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PAPER

Optimization Approach to Minimize Backup Capacity Considering Routing in Primary and Backup Networks for Random Multiple Link Failures

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SUMMARY This paper proposes an optimization approach that designs the backup network with the minimum total capacity to protect the primary network from random multiple link failures with link failure probability. In the conventional approach, the routing in the primary network is not considered as a factor in minimizing the total capacity of the backup network. Considering primary routing as a variable when deciding the backup network can reduce the total capacity in the backup network compared to the conventional approach. The optimization problem examined here employs robust optimization to provide probabilistic survivability guarantees for different link capacities in the primary network. The proposed approach formulates the optimization problem as a mixed integer linear programming (MILP) problem with robust optimization. A heuristic implementation is introduced for the proposed approach as the MILP problem cannot be solved in practical time when the network size increases. Numerical results show that the proposed approach can achieve lower total capacity in the backup network than the conventional approach.
key words: *link failure, optimization problem, backup capacity, probabilistic survivability guarantee*

1. Introduction

Link failure can lead to the loss of huge amounts of data. In order to ensure recovery from failures, a backup network with proper routes must be prepared prior to any link failure [1]–[3]. Link protection, which can recover link failures, has two main categories. The first category prepares the resource on demand, upon request. It has the advantage that the spare capacity is efficiently utilized. However, it takes time to setup the connection since the necessary links must be computed for every incoming request. The second category is pre-planned link restoration, where routes in a backup network are computed for each link failure in advance, and the computed routes are established prior to service commencement. It is superior to the first approach in terms of speed and simplicity of failure recovery, as no additional dynamic routing is necessary at the time of link failure [4], [5].

A spare capacity allocation method to protect the primary network from single link failure was introduced in [6]. This method designs a backup network that offers adequate protection resources to recover the primary network from any single link failure. When a failure occurs, the traffic on the failed link in the primary network is switched to the predetermined routes in the backup network. To reduce the risk of the protection routes failing, it is advantageous to establish the routes in the primary and backup networks on separate resources. The integer linear programming (ILP) problem can be used to design a backup network with the minimum cost [6]; the ratio of backup network capacity to primary network capacity falls as the latter increases.

A protection approach to recover the primary network from random multiple link failures with probabilistic survivability guarantees was presented in [7], [8]. The probability of link failure in the primary is considered in [7], which applies the results of the optimization problem for a single link failure from [6] to design the backup network for link failure. A mixed integer linear programming (MILP) formulation to design the backup network was provided. Simulated annealing (SA) [9], which is a heuristic approach, has been used to solve the problem for large networks and reduce the capacity of the backup network. This work considers only backup network routing as a variable. Primary network routing is not considered as a variable. The work in [8] imposed the condition that the primary network would have discrete capacities for the backup network design problem in [7] to suppress overestimating of the capacity of the backup network. It also considers backup network routing as the only variable.

Network operators design their networks to minimize the total capacity of the backup network by considering the traffic as an input to the design problem [10]. An optimization approach to minimize the over-provisioning overhead for the spare capacity assignment problem was introduced in [11]; traffic conditions were given with some bounds. This work established a backup network with capacity necessary to protect the primary network against a single link failure. A mathematical formulation to design the backup network was introduced by taking into account the traffic conditions. Several types of traffic conditions are considered in [12]–[14].

The works on multiple link failures [7], [8] designed the backup network with minimized capacity by considering that primary network routing is given. All traffic already allocated on each link in the primary network is considered to be protected. Considering the routing in the primary net-

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work as given parameter may not achieve the minimum total capacity in the backup network.

We consider that the total capacity in the backup network can be lower by taking both primary and backup network routing as decision variables rather than by considering only the backup routing as a derision variable. Determining the primary network routing means that the traffic allocation on each link in the primary network is determined.

This paper proposes an optimization approach that designs the backup network with minimum total capacity to protect the primary network from random multiple link failures where the probability of link failure is specified. This paper is an extended version of [15]. The probabilistic survivability guarantee is provided by determining both primary and backup network routing, simultaneously. Robust optimization is introduced to provide probabilistic survivability guarantees for different link capacities in the primary network. We formulate our optimization problem as an MILP problem by using the robust optimization technique. We investigate how the probability of link failure affects both primary and backup network routing. Since the MILP problem cannot be solved in practical time when the primary network is large, a heuristic method is added to the proposal. Numerical results show that the proposed approach yields a backup network with lower total capacity than the conventional approach, in which the routing in the primary network is not considered as a factor in minimizing the total capacity of the backup network, for the link failure probabilities examined in this paper. The results indicate that the proposed approach yields highly efficient primary and backup network routing designs that well reflect the given probability of link failure.

Our proposed approach adopts detour routing. The detour routing uses in the dedicated backup network, which is different from a network that considers only the primary network with redundant capacity on each link. The dedicated backup network uses a lower capacity to provide the protection against link failures than the primary network, as backup resource sharing can be expected, which was observed in [6], [7]. Thanks to the effect of backup resource sharing, links in the backup network can be made more reliable by hardening or shielding than those in the primary network [7]. In the network that considers only the primary network, the reliability of the detour routing and the primary routing are the same. In the network that considers the dedicated backup network, the reliability of the detour routing in the backup network can achieve higher than that of the primary routing. This is a benefit of the dedicated backup network.

The structure of this paper is organized as follows. Section 2 presents the network model with probabilistic failures used in this paper. Section 3 introduces the optimization problem used to find the optimal solution. Section 4 presents the heuristic method. The performance of the proposed approach is evaluated in Section 5. Section 6 concludes our paper.

2. Network model with probabilistic failures

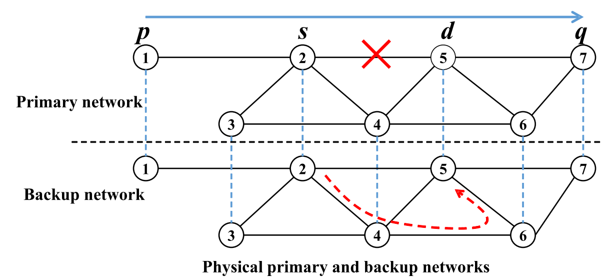


Fig. 1 Network model.

We present the network model with probabilistic failures used in this paper. We basically use the same model presented in [7], except that this paper considers the traffic from each source node to each destination node, which is given, and considers primary network routing as a decision variable. We assume that explicit routing is adopted in both primary and backup networks [16].

Let $G(V, E)$ be a directed graph for the primary network, where V is the set of nodes and E is the set of links. $Q \subseteq V$ is the set of edge nodes through which traffic is allowed into the network. The link in the primary network from node $s \in V$ to node $d \in V \backslash \{s\}$ is denoted as $(s, d) \in E$. P is the set of pairs of source node $p \in Q$ and destination node $q \in Q \backslash \{p\}$ in the primary network. The traffic demand from edge node $p \in Q$ to node $q \in Q \backslash \{p\}$ is denoted as d_{pq}. We assume that d_{pq} is given. Let decision variable w_{sd}^{pq}, where $0 \leq w_{sd}^{pq} \leq 1$, be the portion of traffic demand d_{pq} from node $p \in Q$ to node $q \in Q \backslash \{p\}$ passing through link $(s, d) \in E$. $0 < w_{sd}^{pq} \leq 1$ means that $(s, d) \in E$ is a link on one of the routes for d_{pq}. $w_{sd}^{pq} = 0$ means that $(s, d) \in E$ is not a link on any route for d_{pq}. If $0 < w_{sd}^{pq} < 1$, for at least one (s, d), d_{pq} is split over multiple routes.

Using the same set of nodes V and a new set of links E_b construct backup network $G_b(V, E_b)$, by establishing backup network routes to protect the traffic carried by each primary link $(s, d) \in E$. Sufficient capacity is allocated to every backup link (i, j), where $(i, j) \in E_b$ denotes a backup link from node $i \in V$ to node $j \in V \backslash \{i\}$. A backup network route is chosen if link $(s, d) \in E$ fails; the traffic through link $(s, d) \in E$ in the primary network is switched to the backup network route.

Figure 1 shows the example of a network model with seven nodes, where node 1 and node 7 are edge nodes represented as nodes $p \in Q$ and $q \in Q \backslash \{p\}$, respectively. The links $(s, d) \in E$ for the route from $p = 1$ to $q = 7$ are $(1, 2)$, $(2, 5)$, and $(5, 7)$. To cover the case that primary link $(2, 5)$ in the primary network fails, the backup route in the backup network from source node $s = 2$ and destination node $d = 5$ is designed as the route of $(2 \rightarrow 4 \rightarrow 6 \rightarrow 5)$.

The probability of link failure, κ, in the primary network is given for each link $(s, d) \in E$; each link has independent probability. Let X_{sd} be a random variable in the primary network. X_{sd} is equal to 1 if link $(s,d) \in E$ in the primary network fails, and 0 otherwise. Let binary variable $b_{ij}^{sd} = 1$ if link $(s,d) \in E$ uses backup link $(i,j) \in E_b$ on the backup network, and 0 otherwise. b_{ij}^{sd} represents the route in the backup network for primary link $(s,d) \in E$. We assume that the traffic that is routed through the backup network against each primary link failure is not split. This is because the backup routing operation is required to be simple for fast recovery.

We describe the idea of probabilistic survivability guarantee for the backup network design [7]. Let ϵ denote the probabilistic survivability guarantee parameter, where $\epsilon > 0$. Let Y_{ij} denote a random variable that is the capacity required to completely protect the primary failure-link capacities, where the probability of failure for each link is κ. Let C_{ij}^{B} denote the required backup capacity of $(i,j) \in E$ to protect failed links $(s,d) \in E$; C_{ij}^{B} is a decision variable. In the probabilistic survivability guarantee, for each $(i,j) \in E_{\mathrm{b}}$ the probability that Y_{ij} is larger than C_{ij}^{B} must be less than or equal to ϵ. This work only the case of $\kappa > \epsilon$, since the requirement of backup is not necessary. This reason is as follows. Suppose that each $(s,d) \in E$ in the primary network is protected by each backup route that uses only link $(s,d) \in E_{\mathrm{b}}$ dedicatedly in the backup network. In the probabilistic survivability guarantee, the primary link failure is not required to protect within the probability ϵ. If $\kappa \le \epsilon$, no capacity in $(s,d) \in E_{\mathrm{b}}$ is required. We assume that no link in the backup network fails, which is the same assumption used in [7], for simplicity. To cover the case that a link in the backup network fails, we can adopt the same idea described in Section IV.B of [7].

3. Optimization problem

3.1 Definition of optimization problem

The objective is to minimize the total capacity in the backup network by considering both backup and primary network routing. Backup and primary network routing are determined by decision variables of b_{ij}^{sd} and w_{sd}^{pq}, respectively. We consider the following assumptions. Each primary link is already facilitated. The allowable capacity of each facilitated primary link is sufficiently large to accommodate traffic demands. The backup network is designed to protect the traffic demands routed through each primary link. That is why our objective is to minimize the total capacity in the backup network.

The optimization problem is written as:

$$\min \sum_{(i,j)\in E_b} C_{ij}^{\mathrm{B}} \tag{1a}$$

$$\text{s.t.} \quad \mathrm{P}(Y_{ij} > C_{ij}^{\mathrm{B}}) \le \epsilon, \quad \forall (i,j) \in E_b \tag{1b}$$

$$\sum_{j:(i,j)\in E_b} b_{ij}^{sd} - \sum_{j:(j,i)\in E_b} b_{ji}^{sd} = 1, \quad i = s, \quad \forall (s,d) \in E \tag{1c}$$

$$\sum_{j:(i,j)\in E_b} b_{ij}^{sd} - \sum_{j:(j,i)\in E_b} b_{ji}^{sd} = 0, \quad i \ne s,d, \quad \forall (s,d) \in E \tag{1d}$$

$$\sum_{d:(s,d)\in E} w_{sd}^{pq} - \sum_{d:(d,s)\in E} w_{ds}^{pq} = 1, \quad s = p, \quad \forall (p,q) \in P \tag{1e}$$

$$\sum_{d:(s,d)\in E} w_{sd}^{pq} - \sum_{d:(d,s)\in E} w_{ds}^{pq} = 0, \quad s \ne p,q, \quad \forall (p,q) \in P \tag{1f}$$

$$b_{ij}^{sd} \in \{0,1\}, \quad \forall (i,j) \in E_b, (s,d) \in E \tag{1g}$$

$$0 \le w_{sd}^{pq} \le 1, \quad \forall (p,q) \in P, (s,d) \in E. \tag{1h}$$

Equation (1a) minimizes the objective value, which is the total capacity of the backup network. Equation (1b) expresses the constraint of probabilistic survivability guarantees for each link in the backup network. Equation (1b) demands that the probability of Y_{ij} being larger than C_{ij}^{B} must be less than or equal to probabilistic survivability guarantee parameter ϵ. Equations (1c)-(1d) express the flow constraints for routing in the backup network at each source and each intermediate node, respectively [17]. Equations (1e)-(1f) express flow constraints for routing in the primary network at each source and each intermediate node, respectively. Equations (1g)-(1h) indicate the ranges of decision variables for b_{ij}^{sd} and w_{sd}^{pq}, respectively.

3.2 Unit link capacity

We first assume that the link capacity in the primary network is unitary. The link capacity in the primary network, which is determined by the optimization problem, means the total traffic demands routed through this link. Let the number of primary links $(s,d) \in E$ that use backup link $(i,j) \in E_b$ be denoted as n_{ij}. We have:

$$n_{ij} = \sum_{(s,d)\in E} b_{ij}^{sd}. \tag{2}$$

The number of failed links in the primary network using backup link (i,j) as part of their route in the backup network is equivalent to Y_{ij}, which is expressed by:

$$Y_{ij} = \sum_{(s,d)\in E} b_{ij}^{sd} X_{sd}. \tag{3}$$

The probabilistic constraint, which is the capacity constraint, from which the backup capacities are computed, is given by:

$$\mathrm{P}(Y_{ij} > C_{ij}^{\mathrm{B}}) = \sum_{y=C_{ij}^{B}+1}^{n_{ij}} \binom{n_{ij}}{y} \kappa^{y} (1-\kappa)^{(n_{ij}-y)} \le \epsilon, \quad \forall (i,j) \in E_b. \tag{4}$$

Equation (4) defines the probability that y links out of n_{ij} links have failed. $G(n_{ij},\kappa,\epsilon)$ be the minimum value of C_{ij}^{B} satisfying Eq. (4), which is computed as the capacity allocated to each link $(i,j) \in E_b$ in the backup network.

Since the required capacity in the backup network depends on which links of $(s, d) \in E$ in the primary network are protected, in the case of general link capacity, this approach cannot be applied directly. Fortunately, techniques from the field of robust optimization can be used to formulate the problem of general link capacity.

3.3 Robust optimization for general link capacity

Robust optimization is a technique that can find an optimal solution for a problem where a degree of uncertainty is involved. Robust optimization is employed in our optimization problem to allow consideration of the general link capacities in the primary network.

We denote $\Gamma_{ij} = G(n_{ij}, \kappa, \epsilon)$. In case of unit link capacity, $G(n_{ij}, \kappa, \epsilon)$, which is introduced in section 3.2, is determined by n_{ij}, where κ and ϵ are given parameters. n_{ij} is determined by b_{ij}^{sd} in Eq. (2). In section 3.4, we will give the relationship between Γ_{ij} and b_{ij}^{sd}, in linear form, where κ and ϵ are given. In other words, Γ_{ij} is determined by b_{ij}^{sd}, κ, and ϵ.

Let $L_{ij} = \{(s, d) \mid b_{ij}^{sd} = 1\}$ be the set of primary links $(s, d) \in E$ protected by backup link $(i, j) \in E_b$. S_{ij} is a subset of L_{ij} with the largest capacities, where $|S_{ij}| = \Gamma_{ij}$. For any $(s, d) \in S_{ij}$, we have:

$$\sum_{(p,q)\in P} d_{pq} w_{sd}^{pq} \geq \sum_{(p,q)\in P} d_{pq} w_{s'd'}^{pq}, \forall (s', d') \in L_{ij} \backslash S_{ij}. \quad (5)$$

The required backup capacity to protect against any Γ_{ij} primary link failures is given by,

$$C_{ij}^{\mathrm{B}} = \sum_{(s,d)\in S_{ij}} \sum_{(p,q)\in P} d_{pq} w_{sd}^{pq}. \quad (6)$$

The above constraint can be expressed as the following complete form:

$$C_{ij}^{\mathrm{B}} \geq \max_{S_{ij}|S_{ij}\subseteq E, |S_{ij}|=\Gamma_{ij}} \sum_{(s,d)\in S_{ij}} \sum_{(p,q)\in P} d_{pq} w_{sd}^{pq} b_{ij}^{sd}, \quad \forall (i, j) \in E_b. \quad (7)$$

The probability ensuring full protection from multiple link failure is determined by the value of Γ_{ij}, which is fixed for each link $(i, j) \in E_b$. The probabilistic constraint is replaced by the capacity constraint in Eq. (7). The nonlinear optimization problem is written as follows.

$$\min \sum_{(i,j)\in E_b} C_{ij}^{\mathrm{B}} \quad (8a)$$

$$\text{s.t. } C_{ij}^{\mathrm{B}} \geq \max_{S_{ij}|S_{ij}\in E, |S_{ij}|=\Gamma_{ij}} \sum_{(s,d)\in S_{ij}} \sum_{(p,q)\in P} d_{pq} w_{sd}^{pq} b_{ij}^{sd}, \quad \forall (i, j) \in E_b \quad (8b)$$

$$\text{Eqs. (1c)-(1h).} \quad (8c)$$

Since the backup capacity constraint in Eq. (8b) is nonlinear form, Eqs. (8a)-(8c) also are nonlinear form. We reformulate the backup capacity constraint as a linear programming (LP) problem using a duality technique. For fixed b_{ij}^{sd} and Γ_{ij}, the backup capacity of each link $(i, j) \in E_b$ is given as follows.

$$\beta_{ij}(\mathrm{b}_{ij}, \Gamma_{ij}) = \max_{S_{ij}|S_{ij}\in E, |S_{ij}|=\Gamma_{ij}} \sum_{(s,d)\in S_{ij}} \sum_{(p,q)\in P} d_{pq} w_{sd}^{pq} b_{ij}^{sd}. \quad (9)$$

Equation (9) can be written as the solution to the following LP problem, where z_{ij}^{sd} is the decision variable; b_{ij}^{sd} and $\sum_{(p,q)\in P} d_{pq} w_{sd}^{pq}$ are the given parameters.

$$\beta_{ij}(\mathrm{b}_{ij}, \Gamma_{ij}) = \max \sum_{(s,d)\in E} \sum_{(p,q)\in P} d_{pq} w_{sd}^{pq} b_{ij}^{sd} z_{ij}^{sd} \quad (10a)$$

$$\text{s.t.} \sum_{(s,d)\in E} z_{ij}^{sd} \leq \Gamma_{ij} \quad (10b)$$

$$0 \leq z_{ij}^{sd} \leq 1, \ \forall (s, d) \in E. \quad (10c)$$

The Γ_{ij} primary links with the largest capacities among the primary links $(s, d) \in E$ that satisfy $b_{ij}^{sd} = 1$ are chosen in the LP problem by setting $z_{ij}^{sd} = 1$ for those links $(s, d) \in E$. If the number of primary links $(s, d) \in E$ that satisfy $b_{ij}^{sd} = 1$ is fewer than Γ_{ij}, z_{ij}^{sd} is equal to 1 for each of these links and the other $(s, d) \in E$ satisfying $z_{ij}^{sd} = 1$ are arbitrarily chosen.

We consider Eqs. (10a)-(10c) as a primal problem. It is transformed into the dual problem (see Appendix) [17], [18], which is formulated by:

$$\min \varrho_{ij} \Gamma_{ij} + \sum_{(s,d)\in E} \xi_{ij}^{sd} \quad (11a)$$

$$\text{s.t. } \varrho_{ij} + \xi_{ij}^{sd} \geq \sum_{(p,q)\in P} d_{pq} w_{sd}^{pq} b_{ij}^{sd}, \quad \forall (s, d) \in E \quad (11b)$$

$$\xi_{ij}^{sd} \geq 0, \quad \forall (s, d) \in E \quad (11c)$$

$$\varrho_{ij} \geq 0, \quad (11d)$$

where ϱ_{ij} and ξ_{ij}^{sd} are newly introduced dual decision variables. Dual decision variables of ϱ_{ij} and ξ_{ij}^{sd} are produced in the transformation from the primal problem to the dual problem as a result of mathematical operation. The transformation from the primal problem to the dual problem is explained in the Appendix.

By the duality theorem [17], [19], for a pair of primal and dual problems, if there is an optimum solution of either the primal problem or the dual problem, it is guaranteed that an optimum solution of the other problem exists. Moreover, both optimum values of the objective functions are the same. Therefore, the primal problem in Eqs. (10a)-(10c) and its dual problem in Eqs. (11a)-(11d) have zero duality gap. Function $\beta_{ij}(\mathrm{b}_{ij}, \Gamma_{ij})$ is equal to the optimal objective value of the dual problem. In addition, since the problem in Eqs. (8a)-(8c) minimizes $\beta_{ij}(\mathrm{b}_{ij}, \Gamma_{ij})$ for each $(i, j) \in E_b$, the problem in Eqs. (11a)-(11d) can be substituted into Eqs. (8a)-(8c) to obtain the following optimization problem:

$$\min \sum_{(i,j)\in E_b} C_{ij}^{\mathrm{B}} \quad (12a)$$

$$\text{s.t.}\quad C^{\mathrm{B}}_{ij} \geq \varrho_{ij}\Gamma_{ij} + \sum_{(s,d)\in E} \xi^{sd}_{ij},\quad \forall (i,j) \in E_b \tag{12b}$$

$$\varrho_{ij} + \xi^{sd}_{ij} \geq \sum_{(p,q)\in P} d_{pq} w^{pq}_{sd} b^{sd}_{ij},\quad \forall (s,d) \in E, (i,j) \in E_b \tag{12c}$$

$$\text{Eqs. (1c) - (1h).} \tag{12d}$$

$$\xi^{sd}_{ij} \geq 0,\quad \forall (s,d) \in E, (i,j) \in E_b \tag{12e}$$

$$\varrho_{ij} \geq 0,\quad \forall (i,j) \in E_b. \tag{12f}$$

The derivation of Eq. (12b) is explained below. The right hand side of Eq. (8b) is replaced by Eq. (11a), which must be minimized, with Eqs. (11b) and (11c) as constraints. This is because there is no duality gap in the optimal solution by the duality theorem. To minimize the objective function in Eq. (8a), C^{B}_{ij} must be minimized, which means that the objective function in Eq. (10a) must be minimized. Thus, Eq. (12a) does not include "min", which is covered by "min" in Eq. (12a).

To write the product $w^{pq}_{sd} b^{sd}_{ij}$ in Eq. (12c) in linear form, a set of optimization variables is added. The positive variable denoted as u^{sd}_{ijpq} is introduced that satisfies the following constraints:

$$u^{sd}_{ijpq} \geq w^{pq}_{sd} + b^{sd}_{ij} - 1,\quad \forall (i,j)\in E_b, (s,d)\in E, (p,q) \in P \tag{13a}$$

$$u^{sd}_{ijpq} \leq b^{sd}_{ij},\quad \forall (i,j)\in E_b, (s,d)\in E, (p,q) \in P \tag{13b}$$

$$u^{sd}_{ijpq} \leq w^{pq}_{sd},\quad \forall (i,j)\in E_b, (s,d)\in E, (p,q) \in P \tag{13c}$$

$$u^{sd}_{ijpq} \geq 0,\quad \forall (i,j)\in E_b, (s,d)\in E, (p,q) \in P. \tag{13d}$$

Equation (13a) forces $u^{sd}_{ijpq} \geq w^{pq}_{sd}$ if $b^{sd}_{ij} = 1$. Equations (13b) and (13d) force u^{sd}_{ijpq} to 0 if $b^{sd}_{ij} = 0$. Equation (13c) forces u^{sd}_{ijpq} not to exceed w^{pq}_{sd}. Thus, Eqs. (13a)-(13d) force $u^{sd}_{ijpq} = w^{pq}_{sd}$ if $b^{sd}_{ij} = 1$, and $u^{sd}_{ijpq} = 0$ otherwise.

3.4 Mixed integer linear programming formulation

A table of Γ_{ij} values is numerically computed in which the mth entry, Γ_m equals, the function of m, p, and ϵ as $G(m, p, \epsilon)$. We provide an ILP problem that computes n_{ij} directly by indexing the table. Let $m \in M$, where $M = \{0, \cdots, |E|\}$, denote the number of failed links in the primary network. To compute n_{ij}, let x^m_{ij} be a decision variable that sets $x^m_{ij} = 1$ if $n_{ij} = m$, and 0 otherwise.

The following constraints are introduced:

$$\sum_{m\in M} x^m_{ij} = 1,\quad \forall (i,j) \in E_b. \tag{14}$$

Only one value of m for each backup link $(i,j) \in E_b$ enforces $x^m_{ij} = 1$ in Eq. (14).

$$\sum_{(s,d)\in E} b^{sd}_{ij} = \sum_{m\in M} m x^m_{ij},\quad \forall (i,j) \in E_b. \tag{15}$$

With Eq. (14), Eq. (15) selects only one x^m_{ij} that is set to one so that $\sum_{(s,d)\in E} b^{sd}_{ij} = m$ can be satisfied. x^m_{ij} is zero if $\sum_{(s,d)\in E} b^{sd}_{ij} \neq m$. This selected $m = \sum_{(s,d)\in E} b^{sd}_{ij}$ as above is equivalent to n_{ij} by definition of n_{ij} in Eq. (2). Γ_{ij} is represented as follow.

$$\Gamma_{ij} = G(n_{ij}, p, \epsilon) = \sum_{m\in M} \Gamma_m x^m_{ij},\quad \forall (i,j) \in E_b. \tag{16}$$

The right hand side of Eq. (16) is equivalent to $\Gamma_{n_{ij}}$, where n_{ij} is restricted by Eqs. (2), (14), and (15).

Equation (12b), which is represented as the capacity constraint, can be rewritten as below:

$$C^{\mathrm{B}}_{ij} \geq \sum_{m\in M} \varrho_{ij} x^m_{ij} \Gamma_m + \sum_{(s,d)\in E} \xi^{sd}_{ij},\quad \forall (i,j) \in E_b. \tag{17}$$

To represent the product of $\varrho_{ij} x^m_{ij}$ in linear form, another set of optimization variables is added. Let Ω^m_{ij} be a non-negative variable satisfying the following constraints:

$$\Omega^m_{ij} \geq \varrho_{ij} + D(x^m_{ij} - 1),\quad \forall (i,j) \in E_b, m \in M \tag{18a}$$

$$\Omega^m_{ij} \leq D x^m_{ij},\quad \forall (i,j) \in E_b, m \in M \tag{18b}$$

$$\Omega^m_{ij} \leq \varrho_{ij},\quad \forall (i,j) \in E_b, m \in M \tag{18c}$$

$$\Omega^m_{ij} \geq 0,\quad \forall (i,j) \in E_b, m \in M. \tag{18d}$$

In the above equations, D is a sufficiently large value to satisfy $D \geq \max_{(s,d)\in E} \sum_{(p,q)\in P} w^{pq}_{sd} d_{pq}$. If $x^m_{ij} = 0$, then $D x^m_{ij} = 0$, and the constraints in Eqs. (18b)-(18d) force Ω^m_{ij} to 0. The constraint in Eq. (18a) forces $\Omega^m_{ij} \geq \varrho_{ij}$ if $x^m_{ij} = 1$.

The constraint in Eq. (17) is written as:

$$C^{\mathrm{B}}_{ij} \geq \sum_{m\in M} \Omega^m_{ij} \Gamma_m + \sum_{(s,d)\in E} \xi^{sd}_{ij},\quad \forall (i,j) \in E_b \tag{19a}$$

$$\sum_{m\in M} x^m_{ij} = 1,\quad \forall (i,j) \in E_b \tag{19b}$$

$$\sum_{(s,d)\in E} b^{sd}_{ij} \leq \sum_{m\in M} m x^m_{ij},\quad \forall (i,j) \in E_b \tag{19c}$$

$$x^m_{ij} \in \{0,1\},\quad \forall (i,j) \in E_b, m \in M \tag{19d}$$

$$\text{Eqs. (18a)-(18d).} \tag{19e}$$

The following is the MILP problem to protect against random multiple link failures:

$$\min \sum_{(i,j)\in E_b} C^{\mathrm{B}}_{ij} \tag{20a}$$

$$\text{s.t.}\quad \varrho_{ij} + \xi^{sd}_{ij} \geq \sum_{(p,q)\in P} d_{pq} u^{sd}_{ijpq},\quad \forall (s,d) \in E, (i,j) \in E_b \tag{20b}$$

$$\text{Eqs. (13a)-(13d), (18a)-(18d), (19a)-(19d), (1c)-(1h), (12e)-(12f).} \tag{20c}$$

It should be noted that κ and ϵ do not include in Eqs. (20a)-(20c). These parameters are used to compute Γ_m from Eq. (4) to be a given parameter as an input to Eqs. (20a)-(20c).

4. Heuristic method

The MILP formulation in Eqs. (20a)-(20c) cannot be solved directly in practical time when the network size increases. A heuristic method is employed to minimize the total backup capacity with estimating both primary and backup network routing.

The heuristic algorithm begins with the initial input of backup network routing. A routing in the primary network is obtained from LP problem in Eqs. (21a)-(21c), which is modified from the MILP problem in Eqs. (20a)-(20c). The heuristic algorithm starts by randomly selecting link $(s,d) \in E$ in the primary network and randomly selecting the route in the backup network from node s to node d. We employ the simulated annealing (SA) used in [7] to update the backup network routing. b_{ij}^{sd} and x_{ij}^{m} are updated, and then we solve the LP problem in Eqs. (21a)-(21c) to obtain the primary network routing. The above procedure is iterated by changing selected link $(s,d) \in E$ and solving the LP problem in Eqs. (21a)-(21c) to obtain the primary network routing if b_{ij}^{sd} and x_{ij}^{m} are updated. The solutions at each iteration are both backup and primary network routing. This iteration may reduce the total backup capacity. The iteration is terminated if a condition of convergence is satisfied or if the number of iterations reaches some maximum number of iterations that is given in advance. Finally, the solution is obtained.

The LP problem to determine the primary network routing from the backup network routing is written as,

$$\min \sum_{(i,j)\in E_b} C_{ij}^{\mathrm{B}} \tag{21a}$$

$$\text{s.t.}\quad C_{ij}^{\mathrm{B}} \geq \sum_{m\in M} \varrho_{ij} x_{ij}^{m} \Gamma_m + \sum_{(s,d)\in E} \xi_{ij}^{sd}, \quad \forall (i,j) \in E_b \tag{21b}$$

$$\text{Eqs. (11b), (1e) - (1f), (1h), (12e) - (12f),} \tag{21c}$$

where b_{ij}^{sd} and x_{ij}^{m} are given parameters and w_{sd}^{pq}, ξ_{ij}^{sd} and ϱ_{ij} are decision variables.

The initial state is set by giving the backup network routing as follows.

1. b_{ij}^{sd} and x_{ij}^{m} are set as given parameters.
2. Solve the LP problem in Eqs. (21a)-(21c) to obtain the initial primary network routing and the initial total capacity in the backup network as $C_{\mathrm{total}}^{\mathrm{B}}$, which is given by Eq. (21a).

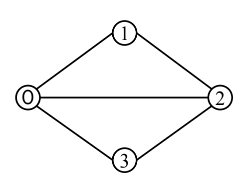
Let k be an index that counts the number of iterations. The heuristic algorithm proceeds as follows.

1. Set k=1.
2. Randomly select link $(s,d) \in E$ in the primary network, and randomly select the route in the backup network from node s to node d. The updated values of b_{ij}^{sd} and x_{ij}^{m} are set as given parameters in Eqs. (21a)-(21c).
3. Solve the LP problem in Eqs. (21a)-(21c) to update the primary network routing and obtain the total backup capacity $C_{\mathrm{total}}^{\mathrm{B}'}$.
4. The new backup routing for $(s,d) \in E$ obtained from step 3 is accepted with probability of min$(q, 1)$, where $q = \exp(\frac{C_{\mathrm{total}}^{\mathrm{B}} - C_{\mathrm{total}}^{\mathrm{B}'}}{\Delta \cdot T})$. A better solution is unconditionally accepted, and a worse solution is accepted with probability of q. Parameter T represents the temperature of the system. T decreases at each iteration from T to $\Delta \cdot T$. Δ is a coefficient to decrease the temperature, where $0 < \Delta < 1$.
5. Increase k by one and repeat from step 2 until a condition of convergence is satisfied or the number of iterations, k, reaches the maximum number of iterations.
6. The algorithm outputs the total backup capacity and the primary and backup network routing, and then terminates.

5. Numerical results

This section evaluates the performance of the proposed approach and compares its results with those of the conventional approach. We use Intel(R) Core(TM) i7-2600K CPU @3.40GHz, 32GB memory for our evaluations. CPLEX is used to solve the MILP formulation. We first compare the results from both the MILP formulation and the heuristic method in a four-node network. The objective values and the computation times of our proposed approach between the MILP formulation and the heuristic method are compared. The MILP formulation in Eqs. (20a)-(20c) is not able to solve in practical time when the number of nodes is increased. We use the introduced heuristic method to demonstrate its performance on larger scale networks.

The four-node network with the given traffic demand on each edge node $(p,q) \in P$ in Fig. 2 is used for the comparison.



(a) Four-node network

$$\{d_{pq}\} = \begin{bmatrix} 0 & 5 & 10 & 5 \\ 5 & 0 & 5 & 5 \\ 10 & 10 & 0 & 10 \\ 10 & 5 & 10 & 0 \end{bmatrix}$$

(b) Traffic demand

Fig. 2 Four-node network and traffic demand.

Table 1 shows the required total backup capacity in the four-node network for different values of probability of link failure κ compared with the results of the conventional approach where $\epsilon = 0.005$ and 0.01. The results show that the proposal yields a smaller backup network than the conventional approach when the value of the probability of link failure κ is less than 0.1 when $\epsilon = 0.01$, and κ is less than

0.075 when $\epsilon = 0.005$. The constraint becomes more lenient and more link failures in the primary network can be tolerated, as ϵ increases. Thus, even lower required backup capacity is expected. As the value of κ increases, the number of backup links needed to protect the failed links in the primary network increases; greater total backup capacity is required, as shown in Table 1. The maximum computation time of MILP is set to 25000 [s]. The symbol of $*$ indicates the best objective value obtained within the maximum computation time; the optimality of solution is not confirmed. The gap between the best objective value and the optimal value is at most 35%, which is obtained by CPLEX. As κ becomes larger and ϵ becomes smaller, no difference in required backup capacity between the proposed and conventional approaches is observed. In this condition, the backup resource sharing under the probabilistic survivability guarantee is not expected to be effective. It should be noted that we consider that κ is greater than ϵ, as is explained at the end of Section 2.

Table 2 shows the computation time results yielded by MILP and the heuristic method. The symbol of * has the same meaning as in Table 1; solution optimality is not confirmed. The results show that the computation time of the heuristic method is less than that of MILP. The reduction effect of the computation time by the proposed approach is significant with ≥ 0.05 for both $\epsilon = 0.005$ and 0.01. We have the following observations on the computation times in MILP and the heuristic method. In MILP, as ϵ decreases and κ increases, the computation time increases. As ϵ decreases and κ increases, Γ_m becomes large. This increases the number of decision variables in MILP. In the heuristic method, as ϵ decreases, the computation time increases. This is because smaller ϵ requires larger Γ_m. This triggers an increase in the computation time in Eqs. (21a)-(21c). It is notable that the computation time does not strongly depend on κ. At each iteration in the heuristic method, the backup network routing is given, i.e., b_{ij}^{sd} and x_{ij}^{m} are set as given parameters, the value of κ does not affect the computation time; the number of decision variables is not varied.

Table 1 Required backup capacity in four-node network.

κ	Method	ϵ=0.005		ϵ=0.01	
		Proposed approach	Conventional approach	Proposed approach	Conventional approach
0.025	MILP	53.75	55.00	45.00	45.00
	Heuristic	53.75	60.00	46.65	50.00
0.05	MILP	72.50	75.00	62.50	65.00
	Heuristic	75.00	75.00	62.50	65.00
0.075	MILP	85.00	85.00	72.50	75.00
	Heuristic	85.00	85.00	75.00	75.00
0.1	MILP	100.00*	100.00*	85.00	85.00
	Heuristic	100.00	100.00	85.00	85.00

*: Solution optimality is not confirmed.

Table 2 Computation times of MILP and heuristic method [s].

κ	ϵ=0.005		ϵ=0.01	
	MILP	Heuristic	MILP	Heuristic
0.025	258.24	8.35	27.23	0.78
0.05	7537.99	7.94	3407.80	0.65
0.075	7371.49	7.71	4003.15	0.67
0.1	25000.00*	7.99	21972.80	0.75

*: Solution optimality is not confirmed.

The required capacities on each link in the backup and primary networks for the four-node network yielded by MILP with the probabilistic survivability guarantee parameter $\epsilon = 0.01$ and different values of κ are shown in Figs. 3 and 4, respectively. Since the routing in the primary network is changed with different values of κ, the capacity on each link in the primary network is changed. The routing in the backup network is changed if capacity of any link in the primary network is changed. The capacity on each link in the backup network is then changed. Figure 3 shows the required capacity on each link in the backup network with different values of κ. The results indicate that different values of κ affect the backup network routing. Figure 4 shows that the required capacity on each link in the primary network, which is determined by the routing, is affected by the value of κ when $\kappa < 0.1$. From the results, the backup network depends on the value of κ. Some values of κ, which changes the required link capacity, affects the routing in the primary network.

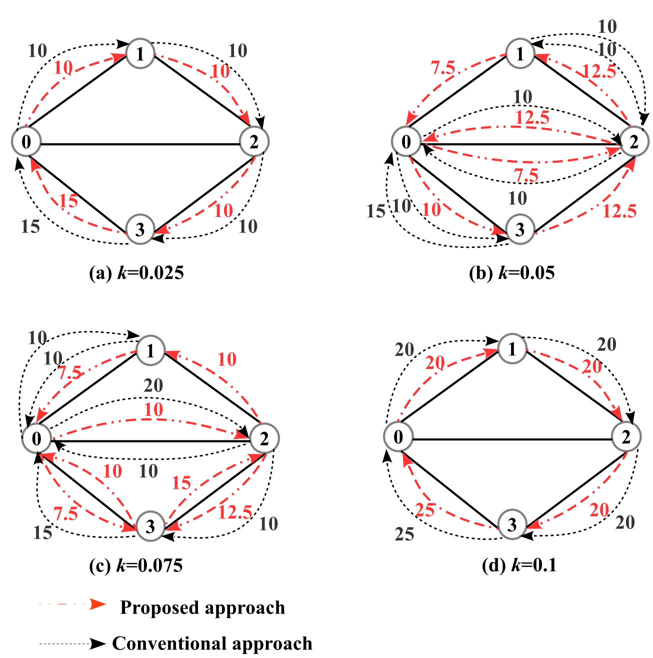


Fig. 3 Required capacity on each link in backup network using $\epsilon = 0.01$ with (a) $\kappa = 0.025$, (b) $\kappa = 0.05$, (c) $\kappa = 0.075$, and (d) $\kappa = 0.1$.

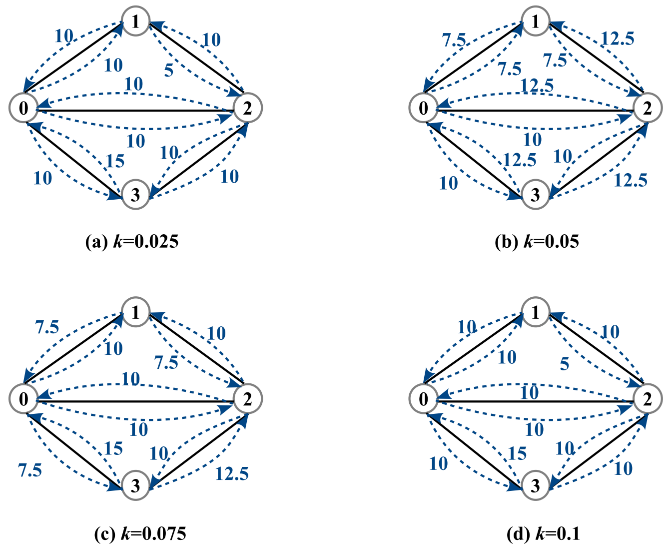


Fig. 4 Required capacity on each link in primary networks using ϵ = 0.01 with (a) κ = 0.025, (b) κ = 0.05, (c) κ = 0.075, and (d) κ = 0.1.

The COST 239 networks and National Science Foundation Network (NSFNET), as shown in Fig. 5, are used to demonstrate the proposed approach. The traffic demand for each pair of source and destination $(p, q) \in P$ is uniformly distributed in the range of 0 to 10.

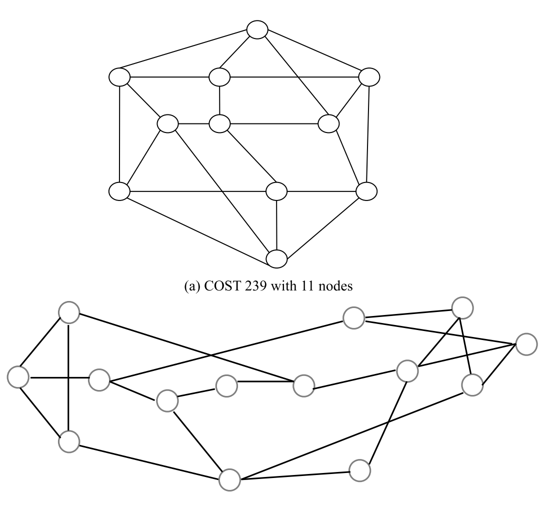


Fig. 5 COST 239 with 11 nodes and NSFNET with 14 nodes.

The required backup capacity in the backup network for COST 239 and for NSFNET are shown in Tables 3 and 4, respectively, with different values of probability of link failure κ, compared with the conventional approach, where ϵ = 0.005, 0.0075, and 0.01. Table 3 shows that, for ϵ = 0.005, 0.0075, and 0.01, when $\kappa \leq 0.05$, $\kappa \leq 0.075$, and $\kappa \leq 0.075$, respectively, the proposed approach achieves lower backup capacity than the conventional approach for COST 239. Table 4 shows that, for ϵ = 0.005, 0.0075, and 0.01, when $\kappa \leq 0.015$, $\kappa \leq 0.015$, and $\kappa \leq 0.05$, respectively, the proposed approach achieves lower backup capacity than the conventional approach for NSFNET. We observe that the trends of backup capacity of these networks in terms of the dependency on κ and ϵ are the same as those observed in Table 1.

The computation times using the heuristic method for COST 239 and NSFNET are shown in Tables 5 and 6, respectively. We observe that the trends in computation time of these networks are, in terms of the dependency on κ and ϵ, the same as those observed in Table 2.

6. Conclusion

This paper proposed an optimization approach that provides probabilistic survivability guarantees while minimizing the total capacity of a backup network. It proceeds by determining both primary and backup network routing, simultaneously. The backup network is designed to protect the primary network from random link failures whose probabilities are given. Our optimization problem introduces robust optimization to provide probabilistic survivability guarantees for different link capacities in the primary network. We formulated our optimization problem as an MILP problem so that the backup network is designed while considering both primary and backup network routing. When the network size increases, the MILP problem cannot be solved in practical time. Thus, a heuristic method was introduced to solve the backup network design problem. Numerical results showed that the proposed approach can reduce the total capacity of the backup network compared to the conventional approach.

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Table 3 Required backup capacity for COST 239.

κ	ϵ=0.005 Proposed approach	ϵ=0.005 Conventional approach	ϵ=0.0075 Proposed approach	ϵ=0.0075 Conventional approach	ϵ=0.01 Proposed approach	ϵ=0.01 Conventional approach
0.01	482.1	652.0	482.1	649.0	-	-
0.015	576.9	733.0	572.6	709.0	532.8	696.0
0.02	738.0	808.0	640.5	754.0	576.9	733.0
0.025	746.9	843.0	717.4	827.0	640.5	754.0
0.05	837.0	853.0	821.8	845.0	810.0	845.0
0.075	853.0	853.0	837.0	853.0	837.0	853.0
0.1	853.0	853.0	853.0	853.0	853.0	853.0

Table 4 Required backup capacity for NSFNET.

κ	ϵ=0.005 Proposed approach	ϵ=0.005 Conventional approach	ϵ=0.0075 Proposed approach	ϵ=0.0075 Conventional approach	ϵ=0.01 Proposed approach	ϵ=0.01 Conventional approach
0.01	1885.2	2064.0	1824.5	1915.0	-	-
0.015	2010.0	2064.0	1973.0	2064.0	1868.0	2008.0
0.02	2064.0	2064.0	2064.0	2064.0	1877.0	2048.0
0.025	2064.0	2064.0	2064.0	2064.0	1880.0	2064.0
0.05	2064.0	2064.0	2064.0	2064.0	1893.0	2064.0
0.075	2064.0	2064.0	2064.0	2064.0	2064.0	2064.0
0.1	2064.0	2064.0	2064.0	2064.0	2064.0	2064.0

Table 5 Computation time for COST 239 [s].

κ	ϵ =0.005	ϵ=0.0075	ϵ=0.01
0.01	944.87	905.20	–
0.015	995.20	909.50	825.50
0.02	995.32	909.68	826.62

Table 6 Computation time for NSFNET [s].

κ	ϵ =0.005	ϵ=0.0075	ϵ=0.01
0.01	1004.20	941.26	–
0.015	990.13	978.64	213.69
0.02	1000.35	948.38	227.99

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Appendix A: Transformation from Eqs. (10a)-(10c) to Eqs. (11a)-(11d)

The following steps are used to derive Eqs. (11a)-(11d) from Eqs. (10a)-(10c) using the duality theorem. Let us express $(s, d) \in E$ with $e \in E$ to simplify all the related notations here. Therefore, z_{ij}^{sd}, b_{ij}^{sd}, and w_{sd}^{pq} are expressed by z_{ij}^{e}, b_{ij}^{e}, and w_{e}^{pq}, respectively.

Equations (10a)-(10c), which is the LP problem of finding $z_{ij} = \{z_{ij}^{1}, \cdots, z_{ij}^{|E|}\}$ that maximizes Γ_{ij} primary links with the largest capacities among the primary links $e \in E$, is represented with a matrix expression by,

$$\max \ \boldsymbol{X}_{ij}^{T} \boldsymbol{z}_{ij} \quad \text{(A· 1a)}$$
$$\text{s.t.} \ \boldsymbol{A}\boldsymbol{z}_{ij} \leq \boldsymbol{C} \quad \text{(A· 1b)}$$
$$\boldsymbol{z}_{ij} \geq 0, \quad \text{(A· 1c)}$$

where

$$\boldsymbol{z}_{ij}^{T} = \left[z_{ij}^{1} \cdots z_{ij}^{|E|} \right] \quad \text{(A· 2a)}$$
$$\chi_{ij}^{e} = \sum_{(p,q)\in P} \sum_{e\in E} d_{pq} w_{e}^{pq} b_{ij}^{e}$$
$$\boldsymbol{X}_{ij}^{T} = \left[\chi_{ij}^{1} \chi_{ij}^{2} \cdots \chi_{ij}^{|E|} \right] \quad \text{(A· 2b)}$$
$$\boldsymbol{A} = \left[\begin{array}{cccc} 1 & 1 & \cdots & 1 \\ \hline 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ & & \cdots & \\ 0 & 0 & \cdots & 1 \end{array} \right] \quad \text{(A· 2c)}$$
$$\boldsymbol{C}^{T} = \left[\Gamma_{ij} | 1 \cdots 1 \right]. \quad \text{(A· 2d)}$$

$\boldsymbol{z}_{ij}$ is an $|E| \times 1$ matrix. $\boldsymbol{X}_{ij}$ is an $|E| \times 1$ matrix. $\boldsymbol{A}$ is a $(1 + |E|) \times |E|$ matrix. $\boldsymbol{C}$ is a $(1 + |E|) \times 1$ matrix. For $\boldsymbol{A}$ and $\boldsymbol{C}$, the first row corresponds to Eq. (10b). The next $|E|$ rows correspond to $z_{ij}^{e} \leq 1, \forall e \in (s, d)$, in Eq. (10c). $z_{ij}^{e} \geq 0$ in Eq. (10c) is a part of the canonical form expression in the maximizing problem [17].

The dual of the LP problem represented by Eqs. (A· 1a)-(A· 2d) for (i, j) is:

$$\min \boldsymbol{C}^{T} \boldsymbol{b}_{ij} \quad \text{(A· 3a)}$$
$$\text{s.t.} \ \boldsymbol{A}^{T} \boldsymbol{b}_{ij} \geq \boldsymbol{X}_{ij} \quad \text{(A· 3b)}$$
$$\boldsymbol{b}_{ij} \geq 0, \quad \text{(A· 3c)}$$

where

$$\boldsymbol{b}_{ij}^{T} = \left[\varrho_{ij} | \xi_{ij}^{1} \cdots \xi_{ij}^{|E|} \right]. \quad \text{(A· 4a)}$$

$\boldsymbol{b}_{ij}$ is a $(1 + |E|) \times 1$ matrix. Eqs. (A· 3a)-(A· 3c), (A· 2b)-(A· 2d) and (A· 4a) are matrix expression of Eqs. (11a)-(11d). ϱ_{ij}, and ξ_{ij}^{e} are newly introduced as dual decision variables.

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