Stochastic analysis of seepage under water-retaining structures

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Abstract

This paper investigated the problem of confined flow under dams and water retaining structures using stochastic modelling. The approach advocated in the study combined a finite elements method based on the equation governing the dynamics of incompressible fluid flow through a porous medium with a random field generated hydraulic conductivity using a lognormal probability distribution. The resulting model was then used to analyse confined flow under a hydraulic structure. Cases for a structure provided with cutoff wall and when the wall did not exist were both tested. Various statistical parameters that reflected different degrees of heterogeneity were examined and the changes in the mean seepage flow, the mean uplift force and the mean exit gradient observed under the structure were analysed. Results reveal that under heterogeneous conditions, the reduction made by the sheetpile in the uplift force and exit hydraulic gradient may be underestimated when deterministic solutions are used.

Keywords: Confined flow; Dams; Water retaining structures; Stochastic analysis; Uplift force, Hydraulic conductivity, Exit gradient.

1. Introduction

The spatial variability in hydraulic conductivity and other soil properties has frequently been observed in real field sites. Through numerous detailed hydraulic conductivity measurements at the Borden site in Ontario, Sudicky (1986) observed that the hydraulic conductivity varies irregularly in three-dimensional space. The study tested 32 cores, each of which is approximately 2 m long and found that the hydraulic conductivity ranged between 6.0×10^{-4} and 2.0×10^{-2} cm/sec, i.e. more than a factor of 30. In another field study, Hicks and Onisiphorou (2005) analysed data from 71 CPTs on sands and found large variability in the statistics and spatial correlation coefficient of shear strength parameters. Other examples of spatial variability of soils have also been reported by Phoon and Kuthway (1999).

Traditional deterministic approaches, for analysing flow problems under water-retaining structures, generally represent soil properties using one single value. At most, the designer assigns different soil properties to individual layers of the soil. Hence, the obtained solutions do not account for the inherent variability of soils. This leads to the conclusion that results obtained using deterministic solutions, which disregard the variability in soil properties, may suffer from serious deficiencies in many cases (Ahmed, 2009, 2013).

Stochastic approaches based on probabilistic distributions of soil properties provide a framework for addressing more effectively the aforementioned major deficiencies of deterministic methods. Freeze (1975) was among the pioneers of stochastic analysis of flow problems in porous media. This seminal work has inspired many other studies dedicated to the analysis of water flow problems using stochastic approaches (e.g. Griffiths and Fenton, 1997, 1998; Ahmed, 2009, 2013, 2014).

The most commonly used approach to account for soil heterogeneity is to assume homogeneous soil formations of several layers, each having its own soil properties. In contrast with some previous studies, which assessed the effectiveness of the cutoff walls using such approach or by assuming homogenous soil formations (e.g. Ahmed and Bazaraa, 2009; Ahmed, 2011), the research work in this paper assumed heterogeneous random soil to investigate the effectiveness of cutoff walls. It is therefore the objective of this paper is to account for the effectiveness of the sheet pile or cutoff walls in reducing the seepage losses, the downstream uplift force, and the exit hydraulic gradient based on heterogeneous random soil. The methodology adopted here reflects the variability in soil properties that exist in real world problems. More specifically, the application problem consisted of a hydraulic structure subject to two different scenarios, namely with and without sheet pile. Various coefficients of variation and correlation length were examined to simulate sites with different degrees of heterogeneity. The hydraulic conductivity resulted from the random field generator was then mapped to a finite element model, which estimates the seepage flow parameters. The corresponding changes in the mean seepage flow, the mean uplift force and the mean exit gradient were analysed. Furthermore, the results produced using the stochastic approach, were contrasted with those obtained using a deterministic method. The obtained results provide some valuable insights for understanding water flow problems, hence enabling a framework for improvement in the design of water-retaining structures.

2. The Finite Element Model

The finite element part of the model used in this study is based on the partial differential equation governing steady incompressible fluid flow through porous media for both confined and free surface flow problems. For a detailed presentation of this part of the model as well as its validation and applications, we refer the reader to Ahmed (2008, 2011) and Ahmed and Bazaraa (2009). The corresponding partial differential equation writes:

(1) $\text{div}(k\ \text{grad}\phi) = 0,$

Where, k is the hydraulic conductivity of the medium, $\phi = \frac{P}{\gamma} + Z$ is the total fluid head, $\frac{P}{\gamma}$ is the pressure head, Z is the elevation head, and γ is the unit weight of the fluid. The pseudo-functional for the steady state flow, denoted U, can be expressed as follows:

(2) $$U = \frac{1}{2} \iiint_V k \left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2 + \left(\frac{\partial\phi}{\partial z}\right)^2 \right] \mathrm{d}x\ \mathrm{d}y\ \mathrm{d}z.$$

Applying the residual flow procedure (Desai and Baseghi, 1988) yields the following element equations:

(3) $[k]^e\{q\} = \{Q_r\}^e,$

where, $[k_s]^e$ is the element hydraulic conductivity matrix at saturation, $\{q\}$ is the vector of nodal fluid heads of the elements, and $\{Q_r\}^e$ is the element residual flow vector.

The assembly over all elements yields the following equation on the entire domain:

(4) $[K_s]\{r\} = \{R_r\},$

where, $[K_s]$ is the overall hydraulic conductivity matrix at saturation, $\{r\}$ is the overall nodal fluid head vector, and $\{R_r\}$ is the overall residual flow vector.

3. The Random Field Model

A lognormal distribution is commonly adopted to describe the probability density function of the soil hydraulic conductivity (e.g. Sudicky, 1986; Ahmed, 2009). The saturated hydraulic conductivity field was obtained through the transformation (Griffths and Fenton, 1997):

(5) $k_i = \exp(\mu_{\ln k} + \sigma_{\ln k}\, g_i),$

where, k_i is the hydraulic conductivity assigned to the i^{th} element, g_i is local average of a standard Gaussian random field g over the domain of the i^{th} element, and $\mu_{\ln k}$ and $\sigma_{\ln k}$ are the mean and standard deviation of the logarithm of k, respectively, obtained via the transformations (Griffiths and Fenton 1997):

$$(6) \qquad \sigma_{\ln k}^2 = \ln\left(1 + \frac{\sigma_k^2}{\mu_k^2}\right),$$

$$(7) \qquad \mu_{\ln k} = \ln(\mu_k) - \frac{1}{2}\,\sigma_{\ln k}^2,$$

where, μ_k and σ_k denote the mean and standard deviation of k, respectively.

The Local Average Subdivision (LAS) technique (Fenton and Vanmarcke, 1990; Ahmed, 2009, 2013) was used to generate correlated local averages, g_i, based on a Gaussian probability distribution function, which has zero mean and unit variance, and a Gauss-Markov spatial correlation function:

$$(8) \qquad \rho(\tau) = \exp\left(-\frac{2}{\theta}|\tau|\right),$$

where, $|\tau|$ is the distance between points in the field. The scale of fluctuation θ is a measure of the distance between adjacent strong or weak zones. Larger value of θ leads to a more spatially uniform hydraulic conductivity field. In contrast, smaller value of θ means that the hydraulic conductivity varies rapidly from point to point in the random field.

Based on the above equations, a random field generator was used to generate the hydraulic conductivity distributions.

4. Description of the Application-Problem and the Analysis Procedure

The application problem, shown in Fig 1, deals with confined seepage under a hydraulic structure with a floor length of 15 m and resting on a pervious stratum of 5 m depth. The upstream water head was 1 m, while the downstream head was zero. The length of the modeled zone was 45 m between two vertical impervious boundaries located 15 m upstream and 15 m downstream of the floor of the structure. The mesh comprised square elements of 0.25 m. This small element size was used to accurately model random fields with a small scale of fluctuation. The mean hydraulic conductivity was held at $1\text{x}10^{-5}$ m/s, the coefficient of variation $\text{COV}={\sigma_k}/{\mu_k}$ ranged from 0.125 to 8, and the scale of fluctuation ranged from 1 to 16 m. There was a sheetpile wall of 3 m depth driven under the midpoint of the floor for part of the analysis.

5. Results and Discussion

5.1 Case 1: Sheetpile is driven under the floor

Fig 3 shows the change of seepage flow as a function of the coefficient of variation for different scales of fluctuation. The deterministic solution of this problem produced normalized seepage flow Q/kH = 0.222, which agrees well with the analytical solution of Harr (1962).

5.1.1 Mean Seepage Flow

The mean seepage flow was significantly reduced as the coefficient of variation increased (Fig 3). Likewise, smaller scales of fluctuation reduced the seepage flow. The explanation of this reduction in seepage flow lies in the fact that, for weakly correlated field having small scale of fluctuation, the hydraulic conductivity is very changeable. Cells that have low hydraulic conductivity behave like blocks in the way of seeping water; and because they are spread all over the domain, the overall seepage flow is reduced. It can be noticed that as θ became higher, the seepage flow moved towards the deterministic analysis. This is expected, since for higher scale of fluctuation the field tends to become uniform; hence the mean seepage flow value of the 2000 realisations moves towards the deterministic value. When θ varied from 8 to 16 m, the change in the seepage flow was slight.

5.1.2 Mean Uplift Pressure

In contrast to seepage flow, as the scale of fluctuation became larger, the uplift force decreased (Fig 4). This reduction in the uplift force was more pronounced for larger values of the coefficient of variation. This can be explained in similar way to seepage flow; for smaller scales of fluctuations, the hydraulic conductivity is very changeable over the field; hence lower hydraulic conductivity cells work as blocks in the way of the flow. As a result, the seeping water accumulates in front of these blocks, which builds up uplift pressure.

This last point may be clarified with reference to the uplift pressure distribution according to the method of Bligh (Leliavsky, 1965) as shown in Fig 5. The plot is for a case of hydraulic structure with sheetpile at the end toe of the floor. This particular case was chosen because it will clarify the point easily. In Fig 5, the value of uplift pressure at the end of the downstream toe was $h=(t+2d)/C_b$ where h is the uplift pressure, t is the floor thickness, d is the sheetpile depth, and C_b is the coefficient of Bligh. When the sheetpile was off, the value of the uplift pressure at the same point was $h=t/C_b$. The value of the uplift pressure at the start point of the floor H for both cases. As a result, the uplift pressure at any point on the floor is greater when the sheetpile was on (Case 1) compared to the case where the sheetpile was off (Case 2). A similar scenario happens when low hydraulic conductivity cells in the field block the way of seeping water; these cells play the role of sheetpile in Fig 5, and this led to a greater uplift force compared to the case without sheetpile.

5.1.2 Mean Exit Hydraulic Gradient

As the scale of fluctuation increased, the exit gradient became greater (Fig 6). However, this happened only for $\theta \leq 8$ m after which the exit gradient declined. Hence, the maximum exit gradient was attained at θ=8 m. Therefore, it appears that there is a value of the scale of fluctuation at which the mean exit hydraulic gradient attains its maximum value.

This result is in agreement with results from other studies (e.g. Griffiths and Fenton, 1998; Ahmed, 2013). Ahmed (2013) observed similar results for anisotropic heterogeneous soil, in which the exit hydraulic gradient attained its maximum value at anisotropic heterogeneity ratio of 3. However, in the work of Griffiths and Fenton (1998), the exit gradient was the greatest at θ =2 m. However the dimensions of the current problem being investigated were different from that of Griffiths and Fenton. In addition, Griffiths and Fenton (1998) investigated a problem without the floor (it was just a simple sheetpile problem with penetration depth equals half of depth of the pervious stratum). We therefore solved our problem for the case when there was no floor and found θ =8 m also produced the greatest exit gradient when there was no floor. This means that the discrepancy between our results and results of Griffiths and Fenton is mainly due to the difference in the geometry and dimensions of the investigated problems. It is interesting to note that the ratio of the long dimension to the scale of fluctuation that produced the highest exit gradient was 12.8/2=6.4 in Griffiths and Fenton's problem whereas it was 45.0/8=5.6 in our problem.

The results of exit gradient, in general, showed that any deviation from homogeneous medium produced greater exit gradient (Fig 6). The exit gradient was higher, for any value of COV and θ, than its value obtained from a deterministic solution which was 0.1435.

5.2 Case 2: No Sheetpile under the Floor
5.2.1 Mean Seepage Flow

Results of seepage under hydraulic structure with no sheetpile under the floor (Case 2) showed different behaviour from the case when the sheetpile was enabled (Case 1). In case 1, the mean seepage flow increased steadily with the increase of the scale of fluctuation. However in Case 2, the value $\theta = 4$ produced the greatest seepage flow under the structure (Fig 7). As in Case 1, increasing the coefficient of variation decreased the seepage flow.

The deterministic solution of the problem shows a reduction of the flow rate *Q/kH* from 0.26 for Case 2 to 0.22 for Case 1, i.e. a reduction of about 15%. This means the sheetpile reduced the flow by somewhat 14%. When both the coefficient of variation and the scale of fluctuation equaled 4, the flow rate dropped from 0.12 to 0.08, i.e. a reduction of about 33% because of the sheetpile derivation. This means that the reductions in seepage losses, due to the sheetpile derivation under the floor of the structure, may not be accurate when the soil is regarded as homogenous.

5.2.2. Mean uplift force

The uplift force showed a different behaviour in Case 2 compared to Case 1. In Case 1, the uplift steadily decreased as the scale of fluctuation θ becomes greater. The difference in the uplift force for different values of coefficient of variation was more pronounced at larger values of the coefficient of fluctuation. In Case 2, the value θ =2 showed greater mean uplift force than other values of θ (Fig 8). The only exception from this is when θ =16, which produced greater mean uplift force.

It appears for Case 2 that the mean seepage flow and uplift force reached their maximum values at some particular values of the scale of fluctuation. In our problem, these values were in the range θ =2 – 4.

5.2.2. Mean exit hydraulic gradient

Results of the mean exit hydraulic gradient for Case 2 (Fig 9) showed a different behaviour compared to those in Case 1 (Fig 6). For each coefficient of variation, the value of the scale of fluctuation θ that resulted in the maximum mean exit hydraulic gradient was within the range $\theta = 2m - 4m$. This is different from Case 1, in which the maximum exit hydraulic gradient was attained at the scale of fluctuation θ =*8*.

The above results confirm the need to consider the soil variability when designing water retaining structures, as recommended by Euro code 7. The case when a sheetpile was driven out below the structure has produced different structural response from the case when there was no sheetpile. This happened even for the same problem.

5.3 Effectiveness of the cutoff wall

The deterministic exit hydraulic gradient below the structure was reduced by about 15% when a cutoff wall was installed at the middle of the floor. This happened also for the case of homogeneous soil formation. However, the stochastic solution of the problem produced different reductions, and was heavily dependent on the scale of fluctuation. For example, when the coefficient of variation equalled 8, different scales of fluctuations showed different reductions in exit hydraulic gradient caused by the sheetpile. The reduction in the hydraulic gradient varied from zero for $\theta = 8$ to 25% for $\theta = 2$. The value θ =2 produced the greatest reductions in the exit hydraulic gradient for all values of coefficient of variation. In contrast, the value $\theta = 8$ produced the smallest reduction regardless of the variation coefficient. As expected, smaller coefficients of variation produced nearly the same reduction as in the case of homogeneous soil, which is about 15%.

The reductions in the uplift force due to the cutoff wall were also found to be significantly dependent on the degree of heterogeneity. A homogenous soil produced about 21% reduction in the uplift force caused by the cutoff wall as shown by deterministic results. As the soil became homogeneous, the reductions in the uplift force due to the cutoff wall appeared to increase. Increasing the coefficient of variation consistently increased the reductions caused by the cutoff wall on the uplift pressure, and these can reach up to about 45% for COV=8 and $\theta = 16$. This means that results, obtained from the deterministic solution, provide extra factor of safety because it produces lower reductions in the uplift force due to the cutoff wall compared to the real heterogeneous soil. The case $\theta = 16$ produced the greatest uplift reductions for all values of the coefficient of variation.

The seepage losses flowing below the structure for homogenous soil was reduced by 14% as a result of cutoff wall. For heterogeneous soil, the flow rate's greatest reduction happened when θ =2. It is the same value of θ that resulted in the greatest reduction in the exit hydraulic gradient. Interestingly, values of $\theta = 1$, 8, and 16 produced seepage losses less than the homogeneous deterministic solution. This happened for larger coefficient of variations i.e. $\text{COV} \geq 2$. The value $\theta = 8$ gave the greatest seepage losses. Obviously, for larger values of θ, the domain is strongly correlated and this creates preferential paths of high permeability for the water to flow.

The above results demonstrate that heterogeneity of the site has a great influence on the design parameters of hydraulic structures such as the uplift force. Ignoring the effect of site heterogeneity at the design stage would result in the use of high factor of safety to account for the uncertainty in the design parameters, which leads to more costs for the structure. For this reason, Euro code 7 has recommended the consideration of variability in soil parameters in geotechnical design. Probabilistic analysis provides the most appropriate framework to account for this variability in soil properties.

6. Summary and Conclusions

Inherent variability of soils is inevitable and the representation of this variability is important to have more realistic understanding of water flow problems. The present study investigated the problem of confined seepage under hydraulic structure using a stochastic approach, which combined both the random field theory and the finite element method. Wide ranges of coefficient of variation as well as the scale of fluctuation were examined. A distinctive feature of the current study is that it enables the consideration of high values of the coefficient of variation, which is not the case in other probabilistic methods such as the perturbation method. The perturbation method is only suitable for cases having a small coefficient of variation less than 20%.

Results of the present study showed that the seepage flow became lower as the coefficient of variation increased. Likewise, smaller scales of fluctuation have also reduced the seepage flow under the structure. This happened when a cutoff wall was driven under the structure. When there was no cutoff wall, the maximum flow occurred when the scale of fluctuation equalled 4.

Different behaviour was observed in the case of uplift force; that is, as the scale of fluctuation became higher, the uplift force decreased. Increasing the coefficient of variation also lowered the uplift force under the structure. Larger values of the coefficient of variation had more impact on the uplift force than smaller coefficients of variation.

Likewise, the exit hydraulic gradient attained its maximum value at different scale of fluctuation for the case when a cutoff wall existed compared to the case without cutoff. In the first case, the exit gradient was greatest when the scale of fluctuation equalled 8 while in the second case, this corresponds to the range $\theta = 2 - 4$.

The effectiveness of the cutoff walls obtained from the deterministic solution was found to be greatly different from the stochastic solution of the problem. The latter can handle the soil heterogeneity that exists in real world problems. In heterogeneous soil, the effectiveness of the cutoff wall was found to be heavily dependent on the coefficient of variation and the scale of fluctuation, which represent different degrees of heterogeneity. Such heterogeneity cannot be reflected when deterministic methods are used. This shows the importance of using probabilistic methods when analysing seepage flow problems through dams and under hydraulic structures.

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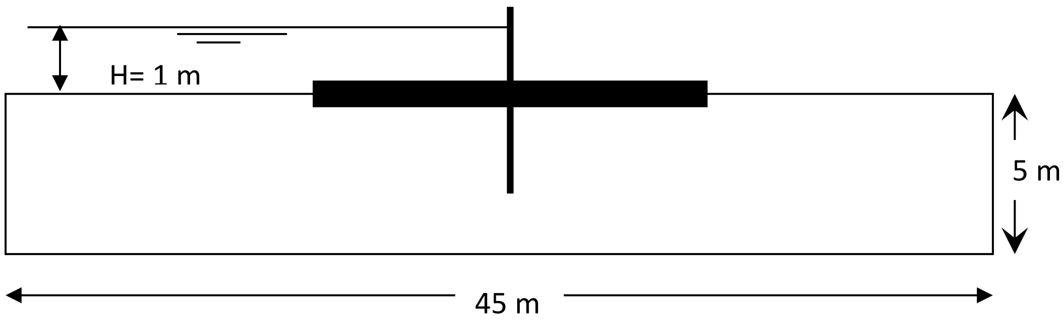

Fig 1. Confined seepage under a hydraulic structure

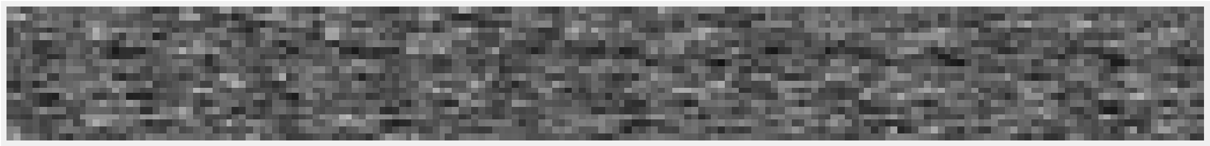

Fig 2. Distribution of hydraulic conductivity for a typical realisation; each element has its own hydraulic conductivity.

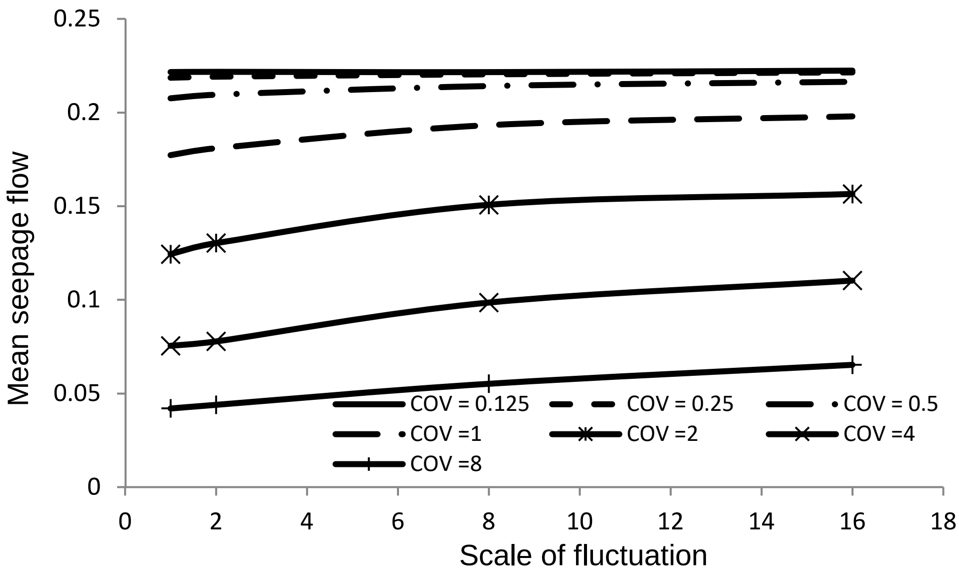

Fig 3.Influence of the scale of fluctuation θ on seepage flow (sheetpile on).

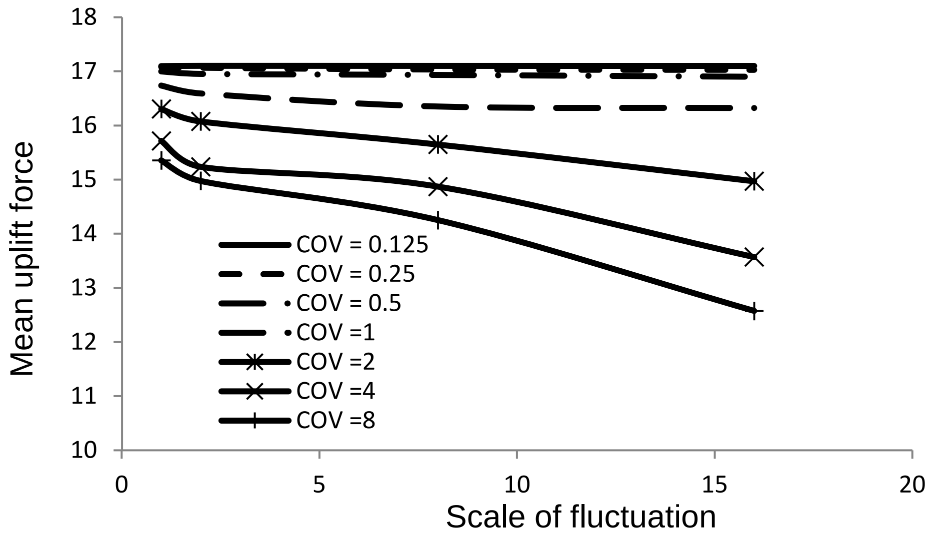

Fig 4.Influence of the scale of fluctuation θ on downstream uplift force (sheetpile on).

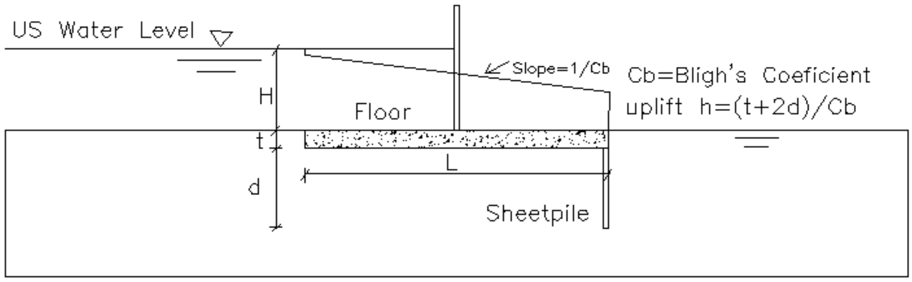

Fig 5. Distribution of the uplift pressure according to the method of Bligh.

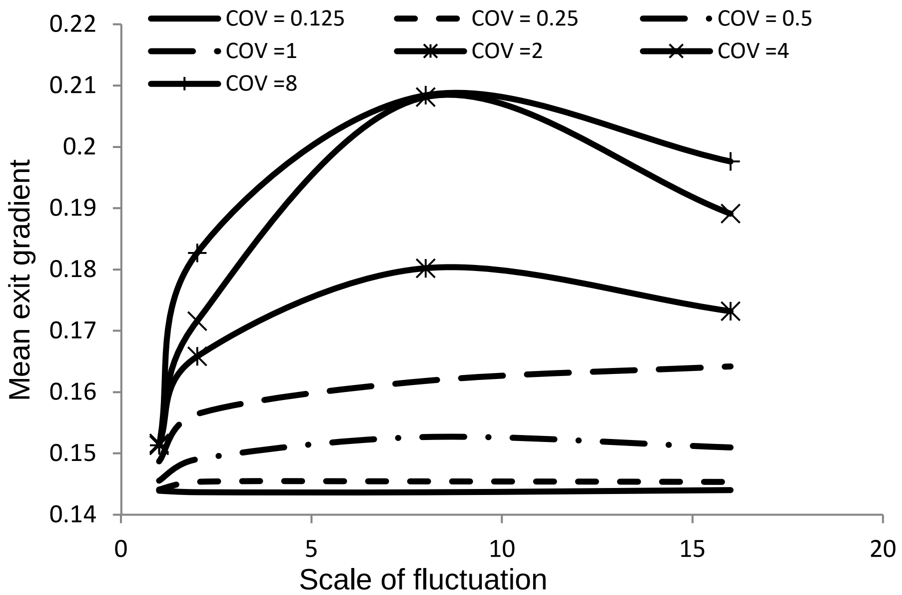

Fig 6.Influence of the scale of fluctuation θ on exit gradient (sheetpile on).

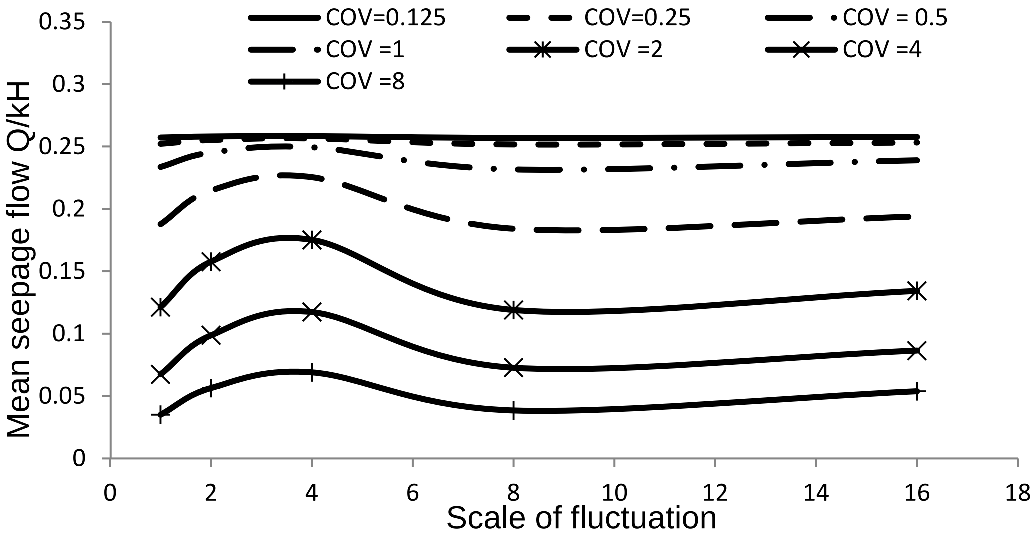

Fig 7.Influence of the scale of fluctuation θ on the mean seepage flow (no sheetpile).

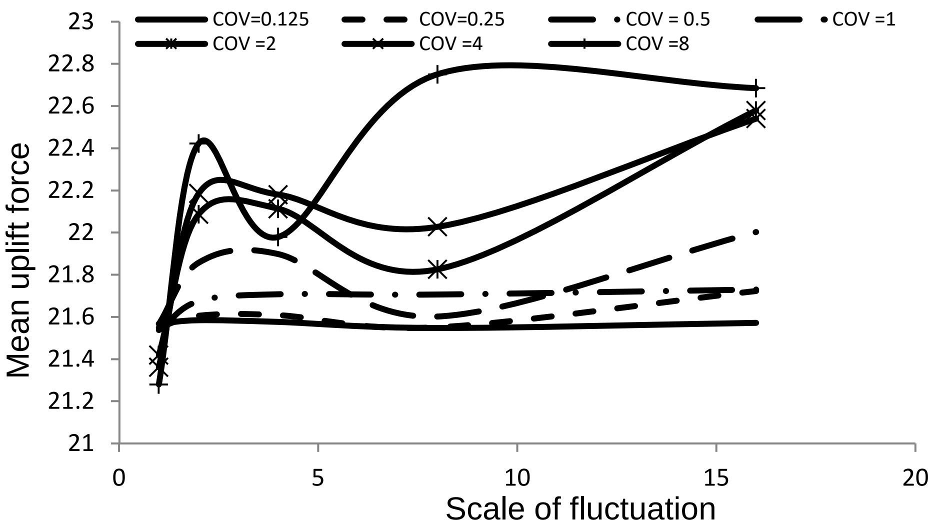

Fig 8.Influence of the scale of fluctuation θ on the uplift pressure (no sheetpile).

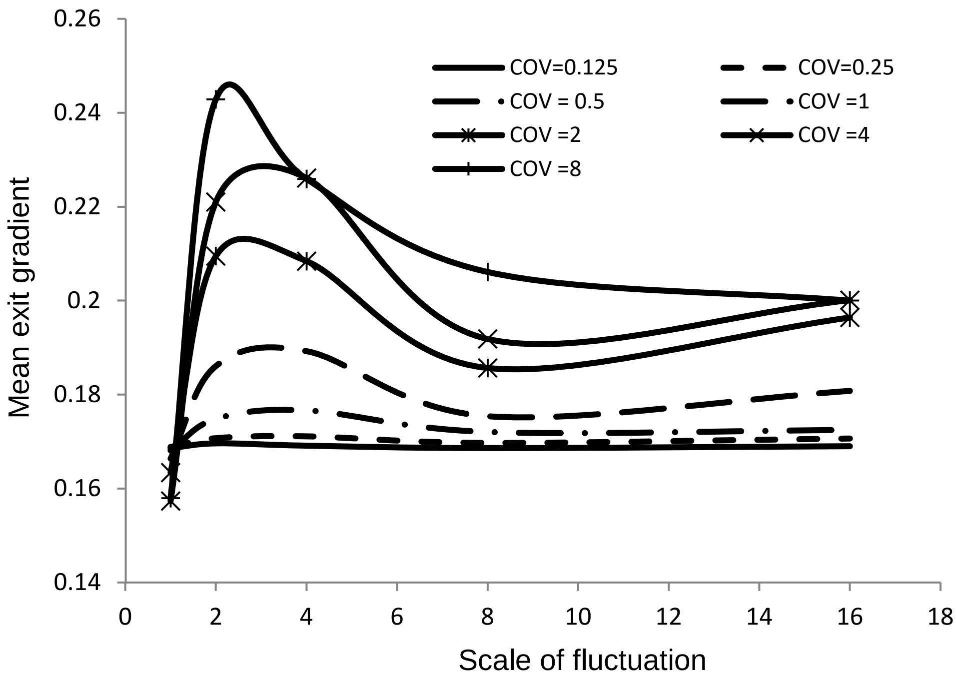

Fig 9.Influence of the scale of fluctuation θ on the exit hydraulic gradient (no sheetpile).