Impact of Unknown Digital Map Errors on Satellite-based Navigation in Railway

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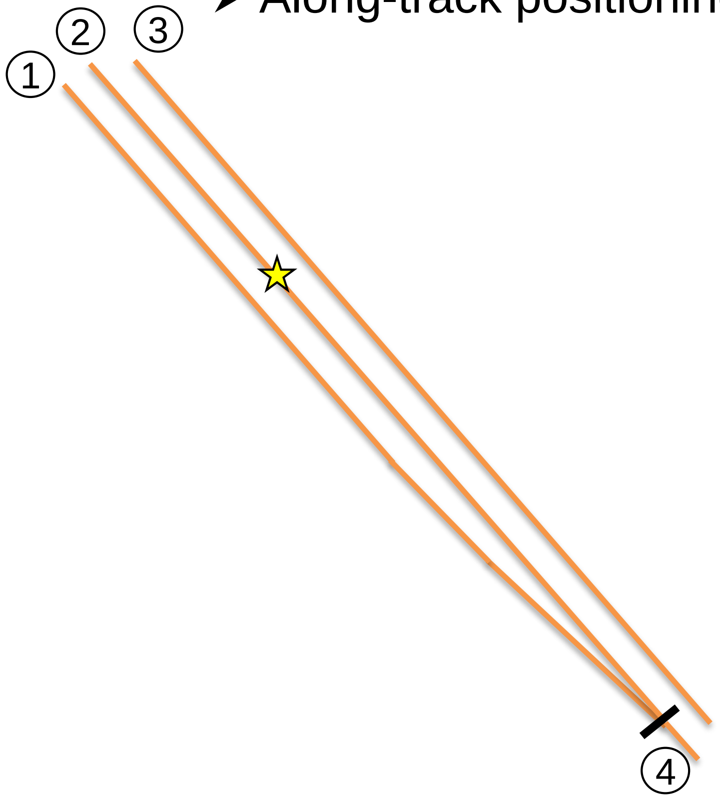
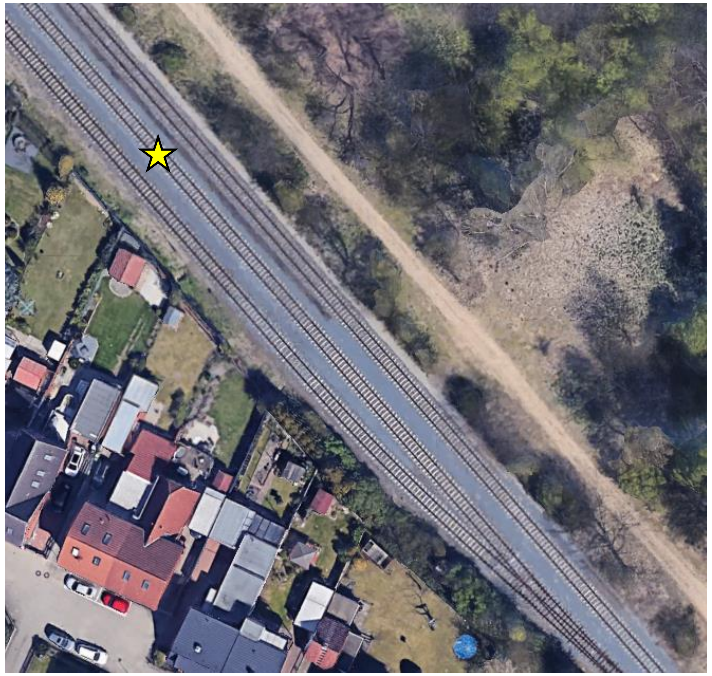
DLR

Merging of GNSS Outcomes and Railway Needs

- Railway localization goal: topological position
  - Track ID identification
  - Along-track positioning



- GNSS provides a geo-referenced position
- Key player for enabling:
  - world-wide operational and
  - railway infrastructure independent navigation



Merging of GNSS Outcomes and Railway Needs

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How to enable GNSS for railway?
Several approaches has been discussed such as the virtual balise concept

All have in common that a **track map** needs to be used together with GNSS

Track map converts topological into geo-referenced positions and vice versa

Research Question

- Majority of investigations assume the track map to be error-free

- **Open question:**
  "What would be the impact of track map error on the GNSS performance, especially if these are not considered by the satellite navigation system?"

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The general impact of map errors on

1. GNSS measurement model
2. GNSS position solution
3. Standard Fault Detection
   - Quantification of the impact on specific reliability measures for GNSS via simulations based on real railway scenario

Digital Track Map

- Projection function relating topological railway localization (k, s) to geo-references position $(\boldsymbol{x} \in \mathbb{R}^3)$

$$\boldsymbol{x}_{\text{map}} = f_{\text{map}}(k, s)$$

- Defining the 3D map error in the geo-references frame as

$$\boldsymbol{e}_m(k, s) \triangleq f_{\text{map}}(k, s) - f_{\text{true}}(k, s)$$

- Our methodology enables the investigation of all kind of map errors
- For this study we concentrate on partially constant errors: shifts/bias

$$\boldsymbol{e}_m(s) \approx \boldsymbol{\mu}_m$$

Short remark :

Focus on along-track position and NOT track identification

Assume perfect track identification

Omit the index k from now

Impact on Multi-constellation GNSS Measurement Model

- Track map constrained GNSS pseudorange measurement model with map errors:

$$\begin{aligned}\rho^{i,j} &= \|\boldsymbol{x}_{\mathrm{s}}^{i} - f_{\mathrm{true}}(s)\|_2 + b_{\mathrm{u}}^{j} + T^{i,j} + I^{i,j} - b_{\mathrm{s}}^{i} + e_{\mathrm{PR}}^{i}, \\ &= \|\boldsymbol{x}_{\mathrm{s}}^{i} - (f_{\mathrm{map}}(s) - \boldsymbol{e}_m)\|_2 + b_{\mathrm{u}}^{j} + T^{i,j} + I^{i,j} - b_{\mathrm{s}}^{i} + e_{\mathrm{PR}}^{i}\end{aligned}$$

- Linearized set of measurement equations (details in the paper):

$$\boldsymbol{y} = \underbrace{\left[\boldsymbol{G}_p \vdots \boldsymbol{G}_b\right]}_{\boldsymbol{G}} \begin{bmatrix} r_m(\Delta s) \\ \Delta \boldsymbol{b}_{\mathrm{u}} \end{bmatrix} + \boxed{\boldsymbol{G}_p \boldsymbol{e}_m} + \boldsymbol{e}_{\mathrm{PR}}$$

- $\Delta \boldsymbol{x} = r_m(\Delta s)$: transition function between two points s_0 and $s_0 + \Delta s$ along the track

Impact on GNSS Positioning

- Track constrained GNSS Least-Square Solution:

$$\hat{\mathbf{x}}_{\text{LS}} = \begin{bmatrix} \Delta\hat{s} \\ \Delta\hat{\boldsymbol{b}}_{\text{u}} \end{bmatrix} = \underbrace{\left(\boldsymbol{G}_{\text{track}}^{T}\boldsymbol{G}_{\text{track}}\right)^{-1}\boldsymbol{G}_{\text{track}}^{T}}_{\mathbf{S}}\boldsymbol{y}$$

$$\boldsymbol{G}_{\text{track}} = \left[\boldsymbol{G}_p\boldsymbol{u}_m \,\vdots\, \boldsymbol{G}_b\right]$$

- Solution mean value:

$$E\left[\hat{\mathbf{x}}_{\text{LS}}\right] = E\left[\boldsymbol{S}\boldsymbol{y}\right] = \boldsymbol{S}\boldsymbol{G}_p\,\boldsymbol{\mu}_m$$

→ Projection of the map errors on pseudorange domain back to topological domain

- Solution covariance matrix

$$E\left[\left(\hat{\mathbf{x}}_{\text{LS}} - E\left[\hat{\mathbf{x}}_{\text{LS}}\right]\right)\left(\hat{\mathbf{x}}_{\text{LS}} - E\left[\hat{\mathbf{x}}_{\text{LS}}\right]\right)^{T}\right] = \boldsymbol{S}\boldsymbol{\Sigma}_y\boldsymbol{S}^{T}$$

→ Projection of the pseudorange covariance matrix to topological domain

Impact on Measurement Consistency Fault Detection Algorithm

- Pseudorange residual with map errors:

$$\boldsymbol{r} = \boldsymbol{y} - \boldsymbol{G}_{\text{track}}\,\hat{\mathrm{x}}_{\text{LS}} = \left(\boldsymbol{I} - \boldsymbol{G}_{\text{track}}\left(\boldsymbol{G}_{\text{track}}^{T}\boldsymbol{G}_{\text{track}}\right)^{-1}\boldsymbol{G}_{\text{track}}^{T}\right)\boldsymbol{y} = (\boldsymbol{I} - \boldsymbol{P})\boldsymbol{y}.$$

- Following a Gaussian distribution with

$$\boldsymbol{\mu_r} = E\left[(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{y}\right] = (\boldsymbol{I} - \boldsymbol{P})\mathrm{G}_p\boldsymbol{\mu}_m$$

$$\boldsymbol{\Sigma_r} = E\left[\left((\boldsymbol{I} - \boldsymbol{P})\boldsymbol{y} - \boldsymbol{\mu_r}\right)\left((\boldsymbol{I} - \boldsymbol{P})\boldsymbol{y} - \boldsymbol{\mu_r}\right)^T\right] = (\boldsymbol{I} - \boldsymbol{P})\boldsymbol{\Sigma_y}(\boldsymbol{I} - \boldsymbol{P})^T$$

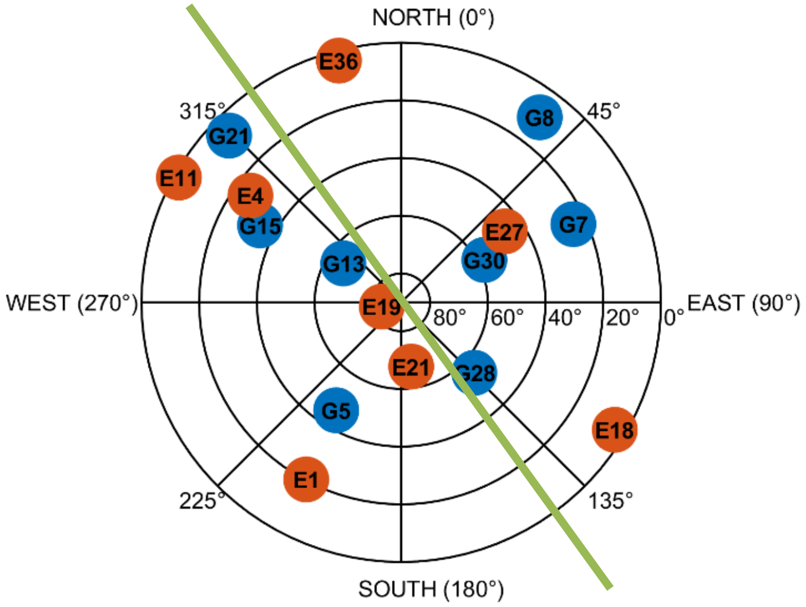
- Test statistic is defined as

$$q = \boldsymbol{r}^T\boldsymbol{\Sigma}_{\boldsymbol{r}}^{-1}\boldsymbol{r}.$$

- Follows a **non-central Chi-square** distribution with N-(L+1) degrees of freedom and non-central parameter

$$\lambda_m = \boldsymbol{\mu}_{\boldsymbol{r}}^T\boldsymbol{\mu_r} = \boldsymbol{\mu}_m^T\boldsymbol{G}_p^T(\boldsymbol{I} - \boldsymbol{P})^T(\boldsymbol{I} - \boldsymbol{P})\boldsymbol{G}_p\boldsymbol{\mu}_m$$

Simulation Environment



- Dual constellation and dual frequency GNSS (maximum of 8 GPS and 8 Galileo satellites in view)
- Considered location is on a track in Braunschweig, Germany
- Track is orientated from north-west to south-east
- Assumed GNSS pseudorange error model

$$\sigma_\rho^2 = \sigma_{\text{URA}}^2 + \sigma_{\text{tropo}}^2 + \sigma_{\text{mp}}^2 + \sigma_{\text{noise}}^2$$

Parameter	Value
σ_{URA}^2	1 m
σ_{tropo}^2	RTCA standard model [1]
$\sigma_{mp}^2 + \sigma_{noise}^2$	ARAIM standard model [2]

[1] RTCA/SC-159, "RTCA/DO-229C: Minimum Operational Performance Standards for Global Positioning System/Wide Area Augmentation System Airborne Equipment", RTCA, Tech. Rep., 2001.
[2] Working Group C - ARAIM Subgroup, "Milestone 3 report", EU/US Cooperation on Satellite Navigation, Tech. Rep., 2016.

Impact of Unknown Track Map Errors to Fault Detection

- Definition of four hypotheses:

	No Track Map Errors	Track Map Errors
No GNSS Faults	$H_0\colon \lambda = 0$	$H_0^{'}\colon \lambda = \lambda_m$
GNSS Faults present	$H_1\colon \lambda = \lambda_f$	$H_1^{'}\colon \lambda = \lambda_f + \lambda_m$

- Investigation of impact of three reliability measures of GNSS positioning

1. Probability of false alarm requirement (P_{fa})
2. Probability of missed detection (P_{md})
3. Minimum Detectable Bias (MDB)

Probability of False Alarm Requirement

- H_0 = Chi-square distribution (standard model)
- H_0' = Non-central chi-square distribution

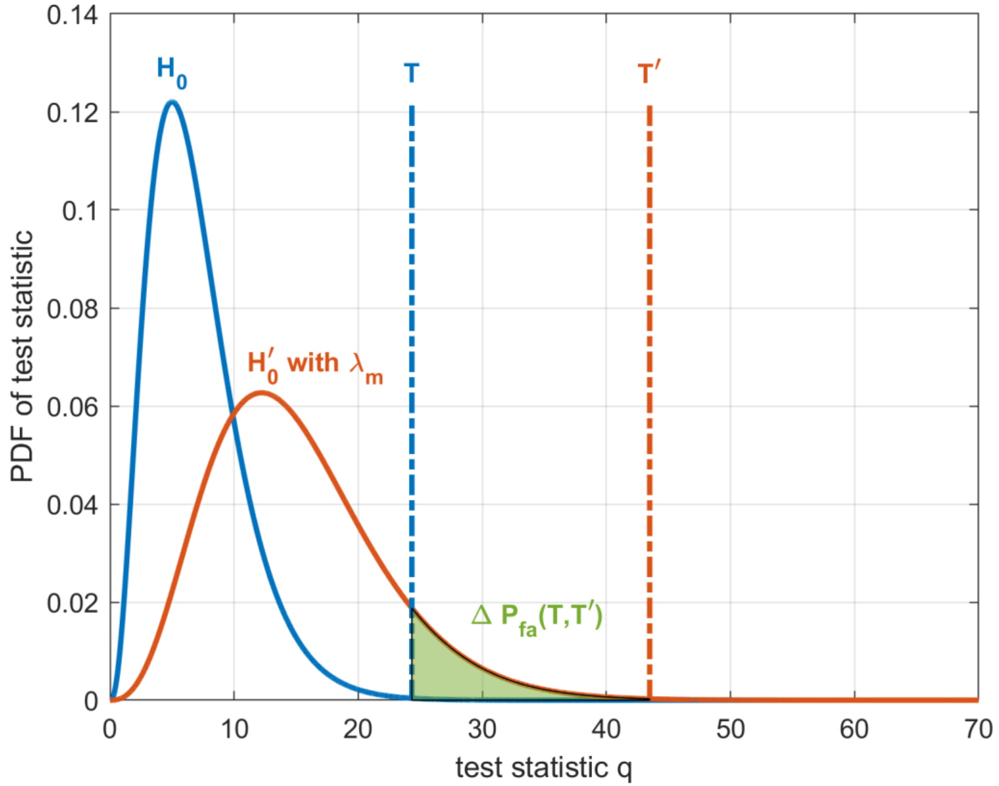
- Probability of false alarm

$$H_0\text{: } T := 1 - P_{fa} = \int_{-\infty}^{T} f_X(x; N-(L+1))\,dx,$$

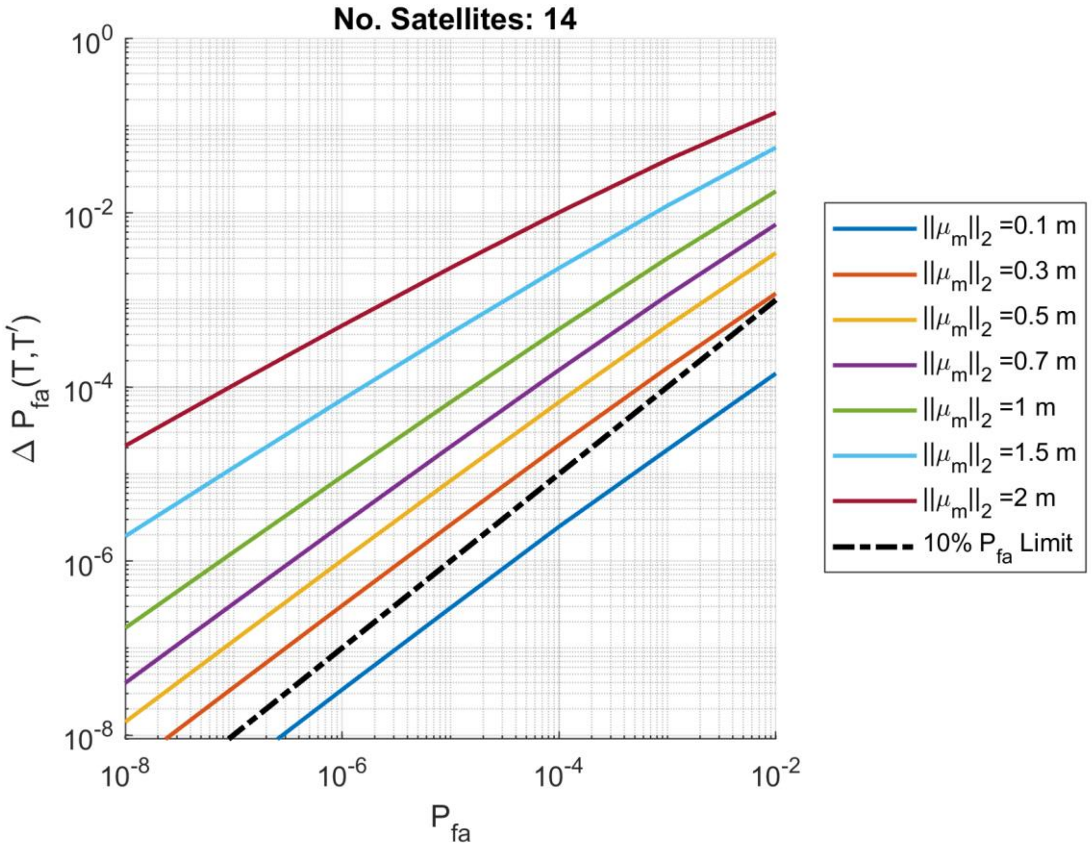
$$H_0'\text{: } T' := 1 - P_{fa} = \int_{-\infty}^{T'} f_X(x; N-(L+1), \lambda_m)\,dx.$$

- Requirement violation:

$$\Delta P_{fa}(T,T') := \int_{T}^{T'} f_X(x; N-(L+1), \lambda_m)\,dx$$



Violation of Probability of False Alarm Requirement



Difference of P_{fa} strongly depends on:

- Map error $||\mu_m||_2$
- Required P_{fa}
- Satellite geometry
- Track orientation

Increase of type I errors

Violation of P_{fa} requirement

Reduction of availability

Test Statistic Distribution for GNSS Faults

- H_1 = hypothesis **NO** track maps and GNSS faults are present (standard model)
- H_1' = hypothesis track errors and GNSS faults are present

- Impact of GNSS faults

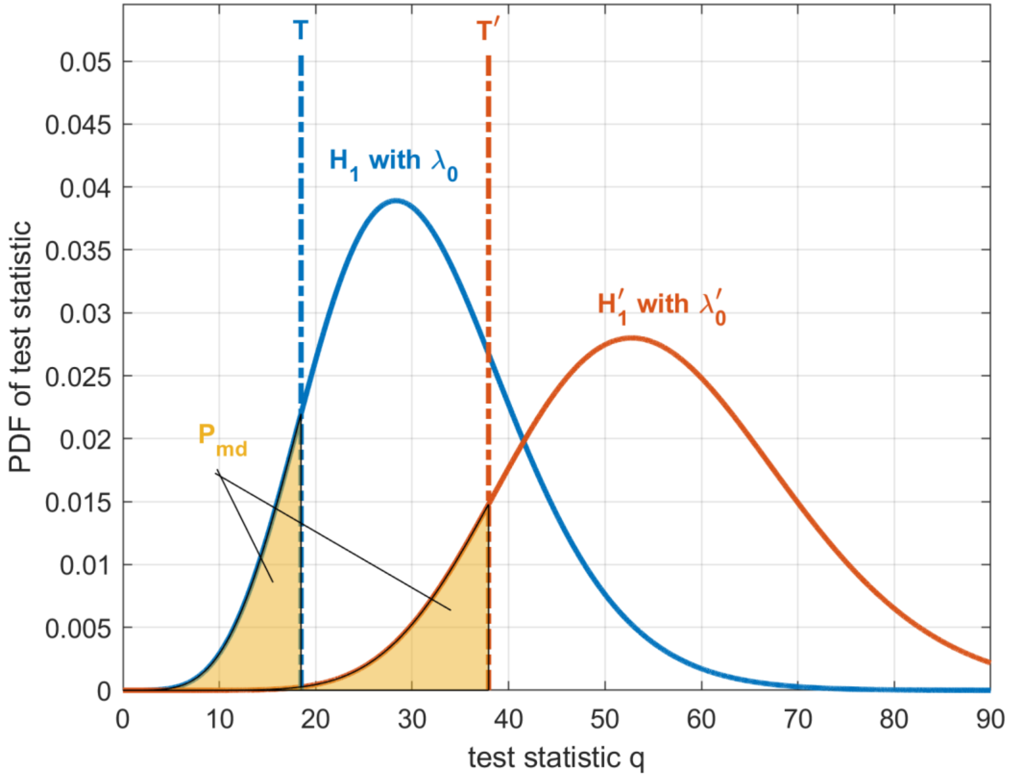
$$H_1 : \lambda_f = \boldsymbol{f}^T(\boldsymbol{I} - \boldsymbol{P})^T \boldsymbol{W} (\boldsymbol{I} - \boldsymbol{P}) \boldsymbol{f}$$

$$H_1' : \lambda_f' = \boldsymbol{f}^T(\boldsymbol{I} - \boldsymbol{P})^T \boldsymbol{W} (\boldsymbol{I} - \boldsymbol{P}) \boldsymbol{f} + \lambda_m.$$

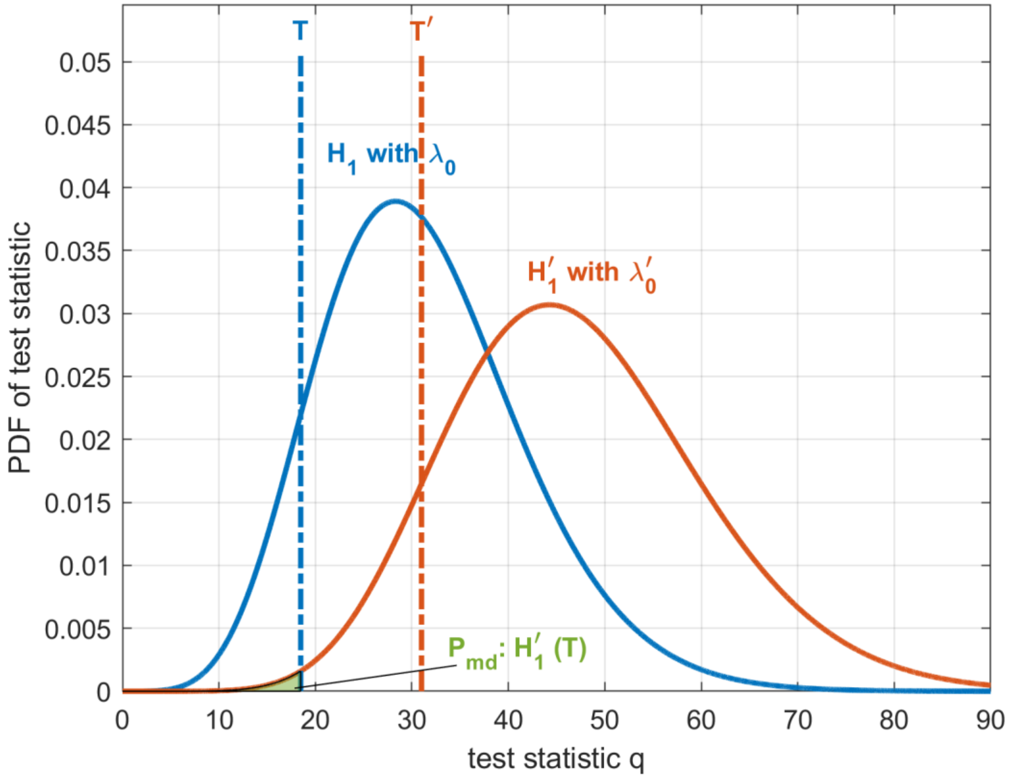
- Probability of missed detection

$$\int_{-\infty}^{T} f_X(x; N - (L+1), \lambda_0)\, dx = P_{md}$$

$$\int_{-\infty}^{T'} f_X(x; N - (L+1), \lambda_0')\, dx = P_{md}$$



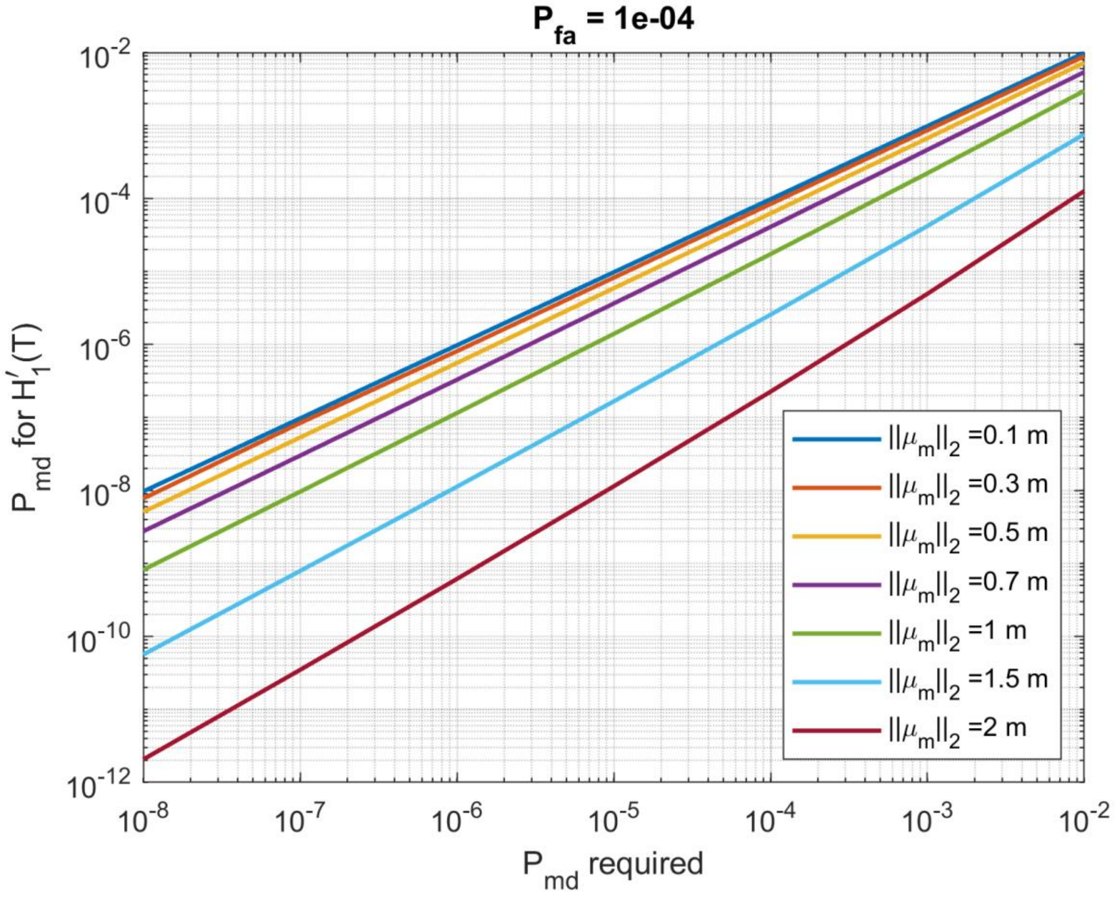
Impact on Probability of Missed Detection



- Track map errors are unknown →
  - Detection threshold T instead of T’
  - Recall: $\lambda_0' = \lambda_f + \lambda_m$

$$\int_{-\infty}^{T} f_X\left(x;\ N-(L+1), \lambda_0'\right) dx < P_{md}$$

Reduction of Probability of Missed Detection



- Track map errors are unknown →
  - Detection threshold T instead of T’
  - Recall: $\lambda_0' = \lambda_0 + \lambda_m$

$$P_{md}|H_1'(T) := \int_{-\infty}^{T} f_X\,(x;\ N-(L+1), \lambda_0')\,dx < P_{md}$$

Detection of type II errors is over conservative

Probability of missed detection requirement is NOT violated

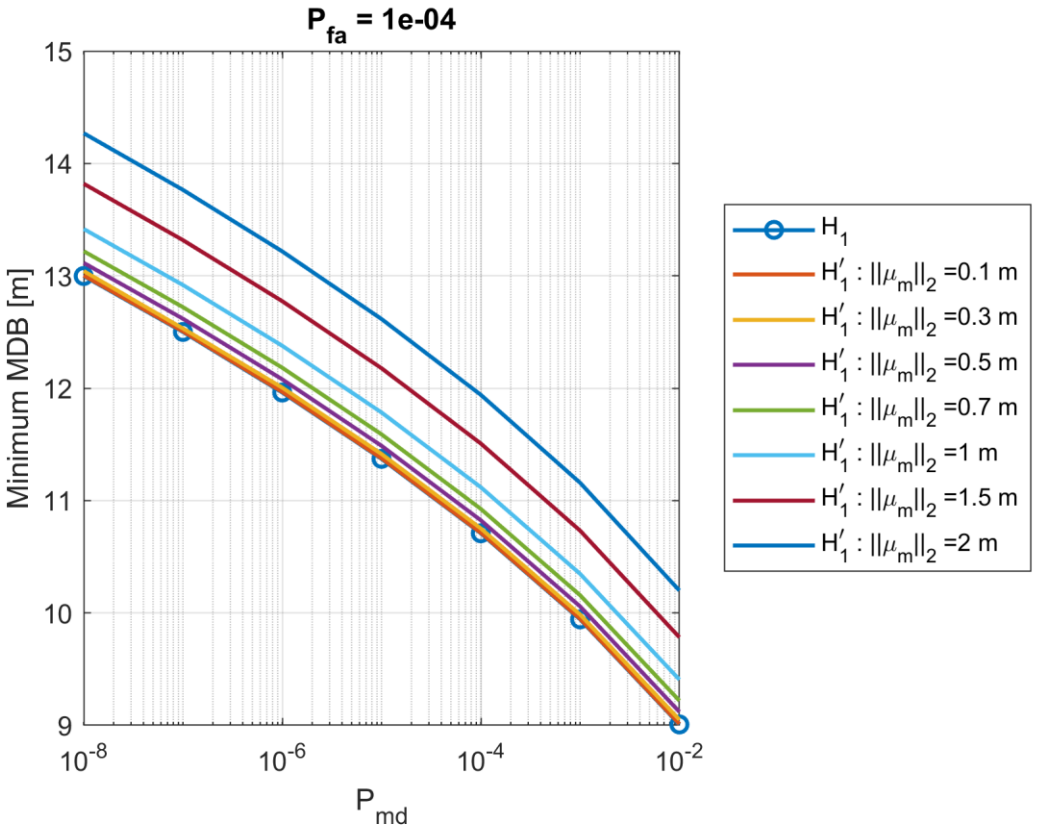
Minimum Detectable Bias with and without Track Map Errors

Under the assumption of single GNSS fault:

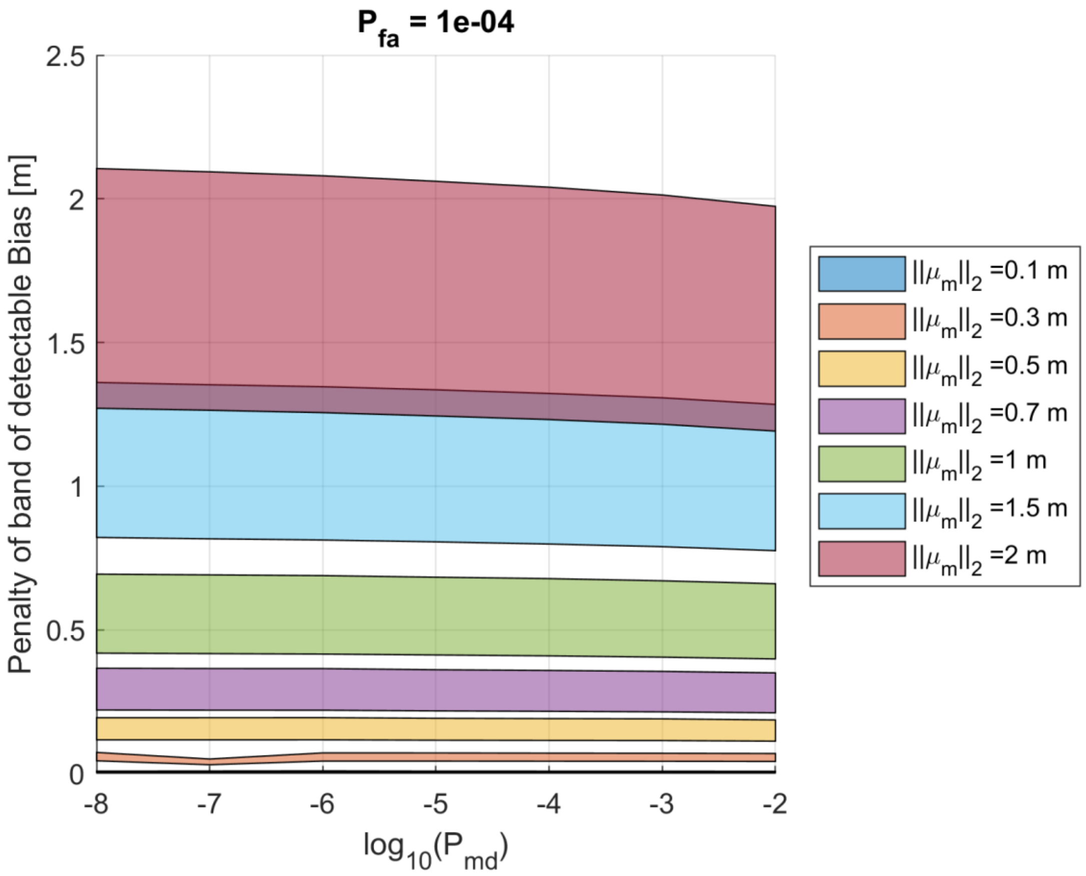
- Minimum Detectable Bias (MDB) = minimum non-central parameter (λ_0, λ_0') projected on the *i*th satellite:

$$H_1 : b_i^2 = \frac{\lambda_0}{\boldsymbol{a}_i^T(\boldsymbol{I}-\boldsymbol{P})^T\boldsymbol{W}(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{a}_i}$$

$$H_1' : b_i'^2 = \frac{\lambda_0' - \lambda_m}{\boldsymbol{a}_i^T(\boldsymbol{I}-\boldsymbol{P})^T\boldsymbol{W}(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{a}_i}$$



Minimum Detectable Bias Capabilities



Track error are present but not considered

- Penalty band describes the detectability loss

- Lower curve is given by

$$\min_{i=1,\dots,N} b_i - \min_{i=1,\dots,N} b_i'$$

- Upper curve is given by

$$\max_{i=1,\dots,N} b_i - \max_{i=1,\dots,N} b_i'$$

→ Not considering the track map would lead to an underestimation of measurement faults, might impact integrity concepts and causes HMI if protection levels are underestimated

Conclusions

In this work, we performed a first study on

- Impact of map errors on the GNSS measurement model
- Effect on the GNSS track constrained position solution and residual fault detection
- Impact on unknown map error on some reliability measures of GNSS positioning
    - Violation of probability of false alarm requirement
    - Over-conservative probability of missed detection
    - Underestimation of Minimum Detectable Bias

Future Investigations

- Considering different type of track map errors
- Exploiting impact on integrity concepts
- Investigation of techniques to compensate/reduce impact of track map errors

Thank you for your attention!

Any questions, comments or suggestions?

Q&A Session D1 Rail Navigation
23.11.2020 18:40 - 19:00

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