Journal of Mechanical Engineering *Vol SI 4(3), 142-154, 2017*

Frequency Based Substructuring Method for the Investigation of the Dynamic Behaviour of a Beam Structure

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ABSTRACT

Theoretically, the frequency based substructuring (FBS) method can be used to predict the dynamic behaviour of an assembled structure by combining some parts of the numerical and experimental frequency response functions (FRFs) of the substructures. However, the dynamic behaviour prediction via this method often leads to huge discrepancies in comparison with the test data. The discrepancies may be as a result of insufficient information on the experimental data, the rotational FRFs in particular which are very challenging to be accurately measured. One way to improve the quality of the dynamic behaviour prediction via the FBS method is by using 3D solid elements for modelling work. This is because the element type does not provide and require the rotational responses. Therefore, this study is aimed to evaluate the FRFs of a beam structure obtained from the FBS method using3D solid elements based finite element models. The test models in this study are simple aluminium beams that are coupled together using the FBS method. The resulting FRFs of the beam structure were compared with the experimental counterparts. The comparison of the FRFs showed good agreement. This indicates that there is a strong possibility of efficiently predicting the dynamic behaviour of an assembled structure using the 3D elements in FBS method.

Keywords: *Dynamic Behaviour, Modal Testing, Finite Element Method, Substructuring Methods, 2D & 3D elements*

ISSN 1823- 5514, eISSN 2550-164X

Received for review: 2017-02-12
Accepted for publication: 2017-05-12
Published: 2017-10-15

Introduction

The importance of determining the dynamic behaviour of a structure remains to be a major consideration for engineering applications nowadays. The rapid growth of computer technology allows the dynamic behaviour of the structures to be economically and efficiently understood and predicted using the computational analysis. Nevertheless, an assembled structure which consists of many components can be analysed by using the dynamic substructuring methods. These methods divide an assembled structure in subsystems that can be analysed individually and then combining them back into a system by an assembly procedure [1]–[4]. One of the most preferable dynamic substructuring methods used is the frequency based substructuring (FBS) method with the capability to versatility assemble the analytically derived FRF with the experimentally derived counterparts [5]. The FBS method has also been called FRFs coupling by [6] or admittance coupling by [7]. The equations for the FRF's of the combined system are derived by balancing the forces and enforcing continuity at the interface which is explained in details in [8].

In recent times, de Klerk [9] has summarized the history, review, classification of techniques and has provided a general framework for dynamic substructuring. The advantages of using the FBS method is discussed in detail in [6]. The latest study related to the FBS method is presented by Law, Rentzsh and Ihlenfeldt [10] which they used the experimental dynamic substructuring to predict the dynamic of model machine tool. The FRFs were measured by using the operating excitation from the machine itself. Apart from machining tools, the FBS method also was used for modelling machinery isolation system as presented in [11]

Some subsystems of assembly structure are very challenging to be modelled adequately in the finite element model. It has become more difficult if there are no specific properties or geometries given which are needed for constructing reliable finite element model that consumes a lot of time. Hence, one can use FRF based substructuring method which combines analytical and experimental models directly. This may sound a lot easier than it looks, however this method is not often successfully applied. The main challenge lies in the FRF based substructuring method is the preparation for experimental modal analysis which is discussed at length in [1] and [12]. The road blocks for experimental model preparation are as follows:

- Applicability of this method is usually restricted to linear and stationary systems with invariable parameters.
- Most measurements do not include the rotational degree of freedom because it is known that rotational is very difficult to measure. Hence, it some cases, the responses are only limited to translational degree of freedom.
- Providing a perfect FRF is hard because FRF is often contaminated by noise which is the inversion of matrix used for the algorithm that will propagate measurement of noise. This will give inaccurate result for the whole complete system.

The assessment of rotational degree of freedoms (DOFs) in the FRF coupling analysis is very important procedure for the FRF based substructuring method. The details of the assessment was covered in [13] in which the analytical model of the substructure was used to couple with experimental counterpart. In the study, the importance of the rotational DOFs is quantitatively described by explicit error functions for both poorly coupled FRF from using substructure with translational FRFs and another substructure with rotational FRFs systems and more general cases. These error functions reveal the composition of the error caused by the absence of rotational DOF-related FRFs.

Williams et al [14], have also performed a study on modal and frequency based substructuring using rotational DOF considerations of a simple beam. The study revealed that the predicted FRF with inducement of rotational DOFs shows better and smooth FRF curves and by using only translational DOFs somehow shows more accurate result. Later, Manzato et. al [15], investigate the effect of noise and rotational FRFs consideration of the effect of coupling and decoupling of two substructures using the FBS method in their latest review paper. The study also revealed that the coupled and decoupled FRFs showed an unacceptable result if the rotational FRFs did not include in the process of coupling. The attempt of coupling the numerical and experimental FRFs is also a failure in this paper because of the low quality of FRFs and the exclusion of rotational FRFs in the measurement.

A study in [16] on coupling an experimental beam to an analytical beam subsystem required a fixture to estimate rotations and moments at interface point in experimental model. The study revealed that the coupling was more accurate when elastic and rigid body modes of the fixture were included. As the rotational FRFs is very difficult to be measured, the modal expansion approach was used to predict the FRFs as presented in recent study by [17]. Coupling the numerical FRFs with the expanded FRFs from the measurement is showing a promising result. Dana Nicgorski and Peter Avitabile [18] discussed on the issue of measuring all the DOFs which drive to their new approach call variability improvement of key inaccurate node groups (VIKING). The VIKING method was used in this work to help remove variation that is known to exist on measured DOF and improve the results overall. However, this method seems to be very difficult to be applied on complex substructures if the variation on the measured DOFs is unknown.

The previous research work related to the frequency based substructuring method has been reviewed and discussed. Despite the frequency based substructuring method’s outstanding versatility as a structural dynamics prediction tool, its accuracy and reliability remains unconvinced and difficult to be attained by the structural dynamics analysts. Therefore, new significant contributions to knowledge are highly required in order to improve and enhance the method especially the use of rotational FRFs in the FBS method. This situation had driven an idea to the author to use the 3D type of element in the process of FRFs coupling, because 3D element did not provide and required the rotational responses, and being used for modal analysis as in [19], [20]. To the best of author’s knowledge and open literature review, there are no detailed study has been carried out for investigation of 3D elements for FRF coupling via FBS method.

Theoretical explanation of FRF Coupling

Consider a system (S) consists of subsystems (A) and (B) which the DOFs are classified either as internal or coupling DOFs as shown in Figure 1. Both subsystems are independent of one another before the coupling. The c-DOFs must be the same for both subsystems.

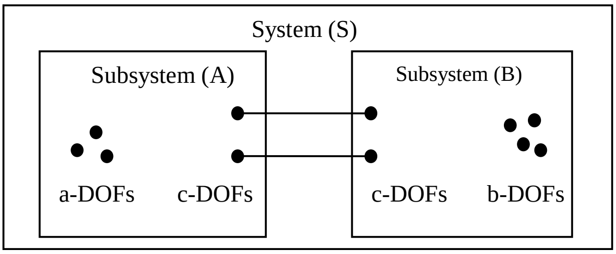

Figure 1: Schematic diagram of systems (s), subsystem (A) and subsystem (B)

The equation of motion for each subsystem in the frequency domain is:

$$\boldsymbol{X}_n = \boldsymbol{H}_{nn}\boldsymbol{F}_n \tag{1}$$

where $\boldsymbol{X}_n$ is the complex displacement, $\boldsymbol{H}_{nn}$ is the complex admittance matrix in the form of displacement, force $\boldsymbol{F}_n$ is the applied force vector and the subscript *n* is the total number of DOF for each subsystem. The basic governing relationship for subsystem A:

$$\left\{\begin{array}{c}\boldsymbol{X}^A{}_a \\ \hline \boldsymbol{X}^A{}_c\end{array}\right\}_n = \begin{bmatrix}\boldsymbol{H}^A{}_{aa} & \boldsymbol{H}^A{}_{ac} \\ \boldsymbol{H}^A{}_{ca} & \boldsymbol{H}^A{}_{cc}\end{bmatrix}\left\{\begin{array}{c}\boldsymbol{F}^A{}_a \\ \hline \boldsymbol{F}^A{}_c\end{array}\right\}_n \tag{2}$$

and for subsystem B:

$$\left\{\begin{array}{c}\boldsymbol{X}^B{}_b \\ \hline \boldsymbol{X}^B{}_c\end{array}\right\}_n = \begin{bmatrix}\boldsymbol{H}^B{}_{bb} & \boldsymbol{H}^B{}_{bc} \\ \boldsymbol{H}^B{}_{cb} & \boldsymbol{H}^B{}_{cc}\end{bmatrix}\left\{\begin{array}{c}\boldsymbol{F}^B{}_b \\ \hline \boldsymbol{F}^B{}_c\end{array}\right\}_n \tag{3}$$

For the rigid connections between subsystems A and B, compatibility implies that

$$\boldsymbol{X}^A{}_c = \boldsymbol{X}^B{}_c = \boldsymbol{X}^C{}_c \tag{4}$$

and the force equilibrium requires that

$$\boldsymbol{F}^A{}_c = \boldsymbol{F}^B{}_c = \boldsymbol{F}^C{}_c \tag{5}$$

The FRFs of the system can be defined as

$$\begin{Bmatrix}\boldsymbol{X}^s{}_a \\ \boldsymbol{X}^s{}_c \\ \boldsymbol{X}^s{}_b\end{Bmatrix}_n = \begin{bmatrix}\boldsymbol{H}^s{}_{aa} & \boldsymbol{H}^s{}_{ac} & \boldsymbol{H}^s{}_{ab} \\ \boldsymbol{H}^s{}_{ca} & \boldsymbol{H}^s{}_{cc} & \boldsymbol{H}^s{}_{cb} \\ \boldsymbol{H}^s{}_{ba} & \boldsymbol{H}^s{}_{bc} & \boldsymbol{H}^s{}_{bb}\end{bmatrix}\begin{Bmatrix}\boldsymbol{F}^s{}_a \\ \boldsymbol{F}^s{}_c \\ \boldsymbol{F}^s{}_b\end{Bmatrix}_n \tag{6}$$

The connection DOF from the partitioned Equations (2) and (3) for subsystems A and B are

$$\boldsymbol{X}^A{}_c = \boldsymbol{H}^A{}_{ca}\boldsymbol{F}^A{}_a + \boldsymbol{H}^A{}_{cc}\boldsymbol{F}^A{}_c \tag{7}$$

$$\boldsymbol{X}^B{}_c = \boldsymbol{H}^B{}_{ca}\boldsymbol{F}^B{}_a + \boldsymbol{H}^B{}_{cc}\boldsymbol{F}^B{}_c \tag{8}$$

Now let $\widetilde{\boldsymbol{F}}^A{}_c$ and $\widetilde{\boldsymbol{F}}^B{}_c$ be the internal transmitted forces at the connection point for subsystems A and B respectively. Note that for the fully coupled system

$$\widetilde{\boldsymbol{F}}^s{}_c = \widetilde{\boldsymbol{F}}^A{}_c + \widetilde{\boldsymbol{F}}^B{}_c \tag{9}$$

$$\widetilde{\boldsymbol{F}}^A{}_c = \widetilde{\boldsymbol{F}}^S{}_c - \widetilde{\boldsymbol{F}}^B{}_c \tag{10}$$

$$\widetilde{\boldsymbol{F}}^{B}{}_{c} = \widetilde{\boldsymbol{F}}^{S}{}_{c} + \widetilde{\boldsymbol{F}}^{A}{}_{c} \tag{11}$$

The c-DOFs from the partitioned equation of equation (2) and (3) are derived and can be equated and solved for the connection force $\widetilde{\boldsymbol{F}}^{A}{}_{c}$ can be derived as

$$\widetilde{\boldsymbol{F}}^{A}{}_{c} = [\boldsymbol{H}^{A}{}_{cc} + \boldsymbol{H}^{B}{}_{cc}]^{-1}\,[\boldsymbol{H}^{B}{}_{cb}\boldsymbol{F}^{B}{}_{c} - \boldsymbol{H}^{A}{}_{ca}\boldsymbol{F}^{A}{}_{a} + \boldsymbol{H}^{B}{}_{cc}\boldsymbol{F}^{S}{}_{c}] \tag{12}$$

with $\widetilde{\boldsymbol{F}}^{A}{}_{c}$ as the internal transmitted force at the interfaces of subsystem A at the coupling. The equation derived above can be used to determine the coupled system FRFs on the basis of the uncoupled FRFs of the individual components.

Description of the Test Structure

In this research, the structure under the investigation was a simple aluminium beam structure with 1050mm length, 50.5mm wide and 4.7mm thickness as shown in Figure 2. The configuration allows an initial study of the efficiency of the FRF substructuring method in predicting the dynamic behaviour of a simple structure such as beams as demonstrated in the previous work [2], [7], [14], [16] to be performed. Firstly, the dynamic behaviour of the aluminium beam was experimentally determined before it was divided into two substructures. Upon successfully measuring the dynamic behaviour, the beam was divided equally into two components or substructures which are named substructures A and B respectively. Secondly, on the finite element work, finite element modelling of each of the substructures using 2D and 3D elements was performed respectively. Having completed the modelling work, the two different types of the finite element models which are 2D based model and 3D based model were then used for calculating the FRFs and FBS analysis. Lastly, the evaluation of the most suitable element for the coupling purposes was carried. This was performed by comparing the resulting FRFs of both types of the finite element modes with those obtained experimentally.

Figure 1 : Assembled structure

Test set-up of the beam structure

Figure 3 shows the experimental set-up for the measurement of the dynamic behaviour of the aluminium beam structure. Using an impact testing method, the structure was tested on free-free boundary conditions. Four rubber bands were used to simulate the free-free boundary conditions to the test structure.

The finite element results were used as guidance for the determination of the FRF of the test structure.

The frequency of interest of the structure was within 0 to 300Hz. An impact hammer was used to excite the structure and accelerometer were used to acquire the dynamic data. The accelerometers were mounted using bee wax. The use of the bee wax was because the frequency of interest in this study was not more than 10,000 Hz which is acceptable for this kind of adhesive mounting. To measure FRFs of the beam, he a reference accelerometer was mounted on in the centre of the beam where the centre is the ideally placed for the connection between the substructures A and B. LMS SCADAS analyser was used to interpret the load and response signals.

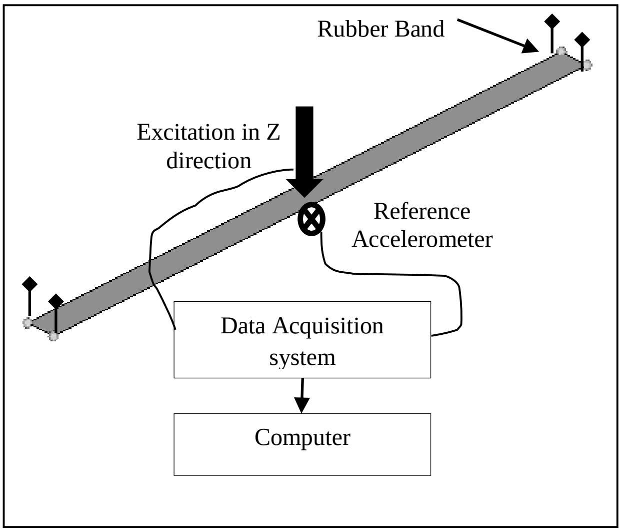

Figure 3: Test set-up for aluminium beam structure

Finite Element Modelling and FRF Based Substructuring Method

The aim of this study was to evaluate the FRFs of the assembled structure from the FBS method using 2D and 3D elements. Each of the substructures was constructed by using two different types of elements. The first one was the 2D based finite element model and the other one was the 3D based finite element models. Prior to performing the FBS assembly analysis of the structure, the FRFs of each of the substructures were needed to be derived. The FRFs of the substructures were extracted by using FRF synthesis method, based on a finite number of natural frequencies and mode shapes of the subsystem. For this method, the synthesized FRF matrix $H_{syn}(\omega_k)$ and mode shapes are expressed by:

$$\mathrm{H_{syn}}(\omega_\mathrm{k}) = \sum_{i=1}^{N} \frac{\{\emptyset\}_i\{\emptyset\}_i^T}{\left(\omega_{n_i}^2 - \omega_k^2\right) + j2\xi_i\omega_k\omega_{n_i}} \tag{13}$$

where *N* is the number of calculated modes, $\{\emptyset\}_i$ is the *i*th mass normalised mode shapes, ω_{n_i} is *i*th natural frequency and ξ_i is the *i*h modal damping ratio.

For the 2D based finite element model of the substructures, there are six DOFs on each of the interface nodes. Since there are merely two interface nodes on each of the 2D based finite element model of the substructures, therefore, the total number of interface nodes involved in the coupling is four nodes, and in terms of the number of DOFs, the 2D based finite element model has 24 DOFs in total. Meanwhile, the 2D based finite element models of the substructures were assembled using a rigid type of connection as shown in Figure 4. The previous studies have revealed that in order to achieve a reliable FBS prediction, each of the DOFs has to be coupled [9]. Therefore, there are two widely used types of coupling configurations used in this study. The types are as follows:

1. Coupling all translation and rotation DOFs
2. Coupling only translation DOFs.

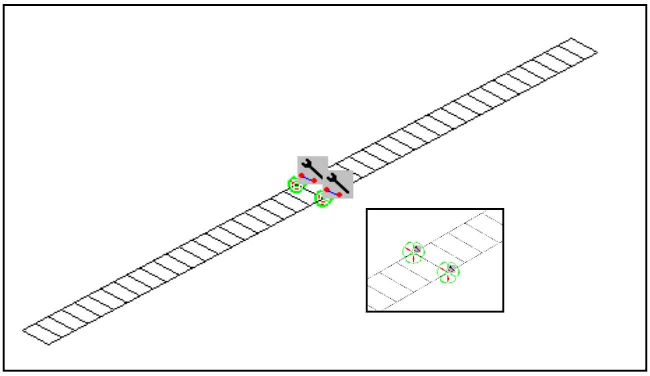

Figure 4: FBS coupling of 2D FE model of substructure A and B

The same method and procedures were used to couple the 3D based finite element models of the substructures as show in Figure 5. There are four nodes on each of the interface nodes of the 3D based finite element models of the substructures. The coupling of the substructures required eight interface nodes. The total number of the coupled DOFs is 24 DOFs. This is because 3D element has only three translational DOFs in comparison with 2D element.

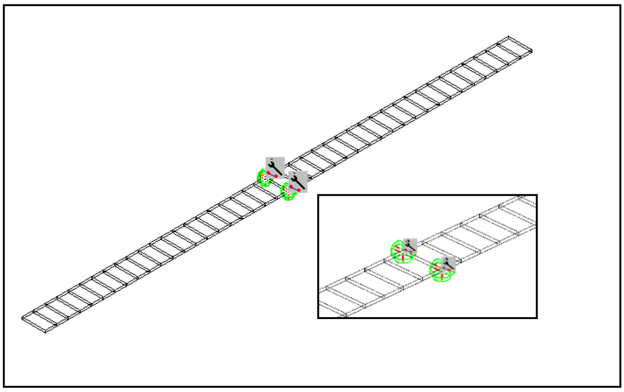

Figure 5: FBS coupling for 3D based FE model of substructure A and B

Results and Discussion

Attempts made by number of researchers [13], [14], [21] in predicting the dynamic behaviour of structures using frequency based substructure (FBS) revealed that the accuracy of the FBS method is highly dependent on the quality of experimental FRFs and the inclusion of rotational FRFs. In other words, poor quality of the experimental FRFs and the lack of information on rotational responses may lead to the different level of accuracy in predicting the dynamic behaviour of the structures. Therefore, the enhancement of the accuracy of the FBS method in terms of producing good quality of experimental FRFs and considering the rotational FRFs has been much attention to researchers [22], [23]. In this research, the FRFs of each of the substructures were obtained from the finite element method and experimental modal analysis. The FBS method was then used to couple the finite element FRFs with the experimental FRFs for the prediction of the dynamic behaviour of the assembled substructure (substructures A and B).

The initial procedure of the FBS method used in this research begins with the FRF at the reference point of the assembled aluminium beam structure was firstly measured. The measured FRF was used for pairing purposes with the FRFs calculated from the FBS analysis. The reference point which is the coupling interface of the two aluminium beam substructures, was located at the centre of the beam. Figure 6 shows two sets of the FRFs of the assembled aluminium obtained from the finite element method and experimental modal analysis. As can be seen in the figure 6, the FE (finite element) FRFs have good agreement with the experimentally obtained FRFs. The frequency of interest selected for the determination of the FRFs was limited from 0 to 300 Hz. This is because firstly the sensitivity of the tri-axial accelerometer used was found to be about 100mv/g on average and secondly the measurement capability of the accelerometer was found to be below 500 Hz.

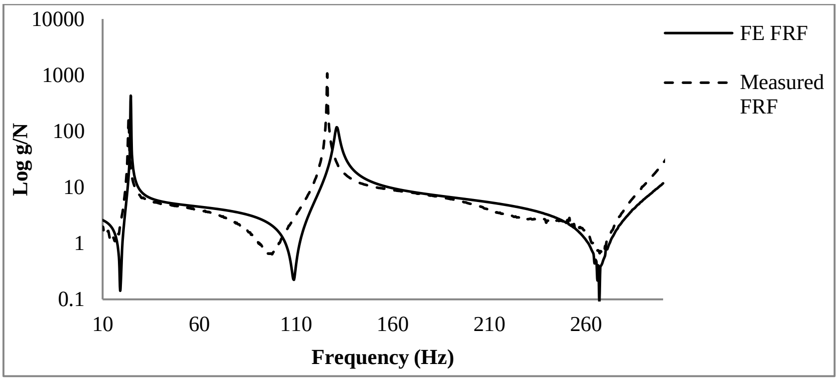

Figure 6: Finite element and measured FRFs of the assembled aluminium beam structure

From Figure 6, it was found that there are four peaks identified within the range of the frequency of interest and they are 23.4 Hz and 126.2 Hz for the measured FRF while 23 Hz and 125.78 Hz for the finite element FRF. It was worth noting that inconsiderable differences were recorded between the measured and FE FRFs. The discrepancies may be because of the invalid assumptions made about the material properties of the finite element model [24], [25]. In addition, the nominal values of the material properties of aluminium T6061 were used in the finite element model. After the FRF measurement of the aluminium beam was performed, the beam was spilt into two substructures namely substructure A and substructure B. The finite element models of substructures A and B were built by using two different types of elements which are 2D shell elements and 3D solid elements. It is worth noting that 2D shell elements consist of four nodes and each of the nodes has six degrees of freedoms (DOFs) which are three translations and three rotations.

In this study, the substructures A and B were coupled by using two coupling configurations. The first one was carried out by combining the substructures using all the DOFs in terms of translational and rotational DOFs. The other one was performed by merely coupling translational DOFs. The decision to limit the coupling procedure to the translational DOFs by not including the rotational DOFs because it was found that the measurements of rotational DOFs are very difficult and the results obtained are often inaccurate [1]. Therefore, in this work, FRF coupling using only the translational DOFs was carried out in order to analyse the effect of excluding the rotational DOFs.

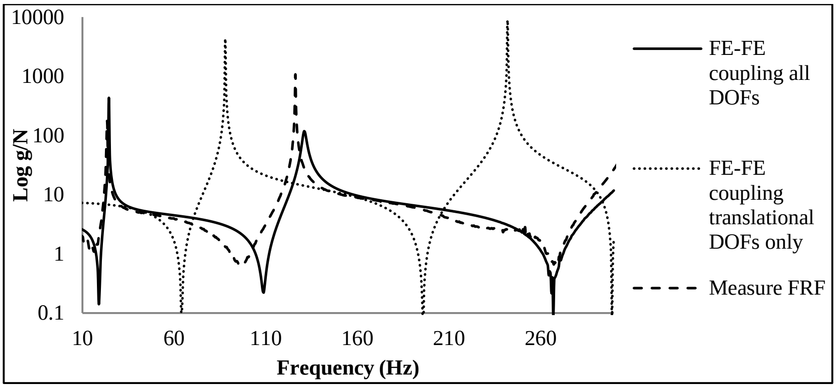

Figure 7: Coupled (using 2D finite element models) and measured FRFs of the combined structure

The FRFs of the assembled structured (substructures A & B) which were obtained from the coupling procedure of all DOFs and translational DOFs are presented in Figure 7. In addition, the measured FRFs are also available and they are used for validation purposes. From the figure, the comparison of the FRFs clearly shows that huge discrepancies between the FE model with only coupling the translational DOFs and measured FRFs were found. This reveals that by completely excluding the rotational DOFs in the coupling procedure has rendered the FRFs predicted from the FE model of the assembled beam incapable to well match with the measured counterparts. Theoretically, 2D elements are built based on the translational and rotational matrices, the theory lies in 2D elements leads to infer that ignoring the rotational matrices in the coupling procedure may lead to the overall matrices become unreliable. Therefore, the second coupling configuration which is translational based coupling is not trustworthy for 2D shell elements. On the other hand, it was found that coupling all the DOFs provides better correlation with the measured FRF. These findings suggest that the rotational DOFs must be included in FRF coupling procedure for 2D elements based finite element models. As a conclusion, including the rotational DOFs in the coupling procedures of the FBS method seems to be a necessary requirement. Therefore, the rotational FRFs of the actual test structure need to be derived experimentally in order to produce accurate results of FBS analysis.

The second type of elements used for the development of the finite element model of both substructures A and B was 3D solid elements. The 3D solid elements only support translational DOFs, in comparison with the 2D elements which consist of translational and rotational DOFs. The FRFs derived from the use of 3D solid elements are presented in Figure 8. From the figure, the FRFs obtained were found to be in good agreement with the experimental counterparts.

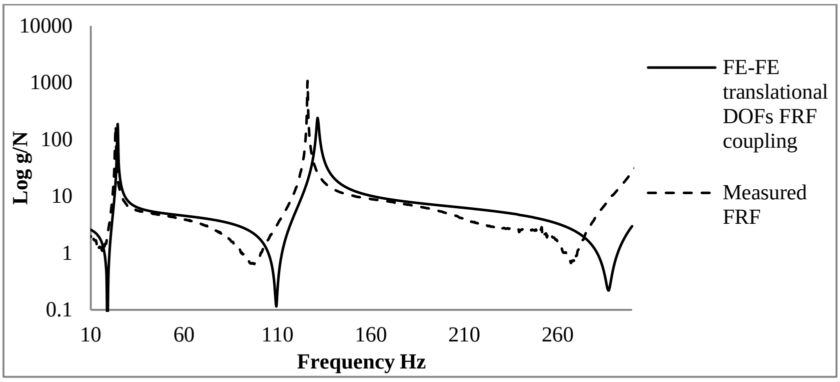

Figure 8: Coupled (using 3D finite element models) and measured FRFs of the combined structure

It was worth noting that the coupling procedure of the FRFs was performed by merely using the translational DOFs. This indicates that firstly coupling the translation DOFs can lead to the acceptable level of the FRF results and secondly the finite element models of the substructures should be built based on 3D elements. In addition, there is a high possibility of obtaining more accurate FRF results by coupling the experimental FRFs with the FRFs obtained from the 3D finite element models of the substructures. However, the application of the FBS method is mostly found in the combination between the numerical and experimental FRFs [15], [26]. Therefore, in order to prove the hypothesis, the finite element and measured FRFs of the substructure were couple together instead of merely using the finite element FRFs in the coupling procedure. In this study, substructure A was selected as the actual test model, which its FRFs were measured using the experimental modal analysis. On the other hand, the FRFs from the substructure B were derived from the 3D finite element model. The finite element and measured FRFs were coupled by using a rigid type of coupling. In this case, only the translational FRFs were used in the coupling procedure. Figure 10 shows the coupled FRFs and measured FRFs of the assembled beam.

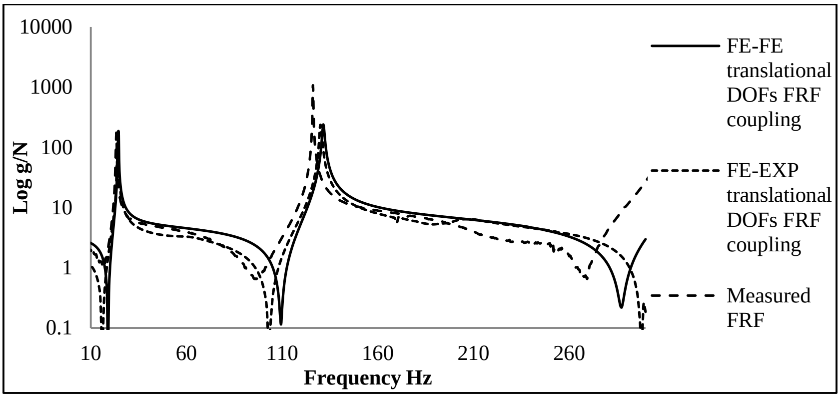

Figure 9: Coupled and measured FRFs of the combined structure

From Figure 9, it is clearly shown that the coupled FRFs from the use of the finite element and experimental FRFs have good agreement with the measured FRF of the assembled structure. It was worth noting that despite excluding the rotational FRFs in the coupling procedure, the resulting coupled

FRFs recorded as almost the same pattern as the measured FRFs. This achievement revealed that coupling the experimental FRFs with the FRFs derived from 3D finite elements has resulted in the acceptable level of accuracy. The slight discrepancies between the measured anti-resonance and FE counterparts as shown in Figure 10 may be due to the invalid assumptions about the finite element model or the effect of the rigid coupling in the coupling process. Furthermore, the discrepancies may also be attributed to the effect of damping which was not included in the finite element model.

Conclusions

The use of the FBS for the prediction of the dynamic behaviour of a simple aluminium beam is presented. . It is clearly shown that the FBS method has been successfully performed for the finite element FRFs of the substructures A and B. This shows that completely excluding the rotational DOFs in the coupling procedure of the 2D finite element model derived FRFs of the substructures A and B has rendered the FRFs predicted from the FE model of the assembled beam incapable to well match with the measured counterparts. Nevertheless, it is contrasted to the FRFs derived from the 3D finite element model of the assembled beam in which good agreement with the experimental counterparts is obtained. Therefore, it is imperative to note that despite completely excluding the rotational FRFs in the coupling procedure of the coupled FRFs, the predicted results have shown good agreement with the experimental counterparts. .

Acknowledgements

The authors gratefully acknowledge the Malaysian Ministry of Higher Education and Research Management Institute (RMI) of Universiti Teknologi MARA (UiTM) for providing financial support for this study through the research acculturation grant scheme (RAGS) (600-RMI/RAGS 5/3 (159/2014) and fundamental research grant scheme (FRGS) (600-RM1/FRGS 5/3 (96/2016), and UiTM Institute of Graduate Studies (IPSIS) for paying the conference fee. They would like to express their heart-felt appreciation for the constant support and help given by Prof. Dr. Hadariah Bahron, Assoc. Prof. Dr. Siti Akmar Abu Samah and Mr Mohd Hakimi of SIRIM.

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