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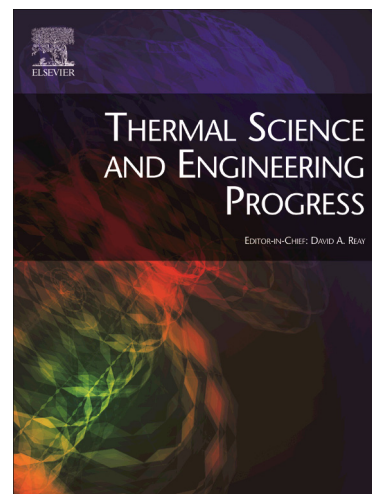
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Mathematical model of sulphate ion concentration in a closed cooling system of a power plant

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Abstract

In commercial power plants, water is used in many processes and its physical and chemical properties have a significant impact on the efficiency of energy and heat production as well as failure-free operation. One of the largest consumers of water in a power unit is the cooling system consisting of condensers and cooling towers. In cooling towers, the main mechanism for the decrease in temperature of the water is its partial evaporation, which causes a gradual decrease in the amount of circulating water and, on the other hand, a continuous increase in the concentration of chemical compounds in the closed system. Among others, an uncontrolled increase in the sulphate ion concentration in cooling water may cause the corrosion of the concrete parts of the hydraulic system as well as an increase in the deposition of calcium salts on the surfaces of the heat exchangers, thereby worsening the heat exchange processes inside the condenser.

The daily demand for fresh water in power plants often reaches tens of thousands of cubic metres and so the amount of wastewater released also has a significant influence on the environment. Therefore the Polish Ministry of Environment and EU directives have introduced, from the beginning of 2016, new limits on the physical and chemical parameters of wastewater released to natural reservoirs. Taking into account the previous regulations, the authors present a mathematical model which allows the prediction of the daily changes of the sulphate ion concentration in the circulating water in a condenser - cooling tower closed cooling system and the calculation of the minimum wastewater flow rate fulfilling legal restrictions.

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1. Introduction

In commercial power plants, water is commonly used in many industrial processes, including, for example, the water-steam cycle in power boilers, the cooling installations in boiler auxiliary equipment or cooling systems operating with condensers [1, 2]. Each of these processes requires continuous access to fresh water with suitable physical and chemical properties. Therefore the quality of water has a significant impact on the efficiency of energy and heat production as well as failure-free operation of power units. Taking into account that water is mainly used in heat transfer processes [3], its chemical composition has an influence on the long-term utilization of all kinds of heat exchangers.

It is a well-documented fact that the efficiency of coal-fired power plants gradually decreases with age [4]. It results in the higher emission of the greenhouse gasses as well as other pollutants (which are mainly SO_2 i NO_x). Abovementioned observation applies not only to power boilers but also to internal combustion engines [5-7].

Keeping in mind that the improvement of heat transfer processes could have a direct influence on the overall efficiency of a power unit, Ryabchikov et al. [8] suggested a number of specific actions, among which the retrofit and investigation of cooling water installations are of the most importance.

One of the largest consumers of water in a power unit is the cooling system consisting of condensers and cooling towers. The influence of weather conditions as well as water consumption on the working parameters of cooling towers was recently investigated by Mariano Martín and Mónica Martín [9]. They pointed to the seasonal and climatic impacts on the limitations in the heat transfer capacity of cooling systems. Weiliang Wang et al. [10] also investigated the possibility of heat exchange improvement of cooling towers considering the

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influence of windbreak installations and enclosures on the cooling performance and the flow field characteristics inside cooling towers. The interest in these elements of cooling system design arises from the fact that improving heat transfer inside cooling towers has a direct influence on a better vacuum inside the condensers of the low pressure turbine resulting in an increase in the power unit efficiency.

The main mechanism of the temperature decrease in the water inside cooling towers is its partial evaporation, which causes a gradual decrease in the amount of circulating water and, consequently, a continuous increase in the concentration of chemical compounds in the closed system [11]. Among others, an uncontrolled increase in the sulphate ion concentration in cooling water may cause corrosion in the concrete parts of the channels and cooling towers [12, 13] as well as an increase in the deposition of calcium salts on the heat exchanger surfaces, thereby worsening the heat exchange processes inside the condenser.

In order to minimize above-mentioned effects, the circulating water is regularly refreshed, which leads to an increase in the demand for fresh water from natural resources. Having in mind that the daily demand for fresh water in power plants often reaches tens of thousands of cubic metres, the amount of wastewater released from the cooling system also has a significant influence on the environment [14].

In response, the Environmental Protection Agency (EPA) in the USA pointed out that power plants are one of the largest consumers of natural water resources in the USA, recommending, in September 2013, new standards for the control of carbon dioxide (CO_2) emissions from new power units burning fossil fuels and regulations concerning the protection of water resources. It is also worth mentioning that the deterioration of the overall power unit efficiency results in a greater consumption of fossil fuels and consequently higher CO_2 emissions (as well as those of other pollutants) [15]. The Polish Ministry of Environment and EU authorities have also noticed this unfavourable effect and introduced, at the beginning of

2016, new limits on the physical and chemical parameters of wastewater released to natural reservoirs [16]. The cooling system's demand for large amounts of water on the one hand, and the recent legal regulations connected with the chemical parameters of wastewater on the other, mean that power plant operators are forced to introduce optimal methods of water resource management.

Efficient wastewater management needs a scientific background and is based on mathematical models. Focusing on the optimization of fresh water consumption, the authors present an original mathematical model, which allows for the prediction of daily changes of sulphate ion concentration in water circulating in a closed cooling system consisting of condensers and cooling towers.

2. Materials and Methods

2.1 Closed cooling system

The closed cooling system is one of the largest fresh water consumers in a power plant. As a result, knowledge about the scientific processes influencing changes in the sulphate ion concentration in the installation is essential for any attempts at establishing a mathematical model. The scheme of a closed cooling water system is presented in Fig. 1.
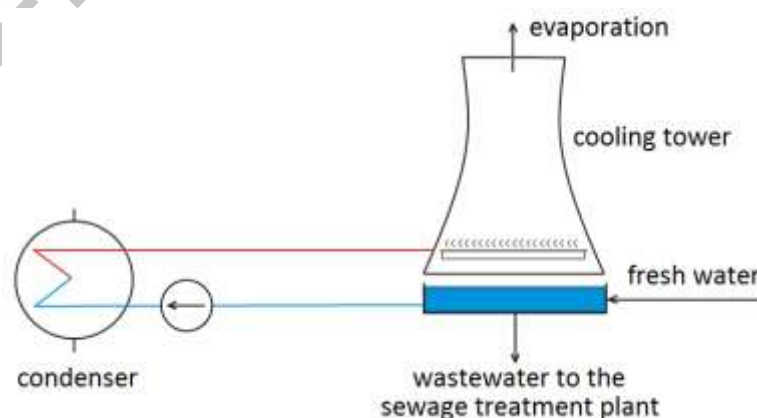
Fig. 1. Scheme of a closed system of cooling water in a power plant.

The water circulating in this system is heated in a condenser by condensing steam from the low pressure turbine, and it is then cooled down in a cooling tower. The main mechanism of

water temperature decrease inside the cooling tower is its partial evaporation in contact with counter flowing air [11]. This phenomenon causes a continuous decrease in the mass of the water in the installation and it has a direct influence on the increase of the concentration of chemical compounds (among others, the above-mentioned sulphate (VI) ions) in the circulating water. Due to the fact that the volume of water circulating in the system is usually at the level of hundreds of thousands of cubic metres, it is impossible to apply any chemical methods to reduce the concentration of sulphate ions. Fresh water, before delivery to the closed cooling system, is only mechanically cleaned of solid impurities by means of a set of sieves. So, the only reasonable method of avoiding the uncontrolled increase in the undesirable SO_4^{-2} ion concentration in cooling water, is the periodic discharge of wastewater to a sewage treatment plant. In order to maintain a constant amount of water in the cooling installation, the system is replenished with water from natural resources. This action leads to the reduction of the total concentration of SO_4^{-2} ions in the closed system because the fresh water usually has a lower concentration of sulphate ions in comparison to its actual concentration in the closed cooling system.

Measurements carried out in a commercial power plant indicate seasonal changes in the concentration of SO_4^{-2} ions (denoted by x, [g/m^3]). Sample results are presented in Table 1.

Table 1. Monthly changes of mean concentration of sulphate ions x and mean temperature of cooling water T at the bottom reservoir of a cooling tower.

Month	I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
T, °C	15.7	14.0	14.5	17.4	19.1	19.9	21.2	22.0	20.6	20.4	16.7	16.9
x, g/m^3	175.0	160.0	190.2	206.4	252.0	303.7	313.0	316.2	253.6	276.4	190.7	195.4

The data presented indicate that the mean monthly concentration of SO_4^{-2} ions changes over the course of the year. There are two significant factors: the high intensity of water

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evaporation in the cooling tower during the summer and the reduction of the operational parameters of the cooling tower in winter. It is observed, that in summer about 90% of the heat transfer is due to evaporation, which causes approximately 1% of the mass flow rate of circulating water to evaporate in the cooling towers [11]. The intensive evaporation of circulating water is also influenced by weather conditions, mainly through the relative humidity or the orientation and strength of local winds [9, 10]. On the other hand, in winter the intensity of water evaporation is limited not only by the low ambient temperature but also for technological reasons. The minimum temperature of the cooling water in the installation is restricted to no less than 11 - 12 °C. In order to maintain the temperature of the circulating water at above 11 °C in winter only part of it is cooled down in the cooling towers with a subsequent mixing process in the bottom reservoirs between this cooler water and the remaining hotter water.

Analyzing the data from Table 1 it is easy to notice that the intensity of water loss is reflected in the highest value of SO_4^{-2} ion concentration in July and August (up to ~320 g/m^3) in contrast with the lowest concentration in February (approx. 160 g/m^3). Keeping in mind that legal regulations allow for a much higher SO_4^{-2} ion concentration in wastewater (maximally $x_{limit} = 500$ g/m^3 [16]), the operator of the closed cooling system releases more sewage than required. This situation provides an opportunity to optimize wastewater management in order to reduce the operating costs of the system.

2.2 Mathematical model

Analysing the thermal−flow processes in a closed cooling system, the authors consider three main physical mechanisms leading to changes in the sulphate ion concentration in the circulating water:

- evaporation of water in cooling towers, $q_{v,ct}$,

- replenishment of fresh water, $q_{v,fw}$,

- discharge of wastewater to a sewage treatment plant, $q_{v,dw}$.

Due to the fact that the total volume of circulating water in the closed system V must be conserved, the conservation law for volumetric flow rates is satisfied:

$$q_{v,fw} = q_{v,ct} + q_{v,dw} \tag{1}$$

The analysis of losses and replenishment of water in the closed system has allowed the authors to formulate a differential equation describing the changes of SO_4^{-2} concentration in circulating water (t is time in days):

$$V\frac{dx(t)}{dt} = -q_{v,dw}(t) \cdot x(t) - q_{v,ct}(t) \cdot x_{ct} + q_{v,fw}(t) \cdot x_{fw}(t) \tag{2}$$

where: $x(t)$ is the concentration of sulphate ions in the circulating water, x_{ct} is the sulphate ion concentration in vapour released in the cooling tower and $x_{fw}(t)$ is the concentration of sulphate ions in fresh water. All the concentrations are in g/m^3, the volumetric flow rates in m^3/day, and the total volume V in m^3.

Equation (2) can be analytically solved under the assumptions that volumetric flow rates $q_{v,dw}$ and $q_{v,ct}$ are constant over time, sulphate ion concentration in fresh water x_{fw} is fixed and vapour leaving the cooling tower is chemically clean ($x_{ct} = 0$ g/m^3). This reduces equation (2) to the form of a first order ordinary differential equation with constant coefficients a and b:

$$\frac{d\,x(t)}{dt} + a \cdot x(t) - b = 0 \tag{3}$$

where: $a = q_{v,dw}/V$ and $b = q_{v,fw} \cdot x_{fw}/V$.

The solution of (3) is the function $x(t)$, which describes a change in SO_4^{-2} concentration over time:

$$x(t) = \frac{b}{a} + \left(x_0 - \frac{b}{a}\right) \cdot e^{-at} \tag{4}$$

where $x_0 = x(t_0)$ is the initial SO_4^{-2} ion concentration in the closed cooling system.

It is worth noting that the solution (4) does not have physical sense for coefficient $b = q_{v,fw} \cdot x_{fw}/V$ equal to zero because in this case:

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- either the mass conservation law (2) is not satisfied (if $q_{v,fw}$ is zero then the volume of circulating water V tends to zero over time)

- or the concentration of sulphate ions in fresh water x_{fw} is zero (which means that fresh water is chemically clean or subjected to chemical purification processes).

In both above-mentioned cases the total volume of circulating water decreases over time due to the partial evaporation and wastewater discharge effects which cannot happen in real cooling systems.

In addition, in the solution (4) the coefficient $a = q_{v,dw}/V$ must also have a nonzero value (which means that $q_{v,dw} \neq 0$), because otherwise the equation (2) reduces to the form:

$$\frac{d\,x(t)}{dt} - b = 0 \tag{5}$$

which has the linear solution $x(t) = b \cdot t + x_0$ and the concentration $x(t)$ tends to infinity over time. Finally, the case when the volumetric flow rate of wastewater $q_{v,dw} = 0$ (therefore $a = 0$) leads either to continuous increase of $x(t)$ (the situation when $q_{v,fw}$ and x_{fw} are not simultaneously equal to zero) or to the constant value of $x(t) = x_0$ in case when $q_{v,fw} \neq 0$ and $x_{fw} = 0$ (and in consequence $b = 0$). These cases also are not observed in real cooling systems of power units.

The SO_4^{-2} ion concentration in fresh water varies between 20 - 40 g/m^3 depending on local chemical conditions; thus, in the model, the authors have used $x_{fw} = 40$ g/m^3. Figure 2 presents an example of solutions (4) obtained for $q_{v,ct} = 50000$ m^3/day and the total volume of the circulating water V equal to 100000 m^3.
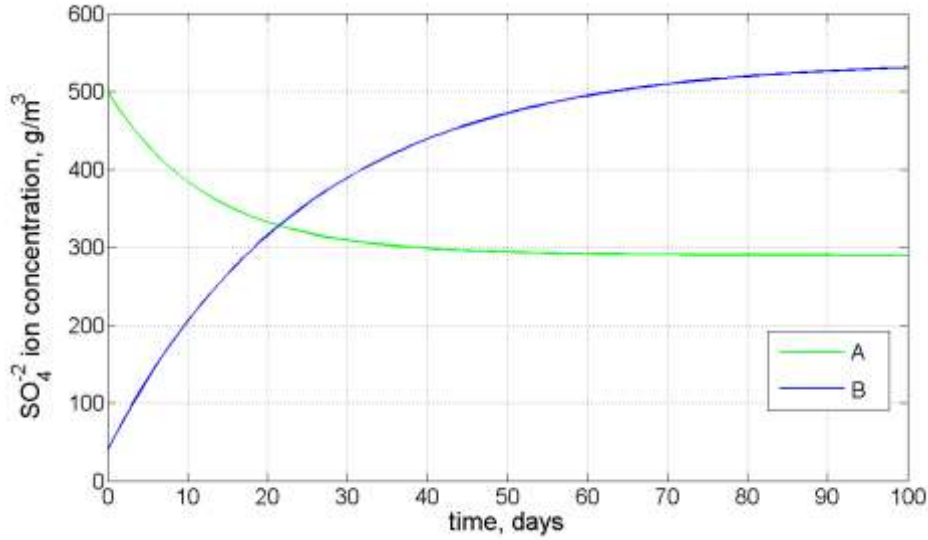
Fig. 2. Example of analytical solutions of (4) for different initial conditions: A (green line) - $x_0 = 500$ g/m^3 and $q_{v,dw} = 8000$ m^3/day; B (blue line) - $x_0 = 40$ g/m^3, $q_{v,dw} = 4000$ m^3/day.

The curves A and B in Figure 2 present two possible solutions of (4) depending on the initial condition x_0 and values of the coefficients a and b. For a fixed volumetric flow rate of evaporated water inside cooling towers $q_{v,ct}$ and a fixed total volume of water circulating in the installation V, the value of the discharge of wastewater to a sewage treatment plant $q_{v,dw}$ has a decisive influence on the shape of the curves. In case A, the flow rate $q_{v,dw} = 8000$ m^3/day causes a decrease in the initial SO$_4^{-2}$ ion concentration over time to the level 290 g/m^3, while in case B the low value of $q_{v,dw} = 4000$ m^3/day causes the increase of $x(t)$ over time to 540 g/m^3. It is worth noting that both curves have the asymptotic solutions x_∞, calculated for both cases from the limit:

$$x_\infty = \lim_{t \to \infty} x(t) = \lim_{t \to \infty} \left[\frac{b}{a} + \left(x_0 - \frac{b}{a}\right) \cdot e^{-at} \right] = \frac{b}{a} \tag{6}$$

From a mathematical point of view, a constant value of x is obtained when the bracket $(x_0 - b/a) = (x_0 - x_\infty)$ in formula (4) is equal to zero. In this case $x(t)$ and x_∞ are simply equal to x_0.

2.3 Comparison of the analytical solution with numerical study

The differential equation (2) discussed in subsection 2.2 has an analytical solution equation (4) with simplifications which assume fixed values of volumetric flow rates $q_{v,dw}$ and $q_{v,ct}$ as

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well as sulphate ion concentration x_{fw} in fresh water. It is worth comparing this analytical approach with a numerical study. A numerical solution of (2) can be found by applying numerical procedures dedicated to solving ordinary differential equation of the first order [17]. Equation (2) was discretized using the Euler method with time step Δt (for all further calculations $\Delta t = 1$ [day]):

$$x(t_{i+1}) = x(t_i) + \frac{\Delta t}{V}\left(\left(q_{v,ct}(t_i) + q_{v,dw}(t_i)\right) \cdot x_{fw}(t_i) - q_{v,dw}(t_i) \cdot x(t_i)\right) \qquad (7)$$

where $t_i = t_0 + \Delta t \cdot i$ ($i = 0, 1, ...$). The volumetric flow rate $q_{v,fw}(t)$ is calculated for each time interval from the mass conservation law (1). Numerical data obtained from (7) for fixed values: $V = 100000$ m^3, $q_{v,dw} = 8000$ m^3/day, $q_{v,ct} = 50000$ m^3/day, $x_{fw} = 40$ g/m^3 and initial sulphate ion concentration $x_0 = 500$ g/m^3 was compared with the analytical solution (4). The result is presented in Figure 3 in the form of a relative error e_i calculated from the formula:

$$e_i = \left| \frac{x(t_i) - x_{an}(t_i)}{x_{an}(t_i)} \right| \cdot 100\% \qquad (8)$$

where: $x(t_i)$ and $x_{an}(t_i)$ are the numerical and analytical solutions respectively.
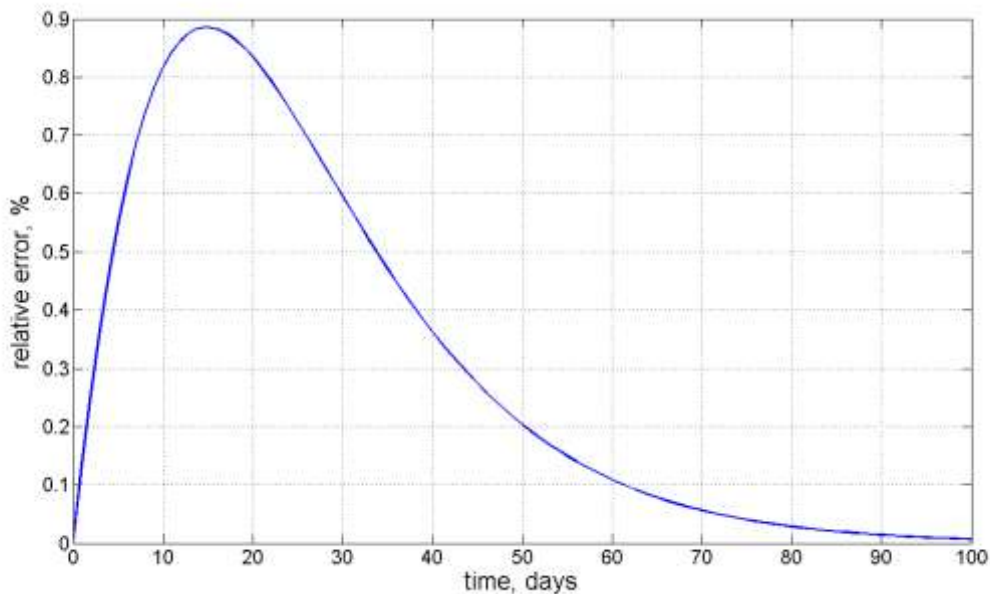
Fig. 3. Relative error e_i calculated from formula (8) from the comparison of analytical solution (4) with numerical data obtained from formula (7). Applied values: $V = 100000$ m^3,

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$q_{v,dw} = 8000$ m^3/day, $q_{v,ct} = 50000$ m^3/day, $x_{fw} = 40$ g/m^3 and initial sulphate ion concentration $x_0 = 500$ g/m^3.

The curve presented in Figure 3 shows that the maximum value of e_i is approximately 0.9 % and this decreases towards the end of the period considered, which indicates high compatibility of the two approaches.

However, measurements conducted on a cooling water installation have shown that despite the fact that mean values of volumetric flow rates are conserved, their daily values can vary by even $\pm 15\%$. It is worth mentioning that the concentration of sulphate ions in fresh water x_{fw} is rather stable and depends on the chemical properties of local reservoirs. In order to assess the suitability of the analytical solution (4) in predicting the change of SO$_4^{-2}$ ion concentration in a real hydraulic system, the numerical scheme (7) was modified to take into account the possibility of daily fluctuations in the flow rates $q_{v,dw}$ and $q_{v,ct}$. In order to estimate the influence of the variability of these parameters on the numerical solution $x(t)$, three variants were investigated:
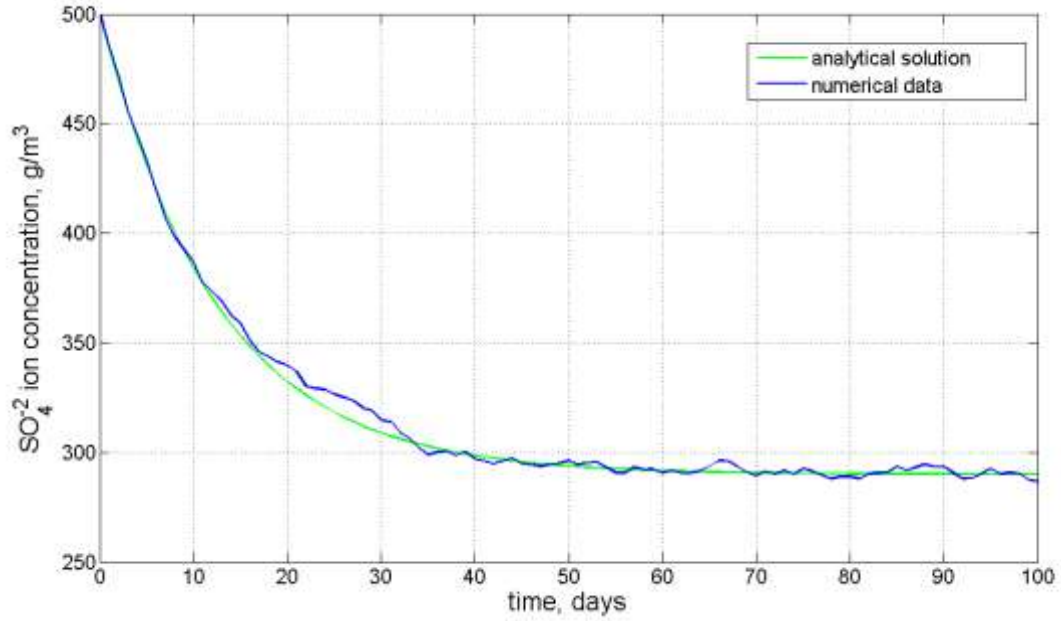
No. 1 - $q_{v,ct}$ randomly changes its value in the range of $\pm 15\%$; $q_{v,dw}$ is fixed,

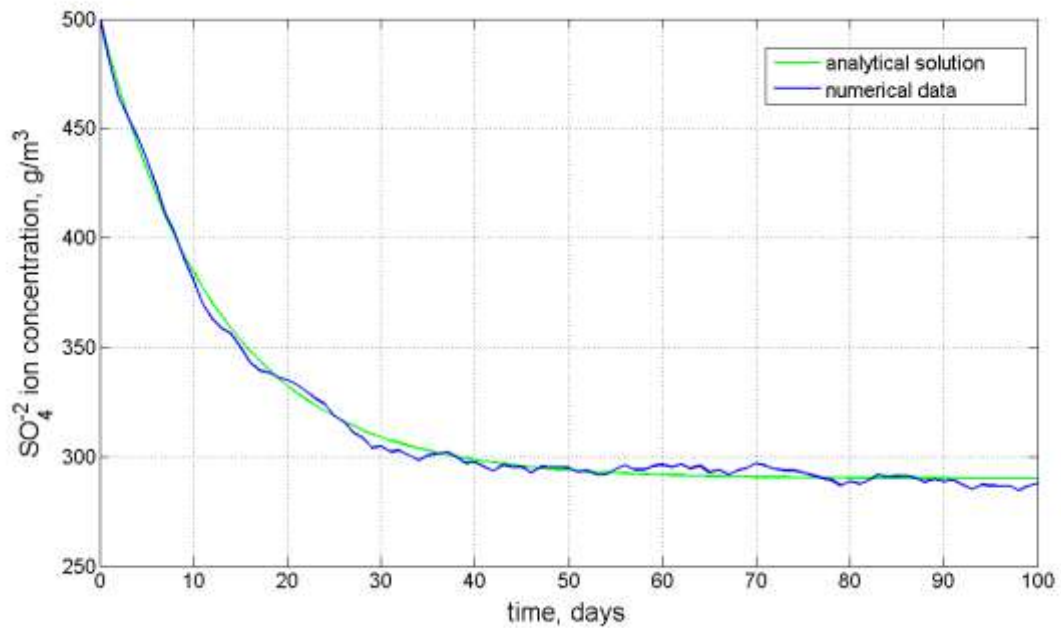No. 2 - $q_{v,dw}$ randomly changes its value in the range of $\pm 15\%$; $q_{v,ct}$ is fixed,

No. 3 - both flow rates, $q_{v,dw}$ and $q_{v,ct}$, randomly change their values in the range of $\pm 15\%$.

It is worth mentioning that such value of daily fluctuation is observed in the cooling system even if the power plant works with constant power demand and is connected mainly with actual weather conditions.

Figures 4 - 6 present numerical data and their comparison with analytical solutions for the three above-mentioned variants. Calculations were made for the following parameters: $V = 100000$ m^3, $q_{v,dw} = 8000$ m^3/day, $q_{v,ct} = 50000$ m^3/day, $x_{fw} = 40$ g/m^3 and initial sulphate ion concentration $x_0 = 500$ g/m^3.

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Fig. 4. Comparision of numerical data with the analytical solution for variant No. 1 ($q_{v,ct}$ randomly changes its value in the range of $\pm15\%$, $q_{v,dw}$ is fixed). The parameters: $V = 100000$ m^3, $q_{v,dw} = 8000$ m^3/day, $q_{v,ct} = 50000$ m^3/day, $x_{fw} = 40$ g/m^3 and $x_0 = 500$ g/m^3.


Fig. 5. Comparision of numerical data with the analytical solution for variant No. 2 ($q_{v,dw}$ randomly changes its value in the range of $\pm15\%$, $q_{v,ct}$ is fixed). The parameters: $V = 100000$ m^3, $q_{v,dw} = 8000$ m^3/day, $q_{v,ct} = 50000$ m^3/day, $x_{fw} = 40$ g/m^3 and $x_0 = 500$ g/m^3.

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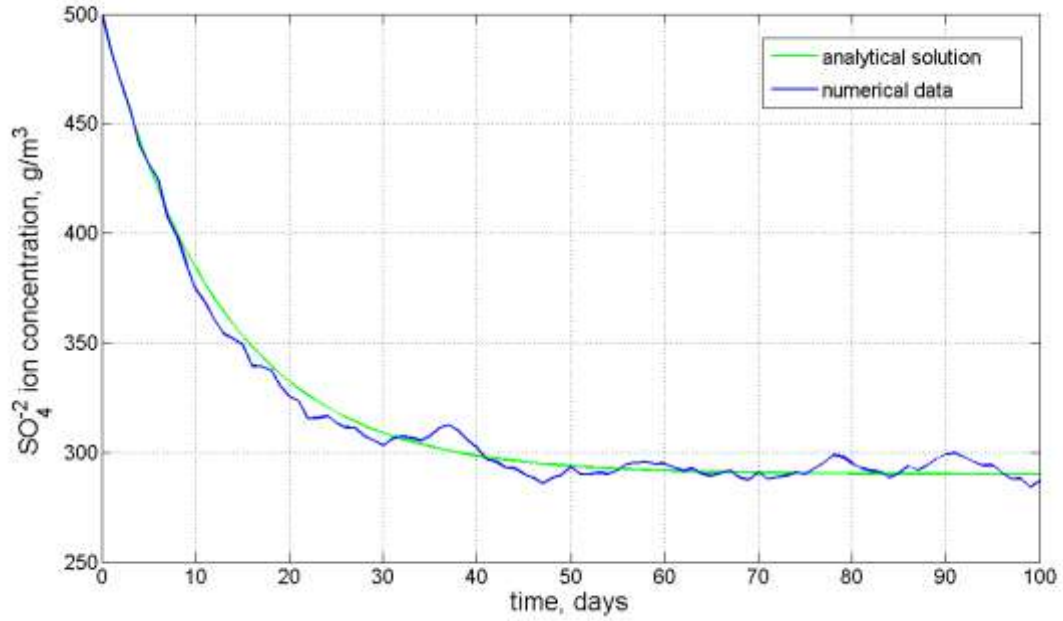
Fig. 6. Comparision of numerical data with the analytical solution for variant No. 3 ($q_{v,ct}$ and $q_{v,dw}$ randomly change their values in the range of $\pm15\%$). The parameters: $V = 100000$ m^3, $q_{v,dw} = 8000$ m^3/day, $q_{v,ct} = 50000$ m^3/day, $x_{fw} = 40$ g/m^3 and $x_0 = 500$ g/m^3.
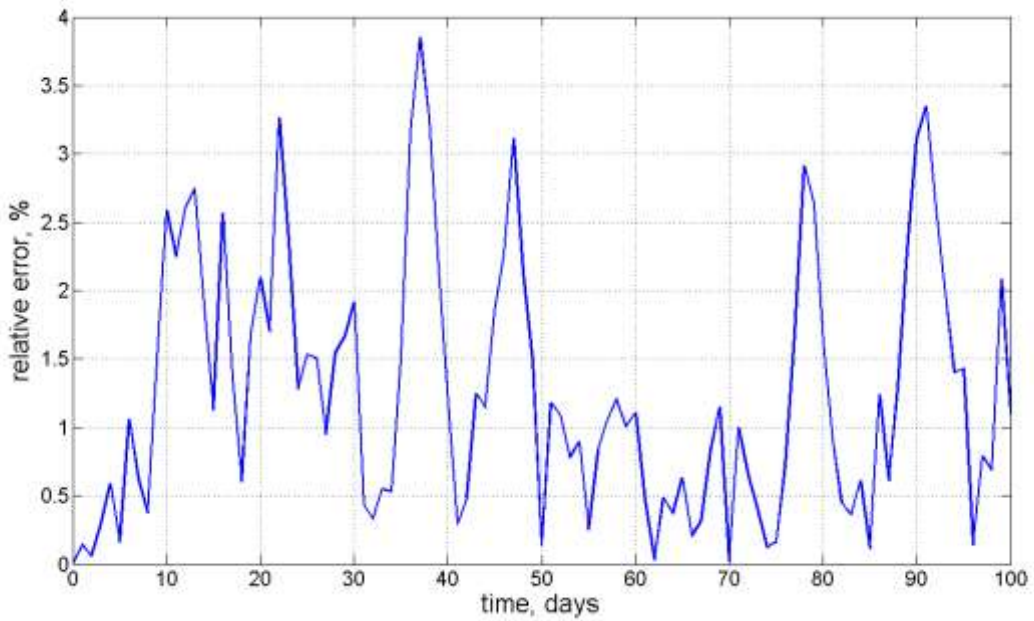
Fig. 7. Relative error e_i calculated from equation (8) in case of variant No. 3 presented in Fig.

6.

Analysis of the numerical data shown in Figs. 4-6 indicates a very high level of agreement between numerical calculations and the analytical solution for all three variants. The maximum relative errors obtained for the variants 1-3 were 3.28%, 3.70% and 3.85%

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respectively. It means that fluctuations in the volumetric flow rates at a level of ±15% do not have a significant influence on the shape of the numerical solutions for all three variants. Variant no. 3 has the largest maximum relative error, which is related to the fact that the fluctuations of volumetric flow rates $\{q_{v,ct}, q_{v,dw}\}$ are considered simultaneously. Table 2 presents the maximum, mean and minimum values of the flow rates $q_{v,ct}$ and $q_{v,dw}$ obtained in the numerical calculation for variant no. 3.

Table 2. The maximum, mean and minimum values of the flow rates $q_{v,ct}$ and $q_{v,dw}$ obtained in numerical calculation for variant No. 3 presented in Fig. 6.

Variant no. 3	Maximum value	Mean value	Minimum value
$q_{v,ct}$, m^3/day	57328	50264	42894
$q_{v,dw}$, m^3/day	9199	8060	6854

3. Results and Discussion

The goal of the mathematical model lies in the fact that its analytical solution (4) gives an estimate of the minimum requirement for the wastewater volumetric flow rate $q_{v,dw}$ in order to satisfy the current legal regulation of $x_{\text{limit}} = 500$ g/m^3. To do this, firstly one can focus on equation (6) derived in subsection 2.2 which describes the asymptotic value of SO_4^{-2} ion concentration x_∞ in circulating water :

$$x_\infty = \frac{b}{a} = \frac{q_{v,fw} \cdot x_{fw}}{q_{v,dw}} = \left(1 + \frac{q_{v,ct}}{q_{v,dw}}\right) \cdot x_{fw} \tag{9}$$

From equation (9), one can notice that the asymptotic value x_∞ depends only on three parameters: SO_4^{-2} ion concentration in fresh water x_{fw} and the volumetric flow rates $\{q_{v,dw}, q_{v,ct}\}$. The power plant operator has an influence on only one of them − $q_{v,dw}$, because the number of cooling towers operating and weather conditions determine $q_{v,ct}$ in an unequivocal way. Also, the chemical parameters of fresh water (among them x_{fw}) are rather fixed for local

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reservoirs. It is an interesting observation, that the asymptotic value x_∞ can reach the level of SO_4^{-2} ion concentration available in fresh water x_{fw} if the volumetric flow rate of wastewater $q_{v,dw}$ tends to infinity. From a wastewater management point of view such an approach to the reduction of sulphate ion concentration is costly and thus uneconomical for power plants. There are no operational reasons to maintain the SO_4^{-2} ion concentration as low as this and the operator will instead tend to hold the sulphate ion concentration close to the value specified in legal regulations, reducing in this way the amount of discharge water (and hence the cost of sewage charges). Sample values of asymptotic sulphate ion concentrations x_∞ for a fixed value of $q_{v,ct} = 50000$ m^3/day are presented in Table 3.

Table 3. Sample values of asymptotic sulphate ion concentrations x_∞ (in g/m^3) for selected values of volumetric flow rates $q_{v,dw}$ and SO_4^{-2} ion concentrations in fresh water x_{fw} (for fixed $q_{v,ct} = 50000$ m^3/day). Red colour marks values exceeding the legal limit $x_{limit} = 500$ g/m^3.

SO_4^{-2} ions concentration x_{fw}, [g/m^3] / flow rate $q_{v,dw}$,[m^3/day]	20	25	30	35	40
2000	520	650	780	910	1040
4000	270	337	405	472	540
6000	187	233	280	327	373
8000	145	181	217	254	290
10000	120	150	180	210	240

Analyzing data from Table 3 one notices that there exist combinations of $q_{v,dw}$ and x_{fw} for which the asymptotic value of x_∞ is below the legally acceptable limit $x_{limit} = 500$ g/m^3. On the other hand one can use the equation (9) to determine the minimum volumetric flow rate of wastewater discharge $q_{v,dw}$ required to satisfy the legal requirement for SO_4^{-2} ion concentration in wastewater releases to natural water reservoirs. If in equation (9) the asymptotic ion concentration x_∞ is replaced by x_{limit}, then one can transform equation (9) to obtain the formula for the minimum volumetric flow rate of wastewater discharge $q_{v,dw}$:

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$$q_{v,dw} = \frac{q_{v,ct} \cdot x_{fw}}{(x_{limit} - x_{fw})} \tag{10}$$

Assuming that SO_4^{-2} ion concentration in fresh water x_{fw} is fixed as well as the volumetric flow rate of evaporated water in cooling towers $q_{v,ct}$ for a given number of working cooling towers, an operator responsible for wastewater management can calculate from equation (10) the minimum volumetric flow rate of wastewater $q_{v,dw}$. Sample values of minimum volumetric flow rates $q_{v,dw}$, parameterized by sulphate ion concentration in fresh water x_{fw} and volumetric flow rate of vapour from cooling towers $q_{v,ct}$, for $x_{limit} = 500$ g/m^3, are presented in Table 4. By maintaining the minimum volumetric flow rate $q_{v,dw}$ which preserves legal requirements for SO_4^{-2} ion concentration in wastewater the operator can both save fresh water resources as well as minimizing the cost of industrial wastewater in the power plant.

Table 4. Minimum values of volumetric flow rates $q_{v,dw}$ (in m^3/day) required to satisfy legal regulations $x_{limit} = 500$ g/m^3 imposed on sulphate ion concentration in wastewater from a power plant.

SO_4^{-2} ions concentration x_{fw}, [g/m^3] / flow rate $q_{v,ct}$,[m^3/day]	20	25	30	35	40
20000	833	1053	1277	1505	1739
30000	1250	1579	1915	2258	2609
40000	1667	2105	2553	3011	3478
50000	2083	2632	3191	3763	4348

The mathematical model presented allows not only the estimation of the minimum requirements for water management in closed cooling water systems but it can also be useful for testing different wastewater strategies. As an example of such possibilities, Fig. 8 presents two different approaches to the problem of wastewater management during a period of 203 days.
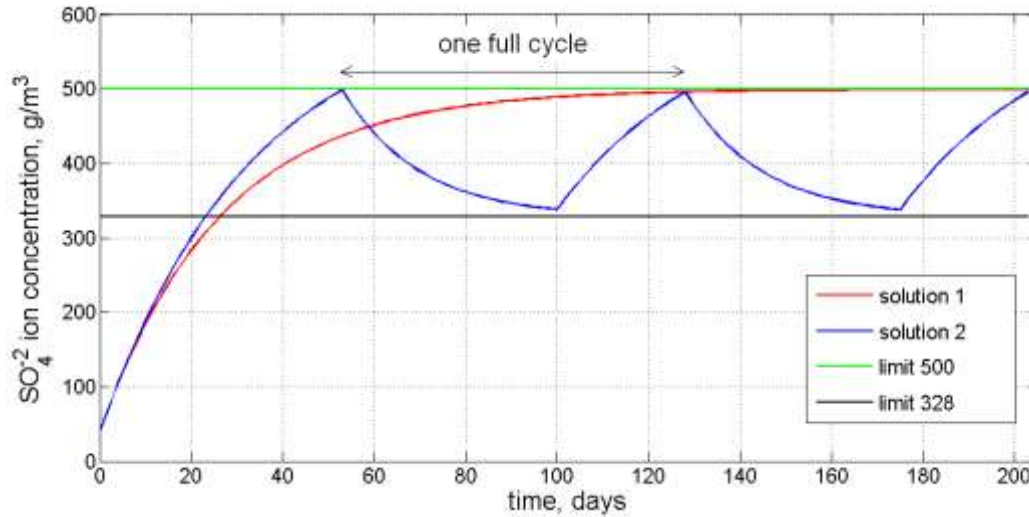
Fig. 8. Example of different solutions of the mathematical model with fixed parameters: $V = 100000$ m^3, $x_0 = 40$ g/m^3, $x_{fw} = 40$ g/m^3 and $q_{v,ct} = 43200$ m^3/day. Red curve: $q_{v,dw} = 3757$ m^3/day; blue curve: cycle with $q_{v,dw} = 3000$ and 6000 m^3/day respectively.

Both curves (red and blue one) are calculated for the same initial condition $x_0 = 40$ g/m^3 and fixed parameters: $V = 100000$ m^3, $x_{fw} = 40$ g/m^3 and $q_{v,ct} = 43200$ m^3/day which reflects the working parameters of the real closed cooling installation. In case of first solution (red curve) the volumetric flow rate of discharged water is fixed during the whole period of 203 days at the level of $q_{v,dw} = 3757$ m^3/day (preserving $x_{limit} = 500$ g/m^3). The blue curve consists of two solutions calculated for $q_{v,dw} = 3000$ m^3/day (first 53 days and periods between 101-128 and 176-203 days) and 6000 m^3/day (period between 54-100 and 129-175 days). Changes in volumetric flow rate $q_{v,dw}$ for the blue curve are forced by the legal limit for the SO_4^{-2} ion concentration. Initially the volumetric flow rate is $q_{v,dw} = 3000$ m^3/day which causes the sulphate ion concentration to tend to cross the legal limit $x_{limit} = 500$ g/m^3. In order to avoid it, after 53 days, the operator has to increase the volumetric flow rate of wastewater up to $q_{v,dw} = 6000$ m^3/day for the next 47 days. When the sulphate ion concentration decreases to approx. 340 g/m^3, the operator again reduces the volumetric flow rate to $q_{v,dw} = 3000$ m^3/day for the next 28 days. Hence the wastewater management operates in a cycle of 75 days with sulphate ion concentration varying between 340 and 500 g/m^3 for $q_{v,dw}$ alternately 3000 and 6000 m^3/day. Both curves presented preserve legal limits but in the first case (solution No. 1) the operator released only

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762671 m^3 of waste water during 203 days when in the other case (solution No. 2) the figure was 891000 m^3. The difference between these two approaches is 128329 m^3. It means that optimal wastewater management uses approximately 16.8% less fresh water than in the second case.

It is worth mentioning that a closed cooling system does not operate under the same hydraulic load all the time. The overall volumetric flow rate in the installation depends on the number of power units operating (hence the number of circulation pumps working). Stopping the operation of one of the power units results in a reduction in the hydraulic load and consequently a decrease in the evaporation of water in cooling towers whichcauses the volumetric flow rate $q_{v,ct}$ to decrease.

The next example examines the possibility of the "on-line" modelling of SO_4^{-2} ion concentration changes when the number of working power units varies over time.
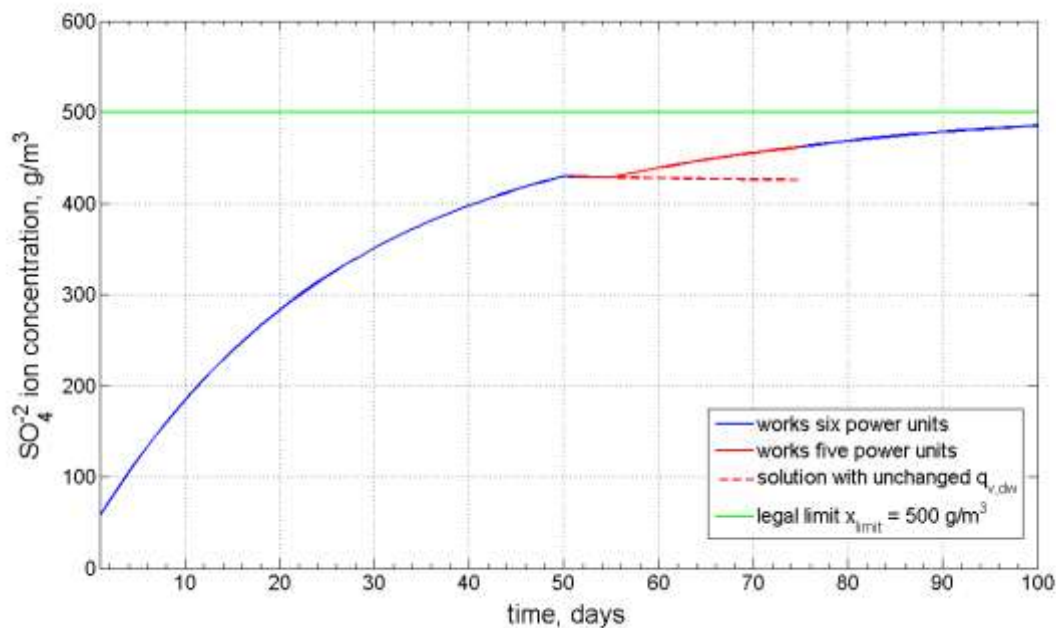
Fig. 9. Simulation of the SO_4^{-2} ion concentration changes in circulating water with a variable number of power units operating (red lines mark the period of 25 days analyzed).

Fig. 9 shows a simulation when, during the first 50 days, six power units generated the volumetric flow rate of evaporated water through the cooling towers at the level of $q_{v,ct} = 43200$ m^3/day. From equation (10) one can calculate $q_{v,dw} = 3757$ m^3/day (assuming $x_{fw} = 40$ g/m^3). Next, one of the operating power units was stopped for 25 days, which caused a

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reduction of $q_{v,ct}$ to the level of 36000 m^3/day. At this point the actual volumetric flow rate of wastewater $q_{v,dw}$ = 3757 m^3/day was too high and the sulphate ion concentration was decreasing. The dashed red line in Fig. 9 visualizes the changes of SO$_4^{-2}$ ion concentration in case of an unchanged value of $q_{v,dw}$ over the next 25 days. It is noted that in this case sulphate ion concentration would decrease to a level of 426 g/m^3. In order to minimize costs, after five days a new volumetric flow rate $q_{v,dw}$ = 3130 m^3/day was applied. This correction allowed the saving of 627 m^3 of fresh water per day over the next 20 days. Finally the sixth power unit returned to the operation and $q_{v,ct}$ increased again to the level of 43200 m^3/day. At this moment the current $q_{v,dw}$ was too low and SO$_4^{-2}$ concentration would exceed the legal limit. In order to prevent this situation the volumetric flow rate of wastewater $q_{v,dw}$ returned to the old level of 3757 m^3/day. As a consequence of an „on-line " reaction to the changes in hydraulic loading of the cooling system, the operator saved 12540 m^3 of fresh water.

4. Conclusions

The mathematical model created by the authors is the first in literature which describes the changes in the sulphate ion concentration in a closed cooling system. The examples discussed prove that the analytical solution of the mathematical model can be used to predict the changes of sulphate ion concentration in water circulating in a closed cooling system as well as to calculate the minimal volumetric flow rate of wastewater required to keep the SO$_4^{-2}$ concentration below legal limits. This knowledge could help operators and decision makers to introduce planned and cost-effective wastewater management in power plants, reducing the consumption of fresh water as well as the amount of industrial waste discharged into the environment. In order to automate the wastewater management process, it is possible to implement the mathematical model into a computer program that could be included in the power plant's monitoring system. It is worth mentioning that the proposed model could be applied, after suitable changes, in different industry and power engineering sectors.

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(top banner) ACCEPTED MANUSCRIPT

Acknowledgments

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Highlights:

- The presented mathematical model can be useful in power plants in cost-effective wastewater management
- The analytical solution of the mathematical model can be used to predict the changes of sulphate ion concentration in water circulating in a closed cooling system
- The analytical solution of the mathematical model can be used to calculate the minimal volumetric flow rate of wastewater required to keep the SO_4^{-2} concentration below legal limits