation of the response of fibre-metal laminates to localised blast loading. *International Journal of Impact Engineering*, Elsevier Ltd 37(6): 766–782. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2009.04.001.

Komarov KL and Nemirovskii Y V. (1986) Dynamic behaviour of rigid-plastic rectangular plates (Translated). *Prikladnaya Mekhanika* 21(7): 683–690.

Langdon GS, Yuen SCK and Nurick GN (2005) Experimental and numerical studies on the response of quadrangular stiffened plates. Part II: Localised blast loading. *International Journal of Impact Engineering* 31(1): 85–111.

Langdon GS, Lee WC and Louca LA (2015) The influence of material type on the response of plates to air-blast loading. *International Journal of Impact Engineering*, Elsevier Ltd 78: 150–160. Available from: http://linkinghub.elsevier.com/retrieve/pii/S0734743X14003066.

Li QM and Huang YG (1989) Dynamic plastic response of thin circular plates with transverse shear and rotatory inertia subjected to rectangular pulse loading. *International Journal of Impact Engineering* 8(3): 219–228.

Li QM and Jones N (2000) On dimensionless numbers for dynamic plastic response of structural members. *Archive of Applied Mechanics* 70(February 1999): 245–254.

Li QM and Jones N (2005) Foundation of Correlation Parameters for Eliminating Pulse Shape Effects on Dynamic Plastic Response of Structures. *Journal of Applied Mechanics* 72(2): 172.

Mehreganian N, Louca LA, Langdon GS, et al. (2018) The response of mild steel and armour steel plates to localised air-blast loading-comparison of numerical modelling techniques. *International Journal of Impact Engineering* 115(May): 81–93.

Micallef K, Fallah a. S, Pope DJ, et al. (2012) The dynamic performance of simply-supported rigid-plastic circular steel plates subjected to localised blast loading. *International Journal of Mechanical Sciences*, Elsevier 65(1): 177–191. Available from: http://dx.doi.org/10.1016/j.ijmecsci.2012.10.001.

Micallef K, Fallah AS, Pope DJ, et al. (2014) Dynamic Performance of Simply Supported Rigid Plastic Circular Thick Steel Plates Subjected to Localized Blast Loading. *Journal of Engineering Mechanics* (January): 159–171.

Micallef K, Soleiman Fallah A, Pope DJ, et al. (2015) On dimensionless loading parameters for close-in blasts. *International Jounral of Multiphysics* 9(2): 171–194.

Micallef K, Fallah AS, Curtis PT, et al. (2016) On the dynamic plastic response of steel membranes subjected to localised blast loading. *International Journal of Impact Engineering*, Elsevier Ltd 89: 25–37. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2015.11.002.

Neuberger A, Peles S and Rittel D (2007) Scaling the response of circular plates subjected to large and close-range spherical explosions. Part II: Buried charges. *International Journal of Impact Engineering* 34(5): 874–882.

Neuberger A, Peles S and Rittel D (2009) Springback of circular clamped armor steel plates subjected to spherical air-blast loading. *International Journal of Impact Engineering* 36(1): 53–60.

Nurick GN and Balden V. (2010) Localised Blast Loaded Circular Plates: an Experimental and Numerical Investigation. *Proceedings of the IMPLAST 2010 Conference* 12(14): 145–152.

Nurick GN and Martin JB (1989) Deformation of thin plates subjected to impulsive loading - A review - Part II - Experimental results. *International Journal of Impact Engineering* 8(2): 171–186.

Nurick GN and Radford AM (1997) Deformation and tearing of clamped circular plates subjected to localised central blast loads. *Recent Developments in Computational and Applied Mechanics (Ed. B.D Reddy)* a volume i: 276–301.

Nurick GN, Mahoi S and Langdon GS (2016) The response of plates subjected to loading arising from the detonation of different shapes of plastic explosive. *International Journal of Impact Engineering*, Elsevier Ltd 89: 102–113. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2015.11.012.

Nwankwo E, Soleiman Fallah A and Louca LA (2013) An investigation of interfacial stresses in adhesively-bonded single lap joints subject to transverse pulse loading. *Journal of Sound and Vibration*, Elsevier 332(7): 1843–1858. Available from: http://dx.doi.org/10.1016/j.jsv.2012.11.008.

Nwankwo E, Soleiman Fallah A, Langdon GS, et al. (2013) Inelastic deformation and failure of partially strengthened profiled blast walls. *Engineering Structures*, Elsevier Ltd 46: 671–686. Available from: http://dx.doi.org/10.1016/j.engstruct.2012.08.019.

Rajendran R and Lee JM (2009) Blast loaded plates. *Marine Structures*, Elsevier Ltd 22(2): 99–127. Available from: http://dx.doi.org/10.1016/j.marstruc.2008.04.001.

Rajendran R and Narasimhan K (2006) A Shock Factor Based Approach for the Damage Assessment of Plane Plates Subjected to Underwater Explosion. *The Journal of Strain Analysis for Engineering Design* 41(6): 417–425.

Schultz E, Wintenberger E and Shepherd J (1999) Investigation of Deflagration to Detonation Transition for Application to Pulse Detonation Engine Ignition Systems. *Proceedings of the 16th JANNAF Propulsion Symposium* 1(c): 175–202.

Timoshenko, S.;Woinosky-Kreiger S (1959) *Theory of Plates and Shells*. New York: McGraw Hill.

Toolabi M, Fallah AS, Baiz P., et al. (2018) Enhanced mixed interpolation XFEM formulations for discontinuous Timoshenko beam and Mindlin-Reissner plate. *International Journal for Numerical Methods in Engineering* (April 2018): 1–24.

Wierzbicki T and Nurick GN (1996) Large deformation of thin plates under localised impulsive loading. *Int. j. Impact Engng* 18(96): 899–918.

Wierzbicki T and Florence AL (1970) Theoretical and Experimental Investigation of Impulsively Loaded Clamped Circular Viscoplastic Plates. *International Journal of Solids and Structures* 6(5): 553–568. Available from: http://dx.doi.org/10.1016/0020-7683(70)90030-2.

Xue Z and Hutchinson JW (2004) A comparative study of impulse-resistant metal sandwich plates. *International Journal of Impact Engineering* 30(10): 1283–1305.

Youngdahl CK (1971) Influence of pulse shape on the final plastic deformation of a circular plate. *International Journal of Solids and Structures* 7(9): 1127–1142.

Yu T. and Chen FL (1992) The large deflection dynamic plate response of rectangular plates. *International Journal of Impact Engineering* 12(4): 603–616.

Yuan Y and Tan PJ (2013) Deformation and failure of rectangular plates subjected to impulsive loadings. *International Journal of Impact Engineering*, Elsevier Ltd 59: 46–59. Available from: http://dx.doi.org/10.1016/j.ijimpeng.2013.03.009.

Yuan Y, Tan PJ, Shojaei KA, et al. (2016) Large deformation, damage evolution and failure of ductile structures to pulse-pressure loading. *International Journal of Solids and Structures*, Elsevier Ltd 96: 320–339. Available from: http://linkinghub.elsevier.com/retrieve/pii/S0020768316301068.

Yuen SCK and Nurick GN (2001) Deformation and Tearing of Uniformly Blast-Loaded Quadrangular Stiffened Plates. In:

Zingoni A (ed.), *Structural Engineering, Mechanics and Computation*, Oxford: Elsevier Science, pp. 1029–1036. Available from: http://www.sciencedirect.com/science/article/pii/B9780080439488501144.

Zheng C, Kong X, Wu W, et al. (2016) The elastic-plastic dynamic response of stiffened plates under confined blast load. *International Journal of Impact Engineering*, Elsevier Ltd 95: 141–153. Available from: http://linkinghub.elsevier.com/retrieve/pii/S0734743X16302639.

Appendix A.

A.1. Dynamic pressure pulse loading

The analyses conducted in sections 0 were limited to the localised blast loads with rectangular temporal pulse shape. A blast peak pressure with dynamic pressure more than 10 times the static collapse can be idealised as rectangular pulse (Jones, 1997), (Rajendran and Lee, 2009),(Yuan et al., 2016). In real-life, the majority of near field explosions are non-impulsive, which may have linear or exponential pulse shapes. Since the pulse shape can significantly affects the structural response (Youngdahl, 1971), it is essential to understand the structural response under various pulse loads, which is the aim of this section. The linearly decaying pulse is defined as in Eqn. A. 1 and Fig. A. 1.

$$p_2(t) = \begin{cases} \left(1 - \frac{t}{\tau}\right) & 0 \le t \le \tau \\ 0 & t \ge \tau \end{cases}, \qquad \textbf{A. 1}$$

And a more general, exponentially decaying temporal variation of the pressure load is represented in the form of Fig. A.2 and Eqn. A. 2.

$$p_3(t) = \begin{cases} Xe^{\frac{-Yt}{\tau}} & 0 \le t \le \tau \\ 0 & t \ge \tau \end{cases} \qquad \textbf{A. 2}$$

which simplifies to Friedlander's equation (Rajendran and Lee, 2009) with a specific choice of parameters, i.e. $X = Y = 1$. Following the procedure outlined in Section 4, the final transverse deflection for the simplified exponential pulse shape is determined as in Eqn. A. 3:

$$W_f = \frac{\left(-\left(12\left((12\beta + \eta)\ln\left(\frac{12\beta}{12\beta + \eta}\right) + \eta\right)\right)\beta\delta + \eta\left(\left(\eta(\gamma_e + 1) + 24\beta\right)e - \eta(\gamma_e + 1)\right)\right)p_c e^{-2}\tau^2}{12\mu\beta} \qquad \textbf{A. 3}$$

where $\gamma_e = \xi_0^3 - \xi_0^2 - \xi_0 - 1 + \frac{12\beta}{\eta(e^{-1} - 1)}$ and δ is defined as:

$$\delta = e^{\frac{(\eta(\gamma_0+1)(1-e^{-1})+24\beta)}{12\beta}} \qquad \textbf{A. 4}$$

The final deflection occurs at:

$$T_f = \tau ln\left(\frac{12e\beta\tau + \eta(\xi_0 + \xi_0^2 - \xi_0^3)}{\tau(12\beta + \eta)}\right) \qquad \textbf{A. 5}$$

For the linearly decaying pulse, the final deformation will be:

$$W_f = -\frac{\eta\ddot{w}\tau^2\left(\left(\gamma_l + \frac{1}{2}\right)\eta + 8\beta\right)}{48\beta^2} \qquad \textbf{A. 6}$$

where $\ddot{w} = p_c\beta/\mu$ also defined in (29a) and $\gamma_l = \xi_0^3 - \xi_0^2 - \xi_0 - 1 - 24\beta/\eta$. The permanent deformation occurs at:

$$T_f = \frac{-\gamma_l\eta\tau}{24\beta} \qquad \textbf{A. 7}$$

Comparing the Eqn. s' A. 6, A. 3 and (60), it is observed that, for non-impulsive dynamic loads the plastic transverse deflection is highly dependent on the pulse shape. However, this effect can be practically eliminated by incorporating Youngdahl's correlation parameters, namely total impulse, effective pressure and mean time (t_m) (Jones, 1997), (Youngdahl, 1971) defined as follows:

$$I = \int_0^T P(t)dt \qquad \textbf{A. 8}$$

$$t_m = \frac{1}{I}\int_0^T t.P(t)dt \qquad \textbf{A. 9}$$

$$p_e = \frac{I}{2t_m} \qquad \textbf{A. 10}$$

where I is the total impulse, $P(t) = p_1p(t)$ and p_e is the effective pressure, T is the time where plastic deformation ceases and t_m is the centroid of the pulse. In the case of rectangular pressure pulse, for instance, $I = p_1\tau$ and $p_e = p_1$ is used in lieu of dynamic pressure p_1 in Eqn. (60). Since Youngdahl's correlation parameters are insensitive to small perturbations of the pulse, it transpires that the various pulse shapes virtually fall onto a single curve, when using $\eta_e = p_e/p_c$. This obviates the need to record the pressure time histories and model the dynamic loading for design purposes, as it is practically difficult to do so by laboratory tests.

Performing the analyses in Section 4 with effective pressure from Eqn.'s A. 8-A. 10, compiled with Eqn.'s A. 1-A. 2, a single plot can be obtained for the (effective) permanent displacement in terms of effective impulse and pressure, as presented in Fig. A. 3. The influence of load parameters β and η on permanent displacement in such case is presented in Fig. A.4.